Chapter 10

Exercise Solutions

E10.1

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on})}{R_1} = \frac{10-0.7}{15}$$

$\underline{I_{REF} = 0.62 \text{ mA}}$

$$I_0 = \frac{I_{REF}}{1+\frac{2}{\beta}} = \frac{0.62}{1+\frac{2}{75}}$$

$\underline{I_0 = 0.604 \text{ mA}}$

E10.2

For $I_0 = 0.75$ mA

$$I_{REF} = I_0\left(1+\frac{2}{\beta}\right) = (0.75)\left(1+\frac{2}{100}\right)$$

$\underline{I_{REF} = 0.765 \text{ mA}}$

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1}$$

$$R_1 = \frac{5-0.7-(-5)}{0.765}$$

$\underline{R_1 = 12.2 \text{ k}\Omega}$

E10.3

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1} = \frac{5-0.7-(-5)}{12}$$

$I_{REF} = 0.775$ mA

$$I_0 = \frac{I_{REF}}{1+\frac{2}{\beta}} = \frac{0.775}{1+\frac{2}{75}} = 0.754 \text{ mA}$$

$$\Delta I_0 = (0.02)(0.754) = 0.0151 \text{ mA}$$

$$\text{and } \Delta I_0 = \frac{1}{r_0}\Delta V_{CE2} \Rightarrow r_0 = \frac{\Delta V_{CE2}}{\Delta I_0}$$

$$r_0 = \frac{4}{0.0151} = 265 \text{ k}\Omega = \frac{V_A}{I_0}$$

$$\Rightarrow V_A = (265)(0.754) \Rightarrow \underline{V_A \cong 200 \text{ V}}$$

E10.4

$$I_{REF} = \frac{V^+ - 2V_{BE}(\text{on})}{R_1} = \frac{9-2(0.7)}{12}$$

$\underline{I_{REF} = 0.6333 \text{ mA}}$

$$I_0 = \frac{I_{REF}}{1+\frac{2}{\beta(1+\beta)}} = \frac{0.6333}{1+\frac{2}{75(76)}} = 0.6331 \text{ mA}$$

$\underline{I_0 = 0.6331 \text{ mA} = I_{C1}}$

$$I_{B1} = I_{B2} = \frac{I_0}{\beta} \Rightarrow \underline{I_{B1} = I_{B2} = 8.44\ \mu\text{A}}$$

$$I_{E3} = I_{B1} + I_{B2} \Rightarrow \underline{I_{E3} = 16.88\ \mu\text{A}}$$

$$I_{B3} = \frac{I_{E3}}{1+\beta} \Rightarrow \underline{I_{B3} = 0.222\ \mu\text{A}}$$

E10.5

$$I_{REF} = \frac{10-(0.7)(2)}{12} = 0.717 \text{ mA}$$

$$I_0 \approx I_{REF} = 0.717$$

$$r_0 = \frac{V_A}{I_0} = \frac{100}{0.717} \Rightarrow \underline{r_0 = 139 \text{ k}\Omega}$$

$$\Delta I_0 = \frac{1}{r_0}\Delta V_{CE2} = \frac{4}{139}$$

$$\Rightarrow \underline{\Delta I_0 = 0.0288 \text{ mA}}$$

E10.6

$$I_0 = I_{REF} \cdot \frac{1}{\left(1+\frac{2}{\beta(1+\beta)}\right)} = \frac{0.50}{\left(1+\frac{2}{50(51)}\right)}$$

$$\Rightarrow \underline{I_0 = 0.4996 \text{ mA}}$$

$$I_{B3} = \frac{I_0}{\beta} \Rightarrow \underline{I_{B3} = 9.99\ \mu\text{A}}$$

$$I_{E3} = \left(\frac{1+\beta}{\beta}\right)I_{C3} \Rightarrow \underline{I_{E3} = 0.5096 \text{ mA}}$$

$$I_{C2} = \frac{I_{E3}}{\left(1+\frac{2}{\beta}\right)} = \frac{0.5096}{\left(1+\frac{2}{50}\right)}$$

$$\Rightarrow \underline{I_{C2} = 0.490 \text{ mA} = I_{C1}}$$

$$I_{B1} = I_{B2} = \frac{I_{C2}}{\beta} \Rightarrow \underline{I_{B1} = I_{B2} = 9.80\ \mu\text{A}}$$

E10.7

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$R_E = \frac{V_T}{I_0}\ln\left(\frac{I_{REF}}{I_0}\right) = \frac{0.026}{0.025}\ln\left(\frac{0.75}{0.025}\right)$$

$$\Rightarrow \underline{R_E = 3.54 \text{ k}\Omega}$$

$$R_1 = \frac{5-0.7}{0.75} \Rightarrow \underline{R_1 = 5.73 \text{ k}\Omega}$$

$$V_{BE1} - V_{BE2} = I_0 R_E = (0.025)(3.54)$$

$$\Rightarrow \underline{V_{BE1} - V_{BE2} = 88.5 \text{ mV}}$$

E10.8

$$I_{REF} = \frac{5-0.7-(-5)}{12} \Rightarrow \underline{I_{REF} = 0.775 \text{ mA}}$$

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$I_0(6) = (0.026)\ln\left(\frac{0.775}{I_0}\right)$$

$$\Rightarrow \underline{I_0 \cong 16.6\ \mu\text{A}}$$

E10.9

$$I_0R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$R_E = \frac{0.026}{0.025}\ln\left(\frac{0.70}{0.025}\right) \Rightarrow R_E = 3.47\text{ k}\Omega$$

$$g_{m2} = \frac{I_0}{V_T} = \frac{0.025}{0.026} \Rightarrow g_{m2} = 0.962\text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_0} = \frac{(150)(0.026)}{0.025} = 156\text{ k}\Omega$$

$$r_{o2} = \frac{V_A}{I_0} = \frac{100}{0.025} = 4000\text{ k}\Omega$$

$$R'_E = R_E \| r_{\pi 2} = 3.47\|156 = 3.39\text{ k}\Omega$$

$$R_o = r_{o2}(1 + g_{m2}R'_E) = 4000[1 + (0.962)(3.39)]$$

$$R_o = 17,045\text{ k}\Omega$$

$$dI_0 = \frac{1}{R_o}\cdot dV_{C2} = \frac{3}{17,045}$$

$$\Rightarrow \underline{dI_0 = 0.176\ \mu\text{A}}$$

E10.10

$$I_{REF} = I_R + I_{BR} + I_{B1} + \ldots + I_{BN}$$

$$I_R = I_{01} = I_{02} = \ldots = I_{0N}$$

and $I_{BR} = I_{B1} = I_{B2} = \ldots = I_{BN} = \frac{I_{01}}{\beta}$

$$I_{REF} = I_{01} + (N+1)\left(\frac{I_{01}}{\beta}\right) = I_{01}\left(1 + \frac{N+1}{\beta}\right)$$

So $I_{01} = I_{02} = \ldots = I_{0N} = \frac{I_{REF}}{1 + \frac{N+1}{\beta}}$

$$\frac{I_{01}}{I_{REF}} = 0.90 = \frac{1}{1 + \frac{N+1}{50}}$$

$$1 + \frac{N+1}{50} = \frac{1}{0.9}$$

$$N + 1 = \left(\frac{1}{0.9} - 1\right)(50)$$

$$N = \left(\frac{1}{0.9} - 1\right)(50) - 1$$

$$N = 4.55 \Rightarrow \underline{N = 4}$$

E10.11

a. From Equation (10.52),

$$V_{GS1} = \frac{\sqrt{\frac{3}{12}}}{1 + \sqrt{\frac{3}{12}}} \times 10 + \left(\frac{1 - \sqrt{\frac{3}{12}}}{1 + \sqrt{\frac{3}{12}}}\right) \times (1.8)$$

$$V_{GS1} = \left(\frac{0.5}{1 + 0.5}\right)(10) + \left(\frac{1 - 0.5}{1 + 0.5}\right) \times (1.8)$$

$V_{GS1} = 3.93$ V also $V_{DS1} = 3.93$ V

$$I_{REF} = (12)(0.020)[3.93 - 1.8]^2[1 + (0.01)(3.93)]$$

$$\Rightarrow \underline{I_{REF} = 1.13\text{ mA}}$$

b. $I_0 = I_{REF} \times \frac{(W/L)_2}{(W/L)_1} \times \frac{(1 + \lambda V_{DS2})}{(1 + \lambda V_{DS1})}$

$$I_0 = (1.13) \times \left(\frac{6}{12}\right) \times \frac{[1 + (0.01)(2)]}{[1 + (0.01)(3.93)]}$$

$$\Rightarrow \underline{I_0 = 0.555\text{ mA}}$$

c. For $V_{DS2} = 6$ V

$$\Rightarrow \underline{I_0 = 0.576\text{ mA}}$$

E10.12

$$K_{n1}(V_{GS1} - V_{TN})^2 = K_{n3}(V_{GS3} - V_{TN})^2$$

$$V_{GS1} - 2 = \left(\sqrt{\frac{0.10}{0.25}}\right)(V_{GS3} - 2)$$

$$V_{GS1} - 2 = (0.632)(V_{GS3} - 2)$$

$$V_{GS3} = 10 - V_{GS1}$$

$$V_{GS1} - 2 = (0.632)(10 - V_{GS1}) - (0.632)(2)$$

$$1.632V_{GS1} = 7.056 \Rightarrow V_{GS1} = 4.32\text{ V}$$

$$I_{REF} = K_{n1}(V_{GS1} - V_{TN})^2 = (0.25)(4.32 - 2)^2 \Rightarrow$$

$$\underline{I_{REF} = 1.35\,mA}$$

$$I_O = 3K_{n2}(V_{GS1} - V_{TN})^2 = 3(0.25)(4.32 - 2)^2$$

$$I_O = 3I_{REF} \Rightarrow \underline{I_O = 4.04\,mA}$$

E10.13

$$V_{DS}(sat) = 1\,V = V_{GS2} - V_{TN} = V_{GS2} - 2$$

$$\Rightarrow V_{GS2} = 3\,V$$

$$I_O = K_{n2}(V_{GS2} - V_{TN})^2 = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2$$

$$0.20 = (0.020)\left(\frac{W}{L}\right)_2 (3-2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 10}$$

$$I_{REF} = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2$$

$$V_{GS1} = V_{GS2}$$

$$0.5 = (0.020)\left(\frac{W}{L}\right)_1 (3-2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 25}$$

$$V_{GS3} = V^+ - V_{GS1} = 10 - 3 = 7\,V$$

$$I_{REF} = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TN})^2$$

$$0.5 = (0.020)\left(\frac{W}{L}\right)_3 (7-2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 1}$$

E10.14

a. $I_{REF} = K_n(V_{GS} - V_{TN})^2$

$$0.020 = 0.080(V_{GS} - 1)^2$$

$\underline{V_{GS} = 1.5\text{ V}}$ all transistors

b. $V_{G4} = V_{GS3} + V_{GS1} + V^- = 1.5 + 1.5 - 5 = -2$ V

$V_{S4} = V_{G4} - V_{GS4} = -2 - 1.5 = -3.5$ V

$V_{D4}(\min) = V_{S4} + V_{DS4}(\text{sat})$

and $V_{DS4}(sat) = V_{GS4} - V_{TN} = 1.5 - 1 = 0.5\,V$

So $V_{D4}(\min) = -3.5 + 0.5$

$\Rightarrow \underline{V_{D4}(\min) = -3.0 \text{ V}}$

c. $R_0 = r_{04} + r_{02}(1 + g_m r_{04})$

$$r_{02} = r_{04} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.020)} = 2500 \text{ k}\Omega$$

$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.080)(1.5 - 1) \Rightarrow$

