# Chapter 12

## Exercise Solutions

### E12.1

a. $A_f = \dfrac{A}{1+A\beta}$

$1 + A\beta = \dfrac{A}{A_f} \Rightarrow A\beta = \dfrac{A}{A_f} - 1$

$\beta = \dfrac{1}{A_f} - \dfrac{1}{A} = \dfrac{1}{20} - \dfrac{1}{10^4} = 0.05 - 0.0001$

$\Rightarrow \underline{\beta = 0.0499}$

b. $\dfrac{A_f}{(1/\beta)} = \dfrac{20}{(1/0.0499)} = \dfrac{20}{20.040} = \underline{0.998}$

### E12.2

$A_f = \dfrac{A}{1+A\beta}$

$A_f + A_f A\beta = A$

$A_f = A(1 - A_f\beta)$

$A = \dfrac{A_f}{1 - A_f\beta} = \dfrac{80}{1-(80)(0.0120)}$

$\underline{A = 2000}$

### E12.3

$A_f = \dfrac{A}{1+A\beta}$

$\beta = \dfrac{1}{A_f} - \dfrac{1}{A} = \dfrac{1}{100} - \dfrac{1}{10^6} = 0.01 - 10^{-6}$

$\beta = 0.009999$

$\dfrac{dA_f}{A_f} = \dfrac{1}{(1+\beta A)} \cdot \dfrac{dA}{A} = \left(\dfrac{A_f}{A}\right) \cdot \dfrac{dA}{A}$

$\dfrac{dA_f}{A_f} = \left(\dfrac{100}{10^6}\right)(20)\%$

$\Rightarrow \underline{\dfrac{dA_f}{A_f} = 0.002\%}$

### E12.4

$\dfrac{dA_f}{A_f} = \dfrac{1}{(1+\beta A)} \cdot \dfrac{dA}{A} = \left(\dfrac{A_f}{A}\right) \cdot \dfrac{dA}{A}$

$\dfrac{dA}{A} = \dfrac{dA_f}{A_f} \cdot \left(\dfrac{A}{A_f}\right) = (0.001)\left(\dfrac{5 \times 10^5}{100}\right)$

$\Rightarrow \underline{\dfrac{dA}{A} = \pm 5\%}$

### E12.5

Bandwidth $= \omega_H(1 + \beta A_0)$

$= \omega_H\left(\dfrac{A_0}{A_f}\right) = (2\pi)(10)\left(\dfrac{10^5}{100}\right)$

$\underline{\omega = (2\pi)(10^4) \text{ rad/sec} \Rightarrow f = 10 \text{ kHz}}$

### E12.6

$A_f \cdot f_H = A_0 \cdot f_1$

$A_f = \dfrac{A_0 \cdot f_1}{f_H} = \dfrac{(10^6)(8)}{250 \times 10^3}$

$\Rightarrow \underline{A_f(0) = 32}$

### E12.7

a. $V_\epsilon = V_S - V_{fb} = 100 - 99 = 1 \text{ mV}$

$V_0 = A_v V_\epsilon \Rightarrow A_v = \dfrac{5}{0.001} \Rightarrow \underline{A_v = 5000 \text{ V/V}}$

$V_{fb} = \beta V_0 \Rightarrow \beta = \dfrac{V_{fb}}{V_0} = \dfrac{0.099}{5} \Rightarrow \underline{\beta = 0.0198 \text{ V/V}}$

$A_{vf} = \dfrac{A_v}{1+\beta A_v} = \dfrac{5000}{1+(0.0198)(5000)}$

$\Rightarrow \underline{A_{vf} = 50 \text{ V/V}}$

b. $R_{if} = R_i(1+\beta A_v) = (5)[1 + (0.0198)(5000)]$

$\Rightarrow \underline{R_{if} = 500 \text{ k}\Omega}$

$R_{of} = \dfrac{R_0}{1+\beta A_v} = \dfrac{4}{1+(0.0198)(5000)}$

$\Rightarrow \underline{R_{of} \Rightarrow 40\ \Omega}$

### E12.8

a. $I_\epsilon = I_S - I_{fb} = 100 - 99 = 1\ \mu\text{A}$

$A_i = \dfrac{I_0}{I_\epsilon} = \dfrac{5}{0.001} \Rightarrow \underline{A_i = 5000 \text{ A/A}}$

$\beta = \dfrac{I_{fb}}{I_0} = \dfrac{0.099}{5} \Rightarrow \underline{\beta = 0.0198 \text{ A/A}}$

$A_{if} = \dfrac{A_i}{1+A_i\beta} = \dfrac{5000}{1+(5000)(0.0198)}$

$\Rightarrow \underline{A_{if} = 50 \text{ A/A}}$

b. $R_{if} = \dfrac{R_i}{1+\beta A_i} = \dfrac{5}{1+(0.0198)(5000)}$

$\Rightarrow \underline{R_{if} \Rightarrow 50\ \Omega}$

$R_{of} = (1+\beta A_i)R_0 = [1 + (0.0198)(5000)](4)$

$\Rightarrow \underline{R_{of} = 400 \text{ k}\Omega}$

### E12.9

$V_\epsilon = V_S - V_{fb} = 100 - 99 = 1 \text{ mV}$

$A_g = \dfrac{I_0}{V_\epsilon} = \dfrac{5 \text{ mA}}{1 \text{ mV}} \Rightarrow \underline{A_g = 5 \text{ A/V}}$

$\beta = \dfrac{V_{fb}}{I_0} = \dfrac{99 \text{ mV}}{5 \text{ mA}} \Rightarrow \underline{\beta = 19.8 \text{ V/A}}$

$A_{gf} = \dfrac{A_g}{1+\beta A_g} = \dfrac{5}{1+(19.8)(5)}$

$\Rightarrow \underline{A_{gf} = 0.05 \text{ A/V} = 50 \text{ mA/V}}$

E12.10

$I_\epsilon = I_S - I_{fb} = 100 - 99 = 1\ \mu A$

$A_z = \dfrac{V_0}{I_\epsilon} = \dfrac{5\ V}{1\ \mu A} \Rightarrow \underline{A_z = 5 \times 10^6\ V/A}$

$\beta = \dfrac{I_{fb}}{V_0} = \dfrac{99\ \mu A}{5\ V} \Rightarrow \underline{\beta = 1.98 \times 10^{-5}\ A/V}$

$$A_{zf} = \frac{A_z}{1+\beta A_z} = \frac{5 \times 10^6}{1+(1.98 \times 10^{-5})(5 \times 10^6)}$$

$\Rightarrow \underline{A_{zf} = 5 \times 10^4\ V/A = 50\ V/mA}$

E12.11

$$A_{vf} = \frac{A_v}{1+\dfrac{A_v}{1+(R_2/R_1)}} = \frac{10^4}{1+\dfrac{10^4}{1+(30/10)}}$$

$\Rightarrow \underline{A_{vf} = 3.9984}$

$$A_{vf} = \frac{10^5}{1+\dfrac{10^5}{1+(30/10)}} = 3.99984$$

$\dfrac{3.99984 - 3.9984}{3.9984} \times 100\% \Rightarrow \underline{0.0360\%}$

E12.12

a. $r_\pi = \dfrac{h_{FE}V_T}{I_{CQ}} = \dfrac{(100)(0.026)}{0.5} = 5.2\ k\Omega$

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.5}{0.026} = 19.23\ mA/V$

$$A_{vf} = \frac{\left(\dfrac{1}{r_\pi}+g_m\right)R_E}{1+\left(\dfrac{1}{r_\pi}+g_m\right)R_E} = \frac{\left(\dfrac{1}{5.2}+19.23\right)(2)}{1+\left(\dfrac{1}{5.2}+19.23\right)(2)}$$

$= \dfrac{(19.42)(2)}{1+(19.42)(2)} \Rightarrow \underline{A_{vf} = 0.97490}$

$R_{if} = r_\pi + (1+h_{FE})R_E = 5.2 + (101)(2)$

$\Rightarrow \underline{R_{if} = 207.2\ k\Omega}$

$R_{of} = R_E \,\Big\|\, \dfrac{r_\pi}{1+h_{FE}} = 2 \,\Big\|\, \dfrac{5.2}{101}$

$\Rightarrow \underline{R_{of} = 0.0502\ k\Omega \Rightarrow 50.2\ \Omega}$

b. $h_{FE} = 150 \Rightarrow r_\pi = 7.8\ k\Omega,\ g_m = 19.23\ mA/V$

$$A_{vf} = \frac{\left(\dfrac{1}{7.8}+19.23\right)(2)}{1+\left(\dfrac{1}{7.8}+19.23\right)(2)} = \frac{(19.36)(2)}{1+(19.36)(2)}$$

$A_{vf} = 0.97482 \Rightarrow \underline{0.0082\%\ \text{change in}\ A_{vf}}$

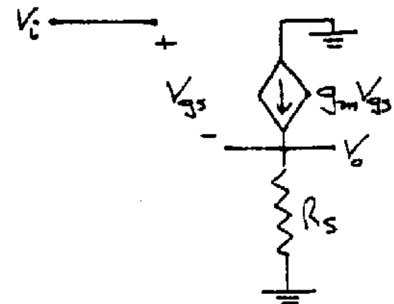
$R_{if} = 7.8 + (101)(2) = 209.8\ k\Omega$

$\Rightarrow \underline{1.25\%\ \text{change in}\ R_{if}}$

$R_{of} = R_E \,\Big\|\, \dfrac{r_\pi}{1+h_{FE}} = 2 \,\Big\|\, \dfrac{7.8}{151} = 2\|0.0517$

$R_{of} = 50.4\ \Omega \Rightarrow \underline{0.397\%\ \text{change in}\ R_{of}}$

E12.13




$V_0 = (g_mV_{gs})R_S$

$V_i = V_{gs} + g_mR_SV_{gs}$

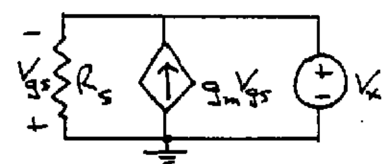
$V_{gs} = \dfrac{V_i}{1+g_mR_S}$

$g_m = 2\sqrt{K_nI_{DQ}} = 2\sqrt{(0.2)(0.25)} = 0.447\ mA/V$

$A_{vf} = \dfrac{V_0}{V_i} = \dfrac{g_mR_S}{1+g_mR_S}$

$A_{vf} = \dfrac{(0.447)(5)}{1+(0.447)(5)} \Rightarrow \underline{A_{vf} = 0.691}$

$\underline{R_{if} = \infty}$




$I_X = g_mV_{gs} = \dfrac{V_X}{R_S}$

$V_{gs} = -V_X$

$I_X = V_X\left(g_m + \dfrac{1}{R_S}\right)$

$R_{of} = \dfrac{1}{g_m} \,\Big\|\, R_S = \dfrac{1}{0.447} \,\Big\|\, 5$

$\underline{R_{of} = 1.55\ k\Omega}$

E12.15

$$i_o = \left(\frac{h_{FE}}{1+h_{FE}}\right)\left(\frac{R_E}{R_E + \dfrac{r_\pi}{1+h_{FE}}}\right)\cdot i_i$$

$r_\pi = \dfrac{(80)(0.026)}{0.5} = 4.16\ k\Omega$

Then

$\dfrac{r_\pi}{1+h_{FE}} = \dfrac{4.16}{81} = 0.0514\ k\Omega$

Then we want

$$\frac{i_o}{i_i} = 0.95 = \left(\frac{80}{81}\right)\left(\frac{R_E}{R_E + 0.0514}\right)$$

or

$$\left(\frac{R_E}{R_E + 0.0514}\right) = 0.9619$$

which yields

$$\underline{R_E(\min) = 1.30\ k\Omega}$$

and

$$V^+ = I_E R_E + 0.7 = \left(\frac{81}{80}\right)(0.5)(1.3) + 0.7 \Rightarrow$$

$$\underline{V^+(\min) = 1.36\ V}$$

E12.19

a. $$A_{gf} = \frac{h_{FE}\cdot A_g}{1+(h_{FE}A_g)R_E} = \frac{(200)(10^3)}{1+(200)(10^3)(10^3)}$$

$$\Rightarrow \underline{A_{gf} = 10^{-3}\text{ A/V} = 1\text{ mA/V}}$$

b. $$A_{gf} = \frac{(200)(10^4)}{1+(200)(10^4)(10^3)} \Rightarrow \underline{A_{gf} \cong 1\text{ mA/V}}$$

Percent change is negligible. $(4.5\times 10^{-7}\%)$

$$\frac{\dfrac{(200)(10^4)}{1+(200)(10^4)(10^3)} - \dfrac{(200)(10^3)}{1+(200)(10^3)(10^3)}}{10^{-3}}$$

$$= \frac{(200)(10^4)\left[1+(200)(10^3)(10^3)\right]}{(10^{-3})(2\times 10^9)(2\times 10^8)} - \frac{(200)(10^3)\left[1+(200)(10^4)(10^3)\right]}{(10^{-3})(2\times 10^9)(2\times 10^8)}$$

$$= \frac{200\times 10^4 - 200\times 10^3}{(10^{-3})(2\times 10^9)(2\times 10^8)} = 4.5\times 10^{-9}$$

E12.20

a. $$V_G = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5 = \left(\frac{20}{20+30}\right)(10) - 5$$

$V_G = -1$ V

$V_S = -1 - V_{GS}$

$$I_D = \frac{V_S - (-5)}{R_S} = K_n(V_{GS} - V_{TN})^2$$

$1 - V_{GS} + 5 = (1.5)(0.4)(V_{GS} - 2)^2$

$4 - V_{GS} = 0.6(V_{GS}^2 - 4V_{GS} + 4)$

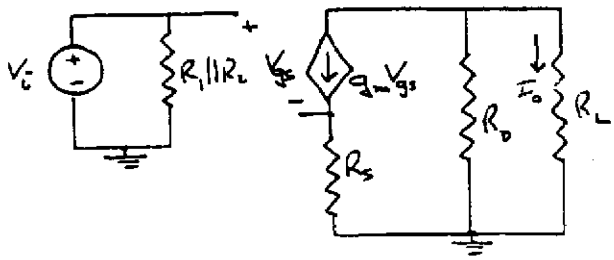
$0.6V_{GS}^2 - 1.4V_{GS} - 1.6 = 0$

$$V_{GS} = \frac{1.4 \pm \sqrt{(1.4)^2 + 4(0.6)(1.6)}}{2(0.6)}$$

$V_{GS} = 3.17$ V

$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1.5)(3.17 - 2)$

$g_m = 3.51$ mA/V




$$I_o = -\left(\frac{R_D}{R_D+R_L}\right)(g_m V_{gs}) = -\left(\frac{2}{2+2}\right)(3.51)V_{gs}$$

$I_o = -1.76V_{gs}$

$$V_i = V_{gs} + g_m V_{gs} R_S \Rightarrow V_{gs} = \frac{V_i}{1+g_m R_S}$$

$$V_{gs} = \frac{V_i}{1+(3.51)(0.4)} = (0.416)V_i$$

$I_o = -(1.76)(0.416)V_i$

$$\Rightarrow \underline{A_{gf} = \frac{I_o}{V_i} = -0.732\text{ mA/V}}$$

b. For $K_n = 1\ mA/V^2$

From dc analysis:

$4 - V_{GS} = (1)(0.4)(V_{GS} - 2)^2$

$4 - V_{GS} = 0.4(V_{GS}^2 - 4V_{GS} + 4)$

$0.4V_{GS}^2 - 0.6V_{GS} - 2.4 = 0$

$$V_{GS} = \frac{0.6 \pm \sqrt{(0.6)^2 + 4(0.4)(2.4)}}{2(0.4)}$$

$V_{GS} = 3.31$ V

$g_m = 2(1)(3.31 - 2) = 2.62$ mA/V

$$I_o = -\left(\frac{2}{2+2}\right)(2.62)V_{gs} = -1.31V_{gs}$$

$$V_{gs} = \frac{V_i}{1+(2.62)(0.4)} = V_i(0.488)$$

$I_o = -(1.31)(0.488)V_i$

$$A_{gf} = \frac{I_o}{V_i} = -0.639\text{ mA/V}$$

$$\%\text{ change} = \frac{0.732 - 0.639}{0.732} \Rightarrow \underline{12.7\%}$$

**E12.22**

dc analysis:

$$\frac{10-V_0}{4.7} = I_D + \frac{V_0}{47+20} \quad (1)$$

$$I_D = K_n(V_{GS} - V_{TN})^2 \quad (2)$$

$$V_{GS} = \left(\frac{20}{20+47}\right)V_0 = 0.2985V_0 \quad (3)$$

$$2.13 - V_0(0.213) = I_D + (0.0149)V_0 \quad (1)$$
$$I_D = 2.13 - V_0(0.2279)$$

From (2):

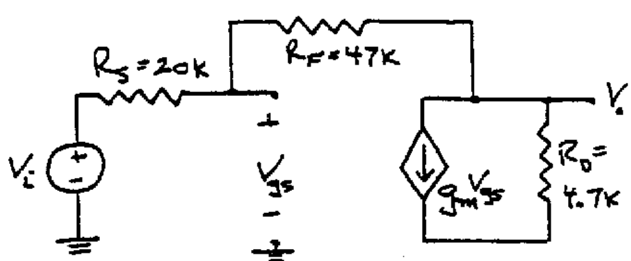
$$2.13 - V_0(0.2279) = 1[(0.2985V_0) - 1.5]^2$$
$$2.13 - V_0(0.2279) = 0.0891V_0^2 - 0.8955V_0 + 2.25$$
$$0.0891V_0^2 - 0.6676V_0 + 0.12 = 0$$

$$V_0 = \frac{0.6676 \pm \sqrt{(0.6676)^2 - 4(0.0891)(0.12)}}{2(0.0891)}$$
$$V_0 = 7.31 \text{ V}$$

$$I_D = \frac{10-7.31}{4.7} - \frac{7.31}{67} = 0.572 - 0.109$$
$$I_D = 0.463 \text{ mA}$$
$$V_{GS} = \sqrt{\frac{0.463}{1}} + 1.5 = 2.18$$

a. $g_m = 2K_n(V_{GS} - V_{TN}) = 2(1)(2.18 - 1.5)$

$$\Rightarrow g_m = 1.36 \text{ mA/V}$$




$$\frac{V_0}{R_D} + g_m V_{gs} + \frac{V_0 - V_{gs}}{R_F} = 0 \quad (1)$$

$$\frac{V_{gs} - V_i}{R_S} + \frac{V_{gs} - V_0}{R_F} = 0 \quad (2)$$

$$V_{gs}\left(\frac{1}{R_S} + \frac{1}{R_F}\right) = \frac{V_0}{R_F} + \frac{V_i}{R_S}$$
$$V_{gs}\left(\frac{1}{20} + \frac{1}{47}\right) = \frac{V_0}{47} + \frac{V_i}{20}$$

$$V_{gs}(0.0713) = V_0(0.0213) + V_i(0.050)$$
$$V_{gs} = V_0(0.299) + V_i(0.701)$$

From (1):

$$\frac{V_0}{4.7} + (1.36)V_{gs} + \frac{V_0}{47} - \frac{V_{gs}}{47} = 0$$
$$V_0(0.213) + (1.36)V_{gs} + V_0(0.0213) - V_{gs}(0.0213) = 0$$
$$V_0(0.234) + V_{gs}(1.34) = 0$$

$$V_0(0.234) + (1.34)[(V_0)(0.299) + (V_i)(0.701)] = 0$$
$$V_0(0.635) + V_i(0.939) = 0$$
$$\Rightarrow \underline{A_{vf} = \frac{V_0}{V_i} = -1.48}$$

b. For $K_n = 1.5\, mA/V^2$

From dc analysis:

$$2.13 - V_0(0.2279) = 1.5[(0.2985V_0) - 1.5]^2$$
$$= 1.5[0.0891V_0^2 - 0.8955V_0 + 2.25]$$
$$= 0.1337V_0^2 - 1.343V_0 + 3.375$$

$$0.1337V_0^2 - 1.115V_0 + 1.245 = 0$$
$$V_0 = \frac{1.115 \pm \sqrt{(1.115)^2 - 4(0.1337)(1.245)}}{2(0.1337)}$$
$$V_0 = \frac{1.115 \pm 0.7597}{2(0.1337)} \Rightarrow V_0 = 7.01 \text{ V}$$

$$I_D = \frac{10-7.01}{4.7} - \frac{7.01}{67} = 0.636 - 0.105$$
$$I_D = 0.531 \text{ mA}$$
$$V_{GS} = \sqrt{\frac{0.531}{1.5}} + 1.5 = 2.09$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1.5)(2.09 - 1.5)$$
$$= 1.77 \text{ mA/V}$$

From ac analysis:

