Chapter 15

Exercise Solutions

E15.1

$$f_{3dB} = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_{3dB}} = \frac{1}{2\pi(10^4)} = 1.59 \times 10^{-5}$$

Let $C = 0.01\ \mu F \Rightarrow \underline{R = 1.59\ k\Omega}$

Then

$$C_1 = 0.03546\ \mu F$$
$$C_2 = 0.01392\ \mu F$$
$$C_3 = 0.002024\ \mu F$$

$$|T| = \frac{1}{\sqrt{1+\left(\frac{f}{f_{3d}}\right)^6}} = \frac{1}{\sqrt{1+\left(\frac{20}{10}\right)^6}}$$

$|T| = 0.124$ or $|T| = -18.1$ dB

E15.2

$$f_{3dB} = \frac{1}{2\pi RC} \Rightarrow RC = \frac{1}{2\pi f_{3dB}}$$

$$RC = \frac{1}{2\pi(50\times 10^3)} = 3.18 \times 10^{-6}$$

Let $C = 0.001\ \mu F = 1$ nF $\Rightarrow \underline{R = 3.18\ k\Omega}$

Then

$$R_1 = 2.94\ k\Omega$$
$$R_2 = 3.44\ k\Omega$$
$$R_3 = 1.22\ k\Omega$$
$$R_4 = 8.31\ k\Omega$$

$$|T| = 0.01 = \frac{1}{\sqrt{1+\left(\frac{f_{3-dB}}{f}\right)^6}}$$

$$1+\left(\frac{f_{3-dB}}{f}\right)^6 = \left(\frac{1}{0.01}\right)^2 = 10^4$$

$$\left(\frac{f_{3-dB}}{f}\right)^2 \cong 10 \Rightarrow f = \frac{f_{3dB}}{\sqrt{10}}$$

$$\Rightarrow \underline{f \cong 15.8\text{ kHz}}$$

E15.3

1-pole $\quad |T| = \dfrac{1}{\sqrt{1+\left(\frac{12}{10}\right)^2}} \Rightarrow -3.87$ dB

2-pole $\quad |T| = \dfrac{1}{\sqrt{1+\left(\frac{12}{10}\right)^4}} \Rightarrow -4.88$ dB

3-pole $\quad |T| = \dfrac{1}{\sqrt{1+\left(\frac{12}{10}\right)^6}} \Rightarrow -6.0$ dB

4-pole $\quad |T| = \dfrac{1}{\sqrt{1+\left(\frac{12}{10}\right)^8}} \Rightarrow -7.24$ dB

E15.4

$$R_{eq} = \frac{1}{f_C C}$$

or $f_C C = \dfrac{1}{R_{eq}} = \dfrac{1}{5\times 10^6} = 2\times 10^{-7}$

If $\underline{C = 10\text{ pF}} \Rightarrow \underline{f_C = 20\text{ kHz}}$

E15.5

Low-frequency gain: $T = -\dfrac{C_1}{C_2} = -\dfrac{30}{5} = \underline{-6}$

$$f_{3dB} = \frac{f_C C_2}{2\pi C_F} = \frac{(100\times 10^3)(5\times 10^{-12})}{2\pi(12\times 10^{-12})}$$

$$\Rightarrow \underline{f_{3dB} = 6.63\text{ kHz}}$$

E15.6

$$f_0 = \frac{1}{2\pi\sqrt{3}RC}$$

$$RC = \frac{1}{2\pi f_0\sqrt{3}} = \frac{1}{2\pi(15\times 10^3)\sqrt{3}} = 6.13\times 10^{-6}$$

Let $\underline{C = 0.001\ \mu F = 1\text{ nF}}$

Then $\underline{R = 6.13\ k\Omega}$ so $\underline{R_2 = 8R = 49\ k\Omega}$

E15.7

$$f_0 = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}(10^4)(100\times 10^{-12})}$$

$$\Rightarrow \underline{f_0 \cong 65\text{ kHz}}$$

$$R_2 = 29R = 29(10^4)$$

$$\Rightarrow \underline{R_2 = 290\ k\Omega}$$

E15.8

$$f_0 = \frac{1}{2\pi RC} \Rightarrow C = \frac{1}{2\pi f_0 R}$$

$$C = \frac{1}{2\pi(800)(10^4)} \Rightarrow \underline{C \cong 0.02\ \mu F}$$

$$R_2 = 2R_1 = 2(10) \Rightarrow \underline{R_2 = 20\ k\Omega}$$

E15.9

$$f_0 = \frac{1}{2\pi\sqrt{L\cdot\left(\frac{C_1C_2}{C_1+C_2}\right)}} = \frac{1}{2\pi\sqrt{(10^{-6})\left[\frac{(10^{-9})^2}{2\times 10^{-9}}\right]}}$$

$$\Rightarrow \underline{f_0 = 7.12\text{ MHz}}$$

$$\frac{C_2}{C_1} = g_m R$$

$$g_m = \frac{C_2}{C_1}\cdot\frac{1}{R} = \frac{1}{4\times 10^3} \Rightarrow \underline{g_m = 0.25\text{ mA/V}}$$

We have

$$g_m = 2\left(\frac{k'}{2}\right)\left(\frac{W}{L}\right)(V_{GS}-V_{Th})$$

$$k' \cong 20\ \mu\text{A/V}^2,\ V_{GS}-V_{Th}\cong 1\text{ V}$$

$$\text{So } \frac{W}{L} = \frac{0.25\times 10^{-3}}{(20\times 10^{-6})(1)} = 12.5$$

and a value of $W/L = 12.5$ is certainly reasonable.

E15.10

$$V_{TH} = \left(\frac{R_1}{R_1+R_2}\right)V_H$$

$$2 = \left(\frac{R_1}{R_1+20}\right)(12)$$

$$2(R_1+20) = 12R_1$$

$$40 = 10R_1 \Rightarrow \underline{R_1 = 4\text{ k}\Omega}$$

E15.11

$$V_{TH} = -\left(\frac{R_1}{R_2}\right)V_L$$

$$0.10 = -\left(\frac{R_1}{R_2}\right)(-10) \Rightarrow \frac{R_1}{R_2} = 0.010$$

Let $\underline{R_1 = 0.10\text{ k}\Omega}$ then $\underline{R_2 = 10\text{ k}\Omega}$

E15.12

a. $$V_S = \left(\frac{R_2}{R_1+R_2}\right)V_{REF} = \left(\frac{10}{1+10}\right)(2)$$

$$\underline{V_S = 1.82\text{ V}}$$

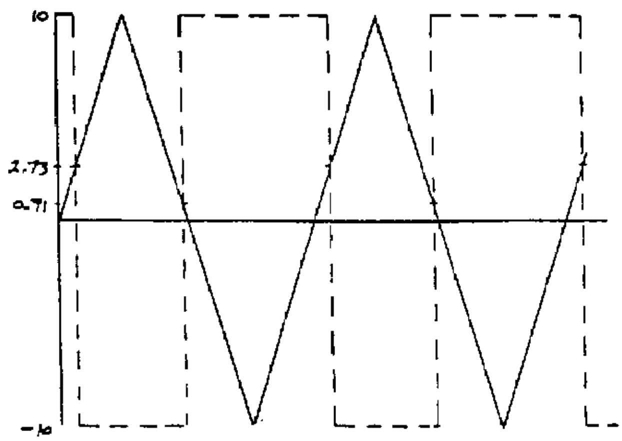
$$V_{TH} = V_S + \left(\frac{R_1}{R_1+R_2}\right)V_H = 1.82 + \left(\frac{1}{1+10}\right)(10)$$

$$\underline{V_{TH} = 2.73\text{ V}}$$

$$V_{TL} = V_S + \left(\frac{R_1}{R_1+R_2}\right)V_L = 1.82 + \left(\frac{1}{1+10}\right)(-10)$$

$$\underline{V_{TL} = 0.91\text{ V}}$$

b.



E15.13

$$V_S = \left(1+\frac{R_1}{R_2}\right)V_{REF}$$

$$V_{TH} = V_S - \left(\frac{R_1}{R_2}\right)V_L \text{ and } V_{TL} = V_S - \left(\frac{R_1}{R_2}\right)V_H$$

$$\text{Hysteresis Width} = V_{TH} - V_{TL} = \left(\frac{R_1}{R_2}\right)(V_H - V_L)$$

$$2.5 = \left(\frac{R_1}{R_2}\right)(5-[-5]) = 10\left(\frac{R_1}{R_2}\right)$$

$$\text{So } \underline{\frac{R_1}{R_2} = 0.25}$$

Then

$$V_S = -1 = \left(1+\frac{R_1}{R_2}\right)V_{REF} = (1+0.25)V_{REF}$$

$$\Rightarrow \underline{V_{REF} = -0.8\text{ V}}$$

Then

$$V_{TH} = -1-(0.25)(-5) \Rightarrow \underline{V_{TH} = 0.25\text{ V}}$$

$$V_{TL} = -1-(0.25)(5) \Rightarrow \underline{V_{TL} = -2.25\text{ V}}$$

E15.14

$$V_{TH} - V_{TL} = \left(\frac{R_1}{R_1+R_2}\right)(V_H - V_L)$$

$$0.10 = \left(\frac{R_1}{R_1+R_2}\right)(10-[-10])$$

$$1+\frac{R_2}{R_1} = \frac{20}{0.10} = 200 \Rightarrow \underline{\frac{R_2}{R_1} = 199}$$

$$V_S = \left(\frac{R_2}{R_1+R_2}\right)V_{REF}$$

$$V_{REF} = \left(1+\frac{R_1}{R_2}\right)V_S = \left(1+\frac{1}{199}\right)(1)$$

$$\Rightarrow \underline{V_{REF} = 1.005\text{ V}}$$

$$I = \frac{V_H - V_{BE}(\text{on}) - V_\gamma}{R + 0.1}$$

$$R + 0.1 = \frac{10 - 0.7 - 0.7}{0.2} = 43\ \text{k}\Omega$$

$$\underline{R = 42.9\ \text{k}\Omega}$$

E15.15

At $t = 0^-$, let $v_0 = -5$ so $v_X = -2.5$. For $t > 0$

$$v_X = 10 + (-2.5 - 10)\exp\left(\frac{-t}{\tau_X}\right)$$

When $v_X = 5.0$, output switches

$$5.0 = 10 - 12.5\exp\left(-\frac{t_1}{\tau_X}\right)$$

$$\exp\left(-\frac{t_1}{\tau_X}\right) = \frac{10 - 5}{12.5} = \frac{5.0}{12.5}$$

$$\exp\left(+\frac{t_1}{\tau_X}\right) = \frac{12.5}{5.0} \Rightarrow t_1 = \tau_X \cdot \ln\left(\frac{12.5}{5.0}\right)$$

$$\Rightarrow t_1 = \tau_X(0.916)$$

During the next part of the cycle

$$v_X = -5 + (5 - [-5])\exp\left(-\frac{t}{\tau_X}\right)$$

When $v_X = -2.5$, output switches

$$-2.5 = -5 + 10\exp\left(-\frac{t_2}{\tau_X}\right)$$

$$\exp\left(-\frac{t_2}{\tau_X}\right) = \frac{5 - 2.5}{10} = \frac{2.5}{10}$$

$$\exp\left(+\frac{t_2}{\tau_X}\right) = \frac{10}{2.5} \Rightarrow t_2 = \tau_X \cdot \ln\left(\frac{10}{2.5}\right)$$

$$\Rightarrow t_2 = \tau_X(1.39)$$

$$\text{Period} = t_1 + t_2 = T = [(0.916) + (1.39)]\tau_X$$

$$= 2.31\tau_X$$

$$\Rightarrow \underline{\text{Frequency} = \frac{1}{2.31\tau_X}}$$

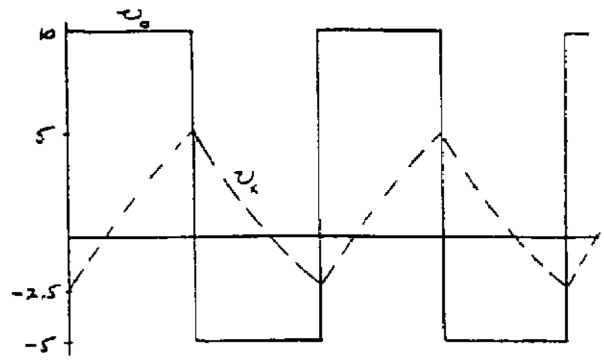
$$\tau_X = (50 \times 10^3)(0.01 \times 10^{-6}) = 5 \times 10^{-4}\ \text{s}$$

$$\Rightarrow \underline{f = 866\ \text{Hz}}$$

$$\text{Duty cycle} = \frac{t_1}{t_1 + t_2} \times 100\%$$

$$= \frac{(0.916)}{(0.916) + (1.39)} \times 100\%$$

$$\Rightarrow \underline{\text{Duty cycle} = 39.7\%}$$



E15.16

$$v_X = \left(\frac{R_1}{R_1 + R_2}\right)v_0 = \left(\frac{10}{10 + 20}\right)v_0 = \frac{1}{3}v_0$$

$$t = 0,\ v_X = -\frac{10}{3}.$$

$$v_X = 10 + \left(-\frac{10}{3} - 10\right)\exp\left(-\frac{t}{\tau_X}\right)$$

Output switches when $v_X = \frac{10}{3}$

$$\frac{10}{3} = 10 - 13.33\exp\left(-\frac{t_1}{\tau_X}\right)$$

$$\exp\left(-\frac{t_1}{\tau_X}\right) = \frac{10 - 3.33}{13.33} = \frac{6.67}{13.33}$$

$$\exp\left(+\frac{t_1}{\tau_X}\right) = \frac{13.33}{6.67} \cong 2$$

$$t_1 = \tau_X \ln(2) = (0.693)\tau_X$$

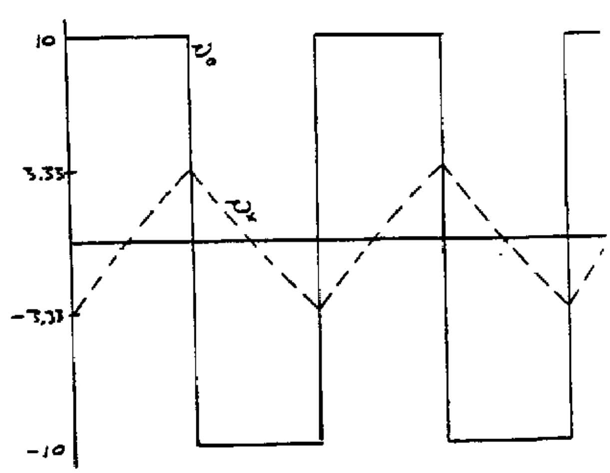
$$T = 2(0.693)\tau_X$$

$$f = \frac{1}{2(0.693)\tau_X}$$

$$\tau_X = R_X C_X = (10^4)(0.1 \times 10^{-6}) = 1 \times 10^{-3}$$

$$\Rightarrow \underline{f = 722\ \text{Hz}}$$

$$\Rightarrow \underline{\text{Duty cycle} = 50\%}$$



E15.17

a. $\tau_X = R_X C_X$

$$v_Y = \left(\frac{R_1}{R_1 + R_2}\right) v_0 = \left(\frac{10}{10+90}\right)(12) = 1.2 \text{ V}$$

