Chapter 6

Exercise Solutions

E6.1

$g_m = 2K_n(V_{GS} - V_{TN})$ and

$I_D = K_n(V_{GS} - V_{TN})^2$

$0.75 = 0.5(V_{GS} - 0.8)^2 \Rightarrow V_{GS} = 2.025\,V$

$g_m = 2(0.5)(2.025 - 0.8) \Rightarrow \underline{g_m = 1.22\,mA/V}$

$r_o = \left[\lambda K_n (V_{GS} - V_{TN})^2\right]^{-1}$

$= \left[(0.01)(0.5)(2.025 - 0.8)^2\right]^{-1} \Rightarrow$

$\underline{r_o = 133\,k\Omega}$

E6.2

$g_m = 2K_n(V_{GS} - V_{TN})$ and $I_D = K_n(V_{GS} - V_{TN})^2$

$\Rightarrow V_{GS} - V_{TN} = \sqrt{\dfrac{I_{DQ}}{K_n}}$ and

$g_m = 2K_n\sqrt{\dfrac{I_{DQ}}{K_n}} = 2\sqrt{K_n I_{DQ}}$

$K_n = \dfrac{g_m^2}{4I_{DQ}} = \dfrac{(3.4)^2}{4(2)} = 1.45\,mA/V$

$K_n = \dfrac{\mu_n C_{ox}}{2} \cdot \dfrac{W}{L}$

$1.45 = (0.018)\left(\dfrac{W}{L}\right) \Rightarrow \underline{\dfrac{W}{L} = 80.6}$

$r_o = \left[\lambda K_n (V_{GS} - V_{TN})^2\right]^{-1} = \left[\lambda I_{DQ}\right]^{-1}$

$r_o = \left[(0.015)(2)\right]^{-1} \Rightarrow \underline{r_o = 33.3\,k\Omega}$

E6.3

a. $I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$0.4 = 0.5(V_{GS} - 2)^2 \Rightarrow \underline{V_{GS} = 2.89\text{ V}}$

$V_{DSQ} = V_{DD} - I_{DQ}R_D = 10 - (0.4)(10)$

$\Rightarrow \underline{V_{DSQ} = 6\text{ V}}$

b. $g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(2.89 - 2)$

$\Rightarrow \underline{g_m = 0.89\text{ mA/V}}$

$r_o = [\lambda I_{DQ}]^{-1}, \quad \lambda = 0 \Rightarrow \underline{r_o = \infty}$

$A_v = \dfrac{v_0}{v_i} = -g_m R_D = -(0.89)(10)$

$\Rightarrow \underline{A_v = -8.9}$

c. $v_i = 0.4 \sin \omega t \Rightarrow v_{ds} = -(8.9)(0.4) \sin \omega t$

$v_{ds} = -3.56 \sin \omega t$

At $V_{DS1} = 6 - 3.56 = 2.44$

$V_{GS1} = 2.89 + 0.4 = 3.29$

$V_{GS1} - V_{TN} = 3.29 - 2 = 1.29$

So $V_{DS1} > V_{GS1} - V_{TN} \Rightarrow$ Biased in saturation region

E6.4

a. $V_{SDQ} = V_{DD} - I_{DQ}R_D$

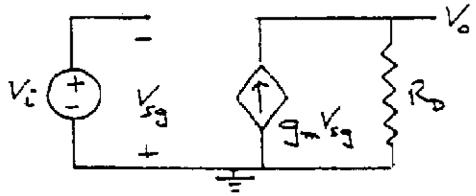
$7 = 12 - I_{DQ}(6) \Rightarrow I_{DQ} = 0.833\text{ mA}$

$I_{DQ} = K_p\left(V_{SG} - |V_{TP}|\right)^2$

$0.833 = 2(V_{SG} - 1)^2 \Rightarrow \underline{V_{SG} = 1.65\text{ V}}$

b. $g_m = 2K_p\left(V_{SG} - |V_{TP}|\right) = 2(2)(1.65 - 1)$

$\Rightarrow \underline{g_m = 2.6\text{ mA/V}}. \quad \underline{r_0 = \infty}$



$A_v = \dfrac{v_0}{v_i} = -g_m R_D = -(2.6)(6)$

$\Rightarrow \underline{A_v = -15.6}$

E6.5

$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow V_{GS} - V_{TN} = \sqrt{\dfrac{I_{DQ}}{K_n}}$

$g_m = 2K_n(V_{GS} - V_{TN}) = 2K_n\sqrt{\dfrac{I_{DQ}}{K_n}}$

So $g_m = 2\sqrt{K_n I_{DQ}}$

E6.6

$\eta = \dfrac{\gamma}{2\sqrt{2\phi_f + v_{SB}}}$

(a) $\eta = \dfrac{0.40}{2\sqrt{2(0.35) + 1}} \Rightarrow \underline{\eta = 0.153}$

(b) $\eta = \dfrac{0.40}{2\sqrt{2(0.35)+3}} \Rightarrow \underline{\eta = 0.104}$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.22\ mA/V$

For (a), $g_{mb} = g_m\eta = (1.22)(0.153) \Rightarrow$
$\underline{g_{mb} = 0.187\ mA/V}$

For (b), $g_{mb} = (1.22)(0.104) \Rightarrow \underline{g_{mb} = 0.127\ mA/V}$

E6.7

a. $I_{DQ} = K_n(V_{GS} - V_{TN})^2 = (0.25)(2-0.8)^2$

$\Rightarrow \underline{I_{DQ} = 0.36\text{ mA}}$

$V_{DSQ} = V_{DD} - I_{DQ}R_D = 5 - (0.36)(5)$

$\Rightarrow \underline{V_{DSQ} = 3.2\text{ V}}$

b. $g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.25)(2-0.8)$

$\Rightarrow \underline{g_m = 0.6\text{ mA/V}}, \quad \underline{r_0 = \infty}$

c. $A_v = \dfrac{v_0}{v_i} = -g_m R_D = -(0.6)(5)$

$\Rightarrow \underline{A_v = -3.0}$

E6.8

$v_i = v_{gs} = 0.1\sin\omega t$

$i_d = g_m v_{gs} = (0.6)(0.1)\sin\omega t$

$i_d = 0.06\sin\omega t\text{ mA}$

$v_{ds} = (-3)(0.1)\sin\omega t = -0.3\sin\omega t$

Then $i_D = I_{DQ} + i_d = 0.36 + 0.06\sin\omega t$

$= i_D\text{ mA}$

$v_{DS} = V_{DSQ} + v_{ds} = \underline{3.2 - 0.3\sin\omega t = v_{DS}}$

E6.9

$V_{SDQ} = 3\text{ V}$ and $I_{DQ} = 0.5\text{ mA}$

$\Rightarrow R_D = \dfrac{5-3}{0.5} \Rightarrow \underline{R_D = 4\text{ k}\Omega}$

$I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2$

$0.5 = 1(V_{SG} - 1)^2 \Rightarrow V_{SG} = 1.71\text{ V}$

$\Rightarrow V_{GG} = 5 - 1.71 \Rightarrow \underline{V_{GG} = 3.29\text{ V}}$

$A_v = -g_m R_D$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.5)}$

$g_m = 1.41\text{ mA/V}$

$A_v = -(1.41)(4) \Rightarrow \underline{A_v = -5.64}$

$A_v = \dfrac{v_0}{v_i} = \dfrac{-v_{sd}}{v_i} = -\dfrac{0.46\sin\omega t}{v_i} = -5.64$

$\Rightarrow \underline{v_i = 0.0816\sin\omega t}$

E6.10

a. $V_{SG} = 9 - I_{DQ}R_S, \quad I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2$

$V_{SG} = 9 - (2)(1.2)(V_{SG} - 2)^2$

$= 9 - 2.4(V_{SG}^2 - 4V_{SG} + 4)$

$2.4V_{SG}^2 - 8.6V_{SG} + 0.6 = 0$

$V_{SG} = \dfrac{8.6 \pm \sqrt{(8.6)^2 - 4(2.4)(0.6)}}{2(2.4)}$

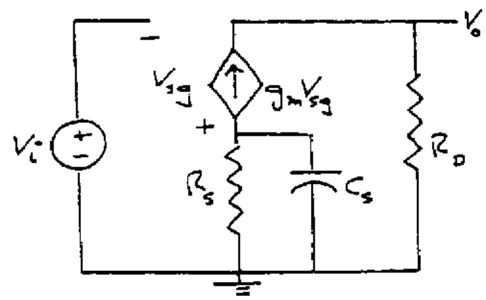
$V_{SG} = 3.51\text{ V}, \quad I_{DQ} = 2(3.51-2)^2$

$\Rightarrow \underline{I_{DQ} = 4.56\text{ mA}}$

$V_{SDQ} = 9 + 9 - I_{DQ}(1.2+1) = 18 - (4.56)(2.2)$

$\Rightarrow \underline{V_{SDQ} = 7.97\text{ V}}$

b.



$V_0 = g_m V_{SG} R_D$

$A_v = -g_m R_D = -(6.04)(1) \Rightarrow \underline{A_v = -6.04}$

E6.11

$I_{DQ} = I_Q = 0.5\text{ mA}$

Let $\dfrac{W}{L} = 25$

$K_n = (20)(25) = 500\ \mu A/V^2$

$V_{GS} = \sqrt{\dfrac{0.5}{0.5}} + 1.5 = 2.5\text{ V} \Rightarrow V_S = -2.5\text{ V}$

$A_v = -g_m R_D$

$g_m = 2(0.5)(2.5 - 1.5) = 1\text{ mA/V}$

For $A_v = -4.0 \Rightarrow R_D = 4\text{ k}\Omega$

$V_D = 5 - (0.5)(4) = 3\text{ V}$

$\Rightarrow V_{DSQ} = 3 - (-2.5) = 5.5\text{ V}$

E6.12

a. With $R_G \Rightarrow V_{GS} = V_{DS} \Rightarrow$ transistor biased in sat. region

$I_D = K_n(V_{GS} - V_{TN})^2 = K_n(V_{DS} - V_{TN})^2$

$V_{DS} = V_{DD} - I_D R_D$

$= V_{DD} - K_n R_D (V_{DS} - V_{TN})^2$

$$V_{DS} = 15 - (0.15)(10)(V_{DS} - 1.8)^2$$
$$= 15 - 1.5(V_{DS}^2 - 3.6V_{DS} + 3.24)$$
$$1.5V_{DS}^2 - 4.4V_{DS} - 10.14 = 0$$
$$V_{DS} = \frac{4.4 \pm \sqrt{(4.4)^2 + (4)(1.5)(10.14)}}{2(1.5)}$$
$$\Rightarrow \underline{V_{DSQ} = 4.45 \text{ V}}$$
$$I_{DQ} = (0.15)(4.45 - 1.8)^2 \Rightarrow \underline{I_{DQ} = 1.05 \text{ mA}}$$

b. Neglecting effect of R_G :

$$A_v = -g_m(R_D \| R_L)$$
$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.15)(4.45 - 1.8)$$
$$\Rightarrow g_m = 0.795 \text{ mA/V}$$
$$A_v = -(0.795)(10\|5) \Rightarrow \underline{A_v = -2.65}$$

c. $R_G \Rightarrow$ establishes $V_{GS} = V_{DS} \Rightarrow$ essentially no effect on small-signal voltage gain.

E6.13

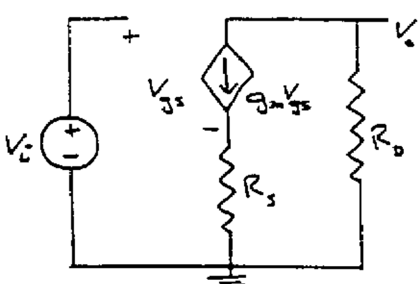
a. $I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$$I_{DQ} = 0.8(2 - V_{SG})^2 = \frac{V_{SG}}{R_S} = \frac{V_{SG}}{4}$$
$$3.2(4 - 4V_{SG} + V_{SG}^2) = V_{SG}$$
$$3.2V_{SG}^2 - 13.8V_{SG} + 12.8 = 0$$
$$V_{SG} = \frac{13.8 \pm \sqrt{(13.8)^2 - 4(3.2)(12.8)}}{2(3.2)}$$
$$V_{SG} = 1.35 \text{ V} \Rightarrow I_{DQ} = 0.8(2 - 1.35)^2$$
$$\Rightarrow \underline{I_{DQ} = 0.338 \text{ mA}}$$

b. $V_{DSQ} = V_{DD} - I_{DQ}(R_D + R_S)$

$$6 = 10 - (0.338)(R_D + 4)$$
$$R_D = \frac{10 - (0.338)(4) - 6}{0.338} \Rightarrow \underline{R_D = 7.83 \text{ k}\Omega}$$

c.



