Chapter 15, Solution 1.

(a) $\cosh(at) = \dfrac{e^{at} + e^{-at}}{2}$

$$L\left[\cosh(at)\right] = \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \underline{\boldsymbol{\frac{s}{s^2 - a^2}}}$$

(b) $\sinh(at) = \dfrac{e^{at} - e^{-at}}{2}$

$$L\left[\sinh(at)\right] = \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \underline{\boldsymbol{\frac{a}{s^2 - a^2}}}$$

Chapter 15, Solution 2.

(a) $f(t) = \cos(\omega t)\cos(\theta) - \sin(\omega t)\sin(\theta)$

$$F(s) = \cos(\theta)\, L\left[\cos(\omega t)\right] - \sin(\theta)\, L\left[\sin(\omega t)\right]$$

$$F(s) = \underline{\boldsymbol{\frac{s\cos(\theta) - \omega\sin(\theta)}{s^2 + \omega^2}}}$$

(b) $f(t) = \sin(\omega t)\cos(\theta) + \cos(\omega t)\sin(\theta)$

$$F(s) = \sin(\theta)\, L\left[\cos(\omega t)\right] + \cos(\theta)\, L\left[\sin(\omega t)\right]$$

$$F(s) = \underline{\boldsymbol{\frac{s\sin(\theta) - \omega\cos(\theta)}{s^2 + \omega^2}}}$$

Chapter 15, Solution 3.

(a) $L\left[e^{-2t}\cos(3t)\,u(t)\right] = \underline{\boldsymbol{\dfrac{s+2}{(s+2)^2 + 9}}}$

(b) $L\left[e^{-2t}\sin(4t)\,u(t)\right] = \underline{\boldsymbol{\dfrac{4}{(s+2)^2 + 16}}}$

(c) Since $\mathcal{L}\left[\cosh(at)\right] = \dfrac{s}{s^2 - a^2}$

$$\mathcal{L}\left[e^{-3t}\cosh(2t)\,u(t)\right] = \underline{\mathbf{\dfrac{s+3}{(s+3)^2 - 4}}}$$

(d) Since $\mathcal{L}\left[\sinh(at)\right] = \dfrac{a}{s^2 - a^2}$

$$\mathcal{L}\left[e^{-4t}\sinh(t)\,u(t)\right] = \underline{\mathbf{\dfrac{1}{(s+4)^2 - 1}}}$$

(e) $\mathcal{L}\left[e^{-t}\sin(2t)\right] = \dfrac{2}{(s+1)^2 + 4}$

If $\quad f(t) \longleftrightarrow F(s)$

$$t\,f(t) \longleftrightarrow \frac{-d}{ds}F(s)$$

Thus, $\mathcal{L}\left[t\,e^{-t}\sin(2t)\right] = \dfrac{-d}{ds}\left[2\left((s+1)^2 + 4\right)^{-1}\right]$

$$= \frac{2}{((s+1)^2 + 4)^2} \cdot 2(s+1)$$

$$\mathcal{L}\left[t\,e^{-t}\sin(2t)\right] = \underline{\mathbf{\dfrac{4(s+1)}{((s+1)^2 + 4)^2}}}$$

Chapter 15, Solution 4.

(a) $$G(s) = 6\frac{s}{s^2 + 4^2}e^{-s} = \underline{\frac{6se^{-s}}{s^2 + 16}}$$

(b) $$F(s) = \underline{\frac{2}{s^2} + 5\frac{e^{-2s}}{s+3}}$$

Chapter 15, Solution 5.

(a) $$\mathcal{L}\left[\cos(2t+30°)\right] = \frac{s\cos(30°) - 2\sin(30°)}{s^2+4}$$

$$\mathcal{L}\left[t^2\cos(2t+30°)\right] = \frac{d^2}{ds^2}\left[\frac{s\cos(30°)-1}{s^2+4}\right]$$

$$= \frac{d}{ds}\frac{d}{ds}\left[\left(\frac{\sqrt{3}}{2}s-1\right)\left(s^2+4\right)^{-1}\right]$$

$$= \frac{d}{ds}\left[\frac{\sqrt{3}}{2}\left(s^2+4\right)^{-1} - 2s\left(\frac{\sqrt{3}}{2}s-1\right)\left(s^2+4\right)^{-2}\right]$$

