Cambridge
International AS and A Level Mathematics

Mechanics

Jean-Paul Muscat

Practice Book

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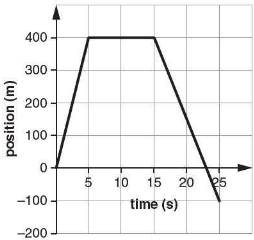
1 Motion in a straight line

The language of motion

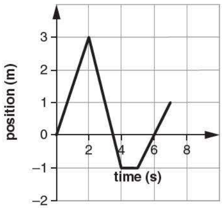
EXERCISE 1.1

1 For the following position–time graphs calculate the total overall displacement and the total distance travelled.

(i)



(ii)



2 A particle is moving from east to west in a straight horizontal line so that its position x at time t is given by $x = 7 - t^2(4 - t)$.

(i) What is the position of the particle at times $t = 0, 0.5, 1, 1.5, 2, 3, 4, 5$?

(ii) What is the displacement from the original position after 5 s?

(iii) At what time is the particle furthest east from its original position and what is its position then?

(iv) What is the total distance travelled by the particle?

3 The position of a particle moving along a straight line is $x = 3t^2 - 14t + 11$ where x is in metres and t is in seconds with $0 < t < 5$.

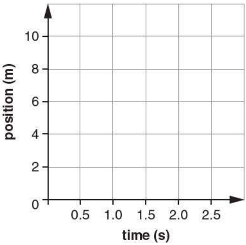
(i) At what times is the particle at O?

(ii) When is the particle furthest from its starting point?

(iii) How far has the particle travelled in the first 5 seconds?

4 A ball is thrown straight up in the air so that its position is given by $x = 2 + 12t - 5t^2$.

(i) Sketch a position–time graph for $0 \leqslant t \leqslant 2.5$.

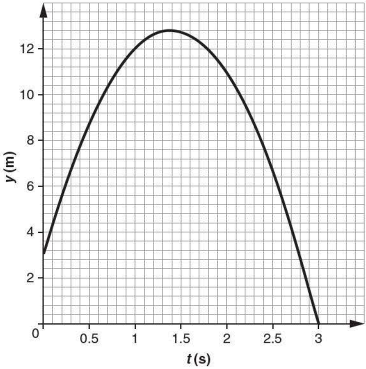


(ii) Find the ball's displacement relative to its starting position after 2.5 s.

(iii) At what time is the highest point reached and how high does the ball go?

(iv) What is the total distance travelled in the 2.5 s?

5 A stone is catapulted vertically upwards so that its position y at time t is as shown in the graph.



(i) Write the position of the stone at times 0, 0.5, 1, 1.5, 2, 2.5 and 3 s.

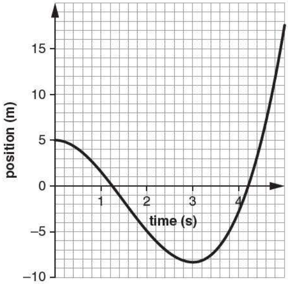
(ii) Find the displacement of the stone relative to its starting position at these times.

(iii) What is the total distance travelled

(a) during the first 1.4 s

(b) during the 3 s of the motion?

6 A particle is moving in a straight line so that its position x m at time t is as shown in the graph.



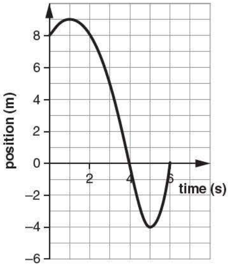
(i) Write the position of the particle at 0.5 s intervals from $t = 0$ to $t = 5$.

(ii) Find the displacement of the particle at these times.

(iii) At what time is the particle furthest from its starting point?

(iv) Find the total distance travelled.

7 A point P is moving in a vertical straight line. The position–time graph is shown below.



(i) Find the greatest displacement of P above its initial position.

(ii) Find the largest distance of P from its initial position.

(iii) Find the time interval in which P is moving downwards.

(iv) Find the times when P is instantaneously at rest.

(v) Find the total distance travelled.

Speed and velocity

EXERCISE 1.2

1 For the position–time graphs **(i)**–**(iv)**, find

(a) the initial and final positions

(b) the total displacement

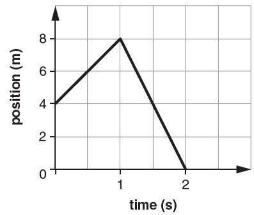
(c) the total distance travelled

(d) the velocity and speed for each part of the journey

(e) the average velocity for the whole journey

(f) the average speed for the whole journey.

(i)



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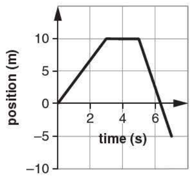

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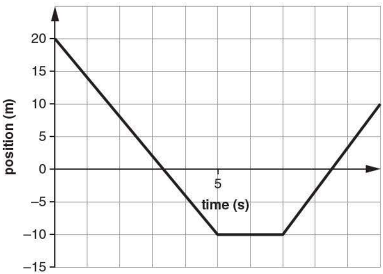

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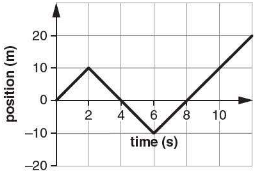
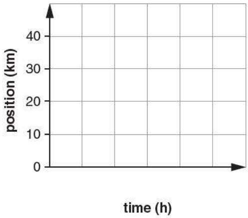


2 A bus leaves town A at 2.00 p.m. and travels to town B, 40 km away, at a speed of 30 km h^{-1}.

A car leaves A at 2.30 p.m. towards B along the same road as the bus and travels at 80 km h^{-1}.

(i) Sketch a position–time graph to show the motion of the two vehicles.



(ii) Find at what time the car catches up with the bus.

(iii) How far from A are they then?

3 A car travels 50 km from A to B at an average speed of $80\,\text{km}\,\text{h}^{-1}$. It stops at B for 30 minutes and then returns to A travelling at an average speed of $60\,\text{km}\,\text{h}^{-1}$.

(i) Find the total time taken for the whole journey.

(ii) Find the average speed for the whole journey.

(iii) Find the car's average velocity.

4 A, B and C are three points on a straight road with AB = 1200 m, BC = 200 m and B is between A and C. A girl cycles from A to B at $10\,\text{m}\,\text{s}^{-1}$, pushes her bike from B to C at an average speed of $0.5\,\text{m}\,\text{s}^{-1}$ and then cycles back from C to B at an average speed of $15\,\text{m}\,\text{s}^{-1}$.

(i) Find the average speed of the girl for the whole journey.

(ii) Find the average velocity of the girl for the whole journey.

5 In the London 2012 Olympic Games 100 m final, Usain Bolt won the gold medal in a time of 9.63 s. The silver medallist Yohan Blake recorded a time of 9.75 s.

(i) Find the average speed for both runners.

(ii) Assuming both were running at uniform speed, find how far Blake was from the finish line when Bolt crossed the line.

6 I first see lightning and then hear the sound of thunder 3 seconds later. Find how far away the lightning struck. Assume the speed of sound is $340\,\mathrm{m\,s^{-1}}$.

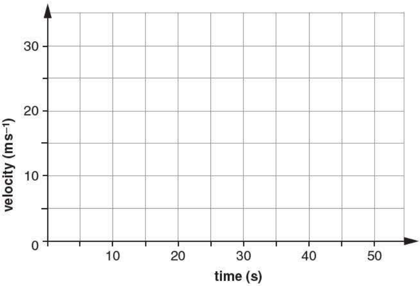
7 A car travels 50 km from A to B at an average speed of $100\,\mathrm{km\,h^{-1}}$. It stops at B for 45 minutes and then returns to A. The average speed for the whole journey is $40\,\mathrm{km\,h^{-1}}$.

(i) Find the average speed from B to A.

(ii) Find the average velocity for the whole journey.

8 A car travels along a straight road. The car starts at rest from point A and accelerates for 30 s at a constant rate until it reaches a speed of 20 m s^{-1}. The car continues at this speed for T s and then decelerates at a constant rate for 20 s until the car slows down to 10 m s^{-1} as it passes point B. The distance AB is 5 km.

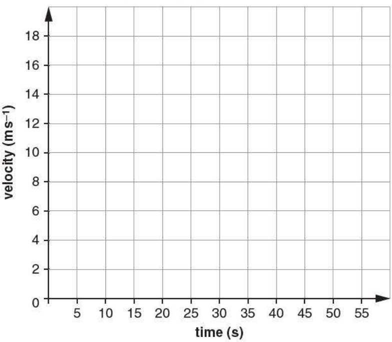
(i) Sketch a velocity–time graph for the journey between A and B.



(ii) Find the total time taken for the journey.

9 The motion of a coach travelling along a straight road follows three stages. Initially travelling at 18 m s^{-1}, the coach decelerates uniformly for 10 s, reaching a velocity of 12 m s^{-1}. During the second stage of the motion, the coach travels at a constant speed of 12 m s^{-1} for 20 s. In the third stage, the coach accelerates uniformly reaching a velocity of 18 m s^{-1} after a further 20 s.

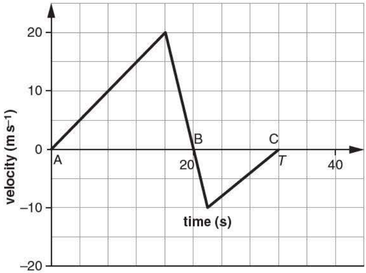
(i) Sketch the velocity–time graph for the motion.



(ii) Find the average speed of the coach.

10 The velocity–time graph shows the motion of a particle along a straight line.

(i) The particle starts at A at $t = 0$ and moves to B in the next 20 s. Find the distance AB.



(ii) T seconds after leaving A the particle is at C, a distance 50 m from B. Find T.

(iii) Find the displacement of C from A.

11 When he won the 2012 Tour de France cycle race, Bradley Wiggins covered the 3496.9 km in a time of 87 hours 34 minutes and 47 seconds, finishing 3 minutes and 21 seconds ahead of the second–place man, Chris Froome. Find the average speed for both riders. Assuming that they were both riding at constant speed, how far behind on the road would Chris Froome have been?

12 A cyclist can cycle uphill at an average speed of $15\,km\,h^{-1}$, he can cycle downhill at an average speed of $60\,km\,h^{-1}$ and on the flat at $45\,km\,h^{-1}$. Find the cyclist's average speed for each of the following stages:

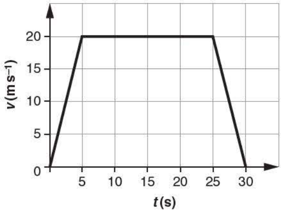
(i) a mountain stage with 80 km on the flat, 50 km uphill and 30 km downhill

(ii) a flat stage comprising of 150 km on the flat, 5 km uphill and 10 km downhill.

Acceleration

EXERCISE 1.3

1 From the velocity–time graph below, find



(i) the acceleration at times 2 s, 10 s and 30 s

(ii) the total distance travelled.

2 A particle starts from rest at time $t = 0$ and moves in a straight line, accelerating as follows:

$a = 2;$ $\quad 0 \leqslant t \leqslant 10$

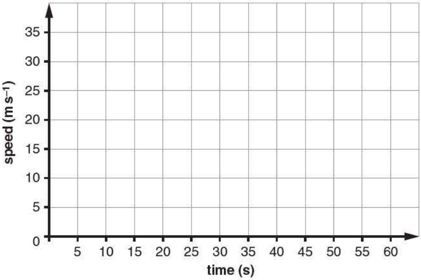
$a = 0.5;$ $\quad 10 < t \leqslant 50$

$a = -3;$ $\quad 50 < t \leqslant 60$

where a is the acceleration in $\mathrm{m\,s^{-2}}$ and t is the time in seconds.

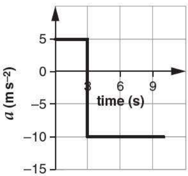
(i) Find the speed of the particle when $t = 10$, 50 and 60.

(ii) Sketch a speed–time graph for the particle in the interval $0 \leqslant t \leqslant 60$.



(iii) Find the total distance travelled by the particle in the interval $0 \leqslant t \leqslant 60$.

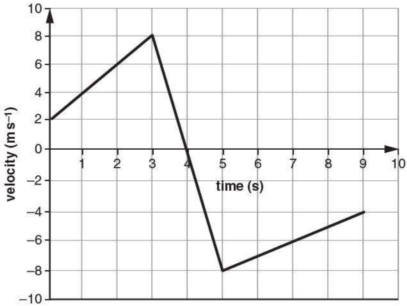
3 The figure shows an acceleration–time graph modelling the motion of a particle moving in a straight line.



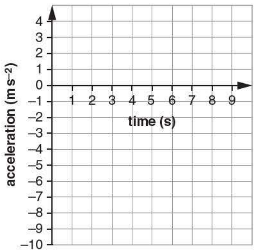
(i) At time $t = 0$ the particle has a velocity of $5\,\text{m}\,\text{s}^{-1}$ in the positive direction. Find the velocity of the particle when $t = 3$.

(ii) At what time is the particle travelling in the negative direction with a speed of $5\,\text{m}\,\text{s}^{-1}$?

4 The velocity–time graph illustrated shows the motion of a particle in a straight line.

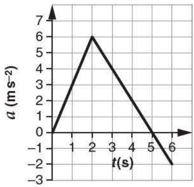


(i) Find the acceleration in the three stages of the motion, then use this to sketch an acceleration–time graph for the motion.



(ii) Find the displacement of the particle at times $t = 3$, $t = 4$, $t = 5$ and $t = 9$.

5 A particle travels along a straight line. Its acceleration in the interval $0 \leqslant t \leqslant 6$ is shown in the acceleration–time graph.



(i) Write down the acceleration at time $t = 2$.

(ii) Given that the particle starts from rest at $t = 0$, find the speed at $t = 2$.

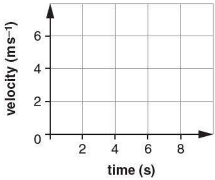
(iii) Find an expression for the acceleration as a function of t in the interval $0 \leqslant t \leqslant 2$.

(iv) At what time is the speed greatest?

(v) Find the change in speed from $t = 2$ to $t = 6$, indicating whether this is an increase or a decrease.

6 A particle moves along a straight line. It starts from rest, accelerates at $3\,\text{m}\,\text{s}^{-2}$ for 2 seconds and then decelerates at a constant rate, coming to rest in a further 6 seconds.

(i) Sketch a velocity–time graph.



(ii) Find the total distance travelled.

(iii) Find the deceleration of the particle.

(iv) Find the average speed for the whole journey.

7 The maximum acceleration for a lift is $2\,\mathrm{m\,s^{-2}}$ and the maximum deceleration is $5\,\mathrm{m\,s^{-2}}$. Use a velocity–time graph to find the minimum time taken for a journey of 40 m

(i) if the maximum speed is $6\,\mathrm{m\,s^{-1}}$

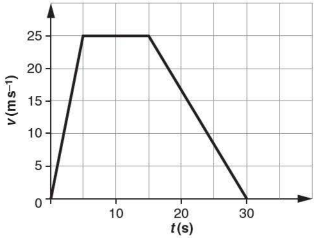
(ii) if there is no restriction on the speed of the lift.

8 A train has a maximum allowed speed of $20\,\mathrm{m\,s^{-1}}$. With its brakes fully applied, it has a deceleration of $0.5\,\mathrm{m\,s^{-2}}$. If it can accelerate at a constant rate of $0.25\,\mathrm{m\,s^{-2}}$ find the shortest time in which it can travel from rest in one station to rest at the next station 10 km away.

Using areas to find distances and displacements

EXERCISE 1.4

1 Use the velocity–time graph below.

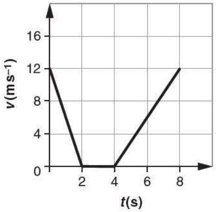


(i) Find the acceleration when $t = 20$.

(ii) Find the distance travelled in the first 10 seconds.

(iii) Find the total distance travelled.

2 A toy car is moving in a straight line. The velocity–time graph is shown below.



(i) Calculate the distance travelled by the car from $t = 0$ to $t = 8$.

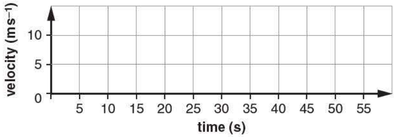
(ii) How much less time would it have taken for the car to travel this distance had it kept its original speed?

(iii) Find the acceleration at times $t = 1$ and $t = 8$.

(iv) From $t = 8$ to $t = 20$, the car travels a further 180 m with a new uniform acceleration a m s^{-2}. Find the velocity at time $t = 20$ and hence find a.

3 A cyclist starts from rest and takes 5 seconds to accelerate at a constant rate up to a speed of $12\,\mathrm{m\,s^{-1}}$. After travelling at this speed for 50 s, the cyclist decelerates to rest at a constant rate over the next 5 s.

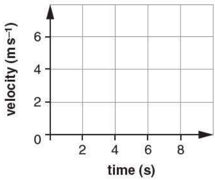
(i) Sketch the velocity–time graph.



(ii) Find the distance travelled.

4 A particle moves along a straight line. It starts from rest, accelerates at $3\,\mathrm{m\,s^{-2}}$ for 2 seconds and then decelerates at a constant rate, coming to rest in a further 6 seconds.

(i) Sketch a velocity–time graph.

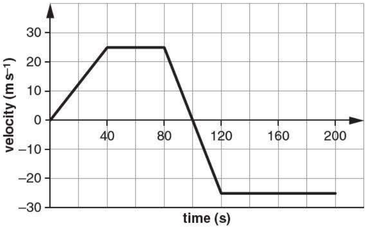


(ii) Find the total distance travelled.

(iii) Find the deceleration of the particle.

(iv) Find the average speed for the whole journey.

5 The figure shows a velocity–time graph for the motion of a particle in a straight line.

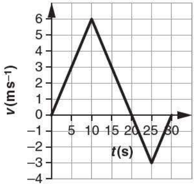


(i) Find the displacement of the particle at times $t = 40$, $t = 80$, $t = 120$ and $t = 200$.

(ii) Find the total distance travelled.

(iii) At what time does the particle pass its starting point?

6 The graph shows how the velocity of a particle varies in a 30-second period as it moves in a straight line.



(i) At what time is the particle furthest from its starting point and how far is it then?

(ii) Find the displacement of the particle at the end of the 30 seconds.

(iii) Find the total distance travelled.

7 A cage goes down a vertical mine-shaft 425 m deep in 48 s. During the first 14 s it is accelerated from rest to its maximum speed. For the next 20 s it moves at this speed. It is then uniformly decelerated to rest. Find the maximum speed it attains.

8 A particle is travelling in a straight line. Its velocity v m s^{-1} at time t s is given by

$$v = 5 + 3t \text{ and } 0 \leqslant t \leqslant 4.$$

(i) Write down the initial velocity and the acceleration.

(ii) Write down the velocity of the particle when $t = 4$ and find the distance travelled in the first 4 seconds.

(iii) For $4 \leqslant t \leqslant 12$ the acceleration of the particle is -2 m s^{-2}. Write down an expression for the velocity in this interval.

(iv) Find the total distance travelled by the particle in the first 12 seconds.

9 A train travels from A to B, a distance of 50 km. Starting from A it takes the train 5 minutes to accelerate uniformly to $40\,\mathrm{m\,s^{-1}}$, maintaining this speed until, with uniform deceleration over 2.5 km, it comes to rest at B.

(i) Find the distance travelled by the train while accelerating.

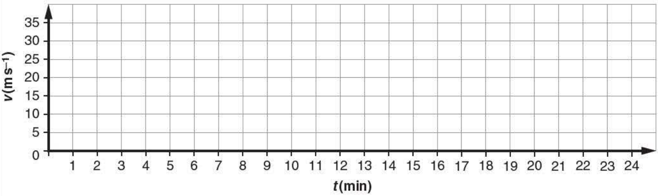
(ii) Find the time taken to decelerate.

(iii) Find the time taken for the journey.

10 A train has a maximum allowed speed of $25\,\mathrm{m\,s^{-1}}$. With its brakes fully applied, it has a deceleration of $1\,\mathrm{m\,s^{-2}}$. If it can accelerate at a constant rate of $0.5\,\mathrm{m\,s^{-2}}$ find the shortest time in which it can travel from rest in one station to rest at the next station 8 km away.

11 The driver of a train travelling at $150\,\text{km}\,\text{h}^{-1}$ on a straight level track sees a signal to stop 1 km ahead and, putting the brakes on fully, comes to rest at the signal. He stops for 2 minutes and then resumes the journey attaining the original speed in a distance of 5 km. Assuming constant acceleration and deceleration, find how much time has been lost owing to the stoppage.

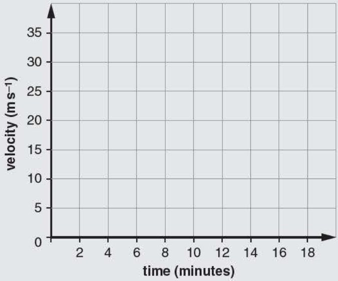
12 A train starts from rest at A and accelerates uniformly at $0.5\,\text{m}\,\text{s}^{-2}$ for 1 minute. It then travels at a constant speed for 20 minutes, after which it is brought to rest at B with a constant deceleration of $2\,\text{m}\,\text{s}^{-2}$. Sketch a velocity–time graph and use it to find the distance AB.



Stretch and challenge

1 A car travels a distance of 20 km from A to B taking 16 minutes. The journey is in three stages. In the first stage the car starts from rest at A and accelerates uniformly to reach a speed of $V\,\mathrm{m\,s^{-1}}$. In the second stage the car travels at constant speed V for 15 minutes. During the third stage of the journey the car decelerates uniformly coming to rest at B.

(i) Sketch the velocity–time graph for the journey.



(ii) Find the value of V.

(iii) Given that the ratio of the acceleration : deceleration is 1 : 2, find how long it takes to accelerate in the first stage of the motion and the distance travelled while the car decelerated in the third stage of the journey.

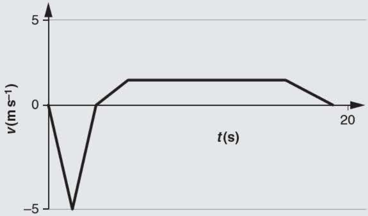
2 A car travelling in a straight line accelerates uniformly from rest for x m. The car then travels a distance of $6x$ m at a constant speed taking 2 minutes, after which it decelerates uniformly until it stops. If the total distance travelled is $7.5x$, calculate the time for the journey.

3 A lift makes the first part of its descent with uniform acceleration a and the remainder with uniform deceleration $2a$. Prove that, if D is the distance travelled and T is the time taken, then $D = \frac{1}{3}aT^2$.

4 A particle moves in a straight line. It starts from rest at A and moves from A to B with constant acceleration $3a$. It then moves from B to C with acceleration a and reaches C with speed V. The times taken in the motion from A to B and from B to C are equal to T. Find T in terms of V and a. Show that the ratio of the distances AB : BC is 3 : 7.

Exam focus

1 The velocity–time graph for a particle P moving in a straight line AB is shown.



The particle starts at rest from a point X halfway between A and B and moves towards A. It comes to rest at A when $t = 3$ s.

(i) Find the distance AX. **[2]**

In the second stage of the motion P is starting from rest at $t = 3$ and moves towards B. The particle takes 15 seconds to travel from A to B and comes to rest at B. For the first 2 seconds of this stage, P is accelerating at $0.6\,\text{m s}^{-2}$, reaching a velocity V which it keeps for T seconds after which it decelerates to rest at B.

(ii) Find V. **[2]**

(iii) Find T. **[3]**

(iv) Find the deceleration. **[2]**

2 The constant acceleration formulae

The constant acceleration formulae

EXERCISE 2.1

1 The following questions involve motion under constant acceleration.

(i) Find v when $u = 7$, $a = 2$, $t = 3$.

(ii) Find s when $u = 5$, $v = 10$ and $t = 5$.

(iii) Find s when $u = 2$, $a = -2$ and $t = 4$.

(iv) Find u when $a = -4$, $s = 2$ and $v = 3$.

(v) Find a when $u = 20$, $v = 50$ and $t = 5$.

(vi) Find a when $u = 20$, $v = 10$ and $s = 100$.

(vii) Find s when $a = 2$, $v = 5$ and $t = 8$.

(viii) Find v when $a = -2$, $s = 100$ and $t = 5$.

2 A cyclist accelerates uniformly along a straight horizontal road so that when she has travelled 20 metres, her velocity has increased from $8\,\mathrm{m\,s^{-1}}$ to $10\,\mathrm{m\,s^{-1}}$. Find the acceleration of the cyclist and the time it takes her to travel that distance.

3 A ball is thrown vertically upwards at a speed of $7.5\,\mathrm{m\,s^{-1}}$ from a height of 1.25 m above ground level. The ball is caught when it returns to its starting position.

(i) Find the time it takes for the ball to reach its maximum height.

(ii) Find the maximum height of the ball above ground level.

(iii) How long is the ball in the air for and what is its speed when it is caught?

4 A sprinter accelerates uniformly from rest for the first 8 metres of a 100-metre race. He takes 1.5 seconds to run the first 8 metres.

(i) Find the acceleration in the first 1.5 seconds of the race.

(ii) Find the speed of the sprinter after 1.5 s.

(iii) The sprinter completes the 100 m travelling at that speed. Find the total time taken to run the 100 metres.

(iv) Calculate the average speed of the sprinter.

5 A car travelling at $30\,\text{m}\,\text{s}^{-1}$ decelerates uniformly to rest in 15 s. Find the deceleration and the distance travelled in this time.

6 A train is travelling at $40\,\text{m}\,\text{s}^{-1}$ when the driver sees an obstacle across the track 100 m ahead. The brakes produce a deceleration of $10\,\text{m}\,\text{s}^{-2}$. With what speed will the train hit the obstacle?

7 The 0–60 mph time for a car is 3 seconds. Find its acceleration (assumed uniform). [Use the conversion factor 1 mile = 1610 m.]

8 A car accelerates uniformly from $10\,\text{m}\,\text{s}^{-1}$ to $30\,\text{m}\,\text{s}^{-1}$ over a distance of 100 m.

(i) Find the acceleration.

(ii) Find the speed of the car when it has travelled half that distance.

9 A pebble is thrown vertically downwards with a speed of $3.5\,\mathrm{m\,s^{-1}}$ from the top of a well, which is 22.5 m deep.