$g_m = 0.080\, mA/V$

$R_0 = 2500 + 2500(1 + (0.080)(2500))$

$\Rightarrow \underline{R_0 = 505 \text{ M}\Omega}$

E10.15

$I_{REF} = 0.20 = K_{n1}(V_{GS1} - V_{TN})^2 = 0.15(V_{GS1} - 1)^2$

$\Rightarrow \underline{V_{GS1} = V_{GS2} = 2.15\,V}$

$I_O = K_{n2}(V_{GS2} - V_{TN})^2 = \dfrac{0.15}{2}(2.15 - 1)^2 \Rightarrow$

$\underline{I_O = 0.10\, mA}$

$I_O = K_{n3}(V_{GS3} - V_{TN})^2$

$0.10 = 0.15(V_{GS3} - 1)^2 \Rightarrow \underline{V_{GS3} = 1.82\,V}$

E10.16

All transistors are identical

$\Rightarrow \underline{I_0 = I_{REF} = 250\ \mu\text{A}}$

$I_{REF} = K_n(V_{GS} - V_{TN})^2$

$0.25 = 0.20(V_{GS} - 1)^2$

$\Rightarrow \underline{V_{GS} = 2.12 \text{ V}}$

E10.17

For Q_2: $v_{DS}(\min) = |V_P| = 2$ V

$\Rightarrow V_S(\min) = v_{DS}(\min) - 5 = 2 - 5$

$\Rightarrow \underline{V_S(\min) = -3 \text{ V}}$

$I_0 = I_{DSS2}(1 + \lambda v_{DS2}) = 0.5(1 + (0.15)(2))$

$\Rightarrow \underline{I_0 = 0.65 \text{ mA}}$

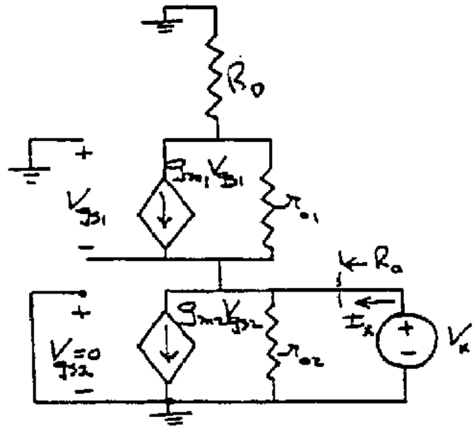
$$I_0 = I_{DSS1}\left(1 - \frac{v_{GS1}}{V_{P1}}\right)^2$$

$$0.65 = 0.80\left(1 - \frac{v_{GS1}}{-2}\right)^2$$

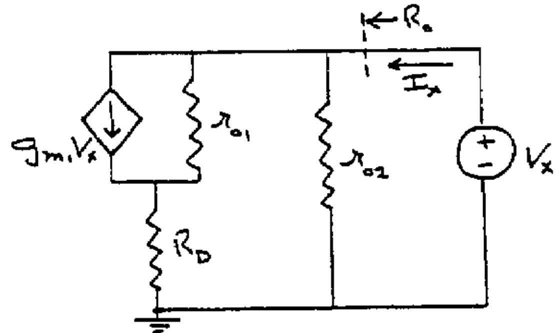
$\dfrac{v_{GS1}}{-2} = 0.0986 \Rightarrow v_{GS1} = -0.197$ V

$v_{GS1} = V_I - V_S$

$-0.197 = V_I - (-3) \Rightarrow \underline{V_I(\min) = -3.2 \text{ V}}$



$V_{gs2} = 0,\ V_{gs1} = -V_X$



$$I_X = \frac{V_X}{r_{02}} + \frac{V_X - V_1}{r_{01}} + g_{m1}V_X \qquad (1)$$

$$\frac{V_1}{R_D} + \frac{V_1 - V_X}{r_{01}} = g_{m1}V_X \qquad (2)$$

$$V_1 = \frac{V_X\left(\frac{1}{r_{01}} + g_{m1}\right)}{\frac{1}{R_D} + \frac{1}{r_{01}}}$$

$$\frac{I_X}{V_X} = \frac{1}{R_0} = \frac{1}{r_{02}} + \frac{1}{r_{01}} + g_{m1} - \frac{\frac{1}{r_{01}}\left(\frac{1}{r_{01}} + g_{m1}\right)}{\frac{1}{R_D} + \frac{1}{r_{01}}}$$

$$= \frac{1}{r_{02}} + \left(\frac{1}{r_{01}} + g_{m1}\right)\left[1 - \frac{\frac{1}{r_{01}}}{\frac{1}{R_D} + \frac{1}{r_{01}}}\right]$$

$$= \frac{1}{r_{02}} + \left(\frac{1}{r_{01}} + g_{m1}\right)\left(\frac{\frac{1}{R_D}}{\frac{1}{R_D} + \frac{1}{r_{01}}}\right)$$

For $R_D \ll r_{o1}$

$$\Rightarrow \frac{1}{R_o} \cong \frac{1}{r_{o2}} + \left(\frac{1}{r_{o1}} + g_{m1}\right)$$

For Q_1 :

$$g_{m1} = \frac{2I_{DSS1}}{|V_P|}\left(1 - \frac{V_{GS1}}{V_P}\right) = \frac{2(0.8)}{2}\left(1 - \frac{-0.197}{-2}\right)$$

$$g_{m1} = 0.721 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_o} = \frac{1}{(0.15)(0.65)} = 10.3 \text{ k}\Omega$$

$$\frac{1}{R_o} = \frac{1}{10.3} + \frac{1}{10.3} + 0.721 = 0.915$$

$$\Rightarrow \underline{R_o = 1.09 \text{ k}\Omega}$$

E10.18

For Q_1 : $i_D = I_{DSS1}(1 + \lambda v_{DS1})$

For Q_2 : $i_D = I_{DSS2}\left(1 - \frac{v_{GS2}}{V_P}\right)^2(1 + \lambda v_{DS2})$

$v_{GS2} = -v_{DS1}$

and $v_{DS2} = V_{DS} - v_{DS1}$

So

$$I_{DSS1}(1 + \lambda v_{DS1}) = I_{DSS2}\left[1 - \frac{-v_{DS1}}{V_P}\right]^2[1 + \lambda(V_{DS} - v_{DS1})]$$

$I_{DSS1} = I_{DSS2}$

$$[1 + (0.1)v_{DS1}] = \left[1 - \frac{v_{DS1}}{2}\right]^2[1 + (0.1)(3) - (0.1)v_{DS1}]$$

$$1 + 0.1v_{DS1} = (1 - v_{DS1} + 0.25v_{DS1}^2)(1.3 - 0.1v_{DS1})$$

This becomes

$$0.025v_{DS1}^3 - 0.425v_{DS1}^2 + 1.5v_{DS1} - 0.3 = 0$$

We find $\underline{v_{DS1} = 0.212 \text{ V}}$, $\underline{v_{DS2} = 2.79 \text{ V}}$,

$\underline{v_{GS2} = -0.212 \text{ V}}$

$$i_D = I_{DSS1}(1 + \lambda v_{DS1}) = 2[1 + (0.1)(0.212)]$$

$\underline{i_D = 2.04 \text{ mA}}$

$$R_o = r_{o2} + r_{o1}(1 + g_{m2}r_{o2})$$

$$g_m = \frac{2I_{DSS}}{-V_P}\left(1 - \frac{v_{GS2}}{V_P}\right) = \frac{2(2)}{2}\left(1 - \frac{-0.212}{-2}\right)$$

$$g_m = 1.79 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{1}{\lambda I_{DSS}} = \frac{1}{(0.1)(2)} = 5 \text{ k}\Omega$$

$$R_o = 5 + 5[1 + (1.79)(5)]$$

$$\Rightarrow \underline{R_o = 54.8 \text{ k}\Omega}$$

E10.19

a. $I_{REF} = I_S \exp\left(\frac{V_{EB2}}{V_T}\right)$

$$V_{EB2} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) = (0.026)\ln\left(\frac{0.5 \times 10^{-3}}{10^{-12}}\right)$$

$$\Rightarrow \underline{V_{EB2} = 0.521 \text{ V}}$$

b. $R_1 = \frac{5 - 0.521}{0.5} \Rightarrow \underline{R_1 = 8.96 \text{ k}\Omega}$

c. From Equation (10.72)

$$I_{S0}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1 + \frac{V_{CE0}}{V_{AN}}\right) = I_{REF} \times \frac{\left(1 + \frac{V_{EC2}}{V_{AP}}\right)}{\left(1 + \frac{V_{EB2}}{V_{AP}}\right)}$$

$$10^{-12}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1 + \frac{2.5}{100}\right) = (0.5 \times 10^{-3})\frac{\left(1 + \frac{2.5}{100}\right)}{\left(1 + \frac{0.521}{100}\right)}$$

$$1.03 \times 10^{-12}\exp\left(\frac{V_I}{V_T}\right) = 5.125 \times 10^{-4}$$

$$\exp\left(\frac{V_I}{V_T}\right) = 4.976 \times 10^8$$

$$\Rightarrow \underline{V_I = 0.521 \text{ V}}$$

d.

$$A_v = \frac{-\left(\frac{1}{V_T}\right)}{\frac{1}{V_{AN}} + \frac{1}{V_{AP}}} = \frac{-\frac{1}{0.026}}{\frac{1}{100} + \frac{1}{100}} = \frac{-38.46}{0.01 + 0.01}$$

$$\Rightarrow \underline{A_v = -1923}$$

E10.20

a. $V_{EB2} = (0.026)\ln\left(\frac{0.1 \times 10^{-3}}{5 \times 10^{-14}}\right)$

$$\Rightarrow \underline{V_{EB2} = 0.557 \text{ V}}$$

b. $R_1 = \frac{5 - 0.557}{0.1} \Rightarrow \underline{R_1 = 44.4 \text{ k}\Omega}$

c.

$$I_{S0}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1 + \frac{V_{CE0}}{V_{AN}}\right) = I_{REF} \times \left(\frac{1 + \frac{V_{EC2}}{V_{AP}}}{1 + \frac{V_{EB2}}{V_{AP}}}\right)$$

$$5 \times 10^{-14}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1+\frac{2.5}{100}\right)$$
$$= (0.1 \times 10^{-3})\left(\frac{1+\frac{2.5}{100}}{1+\frac{0.557}{100}}\right)$$

$$(5.125 \times 10^{-14})\exp\left(\frac{V_I}{V_T}\right) = 1.019 \times 10^{-4}$$
$$\exp\left(\frac{V_I}{V_T}\right) = 1.988 \times 10^{9}$$
$\Rightarrow \underline{V_I = 0.557\text{ V}}$

d. $A_v = \dfrac{-\frac{1}{0.026}}{\frac{1}{100}+\frac{1}{100}} \Rightarrow \underline{A_v = -1923}$

E10.21

a. $I_{REF} = K_{p1}(V_{SG}+V_{TP})^2$

$0.25 = 0.20(V_{SG}-1)^2$

$\Rightarrow \underline{V_{SG} = 2.12\text{ V}}$

b. From Equation (10.89)

$$V_{DSO} = V_o = \frac{[1+\lambda_p(V^+ - V_{SG})]}{\lambda_n+\lambda_p} - \frac{K_n(V_I-V_{TN})^2}{I_{REF}(\lambda_n+\lambda_p)}$$

$$5 = \frac{1+(0.015)(10-2.12)}{0.030} - \frac{(0.2)(V_I-1)^2}{0.25(0.030)}$$
$0.15 = 1.12 - 0.8(V_I-1)^2$

$\Rightarrow V_I = 2.10\text{ V}$

c. $A_v = \dfrac{-2K_n(V_I-V_{TN})}{I_{REF}(\lambda_n+\lambda_p)}$

$A_v = -\dfrac{2(0.2)(2.10-1.0)}{0.25(0.030)}$

$\Rightarrow \underline{A_v = -58.7}$

E10.22

(a) $I_{REF} = K_{p1}(V_{SG}+V_{TP})^2$

$80 = 50(V_{SG}-1)^2 \Rightarrow \underline{V_{SG} = 2.26\text{ V}}$

(b) $V_{DSo} = V_o = \dfrac{[1+\lambda_p(V^+ - V_{SG})]}{\lambda_n+\lambda_p} - \dfrac{K_n(V_I-V_{TN})^2}{I_{REF}(\lambda_n+\lambda_p)}$

$$5 = \frac{[1+(0.015)(10-2.26)]}{0.030} - \frac{(50)(V_I-1)^2}{(80)(0.030)}$$
$20.83(V_I-1)^2 = 32.2 \Rightarrow \underline{V_I = 2.24\text{ V}}$