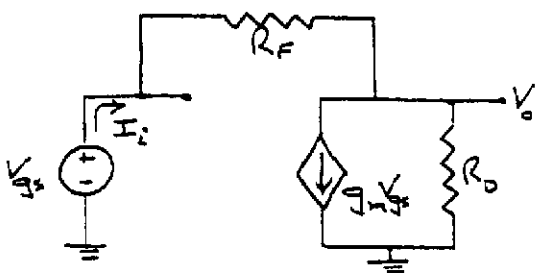
$$\frac{V_0}{4.7} + (1.77)V_{gs} + \frac{V_0}{47} - \frac{V_{gs}}{47} = 0$$
$$V_0(0.213) + (1.77)V_{gs} + V_0(0.0213) - V_{gs}(0.0213) = 0$$
$$V_0(0.234) + V_{gs}(1.75) = 0$$

$$V_0(0.234) + (1.75)[V_0(0.299) + V_i(0.701)] = 0$$
$$V_0(0.757) + V_i(1.23) = 0$$
$$\Rightarrow \underline{A_{vf} = \frac{V_0}{V_i} = -1.62}$$

$$\% \text{ change} = \frac{1.62 - 1.48}{1.48} \Rightarrow \underline{9.46\%}$$

**E12.23**

a. Input resistance.




$$\frac{V_0}{R_D} + g_m V_{gs} + \frac{V_0 - V_{gs}}{R_F} = 0 \qquad (1)$$

$$I_i = \frac{V_{gs} - V_0}{R_F} \qquad (2)$$

So $V_0 = V_{gs} - I_i R_F$

$$V_0\left(\frac{1}{R_D} + \frac{1}{R_F}\right) + V_{gs}\left(g_m - \frac{1}{R_F}\right) = 0 \qquad (1)$$

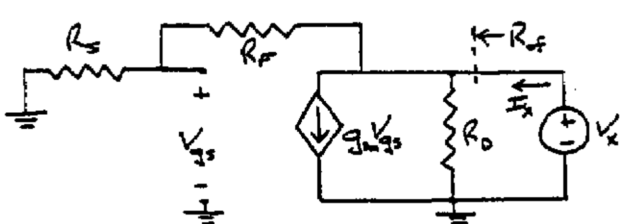
$$[V_{gs} - I_i(47)]\left(\frac{1}{4.7} + \frac{1}{47}\right) + V_{gs}\left(1.36 - \frac{1}{47}\right) = 0$$

$$[V_{gs} - I_i(47)](0.234) + V_{gs}(1.34) = 0$$

$$V_{gs}(1.57) = I_i(11.0)$$

$$\Rightarrow \underline{R_{if} = \frac{V_{gs}}{I_i} = 7.0\ \text{k}\Omega}$$

Output Resistance.




$$I_X = \frac{V_X}{R_D} + g_m V_{gs} + \frac{V_X}{R_S + R_F}$$

$$V_{gs} = \left(\frac{R_S}{R_S + R_F}\right)V_X = \left(\frac{20}{20 + 47}\right)V_X = 0.2985 V_X$$

$$I_X = \frac{V_X}{4.7} + (1.36)(0.2985)V_X + \frac{V_X}{20 + 47}$$

$$I_X = V_X[0.213 + 0.406 + 0.0149]$$

$$R_{of} = \frac{V_X}{I_X} = 1.58\ \text{k}\Omega$$

b. From part (a)

$$[V_{gs} - I_i(47)](0.234) + V_{gs}\left(1.77 - \frac{1}{47}\right) = 0$$

$$V_{gs}(1.98) = I_i(11)$$

$$\Rightarrow \underline{R_{if} = \frac{V_{gs}}{I_i} = 5.56\ \text{k}\Omega}$$

$$I_X = \frac{V_X}{4.7} + (1.77)(0.2985)V_X + \frac{V_X}{20 + 47}$$

$$I_X = V_X[0.213 + 0.528 + 0.0149]$$

$$\Rightarrow \underline{R_{of} = \frac{V_X}{I_X} = 1.32\ \text{k}\Omega}$$

**E12.25**

$$V_{TH} = \left(\frac{5.5}{5.5 + 51}\right)(10) = 0.973\ \text{V}$$

$$R_{TH} = 5.5 \| 51 = 4.96\ \text{k}\Omega$$

$$I_{BQ} = \frac{0.973 - 0.7}{4.96 + (121)(1)} = 0.00217\ \text{mA}$$

$$I_{CQ} = 0.260\ \text{mA}$$

$$r_\pi = 12\ \text{k}\Omega,\ g_m = 10\ \text{mA/V}$$

$$R_{eq} = R_S \| R_1 \| R_2 \| r_\pi = 10{,}000 \| 51 \| 5.5 \| 12 = 3.51\ \text{k}\Omega$$

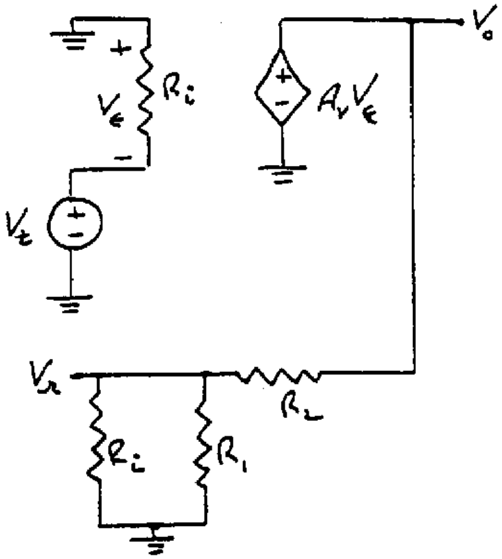
From Equation (12.99(b)):

$$T = (g_m R_C)\left(\frac{R_{eq}}{R_C + R_F + R_{eq}}\right)$$

$$= (10)(10)\left(\frac{3.51}{10 + 82 + 3.51}\right)$$

$$\Rightarrow \underline{T = 3.68}$$

**E12.26**




$$V_\epsilon = -V_t,\ V_0 = A_v V_\epsilon = -A_v V_t$$

$$V_r = \left(\frac{R_1\|R_i}{R_1\|R_i + R_2}\right)V_0 = -\left(\frac{R_1\|R_i}{R_1\|R_i + R_2}\right)(A_v V_t)$$

$$\underline{T = -\frac{V_r}{V_t} = A_v\left(\frac{R_1\|R_i}{R_1\|R_i + R_2}\right)}$$

or

$$\underline{T = \frac{A_v}{1 + \dfrac{R_2}{R_1\|R_i}}}$$

**E12.27**

$$T = A_i\beta = \frac{A_{i0}\beta}{\left(1 + j\cdot\dfrac{f}{f_1}\right)} = \frac{(10^5)(0.01)}{1 + j\cdot\left(\dfrac{f}{10}\right)}$$

$$|T(f_1)| = 1 = \frac{10^3}{\sqrt{1 + \left(\dfrac{f'_E}{10}\right)^2}}$$

$$1 + \left(\frac{f'_E}{10}\right)^2 = 10^6$$

$$f'_E = 10\sqrt{10^6 - 1} \Rightarrow \underline{f'_E \approx 10^4\ \text{Hz}}$$

$$\text{Phase} = \phi = -\tan^{-1}\left(\frac{f'_E}{10}\right) = -\tan^{-1}\left(\frac{10^4}{10}\right)$$
$$= -\tan^{-1}(10^3)$$

$$\phi \approx 90^\circ$$

Phase Margin $= 180 - 90 \Rightarrow$ Phase Margin $= 90^\circ$

**E12.28**

$$T = A_i\beta = \frac{A_{i0}\beta}{\left(1 + j\cdot\dfrac{f}{f_1}\right)\left(1 + j\cdot\dfrac{f}{f_2}\right)}$$

$$\text{Phase} = -\left[\tan^{-1}\left(\frac{f}{f_1}\right) + \tan^{-1}\left(\frac{f}{f_2}\right)\right]$$

Phase Margin $= 60^\circ \Rightarrow$ Phase $= -120^\circ$

$$-120^\circ = -\left[\tan^{-1}\left(\frac{f}{10^4}\right) + \tan^{-1}\left(\frac{f}{10^5}\right)\right]$$

At $f' = 7.66 \times 10^4$ Hz,

$$\text{Phase} = -\left[\tan^{-1}(7.66) + \tan^{-1}(0.766)\right]$$
$$= -[82.56 + 37.45]$$
$$= -120^\circ$$

$$|T(f')| = 1 = \frac{(10^5)\beta}{\sqrt{1 + (7.66)^2} \times \sqrt{1 + (0.766)^2}}$$

$$1 = \frac{(10^5)\beta}{(7.725)(1.26)} \Rightarrow \underline{\beta = 9.73 \times 10^{-5}}$$

**E12.29**

$$\text{Phase} = -180^\circ = -3\tan^{-1}\left(\frac{f'}{10^5}\right)$$

or $\tan^{-1}\left(\dfrac{f'}{10^5}\right) = 60^\circ \Rightarrow f' = 1.732 \times 10^5$ Hz

$$|T(f')| = 1 = \frac{\beta(100)}{\left[\sqrt{1 + \left(\dfrac{f'}{10^5}\right)^2}\right]^3}$$

$$= \frac{\beta(100)}{\left[\sqrt{1 + (1.732)^2}\right]^3}$$

$$\Rightarrow \underline{\beta = 0.08}$$

**E12.30**

Phase Margin $= 60^\circ \Rightarrow$ Phase $= -120^\circ$

$$\text{Phase} = -120^\circ = -3\tan^{-1}\left(\frac{f'}{10^5}\right)$$

$$\tan^{-1}\left(\frac{f'}{10^5}\right) = 40^\circ \Rightarrow f' = 0.839 \times 10^5\ \text{Hz}$$

$$|T(f')| = 1 = \frac{\beta(100)}{\left[\sqrt{1 + \left(\dfrac{f'}{10^5}\right)^2}\right]^3}$$

$$= \frac{\beta(100)}{\left[\sqrt{1 + (0.839)^2}\right]^3}$$

$$\Rightarrow \underline{\beta = 0.0222}$$

**E12.31**

The new loop gain function is

$$T'(f) = \frac{10^5}{\left(1 + j \cdot \frac{f}{f_{PD}}\right)\left(1 + j \cdot \frac{f}{5 \times 10^5}\right)} \times \frac{1}{\left(1 + j \cdot \frac{f}{10^7}\right)\left(1 + j \cdot \frac{f}{5 \times 10^8}\right)}$$

$$\text{Phase} = -\left\{\tan^{-1}\left(\frac{f}{f_{PD}}\right) + \tan^{-1}\left(\frac{f}{5 \times 10^5}\right) + \tan^{-1}\left(\frac{f}{10^7}\right) + \tan^{-1}\left(\frac{f}{5 \times 10^8}\right)\right\}$$

For a phase margin $45^\circ \Rightarrow \text{Phase} = -135^\circ$, the poles are far apart so this will occur at approximately $f' = 5 \times 10^5$ Hz. Then

$$|T'(f)| = 1 = \frac{10^5}{\sqrt{1 + \left(\frac{5 \times 10^5}{f_{PD}}\right)^2} \times \sqrt{1+1} \times \sqrt{1} \times \sqrt{1}}$$

$$1 = \frac{10^5}{(1.414)\sqrt{1 + \left(\frac{5 \times 10^5}{f_{PD}}\right)^2}}$$

$$1 + \left(\frac{5 \times 10^5}{f_{PD}}\right)^2 = 5 \times 10^9$$

$$f_{PD} \cong \frac{5 \times 10^5}{\sqrt{5 \times 10^9}} \Rightarrow \underline{f_{PD} = 7.07 \text{ Hz}}$$

**E12.32**

Phase Margin $= 45^\circ \Rightarrow \text{Phase} = -135^\circ$

This will occur at approximately $f' = 10^7$ Hz

$$|T'(f)| = 1 = \frac{10^5}{\sqrt{1 + \left(\frac{10^7}{f_{PD}}\right)^2} \times \sqrt{2} \times \sqrt{1}}$$

$$1 + \left(\frac{10^7}{f_{PD}}\right)^2 = 5 \times 10^9$$

$$f_{PD} \cong \frac{10^7}{\sqrt{5 \times 10^9}} \Rightarrow \underline{f_{PD} = 141 \text{ Hz}}$$

**E12.33**

$$A_f(0) = \frac{A_0}{1 + \beta A_0} = \frac{2 \times 10^5}{1 + (0.05)(2 \times 10^5)}$$

$$\Rightarrow \underline{A_f(0) \cong 20}$$

$$f_C = f_{PD}(1 + \beta A_0) = 100\left[1 + (0.05)(2 \times 10^5)\right]$$

$$\Rightarrow \underline{f_C \cong 1 \text{ MHz}}$$

# Chapter 12

## Problem Solutions

12.1

a. $A_f = \dfrac{A}{1 + A\beta}$

$80 = \dfrac{10^5}{1+(10^5)\beta} \Rightarrow 1 + (10^5)\beta = \dfrac{10^5}{80}$

$\Rightarrow \beta = \dfrac{\frac{10^5}{80} - 1}{10^5} \Rightarrow \underline{\beta = 0.01249}$

b. $\dfrac{dA_f}{A_f} = \left(\dfrac{A_f}{A}\right) \cdot \dfrac{dA}{A} = \dfrac{80}{10^5}(-20)$

or

$\underline{\dfrac{dA_f}{A_f} = -0.016\%}$

$A_f = 80 - (0.00016)80 \Rightarrow \underline{A_f \cong 79.99}$

c. $80 = \dfrac{10^3}{1+(10^3)\beta}$

$\beta = \dfrac{\frac{10^3}{80} - 1}{10^3} \Rightarrow \underline{\beta = 0.0115}$

$\dfrac{dA_f}{A_f} = \left(\dfrac{80}{10^3}\right)(-20) \Rightarrow \underline{\dfrac{dA_f}{A_f} = -1.6\%}$

$A_f = 80 - (0.016)(80) \Rightarrow \underline{A_f = 78.72}$

12.2

a. $A_f = \dfrac{(A)^3}{1+(A)^3\beta}$

$100 = \dfrac{1000}{1+(1000)\beta} \Rightarrow \beta = \dfrac{\frac{1000}{100} - 1}{1000} \Rightarrow \underline{\beta = 0.009}$

b. $A$ goes from 10 to 11 so

$A_f = \dfrac{(11)^3}{1+(11)^3(0.009)} = \dfrac{1331}{1+(1331)(0.009)}$

or

$A_f = 102.55$

so

$\dfrac{\Delta A_f}{A_f} = \dfrac{2.55}{100} \Rightarrow \underline{2.55\% \text{ change}}$

12.3

(a) $V_O = (-10)(-15)(-20)V_\epsilon = -3000V_\epsilon$

$V_\epsilon = \beta V_O + V_S$

So $V_O = -3000(\beta V_O + V_S)$

We find

$A_{vf} = \dfrac{V_O}{V_S} = \dfrac{-3000}{1+3000\beta}$

For $A_{vf} = -120 = \dfrac{-3000}{1+3000\beta} \Rightarrow \underline{\beta = 0.008}$

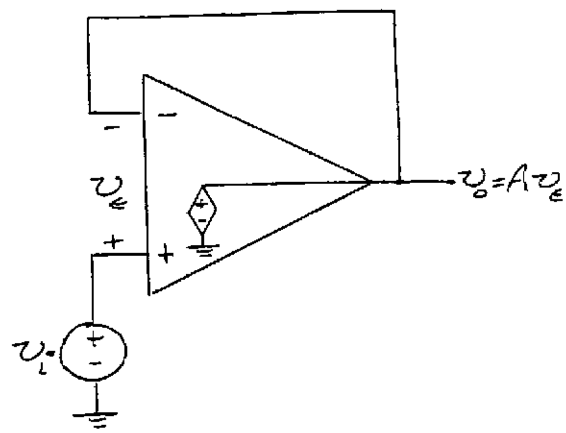
(b) Now $V_O = (-9)(-13.5)(-18)V_\epsilon = -2187V_\epsilon$

Then

$A_{vf} = \dfrac{-2187}{1+2187\beta} = \dfrac{-2187}{1+2187(0.008)} = -118.24$

$\% \text{ change} = \dfrac{120 - 118.24}{120} \text{x}100 \Rightarrow \underline{1.47\% \text{ change}}$

12.4




$v_o = v_i - v_\epsilon \Rightarrow v_\epsilon = v_i - v_o$

Then $v_o = A(v_i - v_o) = Av_i - Av_o$

And

$v_o(1+A) = Av_i$

so

$\dfrac{v_o}{v_i} = \dfrac{A}{1+A} = 0.9998 \Rightarrow \underline{A = 4999}$

12.5

$(10^5)(4) = (50)f_B \Rightarrow \underline{f_B = 8\ kHz}$

12.6

(a) $(50)f_{3-dB} = (10^5)(4) \Rightarrow \underline{f_{3-dB} = 8\ kHz}$

(b) $(10)f_{3-dB} = (10^5)(4) \Rightarrow \underline{f_{3-dB} = 40\ kHz}$

12.7

$(50)(20 \times 10^3) = 5A_0$ so $\underline{A_0 = 2 \times 10^5}$

12.8

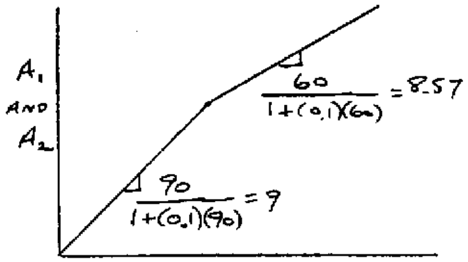
$v_0 = A_1 A_2 v_i + A_1 v_n$

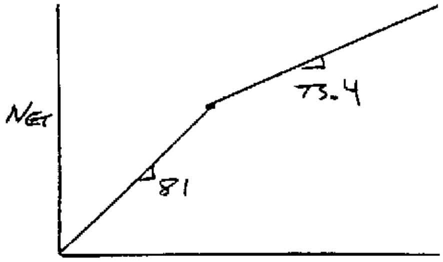
$v_0 = (100)v_i + (1)v_n = (100)(10) + (1)(1)$

$\Rightarrow \underline{\dfrac{S_0}{N_0} = \dfrac{1000}{1} = 1000}$

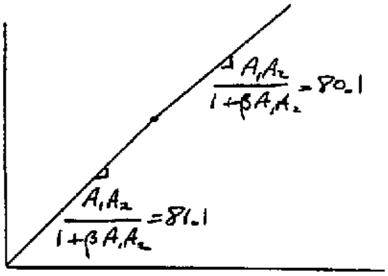
12.9

(a)




(b)




Circuit (b) – less distortion

12.10

(a) Low input $R \Rightarrow$ Shunt input
Low output $R \Rightarrow$ Shunt output
Or a Shunt-Shunt circuit

(b) High input $R \Rightarrow$ Series input
High output $R \Rightarrow$ Series output
Or a Series-Series circuit

(c) Shunt-Series circuit

(d) Series-Shunt circuit

12.11

(a) $R_i(\max) = R_i(1+T) = 10(1+10^4) \Rightarrow$
$\underline{R_i(\max) \cong 10^5 \ k\Omega}$

$R_i(\min) = \dfrac{R_i}{1+T} = \dfrac{10}{1+10^4} \cong 10^{-3} \ k\Omega$
Or $\underline{R_i(\min) = 1 \ \Omega}$

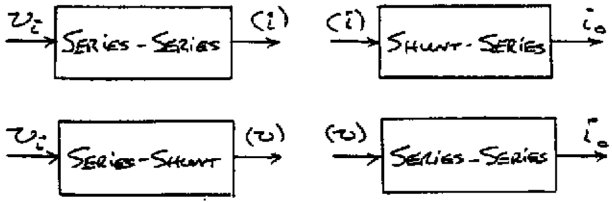
(b) $R_o(\max) = R_o(1+T) = 1(1+10^4) \Rightarrow$
$\underline{R_o(\max) \cong 10^4 \ k\Omega}$

$R_o(\min) = \dfrac{R_o}{1+T} = \dfrac{1}{1+10^4} \cong 10^{-4} \ k\Omega$
Or $\underline{R_o(\min) = 0.1 \ \Omega}$

12.12

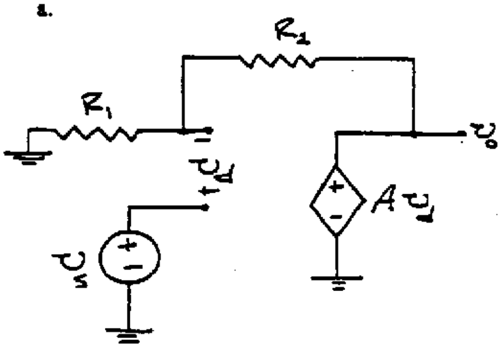
Overall Transconductance Amplifier, $A_g = \dfrac{i_o}{v_i}$

Series output = current signal and Shunt input = current signal. Also, Shunt output = voltage signal and Series input = voltage signal. Two possible solutions are shown.