(i) Calculate the speed of the pebble when it hits the bottom of the well.

(ii) Find the time taken by the pebble to reach the bottom of the well.

10 A ball is thrown vertically upwards and returns to its point of projection after 4 seconds. Calculate its speed of projection and the maximum height reached.

11 A stone is thrown vertically upwards at a speed of $8\,\mathrm{m\,s^{-1}}$. Neglecting air resistance, calculate the speed at which the stone hits the branch of a tree 3 m above the point of projection.

12 A car approaches a bend at a speed of $80\,\mathrm{km\,h^{-1}}$ and has to reduce its speed to $30\,\mathrm{km\,h^{-1}}$ in a distance of 250 m in order to take the bend. Find the required deceleration. After the bend is taken the car regains its speed in 25 seconds, find the distance it travels in doing so.

13 A firework is projected vertically upwards at $20\,\mathrm{m\,s^{-1}}$. Find the length of time for which it is 5 m above the point of projection.

14 The Highway Code states that for a car travelling at 30 mph ($48\,\mathrm{km\,h^{-1}}$) the stopping distance of 23 m is made up of 9 m thinking distance and 14 m braking distance.

(i) Use the thinking distance to work out the average reaction time, T.

(ii) Calculate the deceleration, a, of the car as it comes to rest in 14 m.

(iii) How long does it take for the car to come to rest?

(iv) Using the values found for T and a for a car travelling at 30 mph, find the corresponding values for the thinking distance and the braking distance for a car travelling at 70 mph (112 $\mathrm{km\,h^{-1}}$).

(v) How long does it take for a car travelling at 70 mph to come to rest?

Further examples

EXERCISE 2.2

1 A particle travels with constant acceleration in a straight line. A and B are points on this line, 20 m apart. Initially at A, the particle arrives at B 40 seconds later, with a speed of 2.5 $\mathrm{m\,s^{-1}}$ moving away from A. Find the acceleration of the particle and the initial velocity of the particle making its direction clear.

2 Two stones A and B are initially at points A and B. B is X m directly above A.

B is dropped from rest and at the same instant A is projected vertically upwards with speed $25\,\text{m}\,\text{s}^{-1}$. The stones collide T seconds later and they both have the same speed V.

Show that $T = 1.25$ s and find the values of V and X.

3 A car is driven with constant acceleration $a\,\text{m}\,\text{s}^{-2}$ along a straight road. When it passes a road sign its speed is $u\,\text{m}\,\text{s}^{-1}$. The car travels 200 m in the 10 seconds after passing the sign. After 25 seconds it has a speed of $25\,\text{m}\,\text{s}^{-1}$.

(i) Find u and a.

(ii) What distance does the car travel in the 25 s after passing the sign?

4 A car is travelling at constant acceleration a m s^{-2} along a straight road. Its speed as it passes a sign is u m s^{-1}. The car travels 25 m in the 2 seconds after passing the sign and 35 m in the following 2 seconds.

(i) Find a and u.

(ii) Find the speed of the car after 4 seconds.

(iii) How far will it travel in the next 5 seconds?

5 A particle is projected vertically upwards from a point O at 30 m s^{-1}.

(i) Find the greatest height reached by the particle.

(ii) When at its highest point, a second particle is projected vertically upwards from O at 18 m s^{-1}. Show that the particles collide 2.5 seconds later and determine the height above O at which the collision takes place.

6 A stone is dropped from the top of a tower. One second later another stone is thrown vertically downwards with an initial speed of $12\,\mathrm{m\,s^{-1}}$. The two stones land at the same time.

(i) Find the time taken by each stone to fall to the ground.

(ii) Find the height of the tower.

7 Two particles A and B are moving in a straight line. A starts from rest and has a constant acceleration towards B of $0.8\,\mathrm{m\,s^{-2}}$. B starts 240 m from A at the same time and has a constant speed of $10\,\mathrm{m\,s^{-1}}$ away from A.

(i) Write down expressions for the distances travelled by A and B t seconds after the start of the motion.

(ii) How much time does it take for A to catch up with B and how far has A travelled in this time?

8 A particle travels with constant acceleration along a straight line. A and B are points on the line 10 m apart. Initially at A, the particle reaches B after 40 s with a speed of 2.5 m s^{-1} moving away from A. Calculate the acceleration and initial velocity of the particle, making the directions clear.

9 A ball is thrown vertically upwards at 20 m s^{-1} from a point P. Two seconds later a second ball is also thrown vertically upwards from P with the same speed of 20 m s^{-1}.

(i) Calculate how long the first ball has been in motion when the balls meet.

(ii) Calculate the height above P at which A and B meet.

10 A particle moving in a straight line with constant acceleration a passes through points O, A and B at times $t = 0$, 2 and 4 seconds respectively, where A and B are on the same side of O and OA = 10 m and OB = 50 m. Find a and the initial velocity of the particle when $t = 0$.

11 A particle A is projected vertically upwards, from horizontal ground, with speed 8 m s^{-1}.

(i) Show that the greatest height above ground level is 3.2 m.

A second particle B is projected vertically upwards, from a point 1.4 m above the ground, with speed u m s^{-1}. The greatest height above ground reached by B is also 3.2 m.

(ii) Find the value of u.

It is given that A and B are projected simultaneously.

(iii) Show that, at the instant when A and B are at the same height, they have the same speed and are moving in opposite directions.

Stretch and challenge

1 A car is moving along a straight road with a speed of $10\,\text{m}\,\text{s}^{-1}$. It accelerates uniformly so that during the fifth second of its motion it travels a distance of 46 m.

Find the acceleration and the speed at the end of 5 seconds.

2 A particle A starts from the origin O with velocity $u\,\text{m}\,\text{s}^{-1}$ and moves along the positive x axis with constant acceleration $a\,\text{m}\,\text{s}^{-2}$. Twenty seconds later, another particle B starts from O with velocity u and moves along the positive x axis with acceleration $3a\,\text{m}\,\text{s}^{-2}$.

Find the time that elapses between the start of A's motion and the instant when B has the same velocity as A, and show that A will have then travelled three times as far as B.

3 In order to determine the depth of a well, a stone is dropped into the well and the time taken for the stone to drop is measured. It is found that the sound of the stone hitting the water arrives 5 seconds after the stone is dropped. If the speed of sound is taken as $340\,\mathrm{m\,s^{-1}}$ find the depth of the well.

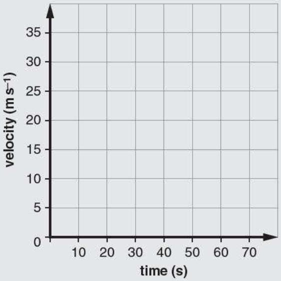
4 A stone is dropped from rest from the top of a tower X m tall. During the last second it travels a distance of $0.36X$. Find the height of the tower.

5 A particle accelerates from rest with acceleration $3\,\text{m}\,\text{s}^{-2}$ to a speed $V\,\text{m}\,\text{s}^{-1}$.
It continues at this speed for T s and then decelerates to rest at $1.5\,\text{m}\,\text{s}^{-2}$.
The total time for the motion is one minute, and the total distance travelled is 1 km.
Find the value of V.

Exam focus

1 Two cars are moving in the same direction along a straight road. At $t = 0$, car A is at rest, accelerates at $3.5\,\text{m s}^{-2}$ for $0 \leqslant t \leqslant 10$, then travels at constant speed. Car B travels at $20\,\text{m s}^{-1}$ for $0 \leqslant t \leqslant 15$ then accelerates at $0.4\,\text{m s}^{-2}$ until it reaches a speed of $30\,\text{m s}^{-1}$ after which it continues at this speed.

(i) Draw a velocity–time graph for the motion of cars A and B for $0 \leqslant t \leqslant 80$. [4]



(ii) Show that in the first 40 seconds A travels 300 m further than B. [4]

(iii) A is 500 m behind B at $t = 0$. At what time does A catch up with B? [2]

2 A train, moving with constant acceleration, is seen to travel 1500 m in one minute and 2500 m in the next.

(i) Find the speed of the train at the start of the first minute. [5]

(ii) Find the acceleration. [2]

(iii) Find the speed of the train at the end of the second minute. [3]

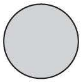
3 Forces and Newton's laws of motion

Force diagrams

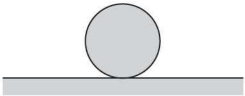
EXERCISE 3.1

1 Draw force diagrams for each of the following situations. In each case, label the forces W (weight), T (tension), P (thrust), F (friction) or R (normal reaction).

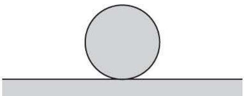
(i) A ball falling to the ground (neglecting air resistance)



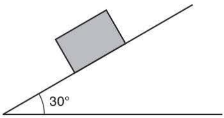
(ii) A ball lying on the ground



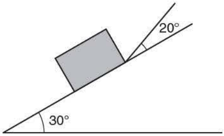
(iii) A ball being kicked on rough ground



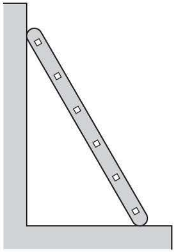
(iv) A box at rest on a slope inclined at 30° to the horizontal



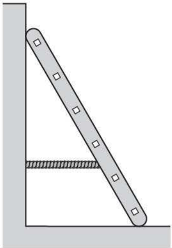
(v) A box being pulled up a slope inclined at 30° to the horizontal by a string inclined at 20° to the slope



(vi) A ladder lying on rough ground, leaning on a rough wall at an angle of 30° to the wall



(vii) A ladder leaning against a rough wall, standing on smooth ground, with a rope tied between the middle of the ladder and the wall



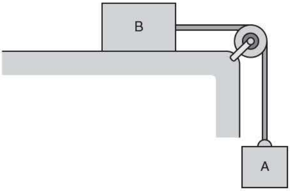
(viii) A plank supported at each end by a vertical cable

Force and motion; Pulleys

EXERCISE 3.2

1 A block of weight W_B lies on a rough horizontal table. A light inextensible string is attached to the block and runs parallel to the table, over a smooth peg at the edge of the table and down to a weight W_A hanging freely.

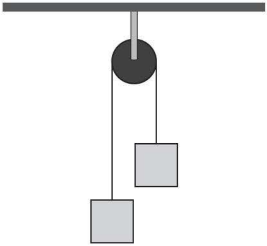
(i) Draw the forces acting on the block and the hanging weight.



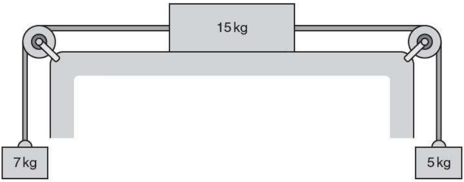
(ii) If the block does not move, write down relations between the different forces.

(iii) If the block is accelerating, write down the resultant force acting on the block.

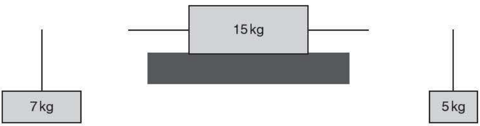
2 A light smooth pulley hangs from the ceiling. A light inextensible string with two equal weights W at either end passes over the pulley. On a diagram show the forces acting on the pulley and the weights. What is the tension in the string? What is the tension in the rod holding the pulley?



3 A block of mass 15 kg lies on a rough table. Two masses of 7 kg and 5 kg are attached to the block by light inextensible strings which pass over smooth pulleys at the edge of the table. The system is in equilibrium.



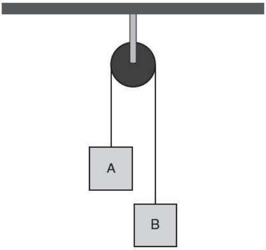
(i) Draw force diagrams for each of the three objects.



(ii) Write down the net force acting on each of these objects.

4 Two particles A and B of masses 5 kg and 4 kg are attached at the ends of a light inextensible string passing over a smooth peg as shown in the diagram.

(i) Draw separate diagrams showing the forces acting on A and B.



(ii) The system is released from rest. What is the net force in the direction of the motion?

5 A box is at rest on the floor of a lift which is moving up and down. The normal reaction of the lift floor on the box is R and its weight is W. Decide in each of the following cases, whether R is greater than, less than or equal to W, and describe the net force acting on the box.

(i) The lift is moving downwards with constant velocity.

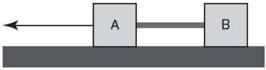
(ii) The lift is moving upwards with increasing speed.

(iii) The lift is moving upwards with decreasing speed.

(iv) The lift is moving downwards with decreasing speed.

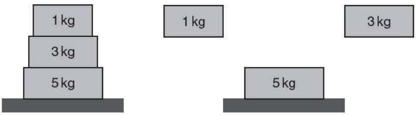
6 Two particles A and B of masses 2 kg and 3 kg lie on rough horizontal ground, connected by a light rigid rod. A force P is applied to A in the direction BA.

(i) Show all forces acting on A and B.

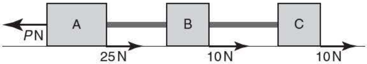


(ii) What is the net force acting on the system?

7 Three boxes of mass 1 kg, 3 kg and 5 kg are placed, one on top of another with the largest at the bottom, in contact with a smooth surface. Show all the forces acting on each of the boxes.



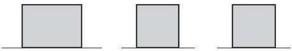
8 Three boxes A, B and C are connected by light, rigid, horizontal rods. The boxes are lying on rough horizontal ground. A force P is applied to A in the direction BA. The frictional force on A is 25 N and that on B and C is 10 N. The boxes are moving at constant speed.



(i) Draw a diagram showing all the forces acting on the system as a whole. Hence, by considering the equilibrium of the system as a whole, find P.

The coupling between A and B exerts a force T_1 on A and the coupling between B and C exerts a force T_2 on B.

(ii) Draw diagrams showing the horizontal forces on each of the boxes.

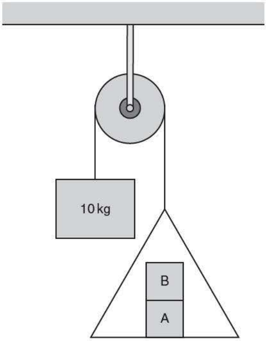


(iii) By considering the equilibrium of A, find T_1.

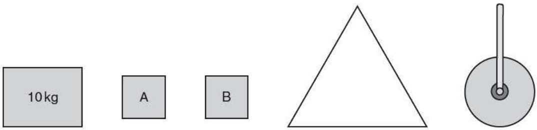
(iv) By considering the equilibrium of C, find T_2.

(v) Show that the forces on B are also in equilibrium.

9 The diagram shows a block of 10 kg connected to a light scale-pan by a light inextensible string which passes over a smooth pulley. The scale-pan holds two blocks A and B each of mass 5 kg, with B resting on top of A. The system is in equilibrium.



(i) On separate diagrams show all the forces acting on each of the three masses, the scale-pan and the pulley.

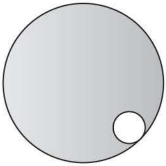


(ii) Find the value of the tension in the string.

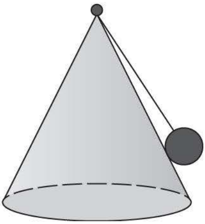
(iii) Find the tension in the rod holding the pulley.

(iv) Find the normal reaction between A and B.

10 A small particle of mass 4 kg is lying on the inside of a hollow sphere. Show the forces acting on the particle.

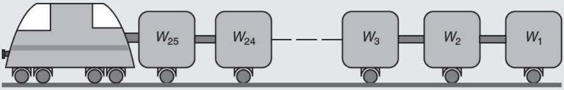


11 A light inextensible string is fixed to the top of a cone. At the other end, a sphere of weight W is describing a horizontal circle on the surface of the cone. Show all the forces acting on the sphere.



Stretch and challenge

1 A train consists of an engine of mass 50 000 kg pulling 25 trucks each of mass 10 000 kg. The force of resistance on the engine is 1500 N and that on each truck is 100 N. The train is travelling at constant speed in a straight line.



(i) Find the driving force of the engine.

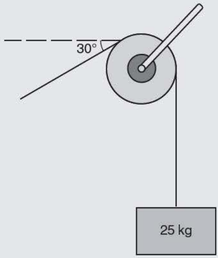
(ii) Find the tension T_1 in the coupling between trucks W_1 and W_2.

(iii) Find the tension T_2 in the coupling between trucks W_2 and W_3.

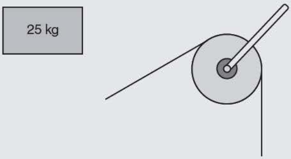
(iv) Derive an expression for T_n, the force in the coupling between trucks W_n and W_{n+1}.

(v) Find the force in the coupling between the engine and truck W_{25}.

2 A smooth fixed pulley is used to lift a 25 kg box. A light inextensible string passes over the pulley and sufficient force is used to keep the box in equilibrium.



(i) Draw force diagrams showing the forces acting on the box and on the pulley.

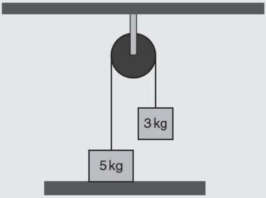


(ii) Find the tension in the string.

(iii) Find the magnitude of the force exerted by the string on the pulley and the direction in which it is acting.

Exam focus

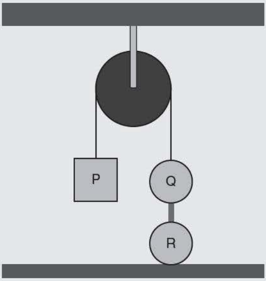
1 Particles of masses 5 kg and 3 kg are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The 5 kg mass is at rest on the ground.



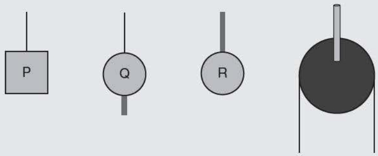
(i) Find the tension in the string. [1]

(ii) Find the normal reaction of the floor on the 5 kg mass. [2]

2 The figure shows a system in equilibrium. A light inextensible string passes over a fixed smooth pulley. Attached to one end of the string is a mass P of 4 kg, attached to the other end is a mass Q of 2 kg, which is itself linked through a rigid rod to a third mass R of 5 kg which is resting on the ground.



(i) Draw separate force diagrams showing all the forces acting on each of the masses P, Q, R and on the pulley.



[3]

(ii) Give a reason as to why the tension in the string is the same throughout. What value is this? [1]

(iii) Find the tension in the rod. [2]

(iv) Find the normal reaction of the ground on mass R. [2]

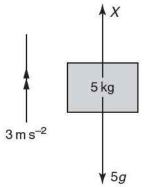
4 Applying Newton's second law along a line

Newton's second law

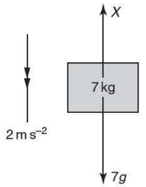
EXERCISE 4.1

1 Each diagram shows the forces acting on an object and the resulting acceleration. In each case, write down the equation of motion and use it to find the quantity marked with a letter.

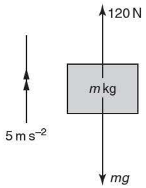
(a)



(b)



(c)



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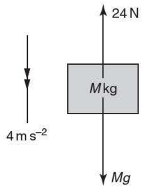

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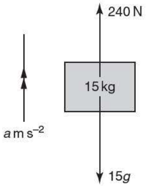
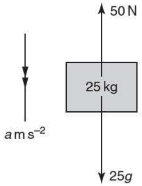


(f)



2 A mass of 5 kg falls with acceleration of $8\,\text{m s}^{-2}$. What resistance is acting on the mass?

3 A box of mass 50 kg is pushed across a rough floor by a horizontal force of 100 N. What is the frictional force if

(i) the box moves with constant velocity?

(ii) the box moves with constant acceleration of $1\,\text{m s}^{-2}$?

4 The total mass of a boy and his sailboard is 70 kg. If the wind produces a force of 200 N and the water a resistance of 95 N, find the acceleration of the sailboard and boy.

5 A hot-air balloon rises from the ground with uniform acceleration. It reaches a height of 200 m in 30 s. If the mass of the balloon and basket is 360 kg, find the lifting force.

6 An empty bottle of mass 2.5 kg is released from a submarine and rises with an acceleration of 0.75 m s^{-1}. The water causes a resistance of 0.5 N. Find the size of the buoyancy force.

7 A mass of 3 kg is placed on a horizontal surface which is moving downwards with uniform acceleration. The normal reaction between the mass and the surface is found to be 8 N. Find the acceleration.

8 A girl of mass 40 kg stands on the floor of a lift which is moving with an upward acceleration of 0.6 m s^{-2}. Calculate the magnitude of the force exerted by the floor on the girl.

9 A woman of mass 65 kg is standing on the floor of a lift of mass 775 kg. The lift is descending with acceleration a m s^{-2}. The tension in the lift cable is 9000 N.

(i) Calculate the value of a.

(ii) Find the reaction of the floor on the woman.

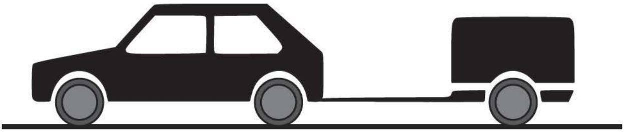
10 A car of mass 1300 kg is travelling in a straight line on a horizontal road. The driving force of 3000 N acts in the direction of motion and a resistance force of 400 N opposes the motion of the car. Find the acceleration of the car.

11 An object is hung from a spring balance suspended from the roof of a lift. When the lift is descending with uniform acceleration of 2 m s^{-2}, the balance indicates a weight of 200 N. When the lift is ascending with uniform acceleration a m s^{-2}, the reading is 300 N. Find a.

Newton's second law applied to connected objects

EXERCISE 4.2

1 A trailer of mass 750 kg is attached to a car of mass 1250 kg by a light rigid tow-bar. The car and trailer are travelling along a level straight road with acceleration of 1.2 m s^{-2}.



(i) Given that the force exerted by the tow-bar on the trailer is 1000 N, find the resistance to motion of the trailer.

(ii) Given also that the driving force of the car is 2800 N, find the resistance to motion of the car.

2 A car is towing a trailer along a straight level road. The masses of the car and the trailer are 1000 kg and 400 kg respectively. The resistance to motion of the car is 550 N and the resistance to motion of the trailer is 200 N.

At one stage of the motion, the pulling force exerted on the trailer is zero.

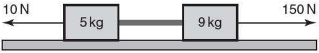
(i) Show that the acceleration of the trailer is $-0.5\,\mathrm{m\,s^{-2}}$.

(ii) Find the driving force exerted by the car.

(iii) Calculate the distance required to reduce the speed of the car and trailer from $22.5\,\mathrm{m\,s^{-1}}$ to $12.5\,\mathrm{m\,s^{-1}}$.

3 In the following question parts the blocks are connected by a light rigid rod and are moving horizontally on a smooth surface. The masses of each block and the forces acting on each block are shown in the diagrams.

(a)

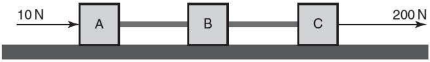


(b)

(i) In each case, draw force diagrams for each block.

(ii) Find the acceleration and the force in each coupling.

4 Three blocks, A of mass 4 kg, B of mass 10 kg and C of mass 16 kg, are linked with rigid rods and are moving horizontally on a smooth surface. Horizontal forces of 200 N and 10 N are acting on C and A respectively.



(i) Draw force diagrams for each block.

(ii) Write down separate equations of motion for each block.

(iii) Find the acceleration and the forces in the couplings.

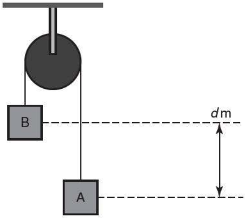
5 A car, of mass 1400 kg, is towing a caravan of mass 800 kg along a straight horizontal road. The caravan is connected to the car by a horizontal tow-bar. Resistance forces of magnitudes 300 N and 550 N act on the car and caravan respectively. The acceleration of the car and caravan is $0.5\,\mathrm{m\,s^{-2}}$.

(i) Find the force in the tow-bar.

(ii) Find the magnitude of the driving force.

6 Two particles, A and B, are connected by a light inextensible string that passes over a smooth fixed pulley, as shown in the diagram.

The mass of A is 4 kg and the mass of B is 5 kg. The particles are released from rest in the position shown, where B is d metres above A.



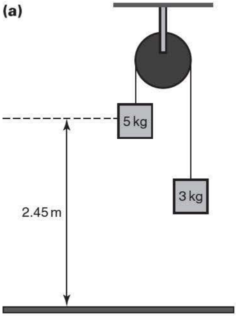
(i) Find the acceleration of each particle.

(ii) When the particles have been moving for 0.5 seconds they are at the same level. Find the speed of the particles at this time.

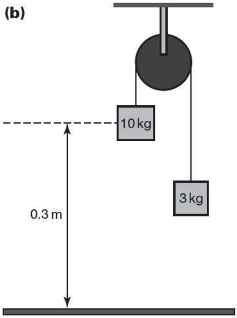
(iii) Find d.

7 In the following examples, two blocks of given masses as shown in the diagrams are connected by a light inextensible string which passes over a smooth fixed pulley. The blocks are released from rest with the heavier of the two masses a given distance above ground level, also shown in the diagrams.

(a)



(b)

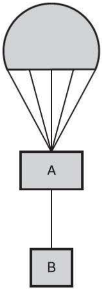


(i) Find the acceleration and the tension in the string in each case.

(ii) Find, in each case, the speed of the masses when the heavier mass hits the ground.

(iii) Find, in each case, how far the lighter mass rises after the heavier mass hits the floor and the string becomes slack.

8 Two boxes are descending vertically, supported by a parachute. Box A has a mass of 60 kg. Box B has a mass of 35 kg and hangs from box A by a light rigid wire. At one stage the boxes are decelerating, with the parachute exerting an upward force of 1050 N on box A. The acceleration of the boxes is a m s^{-2} upwards and the tension in the wire is T N.



(i) Draw separate force diagrams showing all the forces acting on box A and box B.

(ii) Write down separate equations of motion for box A and box B.

(iii) Calculate a and T.

9 A truck of mass 70 kg runs on horizontal rails against a resistance of 50 N. A light rope is attached to the truck and runs horizontally until it passes over a light smooth pulley and drops down a vertical shaft. At the other end of the rope is a bucket of mass 10 kg which is falling down the shaft.