(c) $A_v = \dfrac{-2K_n(V_I-V_{TN})}{I_{REF}(\lambda_n+\lambda_p)} = \dfrac{-2(50)(2.24-1)}{(80)(0.030)} \Rightarrow$

$\underline{A_v = -51.7}$

E10.23

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.8}{0.026} = 30.8\text{ mA/V}$

$r_o = r_{o2} = \dfrac{V_A}{I_{CQ}} = \dfrac{80}{0.8} = 100\text{ k}\Omega$

a. $V_0 = -g_m V_{\pi 1}(r_o \| r_{o2}),\quad V_{\pi 1} = V_i$

$A_v = -g_m(r_o\|r_{o2}) = -(30.8)[100\|100]$

$\Rightarrow \underline{A_v = -1540}$

b. $A_v = -g_m(r_o\|r_{o2}\|R_L)$

$A_v = -\dfrac{1540}{2} = -770$

$-770 = -(30.8)(50\|R_L) \Rightarrow (50\|R_L) = 25$

$\Rightarrow \underline{R_L = 50\text{ k}\Omega}$

E10.24

a. $g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.5}{0.026} \Rightarrow \underline{g_m = 19.2\text{ mA/V}}$

$r_o = \dfrac{V_{AN}}{I_{CQ}} = \dfrac{120}{0.5} \Rightarrow \underline{r_o = 240\text{ k}\Omega}$

$r_{o2} = \dfrac{V_{AP}}{I_{CQ}} = \dfrac{80}{0.5} \Rightarrow \underline{r_{o2} = 160\text{ k}\Omega}$

b. $A_v = -g_m(r_o\|r_{o2}\|R_L) = -(19.2)[240\|160\|50]$

$\Rightarrow \underline{A_v = -631}$

E10.25

(a) Neglecting effect of λ and R_L

$I_O = I_{REF} = K_n\left(V_{IQ} - V_{TN}\right)^2$

$0.40 = 0.25(V_{IQ} - 1)^2$

Then $\underline{V_{IQ} = 2.26 \text{ V}}$

b. $r_o = r_{o2} = \dfrac{1}{\lambda I_o} = \dfrac{1}{(0.02)(0.4)} = 125 \text{ k}\Omega$

$g_m = 2K_n\left(V_{IQ} - V_{TN}\right) = 2(0.25)(2.26 - 1)$

$= 0.63 \text{ mA/V}$

$A_v = -g_m(r_o \| r_{o2}) = -(0.63)(125 \| 125)$

$\Rightarrow \underline{A_v = -39.4}$

c. $A_v = -g_m(r_o \| r_{o2} \| R_L)$

$-\dfrac{39.4}{2} = -(0.63)(62.5 \| R_L)$

$\Rightarrow 62.5 \| R_L = 31.25 \Rightarrow \underline{R_L = 62.5 \text{ k}\Omega}$

E10.26

M_1 and M_2 identical $\Rightarrow I_0 = I_{REF}$

a. $I_O = K_n\left(V_I - V_{TN}\right)^2$

$0.25 = 0.2(V_T - 1)^2$

$V_I = 2.12 \text{ V}$

$g_m = 2K_n\left(V_I - V_{TN}\right) = 2(0.2)(2.12 - 1)$

$\Rightarrow \underline{g_m = 0.448 \text{ mA/V}}$

$r_{on} = \dfrac{1}{\lambda_n I_0} = \dfrac{1}{(0.01)(0.25)} \Rightarrow \underline{r_{on} = 400 \text{ k}\Omega}$

$r_{op} = \dfrac{1}{\lambda_p I_0} = \dfrac{1}{(0.02)(0.25)} \Rightarrow \underline{r_{op} = 200 \text{ k}\Omega}$

b. $A_v = -g_m(r_o \| r_{o2} \| R_L)$

$A_v = -(0.448)[400 \| 200 \| 100]$

$\Rightarrow \underline{A_v = -25.6}$

Chapter 10

Problem Solutions

10.1

a. $I_1 = I_2 = \dfrac{0 - 2V_\gamma - V^-}{R_1 + R_2}$

$2V_\gamma + I_2 R_2 = V_{BE} + I_C R_3$

$2V_\gamma + \dfrac{R_2}{R_1 + R_2}(-2V_\gamma - V^-) = V_{BE} + I_C R_3$

$\underline{I_C = \dfrac{1}{R_3}\left\{2V_\gamma - (2V_\gamma + V^+)\left(\dfrac{R_2}{R_1 + R_2}\right) - V_{BE}\right\}}$

b. $V_\gamma = V_{BE}$ and $R_1 = R_2$

$I_C = \dfrac{1}{R_3}\left\{2V_\gamma - \dfrac{1}{2}(2V_\gamma + V^-) - V_{BE}\right\}$

or $\underline{I_C = \dfrac{-V^-}{2R_3}}$

c. $I_C = 2\text{ mA} = \dfrac{-(-10)}{2R_3} \Rightarrow \underline{R_3 = 2.5\text{ k}\Omega}$

$I_1 = I_2 = 2\text{ mA} = \dfrac{-2(0.7) - (-10)}{R_1 + R_2}$

$\Rightarrow R_1 + R_2 = 4.3\text{ k}\Omega$

$\Rightarrow \underline{R_1 = R_2 = 2.15\text{ k}\Omega}$

10.2

$I_{C2} = \dfrac{I_{REF}}{1 + \frac{2}{\beta}} = \dfrac{1}{1 + \frac{2}{50}}$

$\underline{I_{C1} = I_{C2} = 0.962\text{ mA}}$

$I_{B1} = I_{B2} = \dfrac{I_{C2}}{\beta} \Rightarrow \underline{I_{B1} = I_{B2} = 0.0192\text{ mA}}$

10.3

(a) $I_{REF} = \dfrac{V^+ - V_{BE}(on) - V^-}{R_1}$

or

$R_1 = \dfrac{15 - 0.7 - (-15)}{0.5} \Rightarrow \underline{R_1 = 58.6\text{ k}\Omega}$

(b) $R_1 = \dfrac{V^+ - V_{BE}(on) - V^-}{I_{REF}} = \dfrac{0 - 0.7 - (-15)}{0.5} \Rightarrow$

$\underline{R_1 = 28.6\text{ k}\Omega}$

Advantage: Requires smaller resistance.

(c) For part (a):

$I_O(\max) = \dfrac{29.3}{(58.6)(0.95)} = 0.526\, mA$

$I_O(\min) = \dfrac{29.3}{(58.6)(1.05)} = 0.476\, mA$

$\Delta I_O = 0.526 - 0.476 = 0.05\, mA \Rightarrow \pm 5\%$

For part (b): $I_O(\max) = \dfrac{14.3}{(28.6)(0.95)} = 0.526\, mA$

$I_O(\min) = \dfrac{14.3}{(28.6)(1.05)} = 0.476\, mA$

$\Delta I_O = 0.05\, mA \Rightarrow \underline{\pm 5\%}$

10.4

a. $I_{REF} = I_0\left(1 + \dfrac{2}{\beta}\right) = 2\left(1 + \dfrac{2}{100}\right)$

or $I_{REF} = 2.04$ mA

$R_1 = \dfrac{15 - 0.7}{2.04} \Rightarrow \underline{R_1 = 7.01\text{ k}\Omega}$

b. $r_0 = \dfrac{V_A}{I_0} = \dfrac{80}{2} = 40\text{ k}\Omega$

$\dfrac{\Delta I_0}{\Delta V_{CE}} = \dfrac{1}{r_0} \Rightarrow \Delta I_0 = \left(\dfrac{1}{40}\right)(9.3) = 0.2325\text{ mA}$

$\dfrac{\Delta I_0}{I_0} = \dfrac{0.2325}{2} \Rightarrow \underline{\dfrac{\Delta I_0}{I_0} = 11.6\%}$

10.5

$I_{REF} = I_0\left(1 + \dfrac{2}{\beta}\right) = (0.5)\left(1 + \dfrac{2}{25}\right)$

$= \underline{0.54\text{ mA} = I_{REF}}$

$R_1 = \dfrac{5 - 0.7}{0.54} \Rightarrow \underline{R_1 = 7.96\text{ k}\Omega}$

10.6

a. $I_{REF} = \dfrac{5 - 0.7}{18} = 0.239\text{ mA}$

$I_0 = \dfrac{0.239}{1 + \frac{2}{50}} \Rightarrow \underline{I_0 = 0.230\text{ mA}}$

b. $r_0 = \dfrac{V_A}{I_0} = \dfrac{50}{0.230} = 217\text{ k}\Omega$

$\Delta I_0 = \dfrac{1}{r_0} \cdot \Delta V_{EC} = \left(\dfrac{1}{217}\right)(1.3) = 0.00599\text{ mA}$

$\Rightarrow \underline{I_0 = 0.236\text{ mA}}$

c. $\Delta I_0 = \left(\dfrac{1}{217}\right)(3.3) = 0.0152\text{ mA}$

$\Rightarrow \underline{I_0 = 0.245\text{ mA}}$

10.7

a. $I_{REF} = 1 = \dfrac{5 - 0.7 - (-5)}{R_1}$

$\Rightarrow \underline{R_1 = 9.3\ \text{k}\Omega}$

b. $I_0 = 2I_{REF} \Rightarrow \underline{I_0 = 2\ \text{mA}}$

c. For $V_{EC2}(\min) = 0.7 \Rightarrow R_{C2} = \dfrac{5 - 0.7}{2}$

$\Rightarrow \underline{R_{C2} = 2.15\ \text{k}\Omega}$

10.8

$I_0 = nI_{C1}$

$$I_{REF} = I_{C1} + I_{B1} + I_{B2} = I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_0}{\beta}$$

$$I_{REF} = I_{C1}\left(1 + \frac{1}{\beta} + \frac{n}{\beta}\right) = I_{C1}\left(1 + \frac{1+n}{\beta}\right)$$

$$= \frac{I_0}{n}\left(1 + \frac{1+n}{\beta}\right)$$

or $I_0 = \underline{\dfrac{nI_{REF}}{\left(1 + \dfrac{1+n}{\beta}\right)}}$

10.9

Using the results of Problem 10-8,

$2 = \dfrac{2I_{REF}}{1 + \dfrac{3}{50}} \Rightarrow \underline{I_{REF} = 1.06\ \text{mA}}$

$R_1 = \dfrac{5 - 0.7}{1.06} \Rightarrow \underline{R_1 = 4.06\ \text{k}\Omega}$

10.10

First approximation – BE area of Q_2 is 3 times that of Q_1.

$R_1 \cong \dfrac{5 - 0.7 - (-5)}{0.5} \Rightarrow \quad R_1 = 18.6\ k\Omega$

Second approximation – take into account I_C vs V_{BE} variation.

