Chapter 9, Solution 1.

(a) angular frequency $\omega = \underline{\mathbf{10^3\ rad/s}}$

(b) frequency $f = \dfrac{\omega}{2\pi} = \underline{\mathbf{159.2\ Hz}}$

(c) period $T = \dfrac{1}{f} = \underline{\mathbf{6.283\ ms}}$

(d) Since $\sin(A) = \cos(A - 90°)$,

$v_s = 12\sin(10^3 t + 24°) = 12\cos(10^3 t + 24° - 90°)$

v_s in cosine form is $v_s = \underline{\mathbf{12\cos(10^3 t - 66°)\ V}}$

(e) $v_s(2.5\ \text{ms}) = 12\sin((10^3)(2.5\times10^{-3}) + 24°)$

$= 12\sin(2.5 + 24°) = 12\sin(143.24° + 24°)$

$= \underline{\mathbf{2.65\ V}}$

Chapter 9, Solution 2.

(a) amplitude $= \underline{\mathbf{8\ A}}$

(b) $\omega = 500\pi = \underline{\mathbf{1570.8\ rad/s}}$

(c) $f = \dfrac{\omega}{2\pi} = \underline{\mathbf{250\ Hz}}$

(d) $I_s = 8\angle\text{-}25°\ \text{A}$

$I_s(2\ \text{ms}) = 8\cos((500\pi)(2\times10^{-3}) - 25°)$

$= 8\cos(\pi - 25°) = 8\cos(155°)$

$= \underline{\mathbf{-7.25\ A}}$

Chapter 9, Solution 3.

(a) $4\sin(\omega t - 30°) = 4\cos(\omega t - 30° - 90°) = \underline{\mathbf{4\cos(\omega t - 120°)}}$

(b) $-2\sin(6t) = \underline{\mathbf{2\cos(6t + 90°)}}$

(c) $-10\sin(\omega t + 20°) = 10\cos(\omega t + 20° + 90°) = \underline{\mathbf{10\cos(\omega t + 110°)}}$

Chapter 9, Solution 4.

(a) $v = 8\cos(7t + 15°) = 8\sin(7t + 15° + 90°) = \underline{\mathbf{8\sin(7t + 105°)}}$

(b) $i = -10\sin(3t - 85°) = 10\cos(3t - 85° + 90°) = \underline{\mathbf{10\cos(3t + 5°)}}$

Chapter 9, Solution 5.

$v_1 = 20\sin(\omega t + 60°) = 20\cos(\omega t + 60° - 90°) = 20\cos(\omega t - 30°)$
$v_2 = 60\cos(\omega t - 10°)$

This indicates that the phase angle between the two signals is $\underline{\mathbf{20°}}$ and that $\underline{\mathbf{v_1\ lags\ v_2}}$.

Chapter 9, Solution 6.

(a) $v(t) = 10\cos(4t - 60°)$
$i(t) = 4\sin(4t + 50°) = 4\cos(4t + 50° - 90°) = 4\cos(4t - 40°)$
Thus, $\underline{\mathbf{i(t)\ leads\ v(t)\ by\ 20°}}$.

(b) $v_1(t) = 4\cos(377t + 10°)$
$v_2(t) = -20\cos(377t) = 20\cos(377t + 180°)$
Thus, $\underline{\mathbf{v_2(t)\ leads\ v_1(t)\ by\ 170°}}$.

(c) $x(t) = 13\cos(2t) + 5\sin(2t) = 13\cos(2t) + 5\cos(2t - 90°)$
$\mathbf{X} = 13\angle 0° + 5\angle -90° = 13 - j5 = 13.928\angle -21.04°$
$x(t) = 13.928\cos(2t - 21.04°)$
$y(t) = 15\cos(2t - 11.8°)$
phase difference $= -11.8° + 21.04° = 9.24°$
Thus, $\underline{\mathbf{y(t)\ leads\ x(t)\ by\ 9.24°}}$.

Chapter 9, Solution 7.

If $f(\phi) = \cos\phi + j\sin\phi$,

$$\frac{df}{d\phi} = -\sin\phi + j\cos\phi = j(\cos\phi + j\sin\phi) = jf(\phi)$$

$$\frac{df}{f} = j\,d\phi$$

Integrating both sides

$\ln f = j\phi + \ln A$

$f = Ae^{j\phi} = \cos\phi + j\sin\phi$

$f(0) = A = 1$

i.e. $\underline{\mathbf{f(\phi) = e^{j\phi} = \cos\phi + j\sin\phi}}$

Chapter 9, Solution 8.

(a)
$$\frac{15\angle 45°}{3 - j4} + j2 = \frac{15\angle 45°}{5\angle -53.13°} + j2$$
$$= 3\angle 98.13° + j2$$
$$= -0.4245 + j2.97 + j2$$
$$= \underline{\mathbf{-0.4243 + j4.97}}$$

(b) $(2 + j)(3 - j4) = 6 - j8 + j3 + 4 = 10 - j5 = 11.18\angle -26.57°$
$$\frac{8\angle -20°}{(2 + j)(3 - j4)} + \frac{10}{-5 + j12} = \frac{8\angle -20°}{11.18\angle -26.57°} + \frac{(-5 - j12)(10)}{25 + 144}$$
$$= 0.7156\angle 6.57° - 0.2958 - j0.71$$
$$= 0.7109 + j0.08188 - 0.2958 - j0.71$$
$$= \underline{\mathbf{0.4151 - j0.6281}}$$

(c) $10 + (8\angle 50°)(13\angle -68.38°) = 10 + 104\angle -17.38°$
$$= \underline{\mathbf{109.25 - j31.07}}$$

Chapter 9, Solution 9.

(a)
$$2 + \frac{3 + j4}{5 - j8} = 2 + \frac{(3 + j4)(5 + j8)}{25 + 64}$$
$$= 2 + \frac{15 + j24 + j20 - 32}{89}$$
$$= \underline{\mathbf{1.809 + j0.4944}}$$

(b)
$$4\angle -10° + \frac{1 - j2}{3\angle 6°} = 4\angle -10° + \frac{2.236\angle -63.43°}{3\angle 6°}$$

$$= 4\angle\text{-}10° + 0.7453\angle\text{-}69.43°$$
$$= 3.939 - j0.6946 + 0.2619 - j0.6978$$
$$= \underline{\mathbf{4.201 - j1.392}}$$

(c) $$\frac{8\angle 10° + 6\angle\text{-}20°}{9\angle 80° - 4\angle 50°} = \frac{7.879 + j1.3892 + 5.638 - j2.052}{1.5628 + j8.863 - 2.571 - j3.064}$$
$$= \frac{13.517 - j0.6629}{-1.0083 + j5.799} = \frac{13.533\angle\text{-}2.81°}{5.886\angle 99.86°}$$
$$= 2.299\angle\text{-}102.67°$$
$$= \underline{\mathbf{-0.5043 - j2.243}}$$

Chapter 9, Solution 10.

(a) $z_1 = 6 - j8,\ \ z_2 = 8.66 - j5,$ and $z_3 = -4 - j6.9282$

$z_1 + z_2 + z_3 = \underline{10.66 - j19.93}$

(b) $\frac{z_1 z_2}{z_3} = \underline{9.999 + j7.499}$

Chapter 9, Solution 11.

(a) $$z_1 z_2 = (-3 + j4)(12 + j5)$$
$$= -36 - j15 + j48 - 20$$
$$= \underline{\mathbf{-56 + j33}}$$

(b) $$\frac{z_1}{z_2^*} = \frac{-3 + j4}{12 - j5} = \frac{(-3 + j4)(12 + j5)}{144 + 25} = \underline{\mathbf{-0.3314 + j0.1953}}$$

(c) $$z_1 + z_2 = (-3 + j4) + (12 + j5) = 9 + j9$$
$$z_1 - z_2 = (-3 + j4) - (12 + j5) = -15 - j$$
$$\frac{z_1 + z_2}{z_1 - z_2} = \frac{9(1 + j)}{-(15 + j)} = \frac{-9(1 + j)(15 - j)}{15^2 - 1^2} = \frac{-9(16 + j14)}{226}$$
$$= \underline{\mathbf{-0.6372 - j0.5575}}$$

Chapter 9, Solution 12.

(a) $z_1 z_2 = (-3 + j4)(12 + j5)$
$= -36 - j15 + j48 - 20$
$= \underline{\mathbf{-56 + j33}}$

(b) $$\frac{z_1}{z_2^*} = \frac{-3 + j4}{12 - j5} = \frac{(-3 + j4)(12 + j5)}{144 + 25} = \underline{\mathbf{-0.3314 + j0.1953}}$$

(c) $z_1 + z_2 = (-3 + j4) + (12 + j5) = 9 + j9$
$z_1 - z_2 = (-3 + j4) - (12 + j5) = -15 - j$
$$\frac{z_1 + z_2}{z_1 - z_2} = \frac{9(1 + j)}{-(15 + j)} = \frac{-9(1 + j)(15 - j)}{15^2 - 1^2} = \frac{-9(16 + j14)}{226}$$
$= \underline{\mathbf{-0.6372 - j0.5575}}$

Chapter 9, Solution 13.

(a) $(-0.4324 + j0.4054) + (-0.8425 - j0.2534) = \underline{-1.2749 + j0.1520}$

(b) $$\frac{50\angle -30^o}{24\angle 150^o} = \underline{-2.0833}$$

(c) (2+j3)(8-j5) –(-4) = <u>35 +j14</u>

Chapter 9, Solution 14.

(a) $$\frac{3 - j14}{-15 + j11} = \underline{-0.5751 + j0.5116}$$

(b) $$\frac{(62.116 + j231.82 + 138.56 - j80)(60 - j80)}{(67 + j84)(16.96 + j10.5983)} = \frac{24186 - 6944.9}{246.06 + j2134.7} = \underline{-1.922 - j11.55}$$

(c) $(-2 + j4)^2 \sqrt{(260 - j120)} = \underline{-256.4 - j200.89}$

Chapter 9, Solution 15.

(a) $\begin{vmatrix} 10+j6 & 2-j3 \\ -5 & -1+j \end{vmatrix} = -10 - j6 + j10 - 6 + 10 - j15$

$= \underline{\mathbf{-6 - j11}}$

(b) $\begin{vmatrix} 20\angle-30° & -4\angle-10° \\ 16\angle0° & 3\angle45° \end{vmatrix} = 60\angle15° + 64\angle-10°$

$= 57.96 + j15.529 + 63.03 - j11.114$

$= \underline{\mathbf{120.99 - j4.415}}$

(c) $\begin{vmatrix} 1-j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1+j \\ 1-j & -j & 0 \\ j & 1 & -j \end{vmatrix} = 1 + 1 + 0 - 1 - 0 + j^2(1-j) + j^2(1+j)$

$= 1 - 1(1 - j + 1 + j)$

$= 1 - 2 = \underline{\mathbf{-1}}$

Chapter 9, Solution 16.

(a) $-10\cos(4t + 75°) = 10\cos(4t + 75° - 180°)$

$= 10\cos(4t - 105°)$

The phasor form is $\underline{\mathbf{10\angle-105°}}$

(b) $5\sin(20t - 10°) = 5\cos(20t - 10° - 90°)$

$= 5\cos(20t - 100°)$

The phasor form is $\underline{\mathbf{5\angle-100°}}$

(c) $4\cos(2t) + 3\sin(2t) = 4\cos(2t) + 3\cos(2t - 90°)$

The phasor form is $4\angle0° + 3\angle-90° = 4 - j3 = \underline{\mathbf{5\angle-36.87°}}$

Chapter 9, Solution 17.