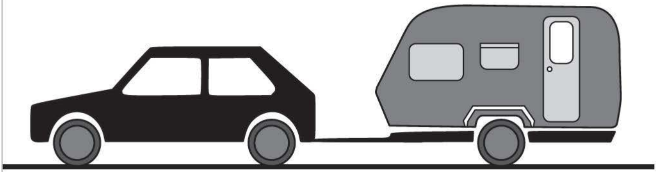
(i) Find the acceleration of the truck and bucket and the tension in the rope.

(ii) If the bucket falls 200m down the shaft, find with what velocity it will hit the bottom.

The truck is now at rest with the bucket at the bottom of the shaft. 25 kg of rock is put into the bucket.

(iii) What horizontal force must be applied to the truck to bring the bucket up with an acceleration of $1.5\,\text{m}\,\text{s}^{-2}$?

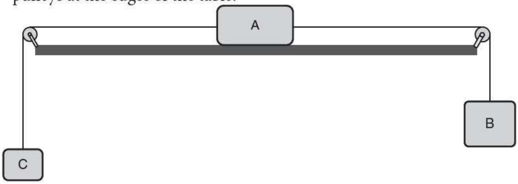
10 A car of mass 1200 kg is towing a caravan of mass 900 kg along a level straight road, using a rigid tow-bar. The resistance to the car's motion is 200 N and the caravan experiences a resistance of 350 N.



(i) If the driving force from the engine is 3 kN, find the tension in the tow-bar and the acceleration.

(ii) The engine is disengaged and the brakes are applied. The braking force is 500 N. Find the deceleration and the force in the tow-bar, stating whether it is a tension or a compression.

11 Block A of mass 25 kg lies on a smooth table. Two blocks, B of mass 12 kg and C of mass 5 kg, are attached to block A by light inextensible strings which pass over smooth pulleys at the edges of the table.



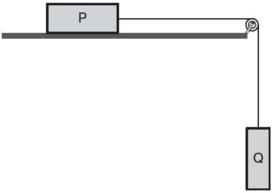
(i) Draw separate force diagrams for A, B and C.

(ii) Write down the equation of motion for each block.

(iii) Find the values of the acceleration and the tensions in the strings.

(iv) The table is not smooth and in fact the acceleration is half the value found in (**iii**). Find the frictional force that would produce this result.

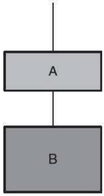
12 A particle P of mass 3 kg is lying on a smooth horizontal table top which is 1.5 m above the floor. A light inextensible string of length 1 m connects P to a particle Q also of mass 3 kg which hangs freely over a smooth pulley at the edge of the table. Initially P is held 0.5 m from the pulley when the system is released from rest.



(i) Find the speed of P when it reaches the pulley.

(ii) Find the tension in the string.

13 The diagram shows a block A of mass 50 kg suspended by a vertical cable. A second block B of mass 200 kg is suspended from A by means of a second vertical cable. The blocks are raised uniformly 10 metres in 10 seconds, starting from rest.

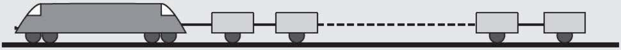


Find the acceleration of the blocks and the tension in each cable.

14 A train has a mass of 250 tonnes. The engine exerts a pull of 50 kN. The resistance to motion is 1% of the weight of the train. The braking force of the engine is 20% of the weight. The train starts from rest and accelerates uniformly until it reaches a speed of $10\,\mathrm{m\,s^{-1}}$. At this point the brakes are applied until the train stops. Find the time taken for the train to stop once the brakes are applied, to the nearest second.

Stretch and challenge

1 The engine of a goods train has mass 50 tonnes and is pulling a convoy of 25 trucks, each of mass 8 tonnes. The resistive forces are 2500 N on the engine and 250 N on each truck. The driving force of the engine is 40 kN and the train is travelling along a level straight track.



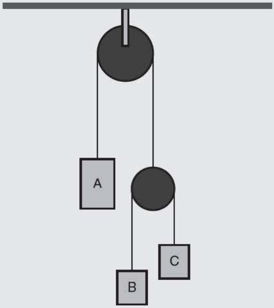
(i) Find the acceleration of the train.

(ii) Find the force in the coupling between the engine and the first truck.

(iii) Show that the force T_n in the coupling between the nth truck and the $(n+1)$th truck is given by $T_n = 31\,250 - 1250n$.

(iv) Hence, or otherwise, find the force in the coupling between the last two trucks.

2 A light inextensible string passes over a smooth pulley and carries at one end a particle A of mass 5 kg and at the other end a light smooth pulley over which passes a second light inextensible string carrying particles B of mass 2 kg and C of mass 3 kg at its ends.



(i) Find the acceleration of A when the particles move vertically under gravity.

(ii) Find by how much the mass of A must be reduced in order that A can remain at rest while the other two particles are in motion.

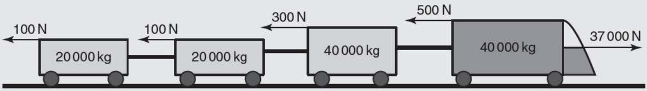
3 A particle P of mass 8 kg rests on a smooth horizontal table and is attached by light inextensible strings to particles Q and R of mass 2 kg and 6 kg respectively. The strings pass over smooth pulleys on opposite edges of the table so that Q and R can hang freely downwards. The system is released from rest.

(i) Obtain equations of motion for each of the particles and use these to find the common acceleration and the tension in the strings.

(ii) After falling a distance of 0.5 m from rest, R strikes the floor and is brought to rest. Find the further distance that Q rises before momentarily coming to rest. (It is assumed that the lengths of the strings are such that P remains on the table and Q does not reach it.)

Exam focus

1 A train consists of an engine and three trucks with masses and resistances to motion as shown in the figure. There is also a driving force of 37 kN. All couplings are light, rigid and horizontal.



(i) Show that the acceleration of the train is $0.3\,\text{m}\,\text{s}^{-2}$. **[3]**

(ii) Calculate the force in the coupling between the last two trucks. **[4]**

With the driving force removed, brakes are applied, so adding an additional resistance of 11 000 N to the total of the resistances shown in the figure.

(iii) Calculate the new acceleration of the train. **[2]**

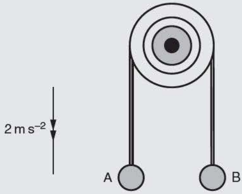
(iv) Calculate the new force in the coupling between the last two trucks if the brakes are applied to

(a) the engine [3]

(b) the last truck. [3]

In each case state whether the force is a tension or a thrust.

2 Particles A and B are attached to the ends of a light inextensible string. The string passes over a smooth pulley. The particles are released from rest, with the string taut, and A and B at the same height above a horizontal floor. In the subsequent motion A descends with acceleration $2\,\text{m}\,\text{s}^{-2}$ and strikes the floor 0.5 s after being released. It is given that B never reaches the pulley.



(i) Calculate the distance A moves before reaching the ground and the speed of A immediately before it hits the floor. [3]

(ii) Show that B rises a further 0.05 m after A strikes the floor and calculate the total length of time during which B is rising. [4]

(iii) Before A strikes the floor the tension in the string is 6 N. Calculate the mass of A and the mass of B. [3]

(iv) The pulley has mass 0.5 kg, and is held in a fixed position by a light vertical rod. Calculate the tension in the rod

(a) immediately before A strikes the floor

(b) immediately after A strikes the floor. [5]

5 Vectors

Adding vectors; Components of a vector; The magnitude and direction of vectors written in component form; Resolving vectors

EXERCISE 5.1

1 Find the magnitude and direction of each of the following vectors, giving the direction from the positive x axis. Vectors **i** and **j** are unit vectors along the x and y directions respectively.

(i) $5\mathbf{i} + 12\mathbf{j}$

(ii) $\mathbf{i} - 2\mathbf{j}$

(iii) $4\mathbf{i} - 3\mathbf{j}$

(iv) $-2\mathbf{i} + 5\mathbf{j}$

2 Vectors **i** and **j** are unit vectors in the east and north directions respectively. Find the magnitude and direction of the following vectors, giving the direction as a bearing.

(i) $\mathbf{i} + \mathbf{j}$

(ii) $3\mathbf{i} - 5\mathbf{j}$

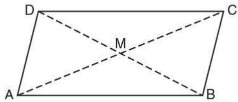
(iii) $20\mathbf{i} - 4.5\mathbf{j}$

(iv) $-30\mathbf{i} + 5.5\mathbf{j}$

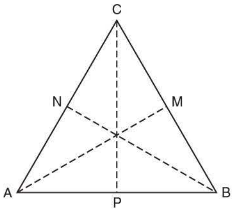
3 ABCD is a parallelogram with $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{BC} = \mathbf{q}$.

Diagonals AC and BD intersect at M.

Find $\overrightarrow{AB}$, $\overrightarrow{AM}$, $\overrightarrow{BD}$ and $\overrightarrow{BM}$.



4 The vertices of a triangle ABC have position vectors **a**, **b** and **c**. Points M, N and P are the midpoints of BC, CA and AB respectively, and G is the point which is $\frac{2}{3}$ of the way along AM.



(i) Find the following in terms of **a**, **b** and **c**.

(a) $\overrightarrow{AM}$

(b) $\overrightarrow{AG}$

(c) $\overrightarrow{BN}$

(d) $\overrightarrow{CP}$

(e) the position vector of G

(ii) Show that G is also $\frac{2}{3}$ of the way along BN and CP.

5 ABCD is an isosceles trapezium such that AB and CD are parallel with AB = 2CD.

(i) Given that $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$, express each of the following vectors in terms of $\mathbf{p}$ and $\mathbf{q}$.

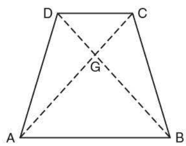
(a) $\overrightarrow{CD}$

(b) $\overrightarrow{BC}$

(c) $\overrightarrow{AC}$

(d) $\overrightarrow{BD}$

(ii) Point G is λ of the way along both diagonals and $0 < \lambda < 1$. Show that λ must equal $\frac{2}{3}$ for G to be the point of intersection of the diagonals.



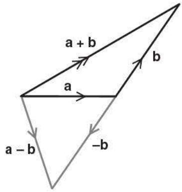
6 The coordinates of A and B are (3, 2) and (–2, 5). Express $\overrightarrow{AB}$ in terms of its magnitude and direction.

7 Find the coordinates of A if $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and B is the point (7, 1).

8 If $\mathbf{p} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ find $|\,\mathbf{p} + \mathbf{q}\,|$ and $|\,\mathbf{p} - \mathbf{q}\,|$.

9 Vectors **a** and **b** are perpendicular with $|\mathbf{a}| = 5$ and $|\mathbf{b}| = 12$. Find $|\,\mathbf{a} + \mathbf{b}\,|$ and $|\,\mathbf{a} - \mathbf{b}\,|$.

10 Vectors **a** and **b** are such that $|\mathbf{a}| = 13$, $|\mathbf{b}| = 19$ and $|\mathbf{a} + \mathbf{b}| = 24$. Find $|\mathbf{a} - \mathbf{b}|$.



11 The three forces $\mathbf{F}_1 = \begin{pmatrix} -7 \\ 4 \end{pmatrix}$, $\mathbf{F}_2 = \begin{pmatrix} 10 \\ -9 \end{pmatrix}$ and $\mathbf{F}_3$ are in equilibrium. Find the magnitude and direction of $\mathbf{F}_3$.

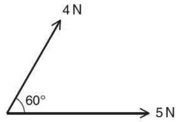
12 Vector **i** is a unit vector pointing due east and **j** is a unit vector pointing due north.

(i) If $\mathbf{F} = 5\mathbf{i} - 8\mathbf{j}$, find the magnitude of **F** and its direction expressed as a bearing.

(ii) If $\mathbf{G} = 7.5\mathbf{i} - 12\mathbf{j}$, express **G** in terms of **F**.

(iii) If $\mathbf{F}_1 = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{F}_2 = 7\mathbf{i} + Q\mathbf{j}$, find Q such that $\mathbf{F}_1 + \mathbf{F}_2$ is parallel to **F**.

13 Two forces of magnitude 5 N and 4 N act at a point.
The angle between the two forces is 60°.



(i) Calculate the magnitude of the resultant force.

(ii) Calculate the angle between the resultant and the 5 N force.

14 Find the magnitude and direction of the resultant of these vectors.

(i) $\mathbf{p} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{q} = 5\mathbf{i} - 7\mathbf{j}$, where $\mathbf{i}$ is a unit vector in the x direction and $\mathbf{j}$ is a unit vector in the y direction.

(ii) $\mathbf{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$

15 Two vectors p and q are given by $\mathbf{p} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 2 \\ x \end{pmatrix}$.

Find x in each of the following cases.

(i) Vectors **p** and **q** are parallel.

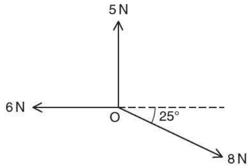
(ii) $2\mathbf{p} + \mathbf{q} = 4\mathbf{q}$

(iii) $|\mathbf{p} + \mathbf{q}| = 2|\mathbf{p}|$

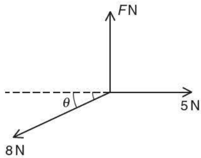
16 Find the magnitude and direction of the resultant of two forces of magnitude 26 N and $5\sqrt{5}$ N acting in the directions $12\mathbf{i} - 5\mathbf{j}$ and $2\mathbf{i} + \mathbf{j}$ respectively.

17 Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act in the directions of $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 13 \\ 3 \end{pmatrix}$ respectively. The resultant of the two forces is given by $\mathbf{F} = \begin{pmatrix} 60 \\ 14 \end{pmatrix}$. Find $\mathbf{F}_1$ and $\mathbf{F}_2$.

18 Three forces of magnitudes 5 N, 6 N and 8 N are acting at the point O as shown in the diagram. Find the magnitude and direction of the resultant of the three forces.



19 Three forces of magnitudes F N, 5 N and 8 N act at a point and are in equilibrium. The forces of magnitude F N and 5 N are at right angles to one another and the 8 N force makes an angle of $(180 - \theta)°$ with the 5 N force.



(i) Find θ.

(ii) Find F.

20 A particle is moving with constant acceleration $\mathbf{a} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$. It passes point O at $t = 0$ with initial velocity $\mathbf{u} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$. The unit vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are pointing due east and due north respectively.

(i) Show that after 1 second the particle is moving in the direction SE and find its speed.

(ii) Calculate the bearing of the particle from O after it has been moving for 2.5 seconds.

21 Two forces $\mathbf{F}_1 = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$ and $\mathbf{F}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ are acting on a particle of mass 2 kg.

(i) Find the resultant force acting on the particle and hence the acceleration of the particle.

(ii) When $t = 10$ seconds, the velocity of the particle is given by $\mathbf{v} = \begin{pmatrix} 30 \\ -10 \end{pmatrix}$. Find the initial velocity $\mathbf{u}$.

(iii) Find the times when the speed of the particle is $10\,\text{m}\,\text{s}^{-1}$.

22 A particle moves with constant acceleration, so that its velocity at time t is given by

$$\mathbf{v} = (5 - 0.1t)\mathbf{i} + (3 + 0.2t)\mathbf{j}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the directions east and north respectively.

(i) Write down the acceleration $\mathbf{a}$ and the initial velocity $\mathbf{u}$.

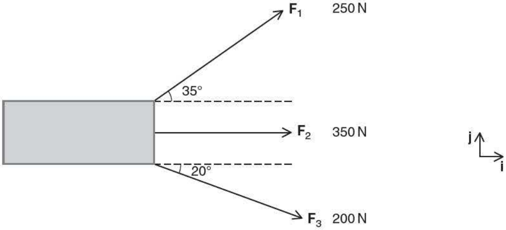
(ii) Find the time t when the particle is travelling in the direction NE.

23 Two forces $\mathbf{X}$ and $\mathbf{Y}$ act at a point O. Force $\mathbf{X}$ has magnitude 25 N and acts along a bearing of 060°. Force $\mathbf{Y}$ has magnitude 20 N and acts along a bearing of 000°.

(i) Calculate the magnitude and bearing of the resultant of $\mathbf{X}$ and $\mathbf{Y}$.

(ii) A third force $\mathbf{Z}$ is now acting on O. The three forces are in equilibrium. Find the magnitude and bearing of $\mathbf{Z}$.

24 Three forces, $\mathbf{F}_1$ of magnitude 250 N at an angle of 35° to the positive x direction, $\mathbf{F}_2$ of magnitude 350 N along the positive x direction and $\mathbf{F}_3$ of magnitude 200 N at an angle of 20° below the positive x direction, are applied to a packing case in order to move it.



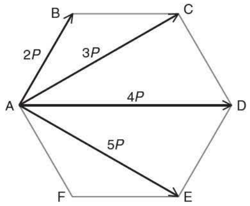
(i) Express each of the three forces in terms of their components.

(ii) Find the magnitude and direction of the resultant force.

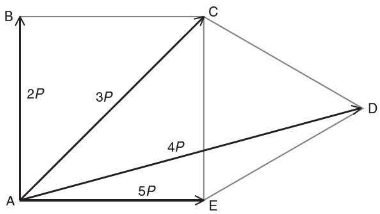
(iii) A fourth force $\mathbf{F}_4 = 120\mathbf{i} + P\mathbf{j}$ is now applied and the case now moves at constant speed along the positive x direction. Find P.

25 Forces of magnitude $2P$, $3P$, $4P$ and $5P$ act along $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$, respectively. Find the magnitude and direction of their resultant in the following cases.

(i) ABCDEF is a regular hexagon.



(ii) ABCE is a square and CDE an equilateral triangle.



Stretch and challenge

1 The magnitude of the resultant of two forces **P** and **Q** is equal to the magnitude of **P**. The magnitude of the resultant of 2**P** and **Q** is equal to $\sqrt{3}$ times the magnitude of **P**.

Find the magnitude of **Q** and show that **Q** makes an angle of 120° with **P**.

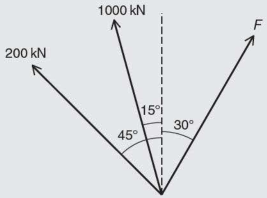
2 Two forces of magnitude 5 N and 8 N act at an angle θ such that $\sin \theta = 0.25$. Find the magnitude of the two possible resultants.

3 Three tugs are pulling a liner due north into a harbour.

The ropes attaching the liner to the tugs are in the direction NW, N 15° W and N 30° E.

The tensions in the first two ropes are 200 kN and 1000 kN.

Find the tension in the third rope and the resultant pull on the liner.



Exam focus

1 Vectors **i** and **j** are unit vectors in directions east and north respectively. A particle is moving with constant acceleration $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$ and initial velocity $\mathbf{u} = -2\mathbf{i}$.

(i) Find the magnitude and direction of the velocity 5 seconds later. [4]

(ii) When the particle is moving north-east find the magnitude of its velocity. [4]

2 A force **P** is given in component form by $\mathbf{P} = \begin{pmatrix} 36 \\ -4.25 \end{pmatrix}$.

(i) Find the magnitude and direction of **P**. [3]

A second force **Q** of magnitude 87 N acts in the same plane at 90° anticlockwise from **P**. The resultant of these two forces has a magnitude R and makes an angle θ with the positive x axis.

(ii) Find R and θ. [4]

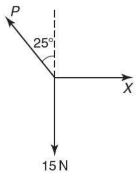
6 Forces in equilibrium and resultant forces

Finding resultant forces; Forces in equilibrium

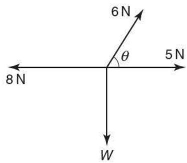
EXERCISE 6.1

1 In each of the force diagrams below, a particle is kept in equilibrium under the action of the forces shown. In each case find the values of the unknown forces and angles marked by letters.

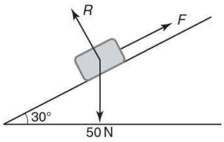
(i)



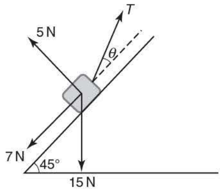
(ii)



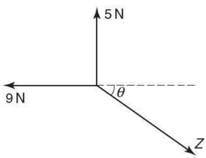
(iii)



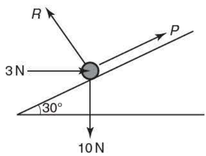
(iv)



(v)



(vi)

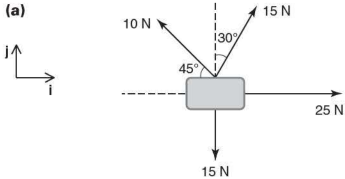


2 For diagrams (**a**) and (**b**) below, find

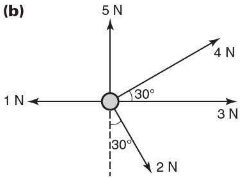
(i) the resultant force in terms of its components

(ii) the magnitude and direction of the resultant.

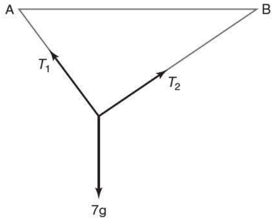
(a)



(b)



3 A particle of mass 7 kg is suspended by two strings of length 0.25 m and 0.6 m to two points A and B which are 0.65 m apart. Find the tensions in the string.



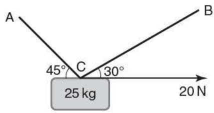
4 A force of 0.5 N making an angle of 20° with the horizontal is applied to a particle of mass 25 grams hanging at the end of a string. If the mass is in equilibrium, find the tension in the string and the angle which the string makes with the vertical.

5 A particle of mass 6 kg is held in equilibrium on a smooth inclined plane at 35° to the horizontal by an inextensible string which makes an angle of 15° with the slope.

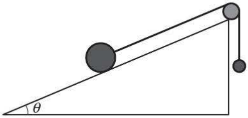
(i) Find the tension in the string.

(ii) Find the normal reaction of the plane on the particle.

6 A box of mass 25 kg is supported by two light strings AC and BC that are tied to the box at C. AC and BC make angles of 45° and 30° with the horizontal. There is also a horizontal force of 20 N acting at C as shown in the diagram. Find the tensions in the strings.

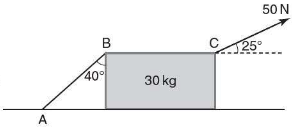


7 A particle of mass 3 kg rests on a smooth plane inclined at an angle θ to the horizontal. It is attached to a light inextensible string which passes over a smooth pulley at the top of the plane. At the other end of the string is a mass of 2 kg which is hanging freely. If the system is in equilibrium, find the angle θ, the tension in the string and the normal reaction between the 3 kg particle and the plane.



8 A particle of mass 7 kg is attached at the end of a string. The other end of the string is fixed. An upward vertical force of 5 N and a horizontal force of 21 N act upon the particle, so that it rests in equilibrium with the string at an angle θ to the vertical. Calculate the tension in the string and the angle θ.

9 A box of mass 30 kg is on a smooth horizontal surface, as shown in the figure. A light string AB is attached to the surface at A and to the box at B. AB makes an angle of 40° to the vertical. Another light string is attached to the box at C, this string is inclined at 25° to the horizontal and the tension in it is 50 N. The box is in equilibrium.



(i) Calculate the tension in the string AB.

(ii) Calculate the normal reaction of the floor on the box.

The string at C is replaced by another string which is inclined at 25° below the horizontal with the same tension of 50 N.

(iii) Explain why this has no effect on the tension in string AB.

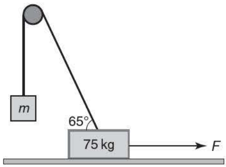
(iv) How much larger is the normal reaction?

10 A particle is suspended by two light inextensible strings and hangs in equilibrium. One string is inclined at 35° to the horizontal and has a tension of 40 N. The second string is inclined at 55° to the horizontal.

(i) Find the tension in the second string.

(ii) Find the mass of the particle.

11 A block of mass 75 kg is in equilibrium on smooth horizontal ground with one end of a light string attached to its upper edge. The string passes over a smooth pulley, with a block of mass m kg attached at the other end.



The part of the string between the pulley and the block makes an angle of 65° with the horizontal. A horizontal force F is also acting on the block.

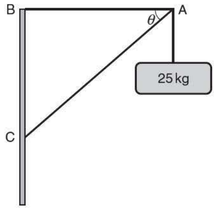
(i) If T is the tension in the string and R is the normal reaction of the floor on the block, find a relationship between T and R.

It is given that the block is on the point of lifting off the ground.

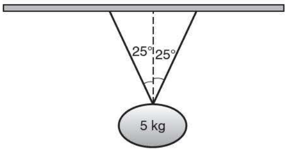
(ii) Find T and m.

(iii) Find F.

12 The diagram shows a sign attached to a point A. It is supported by two rigid light rods AB and AC. AB is horizontal and AC makes an angle θ with the horizontal where $\sin \theta = 0.75$. The mass of the sign is 25 kg. Find the forces in rods AB and AC, stating whether they are in tension or compression.



13 A sign of mass 5 kg is hung from the ceiling of a shop by two light strings, each making an angle of 25° with the vertical as shown in the diagram.



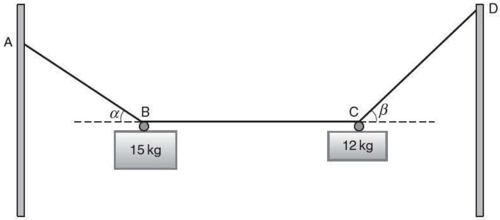
(i) Show that the tension is the same in each string

(ii) Find the tension in each string.

(iii) If the tension in a string exceeds 75 N, the string will break. Find the mass of the heaviest sign which can be hung in this way.

14 A uniform rod AB of weight W rests in equilibrium with the end A in contact with a smooth vertical wall and the end B in contact with a smooth plane inclined at 45° to the wall. Find the reactions at A and B in terms of W.

15 Two boxes of masses 15 kg and 12 kg are held by light strings AB, BC and CD. As shown in the figure, AB makes an angle α with the horizontal and is fixed at A. Angle α is such that $\sin\alpha = 0.6$ and $\cos\alpha = 0.8$. BC is horizontal and CD makes an angle β with the horizontal.



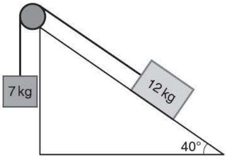
(i) By considering the equilibrium of B, find the tension in string AB and show that the tension in BC is equal to 200 N.