$$\beta = \frac{R_1}{R_1 + R_2} = 0.10$$

$$T = \tau_X \ln\left[\frac{1 + V_\gamma/V_P}{1-\beta}\right] = \tau_X \ln\left[\frac{1+\frac{0.7}{12}}{1-(0.10)}\right]$$

$$T = 50 \times 10^{-6} = \tau_X \ln[1.18] = (0.162)\tau_X$$

$$R_X = \frac{50 \times 10^{-6}}{(0.1 \times 10^{-6})(0.162)} \Rightarrow \underline{R_X = 3.09 \text{ k}\Omega}$$

b. Recovery time

$$v_X = V_P + (-1.2 - V_P)\exp\left(-\frac{t}{\tau_X}\right)$$

When $v_X = V_\gamma$, $t = t_2$

$$0.7 = 12 + (-1.2 - 12)\exp\left(-\frac{t_2}{\tau_X}\right)$$

$$\exp\left(-\frac{t_2}{\tau_X}\right) = \frac{12 - 0.7}{13.2} = 0.856$$

$$t_2 = \tau_X \ln\left(\frac{1}{0.856}\right) = (0.155)\tau_X$$

$$\tau_X = (3.09 \times 10^3)(0.1 \times 10^{-6}) = 3.09 \times 10^{-4}$$

$$\Rightarrow \underline{t_2 = 47.9\ \mu s}$$

E15.18

$$\beta = \left(\frac{R_1}{R_1 + R_2}\right) = \frac{20}{20+40} = 0.333$$

$$\tau_X = R_X C_X = (10^4)(0.01 \times 10^{-6}) = 1 \times 10^{-4}$$

$$T = \tau_X \ln\left(\frac{1 + V_\gamma/V_P}{1-\beta}\right) = (1 \times 10^{-4})\ln\left[\frac{1+\frac{0.7}{8}}{1-0.333}\right]$$

$$\Rightarrow \underline{T = 48.9\ \mu s}$$

Recovery time

$$0.7 = 8 + (-2.66 - 8)\exp\left(-\frac{t_2}{\tau_X}\right)$$

$$\exp\left(-\frac{t_2}{\tau_X}\right) = \frac{8 - 0.7}{10.66} = 0.685$$

$$t_2 = \tau_X \ln\left(\frac{1}{0.685}\right)$$

$$\Rightarrow \underline{t_2 = 37.8\ \mu s}$$

E15.19

$$f = \frac{1}{0.693(R_A + 2R_B)C}$$

$$= \frac{1}{(0.693)[20 + 2(80)] \times 10^3 \times (0.01 \times 10^{-6})}$$

$$\Rightarrow \underline{f = 802 \text{ Hz}}$$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$= \frac{20 + 80}{20 + 2(80)} \times 100\%$$

$\Rightarrow$ <u>Duty cycle = 55.6%</u>

E15.20

$$f = \frac{1}{(0.693)(R_A + R_B)C}$$

$$R_A + R_B = \frac{1}{(0.693)fC}$$

Let $C = 0.01\ \mu\text{F}$, $f = 1$ kHz

$$R_A + R_B = \frac{1}{(0.693)(10^3)(0.01 \times 10^{-6})} = 1.44 \times 10^5$$

$$\text{Duty cycle} = 55 = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$55 = \frac{(1.44 \times 10^5)(100)}{(1.44 \times 10^5) + R_B}$$

$$R_B = \frac{(1.44 \times 10^5)(100 - 55)}{55}$$

$$\Rightarrow \underline{R_B = 118 \text{ k}\Omega} \text{ so } \underline{R_A = 26 \text{ k}\Omega}$$

E15.21

a. $\overline{P} = \frac{1}{2}\cdot\frac{V_P^2}{R_L}$

$$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(8)(1)} \Rightarrow \underline{V_P = 4 \text{ V}}$$

$$I_P = \frac{V_P}{R_L} = \frac{4}{8} \Rightarrow \underline{I_P = 0.5 \text{ A}}$$

b. $V_{CE} = 12 - 4 = 8$ V

$I_C \approx 0.5$ A

So $P = I_C \cdot V_{CE} = (0.5)(8) \Rightarrow \underline{P = 4 \text{ W}}$

E15.22

a. $$\frac{v_{01}}{v_I} = \left(1 + \frac{R_2}{R_1}\right) = \left(1 + \frac{30}{20}\right) = 2.5$$

$$\frac{v_{02}}{v_I} = -\frac{R_4}{R_3} = -\frac{50}{20} = -2.5$$

(b) $$\overline{P} = \frac{1}{2}\cdot\frac{V_L^2}{R_L} = \frac{1}{2}\cdot\frac{[12-(-12)]^2}{12} = 240\ mW$$

Or

$\underline{\overline{P} = 0.24\ W}$

c. $$\frac{12}{2.5} = \underline{V_{pi} = 4.8 \text{ V}}$$

E15.23

$$\text{Line regulation} = \frac{dV_0}{dV^+} = \frac{dV_0}{dV_Z}\cdot\frac{dV_Z}{dV^+}$$

Now

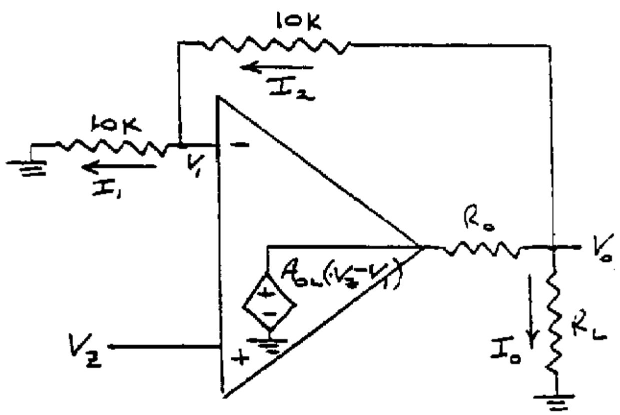
$$\frac{dV_0}{dV_Z} = \left(1+\frac{10}{10}\right) = 2$$

$$\frac{dV_Z}{dV^+} = \left(\frac{r_Z}{r_Z+R_1}\right) = \frac{10}{10+4400} = 0.00227$$

So Line regulation $= (2)(0.00227) = 0.00454$

$\underline{0.454\%}$

E15.24



$$\frac{V_1}{10} = \frac{V_0-V_1}{10} \Rightarrow V_1\left(\frac{1}{10}+\frac{1}{10}\right) = \frac{V_0}{10}$$

$$V_1\left(\frac{2}{10}\right) = \frac{V_0}{10} \Rightarrow V_0 = 2V_1 \Rightarrow V_1 = \frac{V_0}{2}$$

$$\frac{V_0-V_1}{10}+\frac{V_0}{R_L}+\frac{V_0-A_{0L}(V_Z-V_1)}{R_0} = 0$$

$$\frac{V_0}{10}+\frac{V_0}{R_L}+\frac{V_0}{R_0}-\frac{A_{0L}V_Z}{R_0} = \frac{V_1}{10}-\frac{A_{0L}V_1}{R_0}$$

$$= \frac{V_0}{2(10)}-\frac{A_{0L}V_0}{2R_0}$$

$$\frac{V_0}{10}+I_0+\frac{V_0}{0.5}-\frac{1000(6.3)}{0.5} = \frac{V_0}{20}-\frac{(1000)V_0}{2(0.5)}$$

$$V_0[0.10+2.0-0.05+1000]+I_0 = 12,600$$

$$V_0(1002.05)+I_0 = 12,600$$

For $I_0 = 1\text{ mA} \Rightarrow V_0 = 12.5732$

For $I_0 = 100\text{ mA} \Rightarrow V_0 = 12.4744$

$$\text{Load reg} = \frac{V_0(\text{NL})-V_0(\text{FL})}{V_0(\text{NL})}\times 100\%$$

$$= \frac{12.5732-12.4744}{12.5732}\times 100\%$$

$\underline{\text{Load reg} = 0.786\%}$

E15.25

a. $$I_{C3} = \frac{V_Z-3V_{BE}(\text{on})}{R_1+R_2+R_3}$$

$$I_{C3} = \frac{5.6-3(0.6)}{3.9+3.4+0.576} = \frac{3.8}{7.88}$$

$$\Rightarrow \underline{I_{C3} = 0.482\text{ mA}}$$

$$I_{C4}R_4 = V_T\ln\left(\frac{I_{C3}}{I_{C4}}\right)$$

$$I_{C4}(0.1) = (0.026)\ln\left(\frac{0.482}{I_{C4}}\right)$$

By trial and error

$\underline{I_{C4} = 0.213\text{ mA}}$

$$V_{B7} = 2(0.6)+(0.482)(3.9)$$

$$\Rightarrow \underline{V_{B7} = 3.08\text{ V}}$$

b. $$\left(\frac{R_{13}}{R_{13}+R_{12}}\right)V_0 = V_{B8} = V_{B7}$$

$$\left(\frac{2.23}{2.23+R_{12}}\right)(5) = 3.08$$

$$(2.23)(5) = (3.08)(2.23)+(3.08)R_{12}$$

$$11.15 = 6.868 = 3.08R_{12}$$

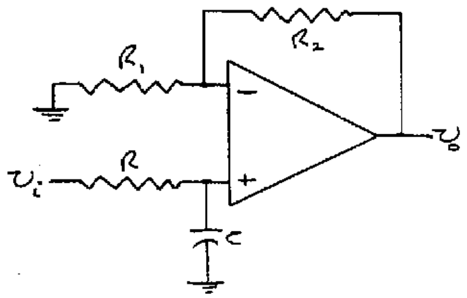
$$\Rightarrow \underline{R_{12} = 1.39\text{ k}\Omega}$$

Chapter 15

Problem Solutions

15.1

(a) For example:

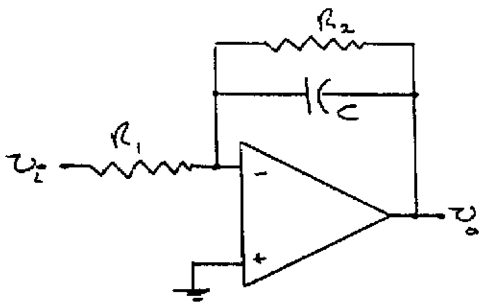


Low-Frequency: $\frac{v_o}{v_i}=\left(1+\frac{R_2}{R_1}\right)=10 \Rightarrow \underline{\frac{R_2}{R_1}=9}$

Corner Frequency:

$f=\frac{1}{2\pi RC}=5x10^3 \Rightarrow \underline{RC=3.18x10^{-5}}$

(b) For Example:



$$\frac{v_o}{v_i}=\frac{-R_2\left\|\frac{1}{j\omega C}\right.}{R_1}=-\frac{R_2}{R_1}\cdot\frac{1}{1+j\omega R_2 C}$$

So, set

$\frac{R_2}{R_1}=15 \Rightarrow$ For example, $\underline{R_1=10\ k\Omega,\quad R_2=150\ k\Omega}$

$$R_2C=\frac{1}{2\pi f_{3-dB}}=\frac{1}{2\pi(15x10^3)}=1.06x10^{-5}$$

Then $\underline{C=70.7\ pF}$

15.2

(a) $|A_v|=\frac{1}{\sqrt{1+\left(\frac{f}{f_{3-dB}}\right)^2}}=\frac{1}{\sqrt{1+(2)^2}}=0.447\Rightarrow$

$\underline{|A_v|=-7\ dB}$

(b) $|A_v|=\frac{1}{\sqrt{1+\left(\frac{f}{f_{3-dB}}\right)^2}}=\frac{1}{\sqrt{1+(2)^4}}=0.2425\Rightarrow$

$\underline{|A_v|=-12.3\ dB}$

(c) $|A_v|=\frac{1}{\sqrt{1+\left(\frac{f}{f_{3-dB}}\right)^2}}=\frac{1}{\sqrt{1+(2)^6}}=0.1240\Rightarrow$

$\underline{|A_v|=-18.1\ dB}$

15.3

Using Figure 15.9(a)

$$f_{3-dB}=\frac{1}{2\pi RC}$$

So

$$RC=\frac{1}{2\pi f_{3-dB}}=\frac{1}{2\pi(10\times10^3)}=1.59\times10^{-5}$$

For example, $C=0.001\ \mu F$, $R=15.9\ k\Omega$
so that $R_3=11.2\ k\Omega$ and $R_4=22.4\ k\Omega$

15.4

Use Figure 15.10(b)

$$f_{3-dB}=\frac{1}{2\pi RC}$$

or

$$RC=\frac{1}{2\pi(50\times10^3)}=3.18\times10^{-6}$$

For example, let $C=100$ pF Then $R=31.8\ k\Omega$
And $R_1=8.97\ k\Omega$
$R_2=22.8\ k\Omega$
$R_3=157\ k\Omega$

From Equation (15.26)

$$|T|=\frac{1}{\sqrt{1+\left(\frac{f_{3dB}}{f}\right)^6}}$$

We find

f kHz	$\|T\|$
30	0.211
35	0.324
40	0.456
45	0.589

15.5

From Equation (15.7),

$$T(s) = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4(Y_1 + Y_2 + Y_3)}$$

For a high-pass filter, let $Y_1 = Y_2 = sC$,

$$Y_3 = \frac{1}{R_3}, \text{ and } Y_4 = \frac{1}{R_4}$$

Then

$$T(s) = \frac{s^2C^2}{s^2C^2 + \frac{1}{R_4}\left(sC + sC + \frac{1}{R_3}\right)}$$
$$= \frac{1}{1 + \frac{1}{sR_4C}\left(2 + \frac{1}{sR_3C}\right)}$$

Define $\tau_3 = R_3C$ and $\tau_4 = R_4C$

$$T(s) = \frac{1}{1 + \frac{1}{s\tau_4}\left(2 + \frac{1}{s\tau_3}\right)}$$

Set $s = j\omega$

$$T(j\omega) = \frac{1}{1 + \frac{1}{j\omega\tau_4}\left(2 + \frac{1}{j\omega\tau_3}\right)}$$
$$= \frac{1}{1 - \frac{j}{\omega\tau_4}\left(2 - \frac{j}{\omega\tau_3}\right)}$$
$$= \frac{1}{\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right) - \frac{2j}{\omega\tau_4}}$$

$$|T(j\omega)| = \left\{\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right)^2 + \frac{4}{\omega^2\tau_4^2}\right\}^{-1/2}$$