(a) Let $\mathbf{A} = 8\angle-30° + 6\angle0°$

$= 12.928 - j4$

$= 13.533\angle-17.19°$

$a(t) = \underline{\mathbf{13.533\cos(5t + 342.81°)}}$

(b) We know that $-\sin\alpha = \cos(\alpha + 90°)$.

Let $\mathbf{B} = 20\angle 45° + 30\angle(20° + 90°)$
$= 14.142 + j14.142 - 10.261 + j28.19$
$= 3.881 + j42.33$
$= 42.51\angle 84.76°$

$b(t) = \underline{\mathbf{42.51 \cos(120\pi t + 84.76°)}}$

(c) Let $\mathbf{C} = 4\angle{-90°} + 3\angle(-10° - 90°)$
$= -j4 - 0.5209 - j2.954$
$= 6.974\angle 265.72°$

$c(t) = \underline{\mathbf{6.974 \cos(8t + 265.72°)}}$

Chapter 9, Solution 18.

(a) $v_1(t) = \underline{\mathbf{60 \cos(t + 15°)}}$

(b) $\mathbf{V}_2 = 6 + j8 = 10\angle 53.13°$

$v_2(t) = \underline{\mathbf{10 \cos(40t + 53.13°)}}$

(c) $i_1(t) = \underline{\mathbf{2.8 \cos(377t - \pi/3)}}$

(d) $\mathbf{I}_2 = -0.5 - j1.2 = 1.3\angle 247.4°$

$i_2(t) = \underline{\mathbf{1.3 \cos(10^3 t + 247.4°)}}$

Chapter 9, Solution 19.

(a) $3\angle 10° - 5\angle{-30°} = 2.954 + j0.5209 - 4.33 + j2.5$
$= -1.376 + j3.021$
$= 3.32\angle 114.49°$

Therefore, $3 \cos(20t + 10°) - 5 \cos(20t - 30°) = \underline{\mathbf{3.32 \cos(20t + 114.49°)}}$

(b) $4\angle{-90°} + 3\angle{-45°} = -j40 + 21.21 - j21.21$
$= 21.21 - j61.21$
$= 64.78\angle{-70.89°}$

Therefore, $40 \sin(50t) + 30 \cos(50t - 45°) = \underline{\mathbf{64.78 \cos(50t - 70.89°)}}$

(c) Using $\sin\alpha = \cos(\alpha - 90°)$,

$20\angle{-90°} + 10\angle 60° - 5\angle{-110°} = -j20 + 5 + j8.66 + 1.7101 + j4.699$
$= 6.7101 - j6.641$
$= 9.44\angle{-44.7°}$

Therefore, $20 \sin(400t) + 10 \cos(400t + 60°) - 5 \sin(400t - 20°)$
$= \underline{\mathbf{9.44 \cos(400t - 44.7°)}}$

Chapter 9, Solution 20.

(a) $V = 4\angle \underline{-60^o - 90^o} - 5\angle 40^o = -3.464 - j2 - 3.83 - j3.2139 = 8.966\angle -4.399^o$

Hence,

$$\underline{v = 8.966\cos(377t - 4.399^o)}$$

(b) $I = 10\angle 0^o + j\omega 8\angle \underline{20^o - 90^o}$, $\quad \omega = 5$, i.e. $I = 10 + 40\angle 20^o = 49.51\angle 16.04^o$

$$\underline{i = 49.51\cos(5t + 16.04^o)}$$

Chapter 9, Solution 21.

(a) $F = 5\angle 15^o - 4\angle \underline{-30^o - 90^o} = 6.8296 + j4.758 = 8.3236\angle 34.86^o$

$$\underline{f(t) = 8.324\cos(30t + 34.86^o)}$$

(b) $G = 8\angle -90^o + 4\angle 50^o = 2.571 - j4.9358 = 5.565\angle -62.49^o$

$$\underline{g(t) = 5.565\cos(t - 62.49^o)}$$

(c) $H = \dfrac{1}{j\omega}\left(10\angle 0^o + 5\angle -90^o\right), \quad \omega = 40$

i.e. $H = 0.25\angle -90^o + 0.125\angle -180^o = -j0.25 - 0.125 = 0.2795\angle -116.6^o$

$$\underline{h(t) = 0.2795\cos(40t - 116.6^o)}$$

Chapter 9, Solution 22.

Let $\quad$ f(t) $= 10v(t) + 4\dfrac{dv}{dt} - 2\displaystyle\int_{-\infty}^{t} v(t)dt$

$F = 10V + j\omega 4V - \dfrac{2V}{j\omega}, \quad \omega = 5, \quad V = 20\angle -30^o$

$F = 10V + j20V - j0.4V = (10 - j19.6)(17.32 - j10) = 440.1\angle -92.97^o$

$$\underline{f(t) = 440.1\cos(5t - 92.97^o)}$$

Chapter 9, Solution 23.

(a) v(t) = **40 cos(ωt – 60°)**

(b) $\mathbf{V}$ = -30∠10° + 50∠60°
= -4.54 + j38.09
= 38.36∠96.8°
v(t) = **38.36 cos(ωt + 96.8°)**

(c) $\mathbf{I}$ = j6∠-10° = 6∠(90° – 10°) = 6∠80°
i(t) = **6 cos(ωt + 80°)**

(d) $\mathbf{I} = \dfrac{2}{j} + 10\angle\text{-}45° = \text{-}j2 + 7.071 - j7.071$

$= 11.5\angle\text{-}52.06°$

i(t) = **11.5 cos(ωt – 52.06°)**

Chapter 9, Solution 24.

(a)

$$\mathbf{V} + \frac{\mathbf{V}}{j\omega} = 10\angle 0°, \quad \omega = 1$$
$$\mathbf{V}\,(1 - j) = 10$$
$$\mathbf{V} = \frac{10}{1 - j} = 5 + j5 = 7.071\angle 45°$$

Therefore, v(t) = **7.071 cos(t + 45°)**

(b)

$$j\omega\mathbf{V} + 5\mathbf{V} + \frac{4\mathbf{V}}{j\omega} = 20\angle(10° - 90°), \quad \omega = 4$$
$$\mathbf{V}\left(j4 + 5 + \frac{4}{j4}\right) = 20\angle\text{-}80°$$
$$\mathbf{V} = \frac{20\angle\text{-}80°}{5 + j3} = 3.43\angle\text{-}110.96°$$

Therefore, v(t) = **3.43 cos(4t – 110.96°)**

Chapter 9, Solution 25.

(a)

$$2j\omega\mathbf{I} + 3\mathbf{I} = 4\angle\text{-}45°, \quad \omega = 2$$

$$\mathbf{I}(3 + j4) = 4\angle\text{-}45°$$

$$I = \frac{4\angle\text{-}45°}{3 + j4} = \frac{4\angle\text{-}45°}{5\angle 53.13°} = 0.8\angle\text{-}98.13°$$

Therefore, i(t) = **<u>0.8 cos(2t – 98.13°)</u>**

(b)

$$10\frac{\mathbf{I}}{j\omega} + j\omega\mathbf{I} + 6\mathbf{I} = 5\angle 22°, \quad \omega = 5$$

$$(\text{-}j2 + j5 + 6)\mathbf{I} = 5\angle 22°$$

$$\mathbf{I} = \frac{5\angle 22°}{6 + j3} = \frac{5\angle 22°}{6.708\angle 26.56°} = 0.745\angle\text{-}4.56°$$

Therefore, i(t) = **<u>0.745 cos(5t – 4.56°)</u>**

Chapter 9, Solution 26.

$$j\omega\mathbf{I} + 2\mathbf{I} + \frac{\mathbf{I}}{j\omega} = 1\angle 0°, \quad \omega = 2$$

$$\mathbf{I}\left(j2 + 2 + \frac{1}{j2}\right) = 1$$

$$\mathbf{I} = \frac{1}{2 + j1.5} = 0.4\angle\text{-}36.87°$$

Therefore, i(t) = **<u>0.4 cos(2t – 36.87°)</u>**

Chapter 9, Solution 27.

$$j\omega\mathbf{V} + 50\mathbf{V} + 100\frac{\mathbf{V}}{j\omega} = 110\angle\text{-}10°, \quad \omega = 377$$

$$\mathbf{V}\left(j377 + 50 - \frac{j100}{377}\right) = 110\angle\text{-}10°$$

$$\mathbf{V}(380.6\angle 82.45°) = 110\angle\text{-}10°$$

$$\mathbf{V} = 0.289\angle\text{-}92.45°$$

Therefore, v(t) = **<u>0.289 cos(377t – 92.45°)</u>**.

Chapter 9, Solution 28.

$$i(t) = \frac{v_s(t)}{R} = \frac{110\cos(377t)}{8} = \underline{\mathbf{13.75\ cos(377t)\ A}}.$$

Chapter 9, Solution 29.

$$\mathbf{Z} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(2\times10^{-6})} = -j0.5$$

$$\mathbf{V} = \mathbf{IZ} = (4\angle25°)(0.5\angle-90°) = 2\angle-65°$$

Therefore $\quad v(t) = \underline{\mathbf{2\ sin(10^6t - 65°)\ V}}$.

Chapter 9, Solution 30.

$$\mathbf{Z} = j\omega L = j(500)(4\times10^{-3}) = j2$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{60\angle-65°}{2\angle90°} = 30\angle-155°$$

Therefore, $\quad i(t) = \underline{\mathbf{30\ cos(500t - 155°)\ A}}$.

Chapter 9, Solution 31.

$i(t) = 10\sin(\omega t + 30°) = 10\cos(\omega t + 30° - 90°) = 10\cos(\omega t - 60°)$

Thus, $\quad \mathbf{I} = 10\angle-60°$

$v(t) = -65\cos(\omega t + 120°) = 65\cos(\omega t + 120° - 180°) = 65\cos(\omega t - 60°)$

Thus, $\quad \mathbf{V} = 65\angle-60°$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{65\angle-60°}{10\angle-60°} = 6.5\ \Omega$$

Since **V** and **I** are in phase, the element is a **<u>resistor</u>** with R = $\underline{\mathbf{6.5\ \Omega}}$.

Chapter 9, Solution 32.

$$\mathbf{V} = 180\angle 10°, \qquad \mathbf{I} = 12\angle\text{-}30°, \qquad \omega = 2$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{180\angle 10°}{12\angle\text{-}30°} = 15\angle 40° = 11.49 + j9.642\ \Omega$$

One element is a resistor with R = **11.49 Ω**.
The other element is an inductor with ωL = 9.642 or L = **4.821 H**.

Chapter 9, Solution 33.

$$110 = \sqrt{v_R^2 + v_L^2}$$
$$v_L = \sqrt{110^2 - v_R^2}$$
$$v_L = \sqrt{110^2 - 85^2} = \underline{\mathbf{69.82\ V}}$$

Chapter 9, Solution 34.

$$v_o = 0 \text{ if } \omega L = \frac{1}{\omega C} \longrightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{(5\times 10^{-3})(2\times 10^{-3})}} = \underline{\mathbf{100\ rad/s}}$$

Chapter 9, Solution 35.

$$\mathbf{V}_s = 5\angle 0°$$
$$j\omega L = j(2)(1) = j2$$
$$\frac{1}{j\omega C} = \frac{1}{j(2)(0.25)} = \text{-}j2$$

$$\mathbf{V}_o = \frac{j2}{2 - j2 + j2}\mathbf{V}_s = \frac{j2}{2} 5\angle 0° = (1\angle 90°)(5\angle 0°) = 5\angle 90°$$

Thus, $v_o(t) = 5\cos(2t + 90°) = \underline{\mathbf{-5\sin(2t)\ V}}$

Chapter 9, Solution 36.