(ii) By considering the equilibrium of point C, find β and the tension in CD.

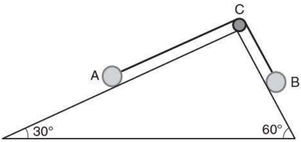
Newton's second law in two dimensions

EXERCISE 6.2

1 A block of mass 12 kg is placed on a smooth plane inclined at 40° to the horizontal. It is connected by a light inextensible string, which passes over a smooth pulley at the top of the plane, to a mass of 7 kg hanging freely. Find the common acceleration and the tension in the string.



2 Two particles A and B rest on the inclined faces of a fixed triangular wedge as shown in the diagram. A and B are connected by a light inextensible string which passes over a light smooth pulley at C. The faces of the wedge are smooth. A and B have the same mass, 5 kg.



Find the acceleration of the system, the tension in the string and the force exerted by the string on the pulley at C.

3 A car is towing a trailer along a straight road down a slope inclined at 5° to the horizontal. The masses of the car and trailer are 950 kg and 350 kg respectively. The resistance to motion for the car is 500 N and that for the trailer is 200 N. The driving force of the car is 1000 N.

(i) Find the acceleration of the car.

(ii) Find the pulling force exerted on the trailer.

4 A car of mass 1050 kg pulls a trailer of mass 450 kg up a road inclined at an angle of θ to the horizontal, where $\sin\theta = 0.125$. The resistance to motion for both car and trailer is 0.25 N per kg.

(i) Find the driving force exerted by the engine and the tension in the coupling if the car and trailer are travelling at constant speed.

(ii) Find the driving force exerted by the engine and the tension in the coupling if the car and trailer are accelerating at $0.8\,\text{m}\,\text{s}^{-2}$.

5 A slide consists of a sloping section followed by a horizontal section. Both sections are 5 m long. The sloping section is inclined at an angle θ to the horizontal, with $\sin\theta = 0.28$.

A child of mass 25 kg starts from rest at the top of the slide and is subject to a constant resistance of 35 N along both sections of the slide.

(i) Calculate the child's acceleration down the slope.

(ii) Calculate the speed of the child at the bottom of the slope.

(iii) Calculate the child's deceleration on the horizontal section.

(iv) Calculate the speed of the child at the end of the horizontal section.

6 An engine of mass 35 tonnes is pulling two trucks each of mass 15 tonnes along a horizontal straight track. The engine is subject to a resistance of 1000 N and each truck to a resistance of 400 N.

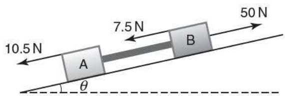
(i) Find the driving force of the engine if the train is travelling at constant speed.

(ii) Find the force in each of the couplings.

The train now comes to an incline of 5° to the horizontal and starts to ascend.

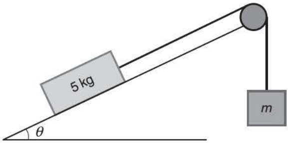
(iii) If the engine maintains the same driving force, calculate the deceleration of the train and the new forces in the couplings.

7 Two boxes, A of mass 4 kg and B of mass 2.5 kg, are linked by a light rigid rod. A force of 50 N is pulling the boxes up a slope inclined at an angle θ to the horizontal such that $\sin\theta = 0.1$. Resistances to the motion of the boxes are 10.5 N for A and 7.5 N for B.



Find the acceleration of the boxes and the force in the coupling, stating whether it is a tension or a thrust.

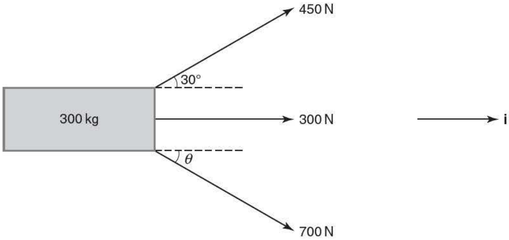
8 A block of mass 5 kg is being accelerated at a rate of $1\,\mathrm{m\,s^{-2}}$ up a smooth plane inclined at an angle θ to the horizontal, such that $\sin\theta = 0.26$. A light inelastic string is attached to the block, passes over a smooth pulley and supports a mass of m kg which is hanging freely.



(i) Find m and the tension in the string.

(ii) If $m = 3.5$ kg, find the acceleration and the tension in the string in this case.

9 A large box of mass 300 kg is being pulled by three forces as shown in the diagram. The angle θ is chosen so that the resultant of the three forces acts along the **i** direction.



(i) Find θ and the resultant of the three forces.

With this resultant force, the box moves with constant acceleration and travels 1 m from rest in 5 s.

(ii) Find the magnitude of the frictional force.

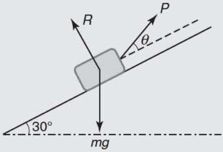
When the speed of the box is $0.5\,\text{m}\,\text{s}^{-1}$ it comes to a point on the floor where the friction is 250 N smaller. The pulling forces are the same.

(iii) Find the velocity of the box when it has moved a further 1.5 m.

Stretch and challenge

1 A particle of mass m rests on a smooth plane inclined at 30° to the horizontal. A force P, inclined up the plane at an angle of θ to the plane, holds the particle in equilibrium.

If the normal reaction is given by $R = \frac{mg}{\sqrt{3}}$, find the value of θ and show that $P = \frac{mg}{\sqrt{3}}$.



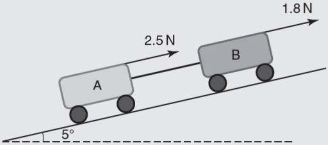
2 A sleigh is found to travel with uniform speed down a slope of 1 in 50. If the sleigh starts from the bottom of the same slope with a speed of $2\,\mathrm{m\,s^{-1}}$ how far will it travel up the slope before coming to rest?

3 Two toy trucks are travelling down a slope inclined at an angle of 5° to the horizontal. Truck A has a mass of 0.5 kg, truck B has a mass of 0.35 kg.

The trucks are linked by a light rigid rod which is parallel to the slope.

The resistances to motion of the trucks are 2.5 N for truck A and 1.8 N for truck B.

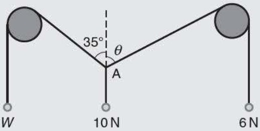
The initial speed of the trucks is $2\,m\,s^{-1}$.



Calculate the speed of the trucks after 3 seconds and also the force in the rod connecting the trucks, stating whether the rod is in tension or in thrust.

Exam focus

1 Three light inextensible strings have a particle attached to one of their ends. The other ends are tied together at A. The strings are in equilibrium with two of them passing over smooth pulleys and the particles are hanging freely.
The weight of the particles and the angles between the sloping parts and the vertical are shown in the figure.

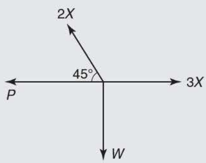


Find W and θ. **[5]**

2 A particle is in equilibrium under the set of forces shown in the diagram.

Show that

$$P = (3\sqrt{2} - 2)\frac{W}{2}$$



[5]

3 A box of mass 50 kg is sliding down a slope inclined at 20° to the horizontal. The box starts from rest and reaches a speed of 4.5 m s^{-1} after 2 seconds.

Calculate the frictional force between the box and the slope. **[6]**

4 A car of mass 900 kg is towing a trailer of mass 350 kg along a slope of 1 in 100 (i.e. at an angle θ to the horizontal and $\sin\theta = \frac{1}{100}$). The driving force of the engine is 2000 N and there are resistances to the motion of both the car, 500 N, and the trailer, 200 N.

Find the acceleration and the tension in the tow-bar. **[6]**

7 General motion in a straight line

Using differentiation; Finding displacement from velocity; The area under a velocity–time graph; Finding velocity from acceleration; The constant acceleration formulae revisited

EXERCISE 7.1

1 A particle moves along the x axis. Its position at time t is given by

$$x = 21 - 9t + 6t^2 - t^3$$

(i) Find an expression for the velocity at time t.

(ii) Show that for the first second the particle moves in the negative direction, and for the next two seconds it moves in the positive direction.

(iii) Find also the acceleration at the instants when the particle is at rest.

2 The displacement x m of a moving point A at time t seconds is given by the formula

$$x = 2t^3 - 3t^2 + 4t - 10$$

Find the velocity and the acceleration of A at the instant $t = 4$.

3 A particle is moving in a straight line and its displacement x from a fixed point O is given by the equation

$$x = 18t - 21t^2 + 4t^3$$

(i) Find the velocity and acceleration after 4 s.

(ii) Find the distance travelled between the two times when the velocity is instantaneously zero.

(iii) Find the total distance travelled in the interval $0 < t < 4$.

4 Find the displacement x m of a particle at time t s if its velocity v m s^{-1} is given by

(i) $v = 5t - 2$, and $x = 3$ when $t = 0$

(ii) $v = 3t^2 + 4t$, and $x = 2$ when $t = 1$

(iii) $v = 1 - 3t$, and $x = -2$ when $t = 0$.

5 Find the velocity v m s^{-1} and displacement x m of a particle at time t s, if its acceleration a m s^{-2} is given by

(i) $a = 12t - 8$, and when $t = 0$, $x = 5$ and $v = 3$

(ii) $a = 3.5t^{1.5} - 3$, and when $t = 1$, $x = 3$ and $v = -2$

(iii) $a = 0.6t^2 - 0.3t + 1$, and when $t = 1$, $x = 0.5$ and $v = 1$.

6 A particle starts from rest at O and moves along a straight line. After t seconds its velocity is v m s^{-1}, where $v = t^2 - t^3$.

(i) Show that the particle is momentarily at rest after 1 s and find its distance from O at this time.

(ii) Find the maximum velocity of the particle in the first second of the motion.

7 A particle starts from rest at a point 5 m from O and moves in a straight line away from O with velocity v m s^{-1} at time t s given by $v = 3t - \frac{1}{12}t^2$.

(i) Find its acceleration and distance from O, each in terms of t.

(ii) Find the time at which it begins to return, and the time at which it again reaches its starting point.

8 Starting from rest at O, a particle travelling in a straight line is subject to an acceleration a m s^{-2} given by $a = 15 - 0.75t$.

(i) Find the particle's velocity after 15 seconds.

(ii) Find the distance travelled by the particle after 15 seconds.

(iii) At what time is the particle's maximum speed reached and what is that maximum speed?

9 The acceleration, a m s^{-2}, of a particle t s after starting from rest is given by $a = 3t - 2$.

(i) Show that the particle returns to its starting point after 2 s, and find the distance of the particle from the starting point after a further 2 s.

(ii) Find at what time the particle's velocity is zero.

(iii) Find the total distance travelled by the particle in the first 4 seconds.

10 The acceleration, a m s^{-2}, of a car t seconds after starting from rest is

$$a = 2 + 0.8t - 0.1t^2$$

until $a = 0$. After that time the speed remains constant.

(i) Find the car's maximum acceleration.

(ii) Find the time taken by the car to attain the greatest speed.

(iii) Find the greatest speed attained.

(iv) Find the distance travelled by the car in the first 20 seconds.

11 A particle moves in a straight line with an acceleration $a = 6t - 2$. If the particle has an initial velocity of $3\,\text{m s}^{-1}$, find the distance travelled by the particle in the first second of its motion.

12 A particle P is moving in a straight line. At time t seconds after starting from O, its velocity v is given by

$$v = t^2(3 - t)$$

(i) Find the values of t when the acceleration is zero.

(ii) After what time does the particle come to instantaneous rest?

At that time the particle has reached its furthest point A from O and then reverses back towards O.

(iii) Find the distance OA.

(iv) Find the time taken for the particle to return to O.

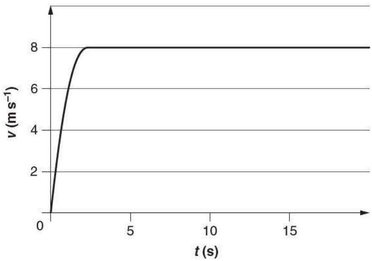
13 The velocity of a particle is given by $v = 5t - t^3$. Find the maximum displacement of the particle.

14 During braking the speed of a car is given by $v = 12 - 3t^2$ until it stops moving. Find the distance travelled from the time that the braking starts.

15 The velocity of a sprinter at the start of a race is shown below and in the figure:

$v = 8t - 2t^2$; $\quad 0 \leqslant t \leqslant 2$

$v = 8$; $\quad t \geqslant 2$



(i) Find the acceleration of the sprinter at $t = 2$. Hence write down the maximum speed.

(ii) How far does the sprinter run in the first 2 seconds?

(iii) How long does the sprinter take to run 100 m?

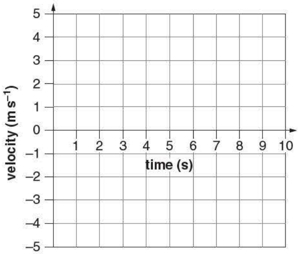
16 A particle moves in a straight line. The velocity, v m s^{-1}, of the particle is given by

$$v = t^2 - 4t, \quad 0 \leqslant t \leqslant 5$$
$$v = c, \quad 5 \leqslant t \leqslant 8$$
$$v = at + b, \quad 8 \leqslant t \leqslant 10$$
$$v = 0 \text{ when } t = 10$$

(i) Show that $c = 5$.

(ii) Calculate a and b, and describe the motion of the particle in the interval $8 \leqslant t \leqslant 10$.

(iii) Sketch the velocity–time graph for the motion of the particle for $0 \leqslant t \leqslant 10$.



(iv) Calculate the distance travelled.

17 A sprinter starts from rest at time $t = 0$ and runs in a straight line. For $0 \leqslant t \leqslant 3$ the sprinter has a velocity given by $v = 2.7t^2 - 0.6t^3$. For $3 \leqslant t \leqslant 23$, the sprinter runs at constant speed of $8.1\,\text{m}\,\text{s}^{-1}$. For $t > 23$ the sprinter decelerates at a constant rate of $0.2\,\text{m}\,\text{s}^{-2}$.

(i) Find the distance travelled by the sprinter in the first 3 seconds.

(ii) Find the time the sprinter takes to run 100 metres.

(iii) Find the time the sprinter takes to run 200 metres.

Stretch and challenge

1 Two sprinters are having a race over 100 m. Their accelerations in $m\,s^{-2}$ are as follows:

Sprinter A		**Sprinter B**	
$a = 5.4t - 1.8t^2$;	$0 \leqslant t \leqslant 3$	$a = 3.24t - 0.81t^2$;	$0 \leqslant t \leqslant 4$
$a = 0$;	$t > 3$	$a = 0$;	$t > 4$

(i) Find the greatest speed of each sprinter.

(ii) Find the distance run by each sprinter while reaching their greatest speed.

(iii) How long does each take to finish the race?

(iv) Who wins the race, by what time margin and by what distance?

2 A particle is moving in a straight line. Starting from rest, it has an acceleration given by

$$a = 1 - 0.2t; \qquad 0 \leqslant t \leqslant 4$$

$$a = 0.2; \qquad 4 \leqslant t \leqslant 10$$

(i) Find the speed of the particle when $t = 4$.

(ii) Find the distance travelled in the interval $0 \leqslant t \leqslant 4$.

(iii) Find the total distance travelled.

3 A particle is moving in a straight line. The position x of the particle at time t is given by

$$x = 18 - 24t + 9t^2 - t^3; \qquad 0 \leqslant t \leqslant 5$$

(i) Find the velocity v at time t and the values of t for which $v = 0$.

(ii) Find the position of the particle at those times.

(iii) Find the total distance travelled by the particle in the interval $0 \leqslant t \leqslant 5$.

Exam focus

1 The displacement x m of a particle from the origin O is given by

$$x = 15 + 12t + 3t^2 - 2t^3; \qquad -3 \leqslant t \leqslant 5.$$

(i) Write down the displacement of the particle when $t = 0$. [1]

(ii) Find an expression in terms of t for the velocity v m s^{-1}. [2]

(iii) Find an expression in terms of t for the acceleration a m s^{-2}. [2]

(iv) Find the maximum value of v in the interval $-3 \leqslant t \leqslant 5$. [3]

(v) Determine the number of times the particle passes through the origin. [2]

(vi) Find the total distance travelled in the interval $-3 \leqslant t \leqslant 5$. [5]

2 A particle moves in a straight line such that its velocity v at time t is given by

$$v = 18 - 27t + 10t^2 - t^3; \qquad 0 \leqslant t \leqslant 5.$$

(i) Calculate the acceleration when $t = 2$. [2]

(ii) Find an expression in terms of t for the displacement of the particle.
Hence find the displacement of the particle when $t = 5$. [3]

(iii) Explain what happens to the motion of the particle between $t = 1$ and $t = 3$. [2]

(iv) Find the total distance travelled in the interval $0 \leqslant t \leqslant 5$. [4]

8 A model for friction

Modelling with friction

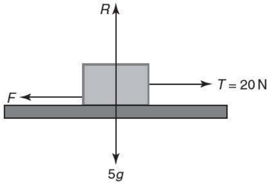
EXERCISE 8.1

1 Each diagram shows a block of mass 5 kg resting on a rough horizontal surface. The block is being pulled by an inextensible string with tension T. Given that the block is on the point of sliding in each case, find:

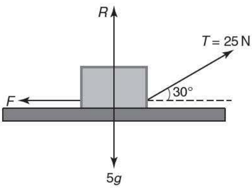
(i) the normal reaction between the block and the surface

(ii) the coefficient of friction.

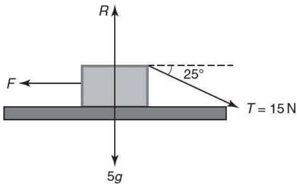
(a)



(b)



(c)



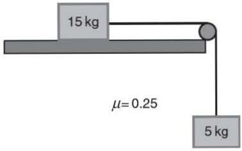
2 In each of the following situations:

(i) find the acceleration

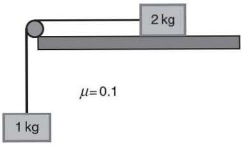
(ii) find the tension in each string

(iii) find the magnitude of the frictional force.

(a)



(b)



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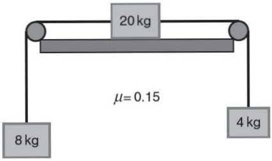
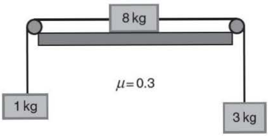
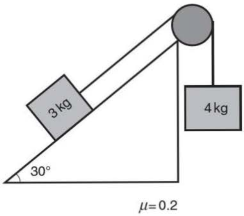


(d)



(e)



3 A car travelling at $15\,\text{m}\,\text{s}^{-1}$ skids to a halt in a distance of 30 m.

(i) Find its deceleration.

(ii) Find the coefficient of friction.

(iii) Find the stopping distance from a speed of $25\,\text{m}\,\text{s}^{-1}$.

4 A stone is sliding in a straight line across a horizontal ice rink. Given that the initial speed of the stone is $5\,\text{m}\,\text{s}^{-1}$ and that it slides 20 m before coming to rest, calculate the coefficient of friction between the stone and the ice.

5 A mass of 3 kg is pulled along a horizontal floor by means of a force of 50 N acting at 15° above the horizontal.

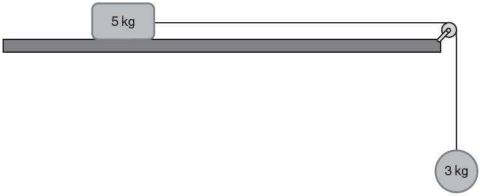
(i) If the coefficient of friction μ is equal to 0.3, find the acceleration of the mass.

(ii) What would the acceleration of the mass be if the force acted downwards at 15° to the horizontal?

6 A child at a water sports centre is sliding down a chute which is 12 m long and inclined at an angle of 30° to the horizontal. If the coefficient of friction between the chute and the child is 0.2, find the child's acceleration down the chute and speed on leaving the chute.

7 Find the least force that will move a block of mass 75 kg up a rough plane inclined at 20° to the horizontal when the coefficient of friction is 0.6.

8 A block of mass 5 kg lies on a rough horizontal table. The coefficient of friction between the table and the block is 0.2. The block is attached by a light inextensible string passing over a smooth pulley at the edge of the table to a mass of 3 kg hanging freely. The 5 kg mass is 2 m from the pulley and the 3 kg mass is 1.5 m from the floor.



The system is released from rest. Find:

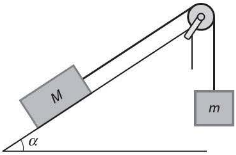
(i) the acceleration of the system

(ii) the time taken for the 3 kg mass to reach the floor

(iii) the velocity with which the 5 kg mass hits the pulley.

9 A block of mass M is lying on a rough inclined plane at an angle α to the horizontal. It is connected by a light inextensible string, which passes over a smooth pulley at the top of the plane, to a mass m which is hanging freely.

The coefficient of friction between the block and the plane is μ.



When the system is free to move, find the acceleration and the tension in the string if:

(i) the hanging mass descends

(ii) the hanging mass ascends.

10 A block of mass 25 kg rests on a rough plane inclined at an angle θ to the horizontal with $\sin \theta = 0.35$. The coefficient of friction between the block and the plane is 0.2.

(i) Find the acceleration of the block when a force of 40 N acts on it. The force is parallel to the plane and down the plane.

(ii) What force acting parallel to the plane would be required to give an equal acceleration up the plane?

11 A stone is released from rest on a rough inclined plane, making an angle of 30° to the horizontal, and slides downhill for one metre in one second. Find the coefficient of friction between the stone and the plane.

12 A block of mass 1.2 kg is pulled across a horizontal surface by a force of 10 N inclined at an angle of 60° to the vertical. The coefficient of friction between the block and the surface is 0.3.

(i) Calculate the vertical component of the force exerted by the surface on the block.

(ii) Calculate the acceleration of the block.

The 10 N force is removed when the speed of the block is $3\,\text{m}\,\text{s}^{-1}$.

(iii) Calculate the time taken for the block to decelerate from a speed of $3\,\text{m}\,\text{s}^{-1}$ to rest.

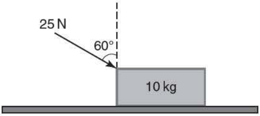
13 A block of mass 10 kg is placed on a rough plane inclined at 20° to the horizontal. A force P of magnitude 25 N acting parallel to the plane is just enough to prevent the block from sliding down the plane.

(i) Find the coefficient of friction μ between the block and the plane.

(ii) P is now increased until the block is about to slide up the plane. Find the value of P.

(iii) P is now increased to 100 N. What is the acceleration of the block up the plane?

14 A box of mass 10 kg is being pushed along uniform rough ground by means of a downward force of 25 N at 60° to the vertical as shown in the figure.



The box is initially at rest and is travelling at $0.5\,\mathrm{m\,s^{-1}}$ after it has slid 4 m.

(i) Find the frictional force.

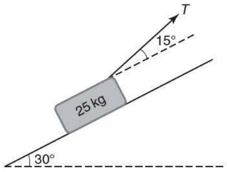
(ii) Find the coefficient of friction.

When the box is moving at $0.5\,\mathrm{m\,s^{-1}}$, the force is removed.

(iii) How far does the box slide before coming to rest?

(iv) If the force had been 25 N upwards at 60° to the vertical, would the box have been travelling at the same speed, or faster or slower, after sliding for 4 m?

15 The figure shows a mass of 25 kg on a slope making an angle of 30° with the horizontal. The mass is being pulled by a rope making an angle of 15° with the slope. The tension in the rope is T. The coefficient of friction between the mass and the slope is 0.3.



Find T if:

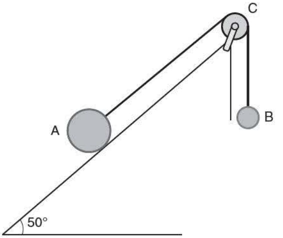
(i) the mass is about to move down the slope

(ii) the mass is about to move up the slope

(iii) the mass is accelerating at $2\,\text{m}\,\text{s}^{-2}$ up the slope.

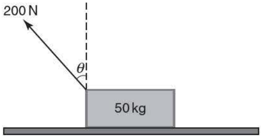
16 Two particles A and B of masses 10 kg and 4 kg, respectively, are attached to the ends of a light inextensible string which passes over a smooth pulley C at the top of a fixed rough plane inclined at an angle of 50° to the horizontal.

The particles are at rest with the 10 kg mass in contact with the plane and the 4 kg mass hanging freely.



Given that A is in limiting equilibrium and on the point of moving down the plane, find μ, the coefficient of friction.

17 A box of mass 50 kg is at rest on rough horizontal ground. A force of magnitude 200 N acts upwards on the box at an angle θ to the vertical, where $\tan\theta = 0.75$.



The box is about to slip. Find μ, the coefficient of friction between the box and the ground.

18 A force of $3X$ is applied to a block of mass m which is lying on a rough plane inclined at an angle α to the horizontal, in a direction parallel to the plane. This causes the mass to be on the point of moving up the plane.

A force X up the plane is applied to the same mass and is just sufficient to prevent the mass from sliding down the plane.

If μ is the coefficient of friction, show that $\mu = 0.5 \tan\alpha$.

Stretch and challenge

1 A particle slides down a rough plane inclined at an angle θ to the horizontal such that $\sin\theta = 0.6$. The coefficient of friction between the particle and the plane is $\frac{9}{16}$.

Show that the time to descend any distance X is twice the time that would be taken if the plane was smooth.

2 A box of mass 850 kg is placed on a rough plane inclined at an angle $\alpha = \sin^{-1}\left(\frac{13}{85}\right)$. The coefficient of friction between the box and the plane is $\frac{1}{7}$. A rope is attached to the box and the direction of the rope makes an angle $\beta = \sin^{-1}\left(\frac{7}{25}\right)$ with the surface of the plane.

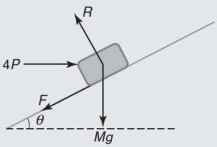
If the tension in the rope is T, then show that, for the box to remain in equilibrium, $109 \leqslant T \leqslant 2500$.

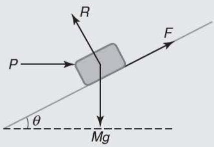
3 A horizontal force $4P$ applied to a mass M on a rough plane of inclination θ causes the mass to be on the point of moving up the plane.