For a maximally flat filter, we want

$$\left.\frac{d|T|}{d\omega}\right|_{\omega\to\infty} = 0$$

Taking the derivative, we find

$$\frac{d|T(j\omega)|}{d\omega} = -\frac{1}{2}\left\{\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right)^2 + \frac{4}{\omega^2\tau_4^2}\right\}^{-3/2}$$
$$\times\left[2\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right)\left(\frac{2}{\omega^3\tau_3\tau_4}\right) + \frac{4(-2)}{\omega^3\tau_4^2}\right]$$

or

$$\left.\frac{d|T(j\omega)|}{d\omega}\right|_{\omega\to\infty} = 0$$
$$= \left[\left(\frac{4}{\omega^3\tau_3\tau_4}\right)\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right) - \frac{8}{\omega^3\tau_4^2}\right]$$
$$= \frac{4}{\omega^3}\left[\frac{1}{\tau_3\tau_4}\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right) - \frac{2}{\tau_4^2}\right]$$

Then

$$\left.\left[\frac{1}{\tau_3\tau_4}\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right) - \frac{2}{\tau_4^2}\right]\right|_{\omega\to\infty} = 0$$

So that $\frac{1}{\tau_3} = \frac{2}{\tau_4} \Rightarrow 2\tau_3 = \tau_4$

Then the transfer function can be written as:

$$|T(j\omega)| = \left\{\left[1 - \frac{1}{\omega^2(2\tau_3^2)}\right]^2 + \frac{4}{\omega^2(4\tau_3^2)}\right\}^{-1/2}$$
$$= \left\{1 - \frac{1}{\omega^2\tau_3^2} + \frac{1}{4(\omega^2\tau_3^2)^2} + \frac{1}{\omega^2\tau_3^2}\right\}^{-1/2}$$
$$= \left\{1 + \frac{1}{4(\omega^2\tau_3^2)^2}\right\}^{-1/2}$$

$3 - dB$ frequency

$$2\omega^2\tau_3^2 = 1 \text{ or } \omega = \frac{1}{\sqrt{2}\tau_3} = \frac{1}{\sqrt{2}R_3C}$$

Define

$$\omega = \frac{1}{RC}$$

So that

$$\underline{R_3 = \frac{R}{\sqrt{2}}}$$

We had $2\tau_3 = \tau_4$ or $2(R_3C) = R_4C \Rightarrow R_4 = 2R_3$

So that $\underline{R_4 = \sqrt{2}\cdot R}$

15.6

From Equation (15.25)

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3-dB}}\right)^{2N}}}$$

-25 dB $\Rightarrow |T| = 0.0562$

$$\frac{f}{f_{3-dB}} = \frac{20}{10} = 2$$

So

$$0.0562 = \frac{1}{\sqrt{1+(2)^{2N}}}$$

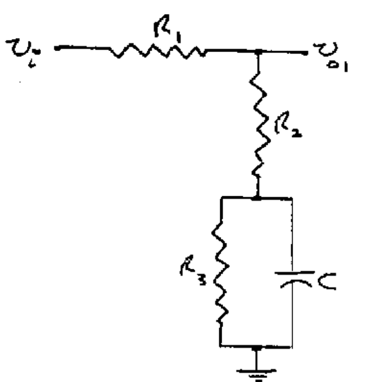
$1+(2)^{2N} = 316.6 \Rightarrow (2)^{2N} = 315.6$

$2N \cdot \ln(2) = \ln(315.6)$

$\Rightarrow N = 4.15 \Rightarrow \underline{N=5}$ A 5-pole filter

15.7

Consider

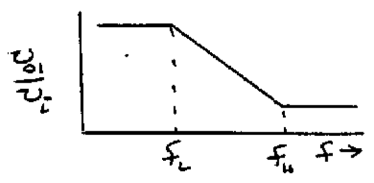


For low-frequency: $\dfrac{v_o}{v_i} = \dfrac{R_2+R_3}{R_1+R_2+R_3}$

For high-frequency: $\dfrac{v_o}{v_i} = \dfrac{R_2}{R_1+R_2}$

So we need

$$\frac{R_2+R_3}{R_1+R_2+R_3} = 25\left(\frac{R_2}{R_1+R_2}\right)$$

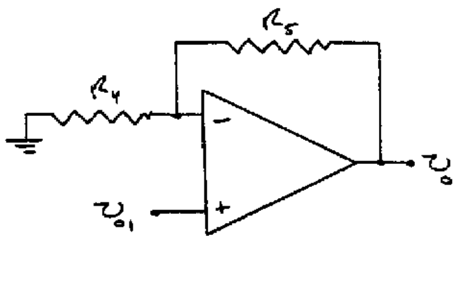


Let $R_1+R_2 = 50\,k\Omega$ and $\underline{R_2 = 1.5\,k\Omega} \Rightarrow$ $\underline{R_1 = 48.5\,k\Omega}$

Then

$$\frac{1.5+R_3}{50+R_3} = 25\left(\frac{1.5}{50}\right) \Rightarrow \underline{R_3 = 144\,k\Omega}$$

Connect the output of this circuit to a non-inverting op-amp circuit.



At low-frequency:

$$v_{o1} = \frac{R_2+R_3}{R_1+R_2+R_3}\cdot v_i = \frac{1.5+144}{48.5+1.5+144}\cdot v_i = 0.75 v_i$$

Need to have $v_o = 25$.

$$v_o = 25 = \left(1+\frac{R_5}{R_4}\right)\cdot v_{o1} = \left(1+\frac{R_5}{R_4}\right)(0.75)v_i \Rightarrow$$

$$\frac{R_5}{R_4} = 32.3$$

To check at high-frequency.

$$v_{o1} = \frac{R_2}{R_1+R_2}v_i = \frac{1.5}{1.5+48.5}v_i = 0.03 v_i$$

$$v_o = (1+32.3)v_{o1} = (33.3)(0.03)v_i = (1.0)v_i$$

which meets the design specification

Consider the frequency response.

$$\frac{v_{o1}}{v_i} = \frac{R_2 + R_3 \| \frac{1}{sC}}{R_1+R_2+R_3\| \frac{1}{sC}}$$

Now

$$R_3 \| \frac{1}{sC} = \frac{R_3}{1+sR_3C}$$

Then, we find

$$\frac{v_{o1}}{v_i} = \frac{R_3 + R_2(1+sR_3C)}{R_3+(R_1+R_2)(1+sR_3C)}$$

which can be rearranged as

$$\frac{v_{o1}}{v_i} = \frac{(R_2+R_3)(1+s(R_2\|R_3)C)}{(R_1+R_2+R_3)(1+s(R_3\|(R_1+R_2))C)}$$

So

$$f_L \cong \frac{1}{2\pi(R_2\|R_3)C} = \frac{1}{2\pi(1.5\|144)x10^3C} = \frac{1}{(9.33x10^3)C}$$

$$f_H \cong \frac{1}{2\pi(R_3\|(R_1+R_2))C} = \frac{1}{2\pi(144\|50)x10^3C}$$

$$= \frac{1}{(2.33x10^5)C}$$

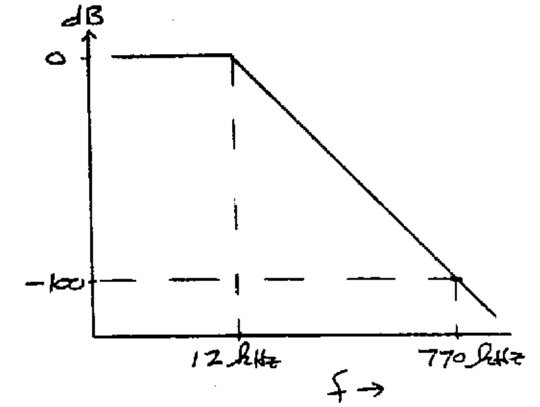
Set

$$25\,kHz = \frac{f_L+f_H}{2} = \frac{1}{2}\left[\frac{1}{(9.33x10^3)C}+\frac{1}{(2.33x10^5)C}\right]$$

Which yields

$\underline{C = 2.23\,nF}$

15.8



$$|A_v| = \frac{1}{\sqrt{1+\left(\frac{f}{f_{3-dB}}\right)^{2N}}}$$

$-100\ dB \Rightarrow 10^{-5}$

So

$$10^{-5} = \frac{1}{\sqrt{1+\left(\frac{770}{12}\right)^{2N}}}$$

or

$$1+(64.2)^{2N} = \left(\frac{1}{10^{-5}}\right)^2 = 10^{10}$$

or

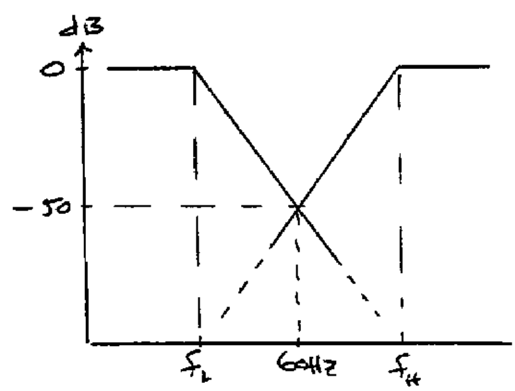
$$(64.2)^{2N} \cong 10^{10}$$

Now

N	Left Side
1	4.112×10^3
2	1.7×10^7
3	7×10^{10}

So, we need a 3rd order filter.

15.9



Low-pass: $-50\ dB \Rightarrow 3.16 \times 10^{-3}$

Then

$$3.16 \times 10^{-3} = \frac{1}{\sqrt{1+\left(\frac{f}{f_L}\right)^4}} = \frac{1}{\sqrt{1+\left(\frac{60}{f_L}\right)^4}}$$

We find $f_L = 3.37\ Hz$

High Pass:

$$3.16 \times 10^{-3} = \frac{1}{\sqrt{1+\left(\frac{f_H}{f}\right)^4}} = \frac{1}{\sqrt{1+\left(\frac{f_H}{60}\right)^4}}$$

We find $f_H = 1067\ Hz$

Bandwidth: $BW = f_H - f_L = 1067 - 3.37 \Rightarrow$

$\underline{BW \cong 1064\ Hz}$

15.10

a.

$$\frac{v_I}{R_4} = -\frac{v_{02}}{R_3} - \frac{v_0}{R_1 \parallel \left(\frac{1}{sC}\right)} \qquad (1)$$

$$\frac{v_0}{R_2} = -\frac{v_{01}}{\left(\frac{1}{sC}\right)} \qquad (2)$$

$$\frac{v_{01}}{R_5} = -\frac{v_{02}}{R_5} \Rightarrow v_{01} = -v_{02} \qquad (3)$$

Then

$$\frac{v_0}{R_2} = +\frac{v_{02}}{\left(\frac{1}{sC}\right)} \text{ or } v_{02} = v_0\left(\frac{1}{sR_2C}\right) \qquad (2)$$

And

$$\frac{v_I}{R_4} = -\frac{v_0}{R_3}\cdot\left(\frac{1}{sR_2C}\right) - \frac{v_0}{R_1 \parallel \left(\frac{1}{sC}\right)}$$

$$= -v_0\left[\frac{1}{R_3(sR_2C)} + \frac{1}{\frac{R_1\cdot(1/sC)}{R_1+(1/sC)}}\right] \qquad (1)$$

$$= -v_0\left[\frac{1}{R_3(sR_2C)} + \frac{1+sR_1C}{R_1}\right]$$

$$= -v_0\left[\frac{R_1+(1+sR_1C)(sR_2R_3C)}{(sC)R_1R_2R_3}\right]$$

Then

$$\frac{v_0}{v_I} = -\frac{1}{R_4}\left[\frac{(sC)(R_1R_2R_3)}{R_1+sR_2R_3C+s^2R_1R_2R_3C^2}\right]$$

or

$$A_v(s) = \frac{v_0}{v_I} = \frac{-\frac{1}{R_4}}{\frac{1}{R_1}+sC+\frac{1}{sCR_2R_3}}$$

b. $A_v(j\omega) = \dfrac{-\dfrac{1}{R_4}}{\dfrac{1}{R_1} + j\omega C + \dfrac{1}{j\omega C R_2 R_3}}$

or

$$A_v(j\omega) = \frac{-\dfrac{1}{R_4}}{\dfrac{1}{R_1} + j\left[\omega C - \dfrac{1}{\omega C R_2 R_3}\right]}$$
$$= -\frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + j\left[\omega R_1 C - \dfrac{R_1}{\omega C R_2 R_3}\right]\right\}}$$

$$|A_v(j\omega)| = \frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + \left[\omega R_1 C - \dfrac{R_1}{\omega C R_2 R_3}\right]^2\right\}^{1/2}}$$

$|A_v|_{max}$ when $\left[\omega R_1 C - \dfrac{R_1}{\omega C R_2 R_3}\right] = 0$

Then

$|A_v|_{max} = \dfrac{R_1}{R_4} = \dfrac{85}{3} \Rightarrow \underline{|A_v|_{max} = 28.3}$

Now

$\omega R_1 C\left[1 - \dfrac{1}{\omega^2 C^2 R_2 R_3}\right] = 0$ or $\omega = \dfrac{1}{C\sqrt{R_2 R_3}}$

Then

$$f = \frac{1}{2\pi C\sqrt{R_2 R_3}} = \frac{1}{2\pi(0.1 \times 10^{-6})\sqrt{(300)^2}}$$

So

$\underline{f = 5.305 \text{ kHz}}$

To find the two $3 - dB$ frequencies,

$$\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right] = \pm 1$$
$$\omega^2 R_1 R_2 R_3 C^2 - R_1 = \pm\omega R_2 R_3 C$$

$$\omega^2(85 \times 10^3)(300)^2(0.1 \times 10^{-6})^2 - 85 \times 10^3$$
$$= \pm\omega(300)^2(0.1 \times 10^{-6})$$
$$\omega^2(7.65 \times 10^{-5}) - 85 \times 10^3 = \pm\omega(9 \times 10^{-3})$$
$$\omega^2(7.65 \times 10^{-5}) \pm \omega(9 \times 10^{-3}) - 85 \times 10^3 = 0$$

$$\omega = \frac{\pm(9 \times 10^{-3})}{2(7.65 \times 10^{-5})} \pm \frac{\sqrt{(9 \times 10^{-3})^2 + 4(7.65 \times 10^{-5})(85 \times 10^{-3})}}{2(7.65 \times 10^{-5})}$$

We find $f = 5.315$ kHz and $f = 5.296$ kHz

15.11

a.

$$\frac{v_I - v_A}{R} = \frac{v_A}{\left(\dfrac{1}{sC}\right)} \quad (1)$$

$$\frac{v_I - v_B}{R} = \frac{v_B - v_O}{R} \quad (2)$$

and $v_A = v_B$

So

$$\frac{v_I}{R} = v_A\left(\frac{1}{R} + sC\right) = v_A\left(\frac{1 + sRC}{R}\right) \quad (1)$$

or

$$v_A = \frac{v_I}{1 + sRC}$$

Then

$$v_I + v_O = 2v_B = 2v_A = \frac{2v_I}{1 + sRC} \quad (2)$$

$$v_O = v_I\left[\frac{2}{1 + sRC} - 1\right] = v_I\left[\frac{1 - sRC}{1 + sRC}\right]$$