Let Z be the input impedance at the source.

100 mH $\longrightarrow$ $j\omega L = j200x100x10^{-3} = j20$

10μF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j10x10^{-6}x200} = -j500$

1000//-j500 = 200 –j400
1000//(j20 + 200 –j400) = 242.62 –j239.84

$Z = 2242.62 - j239.84 = 2255\angle -6.104^o$

$$I = \frac{60\angle -10^o}{2255\angle -6.104^o} = 26.61\angle -3.896^o \text{ mA}$$

$i = \underline{266.1\cos(200t - 3.896^o)}$

Chapter 9, Solution 37.

$$j\omega L = j(5)(1) = j5$$

$$\frac{1}{j\omega C} = \frac{1}{j(5)(0.2)} = -j$$

Let $\mathbf{Z}_1 = -j$, $\mathbf{Z}_2 = 2 \,||\, j5 = \dfrac{(2)(j5)}{2 + j5} = \dfrac{j10}{2 + j5}$

Then, $\mathbf{I}_x = \dfrac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}_s$, where $\mathbf{I}_s = 2\angle 0°$

$$\mathbf{I}_x = \frac{\dfrac{j10}{2 + j5}}{-j + \dfrac{j10}{2 + j5}}(2) = \frac{j20}{5 + j8} = 2.12\angle 32°$$

Therefore, $i_x(t) =$ **<u>2.12 sin(5t + 32°) A</u>**

Chapter 9, Solution 38.

(a) $$\frac{1}{6}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = \text{-}j2$$

$$\mathbf{I} = \frac{\text{-}j2}{4 - j2}(10\angle 45°) = 4.472\angle\text{-}18.43°$$

Hence, i(t) = **<u>4.472 cos(3t – 18.43°) A</u>**

$$\mathbf{V} = 4\mathbf{I} = (4)(4.472\angle\text{-}18.43°) = 17.89\angle\text{-}18.43°$$

Hence, v(t) = **<u>17.89 cos(3t – 18.43°) V</u>**

(b) $$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = \text{-}j3$$

$$3\text{ H} \longrightarrow j\omega L = j(4)(3) = j12$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\angle 0°}{4 - j3} = 10\angle 36.87°$$

Hence, i(t) = **<u>10 cos(4t + 36.87°) A</u>**

$$\mathbf{V} = \frac{j12}{8 + j12}(50\angle 0°) = 41.6\angle 33.69°$$

Hence, v(t) = **<u>41.6 cos(4t + 33.69°) V</u>**

Chapter 9, Solution 39.

$$\mathbf{Z} = 8 + j5 \,||\, (\text{-}j10) = 8 + \frac{(j5)(\text{-}j10)}{j5 - j10} = 8 + j10$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{40\angle 0°}{8 + j10} = \frac{20}{6.403\angle 51.34°} = 3.124\angle\text{-}51.34°$$

$$\mathbf{I}_1 = \frac{\text{-}j10}{j5 - j10}\mathbf{I} = 2\mathbf{I} = 6.248\angle\text{-}51.34°$$

$$\mathbf{I}_2 = \frac{j5}{\text{-}j5}\mathbf{I} = \text{-}\mathbf{I} = 3.124\angle 128.66°$$

Therefore, $i_1(t) =$ **<u>6.248 cos(120πt – 51.34°) A</u>**

$i_2(t) =$ **<u>3.124 cos(120πt + 128.66°) A</u>**

Chapter 9, Solution 40.

(a) For $\omega = 1$,

$1\text{ H} \longrightarrow j\omega L = j(1)(1) = j$

$0.05\text{ F} \longrightarrow \dfrac{1}{j\omega C} = \dfrac{1}{j(1)(0.05)} = \text{-}j20$

$$\mathbf{Z} = j + 2 \parallel (\text{-}j20) = j + \frac{\text{-}j40}{2 - j20} = 1.98 + j0.802$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0°}{1.98 + j0.802} = \frac{4\angle 0°}{2.136\angle 22.05°} = 1.872\angle\text{-}22.05°$$

Hence, $i_o(t) =$ **<u>1.872 cos(t – 22.05°) A</u>**

(b) For $\omega = 5$,

$1\text{ H} \longrightarrow j\omega L = j(5)(1) = j5$

$0.05\text{ F} \longrightarrow \dfrac{1}{j\omega C} = \dfrac{1}{j(5)(0.05)} = \text{-}j4$

$$\mathbf{Z} = j5 + 2 \parallel (\text{-}j4) = j5 + \frac{\text{-}j4}{1 - j2} = 1.6 + j4.2$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0°}{1.6 + j4} = \frac{4\angle 0°}{4.494\angle 69.14°} = 0.89\angle\text{-}69.14°$$

Hence, $i_o(t) =$ **<u>0.89 cos(5t – 69.14°) A</u>**

(c) For $\omega = 10$,

$1\text{ H} \longrightarrow j\omega L = j(10)(1) = j10$

$0.05\text{ F} \longrightarrow \dfrac{1}{j\omega C} = \dfrac{1}{j(10)(0.05)} = \text{-}j2$

$$\mathbf{Z} = j10 + 2 \parallel (\text{-}j2) = j10 + \frac{\text{-}j4}{2 - j2} = 1 + j9$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0°}{1 + j9} = \frac{4\angle 0°}{9.055\angle 83.66°} = 0.4417\angle\text{-}83.66°$$

Hence, $i_o(t) =$ **<u>0.4417 cos(10t – 83.66°) A</u>**

Chapter 9, Solution 41.

$$\omega = 1,$$
$$1\text{ H} \longrightarrow \quad j\omega L = j(1)(1) = j$$

$$1\text{ F} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(1)(1)} = \text{-}j$$

$$\mathbf{Z} = 1 + (1 + j) \,||\, (\text{-}j) = 1 + \frac{\text{-}j + 1}{1} = 2 - j$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10}{2 - j}, \qquad \mathbf{I}_c = (1 + j)\,\mathbf{I}$$

$$\mathbf{V} = (\text{-}j)(1 + j)\,\mathbf{I} = (1 - j)\,\mathbf{I} = \frac{(1 - j)(10)}{2 - j} = 6.325\angle\text{-}18.43°$$

Thus, $\quad v(t) = \underline{\mathbf{6.325\cos(t - 18.43°)\ V}}$

Chapter 9, Solution 42.

$$\omega = 200$$
$$50\ \mu\text{F} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(200)(50 \times 10^{-6})} = \text{-}j100$$

$$0.1\text{ H} \longrightarrow \quad j\omega L = j(200)(0.1) = j20$$

$$50 \,||\, \text{-}j100 = \frac{(50)(\text{-}j100)}{50 - j100} = \frac{\text{-}j100}{1\text{-}j2} = 40 - j20$$

$$\mathbf{V}_o = \frac{j20}{j20 + 30 + 40 - j20}(60\angle 0°) = \frac{j20}{70}(60\angle 0°) = 17.14\angle 90°$$

Thus, $\quad v_o(t) = \underline{\mathbf{17.14\sin(200t + 90°)\ V}}$

or $\quad v_o(t) = \underline{\mathbf{17.14\cos(200t)\ V}}$

Chapter 9, Solution 43.

$$\omega = 2$$

$$1\text{ H} \longrightarrow \quad j\omega L = j(2)(1) = j2$$

$$1\text{ F} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(2)(1)} = -j0.5$$

$$\mathbf{I}_o = \frac{j2 - j0.5}{j2 - j0.5 + 1}\mathbf{I} = \frac{j1.5}{1 + j1.5} 4\angle 0° = 3.328\angle 33.69°$$

Thus, $i_o(t) =$ **<u>3.328 cos(2t + 33.69°) A</u>**

Chapter 9, Solution 44.

$$\omega = 200$$

$$10\text{ mH} \longrightarrow \quad j\omega L = j(200)(10 \times 10^{-3}) = j2$$

$$5\text{ mF} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(200)(5 \times 10^{-3})} = -j$$

$$\mathbf{Y} = \frac{1}{4} + \frac{1}{j2} + \frac{1}{3 - j} = 0.25 - j0.5 + \frac{3 + j}{10} = 0.55 - j0.4$$

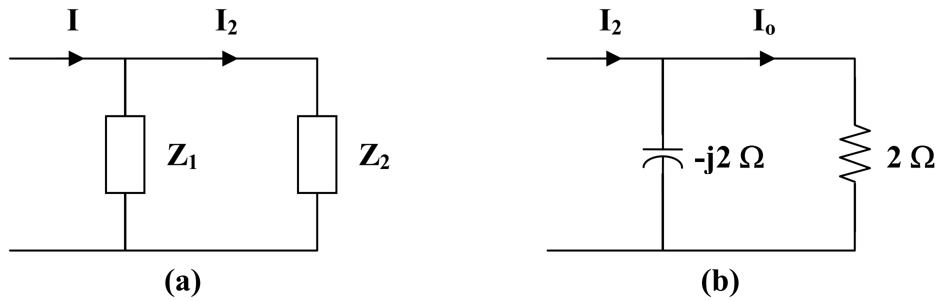
$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{0.55 - j0.4} = 1.1892 + j0.865$$

$$\mathbf{I} = \frac{6\angle 0°}{5 + \mathbf{Z}} = \frac{6\angle 0°}{6.1892 + j0.865} = 0.96\angle -7.956°$$

Thus, i(t) = **<u>0.96 cos(200t – 7.956°) A</u>**

Chapter 9, Solution 45.

We obtain I_o by applying the principle of current division twice.



$$\mathbf{Z}_1 = \text{-j}2\,, \qquad \mathbf{Z}_2 = \text{j}4 + (\text{-j}2)\,||\,2 = \text{j}4 + \frac{\text{-j}4}{2\text{ - j}2} = 1 + \text{j}3$$

$$\mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I} = \frac{\text{-j}2}{\text{-j}2 + 1 + \text{j}3}(5\angle 0°) = \frac{\text{-j}10}{1 + \text{j}}$$

$$\mathbf{I}_o = \frac{\text{-j}2}{2\text{ - j}2}\mathbf{I}_2 = \left(\frac{\text{-j}}{1\text{ - j}}\right)\left(\frac{\text{-j}10}{1 + \text{j}}\right) = \frac{-10}{1+1} = \underline{\textbf{-5 A}}$$

Chapter 9, Solution 46.

$$i_s = 5\cos(10t + 40°) \longrightarrow \mathbf{I}_s = 5\angle 40°$$

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.1)} = \text{-j}$$

$$0.2\text{ H} \longrightarrow j\omega L = j(10)(0.2) = j2$$

Let $\mathbf{Z}_1 = 4\,||\,j2 = \dfrac{j8}{4 + j2} = 0.8 + j1.6\,,\qquad \mathbf{Z}_2 = 3 - j$

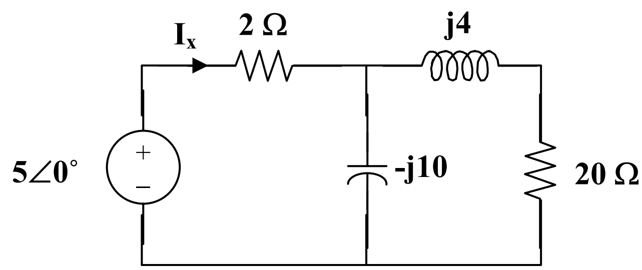
$$\mathbf{I}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}_s = \frac{0.8 + j1.6}{3.8 + j0.6}(5\angle 40°)$$

$$\mathbf{I}_o = \frac{(1.789\angle 63.43°)(5\angle 40°)}{3.847\angle 8.97°} = 2.325\angle 94.46°$$

Thus, $i_o(t) =$ $\underline{\textbf{2.325 cos(10t + 94.46°) A}}$

Chapter 9, Solution 47.