A horizontal force P applied to the same mass on the same plane is just sufficient to prevent the mass from sliding down the plane.

μ is the coefficient of friction between the mass and the plane.

Show that $\mu = 0.6(1 + \mu^2) \sin \theta \cos \theta$.





Exam focus

1 A particle of mass 2 kg is projected up an inclined plane, making an angle of 20° with the horizontal, with a speed of $6\,\mathrm{m\,s^{-1}}$. The particle comes to rest after 4 m.

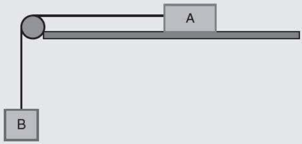
(i) Find the deceleration of the particle. [2]

(ii) Find the frictional force F and the normal reaction R, and hence deduce the coefficient of friction between the particle and the plane. [4]

The particle then starts to move down the plane with acceleration $a\,\mathrm{m\,s^{-2}}$.

(iii) Find a and the speed of the particle as it passes its starting point. [4]

2 Particles A and B are attached to opposite ends of a light inextensible string. Particle A, of mass m kg, is at rest on a rough horizontal table. The string passes over a smooth pulley fixed at the edge of the table. Particle B of mass 2 kg hangs vertically below the pulley. The coefficient of friction between particle A and the table is 0.45. Particle A is on the point of slipping.



(i) Find T, the tension in the string. [1]

(ii) Find m. [4]

A particle of mass 0.5 kg is now attached to B and the system starts to move.

(iii) Find the tension in the string. [3]

3 A box of mass 20 kg rests on a rough horizontal surface. The coefficient of friction between the box and the surface is 0.25. A light inextensible string is attached to the box in order to pull it. T is the tension in the string.

(i) Find the minimum value of T for the box to move when

(a) the string is horizontal [2]

(b) the string makes an angle of 20° above the horizontal [3]

(c) the string makes an angle of 20° below the horizontal. [2]

(ii) If the string is pulled at 30° above the horizontal, what value of T would make the box move with an acceleration of $2\,\mathrm{m\,s^{-2}}$? [3]

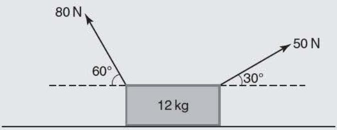
4 A block of mass 25 kg is at rest on a plane inclined at 15° to the horizontal. A force acts on the block in a direction parallel to the line of greatest slope of the plane. The coefficient of friction between the block and the plane is 0.35.

Find the least magnitude of the force necessary to move the block

(i) given that the force acts up the plane [3]

(ii) given instead that the force acts down the plane. [3]

5 A box of mass 12 kg is held in equilibrium on a rough horizontal table by two light inextensible strings. The strings make angles of 60° and 30° with the horizontal on either side of the box and the tensions in the strings are 80 N and 50 N respectively.



The box is on the point of slipping to the right. Find the coefficient of friction between the box and the table. [4]

9 Energy, work and power

Work and energy

EXERCISE 9.1

1 A ball of mass 0.25 kg is thrown vertically upwards from a point 1 m above the ground with an initial speed of $10\,\text{m}\,\text{s}^{-1}$.

(i) Calculate the initial kinetic energy of the ball.

(ii) Assuming that there is no air resistance, use an energy method to find the greatest height above ground reached by the ball.

An experiment shows that the maximum height reached is 5.5 m above ground level.

(iii) Find the work done against the air resistance that acts on the ball as it moves.

(iv) Assuming that the resistance force is constant, find its magnitude.

(v) Find the speed of the ball as it hits the ground.

2 A cyclist and her bicycle have a combined mass of 55 kg. The cyclist ascends a straight hill of constant slope making an angle θ with the horizontal with $\sin \theta = 0.25$ and of length 50 m. Starting from rest at A she reaches a speed of $6\,\text{m}\,\text{s}^{-1}$ at B. The resistance to motion is constant and has a magnitude of 50 N.

(i) Find the gain in kinetic energy.

(ii) Find the increase in gravitational potential energy.

(iii) Find the total work done by the cyclist.

3 A sledge slides down a straight track of length 120 m which drops down a vertical distance of 20 m. The mass of the sledge is 25 kg. The sledge's initial speed is $2\,\text{m}\,\text{s}^{-1}$ and its final speed is $10\,\text{m}\,\text{s}^{-1}$. The resistance force of magnitude F is constant. Use an energy method to find F.

4 A block of mass 20 kg is dragged 12 m up a slope inclined at an angle of 10° to the horizontal by a rope inclined at 15° to the slope. The tension in the rope is 100 N and the resistance to motion of the block is 50 N.

(i) Calculate the work done by the tension in the rope.

(ii) Calculate the change in potential energy of the block.

(iii) Find the speed of the block after it has moved 12 m up the slope.

5 A mass of 5 kg is projected with a speed of 5 m s^{-1} up a smooth slope inclined at 40° to the horizontal. How far up the slope does the mass travel?

6 A ball of mass 5 kg ascends a rough slope inclined at 30° to the horizontal. The initial speed of the ball is $5\,\text{m}\,\text{s}^{-1}$ and the coefficient of friction between the slope and the ball is 0.4.

(i) When the ball has travelled 1.2 m up the slope, find

(a) the work done against friction

(b) the gain in gravitational potential energy

(c) the loss in kinetic energy

(d) the speed of the ball.

(ii) Find the total distance up the slope travelled by the ball before it comes to rest.

7 A particle of mass 2.5 kg slides 2 m down a rough slope making an angle of 30° with the horizontal. Starting from rest, it has a speed of $4\,\text{m}\,\text{s}^{-1}$ at the end of the slope.

(i) Find the loss in gravitational potential energy.

(ii) Find the gain in kinetic energy.

(iii) Calculate the work done against friction.

(iv) Find the coefficient of friction between the particle and the slope.

8 A sledge of mass 25 kg is being pulled with a force of 50 N against a resistance of 35 N. If the sledge starts from rest, what is its speed after it has travelled 25 m?

9 A truck of mass 3500 kg is moving along a straight horizontal track with a speed of $5\,\mathrm{m\,s^{-1}}$. The truck is then brought to rest by a uniform force of magnitude P N.

(i) Find the work done by the force.

(ii) If the truck travels a distance of 50 m before coming to rest, find the value of P.

10 A sledge of mass 20 kg is being pulled up a slope which makes an angle of 10° to the horizontal. The sledge is being pulled by a rope which makes an angle of 15° to the slope. The sledge starts from rest and after travelling 8 m has a speed of $1.5\,\mathrm{m\,s^{-1}}$.

If resistances to motion are ignored, find:

(i) the gain in gravitational potential energy

(ii) the gain in kinetic energy

(iii) the tension in the rope.

11 A bullet of mass 12 grams passes horizontally through a fixed board of thickness 5 cm. The speed of the bullet is reduced from $200\,\mathrm{m\,s^{-1}}$ to $120\,\mathrm{m\,s^{-1}}$ as it passes through the board. If the board exerts a constant resistive force on the bullet, find the magnitude of that force.

12 A box of mass 12 kg is placed on a rough slope inclined at an angle θ to the horizontal and $\cos\theta = 0.8$. The coefficient of friction between the box and the slope is 0.15.

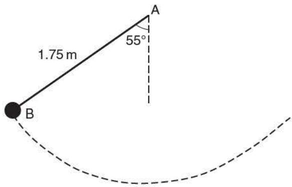
The box is projected from a point A with initial speed $u\,\mathrm{m\,s^{-1}}$. It travels a distance of 1.2 m along the slope before coming to rest.

(i) Calculate the value of u.

As the box slides back down the slope it passes through the point of projection A and later reaches its initial speed u at a point B.

(ii) Calculate the distance AB.

13 A small sphere of mass 0.25 kg is attached at one end, B, to a light inextensible string of length 1.75 m. The other end of the string, A, is fixed and the string can swing freely. The sphere swings with the string taut from a point where the string makes an angle of 55° with the vertical.



(i) Find how far down the sphere has dropped when it is at its lowest point of its swing and calculate the amount of gravitational potential energy lost at this point.

(ii) Assuming no air resistance and that the sphere starts from rest, calculate the speed of the sphere at the lowest point of its swing.

There is now a force opposing the motion that results in an energy loss of 0.5 J for every metre travelled by the sphere. The sphere is given an initial speed of $3\,m\,s^{-1}$ and is descending with AB at 55° to the vertical.

(iii) Calculate the speed of the sphere at the lowest point of the swing.

14 A block C of mass 15 kg lies on a smooth horizontal table. Each side of the block is connected to a small sphere by means of a light inextensible string passing over a smooth pulley as shown in the figure. Sphere A has mass 5 kg and sphere B has mass 12.5 kg. The spheres hang freely.

With the block at point O, the system is released from rest with both strings taut. The block reaches a speed of $0.75\,\mathrm{m\,s^{-1}}$ at point P.

(i) Calculate the change in gravitational potential energy of the system, stating whether it is a gain or loss.

(ii) Find the distance OP.

Power

EXERCISE 9.2

1 A car of mass 900 kg is travelling along a straight level road. The car's engine is developing a constant power of 20 kW and there is a constant resistance of 1000 N.

(i) Calculate the maximum possible steady speed of the car.

(ii) Find the driving force and the acceleration of the car when the speed is $10\,\mathrm{m\,s^{-1}}$.

When the car is travelling at $15\,\mathrm{m\,s^{-1}}$ up a constant slope inclined at $\sin^{-1}(0.1)$ to the horizontal, the driving force is removed. Subsequently, the resistance to the motion of the car remains constant and equal to 1000 N.

(iii) How far up the slope does the car go before coming to rest?

2 A cyclist and his bicycle have a combined mass of 75 kg.

(i) The cyclist freewheels down a hill. His speed increases from 3.5 m s^{-1} to 10.5 m s^{-1}. The total work done against all the resistances to motion is 1500 J. The drop in vertical height is x m. Find x.

(ii) The cyclist then reaches a horizontal stretch of road and there is now a constant resistance to motion of 50 N. When the cyclist is developing a constant power of 250 W, find the constant speed which he can maintain.

3 The resistance to the motion of a train of total mass 200 tonnes is 16 000 N. The greatest driving force which the engine can exert is 25 000 N and the greatest power is 400 kW. The train starts from rest and moves along a level track with the greatest possible acceleration.

(i) Show that the engine first develops its maximum power when the speed is 16 m s^{-1}.

(ii) Show also that the speed of the train cannot exceed 25 m s^{-1}.

(iii) Find the acceleration of the train when its speed is 20 m s^{-1}.

4 A car of mass 1250 kg travelling at speed v experiences a resistance force of magnitude $25v$ N. The car's maximum speed on a straight horizontal road is $45\,\text{m}\,\text{s}^{-1}$.

(i) Find the maximum power of the car.

(ii) Find the maximum possible acceleration of the car when it is travelling at $25\,\text{m}\,\text{s}^{-1}$.

The car starts to descend a hill on a straight road which is inclined at an angle θ to the horizontal with $\sin\theta = 0.05$.

(iii) Find the maximum possible constant speed of the car as it travels on this road down the hill.

5 The resistance to the motion of a car of mass 1000 kg is kv N, where v is the car's speed and k is a constant. The car ascends a hill of angle θ to the horizontal and $\sin\theta = 0.05$. The power exerted by the car's engine is 15 kW and the car has a constant speed of 20 m s^{-1}.

(i) Show that $k = 12.5$.

The power exerted by the engine is now increased to 20 kW.

(ii) Calculate the maximum speed of the car while ascending the hill.

6 A car of mass 1250 kg has a maximum power of 50 kW. Resistive forces have a constant magnitude of 1500 N.

(i) Find the maximum speed of the car on level ground.

The car is now ascending a hill with inclination θ and $\sin\theta = 0.1$.

(ii) Calculate the maximum steady speed of the car when ascending the hill.

(iii) Calculate the acceleration of the car when it is descending the hill at 20 m s^{-1} working at half the maximum power.

Stretch and challenge

1 A railway truck runs down an incline of angle θ, such that $\sin \theta = 0.01$. At the bottom of the incline, the truck runs along a level track.

Find how far it will run on the level if the speed was a constant $8\,\mathrm{m\,s^{-1}}$ on the incline and the resistance is unchanged on the level.

2 The resistance to motion of a car is proportional to the square of its speed. The car has a mass of 1250 kg and can maintain a constant speed of $25\,\mathrm{m\,s^{-1}}$ when it is ascending a hill inclined at $\sin^{-1}(0.075)$ to the horizontal, with the engine working at 60 kW.

Find the acceleration of the car when it is travelling down the same hill with the engine working at 40 kW at the instant when the speed is $20\,\mathrm{m\,s^{-1}}$.

3 A block of mass m rests on a rough table. The coefficient of friction between the block and the table is μ. The block is connected by a light inextensible string, which passes over a smooth pulley at the edge of the table and carries a mass M hanging vertically.

Find the velocity of the block when it has moved a distance d across the table.

4 The resistance to the motion of a van of mass M kg is proportional to the square of the speed of the van. If the engine is working at P W, the van can reach a maximum speed of $V\,\mathrm{m\,s^{-1}}$ up an incline making an angle θ with the horizontal.

Show that the resistance when the speed is V is given by $\frac{P}{V} - Mg\sin\theta$.

Find the acceleration when the speed is $\frac{V}{2}$.

Exam focus

1 A block of mass 80 kg is pulled up a straight hill and passes through points A and B with speeds of $10\,\mathrm{m\,s^{-1}}$ and $4\,\mathrm{m\,s^{-1}}$ respectively. The distance AB is 350 m and B is 20 m higher than A. For the motion of the block from A to B, find

(i) the loss in kinetic energy of the block [2]

(ii) the gain in potential energy of the block. [2]

The resistance to motion of the block has magnitude 12.5 N.

(iii) Find the work done by the pulling force acting on the block. [2]

The pulling force of constant magnitude 60 N acts at an angle θ upwards from the hill.

(iv) Find the value of θ. [3]

2 A load of mass 280 kg is lifted vertically by a crane, with constant acceleration. The load starts from rest at the point O. After 12 s, it passes through the point P with speed 1.5 m s^{-1}. By considering energy, find the work done by the crane in moving the load from O to P. **[5]**

3 A car of mass 1250 kg travels up a straight hill inclined at 2° to the horizontal. The resistance to motion of the car is 1175 N. Find the acceleration of the car at an instant when it is moving with speed 24 m s^{-1} and the engine is working at a power of 45 kW. **[5]**

10 Motion of a projectile

Projectile problems

EXERCISE 10.1

1 In each of the following cases you are given the initial position vector $\mathbf{r}_0$ and the initial velocity $\mathbf{u}$ of the projectile.

(a) Write down the velocity of the projectile at time *t*.

(b) Write down the position of the projectile at time *t*.

(i) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; $\mathbf{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(ii) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$; $\mathbf{u} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$

(iii) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; initial velocity is $15\,\text{m}\,\text{s}^{-1}$ at 30° above the horizontal

(iv) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$; initial velocity is $30\,\text{m}\,\text{s}^{-1}$ at 15° below the horizontal

2 In each case, you are given the initial position vector and initial velocity of the projectile. Find the time taken for the projectile to reach its highest point and the maximum height reached.

(i) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; $\mathbf{u} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

(ii) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$; $\mathbf{u} = \begin{pmatrix} 7 \\ 24 \end{pmatrix}$

(iii) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; initial velocity is $20\,\text{m s}^{-1}$ at $60°$ above the horizontal

3 In each case, you are given the initial position vector and initial velocity of the projectile. Find the time of flight and the horizontal range of the projectile.

(i) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; $\mathbf{u} = \begin{pmatrix} 15 \\ 8 \end{pmatrix}$

(ii) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$; $\mathbf{u} = \begin{pmatrix} 15 \\ 15 \end{pmatrix}$

(iii) $\mathbf{r}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; initial velocity is $15\,\text{m}\,\text{s}^{-1}$ at 25° above the horizontal

4 A stone is thrown horizontally from a cliff 30 m high. It travels 45 m horizontally before hitting the water.

(i) Find the time in the air.

(ii) Find the stone's initial speed.

(iii) Find the stone's speed when it hits the water.

(iv) Find the angle at which it hits the water.

5 A stone is projected from ground level. The maximum height of the stone above horizontal ground is 45 m.

(i) Show that the vertical component of the stone's initial velocity is $30\,\text{m}\,\text{s}^{-1}$.

The speed of projection is $40\,\mathrm{m\,s^{-1}}$.

(ii) Find the angle of projection of the stone.

(iii) Find the horizontal range of the stone.

6 A particle is projected over horizontal ground from a point O at ground level.
The initial speed of the particle is $20\,\mathrm{m\,s^{-1}}$ and is directed at an angle θ to the horizontal.
The particle is at a height of 12 m above ground after 2 seconds.

(i) Write down an expression for the height of the particle in terms of t and θ.

(ii) Show that $\sin\theta = 0.8$ and find the corresponding value for $\cos\theta$.

(iii) Calculate the greatest height reached by the particle.

(iv) Find the horizontal range of the particle.

7 A golfer strikes a ball so that its initial velocity makes an angle $\theta = \tan^{-1}(2)$ with the horizontal.

(i) Write down expressions for the horizontal and vertical displacements at time t.

The horizontal range of the ball is 120 m.

(ii) Find the speed of projection.

(iii) Find the time of flight.

(iv) Find the greatest height of the ball above the ground.

8 A ball is struck so that it leaves the ground travelling at $25\,\mathrm{m\,s^{-1}}$, at an angle θ above the horizontal.

(i) Find the time taken by the ball to reach its maximum height.

The ball reaches a maximum height of 5 metres.

(ii) Find θ.

(iii) Find the horizontal range of the ball.

9 A shell is fired over horizontal ground from a point O at ground level and makes a loud noise on landing. The shell has an initial speed of $75\,\text{m s}^{-1}$ at an angle of 40° to the horizontal.

Assuming air resistance is neglected and that the speed of sound is $343\,\text{m s}^{-1}$, calculate how long after projection the noise is heard at O.

10 A cricketer hits the ball at ground level, and it is in the air for 2.35 s before bouncing on the ground 64 m away.

(i) Find the initial speed and direction of the ball.

(ii) Find the maximum height above ground reached by the ball.

11 A ball is hit from a point P on level ground and hits a goalpost at a point 4 m above the ground. The goalpost is at a distance of 32 m from P. Initially the velocity of the ball makes an angle of $\theta = \tan^{-1}(0.75)$ with the ground.

(i) Show that the initial speed of the ball is $20\,\mathrm{m\,s^{-1}}$.

(ii) Find the speed of the ball when it hits the goalpost.

12 A particle is projected from O on level ground and strikes the ground again at a distance of 200 m from O after a time of 10 seconds.

(i) Find the horizontal and vertical components of the initial velocity of the particle.

The particle passes through a point P, whose horizontal displacement from O is 50 m.

(ii) Find the height of P above ground.

(iii) Find the speed and the direction of motion of the particle at P.

(iv) Find the horizontal distance from O of the point at which the particle is next at the same height as P.

13 A stone is thrown upwards from the top of a vertical cliff 56 m high. It falls into the sea 4 seconds later, 32 m from the foot of the cliff.

(i) Find the speed and direction of projection.

A second stone is thrown at the same time, in the same vertical plane, at the same speed and at the same angle to the horizontal, but downwards.

(ii) Find how long it will take to reach the sea and the distance between the points of entry of the stones into the water.

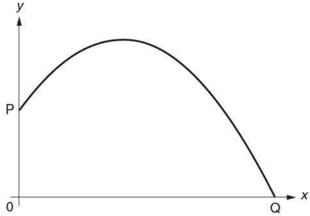
The path of a projectile

EXERCISE 10.2

1 The equation of the trajectory of a projectile which is projected from a point P is given by

$$y = 1 + 0.16x - 0.008x^2$$

where y is the height of the projectile above horizontal ground and x is the horizontal displacement of the projectile from P.
The projectile hits the ground at a point Q.



(i) Write down the height of P and find the coordinates of Q.

(ii) Find the horizontal distance x from P of the highest point of the trajectory and show that this point is 1.8 m above ground.

(iii) Find the time taken for the projectile to fall from its highest point to the ground.

(iv) Find the horizontal component of the velocity. Deduce from this the time of flight for the projectile to travel from P to Q.

(v) Calculate the speed of the projectile when it hits the ground.

2 A stone is projected from a point O on the edge of a vertical cliff. The components of the initial velocity are $10\,\mathrm{m\,s^{-1}}$ and $20\,\mathrm{m\,s^{-1}}$ in the horizontal and vertical directions respectively. At time t seconds after projection the stone is at $P(x, y)$ referred to horizontal and vertical axes through O.

(i) Express x and y in terms of t, and hence show that $y = 2x - 0.05x^2$.

The stone hits the sea at a point which is 60 m below the level of O.

(ii) Find the horizontal distance between the cliff and the point where the stone hits the sea.

(iii) Find the angle that the trajectory makes with the horizontal as the stone enters the sea.

3 A particle is projected from a point A, 2 m above ground level with speed u m s^{-1} and direction of 45° above the horizontal. At time t the horizontal and vertical displacements of the particle from O are x and y respectively.

(i) Write down expressions for x and y in terms of t and u. Hence show that the equation of the trajectory is

$$y = 2 + x - \frac{10x^2}{u^2}$$

The particle hits the ground at a point B which is 8 m from O.

(ii) Find the value of u.

4 A ball is kicked from ground level over horizontal ground. It leaves the ground at a speed of $41\,\mathrm{m\,s^{-1}}$ at an angle θ to the horizontal such that $\tan\theta = \frac{9}{40}$.

(i) Show that the height, y, of the ball above the ground after t seconds is given by $y = 9t - 5t^2$. Show also that the horizontal distance, x, travelled by the ball is given by $x = 40t$.

(ii) Calculate the maximum height reached by the ball.

(iii) Calculate the times at which the ball is at half its maximum height. Find the horizontal distance travelled by the ball between these times.

(iv) Find the speed of the ball when $t = 1.25$ s.

(v) Find the equation of the trajectory of the ball. Hence or otherwise find the range of the ball.

5 A golfer strikes the ball with a speed of $50\,\mathrm{m\,s^{-1}}$ at an angle θ to the horizontal.

(i) Show that the equation of the trajectory of the ball is given by

$$y = x\tan\theta - 0.002x^2(1 + \tan^2\theta)$$

(ii) The golfer is standing 200 m from the hole. Find two values of θ for which the ball lands straight in the hole.

6 A particle is projected from a point O with initial velocity having components u_x and u_y along the horizontal and vertical directions respectively.

(i) If (x, y) is a point on the trajectory of the projectile, show that

$$yu_x^2 - u_x u_y x + 5x^2 = 0$$

(ii) Given that the particle passes through the points with coordinates (5, 2.5) and (12, 1.8), show that the velocity of projection is $12.5\,\mathrm{m\,s^{-1}}$ at an elevation of $\tan^{-1}(0.75)$.

Stretch and challenge

1 A boy is firing small stones from a catapult at a target on the top of a wall. The stones are projected from a point which is 5 m from the wall and 1 m above ground level. The target is on top of the wall which is 3 m high. The stones are projected at a speed of $15\,\text{m}\,\text{s}^{-1}$ at an angle of θ with the horizontal.

(i) If the stone hits the target, use the identity $\sec^2\theta = 1 + \tan^2\theta$ to show that θ must satisfy the equation

$$5\tan^2\theta - 45\tan\theta + 23 = 0$$

(ii) Find the two values of θ.

2 Find the greatest possible range of a projectile inside a tunnel which is 4 m high, if the projectile's initial velocity is 75 m s^{-1}.

3 A particle is projected from a point O and passes through a point P on its trajectory when it is travelling horizontally. The coordinates of P are (16, 12). Find the angle of projection and the magnitude of the initial velocity.

Exam focus

1 A golf ball is hit over horizontal ground from a point O on the ground. The velocity of projection is $25\,\mathrm{m\,s^{-1}}$ at an angle of $45°$ to the horizontal.

(i) Write down an expression for the height of the ball at time t and use it to find the time at which the ball first hits the ground. **[4]**

The ball passes directly over a tree which is a horizontal distance of 42 m from O.

(ii) Find the speed of the ball as it passes over the tree. **[5]**

(iii) Find the angle between the direction of motion and the horizontal as the ball passes over the tree, stating whether it is rising or falling. **[2]**

2 A particle is projected from horizontal ground with speed u m s^{-1} at an angle 40° above the horizontal. At time T it passes the point A whose horizontal and vertical displacements from O are 12 m and 5 m respectively.

(i) Find the speed of projection of the particle from O and the value of T. [5]

(ii) Find the angle between the direction of motion of the particle at A and the horizontal. [3]

3 A particle is projected from a point O on horizontal ground. The velocity of projection is $25\,\text{m}\,\text{s}^{-1}$ at an angle of θ to the horizontal. The particle passes through the point P which is 15 m above the ground and 25 m horizontally from O.

(i) Show that the equation of the trajectory may be written as

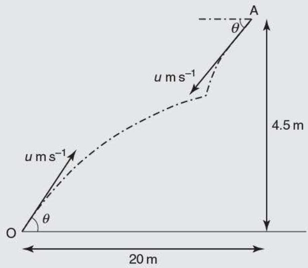
$$y = x \tan \theta - 0.008(1 + \tan^2 \theta)x^2$$ [4]

(ii) Find the two possible values of θ. [4]

The particle hits the ground at the point A.

(iii) Find the distance OA for each of the values of θ. [3]

4 A particle P is projected from a point O on the ground with initial speed $u\,\mathrm{m\,s^{-1}}$ and initial direction making an angle θ above the horizontal. At the same time a second particle Q is projected from a point A, 4.5 m above ground with the horizontal distance between O and A being 20 m. Q is projected towards O with initial speed $u\,\mathrm{m\,s^{-1}}$ and initial direction making an angle θ below the horizontal as shown in the diagram.



(i) Write down expressions in terms of u, θ and t for

(a) the horizontal distances of P and Q from O at time t [2]

(b) the vertical height of P and Q above ground at time t. [2]

At time $t = T$, the particles collide at a point above ground level.