Now

$$\frac{v_O}{v_I} = A(j\omega) = \frac{1 - j\omega RC}{1 + j\omega RC}$$
$$|A| = \frac{\sqrt{1 + \omega^2 R^2 C^2}}{\sqrt{1 + \omega^2 R^2 C^2}} \Rightarrow |A| = 1$$

Phase:

$$\phi = -2\tan^{-1}(\omega RC)$$

b. $RC = (10^4)(15.9 \times 10^{-9}) = 1.59 \times 10^{-4}$

f	ϕ
0	0
10^2	-11.4
5×10^2	-53.1
$1/2\pi RC = 10^3$ Hz	$-90°$
5×10^3	-157
10^4	-169

15.12

a. $\dfrac{V_i}{R_1} + \dfrac{V_i - V_0}{R_2\|(1/sC)} = 0$

$$\frac{V_i}{R_1} + \frac{V_i - V_0}{\left[\dfrac{R_2}{1+sR_2C}\right]} = 0$$

$$\frac{R_2}{R_1}\cdot\frac{1}{1+sR_2C}(V_i) + V_i = V_0$$

$$\frac{V_0}{V_i} = \frac{R_2 + R_1(1+sR_2C)}{R_1(1+sR_2C)}$$

$$= \frac{(R_2+R_1)[1+s(R_1\|R_2)C]}{R_1(1+sR_2C)}$$

$$\Rightarrow \frac{V_0}{V_i} = \left(1+\frac{R_2}{R_1}\right)\left[\frac{1+s(R_1\|R_2)C}{(1+sR_2C)}\right]$$

$$\Rightarrow f_{3dB_1} = \frac{1}{2\pi R_2 C}$$

$$\Rightarrow f_{3dB_2} = \frac{1}{2\pi (R_1\|R_2)C}$$

b. $\dfrac{V_i}{R_1\|(1/sC)} + \dfrac{V_i - V_0}{R_2} = 0$

$$\frac{V_i}{\left(\dfrac{R_1}{1+sR_1C}\right)} + \frac{V_i}{R_2} = \frac{V_0}{R_2}$$

$$V_i\left[\frac{R_2}{R_1}\cdot(1+sR_1C)+1\right] = V_0$$

$$\frac{V_i}{R_1}\cdot[R_2 + R_1 + sR_1R_2C] = V_0$$

$$\frac{V_0}{V_i} = \frac{R_2+R_1}{R_1}\cdot[1+s(R_1\|R_2)C]$$

$$\Rightarrow \frac{V_0}{V_i} = \left(1+\frac{R_2}{R_1}\right)[1+s(R_1\|R_2)C]$$

$$\Rightarrow f_{3dB} = \frac{1}{2\pi(R_1\|R_2)C}$$

15.13

a. $\dfrac{V_i}{R_1 + (1/sC_1)} = \dfrac{-V_0}{R_2\|(1/sC_2)}$

$$V_i\left(\frac{sC_1}{1+sR_1C_1}\right) = -V_0\left(\frac{1+sR_2C_2}{sC_2}\right)$$

$$\frac{V_0}{V_i} = \frac{-sR_2C_1}{(1+sR_1C_1)(1+sR_2C_2)}$$

$$= \frac{-sR_2C_1}{1+sR_1C_1+sR_2C_2+s^2R_1R_2C_1C_2}$$

$$\frac{V_0}{V_i} = -\frac{R_2}{R_1}\times$$

$$\times\left[\frac{sC_1}{\dfrac{1}{R_1} + sC_1\left(1+\dfrac{R_2}{R_1}\cdot\dfrac{C_2}{C_1}\right) + s^2R_2C_1C_2}\right]$$

or

$$T(s) = \frac{V_0}{V_i}$$

$$= -\frac{R_2}{R_1}\cdot\left[\frac{1}{\dfrac{1}{sR_1C_1} + \left(1+\dfrac{R_2}{R_1}\cdot\dfrac{C_2}{C_1}\right) + sR_2C_2}\right]$$

b.

$$|T(j\omega)| = -\frac{R_2}{R_1}\times$$

$$\times\frac{1}{\left\{\left(1+\dfrac{R_2}{R_1}\cdot\dfrac{C_2}{C_1}\right)^2 + \left(\omega R_2C_2 - \dfrac{1}{\omega R_1C_1}\right)^2\right\}^{1/2}}$$

when $\left(\omega R_2C_2 - \dfrac{1}{\omega R_1C_1}\right) = 0$, we want

$$|T(j\omega)| = 50 = \frac{R_2}{R_1}\cdot\frac{1}{\left(1+\dfrac{R_2}{R_1}\cdot\dfrac{C_2}{C_1}\right)}$$

At the $3-dB$ frequencies, we want

$$\left(\omega R_2C_2 - \frac{1}{\omega R_1C_1}\right) = \pm\left(1+\frac{R_2}{R_1}\cdot\frac{C_2}{C_1}\right)$$

For $f = 5$ kHz, use + sign and for $f = 200$ Hz, use − sign.

$$\omega_1 = 2\pi(200) = 1257$$

$$\omega_2 = 2\pi(5\times10^3) = 3.142\times10^4$$

Define $\tau_2 = R_2C_2$ and $\tau_1 = R_1C_1$
Then

$$50 = \frac{R_2}{R_1}\cdot\frac{1}{1+\dfrac{\tau_2}{\tau_1}} \quad (1)$$

$$\left(\omega_2\tau_2 - \frac{1}{\omega_2\tau_1}\right) = +\left(1+\frac{\tau_2}{\tau_1}\right) \quad (2)$$

$$\left(\omega_1\tau_2 - \frac{1}{\omega_1\tau_1}\right) = -\left(1+\frac{\tau_2}{\tau_1}\right) \quad (3)$$

From (2)

$$\frac{\omega_2^2\tau_1\tau_2 - 1}{\omega_2\tau_1} = \frac{\tau_1+\tau_2}{\tau_1}$$

or

$$\omega_2\tau_1\tau_2 - \frac{1}{\omega_2} = \tau_1 + \tau_2$$

$$\tau_1(\omega_2\tau_2 - 1) = \frac{1}{\omega_2} + \tau_2$$

So

$$\tau_1 = \frac{\frac{1}{\omega_2} + \tau_2}{\omega_2 \tau_2 - 1}$$

Substituting into (3), we find

$$\omega_1 \tau_2 - \frac{1}{\omega_1 \left[\frac{\frac{1}{\omega_2} + \tau_2}{\omega_2 \tau_2 - 1} \right]} = -\left[1 + \frac{\tau_2(\omega_2 \tau_2 - 1)}{\frac{1}{\omega_2} + \tau_2} \right]$$

$$\omega_1 \tau_2 \left[\frac{1}{\omega_2} + \tau_2 \right] - \frac{1}{\omega_1}(\omega_2 \tau_2 - 1)$$
$$= -\left[\left(\frac{1}{\omega_2} + \tau_2 \right) + \tau_2(\omega_2 \tau_2 - 1) \right]$$

$$\frac{\omega_1}{\omega_2} \cdot \tau_2 + \omega_1 \tau_2^2 - \frac{\omega_2}{\omega_1} \cdot \tau_2 + \frac{1}{\omega_1}$$
$$= -\frac{1}{\omega_2} - \tau_2 - \omega_2 \tau_2^2 + \tau_2$$

$$(\omega_1 + \omega_2)\tau_2^2 + \left(\frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1} \right) \tau_2 + \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} \right) = 0$$
$$(3.2677 \times 10^4)\tau_2^2 - 24.96\tau_2 + 8.273 \times 10^{-4} = 0$$

$$\tau_2 = \frac{24.96}{2(3.2677 \times 10^4)} \pm \frac{\sqrt{(24.96)^2 - 4(3.2677 \times 10^4)(8.273 \times 10^{-4})}}{2(3.2677 \times 10^4)}$$

Since ω_2 is large, τ_2 should be small so use minus sign:

$$\tau_2 = 3.47 \times 10^{-5}$$

Then

$$\tau_1 = \frac{3.18 \times 10^{-5} + 3.47 \times 10^{-5}}{9.09 \times 10^{-2}} \Rightarrow \tau_1 = 7.32 \times 10^{-4}$$

Now

$$50 = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{3.47 \times 10^{-5}}{7.32 \times 10^{-4}}}$$

Then

$$\frac{R_2}{R_1} = 52.37 \text{ or } \underline{R_2 = 524 \text{ k}\Omega}$$

Also

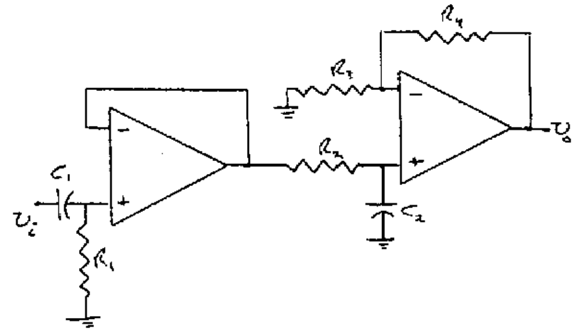
$\tau_1 = R_1 C_1$ so that $C_1 = 0.0732\ \mu\text{F}$

$\tau_2 = R_2 C_2$ so that $C_2 = 66.3$ pF

15.14

Gain = 10 dB $\Rightarrow$ Gain = 3.162

For example, we may have



Want $\dfrac{R_4}{R_3} = 2.162$

For example, let $R_3 = 50$ kΩ,

$R_4 = 108$ kΩ

$$f_1 = \frac{1}{2\pi R_1 C_1} = 200$$

So

$$R_1 C_1 = \frac{1}{2\pi(200)} = 0.796 \times 10^{-3}$$

For example, let $R_1 = 200$ kΩ $\Rightarrow$ A large input resistance

$$C_1 = 0.00398\ \mu\text{F}$$

$$f_2 = \frac{1}{2\pi R_2 C_2} = 50 \times 10^3$$
$$\Rightarrow R_2 C_2 = \frac{1}{2\pi(50 \times 10^3)} = 3.18 \times 10^{-6}$$

For example, let

$R_2 = 10$ kΩ and $C_2 = 318$ pF

15.15

$f_C = 100$ kHz

$$R_{eq} = \frac{1}{f_C C}$$

a. For $C = 1$ pF, $R_{eq} = 10$ MΩ
b. For $C = 10$ pF, $R_{eq} = 1$ MΩ
c. For $C = 30$ pF, $R_{eq} = 333$ kΩ

15.16

a. From Equation (15.28),

$$Q = \frac{V_1 - V_2}{R_{eq}} \cdot T_C$$

and $f_C = 100$ kHz so that $T_C = \frac{1}{100 \times 10^3} \Rightarrow 10\ \mu s$

Now

$$R_{eq} = \frac{1}{f_C C} = \frac{1}{(100 \times 10^3)(10 \times 10^{-12})} \Rightarrow 1\ M\Omega$$

So

$$Q = \frac{(2-1)(10 \times 10^{-6})}{10^6} = 10 \times 10^{-12}\ C$$

or

$$\underline{Q = 10\ pC}$$

b. $I_{eq} = \frac{Q}{T_C} = \frac{10 \times 10^{-12}}{10 \times 10^{-6}}$ or $\underline{I_{eq} = 1\ \mu A}$

c.

$Q = CV$ so find the time that V_0 reaches 99% of its full value.

$V_0 = V_1\left(1 - e^{-t/\tau}\right)$ where $\tau = RC$

Then $0.99 = 1 - e^{-t/\tau}$ or $e^{-t/\tau} = 0.01$

or $t = \tau \ln(100)$

$$\tau = RC = (10^3)(10 \times 10^{-12}) = 10^{-8}\ s$$

Then

$$\underline{t = 4.61 \times 10^{-8}\ s}$$

15.17

Low frequency gain $= -10 \Rightarrow \frac{C_1}{C_2} = 10$

$f_{3dB} = 10 \times 10^3\ \text{Hz} = \frac{f_C C_2}{2\pi C_F}$

Set

$$f_C = 10 f_{3dB} = 100\ \text{kHz}$$

Then

$$\frac{C_2}{C_F} = \frac{2\pi(10 \times 10^3)}{100 \times 10^3} = 0.628$$

The largest capacitor is C_1, so let

$$\underline{C_1 = 30\ pF}$$

Then

$$\underline{C_2 = 3\ pF}$$

and

$$\underline{C_F = 4.78\ pF}$$

15.18

a. Time constant $= R_{eq} \cdot C_F = \tau$ where

$$R_{eq} = \frac{1}{f_C C_1} = \frac{1}{(100 \times 10^3)(5 \times 10^{-12})} = 2 \times 10^6\ \Omega$$

Then

$$\tau = (2 \times 10^6)(30 \times 10^{-12})$$

or

$$\underline{\tau = 60\ \mu s}$$

b. $v_0 = -\frac{1}{\tau}\int v_I \cdot dt$

or

$$\Delta v_0 = \frac{(1)T_C}{\tau},\quad T_C = \frac{1}{f_C}$$

So

$$\Delta v_0 = \frac{1}{(60 \times 10^{-6})(100 \times 10^3)}$$

or

$$\underline{\Delta v_0 = 0.167\ V}$$

c. Now $\Delta v_0 = 13 = N(0.167)$

or

$\underline{N = 78}$ clock pulses

15.19

Using Equation (15.41)

$$f_0 = \frac{1}{2\pi\sqrt{3}RC} = \frac{1}{2\pi\sqrt{3}(4 \times 10^3)(10 \times 10^{-9})}$$

or

$$\underline{f_0 = 2.3\ \text{kHz}}$$

$\frac{R_2}{R} = 8$ so that $R_2 = 8(4 \times 10^3)$

$\Rightarrow \underline{R_2 = 32\ k\Omega}$

15.20

a. $v_1 = \frac{R}{R + (1/sC_V)} \cdot v_0 = \left(\frac{sRC_V}{1 + sRC_V}\right) \cdot v_0$

$$v_2 = \frac{R}{R + \frac{1}{sC}} \cdot v_1 = \left(\frac{sRC}{1 + sRC}\right) \cdot v_1$$

$$v_3 = \frac{R}{R + \frac{1}{sC}} \cdot v_2 = \left(\frac{sRC}{1 + sRC}\right) \cdot v_2$$

$$v_0 = -\frac{R_2}{R} \cdot v_3$$

Then

$$v_0 = -\frac{R_2}{R}\left(\frac{sRC}{1+sRC}\right)^2\left(\frac{sRC_V}{1+sRC_V}\right)v_0$$

Set $s = j\omega$

$$1 = -\frac{R_2}{R}\left(\frac{-\omega^2R^2C^2}{1+2j\omega RC-\omega^2R^2C^2}\right)\left(\frac{j\omega RC_V}{1+j\omega RC_V}\right)$$

The real part of the denominator must be zero.

$$1-\omega^2R^2C^2-2\omega^2R^2CC_V=0$$

so

$$\underline{\omega_0 = \frac{1}{R\sqrt{C(C+2C_V)}}}$$

b. $$f_{0,\max} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11}+2[10^{-11}])}}$$