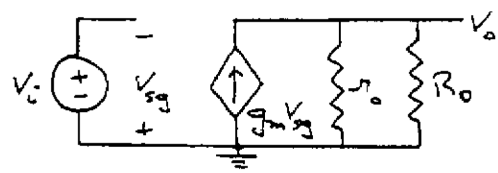
$$V_i = V_{gs} + g_m V_{gs} R_S \Rightarrow V_{gs} = \frac{V_i}{1 + g_m R_S}$$
$$V_0 = -g_m V_{gs} R_D$$
$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.8)(-1.35 + 2)$$
$$= 1.04 \text{ mA/V}$$
$$A_v = \frac{V_0}{V_i} = \frac{-g_m R_D}{1 + g_m R_S} = \frac{-(1.04)(7.83)}{1 + (1.04)(4)}$$
$$\Rightarrow \underline{A_v = -1.58}$$

E6.14

a. $5 = I_{DQ}R_S + V_{SG}$ and

$$I_{DQ} = K_p(V_{SG} + V_{TP})^2$$
$$0.8 = 0.5(V_{SG} + 0.8)^2 \Rightarrow V_{SG} = 0.465 \text{ V}$$
$$5 = (0.8)R_S + 0.465 \Rightarrow \underline{R_S = 5.67 \text{ k}\Omega}$$
$$V_{SDQ} = 10 - I_{DQ}(R_S + R_D)$$
$$3 = 10 - (0.8)(5.67 + R_D)$$
$$R_D = \frac{10 - (0.8)(5.67) - 3}{0.8} \Rightarrow \underline{R_D = 3.08 \text{ k}\Omega}$$

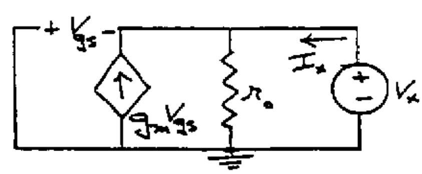
b.



$$V_0 = g_m V_{sg}(R_D \| r_0) = -g_m V_i(R_D \| r_0)$$
$$A_v = \frac{V_0}{V_i} = -g_m(R_D \| r_0)$$
$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(0.5)(0.465 + 0.8)$$
$$= 1.27 \text{ mA/V}$$
$$r_0 = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.8)} = 62.5 \text{ k}\Omega$$
$$A_v = -(1.27)(3.08\|62.5) \Rightarrow \underline{A_v = -3.73}$$

E6.15

$$V_0 = g_m V_{gs} r_0$$
$$V_i = V_{gs} + V_0 \Rightarrow V_{gs} = V_i - V_0$$
$$\text{So } V_0 = g_m r_0 (V_i - V_0)$$
$$A_v = \frac{V_0}{V_i} = \frac{g_m r_0}{1 + g_m r_0} = \frac{(4)(50)}{1 + (4)(50)}$$
$$\Rightarrow \underline{A_v = 0.995}$$



$I_x + g_m V_{gs} = \frac{V_x}{r_o}$ and $V_{gs} = -V_x$

$I_x = g_m V_x + \frac{V_x}{r_o} \Rightarrow R_o = r_o \,\|\, \frac{1}{g_m} = 50 \,\|\, \frac{1}{4}$

$\Rightarrow \underline{R_o \cong 0.25\ \text{k}\Omega}$

With $R_S = 4\ \text{k}\Omega \Rightarrow A_v = \frac{g_m(r_o \| R_S)}{1 + g_m(r_o \| R_S)}$

$r_o \| R_S = 50 \| 4 = 3.7\ \text{k}\Omega \Rightarrow A_v = \frac{(4)(3.7)}{1 + (4)(3.7)}$

$\Rightarrow \underline{A_v = 0.937}$

E6.16

$V_{DSQ} = V_{DD} - I_{DQ} R_S$

$5 = 10 - (1.5) R_S \Rightarrow \underline{R_S = 3.33\ \text{k}\Omega}$

$I_{DQ} = K_n (V_{GS} - V_{TN})^2 \Rightarrow 1.5 = (1)(V_{GS} - 0.8)^2$

$V_{GS} = 2.02\ \text{V} = V_G - V_S = V_G - 5$

$\Rightarrow V_G = 7.02\ \text{V} = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{R_2}{400} \cdot 10$

So $\underline{R_2 = 280.8\ \text{k}\Omega}$, $\underline{R_1 = 119.2\ \text{k}\Omega}$

Neglecting R_{Si},

$A_v = \frac{g_m (R_S \| r_o)}{1 + g_m (R_S \| r_o)}$

$r_o = [\lambda I_{DQ}]^{-1} = [(0.015)(1.5)]^{-1} = 44.4\ \text{k}\Omega$

$R_S \| r_o = 3.33 \| 44.4 = 3.1\ \text{k}\Omega$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(1.5)} = 2.45\ mA/V$

$A_v = \frac{(2.45)(3.1)}{1 + (2.45)(3.1)} \Rightarrow \underline{A_v = 0.884}$

$R_o = \frac{1}{g_m} \,\|\, R_S \,\|\, r_o = \frac{1}{2.45} \,\|\, 3.33 \,\|\, 44.4$

$= 0.408 \| 3.1$

$\Rightarrow \underline{R_o = 0.36\ \text{k}\Omega}$

E6.17

$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \left(\frac{9.3}{70.7 + 9.3}\right)(5)$

$= 0.581\ \text{V}$

$I_{DQ} = K_p (V_{SG} - |V_{TP}|)^2 = K_p (V_s - V_G - |V_{TP}|)^2$

$= \frac{5 - V_S}{R_S}$

Then $(0.4)(5)(V_S - 0.581 - 0.8)^2 = 5 - V_S$

$2(V_S - 1.381)^2 = 5 - V_S$

$2(V_S^2 - 2.762 V_S + 1.907) = 5 - V_S$

$2V_S^2 - 4.52 V_S - 1.19 = 0$

$V_S = \frac{4.52 \pm \sqrt{(4.52)^2 + 4(2)(1.19)}}{2(2)}$

$V_S = 2.50\ \text{V} \Rightarrow I_{DQ} = \frac{5 - 2.5}{5} = 0.5\ \text{mA}$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.4)(0.5)} = 0.894\ mA/V$

$A_v = \frac{g_m R_S}{1 + g_m R_S} \cdot \frac{R_1 \| R_2}{R_1 \| R_2 + R_{Si}}$

$= \frac{(0.894)(5)}{1 + (0.894)(5)} \cdot \frac{70.7 \| 9.3}{70.7 \| 9.3 + 0.5} \Rightarrow \underline{A_v = 0.770}$

Neglecting R_{Si}, $A_v = 0.817$

$R_o = R_S \,\|\, \frac{1}{g_m} = 5 \,\|\, \frac{1}{0.894} = 5 \| 1.12$

$\Rightarrow \underline{R_o = 0.915\ \text{k}\Omega}$

E6.18

(a) $g_m = 2\sqrt{K_n I_{DQ}} \Rightarrow 2 = 2\sqrt{K_n (0.8)} \Rightarrow$

$K_n = 1.25\ mA/V^2$

$K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \Rightarrow 1.25 = (0.020)\left(\frac{W}{L}\right)$

So $\underline{\frac{W}{L} = 62.5}$

$I_{DQ} = K_n (V_{GS} - V_{TN})^2 \Rightarrow 0.8 = 1.25 (V_{GS} - 2)^2$

$\Rightarrow \underline{V_{GS} = 2.8\ \text{V}}$

b. $r_o = [\lambda I_{DQ}]^{-1} = [(0.01)(0.8)]^{-1} = 125\ \text{k}\Omega$

$A_v = \frac{g_m (r_o \| R_L)}{1 + g_m (r_o \| R_L)}$

$r_o \| R_L = 125 \| 4 = 3.88$

$A_v = \frac{(2)(3.88)}{1 + (2)(3.88)} \Rightarrow \underline{A_v = 0.886}$

$R_o = \frac{1}{g_m} \,\|\, r_o = \frac{1}{2} \,\|\, 125 \Rightarrow \underline{R_o \approx 0.5\ \text{k}\Omega}$

E6.19

$I_{DQ} = K_p (V_{SG} - |V_{TP}|)^2$

$3 = 2(V_{SG} - 2)^2 \Rightarrow V_{SG} = 3.22\ \text{V}$

$I_{DQ} = \frac{5 - V_{SG}}{R_S} \Rightarrow 3 = \frac{5 - 3.22}{R_S}$

$\Rightarrow \underline{R_S = 0.593\ \text{k}\Omega}$

$r_o = [\lambda I_{DQ}]^{-1} = [(0.02)(3)]^{-1} = 16.7\ \text{k}\Omega$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(3)} = 4.9\ mA/V$

For $R_L = \infty$, $A_v = \frac{g_m (r_o \| R_S)}{1 + g_m (r_o \| R_S)}$

$r_o \| R_S = 16.7 \| 0.593 = 0.573\ \text{k}\Omega$

$A_v = \frac{(4.9)(0.573)}{1 + (4.9)(0.573)} \Rightarrow \underline{A_v = 0.737}$

If A_v is reduced by 10%

$\Rightarrow A_v = 0.737 - 0.0737 = 0.663$

$A_v = \frac{g_m (r_o \| R_S \| R_L)}{1 + g_m (r_o \| R_S \| R_L)}$

Let $r_o \| R_S \| R_L = x$

$$0.663 = \frac{(4.9)x}{1+(4.9)x} \Rightarrow 0.663 = 4.9x(1-0.663)$$

$$x = 0.402 = 0.573 \| R_L$$

$$\frac{0.573 R_L}{R_L + 0.573} = 0.402$$

$$\Rightarrow (0.573 - 0.402)R_L = (0.402)(0.573)$$

$$\Rightarrow \underline{R_L = 1.35\ \text{k}\Omega}$$

E6.20

$$R_{in} = \frac{1}{g_m} = 0.35\ \text{k}\Omega \Rightarrow \underline{g_m = 2.86\ \text{mA/V}}$$

$$\frac{V_0}{I_i} = R_D \| R_L = 2.4 = R_D \| 4 = \frac{4R_D}{4+R_D}$$

$$(4-2.4)R_D = (2.4)(4) \Rightarrow \underline{R_D = 6\ \text{k}\Omega}$$

$$g_m = 2\sqrt{K_n I_{DQ}}$$

$$2.86 = 2\sqrt{K_n(0.5)} \Rightarrow K_n = 4.09\ mA/V^2$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$0.5 = 4.09(V_{GS}-1)^2 \Rightarrow V_{GS} = 1.35\ \text{V}$$

$$\Rightarrow V_S = -1.35\ \text{V}, \quad V_D = 5 - (0.5)(6) = 2\ \text{V}$$

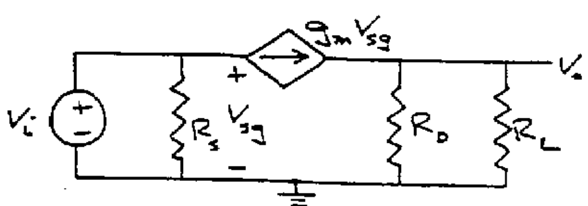
$$V_{DS} = V_D - V_S = 2 - (-1.35) = 3.35\ \text{V}$$

We have

$V_{DS} = 3.35 > V_{GS} - V_{TN} = 1.35 - 1 = 0.35\,V$

$\Rightarrow$ Biased in the saturation region

E6.21



$V_0 = g_m V_{sg}(R_D \| R_L)$ and $V_{sg} = V_i$

$$A_v = g_m(R_D \| R_L)$$

$$I_{DQ} = \frac{5 - V_{SG}}{R_S} = K_p(V_{SG} - |V_{TP}|)^2$$

$$5 - V_{SG} = (1)(4)(V_{SG} - 0.8)^2$$

$$5 - V_{SG} = 4(V_{SG}^2 - 1.6V_{SG} + 0.64)$$

$$4V_{SG}^2 - 5.4V_{SG} - 2.44 = 0$$

$$V_{SG} = \frac{5.4 \pm \sqrt{(5.4)^2 + (4)(4)(2.44)}}{2(4)}$$

$$V_{SG} = 1.71\ \text{V}$$

$$I_{DQ} = \frac{5 - 1.71}{4} = 0.822\ \text{mA}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.822)} = 1.81\ mA/V$$

$$A_v = (1.81)(2\|4) = (1.81)(1.33) \Rightarrow \underline{A_v = 2.41}$$

$$R_{in} = R_S \left\| \frac{1}{g_m} = 4 \right\| \frac{1}{1.81} = 4 \| 0.552$$

$$\Rightarrow \underline{R_{in} = 0.485\ \text{k}\Omega}$$

E6.22

$$K_{n1} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L}\right)_1 = (0.020)(80) = 1.6\ mA/V^2$$

$$K_{n2} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L}\right)_2 = (0.020)(1) = 0.020\ mA/V^2$$

$$A_v = -\sqrt{\frac{K_{n1}}{K_{n2}}} = -\sqrt{\frac{1.6}{0.020}} \Rightarrow \underline{A_v = -8.94}$$

The transition point is determined from

$$v_{GSt} - V_{TND} = V_{DD} - V_{TNL} - \sqrt{\frac{K_{n1}}{K_{n2}}}(v_{GSt} - V_{TND})$$

$$v_{GSt} - 0.8 = (5 - 0.8) - (8.94)(v_{GSt} - 0.8)$$

$$v_{GSt} = \frac{(5-0.8) + (8.94)(0.8) + 0.8}{1 + 8.94}$$

$$v_{GSt} = 1.22\ \text{V}$$

For Q-point in middle of saturation region

$$V_{GS} = \frac{1.22 - 0.8}{2} + 0.8 \Rightarrow \underline{V_{GS} = 1.01\ \text{V}}$$

E6.23

$$K_{n2} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L}\right)_2 = (0.015)(2) = 0.030\ mA/V^2$$

$$A_v = -\sqrt{\frac{K_{n1}}{K_{n2}}} = -6 \Rightarrow \frac{K_{n1}}{K_{n2}} = 36$$

$$K_{n1} = (36)(0.030) = 1.08\ mA/V^2$$

$$1.08 = (0.015)\left(\frac{W}{L}\right)_1 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 72}$$

The transition point is found from

$$v_{GSt} - 1 = (10 - 1) - (6)(v_{GSt} - 1)$$

$$v_{GSt} = \frac{10 - 1 + 6 + 1}{1 + 6} = 2.29\ \text{V}$$

For Q-point in middle of saturation region

$$V_{GS} = \frac{2.29 - 1}{2} + 1 \Rightarrow \underline{V_{GS} = 1.645\ \text{V}}$$

E6.24

(a) Transition points:

For M_2: $v_{OB} = V_{DD} - |V_{TNL}| = 5 - 1.2 = 3.8\,V$

For M_1:

$$K_{n1}\left[(v_{OA})^2(1 + \lambda v_{OA})\right] = K_{n2}\left[(V_{TNL})^2\left(1 + \lambda_2\left[V_{DD} - v_{OA}\right]\right)\right]$$

$$250\left[v_{OA}^2 + (0.01)v_{OA}^3\right] = 25\left[(1.2)^2\left(1 + (0.01)(5) - (0.01)v_{OA}\right)\right]$$

$$10\left[v_{OA}^2+(0.01)v_{OA}^3\right]=1.512-0.0144v_{OA}$$
$$(0.01)v_{OA}^3+v_{OA}^2+0.00144v_{OA}-0.512=0$$
which yields $v_{OA}\cong 0.388\,V$

Then middle of saturation region

$$v_{OQ}=\frac{3.8-0.388}{2}+0.388 \Rightarrow V_{DSQ1}=2.094\text{ V}$$

$$K_{n1}\left[(V_{GS1}-V_{TND})^2(1+\lambda_1 v_O)\right]$$
$$=K_{n2}\left[(V_{TNL})^2(1+\lambda_2[V_{DD}-v_O])\right]$$
$$250\left[(V_{GS1}-0.8)^2(1+[0.01][2.094])\right]$$
$$=25\left[(1.2)^2(1+[0.01][5-2.094])\right]$$
$$10\left[(V_{GS1}-0.8)^2(1.0209)\right]=1.482$$
$$(V_{GS1}-0.8)^2=0.145 \Rightarrow \underline{V_{GS1}=1.18\text{ V}}$$

b. $I_{DQ}=K_{n1}\left[(V_{GS1}-0.8)^2(1+(0.01)(2.094))\right]$

$$I_{DQ}=(0.25)\left[(0.145)^2(1.02094)\right]$$
$$\Rightarrow \underline{I_{DQ}=37.0\ \mu\text{A}}$$

c. $A_v=\dfrac{-g_{m1}}{I_{DQ}(\lambda_1+\lambda_2)}=-g_{m1}(r_{o1}\|r_{o2})$

$$g_{m1}=2K_{n1}(V_{GS1}-V_{TND})$$
$$=2(0.25)(1.18-0.8)=0.19\text{ mA/V}$$
$$A_v=\frac{-0.19}{(0.037)(0.01+0.01)} \Rightarrow \underline{A_v=-257}$$