(ii) Show that $\tan\theta = 0.225$ and $uT = 10.25$. [4]

(iii) Hence deduce that $36u^2 > 8405$. [3]

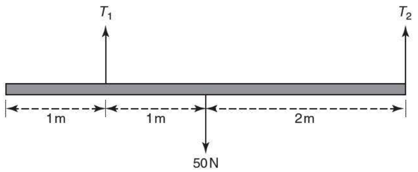
11 Moments of forces

Rigid bodies; Moments; Couples; Equilibrium revisited

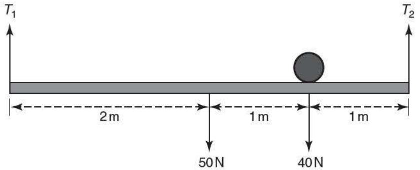
EXERCISE 11.1

1 Each of the diagrams shows a uniform beam, of weight 50 N, held in equilibrium by two light inextensible strings. Find the tension in each string.

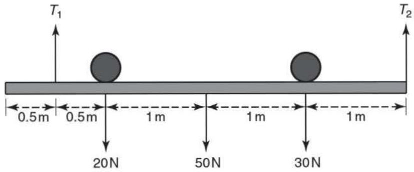
(a)



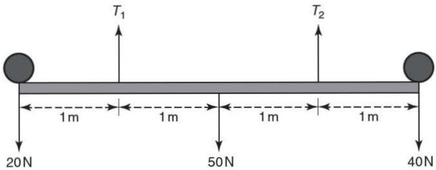
(b)



(c)



(d)



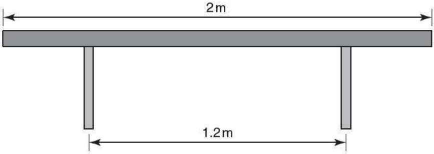
2 A uniform bar AB of length 4.2 m and weight 120 N has loads of 30 N and 50 N attached at A and B respectively. If the bar balances in a horizontal position when smoothly supported at C, find the distance of C from A.

3 A uniform bar AB of length 1 m can be balanced about a point 0.2 m from A by hanging a weight of 5 N at A.

(i) Find the weight of the bar.

(ii) What additional weight should be hung from A if the point of support is moved 0.1 m nearer to A?

4 The figure shows a park bench of mass 40 kg consisting of a horizontal plank of wood of length 2 m resting on two supports, each being 0.6 m from the centre of the plank.



Ben sits on the bench at its midpoint and his father Steven sits at one end. Their respective masses are 35 kg and 80 kg.

(i) By modelling the bench as a uniform rod and Ben and Steven as particles, find the reaction at each of the two supports.

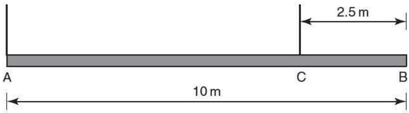
(ii) Ben now moves closer to his father. Find how close Ben can get to Steven before the reaction at one of the supports becomes zero.

(iii) What is the significance of a zero reaction at one of the supports?

5 A uniform bar AB of length 1.2 m and weight 25 N is supported in a horizontal position by two vertical strings, one at point C where AC = 0.3 m and one at B. The bar carries loads of 10 N at A and 15 N at D, such that DB = 0.4 m.

Find the tensions in the strings.

6 A uniform plank AB is 10 m long and has a mass of 50 kg. It is supported in equilibrium in a horizontal position by two vertical inextensible ropes. One of the ropes is attached to the plank at A and the other rope to the point C, where BC = 2.5 m, as shown in the diagram.



Find the tension in each rope.

7 A uniform beam AB of length 4 m has a load of mass 35 kg placed at A. The beam is kept in equilibrium by a support at a point 0.75 m from A.

Find the mass of the beam.

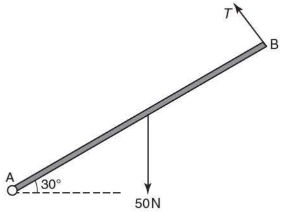
8 A uniform rod, of length 150 cm and mass 3 kg, has masses of 1 kg, 2 kg, 3 kg and 4 kg suspended from it at distances of 30 cm, 60 cm, 90 cm and 120 cm respectively from one end.

Find the position about which the rod will balance.

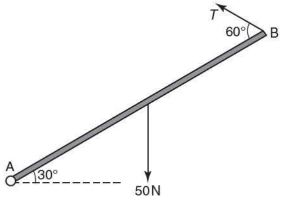
9 In each of the following diagrams, a uniform rod AB, of mass 5 kg and length 1 m, is freely hinged at A. The rod is held in equilibrium by a string attached at B.

Find the tension in the string in each case.

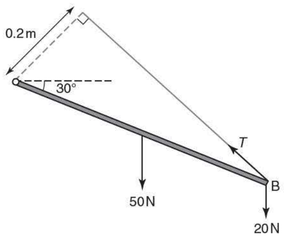
(a)



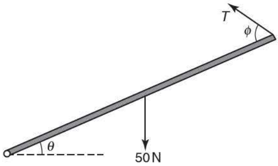
(b)



(c)

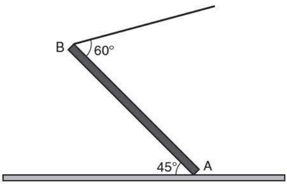


(d)



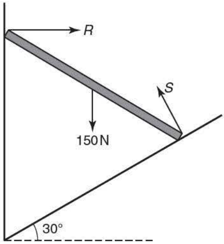
10 A uniform rod AB, of weight W and length L, has its end A resting on rough horizontal ground and is kept in equilibrium by a string attached to B.

If the rod makes an angle of 45° with the horizontal and the string makes an angle of 60° with BA, show that the tension in the string is $W/\sqrt{6}$.



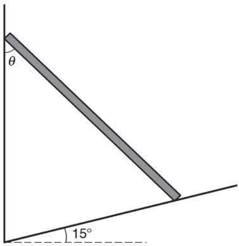
11 A uniform rod of weight 150 N rests with one end against a smooth vertical wall and the other end on a smooth plane inclined at 30° to the horizontal.

Find θ, the angle the rod makes with the vertical, and the normal reactions R and S at each end of the rod.

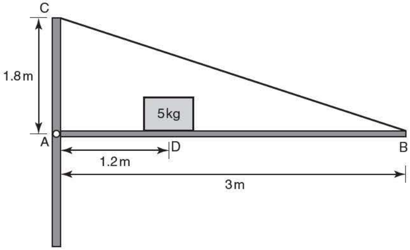


12 A uniform ladder rests in equilibrium with its top end against a smooth vertical wall and its foot resting on a smooth plane inclined at 15° to the horizontal. The ladder makes an angle θ with the vertical.

Find the value of θ.



13 A uniform rigid rod AB, of mass 20 kg and length 3 m, is hinged at A to a vertical wall. The rod is kept horizontal by a chain attached to B and to a point C on the wall, 1.8 m above A. The rod carries an additional mass of 5 kg at D, 1.2 m from A.

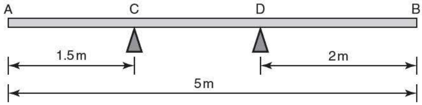


(i) Find the tension in the chain.

(ii) Find the magnitude of the reaction force at A.

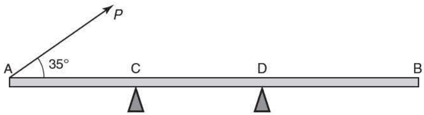
(iii) Find the direction of the reaction force at A.

14 A uniform beam AB is 5 m long and has a weight of 200 N. Initially, the beam is in equilibrium on two supports at C and D, as shown in the figure. The beam is horizontal.



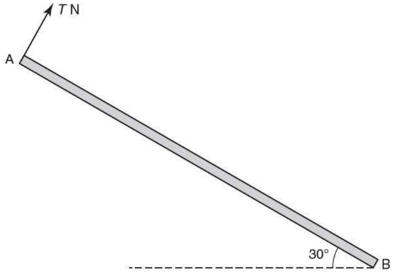
(i) Calculate the forces acting on the beam at C and D.

A force *P* is applied at A, at 35° to the beam, to try to move it. The beam remains in horizontal equilibrium but the reaction at C is now zero.



(ii) Calculate the value of *P*.

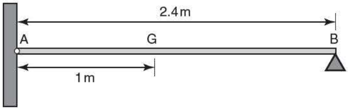
The beam is now supported by a light rope attached to the beam at A, with B on rough horizontal ground. The rope is at right angles to the beam, which is at 30° to the horizontal, as shown in the figure. The tension in the rope is T N. The beam is in equilibrium, on the point of sliding.



(iii) Show that $T = 50\sqrt{3}$ and hence, or otherwise, find the frictional force between the beam and the ground.

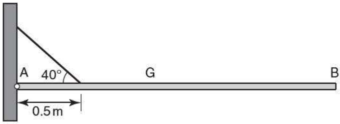
(iv) Calculate the coefficient of friction between the beam and the ground.

15 A thin straight beam AB is 2.4 m long. It has a weight of 500 N and its centre of mass G is 1 m from A. The beam is freely hinged at A. The beam is held horizontally in equilibrium. Initially this is done by means of a support at B, as shown in the figure.



(i) Calculate the force on the beam due to the support at B.

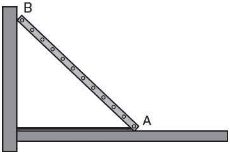
The support at B is now removed and replaced by a wire attached to the beam 0.5 m from A and inclined at 40° to the beam, as shown in the figure. The beam is still horizontal and in equilibrium.



(ii) Calculate the tension in the wire.

(iii) Calculate the magnitude and direction of the force on the beam at A due to the hinge.

16 A uniform ladder AB, of mass 25 kg and length 7.5 m, rests with A on a smooth horizontal floor and B against a smooth vertical wall. The end A is attached to the junction of the wall and the floor by a light inextensible string of length 2.1 m.



(i) Find the tension in the string.

(ii) If the tension in the string is not to exceed 260 N, find how far someone of mass 80 kg can ascend the ladder.

Stretch and challenge

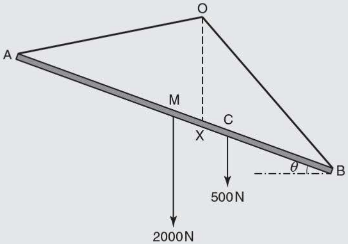
1 A uniform ladder PQ of weight W and length $2L$ rests in equilibrium with P against a smooth vertical wall and Q on a rough horizontal floor, at an angle θ to the horizontal. If F is the frictional force, show that $2F \tan \theta = W$.

2 A uniform ladder rests inclined at 60° to the horizontal against a smooth vertical wall and on rough horizontal ground. The coefficient of friction between the ladder and the ground is 0.25. The ladder is 8 m long and has a mass of 40 kg. A man weighing 80 kg begins to climb.

(i) How far can he ascend before the ladder slips?

(ii) What value of the coefficient of friction is required for the man to get to the top of the ladder?

3 A uniform rod AB of length 6 m and weight 2000 N is hung from a point O by two strings, each of length 5 m, attached to each end of the rod. A weight of 500 N is placed at a point C, 2 m from B. The tension in string AO is T_1 and that in string BO is T_2. The rod rests in equilibrium at an angle θ to the horizontal. The point X is directly below O and M is the midpoint of the rod.



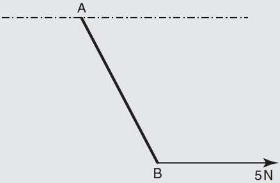
(i) By taking moments about O, find the distances MX and XC.

(ii) Find the angle θ.

(iii) By taking moments about each end of the rod, show that the ratio of the tensions in the strings is $T_1 : T_2 = 7 : 8$ and use this to find T_1 and T_2.

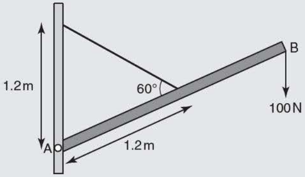
Exam focus

1 A uniform rod AB of weight 25 N is freely hinged at A to a fixed point. A force of magnitude 5 N acting horizontally is applied at B.



If the rod is in equilibrium, find the angle that the rod makes with the horizontal. **[4]**

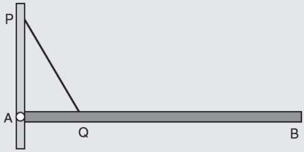
2 A uniform rod AB of length 2 m and weight 50 N is hinged at a fixed point A on a vertical wall. The rod is held in equilibrium by a light inextensible string. One end of the string is attached to the wall at a point 1.2 m vertically above the hinge. The other end of the string is attached to the rod at a point 1.2 m from A. The string makes an angle of 60° with AB. The rod carries a load of 100 N at B.



(i) Find the tension in the string. [4]

(ii) Find the magnitude and direction of the force exerted on the rod at A. [5]

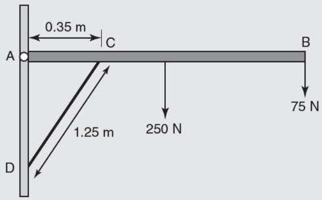
3 A uniform rod AB of weight 100 N and length 3 m is freely hinged to a vertical wall at A. The rod is held in equilibrium in a horizontal position by a chain PQ of length 0.5 m. P is fixed to the wall at a point 0.4 m vertically above A.



(i) Find the tension in the chain. [4]

(ii) Find the magnitude and direction of the force exerted on the rod at A. [5]

4 A uniform beam of weight 250 N and length 3.2 m is freely hinged to a vertical wall at A and is supported in a horizontal position by a light rigid rod CD of length 1.25 m. One end of the rod is attached to the beam at C, 0.35 m from A, and the other end is attached to the wall at D, vertically below A. The rod exerts a force on the beam in the direction DC. The beam carries a load of 75 N at B.



(i) Calculate the magnitude of the force exerted by the rod on the beam. [4]

(ii) Calculate the magnitude and direction of the force acting on the beam at A. [5]

12 Centre of mass

Composite bodies; Centre of mass for two- and three-dimensional bodies; Sliding and toppling

EXERCISE 12.1

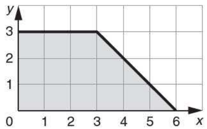
1 Find the position of the centre of mass of five particles of mass 1 kg, 2 kg, 3 kg, 4 kg and 5 kg placed along the x axis at $x = -5, 0, 2, 6$ and 10 respectively.

2 Find the position of the centre of mass of four particles of mass 3 kg, 4 kg, 6 kg and 10 kg placed at the points (–2, 3), (4, 1), (7, –2) and (5, 0) respectively.

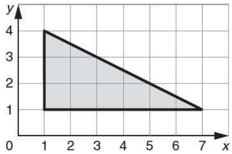
3 Particles of mass 6 kg, 5 kg and 2 kg are placed at points (–2, 2), (3, 1) and (2.5, –0.5) respectively. Where must a fourth particle of mass 4 kg be placed so that the centre of mass of the system is at the origin?

4 Find the coordinates of the centre of mass of each of the following uniform laminae.

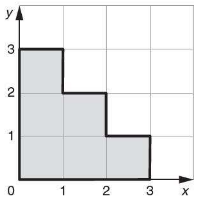
(a)



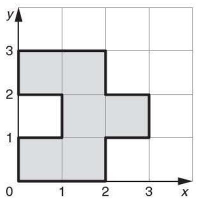
(b)



(c)

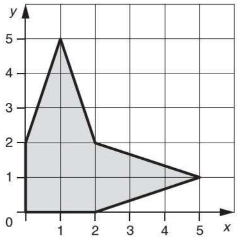


(d)

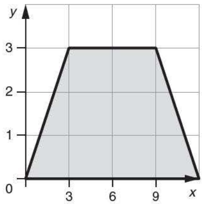


5 Find the coordinates of the centre of mass of each of the following uniform laminae.

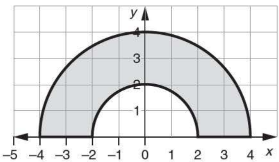
(a)



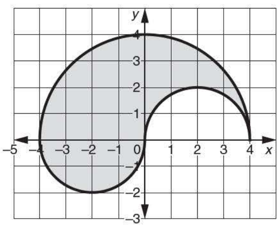
(b)



(c)

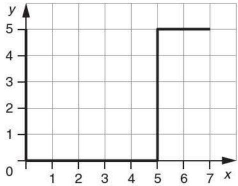


(d)

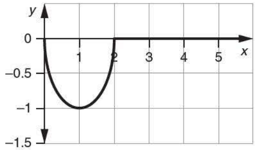


6 Find the centre of mass of the following uniform wires.

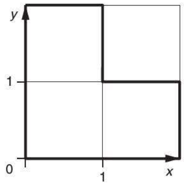
(a)



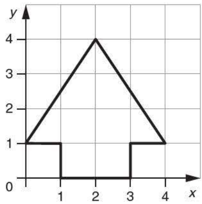
(b)



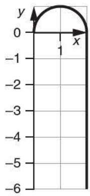
(c)



(d)



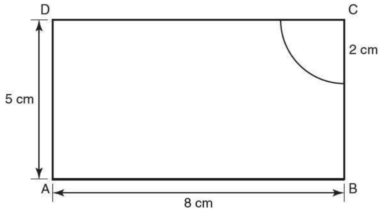
(e)



7 ABCD is a uniform rectangular lamina with AB = 8 cm and AD = 5 cm.

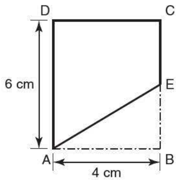
A quadrant of a circle with centre C and radius 2 cm is removed.

Find the distance of the centre of mass of the remainder from AD and AB.



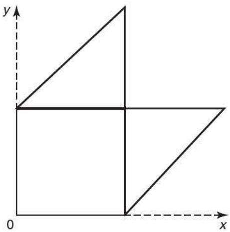
8 ABCD is a rectangular plate, with AB = 4 cm and AD = 6 cm. E is the midpoint of BC.

The triangular portion ABE is removed and the remainder is suspended from A.
Find the angle that AD makes with the vertical.



9 A uniform lamina is composed of a square with two isosceles right-angled triangles attached to two adjacent edges.

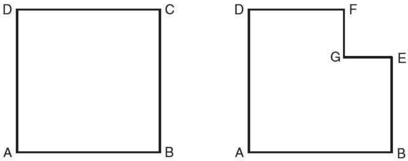
If the square is of side length L, find the position of the centre of mass of the composite lamina.



10 A uniform lamina is in the form of a square ABCD of side length $3L$.

E is a point on BC and F a point on DC such that $CE = CF = L$.

A square FCEG is removed from the lamina.



(i) Find the centre of mass of the remainder ABEGFD.

(ii) A uniform wire is bent in the form of the shape ABEGFD. Find the centre of mass of the wire.

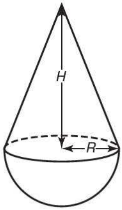
(iii) The wire is hung from point D. Find the angle that AD makes with the vertical.

11 A toy consists of a solid hemisphere of radius R joined to a solid right cone of radius R and height H. The hemispherical base is made of a material which is twice as dense as the conical top.

Show that the centre of mass of the toy lies at a distance

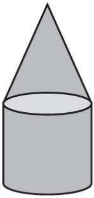
$$\frac{H^2 - 6R^2}{4(H + 4R)}$$

from the common face of the two solids.



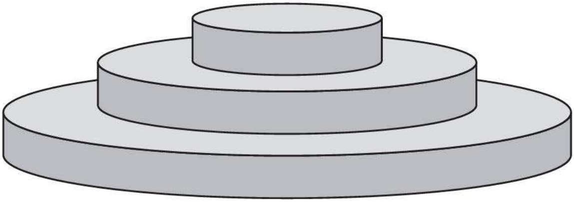
12 A solid cylinder of radius 10 cm and height 25 cm is attached to a solid right cone, made of the same material and with the same radius. The cone has a height of 20 cm.

Find the distance of the centre of mass of the composite body from the base of the cylinder.



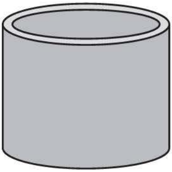
13 Three uniform cylinders, all made from the same material, and with radii 20 cm, 10 cm and 5 cm, are joined together, with their axes of symmetry in line, to form the solid shown in the diagram. All cylinders are 10 cm high.

Find the height of the centre of mass of the composite solid above ground level.

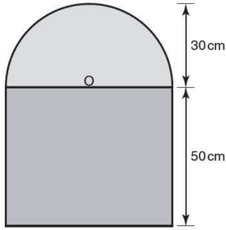


14 A vase is made from a uniform solid cylinder of height 25 cm and radius 10 cm by removing a smaller cylinder of height 22 cm and radius 9 cm from it so that there is an axis of symmetry vertically through the centre of the base.

Find the centre of mass of the vase.

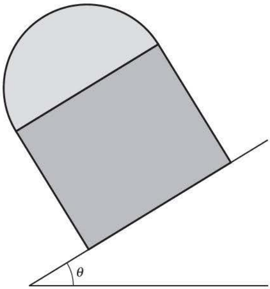


15 A solid consists of a uniform solid right cylinder of height 50 cm and radius 30 cm joined to a solid hemisphere of radius 30 cm. The plane face of the hemisphere coincides with the top circular end of the cylinder and has centre O. The density of the hemisphere is three times the density of the cylinder.



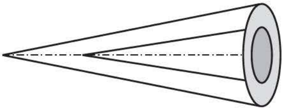
(i) Find the distance of the centre of mass of the solid from O.

The solid is now placed with its circular face on a plane inclined at an angle θ to the horizontal, as shown in the figure. The plane is sufficiently rough to prevent the solid slipping. The solid is on the point of toppling.



(ii) Find the value of θ.

16 A hole in the shape of a right circular cone of base radius 5 cm and height 20 cm is bored out of a solid cone of radius 10 cm and height 30 cm. The axes of the two cones coincide as shown in the diagram.



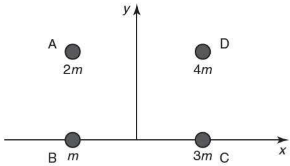
(i) Find the distance of the centre of mass of the solid from the vertex of the original cone.

The solid is placed with its flat surface on a plane inclined at an angle θ with the horizontal. The plane is sufficiently rough to prevent slipping.

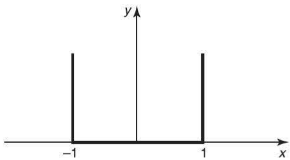
(ii) Find the value of θ for which the solid is about to topple.

17 Four particles of masses $2m$, m, $3m$ and $4m$ are located at the points A (–1, 2), B (–1, 0), C (1, 0) and D (1, 2) respectively.

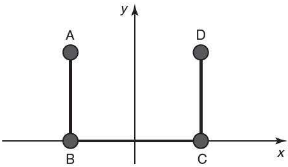
(i) Find the centre of mass of the particles.



(ii) A uniform wire of total mass $6m$ is made up of three rods of length 2 units arranged as in the diagram. Find the centre of mass of the wire.



(iii) Find the centre of mass of the combined solid made up of the three rods and the four particles attached at A, B, C and D.



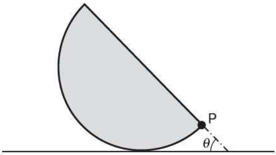
A is connected to D by means of a further straight wire of negligible mass. The combined system of wires with the attached particles at A, B, C and D is suspended freely from the midpoint of AD.

(iv) What extra mass must be added at A if the system is to stay in equilibrium with AD horizontal?

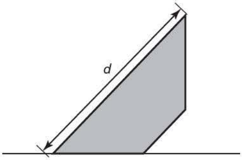
18 A uniform solid hemisphere of weight W and radius R is placed with its spherical surface on a smooth horizontal plane. A particle P of weight $\frac{1}{2}W$ is attached to its rim.

In the position of equilibrium, the plane face of the hemisphere is inclined at an angle θ to the horizontal as shown in the diagram.

Find θ.



19 A frustum is cut from a cone of height 20 cm by a plane parallel to the base and 10 cm from the base. If the frustum can just rest with its curved surface on a horizontal plane, find the diameter d of the base of the cone.

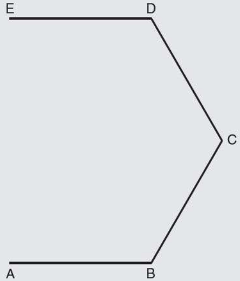


Stretch and challenge

1 An isosceles triangle is cut off a corner of a square lamina. Show that the remainder can stand on a shortened edge if the part cut off is 50% of an edge, but not if it is 60% of an edge.

2 A rigid framework ABCDE of four equal rods, forming part of a regular hexagon, is suspended from A. Show that the angle made by AB with the vertical is

$\tan^{-1}\left(4\sqrt{\frac{3}{7}}\right)$.



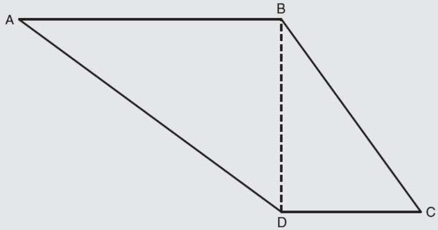
3 A hollow cylindrical can, which has a base but no top, is made from uniform thin material. The radius of the base of the can is 5 cm and its height is 10 cm.

(i) Show that the centre of mass of the can is at a distance of 4 cm above the base.

(ii) The can rests with its base on a horizontal table and water is poured into the can until the depth of the water is h cm. The material of the can has mass 0.5 g cm^{-2} and the water has mass 1 g cm^{-3}. Given that the centre of mass of the water and the can together is on the water surface, find the value of h.

Exam focus

1 A uniform lamina ABCD is in the form of a trapezium with parallel sides AB and DC of length 4 cm and 2 cm respectively. BD is perpendicular to AB and CD and has length 3 cm.

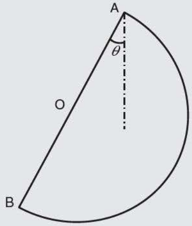


(i) Find the distance of the centre of mass from BD. [4]

The lamina has weight W and is resting on its side DC.

(ii) Find the force that is required at C to stop the lamina from toppling over. [3]

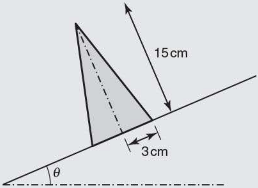
2 A uniform rigid wire is in the form of a semicircle of radius 10 cm and centre O. The wire is in equilibrium, freely suspended from A and AB makes an angle θ with the vertical.