12.13

a.




$$\frac{v_S - v_d}{R_1} = \frac{v_O - (v_S - v_d)}{R_2} \text{ and } v_d = \frac{v_O}{A}$$

$$\frac{v_S}{R_1} + \frac{v_S}{R_2} = \frac{v_O}{R_2} + v_d\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{v_O}{R_2} + \frac{v_O}{A}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$v_S\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{v_0}{R_2}\left[1+\frac{1}{A}\left(1+\frac{R_2}{R_1}\right)\right]$$

$$\frac{v_0}{v_S} = \frac{\left(1+\frac{R_2}{R_1}\right)}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}\right)}$$

which can be written as

$$A_{vf} = \frac{v_0}{v_S} = \frac{A}{1+\left[A/\left(1+\frac{R_2}{R_1}\right)\right]}$$

b. $\beta = \dfrac{1}{1+\dfrac{R_2}{R_1}}$

c. $20 = \dfrac{10^5}{1+(10^5)\beta}$

So $\beta = \dfrac{\dfrac{10^5}{20}-1}{10^5} \Rightarrow \underline{\beta = 0.04999}$

Then $\dfrac{R_2}{R_1} = \dfrac{1}{\beta} - 1 = \dfrac{1}{0.04999} - 1$

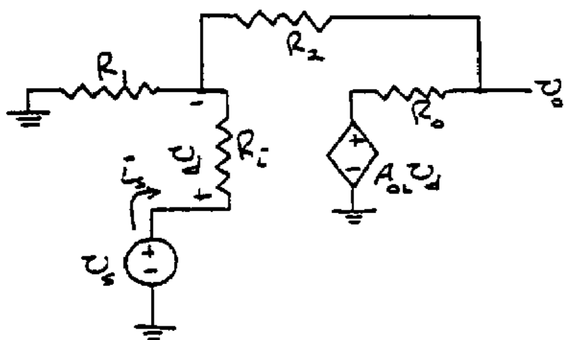
$$\Rightarrow \underline{\frac{R_2}{R_1} = 19.004}$$

d. $A \to 8 \times 10^4$

$$A_f = \frac{8\times 10^4}{1+(8\times 10^4)(0.04999)} = 19.999$$

$$\frac{\Delta A_f}{A_f} = -\frac{0.001}{20} \Rightarrow \underline{\frac{\Delta A_f}{A_f} = -0.005\%}$$

12.14




$$A_{vf} \approx \left(1+\frac{R_2}{R_1}\right) = 20 \Rightarrow \frac{R_2}{R_1} = 19$$

$v_d = i_S R_i$

$$i_S = \frac{v_S - v_d}{R_1} + \frac{(v_S - v_d) - v_0}{R_2} \qquad (1)$$

$$\frac{v_0 - A_{0L}v_d}{R_0} + \frac{v_0 - (v_S - v_d)}{R_2} = 0 \qquad (2)$$

$$v_0\left(\frac{1}{R_0}+\frac{1}{R_2}\right) = \frac{A_{0L}v_d}{R_0} + \frac{(v_S - v_d)}{R_2}$$

$$v_0 = \frac{\dfrac{A_{0L}v_d}{R_0} + \dfrac{(v_S-v_d)}{R_2}}{\left(\dfrac{1}{R_0}+\dfrac{1}{R_2}\right)}$$

From (1):

$$i_S = \frac{v_S - v_d}{R_1} + \frac{v_S - v_d}{R_2} - \frac{\dfrac{1}{R_2}\cdot\left[\dfrac{A_{0L}v_d}{R_0} + \dfrac{(v_S - v_d)}{R_2}\right]}{\left(\dfrac{1}{R_0}+\dfrac{1}{R_2}\right)}$$

$$i_S = v_S\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{\dfrac{1}{R_2}}{1+\dfrac{R_2}{R_0}}\right) - v_d\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{\dfrac{A_{0L}}{R_0} - \dfrac{1}{R_2}}{1+\dfrac{R_2}{R_0}}\right)$$

$v_d = i_S R_i$

$$i_S\left\{1 + \frac{R_i\left[\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)\left(1+\dfrac{R_2}{R_0}\right) + \dfrac{A_{0L}}{R_0} - \dfrac{1}{R_2}\right]}{1+\dfrac{R_2}{R_0}}\right\} = v_S\left[\frac{\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)\left(1+\dfrac{R_2}{R_0}\right) - \dfrac{1}{R_2}}{1+\dfrac{R_2}{R_0}}\right]$$

$$i_S\left\{1+\frac{R_2}{R_0} + R_i\left[\frac{1}{R_1}+\frac{R_2}{R_1}\cdot\frac{1}{R_0}+\frac{1}{R_0}+\frac{A_{0L}}{R_0}\right]\right\} = v_S\left[\frac{1}{R_1}+\frac{R_2}{R_1}\cdot\frac{1}{R_0}+\frac{1}{R_0}\right]$$

$$i_S\left\{R_0 + R_2 + R_i\left[\frac{R_0}{R_1} + \left(1+\frac{R_2}{R_1}\right) + A_{0L}\right]\right\} = v_S\left[\frac{R_0}{R_1} + \left(1+\frac{R_2}{R_1}\right)\right] \qquad (1)$$

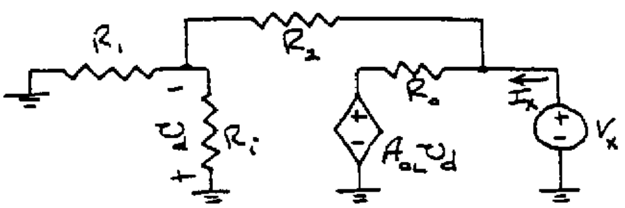
Let $R_2 = 190\ \text{k}\Omega,\ R_i = 10\ \text{k}\Omega$

$$i_S\left\{0.1 + 190 + 100\cdot\left[\frac{0.1}{10} + 20 + 10^5\right]\right\} = v_S\left[\frac{0.1}{10}+20\right]$$

$$i_S(1.000219\times 10^7) = v_S(20.01)$$

$$R_{if} = \frac{v_S}{i_S} \cong 5\times 10^5\ \text{k}\Omega \Rightarrow \underline{R_{if} \cong 500\ \text{M}\Omega}$$

Output Resistance




$$I_X = \frac{V_X - A_{0L}\nu_d}{R_0} + \frac{V_X}{R_2 + R_1 \| R_i}$$

$$\nu_d = \frac{-R_1\|R_i}{R_1\|R_i + R_2} \cdot V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_{0f}} = \frac{1}{R_0} + \frac{A_{0L} \cdot R_1\|R_i}{R_0(R_1\|R_i + R_2)} + \frac{1}{R_2 + R_1\|R_i}$$

$$R_1\|R_i = 10\|100 = 9.09$$

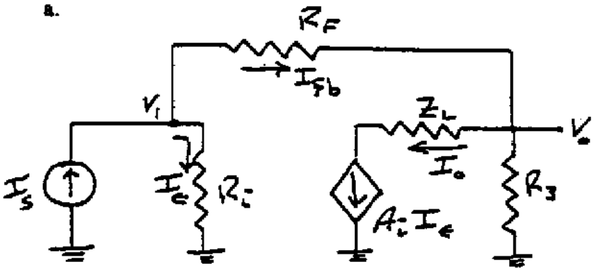
$$\frac{1}{R_{0f}} = \frac{1}{0.1} + \frac{10^5}{0.1} \cdot \left(\frac{9.09}{9.09 + 190}\right) + \frac{1}{190 + 9.09}$$

$$= 10 + 4.566 \times 10^4 + 0.00502$$

$$R_{0f} = 2.19 \times 10^{-5}\ \text{k}\Omega \Rightarrow \underline{R_{0f} = 0.0219\ \Omega}$$

12.15

a.




Assume that $V_1$ is at virtual ground.

$$V_0 = -I_{fb}R_F$$

Now

$$I_{fb} = I_0 + \frac{V_0}{R_3} = I_0 - \frac{I_{fb}R_F}{R_3}$$

$$I_{fb} = I_S - I_e$$

and

$$I_0 = A_iI_e \Rightarrow I_e = \frac{I_0}{A_i}$$

so

$$I_{fb} = I_S - \frac{I_0}{A_i}$$

From above

$$I_{fb}\left(1 + \frac{R_F}{R_3}\right) = I_0$$

$$\left(I_S - \frac{I_0}{A_i}\right)\left(1 + \frac{R_F}{R_3}\right) = I_0$$

$$I_S\left(1 + \frac{R_F}{R_3}\right) = I_0\left[1 + \frac{1}{A_i}\left(1 + \frac{R_F}{R_3}\right)\right]$$

or

$$A_{if} = \frac{I_0}{I_S} = \frac{\left(1 + \frac{R_F}{R_3}\right)}{\left[1 + \frac{1}{A_i}\left(1 + \frac{R_F}{R_3}\right)\right]}$$

$$= \underline{\frac{A_i}{1 + \frac{A_i}{\left(1 + \frac{R_F}{R_3}\right)}} = A_{if}}$$

b. $\beta_i = \dfrac{1}{\left(1 + \dfrac{R_F}{R_3}\right)}$

c. $25 = \dfrac{10^5}{1 + (10^5)\beta_i}$

$$\text{so } \beta_i = \frac{\frac{10^5}{25} - 1}{10^5} \Rightarrow \underline{\beta_i = 0.03999}$$

$$\text{so } \frac{R_F}{R_3} = \frac{1}{\beta_i} - 1 = \frac{1}{0.03999} - 1 \Rightarrow \underline{\frac{R_F}{R_3} = 24.0}$$

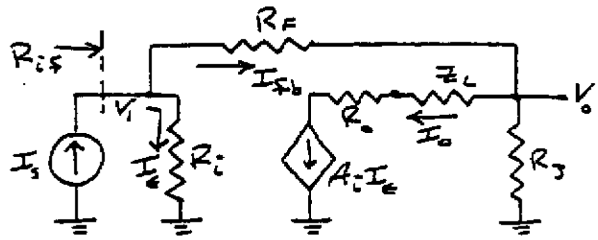
d. $A_i = 10^5 - (0.15)(10^5) = 8.5 \times 10^4$

$$\text{so } A_{if} = \frac{8.5 \times 10^4}{1 + (8.5 \times 10^4)(0.03999)} = 24.9989$$

$$\text{so } \frac{\Delta A_{if}}{A_{if}} = -\frac{1.10 \times 10^{-3}}{25} = -4.41 \times 10^{-5}$$

$$\Rightarrow \underline{-4.41 \times 10^{-3}\%}$$

12.16




$$I_S = I_e + I_{fb},\quad V_1 = I_eR_i$$

$$I_{fb} = I_0 + \frac{V_0}{R_3} \text{ and } V_0 = V_1 - I_{fb}R_F$$

$$I_0 = A_iI_e \Rightarrow I_e = \frac{I_0}{A_i}$$

Now

$$I_{fb} = A_i I_e + \frac{1}{R_3}(V_1 - I_{fb}R_F)$$

$$I_{fb}\left[1 + \frac{R_F}{R_3}\right] = A_i I_e + \frac{V_1}{R_3}$$

$$I_{fb} = I_S - I_e$$

$$(I_S - I_e)\left[1 + \frac{R_F}{R_3}\right] = A_i I_e + \frac{V_1}{R_3}$$

$$I_S\left[1 + \frac{R_F}{R_3}\right] = I_e\left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{V_1}{R_3}$$

$$I_e = \frac{V_1}{R_i}$$

$$I_S\left[1 + \frac{R_F}{R_3}\right] = V_1\left\{\frac{1}{R_i}\cdot\left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{1}{R_3}\right\}$$

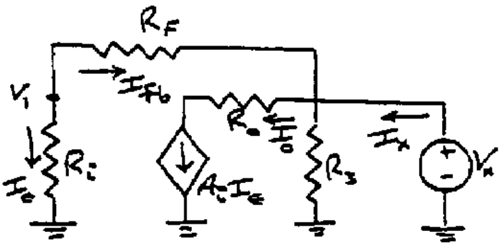
$$R_{if} = \frac{V_1}{I_S} = \frac{\left(1 + \frac{R_F}{R_3}\right)}{\left\{\frac{1}{R_i}\cdot\left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{1}{R_3}\right\}}$$

The $1/R_3$ term in the denominator will be negligible.
Using the results of Problem 12.15:

$$R_{if} = \frac{25}{\left\{\frac{1}{2}[(25) + 10^5]\right\}}$$

$R_{if} \cong 5 \times 10^{-4}$ k$\Omega \Rightarrow \underline{R_{if} = 0.5\ \Omega}$

Output Resistance (Let $Z_L = 0$)




$$I_X = \frac{V_X}{R_3} + A_i I_e + \frac{V_X}{R_F + R_i}$$

$$I_e = \frac{V_X}{R_F + R_i}$$

so

$$\frac{I_X}{V_X} = \frac{1}{R_{of}} = \frac{1}{R_3} + \frac{A_i + 1}{R_F + R_i},\ \frac{R_F}{R_3} = 24$$

Let $R_F = 240$ k$\Omega$, $R_3 = 10$ k$\Omega$

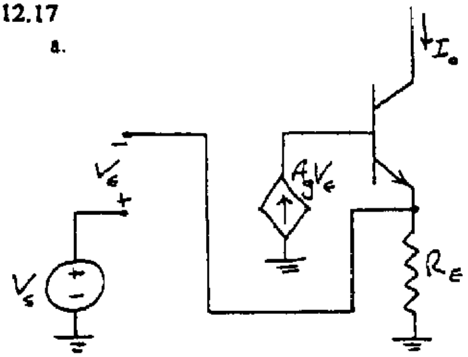
$$\frac{1}{R_{of}} = \frac{1}{10} + \frac{10^5 + 1}{240 + 2}$$

so $R_{of} \approx \dfrac{R_F + R_i}{A_i + 1} = \dfrac{240 + 2}{10^5 + 1}$

$\Rightarrow R_{of} \approx 2.42 \times 10^{-3}$ k$\Omega$ or $\underline{R_{of} \approx 2.42\ \Omega}$

**12.17**

a.




$$I_E = \frac{(1 + h_{FE})}{h_{FE}}\cdot I_0 = \frac{V_S - V_e}{R_E}$$

Also $I_0 = h_{FE}(A_g V_e)$ so $V_e = \dfrac{I_0}{h_{FE}A_g}$

Then

$$\frac{1 + h_{FE}}{h_{FE}}\cdot I_0 = \frac{V_S}{R_E} - \frac{I_0}{h_{FE}A_g R_E}$$

$$\left[\frac{1 + h_{FE}}{h_{FE}} + \frac{1}{h_{FE}A_g R_E}\right]I_0 = \frac{V_S}{R_E}$$

$$\left[\frac{A_g(1 + h_{FE})R_E + 1}{h_{FE}A_g R_E}\right]I_0 = \frac{V_S}{R_E}$$

$$\frac{I_0}{V_S} = \frac{1}{R_E}\cdot\left[\frac{h_{FE}A_g R_E}{1 + A_g(1 + h_{FE})R_E}\right]$$

$$\Rightarrow \underline{\frac{I_0}{V_S} \approx \frac{h_{FE}A_g}{1 + (h_{FE}A_g)R_E}}$$

b. $\underline{\beta_z = R_E}$

c. $10 = \dfrac{5 \times 10^5}{1 + (5 \times 10^5)\beta_z}$

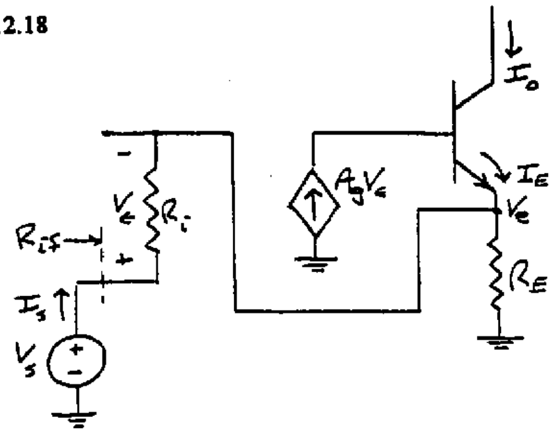
$$\beta_z = \frac{\frac{5 \times 10^5}{10} - 1}{5 \times 10^5} \Rightarrow \underline{\beta_z = R_E = 0.099998\ \text{k}\Omega}$$

d. If $A_g \to 5.5 \times 10^5$ then

$$A_{gf} = \frac{5.5 \times 10^5}{1 + (5.5 \times 10^5)(0.099998)} = 10.0000182$$

$$\frac{\Delta A_{gf}}{A_{gf}} = \frac{1.82 \times 10^{-5}}{10} \Rightarrow \underline{1.82 \times 10^{-4}\%}$$

12.18




$I_E = (1+h_{FE})A_g V_\epsilon,\ I_E = \frac{V_e}{R_E} - I_S$ and $V_\epsilon = I_S R_i,$

$V_e = V_S - V_\epsilon = V_S - I_S R_i$

Now $(1+h_{FE})A_g I_S R_i = \frac{1}{R_E}\cdot(V_S - I_S R_i) - I_S$

$$\left[(1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1\right] I_S = \frac{V_S}{R_E}$$

$$R_{if} = \frac{V_S}{I_S} = R_E\left[(1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1\right]$$

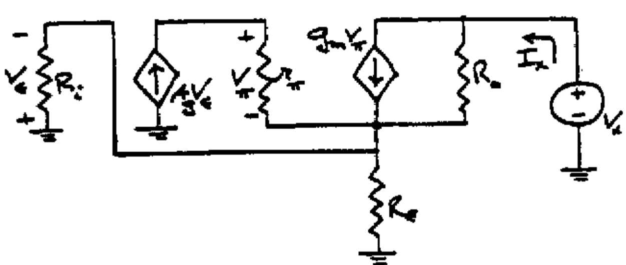
From Problem 12.16:

$(1+h_{FE})A_g \approx h_{FE}A_g = 5\times 10^5$ mS

$R_E \approx 0.1$ kΩ

so $R_{if} = (0.1)\left[(5\times 10^5)(20) + \frac{20}{0.1} + 1\right]$

or $\underline{R_{if} = 10^6\ \text{k}\Omega}$




$$\frac{V_\pi}{r_\pi} = A_g V_\epsilon$$

$$I_X = g_m V_\pi + \frac{V_X - (-V_\epsilon)}{R_0} \quad (1)$$

$$V_\epsilon = -(I_X + A_g V_\epsilon)(R_E\|R_i) \quad (2)$$

or $V_\epsilon = [1 + A_g(R_E\|R_i)] = -I_X(R_E\|R_i)$

Now

$$I_X = g_m A_g r_\pi V_\epsilon + \frac{V_X}{R_0} + \frac{V_\epsilon}{R_0} \quad (1)$$

$$I_X = \left(g_m A_g r_\pi + \frac{1}{R_0}\right)\left[\frac{-I_X(R_E\|R_i)}{1 + A_g(R_E\|R_i)}\right] + \frac{V_X}{R_0}$$

$$R_{0f} = \frac{V_X}{I_X}$$

$$= R_0\left\{1 + \left(g_m A_g r_\pi + \frac{1}{R_0}\right)\left[\frac{(R_E\|R_i)}{1 + A_g(R_E\|R_i)}\right]\right\}$$

$g_m r_\pi A_g = h_{FE}A_g = 5\times 10^5$ mS

Let $h_{FE} = 100$ so $A_g = 5\times 10^3$ mS

$R_E\|R_i = 0.1\|20 \approx 0.1$ kΩ

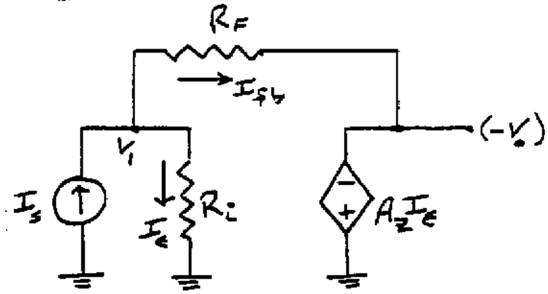
Then

$$R_{0f} = 50\left\{1 + \left(5\times 10^5 + \frac{1}{50}\right)\left[\frac{0.1}{1 + (5\times 10^3)(0.1)}\right]\right\}$$

or $\underline{R_{0f} = 5.04\ \text{M}\Omega}$

12.19

a.