For Q_1:

$$\frac{I_{C1}}{I_{C2}} = \exp\left(\frac{V_{BE1} - V_{BE2}}{V_T}\right)$$

or

$$V_{BE1} - V_{BE2} = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right)$$

$0.7 - V_{BE2} = 0.026 \ln\left(\dfrac{1}{0.5}\right) \Rightarrow V_{BE} = 0.682\,V$ for $I_C = 0.5\,mA$

Then

$R_1 = \dfrac{5 - 0.682 - (-5)}{0.5} \Rightarrow \quad \underline{R_1 = 18.64\ k\Omega}$

Now

$$I_{S1} = \frac{I_C}{\exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{1\text{x}10^{-3}}{\exp\left(\frac{0.7}{0.026}\right)} \Rightarrow I_{S1} = 2.03\text{x}10^{-15}\ A$$

For Q_2:

$$I_{S2} = \frac{I_{C2}}{\exp\left(\frac{V_{BE2}}{V_T}\right)} = \frac{1.5\text{x}10^{-3}}{\exp\left(\frac{0.682}{0.026}\right)} = 6.09\text{x}10^{-15}\ A$$

10.11

$I_2 = 2I_1$ and $I_3 = 3I_1$

(a) $I_2 = 1.0\,mA$, $I_3 = 1.5\,mA$

(b) $I_1 = 0.25\,mA$, $I_3 = 0.75\,mA$

(c) $I_1 = 0.167\,mA$, $I_2 = 0.333\,mA$

10.12

a. $I_0 = I_{C1}$ and $I_{REF} = I_{C1} + I_{B3} = I_{C1} + \dfrac{I_{E3}}{1+\beta}$

$$I_{E3} = I_{B1} + I_{B2} + \frac{V_{BE}}{R_2} = \frac{2I_{C1}}{\beta} + \frac{V_{BE}}{R_2}$$

$$I_{REF} = I_{C1} + \frac{2I_{C1}}{\beta(1+\beta)} + \frac{V_{BE}}{(1+\beta)R_2}$$

$$I_{REF} - \frac{V_{BE}}{(1+\beta)R_2} = I_0\left(1 + \frac{2}{\beta(1+\beta)}\right)$$

$$\underline{I_0 = \frac{I_{REF} - \dfrac{V_{BE}}{(1+\beta)R_2}}{\left(1 + \dfrac{2}{\beta(1+\beta)}\right)}}$$

b. $I_{REF} = (0.70)\left(1 + \dfrac{2}{(80)(81)}\right) + \dfrac{0.7}{(81)(10)}$

$I_{REF} = 0.700216 + 0.000864$

$\underline{I_{REF} = 0.7011\ \text{mA}} = \dfrac{10 - 2(0.7)}{R_1}$

$\Rightarrow \underline{R_1 = 12.27\ \text{k}\Omega}$

10.13

a. $I_{0i} = I_{CR}$ and $I_{REF} = I_{CR} + I_{BS} = I_{CR} + \dfrac{I_{ES}}{1+\beta}$

$$I_{ES} = I_{BR} + I_{B1} + I_{B2} + \ldots + I_{BN} = (1+N)I_{BR}$$

$$= \frac{(1+N)I_{CR}}{\beta}$$

Then $I_{REF} = I_{CR} + \dfrac{(1+N)I_{CR}}{\beta(1+\beta)}$

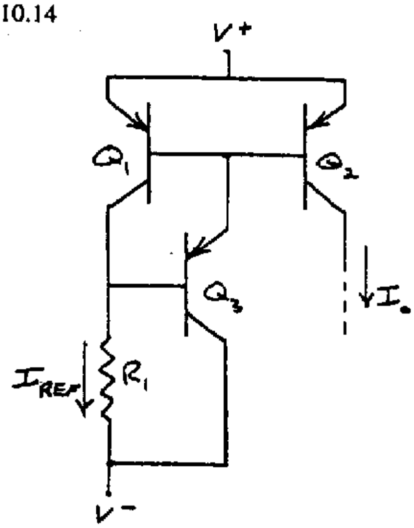
or $I_{0i} = \underline{\dfrac{I_{REF}}{\left(1 + \dfrac{(1+N)}{\beta(1+\beta)}\right)}}$

b. $I_{REF} = (0.5)\left[1 + \dfrac{6}{(50)(51)}\right] = 0.5012\ \text{mA}$

$R_1 = \dfrac{5 - 2(0.7) - (-5)}{0.5012}$

$\Rightarrow \underline{R_1 = 17.16\ \text{k}\Omega}$

10.14

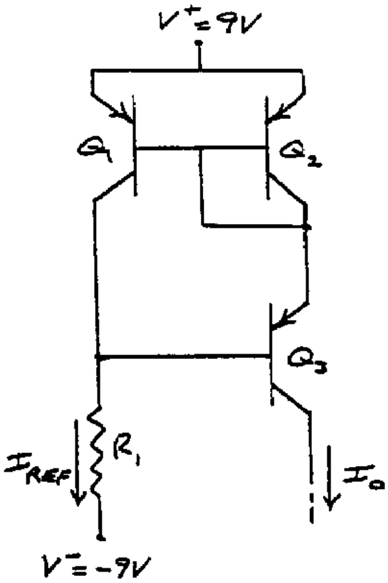


$$I_{REF} = I_0\left(1 + \frac{2}{\beta(1+\beta)}\right) = (0.5)\left[1 + \frac{2}{(50)(51)}\right]$$

$\Rightarrow \underline{I_{REF} = 0.5004 \text{ mA}}$

$$R_1 = \frac{5 - 2(0.7) - (-5)}{0.5004} \Rightarrow \underline{R_1 = 17.19 \text{ k}\Omega}$$

10.15



$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2+\beta)}\right)}$$

For $I_0 = 0.8$ mA

$$I_{REF} = (0.8)\left(1 + \frac{2}{25(27)}\right)$$

$\Rightarrow \underline{I_{REF} = 0.8024 \text{ mA}}$

$$R_1 = \frac{18 - 2(0.7)}{0.8024} \Rightarrow \underline{R_1 = 20.69 \text{ k}\Omega}$$

10.16

The analysis is exactly the same as in the text. We have

$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2+\beta)}\right)}$$

10.17

$$I_0 = 2 \text{ mA}, \ I_{B2} = \frac{2}{75} = 0.0267 \text{ mA}$$
$$I_{C1} = 1 \text{ mA}, \ I_{B1} = \frac{1}{75} = 0.0133 \text{ mA}$$
$$I_{E3} = I_{B1} + I_{B2} = 0.0133 + 0.0267 = 0.04 \text{ mA}$$
$$I_{B3} = \frac{I_{E3}}{1+\beta} = \frac{0.04}{76} = 0.000526 \text{ mA}$$
$$I_{REF} = I_{C1} + I_{B3} \Rightarrow \underline{I_{REF} = 1.000526 \approx 1 \text{ mA}}$$
$$R_1 = \frac{10 - 2(0.7)}{I_{REF}} = \frac{8.6}{1} \Rightarrow \underline{R_1 = 8.6 \text{ k}\Omega}$$

10.18

a. We have

$$R_0 \cong \frac{\beta r_{03}}{2}$$
$$r_{03} = \frac{V_A}{I_0} \approx \frac{V_A}{I_{REF}} = \frac{80}{0.5} = 160 \text{ k}\Omega$$

Then

$$R_0 \approx \frac{(80)(160)}{2} \Rightarrow \underline{R_0 \approx 6.4 \text{ M}\Omega}$$

b. $\Delta I_0 = \frac{1}{R_0} \cdot \Delta V_C = \frac{5}{6.4} \Rightarrow \underline{\Delta I_0 = 0.781 \ \mu\text{A}}$

10.19

$$V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$
$$0.7 = (0.026)\ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$
$$\text{At 2 mA, } V_{BE} = (0.026)\ln\left(\frac{2 \times 10^{-3}}{2.03 \times 10^{-15}}\right)$$
$$= 0.718 \text{ V}$$

$$R_1 = \frac{15 - 0.718}{2} \Rightarrow \underline{R_1 = 7.14 \text{ k}\Omega}$$
$$R_E = \frac{V_T}{I_0}\ln\left(\frac{I_{REF}}{I_0}\right) = \frac{0.026}{0.050} \cdot \ln\left(\frac{2}{0.050}\right)$$
$\Rightarrow \underline{R_E = 1.92 \text{ k}\Omega}$

10.20

a. $I_{REF} \approx \frac{10-0.7}{20} = 0.465 \text{ mA}$

Let $\underline{V^- = 0}$

$V_{BE} \cong V_T \ln\left(\frac{I_{REF}}{I_S}\right)$

$0.7 = (0.026)\ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$

Then

$V_{BE} \cong (0.026)\ln\left(\frac{0.465 \times 10^{-3}}{2.03 \times 10^{-15}}\right) = 0.680 \text{ V}$

Then

$I_{REF} \cong \frac{10-0.680}{20} \Rightarrow \underline{I_{REF} = 0.466 \text{ mA}}$

b. $R_E = \frac{V_T}{I_0}\ln\left(\frac{I_{REF}}{I_0}\right) = \frac{0.026}{0.10} \cdot \ln\left(\frac{0.466}{0.10}\right)$

$\Rightarrow \underline{R_E = 400\Omega}$

10.21

(a) $I_{REF} = \frac{V^+ - V_{BE1} - V^-}{R_1} = \frac{5-0.7-(-5)}{100} \Rightarrow$

$\underline{I_{REF} = 93\ \mu A}$

$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right) \Rightarrow I_O(10) = 0.026\ln\left(\frac{93x10^{-3}mA}{I_O}\right)$

By trial and error, $\underline{I_O \cong 6.8\ \mu A}$

$R_o = r_{o2}(1 + g_{m2}R_E')$

Now

$r_{o2} = \frac{30}{6.8} = 4.41\ M\Omega$

$g_{m2} = \frac{0.0068}{0.026} = 0.262\ mA/V$

$r_{\pi 2} = \frac{(100)(0.026)}{0.0068} = 382\ k\Omega$

So

$R_E' = r_{\pi 2} \| R_E = 382\|10 = 9.74\ k\Omega$

Then

$R_o = 4.41[1+(0.262)(9.74)] \Rightarrow \underline{R_o = 15.7\ M\Omega}$

(d) $V_{BE1} - V_{BE2} = I_O R_E = (0.0068)(10) \Rightarrow$

$\underline{V_{BE1} - V_{BE2} = 0.068\ V}$

10.22

$\Delta I_0 = \frac{1}{R_0} \cdot \Delta V_C$

$R_0 = r_{02}(1 + g_{m2}R_E')$

$r_{02} = \frac{V_A}{I_0} = \frac{80}{17.4} = 4.6 \text{ M}\Omega$

$g_{m2} = \frac{I_0}{V_T} = \frac{0.0174}{0.026} = 0.669 \text{ mA/V}$

$r_{\pi 2} = \frac{(80)(0.026)}{0.0174} = 119.5 \text{ k}\Omega$

$R_E' = R_E \| r_{\pi 2} = 7\|119.5$

$R_E' = 6.61 \text{ k}\Omega$

$R_0 = (4.6)[1 + (0.669)(6.61)] \Rightarrow R_0 = 24.9 \text{ M}\Omega$

Now

$\Delta I_0 = \left(\frac{1}{24.9}\right)(5) \Rightarrow \underline{\Delta I_0 = 0.201\ \mu\text{A}}$

10.23

$R_0 = r_{02}(1 + g_{m2}R_E')$ where $R_E' = R_E \| r_{\pi 2}$

$r_{02} = \frac{V_A}{I_0} = \frac{75}{25} = 3 \text{ M}\Omega$

$g_{m2} = \frac{I_0}{V_T} = \frac{0.025}{0.026} = 0.962 \text{ mA/V}$

$r_{\pi 2} = \frac{\beta V_T}{I_0} = \frac{(80)(0.026)}{0.025} = 83.2 \text{ k}\Omega$

$R_E = \frac{V_T}{I_0}\ln\left(\frac{I_{REF}}{I_0}\right) = \frac{0.026}{0.025} \cdot \ln\left(\frac{0.75}{0.025}\right)$

$= 3.54 \text{ k}\Omega$

$R_E' = 3.54\|83.2 = 3.40 \text{ k}\Omega$

$R_0 = 3[1 + (0.962)(3.4)] = 12.8 \text{ M}\Omega$

$\Delta I_0 = \frac{1}{R_0} \cdot \Delta V_{C2} = \frac{3}{12.8} = 0.234\ \mu\text{A}$