$$= \frac{\frac{\sqrt{3}}{2}(-2s)}{\left(s^2+4\right)^2} - \frac{2\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^2} - \frac{2s\left(\frac{\sqrt{3}}{2}\right)}{\left(s^2+4\right)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^3}$$

$$= \frac{-\sqrt{3}s - \sqrt{3}s + 2 - \sqrt{3}s}{\left(s^2+4\right)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^3}$$

$$= \frac{(-3\sqrt{3}s+2)(s^2+4)}{\left(s^2+4\right)^3} + \frac{4\sqrt{3}s^3 - 8s^2}{\left(s^2+4\right)^3}$$

$$\mathcal{L}\left[t^2\cos(2t+30°)\right] = \underline{\frac{\mathbf{8 - 12\sqrt{3}s - 6s^2 + \sqrt{3}s^3}}{\mathbf{\left(s^2+4\right)^3}}}$$

(b) $$\mathcal{L}\left[30t^4e^{-t}\right] = 30\cdot\frac{4!}{(s+2)^5} = \underline{\frac{\mathbf{720}}{\mathbf{(s+2)^5}}}$$

(c) $$\mathcal{L}\left[2t\,u(t) - 4\frac{d}{dt}\delta(t)\right] = \frac{2}{s^2} - 4(s\cdot 1 - 0) = \underline{\mathbf{\frac{2}{s^2} - 4s}}$$

(d) $2e^{-(t-1)}u(t) = 2e^{-t}u(t)$

$$\mathcal{L}\left[2e^{-(t-1)}u(t)\right] = \underline{\frac{\mathbf{2e}}{\mathbf{s+1}}}$$

(e) Using the scaling property,

$$\mathcal{L}\left[5u(t/2)\right] = 5\cdot\frac{1}{1/2}\cdot\frac{1}{s/(1/2)} = 5\cdot 2\cdot\frac{1}{2s} = \underline{\frac{\mathbf{5}}{\mathbf{s}}}$$

(f) $$\mathcal{L}\left[6e^{-t/3}u(t)\right] = \frac{6}{s+1/3} = \underline{\frac{\mathbf{18}}{\mathbf{3s+1}}}$$

(g) Let $f(t) = \delta(t)$. Then, $F(s) = 1$.

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots$$

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^n\cdot 1 - s^{n-1}\cdot 0 - s^{n-2}\cdot 0 - \cdots$$

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \underline{\mathbf{s^n}}$$

Chapter 15, Solution 6.

(a) $$\mathcal{L}\left[2\delta(t-1)\right] = \underline{\mathbf{2e^{-s}}}$$

(b) $$\mathcal{L}\left[10u(t-2)\right] = \underline{\frac{\mathbf{10}}{\mathbf{s}}\mathbf{e^{-2s}}}$$

(c) $$\mathcal{L}\left[(t+4)u(t)\right] = \underline{\frac{\mathbf{1}}{\mathbf{s^2}} + \frac{\mathbf{4}}{\mathbf{s}}}$$

(d) $$\mathcal{L}\left[2e^{-t}u(t-4)\right] = \mathcal{L}\left[2e^{-4}e^{-(t-4)}u(t-4)\right] = \underline{\frac{\mathbf{2e^{-4s}}}{\mathbf{e^4(s+1)}}}$$

Chapter 15, Solution 7.

(a) Since $\mathcal{L}\left[\cos(4t)\right] = \dfrac{s}{s^2+4^2}$, we use the linearity and shift properties to obtain $\mathcal{L}\left[10\cos(4(t-1))u(t-1)\right] = \underline{\dfrac{\mathbf{10\,s\,e^{-s}}}{\mathbf{s^2+16}}}$

(b) Since $\mathcal{L}\left[t^2\right] = \dfrac{2}{s^3}$, $\mathcal{L}\left[u(t)\right] = \dfrac{1}{s}$,

$\mathcal{L}\left[t^2\,e^{-2t}\right] = \dfrac{2}{(s+2)^3}$, and $\mathcal{L}\left[u(t-3)\right] = \dfrac{e^{-3s}}{s}$

$$\mathcal{L}\left[t^2\,e^{-2t}\,u(t) + u(t-3)\right] = \underline{\frac{\mathbf{2}}{\mathbf{(s+2)^3}} + \frac{\mathbf{e^{-3s}}}{\mathbf{s}}}$$

Chapter 15, Solution 8.