Assuming $V_1$ is at virtual ground

$(-V_0) = -I_{fb}R_F$ and $(-V_0) = -A_z I_\epsilon \Rightarrow I_\epsilon = \frac{V_0}{A_z}$

$I_{fb} = I_S - I_\epsilon$

So $V_0 = (I_S - I_\epsilon)R_F = I_S R_F - \left(\frac{V_0}{A_z}\right)R_F$

$$V_0\left[1 + \frac{R_F}{A_z}\right] = I_S R_F$$

$$\text{so } A_{zf} = \frac{V_0}{I_S} = \frac{R_F}{\left[1 + \frac{R_F}{A_z}\right]} = \frac{A_z R_F}{A_z + R_F}$$

$$\text{or } A_{zf} = \frac{A_z}{1 + A_z\left(\frac{1}{R_F}\right)} = \frac{A_z}{1 + A_z\beta_g}$$

b. $\beta_g = \frac{1}{R_F}$

c. $5 \times 10^4 = \dfrac{5 \times 10^6}{1 + (5 \times 10^6)\beta_g}$

$$\beta_g = \frac{\dfrac{5 \times 10^6}{5 \times 10^4} - 1}{5 \times 10^6} \Rightarrow \underline{\beta_g = 1.98 \times 10^{-5}}$$

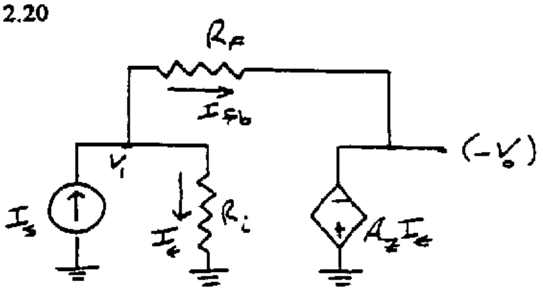
$$R_F = \frac{1}{\beta_g} \Rightarrow \underline{R_F = 50.5 \text{ k}\Omega}$$

d. $A_z = (0.9)(5 \times 10^6) = 4.5 \times 10^6$

$$A_{zf} = \frac{4.5 \times 10^6}{1 + (4.5 \times 10^6)(1.98 \times 10^{-5})} = 4.994 \times 10^4$$

$$\frac{\Delta A_{zf}}{A_{zf}} = -\frac{55.4939}{5 \times 10^4} = -1.11 \times 10^{-3} \Rightarrow \underline{-0.111\%}$$

12.20




$V_1 = I_e R_i, \quad -V_0 = -A_z I_e \Rightarrow V_0 = A_z I_e$

$I_{fb} = I_S - I_e$ and $-V_0 = V_1 - I_{fb} R_F$

$$-A_z I_e = V_1 - (I_S - I_e) R_F$$

$$-A_z\left(\frac{V_1}{R_i}\right) = V_1 - I_S R_F + \left(\frac{V_1}{R_i}\right) R_F$$

$$I_S R_F = V_1\left[1 + \frac{A_z}{R_i} + \frac{R_F}{R_i}\right]$$

$$R_{if} = \frac{V_1}{I_S} = \frac{R_F}{\left[1 + \dfrac{A_z}{R_i} + \dfrac{R_F}{R_i}\right]}$$

Or, using the results from Problem 12.18.

$$R_{if} = \frac{50.5 \times 10^3}{\left[1 + \dfrac{5 \times 10^6}{10 \times 10^3} + \dfrac{50.5 \times 10^3}{10 \times 10^3}\right]}$$

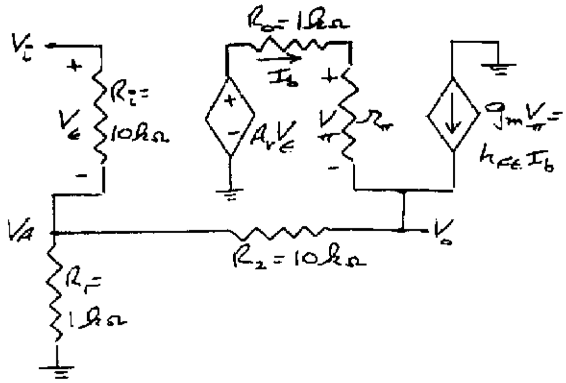
$$= \frac{50.5 \times 10^3}{[1 + 500 + 5.05]}$$

$$\Rightarrow \underline{R_{if} = 99.79\ \Omega}$$

12.21

Assume $I_{CQ} = 0.2\ mA$

Then $r_\pi = \dfrac{(100)(0.026)}{0.2} = 13\ k\Omega$




(1) $\dfrac{V_i - V_A}{R_i} = \dfrac{V_A}{R_1} + \dfrac{V_A - V_o}{R_2} \Rightarrow$

$$\frac{V_i}{R_i} + \frac{V_o}{R_2} = V_A\left(\frac{1}{R_i} + \frac{1}{R_1} + \frac{1}{R_2}\right)$$

Now

$$\frac{V_i}{10} + \frac{V_o}{10} = V_A\left(\frac{1}{10} + \frac{1}{1} + \frac{1}{10}\right) \Rightarrow V_i + V_o = V_A(12)$$

or $V_A = \dfrac{1}{12}(V_i + V_o)$

(2) $\left(\dfrac{A_v V_\epsilon - V_o}{R_o + r_\pi}\right)(1 + h_{FE}) = \dfrac{V_o - V_A}{R_2}$

where $V_\epsilon = V_i - V_A$

Then

$$\left(\frac{A_v(V_i - V_A) - V_o}{R_o + r_\pi}\right)(1 + h_{FE}) = \frac{V_o - V_A}{R_2}$$

we find

$$\frac{A_v V_i(1 + h_{FE})}{R_o + r_\pi} - \frac{V_o(1 + h_{FE})}{R_o + r_\pi} - \frac{V_o}{R_2} = \frac{A_v V_A(1 + h_{FE})}{R_o + r_\pi} - \frac{V_A}{R_2}$$

Then

$$\frac{(5x10^3)(101)V_i}{14} - \frac{V_o(101)}{14} - \frac{V_o}{10}$$

$$= \left(\frac{(5x10^3)(101)}{14} - \frac{1}{10}\right)V_A$$

Rearranging terms, we find

$$\underline{A_{vf} = \frac{V_o}{V_i} = 10.97}$$

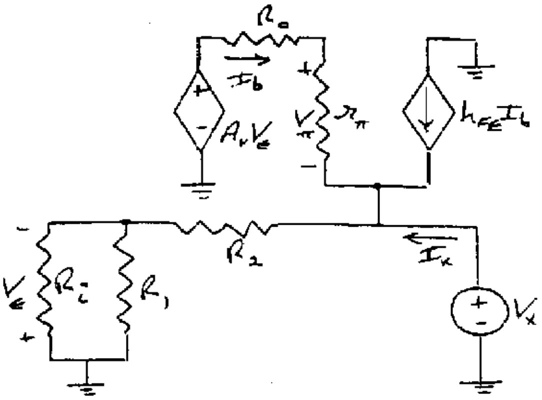
$$R_{if} = \frac{V_i}{I_i} = \frac{V_i}{\left(\dfrac{V_i - V_A}{R_i}\right)} = \left(\frac{V_i}{V_i - V_A}\right) R_i$$

$$V_A = \frac{1}{12}(V_i + V_o) = \frac{1}{12}(V_i + 10.97 V_i) = 0.9975 V_i$$

Then

$$R_{if} = \left(\frac{1}{1 - 0.9975}\right)(10\ k\Omega) \Rightarrow \underline{R_{if} = 4\ M\Omega}$$

To find the output resistance:



$$I_x + \frac{(A_v V_\epsilon - V_x)(1+h_{FE})}{R_o + r_\pi} = \frac{V_x}{R_2 + R_1 \| R_i}$$

$$V_\epsilon = -\left(\frac{R_1\|R_i}{R_1\|R_i + R_2}\right)\cdot V_x$$

Now

$R_1\|R_i = 1\|10 = 0.909$

Then

$V_\epsilon = -0.0833 V_x$

Now

$$I_x = V_x\left\{\left[\frac{(5x10^3)(0.0833)+1}{1+13}\right](101) + \frac{1}{10+0.909}\right\}$$

$$= V_x\{3.012x10^3 + 0.0917\}$$

Or

$\frac{V_x}{I_x} = R_{of} = 3.32x10^{-4}\ k\Omega \Rightarrow \underline{R_{of} = 0.332\ \Omega}$

**12.22**

a. Neglecting base currents

$I_{C2} = 0.5$ mA, $V_{C2} = 12 - (0.5)(22.6) = 0.7$ V

$I_{C1} = 0.5$ mA

$\Rightarrow \underline{v_0 = 0}$

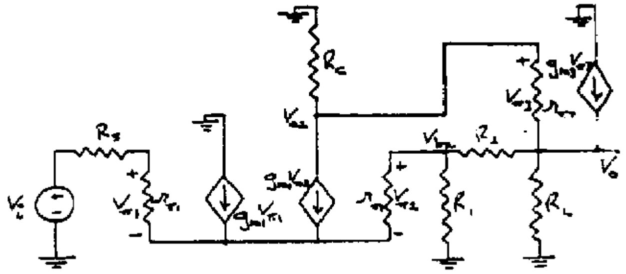
Then $I_{C3} = 2$ mA

b. $r_{\pi 1} = r_{\pi 2} = \frac{h_{FE}\cdot V_T}{I_{C1}} = \frac{(100)(0.026)}{0.5} = 5.2\ k\Omega$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23\ mA/V$$

$$r_{\pi 3} = \frac{(100)(0.026)}{2} = 1.3\ k\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92\ mA/V$$



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} + g_{m1}V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0$$

$$(V_{\pi 1} + V_{\pi 2})\left(\frac{1}{r_{\pi 1}} + g_{m1}\right) = 0$$

$$\Rightarrow V_{\pi 1} = -V_{\pi 2} \qquad (1)$$

$$V_i = \frac{V_{\pi 1}}{r_{\pi 1}}(R_S + r_{\pi 1}) - V_{\pi 2} + V_{b2}$$

or

$$V_i = V_{\pi 1}\left(1 + \frac{R_S}{r_{\pi 1}}\right) - V_{\pi 2} + V_{b2}$$

But $V_{\pi 2} = -V_{\pi 1}$

so

$$V_i = V_{\pi 1}\left(2 + \frac{R_S}{r_{\pi 1}}\right) + V_{b2} \qquad (2)$$

$$\frac{V_{02}}{R_C} + g_{m1}V_{\pi 2} + \frac{V_{02} - V_0}{r_{\pi 3}} = 0 \qquad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_0}{R_L} + \frac{V_0 - V_{b2}}{R_2}$$

$V_{\pi 3} = V_{02} - V_0$

so

$$(V_{02} - V_0)\left(\frac{1 + h_{FE}}{r_{\pi 3}}\right) = V_0\left(\frac{1}{R_L} + \frac{1}{R_2}\right) - \frac{V_{b2}}{R_2} \qquad (4)$$

$$\frac{V_{b2} - V_0}{R_2} + \frac{V_{b2}}{R_1} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0 \qquad (5)$$

Substitute numbers into (2), (3), (4) and (5):

$$V_i = -V_{\pi 2}\left(2 + \frac{1}{5.2}\right) + V_{b2}$$

$$V_i = -V_{\pi 2}(2.192) + V_{b2} \qquad (2)$$

$$V_{02}\left(\frac{1}{22.6} + \frac{1}{1.3}\right) + (19.23)V_{\pi 2} - V_0\left(\frac{1}{1.3}\right) = 0$$

$$V_{02}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \qquad (3)$$

$$V_{02}\left(\frac{101}{1.3}\right) = V_0\left(\frac{101}{1.3}+\frac{1}{4}+\frac{1}{50}\right) - V_{b2}\left(\frac{1}{50}\right)$$

$$V_{02}(77.69) = V_0(77.96) - V_{b2}(0.02) \qquad (4)$$

$$V_{b2}\left(\frac{1}{50}+\frac{1}{10}\right) - V_0\left(\frac{1}{50}\right) + V_{\pi 2}\left(\frac{1}{5.2}\right) = 0$$

$$V_{b2}(0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \qquad (5)$$

From (2): $V_{b2} = V_i + V_{\pi 2}(2.192)$. Substitute in (4) and (5) to obtain:

$$V_{02}(77.69) = V_0(77.96) - [V_i + V_{\pi 2}(2.192)](0.02) \qquad (4')$$

$$[V_i + V_{\pi 2}(2.192)](0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \qquad (5')$$

So we now have the following three equations:

$$V_{02}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \qquad (3)$$

$$V_{02}(77.69) = V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384) \qquad (4')$$

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0 \qquad (5')$$

From (3): $V_{02} = V_0(0.9455) - V_{\pi 2}(23.64)$. Substitute for $V_{02}$ in (4′) to obtain:

$$(77.69)[V_0(0.9455) - V_{\pi 2}(23.64)] = V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384)$$

or

$$0 = V_0(4.504) - V_i(0.02) + V_{\pi 2}(1836.5)$$

Next, solve (5′) for $V_{\pi 2}$:

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0$$

$$V_{\pi 2} = V_0(0.04393) - V_i(0.2636)$$

Finally,

$$0 = V_0(4.504) - V_i(0.02) + (1836.5)[V_0(0.04393) - V_i(0.2636)]$$

$$0 = V_0(85.18) - V_i(484.12)$$

So

$$A_{vf} = \frac{V_0}{V_i} = \frac{484.12}{85.18} \Rightarrow \underline{A_{vf} = 5.68}$$

12.23

a. $R_{TH} = R_1 \| R_2 = 400 \| 75 = 63.2$ kΩ

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{75}{75+400}\right)(10) = 1.579 \text{ V}$$

$$I_{BQ1} = \frac{1.579 - 0.7}{63.2 + (121)(0.5)} = 0.007106 \text{ mA}$$

$$I_{CQ1} = 0.853 \text{ mA}$$

$$V_{C1} = 10 - (0.853)(8.8) = 2.49 \text{ V}$$

$$I_{C2} \approx \frac{2.49 - 0.7}{3.6} = 0.497 \text{ mA}$$

$$V_{C2} = 10 - (0.497)(13) = 3.54 \text{ V}$$

$$I_{C3} \approx \frac{3.54 - 0.7}{1.4} = 2.03 \text{ mA}$$

Then

$$r_{\pi 1} = \frac{(120)(0.026)}{0.853} = 3.66 \text{ k}\Omega$$

$$g_{m1} = \frac{0.853}{0.026} = 32.81 \text{ mA/V}$$

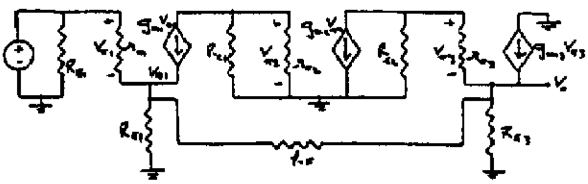
$$r_{\pi 2} = \frac{(120)(0.026)}{0.497} = 6.28 \text{ k}\Omega$$

$$g_{m2} = \frac{0.497}{0.026} = 19.12 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{2.03} = 1.54 \text{ k}\Omega$$

$$g_{m3} = \frac{2.03}{0.026} = 78.08 \text{ mA/V}$$

b.



$$V_i = V_{\pi 1} + V_{e1} \Rightarrow V_{e1} = V_i - V_{\pi 1} \qquad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_0}{R_F} \qquad (2)$$

$$V_{\pi 2} = -(g_{m1}V_{\pi 1})(R_{C1} \| r_{\pi 2}) \qquad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3} + V_0}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \qquad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{e1}}{R_F} \qquad (5)$$

Substitute numbers in (2), (3), (4) and (5):

$$V_{\pi1}\left(\frac{1}{3.66}+32.81\right)=(V_i-V_{\pi1})\left(\frac{1}{0.5}+\frac{1}{10}\right)-\frac{V_0}{10}$$
or $V_{\pi1}(35.18)=V_i(2.10)-V_0(0.10)$ (2)

$V_{\pi2}=-(32.81)V_{\pi1}(88\|6.28)$
or $V_{\pi2}=-V_{\pi1}(120.2)$ (3)

$$(19.12)V_{\pi2}+\frac{V_{\pi3}}{13}+\frac{V_0}{13}+\frac{V_{\pi3}}{1.54}=0$$
or
$V_{\pi2}(19.12)+V_{\pi3}(0.7263)+V_0(0.07692)=0$ (4)

$$V_{\pi3}\left(\frac{1}{1.54}+78.08\right)=V_0\left(\frac{1}{1.4}+\frac{1}{10}\right)-\frac{V_i-V_{\pi1}}{10}$$
or
$V_{\pi3}(78.73)$
$=V_0(0.8143)-V_i(0.10)+V_{\pi1}(0.10)$ (5)

Now substituting $V_{\pi2}=-V_{\pi1}(120.2)$ in (4):

$(19.12)[-V_{\pi1}(120.2)]+V_{\pi3}(0.7263)$
$+V_0(0.07692)=0$

or
$-V_{\pi1}(2298.2)+V_{\pi3}(0.7263)+V_0(0.07692)=0$
Then
$V_{\pi3}=V_{\pi1}(3164.3)-V_0(0.1059)$

Substituting $V_{\pi3}=V_{\pi1}(3164.3)-V_0(0.1059)$ in (5):

$(78.73)[V_{\pi1}(3164.3)-V_0(0.1059)]$
$=V_0(0.8143)-V_i(0.10)+V_{\pi1}(0.10)$
or
$V_{\pi1}(2.49\times10^5)-V_0(9.152)=-V_i(0.10)$
Then
$V_{\pi1}=V_0(3.674\times10^{-5})-V_i(4.014\times10^{-7})$

Now substituting $V_{\pi1}=V_0(3.674\times10^{-5})$ $-V_i(4.014\times10^{-7})$ in (2):

$(35.18)[V_0(3.674\times10^{-5})-V_i(4.014\times10^{-7})]$
$=V_i(2.10)-V_0(0.10)$
or $V_0(0.1013)=V_i(2.10)$

So $\frac{V_0}{V_i}=20.7$

c. $R_{if}=\frac{V_i}{I_i}$ and $I_i=I_{RB1}+I_{b1}$

$I_{RB1}=\frac{V_i}{R_{B1}}$
$I_{b1}=\frac{V_{\pi1}}{r_{\pi1}}$

Now
$V_{\pi1}=(20.7V_i)(3.674\times10^{-5})-V_i(4.014\times10^{-7})$
$V_{\pi1}=V_i(7.60\times10^{-4})$

Then

$$R_{if}=\frac{V_i}{\frac{V_i}{63.2}+\frac{V_i(7.60\times10^{-4})}{3.66}}=\frac{1}{0.01582+2.077\times10^{-4}}$$
or $\underline{R_{if}=62.4\text{ k}\Omega}$

d. To determine $R_{of}$:

Equation (1) is modified to $V_{\pi1}+V_{e1}=0$ $(V_i=0)$
Equation (5) is modified to:

$V_{\pi3}(78.73)+I_X=V_0(0.8143)+V_{\pi1}(0.10)$ (5)

Now

$V_{\pi1}(35.18)=-V_0(0.10)$ (2)

$V_{\pi2}=-V_{\pi1}(120.2)$ (3)

$V_{\pi2}(19.12)+V_{\pi3}(0.7263)+V_0(0.07692)=0$ (4)

Now

$V_{\pi1}=-V_0(0.002843)$
so
$V_{\pi2}=-(-V_0)(0.002843)(120.2)$
$V_{\pi2}=V_0(0.3417)$
Then

$V_0(0.3417)(19.12)+V_{\pi3}(0.7263)+V_0(0.07692)=0$
or
$V_{\pi3}=-V_0(9.101)$ (4)

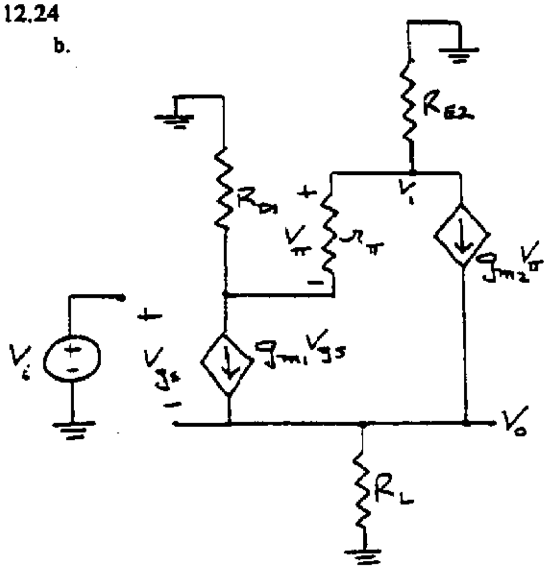
So then

$-V_0(9.101)(78.73)+I_X$
$=V_0(0.8143)+(0.10)(-V_0)(0.002843)$
or
$I_X=V_0(717.3)$ (5)
or

$R_{of}=\frac{V_0}{I_X}=0.00139\text{ k}\Omega\Rightarrow\underline{R_{of}=1.39\ \Omega}$

**12.24**

b.




$$V_0 = (g_{m1}V_{gs} + g_{m2}V_\pi)R_L \qquad (1)$$

$$V_i = V_{gs} + V_0 \qquad (2)$$

$$\frac{V_\pi}{r_\pi} + g_{m2}V_\pi + \frac{V_1}{R_{E2}} = 0$$

$$\Rightarrow V_\pi\left(\frac{1+h_{FE}}{r_\pi}\right) + \frac{V_1}{R_{E2}} = 0 \qquad (3)$$

$$\frac{V_\pi}{r_\pi} = g_{m1}V_{gs} + \frac{V_1 - V_\pi}{R_{D1}} = 0$$

or

$$V_1 = R_{D1}\left[\frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_{D1}} - g_{m1}V_{gs}\right] \qquad (4)$$

Then

$$V_\pi\left(\frac{1+h_{FE}}{r_\pi}\right) + \frac{R_{D1}}{R_{E2}}\left[\frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_{D1}} - g_{m1}V_{gs}\right] = 0 \qquad (3)$$

$$V_\pi\left\{\left(\frac{1+h_{FE}}{r_\pi}\right) + \frac{R_{D1}}{R_{E2}r_\pi} + \frac{1}{R_{E2}}\right\} = g_{m1}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs}$$