First, we convert the circuit into the frequency domain.

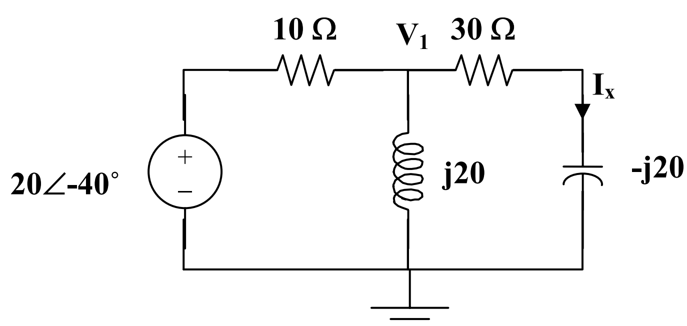


$$I_x = \frac{5}{2 + \frac{-j10(20 + j4)}{-j10 + 20 + j4}} = \frac{5}{2 + 4.588 - j8.626} = \frac{5}{10.854\angle -52.63^\circ} = 0.4607\angle 52.63^\circ$$

$i_s(t) =$ **$0.4607\cos(2000t + 52.63^\circ)$ A**

Chapter 9, Solution 48.

Converting the circuit to the frequency domain, we get:



We can solve this using nodal analysis.

$$\frac{V_1 - 20\angle -40°}{10} + \frac{V_1 - 0}{j20} + \frac{V_1 - 0}{30 - j20} = 0$$

$$V_1(0.1 - j0.05 + 0.02307 + j0.01538) = 2\angle -40°$$

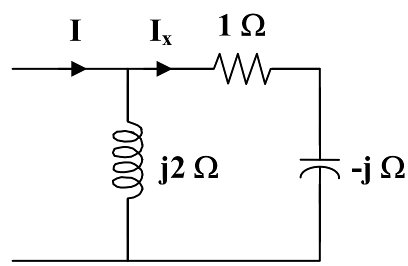
$$V_1 = \frac{2\angle 40°}{0.12307 - j0.03462} = 15.643\angle -24.29°$$

$$I_x = \frac{15.643\angle -24.29°}{30 - j20} = 0.4338\angle 9.4°$$

$$i_x = \underline{0.4338\sin(100t + 9.4°)\,A}$$

Chapter 9, Solution 49.

$$\mathbf{Z}_T = 2 + j2 \,\|\, (1 - j) = 2 + \frac{(j2)(1 - j)}{1 + j} = 4$$



$$\mathbf{I}_x = \frac{j2}{j2 + 1 - j}\mathbf{I} = \frac{j2}{1 + j}\mathbf{I}, \qquad \text{where } \mathbf{I}_x = 0.5\angle 0° = \frac{1}{2}$$

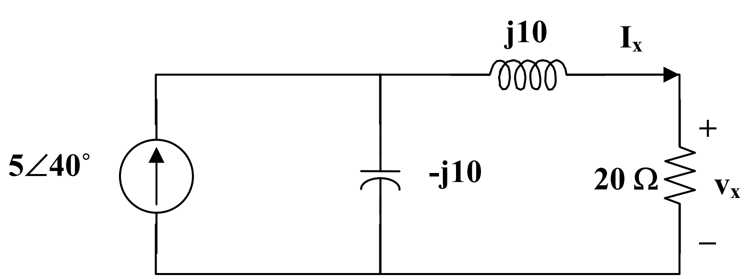
$$\mathbf{I} = \frac{1 + j}{j2}\mathbf{I}_x = \frac{1 + j}{j4}$$

$$\mathbf{V}_s = \mathbf{I}\,\mathbf{Z}_T = \frac{1 + j}{j4}(4) = \frac{1 + j}{j} = 1 - j = 1.414\angle -45°$$

$v_s(t) =$ **<u>1.414 sin(200t – 45°) V</u>**

Chapter 9, Solution 50.

Since ω = 100, the inductor = j100x0.1 = j10 Ω and the capacitor = 1/(j100x10^{-3}) = -j10Ω.



Using the current dividing rule:

$$I_x = \frac{-j10}{-j10+20+j10} 5\angle 40° = -j2.5\angle 40° = 2.5\angle -50°$$
$$V_x = 20 I_x = 50\angle -50°$$
$$v_x = \underline{50\cos(100t - 50°)\,V}$$

Chapter 9, Solution 51.

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.1)} = \text{-}j5$$
$$0.5\text{ H} \longrightarrow j\omega L = j(2)(0.5) = j$$

The current **I** through the 2-Ω resistor is

$$\mathbf{I} = \frac{1}{1 - j5 + j + 2}\mathbf{I}_s = \frac{\mathbf{I}_s}{3 - j4}, \qquad \text{where } \mathbf{I} = 10\angle 0°$$
$$\mathbf{I}_s = (10)(3 - j4) = 50\angle\text{-}53.13°$$

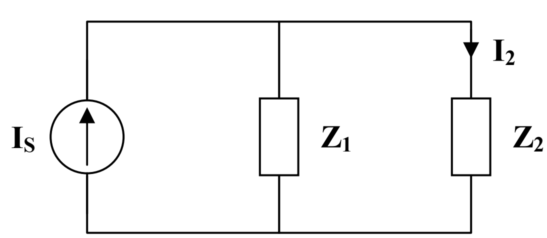
Therefore,

$i_s(t) =$ **<u>50 cos(2t – 53.13°) A</u>**

Chapter 9, Solution 52.

$$5 \,||\, j5 = \frac{j25}{5 + j5} = \frac{j5}{1 + j} = 2.5 + j2.5$$

$$\mathbf{Z}_1 = 10, \qquad \mathbf{Z}_2 = \text{-}j5 + 2.5 + j2.5 = 2.5 - j2.5$$



$$\mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}_s = \frac{10}{12.5 - j2.5}\mathbf{I}_s = \frac{4}{5 - j}\mathbf{I}_s$$

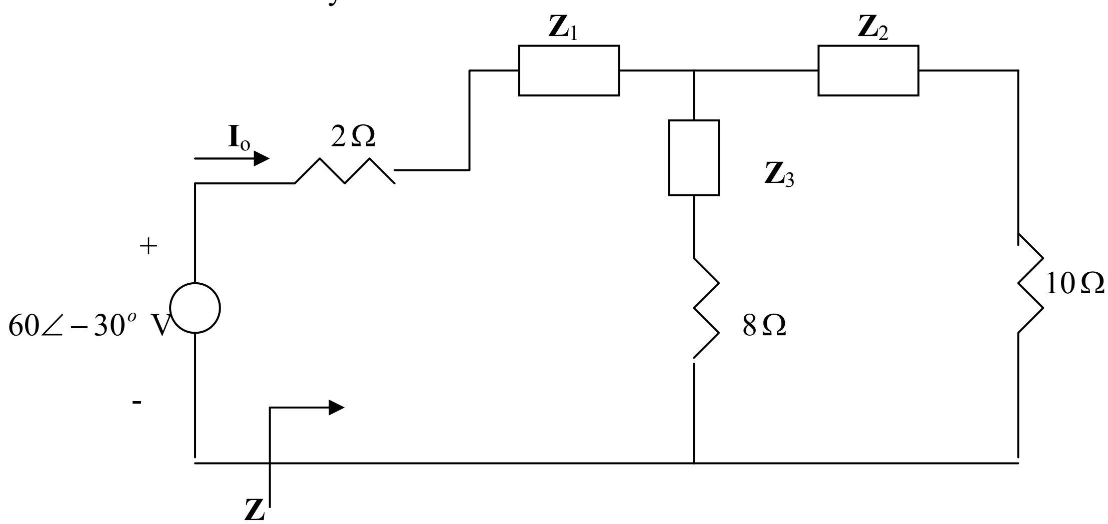
$$\mathbf{V}_o = \mathbf{I}_2\,(2.5 + j2.5)$$

$$8\angle 30° = \left(\frac{4}{5 - j}\right)\mathbf{I}_s\,(2.5)(1 + j) = \frac{10\,(1 + j)}{5 - j}\mathbf{I}_s$$

$$\mathbf{I}_s = \frac{(8\angle 30°)(5 - j)}{10\,(1 + j)} = \underline{\mathbf{2.884\angle\text{-}26.31°\ A}}$$

Chapter 9, Solution 53.

Convert the delta to wye subnetwork as shown below.



$$Z_1 = \frac{-j2x4}{10 - j2} = 0.1532 - j0.7692, \qquad Z_2 = \frac{j6x4}{10 - j2} = -0.4615 + j2.3077,$$

$$Z_3 = \frac{12}{10 - j2} = 1.1538 + j0.2308$$

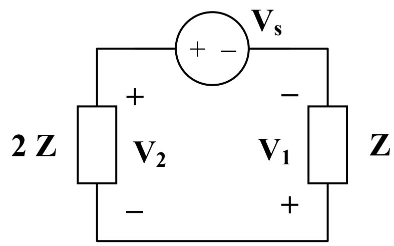
$$(Z_3 + 8)//(Z_2 + 10) = (9.1538 + j0.2308)//(9.5385 + j2.3077) = 4.726 + j0.6062$$

$$Z = 2 + Z_1 + 4.726 + j0.6062 = 6.878 - j0.163$$

$$\mathrm{I_o} = \frac{60\angle -30^{\mathrm{o}}}{\mathrm{Z}} = \frac{60\angle -30^{\mathrm{o}}}{6.88\angle -1.3575^{\mathrm{o}}} = \underline{8.721\angle -28.64^{\mathrm{o}}\ \ \mathrm{A}}$$

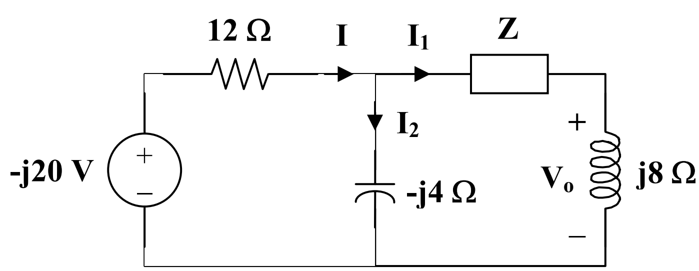
Chapter 9, Solution 54.

Since the left portion of the circuit is twice as large as the right portion, the equivalent circuit is shown below.



$$\mathbf{V}_1 = \mathbf{I}_o(1-j) = 2(1-j)$$
$$\mathbf{V}_2 = 2\mathbf{V}_1 = 4(1-j)$$
$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{V}_2 = 6(1-j)$$
$$\mathbf{V}_s = \underline{\mathbf{8.485\angle\text{-}45° \ V}}$$

Chapter 9, Solution 55.