E6.25

$$R_o=R_{S2}\,\Big\|\,\frac{1}{g_{m2}}$$
$$g_{m2}=0.632\text{ mA/V},\quad R_{S2}=8\text{ k}\Omega$$
$$R_o=8\,\Big\|\,\frac{1}{0.632}=8\|1.58 \Rightarrow \underline{R_o=1.32\text{ k}\Omega}$$

E6.26

a. $I_{DQ2}=2mA,\quad V_{SDQ2}=10\text{ V}$

$$I_{DQ2}\cdot R_{S2}=10=2R_{S2} \Rightarrow \underline{R_{S2}=5\text{ k}\Omega}$$
$$I_{DQ2}=K_{n2}(V_{GS2}-V_{TN2})^2$$
$$2=1(V_{GS2}-2)^2 \Rightarrow V_{SG2}=3.41\text{ V}$$
$$\Rightarrow V_{G2}=3.41\text{ V}$$

Then $R_{D1}=\dfrac{10-3.41}{2} \Rightarrow \underline{R_{D1}=3.3\text{ k}\Omega}$

For $V_{DSQ1}=10\text{ V} \Rightarrow V_{S1}=3.41-10=-6.59\text{ V}$

Then $R_{S1}=\dfrac{-6.59-(-10)}{2} \Rightarrow \underline{R_{S1}=1.71\text{ k}\Omega}$

$$I_{D1}=K_{n1}(V_{GS1}-V_{TN1})^2$$
$$2=1(V_{GS1}-2)^2 \Rightarrow V_{GS1}=3.41\text{ V}$$

$$V_{GS1}=\left(\frac{R_2}{R_1+R_2}\right)(20)-I_{DQ1}R_{S1}$$
$$\frac{R_2}{R_1+R_2}=\frac{1}{R_1}\cdot R_{in}$$
$$3.41=\frac{1}{R_1}(200)(20)-(2)(1.71)$$
$$\Rightarrow \underline{R_1=586\text{ k}\Omega}$$
$$\frac{586R_2}{586+R_2}=200 \Rightarrow (586-200)R_2=(200)(586)$$
$$\Rightarrow \underline{R_2=304\text{ k}\Omega}$$

b. $g_{m1}=2\sqrt{K_{n1}I_{DQ1}}=2\sqrt{(1)(2)}$

$$\Rightarrow g_{m1}=g_{m2}=2.83\text{ mA/V}$$

From Example 6-16

$$A_v=\frac{-g_{m1}g_{m2}R_{D1}(R_{S2}\|R_L)}{1+g_{m2}(R_{S2}\|R_L)}$$
$$R_{S2}\|R_L=5\|4=2.22\text{ k}\Omega$$
$$A_v=\frac{-(2.83)(2.83)(3.3)(2.22)}{1+(2.83)(2.22)}$$
$$\Rightarrow \underline{A_v=-8.06}$$

$$R_o=\frac{1}{g_{m2}}\,\Big\|\,R_{S2}=\frac{1}{2.83}\,\Big\|\,5=0.353\|5$$
$$\Rightarrow \underline{R_o=0.330\text{ k}\Omega}$$

E6.27

a. $I_{DQ1}=K_{n1}(V_{GS1}-V_{TN1})^2$

$$1=1.2(V_{GS1}-2)^2 \Rightarrow V_{GS1}=V_{GS2}=2.91\text{ V}$$
$$R_S=10\text{ k}\Omega \Rightarrow V_{S1}=I_{DQ}R_S-10$$
$$=(1)(10)-10=0$$
$$V_{G1}=2.91=\left(\frac{R_3}{R_1+R_2+R_3}\right)(10)$$
$$=\left(\frac{R_3}{500}\right)(10)$$
$$\Rightarrow \underline{R_3=145.5\text{ k}\Omega}$$

$$V_{DSQ1}=3.5 \Rightarrow V_{S2}=3.5\text{ V} \Rightarrow 3.5+2.91$$
$$\Rightarrow V_{G2}=6.41$$
$$V_{G2}=\left(\frac{R_2+R_3}{R_1+R_2+R_3}\right)(10)=6.41$$
$$=\left(\frac{R_2+R_3}{500}\right)(10)$$
$$R_2+R_3=320.5=R_2+145.5 \Rightarrow \underline{R_2=175\text{ k}\Omega}$$

Then $R_1+R_2+R_3=500=R_1+175+145.5$

$$\Rightarrow \underline{R_1=179.5\text{ k}\Omega}$$

Now $V_{S2}=3.5 \Rightarrow V_{D2}=V_{S2}+V_{SDQ2}$

$$=3.5+3.5=7\text{ V}$$

So $R_D = \frac{10-7}{1} \Rightarrow \underline{R_D = 3\ \text{k}\Omega}$

b. From Example 6-18:

$A_v = -g_{m1} R_D$

$g_{m1} = 2\sqrt{K_{n1} I_{DQ}} = 2\sqrt{(1.2)(1)} = 2.19\ mA/V$

$A_v = -(2.19)(3) \Rightarrow \underline{A_v = -6.57}$

E6.28

From Example 6-18:

$g_m = 2.98$ mA/V, $r_o = 42.1$ kΩ

$R_1 \| R_2 = 420 \| 180 = 126$ kΩ

$$V_{gs} = \left(\frac{R_1\|R_2}{R_1\|R_2 + R_i}\right)V_i$$

$$= \left(\frac{126}{126+20}\right)V_i = 0.863V_i$$

$$A_v = \frac{-g_m V_{gs}(r_o\|R_D\|R_L)}{V_i}$$

$$= -(2.98)(0.863)(42.1\|2.7\|4)$$

$$= -(2.57)(42.1\|1.61) = -(2.57)(1.55)$$

$\Rightarrow \underline{A_v = -3.98}$

E6.29

$V_S = I_{DQ}R_S = (1.2)(2.7) = 3.24$ V

$V_D = V_S + V_{DSQ} = 3.24 + 12 = 15.24$

$R_D = \frac{20 - 15.24}{1.2} \Rightarrow \underline{R_D = 3.97\ \text{k}\Omega}$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$1.2 = 4\left(1 - \frac{V_{GS}}{V_P}\right)^2 \Rightarrow \frac{V_{GS}}{V_P} = 0.452$$

$V_{GS} = (0.452)(-3) = -1.356$

$V_G = V_S + V_{GS} = 3.24 - 1.356 = 1.88$ V

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)(20) = \left(\frac{R_2}{500}\right)(20) = 1.88$$

$\Rightarrow \underline{R_2 = 47\ \text{k}\Omega}$. $\underline{R_1 = 453\ \text{k}\Omega}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.005)(1.2)} = 167\ \text{k}\Omega$$

$$g_m = \frac{2I_{DSS}}{(-V_P)}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(4)}{3}\left(1 - \frac{1.356}{3}\right)$$

$$= 1.46\ \text{mA/V}$$

$A_v = -g_m(r_o\|R_D\|R_L) = -(146)(167\|3.97\|4)$

$\Rightarrow \underline{A_v = -2.87}$

E6.30

a. $V_{G1} = \left(\frac{R_2}{R_1+R_2}\right)(V_{DD})$

$$V_{G1} = \left(\frac{430}{430+70}\right)(20) = 17.2\ \text{V}$$

$$I_{DQ1} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 6\left(1 - \frac{V_{G1} - V_{S1}}{2}\right)^2$$

$$= 6\left(1 - \frac{17.2}{2} + \frac{V_{S1}}{2}\right)^2 = 6\left(\frac{V_{S1}}{2} - 7.6\right)^2$$

and $I_{DQ1} = \frac{20 - V_{S1}}{4}$

Then $\frac{20 - V_{S1}}{4} = 6\left(\frac{V_{S1}}{2} - 7.6\right)^2$

$$20 - V_{S1} = 24\left(\frac{V_{S1}^2}{4} - 7.6V_{S1} + 57.76\right)$$

$$= 6V_{S1}^2 - 182.4V_{S1} + 1386.24$$

$6V_{S1}^2 - 181.4V_{S1} + 1366.24 = 0$

$$V_{S1} = \frac{181.4 \pm \sqrt{(181.4)^2 - 4(6)(1366.24)}}{2(6)}$$

$V_{S1} = 14.2$ V $\Rightarrow V_{GS1} = 17.2 - 14.2$

$= 3$ V $> V_P$

So $V_{S1} = 16.0 \Rightarrow V_{GS1} = 17.2 - 16$

$= 1.2 < V_P - Q$

on $I_{DQ1} = \frac{20-16}{4} \Rightarrow \underline{I_{DQ1} = 1\ \text{mA}}$

$V_{SDQ1} = 20 - I_{DQ1}(R_{S1} + R_{D1})$

$= 20 - (1)(8)$

$\Rightarrow \underline{V_{SDQ1} = 12\ \text{V}}$

$V_{D1} = I_{DQ1}R_{D1} = (1)(4) = 4$ V $= V_{G2}$

$$I_{DQ2} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 6\left(1 - \frac{V_{G2} - V_{S2}}{(-2)}\right)^2$$

$$= 6\left(1 + \frac{4}{2} - \frac{V_{S2}}{2}\right)^2 = 6\left(3 - \frac{V_{S2}}{2}\right)^2$$

and $I_{DQ2} = \frac{V_{S2}}{R_{S2}} = \frac{V_{S2}}{4}$

Then

$$\frac{V_{S2}}{4} = 6\left(3 - \frac{V_{S2}}{2}\right)^2$$

$$V_{S2} = 24\left(9 - 3V_{S2} + \frac{V_{S2}^2}{4}\right)$$

$6V_{S2}^2 - 73V_{S2} + 216 = 0$

$$V_{S2} = \frac{73 \pm \sqrt{(73)^2 - 4(6)(216)}}{2(6)}$$

$\Rightarrow V_{S2} = 7.09$ V or $= 5.08$ V

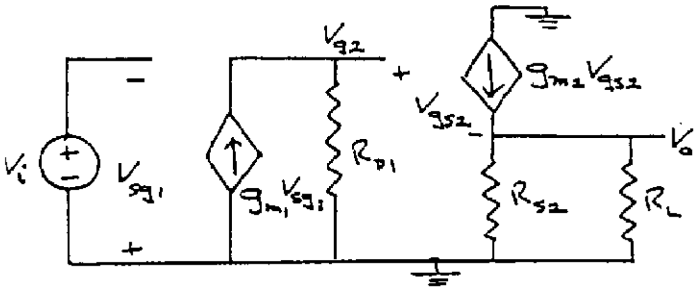
For $V_{S2} = 5.08$ V

$\Rightarrow V_{GS2} = 4 - 5.08 = -1.08$ transistor biased ON

$I_{DQ2} = \frac{5.08}{4} \Rightarrow \underline{I_{DQ2} = 1.27 \text{ mA}}$

$V_{DS2} = 20 - V_{S2} = 20 - 5.08 \Rightarrow \underline{V_{DS2} = 14.9 \text{ V}}$

b.



$V_{g2} = g_{m1}V_{sg1}R_{D1} = -g_{m1}V_iR_{D1}$

$V_0 = g_{m2}V_{gs2}(R_{S2}\|R_L)$

$V_{g2} = V_{gs2} + V_0 \Rightarrow V_{gs2} = \frac{V_{g2}}{1 + g_{m2}(R_{S2}\|R_L)}$

$A_v = \frac{V_0}{V_i} = \frac{-g_{m1}R_{D1}}{1 + g_{m2}(R_{S2}\|R_L)}$

$g_{m1} = \frac{2I_{DSS}}{|V_P|}\left(1 - \frac{V_{GS}}{V_P}\right)$

$= \frac{2(6)}{2}\left(1 - \frac{1.2}{2}\right) = 2.4 \text{ mA/V}$

$g_{m2} = \frac{2(6)}{2}\left(1 - \frac{1.08}{2}\right) = 2.76 \text{ mA/V}$

Then $A_v = \frac{-(2.4)(4)}{1 + (2.76)(4)(2)} = \underline{-2.05 = A_v}$

E6.31

a. $I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

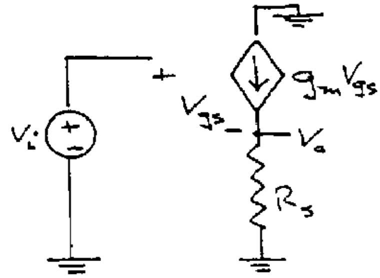
$2 = 8\left(1 - \frac{V_{GS}}{V_P}\right)^2 \Rightarrow \frac{V_{GS}}{V_P} = 0.5$

$V_{GS} = (0.5)(-3.5) \Rightarrow V_{GS} = -1.75$

Also $I_{DQ} = \frac{-V_{GS} - (-10)}{R_S}$

$2 = \frac{1.75 + 10}{R_S} \Rightarrow \underline{R_S = 5.88 \text{ k}\Omega}$

b.