(a) $\mathcal{L}\left[2\delta(3t) + 6u(2t) + 4e^{-2t} - 10e^{-3t}\right]$

$$= 2\cdot\frac{1}{3} + 6\cdot\frac{1}{2}\cdot\frac{1}{s/2} + \frac{4}{s+2} - \frac{10}{s+3}$$

$$= \underline{\frac{\mathbf{2}}{\mathbf{3}} + \frac{\mathbf{6}}{\mathbf{s}} + \frac{\mathbf{4}}{\mathbf{s+2}} - \frac{\mathbf{10}}{\mathbf{s+3}}}$$

(b) $t\,e^{-t}\,u(t-1) = (t-1)\,e^{-t}\,u(t-1) + e^{-t}\,u(t-1)$
$t\,e^{-t}\,u(t-1) = (t-1)\,e^{-(t-1)}\,e^{-1}\,u(t-1) + e^{-(t-1)}\,e^{-1}\,u(t-1)$

$$\mathcal{L}\left[t\,e^{-t}\,u(t-1)\right] = \frac{e^{-1}\,e^{-s}}{(s+1)^2} + \frac{e^{-1}\,e^{-s}}{s+1} = \underline{\frac{\mathbf{e^{-(s+1)}}}{\mathbf{(s+1)^2}} + \frac{\mathbf{e^{-(s+1)}}}{\mathbf{s+1}}}$$

(c) $\mathcal{L}\left[\cos(2(t-1))\,u(t-1)\right] = \underline{\dfrac{\mathbf{s\,e^{-s}}}{\mathbf{s^2+4}}}$

(d) Since $\sin(4(t-\pi)) = \sin(4t)\cos(4\pi) - \sin(4\pi)\cos(4t) = \sin(4t)$

$\sin(4t)\,u(t-\pi) = \sin(4(t-\pi))\,u(t-\pi)$

$$\mathrm{L}\left[\sin(4t)\left[u(t)-u(t-\pi)\right]\right]$$
$$= \mathrm{L}\left[\sin(4t)\,u(t)\right] - \mathrm{L}\left[\sin(4(t-\pi))\,u(t-\pi)\right]$$

$$= \frac{4}{s^2+16} - \frac{4e^{-\pi s}}{s^2+16} = \underline{\mathbf{\frac{4}{s^2+16}\cdot(1-e^{-\pi s})}}$$

Chapter 15, Solution 9.

(a) $f(t) = (t-4)\,u(t-2) = (t-2)\,u(t-2) - 2\,u(t-2)$

$$\underline{F(s) = \mathbf{\frac{e^{-2s}}{s^2} - \frac{2e^{-2s}}{s^2}}}$$

(b) $g(t) = 2e^{-4t}\,u(t-1) = 2e^{-4}\,e^{-4(t-1)}\,u(t-1)$

$$\underline{G(s) = \mathbf{\frac{2e^{-s}}{e^4(s+4)}}}$$

(c) $h(t) = 5\cos(2t-1)\,u(t)$

$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

$\cos(2t-1) = \cos(2t)\cos(1) + \sin(2t)\sin(1)$

$h(t) = 5\cos(1)\cos(2t)\,u(t) + 5\sin(1)\sin(2t)\,u(t)$

$$H(s) = 5\cos(1)\cdot\frac{s}{s^2+4} + 5\sin(1)\cdot\frac{2}{s^2+4}$$

$$\underline{H(s) = \mathbf{\frac{2.702\,s}{s^2+4} + \frac{8.415}{s^2+4}}}$$

(d) $p(t) = 6u(t-2) - 6u(t-4)$

$$\underline{P(s) = \mathbf{\frac{6}{s}e^{-2s} - \frac{6}{s}e^{-4s}}}$$

Chapter 15, Solution 10.