$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{j8} = \frac{4}{j8} = \text{-}j0.5$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1(\mathbf{Z}+j8)}{\text{-}j4} = \frac{(\text{-}j0.5)(\mathbf{Z}+j8)}{\text{-}j4} = \frac{\mathbf{Z}}{8} + j$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \text{-}j0.5 + \frac{\mathbf{Z}}{8} + j = \frac{\mathbf{Z}}{8} + j0.5$$

$$\text{-}j20 = 12\mathbf{I} + \mathbf{I}_1(\mathbf{Z}+j8)$$

$$\text{-}j20 = 12\left(\frac{\mathbf{Z}}{8} + \frac{j}{2}\right) + \frac{\text{-}j}{2}(\mathbf{Z}+j8)$$

$$-4 - j26 = \mathbf{Z}\left(\frac{3}{2} - j\frac{1}{2}\right)$$

$$\mathbf{Z} = \frac{-4 - j26}{\frac{3}{2} - j\frac{1}{2}} = \frac{26.31\angle 261.25°}{1.5811\angle -18.43°} = 16.64\angle 279.68°$$

$$\mathbf{Z} = \underline{\mathbf{2.798 - j16.403\ \Omega}}$$

Chapter 9, Solution 56.

$$3\text{H} \longrightarrow j\omega L = j30$$

$$3\text{F} \longrightarrow \frac{1}{j\omega C} = -j/30$$

$$1.5\text{F} \longrightarrow \frac{1}{j\omega C} = -j/15$$

$$j30//(-j/15) = \frac{j30x\frac{-j}{15}}{j30 - \frac{j}{15}} = -j0.06681$$

$$Z = \frac{-j}{30}//(2 - j0.06681) = \frac{-j0.033(2 - j0.06681)}{-j0.033 + 2 - j0.06681} = \underline{6 - j333\ \text{m}\Omega}$$

Chapter 9, Solution 57.

$$2\text{H} \longrightarrow j\omega L = j2$$

$$1\text{F} \longrightarrow \frac{1}{j\omega C} = -j$$

$$Z = 1 + j2//(2 - j) = 1 + \frac{j2(2 - j)}{j2 + 2 - j} = 2.6 + j1.2$$

$$Y = 1/Z = \underline{0.3171 - j0.1463\ \text{S}}$$

Chapter 9, Solution 58.

(a) 10 mF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(50)(10\times 10^{-3})} = -j2$

10 mH $\longrightarrow$ $j\omega L = j(50)(10\times 10^{-3}) = j0.5$

$$\mathbf{Z}_{in} = j0.5 + 1 \,||\, (1 - j2)$$
$$\mathbf{Z}_{in} = j0.5 + \frac{1 - j2}{2 - j2}$$
$$\mathbf{Z}_{in} = j0.5 + 0.25(3 - j)$$
$$\mathbf{Z}_{in} = \underline{\mathbf{0.75 + j0.25\ \Omega}}$$

(b) 0.4 H $\longrightarrow$ $j\omega L = j(50)(0.4) = j20$

0.2 H $\longrightarrow$ $j\omega L = j(50)(0.2) = j10$

1 mF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(50)(1\times 10^{-3})} = -j20$

For the parallel elements,

$$\frac{1}{\mathbf{Z}_p} = \frac{1}{20} + \frac{1}{j10} + \frac{1}{-j20}$$
$$\mathbf{Z}_p = 10 + j10$$

Then,

$$\mathbf{Z}_{in} = 10 + j20 + \mathbf{Z}_p = \underline{\mathbf{20 + j30\ \Omega}}$$

Chapter 9, Solution 59.

$$\mathbf{Z}_{eq} = 6 + (1 - j2) \,||\, (2 + j4)$$

$$\mathbf{Z}_{eq} = 6 + \frac{(1 - j2)(2 + j4)}{(1 - j2) + (2 + j4)}$$

$$\mathbf{Z}_{eq} = 6 + 2.308 - j1.5385$$

$$\mathbf{Z}_{eq} = \underline{\mathbf{8.308 - j1.5385\ \Omega}}$$

Chapter 9, Solution 60.

$$Z = (25 + j15) + (20 - j50) // (30 + j10) = 25 + j15 + 26.097 - j5.122 = \underline{51.1 + j9.878\Omega}$$

Chapter 9, Solution 61.

All of the impedances are in parallel.

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{1-j} + \frac{1}{1+j2} + \frac{1}{j5} + \frac{1}{1+j3}$$

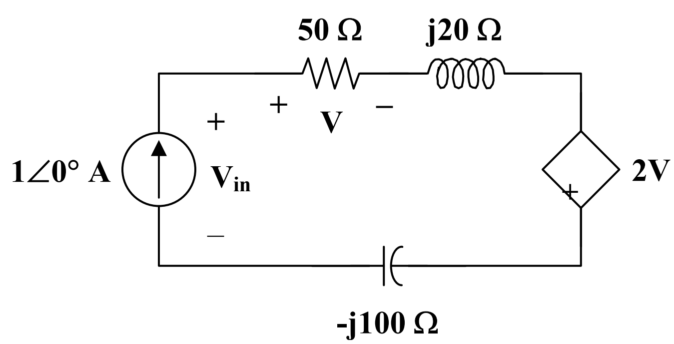
$$\frac{1}{\mathbf{Z}_{eq}} = (0.5 + j0.5) + (0.2 - j0.4) + (\text{-}j0.2) + (0.1 - j0.3) = 0.8 - j0.4$$

$$\mathbf{Z}_{eq} = \frac{1}{0.8 - j0.4} = \underline{\mathbf{1 + j0.5\ \Omega}}$$

Chapter 9, Solution 62.

$$2\text{ mH} \longrightarrow \quad j\omega L = j(10\times10^{3})(2\times10^{-3}) = j20$$

$$1\ \mu\text{F} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(10\times10^{3})(1\times10^{-6})} = \text{-}j100$$



$$\mathbf{V} = (1\angle0°)(50) = 50$$

$$\mathbf{V}_{in} = (1\angle0°)(50 + j20 - j100) + (2)(50)$$
$$\mathbf{V}_{in} = 50 - j80 + 100 = 150 - j80$$

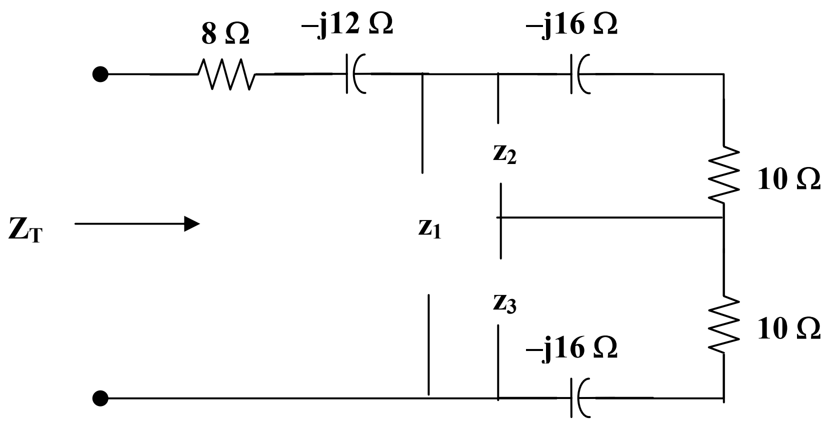
$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{1\angle0°} = \underline{\mathbf{150 - j80\ \Omega}}$$

Chapter 9, Solution 63.

First, replace the wye composed of the 20-ohm, 10-ohm, and j15-ohm impedances with the corresponding delta.

$$z_1 = \frac{200 + j150 + j300}{10} = 20 + j45$$

$$z_2 = \frac{200 + j450}{j15} = 30 - j13.333, \; z_3 = \frac{200 + j450}{20} = 10 + j22.5$$



Now all we need to do is to combine impedances.

$$z_2 \| (10 - j16) = \frac{(30 - j13.333)(10 - j16)}{40 - j29.33} = 8.721 - j8.938$$

$$z_3 \| (10 - j16) = 21.70 - j3.821$$

$$Z_T = 8 - j12 + z_1 \| (8.721 - j8.938 + 21.7 - j3.821) = \underline{34.69 - j6.93\Omega}$$

Chapter 9, Solution 64.

$$Z_T = 4 + \frac{-j10(6 + j8)}{6 - j2} = \underline{19 - j5\Omega}$$

$$I = \frac{30\angle 90^\circ}{Z_T} = -0.3866 + j1.4767 = \underline{1.527\angle 104.7^\circ\,A}$$

Chapter 9, Solution 65.

$$\mathbf{Z}_T = 2 + (4 - j6) \parallel (3 + j4)$$

$$\mathbf{Z}_T = 2 + \frac{(4 - j6)(3 + j4)}{7 - j2}$$

$$\mathbf{Z}_T = \underline{\mathbf{6.83 + j1.094\ \Omega}} = 6.917\angle 9.1°\ \Omega$$

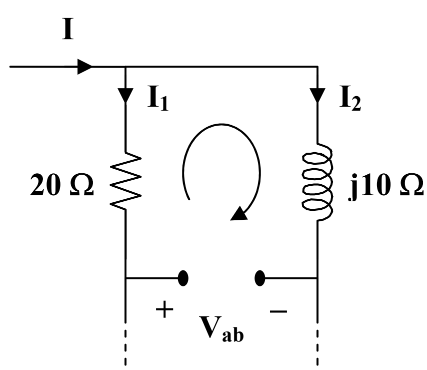
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{120\angle 10°}{6.917\angle 9.1°} = \underline{\mathbf{17.35\angle 0.9°\ A}}$$

Chapter 9, Solution 66.

$$\mathbf{Z}_T = (20 - j5) \parallel (40 + j10) = \frac{(20 - j5)(40 + j10)}{60 + j5} = \frac{170}{145}(12 - j)$$

$$\mathbf{Z}_T = \underline{\mathbf{14.069 - j1.172\ \Omega}} = 14.118\angle\text{-}4.76°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{60\angle 90°}{14.118\angle\text{-}4.76°} = 4.25\angle 94.76°$$



$$\mathbf{I}_1 = \frac{40 + j10}{60 + j5}\mathbf{I} = \frac{8 + j2}{12 + j}\mathbf{I}$$

$$\mathbf{I}_2 = \frac{20 - j5}{60 + j5}\mathbf{I} = \frac{4 - j}{12 + j}\mathbf{I}$$

$$\mathbf{V}_{ab} = \text{-}20\mathbf{I}_1 + j10\mathbf{I}_2$$

$$\mathbf{V}_{ab} = \frac{-(160 + j40)}{12 + j}\mathbf{I} + \frac{10 + j40}{12 + j}\mathbf{I}$$

$$\mathbf{V}_{ab} = \frac{-150}{12 + j}\mathbf{I} = \frac{(-12 + j)(150)}{145}\mathbf{I}$$

$$\mathbf{V}_{ab} = (12.457\angle 175.24°)(4.25\angle 97.76°)$$

$$\mathbf{V}_{ab} = \underline{\mathbf{52.94\angle 273°\ V}}$$

Chapter 9, Solution 67.