$g_m = \frac{2I_{DSS}}{|V_P|}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(8)}{3.5}\left(1 - \frac{1.75}{3.5}\right)$

$= 2.29 \text{ mA/V}$

$r_0 = \frac{1}{(0.01)(2)} = 50 \text{ k}\Omega$

$V_i = V_{gs} + g_mR_SV_{gs} \Rightarrow V_{gs} = \frac{V_i}{1 + g_mR_S}$

$A_v = \frac{V_0}{V_i} = \frac{g_mR_S\|r_0}{1 + g_mR_S\|r_0} = \frac{(2.29)[5.88\|50]}{1 + (2.29)[5.88\|50]}$

$\Rightarrow \underline{A_v = 0.923}$

c. $A_v = \frac{g_m(R_S\|R_L)}{1 + g_m(R_S\|R_L)} = 0.931 - 0.186 = 0.745$

$\frac{(2.29)(R_S\|R_L)}{1 + (2.29)(R_S\|R_L)} = 0.745$

$(2.29)(R_S\|R_L)(1 - 0.745) = 0.745$

$\Rightarrow R_S\|R_L = 1.28 \text{ k}\Omega$

$\frac{5.88R_L}{5.88 + R_L} = 1.28$

$\Rightarrow (5.88 - 1.28)R_L = (1.28)(5.88)$

$\Rightarrow \underline{R_L = 1.64 \text{ k}\Omega}$

Chapter 6

Problem Solutions

6.1

(a) $g_m = 2\sqrt{K_n I_D} = 2\sqrt{\left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right) I_D}$

$0.5 = 2\sqrt{(0.020)\left(\frac{W}{L}\right)(0.5)} \Rightarrow \underline{\frac{W}{L} = 12.5}$

(b) $I_D = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$

$0.5 = (0.02)(12.5)(V_{GS} - 0.8)^2$

$\Rightarrow \underline{V_{GS} = 2.21 \text{ V}}$

6.2

(a) $g_m = 2\sqrt{K_p I_D} = 2\sqrt{\left(\frac{\mu_p C_{ox}}{2}\right)\left(\frac{W}{L}\right) I_D}$

$\left(\frac{50}{2}\right)^2 = (10)\left(\frac{W}{L}\right)(100) \Rightarrow \underline{\frac{W}{L} = 0.625}$

(b) $I_D = \left(\frac{\mu_p C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$

$100 = (10)(0.625)(V_{SG} - 1.2)^2$

$\Rightarrow \underline{V_{SG} = 4.2 \text{ V}}$

6.3

$I_D = K_n(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$

$\frac{I_{D1}}{I_{D2}} = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} \Rightarrow \frac{3.4}{3.0} = \frac{1 + \lambda(10)}{1 + \lambda(5)}$

$3.4[1 + 5\lambda] = 3.0[1 + 10\lambda]$

$3.4 - 3.0 = \lambda(3 \cdot 10 - (3.4) \cdot 5)$

$\Rightarrow \underline{\lambda = 0.0308}$

$r_0 \approx \frac{1}{\lambda I_D} = \frac{1}{(0.0308)(3)} \Rightarrow \underline{r_0 = 10.8 \text{ k}\Omega}$

6.4

$r_0 = \frac{1}{\lambda I_D}$

$I_D = \frac{1}{\lambda r_0} = \frac{1}{(0.012)(100)} \Rightarrow \underline{I_D = 0.833 \text{ mA}}$

6.5

$A_v = -g_m(r_0 \| R_D) = -(1)(50\|10)$

$\Rightarrow \underline{A_v = -8.33}$

6.6

a. $R_D = \frac{V_{DD} - V_{DSQ}}{I_{DQ}} = \frac{10 - 6}{0.5} \Rightarrow \underline{R_D = 8 \text{ k}\Omega}$

For $V_{GSQ} = 2$ V, for example,

$I_{DQ} = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$

$0.5 = (0.030)\left(\frac{W}{L}\right)(2 - 0.8)^2$

$\Rightarrow \underline{\frac{W}{L} = 11.6}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2K_n(V_{GSQ} - V_{TN})$

$g_m = 2(0.030)(11.6)(2 - 0.8)$

$\Rightarrow \underline{g_m = 0.835 \text{ mA/V}}$

$I_{DQ} = (0.030)(11.6)(2 - 0.8)^2 = 0.501$ mA

$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(0.501)} \Rightarrow \underline{r_0 = 133 \text{ k}\Omega}$

c. $A_v = -g_m(r_0 \| R_D) = -(0.835)(133\|8)$

$\Rightarrow \underline{A_v = -6.30}$

6.7

$K_n v_{gs}^2 = K_n[V_{gs} \sin \omega t]^2 = K_n V_{gs}^2 \sin^2 \omega t$

$\sin^2 \omega t = \frac{1}{2}[1 - \cos 2\omega t]$

So $K_n v_{gs}^2 = \frac{K_n V_{gs}^2}{2}[1 - \cos 2\omega t]$

Ratio of signal at 2ω to that at ω:

$$\frac{\frac{K_n V_{gs}^2}{2} \cdot \cos 2\omega t}{2K_n(V_{GSQ} - V_{TN})V_{gs} \cdot \sin \omega t}$$

The coefficient of this expression is then:

$$\underline{\frac{V_{gs}}{4(V_{GSQ} - V_{TN})}}$$

6.8

$0.01 = \frac{V_{gs}}{4(V_{GSQ} - V_{TN})}$

So $V_{gs} = (0.01)(4)(3 - 1)$

$\Rightarrow \underline{V_{gs} = 0.08 \text{ V}}$

6.9

$V_o = -g_m V_{gs}(r_o \| R_D)$

$V_{gs} = \frac{R_1 \| R_2}{R_1 \| R_2 + R_{Si}} \cdot V_i = \left(\frac{50}{50 + 2}\right) \cdot V_i = (0.962)V_i$

$A_v = -g_m(0.962)(r_o \| R_D) = -(1)(0.962)(50\|10) \Rightarrow$

$\underline{A_v = -8.02}$

6.10

$A_v = -g_m(r_o \| R_D)$

$-10 = -g_m(100\|5)$

$\Rightarrow \underline{g_m = 2.1 \text{ mA/V}}$

6.11

a. $V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD}$

$V_G = \left(\frac{200}{200 + 300}\right)(12) = 4.8 \text{ V}$

$I_D = \frac{V_G - V_{GS}}{R_S} = K_n(V_{GS} - V_{TN})^2$

$4.8 - V_{GS} = (1)(2)(V_{GS}^2 - 4V_{GS} + 4)$

$2V_{GS}^2 - 7V_{GS} + 3.2 = 0$

$$V_{GS} = \frac{7 \pm \sqrt{(7)^2 - 4(2)(3.2)}}{2(2)} = 2.96 \text{ V}$$

$I_D = (1)(2.96 - 2)^2 \Rightarrow \underline{I_D = 0.920 \text{ mA}}$

$V_{DS} = V_{DD} - I_D(R_D + R_S)$

$= 12 - (0.92)(3 + 2)$

$\Rightarrow \underline{V_{DS} = 7.4 \text{ V}}$

(b) $V_o = \frac{-g_m V_{gs}(R_D \| R_L)}{1 + g_m R_S}$

where $V_{gs} = \frac{R_1 \| R_2}{R_1 \| R_2 + R_{Si}} \cdot V_i = \frac{300\|200}{300\|200 + 2} \cdot V_i$

$= \frac{120}{120 + 2} \cdot V_i = (0.984)V_i$

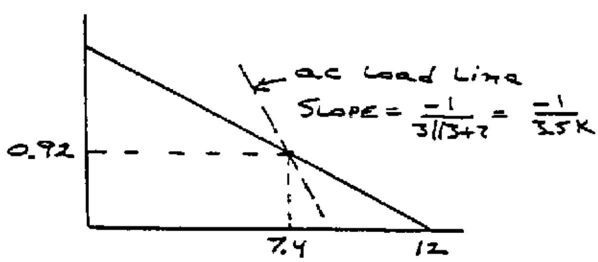
Then

$A_v = \frac{-g_m(0.984)(R_D \| R_L)}{1 + g_m R_S}$

$g_m = 2(1)(2.96 - 2) = 1.92 \ mA/V$

So $A_v = \frac{-(1.92)(0.984)(3\|3)}{1 + (1.92)(2)} \Rightarrow \underline{A_v = -0.586}$

c.



$\Delta i_D = \frac{-1}{3.5 \text{ k}\Omega} \cdot \Delta v_{ds}$

$\Delta i_D = 0.92 \text{ mA}$

$\Rightarrow |\Delta v_{ds}| = (0.92)(3.5) = 3.22$

$\Rightarrow$ <u>6.44 V peak-to-peak</u>

6.12

a. $I_{DQ} = 3 \text{ mA} \Rightarrow V_S = I_{DQ}R_S = (3)(0.5) = 1.5 \text{ V}$

$I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$3 = (2)(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.22 \text{ V}$

$V_G = V_{GS} + V_S = 3.22 + 1.5 = 4.72 \text{ V}$

$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$4.72 = \frac{1}{R_1}(200)(15) \Rightarrow \underline{R_1 = 636 \text{ k}\Omega}$

$\frac{636 R_2}{636 + R_2} = 200 \Rightarrow \underline{R_2 = 292 \text{ k}\Omega}$

b. $A_v = \frac{-g_m(R_D \| R_L)}{1 + g_m R_S}$

$g_m = 2(2)(3.22 - 2) = 4.88 \text{ mA/V}$

$A_v = \frac{-(4.88)(2\|5)}{1 + (4.88)(0.5)} \Rightarrow \underline{A_v = -2.03}$

6.13

From Problem 6-11: $I_D = 0.920$ mA

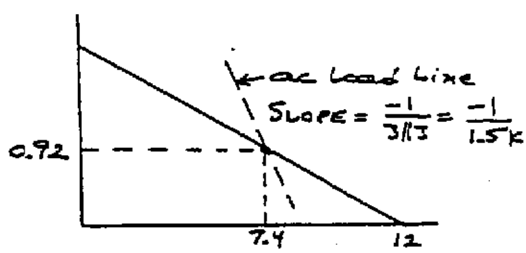
$V_{DS} = 7.4$ V

$g_m = 1.92$ mA/V

$A_v = -g_m(R_D \| R_L)\left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{Si}}\right)$

$= -(1.92)(3\|3)\left(\frac{200\|300}{200\|300 + 2}\right) = -(2.88)(0.984)$

$\underline{A_v = -2.83}$



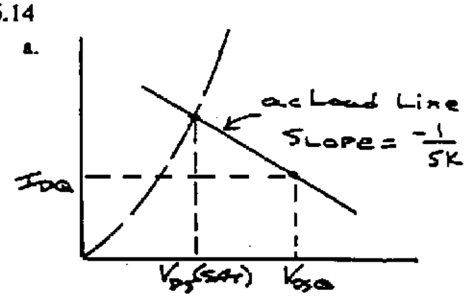
$\Delta i_D = \frac{-1}{1.5 \text{ k}\Omega} \cdot \Delta v_{DS}, \quad \Delta i_D = 0.92 \text{ mA}$

$\Rightarrow |\Delta v_{DS}| = (0.92)(1.5) = 1.38$

$\Rightarrow$ 2.76 V peak-to-peak output voltage swing

6.14

a.



$V_{DSQ} = V^+ - I_{DQ}R_D - (-V_{GSQ})$

Output Voltage Swing $= V_{DSQ} - V_{DS}(\text{sat})$

$= \left[V^+ - I_{DQ}R_D + V_{GSQ}\right] - \left(V_{GSQ} - V_{TN}\right)$

$= V^+ - I_{DQ}R_D + V_{TN}$

So $|\Delta I_D| = I_{DQ} = \frac{1}{5\,k\Omega}\cdot|\Delta V_{DS}|$

$= \frac{1}{5\,k\Omega}\left[V^+ - I_{DQ}R_D + V_{TN}\right]$

$I_{DQ} = \frac{1}{5\text{ k}\Omega}[5 - I_{DQ}(10) + 1]$

$= 1.2 - 2I_{DQ} = I_{DQ}$

$\Rightarrow \underline{I_{DQ} = 0.4\text{ mA} = I_Q}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.4)} = 0.894\ mA/V$

$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(0.4)} = 250\text{ k}\Omega$

$A_v = -g_m(R_D\|R_L\|r_o)$

$= -(0.894)(10\|10\|250)$

$\Rightarrow \underline{A_v = -4.38}$

6.15

a. $I_D = K_n\left(V_{GS} - V_{TN}\right)^2$

$2 = 4(V_{GS} - (-1))^2$

$V_{GS} = -0.293\text{ V}$

$\Rightarrow V_S = 0.293\text{ V} = I_D R_S = (2)R_S$

$\Rightarrow \underline{R_S = 0.146\text{ k}\Omega}$

$V_D = V_{DS} + V_S = 6 + 0.293 = 6.293$

$R_D = \frac{10 - 6.293}{2} \Rightarrow \underline{R_D = 1.85\text{ k}\Omega}$

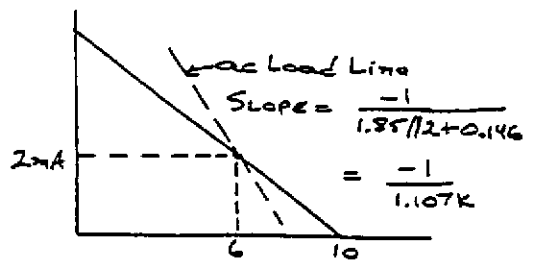
b. $A_v = \frac{-g_m(R_D\|R_L)}{1 + g_m R_S}$

$g_m = 2K_n\left(V_{GS} - V_{TN}\right)$

$g_m = 2(4)(-0.293 + 1) = 5.66\text{ mA/V}$

$A_v = \frac{-(5.66)(1.85\|2)}{1 + (5.66)(0.146)} \Rightarrow \underline{A_v = -2.98}$

c.



$\Delta v_0 = (\Delta i_D)(1.85\|2) = (2)(1.85\|2)$

$= 1.92\text{ V}$

$\Delta v_i = \frac{1.92}{2.98} = 0.645 \Rightarrow \underline{V_i = 0.645\text{ V}}$

6.16

a. $V_{DSQ} = V_{DD} - I_{DQ}(R_D + R_S)$

$2.5 = 5 - I_{DQ}(2 + R_S)$

$I_{DQ} = \frac{2.5}{2 + R_S}$

$I_{DQ} = K_n\left(V_{GS} - V_{TN}\right)^2 = \frac{-V_{GS}}{R_S}$

$\Rightarrow V_{GS} = -I_{DQ}R_S = \frac{-2.5R_S}{2 + R_S}$

$K_n\left[\frac{-2.5R_S}{2+R_S} - V_{TN}\right]^2 = \frac{2.5}{2+R_S}$

$4\left[1 - \frac{2.5R_S}{2 + R_S}\right]^2 = \frac{2.5}{2 + R_S}$

$4\left[\frac{2 + R_S - 2.5R_S}{2 + R_S}\right]^2 = \frac{2.5}{2 + R_S}$

$4\frac{(2 - 1.5R_S)^2}{2 + R_S} = 2.5$

$4(4 - 6R_S + 2.25R_S^2) = 2.5(2 + R_S)$

$9R_S^2 - 26.5R_S + 11 = 0$

$R_S = \frac{26.5 \pm \sqrt{(26.5)^2 - 4(9)(11)}}{2(9)}$

$R_S = 0.5\text{ k}\Omega$ or $2.44\text{ k}\Omega$

But $R_S = 2.44\text{ k}\Omega \Rightarrow V_{GS} = -1.37$ Cut off.