(a) By taking the derivative in the time domain,

$$g(t) = (-t e^{-t} + e^{-t})\cos(t) - t e^{-t} \sin(t)$$

$$g(t) = e^{-t} \cos(t) - t e^{-t} \cos(t) - t e^{-t} \sin(t)$$

$$G(s) = \frac{s+1}{(s+1)^2+1} + \frac{d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] + \frac{d}{ds}\left[\frac{1}{(s+1)^2+1}\right]$$

$$G(s) = \frac{s+1}{s^2+2s+2} - \frac{s^2+2s}{(s^2+2s+2)^2} - \frac{2s+2}{(s^2+2s+2)^2} = \underline{\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}}$$

(b) By applying the time differentiation property,

$$G(s) = sF(s) - f(0)$$

where $f(t) = t e^{-t} \cos(t)$, $f(0) = 0$

$$G(s) = (s) \cdot \frac{-d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] = \frac{(s)(s^2+2s)}{(s^2+2s+2)^2} = \underline{\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}}$$

Chapter 15, Solution 11.

(a) Since $\mathcal{L}\left[\cosh(at)\right] = \frac{s}{s^2-a^2}$

$$F(s) = \frac{6(s+1)}{(s+1)^2-4} = \underline{\mathbf{\frac{6(s+1)}{s^2+2s-3}}}$$

(b) Since $\mathcal{L}\left[\sinh(at)\right] = \frac{a}{s^2-a^2}$

$$\mathcal{L}\left[3e^{-2t}\sinh(4t)\right] = \frac{(3)(4)}{(s+2)^2-16} = \frac{12}{s^2+4s-12}$$

$$F(s) = \mathcal{L}\left[t \cdot 3e^{-2t}\sinh(4t)\right] = \frac{-d}{ds}\left[12(s^2+4s-12)^{-1}\right]$$

$$F(s) = (12)(2s+4)(s^2+4s-12)^{-2} = \underline{\mathbf{\frac{24(s+2)}{(s^2+4s-12)^2}}}$$

(c) $\cosh(t) = \frac{1}{2}\cdot(e^{t} + e^{-t})$

$$f(t) = 8e^{-3t}\cdot\frac{1}{2}\cdot(e^{t} + e^{-t})u(t-2)$$

$$= 4e^{-2t}u(t-2) + 4e^{-4t}u(t-2)$$
$$= 4e^{-4}e^{-2(t-2)}u(t-2) + 4e^{-8}e^{-4(t-2)}u(t-2)$$

$$L\left[4e^{-4}e^{-2(t-2)}u(t-2)\right] = 4e^{-4}e^{-2s}\cdot L\left[e^{-2}u(t)\right]$$

$$L\left[4e^{-4}e^{-2(t-2)}u(t-2)\right] = \frac{4e^{-(2s+4)}}{s+2}$$

Similarly, $L\left[4e^{-8}e^{-4(t-2)}u(t-2)\right] = \dfrac{4e^{-(2s+8)}}{s+4}$

Therefore,

$$F(s) = \frac{4e^{-(2s+4)}}{s+2} + \frac{4e^{-(2s+8)}}{s+4} = \underline{\frac{\mathbf{e^{-(2s+6)}\left[(4e^{2}+4e^{-2})s+(16e^{2}+8e^{-2})\right]}}{\mathbf{s^{2}+6s+8}}}$$

Chapter 15, Solution 12.

$$f(t) = te^{-2(t-1)}e^{-2}u(t-1) = (t-1)e^{-2}e^{-2(t-1)}u(t-1) + e^{-2}e^{-2(t-1)}u(t-1)$$

$$f(s) = e^{-s}\frac{e^{-2}}{(s+2)^{2}} + e^{-2}\frac{e^{-s}}{s+2} = \frac{e^{-(s+2)}}{s+2}\left(1+\frac{1}{s+2}\right) = \underline{\frac{s+3}{(s+2)^{2}}e^{-(s+2)}}$$

Chapter 15, Solution 13.