So

$\frac{\Delta I_0}{I_0} = \frac{0.234}{25} \Rightarrow \underline{0.936\%}$

10.24

Let $\underline{R_1 = 5\ k\Omega}$, Then

$I_{REF} = \frac{12-0.7-(-12)}{5} \Rightarrow I_{REF} = 4.66\ mA$

Now

$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right) \Rightarrow$

$R_E = \frac{0.026}{0.10}\ln\left(\frac{4.66}{0.10}\right) \Rightarrow \underline{R_E \cong 1\ k\Omega}$

10.25

$$I_{REF} \approx \frac{10 - 0.7 - (-10)}{40} = 0.4825 \text{ mA}$$

$$V_{BE} \cong V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$0.7 = (0.026)\ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

Now

$$V_{BE} = (0.026)\ln\left(\frac{0.4825 \times 10^{-3}}{2.03 \times 10^{-15}}\right) = 0.681 \text{ V}$$

$\underline{V_{BE1} = 0.681 \text{ V}}$

So

$$I_{REF} \cong \frac{10 - 0.681 - (-10)}{40}$$

$\Rightarrow \underline{I_{REF} = 0.483 \text{ mA}}$

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$I_0(12) = (0.026)\ln\left(\frac{0.483}{I_0}\right)$$

By trial and error,

$\Rightarrow \underline{I_0 \cong 8.7\ \mu\text{A}}$

$$V_{BE2} = V_{BE1} - I_0 R_E = 0.681 - (0.0087)(12)$$

$\Rightarrow \underline{V_{BE2} = 0.5766 \text{ V}}$

10.26

$$V_{BE1} + I_{REF}R_{E1} = V_{BE2} + I_0 R_{E2}$$

$$V_{BE1} - V_{BE2} = I_0 R_{E2} - I_{REF}R_{E1}$$

For matched transistors

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$V_{BE2} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

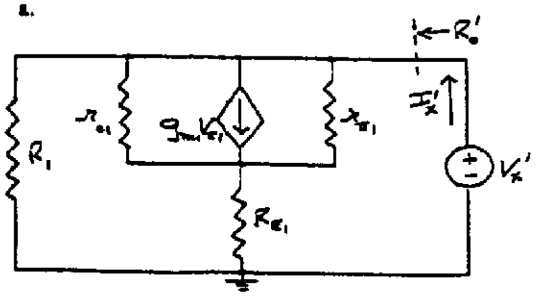
Then

$$\underline{V_T \ln\left(\frac{I_{REF}}{I_0}\right) = I_0 R_{E2} - I_{REF}R_{E1}}$$

Output resistance looking into the collector of Q_2 is increased.

10.27

a.



$$I_X' = \frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} + \frac{V_{\pi1}}{r_{01}} + \frac{V_X'}{R_1} \quad (1)$$

$$V_X' = V_{\pi1} + \left(\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} + \frac{V_{\pi1}}{r_{01}}\right)R_{E1} \quad (2)$$

$$V_X' = V_{\pi1}\left[1 + \left(\frac{1}{r_{\pi1}} + g_{m1} + \frac{1}{r_{01}}\right)R_{E1}\right]$$

$$I_{REF} \cong \frac{10 - 0.7}{13.6 + 5} = 0.5 \text{ mA}$$

$$r_{\pi1} = \frac{(50)(0.026)}{0.5} = 2.6 \text{ k}\Omega$$

$$r_{01} = \frac{75}{0.5} = 150 \text{ k}\Omega$$

$$g_{m1} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

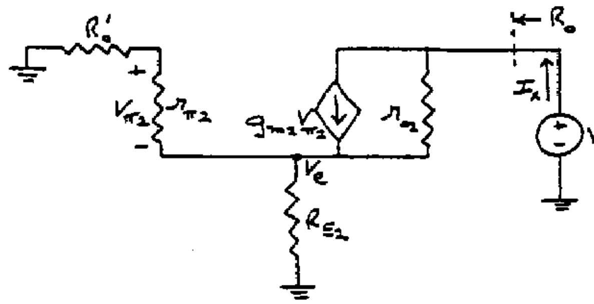
$$V_X' = V_{\pi1}\left[1 + \left(\frac{1}{2.6} + 19.23 + \frac{1}{150}\right)(5)\right]$$

$$\Rightarrow V_{\pi1} = V_X'(0.01009)$$

$$\text{Then } I_X' = V_X'(0.01009)\left[\frac{1}{2.6} + 19.23 + \frac{1}{150}\right] + \frac{V_X'}{13.6}$$

$$I_X' = V_X'(0.1980) + V_X'(0.0735)$$

$$\Rightarrow R_0' = \frac{V_X'}{I_X'} = 3.683 \text{ k}\Omega$$



$$I_X = \frac{V_X - V_e}{r_{02}} + g_{m2}V_{\pi2}$$

$$V_e = I_X\left[R_{E2}\|(r_{\pi2} + R_0')\right]$$

$$V_{\pi2} = -\left(\frac{r_{\pi2}}{r_{\pi2} + R_0'}\right)V_e$$

Then

$$I_X = \frac{V_X}{r_{02}} - \frac{I_X}{r_{02}}\left[R_{E2}\|(r_{\pi2} + R_0')\right] - g_{m2}\left(\frac{r_{\pi2}}{r_{\pi2} + R_0'}\right)(I_X)\left[R_{E2}\|(r_{\pi2} + R_0')\right]$$

$$R_{E1} = R_{E2} \Rightarrow I_{REF} = I_0 \Rightarrow r_{\pi2} = 2.6 \text{ k}\Omega$$

$$r_{02} = 150 \text{ k}\Omega$$

$$g_{m2} = 19.23 \text{ mA/V}$$

$$I_X = \frac{V_X}{150} - \frac{I_X}{150}[5\|(2.6+3.68)] - (19.23)\left(\frac{2.6}{2.6+3.68}\right)(I_X)[5\|(2.6+3.68)]$$

$I_X = V_X(0.00666) - I_X(0.01853) - I_X(22.13)$

$I_X(23.148) = V_X(0.00666)$

$\Rightarrow \underline{R_0 = \frac{V_X}{I_X} = 3.48\ \text{M}\Omega}$

b. When $R_{E1} = R_{E2} = 0$

$\underline{R_0 \cong r_{02} = 150\ \text{k}\Omega}$

10.28

Assume all transistors are matched.

a.

$2V_{BE1} = V_{BE3} + I_0 R_E$

$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$

$V_{BE3} = V_T \ln\left(\frac{I_0}{I_S}\right)$

$2V_T \ln\left(\frac{I_{REF}}{I_S}\right) - V_T \ln\left(\frac{I_0}{I_S}\right) = I_0 R_E$

$V_T\left[\ln\left(\frac{I_{REF}}{I_S}\right)^2 - \ln\left(\frac{I_0}{I_S}\right)\right] = I_0 R_E$

$\underline{V_T \ln\left(\frac{I_{REF}^2}{I_0 I_S}\right) = I_0 R_E}$

b. $V_{BE} = 0.7$ V at 1 mA $\Rightarrow 10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right)$

or $I_S = 2.03 \times 10^{-15}$ A

V_{BE} at 0.1 mA

$\Rightarrow V_{BE} = (0.026)\ln\left(\frac{0.1\times 10^{-3}}{2.03\times 10^{-15}}\right) = 0.640$ V

Since $I_0 = I_{REF}$, then

$V_{BE} = I_0 R_E \Rightarrow R_E = \frac{0.640}{0.1}$

or $\underline{R_E = 6.4\ \text{k}\Omega}$

10.29

$I_{REF} = \frac{10-0.7}{R_1} = 0.5 \Rightarrow \underline{R_1 = 18.6\ \text{k}\Omega}$

$I_{02}R_{E2} = V_T \ln\left(\frac{I_{REF}}{I_{01}}\right)$

$R_{E2} = \frac{0.026}{0.010}\cdot\ln\left(\frac{0.50}{0.01}\right) \Rightarrow \underline{R_{E2} = 10.17\ \text{k}\Omega}$

$R_{E3} = \frac{0.026}{0.030}\cdot\ln\left(\frac{0.50}{0.03}\right) \Rightarrow \underline{R_{E3} = 2.438\ \text{k}\Omega}$

$V_{BE2} = 0.7 - I_{02}R_{E2} = 0.7 - (0.01)(10.17)$

$\Rightarrow \underline{V_{BE2} = 0.598\ \text{V}}$

$V_{BE3} = 0.7 - I_{03}R_{E3} = 0.7 - (0.03)(2.438)$

$\Rightarrow \underline{V_{BE3} = 0.627\ \text{V}}$

10.30

(a) $V_{BE1} = V_{BE2}$

$I_{REF} = \frac{V^+ - 2V_{BE1} - V^-}{R_1 + R_2}$

Now

$2V_{BE1} + I_{REF}R_2 = V_{BE3} + I_O R_E$

or

$\underline{I_O R_E = 2V_{BE1} - V_{BE3} + I_{REF}R_2}$

We have

$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$ and $V_{BE3} = V_T \ln\left(\frac{I_O}{I_S}\right)$

(b) Let $R_1 = R_2$ and $I_O = I_{REF} \Rightarrow V_{BE1} = V_{BE3} \equiv V_{BE}$

Then

$V_{BE} = I_O R_E - I_{REF}R_2 = I_O(R_E - R_2)$

so

$I_{REF} = I_O = \frac{V^+ - V^- - 2I_O(R_E - R_2)}{2R_2}$

$= \frac{V^+ - V^-}{2R_2} - I_O\left(\frac{R_E}{R_2}\right) + I_O$

Then

$I_O = \frac{V^+ - V^-}{2R_E}$

(c) Want $I_O = 0.5\ mA$

So $R_E = \frac{5-(-5)}{2(0.5)} \Rightarrow \underline{R_E = 10\ k\Omega}$

$2R_2 = \frac{5 - 2(0.7) - (-5)}{0.5} = 17.2\ k\Omega$

Then $\underline{R_1 = R_2 = 8.6\ k\Omega}$

10.31

a. $I_{REF} = \frac{20 - 0.7 - 0.7}{12} = 1.55$ mA

$I_{01} = 2I_{REF} = 3.1$ mA

$I_{02} = I_{REF} = 1.55$ mA

$I_{03} = 3I_{REF} = 4.65$ mA

b. $V_{CE1} = -I_{01}R_{C1} - (-10) = -(3.1)(2) + 10$

$\Rightarrow \underline{V_{CE1} = 3.8\ \text{V}}$

$V_{EC2} = 10 - I_{02}R_{C2} = 10 - (1.55)(3)$

$\Rightarrow \underline{V_{EC2} = 5.35\ \text{V}}$

$V_{EC3} = 10 - I_{03}R_{C3} = 10 - (4.65)(1)$

$\Rightarrow \underline{V_{EC3} = 5.35\ \text{V}}$

10.32

a. 1st approximation

$I_{REF} \cong \frac{20-1.4}{8} = 2.325 \text{ mA}$

Now $V_{BE} - 0.7 = (0.026)\ln\left(\frac{2.32}{1}\right)$

$\Rightarrow V_{BE} = V_{EB} = 0.722 \text{ V}$

Then 2nd approximation

$I_{REF} \cong \frac{20-2(0.722)}{8} = 2.32 \text{ mA}$

$I_{01} = 2I_{REF} = 4.64 \text{ mA}$

$I_{02} = I_{REF} = 2.32 \text{ mA}$

$I_{03} = 3I_{REF} = 6.96 \text{ mA}$

b. At the edge of saturation, $V_{CE} = V_{BE} = 0.722 \text{ V}$

$R_{C1} = \frac{0-0.722-(-10)}{4.64} \Rightarrow \underline{R_{C1} = 2.0 \text{ k}\Omega}$

$R_{C2} = \frac{10-0.722}{2.32} \Rightarrow \underline{R_{C2} = 4.0 \text{ k}\Omega}$

$R_{C3} = \frac{10-0.722}{6.96} \Rightarrow \underline{R_{C3} = 1.33 \text{ k}\Omega}$