(i) Find the distance of the centre of mass from O. **[3]**

(ii) Find the value of θ. **[3]**

3 A uniform solid cone of radius 3 cm and height 15 cm is placed with its flat surface on a rough plane inclined at an angle θ to the horizontal. The coefficient of friction between the cone and the plane is 0.25.

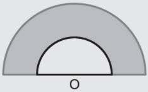


(i) Assuming that it does not topple, for what value of θ does the cone slide down the plane? **[3]**

(ii) Assuming that it does not slide, for what value of θ does the cone topple? **[3]**

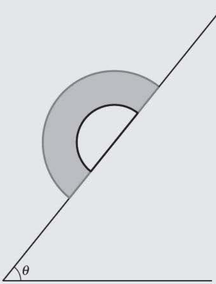
(iii) θ is increased slowly from an initial value of 0°. Does the cone slide or topple? **[1]**

4 A lamina is formed when a semicircle of radius 5 cm is removed from a larger semicircle of radius 10 cm. The centres of the semicircles coincide at O.



(i) Find the distance of the centre of mass of the lamina from O. [3]

The lamina is placed on a plane inclined at an angle θ to the horizontal, with the flat face on a line of greatest slope of the plane. The plane is sufficiently rough to prevent the lamina from sliding. The lamina is on the point of toppling.



(ii) Find the value of θ. [3]

13 Uniform motion in a circle

Notation; Angular speed; Velocity and acceleration; The forces required for circular motion; Examples of circular motion

EXERCISE 13.1

1 The Earth is almost spherical with a mean radius of 6371 km and it spins on its axis through its two poles.

(i) Calculate the Earth's angular speed in radians per hour to four significant figures.

(ii) Find the velocities (in km h^{-1}) due to the rotation of the Earth of the following cities:

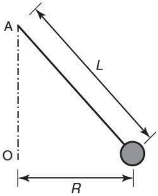
(a) Quito at latitude 0.2186° S

(b) Rio de Janeiro at latitude 22.9083° S

(c) Kiruna at latitude 67.8500° N.

(iii) Show that the acceleration of Kiruna is approximately $165\,\text{km h}^{-2}$.

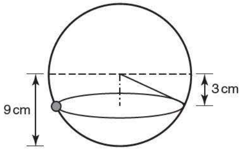
2 A particle of mass M describes a horizontal circle of radius R at the end of a light string of length L attached at a point A above the centre O of the circle.



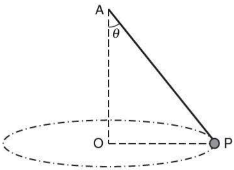
(i) Find the tension in the string.

(ii) Find the velocity of the particle.

3 A small ball moves in a horizontal circle on the smooth inner surface of a fixed spherical bowl of radius 9 cm. If the depth of the circle below the centre of the sphere is 3 cm, find the speed of the particle.



4 One end of a light inextensible string is attached to a fixed point A. The other end is attached to a particle P of mass 0.5 kg. P moves in a horizontal circle with centre O, which is vertically below point A. P moves with a uniform angular speed of 3 radians per second. The tension in the string is 9 N.



(i) Find the angle θ which the string makes with the vertical.

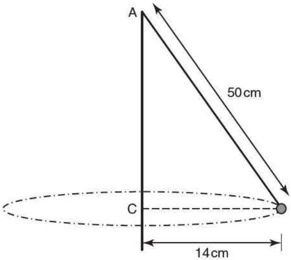
(ii) Find the length of the string.

5 A particle P of mass 0.2 kg is attached to one end of a light inextensible string of length 1.2 m. The other end of the string is attached to a fixed point A. The particle is moving, with the string taut, in a horizontal circle with centre O vertically below A. The particle is moving with constant angular speed of 5 rad s^{-1}.

(i) Find the tension in the string.

(ii) Find the angle that AP makes with the downward vertical.

6 A small ball of mass 0.2 kg is attached to one end of a light inextensible string of length 50 cm. The other end of the string is fixed to a point A on a vertical pole.
The ball is moving at constant speed in a horizontal circle of radius 14 cm and centre C, where C is vertically below A.

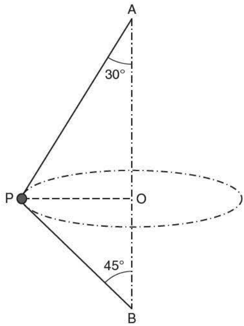


(i) Find the tension in the string.

(ii) Find the speed of the ball.

7 A particle P of mass 0.3 kg is attached to one end of each of two light inextensible strings which are both taut. The other end of the longer string is attached to a fixed point A, and the other end of the shorter string is attached to a fixed point B which is vertically below A. String AP makes an angle of 30° with the vertical and is 0.5 m long. String BP makes an angle of 45° with the vertical.

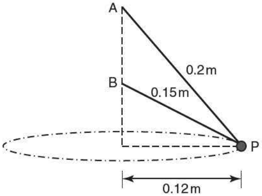
P moves with constant angular speed of 5 radians per second in a horizontal circle with centre O between A and B.



(i) Resolve vertically to obtain a relation between the tension, T, in AP and the tension, S, in BP.

(ii) Find another equation connecting T and S and hence calculate T and S.

8 A particle P of mass 0.25 kg is attached to one end of each of two light inextensible strings which are both taut. The other end of the longer string is attached to a fixed point A, and the other end of the shorter string is attached to a fixed point B which is vertically below A. String AP is 0.2 m long and BP is 0.15 m long. The particle P moves in a horizontal circle of radius 0.12 m with constant angular speed of 10 rad s^{-1}. Both strings are taut; T_1 is the tension in AP and T_2 is the tension in BP.



(i) Resolve vertically to show that $8T_1 + 6T_2 = 25$.

(ii) Find another equation connecting T_1 and T_2 and hence calculate T_1 and T_2.

9 A particle P of mass 0.25 kg is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is fixed at a point A which is 0.3 m above a smooth horizontal table. The particle P moves on the table in a circular path whose centre O is vertically below A.

(i) Given that the angular speed of P is 3.5 rad s^{-1}, find

(a) the tension in the string

(b) the normal reaction between the particle and the table.

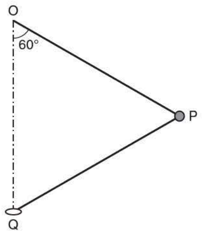
(ii) Find the greatest possible speed of P, given that it remains in contact with the table.

10 A particle P of mass 2.5 kg is attached to fixed points A and B by light inextensible strings, each of length 50 cm. A and B are 96 cm apart with A vertically above B. Particle P moves in a horizontal circle with centre at the midpoint of AB.

(i) Find the tension in each string when the angular speed of P is 6 rad s^{-1}.

(ii) Find the least possible speed of P.

11 A particle P of mass 0.4 kg is attached by a light inextensible string of length 0.3 m to a fixed point O and is also attached by another light inextensible string of length 0.3 m to a small ring Q of mass 0.6 kg which can slide on a fixed smooth vertical wire passing through O. Particle P describes a horizontal circle with OP inclined at an angle 60° with the downward vertical.

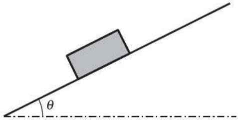


(i) Find the tensions in the strings OP and PQ.

(ii) Show that the speed of P is $3\sqrt{2}$ m s^{-1}.

(iii) Find the period of revolution of the system.

12 A car takes a bend of radius 100 m which is banked at an angle θ to the horizontal, such that $\tan\theta = 0.1$.

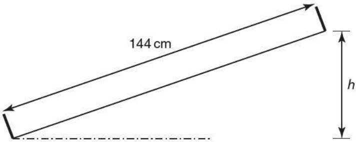


(i) Find the greatest speed at which the car can be driven without slipping occurring if the coefficient of friction between the road and the tyres is 0.8.

(ii) Find the least speed at which the car can be driven without slipping.

13 The distance between the rails on a track is 144 cm and the line runs along an arc of circle of radius 500 m. The average speed of trains on the line is 90 km h^{-1}.

What should be the height of the outer rail above the inner rail?

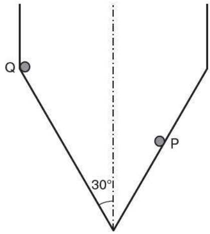


14 A train of mass 250 tonnes is moving at 15 m s^{-1} round a curve of radius 1 km.

(i) If the track is level, find the lateral thrust on the rails.

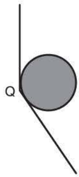
(ii) If the width of the track is 144 cm, find the height to which the outer rail must be raised above the inner rail if there is to be no lateral thrust on the rails at a speed of 20 m s^{-1}.

15 A container is constructed from a hollow cylindrical shell and a hollow cone which are joined together along their circumferences. The cylinder has a radius of 25 cm and the cone has a semi-vertical angle of 30°. Two identical particles P and Q are moving independently in horizontal circles on the smooth inner surface of the container. Each particle has a mass of 0.2 kg.



(i) P moves in a circle of radius 10 cm and is in contact with the conical part of the container. Calculate the angular speed of P.

(ii) Q moves with speed 2.5 m s^{-1} and is in contact with both the cylindrical and conical surfaces of the container. Calculate the magnitude of the force which the cylindrical shell exerts on the sphere.



Stretch and challenge

1 A particle suspended by a light inextensible string from a fixed point O describes a circle in a horizontal plane. If the particle makes five complete revolutions in two seconds, show that the vertical depth of the circle below O is approximately 4 cm.

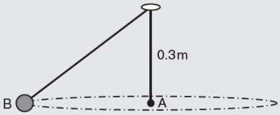
2 A circular cone of semi-vertical angle θ is fixed with its axis vertical and its vertex, O, at the lowest point.

A particle P of mass m moves on the inner surface of the cone, which is smooth. The particle is joined to O by a light inextensible string of length L.

The particle moves in a horizontal circle with constant speed v, with the string taut.

Find the reaction exerted on P by the cone. Find the tension in the string and show that the motion is possible only if $v^2 > gL \cos \theta$.

3 A light inextensible string of length 0.8 m is threaded through a smooth ring and carries a particle at each end. One particle A of mass m kg is at rest at a distance 0.3 m below the ring. The other particle B of mass M kg is rotating in a horizontal circle whose centre is A.



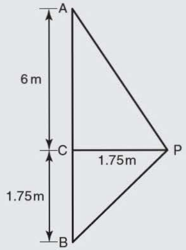
(i) Express M in terms of m.

(ii) Find the angular velocity of B.

Exam focus

1 A light inextensible string has its ends attached to two fixed points A and B. A is vertically above B and AB = 7.75 m.

A particle P of mass 0.25 kg is fixed to the string and moves in a horizontal circle of radius 1.75 m with angular speed ω. The centre of the circle is C, and AC = 6 m, as shown in the figure.



(i) Express the tension in AP in terms of ω^2. [4]

(ii) Find the tension in BP. [4]

(iii) Deduce that $\omega \geqslant \sqrt{\frac{5}{3}}$. [2]

2 A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 70 cm. The other end of the string is fixed to a point A vertically above O on a smooth horizontal table. P remains in contact with the surface of the table and moves in a horizontal circle with centre O and angular speed of 3 rad s^{-1}. Throughout the motion the string remains taut and inclined at an angle $\theta = \tan^{-1}(0.75)$ to the vertical.

(i) Find the tension in the string. [4]

(ii) Find the normal contact force exerted by the table on P. [3]

(iii) The angular speed is now increased to 5 rad s^{-1}. P now moves in a horizontal circle above the table, with centre X. Find the distance AX. [5]

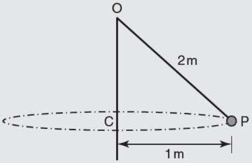
3 A particle P, of mass 2 kg, is attached to fixed points A and B by light inextensible strings, each of length 3 m. A and B are 2 m apart with A vertically above B.

P moves in a horizontal circle with centre at the midpoint of AB.

(i) Find the tension in each string when the angular speed of P is 5 rad s^{-1}. [6]

(ii) Find the least possible speed of P. [5]

4 A ball P of mass 0.15 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is fixed to a point O on a vertical pole as shown in the figure. The ball is hit and moves with constant speed in a horizontal circle of radius 1 m and centre C, with C vertically below O.



(i) Find the tension in the string. [3]

(ii) Find the speed of the ball. [3]

14 Hooke's law

Strings and springs; Hooke's law; Using Hooke's law with more than one spring or string; Work and energy; Vertical motion

EXERCISE 14.1

1 In each of parts (**i**) to (**vi**), a force F N is applied to a string of natural length l_0 m, with modulus of elasticity λ N, which extends the string by an amount x m to a new length L m. Find the missing quantity in each case.

(i) $F = 10$ N, $\lambda = 9$ N, $x = 0.2$ m; find l_0.

(ii) $F = 5$ N, $l_0 = 0.5$ m, $x = 0.1$ m; find λ.

(iii) $\lambda = 18$ N, $l_0 = 1.2$ m, $x = 0.2$ m; find F.

(iv) $F = 1.5$ N, $l_0 = 2$ m, $\lambda = 12$ N; find L.

(v) $F = 2.5$ N, $\lambda = 10$ N, $L = 2$ m; find l_0.

(vi) $F = 8$ N, $L = 2l_0$; find λ.

2 An elastic string of natural length 0.8 m and modulus of elasticity λ N is extended by 0.2 m when a fixed force of 20 N is applied to it.

(i) Find λ.

(ii) Find the force required to increase the natural length of the string by 60%.

3 A spring AB of natural length 0.5 m and modulus of elasticity 25 N is fixed at A. The other end is joined to another spring BC of natural length 0.8 m and modulus of elasticity 40 N.

A weight W N is attached at C and the system hangs vertically in equilibrium so that AC = 2 m.

(i) Find the extension of the two springs.

(ii) Find W.

4 A light elastic spring AB hangs vertically with its upper end fixed and a mass of 0.5 kg attached to its other end B. The natural length of the spring is 1.2 m and the mass hangs in equilibrium 2 m below A.

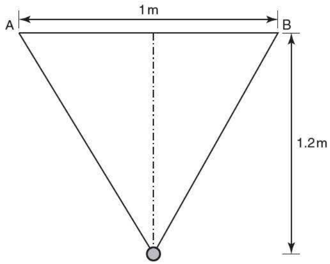
(i) Find the modulus of elasticity of the spring.

The mass is pulled down a distance of 0.4 m below the equilibrium position and then released.

(ii) Find its initial acceleration.

5 The ends of an elastic string of natural length 1.5 m are fixed to points A and B on the same horizontal level where AB = 1 m. Particle P of weight 5 N is attached to the midpoint of the string and P hangs in equilibrium at a depth of 1.2 m below the level of AB.

Find the modulus of elasticity of the string.



6 One end of a light elastic string of natural length 2.5 m and modulus of elasticity 300 N, is attached to a fixed point O. The other end of the string is attached to a particle of mass 5 kg. The particle hangs in equilibrium vertically below O.

(i) Calculate the extension of the string.

(ii) Find the elastic energy stored in the string.

7 A particle of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.4 m. The other end is attached to a fixed point O. The particle is released from rest at O and comes to instantaneous rest 0.8 m below O. Find the modulus of elasticity of the string.

8 A light elastic string has natural length 3 m and modulus of elasticity 50 N. A particle P of mass $\frac{2}{3}$ kg is attached to one end of the string. The other end of the string is attached to a point A. The particle is released from rest at A and falls vertically.

(i) Find the distance travelled by P before it immediately comes to instantaneous rest for the first time.

The particle is now held at a point 6 m vertically below A and released from rest.

(ii) Find the speed of the particle when the string first becomes slack.

9 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 1.5 m and modulus of elasticity 3.75 N. The other end of the string is attached to a fixed point O on a smooth plane inclined at 30° to the horizontal. P is released from rest at O and moves down the plane.

(i) Show that the maximum speed of P is reached when the extension of the string is 1 m.

(ii) Find the maximum speed of P.

(iii) Find the maximum displacement of P from O.

10 A particle of mass 0.3 kg is suspended by two identical strings of length 1.5 m and modulus of elasticity λ N. The other ends of the strings are fixed to two points A and B on a horizontal ceiling where AB = 3 m. P is released from rest at the midpoint of AB and falls vertically until it is instantaneously at rest at a point 1 m below the level of the ceiling.

(i) Calculate the value of λ.

(ii) Find the speed of the particle when it is 0.5 m below the level of the ceiling.

11 A particle of mass 0.8 kg is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 8 N. One end of the string is fixed to a point A on a rough plane inclined at 30° to the horizontal. P is held at rest on the plane below A with AP along a line of greatest slope of the plane and AP = 0.5 m. The coefficient of friction between P and the plane is 0.3. The particle P is released from rest.

(i) Calculate the distance P moves down the plane before coming to instantaneous rest.

(ii) Find the speed of P when AP = 0.6 m.

12 A light elastic string has natural length of 2 m and modulus of elasticity 150 N. One end of the string is attached to a fixed point O and a mass of 3 kg hangs in equilibrium from the other end. The mass is pulled down by 10 cm and then released.

(i) Find the extension of the string when the mass is in the equilibrium position.

(ii) Find the energy stored in the string just before release.

(iii) Find the speed of the particle as it passes through the equilibrium position.

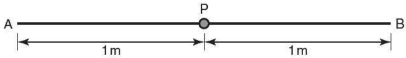
13 A jack-in-the-box is made using a spring of natural length 0.2 m and modulus of elasticity 100 N and a jack of mass 0.5 kg. When the lid is closed the spring is compressed to a length of 0.1 m. Assuming the spring to be vertical throughout, calculate the maximum distance that the jack will rise when the lid is suddenly opened.

14 A and B are two points on a smooth horizontal table. The distance AB is 2 m.

An elastic string of natural length 0.5 m and modulus of elasticity 25 N has one end attached to A and the other to a particle P of mass 1.25 kg.

Another elastic string of natural length 0.8 m and modulus of elasticity 20 N has one end attached to B and the other to the particle P.

The particle P is held at rest at the midpoint of AB.



(i) Find the tensions in the two strings.

The particle is released from rest.

(ii) Find the acceleration of the particle, immediately after its release.

(iii) P reaches its maximum speed at a point C. Find AC.

15 A particle of mass 0.3 kg is attached to the end of an elastic string of natural length 0.8 m and modulus of elasticity 12 N. The string is suspended from A. Find the maximum length of the string, when

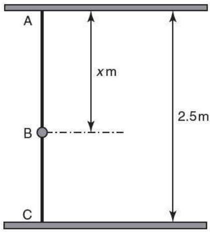
(i) the mass is held 0.8 m below A and let go

(ii) the mass is held at A and let go.

16 A light elastic string has natural length 0.6 m and modulus of elasticity 60 N. The end A is attached to a point on the ceiling. A particle of mass 0.5 kg is attached to the end B of the string and hangs in equilibrium.

(i) Calculate the length AB.

A second string, identical to the first one, is now attached to the particle at B and to a point C on the floor, 2.5 m vertically below A. The system is in equilibrium with B a distance x m below A, as shown in the figure.



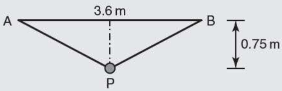
(ii) Find the tension in each of the strings in terms of x and hence show that $x = 1.275$ m.

(iii) Calculate the elastic potential energy in the strings when the particle hangs in equilibrium.

(iv) The particle is now pulled down 0.1 m from its equilibrium position and released from rest. Calculate the speed of the particle when it passes through the equilibrium position.

Stretch and challenge

1 A light elastic string of natural length 3.6 m and modulus of elasticity λ N has its ends attached to two points A and B, where AB = 3.6 m and AB is horizontal. A particle P of mass 0.5 kg is attached to the midpoint of the string. P rests in equilibrium at a distance of 0.75 m below the line AB as shown in the figure.



(i) Show that $\lambda = 78$ N.

The particle is pulled downwards from its equilibrium position until the total length of the elastic string is 6 m. The particle is released from rest.

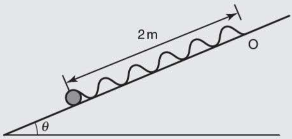
(ii) Find the speed of P when it passes the line AB.

2 A particle of mass 0.4 kg is attached to one end of a light elastic spring of natural length 1.2 m and modulus of elasticity λ N. The other end of the spring is attached to a fixed point O on a rough plane inclined at an angle of θ to the horizontal, where $\sin\theta = 0.25$. The coefficient of friction between the particle and the plane is 0.3.

The particle is held on the plane at a point which is 2 m down the line of greatest slope from O, as shown in the figure.

The particle is released from rest and first comes to rest again after moving 1 m up the plane.

Find the value of λ.



3 One end of a light elastic string of natural length L and modulus of elasticity 20 N is attached to a fixed point O. The other end is attached to a particle P of mass 0.5 kg. The particle is projected vertically downwards with speed $\sqrt{40L}$.

(i) Find the speed of P when P is at a depth x below O and $x > L$, and show that the greatest depth of P below O is $\frac{5}{2}L$.

(ii) Find the maximum speed of P.

(iii) Show that the particle subsequently rises to a maximum height of $\frac{3}{2}L$ above O.

Exam focus

1 A light elastic string, of natural length 2.5 m and modulus of elasticity 100 N, has one end attached to a fixed point O and the other end attached to a particle P, of mass 5 kg. Initially the particle is at rest at a point 0.8 m below O and is allowed to fall.

(i) Calculate the tension in the string when the length of the string is 3 m. [3]

(ii) Find the speed of P when the length of the string is 3 m. [3]

2 A light elastic string, of natural length 0.5 m and modulus of elasticity 16 N, has one end attached to a fixed point A. A particle P, of mass 0.4 kg, is attached to the other end of the string.

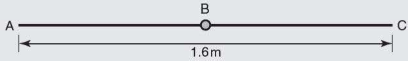
(i) Find the length of the string when the particle hangs in equilibrium. [3]

The particle is released from rest at A and falls vertically. Use the work–energy principle to calculate

(ii) the speed of P at a distance of 0.625 m below A [4]

(iii) the greatest length of the string. [3]

3 Two light elastic strings are joined and stretched between two fixed points A and C on a smooth horizontal table which are 1.6 m apart as shown in the diagram.



The string AB has natural length 0.4 m and modulus of elasticity 12 N. The string BC has natural length 0.5 m and modulus of elasticity 7.5 N. The system is in equilibrium.

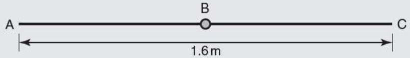
(i) Find the distance AB and the tension in each string. **[4]**

(ii) Find the elastic potential energy stored in the strings after they have been stretched from their natural lengths to be connected as described above. **[2]**

A particle of mass 0.5 kg is attached at B. The particle is released from rest at the midpoint of AC.

(iii) Find the speed of the particle when it passes through the equilibrium position of the system. **[4]**

4 Two light elastic strings AB and BC, both have modulus of elasticity equal to 24 N. String AB has natural length 0.4 m and string BC has natural length 0.6 m. The strings are both attached at B to a particle of mass 0.8 kg. The ends A and C are fixed to points on a smooth horizontal table, such that AC = 1.6 m, as shown in the figure.



Initially the particle is held at the midpoint of AC and released from rest.

(i) Find the tension in each string before release and calculate the acceleration of the particle immediately after it is released. **[3]**

The particle is now moved to the position where it is in equilibrium. The extension in AB is x m.

(ii) Calculate x. **[3]**

The particle is now held at A and released from rest.

(iii) Show that in the subsequent motion BC becomes slack. Calculate the furthest distance of the particle from A. **[4]**

15 Linear motion under a variable force

Newton's second law as a differential equation; Variable force examples

EXERCISE 15.1

1 In each of the following cases, a particle of mass 1 kg is under the influence of a single force F N in a constant direction but with a variable magnitude given as a function of velocity, v m s^{-1}, displacement, x m, or time t s.

In each case express $F = ma$ as a differential equation using either $a = \frac{dv}{dt}$ or $a = v\frac{dv}{dx}$ as appropriate. Separate the variables and integrate, giving the result in the required form with an arbitrary constant in the answer.

(i) $F = -3v$ Express x in terms of v.

(ii) $F = 2v + 1$ Express v in terms of t.

(iii) $F = 2e^{-t}$ Express v in terms of t.

(iv) $F = -3v^2$ Express v in terms of t.

(v) $F = \dfrac{1}{1+x}$ Express x in terms of v.

(vi) $F = \dfrac{5}{v}$ Express v in terms of x.

2 In each of the following cases, a particle of mass 1 kg is under the influence of a single force F N in a constant direction but with a variable magnitude given as a function of velocity v m s^{-1}, displacement x m, or time t s. The particle is initially at rest at the origin.

In each case, write down the equation of motion and solve it to supply the required information.

(i) $F = 3t + 5$ Find v when $t = 1$.

(ii) $F = \dfrac{1}{3+x}$ Find v when $x = 2$.

(iii) $F = \dfrac{1}{(x+2)^3}$ Find v when $x = 1$.

(iv) $F = 4 - v^2$ Find t when $v = 1$.

(v) $F = 1 + v^2$ Find t when $v = 1$.

(vi) $F = 3 + v^2$ Find x when $v = 4$.

3 A particle of mass m kg is held at rest at a point O on a fixed plane inclined at an angle $\theta = \sin^{-1}(0.4)$ to the horizontal. P is released and moves down the plane. The total resistance on P is $0.25mv$ N where v m s^{-1} is the velocity of P at time t s after leaving O.

(i) Show that $4\frac{dv}{dt} = 16 - v$ and hence find an expression for v in terms of t.

(ii) Find the acceleration of P at time $t = 4$ s.

4 A particle P of mass 0.5 kg moves along the positive x axis away from O under the action of a single force F directed away from O. When OP = x m the magnitude of F is $\frac{5}{(x+1)^5}$ N and the speed of P is v m s^{-1}. Initially P is at rest at O.

(i) Show that $v^2 = 5\left[1 - \frac{1}{(x+1)^4}\right]$.

(ii) Find the acceleration of P when $x = 2$.

5 A ball P of mass 0.25 kg is projected vertically upwards from ground level with an initial speed of $20\ \text{m s}^{-1}$. A resisting force of magnitude $0.05v$ N acts on P during its ascent, where $v\ \text{m s}^{-1}$ is the speed and x m is the displacement of the ball at time t s after it starts to move.

(i) Show that $\dfrac{dv}{dt} = -0.2(50 + v)$.

(ii) Find an expression for v as a function of t.