(a) $20\text{ mH} \longrightarrow j\omega L = j(10^3)(20 \times 10^{-3}) = j20$

$12.5\ \mu\text{F} \longrightarrow \dfrac{1}{j\omega C} = \dfrac{1}{j(10^3)(12.5 \times 10^{-6})} = -j80$

$$\mathbf{Z}_{in} = 60 + j20 \,||\, (60 - j80)$$
$$\mathbf{Z}_{in} = 60 + \frac{(j20)(60 - j80)}{60 - j60}$$
$$\mathbf{Z}_{in} = 63.33 + j23.33 = 67.494\angle 20.22°$$

$$\mathbf{Y}_{in} = \frac{1}{\mathbf{Z}_{in}} = \underline{\mathbf{0.0148\angle\text{-}20.22°\ S}}$$

(b) $10\text{ mH} \longrightarrow j\omega L = j(10^3)(10 \times 10^{-3}) = j10$

$20\ \mu\text{F} \longrightarrow \dfrac{1}{j\omega C} = \dfrac{1}{j(10^3)(20 \times 10^{-6})} = -j50$

$30 \,||\, 60 = 20$

$$\mathbf{Z}_{in} = -j50 + 20 \,||\, (40 + j10)$$
$$\mathbf{Z}_{in} = -j50 + \frac{(20)(40 + j10)}{60 + j10}$$
$$\mathbf{Z}_{in} = 13.5 - j48.92 = 50.75\angle -74.56°$$

$$\mathbf{Y}_{in} = \frac{1}{\mathbf{Z}_{in}} = \underline{\mathbf{0.0197\angle 74.56°\ S}} = 5.24 + j18.99\text{ mS}$$

Chapter 9, Solution 68.

$$\mathbf{Y}_{eq} = \frac{1}{5 - j2} + \frac{1}{3 + j} + \frac{1}{-j4}$$

$$\mathbf{Y}_{eq} = (0.1724 + j0.069) + (0.3 - j0.1) + (j0.25)$$

$$\mathbf{Y}_{eq} = \underline{\mathbf{0.4724 + j0.219\ S}}$$

Chapter 9, Solution 69.

$$\frac{1}{\mathbf{Y}_o} = \frac{1}{4} + \frac{1}{-j2} = \frac{1}{4}(1 + j2)$$

$$\mathbf{Y}_o = \frac{4}{1 + j2} = \frac{(4)(1 - j2)}{5} = 0.8 - j1.6$$

$$\mathbf{Y}_o + j = 0.8 - j0.6$$

$$\frac{1}{\mathbf{Y}_o'} = \frac{1}{1} + \frac{1}{-j3} + \frac{1}{0.8 - j0.6} = (1) + (j0.333) + (0.8 + j0.6)$$

$$\frac{1}{\mathbf{Y}_o'} = 1.8 + j0.933 = 2.028\angle 27.41°$$

$$\mathbf{Y}_o' = 0.4932\angle -27.41° = 0.4378 - j0.2271$$

$$\mathbf{Y}_o' + j5 = 0.4378 + j4.773$$

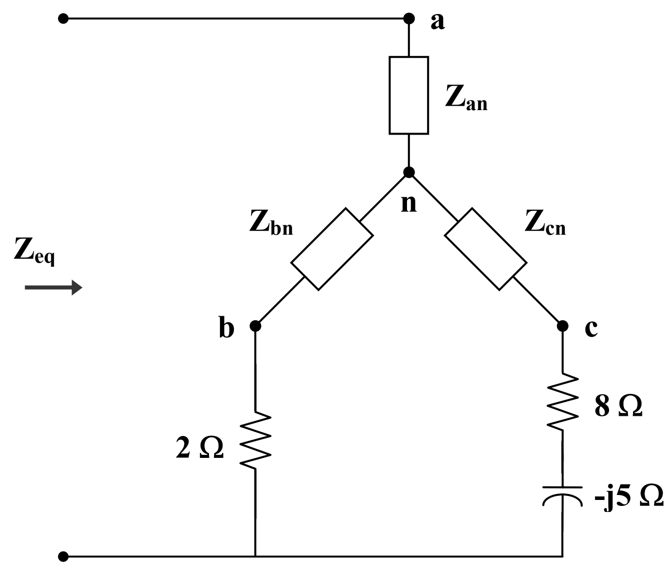
$$\frac{1}{\mathbf{Y}_{eq}} = \frac{1}{2} + \frac{1}{0.4378 + j4.773} = 0.5 + \frac{0.4378 - j4.773}{22.97}$$

$$\frac{1}{\mathbf{Y}_{eq}} = 0.5191 - j0.2078$$

$$\mathbf{Y}_{eq} = \frac{0.5191 - j0.2078}{0.3126} = \underline{\mathbf{1.661 + j0.6647\ S}}$$

Chapter 9, Solution 70.

Make a delta-to-wye transformation as shown in the figure below.



$$\mathbf{Z}_{an} = \frac{(\text{-j}10)(10+\text{j}15)}{5-\text{j}10+10+\text{j}15} = \frac{(10)(15-\text{j}10)}{15+\text{j}5} = 7-\text{j}9$$

$$\mathbf{Z}_{bn} = \frac{(5)(10+\text{j}15)}{15+\text{j}5} = 4.5+\text{j}3.5$$

$$\mathbf{Z}_{cn} = \frac{(5)(\text{-j}10)}{15+\text{j}5} = \text{-}1-\text{j}3$$

$$\mathbf{Z}_{eq} = \mathbf{Z}_{an} + (\mathbf{Z}_{bn}+2) \,\|\, (\mathbf{Z}_{cn}+8-\text{j}5)$$

$$\mathbf{Z}_{eq} = 7-\text{j}9+(6.5+\text{j}3.5) \,\|\, (7-\text{j}8)$$

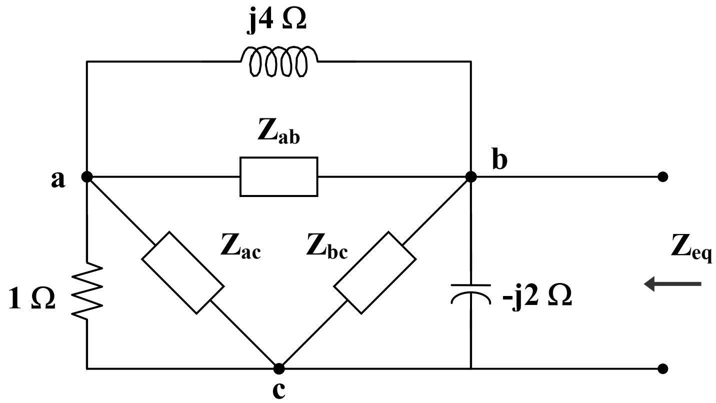
$$\mathbf{Z}_{eq} = 7-\text{j}9+\frac{(6.5+\text{j}3.5)(7-\text{j}8)}{13.5-\text{j}4.5}$$

$$\mathbf{Z}_{eq} = 7-\text{j}9+5.511-\text{j}0.2$$

$$\mathbf{Z}_{eq} = 12.51-\text{j}9.2 = \underline{\mathbf{15.53\angle\text{-}36.33°\ \Omega}}$$

Chapter 9, Solution 71.

We apply a wye-to-delta transformation.



$$\mathbf{Z}_{ab} = \frac{2 - j2 + j4}{j2} = \frac{2 + j2}{j2} = 1 - j$$

$$\mathbf{Z}_{ac} = \frac{2 + j2}{2} = 1 + j$$

$$\mathbf{Z}_{bc} = \frac{2 + j2}{-j} = -2 + j2$$

$$j4 \,||\, \mathbf{Z}_{ab} = j4 \,||\, (1 - j) = \frac{(j4)(1 - j)}{1 + j3} = 1.6 - j0.8$$

$$1 \,||\, \mathbf{Z}_{ac} = 1 \,||\, (1 + j) = \frac{(1)(1 + j)}{2 + j} = 0.6 + j0.2$$

$$j4 \,||\, \mathbf{Z}_{ab} + 1 \,||\, \mathbf{Z}_{ac} = 2.2 - j0.6$$

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{-j2} + \frac{1}{-2 + j2} + \frac{1}{2.2 - j0.6}$$

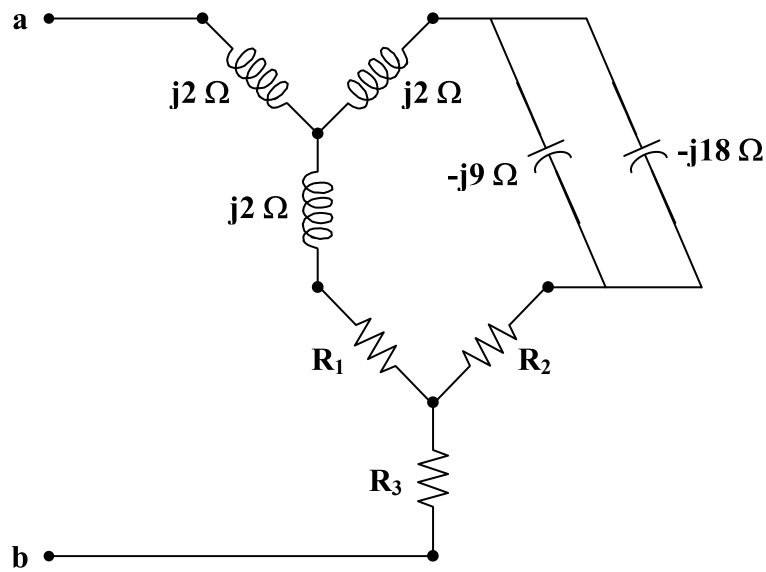
$$= j0.5 - 0.25 - j0.25 + 0.4231 + j0.1154$$

$$= 0.173 + j0.3654 = 0.4043\angle 64.66°$$

$$\mathbf{Z}_{eq} = 2.473\angle\text{-}64.66°\ \Omega = \underline{\mathbf{1.058 - j2.235\ \Omega}}$$

Chapter 9, Solution 72.

Transform the delta connections to wye connections as shown below.



$$- j9 \,||\, -j18 = -j6,$$

$$R_1 = \frac{(20)(20)}{20+20+10} = 8\,\Omega, \qquad R_2 = \frac{(20)(10)}{50} = 4\,\Omega, \qquad R_3 = \frac{(20)(10)}{50} = 4\,\Omega$$

$$\mathbf{Z}_{ab} = j2 + (j2+8) \,||\, (j2 - j6 + 4) + 4$$

$$\mathbf{Z}_{ab} = 4 + j2 + (8 + j2) \,||\, (4 - j4)$$

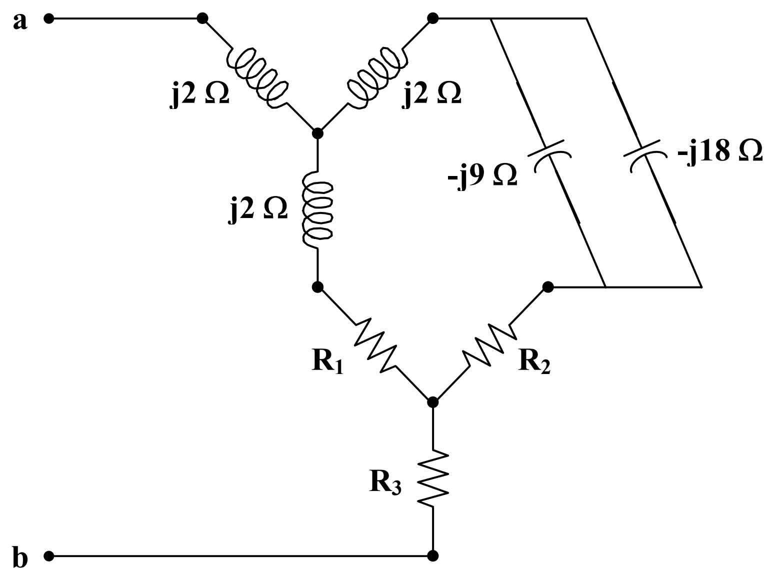
$$\mathbf{Z}_{ab} = 4 + j2 + \frac{(8+j2)(4-j4)}{12 - j2}$$

$$\mathbf{Z}_{ab} = 4 + j2 + 3.567 - j1.4054$$

$$\mathbf{Z}_{ab} = \underline{\mathbf{7.567 + j0.5946\ \Omega}}$$

Chapter 9, Solution 73.