$\Rightarrow \underline{R_S = 0.5\text{ k}\Omega},\quad \underline{I_{DQ} = 1.0\text{ mA}}$

b. $A_v = \frac{-g_m(R_D\|R_L)}{1 + g_m R_S}$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(1)} = 4\ mA/V$

$A_v = \frac{-(4)(2\|2)}{1 + (4)(0.5)} \Rightarrow \underline{A_v = -1.33}$

6.17

a. $5 = I_{DQ}R_S + V_{SDQ} + I_{DQ}R_D - 5$

$5 = I_{DQ}R_S + 6 + I_{DQ}(10) - 5$

1. $\underline{I_{DQ} = \frac{4}{R_S + 10}}$

$V_S = V_{SDQ} + I_{DQ}R_D - 5 = V_{SGQ}$

2. $1 + I_{DQ}(10) = V_{SGQ}$
3. $I_{DQ} = K_p\left(V_{SGQ} - 2\right)^2$

Choose $\underline{R_S = 10\ \text{k}\Omega} \Rightarrow I_{DQ} = \frac{4}{20} = 0.20\ \text{mA}$

$V_{SGQ} = 1 + (0.2)(10) = 3\ \text{V}$

$0.20 = K_p(3-2)^2 \Rightarrow \underline{K_p = 0.20\ mA/V^2}$

b. $I_{DQ} = (0.20)(3-2)^2 = 0.20\ \text{mA}$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.2)(0.2)} = 0.4\ mA/V$

$A_v = -g_m(R_D \| R_L) = -(0.4)(10\|10)$

$\Rightarrow \underline{A_v = -2.0}$

c. Choose $\underline{R_S = 20\ \text{k}\Omega} \Rightarrow I_{DQ} = \frac{4}{30} = 0.133\ \text{mA}$

$V_{SGQ} = 1 + (0.133)(10) = 2.33\ \text{V}$

$0.133 = K_p(2.33-2)^2 \Rightarrow \underline{K_p = 1.22\ mA/V^2}$

$g_m = 2\sqrt{(1.22)(0.133)} = 0.806\ \text{mA/V}$

$A_v = -(0.806)(10\|10) \Rightarrow \underline{A_v = -4.03}$

A larger gain can be achieved.

6.18

a. $I_{DQ} = 1 = K_p(V_{SGQ} + V_{TP})^2$

$1 = 5(V_{SGQ} - 1.5)^2 \Rightarrow V_{SGQ} = 1.95\ \text{V}$

$R_S = \frac{5 - 1.95}{1} \Rightarrow \underline{R_S = 3.05\ \text{k}\Omega}$

$V_{SDQ} = 10 - I_{DQ}(R_S + R_D)$

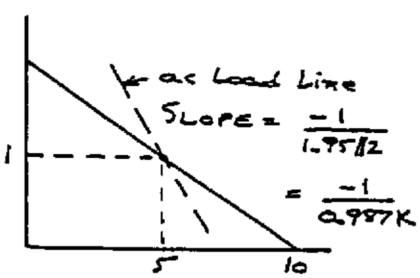
$5 = 10 - (1)(3.05 + R_D) \Rightarrow \underline{R_D = 1.95\ \text{k}\Omega}$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(5)(1)} = 4.47\ mA/V$

$A_v = -g_m(R_D \| R_L) = -(4.47)(1.95\|2)$

$\Rightarrow \underline{A_v = -4.41}$

c.



$\Delta i_D = -\frac{1}{0.987\ \text{k}\Omega} \cdot \Delta v_{DS}$

$\Rightarrow |\Delta v_{DS}| = (1)(0.987) = 0.987\ \text{V}$

Swing in output voltage = 1.97 V peak-to-peak

6.19

$I_{DQ} = K_n(V_{GSQ} - V_{TN})^2$

$g_m = 2\sqrt{K_n I_{DQ}}$

$2.2 = 2\sqrt{K_n(6)} \Rightarrow \underline{K_n = 0.202\ mA/V^2}$

$6 = 0.202(2.8 - V_{TN})^2 \Rightarrow \underline{V_{TN} = -2.65\ V}$

$V_{DSQ} = 18 - I_{DQ}(R_S + R_D)$

$R_S + R_D = \frac{18 - 10}{6} = 1.33\ \text{k}\Omega \Rightarrow R_S = 1.33 - R_D$

$$A_v = -\frac{g_m(R_D \| R_L)}{1 + g_m R_S}$$

$$-1 = \frac{-2.2\left(\frac{R_D \cdot 1}{R_D + 1}\right)}{1 + (2.2)(1.33 - R_D)}$$

$$1 + 2.93 - 2.2R_D = \frac{2.2R_D}{1 + R_D}$$

$(3.93 - 2.2R_D)(1 + R_D) = 2.2R_D$

$3.93 + 1.73R_D - 2.2R_D^2 = 2.2R_D$

$2.2R_D^2 + 0.47R_D - 3.93 = 0$

$$R_D = \frac{-0.47 + \sqrt{(0.47)^2 + 4(2.2)(3.93)}}{2(2.2)}$$

$\Rightarrow \underline{R_D = 1.23\ \text{k}\Omega}, \quad \underline{R_S = 0.10\ \text{k}\Omega}$

$V_G = V_{GS} + V_S = 2.8 + (6)(0.1) = 3.4\ \text{V}$

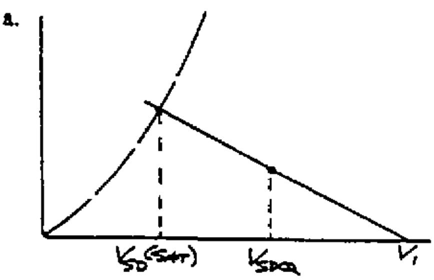
$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} = \frac{1}{R_1}(100)(18) = 3.4$

$\Rightarrow \underline{R_1 = 529\ \text{k}\Omega}$

$\frac{529 R_2}{529 + R_2} = 100 \Rightarrow \underline{R_2 = 123\ \text{k}\Omega}$

6.20

a.



$V_1 = 9 + V_{SG}, \quad V_{SD}(sat) = V_{SG} + V_{TP}$

$$V_{SDQ} = \frac{V_1 - V_{SD}(sat)}{2} + V_{SD}(sat)$$

$$= \frac{(9 + V_{SG}) - (V_{SG} + V_{TP})}{2} + (V_{SG} + V_{TP})$$

$$= \frac{9 + 1.5}{2} + V_{SG} - 1.5$$

$V_{SDQ} = 3.75 + V_{SG} = 9 + V_{SG} - I_{DQ}R_D$

$I_{DQ} = K_p(V_{SG} + V_{TP})^2$

Set $R_D = 0.1R_L = 2\ \text{k}\Omega$

$3.75 = 9 - I_{DQ}(2) \Rightarrow I_{DQ} = 2.625\ \text{mA}$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(2.625)} = 4.58\ mA/V$

$r_o = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.01)(2.625)} = 38.1\ \text{k}\Omega$

Open circuit.

$A_v = -g_m(R_D \| r_o)$

$A_v = -4.58(2\|38.1) \Rightarrow \underline{A_v = -8.70}$

c. With R_L

$A_v = -4.58(2\|20\|38.1) \Rightarrow \underline{A_v = -7.95}$

$\Rightarrow$ Change $= 8.62\%$

6.21

a. $I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$2 = (0.5)(V_{SG}+2)^2 \Rightarrow V_{SG} = 0\ \text{V}$

$R_S = \dfrac{10-0}{2} \Rightarrow \underline{R_S = 5\ \text{k}\Omega}$

$V_D = 0 - V_{SDQ} = 0 - 6 \Rightarrow R_D = \dfrac{-6-(-10)}{2}$

$\Rightarrow \underline{R_D = 2\ \text{k}\Omega}$

$A_v = -g_m R_D$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(2)} = 2\ mA/V$

$A_v = -(2)(2) \Rightarrow \underline{A_v = -4.0}$

6.22

$A_v = -g_m(R_D \| R_L)$

$V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$

$10 = 20 - (1)(R_S + R_D) \Rightarrow R_S + R_D = 10\ \text{k}\Omega$

Let $\underline{R_D = 8\ \text{k}\Omega}$. $\underline{R_S = 2\ \text{k}\Omega}$

$A_v = -10 = -g_m(8\|20)$

$g_m = 1.75\ mA/V = 2\sqrt{K_n I_{DQ}} = 2\sqrt{K_n(1)} \Rightarrow$

$\underline{K_n = 0.766\ mA/V^2}$

$V_S = I_{DQ}R_S = (1)(2) = 2\ \text{V}$

$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow 1 = 0.766(V_{GS}-2)^2$

$\Rightarrow V_{GS} = 3.31\ \text{V}$

$V_G = V_{GS} + V_S = 3.31 + 2 = 5.31$

$V_G = \dfrac{1}{R_1}\cdot R_{in}\cdot V_{DD}$

$\Rightarrow \dfrac{1}{R_1}(200)(20) = 5.31 \Rightarrow \underline{R_1 = 753\ \text{k}\Omega}$

$\dfrac{753R_2}{753 + R_2} = 200 \Rightarrow \underline{R_2 = 272\ \text{k}\Omega}$

6.23

$A_v = \dfrac{g_m r_o}{1 + g_m r_o} = \dfrac{(4)(50)}{1+(4)(50)} \Rightarrow \underline{A_v = 0.995}$

$R_o = \dfrac{1}{g_m}\Big\| r_o = \dfrac{1}{4}\Big\| 50 \Rightarrow \underline{R_o = 0.249\ \text{k}\Omega}$

$A_v = \dfrac{g_m(r_o\|R_S)}{1 + g_m(r_o\|R_S)} = \dfrac{4(50\|2.5)}{1 + 4(50\|2.5)}$

$= \dfrac{4(2.38)}{1+4(2.38)} \Rightarrow \underline{A_v = 0.905}$

$R_o = \dfrac{1}{g_m}\Big\| r_o \Big\| R_S = \dfrac{1}{4}\Big\| 50 \Big\| 2.5$

$\Rightarrow \underline{R_o = 0.226\ \text{k}\Omega}$

6.24

a. $V_G = \left(\dfrac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\dfrac{396}{396+1240}\right)(10)$

$\Rightarrow V_G = 2.42\ \text{V}$

$I_{DQ} = \dfrac{10 - (V_{SG} + 2.42)}{R_S} = K_p(V_{SG} + V_{TP})^2$

$7.58 - V_{SG} = (2)(4)(V_{SG}^2 - 4V_{SG} + 4)$

$8V_{SG}^2 - 31V_{SG} + 24.4 = 0$

$V_{SG} = \dfrac{31 \pm \sqrt{(31)^2 - 4(8)(24.4)}}{2(8)}$

$\Rightarrow V_{SG} = 2.78\ \text{V}$

$I_{DQ} = (2)(2.78-2)^2 \Rightarrow \underline{I_{DQ} = 1.21\ \text{mA}}$

$V_{SDQ} = 10 - I_{DQ}R_S = 10 - (1.21)(4)$

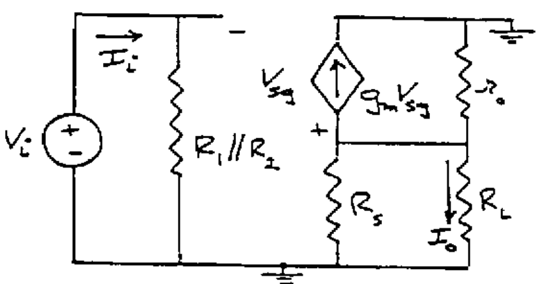
$\Rightarrow \underline{V_{SDQ} = 5.16\ \text{V}}$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(1.21)} = 3.11\ mA/V$

$r_o = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.02)(1.21)} = 41.3\ \text{k}\Omega$

$A_v = \dfrac{g_m(R_S\|R_L\|r_o)}{1 + g_m(R_S\|R_L\|r_o)}$

$= \dfrac{3.11(4\|4\|41.3)}{1 + (3.11)(4\|4\|41.3)} \Rightarrow \underline{A_v = 0.886}$



$I_o = -(g_m V_{sg})\left(\dfrac{R_S\|r_o}{R_S\|r_o + R_L}\right)$

$V_{sg} = \dfrac{-V_i}{1 + g_m(R_S\|R_L\|r_o)}$

$V_i = I_i(R_1\|R_2)$

$$A_i = \frac{I_o}{I_i} = \frac{g_m(R_1\|R_2)}{1+g_m(R_S\|R_L\|r_o)} \cdot \frac{R_S\|r_o}{R_S\|r_o + R_L}$$
$$= \frac{(3.11)(396\|1240)}{1+(3.11)(4\|4\|41.3)} \cdot \frac{4\|41.3}{4\|41.3+4}$$
$$= \frac{(3.11)(300)}{1+(3.11)(1.908)} \cdot \frac{3.647}{3.647+4}$$
$$\Rightarrow \underline{A_v = 64.2}$$
$$R_o = \frac{1}{g_m}\left\|R_S\right\|R_L\left\|r_o = \frac{1}{3.11}\right\|4\|4\|41.3$$
$$\Rightarrow \underline{R_o = 0.275\ \text{k}\Omega}$$

6.25

$$g_m = 2\sqrt{K_n I_Q} = 2\sqrt{(1)(1)} = 2\ mA/V$$
$$r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(1)} = 100\ \text{k}\Omega$$
$$A_v = \frac{g_m(r_o\|R_L)}{1+g_m(r_o\|R_L)} = \frac{2(100\|4)}{1+2(100\|4)}$$
$$\Rightarrow \underline{A_v = 0.885}$$
$$R_o = \frac{1}{g_m}\left\| r_o = \frac{1}{2}\right\| 100 \Rightarrow \underline{R_o = 0.490\ \text{k}\Omega}$$

6.26

a. $$A_v = \frac{g_m R_L}{1+g_m R_L} \Rightarrow 0.95 = \frac{g_m(4)}{1+g_m(4)}$$
$$0.95 = 4(1-0.95)g_m \Rightarrow g_m = 4.75\ \text{mA/V}$$
$$g_m = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_Q}$$
$$4.75 = 2\sqrt{(0.030)\left(\frac{W}{L}\right)(4)} \Rightarrow \underline{\frac{W}{L} = 47.0}$$

b. $$g_m = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_Q}$$
$$4.75 = 2\sqrt{(0.030)(60)I_Q} \Rightarrow \underline{I_Q = 3.13\ \text{mA}}$$

6.27

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$
$$5 = 5(V_{GS}+2)^2 \Rightarrow V_{GS} = -1\ \text{V}$$
$$\Rightarrow V_S = -V_{GS} = 1\ \text{V}$$
$$I_{DQ} = \frac{V_S - (-5)}{R_S} \Rightarrow R_S = \frac{1+5}{5} \Rightarrow \underline{R_S = 1.2\ \text{k}\Omega}$$
$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(5)(5)} = 10\ mA/V$$
$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20\ \text{k}\Omega$$
$$A_v = \frac{g_m(r_o\|R_S\|R_L)}{1+g_m(r_o\|R_S\|R_L)}$$
$$= \frac{(10)(20\|1.2\|2)}{1+(10)(20\|1.2\|2)} \Rightarrow \underline{A_v = 0.878}$$
$$R_o = \frac{1}{g_m}\left\|r_o\right\|R_S = \frac{1}{10}\|20\|1.2$$
$$\Rightarrow \underline{R_o = 91.9\ \Omega}$$