(iii) Find the displacement x as a function of t.

(iv) Find the greatest height reached by the ball.

6 A particle of mass 0.6 kg is moving in a straight line under the action of a force of constant direction and magnitude $\frac{3}{v}$ N, where v is the velocity of the particle at time t. Initially the particle is at a fixed point O, and travelling with speed $3\,\mathrm{m\,s^{-1}}$.

(i) Find the velocity of the particle at time t.

(ii) Find the velocity when the displacement from O is 5 m.

7 A particle of mass 0.75 kg moves in a straight line under the action of a force of uniform direction and magnitude $(3x^2 + 2)$ N, where x is the displacement from a fixed point O on the line, at time t. When the displacement is -4 m, its velocity is 0. Find the velocity of the particle when it passes through O.

8 A particle of mass 2 kg starts with speed 20 m s^{-1} and moves in a straight line. The particle is subject to a resistive force F, which increases uniformly with the distance moved. Initially F is 4 N, and after the particle has moved 10 m, it has a value of 9 N.

(i) If the particle has speed v m s^{-1} after moving a distance x m, show that while the particle is in motion $v\dfrac{dv}{dx} = -0.25(8 + x)$.

(ii) Find the distance moved by the particle in coming to rest.

9 A particle P starts from O and moves in a straight line. When the displacement is x m its velocity is v m s^{-1} and its acceleration is $\dfrac{2}{x+1}$.

(i) Given that $v = 3$ when $x = 0$, show that $v^2 = 4 \ln |x + 1| + 9$.

(ii) Find the value of v when the acceleration is $\frac{2}{5}$.

10 A particle of mass 0.5 kg moves in a straight line on a smooth horizontal surface. A variable resisting force acts on the particle. At time t s, the displacement of the particle from a point O on the line is x m, and its velocity is $4 - x^2$ m s^{-1}. It is given that $x = 0$ when $t = 0$.

(i) Find the acceleration of the particle in terms of x, and hence find the magnitude of the resisting force when $x = 1.5$.

(ii) Find an expression for x in terms of t.

(iii) How far from O can the particle ever get?

11 A particle of mass 0.5 kg falls from rest in a medium offering a resistance of $0.05v^2$.

(i) Find the relation between the velocity and the time t.

(ii) Find the limiting velocity.

12 A particle P of mass m is held at rest at a point O on a fixed inclined plane at an angle of $\sin^{-1}\left(\frac{2}{5}\right)$ to the horizontal. P is released from rest and moves down the plane. The total resistance acting on P is $0.1\,mv$ N, where $v\ \mathrm{m\,s^{-1}}$ is the velocity of P t seconds after leaving O.

(i) Show that $10\dfrac{\mathrm{d}v}{\mathrm{d}t} = 40 - v$ and hence find an expression for v in terms of t.

(ii) Find the distance travelled in 5 s.

13 A particle P of mass 0.5 kg moves on a horizontal surface along the straight line OA, in the direction from O to A. The coefficient of friction between P and the surface is 0.2. Air resistance of magnitude $0.1v^2$ opposes the motion, where v m s^{-1} is the speed of P at time t seconds. The particle passes through O with speed 5 m s^{-1}.

(i) Show that $5v\dfrac{dv}{dx} = -(v^2 + 10)$ and hence find the value of x when $v = 0$.

(ii) Find v when $x = 2$.

14 A car of mass 1000 kg passes the point O on a straight horizontal road with speed $24.8\,\text{m}\,\text{s}^{-1}$. The car is subject to a resistance of $(500 + 2500v)$ N, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of the car at time t.

(i) Show that v satisfies the equation $\frac{dv}{dt} = -2.5(0.2 + v)$.

(ii) Find an expression for v in terms of t.

(iii) Find the time when $v = 0$.

(iv) Find the distance travelled by the car in that time.

Stretch and challenge

1 A rocket of mass 1000 kg is launched from rest at ground level and travels vertically upwards. The mass of the rocket is constant and the only forces acting on it are its weight, a driving force of 20 000 N and a resistance force $5v$.

(i) Show that $\frac{dv}{dt} = 10 - 0.005v$.

(ii) Find v in terms of t.

(iii) Find the distance travelled by the rocket in the first 5 seconds of its motion.

2 A particle is projected with speed U at time $t = 0$ and moves in a straight line. At time t, its velocity is v and the distance travelled is x. The acceleration of the particle is $-k\sqrt{v}$ where k is a constant.

(i) Show that the particle will come to rest when $t = \frac{2\sqrt{U}}{k}$

(ii) Find in terms of k and U the distance travelled while the particle comes to rest.

3 A particle of mass 0.4 kg is projected vertically upwards with a speed of $20\,\mathrm{m\,s^{-1}}$.
The particle experiences a resistance of $0.5v$, where v is the velocity of the particle.

(i) Find the time taken for the particle to come to instantaneous rest.

(ii) Find the greatest height attained by the particle.

Having reached its highest point, the particle then drops down against a resistance of $0.5v$.

(iii) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 1.25(8 - v)$ and use it to find an expression for v as a function of t.

(iv) Find the distance travelled as a function of t and hence show that the time taken for the particle to drop down to ground level is greater than the time taken to reach its highest point.

Exam focus

1 A particle P of mass 2.5 kg is moving in a straight line. The only force acting on P is a resistance to motion of magnitude $\left(10+\frac{25}{(t+1)^2}\right)$ N. The velocity of P at time t is v m s^{-1}. The particle is at rest when $t = 3$.

Find v when $t = 0$. **[5]**

2 A car of mass 1200 kg moves along a horizontal road. At time t seconds, the resultant force acting on the car has magnitude $\frac{60\,000}{(t+1)^3}$ N in the direction of the motion of the car. When $t = 0$, the car is at rest.

(i) Show that the speed of the car approaches a limiting value as t increases and find that value. **[6]**

(ii) Find the distance moved by the car in the first 10 seconds of its motion. **[4]**

3 A particle of mass 0.3 kg moves in a straight line on a smooth horizontal table. When P is a distance x m from a fixed point O, it experiences a force of magnitude $\frac{3}{x^2}$ N away from O in the direction OP. Initially P is at a point 5 m from O and is moving towards O with speed $2\,\text{m}\,\text{s}^{-1}$.

Find the distance of P from O when P first comes to rest. [5]

4 A particle P is moving in a straight line. At time t seconds, P is a distance x m from a fixed point O on the line and is moving away from O with speed $\frac{8}{x+4}\ \text{m}\,\text{s}^{-1}$.

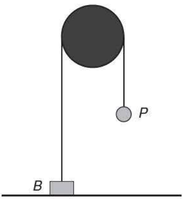
(i) Find the acceleration of P when $x = 4$. [4]

(ii) Given that $x = 4$ when $t = 0.5$, find the value of t when $x = 10$. [4]

PAST EXAMINATION QUESTIONS

Mechanics 1 (M1) Paper 1

1



A block B of mass 5 kg is attached to one end of a light inextensible string. A particle P of mass 4 kg is attached to the other end of the string. The string passes over a smooth pulley. The system is in equilibrium with the string taut and its straight parts vertical. B is at rest on the ground (see diagram). State the tension in the string and find the force exerted on B by the ground. **[3]**

(Cambridge International AS & A level Mathematics, 9709/04 June 2009 Q1)

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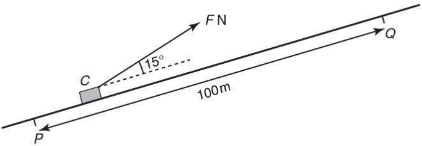

A crate C is pulled at constant speed up a straight inclined path by a constant force of magnitude F N, acting upwards at an angle of 15° to the path. C passes through points P and Q which are 100 m apart (see diagram). As C travels from P to Q the work done against resistance to C's motion is 900 J, and the gain in C's potential energy is 2100 J. Write down the work done by the pulling force as C travels from P to Q, and hence find the value of F. **[3]**

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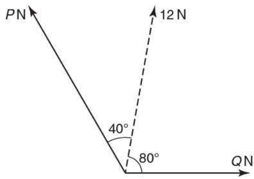
(Cambridge International AS & A level Mathematics, 9709/04 June 2009 Q2)

3 Two forces have magnitudes P N and Q N. The resultant of the two forces has magnitude 12 N and acts in a direction 40° clockwise from the force of magnitude P N and 80° anticlockwise from the force of magnitude Q N (see diagram). Find the value of Q. **[4]**



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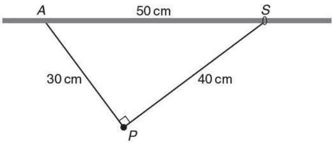
(Cambridge International AS & A level Mathematics, 9709/41 Oct/Nov 2009 Q3)

4 A particle P of weight 5 N is attached to one end of each of two light inextensible strings of lengths 30 cm and 40 cm. The other end of the shorter string is attached to a fixed point A of a rough rod which is fixed horizontally.



A small ring S of weight W N is attached to the other end of the longer string and is threaded on to the rod. The system is in equilibrium with the strings taut and $AS = 50$ cm (see diagram).

(i) By resolving the forces acting on P in the direction of PS, or otherwise, find the tension in the longer string. **[3]**

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(ii) Find the magnitude of the frictional force acting on S. **[2]**

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(iii) Given that the coefficient of friction between S and the rod is 0.75, and that S is in limiting equilibrium, find the value of W. **[3]**

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(Cambridge International AS & A level Mathematics, 9709/41 Oct/Nov 2009 Q4)

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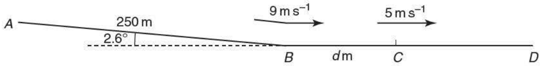

A cyclist and his machine have a total mass of 80 kg. The cyclist starts from rest at the top A of a straight path AB, and freewheels (moves without pedalling or braking) down the path to B. The path AB is inclined at 2.6° to the horizontal and is of length 250 m (see diagram).

(i) Given that the cyclist passes through B with speed $9\,\mathrm{m\,s^{-1}}$, find the gain in kinetic energy and the loss in potential energy of the cyclist and his machine. Hence find the work done against the resistance to motion of the cyclist and his machine. **[3]**

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The cyclist continues to freewheel along a straight path BD until he reaches the point C, where the distance BC is d m. His speed at C is $5\,\mathrm{m\,s^{-1}}$. The resistance to motion is constant, and is the same on BD as on AB.

(ii) Find the value of d. **[3]**

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The cyclist starts to pedal at C, generating 425 W of power.

(iii) Find the acceleration of the cyclist immediately after passing through C. **[3]**

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(Cambridge International AS & A level Mathematics, 9709/04 June 2009 Q5)

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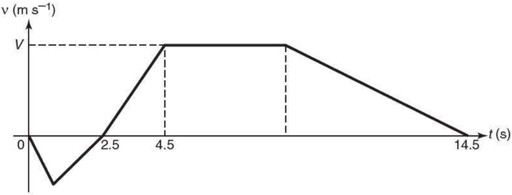

The diagram shows the velocity–time graph for a particle P which travels on a straight line AB, where v m s^{-1} is the velocity of P at time t s. The graph consists of five straight line segments. The particle starts from rest when $t = 0$ at a point X on the line between A and B and moves towards A. The particle comes to rest at A when $t = 2.5$.

(i) Given that the distance XA is 4 m, find the greatest speed reached by P during this stage of the motion. **[2]**

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In the second stage, P starts from rest at A when $t = 2.5$ and moves towards B. The distance AB is 48 m. The particle takes 12 s to travel from A to B and comes to rest at B. For the first 2 s of this stage P accelerates at 3 m s^{-2}, reaching a velocity of V m s^{-1}. Find

(ii) the value of V **[2]**

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(iii) the value of t at which P starts to decelerate during this stage **[3]**

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(iv) the deceleration of P immediately before it reaches B. **[2]**

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(Cambridge International AS & A level Mathematics, 9709/42 Oct/Nov 2010 Q6)

7 A particle P travels in a straight line from A to D, passing through the points B and C. For the section AB the velocity of the particle is $(0.5t - 0.01t^2)$ m s^{-1}, where t is the time after leaving A.

(i) Given that the acceleration of P at B is 0.1 m s^{-2}, find the time taken for P to travel from A to B. **[3]**

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The acceleration of P from B to C is constant and equal to 0.1 m s^{-2}.

(ii) Given that P reaches C with speed 14 m s^{-1}, find the time taken for P to travel from B to C. **[3]**

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P travels with constant deceleration 0.3 m s^{-2} from C to D. Given that the distance CD is 300 m, find

(iii) the speed with which P reaches D **[2]**

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(iv) the distance AD. **[6]**

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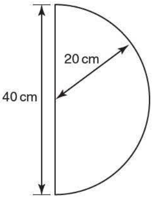
(Cambridge International AS & A level Mathematics, 9709/04 June 2009 Q7)

Mechanics 2 (M2) Paper 1

1



A frame consists of a uniform semicircular wire of radius 20 cm and mass 2 kg, and a uniform straight wire of length 40 cm and mass 0.9 kg. The ends of the semicircular wire are attached to the ends of the straight wire (see diagram). Find the distance of the centre of mass of the frame from the straight wire. **[4]**

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(Cambridge International AS & A level Mathematics, 9709/51 June 2010 Q1)

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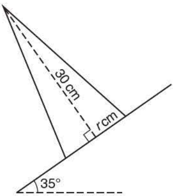

A uniform solid cone has height 30 cm and base radius r cm. The cone is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted and the cone remains in equilibrium until the angle of inclination of the plane reaches 35°, when the cone topples. The diagram shows a cross-section of the cone.

(i) Find the value of r. **[3]**

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(ii) Show that the coefficient of friction between the cone and the plane is greater than 0.7. **[2]**

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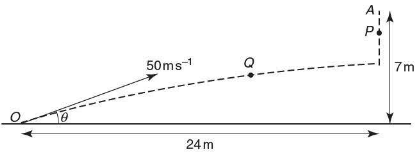
(Cambridge International AS & A level Mathematics, 9709/51 June 2010 Q2)

3 A particle P is released from rest at a point A which is 7 m above horizontal ground. At the same instant that P is released a particle Q is projected from a point O on the ground. The horizontal distance of O from A is 24 m. Particle Q moves in the vertical plane containing O and A, with initial speed 50 m s^{-1} and initial direction making an angle θ above the horizontal, where $\tan\theta = \frac{7}{24}$ (see diagram). Show that the particles collide. **[6]**



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(Cambridge International AS & A level Mathematics, 9709/52 Oct/Nov 2009 Q3)

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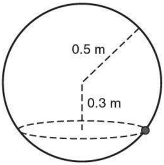

A particle of mass 0.12 kg is moving on the smooth inside surface of a fixed hollow sphere of radius 0.5 m. The particle moves in a horizontal circle whose centre is 0.3 m below the centre of the sphere (see diagram).

(i) Show that the force exerted by the sphere on the particle has magnitude 2 N. **[2]**

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(ii) Find the speed of the particle. **[3]**

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(iii) Find the time taken for the particle to complete one revolution. **[2]**

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(Cambridge International AS & A level Mathematics, 9709/05 June 2009 Q4)

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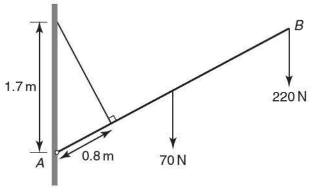

A uniform beam AB has length 2 m and weight 70 N. The beam is hinged at A to a fixed point on a vertical wall, and is held in equilibrium by a light inextensible rope. One end of the rope is attached to the wall at a point 1.7 m vertically above the hinge. The other end of the rope is attached to the beam at a point 0.8 m from A. The rope is at right angles to AB. The beam carries a load of weight 220 N at B (see diagram).

(i) Find the tension in the rope. **[3]**

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(ii) Find the direction of the force exerted on the beam at A. **[4]**

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(Cambridge International AS & A level Mathematics, 9709/51 Oct/Nov 2010 Q4)

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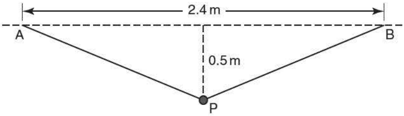

A light elastic string has natural length 2 m and a modulus of elasticity λ N. The ends of the string are attached to fixed points A and B which are at the same horizontal level and 2.4 m apart. A particle P of mass 0.6 kg is attached to the midpoint of the string and hangs in equilibrium at a point 0.5 m below AB (see diagram).

(i) Show that $\lambda = 26$. **[4]**

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P is projected vertically downwards from the equilibrium position, and comes to instantaneous rest at a point 0.9 m below AB.

(ii) Calculate the speed of projection of P. **[5]**

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(Cambridge International AS & A level Mathematics, 9709/51 June 2010 Q6)

7 A cyclist and his bicycle have a total mass of 81 kg. The cyclist starts from rest and rides in a straight line. The cyclist exerts a constant force of 135 N and the motion is opposed by a resistance of magnitude $9v$ N, where v m s^{-1} is the cyclist's speed at time t s after starting.

(i) Show that $\frac{9}{15-v}\frac{dv}{dt}=1$. [2]

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(ii) Solve this differential equation to show that $v = 15(1 - e^{-1/9t})$. [4]

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(iii) Find the distance travelled by the cyclist in the first 9 s of the motion. [4]

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(Cambridge International AS & A level Mathematics, 9709/51 Oct/Nov 2010 Q6)

Mechanics 1 (M1) Paper 2

1 A car of mass 700 kg is travelling along a straight horizontal road. The resistance to the motion is constant and equal to 600 N.

(i) Find the driving force of the car's engine at an instant when the acceleration is $2\,\text{m}\,\text{s}^{-2}$. **[2]**

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(ii) Given that the car's speed at this instant is $15\,\text{m}\,\text{s}^{-1}$, find the rate at which the car's engine is working. **[2]**

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(Cambridge International AS & A level Mathematics, 9709/41 June 2011 Q1)

2 A load of mass 1250 kg is raised by a crane from rest on horizontal ground, to rest at a height of 1.54 m above the ground. The work done against the resistance to motion is 5750 J.

(i) Find the work done by the crane. **[3]**

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(ii) Assuming the power output of the crane is constant and equal to 1.25 kW, find the time taken to raise the load. **[2]**

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(Cambridge International AS & A level Mathematics, 9709/41 June 2011 Q2)

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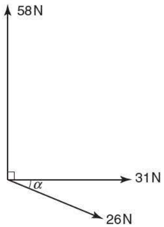

Coplanar forces of magnitudes 58 N, 31 N and 26 N act at a point in the directions shown in the diagram. Given that $\tan \alpha = \frac{5}{12}$, find the magnitude and direction of the resultant of the three forces. **[6]**

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(Cambridge International AS & A level Mathematics, 9709/43 Oct/Nov 2011 Q2)

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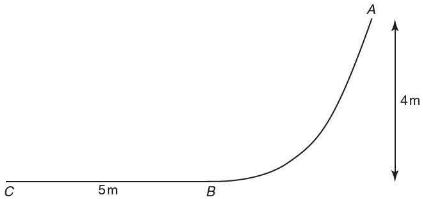

ABC is a vertical cross-section of a surface. The part of the surface containing AB is smooth and A is 4 m higher than B. The part of the surface containing BC is horizontal and the distance BC is 5 m (see diagram). A particle of mass 0.8 kg is released from rest at A and slides along ABC. Find the speed of the particle at C in each of the following cases.

(i) The horizontal part of the surface is smooth. **[3]**

(ii) The coefficient of friction between the particle and the horizontal part of the surface is 0.3. **[3]**

(Cambridge International AS & A level Mathematics, 9709/43 Oct/Nov 2011 Q4)

5 A particle P moves in a straight line. It starts from rest at A and comes to rest instantaneously at B. The velocity of P at time t seconds after leaving A is $v\,\text{m s}^{-1}$, where $v = 6t^2 - kt^3$ and k is a constant.

(i) Find an expression for the displacement of P from A in terms of t and k. [2]

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(ii) Find an expression for t in terms of k when P is at B. [1]

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Given that the distance AB is 108 m, find

(iii) the value of k [2]

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(iv) the maximum value of v when the particle is moving from A towards B. [3]

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(Cambridge International AS & A level Mathematics, 9709/43 Oct/Nov 2011 Q5)

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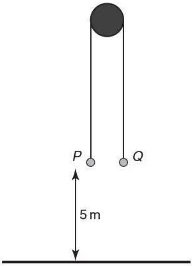

Particles P and Q, of masses 0.55 kg and 0.45 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. The particles are held at rest with the string taut and its straight parts vertical. Both particles are at a height of 5 m above the ground (see diagram). The system is released.

(i) Find the acceleration with which P starts to move. **[3]**

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The string breaks after 2 s and in the subsequent motion P and Q move vertically under gravity.

(ii) At the instant that the string breaks, find

(a) the height above the ground of P and of Q **[2]**

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(b) the speed of the particles. **[1]**

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(iii) Show that Q reaches the ground 0.8 s later than P. **[4]**

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(Cambridge International AS & A level Mathematics, 9709/41 Oct/Nov 2009 Q6)

7 A particle P starts from rest at the point A at time $t = 0$, where t is in seconds, and moves in a straight line with constant acceleration a m s^{-2} for 10 s. For $10 \leqslant t \leqslant 20$, P continues to move along the line with velocity v m s^{-1}, where $v = \frac{800}{t^2} - 2$.

(i) Find the speed of P when $t = 10$, and the value of a. **[2]**

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(ii) Find the value of t for which the acceleration of P is $-a$ m s^{-2}. **[4]**

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(iii) Find the displacement of P from A when $t = 20$. **[6]**

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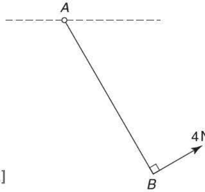
(Cambridge International AS & A level Mathematics, 9709/41 Oct/Nov 2009 Q7)

Mechanics 2 (M2) Paper 2

1 A uniform rod AB of weight 16 N is freely hinged at A to a fixed point. A force of magnitude 4 N acting perpendicular to the rod is applied at *B* (see diagram). Given that the rod is in equilibrium,

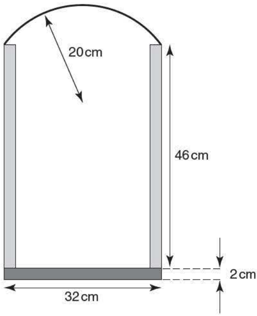


(i) calculate the angle the rod makes with the horizontal **[2]**

(ii) find the magnitude and direction of the force exerted on the rod at A. **[4]**

(Cambridge International AS & A level Mathematics, 9709/52 June 2011 Q1)

2 A bucket that consists of three parts stands on horizontal ground. The base is in the form of a uniform circular disc of diameter 32 cm and thickness 2 cm. The body is in the form of a uniform hollow cylinder of outer diameter 32 cm and height 46 cm. The handle is in a vertical plane, attached at opposite ends of an outer diameter at the top of the cylinder. The handle is in the form of a uniform circular arc of radius 20 cm. The diagram shows the cross-section of the bucket in the plane of the handle.



(i) Show that the centre of mass of the handle is 53.25 cm above the ground, correct to 4 significant figures. **[3]**

The weights of the base, body and handle are 50 N, 100 N and 25 N respectively.

(ii) Find the height of the centre of mass of the bucket above the ground. [2]

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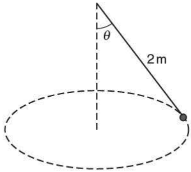
(Cambridge International AS & A level Mathematics, 9709/52 Oct/Nov 2009 Q2)

3 A particle of mass 0.24 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point. The particle moves with constant speed in a horizontal circle. The string makes an angle θ with the vertical (see diagram), and the tension in the string is T N. The acceleration of the particle has magnitude 7.5 m s^{-2}.



(i) Show that $\tan\theta = 0.75$ and find the value of T. [4]

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(ii) Find the speed of the particle. [2]

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(Cambridge International AS & A level Mathematics, 9709/51 June 2010 Q3)

4 One end of a light elastic string of natural length 3 m and modulus of elasticity 15 m N is attached to a fixed point O. A particle P of mass m kg is attached to the other end of the string. P is released from rest at O and moves vertically downwards. When the extension of the string is x m the velocity of P is v m s^{-1}.

(i) Show that $v^2 = 5(12 + 4x - x^2)$. **[4]**

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(ii) Find the magnitude of the acceleration of P when it is at its lowest point, and state the direction of this acceleration. **[3]**

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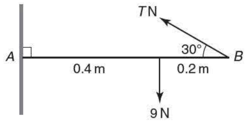
(Cambridge International AS & A level Mathematics, 9709/52 Oct/Nov 2009 Q4)

5 A non-uniform rod AB, of length 0.6 m and weight 9 N, has its centre of mass 0.4 m from A. The end A of the rod is in contact with a rough vertical wall. The rod is held in equilibrium, perpendicular to the wall, by means of a light string attached to B. The string is inclined at 30° to the horizontal. The tension in the string is T N (see diagram).



(i) Calculate T. **[2]**

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(ii) Find the least possible value of the coefficient of friction at A. **[3]**

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(Cambridge International AS & A level Mathematics, 9709/51 Oct/Nov 2011 Q1)

6 A particle P is projected with speed $26\,\mathrm{m\,s^{-1}}$ at an angle of $30°$ below the horizontal, from a point O which is 80 m above horizontal ground.

(i) Calculate the distance from O of the particle 2.3 s after projection. **[4]**

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(ii) Find the horizontal distance travelled by P before it reaches the ground. **[3]**

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(iii) Calculate the speed and direction of motion of P immediately before it reaches the ground. **[4]**

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(Cambridge International AS & A level Mathematics, 9709/52 June 2011 Q6)

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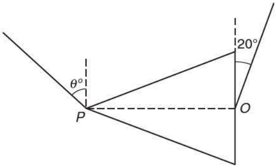

P is the vertex of a uniform solid cone of mass 5 kg, and O is the centre of its base. Strings are attached to the cone at P and at O. The cone hangs in equilibrium with PO horizontal and the strings taut. The strings attached at P and O make angles of $\theta°$ and 20°, respectively, with the vertical (see diagram, which shows a cross-section).

(i) By taking moments about P for the cone, find the tension in the string attached at O. **[4]**

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(ii) Find the value of θ and the tension in the string attached at P. **[6]**

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(Cambridge International AS & A level Mathematics, 9709/52 Oct/Nov 2009 Q6)