Let

$$V_\pi \cdot \frac{1}{R_{eq}} = g_{m1}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs}$$

so $V_\pi = g_{m1}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs}$

Then

$$V_0 = \left[g_{m1}V_{gs} + g_{m1}g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs}\right]R_L \qquad (1)$$

so

$$V_0 = g_{m1}R_L\left[1 + g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)\right](V_i - V_0)$$

so

$$A_v = \frac{V_0}{V_i} = \frac{g_{m1}R_L\left[1 + g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)\right]}{1 + g_{m1}R_L\left[1 + g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)\right]}$$

c. Set $V_i = 0$

$$I_X + g_{m1}V_{gs} + g_{m2}V_\pi = \frac{V_X}{R_L}$$

$$V_{gs} = -V_X$$

From part (b), we have

$$V_\pi = g_{m1}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs} = -g_{m1}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)V_X$$

Then

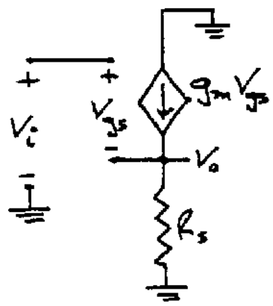
$$\frac{I_X}{V_X} = \frac{1}{R_o} = \frac{1}{R_L} + g_{m1}g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)$$

or

$$R_o = R_L \,\Big\|\, \frac{1}{g_{m1}} \,\Big\|\, \frac{1}{g_{m1}g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)}$$

**12.25**

a. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1\ mA/V$




$$V_0 = (g_m V_{gs})R_S$$

$$V_i = V_{gs} + V_0 \text{ so } V_{gs} = V_i - V_0$$

Then

$$V_0 = g_m R_S(V_i - V_0)$$
$$A_v = \frac{g_m R_S}{1 + g_m R_S} = \frac{1(2)}{1+(1)(2)} \Rightarrow \underline{A_v = 0.667}$$

To determine $R_{of}$

$$I_X + g_m V_{gs} = \frac{V_X}{R_S} \text{ and } V_{gs} = -V_X$$
$$\frac{I_X}{V_X} = \frac{1}{R_{of}} = g_m + \frac{1}{R_S}$$
$$\text{so } \underline{R_{of} = \frac{1}{g_m}} \,\Big\|\, R_S = \frac{1}{1} \,\Big\|\, 2 \Rightarrow \underline{R_{of} = 0.667\ \text{k}\Omega}$$

b. For $K_n = 0.8\ mA/V^2$

$$g_m = 2\sqrt{(0.8)(0.5)} = 1.265\ \text{mA/V}$$
$$A_v = \frac{(1.265)(2)}{1+(1.265)(2)} = 0.7167$$
$$\frac{\Delta A_f}{A_f} = \frac{0.7167 - 0.667}{0.667} \Rightarrow \underline{7.45\%\ \text{increase}}$$

$$R_{of} = \frac{1}{1.265} \,\Big\|\, 2 = 0.7905\|2$$
$$R_{of} = 0.5666$$
$$\frac{\Delta R_{of}}{R_{of}} = \frac{0.5666 - 0.667}{0.667} \Rightarrow \underline{15.05\%\ \text{decrease}}$$

12.26

dc analysis:

$$R_{TH1} = 150\|47 = 35.8\ \text{k}\Omega,$$
$$V_{TH1} = \left(\frac{47}{47+150}\right)(25) = 5.96\ \text{V}$$

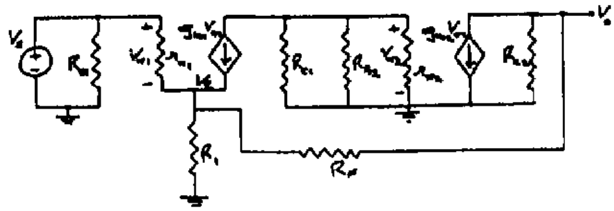
$$R_{TH2} = 33\|47 = 19.4\ \text{k}\Omega,$$
$$V_{TH2} = \left(\frac{33}{33+47}\right)(25) = 10.3\ \text{V}$$

$$I_{B1} = \frac{5.96 - 0.7}{35.8 + (51)(4.8)} = 0.0187\ \text{mA}$$
$$I_{C1} = (50)(0.0187) = 0.935\ \text{mA}$$

$$I_{B2} = \frac{10.3 - 0.7}{19.4 + (51)(4.7)} = 0.03705\ \text{mA}$$
$$I_{C2} = (50)(0.03705) = 1.85\ \text{mA}$$

$$r_{\pi1} = \frac{(50)(0.026)}{0.935} = 1.39\ \text{k}\Omega;$$
$$r_{\pi2} = \frac{(50)(0.026)}{1.85} = 0.703\ \text{k}\Omega$$

$$g_{m1} = \frac{0.935}{0.026} = 35.96\ \text{mA/V}$$
$$g_{m2} = \frac{1.85}{0.026} = 71.15\ \text{mA/V}$$



$$V_S = V_{\pi1} + V_e \tag{1}$$

$$\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} = \frac{V_e}{R_1} + \frac{V_e - V_0}{R_F} \tag{2}$$

$$g_{m1}V_{\pi1} + \frac{V_{\pi2}}{R_{C1}} + \frac{V_{\pi2}}{R_{B2}} + \frac{V_{\pi2}}{r_{\pi2}} = 0 \tag{3}$$

$$g_{m2}V_{\pi2} + \frac{V_0}{R_{C2}} + \frac{V_0 - V_e}{R_F} = 0 \tag{4}$$

Substitute numerical values in (2), (3) and (4):

$$V_e = V_S - V_{\pi1} \tag{1}$$

$$\frac{V_{\pi1}}{1.39} + (35.96)V_{\pi1} = (V_S - V_{\pi1})\left(\frac{1}{0.1} + \frac{1}{4.7}\right) - V_0\left(\frac{1}{4.7}\right)$$

or

$$V_{\pi1}(46.89) = V_S(10.213) - V_0(0.2128) \tag{2}$$

$$(35.96)V_{\pi1} + V_{\pi2}\left(\frac{1}{10} + \frac{1}{19.4} + \frac{1}{0.703}\right) = 0$$

or

$$(35.96)V_{\pi1} + V_{\pi2}(1.574) = 0 \tag{3}$$

$$(71.15)V_{\pi2} + V_0\left(\frac{1}{4.7} + \frac{1}{4.7}\right) - (V_S - V_{\pi1})\left(\frac{1}{4.7}\right) = 0$$

or

$$(71.15)V_{\pi2} + V_0(0.4255) - V_S(0.2128) + V_{\pi1}(0.2128) = 0 \tag{4}$$

From (3): $V_{\pi2} = -V_{\pi1}(22.85)$

Then substitute in (4):

$$-(71.15)V_{\pi1}(22.85) + V_0(0.4255) - V_S(0.2128) + V_{\pi1}(0.2128) = 0$$

or

$$-V_{\pi1}(1625.6) + V_0(0.4255) - V_S(0.2128) = 0$$

From (2): $V_{\pi1} = V_S(0.2178) - V_0(0.004538)$

Then

$$-(1625.6)[V_S(0.2178) - V_0(0.004538)] + V_0(0.4255) - V_S(0.2128) = 0$$

or $-V_S(354.3) + V_0(7.802) = 0$

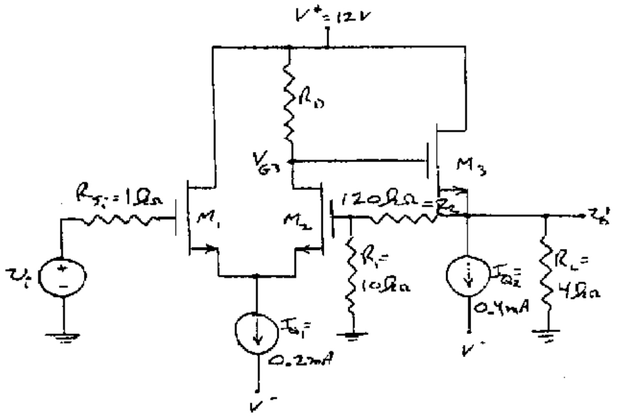
Finally

$$\Rightarrow \frac{V_0}{V_S} = 45.4$$

12.27

For example, use the circuit shown in Figure 12.23

12.28




For $M_3$: $K_{n3} = \frac{k_n'}{2}\cdot\left(\frac{W}{L}\right)_3$ Let $\left(\frac{W}{L}\right)_3 = 25$

Then $K_{n3} = \left(\frac{0.080}{2}\right)(25) = 1\ mA/V^2$

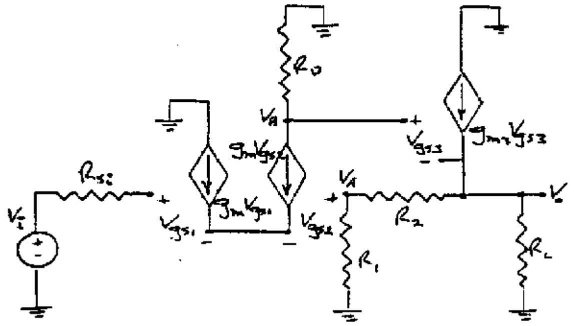
Want $v_o = 0$ for $v_i = 0$, so that

$I_{D3} = I_{Q2} = 0.4 = 1\cdot(V_{GS3} - V_{TN})^2$

Then $V_{GS3} = \sqrt{\frac{0.4}{1}} + 2 = 2.63\ V$

For $V_{GS3} = 2.63\ V \Rightarrow V_{G3} = 12 - I_{D2}R_D$

Or $2.63 = 12 - (0.1)R_D \Rightarrow$ $R_D = 93.7\ k\Omega$




$g_{m3} = 2\sqrt{K_{n3}I_{D3}} = 2\sqrt{(1)(0.4)} = 1.26\ mA/V$

$V_A = \left(\frac{R_1}{R_1+R_2}\right)(V_o) = \left(\frac{10}{120+10}\right)(V_o) = 0.0769V_o$

(Small amount of feedback)

(1) $V_i = V_{gs1} - V_{gs2} + V_A$

(2) $g_mV_{gs1} + g_mV_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$

Then

$V_i = -2V_{gs2} + V_A \Rightarrow V_{gs2} = \frac{1}{2}(V_A - V_i)$

$V_{gs2} = 0.03846V_o - 0.5V_i$

(3) $V_B = -g_mV_{gs2}R_D = -g_mR_D[0.03846V_o - 0.5V_i]$

(4) $V_{gs3} = V_B - V_o$ and $V_o = g_{m3}V_{gs3}[R_L\|(R_1+R_2)]$

So

$V_o = g_{m3}[R_L\|(R_1+R_2)](V_B - V_o)$

Then

$V_o = g_{m3}[R_L\|(R_1+R_2)][-g_mR_D(0.03846V_o - 0.5V_i) - V_o]$

Or

$V_o\{1 + g_{m3}[R_L\|(R_1+R_2)][g_mR_D(0.03846)+1]\}$

$\quad = g_{m3}[R_L\|(R_1+R_2)][0.5g_mR_D]\cdot V_i$

Now

$R_L\|(R_1+R_2) = 4\|130 = 3.88$

So

$V_o[1 + (1.26)(3.88)[g_m(93.7)(0.03846)+1]]$

$\quad = (1.26)(3.88)(0.5)g_m(93.7)V_i$

Rearranging terms, we find

$\frac{V_o}{V_i} = \frac{229g_m}{5.89 + 17.6g_m} = 10 \Rightarrow g_m = 1.11\ mA/V$

We have

$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_D} \Rightarrow$

$1.11 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.1)} \Rightarrow$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 77$

12.29

Assuming an ideal op-amp, then from Equation (12.58)

$$\frac{I_0}{I_S} = 1 + \frac{R_1}{R_2} = \frac{20}{0.2} = 100$$

Then $\frac{R_1}{R_2} = 99$

For example, set $R_2 = 5\ k\Omega$ and $R_1 = 495\ k\Omega$

12.30

(a) $I_{C1} = \left(\frac{h_{FE}}{1+h_{FE}}\right)I_{E1} = \left(\frac{100}{101}\right)(0.2) = 0.198\ mA$

$V_{C1} = 10 - (0.198)(40) = 2.08\ V$

$I_{E2} = \frac{2.08 - 0.7}{1} = 1.38\ mA$

$I_{C2} = \left(\frac{100}{101}\right)(1.38) = 1.37\ mA$

For $Q_1$:

$$r_{\pi1} = \frac{(100)(0.026)}{0.198} = 13.1\ k\Omega$$

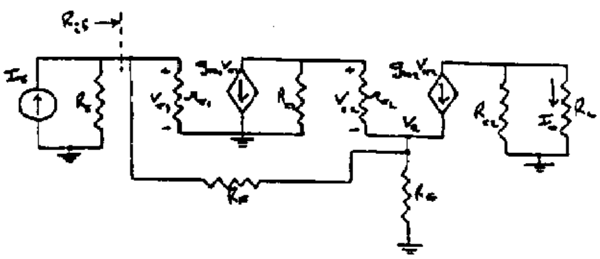
$$g_{m1} = \frac{0.198}{0.026} = 7.62\ mA/V$$

For $Q_2$:

$$r_{\pi2} = \frac{(100)(0.026)}{1.37} = 1.90\ k\Omega$$

$$g_{m2} = \frac{1.37}{0.026} = 52.7\ mA/V$$

(b)



$$I_S = \frac{V_{\pi1}}{R_S} + \frac{V_{\pi1}}{r_{\pi1}} + \frac{V_{\pi1} - V_e}{R_F} \quad (1)$$

$$g_{m1}V_{\pi1} + \frac{V_{\pi2} + V_e}{R_{C1}} + \frac{V_{\pi2}}{r_{\pi2}} = 0 \quad (2)$$

$$\frac{V_{\pi2}}{r_{\pi2}} + g_{m2}V_{\pi2} = \frac{V_e}{R_E} + \frac{V_e - V_{\pi1}}{R_F} \quad (3)$$

Substitute numerical values in (1), (2), and (3):

$$I_S = V_{\pi1}\left(\frac{1}{10} + \frac{1}{13.1} + \frac{1}{10}\right) - V_e\left(\frac{1}{10}\right)$$

$$I_S = V_{\pi1}(0.2763) - V_e(0.10) \quad (1)$$

$$(7.62)V_{\pi1} + V_{\pi2}\left(\frac{1}{40} + \frac{1}{1.90}\right) + V_e\left(\frac{1}{40}\right) = 0$$

$$(7.62)V_{\pi1} + V_{\pi2}(0.5513) + V_e(0.025) = 0 \quad (2)$$

$$V_{\pi2}\left(\frac{1}{1.90} + 52.7\right) = V_e\left(\frac{1}{1} + \frac{1}{10}\right) - V_{\pi1}\left(\frac{1}{10}\right)$$

$$V_{\pi2}(53.23) = V_e(1.10) - V_{\pi1}(0.10) \quad (3)$$

From (3),

$$V_e = V_{\pi2}(48.39) + V_{\pi1}(0.0909)$$

Substituting into (1),

$$I_S = V_{\pi1}(0.2763) - (0.10)\left[V_{\pi2}(48.39) + V_{\pi1}(0.0909)\right]$$

or

$$I_S = V_{\pi1}(0.2672) - V_{\pi2}(4.839) \quad (1')$$

and substituting into (2),

$$(7.62)V_{\pi1} + V_{\pi2}(0.5513) + (0.025)\left[V_{\pi2}(48.39) + V_{\pi1}(0.0909)\right] = 0$$

or

$$(7.622)V_{\pi1} + V_{\pi2}(1.761) = 0$$

$$\Rightarrow V_{\pi1} = -V_{\pi2}(0.2310) \quad (2')$$

Then substituting (2') into (1'), we obtain

$$I_S = (0.2672)(-V_{\pi2})(0.2310) - V_{\pi2}(4.839)$$

or

$$I_S = -V_{\pi2}(4.901)$$

Now

$$I_O = -g_{m2}V_{\pi2}\left(\frac{R_{C2}}{R_{C2} + R_L}\right)$$

$$= -(52.7)\left(\frac{2}{2 + 0.5}\right)V_{\pi2} = -(42.16)V_{\pi2}$$

Then

$$I_O = -(42.16)\left(\frac{-I_S}{4.901}\right)$$

or

$$\underline{A_{if} = \frac{I_O}{I_S} = 8.60}$$

(c) $R_i = \dfrac{V_{\pi1}}{I_S}$ and $R_i = R_S \| R_{if}$

We had

$V_{\pi1} = -V_{\pi2}(0.2310)$ and $I_S = -V_{\pi2}(4.901)$

so

$$I_S = -\left(\frac{-V_{\pi1}}{0.2310}\right)(4.901) = V_{\pi1}(21.22)$$

Then

$$R_i = \frac{V_{\pi1}}{I_S} = \frac{1}{21.22} = 0.04713$$

Finally

$$0.04713 = \frac{10R_{if}}{10 + R_{if}} \Rightarrow$$

$$\underline{R_{if} = 47.4\ \Omega}$$

12.31

**(a) Using Figure 12.25**

$$I_i = \frac{V_{\pi1}}{R_S \| R_{B1} \| r_{\pi1}} + \frac{V_{\pi1} - V_{e2}}{R_F} \quad (1)$$

$$g_{m1}V_{\pi1} + \frac{V_{\pi2}}{R_{C1} \| R_{B2}} + \frac{V_{\pi2}}{r_{\pi2}} = 0$$

$$= g_{m1}V_{\pi1} + \frac{V_{\pi2}}{R_{C1} \| R_{B2} \| r_{\pi2}} \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F} \qquad (3)$$

$$I_o = -\left(\frac{R_{C2}}{R_{C2} + R_L}\right)(g_{m2}V_{\pi 2}) \qquad (4)$$

$$I_i = \frac{V_{\pi 1}}{R_S \| R_{B1} \| r_{\pi 1} \| R_F} - \frac{V_{e2}}{R_F} \qquad (1')$$

so that

$$V_{e2} = \left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right)V_{\pi 1} - R_F I_i$$

Then

$$V_{\pi 2}\left(\frac{1+\beta_2}{r_{\pi 2}}\right) = \left(\frac{1}{R_{E2}} + \frac{1}{R_F}\right) \times \left\{\left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right)V_{\pi 1} - R_F I_i\right\} - \frac{V_{\pi 1}}{R_F} \qquad (3')$$

From (2):

$$V_{\pi 1} = -\frac{V_{\pi 2}}{g_{m1}} \cdot \frac{1}{(R_{C1} \| R_{B2} \| r_{\pi 2})}$$

Then

$$V_{\pi 2}\left(\frac{1+\beta_2}{r_{\pi 2}}\right) = \frac{V_{\pi 2}}{g_{m1}R_F} \cdot \frac{1}{R_{C1} \| R_{B2} \| r_{\pi 2}} \times \left\{1 - \left(1 + \frac{R_F}{R_{E2}}\right)\left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right)\right\} - \left(1 + \frac{R_F}{R_{E2}}\right)I_i$$

Solve for $V_{\pi 2}$ and substitute into Equation 4.

(b) $R_{TH1} = 20 \| 80 = 16\ k\Omega = R_{B1}$

$$V_{TH1} = \left(\frac{20}{100}\right)(10) = 2\ V$$

$$I_{BQ1} = \frac{2 - 0.7}{16 + (101)(1)} = 0.0111\ mA \Rightarrow$$

$I_{CQ1} = 1.11\ mA$

$R_{TH2} = 15 \| 85 = 12.75\ k\Omega$

$$V_{TH2} = \left(\frac{15}{15+85}\right)(10) = 1.5\ V$$

$$I_{BQ2} = \frac{1.5 - 0.7}{12.75 + (101)(0.5)} = 0.0126\ mA \Rightarrow$$

$I_{CQ2} = 1.26\ mA$

$$g_{m1} = \frac{1.11}{0.026} = 42.69\ mA/V$$

$$g_{m2} = \frac{1.26}{0.026} = 48.46\ mA/V$$

$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} = 2.34\ k\Omega$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.26} = 2.06\ k\Omega$$

From part (a)

$$V_{\pi 2}\left(\frac{101}{2.06}\right) = \frac{V_{\pi 2}}{(42.69)(10)} \cdot \frac{1}{2 \| 12.75 \| 2.06} \times \left\{1 - \left(1 + \frac{10}{0.5}\right)\left(\frac{10}{10000 \| 16 \| 2.34 \| 10}\right)\right\} - \left(1 + \frac{10}{0.5}\right)I_i$$

So

$V_{\pi 2}(49.34) = -(21)I_i$

or

$V_{\pi 2} = -0.4256 I_i$

Now

$$I_o = -\left(\frac{4}{4+4}\right)(48.46)(-0.4256)I_i$$

or

$$A_i = \frac{I_o}{I_i} = 10.3$$

From Example 12.9, computer analysis showed $A_i = 9.58$. The difference in results is usually in the calculation of quiescent currents which leads to slight differences in the small-signal parameter values.