10.33

1st approximation

$I_{REF} = \frac{10-0.7}{6.3+3} = 1 \text{ mA}$

$\Rightarrow V_B = 0.7 \text{ V}$ as assumed

$V_{RER} = I_{REF} \cdot R_{ER} = (1)(3) = 3 \text{ V}$

$V_{RE1} = 3 \text{ V} \Rightarrow R_{E1} = \frac{V_{RE1}}{I_{01}} = \frac{3}{1} \Rightarrow \underline{R_{E1} = 3 \text{ k}\Omega}$

$V_{RE2} = 3 \text{ V} \Rightarrow R_{E2} = \frac{V_{RE2}}{I_{02}} = \frac{3}{2} \Rightarrow \underline{R_{E2} = 1.5 \text{ k}\Omega}$

$V_{RE3} = 3 \text{ V} \Rightarrow R_{E3} = \frac{V_{RE3}}{I_{03}} = \frac{3}{4} \Rightarrow \underline{R_{E3} = 0.75 \text{ k}\Omega}$

$I_{01} = 1 \text{ mA}$

$I_{02} = 2 \text{ mA}$

$I_{03} = 4 \text{ mA}$

10.34

$V_{GS} = V_{TN1} + \sqrt{\frac{I_{REF}}{K_{n1}}} = 1 + \sqrt{\frac{200}{250}} = 1.89 \; V = V_{DS1}$

$\frac{I_0}{I_{REF}} = \frac{1+\lambda V_{DS2}}{1+\lambda V_{DS1}}$

a. $V_{DS2} = 2 \text{ V}$

$I_0 = (200)\left[\frac{1+(0.02)(2)}{1+(0.02)(1.89)}\right] \Rightarrow \underline{I_0 \cong 200 \; \mu\text{A}}$

b. $V_{DS2} = 4 \text{ V}$

$I_0 = (200)\left[\frac{1+(0.02)(4)}{1+(0.02)(1.89)}\right] \Rightarrow \underline{I_0 \cong 208 \; \mu\text{A}}$

c. $V_{DS2} = 6 \text{ V}$

$I_0 = (200)\left[\frac{1+(0.02)(6)}{1+(0.02)(1.89)}\right] \Rightarrow \underline{I_0 \cong 216 \; \mu\text{A}}$

10.35

(a) $V_{GS} = V_{TN1} + \sqrt{\frac{I_{REF}}{K_{n1}}} = 1 + \sqrt{\frac{0.5}{0.5}} = 2 \; V$

$I_O = K_{n2}\left(\sqrt{\frac{I_{REF}}{K_{n1}}}\right)^2 = K_{n2}\left(\frac{I_{REF}}{K_{n1}}\right)$

$I_0(\max) = (0.5)(1.05)\left(\frac{0.5}{0.5}\right) \Rightarrow I_0(\max) = 0.525 \text{ mA}$

$I_0(\min) = (0.5)(0.95)\left(\frac{0.5}{0.5}\right) \Rightarrow I_0(\min) = 0.475 \text{ mA}$

So

$0.475 \le I_0 \le 0.525 \text{ mA}$

(b) $I_O = K_{n2}\left[\sqrt{\frac{I_{REF}}{K_{n1}}} + V_{TN1} - V_{TN2}\right]^2$

$I_0(\min) = (0.5)\left[\sqrt{\frac{0.5}{0.5}} + 1 - 1.05\right]^2$

$\Rightarrow I_0(\min) = 0.451 \text{ mA}$

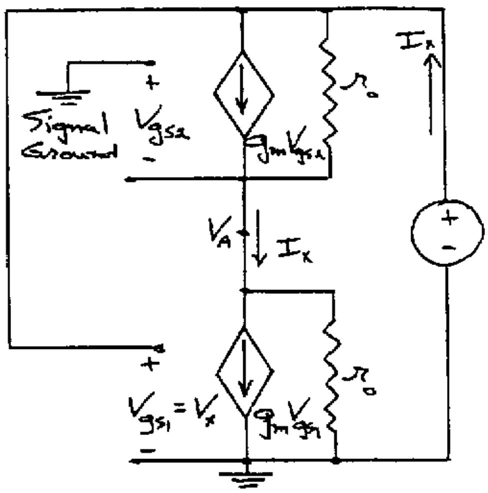
$I_0(\max) = (0.5)\left[\sqrt{\frac{0.5}{0.5}} + 1 - 0.95\right]^2$

$\Rightarrow I_0(\max) = 0.551 \text{ mA}$

So

$0.451 \le I_0 \le 0.551 \text{ mA}$

10.36



(1) $I_x = \frac{V_x - V_A}{r_o} + g_m V_{gs2}$

(2) $I_x = \frac{V_A}{r_o} + g_m V_{gs1}$

$V_{gs1} = V_x$, $V_{gs2} = -V_A$

So

(1) $I_x = \frac{V_x}{r_o} - V_A\left(\frac{1}{r_o} + g_m\right)$

(2) $I_x = \frac{V_A}{r_o} + g_m V_x \Rightarrow V_A = r_o[I_x - g_m V_x]$

Then

$$I_x = \frac{V_x}{r_o} - r_o(I_x - g_m V_x)\left(\frac{1}{r_o} + g_m\right)$$

$$I_x = \frac{V_x}{r_o} - r_o\left[\frac{I_x}{r_o} + g_m I_x - \frac{g_m}{r_o} \cdot V_x - g_m^2 V_x\right]$$

$$I_x = \frac{V_x}{r_o} - I_x - g_m r_o I_x + g_m V_x + g_m^2 r_o V_x$$

$$I_x[2 + g_m r_o] = V_x\left[\frac{1}{r_o} + g_m + g_m^2 r_o\right]$$

Since $g_m >> \frac{1}{r_o}$

$I_x[2 + g_m r_o] \cong V_x(g_m)(1 + g_m r_o)$

Then

$$\frac{V_x}{I_x} = R_o = \frac{2 + g_m r_o}{g_m(1 + g_m r_o)}$$

Usually, $g_m r_o >> 2$, so that

$$R_o \cong \frac{1}{g_m}$$

10.37

(a) $V_{DS}(sat) = V_{GS} - V_{TN}$

or $V_{GS} = V_{DS}(sat) + V_{TN} = 0.2 + 0.8 = 1.0$

$$I_D = \frac{k_n'}{2}\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$$

$$50 = 48\left(\frac{W}{L}\right)(0.2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right) = 26}$$

(b) $V_{GS5} - V_{TN} = 2(V_{GS} - V_{TN})$

$V_{GS5} = 0.8 + 2(0.2) \Rightarrow \underline{V_{GS5} = 1.2\ V}$

(c) $V_{D1}(\min) = 2V_{DS}(sat) = 2(0.2) \Rightarrow$

$\underline{V_{D1}(\min) = 0.4\ V}$

10.38

$V_{DS2}(sat) = 2\ V = V_{GS2} - V_{TN2} = V_{GS2} - 1.5 \Rightarrow$

$V_{GS2} = 3.5\ V$

$$I_O = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN2})^2$$

$$250 = (20)\left(\frac{W}{L}\right)_1 (3.5 - 1.5)^2 \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_2 = 3.125}$$

$$I_{REF} = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_1 (V_{GS2} - V_{TN1})$$

$$100 = (20)\left(\frac{W}{L}\right)_2 (3.5 - 1.5)^2 \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_1 = 1.25}$$

Now $V_{GS3} = 10 - V_{GS2} = 10 - 3.5 = 6.5\ V$

So $100 = (20)\left(\frac{W}{L}\right)_3 (6.5 - 1.5)^2 \Rightarrow$

$$\underline{\left(\frac{W}{L}\right)_3 = 0.2}$$

10.39

a. From Equation (10.50),

$$V_{GS1} = V_{GS2} = \left(\frac{\sqrt{\frac{5}{25}}}{1 + \sqrt{\frac{5}{25}}}\right)(5) + \left(\frac{1 - \sqrt{\frac{5}{25}}}{1 + \sqrt{\frac{5}{25}}}\right)(0.5)$$

$$= \left(\frac{0.447}{1 + 0.447}\right)(5) + \left(\frac{1 - 0.447}{1 + 0.447}\right)(0.5)$$

$$V_{GS1} = V_{GS2} = 1.74\ V$$

$I_{REF} \cong K_{n1}(V_{GS1} - V_{TN})^2 = (18)(25)(1.74 - 0.5)^2 \Rightarrow$
$\underline{I_{REF} = 0.692\ mA}$

b. $I_O = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2(1 + \lambda V_{DS2})$

$I_O = (18)(15)(1.74 - 0.5)^2[1 + (0.02)(2)]$
$= (415)(104) \quad \Rightarrow \underline{I_O = 0.432\ \text{mA}}$

c. $I_O = (415)[1 + (0.02)(4)]$

$\Rightarrow \underline{I_O = 0.448\ \text{mA}}$

10.40

(a) $I_{REF} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_1 (V_{SG1} + V_{TP})^2$
$= \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$

But $V_{SG3} = 3 - V_{SG1}$
So

$25(V_{SG1} - 0.4)^2 = 5(3 - V_{SG1} - 0.4)^2$
which yields $\underline{V_{SG1} = 1.08\ V}$ and $\underline{V_{SG3} = 1.92\ V}$

$I_{REF} = 20(25)(1.08 - 0.4)^2 \Rightarrow \underline{I_{REF} = 231\ \mu A}$

$\frac{I_O}{I_{REF}} = \frac{(W/L)_2}{(W/L)_1} = \frac{15}{25} = 0.6$

Then $\underline{I_O = (0.6)(231) = 139\ \mu A}$

(b) $V_{DS2}(sat) = 1.08 - 0.4 = 0.68\ V$
$V_R = 3 - 0.68 = 2.32 = I_O R$
then
$R = \frac{2.32}{0.139} \Rightarrow \underline{R = 16.7\ k\Omega}$

10.41

$V_{SD2}(sat) = 0.25 = V_{SG} + V_{TP} = V_{SG} - 0.4 \Rightarrow$
$V_{SG2} = 0.65\ V$

$I_O = \frac{k_p'}{2}\left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2$

$25 = \frac{40}{2}\left(\frac{W}{L}\right)_2 (0.65 - 0.4)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 20}$

$I_{REF} = 75\ \mu A = \frac{(W/L)_1}{(W/L)_2} \cdot I_O \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 60}$

$I_{REF} = \frac{k_p'}{2}\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$

$V_{SG3} = 3 - 0.65 = 2.35\ V$
Then

$75 = \frac{40}{2}\left(\frac{W}{L}\right)_3 (2.35 - 0.4)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 0.986}$

10.42

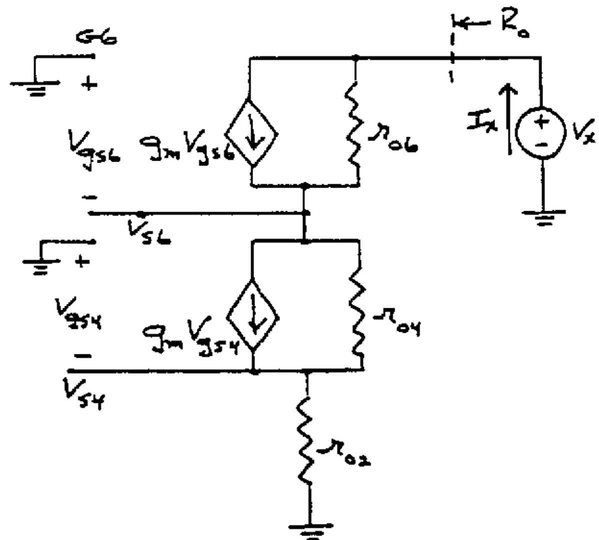
a. $I_{REF} = K_n(V_{GS} - V_{TN})^2$

$100 = 100(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3\ \text{V}$
For $V_{D4} = -3\ \text{V}$, $\underline{I_O \approx 100\ \mu\text{A}}$

b. $R_O = r_{o4} + r_{o2}(1 + g_m r_{o4})$

$r_{o2} = r_{o4} = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(0.1)} = 500\ \text{k}\Omega$
$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.1)(3 - 2) = 0.2\ mA/V$
$R_O = 500 + 500[1 + (0.2)(500)]$
$R_O = 51\ \text{M}\Omega$
$\Delta I_O = \frac{1}{R_O} \cdot \Delta V_{D4} = \frac{6}{51} \Rightarrow \underline{\Delta I_O = 0.118\ \mu\text{A}}$