Transform the delta connection to a wye connection as in Fig. (a) and then transform the wye connection to a delta connection as in Fig. (b).



$$\mathbf{Z}_1 = \frac{(j8)(\text{-}j6)}{j8 + j8 - j6} = \frac{48}{j10} = \text{-}j4.8$$

$$\mathbf{Z}_2 = \mathbf{Z}_1 = \text{-}j4.8$$

$$\mathbf{Z}_3 = \frac{(j8)(j8)}{j10} = \frac{-64}{j10} = j6.4$$

$$(2 + \mathbf{Z}_1)(4 + \mathbf{Z}_2) + (4 + \mathbf{Z}_2)(\mathbf{Z}_3) + (2 + \mathbf{Z}_1)(\mathbf{Z}_3) =$$
$$(2 - j4.8)(4 - j4.8) + (4 - j4.8)(j6.4) + (2 - j4.8)(j6.4) = 46.4 + j9.6$$

$$\mathbf{Z}_a = \frac{46.4 + j9.6}{j6.4} = 1.5 - j7.25$$

$$\mathbf{Z}_b = \frac{46.4 + j9.6}{4 - j4.8} = 3.574 + j6.688$$

$$\mathbf{Z}_c = \frac{46.4 + j9.6}{2 - j4.8} = 1.727 + j8.945$$

$$j6 \,||\, \mathbf{Z}_b = \frac{(6\angle 90°)(7.583\angle 61.88°)}{3.574 + j12.688} = 07407 + j3.3716$$

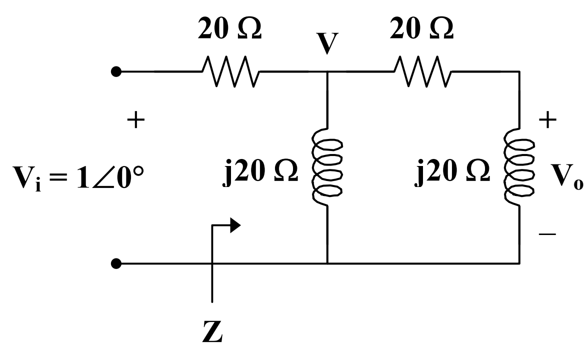
$$\text{-}j4 \,||\, \mathbf{Z}_a = \frac{(\text{-}j4)(1.5 - j7.25)}{1.5 - j11.25} = 0.186 - j2.602$$

$$j12 \,\|\, \mathbf{Z}_c = \frac{(12\angle 90°)(9.11\angle 79.07°)}{1.727 + j20.945} = 0.5634 + j5.1693$$

$$\mathbf{Z}_{eq} = (j6 \,\|\, \mathbf{Z}_b) \,\|\, (\text{-}j4 \,\|\, \mathbf{Z}_a + j12 \,\|\, \mathbf{Z}_c)$$
$$\mathbf{Z}_{eq} = (0.7407 + j3.3716) \,\|\, (0.7494 + j2.5673)$$
$$\mathbf{Z}_{eq} = 1.508\angle 75.42°\ \Omega = \underline{\mathbf{0.3796 + j1.46\ \Omega}}$$

Chapter 9, Solution 74.

<u>One such RL circuit is shown below</u>.



We now want to show that this circuit will produce a 90° phase shift.

$$\mathbf{Z} = j20 \,\|\, (20 + j20) = \frac{(j20)(20 + j20)}{20 + j40} = \frac{-20 + j20}{1 + j2} = 4(1 + j3)$$

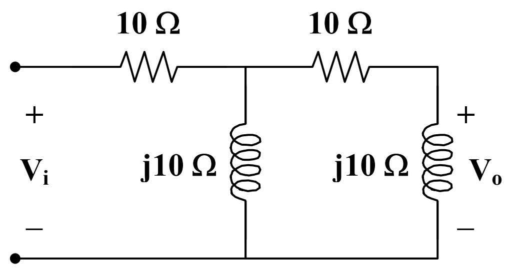
$$\mathbf{V} = \frac{\mathbf{Z}}{\mathbf{Z} + 20}\mathbf{V}_i = \frac{4 + j12}{24 + j12}(1\angle 0°) = \frac{1 + j3}{6 + j3} = \frac{1}{3}(1 + j)$$

$$\mathbf{V}_o = \frac{j20}{20 + j20}\mathbf{V} = \left(\frac{j}{1 + j}\right)\left(\frac{1}{3}(1 + j)\right) = \frac{j}{3} = 0.3333\angle 90°$$

This shows that the output leads the input by 90°.

Chapter 9, Solution 75.

Since $\cos(\omega t) = \sin(\omega t + 90°)$, we need a phase shift circuit that will cause the output to lead the input by 90°. **<u>This is achieved by the RL circuit shown below, as explained in the previous problem</u>**.



This can also be obtained by an RC circuit.

Chapter 9, Solution 76.

Let $Z = R - jX$, where $X = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$$|Z| = \sqrt{R^2 + X^2} \quad \longrightarrow \quad X = \sqrt{|Z|^2 - R^2} = \sqrt{116^2 = 66^2} = 95.394$$

$$C = \frac{1}{2\pi f X} = \frac{1}{2\pi x 60 x 95.394} = \underline{27.81\mu F}$$

Chapter 9, Solution 77.

(a) $$\mathbf{V}_o = \frac{-jX_c}{R - jX_c}\mathbf{V}_i$$

where $$X_c = \frac{1}{\omega C} = \frac{1}{(2\pi)(2\times 10^6)(20\times 10^{-9})} = 3.979$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-j3.979}{5 - j3.979} = \frac{3.979}{\sqrt{5^2 + 3.979^2}}\angle(-90° + \tan^{-1}(3.979/5))$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{3.979}{\sqrt{25 + 15.83}}\angle(-90° - 38.51°)$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = 0.6227\angle -51.49°$$

Therefore, the phase shift is **<u>51.49° lagging</u>**

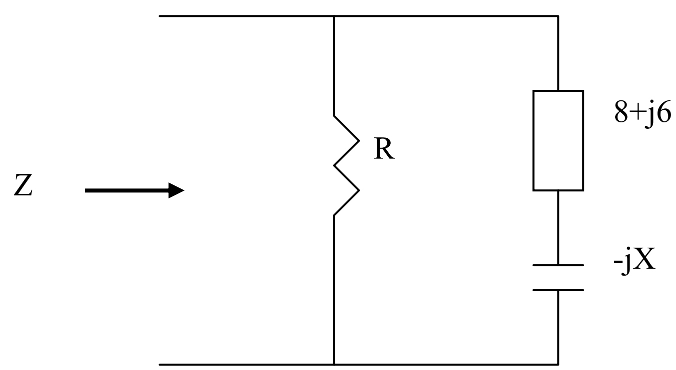
(b) $\theta = -45° = -90° + \tan^{-1}(X_c/R)$

$$45° = \tan^{-1}(X_c/R) \longrightarrow R = X_c = \frac{1}{\omega C}$$

$$\omega = 2\pi f = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(5)(20\times 10^{-9})} = \underline{\mathbf{1.5915\ MHz}}$$

Chapter 9, Solution 78.



$$Z = R//[8 + j(6-X)] = \frac{R[8 + j(6-X)]}{R + 8 + j(6-X)} = 5$$

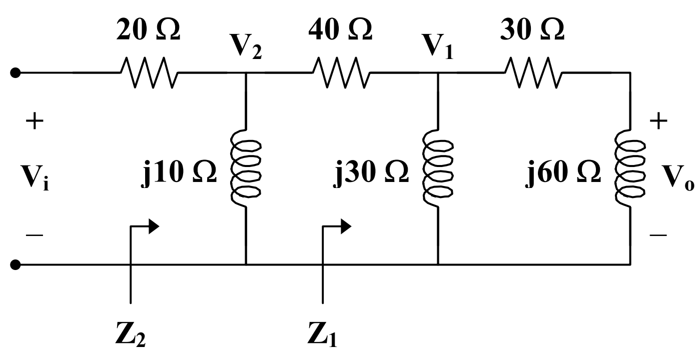
i.e $8R + j6R - jXR = 5R + 40 + j30 - j5X$

Equating real and imaginary parts:

$8R = 5R + 40$ which leads to $\mathbf{R=\underline{13.33\Omega}}$
$6R-XR = 30-5$ which leads to $\mathbf{X=\underline{4.125\Omega}}$.

Chapter 9, Solution 79.

(a) Consider the circuit as shown.



$$\mathbf{Z}_1 = j30 \,||\, (30 + j60) = \frac{(j30)(30 + j60)}{30 + j90} = 3 + j21$$

$$\mathbf{Z}_2 = j10 \,||\, (40 + \mathbf{Z}_1) = \frac{(j10)(43 + j21)}{43 + j31} = 1.535 + j8.896 = 9.028\angle 80.21°$$

Let $\mathbf{V}_i = 1\angle 0°$.

$$\mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + 20}\mathbf{V}_i = \frac{(9.028\angle 80.21°)(1\angle 0°)}{21.535 + j8.896}$$

$$\mathbf{V}_2 = 0.3875\angle 57.77°$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + 40}\mathbf{V}_2 = \frac{3 + j21}{43 + j21}\mathbf{V}_2 = \frac{(21.213\angle 81.87°)(0.3875\angle 57.77°)}{47.85\angle 26.03°}$$

$$\mathbf{V}_1 = 0.1718\angle 113.61°$$

$$\mathbf{V}_o = \frac{j60}{30 + j60}\mathbf{V}_1 = \frac{j2}{1 + j2}\mathbf{V}_1 = \frac{2}{5}(2 + j)\mathbf{V}_1$$

$$\mathbf{V}_o = (0.8944\angle 26.56°)(0.1718\angle 113.6°)$$

$$\mathbf{V}_o = 0.1536\angle 140.2°$$

Therefore, the phase shift is <u>**140.2°**</u>

(b) The phase shift is <u>**leading**</u>.

(c) If $\mathbf{V}_i = 120\text{ V}$, then

$$\mathbf{V}_o = (120)(0.1536\angle 140.2°) = 18.43\angle 140.2°\text{ V}$$

and the magnitude is <u>**18.43 V**</u>.

Chapter 9, Solution 80.

$$200\text{ mH} \longrightarrow j\omega L = j(2\pi)(60)(200 \times 10^{-3}) = j75.4\ \Omega$$

$$\mathbf{V}_o = \frac{j75.4}{R + 50 + j75.4}\mathbf{V}_i = \frac{j75.4}{R + 50 + j75.4}(120\angle 0°)$$

(a) When $R = 100\ \Omega$,

$$\mathbf{V}_o = \frac{j75.4}{150 + j75.4}(120\angle 0°) = \frac{(75.4\angle 90°)(120\angle 0°)}{167.88\angle 26.69°}$$

$$\mathbf{V}_o = \underline{\mathbf{53.89\angle 63.31°\ V}}$$

(b) When $R = 0\,\Omega$,

$$\mathbf{V}_o = \frac{j75.4}{50 + j75.4}(120\angle 0°) = \frac{(75.4\angle 90°)(120\angle 0°)}{90.47\angle 56.45°}$$

$$\mathbf{V}_o = \underline{\mathbf{100\angle 33.55°\ V}}$$

(c) To produce a phase shift of 45°, the phase of $\mathbf{V}_o = 90° + 0° - \alpha = 45°$.
Hence, α = phase of $(R + 50 + j75.4) = 45°$.
For α to be 45°, $R + 50 = 75.4$
Therefore, $R = \underline{\mathbf{25.4\ \Omega}}$

Chapter 9, Solution 81.