6.28

$$A_v = \frac{g_m R_S}{1+g_m R_S} \Rightarrow 0.90 = \frac{g_m(10)}{1+g_m(10)}$$
$$0.90 = 10(1-0.90)g_m \Rightarrow \underline{g_m = 0.90\ \text{mA/V}}$$
$$R_o = \frac{1}{g_m}\left\| R_S = \frac{1}{0.90}\right\| 10 \Rightarrow \underline{R_o = 1\ \text{k}\Omega}$$

With R_L:

$$A_v = \frac{g_m(R_S\|R_L)}{1+g_m(R_S\|R_L)} = \frac{(0.90)(10\|2)}{1+(0.90)(10\|2)}$$
$$\Rightarrow \underline{A_v = 0.60}$$

6.29

$$R_o = \frac{1}{g_m}\| R_S$$

Output resistance determined primarily by g_m

Set $\frac{1}{g_m} = 0.2\ \text{k}\Omega \Rightarrow g_m = 5\ \text{mA/V}$

$$g_m = 2\sqrt{K_n I_{DQ}}$$
$$\Rightarrow 5 = 2\sqrt{(4)I_{DQ}} \Rightarrow I_{DQ} = 1.56\ \text{mA}$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$
$$1.56 = 4(V_{GS}+2)^2$$
$$V_{GS} = -1.38\ \text{V},\quad V_S = -V_{GS} = 1.38\ \text{V}$$
$$R_S = \frac{1.38-(-5)}{1.56} \Rightarrow \underline{R_S = 4.09\ \text{k}\Omega}$$
$$A_v = \frac{g_m(R_S\|R_L)}{1+g_m(R_S\|R_L)} = \frac{5(4.09\|2)}{1+5(4.09\|2)}$$
$$\Rightarrow \underline{A_v = 0.870}$$

6.30

a. $$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(5)(5)} = 10\ mA/V$$
$$R_o = \frac{1}{g_m} = \frac{1}{10} \Rightarrow \underline{R_o = 100\ \Omega}$$

b. Open-circuit gain

$$A_v = \frac{g_m r_o}{1+g_m r_o}$$ But $r_o = \infty$ so $A_v = 1.0$

With R_L:

$$A_v = \frac{g_m R_L}{1+g_m R_L}$$
$$0.50 = \frac{10R_L}{1+10R_L} \Rightarrow 0.50 = 10(1-0.5)R_L$$
$$\Rightarrow \underline{R_L = 0.1\ \text{k}\Omega}$$

6.31

$|\Delta i_D| = I_{DQ} = \frac{-1}{R_S \| R_L} \cdot \Delta v_0$

$\Delta v_0 = -I_{DQ} \cdot R_S \| R_L = -\frac{I_{DQ} \cdot R_S R_L}{R_S + R_L}$

$\underline{v_0(\min) = -\frac{I_{DQ} R_S}{1 + \frac{R_S}{R_L}}}$

$A_v = \frac{g_m(R_S \| R_L)}{1 + g_m(R_S \| R_L)} = \frac{v_0}{v_i}$

$v_i = \frac{-I_{DQ}(R_S \| R_L)[1 + g_m(R_S \| R_L)]}{g_m(R_S \| R_L)}$

$\underline{v_i(\min) = -\frac{I_{DQ}}{g_m}[1 + g_m(R_S \| R_L)]}$

6.32

a. $V_{DSQ} = V_{DD} - I_{DQ} R_S$

$3 = 5 - (1.7)R_S \Rightarrow \underline{R_S = 1.18\ \text{k}\Omega}$

$I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$1.7 = (1)(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.30\ \text{V}$

$V_S = V_{DD} - V_{DSQ} = 5 - 3 = 2\ \text{V}$

$V_G = V_S + V_{GS} = 2 + 2.30 = 4.30\ \text{V}$

$V_G = \frac{1}{R_1} \cdot R_i \cdot V_{DD} = \frac{1}{R_1}(300)(5) = 4.30$

$\Rightarrow \underline{R_1 = 349\ \text{k}\Omega}$

$\frac{349 R_2}{349 + R_2} = 300 \Rightarrow \underline{R_2 = 2.14\ \text{M}\Omega}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(1.7)} = 2.61\ mA/V$

$A_v = \frac{g_m R_S}{1 + g_m R_S} = \frac{2.61(1.18)}{1 + (2.61)(1.18)}$

$\Rightarrow \underline{A_v = 0.755}$

$R_0 = \frac{1}{g_m} \| R_S = \frac{1}{2.61} \| 1.18 = 0.383 \| 1.18$

$\Rightarrow \underline{R_0 = 0.289\ \text{k}\Omega}$

6.33

a. $V_{GS} + I_{DQ} R_S = 5$

$I_{DQ} = \frac{5 - V_{GS}}{R_S} = K_n(V_{GS} - V_{TN})^2$

$5 - V_{GS} = (10)(3)(V_{GS}^2 - 2V_{GS} + 1)$

$30V_{GS}^2 - 59V_{GS} + 25 = 0$

$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow \underline{V_{GS} = 1.35\ \text{V}}$

$I_{DQ} = (3)(1.35 - 1)^2 \Rightarrow \underline{I_{DQ} = 0.365\ \text{mA}}$

$V_{DSQ} = 10 - (0.365)(5 + 10) \Rightarrow \underline{V_{DSQ} = 4.53\ \text{V}}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3)(0.365)} \Rightarrow$

$\underline{g_m = 2.09\ mA/V}$

$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0)(0.365)} \Rightarrow \underline{r_0 = \infty}$

c. $A_v = g_m(R_D \| R_L) = (2.09)(5 \| 4)$

$\Rightarrow \underline{A_v = 4.64}$

6.34

a. $I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$0.75 = (0.5)(V_{SG} - 1)^2 \Rightarrow V_{SG} = 2.22\ \text{V}$

$5 = I_{DQ} R_S + V_{SG} \Rightarrow R_S = \frac{5 - 2.22}{0.75}$

$\Rightarrow \underline{R_S = 3.71\ \text{k}\Omega}$

$V_{SDQ} = 10 - I_{DQ}(R_S + R_D)$

$6 = 10 - (0.75)(3.71 + R_D) \Rightarrow \underline{R_D = 1.62\ \text{k}\Omega}$

b. $R_i = \frac{1}{g_m}$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.22\ mA/V$

$R_i = \frac{1}{1.22} \Rightarrow \underline{R_i = 0.816\ \text{k}\Omega}$

$R_0 = R_D \Rightarrow \underline{R_0 = 1.62\ \text{k}\Omega}$

c. $i_0 = \left(\frac{R_D}{R_D + R_L}\right)\left(\frac{R_S}{R_S + [1/g_m]}\right) \cdot i_i$

$i_0 = \left(\frac{1.62}{1.62 + 2}\right)\left(\frac{3.71}{3.71 + 0.816}\right) i_i$

$i_0 = 0.367 i_i \Rightarrow \underline{i_0 = 1.84 \sin \omega t\ (\mu\text{A})}$

$v_0 = i_0 R_L = (1.84)(2) \sin \omega t$

$\Rightarrow \underline{v_0 = 3.68 \sin \omega t\ (\text{mV})}$

6.35

a. $I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$5 = 4(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.12\ \text{V}$

$V^+ = I_{DQ} R_D + V_{DSQ} - V_{GS}$

$10 = (5)R_D + 12 - 3.12 \Rightarrow \underline{R_D = 0.224\ \text{k}\Omega}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(5)} \Rightarrow \underline{g_m = 8.94\ mA/V}$

$R_i = \frac{1}{g_m} = \frac{1}{8.94} \Rightarrow \underline{R_i = 0.112\ \text{k}\Omega}$

c. $A_v = g_m(R_D \| R_L) = (8.94)(0.224 \| 2)$

$\Rightarrow \underline{A_v = 1.80}$

6.36

a. $I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$3 = 2(V_{SG} - 2)^2 \Rightarrow V_{SG} = 3.22$ V

$V^+ = I_{DQ}R_S + V_{SG}$

$R_S = \dfrac{10 - 3.22}{3} \Rightarrow \underline{R_S = 2.26\ k\Omega}$

$V_{SDQ} = 20 - I_{DQ}(R_S + R_D)$

$10 = 20 - (3)(2.26 + R_D) \Rightarrow \underline{R_D = 1.07\ k\Omega}$

b. $A_v = g_m(R_D \| R_L)$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(3)} = 4.90\ mA/V$

$A_v = (4.90)(1.07\|10) \Rightarrow \underline{A_v = 4.74}$

6.37

a. $|A_v| = \sqrt{\dfrac{(W/L)_D}{(W/L)_L}} = 5$

$\Rightarrow (W/L)_D = 25(0.5) \Rightarrow \underline{(W/L)_D = 12.5}$

$K_D = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)_D = (30)(12.5) = 375\ \mu A/V^2$

$K_L = (30)(0.5) = 15\ \mu A/V^2$

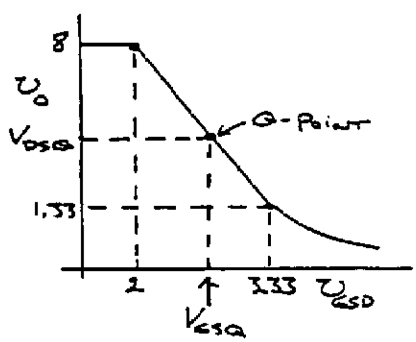
Transition point:

$v_{GSD} - V_{TND} = (V_{DD} - V_{TNL}) - \sqrt{\dfrac{K_D}{K_L}}(v_{GSD} - V_{TND})$

$v_{GSD} - 2 = (10 - 2) - \sqrt{\dfrac{375}{15}}\,(v_{GSD} - 2)$

$v_{GSD}(1 + 5) = (10 - 2) + 2 + 5(2)$

$v_{GSD} = 3.33$ V and $v_{DSD} = 1.33$ V



$V_{GSQ} = \dfrac{3.33 - 2}{2} + 2 \Rightarrow \underline{V_{GSQ} = 2.67\ V}$

b. $I_{DQ} = K_D(V_{GSQ} - V_{TND})^2$

$I_{DQ} = 0.375(2.67 - 2)^2 \Rightarrow \underline{I_{DQ} = 0.167\ mA}$

$V_{DSQ} = \dfrac{8 - 1.33}{2} + 1.33 \Rightarrow \underline{V_{DSQ} = 4.67\ V}$

6.38

(a) Transition point: Load:

$v_{OB} = V_{DD} - |V_{TNL}| = 10 - 2 = 8\ V$

Driver:

$$K_D\left[(v_{OA})^2(1 + \lambda_D v_{OA})\right] = K_L\left[(-V_{TNL})^2(1 + \lambda_L(V_{DD} - v_{OA}))\right]$$

$$0.5[v_{OtA}^2 + (0.02)v_{OtA}^3] = 0.1[(4)(1 + 0.02(10 - v_{OtA}))]$$

We have

$$0.01v_{OtA}^3 + 0.5v_{OtA}^2 + 0.008v_{OtA} - 0.48 = 0$$

Therefore $v_{OtA} = 0.963$ V

Now

$$v_{OQ} = \frac{8 - 0.963}{2} + 0.963 = 4.48\ \text{V} = V_{DSQ}$$

Then

$$K_D\left[(V_{GSD} - V_{TND})^2(1 + \lambda_D v_{OQ})\right] = K_L\left[(-V_{TNL})^2(1 + \lambda_L(V_{DD} - v_{OQ}))\right]$$

$$0.5[(V_{GSD} - 1.2)^2(1 + (0.02)(4.48))] = 0.1[(4)(1 + 0.02(10 - 4.48))]$$

which yields $\underline{V_{GSD} = 2.103\ V}$

b. $I_{DQ} = K_D\left[(V_{GSD} - V_{TND})^2(1 + \lambda_D v_{OQ})\right]$

$I_{DQ} = 0.5[(2.103 - 1.2)^2(1 + (0.02)(4.48))]$

So $\underline{I_{DQ} = 0.444\ mA}$

c. $r_{oD} = r_{oL} = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.02)(0.444)} = 113\ k\Omega$

$g_{mD} = 2K_D(V_{GSD} - V_{TND}) = 2(0.5)(2.103 - 1.2) \Rightarrow$

$g_{mD} = 0.903\ mA/V$

Then

$A_v = -g_{mD}(r_{oD}\|r_{oL}) = -(0.903)(113\|113)$

or $\underline{A_v = -51.0}$

6.39

$I_D = K_n(V_{GS} - V_{TN})^2, \quad V_{DS} = V_{GS}$

$I_D = 0$ when $V_{DS} = 1.5\,V \Rightarrow V_{TN} = 1.5\,V$

$0.8 = K_n(3 - 1.5)^2 \Rightarrow K_n = 0.356\,mA/V^2$

$g_n = \dfrac{dI_D}{dV_{DS}} = \dfrac{1}{R_o} = 2K_n(V_{DS} - V_{TN})$

$= 2(0.356)(3 - 1.5)$

$\Rightarrow \underline{R_o = 0.936\,k\Omega}$

6.40

a.