10.43



$V_{gs4} = -I_X r_{o2}$
$V_{S6} = (I_X - g_m V_{gs4})r_{o4} + I_X r_{o2}$
$= (I_X + g_m I_X r_{o2})r_{o4} + I_X r_{o2}$
$V_{S6} = I_X[r_{o2} + (1 + g_m r_{o2})r_{o4}] = -V_{gs6}$

$I_X = g_m V_{gs6} + \frac{V_X - V_{S6}}{r_{o6}} = \frac{V_X}{r_{o6}} - V_{S6}\left(g_m + \frac{1}{r_{o6}}\right)$

$I_X = \frac{V_X}{r_{o6}} - I_X\left(g_m + \frac{1}{r_{o6}}\right)[r_{o2} + (1 + g_m r_{o2})r_{o4}]$

$I_X\left\{1 + \left(g_m + \frac{1}{r_{o6}}\right)[r_{o2} + (1 + g_m r_{o2})r_{o4}]\right\} = \frac{V_X}{r_{o6}}$

$\frac{V_X}{I_X} = R_O = r_{o6} + (1 + g_m r_{o6})[r_{o2} + (1 + g_m r_{o2})r_{o4}]$
$I_O \approx I_{REF} = 0.2\ \text{mA} = 0.2(V_{GS} - 1)^2$
$V_{GS} = 2\ \text{V}$

$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.2)(2-1) = 0.4\ mA/V$

$r_{o2} = r_{o4} = r_{o6} = \dfrac{1}{\lambda I_0} = \dfrac{1}{(0.02)(0.2)} = 250\ \text{k}\Omega$

$R_o = 250 + [1 + (0.4)(250)]$

$\times \{250 + [1 + (0.4)(250)](250)\}$

$R_o = 2575750\ \text{k}\Omega$

$\Rightarrow \underline{R_o = 2.58 \times 10^9\ \Omega}$

10.44

$$\frac{k_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 = \frac{k_n'}{2}\left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TN})^2$$

$$= \frac{k_p'}{2}\left(\frac{W}{L}\right)_4 (V_{SG4} + V_{TP})^2$$

(1) $50(20)(V_{GS1} - 0.5)^2 = 50(5)(V_{GS3} - 0.5)^2$

(2) $50(20)(V_{GS1} - 0.5)^2 = 20(10)(V_{SG4} - 0.5)^2$

(3) $V_{SG4} + V_{GS3} + V_{GS1} = 6$

From (1) $4(V_{GS1} - 0.5)^2 = (V_{GS3} - 0.5)^2 \Rightarrow$

$V_{GS3} = 2(V_{GS1} - 0.5) + 0.5$

From (2) $5(V_{GS1} - 0.5)^2 = (V_{GS4} - 0.5)^2 \Rightarrow$

$V_{GS4} = \sqrt{5}(V_{GS1} - 0.5) + 0.5$

Then (3) becomes

$\sqrt{5}(V_{GS1} - 0.5) + 0.5 + 2(V_{GS1} - 0.5) + 0.5 + V_{GS1} = 6$

which yields $V_{GS1} = 1.36\,V$ and

$V_{GS3} = 2.22\,V$, $V_{SG4} = 2.42\,V$

Then

$$I_{REF} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 = 50(20)(1.36 - 0.5)^2$$

or $\underline{I_{REF} = I_O = 0.740\ mA}$

$V_{GS1} = V_{GS2} = 1.36\,V$

$V_{DS2}(sat) = V_{GS2} - V_{TN} = 1.36 - 0.5 \Rightarrow$

$\underline{V_{DS2}(sat) = 0.86\,V}$

10.45

$V_{DS2}(sat) = 0.5\,V = V_{GS2} - V_{TN} = V_{GS2} - 0.5 \Rightarrow$

$V_{GS2} = 1\,V$

$$I_O = 50\ \mu A = \frac{k_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2$$

$$= 50\left(\frac{W}{L}\right)_2 (1 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 4}$$

$V_{GS1} = V_{GS2} = 1\,V \Rightarrow$

$$I_{REF} = 150 = \frac{k_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2$$

$$= 50\left(\frac{W}{L}\right)_1 (1 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 12}$$

$V_{GS3} + V_{SG4} + V_{GS1} = 6$

$2V_{GS3} = 6 - 1 = 5\,V \Rightarrow V_{GS3} = 2.5\,V$

$$I_{REF} = 150 = 50\left(\frac{W}{L}\right)_3 (2.5 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 0.75}$$

$$I_{REF} = \frac{k_p'}{2}\left(\frac{W}{L}\right)_4 (V_{SG4} + V_{TP})^2$$

$$150 = 20\left(\frac{W}{L}\right)_4 (2.5 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_4 1.88}$$

10.46

a. As a first approximation

$I_{REF} = 80 = 80(V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 2$ V

Then $V_{DS1} \cong 2(2) = 4$ V

The second approximation

$80 = 80(V_{GS1} - 1)^2[1 + (0.02)(4)]$

Or $\dfrac{80}{86.4} = (V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 1.962$

Then

$I_O = K_n(V_{GS1} - V_{TN})^2(1 + \lambda_n V_{GS1})$

$= 80(1.962 - 1)^2[1 + (0.02)(1.962)]$

Or $\underline{I_0 = 76.94\ \mu\text{A}}$

b. From a PSpice analysis, $I_0 = 77.09\ \mu$A for $V_{D3} = -1$ V and $I_0 = 77.14\ \mu$A for $V_{D3} = 3$ V. The change is $\Delta I_0 \approx 0.05\ \mu$A or 0.065%.

10.47

a. For a first approximation,

$I_{REF} = 80 = 80(V_{GS4} - 1)^2 \Rightarrow V_{GS4} = 2$ V

As a second approximation

$I_{REF} = 80 = 80(V_{GS4} - 1)^2[1 + (0.02)(2)]$

Or $V_{GS4} = 1.98$ V $= V_{GS1}$

$I_O = K_n(V_{GS2} - V_{TN})^2(1 + \lambda V_{GS2})$

To a very good approximation

$I_0 = 80\ \mu$A

b. From a PSpice analysis, $I_0 = 80.00\ \mu$A for $V_{D3} = -1$ V and the output resistance is $R_0 = 76.9$ MΩ. Then

$$\Delta I_0 = \frac{1}{R_0} \cdot V_{D3} = \frac{4}{76.9} = 0.052\ \mu\text{A}$$

or a change of 0.065%.

10.48

(a) $K_{n1} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_1 = 50(5) = 250\ \mu A/V^2$

$$R = \frac{1}{\sqrt{K_{n1}I_{D1}}}\left(1 - \sqrt{\frac{(W/L)_1}{(W/L)_2}}\right)$$

$$= \frac{1}{\sqrt{(0.25)(0.05)}}\left(1 - \sqrt{\frac{5}{50}}\right) = (8.94)(0.684)$$

$R = 6.11\ k\Omega$

(b) $V^+ - V^- = V_{SD3}(sat) + V_{GS1}$

$V_{SD3}(sat) = V_{SG3} + V_{TP}$

$I_{D1} = 50 = 20(5)(V_{SG3} - 0.5)^2 \Rightarrow\ V_{SG3} = 1.21\ V$

Then

$V_{SD3}(sat) = 1.21 - 0.5 = 0.71\ V$

Also

$I_{D1} = 50 = 50(5)(V_{GS1} - 0.5)^2 \Rightarrow\ V_{GS1} = 0.947\ V$

Then

$\underline{(V^+ - V^-)_{min} = 0.71 + 0.947 = 1.66\ V}$

(c) $I_{O1} = 25 = 50\left(\frac{W}{L}\right)_5 (0.947 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_5 = 2.5}$

$I_{O2} = 75 = 20\left(\frac{W}{L}\right)_6 (1.21 - 0.5)^2 \Rightarrow\ \underline{\left(\frac{W}{L}\right)_6 = 7.44}$

10.49

$V_{GS3} = \frac{1}{3}(5) = 1.667$ V

$$I_{REF} = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TN})^2$$

$$100 = (20)\left(\frac{W}{L}\right)_3 (1.667 - 1)^2$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = 11.2}$$

$$I_{O1} = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_1 (V_{GS3} - V_{TN})^2$$

Or $\frac{I_{REF}}{I_{O1}} = \frac{\left(\frac{W}{L}\right)_3}{\left(\frac{W}{L}\right)_1}$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{I_{O1}}{I_{REF}}\right)\left(\frac{W}{L}\right)_3 = \left(\frac{0.2}{0.1}\right)(11.2)$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 22.4}$$

And $\left(\frac{W}{L}\right)_2 = \left(\frac{I_{O2}}{I_{REF}}\right)\left(\frac{W}{L}\right)_3 = \left(\frac{0.3}{0.1}\right)(11.2)$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 33.6}$$

10.50

$$I_{REF} = \frac{24 - V_{SGP} - V_{GSN}}{R}$$

Also

$I_{REF} = 40(1)(V_{GSN} - 1.2)^2$

$I_{REF} = 18(1)(V_{SGP} - 1.2)^2$

Then

$\sqrt{40}(V_{GSN} - 1.2) = \sqrt{18}(V_{SGP} - 1.2)$

which yields

$$V_{SGP} = \frac{6.325}{4.243}(V_{GSN} - 1.2) + 1.2$$

Then

$$\left[0.040(V_{GSN} - 1.2)^2\right] \cdot R = 24 - V_{GSN} - 1.49(V_{GSN} - 1.2) - 1.2$$

which yields

$V_{GSN} = 2.69\ V$ and $V_{SGP} = 3.42\ V$

Now

$$I_{REF} = \frac{24 - 3.42 - 2.69}{200} \Rightarrow\ I_{REF} = 89.5\ \mu A$$

$I_1 = \frac{89.5}{5} = 17.9\ \mu A$

$I_2 = (1.25)(89.5) = 112\ \mu A$

$I_3 = (0.8)(89.5) = 71.6\ \mu A$

$I_4 = 4(89.5) = 358\ \mu A$

10.51

We have $V_{GSN} = 2.69\ V$ and $V_{SGP} = 3.42\ V$

So

$$I_{REF} = \frac{10 - 2.69 - 3.42}{R} = \frac{3.89}{200} \Rightarrow\ I_{REF} = 19.45\ \mu A$$

Then

$I_1 = (0.2)(19.45) = 3.89\ \mu A$

$I_2 = (1.25)(19.45) = 24.3\ \mu A$

$I_3 = (0.8)(19.45) = 15.56\ \mu A$

$I_4 = 4(19.45) = 77.8\ \mu A$

10.52

For $v_{GS} = 0$, $i_D = I_{DSS}(1 + \lambda v_{DS})$

a. $V_D = -5$ V, $v_{DS} = 5$

$i_D = (2)[1 + (0.05)(5)]$

$\Rightarrow \underline{i_D = 2.5\text{ mA}}$

b. $V_D = 0$, $v_{DS} = 10$

$i_D = (2)[1 + (0.05)(10)]$

$\Rightarrow \underline{i_D = 3\text{ mA}}$

c. $V_D = 5$ V, $v_{DS} = 15$ V

$i_D = (2)[1 + (0.05)(15)]$

$\Rightarrow \underline{i_D = 3.5\text{ mA}}$

10.54

$$I_0 = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2 = 4\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\frac{V_{GS}}{V_P} = 1 - \sqrt{\frac{2}{4}} = 0.293$$