12.32

a. $R_{TH} = 13.5 \| 38.3 = 9.98\ k\Omega$

$$V_{TH} = \left(\frac{13.5}{13.5 + 38.3}\right)(10) = 2.606\ V$$

$$I_{C1} = \frac{(120)(2.606 - 0.7)}{9.98 + (121)(1)} = 1.75\ mA$$

$V_{C1} = 10 - (1.75)(3) = 4.75\ V$

$$I_{C2} \approx \frac{4.75 - 0.7}{8.1} = 0.50\ mA$$

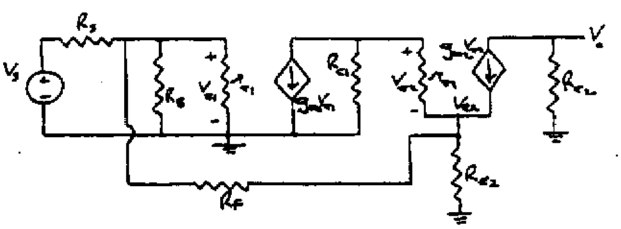
$$r_{\pi 1} = \frac{(120)(0.026)}{1.75} = 1.78\ k\Omega$$

$$g_{m1} = \frac{1.75}{0.026} = 67.31\ mA/V$$

$$r_{\pi 2} = \frac{(120)(0.026)}{0.50} = 6.24\ k\Omega$$

$$g_{m2} = \frac{0.50}{0.026} = 19.23\ mA/V$$

b.



$$\frac{V_S - V_{\pi 1}}{R_S} = \frac{V_{\pi 1}}{R_B \| r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F} \quad (3)$$

and

$$V_0 = -(g_{m2} V_{\pi 2}) R_{C2} \quad (4)$$

Substitute numerical values in (1), (2), and (3):

$$\frac{V_S}{0.6} = V_{\pi 1}\left[\frac{1}{0.6} + \frac{V_{\pi 1}}{9.98 \| 1.78} + \frac{1}{1.2}\right] - \frac{V_{e2}}{1.2}$$

or

$$V_S(1.667) = V_{\pi 1}(4.011) - V_{e2}(0.8333) \quad (1)$$

$$(67.31)V_{\pi 1} + V_{\pi 2}\left(\frac{1}{3} + \frac{1}{6.24}\right) + \frac{V_{e2}}{3} = 0$$

or

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + V_{e2}(0.3333) = 0 \quad (2)$$

$$V_{\pi 2}\left(\frac{1}{6.24} + 19.23\right) = \frac{V_{e2}}{8.1} + \frac{V_{e2}}{1.2} - \frac{V_{\pi 1}}{1.2}$$

or

$$V_{\pi 2}(19.39) = V_{e2}(0.9568) - V_{\pi 1}(0.8333) \quad (3)$$

From (1)

$$V_{e2} = V_{\pi 1}(4.813) - V_S(2.00)$$

Then

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + (0.3333)[V_{\pi 1}(4.813) - V_S(2.00)] = 0$$

or

$$V_{\pi 1}(68.91) + V_{\pi 2}(0.4936) - V_S(0.6666) = 0 \quad (2')$$

and

$$V_{\pi 2}(19.39) = (0.9568)[V_{\pi 1}(4.813) - V_S(2.00)] - V_{\pi 1}(0.8333)$$

or

$$V_{\pi 2}(19.39) = V_{\pi 1}(3.772) - V_S(1.914) \quad (3')$$

We find

$$V_{\pi 1} = V_S(0.009673) - V_{\pi 2}(0.007163)$$

Then

$$V_{\pi 2}(19.39) = (3.772)[V_S(0.009673) - V_{\pi 2}(0.007163)] - V_S(1.914)$$

$$V_{\pi 2}(19.42) = V_S(-1.878) \text{ or } V_{\pi 2} = -V_S(0.09670)$$

so that

$$V_0 = -(19.23)(4)(-V_S)(0.09670)$$

Then

$$\underline{\frac{V_0}{V_S} = 1.86}$$

12.33

Using the circuit from Problem 12.32, we have $R_{if} = \frac{V_{\pi 1}}{I_s}$

where $I_S = \frac{V_S - V_{\pi 1}}{R_S}$.

From Problem 12.32

$$V_{\pi 1} = V_S(0.009673) - V_{\pi 2}(0.007163)$$
$$= V_S(0.009673) - (0.007163)(-V_S)(0.09670)$$
$$= V_S(0.01037)$$

So

$$R_{if} = \frac{V_S(0.01037) \cdot (0.6)}{V_S - V_S(0.01037)} = 0.00629 \text{ k}\Omega$$

or

$$\underline{R_{if} = 6.29\ \Omega}$$

12.34

$$R_{TH} = 1.4 \| 17.9 = 1.298 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{1.4}{1.4 + 17.9}\right)(10) = 0.7254 \text{ V}$$

$$I_{B1} = \frac{0.7254 - 0.7}{1.298} = 0.0196 \text{ mA}$$

$$I_{C1} = (50)(0.0196) = 0.98 \text{ mA}$$

Neglecting dc base currents,

$$V_{B2} = 10 - (0.98)(7) = 3.14 \text{ V}$$

$$I_{E2} = \frac{3.14 - 0.7}{0.25 + 0.5} = 3.25 \text{ mA}$$

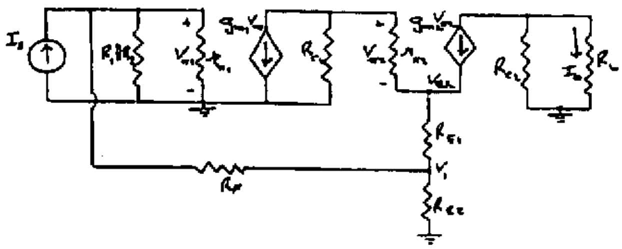
$$I_{C2} = \left(\frac{50}{51}\right)(3.25) = 3.19 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.98} = 1.33 \text{ k}\Omega$$

$$g_{m1} = \frac{0.98}{0.026} = 37.7 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(50)(0.026)}{3.19} = 0.408 \text{ k}\Omega$$

$$g_{m2} = \frac{3.19}{0.026} = 123 \text{ mA/V}$$



$$I_S = \frac{V_{\pi 1}}{R_1 \| R_2 \| r_{\pi 1}} + \frac{V_{\pi 1} - V_1}{R_F} \qquad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} = 0 \qquad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2} - V_1}{R_{E1}} \qquad (3)$$

$$\frac{V_{e2} - V_{\pi 1}}{R_{E1}} = \frac{V_1}{R_{E2}} + \frac{V_1 - V_{\pi 1}}{R_F} \qquad (4)$$

Enter numerical values in (1), (2), (3) and (4):

$$I_S = \frac{V_{\pi 1}}{17.9 \| 1.4 \| 1.33} + \frac{V_{\pi 1} - V_1}{5}$$

or

$$I_S = V_{\pi 1}(1.722) - V_1(0.20) \qquad (1)$$

$$(37.7)V_{\pi 1} + \frac{V_{\pi 2}}{0.408} + \frac{V_{\pi 2} + V_{e2}}{7} = 0$$

or

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + V_{e2}(0.1429) = 0 \qquad (2)$$

$$\frac{V_{\pi 2}}{0.408} + (123)V_{\pi 2} = \frac{V_{e2} - V_1}{0.25}$$

or

$$V_{\pi 2}(125.5) = V_{e2}(4) - V_1(4) \qquad (3)$$

$$\frac{V_{e2} - V_1}{0.25} = \frac{V_1}{0.50} + \frac{V_1 - V_{\pi 1}}{5}$$

or

$$V_{e2}(4) = V_1(6.20) - V_{\pi 1}(0.20) \qquad (4)$$

From (4):

$$V_{e2} = V_1(1.55) - V_{\pi 1}(0.05)$$

Then substituting in (3):

$$V_{\pi 2}(125.5) = (4)[V_1(1.55) - V_{\pi 1}(0.05)] - V_1(4)$$

or

$$V_{\pi 2}(125.5) = V_1(2.20) - V_{\pi 1}(0.20) \qquad (3')$$

and substituting in (2):

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + (0.1429)[V_1(1.55) - V_{\pi 1}(0.05)] = 0$$

or

$$V_{\pi 1}(37.69) + V_{\pi 2}(2.594) + V_1(0.2215) = 0$$

Now

$$V_1 = -V_{\pi 1}(170.16) - V_{\pi 2}(11.71)$$

Then substituting in (1):

$$I_S = V_{\pi 1}(1.722) - (0.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)]$$

or

$$I_S = V_{\pi 1}(35.75) + V_{\pi 2}(2.342)$$

and substituting in (3'):

$$V_{\pi 2}(125.5) = (2.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)] - V_{\pi 1}(0.20)$$

or $V_{\pi 2}(151.3) = -V_{\pi 1}(374.55)$ so that

$$V_{\pi 1} = -V_{\pi 2}(0.4040)$$

Then

$$I_S = (35.75)[-V_{\pi 2}(0.4040)] + V_{\pi 2}(2.342)$$

$$I_S = -V_{\pi 2}(12.10)$$

$$I_0 = -(g_{m2} V_{\pi 2})\left(\frac{R_{C2}}{R_{C2} + R_L}\right)$$

$$= -(123)\left(\frac{2.2}{2.2 + 2}\right)V_{\pi 2} = -(64.43)V_{\pi 2}$$

or $V_{\pi 2} = -(0.01552)I_0$

Then

$$\frac{I_0}{I_S} = \frac{1}{(0.01552)(12.10)} \Rightarrow \underline{\frac{I_0}{I_S} = 5.33}$$

12.35

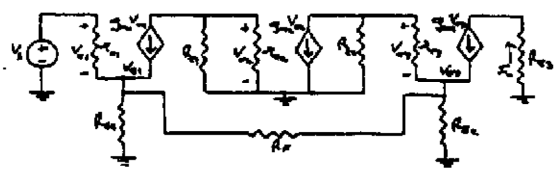
For example, use the circuit shown in Figure P12.30

12.36

$r_{\pi1} = 6.24\ \text{k}\Omega,\ r_{\pi2} = 3.12\ \text{k}\Omega,\ r_{\pi3} = 1.56\ \text{k}\Omega$

$g_{m1} = 19.23\ \text{mA/V},\ g_{m2} = 38.46\ \text{mA/V},$

$g_{m3} = 76.92\ \text{mA/V}$



$$V_S = V_{\pi1} + V_{e1} \qquad (1)$$

$$\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_{e3}}{R_F} \qquad (2)$$

$$V_{\pi2} = -g_{m1}V_{\pi1}(R_{C1}\|r_{\pi2}) \qquad (3)$$

$$g_{m2}V_{\pi2} + \frac{V_{\pi3} + V_{e3}}{R_{C2}} + \frac{V_{\pi3}}{r_{\pi3}} = 0 \qquad (4)$$

$$\frac{V_{\pi3}}{r_{\pi3}} + g_{m3}V_{\pi3} = \frac{V_{e3}}{R_{E2}} + \frac{V_{e3} - V_{e1}}{R_F} \qquad (5)$$

Enter numerical values in (2)-(5):

$$\frac{V_{\pi1}}{6.24} + (19.23)V_{\pi1} = V_{e1}\left(\frac{1}{0.1} + \frac{1}{0.8}\right) - V_{e3}\left(\frac{1}{0.8}\right)$$

or

$$V_{\pi1}(19.39) = V_{e1}(11.25) - V_{e3}(1.25) \qquad (2)$$

$$V_{\pi2} = -(19.23)V_{\pi1}(5\|3.12) = -(36.94)V_{\pi1} \qquad (3)$$

$$(38.46)V_{\pi2} + V_{\pi3}\left(\frac{1}{2} + \frac{1}{1.56}\right) + V_{e3}\left(\frac{1}{2}\right) = 0$$

or

$$V_{\pi2}(38.46) + V_{\pi3}(1.141) + V_{e3}(0.5) = 0 \qquad (4)$$

$$V_{\pi3}\left(\frac{1}{1.56} + 76.92\right) = V_{e3}\left(\frac{1}{0.1} + \frac{1}{0.8}\right) - V_{e1}\left(\frac{1}{0.8}\right)$$

or

$$V_{\pi3}(77.56) = V_{e3}(11.25) - V_{e1}(1.25) \qquad (5)$$

From (1) $V_{\pi1} = V_S - V_{e1}$

Then

$$(V_S - V_{e1})(19.39) = V_{e1}(11.25) - V_{e3}(1.25)$$

or

$$V_S(19.39) = V_{e1}(30.64) - V_{e3}(1.25) \qquad (2')$$

$$V_{\pi2} = -V_S(36.94) + V_{e1}(36.94) \qquad (3')$$

$$(38.46)[-V_S(36.94) + V_{e1}(36.94)] + V_{\pi3}(1.141) + V_{e3}(0.5) = 0 \qquad (4')$$

From (5): $V_{e3} = V_{\pi3}(6.894) + V_{e1}(0.1111)$

Then

$$V_S(19.39) = V_{e1}(30.64) - (1.25)[V_{\pi3}(6.894) + V_{e1}(0.1111)]$$

or

$$V_S(19.39) = V_{e1}(30.50) - V_{\pi3}(8.6175) \qquad (2'')$$

and

$$-V_S(1420.7) + V_{e1}(1420.7) + V_{\pi3}(1.141) + (0.5)[V_{\pi3}(6.894) + V_{e1}(0.1111)] = 0$$

or

$$-V_S(1420.7) + V_{e1}(1420.76) + V_{\pi3}(4.588) = 0 \qquad (4'')$$

From $(2'')$:

$$V_{e1} = V_S(0.6357) + V_{\pi3}(0.2825)$$

Then substituting in $(4'')$:

$$-V_S(1420.7) + (1420.76)[V_S(0.6357) + V_{\pi3}(0.2825)] + V_{\pi3}(4.588) = 0$$

$$-V_S(517.5) + V_{\pi3}(405.95) = 0$$

Now

$I_0 = g_{m3}V_{\pi3} = 76.92V_{\pi3}$ or $V_{\pi3} = I_0(0.0130)$

Then $-V_S(517.5) + I_0(0.0130)(405.95) = 0$

or

$$\underline{\frac{I_0}{V_S} = 98.06\ \text{mA/V}}$$

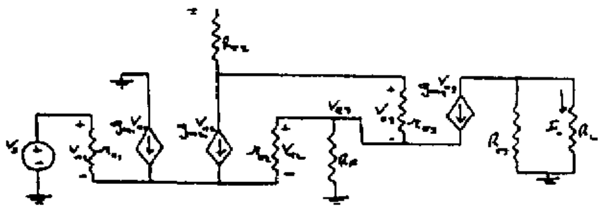
12.38

$$r_{\pi 1} = r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (1)$$

Since $r_{\pi 1} = r_{\pi 2}$ and $g_{m1} = g_{m2}$, then $V_{\pi 1} = -V_{\pi 2}$

$$V_S = V_{\pi 1} - V_{\pi 2} + V_{e3} = -2V_{\pi 2} + V_{e3} \quad (2)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3}}{r_{\pi 3}} + \frac{V_{\pi 3} + V_{e3}}{R_{C2}} = 0 \quad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_{e3}}{R_F} + \frac{V_{\pi 2}}{r_{\pi 2}} \quad (4)$$

$$I_0 = -\left(\frac{R_{C3}}{R_{C3} + R_L}\right)(g_{m3}V_{\pi 3}) \quad (5)$$

From (2): $V_{e3} = V_S + 2V_{\pi 2}$

$$(19.23)V_{\pi 2} + \frac{V_{\pi 3}}{1.3} + \frac{V_{\pi 3}}{18.6} + \frac{1}{18.6}(V_S + 2V_{\pi 2}) = 0$$

or

$$(19.23)V_{\pi 2} + (0.8230)V_{\pi 3} + (0.05376)V_S = 0 \quad (3')$$

$$V_{\pi 3}\left(\frac{1}{1.3} + 76.92\right) = \left(\frac{1}{10}\right)(V_S + 2V_{\pi 2}) + \frac{V_{\pi 2}}{5.2}$$

or

$$(77.69)V_{\pi 3} = (0.3923)V_{\pi 2} + (0.1)V_S \quad (4')$$

$$I_0 = -\left(\frac{2}{2+1}\right)(76.92)V_{\pi 3} = -(51.28)V_{\pi 3} \quad (5')$$

From (3'):

$$V_{\pi 2} = -(0.04255)V_{\pi 3} - (0.002780)V_S$$

Then

$$(77.69)V_{\pi 3} = (0.3923)[-(0.04255)V_{\pi 3} - (0.002780)V_S] + (0.1)V_S$$

$$(77.71)V_{\pi 3} = (0.0989)V_S$$

or

$$V_{\pi 3} = (0.001273)V_S$$

so that

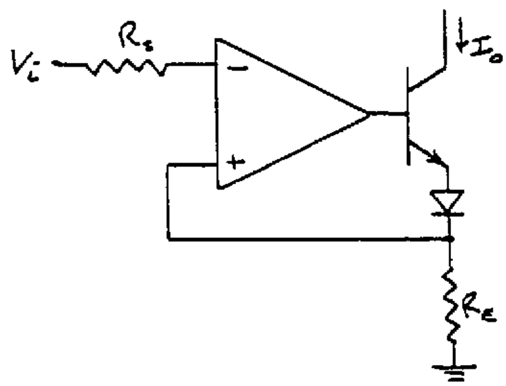
$$I_0 = -(51.28)(0.001273)V_S$$

or

$$\frac{I_0}{V_S} = -(0.0653) \text{ mA/V}$$

12.39

Use the basic circuit shown in Figure 12.27.




For the ideal case

$$\frac{I_0}{V_i} = \frac{1}{R_E}$$

we want

$$\frac{I_0}{V_i} = 10^{-3} \text{ A/V} = 1 \text{ mA/V}$$

Set $R_E = 1$ k$\Omega$

Since the op-amp has a finite gain, finite input resistance, and finite output resistance, the closed-loop gain is slightly less than the ideal. $R_E$ will need to be slightly decreased to increase the gain.

12.40

dc analysis

$$I_E R_E + V_{EB}(\text{on}) + I_B R_B + V_{CC} = ?$$

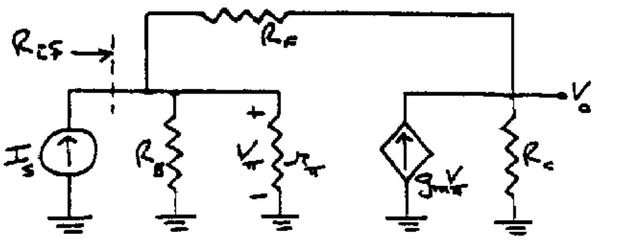
$$I_B = \frac{5 - 0.7}{100 + (51)(0.5)} = 0.0343$$

$$I_C = (50)(0.0343) = 1.71 \text{ mA}$$

Then $r_\pi = \dfrac{(50)(0.026)}{1.71} = 0.760$ k$\Omega$

$$g_m = \frac{1.71}{0.026} = 65.77 \text{ mA/V}$$

a.



To determine $R_{if}$:

$$I_S + \frac{V_\pi}{R_B \| r_\pi} + \frac{V_0 - (-V_\pi)}{R_F} = 0 \qquad (1)$$

$$g_m V_\pi = \frac{V_0}{R_C} + \frac{V_0 - (-V_\pi)}{R_F} \qquad (2)$$

Now from (2):

$$(65.77)V_\pi - \frac{V_\pi}{10} = V_0\left(\frac{1}{1} + \frac{1}{10}\right)$$

$$(65.67)V_\pi = V_0(1.10)$$

or

$$V_0 = (59.7)V_\pi$$

and from (1):

$$I_S + \frac{V_\pi}{100\|0.760} + \frac{V_\pi}{10} + \frac{V_0}{10} = 0$$

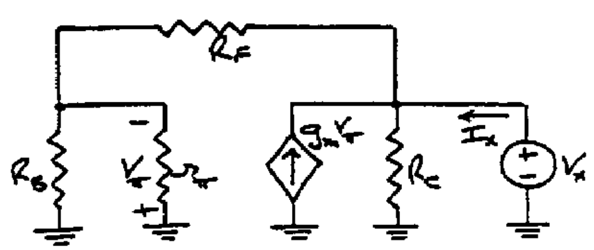
$$I_S + V_\pi(0.8543) + (0.1)(59.7)V_\pi = 0$$

$$I_S = -V_\pi(6.824)$$

Now

$$R_{if} = \frac{(-V_\pi)}{I_S} \Rightarrow \underline{R_{if} = 147\ \Omega}$$

To determine $R_{of}$:



$$I_X = \frac{V_X}{R_C} + \frac{V_X}{R_F + R_B \| r_\pi} - g_m V_\pi \qquad (3)$$

$$V_\pi = \left(\frac{-(R_B \| r_\pi)}{(R_B \| r_\pi) + R_F}\right)(V_X) \qquad (4)$$

Now

$$V_\pi = \left(\frac{-(100\|0.760)}{(100\|0.760) + 10}\right)(V_X) = -(0.07014)V_X$$

so

$$I_X = V_X\left(\frac{1}{1} + \frac{1}{10.754} + (65.77)(0.07014)\right)$$

$$R_{of} = \frac{V_X}{I_X} \Rightarrow \underline{R_{of} = 175\ \Omega}$$

b. From part (a), we find

$$V_\pi = -\frac{I_S}{6.824}$$

then

$$V_0 = (59.7)\left(\frac{-I_S}{6.824}\right)$$

or

$$\underline{\frac{V_0}{V_S} = -8.75\ \text{k}\Omega}$$

c. If capacitance is finite, a phase shift will be introduced.