$I_{DQ} = K_{nD}(V_{GS} - V_{TND})^2 = (0.5)(0-(-1))^2$

$\underline{I_{DQ} = 0.5\,mA}$

$I_{DQ} = K_{nL}(V_{GSL} - V_{TNL})^2 = K_{nL}(V_{DD} - V_O - V_{TNL})^2$

$0.5 = 0.030(10 - V_0 - 1)^2$

$\sqrt{\dfrac{0.5}{0.030}} = 9 - V_0 \Rightarrow \underline{V_0 = 4.92\text{ V}}$

b. $I_{DD} = I_{DL}$

$K_{nD}(V_i - V_{TND})^2 = K_{nL}(V_{DD} - V_o - V_{TNL})^2$

$\sqrt{\dfrac{K_{nD}}{K_{nL}}}(V_i - V_{TND}) = V_{DD} - V_o - V_{TNL}$

$V_o = V_{DD} - V_{TNL} - \sqrt{\dfrac{K_{nD}}{K_{nL}}}(V_i - V_{TND})$

$A_v = \dfrac{dV_o}{dV_i} = -\sqrt{\dfrac{K_{nD}}{K_{nL}}} = -\sqrt{\dfrac{(W/L)_D}{(W/L)_L}}$

$A_v = -\sqrt{\dfrac{500}{30}} \Rightarrow \underline{A_v = -4.08}$

6.41

(a) $I_{DQ} = K_L(V_{GSL} - V_{TNL})^2 = K_L(V_{DSL} - V_{TNL})^2$

$I_D = (0.1)(4-1)^2 = 0.9\,mA$

$I_{DQ} = K_D(V_{GSD} - V_{TND})^2$

$0.9 = (1)(V_{GSD} - 1)^2 \Rightarrow V_{GSD} = 1.95\text{ V}$

$V_{GG} = V_{GSD} + V_{DSL} = 1.95 + 4$

$\Rightarrow \underline{V_{GG} = 5.95\text{ V}}$

b. $I_{DD} = I_{DL}$

$K_D(V_{GSD} - V_{TND})^2 = K_L(V_{GSL} - V_{TNL})^2$

$\sqrt{\dfrac{K_D}{K_L}}(V_{GG} + V_i - V_o - V_{TND}) = V_o - V_{TNL}$

$V_o\left(1 + \sqrt{\dfrac{K_D}{K_L}}\right) = \sqrt{\dfrac{K_D}{K_L}}(V_{GG} + V_i - V_{TND}) + V_{TNL}$

$A_v = \dfrac{dV_o}{dV_i} = \dfrac{\sqrt{K_D/K_L}}{1 + \sqrt{K_D/K_L}} \Rightarrow \underline{A_v = \dfrac{1}{1 + \sqrt{K_L/K_D}}}$

(c) From Problem 6.39:

$R_{LD} = \dfrac{1}{2K_L(V_{DSL} - V_{TNL})}$

$= \dfrac{1}{2(0.1)(4-1)} = 1.67\,k\Omega$

$g_m = 2\sqrt{K_D I_{DQ}} = 2\sqrt{(1)(0.9)} = 1.90\,mA/V$

$A_v = \dfrac{g_m(R_{LD}\|R_L)}{1 + g_m(R_{LD}\|R_L)} = \dfrac{(1.90)(1.67\|4)}{1 + (1.90)(1.67\|4)}$

$\Rightarrow \underline{A_v = 0.691}$

6.42

a. From Problem 6-41:

$A_v = \dfrac{g_m(R_{LD}\|R_L)}{1 + g_m(R_{LD}\|R_L)} = \dfrac{(1.90)(1.67\|10)}{1 + (1.90)(1.67\|10)}$

$\underline{A_v = 0.731}$

b. $R_0 = \dfrac{1}{g_m} \,\Big\|\, R_{LD} = \dfrac{1}{1.90} \,\Big\|\, 1.67 = 0.526\|1.67$

$\underline{R_0 = 0.40\text{ k}\Omega}$

6.43

a. $I_{D1} = K_1(V_{GS1} - V_{TN1})^2$

$0.4 = 0.1(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 4\text{ V}$

$V_{S1} = I_{D1}R_{S1} = (0.4)(4) = 1.6\text{ V}$

$V_{G1} = V_{S1} + V_{GS1} = 1.6 + 4 = 5.6\text{ V}$

$V_{G1} = \left(\dfrac{R_2}{R_1 + R_2}\right)V_{DD} = \dfrac{1}{R_1}\cdot R_{in}\cdot V_{DD}$

$5.6 = \dfrac{1}{R_1}(200)(10) \Rightarrow \underline{R_1 = 357\text{ k}\Omega}$

$\dfrac{357R_2}{357 + R_2} = 200 \Rightarrow \underline{R_2 = 455\text{ k}\Omega}$

$V_{DS1} = V_{DD} - I_{D1}(R_{S1} + R_{D1})$

$4 = 10 - (0.4)(4 + R_{D1}) \Rightarrow \underline{R_{D1} = 11\text{ k}\Omega}$

$V_{D1} = 10 - (0.4)(11) = 5.6\text{ V}$

$I_{D2} = K_2(V_{SG2} + V_{TP2})^2$

$2 = 1(V_{SG2} - 2)^2 \Rightarrow V_{SG2} = 3.41\text{ V}$

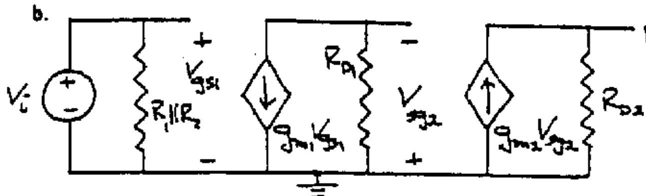
$V_{S2} = V_{D1} + V_{SG2} = 5.6 + 3.41 = 9.01$

$R_{S2} = \dfrac{10 - 9.01}{2} \Rightarrow \underline{R_{S2} = 0.495\text{ k}\Omega}$

$V_{SD2} = V_{DD} - I_{D2}(R_{S2} + R_{D2})$

$5 = 10 - (2)(0.495 + R_{D2}) \Rightarrow \underline{R_{D2} = 2.01\text{ k}\Omega}$

b.



$V_0 = g_{m2}V_{sg2}R_{D2} = (g_{m2}R_{D2})(g_{m1}V_{gs1}R_{D1})$

$V_{gs1} = V_i$

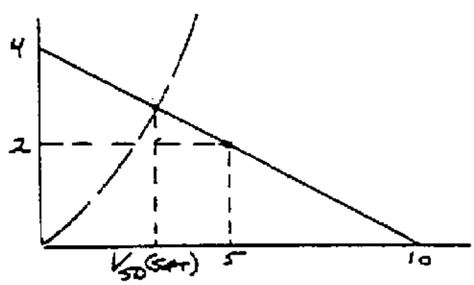
$A_v = \dfrac{V_0}{V_i} = g_{m1}g_{m2}R_{D1}R_{D2}$

$g_{m1} = 2\sqrt{K_1 I_{D1}} = 2\sqrt{(0.1)(0.4)} = 0.4\ mA/V$

$g_{m2} = 2\sqrt{K_2 I_{D2}} = 2\sqrt{(1)(2)} = 2.83\ mA/V$

$A_v = (0.4)(2.83)(11)(2.01) \Rightarrow \underline{A_v = 25.0}$

c.



$$V_{SD}(\text{sat}) = V_{SG} + V_{Th}$$
$$= V_{DD} - I_{D2}(R_{D2} + R_{S2})$$
$$= V_{DD} - k_{p2}(R_{D2} + R_{S2})V_{SD}^2(\text{sat})$$

So

$(1)(2.01 + 0.495)V_{SD}^2(\text{sat}) + V_{SD}(\text{sat}) - V_{DD} = 0$

$2.505V_{SD}^2(\text{sat}) + V_{SD}(\text{sat}) - 10 = 0$

$V_{SD}(\text{sat}) = \dfrac{-1 \pm \sqrt{1 + 4(2.505)(10)}}{2(2.505)}$

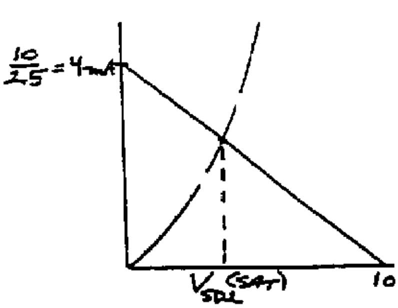
$V_{SD}(\text{sat}) = 1.81$ V

$5 - 1.81 = 3.19$

$\Rightarrow$ Max. output swing = 6.38 V peak-to-peak

6.44

a.



$V_{SD2}(sat) = V_{DD} - I_{D2}(R_{D2} + R_{S2})$
$= V_{DD} - K_{p2}(R_{D2} + R_{S2})V_{SD2}^2(sat)$

$(1)(2 + 0.5)V_{SD2}^2(\text{sat}) + V_{SD2}(\text{sat}) - 10 = 0$

$2.5V_{SD2}^2(\text{sat}) + V_{SD2}(\text{sat}) - 10 = 0$

$V_{SD2}(\text{sat}) = \dfrac{-1 \pm \sqrt{1 + 4(2.5)(10)}}{2(2.5)} = 1.81$ V

$V_{SDQ2} = \dfrac{10 - 1.81}{2} + 1.81 \Rightarrow V_{SDQ2} = 5.91$ V

$V_{SDQ2} = V_{DD} - I_{DQ2}(R_{D2} + R_{S2})$

$5.91 = 10 - I_{DQ2}(2 + 0.5) \Rightarrow I_{DQ2} = 1.64$ mA

$V_{S2} = 10 - (1.64)(0.5) = 9.18$ V

$I_{DQ2} = K_{p2}(V_{SG2} + V_{TP2})^2$

$1.64 = (1)(V_{SG2} - 2)^2 \Rightarrow V_{SG2} = 3.28$ V

$V_{D1} = V_{S2} - V_{SG2} = 9.18 - 3.28 = 5.90$ V

$R_{D1} = \dfrac{10 - 5.90}{0.4} \Rightarrow \underline{R_{D1} = 10.25\ k\Omega}$

$I_{DQ1} = K_{n1}(V_{GS1} - V_{TN1})^2$

$0.4 = (0.1)(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 4$ V

$V_{S1} = (0.4)(1) = 0.4$ V

$V_{G1} = V_{S1} + V_{GS1} = 0.4 + 4 = 4.4$ V

$V_{G1} = \left(\dfrac{R_2}{R_1 + R_2}\right)V_{DD} = \dfrac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$4.4 = \dfrac{1}{R_1} \cdot (200)(10) \Rightarrow \underline{R_1 = 455\ k\Omega}$

$\dfrac{455R_2}{455 + R_2} = 200 \Rightarrow \underline{R_2 = 357\ k\Omega}$

b. $\underline{I_{DQ2} = 1.64\ \text{mA}}$

$V_{SDQ2} = 10 - (1.64)(2 + 0.5) \Rightarrow \underline{V_{SDQ2} = 5.90\ V}$

$V_{DSQ1} = 10 - (0.4)(10.25 + 1) \Rightarrow \underline{V_{DSQ1} = 5.50\ V}$

(c) $g_{m1} = 2\sqrt{K_{n1} I_{DQ1}} = 2\sqrt{(0.1)(0.4)} = 0.4\ mA/V$

$g_{m2} = 2\sqrt{K_{p2} I_{DQ2}} = 2\sqrt{(1)(1.64)} = 2.56\ mA/V$

$A_v = g_{m1}g_{m2}R_{D1}R_{D2} = (0.4)(2.56)(10.25)(2)$

$\Rightarrow \underline{A_v = 21.0}$

6.45

a. $V_{DSQ2} = 7 = V_{DD} - I_{DQ2}R_{S2} = 10 - I_{DQ2}(6)$

$I_{DQ2} = 0.5$ mA

$I_{DQ2} = K_2(V_{GS2} - V_{TN2})^2$

$0.5 = (0.2)(V_{GS2} - 0.8)^2 \Rightarrow V_{GS2} = 2.38$ V

$V_{S1} = V_{S2} + V_{GS2} = 3 + 2.38 = 5.38$

$I_{DQ1} = \dfrac{V_{S1}}{R_{S1}} = \dfrac{5.38}{20} = 0.269$ mA

$I_{DQ1} = K_1(V_{GS1} - V_{TN1})^2$

$0.269 = (0.2)(V_{GS1} - 0.8)^2 \Rightarrow V_{GS1} = 1.96$ V

$V_{G1} = V_{S1} + V_{GS1} = 5.38 + 1.96 = 7.34$ V

$$V_{G1} = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$7.34 = \frac{1}{R_1}(400)(10) \Rightarrow \underline{R_1 = 545\ \text{k}\Omega}$$

$$\frac{545 R_2}{545 + R_2} = 400 \Rightarrow \underline{R_2 = 1.50\ \text{M}\Omega}$$

b. $\underline{I_{DQ1} = 0.269\ \text{mA}}$, $\underline{I_{DQ2} = 0.5\ \text{mA}}$

$$V_{DSQ1} = 10 - (0.269)(20) \Rightarrow \underline{V_{DSQ1} = 4.62\ \text{V}}$$

c. $A_v = \dfrac{g_{m1} R_{S1}}{1 + g_{m1} R_{S1}} \cdot \dfrac{g_{m2} R_{S2}}{1 + g_{m2} R_{S2}}$

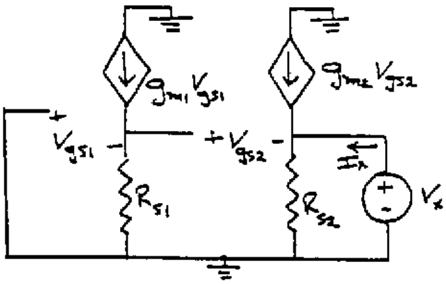
$$g_{m1} = 2\sqrt{K_1 I_{DQ1}} = 2\sqrt{(0.2)(0.269)} = 0.464\ mA/V$$

$$g_{m2} = 2\sqrt{K_2 I_{DQ2}} = 2\sqrt{(0.2)(0.5)} = 0.632\ mA/V$$

$$A_v = \frac{(0.464)(20)}{1 + (0.464)(20)} \cdot \frac{(0.632)(6)}{1 + (0.632)(6)}$$

$$= (0.9027)(0.7913)$$

$$= \underline{A_v = 0.714}$$



$$R_o = \frac{1}{g_{m2}} \parallel R_{S2} = \frac{1}{0.632} \parallel 6 = 1.582 \| 6$$

$$\Rightarrow \underline{R_o = 1.25\ \text{k}\Omega}$$

6.46

(a) $I_{DQ1} = \dfrac{10 - V_{GS1}}{R_{S2}} = K_{n1}(V_{GS1} - V_{TN1})^2$

$$10 - V_{GS1} = (4)(10)(V_{GS1}^2 - 4V_{GS1} + 4)$$

$$40V_{GS1}^2 - 159V_{GS1} + 150 = 0$$

$$V_{GS1} = \frac{159 \pm \sqrt{(159)^2 - 4(40)(150)}}{2(40)}$$

$$\Rightarrow V_{GS1} = 2.435\ \text{V}$$

$$I_{DQ1} = (4)(2.435 - 2)^2 \Rightarrow \underline{I_{DQ1} = 0.757\ \text{mA}}$$

$$V_{DSQ1} = 20 - (0.757)(10) \Rightarrow \underline{V_{DSQ1} = 12.4\ \text{V}}$$

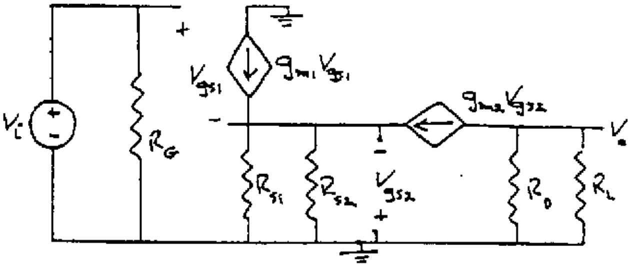
Also $\underline{I_{DQ2} = 0.757\ \text{mA}}$

$$V_{DSQ2} = 20 - (0.757)(10 + 5) \Rightarrow \underline{V_{DSQ2} = 8.65\ \text{V}}$$

(b) $g_{m1} = g_{m2} = 2\sqrt{KI_{DQ}} = 2\sqrt{(4)(0.757)} \Rightarrow$

$$\underline{g_{m1} = g_{m2} = 3.48\ mA/V}$$

c.