Let $\mathbf{Z}_1 = R_1$, $\mathbf{Z}_2 = R_2 + \dfrac{1}{j\omega C_2}$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_x = R_x + \dfrac{1}{j\omega C_x}$.

$$\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$$

$$R_x + \frac{1}{j\omega C_x} = \frac{R_3}{R_1}\left(R_2 + \frac{1}{j\omega C_2}\right)$$

$$R_x = \frac{R_3}{R_1}R_2 = \frac{1200}{400}(600) = \underline{\mathbf{1.8\ k\Omega}}$$

$$\frac{1}{C_x} = \left(\frac{R_3}{R_1}\right)\left(\frac{1}{C_2}\right) \longrightarrow C_x = \frac{R_1}{R_3}C_2 = \left(\frac{400}{1200}\right)(0.3\times 10^{-6}) = \underline{\mathbf{0.1\ \mu F}}$$

Chapter 9, Solution 82.

$$C_x = \frac{R_1}{R_2}C_s = \left(\frac{100}{2000}\right)(40\times 10^{-6}) = \underline{\mathbf{2\ \mu F}}$$

Chapter 9, Solution 83.

$$L_x = \frac{R_2}{R_1}L_s = \left(\frac{500}{1200}\right)(250\times 10^{-3}) = \underline{\mathbf{104.17\ mH}}$$

Chapter 9, Solution 84.

Let $\mathbf{Z}_1 = R_1 \parallel \dfrac{1}{j\omega C_s}$, $\mathbf{Z}_2 = R_2$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_x = R_x + j\omega L_x$.

$$\mathbf{Z}_1 = \frac{\dfrac{R_1}{j\omega C_s}}{R_1 + \dfrac{1}{j\omega C_s}} = \frac{R_1}{j\omega R_1 C_s + 1}$$

Since $\mathbf{Z}_x = \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$,

$$R_x + j\omega L_x = R_2 R_3 \frac{j\omega R_1 C_s + 1}{R_1} = \frac{R_2 R_3}{R_1}(1 + j\omega R_1 C_s)$$

Equating the real and imaginary components,

$$\underline{\mathbf{R_x = \frac{R_2 R_3}{R_1}}}$$

$\omega L_x = \dfrac{R_2 R_3}{R_1}(\omega R_1 C_s)$ implies that

$$\underline{\mathbf{L_x = R_2 R_3 C_s}}$$

Given that $R_1 = 40\ \text{k}\Omega$, $R_2 = 1.6\ \text{k}\Omega$, $R_3 = 4\ \text{k}\Omega$, and $C_s = 0.45\ \mu\text{F}$

$$R_x = \frac{R_2 R_3}{R_1} = \frac{(1.6)(4)}{40}\ \text{k}\Omega = 0.16\ \text{k}\Omega = \underline{\mathbf{160\ \Omega}}$$
$$L_x = R_2 R_3 C_s = (1.6)(4)(0.45) = \underline{\mathbf{2.88\ H}}$$

Chapter 9, Solution 85.

Let $\mathbf{Z}_1 = R_1$, $\mathbf{Z}_2 = R_2 + \dfrac{1}{j\omega C_2}$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_4 = R_4 \parallel \dfrac{1}{j\omega C_4}$.

$$\mathbf{Z}_4 = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{-jR_4}{\omega R_4 C_4 - j}$$

Since $\mathbf{Z}_4 = \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2 \longrightarrow \mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$,

$$\frac{-jR_4R_1}{\omega R_4C_4 - j} = R_3\left(R_2 - \frac{j}{\omega C_2}\right)$$

$$\frac{-jR_4R_1(\omega R_4C_4 + j)}{\omega^2 R_4^2 C_4^2 + 1} = R_3R_2 - \frac{jR_3}{\omega C_2}$$

Equating the real and imaginary components,

$$\frac{R_1R_4}{\omega^2 R_4^2 C_4^2 + 1} = R_2R_3 \qquad (1)$$

$$\frac{\omega R_1 R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1} = \frac{R_3}{\omega C_2} \qquad (2)$$

Dividing (1) by (2),

$$\frac{1}{\omega R_4C_4} = \omega R_2C_2$$

$$\omega^2 = \frac{1}{R_2C_2R_4C_4}$$

$$\omega = 2\pi f = \frac{1}{\sqrt{R_2C_2R_4C_4}}$$

$$\underline{\mathbf{f} = \frac{\mathbf{1}}{\mathbf{2\pi\sqrt{R_2R_4C_2C_4}}}}$$

Chapter 9, Solution 86.

$$\mathbf{Y} = \frac{1}{240} + \frac{1}{j95} + \frac{1}{-j84}$$

$$\mathbf{Y} = 4.1667\times10^{-3} - j0.01053 + j0.0119$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1000}{4.1667 + j1.37} = \frac{1000}{4.3861\angle 18.2°}$$

$\mathbf{Z}$ = $\underline{\mathbf{228\angle -18.2° \ \Omega}}$

Chapter 9, Solution 87.

$$\mathbf{Z}_1 = 50 + \frac{1}{j\omega C} = 50 + \frac{-j}{(2\pi)(2\times10^{3})(2\times10^{-6})}$$

$$\mathbf{Z}_1 = 50 - j39.79$$

$$\mathbf{Z}_2 = 80 + j\omega L = 80 + j(2\pi)(2\times10^{3})(10\times10^{-3})$$

$$\mathbf{Z}_2 = 80 + j125.66$$

$$\mathbf{Z}_3 = 100$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{100} + \frac{1}{50 - j39.79} + \frac{1}{80 + j125.66}$$

$$\frac{1}{\mathbf{Z}} = 10^{-3}\,(10 + 12.24 + j9.745 + 3.605 - j5.663)$$

$$= (25.85 + j4.082)\times10^{-3}$$

$$= 26.17\times10^{-3}\,\angle 8.97^\circ$$

$\mathbf{Z} = \underline{\mathbf{38.21\angle{-8.97^\circ}\ \Omega}}$

Chapter 9, Solution 88.

(a) $\mathbf{Z} = -j20 + j30 + 120 - j20$

$\mathbf{Z} = \underline{\mathbf{120 - j10\ \Omega}}$

(b) If the frequency were halved, $\dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$ would cause the capacitive impedance to double, while $\omega L = 2\pi f L$ would cause the inductive impedance to halve. Thus,

$\mathbf{Z} = -j40 + j15 + 120 - j40$

$\mathbf{Z} = \underline{\mathbf{120 - j65\ \Omega}}$

Chapter 9, Solution 89.

$$\mathbf{Z}_{in} = j\omega L \,||\, \left(R + \frac{1}{j\omega C} \right)$$

$$\mathbf{Z}_{in} = \frac{j\omega L \left(R + \dfrac{1}{j\omega C} \right)}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{\dfrac{L}{C} + j\omega L\, R}{R + j\left(\omega L - \dfrac{1}{\omega C} \right)}$$

$$\mathbf{Z}_{in} = \frac{\left(\dfrac{L}{C} + j\omega L\, R \right)\left(R - j\left(\omega L - \dfrac{1}{\omega C} \right)\right)}{R^2 + \left(\omega L - \dfrac{1}{\omega C} \right)^2}$$

To have a resistive impedance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$. Hence,

$$\omega L\, R^2 - \left(\frac{L}{C} \right)\left(\omega L - \frac{1}{\omega C} \right) = 0$$

$$\omega R^2 C = \omega L - \frac{1}{\omega C}$$

$$\omega^2 R^2 C^2 = \omega^2 LC - 1$$

$$L = \frac{\omega^2 R^2 C^2 + 1}{\omega^2 C} \qquad (1)$$

Ignoring the +1 in the numerator in (1),

$$L = R^2 C = (200)^2 (50 \times 10^{-9}) = \underline{\mathbf{2\ mH}}$$

Chapter 9, Solution 90.

Let $\quad \mathbf{V}_s = 145\angle 0°, \qquad X = j\omega L = j(2\pi)(60)\,L = j377\,L$

$$\mathbf{I} = \frac{\mathbf{V}_s}{80 + R + jX} = \frac{145\angle 0°}{80 + R + jX}$$

$$\mathbf{V}_1 = 80\,\mathbf{I} = \frac{(80)(145)}{80 + R + jX}$$

$$50 = \left| \frac{(80)(145)}{80 + R + jX} \right| \qquad (1)$$

$$\mathbf{V}_o = (R + jX)\,\mathbf{I} = \frac{(R + jX)(145\angle 0°)}{80 + R + jX}$$

$$110 = \left| \frac{(R + jX)(145)}{80 + R + jX} \right| \qquad (2)$$

From (1) and (2),

$$\frac{50}{110} = \frac{80}{\left| R + jX \right|}$$

$$\left| R + jX \right| = (80)\left(\frac{11}{5} \right)$$

$$R^2 + X^2 = 30976 \qquad (3)$$

From (1),

$$\left| 80 + R + jX \right| = \frac{(80)(145)}{50} = 232$$

$$6400 + 160R + R^2 + X^2 = 53824$$

$$160R + R^2 + X^2 = 47424 \qquad (4)$$

Subtracting (3) from (4),

$$160R = 16448 \longrightarrow R = \underline{\mathbf{102.8\ \Omega}}$$

From (3),

$$X^2 = 30976 - 10568 = 20408$$

$$X = 142.86 = 377L \longrightarrow L = \underline{\mathbf{0.3789\ H}}$$

Chapter 9, Solution 91.

$$\mathbf{Z}_{in} = \frac{1}{j\omega C} + R \parallel j\omega L$$

$$\mathbf{Z}_{in} = \frac{-j}{\omega C} + \frac{j\omega LR}{R + j\omega L}$$

$$= \frac{-j}{\omega C} + \frac{\omega^2 L^2 R + j\omega LR^2}{R^2 + \omega^2 L^2}$$

To have a resistive impedance, $\text{Im}(\mathbf{Z}_{in}) = 0$.
Hence,

$$\frac{-1}{\omega C} + \frac{\omega LR^2}{R^2 + \omega^2 L^2} = 0$$

$$\frac{1}{\omega C} = \frac{\omega LR^2}{R^2 + \omega^2 L^2}$$

$$C = \frac{R^2 + \omega^2 L^2}{\omega^2 LR^2}$$

where $\omega = 2\pi f = 2\pi \times 10^7$

$$C = \frac{9 \times 10^4 + (4\pi^2 \times 10^{14})(400 \times 10^{-12})}{(4\pi^2 \times 10^{14})(20 \times 10^{-6})(9 \times 10^4)}$$

$$C = \frac{9 + 16\pi^2}{72\pi^2} \text{ nF}$$

C = **<u>235 pF</u>**

Chapter 9, Solution 92.

(a) $Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{100\angle 75^o}{450\angle 48^o x10^{-6}}} = \underline{471.4\angle 13.5^o \,\Omega}$

(b) $\gamma = \sqrt{ZY} = \sqrt{100\angle 75^o x450\angle 48^o x10^{-6}} = \underline{0.2121\angle 61.5^o}$

Chapter 9, Solution 93.

$$\mathbf{Z} = \mathbf{Z}_s + 2\mathbf{Z}_\ell + \mathbf{Z}_L$$
$$\mathbf{Z} = (1 + 0.8 + 23.2) + j(0.5 + 0.6 + 18.9)$$
$$\mathbf{Z} = 25 + j20$$

$$\mathbf{I}_L = \frac{\mathbf{V}_S}{\mathbf{Z}} = \frac{115\angle 0°}{32.02\angle 38.66°}$$

$$\mathbf{I}_L = \underline{\mathbf{3.592\angle\text{-}38.66° \ A}}$$