12.41

dc analysis: $V_{GS} = V_{DS}$

$$I_D = \frac{V_{DD} - V_{GS}}{R_D} = K_n(V_{GS} - V_{TN})^2$$

$$10 - V_{GS} = (0.20)(8)(V_{GS} - 2)^2$$

$$10 - V_{GS} = 1.6(V_{GS}^2 - 4V_{GS} + 4)$$

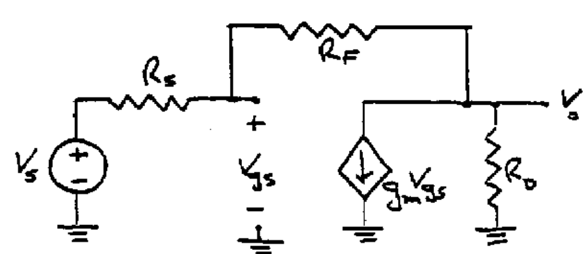
$$1.6V_{GS}^2 - 5.4V_{GS} - 3.6 = 0$$

$$V_{GS} = \frac{5.4 \pm \sqrt{(5.4)^2 + 4(1.6)(3.6)}}{2(1.6)} = 3.95\ \text{V}$$

$$I_D = \frac{10 - 3.95}{8} = 0.756\ \text{mA}$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.2)(0.756)}$$

$$\Rightarrow g_m = 0.778\ \text{mA/V}$$



a.

$$\frac{V_{gs} - V_S}{R_S} + \frac{V_{gs} - V_0}{R_F} = 0$$

$$V_{gs}\left(\frac{1}{R_S} + \frac{1}{R_F}\right) = \frac{V_S}{R_S} + \frac{V_0}{R_F} \qquad (1)$$

$$\frac{V_0}{R_D} + g_m V_{gs} + \frac{V_0 - V_{gs}}{R_F} = 0$$

$$V_0\left(\frac{1}{R_D} + \frac{1}{R_F}\right) = V_{gs}\left(\frac{1}{R_F} - g_m\right) \qquad (2)$$

So from (1):

$$V_{gs}\left(\frac{1}{10} + \frac{1}{100}\right) = \frac{V_S}{10} + \frac{V_0}{100}$$

or

$$V_{gs}(0.11) = V_S(0.10) + V_0(0.010)$$
$$V_{gs} = V_S(0.909) + V_0(0.0909)$$

Then from (2):

$$V_0\left(\frac{1}{8} + \frac{1}{100}\right) = V_{gs}\left(\frac{1}{100} - 0.778\right)$$
$$V_0(0.135) = V_{gs}(-0.768)$$
$$= (-0.768)[V_S(0.909) + V_0(0.0909)]$$

$$V_0(0.2048) = -V_S(0.6981)$$

so

$$\underline{A_v = \frac{V_0}{V_S} = -3.41}$$

b. We have

$$V_{gs} = V_S(0.909) + V_0(0.0909)$$
$$= V_S(0.909) + (0.0909)(-3.41V_S)$$
$$= 0.599V_S$$

Now

$$A_{zf} = \frac{V_0}{I_S} = \frac{V_0}{\frac{V_S - V_{gs}}{R_S}} = \frac{(-3.41V_S)R_S}{V_S - 0.599V_S}$$

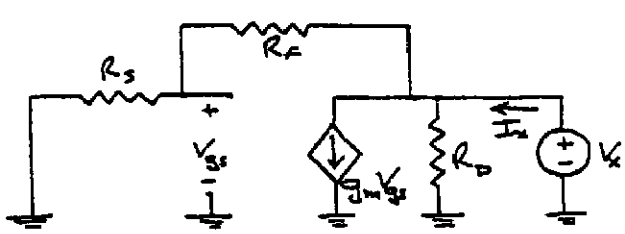
or

$$A_{zf} = \frac{(-3.41)(10)}{0.401} \Rightarrow \underline{A_{zf} = -85.0\ \text{V/ma}}$$

c. $$R_{if} = \frac{V_{gs}}{I_S} = \frac{V_{gs}}{\frac{0.401V_S}{R_S}} = \frac{0.599V_S}{0.401V_S}\cdot(10)$$

$$\Rightarrow \underline{R_{if} = 14.9\ \text{k}\Omega}$$

d.




$$I_X = \frac{V_X}{R_D} + g_m V_{gs} + \frac{V_X}{R_S + R_F}$$
$$V_{gs} = \left(\frac{R_S}{R_S + R_F}\right)V_X = \left(\frac{10}{10 + 100}\right)V_X$$
$$= (0.0909)V_X$$

$$I_X = V_X\left[\frac{1}{8} + (0.778)(0.0909) + \frac{1}{10 + 100}\right]$$
$$\frac{I_X}{V_X} = \frac{1}{R_{0f}} = 0.2048 \Rightarrow \underline{R_{0f} = 4.88\ \text{k}\Omega}$$

12.42

As $g_m \to \infty$, $\dfrac{V_0}{V_S} = \dfrac{-R_F}{R_S} = \dfrac{-100}{10} = -10$

To be within 10% of ideal,

$$\frac{V_0}{V_S} = -10(0.9) = -9$$

From Problem 12.41, we had

$$V_{gs} = V_S(0.909) + V_0(0.0909)$$
$$= V_S(0.909) + (-9V_S)(0.0909)$$
$$= 0.0909V_S$$

Also from Problem 12.41, we had

$$V_0(0.135) = V_{gs}(0.010 - g_m)$$

or

$$(-9V_S)(0.135) = (0.0909)V_S(0.010 - g_m)$$

$$-1.215 = 0.000909 - 0.0909g_m$$

or

$$\underline{g_m = 13.36\ \text{mA/V}}$$

12.43

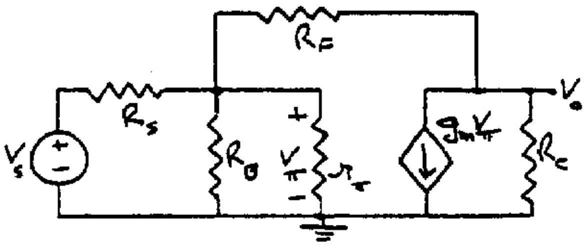
dc analysis

$$R_{TH} = 24\|150 = 20.7\ \text{k}\Omega$$
$$V_{TH} = \left(\frac{24}{24 + 150}\right)(12) = 1.655\ \text{V}$$

$$I_{BQ} = \frac{1.655 - 0.7}{20.7 + (151)(1)} = 0.00556\ \text{mA}$$

so $I_{CQ} = 0.834$ mA

$$r_\pi = \frac{(150)(0.026)}{0.834} = 4.68\ \text{k}\Omega$$
$$g_m = \frac{0.834}{0.026} = 32.08\ \text{mA/V}$$



$$\frac{V_S - V_\pi}{R_S} = \frac{V_\pi}{R_B \| r_\pi} + \frac{V_\pi - V_0}{R_F} \quad (1)$$

$$g_m V_\pi + \frac{V_0}{R_C} + \frac{V_0 - V_\pi}{R_F} = 0 \quad (2)$$

From (1):

$$\frac{V_S}{5} = V_\pi\left[\frac{1}{5} + \frac{1}{20.7\|4.68} + \frac{1}{R_F}\right] - \frac{V_0}{R_F}$$

or

$$V_S(0.20) = V_\pi\left(0.4620 + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

From (2):

$$\left(32.08 - \frac{1}{R_F}\right)V_\pi + V_0\left(\frac{1}{6} + \frac{1}{R_F}\right) = 0$$

so

$$V_\pi = \frac{-V_0\left(0.1667 + \frac{1}{R_F}\right)}{\left(32.08 - \frac{1}{R_F}\right)} \quad (2)$$

Then

$V_S(0.20)$

$$= \left(0.4620 + \frac{1}{R_F}\right)\left[\frac{-V_0\left(0.1667 + \frac{1}{R_F}\right)}{\left(32.08 - \frac{1}{R_F}\right)}\right] - \frac{V_0}{R_F}$$

Neglect the $R_F$ in the denominator term. Now

$$\frac{V_0}{V_S} = -5 \Rightarrow V_S = -\frac{V_0}{5} = -V_0(0.20)$$

$$-V_0(0.20)(0.20)R_F$$
$$= (0.4620R_F + 1)\left[\frac{-V_0(0.1667R_F + 1)}{32.08R_F}\right] - V_0$$

$$-1.283R_F^2 = -(0.4620R_F + 1)(0.1567R_F + 1) - 32.08R_F$$

$$1.206R_F^2 - 32.71R_F - 1 = 0$$

$$R_F = \frac{32.71 \pm \sqrt{(32.71)^2 + 4(1.206)(1)}}{2(1.206)}$$

so that

$$\underline{R_F = 27.2\text{ k}\Omega}$$

12.44

dc analysis

$R_{TH} = 4\|15 = 3.16\text{ k}\Omega = R_B$

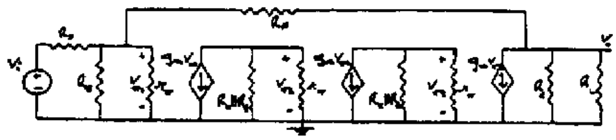
$$V_{TH} = \left(\frac{4}{4+15}\right)12 = 2.526\text{ V}$$

$$I_{BQ} = \frac{2.526 - 0.7}{3.16 + (181)(4)} = 0.00251$$

$I_{CQ} = 0.452\text{ mA}$

$$r_\pi = \frac{(180)(0.026)}{0.452} = 10.4\text{ k}\Omega$$

$$g_m = \frac{0.452}{0.026} = 17.4\text{ mA/V}$$



$$\frac{V_i - V_{\pi1}}{R_S} = \frac{V_{\pi1}}{R_B\|r_\pi} + \frac{V_{\pi1} - V_0}{R_F} \quad (1)$$

$$g_m V_{\pi1} + \frac{V_{\pi2}}{R_C\|R_B\|r_\pi} = 0 \quad (2)$$

$$g_m V_{\pi2} + \frac{V_{\pi3}}{R_C\|R_B\|r_\pi} = 0 \quad (3)$$

$$g_m V_{\pi3} + \frac{V_0}{R_C} + \frac{V_0}{R_L} + \frac{V_0 - V_{\pi1}}{R_F} = 0 \quad (4)$$

Now

$R_C\|R_B\|r_\pi = 8\|3.16\|10.4 = 1.86\text{ k}\Omega$

$R_B\|r_\pi = 3.16\|10.4 = 2.42\text{ k}\Omega$

Now substituting in (2):

$$(17.4)V_{\pi1} + \frac{V_{\pi2}}{1.86} = 0 \text{ or } V_{\pi2} = -(32.36)V_{\pi1}$$

and substituting in (3):

$$(17.4)V_{\pi2} + \frac{V_{\pi3}}{1.86} = 0$$

$$(17.4)[-(32.36)V_{\pi1}] + \frac{V_{\pi3}}{1.86} = 0$$

or $V_{\pi3} = (1047.3)V_{\pi1}$

Substitute numerical values in (1):

$$\frac{V_i}{10} = V_{\pi 1}\left(\frac{1}{10} + \frac{1}{2.42} + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

or

$$V_i(0.10) = V_{\pi 1}\left(0.513 + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

Substitute numerical values in (4):

$$(17.4)(1047.3)V_{\pi 1} + V_0\left(\frac{1}{8} + \frac{1}{4} + \frac{1}{R_F}\right) - \frac{V_{\pi 1}}{R_F} = 0$$

$$V_{\pi 1}\left(1.822 \times 10^4 - \frac{1}{R_F}\right) + V_0\left(0.375 + \frac{1}{R_F}\right) = 0$$

$$V_{\pi 1} = \frac{-V_0\left(0.375 + \frac{1}{R_F}\right)}{1.822 \times 10^4 - \frac{1}{R_F}}$$

so that

$$V_i(0.10)$$

$$= \left(0.513 + \frac{1}{R_F}\right)\left[\frac{-V_0\left(0.375 + \frac{1}{R_F}\right)}{1.822 \times 10^4 - \frac{1}{R_F}}\right] - \frac{V_0}{R_F}$$

We have $\frac{V_0}{V_i} = -80$ or $V_i = -\frac{V_0}{80}$

$$-\frac{(0.10)}{80}$$

$$= \left(0.513 + \frac{1}{R_F}\right)\left[\frac{-\left(0.375 + \frac{1}{R_F}\right)}{1.822 \times 10^4 - \frac{1}{R_F}}\right] - \frac{1}{R_F}$$

Neglect the $1/R_F$ term in the denominator.

$$-(0.00125R_F) = -\frac{(0.513R_F + 1)(0.375R_F + 1)}{1.822 \times 10^4 R_F} - 1$$

$$22.775R_F^2$$
$$= (0.513R_F + 1)(0.375R_F + 1) + 1.822 \times 10^4 R_F$$

We find

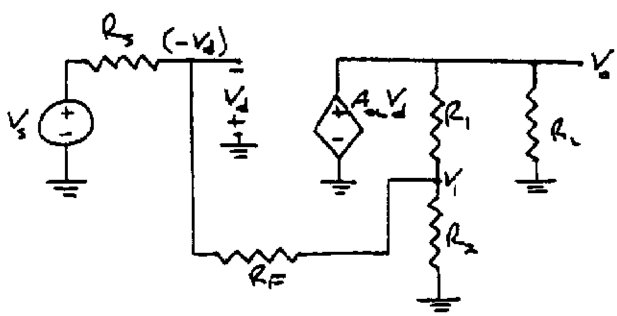
$$22.58R_F^2 - 1.822 \times 10^4 R_F - 1 = 0$$

$$R_F = \frac{1.822 \times 10^4 \pm \sqrt{(1.822 \times 10^4)^2 + 4(22.58)(1)}}{2(22.58)}$$

or

$$\underline{R_F = 0.807\ \text{M}\Omega}$$

12.45




a.

$$\frac{V_S - (-V_d)}{R_S} = \frac{-V_d - V_1}{R_F}$$

or

$$V_d\left(\frac{1}{R_S} + \frac{1}{R_F}\right) + \frac{V_S}{R_S} + \frac{V_1}{R_F} = 0 \qquad (1)$$

$$\frac{V_0 - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1 - (-V_d)}{R_F}$$

or

$$\frac{V_0}{R_1} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F}\right) + \frac{V_d}{R_F} \qquad (2)$$

and $V_0 = A_{OL}V_d$ or $V_d = \frac{V_0}{A_{OL}}$

Substitute numerical values in (1) and (2):

$$\frac{V_0}{10^4}\cdot\left(\frac{1}{5} + \frac{1}{10}\right) + \frac{V_S}{5} + \frac{V_1}{10} = 0$$

or

$$V_0(0.3 \times 10^{-4}) + V_S(0.20) + V_1(0.10) = 0 \qquad (1)$$

$$\frac{V_0}{50} = V_1\left(\frac{1}{50} + \frac{1}{10} + \frac{1}{10}\right) + \frac{V_0}{10^4}\cdot\left(\frac{1}{10}\right)$$

or

$$V_0(0.02 - 10^{-5}) = V_1(0.22) \qquad (2)$$

Then $V_1 = V_0\left(\frac{0.02 - 10^{-5}}{0.22}\right)$

and

$$V_0(0.3 \times 10^{-4}) + V_S(0.20)$$
$$+ (0.10)\left[V_0\left(\frac{0.02 - 10^{-5}}{0.22}\right)\right] = 0$$

$$V_0\left[0.3\times10^{-4} - 0.4545\times10^{-5} + 0.00909\right] + V_S(0.20) = 0$$

Then $\dfrac{V_0}{V_S} = \dfrac{-0.20}{9.115\times10^{-3}} \Rightarrow \underline{\dfrac{V_0}{V_S} = -21.94}$

b. $R_{if} = \dfrac{-V_d}{\dfrac{V_S-(-V_d)}{R_S}} = \dfrac{-V_d\cdot R_S}{V_S+V_d}$

Now $V_d = \dfrac{V_0}{A_{0L}} = -\dfrac{21.94V_S}{10^4}$

Then $R_{if} = \dfrac{(21.94\times10^{-4})(5)}{1-21.94\times10^{-4}}$

or $R_{if} = 1.099\times10^{-2}\ \text{k}\Omega$

$\Rightarrow \underline{R_{if} = 10.99\ \Omega}$

c. Because of the $A_{0L}V_d$ source,

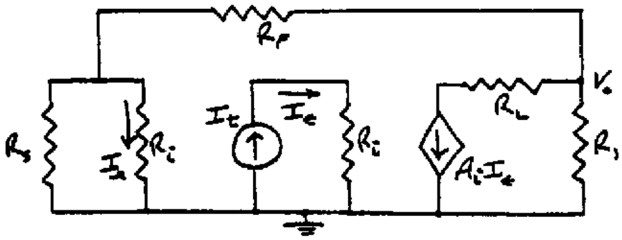
$\underline{R_{0f} = 0}$

12.46

For example, use the circuit shown in Figure 12.41

12.47

Break the loop



$I_t = I_\epsilon$

Now $A_iI_t + \dfrac{V_0}{R_1} + \dfrac{V_0}{R_F + R_S\|R_i} = 0$

$I_r = \left(\dfrac{R_S}{R_S+R_i}\right)\cdot\dfrac{V_0}{R_F+R_S\|R_i}$

or $V_0 = I_r\left(\dfrac{R_S+R_i}{R_S}\right)\cdot(R_F+R_S\|R_i)$

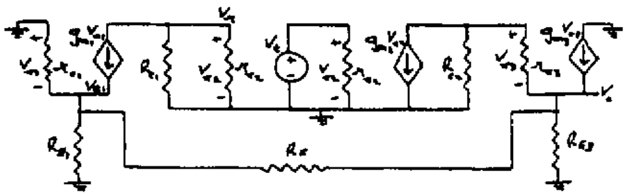
Then

$$A_iI_t + \left(\frac{1}{R_1} + \frac{1}{R_F+R_S\|R_i}\right) \times\left[I_r\left(\frac{R_S+R_i}{R_S}\right)(R_F+R_S\|R_i)\right] = 0$$

$T = -\dfrac{I_r}{I_t} \Rightarrow$

$$\underline{T = \frac{A_i}{\left[\dfrac{1}{R_1}+\dfrac{1}{R_F+R_S\|R_i}\right]\left(\dfrac{R_S+R_i}{R_S}\right)(R_F+R_S\|R_i)}}$$

12.48



$$\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1}-V_0}{R_F} \qquad (1)$$

$$g_{m1}V_{\pi1} + \frac{V_r}{R_{C1}\|r_{\pi2}} = 0$$

$$\Rightarrow V_r = -(g_{m1}V_{\pi1})(R_{C1}\|r_{\pi2}) \qquad (2)$$

$V_{\pi2} = V_t$ so that

$$g_{m2}V_t + \frac{V_{\pi3}+V_0}{R_{C2}} + \frac{V_{\pi3}}{r_{\pi3}} = 0 \qquad (3)$$

$$\frac{V_{\pi3}}{r_{\pi3}} + g_{m3}V_{\pi3} = \frac{V_0}{R_{E3}} + \frac{V_0-V_{e1}}{R_F} \qquad (4)$$

From (4):

$$V_0\left(\frac{1}{R_{E3}}+\frac{1}{R_F}\right) = V_{\pi3}\left(\frac{1}{r_{\pi3}}+g_{m3}\right) + \frac{V_{e1}}{R_F}$$

But $V_{e1} = -V_{\pi1}$

$$\text{so } V_0 = \frac{V_{\pi3}\left(\dfrac{1}{r_{\pi3}}+g_{m3}\right) - \dfrac{V_{\pi1}}{R_F}}{\left(\dfrac{1}{R_{E3}}+\dfrac{1}{R_F}\right)}$$

Then

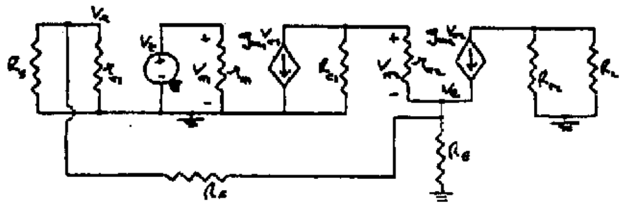
$$V_{\pi1}\left[\left(\frac{1}{r_{\pi1}}+g_{m1}\right) - \left(\frac{1}{R_{E1}}+\frac{1}{R_F}\right)\right] = \frac{-V_{\pi3}\left(\dfrac{1}{r_{\pi3}}+g_{m3}\right)+\dfrac{V_{\pi1}}{R_F}}{R_F\cdot\left(\dfrac{1}{R_{E3}}+\dfrac{1}{R_F}\right)} \qquad (1')$$

and

$$g_{m2}V_t + V_{\pi3}\left(\frac{1}{R_{C2}}+\frac{1}{r_{\pi3}}\right) + \frac{V_{\pi3}\left(\dfrac{1}{r_{\pi3}}+g_{m3}\right)-\dfrac{V_{\pi1}}{R_F}}{R_{C2}\cdot\left(\dfrac{1}{R_{E3}}+\dfrac{1}{R_F}\right)} = 0 \qquad (3')$$

From $(3')$, solve for $V_{\pi3}$ and substitute into $(1')$. Then from $(1')$, solve for $V_{\pi1}$ and substitute into (2). Then $T = -\dfrac{V_r}{V_t}$.