$$V_0 = -(g_{m2} V_{gs2})(R_D \| R_L)$$

$$V_{gs2} = (-g_{m1} V_{gs1} - g_{m2} V_{gs2})(R_{S1} \| R_{S2})$$

$$V_i = V_{gs1} - V_{gs2} \Rightarrow V_{gs1} = V_i + V_{gs2}$$

$$V_{gs2} + g_{m2} V_{gs2}(R_{S1} \| R_{S2})$$
$$= -g_{m1}(V_i + V_{gs2})(R_{S1} \| R_{S2})$$

$$V_{gs2} + g_{m2} V_{gs2}(R_{S1} \| R_{S2}) + g_{m1} V_{gs2}(R_{S1} \| R_{S2})$$
$$= -g_{m1} V_i (R_{S1} \| R_{S2})$$

$$V_{gs2} = \frac{-g_{m1} V_i (R_{S1} \| R_{S2})}{1 + g_{m2}(R_{S1} \| R_{S2}) + g_{m1}(R_{S1} \| R_{S2})}$$

$$A_v = \frac{V_0}{V_i} = \frac{g_{m1} g_{m2} (R_{S1} \| R_{S2})(R_D \| R_L)}{1 + (g_{m1} + g_{m2})(R_{S1} \| R_{S2})}$$

$$A_v = \frac{(3.48)^2 (10\|10)(5\|2)}{1 + (3.48 + 3.48)(10\|10)}$$

$$\Rightarrow \underline{A_v = 2.42}$$

6.47

a. $I_{DQ} = 3\ \text{mA}$

$$V_{S1} = I_{DQ} R_S - 5 = (3)(1.2) - 5 = -1.4\ \text{V}$$

$$I_{DQ} = K_1 (V_{GS} - V_{TN})^2$$

$$3 = 2(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.22\ \text{V}$$

$$V_{G1} = V_{GS} + V_{S1} = 2.22 - 1.4 = 0.82\ \text{V}$$

$$V_{G1} = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)(5) \Rightarrow 0.82 = \left(\frac{R_3}{500}\right)(5)$$

$$\Rightarrow \underline{R_3 = 82\ \text{k}\Omega}$$

$$V_{D1} = V_{S1} + V_{DSQ1} = -1.4 + 2.5 = 1.1\ \text{V}$$

$$V_{G2} = V_{D1} + V_{GS} = 1.1 + 2.22 = 3.32\ \text{V}$$

$$V_{G2} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)(5)$$

$$\Rightarrow 3.32 = \left(\frac{R_2 + R_3}{500}\right)(5)$$

$$R_2 + R_3 = 332 \Rightarrow \underline{R_2 = 250\ \text{k}\Omega}$$

$R_1 = 500 - 250 - 82 \Rightarrow \underline{R_1 = 168\ \text{k}\Omega}$

$V_{D2} = V_{D1} + V_{DSQ2} = 1.1 + 2.5 = 3.6\ \text{V}$

$R_D = \dfrac{5 - 3.6}{3} \Rightarrow \underline{R_D = 0.467\ \text{k}\Omega}$

b. $A_v = -g_{m1} R_D$

$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(3)} = 4.90\ mA/V$

$A_v = -(4.90)(0.467) \Rightarrow \underline{A_v = -2.29}$

6.48

a. $V_{S1} = I_{DQ} R_S - 10 = (5)(2) - 10 \Rightarrow V_{S1} = 0$

$I_{DQ} = K_1 (V_{GS1} - V_{TN})^2$

$5 = 4(V_{GS1} - 1.5)^2 \Rightarrow V_{GS1} = 2.618\ \text{V}$

$V_{G1} = V_{GS1} + V_{S1} = 2.618\ \text{V} = I R_3 = (0.1) R_3$

$\Rightarrow \underline{R_3 = 26.2\ \text{k}\Omega}$

$V_{D1} = V_{S1} + V_{DSQ1} = 0 + 3.5 = 3.5\ \text{V}$

$V_{G2} = V_{D1} + V_{GS} = 3.5 + 2.62 = 6.12\ \text{V}$

$= (0.1)(R_2 + R_3)$

$R_2 + R_3 = 61.2\ \text{k}\Omega \Rightarrow \underline{R_2 = 35\ \text{k}\Omega}$

$V_{D2} = V_{D1} + V_{DSQ2} = 3.5 + 3.5 = 7.0\ \text{V}$

$R_D = \dfrac{10 - 7}{5} \Rightarrow \underline{R_D = 0.6\ \text{k}\Omega}$

$R_1 = \dfrac{10 - 6.12}{0.1} \Rightarrow \underline{R_1 = 38.8\ \text{k}\Omega}$

b. $A_v = -g_{m1} R_D$

$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(5)} = 8.94\ mA/V$

$A_v = -(8.94)(0.6) \Rightarrow \underline{A_v = -5.36}$

6.49

a. $I_{DQ} = I_{DSS}\left(1 - \dfrac{V_{GS}}{V_P}\right)^2$

$4 = 6\left(1 - \dfrac{V_{GS}}{(-3)}\right)^2$

$V_{GS} = (-3)\left[1 - \sqrt{\dfrac{4}{6}}\right] \Rightarrow \underline{V_{GS} = -0.551\ \text{V}}$

$V_{DSQ} = V_{DD} - I_{DQ} R_D$

$6 = 10 - (4) R_D \Rightarrow \underline{R_D = 1\ \text{k}\Omega}$

b. $g_m = \dfrac{2 I_{DSS}}{(-V_P)}\left(1 - \dfrac{V_{GS}}{V_P}\right) = \dfrac{2(6)}{3}\left(1 - \dfrac{-0.551}{-3}\right)$

$\Rightarrow \underline{g_m = 3.27\ \text{mA/V}}$

$r_o = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.01)(4)} \Rightarrow \underline{r_o = 25\ \text{k}\Omega}$

c. $A_v = -g_m(r_o \| R_D) = -(3.27)(25\|1)$

$\Rightarrow \underline{A_v = -3.14}$

6.50

$V_{GS} + I_{DQ}(R_{S1} + R_{S2}) = 0$

$I_{DQ} = I_{DSS}\left(1 - \dfrac{V_{GS}}{V_P}\right)^2$

$V_{GS} + I_{DSS}(R_{S1} + R_{S2})\left(1 - \dfrac{V_{GS}}{V_P}\right)^2 = 0$

$V_{GS} + (2)(0.1 + 0.25)\left(1 - \dfrac{V_{GS}}{V_P}\right)^2 = 0$

$V_{GS} + 0.7\left(1 - \dfrac{2V_{GS}}{(-2)} + \dfrac{V_{GS}^2}{(-2)^2}\right) = 0$

$0.175 V_{GS}^2 + 1.7 V_{GS} + 0.7 = 0$

$V_{GS} = \dfrac{-1.7 \pm \sqrt{(1.7)^2 - 4(0.175)(0.7)}}{2(0.175)}$

$\Rightarrow V_{GS} = -0.431\ \text{V}$

$g_m = \dfrac{2 I_{DSS}}{-V_P}\left(1 - \dfrac{V_{GS}}{V_P}\right) = \dfrac{2(2)}{2}\left(1 - \dfrac{-0.431}{-2}\right)$

$\Rightarrow \underline{g_m = 1.57\ \text{mA/V}}$

$A_v = \dfrac{-g_m(R_D \| R_L)}{1 + g_m R_{S1}} = \dfrac{-(1.57)(8\|4)}{1 + (1.57)(0.1)}$

$\Rightarrow \underline{A_v = -3.62}$

$A_i = \dfrac{i_0}{i_i} = \dfrac{(v_0/R_L)}{(v_i/R_G)} = \dfrac{v_0}{v_i} \cdot \dfrac{R_G}{R_L} = (-3.62)\left(\dfrac{50}{4}\right)$

$\Rightarrow \underline{A_i = -45.3}$

6.51

$I_{DQ} = \dfrac{I_{DSS}}{2} = 4\ \text{mA}$

$V_{DSQ} = \dfrac{V_{DD}}{2} = 10\ \text{V}$

$V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$

$10 = 20 - (4)(R_S + R_D) \Rightarrow R_S + R_D = 2.5\ \text{k}\Omega$

$V_S = 2\ \text{V} = I_{DQ} R_S = 4 R_S$

$\Rightarrow \underline{R_S = 0.5\ \text{k}\Omega}, \quad \underline{R_D = 2.0\ \text{k}\Omega}$

$I_{DQ} = I_{DSS}\left(1 - \dfrac{V_{GS}}{V_P}\right)^2$

$4 = 8\left(1 - \dfrac{V_{GS}}{(-4.2)}\right)^2 \Rightarrow V_{GS} = (-4.2)\left(1 - \sqrt{\dfrac{4}{8}}\right)$

$\Rightarrow V_{GS} = -1.23\ \text{V}$

$V_G = V_S + V_{GS} = 2 - 1.23$

$V_G = 0.77\ \text{V} = \left(\dfrac{R_2}{R_1 + R_2}\right)(20) = \left(\dfrac{R_2}{100}\right)(20)$

$\Rightarrow \underline{R_2 = 3.85\ \text{k}\Omega}, \quad \underline{R_1 = 96.2\ \text{k}\Omega}$

6.52

a. $I_{DQ} = \frac{I_{DSS}}{2} = 5 \text{ mA}$

$V_{DSQ} = \frac{V_{DD}}{2} = \frac{12}{2} = 6 \text{ V}$

$R_S = \frac{12-6}{5} \Rightarrow \underline{R_S = 1.2 \text{ k}\Omega}$

$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$5 = 10\left(1 - \frac{V_{GS}}{(-5)}\right)^2 \Rightarrow V_{GS} = (-5)\left(1 - \sqrt{\frac{5}{10}}\right)$

$\Rightarrow V_{GS} = -1.46 \text{ V}$

$V_G = V_S + V_{GS} = 6 - 1.46 = 4.54 \text{ V}$

$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$4.54 = \frac{1}{R_1}(100)(12) \Rightarrow \underline{R_1 = 264 \text{ k}\Omega}$

$\frac{264R_2}{264 + R_2} = 100 \Rightarrow \underline{R_2 = 161 \text{ k}\Omega}$

b. $g_m = \frac{2I_{DSS}}{(-V_P)}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(10)}{5}\left(1 - \frac{-1.46}{-5}\right)$

$\Rightarrow g_m = 2.83 \text{ mA/V}$

$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$

$A_v = \frac{g_m(r_0 \| R_S \| R_L)}{1 + g_m(r_0 \| R_S \| R_L)}$

$A_v = \frac{(2.83)(20 \| 1.2 \| 0.5)}{1 + (2.83)(20 \| 1.2 \| 0.5)}$

$\Rightarrow \underline{A_v = 0.495}$

$R_o = \frac{1}{g_m} \left\| R_S = \frac{1}{2.83} \right\| 1.2 = 0.353 \| 1.2$

$\Rightarrow \underline{R_o = 0.273 \text{ k}\Omega}$

6.53

a. $V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\frac{110}{110 + 90}\right)(10) = 5.5 \text{ V}$

$I_{DQ} = \frac{10 - (V_G - V_{GS})}{R_S} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$10 - 5.5 + V_{GS} = (2)(5)\left(1 - \frac{V_{GS}}{1.75}\right)^2$

$4.5 + V_{GS} = 10(1 - 1.143V_{GS} + 0.3265V_{GS}^2)$

$3.265V_{GS}^2 - 12.43V_{GS} + 5.5 = 0$

$V_{GS} = \frac{12.43 \pm \sqrt{(12.43)^2 - 4(3.265)(5.5)}}{2(3.265)}$

$\Rightarrow V_{GS} = 0.511 \text{ V}$

$I_{DQ} = (2)\left(1 - \frac{0.511}{1.75}\right)^2 \Rightarrow \underline{I_{DQ} = 1.42 \text{ mA}}$

$V_{SDQ} = 10 - (1.42)(5) \Rightarrow \underline{V_{SDQ} = 2.9 \text{ V}}$

b. $g_m = \frac{2I_{DSS}}{V_P}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(2)}{1.75}\left(1 - \frac{0.511}{1.75}\right)$

$\Rightarrow g_m = 1.62 \text{ mA/V}$

$A_v = \frac{g_m(R_S \| R_L)}{1 + g_m(R_S \| R_L)} = \frac{(1.62)(5 \| 10)}{1 + (1.62)(5 \| 10)}$

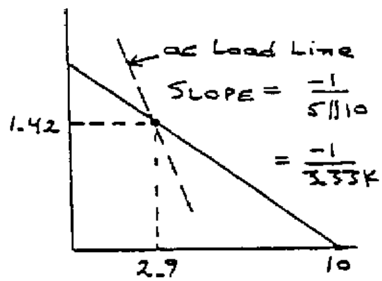
$\Rightarrow \underline{A_v = 0.844}$

$A_i = \frac{i_0}{i_i} = \frac{(v_0/R_L)}{(v_i/R_i)} = A_v \cdot \left(\frac{R_i}{R_L}\right)$

$R_i = R_1 \| R_2 = 90 \| 110 = 49.5 \text{ k}\Omega$

$A_i = (0.844)\left(\frac{49.5}{10}\right) \Rightarrow \underline{A_i = 4.18}$

c.



Maximum swing in output voltage
= 5.8 V peak-to-peak

6.54

$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$4 = 8\left(1 - \frac{V_{GS}}{4}\right)^2 \Rightarrow V_{GS} = 4\left(1 - \sqrt{\frac{4}{8}}\right)$

$\Rightarrow V_{GS} = 1.17 \text{ V}$

$V_{SDQ} = V_{DD} - I_{DQ}(R_S + R_D)$

$7.5 = 20 - 4(R_S + R_D) \Rightarrow R_S + R_D = 3.125 \text{ k}\Omega$

$g_m = \frac{2I_{DSS}}{V_P}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(8)}{4}\left(1 - \frac{1.17}{4}\right)$

$\Rightarrow g_m = 2.83 \text{ mA/V}$

$R_S = 3.125 - R_D$

$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

$$-3(1 + g_m R_S) = -g_m R_D$$

$$3[1 + (2.83)(3.125 - R_D)] = (2.83) R_D$$

$$9.844 - 2.83 R_D = 0.7075 R_D \Rightarrow \underline{R_D = 2.78\ \text{k}\Omega}$$

$$\underline{R_S = 0.345\ \text{k}\Omega}$$

$$V_S = 20 - (4)(0.345) \Rightarrow V_S = 18.6\ \text{V}$$

$$V_G = V_S - V_{GS} = 18.6 - 1.17 = 17.4\ \text{V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \left(\frac{R_2}{400}\right)(20)$$

$$\Rightarrow \underline{R_2 = 348\ \text{k}\Omega}, \quad \underline{R_1 = 52\ \text{k}\Omega}$$