$f_{0,\max} = 919$ kHz

$$f_{0,\min} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11}+2[50\times10^{-12}])}}$$

$f_{0,\min} = 480$ kHz

15.21

From Equation (15.46)

$$f_0 = \frac{1}{2\pi\sqrt{6}RC}$$

so $R = \dfrac{1}{2\pi\sqrt{6}(80\times10^3)(100\times10^{-12})}$ or

$\underline{R = 8.12\ \text{k}\Omega}$

We need

$$\frac{R_2}{R} = 29$$

so that

$\underline{R_2 = 236\ \text{k}\Omega}$

15.22

$$\frac{v_0-v_1}{\frac{1}{sC}} = \frac{v_1}{R}+\frac{v_1-v_2}{\frac{1}{sC}} \qquad (1)$$

or $(v_0-v_1)sC = \dfrac{v_1}{R}+(v_1-v_2)sC$

$$\frac{v_1-v_2}{\frac{1}{sC}} = \frac{v_2}{R}+\frac{v_2}{\frac{1}{sC}+R} \qquad (2)$$

or $(v_1-v_2)sC = \dfrac{v_2}{R}+\dfrac{v_2(sC)}{1+sRC}$

$$\frac{v_2}{\frac{1}{sC}+R} = -\frac{v_0}{R_2} \qquad (3)$$

or $\dfrac{v_2sC}{1+sRC} = -\dfrac{v_0}{R_2}$

so

$$v_2 = \frac{-v_0}{sR_2C}(1+sRC)$$

From (2)

$$v_1(sC) = v_2\left[sC+\frac{1}{R}+\frac{sC}{1+sRC}\right]$$

or

$$v_1 = -\frac{v_0(1+sRC)}{sR_2C}\cdot\left[1+\frac{1}{sRC}+\frac{1}{1+sRC}\right]$$

From (1)

$$v_0(sC) = v_1\left[sC+\frac{1}{R}+sC\right]-v_2(sC)$$

Then

$$v_0 = \left[2+\frac{1}{sRC}\right]\left[\frac{-v_0(1+sRC)}{sR_2C}\right]\times$$
$$\times\left[\frac{1+sRC}{sRC}+\frac{1}{1+sRC}\right]+\frac{v_0}{sR_2C}\cdot(1+sRC)$$

$$-1 = \left[\frac{1+2sRC}{sRC}\right]\left[\frac{1+sRC}{sR_2C}\right]\left[\frac{(1+sRC)^2+sRC}{(sRC)(1+sRC)}\right] - \frac{1+sRC}{sR_2C}$$

$$-1 = \frac{(1+2sRC)(1+2sRC+s^2R^2C^2+sRC)}{(sRC)^2(sR_2C)} - \frac{(1+sRC)(sRC)^2}{(sRC)^2(sR_2C)}$$

Set $s = j\omega$

$$-1 = \frac{(1+2j\omega RC)(1+3j\omega RC+\omega^2R^2C^2)}{(-\omega^2R^2C^2)(j\omega R_2C)} - \frac{(1+j\omega RC)(-\omega^2R^2C^2)}{(-\omega^2R^2C^2)(j\omega R_2C)}$$

The real part of the numerator must be zero.

$$1-\omega^2R^2C^2-6\omega^2R^2C^2+\omega^2R^2C^2=0$$

$$6\omega^2R^2C^2=1$$

so that

$$\underline{\omega_0 = \frac{1}{\sqrt{6}RC}}$$

Condition for oscillation:

$$-1 = \frac{2j\omega RC+3j\omega RC-2j\omega^3R^3C^3+j\omega^3R^3C^3}{(-\omega^2R^2C^2)(j\omega R_2C)}$$

$$1 = \frac{5-\omega^2R^2C^2}{(\omega RC)(\omega R_2C)}$$

But

$$\omega = \omega_0 = \frac{1}{\sqrt{6}RC}$$

Then

$$1 = \frac{5-\frac{1}{6}}{\frac{(RC)(R_2C)}{6R^2C^2}} = \frac{\left(5-\frac{1}{6}\right)(6R^2C^2)}{RR_2C^2}$$

$$1 = \frac{\left(\frac{29}{6}\right)(6R)}{R_2} \text{ or } \underline{\frac{R_2}{R} = 29}$$

15.23

a.

$$v_{01} = \left(1+\frac{R_{F1}}{R_{A1}}\right)\left(\frac{\frac{1}{sC_1}}{\frac{1}{sC_1}+R_1}\right)\cdot v_0 \quad (1)$$

$$v_{02} = \left(1+\frac{R_{F2}}{R_{A2}}\right)\left(\frac{\frac{1}{sC_2}}{\frac{1}{sC_2}+R_2}\right)\cdot v_{01} \quad (2)$$

$$v_{03} = \left(\frac{R_{A3}\,\|\,\frac{1}{sC_3}}{R_{A3}\,\|\,\frac{1}{sC_3}+R_3}\right)\cdot v_{02} \quad (3)$$

$$v_0 = -\frac{R_{F3}}{R_{A3}}\cdot v_{03} \quad (4)$$

With all resistors equal and all capacitors equal, we find:

$$v_{01} = (2)\left(\frac{1}{1+sRC}\right)v_0 \quad (1)$$

$$v_{02} = (2)\left(\frac{1}{1+sRC}\right)v_{01} \quad (2)$$

$$v_{03} = (2)\left(\frac{\frac{R}{1+sRC}}{\frac{R}{1+sRC}+R}\right)v_{02}$$

$$= \left[\frac{R}{R+R(1+sRC)}\right]v_{02}$$

$$v_{03} = \left(\frac{1}{2+sRC}\right)v_{02} \quad (3)$$

and

$$v_0 = -v_{03} \quad (4)$$

Then

$$v_0 = -\left(\frac{1}{2+sRC}\right)(2)\left(\frac{1}{1+sRC}\right)(2)\left(\frac{1}{1+sRC}\right)v_0$$

Let $s = j\omega$

$$(2+j\omega RC)(1+j\omega RC)(1+j\omega RC) = -4$$

$$(2+j\omega RC)(1+2j\omega RC-\omega^2R^2C^2) = -4 \quad (A)$$

The imaginary term on the left must be zero.

$$4j\omega RC + j\omega RC - j\omega^3R^3C^3 = 0$$

$$\omega RC(5-\omega^2R^2C^2) = 0$$

or

$$\underline{\omega = \frac{\sqrt{5}}{RC}} \quad \text{(Not the same as in book)}$$

15.24

a.

$$\frac{v_0 - v_{01}}{R} = \frac{v_{01}}{\left(\frac{1}{sC}\right)} + \frac{v_{01}-v_{02}}{R} \quad (1)$$

$$\frac{v_{01} - v_{02}}{R} = \frac{v_{02}}{\left(\frac{1}{sC}\right)} + \frac{v_{02}-v_{03}}{R} \quad (2)$$

$$\frac{v_{02} - v_{03}}{R} = \frac{v_{03}}{\left(\frac{1}{sC}\right)} + \frac{v_{03}}{R} \quad (3)$$

$$v_0 = -\frac{R_F}{R}\cdot v_{03} \quad (4)$$

We can write the equations as

$$v_0 - v_{01} = v_{01}(sRC) + v_{01} - v_{02} \quad (1)$$

$$v_{01} - v_{02} = v_{02}(sRC) + v_{02} - v_{03} \quad (2)$$

$$v_{02} - v_{03} = v_{03}(sRC) + v_{03} \quad (3)$$

and

$$v_0 = -\frac{R_F}{R}\cdot v_{03} \quad (4)$$

Combining terms, we find

$$v_0 = v_{01}(2+sRC) - v_{02} \quad (1)$$

$$v_{01} = v_{02}(2+sRC) - v_{03} \quad (2)$$

$$v_{02} = v_{03}(2+sRC) \quad (3)$$

and

$$v_0 = -\frac{R_F}{R}\cdot v_{03} \quad (4)$$

Combining Equations (3) and (2)

$$v_{01} = v_{03}(2+sRC)^2 - v_{03} = v_{03}\left[(2+sRC)^2 - 1\right] \quad (2)$$

Then Equation (1) is

$$v_0 = v_{03}\left[(2+sRC)^2-1\right](2+sRC) - v_{03}(2+sRC)$$

Using Equation (4), we find

$$-\frac{R_F}{R}\cdot v_{03} = v_{03}\left\{\left[(2+sRC)^2-1\right](2+sRC) -(2+sRC)\right\}$$

To find the frequency of oscillation, set $s = j\omega$ and set the imaginary part of the right side of the equation to zero.

We will have

$$-\frac{R_F}{R} = (2+j\omega RC)\left[4+4j\omega RC-\omega^2R^2C^2-1-1\right]$$

Then

$$j\omega RC(2-\omega^2R^2C^2)+8j\omega RC=0$$

or

$$j\omega RC\left[2-\omega^2R^2C^2+8\right]=0$$

Then the frequency of oscillation is

$$\underline{f_0=\frac{1}{2\pi}\cdot\frac{\sqrt{10}}{RC}}$$

The condition to sustain oscillations is determined from

$$-\frac{R_F}{R}=2\left[2-\omega^2R^2C^2\right]-4\omega^2R^2C^2$$

or

$$-\frac{R_F}{R}=4-6\omega^2R^2C^2$$

Setting $\omega^2=\dfrac{10}{R^2C^2}$, we have

$$-\frac{R_F}{R}=4-6(10)$$

or

$$\underline{\frac{R_F}{R}=56}$$

b. For $R = 5$ kΩ and $f_0 = 5$ kHz, we find

$$C=\frac{\sqrt{10}}{2\pi(5\times10^3)(5\times10^3)}\Rightarrow \underline{C=0.02\ \mu\text{F}}$$

and $R_F = 56(5) \Rightarrow \underline{R_F = 280\ \text{k}\Omega}$

15.25

a. We can write

$$v_A=\left(\frac{R_1}{R_1+R_2}\right)v_0 \text{ and } v_B=\left(\frac{Z_p}{Z_p+Z_s}\right)v_0$$

where $Z_p = R_B \,\Big\|\, \dfrac{1}{sC_B}=\dfrac{R_B}{1+sR_BC_B}$

and $Z_s = R_A+\dfrac{1}{sC_A}=\dfrac{1+sR_AC_A}{sC_A}$

Setting $v_A = v_B$, we have

$$\frac{R_1}{R_1+R_2}=\frac{\dfrac{R_B}{1+sR_BC_B}}{\dfrac{R_B}{1+sR_BC_B}+\dfrac{1+sR_AC_A}{sC_A}}$$

$$\frac{R_1}{R_1+R_2} = \frac{sR_BC_A}{sR_BC_A+(1+sR_AC_A)(1+sR_BC_B)} \qquad (1)$$

To find the frequency of oscillation, set $s = j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is

$$j\omega R_BC_A+(1+j\omega R_AC_A)(1+j\omega R_BC_B)$$

or

$$j\omega R_BC_A+1+j\omega R_AC_A+j\omega R_BC_B -\omega^2R_AR_BC_AC_B \qquad (2)$$

Then from (2), we must have

$$1-\omega_0^2R_AR_BC_AC_B=0$$

or

$$\underline{f_0=\frac{1}{2\pi\sqrt{R_AR_BC_AC_B}}}$$

b. To find the condition for sustained oscillation, combine Equations (1) and (2). Then

$$\frac{R_1}{R_1+R_2}=\frac{j\omega R_BC_A}{j\omega R_BC_A+j\omega R_AC_A+j\omega R_BC_B}$$

or

$$1+\frac{R_2}{R_1}=1+\frac{R_A}{R_B}+\frac{C_B}{C_A}$$

Then

$$\underline{\frac{R_2}{R_1}=\frac{R_A}{R_B}+\frac{C_B}{C_A}}$$

15.26

a. We can write

$$v_A = \left(\frac{R_1}{R_1 + R_2}\right) v_0$$

and

$$v_B = \left(\frac{R \| sL}{R \| sL + R + sL}\right) v_0$$

Setting $v_A = v_B$, we have

$$\frac{R_1}{R_1 + R_2} = \left[\frac{\frac{sRL}{R + sL}}{\frac{sRL}{R + sL} + R + sL}\right] \cdot v_0$$

$$\frac{R_1}{R_1 + R_2} = \frac{sRL}{sRL + (R + sL)^2} \qquad (1)$$

To find the frequency of oscillation, set $s = j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is:

$$j\omega RL + (R + j\omega L)^2$$

or

$$j\omega RL + R^2 + 2j\omega RL - \omega^2 L^2 \qquad (2)$$

Then

$$R^2 - \omega_0^2 L^2 = 0$$

or

$$\underline{f_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{R}{L}}}$$

b. To find the condition for sustained oscillations, combine Equations (1) and (2).

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega RL}{j\omega RL + 2j\omega RL} = \frac{1}{3}$$

Then

$$1 + \frac{R_2}{R_1} = 3$$

so that

$$\underline{\frac{R_2}{R_1} = 2}$$

15.27

From Equation (15.52(b))

$$f_0 = \frac{1}{2\pi RC}$$

or

$$RC = \frac{1}{2\pi f} = \frac{1}{2\pi(80 \times 10^3)}$$

$$RC \cong 2 \times 10^{-6}$$

Set $R = 20\ \text{k}\Omega$ and $C = 100\ \text{pF}$

We must have

$$\frac{R_2}{R_1} = 2$$

Set $R_2 = 40\ \text{k}\Omega$ and $R_1 = 20\ \text{k}\Omega$, for example.