12.49



$$\frac{V_r}{R_S}+\frac{V_r}{r_{\pi 1}}+\frac{V_r-V_e}{R_F}=0 \qquad (1)$$

$$g_{m1}V_t+\frac{V_{\pi 2}+V_e}{R_{C1}}+\frac{V_{\pi 2}}{r_{\pi 2}}=0 \qquad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}}+g_{m2}V_{\pi 2}=\frac{V_e}{R_E}+\frac{V_e-V_r}{R_F} \qquad (3)$$

Using the parameters from Problem 12.29, we obtain

$$V_r\left(\frac{1}{10}+\frac{1}{15.8}+\frac{1}{10}\right)-\frac{V_e}{10}=0$$

or

$$V_r(0.2633)=V_e(0.10) \qquad (1)$$

$$(7.62)V_t+V_{\pi 2}\left(\frac{1}{40}+\frac{1}{2.28}\right)+\frac{V_e}{40}=0$$

or

$$V_t(7.62)+V_{\pi 2}(0.4636)+V_e(0.025)=0 \qquad (2)$$

$$V_{\pi 2}\left(\frac{1}{2.28}+52.7\right)=V_e\left(\frac{1}{1}+\frac{1}{10}\right)-\frac{V_r}{10}$$

or

$V_{\pi 2}(53.14)=V_e(1.10)-V_r(0.10)$

Then

$$V_{\pi 2}=V_e(0.0207)-V_r(0.001882) \qquad (3)$$

Substituting in (2):

$$V_t(7.62)+(0.4636)[V_e(0.0207)-V_r(0.001882)]+V_e(0.025)=0$$

or

$$V_t(7.62)+V_e(0.03460)-V_r(0.0008725)=0$$

From (1) $V_e = V_r(2.633)$

Then

$$V_t(7.62)+V_r(2.633)(0.03460)-V_r(0.0008725)=0$$

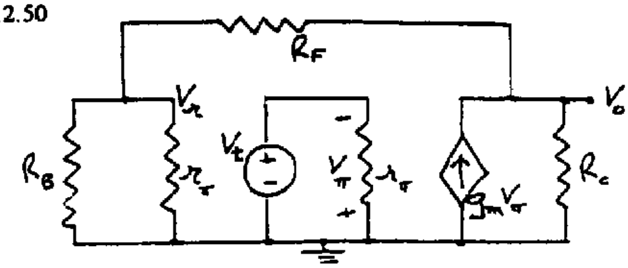
$$V_t(7.62)+V_r(0.09023)=0$$

or $\dfrac{V_r}{V_t}=-84.45$

Now

$$T=-\frac{V_r}{V_t}\Rightarrow \underline{T=84.45}$$

12.50




$V_\pi=-V_t$

$$g_mV_\pi=\frac{V_0}{R_C}+\frac{V_0}{R_F+R_B\|r_\pi} \qquad (1)$$

and

$$V_r=\left(\frac{R_B\|r_\pi}{R_B\|r_\pi+R_F}\right)V_0 \qquad (2)$$

Now

$$(65.77)V_\pi=V_0\left(\frac{1}{1}+\frac{1}{10+100\|0.760}\right)$$

or $(65.77)V_\pi=V_0(1.0930)$

and

$$V_r=\left(\frac{0.754}{10+0.754}\right)V_0=(0.07011)V_0$$

so $V_0=(14.26)V_r$

Then $(65.77)(-V_t)=(14.26)V_r(1.0930)$

$\dfrac{V_r}{V_t}=-4.22$ so that $\underline{T=4.22}$

12.51

a. $$\phi=-\tan^{-1}\left(\frac{f}{5\times 10^2}\right)-2\tan^{-1}\left(\frac{f}{10^4}\right)$$

or

$$-180=-\tan^{-1}\left(\frac{f_{180}}{5\times 10^2}\right)-2\tan^{-1}\left(\frac{f_{180}}{10^4}\right)$$

$$\Rightarrow \underline{f_{180}\approx 1.05\times 10^4\ \text{Hz}}$$

b.

$|T(f_{180})|=1$

$$=\frac{\beta(10^5)}{\sqrt{1+\left(\frac{1.05\times 10^4}{5\times 10^2}\right)^2}\left[1+\left(\frac{1.05\times 10^4}{10^4}\right)^2\right]}$$

$$1=\frac{\beta(10^5)}{(21.02)(2.105)} \qquad \text{or}$$

$$\underline{\beta=4.42\times 10^{-4}}$$

12.52

$$A=\frac{5x10^3}{\left(1+j\frac{f}{10^4}\right)\left(1+j\frac{f}{10^5}\right)^2}$$

Phase $=\phi=-\tan^{-1}\left(\frac{f}{10^4}\right)-2\tan^{-1}\left(\frac{f}{10^5}\right)$

By trial and error, when $f=1.095x10^5\ Hz$,
$\phi\cong 180^\circ$

For $|T|=1$ at $f=1.095x10^5\ Hz$,

$$1=\frac{\beta(5x10^3)}{\sqrt{1+\left(\frac{f}{10^4}\right)^2}\cdot\left[1+\left(\frac{f}{10^5}\right)^2\right]}\Rightarrow$$

$$1=\frac{\beta(5x10^3)}{(10.996)(2.199)}\Rightarrow \quad \underline{\beta=4.84x10^{-3}}$$

12.53

$$\phi=-\tan^{-1}\left(\frac{f}{10^4}\right)-\tan^{-1}\left(\frac{f}{5\times10^4}\right)-\tan^{-1}\left(\frac{f}{10^5}\right)$$

At $f=8.1\times10^4$ Hz, $\phi=-180.28^\circ$

Determine $|T(f)|$ at this frequency.

$|T|$

$$=\beta(10^3)\times\frac{1}{\sqrt{1+\left(\frac{8.1\times10^4}{10^4}\right)^2}}\times\frac{1}{\sqrt{1+\left(\frac{8.1\times10^4}{5\times10^4}\right)^2}}\times\frac{1}{\sqrt{1+\left(\frac{8.1\times10^4}{10^5}\right)^2}}$$

$$=\frac{\beta(10^3)}{(8.161)(1.904)(1.287)}$$

a. For $\beta=0.005$

$|T(f)|=0.250<1\Rightarrow$ Stable

b. For $\beta=0.05$

$|T(f)|=2.50>1\Rightarrow$ Unstable

12.54

(b)Phase margin $=80^\circ\Rightarrow\phi=-100^\circ$

$$\phi=-100=-2\tan^{-1}\left(\frac{f}{10^3}\right)-\tan^{-1}\left(\frac{f}{5x10^4}\right)$$

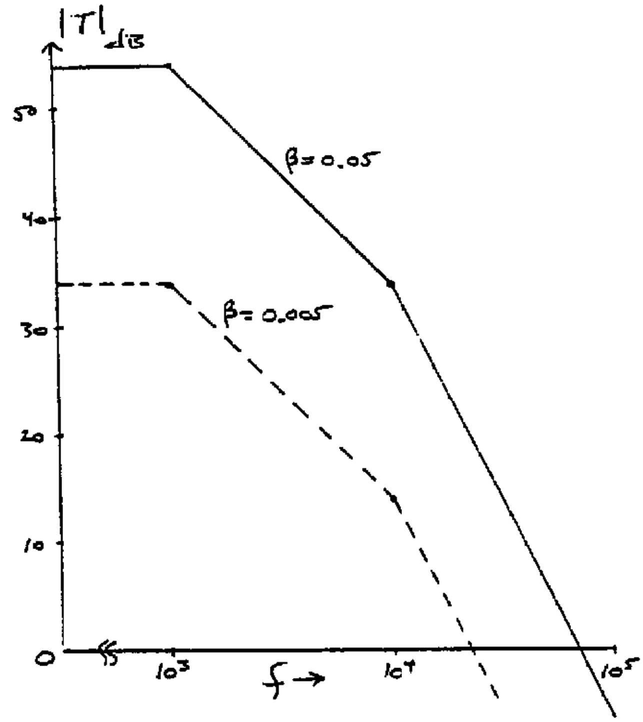
By trial and error, $f=1.16x10^3\ Hz$

Then

$$|T|=1=\frac{\beta(5x10^3)}{\left(\sqrt{1+\left(\frac{1.16x10^3}{10^3}\right)^2}\right)^2\cdot\sqrt{1+\left(\frac{1.16x10^3}{5x10^4}\right)^2}}$$

$$=\frac{\beta(5x10^3)}{(2.35)(1.00)}\Rightarrow\quad\underline{\beta=4.7x10^{-4}}$$

12.55




c. For $\beta=0.005$,

$|T(f)|=1$ (0 dB) at $f\approx2.24\times10^4$ Hz

Then

$$\phi=-\tan^{-1}\left(\frac{2.24\times10^4}{10^3}\right)-\tan^{-1}\left(\frac{2.24\times10^4}{10^4}\right)-\tan^{-1}\left(\frac{2.24\times10^4}{10^5}\right)$$

$$=-87.44-65.94-12.63$$

or

$\phi=-166^\circ$ System is stable.

Phase margin $=14^\circ$

For $\beta = 0.05$,

$|T(f)| = 1$ (0 dB) at $f \approx 7.08 \times 10^4$ Hz

Then

$$\phi = -\tan^{-1}\left(\frac{7.08 \times 10^4}{10^3}\right) - \tan^{-1}\left(\frac{7.08 \times 10^4}{10^4}\right) - \tan^{-1}\left(\frac{7.08 \times 10^4}{10^5}\right)$$

$$= -89.19 - 81.96 - 35.30$$

or

$\phi = -206.45° \Rightarrow$ System is unstable.

12.56

$$T = A\beta = \frac{\beta(10^5)}{\left(1+j\frac{f}{5x10^4}\right)\left(1+j\frac{f}{10^5}\right)\left(1+j\frac{f}{5x10^5}\right)}$$

Phase Margin $= 60° \Rightarrow \phi = -120°$

So

$$-120 = -\tan^{-1}\left(\frac{f}{5x10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{5x10^5}\right)$$

By trial and error, at $f = 10^5\ Hz$, $\phi \cong -120°$

Then

$$|T| = 1 = \frac{\beta(10^5)}{\sqrt{1+\left(\frac{10^5}{5x10^4}\right)^2} \cdot \sqrt{1+\left(\frac{10^5}{10^5}\right)^2} \cdot \sqrt{1+\left(\frac{10^5}{5x10^5}\right)^2}}$$

$$1 = \frac{\beta(10^5)}{(2.236)(1.414)(1.02)} \Rightarrow \quad \underline{\beta = 3.22x10^{-5}}$$

12.57

a. Phase Margin $= 60° \Rightarrow \phi = -120°$

Then

$$\phi = -120° = -2\tan^{-1}\left(\frac{f}{10^3}\right)$$

or $f = 1.732 \times 10^3$ Hz

Then

$$|T(f)| = 1 = \frac{\beta(10^3)}{\left[1+\left(\frac{1.732 \times 10^3}{10^3}\right)^2\right]}$$

which yields

$\underline{\beta = 4 \times 10^{-3}}$

12.58

$$T(0) = A(0)\beta = (500)(0.6) = 300$$

$$T(f) = \frac{300}{\left(1+j\frac{f}{10^4}\right)\left(1+j\frac{f}{10^5}\right)\left(1+j\frac{f}{10^6}\right)}$$

Find $f$ at which $|T = 1|$

$$1 = \frac{300}{\sqrt{1+\left(\frac{f_1}{10^4}\right)^2} \cdot \sqrt{1+\left(\frac{f_1}{10^5}\right)^2} \cdot \sqrt{1+\left(\frac{f_1}{10^6}\right)^2}}$$

By trial and error, $f_1 = 5.12x10^5\ Hz$

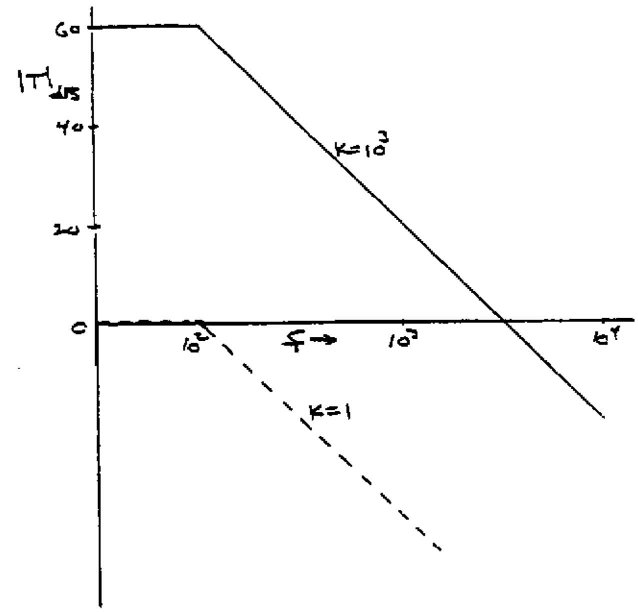
Then

$$\phi = -\tan^{-1}\left(\frac{f_1}{10^4}\right) - \tan^{-1}\left(\frac{f_1}{10^5}\right) - \tan^{-1}\left(\frac{f_1}{10^6}\right)$$

$$= -88.88 - 78.95 - 27.1 = -194.9°$$

System is unstable, Phase margin is not defined.

12.59




12.60

Phase Margin $= 45° \Rightarrow \phi = -135°$

$$\phi = -135° = -\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^6}\right)$$

At $f = 10^4$ Hz, $\phi = -135.6°$

$|T| = 1 =$

$$= \beta(10^3) \times \frac{1}{\sqrt{1+\left(\frac{10^4}{10^3}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{10^4}{10^4}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{10^4}{10^5}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{10^4}{10^6}\right)^2}}$$

$$1 = \frac{\beta(10^3)}{(10.05)(1.414)(1.005)(1.00)}$$

or

$$\underline{\beta = 0.01428}$$

12.61

$$T = 5000 \times \frac{1}{\left(1+j\frac{f}{f_{PD}}\right)} \times \frac{1}{\left(1+j\frac{f}{300\times10^3}\right)} \times \frac{1}{\left(1+j\frac{f}{2\times10^6}\right)} \times \frac{1}{\left(1+j\frac{f}{25\times10^6}\right)}$$

Phase Margin $= 45° \Rightarrow \phi = -135°$ at $f = 300$ kHz

$$-135° = -\tan^{-1}\left(\frac{300\times10^3}{f_{PD}}\right) - \tan^{-1}\left(\frac{300\times10^3}{300\times10^3}\right) - 0 - 0$$

$$= -90° - 45°$$

Now

$|T| = 1 @ f = 300$ kHz

$$|T| = 1 \approx \frac{5000}{\sqrt{1+\left(\frac{300\times10^3}{f_{PD}}\right)^2}\cdot\sqrt{2}\cdot1\cdot1}$$

$$1+\left(\frac{300\times10^3}{f_{PD}}\right)^2 = \left(\frac{5000}{\sqrt{2}}\right)^2$$

$$f_{PD} \approx \frac{300\times10^3\sqrt{2}}{5000}$$

$$\Rightarrow \underline{f_{PD} = 84.8 \text{ Hz}}$$

12.62

a. $T(0) = 100$ dB $\Rightarrow T(0) = 10^5$

$T(f)$

$$= \frac{10^5}{\left(1+j\frac{f}{10}\right)\left(1+j\frac{f}{5\times10^6}\right)\left(1+j\frac{f}{10\times10^6}\right)}$$

$|T| = 1 =$

$$= 10^5 \times \frac{1}{\sqrt{1+\left(\frac{f}{10}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{5\times10^6}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{10\times10^6}\right)^2}}$$

By trial and error

$$\underline{f = 0.976 \text{ MHz}}$$

$$\phi = -\tan^{-1}\left(\frac{0.976\times10^6}{10}\right) - \tan^{-1}\left(\frac{0.976}{5}\right) - \tan^{-1}\left(\frac{0.976}{10}\right)$$

$$= -90° - 11.05° - 5.574° = -106.6°$$

Phase Margin $= 180° - 106.6° = \underline{73.4°}$

b. $f'_{P1} \propto \frac{1}{C_F}$ so $\frac{10}{f'_{P1}} = \frac{75}{20}$

or

$$\underline{f'_{P1} = 2.67 \text{ Hz}}$$

Now

$|T| = 1 =$

$$= 10^5 \times \frac{1}{\sqrt{1+\left(\frac{f}{2.67}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{5\times10^6}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{10\times10^6}\right)^2}}$$

By trial and error

$$f \approx 2.66\times10^5 \text{ Hz}$$

then

$$\phi = -\tan^{-1}\left(\frac{2.66\times10^5}{2.67}\right) - \tan^{-1}\left(\frac{0.266}{5}\right) - \tan^{-1}\left(\frac{0.266}{10}\right)$$

$$= -90° - 3.045° - 1.524° = -94.57°$$

Phase Margin $= 180° - 94.57° = \underline{85.4°}$

12.63

(a) $f_{3-dB} = \dfrac{1}{2\pi\tau}$ where $\tau = (R_{o1} \| R_{i2})C_i$

$= (500\|1000)x10^3 x2x10^{-12} \Rightarrow \quad \tau = 6.67x10^{-7}\ s$

Then

$f_{3-dB} = \dfrac{1}{2\pi(6.67x10^{-7})} \Rightarrow \quad \underline{f_{3-dB} = 239\ kHz}$

(b) For

$f_{PD} = 10\ Hz, \quad \tau = \dfrac{1}{2\pi f_{PD}} = \dfrac{1}{2\pi(10)} = 0.0159\ s$

Then $\tau = (R_{o1} \| R_{i2})(C_i + C_M)$

$0.0159 = (500\|1000)x10^3 x(C_i + C_M)$

or

$(C_i + C_M) = 4.77x10^{-8} = 2x10^{-12} + C_M \Rightarrow$

$\underline{C_M = 477\ \mu F}$

12.64

Want $f_1 = 12\ MHz$ for a phase margin of $45°$

$T_{dB}(0) = 80\ dB \Rightarrow T(0) = 10^4$

Then

$$T(f) = \frac{T(0)}{\left(1 + j\dfrac{f}{f_{PD}}\right)\left(1 + j\dfrac{f}{12x10^6}\right)}$$

Set $f = f_1$ and $|T| = 1$

So

$$|T| = 1 = \frac{10^4}{\sqrt{1 + \left(\dfrac{12x10^6}{f_{PD}}\right)^2} \cdot \sqrt{2}}$$

which yields

$\dfrac{12x10^6}{f_{PD}} = \dfrac{10^4}{\sqrt{2}} \Rightarrow \quad \underline{f_{PD} = 1.70\ kHz}$

12.65

$A_0 = 80\ \text{dB} \Rightarrow A_0 = 10^4$

$A_f(0) = \dfrac{A_0}{1 + \beta A_0}$

or $5 = \dfrac{10^4}{1 + \beta(10^4)} \Rightarrow \beta \approx 0.2$

Then $T(0) = \beta A_0 = 0.2 \times 10^4$

Inserting a dominate pole

$$\phi = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - \tan^{-1}\left(\frac{f}{10^6}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$$

If we want a phase margin of 45°, then

$$-135° \approx -90° - \tan^{-1}\left(\frac{f}{10^6}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$$

By trial and error, $f \approx 0.845$ MHz

Then

$$|T| = 1 = \frac{0.2 \times 10^4}{\sqrt{1 + \left(\dfrac{0.845 \times 10^6}{f_{PD}}\right)^2}\sqrt{1 + \left(\dfrac{0.845}{1}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\dfrac{0.845}{10}\right)^2}}$$

$\dfrac{0.845 \times 10^6}{f_{PD}} \approx \dfrac{0.2 \times 10^4}{(1.309)(1.0036)}$

so $\underline{f_{PD} = 555\ \text{Hz}}$

12.66

Assuming a phase margin of 45°,

$$-135° \approx -90° - \tan^{-1}\left(\frac{f}{2 \times 10^6}\right) - \tan^{-1}\left(\frac{f}{25 \times 10^6}\right)$$

By trial and error, $f \approx 1.74$ MHz

Then

$$|T| = 1 = 5000 \times \frac{1}{\sqrt{1 + \left(\dfrac{1.74 \times 10^6}{f_{PD}}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\dfrac{1.74}{2}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\dfrac{1.74}{25}\right)^2}}$$

or

$\dfrac{1.74 \times 10^6}{f_{PD}} \approx \dfrac{5000}{(1.325)(1.0024)}$

so $\underline{f_{PD} = 462\ \text{Hz}}$