So $V_{GS} = (0.293)(-4) = -1.17$ V

Then $I_0 = \frac{V_S}{R}$ and $V_S = -V_{GS}$

$$R = \frac{-V_{GS}}{I_0} = -\frac{(-1.17)}{2} \Rightarrow \underline{R = 0.585\ \text{k}\Omega}$$

10.55

a. $I_{REF} = I_{S1}\exp\left(\frac{V_{EB1}}{V_T}\right)$

or $V_{EB1} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026)\ln\left(\frac{1\times 10^{-3}}{5\times 10^{-13}}\right)$

$\Rightarrow \underline{V_{EB1} = 0.5568}$

b. $R_1 = \frac{5 - 0.5568}{1} \Rightarrow \underline{R_1 = 4.44\ \text{k}\Omega}$

c. From Equation (10.72) and letting $V_{CE0} = V_{EC2} = 2.5$ V

$$10^{-12}\exp\left(\frac{V_I}{V_T}\right)\left[1+\frac{2.5}{120}\right] = 10^{-3}\left(\frac{1+\frac{2.5}{80}}{1+\frac{0.5568}{80}}\right)$$

$$1.0208x10^{-12}\exp\left(\frac{V_I}{V_T}\right) = \left(10^{-3}\right)\left(\frac{1.03125}{1.00696}\right)$$

Then

$V_I = 0.026\ln\left(1.003613x10^{9}\right)$

So $V_I = 0.5389\ V$

d. $A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120}+\frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

$\underline{A_v = -1846}$

10.56

a. $V_{BE} = V_T\ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026)\ln\left(\frac{0.5\times 10^{-3}}{10^{-12}}\right)$

$\Rightarrow \underline{V_{BE} = 0.5208}$

b. $R_1 = \frac{5 - 0.5208}{0.5} \Rightarrow \underline{R_1 = 8.96\ \text{k}\Omega}$

c. From Equation (10.72) applies with slight modifications

$$I_{S0}\exp\left(\frac{V_{EB0}}{V_T}\right)\left[1 + \frac{V_{EC0}}{V_{AP}}\right] = I_{REF}\cdot\left(\frac{1+\frac{V_{CE2}}{V_{AN}}}{1+\frac{V_{BE2}}{V_{AN}}}\right)$$

$$\left(5\times 10^{-13}\right)\left[\exp\left(\frac{V_{EB0}}{V_T}\right)\right]\left(1+\frac{2.5}{80}\right)$$

$$= \left(0.5\times 10^{-3}\right)\cdot\left(\frac{1+\frac{2.5}{120}}{1+\frac{0.5208}{120}}\right)$$

$$5.15625\times 10^{-13}\exp\left(\frac{V_{EB0}}{V_T}\right) = \left(0.5\times 10^{-3}\right)\cdot\frac{1.02083}{1.00434}$$

$V_{EB0} = 0.5384 \Rightarrow V_I = 5 - 0.5384$

$\Rightarrow \underline{V_I = 4.462\ \text{V}}$

d. $A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120}+\frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

$\underline{A_v = -1846}$

10.57

Ignore $(W/L)_3 = 5$ specification

a. M_1 and M_2 matched, so we must have

$V_{SD2} = V_{SG} = V_{SG3} = V_{DS0} = 2.5$ V

For M_1 and M_3:

$$I_{REF} = \left(\frac{1}{2}\mu_p C_{ox}\right)\left(\frac{W}{L}\right)_1 (V_{SG} + V_{TP})^2(1+\lambda_p V_{SD})$$

$$100 = 10\left(\frac{W}{L}\right)_1 (2.5-1)^2[1 + (0.02)(2.5)]$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 4.23 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_2}$$

For M_0:

$$I_O = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_0 (V_{GS} - V_{TN})^2(1+\lambda_n V_{DS})$$

$$100 = 20\left(\frac{W}{L}\right)_0 (2-1)^2[1 + (0.02)(2.5)]$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_0 = 4.76}$$

b. $r_{on} = r_{op} = \dfrac{1}{\lambda I_0} = \dfrac{1}{(0.02)(0.1)} = 500\ \text{k}\Omega$

$$g_m = 2\sqrt{K_n I_O} = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_o I_O}$$
$$= 2\sqrt{(0.02)(4.76)(0.1)}$$
$$g_m = 0.195\ \text{mA/V}$$
$$A_v = -g_m(r_{on}\|r_{op}) = -(0.195)(500\|500)$$
$$\Rightarrow \underline{A_v = -48.75}$$

10.58

a. $I_{REF} = K_{p1}(V_{SG} + V_{TP})^2$

$$100 = 100(V_{SG} - 1)^2 \Rightarrow \underline{V_{SG} = 2\ \text{V}}$$

b. $V_{DS0} = V_{DS2} = 5\ \text{V}$

From Equation (10.87)

$$100(V_I - 1)^2 = 100[1 + (0.02)(5)] \times [1 - (0.02)(2)][1 - (0.02)(5)]$$
$$(V_I - 1)^2 = (1.1)(0.96)(0.90) = 0.9504$$
$$\Rightarrow \underline{V_I = 1.975\ \text{V}}$$

c. $A_v = -g_m(r_{on}\|r_{op})$

$$r_{on} = r_{op} = \frac{1}{\lambda I_{REF}} = \frac{1}{(0.02)(0.1)} = 500\ \text{k}\Omega$$
$$g_m = 2\sqrt{K_n I_{REF}} = 2\sqrt{(0.1)(0.1)} = 0.2\ mA/V$$
$$A_v = -(0.2)(500\|500)$$
$$\Rightarrow \underline{A_v = -50}$$

10.59

a. Using the results of problem 10.27, we find the resistance looking into the collector of Q_2 to be

$$R_o = r_{o2}\left[1 + \frac{R_E\|(r_{\pi 2} + R_o')}{r_{o2}} + g_{m2}\left(\frac{r_{\pi 2}}{r_{\pi 2} + R_o'}\right)[R_E\|(r_{\pi 2} + R_o')]\right]$$

where R_o' is the resistance from the base of Q_2 toward Q_1. We found

$$\frac{1}{R_o'} = \frac{1}{R_1} + \frac{\dfrac{1}{r_{\pi 1}} + g_{m1} + \dfrac{1}{r_{o1}}}{1 + \left(\dfrac{1}{r_{\pi 1}} + g_{m1} + \dfrac{1}{r_{o1}}\right)(R_E)}$$

b. $A_v = -g_{mo}(r_o\|R_L\|R_o)$

10.60

Output resistance of Wilson source

$$R_o \cong \frac{\beta r_{o3}}{2}$$

Then

$$A_v = -g_m(r_o\|R_o)$$

$$r_{o3} = \frac{V_{AP}}{I_{REF}} = \frac{80}{0.2} = 400\ \text{k}\Omega$$
$$r_o = \frac{V_{AN}}{I_{REF}} = \frac{120}{0.2} = 600\ \text{k}\Omega$$
$$g_m = \frac{I_{REF}}{V_T} = \frac{0.2}{0.026} = 7.69\ \text{mA/V}$$

$$A_v = -7.69\left[600 \,\Big\|\, \frac{(80)(400)}{2}\right] = -7.69[600\|16,000]$$
$$\Rightarrow \underline{A_v = -4447}$$

10.61

a. $g_m(M_o) = 2\sqrt{K_n I_{REF}}$

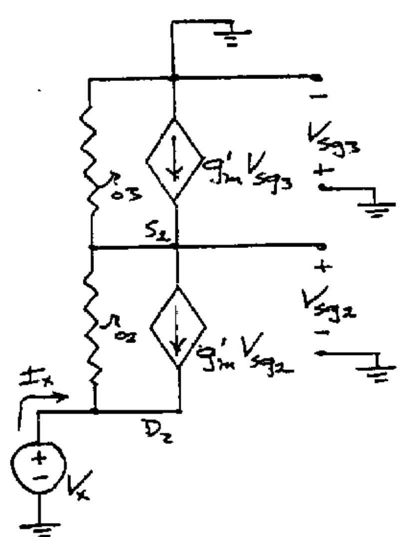
$$g_m(M_o) = 2\sqrt{(0.25)(0.2)}$$
$$\Rightarrow \underline{g_m(M_o) = 0.447\ \text{mA/V}}$$
$$r_{on} = \frac{1}{\lambda_n I_{REF}} = \frac{1}{(0.02)(0.2)} \Rightarrow \underline{r_{on} = 250\ \text{k}\Omega}$$
$$r_{op} = \frac{1}{\lambda_p I_{REF}} = \frac{1}{(0.03)(0.2)} \Rightarrow \underline{r_{op} = 167\ \text{k}\Omega}$$

b. $A_v = -g_m(r_{on}\|r_{op}) = -(0.447)(250\|167)$

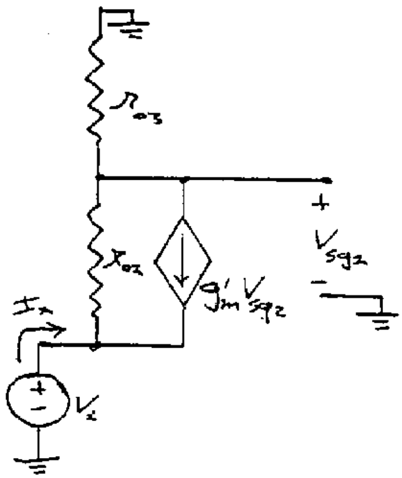
$$\Rightarrow \underline{A_v = -44.8}$$

c. $R_L = 250\|167 = r_{on}\|r_{op}$ or $\underline{R_L = 100\ \text{k}\Omega}$

10.62



Since $V_{sg3} = 0$, the circuit becomes



$$I_x = -g'_m V_{sg2} + \frac{V_x - V_{sg2}}{r_{o2}} \quad \text{and} \quad V_{sg2} = I_x r_{o3}$$

Then

$$I_x\left(1 + g'_m r_{o3} + \frac{r_{o3}}{r_{o2}}\right) = \frac{V_x}{r_{o2}}$$

so that

$$\frac{V_x}{I_x} = R_o = r_{o2}\left(1 + g'_m r_{o3} + \frac{r_{o3}}{r_{o2}}\right)$$

or

$$R_o = r_{o2} + r_{o3}(1 + g'_m r_{o2})$$

$$A_v = \frac{v_o}{v_i} = -g_{m1}(r_{o1} \| R_o)$$

Now

$$g_{m1} = 2\sqrt{(0.050)(20)(0.10)} = 0.632\ mA/V$$

$$r_{o1} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.02)(0.10)} = 500\ k\Omega$$

$$g'_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.020)(80)(0.1)} = 0.80\ mA/V$$

$$r_{o2} = r_{o3} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.020)(0.1)} = 500\ k\Omega$$

Then

$$R_o = 500 + 500[1 + (0.8)(500)] \Rightarrow 201\ M\Omega$$

$$A_v = -(0.632)(500 \| 201000) \Rightarrow \underline{A_v = -315}$$

10.63

$$A_v = -g_{m1}(R_{o2} \| R_{o3})$$

From the results of JFETs:

$$R_{o2} = r_{o1} + r_{o2}(1 + g'_m r_{o1})$$

From results of Problem 10.62

$$R_{o3} = r_{o3} + r_{o4}(1 + g'_m r_{o3})$$

We find

$$g_{m1} = 2\sqrt{(0.05)(20)(0.08)} = 0.566\ mA/V$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.02)(0.08)} = 625\ k\Omega$$

$$r_{o3} = r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.08)} = 625\ k\Omega$$

$$g'_m = 2\sqrt{(0.02)(40)(0.08)} = 0.506\ mA/V$$

Then

$$R_{o2} = 625 + 625[1 + (0.506)(625)] \Rightarrow 199\ M\Omega$$

$$R_{o3} = 625 + 625[1 + (0.506)(625)] \Rightarrow 199\ M\Omega$$

Then

$$A_v = -(0.566)[199000 \| 199000] \Rightarrow \underline{A_v = -56{,}317}$$