15.28

From Equation (15.59)

$$f_0 = \frac{1}{2\pi\sqrt{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}}$$

and from Equation (15.61)

$$\frac{C_2}{C_1} = g_m R$$

Now,

$$g_m = 2\sqrt{k_n I_{DQ}} = 2\sqrt{(0.5)(1)} = 1.414\ \text{mA/V}$$

We have $C_1 = 0.01\ \mu\text{F}$, $R = 4\ \text{k}\Omega$, $f_0 = 400\ \text{kHz}$

So

$$C_2 = g_m R C_1 = (1.414)(4)(0.01)$$

or

$$\underline{C_2 = 0.0566\ \mu\text{F}}$$

and

$$400 \times 10^3 = \frac{1}{2\pi\sqrt{L\left[\frac{(0.01)(0.0566)}{0.01 + 0.0566}\right] \times 10^{-6}}}$$

$$L(8.5 \times 10^{-9}) = \left[\frac{1}{2\pi(400 \times 10^3)}\right]^2 = 1.58 \times 10^{-13}$$

Then

$$\underline{L = 18.6\ \mu\text{H}}$$

15.29

$$V_\pi = -V_0$$

$$\frac{V_0}{\left(\frac{1}{sC_2}\right)} + \frac{V_0}{R_L} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} = g_m V_\pi = -g_m V_0$$

$$V_0\left[sC_2 + sC_1 + \frac{1}{R_L} + g_m\right] = V_1(sC_1) \qquad (1)$$

$$\frac{V_1}{sL} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} + g_m V_\pi = 0 \qquad (2)$$

$$V_1\left(\frac{1}{sL} + sC_1\right) = V_0(sC_1 + g_m)$$

$$V_1 = \frac{V_0(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1\right)}$$

Then

$$V_0\left[s(C_1 + C_2) + \frac{1}{R_L} + g_m\right] = \frac{V_0(sC_1)(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1\right)}$$

$$\left[s(C_1 + C_2) + \frac{1}{R_L} + g_m\right]\left(\frac{1}{sL} + sC_1\right) = sC_1(sC_1 + g_m)$$

$$\frac{C_1 + C_2}{L} + s^2C_1(C_1 + C_2) + \frac{1}{sR_LL} + \frac{sC_1}{R_L} + sg_mC_1 + \frac{g_m}{sL} = s^2C_1^2 + sg_mC_1$$

$$\frac{C_1 + C_2}{L} + s^2C_1C_2 + \frac{1}{sR_LL} + \frac{sC_1}{R_L} + \frac{g_m}{sL} = 0$$

Set $s = j\omega$

$$\frac{C_1 + C_2}{L} - \omega^2C_1C_2 + \frac{1}{j\omega R_LL} + \frac{j\omega C_1}{R_L} + \frac{g_m}{j\omega L} = 0$$

Then

$$\omega^2 = \frac{C_1 + C_2}{C_1C_2L} \Rightarrow \omega_0 = \sqrt{\frac{C_1 + C_2}{C_1C_2L}}$$

and

$$\frac{g_m}{\omega L} + \frac{1}{\omega R_LL} = \frac{\omega C_1}{R_L}$$

Then

$$\frac{g_m}{L} + \frac{1}{R_LL} = \frac{(C_1 + C_2)C_1}{C_1C_2LR_L}$$

$$g_m + \frac{1}{R_L} = \frac{C_1 + C_2}{C_2R_L}$$

$$g_mR_L + 1 = \frac{C_1}{C_2} + 1 \text{ or } \underline{\frac{C_1}{C_2} = g_mR_L}$$

15.30

a.

$$\frac{V_0}{sL_1} + \frac{V_0}{R} + g_mV_\pi + \frac{V_0}{\frac{1}{sC} + sL_2} = 0 \qquad (1)$$

$$V_\pi = \left(\frac{sL_2}{\frac{1}{sC} + sL_2}\right)V_0 \qquad (2)$$

Then

$$V_0\left\{\frac{1}{sL_1} + \frac{1}{R} + \frac{sC}{1 + s^2L_2C} + \frac{g_m(s^2L_2C)}{1 + s^2L_2C}\right\} = 0$$

$$\left\{\frac{R(1 + s^2L_2C) + (sL_1)(1 + s^2L_2C)}{(sRL_1)(1 + s^2L_2C)} + \frac{s^2RL_1C + g_m(sRL_1)(s^2L_2C)}{(sRL_1)(1 + s^2L_2C)}\right\} = 0$$

Set $s = j\omega$. Both real and imaginary parts of the numerator must be zero.

$$R(1 - \omega^2L_2C) + j\omega L_1(1 - \omega^2L_2C) - \omega^2RL_1C + (j\omega g_mRL_1)(-\omega^2L_2C) = 0$$

Real part

$$R(1 - \omega^2L_2C) - \omega^2RL_1C = 0$$

$$R = \omega^2RC(L_1 + L_2)$$

or

$$\underline{\omega_0 = \frac{1}{\sqrt{C(L_1 + L_2)}}}$$

b. Imaginary part

$$j\omega L_1(1 - \omega^2L_2C) - j\omega g_mRL_1(\omega^2L_2C) = 0$$

$$L_1 = \omega^2L_1L_2C + g_mRL_1(\omega^2L_2C)$$

$$\text{Now } \omega^2 = \frac{1}{(L_1 + L_2)}$$

$$1 = \frac{1}{C(L_1 + L_2)}[L_2C + g_mRL_2C]$$

$$1 = \frac{L_2}{L_1 + L_2}(1 + g_mR) \Rightarrow \frac{L_1}{L_2} = (1 + g_mR) - 1$$

or

$$\underline{\frac{L_1}{L_2} = g_mR}$$

15.31

$$\omega_0 = 2\pi(800 \times 10^3) = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

or

$$C(L_1 + L_2) = 3.96 \times 10^{-14}$$

Also $\frac{L_1}{L_2} = g_m R$

For example, if $R = 1\text{ k}\Omega$, then $\frac{L_1}{L_2} = (20)(1) = 20$

So

$$L_1 = 20L_2$$

Then

$C(21L_2) = 3.96 \times 10^{-14}$ or $CL_2 = 1.89 \times 10^{-15}$

If $\underline{C = 0.01\ \mu\text{F}}$

then $\underline{L_2 = 0.189\ \mu\text{H}}$

and $\underline{L_1 = 3.78\ \mu\text{H}}$

15.32

$$\frac{v_0 - v_1}{\left(\frac{1}{sC}\right)} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \qquad (1)$$

and

$$\frac{v_B}{\left(\frac{1}{sC}\right)} + \frac{v_B - v_1}{R} = 0 \qquad (2)$$

or

$$v_B\left(sC + \frac{1}{R}\right) = \frac{v_1}{R} \Rightarrow v_1 = v_B(1 + sRC)$$

From (1)

$$v_0(sC) = v_1\left(sC + \frac{2}{R}\right) - \frac{v_B}{R}$$

or

$$v_0(sRC) = v_B(1 + sRC)(2 + sRC) - v_B$$
$$= v_B[(1 + sRC)(2 + sRC) - 1]$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{(1 + sRC)(2 + sRC) - 1}\right]$$
$$= \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{2 + 3sRC + s^2R^2C^2 - 1}\right]$$

or

$$\underline{T(s) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{s^2R^2C^2 + 3sRC + 1}\right]}$$

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{j\omega RC}{1 - \omega^2R^2C^2 + 3j\omega RC}\right]$$

Frequency of oscillation:

$$\underline{f_0 = \frac{1}{2\pi RC}}$$

Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{j\omega RC}{3j\omega RC}\right]$$

or

$$\underline{\frac{R_2}{R_1} = 2}$$

15.33

$$\frac{v_0 - v_1}{sL} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \qquad (1)$$

$$v_B = \left(\frac{sL}{R + sL}\right)v_1 \qquad (2)$$

or

$$v_1 = \left(\frac{R + sL}{sL}\right)v_B$$

Then

$$\frac{v_0}{sL} = v_1\left(\frac{1}{sL} + \frac{2}{R}\right) - \frac{v_B}{R}$$

or

$$\frac{v_0}{sL} = \left(\frac{R + sL}{sL}\right)\left(\frac{1}{sL} + \frac{2}{R}\right)v_B - \frac{v_B}{R} \qquad (1)$$
$$= v_B\left\{\left(\frac{R + sL}{sL}\right)\left(\frac{R + 2sL}{sRL}\right) - \frac{1}{R}\right\}$$

Then

$$v_B = \frac{v_0}{sL} \cdot \frac{1}{\left\{\frac{(R + sL)(R + 2sL) - (sL)^2}{(sL)(sRL)}\right\}}$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{sRL}{R^2 + 3sRL + 2s^2L^2 - s^2L^2}\right)$$

or

$$\underline{T(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{sRL}{s^2L^2 + 3sRL + R^2}\right)}$$

And

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{j\omega RL}{R^2 - \omega^2L^2 + 3j\omega RL}\right)$$

Frequency of oscillation: $\underline{f_0 = \frac{R}{2\pi L}}$

Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3}\right)$$

or

$$\underline{\frac{R_2}{R_1} = 2}$$

15.34

From Equation (15.65(b)), the crossover voltage is

$$v_I = -\frac{R_2}{R_1}\cdot V_{REF}$$

Let $R_2 = R_{VAR} + R_F$ where R_{VAR} is the potentiometer and R_F is the fixed resistor.

Let $\underline{V_{REF} = -5\text{ V}}$, $\underline{R_F = 10\text{ k}\Omega}$, and $\underline{R_{VAR} = 40\text{ k}\Omega}$

Then we have

$$v_I = -\frac{R_F}{R_1}\cdot V_{REF} = -\left(\frac{10}{50}\right)(-5) = 1\text{ V}$$

and

$$v_I = -\left(\frac{50}{50}\right)(-5) = 5\text{ V}$$

15.35

$$i_{max} = \frac{10}{R_1 + R_2} = 0.1 \Rightarrow R_1 + R_2 = 100\text{ k}\Omega$$

Hysteresis width

$$\Delta V = V_{TH} - V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)(V_H - V_L)$$

or $0.1 = \left(\frac{R_1}{100}\right)(20)$

so that

$$\underline{R_1 = 0.5\text{ k}\Omega}$$

$$\underline{R_2 = 99.5\text{ k}\Omega}$$

15.36

a. $V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)V_H = \left(\frac{10}{10+40}\right)(10)$

so $\underline{V_{TH} = 2\text{ V}}$

$V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)V_L = \left(\frac{10}{10+40}\right)(-10)$

so $\underline{V_{TL} = -2\text{ V}}$

b. $v_I = 5\sin\omega t$

15.37

a. Upper crossover voltage when $v_0 = +V_P$.

Now

$$v_B = \left(\frac{R_1}{R_1 + R_2}\right)(+V_P)$$

and

$$v_A = \left(\frac{R_A}{R_A + R_B}\right)V_{REF} + \left(\frac{R_B}{R_A + R_B}\right)V_{TH}$$

$v_A = v_B$ so that

$$\left(\frac{R_1}{R_1 + R_2}\right)V_P = \left(\frac{R_A}{R_A + R_B}\right)V_{REF} + \left(\frac{R_B}{R_A + R_B}\right)V_{TH}$$

or

$$V_{TH} = \left(\frac{R_A + R_B}{R_1 + R_2}\right)\left(\frac{R_1}{R_B}\right)V_P - \left(\frac{R_A}{R_B}\right)V_{REF}$$

Lower crossover voltage when $v_0 = -V_P$

So

$$V_{TL} = -\left(\frac{R_A + R_B}{R_1 + R_2}\right)\left(\frac{R_1}{R_B}\right)V_P - \left(\frac{R_A}{R_B}\right)V_{REF}$$

b. $V_{TH} = \left(\frac{10+20}{5+20}\right)\left(\frac{5}{20}\right)(10) - \left(\frac{10}{20}\right)(2)$

or $\underline{V_{TH} = 2\text{ V}}$

and

$$V_{TL} = -\left(\frac{10+20}{5+20}\right)\left(\frac{5}{20}\right)(10) - 1 \Rightarrow \underline{V_{TL} = -4\text{ V}}$$

15.38

a. $\frac{v_B}{R_1} = \frac{V_{REF} - v_B}{R_3} + \frac{v_0 - v_B}{R_2}$

$$v_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_{REF}}{R_3} + \frac{v_0}{R_2}$$

$V_{TH} = v_B$ when $v_0 = +V_P$ and $V_{TL} = v_B$ when $v_0 = -V_P$

So

$$V_{TH} = \frac{\frac{V_{REF}}{R_3} + \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

and

$$V_{TL} = \frac{\frac{V_{REF}}{R_3} - \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

b.

$$V_S = \frac{V_{REF}}{R_3\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$-5 = \frac{-10}{10\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{10}\right)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} - \frac{1}{10} = 0.10$$

$$\Delta V_T = V_{TH} - V_{TL} = \frac{\frac{2V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$0.2 = \frac{2(12)}{R_2(0.10 + 0.10)}$$

So $\underline{R_2 = 600\ \text{k}\Omega}$

Then

$$\frac{1}{R_1} + \frac{1}{R_2} = 0.10$$
$$\frac{1}{R_1} + \frac{1}{600} = 0.10 \Rightarrow \underline{R_1 = 10.17\ \text{k}\Omega}$$

c. $V_{TH} = -5 + 0.1 = -4.9$

$V_{TL} = -5 - 0.1 = -5.1$

15.39

a. If the saturated output voltage is $|V_P| < 6.2$ V, then the circuit behaves as a comparator

where $|v_0| < 6.2$ V.

If the saturated output voltage is $|V_P| > 6.2$ V, the output will flip to either $+V_P$ or $-V_P$ and the input has no control.

b. Same as part (a) except the curve at $v_I \approx 0$ will have a finite slope.

c.

Circuit works as a comparator as long as $v_{01} < 8.7$ V and $v_{02} > -3.7$ V. Otherwise the input has no control.

15.40

a. Switching point is when $v_0 = 0$. Then

$$v_+ = v_I \equiv V_S = \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

V_{TH} occurs when $v_0 = V_H$, then by superposition

$$v_+ = V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right) V_H + \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

or

$$V_{TH} = V_S + \left(\frac{R_1}{R_1 + R_2}\right) V_H$$

V_{TL} occurs when $v_0 = V_L$, then by superposition

$$v_+ = V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right) V_L + \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

or

$$V_{TL} = V_S + \left(\frac{R_1}{R_1 + R_2}\right) V_L$$

b. For $V_{TH} = 2$ V and $V_{TL} = 1$ V, then $V_S = 1.5$ V

Now

$$2 = 1.5 + \left(\frac{10}{10 + R_2}\right)(10)$$
$$\frac{0.5}{10} = \frac{10}{10 + R_2} \Rightarrow \underline{R_2 = 190\ \text{k}\Omega}$$

Now $V_S = 1.5 = \left(\frac{190}{10 + 190}\right) V_{REF}$

so

$$\underline{V_{REF} = 1.579\ \text{V}}$$

15.41

a. Switching point when $v_0 = 0$.

Now

$$v_+ = V_{REF} = \left(\frac{R_2}{R_1 + R_2}\right) v_I \text{ where } v_I = V_S.$$

Then

$$V_S = \left(\frac{R_1 + R_2}{R_2}\right) V_{REF} = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$

Now upper crossover voltage for v_I occurs when $v_0 = V_L$ and $v_+ = V_{REF}$. Then

$$\frac{V_{TH} - V_{REF}}{R_1} = \frac{V_{REF} - V_L}{R_2}$$
$$\text{or } V_{TH} = -\frac{R_1}{R_2} \cdot V_L + V_{REF}\left(1 + \frac{R_1}{R_2}\right)$$
$$\text{or } V_{TH} = V_S - \frac{R_1}{R_2} \cdot V_L$$

Lower crossover voltage for v_I occurs when $v_0 = V_H$ and $v_I = V_{REF}$. Then

$$\frac{V_H - V_{REF}}{R_2} = \frac{V_{REF} - V_{TL}}{R_1}$$
$$\text{or } V_{TL} = -\frac{R_1}{R_2} \cdot V_H + V_{REF}\left(1 + \frac{R_1}{R_2}\right)$$
$$\text{or } V_{TL} = V_S - \frac{R_1}{R_2} \cdot V_H$$

b. For $V_{TH} = -1$ and $V_{TL} = -2$, $V_S = -1.5$ V.

Then $V_{TL} = V_S - \frac{R_1}{R_2} \cdot V_H \Rightarrow -2 = -1.5 - \frac{R_1}{20}(12)$
so that $\underline{R_1 = 0.833 \text{ k}\Omega}$

Now

$$V_S = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$
$$-1.5 = \left(1 + \frac{0.833}{20}\right) V_{REF}$$
which gives
$$\underline{V_{REF} = -1.44 \text{ V}}$$

15.42

a. $V_H = 5.6 + 0.7 = 6.3$ V and $V_L = -6.3$ V

From Equation (15.72(b)),
$$V_{TH} = -\left(\frac{R_1}{R_2}\right) V_L$$
and from Equation (15.75(b)),
$$V_{TL} = -\left(\frac{R_1}{R_2}\right) V_H$$

$$V_{TH} - V_{TL} = 1 = -\left(\frac{R_1}{R_2}\right)(V_L - V_H)$$
$$= +\left(\frac{R_1}{R_2}\right)(2)(6.3)$$
so $\frac{R_1}{R_2} = 0.07937$
Then $R_2 = \frac{1}{0.07937} \Rightarrow \underline{R_2 = 12.6 \text{ k}\Omega}$

Now $V_{TH} = -(0.07937)(-6.3) = 0.5$,
$V_{TL} = -0.5$

b. For v_0 high, we have

$I = I_D + I_R$, I is fixed for a given R.
Assume v_I varies between ± 12 V

$$I_R(\text{max}) = \frac{6.3 - (-12)}{13.6} = 1.35 \text{ mA}$$
$$I_R(\text{min}) = \frac{6.3 - 0.5}{13.6} = 0.426 \text{ mA}$$
$$I_R(\text{avg}) = \frac{1.35 + 0.426}{2} = 0.888 \text{ mA}$$

Then we want

$$I = I_D(\text{avg}) + I_R(\text{avg}) = 1 + 0.888 = 1.888 \text{ mA}$$
Then
$$R = \frac{12 - 6.3}{1.888} \Rightarrow \underline{R = 3.02 \text{ k}\Omega}$$

15.43

a. $v_0 = V_{REF} + 2V_\gamma$

$5 = V_{REF} + 2(0.7)$
or
$$\underline{V_{REF} = 3.6 \text{ V}}$$

b. $V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_\gamma)$

$$0.5 = \left(\frac{R_1}{R_1 + R_2}\right)(5)$$
or $1 + \frac{R_2}{R_1} = 10 \Rightarrow \frac{R_2}{R_1} = 9$
For example, let $R_2 = 90$ kΩ and $R_1 = 10$ kΩ

c. For $v_I = 10$ V, and v_0 is in its low state. D_1 is on and D_2 is off.

$$\frac{v_I - (v_1 + 0.7)}{100} + \frac{V_{REF} - v_1}{1} = \frac{v_1 - v_0}{1}$$
For $v_1 = -0.7$, then
$$\frac{10 - 0}{100} + \frac{3.6 - (-0.7)}{1} = \frac{-0.7 - v_0}{1}$$
or
$$\underline{v_0 = -5.1 \text{ V}}$$

15.44

For $v_0 = \text{High} = (V_{REF} + 2V_\gamma)$. Then switching point is when.

$$v_I = v_B = \left(\frac{R_1}{R_1 + R_2}\right) v_0$$
or $V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_\gamma)$

Lower switching point is when

$$v_1 = v_B = \left(\frac{R_1}{R_1 + R_2}\right) v_0 \text{ and } v_0 = -(V_{REF} + 2V_\gamma)$$
so
$$V_{TL} = -\left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_\gamma)$$

15.45

By symmetry, inverting terminal switches about zero.

Now, for v_0 low, upper diode is on.

$V_{REF} - v_1 = v_1 - v_0$
$v_0 = 2v_1 - V_{REF}$ where $v_1 = -V_\gamma$
so
$$\underline{v_0 = -(V_{REF} + 2V_\gamma)}$$

Similarly, in the high state
$$\underline{v_0 = (V_{REF} + 2V_\gamma)}$$

Switching occurs when non-inverting terminal is zero.

So for v_0 low,

$$\frac{V_{TH}-0}{R_1}=\frac{0-[-(V_{REF}+2V_\gamma)]}{R_2}$$

or $V_{TH}=\frac{R_1}{R_2}\cdot(V_{REF}+2V_\gamma)$

By symmetry

$$V_{TL}=-\frac{R_1}{R_2}\cdot(V_{REF}+2V_\gamma)$$

15.46

$f_0 = 5$ kHz and 50% duty cycle.

From Equation (15.88)

$$f=\frac{1}{2.2R_XC_X}$$

so

$$R_XC_X=\frac{1}{2.2(5\times10^3)}=9.09\times10^{-5}$$

Let $C_X = 0.01\ \mu$F. Then $R_X = 9.09$ kΩ.

Also let $R_1 = R_2 = 10$ kΩ

15.47

Switching point occurs when

$$v_X=\left(\frac{R_1}{R_1+R_2}\right)V_P=\left(\frac{30}{30+10}\right)(10)$$

$$\Rightarrow v_X=\pm7.5\text{ V}$$

b. Duty cycle = 50%

From Equation (15.83(a)), we can write

$$v_X=V_P+\left(-\frac{3}{4}V_P-V_P\right)e^{-t/\tau_X}$$

At time $t=t_1$ (one-half period) $v_X=\frac{3}{4}\cdot V_P$

So

$$\frac{3}{4}\cdot V_P=V_P-\frac{7}{4}\cdot V_Pe^{-t_1/\tau_X}$$

$$1=7e^{-t_1/\tau_X}$$

or $t_1=\tau_X\ln(7)$

One period is $T=2\tau_X\ln(7)=3.89\tau_X$ or the frequency is

$$f=\frac{1}{3.89R_XC_X}$$

Then

$$f=\frac{1}{(3.89)(10^4)(0.1\times10^{-6})}\Rightarrow \underline{f=257\text{ Hz}}$$

15.48

Only change from Problem (15.47) is that maximum output is ±15 V and the v_X switching voltages are ±11.25 V.

15.49

$$t_1=1.1R_XC_X=(1.1)(10^4)(0.1\times10^{-6})$$

$$\Rightarrow t_1=1.1\text{ ms}$$

$$0<t<t_1,\ v_Y=10\left(1-e^{-t/\tau_Y}\right)$$

$$\tau_Y=R_YC_Y=(2\times10^3)(0.02\times10^{-6})$$

$$=4\times10^{-5}\text{ s}$$

Now $\frac{t_1}{\tau_Y}=2.75$

$\Rightarrow C_Y$ completely charges during each cycle.

15.50

a. Switching voltage

$$v_X=\left(\frac{R_1+R_3}{R_1+R_3+R_2}\right)\cdot V_P=\left(\frac{10+10}{10+10+10}\right)(\pm10)$$

So $v_X=\pm6.667$ V

Using Equation (15.83(b))

$$v_X=V_P+\left(-\frac{2}{3}V_P-V_P\right)e^{-t_1/\tau_X}=\frac{2}{3}V_P$$

Then $1-\frac{5}{3}\cdot e^{-t_1/\tau_X}=\frac{2}{3}$

$\frac{1}{3}=\frac{5}{3}\cdot e^{-t_1/\tau_X}$ or $t_1=\tau_X\ln(5)$

$$t_1=\frac{T}{2}=\frac{1}{2f}=\frac{1}{2(500)}\Rightarrow t_1=0.001\text{ s}$$

$$10^{-3}=\tau_X\ln(5)\Rightarrow\tau_X=6.21\times10^{-4}$$

$$=R_X(0.01\times10^{-6})$$

So $\underline{R_X=62.1\text{ k}\Omega}$

b. Switching voltage

$$v_X=\left(\frac{R_1}{R_1+R_3+R_2}\right)(\pm V_P)$$

$$=\left(\frac{10}{10+10+10}\right)(\pm V_P)=\frac{1}{3}\cdot(\pm V_P)$$

Using Equation (15.83(b))

$$v_X=V_P+\left(-\frac{1}{3}V_P-V_P\right)e^{-t_1/\tau_X}=\frac{1}{3}V_P$$

Then $1-\frac{4}{3}\cdot e^{-t_1/\tau_X}=\frac{1}{3}$

$\frac{2}{3}=\frac{4}{3}\cdot e^{-t_1/\tau_X}$

$t_1=\tau_X\ln(2)=(6.21\times10^{-4})\ln(2)=4.30\times10^{-4}$ s

$T=2t_1=8.6\times10^{-4}$ s

$f=\frac{1}{T}\Rightarrow \underline{f=1.16\text{ kHz}}$

15.51

From Equation (15.92)

$$T = \tau_X \ln\left(\frac{1+\left(\frac{V_\gamma}{V_P}\right)}{1-\beta}\right)$$

where $\beta = \dfrac{R_1}{R_1+R_2} = \dfrac{10}{10+25} = 0.2857$

so

$$100 = \tau_X \ln\left[\frac{1+\frac{0.7}{5}}{1-0.2857}\right]$$

so $\tau_X = 213.9\ \mu s = R_X C_X$

For example, $R_X = 10\ k\Omega$, $C_X = 0.0214\ \mu F$

$$v_Y = \left(\frac{R_1}{R_1+R_2}\right)V_P = \left(\frac{10}{10+25}\right)(5) = 1.43\ V$$

and $v_X = 0.7\ V$

To trigger the circuit, v_Y must be brought to a voltage less than v_X.

Therefore minimum triggering pulse is -0.73 V.

Using Equation (15.82) for $T < t < T'$

$v_X = V_P + (-0.2857V_P - V_P)e^{-t'/\tau_X}$

Recovery period is when $v_X = V_\gamma = 0.7$ V.

$0.7 = 5 + (-6.43)e^{-t'/\tau_X}$

$6.43e^{-t'/\tau_X} = 4.3$

or $t' = \tau_X \ln(1.495)$

$\tau_X = 213.9\ \mu s$

so

$$\underline{t' = T' - T = 86.1\ \mu s}$$

15.52

From Equation (15.92), the pulse width

$$T = \tau_X \ln\left(\frac{1+\left(\frac{V_\gamma}{V_P}\right)}{1-\beta}\right)$$

where $\beta = \dfrac{R_1}{R_1+R_2} = \dfrac{20}{20+20} = 0.5$

$\tau_X = R_X C_X = (50 \times 10^3)(0.1 \times 10^{-6}) = 5$ ms

So $T = 5 \ln\left[\dfrac{1+\frac{0.7}{10}}{1-0.5}\right] \Rightarrow \underline{T = 3.80\ ms}$

Recovery time $\approx 0.4\tau_X = \underline{2\ ms}$

15.53

a. From Equation (15.95)

$T = 1.1RC$

For $T = 60$ s $= 1.1$RC

then $RC = 54.55$ s

For example, let

$\underline{C = 50\ \mu F}$ and $\underline{R = 1.09\ M\Omega}$

b. Recovery time: capacitor is discharged by current through the discharge transistor.

If $V^+ = 5$ V, then $I_B \cong \dfrac{5-0.7}{100} = 0.043$ mA

If $\beta = 100$, $I_C = 4.3$ mA

$V_C = \dfrac{1}{C}\displaystyle\int I_C\,dt = \dfrac{I_C}{C}\cdot t$

Capacitor has charged to $\dfrac{2}{3}\cdot V^+ = 3.33$ V

So that $t = \dfrac{V_C \cdot C}{I_C} = \dfrac{(3.33)(50\times 10^{-6})}{4.3\times 10^{-3}}$

So recovery time $\underline{t \approx 38.7\ ms}$

15.54

$T = 1.1RC$

$5 \times 10^{-6} = 1.1RC$

so $RC = 4.545 \times 10^{-6}$ s

For example, let

$\underline{C = 100\ pF}$ and $\underline{R = 45.5\ k\Omega}$

From Problem (15.53), recovery time

$$t \cong \frac{V_C \cdot C}{I_C} = \frac{(3.33)(100\times 10^{-12})}{4.3\times 10^{-3}}$$

or

$$\underline{t = 77.4\ ns}$$

15.55

From Equation (15.102),

$$f = \frac{1}{(0.693)(20+2(20))\times 10^3 \times (0.1\times 10^{-6})}$$

or $\underline{f = 240.5\ Hz}$

Duty cycle $= \dfrac{20+20}{20+2(20)} \times 100\% = \underline{66.7\%}$

15.56

$$f = \frac{1}{(0.693)(R_A + 2R_B)C}$$

$R_A = R_1 = 10\ k\Omega$, $R_B = R_2 + xR_3$

So $10\ k\Omega \le R_B \le 110\ k\Omega$

$$f_{min} = \frac{1}{(0.693)(10+2(110))\times 10^3 \times (0.01\times 10^{-6})}$$
$$= 627 \text{ Hz}$$
$$f_{max} = \frac{1}{(0.693)(10+2(10))\times 10^3 \times (0.01\times 10^{-6})}$$
$$= 4.81 \text{ kHz}$$

So $627 \text{ Hz} \le f \le 4.81 \text{ kHz}$

$$\text{Duty cycle} = \frac{R_A+R_B}{R_A+2R_B}\times 100\%$$

Now

$$\frac{10+10}{10+2(10)}\times 100\% = \underline{66.7\%}$$

and

$$\frac{10+110}{10+2(110)}\times 100\% = \underline{52.2\%}$$

So $52.2 \le$ Duty cycle $\le 66.7\%$

15.57

$1\text{ k}\Omega \le R_A \le 51\text{ k}\Omega$

$1\text{ k}\Omega \le R_B \le 51\text{ k}\Omega$

$$f_{min} = \frac{1}{(0.693)(1+2(51))\times 10^3 \times (0.01\times 10^{-6})}$$
$$= 1.40 \text{ kHz}$$
$$f_{max} = \frac{1}{(0.693)(51+2(1))\times 10^3 \times (0.01\times 10^{-6})}$$
$$= 2.72 \text{ kHz}$$

or $1.40 \text{ kHz} \le f \le 2.72 \text{ kHz}$

$$\text{Duty cycle} = \frac{R_A+R_B}{R_A+2R_B}\times 100\%$$

$$\frac{1+51}{1+2(51)}\times 100\% = \underline{50.5\%}$$

or

$$\frac{51+1}{51+2(1)}\times 100\% = \underline{98.1\%}$$

or $\underline{50.5\% \le \text{Duty cycle} \le 98.1\%}$

15.58

a. $I_{E3} = I_{E4} = \dfrac{V^+ - 3V_{EB}}{R_{1A}+R_{1B}}$

Assume $V_{EB} = 0.7$

$$I_{E3} = I_{E4} = \frac{22-3(0.7)}{25+25} = 0.398 \text{ mA}$$

Now

$$I_{C3} = I_{C4} = I_{C5} = I_{C6} = \left(\frac{20}{21}\right)(0.398)$$
$$\underline{I_{C3} = I_{C4} = I_{C5} = I_{C6} = 0.379 \text{ mA}}$$
$$I_{C1} = I_{C2} = \frac{0.398}{21}\left(\frac{20}{21}\right) \Rightarrow \underline{I_{C1} = I_{C2} = 0.018 \text{ mA}}$$

b. $\underline{I_D = 0.398 \text{ mA}}$, current in D_1 and D_2

$$V_{BB} = 2V_D = 2V_T \ln\left(\frac{I_D}{I_S}\right)$$
$$= 2(0.026)\ln\left(\frac{0.398\times 10^{-3}}{10^{-13}}\right)$$

or $V_{BB} = 1.149 \text{ V} = V_{BE7} + V_{EB8}$

Now

$$I_{C7} \approx I_{C4} + I_{C9} + I_{E8}$$
$$I_{C4} = 0.379 \text{ mA}$$
$$I_{B9} = I_{C8} = \left(\frac{20}{21}\right) I_{E8}$$

So

$$I_{E8} = 1.05 I_{B9} = 1.05\left(\frac{I_{C9}}{100}\right)$$
$$I_{C7} = I_{C4} + \left(\frac{100}{1.05}\right) I_{E8} + I_{E8}$$
$$= I_{C4} + (96.24)\left(\frac{21}{20}\right) I_{C8}$$

So $I_{C7} = 0.379 \text{ mA} + 101 I_{C8}$

and

$$V_{BE7} = V_T \ln\left(\frac{I_{C7}}{I_S}\right);\ V_{EB8} = V_T \ln\left(\frac{I_{C8}}{I_S}\right)$$

Then

$$1.149 = 0.026\left[\ln\left(\frac{I_{C7}}{I_S}\right) + \ln\left(\frac{I_{C8}}{I_S}\right)\right]$$
$$44.19 = \ln\left[\frac{I_{C8}(0.379\times 10^{-3}) + 101 I_{C8}}{(10^{-13})^2}\right]$$
$$(10^{-13})^2 \exp(44.19) = 101 I_{C8}^2 + 3.79\times 10^{-4} I_{C8}$$

$$1.554\times 10^{-7} = 101 I_{C8}^2 + 3.79\times 10^{-4} I_{C8}$$
$$I_{C8} = \frac{-3.79\times 10^{-4}}{2(101)} \pm \frac{\sqrt{(3.79\times 10^{-4})^2 + 4(101)(1.554\times 10^{-7})}}{2(101)}$$
$$\underline{I_{C8} = 37.4\ \mu\text{A}}$$

$$I_{C7} = 0.379 + 101(0.0374) \Rightarrow \underline{I_{C7} = 4.16 \text{ mA}}$$
$$I_{C9} = 4.16 - 0.379 - 0.0374\left(\frac{21}{20}\right)$$
$$\underline{I_{C9} = 3.74 \text{ mA}}$$

c. $P = (0.398 + 0.398 + 4.16)(22) \Rightarrow \underline{P = 109 \text{ mW}}$

15.59

a. From Figure 15.47, 3.7 W to the load

b. $\underline{V^+ \approx 19\text{ V}}$

c. $\overline{P} = \frac{1}{2}\frac{V_P^2}{R_L}$

or

$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(10)(3.7)} \Rightarrow \underline{V_P = 8.6\text{ V}}$

15.60

$\overline{P} = \frac{1}{2}\frac{V_P^2}{R_L}$

so $V_P = \sqrt{2R\overline{P}} = \sqrt{2(10)(20)} = 20\text{ V}$

peak-to-peak output voltage

Maximum output voltage of each op-amp $= \pm 10$ V. Current is $(20/10) = 2$ A. Bias op-amps at ± 12 V.

For A_1, $\frac{v_{01}}{v_I} = \left(1 + \frac{R_2}{R_1}\right) = 15 \Rightarrow \frac{R_2}{R_1} = 14$

For A_2, $\left|\frac{v_{02}}{v_I}\right| = \frac{R_4}{R_3} = 15$

For example, let $R_1 = R_3 = 10\text{ k}\Omega$, and $R_2 = 140\text{ k}\Omega$ and $R_4 = 150\text{ k}\Omega$.

15.61

a. $v_{01} = iR_2 + v_I$ where $i = \frac{v_I}{R_1}$

Then

$v_{01} = v_I\left(1 + \frac{R_2}{R_1}\right)$

Now

$v_{02} = -iR_3 = -v_I\left(\frac{R_3}{R_1}\right)$

So

$v_L = v_{01} - v_{02} = v_I\left(1 + \frac{R_2}{R_1}\right) - \left[-v_I\left(\frac{R_3}{R_1}\right)\right]$

$$\underline{A_v = \frac{v_L}{v_I} = 1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}}$$

b. Want $A_v = 10 \Rightarrow \frac{R_2}{R_1} + \frac{R_3}{R_1} = 9$

Also want $\left(1 + \frac{R_2}{R_1}\right) = \frac{R_3}{R_1}$

Then $\frac{R_2}{R_1} + \left(1 + \frac{R_2}{R_1}\right) = 9$ so $\frac{R_2}{R_1} = 4$

For $R_1 = 50\text{ k}\Omega$, $\underline{R_2 = 200\text{ k}\Omega}$

and

$\frac{R_3}{R_1} = 5$ so $\underline{R_3 = 250\text{ k}\Omega}$

c. $\overline{P} = \frac{1}{2}\frac{V_P^2}{R_L}$

or

$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(20)(10)} = 20\text{ V}$

So peak values of output voltages are

$|v_{01}| = |v_{02}| = 10\text{ V}$

Peak load current $= \frac{20}{20} = 1\text{ A}$

15.62

a. $v_{01} = \left(1 + \frac{R_2}{R_1}\right)v_I$

$v_{02} = -\frac{R_4}{R_3}\cdot v_{01} = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)v_I$

$v_L = v_{01} - v_{02} = \left(1 + \frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right)v_I$

so

$$\underline{A_v = \left(1 + \frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right)}$$

b. Want $\left(1 + \frac{R_2}{R_1}\right)v_I = |v_{01}| = |v_{02}| \Rightarrow \underline{R_4 = R_3}$

Then

$\left(1 + \frac{R_2}{R_1}\right)(2) = 15 \Rightarrow \underline{\frac{R_2}{R_1} = 6.5}$

c. $\overline{P} = \frac{1}{2}\cdot\frac{V_P^2}{R_L}$

or

$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(8)(50)} = 28.3\text{ V}$

= peak-to-peak load voltage

Then

$\underline{|v_{01}| = |v_{02}| = 14.15\text{ V}}$

Load current $= \frac{28.3}{8} = \underline{3.54\text{ A}}$

15.63

Line regulation $= \frac{\Delta V_0}{\Delta V+}$

Now

$\Delta I = \frac{\Delta V^+}{R_1}$ and $\Delta V_Z = r_Z \cdot \Delta I$ and $\Delta V_0 = 10\Delta V_Z$

So

$\Delta V_0 = 10 \cdot r_Z \cdot \frac{\Delta V^+}{R_1}$

So

Line regulation $= \frac{\Delta V_0}{\Delta V^+} = \frac{10(15)}{9300}$

$\Rightarrow 1.61\%$

15.64

$$R_{of} = -\frac{\Delta V_0}{\Delta I_0}$$

So $R_{of} = \dfrac{-(-10 \times 10^{-3})}{1}$

or

$$\underline{R_{of} = 10\ \text{m}\Omega}$$

15.65

For $V_0 = 8$ V

$$V^+(\min) = V_0 + I_0(\max)R_{11} + V_{BE11} + V_{BE10} + V_{EB5}$$

This assumes $V_{BC5} = 0$.

Then

$$V^+(\min) = 8 + (0.1)(1.9) + 0.6 + 0.6 + 0.6$$

$$\underline{V^+(\min) = 9.99\ \text{V}}$$

15.66

a. $I_{C3} = I_{C5} = \dfrac{V_Z - 3V_{BE}(\text{npn})}{R_1 + R_2 + R_3}$

$$I_{C3} = I_{C5} = \frac{6.3 - 3(0.6)}{0.576 + 3.4 + 3.9} = 0.571\ \text{mA}$$

$$I_{C8} = \frac{1}{2}\left(\frac{0.6}{2.84}\right) = 0.106\ \text{mA}$$

Neglecting current in Q_9, total collector current and emitter current in Q_5 is

$$0.571 + 0.106 = 0.677$$

Now

$$I_{Z2}R_4 + V_{EB4} = V_{EB5}$$

$$V_{EB4} = V_T \ln\left(\frac{I_{Z2}}{I_S}\right)$$

$$V_{EB5} = V_T \ln\left(\frac{I_{C5}}{2I_S}\right)$$

Then $I_{Z2}R_4 = V_T \ln\left(\dfrac{I_{C5}}{2I_{Z2}}\right)$

$$R_4 = \frac{0.026}{0.25} \cdot \ln\left(\frac{0.677}{2(0.25)}\right)$$

or

$$\underline{R_4 = 31.5\ \Omega}$$

b. From Example 15.16, $V_{B7} = 3.43V$. Then

$$\left(\frac{R_{13}}{R_{12} + R_{13}}\right)V_0 = V_{B8} = V_{B7}$$

or

$$\left(\frac{2.23}{2.23 + R_{12}}\right)(12) = 3.43$$

$$3.43(2.23 + R_{12}) = (2.23)(12)$$

which yields

$$\underline{R_{12} = 5.57\ \text{k}\Omega}$$

15.67

Line regulation $= \dfrac{\Delta V_0}{\Delta V^+}$

Now

$$\Delta V_{B7} = \Delta I_{C3} \cdot R_1$$

and $\left(\dfrac{R_{13}}{R_{12} + R_{13}}\right)(\Delta V_0) = \Delta V_{B7} = \Delta I_{C3} R_1$

and $\Delta I_{C3} = \dfrac{\Delta V_Z}{R_1 + R_2 + R_3} = \dfrac{\Delta I_Z \cdot r_Z}{R_1 + R_2 + R_3}$

and $\Delta I_Z = \dfrac{\Delta V^+}{r_0}$ where $r_0 = \dfrac{V_A}{I_Z}$

Then

$$(0.4288)(\Delta V_0) = \Delta I_{C3}(3.9)$$

$$= (3.9)\Delta I_Z \left(\frac{0.015}{7.876}\right)$$

$$r_0 = \frac{50}{0.571} = 87.6\ \text{k}\Omega$$

Then

$$(0.4288)(\Delta V_0) = (0.00743)\left(\frac{\Delta V^+}{87.6}\right)$$

So

$$\underline{\frac{\Delta V_0}{\Delta V^+} = 0.0198\%}$$

15.68

a. $I_Z = \dfrac{25 - 5}{R_1 + r_Z} = 10$

So $R_1 + r_Z = \dfrac{20}{10} = 2\ \text{k}\Omega = R_1 + 0.01$

$$\Rightarrow \underline{R_1 = 1.99\ \text{k}\Omega}$$

b. In the ideal case;

$$\left(\frac{R_3 + R_4}{R_2 + R_3 + R_4}\right)V_0 = V_Z$$

$$\left(\frac{2 + 1}{2 + 1 + 1}\right)V_0 = 5 \Rightarrow V_0 = 6.67\ \text{V}$$

and

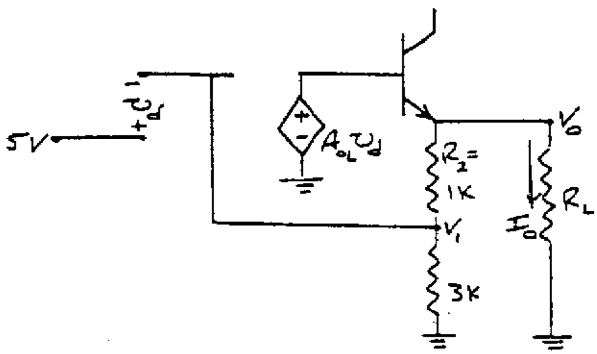
$$\left(\frac{R_4}{R_2 + R_3 + R_4}\right)V_0 = V_Z$$

$$\left(\frac{1}{2 + 1 + 1}\right)V_0 = 5 \Rightarrow V_0 = 20\ \text{V}$$

So

$$\underline{6.67 \le V_0 \le 20\ \text{V}}$$

c.



$V_1 = \frac{3}{4} \cdot V_0$ so $v_d = 5 - \frac{3}{4} \cdot V_0$

and $V_0 = A_{0L} v_d - V_{BE}$

and $V_{BE} = V_T \ln\left(\frac{I_0}{I_S}\right)$

Now

$$V_0 = A_{0L}\left(5 - \frac{3}{4} \cdot V_0\right) - V_{BE}$$

$$V_0\left(1 + \frac{3}{4} \cdot A_{0L}\right) = 5A_{0L} - V_{BE}$$

$$V_0 = \frac{5A_{0L} - V_{BE}}{1 + \frac{3}{4} \cdot A_{0L}}.$$

$$\text{Load regulation} = \frac{V_0(\text{NL}) - V_0(\text{FL})}{V_0(\text{NL})}$$

$$= \frac{\dfrac{5A_{0L} - V_{BE}(\text{NL})}{(1 + \frac{3}{4}A_{0L})} - \dfrac{5A_{0L} - V_{BE}(\text{FL})}{(1 + \frac{3}{4}A_{0L})}}{\dfrac{5A_{0L} - V_{BE}(\text{NL})}{(1 + \frac{3}{4}A_{0L})}}$$

$$= \frac{V_{BE}(\text{FL}) - V_{BE}(\text{NL})}{5A_{0L} - V_{BE}(\text{NL})} = \frac{V_T \ln\left(\frac{I_0(\text{FL})}{I_0(\text{NL})}\right)}{5A_{0L} - V_{BE}(\text{NL})}$$

$$I_0(\text{FL}) = 1 \text{ A}, \quad I_0(\text{NL}) = \frac{V_0}{4\text{ k}\Omega} = \frac{6.67}{4\text{ k}\Omega} = 1.67 \text{ mA}$$

$$\text{Load regulation} = \frac{(0.026)\ln\left(\frac{1}{1.67 \times 10^{-3}}\right)}{5(10^4) - 0.7}$$

$\Rightarrow \underline{3.33 \times 10^{-4}\%}$

15.69

$$I_E = \frac{V_Z}{R_2} = \frac{5.6}{5} = 1.12 \text{ mA}$$

$$I_0 = \frac{\beta}{1+\beta} \cdot I_E = \left(\frac{100}{101}\right)(1.12)$$

$\Rightarrow \underline{I_0 = 1.109 \text{ mA}}$ Load current

For

$$V_{BC} = 0 \Rightarrow V_0 = 20 - V_Z - 0.6$$

$$= 20 - 5.6 - 0.6$$

or

$$V_0 = 13.8 \text{ V}$$

Then

$$R_L = \frac{V_0}{I_0} = \frac{13.8}{1.109} \Rightarrow R_L = 12.4 \text{ k}\Omega$$

So

$\underline{0 \le R_L \le 12.4 \text{ k}\Omega}$