(a) $tf(t) \quad \longleftrightarrow \quad -\frac{d}{ds}F(s)$

If f(t) = cost, then $F(s) = \dfrac{s}{s^{2}+1}$ and $\dfrac{d}{ds}F(s) = \dfrac{(s^{2}+1)(1) - s(2s+1)}{(s^{2}+1)^{2}}$

$$\underline{L(t\cos t) = \frac{s^{2}+s-1}{(s^{2}+1)^{2}}}$$

(b) Let f(t) = e^{-t} sin t.

$$F(s) = \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$$

$$\frac{dF}{ds} = \frac{(s^2+2s+2)(0)-(1)(2s+2)}{(s^2+2s+2)^2}$$

$$\underline{L(e^{-t}t\sin t) = -\frac{dF}{ds} = \frac{2(s+1)}{(s^2+2s+2)^2}}$$

(c) $\frac{f(t)}{t} \quad \longleftrightarrow \quad \int_s^\infty F(s)ds$

Let $f(t) = \sin\beta t$, then $F(s) = \frac{\beta}{s^2+\beta^2}$

$$L\left[\frac{\sin\beta t}{t}\right] = \int_s^\infty \frac{\beta}{s^2+\beta^2}ds = \beta\frac{1}{\beta}\tan^{-1}\frac{s}{\beta}\Big|_s^\infty = \frac{\pi}{2} - \tan^{-1}\frac{s}{\beta} = \underline{\tan^{-1}\frac{\beta}{s}}$$

Chapter 15, Solution 14.

$$f(t) = \begin{cases} 5t & 0 < t < 1 \\ 10-5t & 1 < t < 2 \end{cases}$$

We may write f(t) as

$$\begin{aligned} f(t) &= 5t\left[u(t)-u(t-1)\right] + (10-5t)\left[u(t-1)-u(t-2)\right] \\ &= 5t\,u(t) - 10(t-1)u(t-1) + 5(t-2)u(t-2) \end{aligned}$$

$$F(s) = \frac{5}{s^2} - \frac{10}{s^2}e^{-s} + \frac{5}{s^2}e^{-2s}$$

$$\underline{\mathbf{F(s) = \frac{5}{s^2}(1 - 2e^{-s} + e^{-2s})}}$$

Chapter 15, Solution 15.

$$f(t) = 10\left[u(t) - u(t-1) - u(t-1) + u(t-2)\right]$$

$$F(s) = 10\left[\frac{1}{s} - \frac{2}{s}e^{-s} + \frac{e^{-2s}}{s}\right] = \underline{\mathbf{\frac{10}{s}(1 - e^{-s})^2}}$$

Chapter 15, Solution 16.

$$f(t) = 5u(t) - 3u(t-1) + 3u(t-3) - 5u(t-4)$$

$$F(s) = \underline{\mathbf{\frac{1}{s}\left[5 - 3e^{-s} + 3e^{-3s} - 5e^{-4s}\right]}}$$

Chapter 15, Solution 17.

$$f(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 < t < 1 \\ 1 & 1 < t < 3 \\ 0 & t > 3 \end{cases}$$

$$\begin{aligned} f(t) &= t^2\left[u(t) - u(t-1)\right] + 1\left[u(t-1) - u(t-3)\right] \\ &= t^2 u(t) - (t-1)^2 u(t-1) + (-2t+1)u(t-1) + u(t-1) - u(t-3) \\ &= t^2 u(t) - (t-1)^2 u(t-1) - 2(t-1)u(t-1) - u(t-3) \end{aligned}$$

$$F(s) = \underline{\mathbf{\frac{2}{s^3}(1 - e^{-s}) - \frac{2}{s^2}e^{-s} - \frac{e^{-3s}}{s}}}$$

Chapter 15, Solution 18.

(a)
$$\begin{aligned} g(t) &= u(t) - u(t-1) + 2\left[u(t-1) - u(t-2)\right] + 3\left[u(t-2) - u(t-3)\right] \\ &= u(t) + u(t-1) + u(t-2) - 3u(t-3) \end{aligned}$$

$$G(s) = \underline{\mathbf{\frac{1}{s}(1 + e^{-s} + e^{-2s} - 3e^{-3s})}}$$

(b) $$h(t) = 2t[u(t) - u(t-1)] + 2[u(t-1) - u(t-3)] + (8-2t)[u(t-3) - u(t-4)]$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 2u(t-1) + 2u(t-1) - 2u(t-3) - 2(t-3)u(t-3) + 2u(t-3) + 2(t-4)u(t-4)$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 2(t-3)u(t-3) + 2(t-4)u(t-4)$$

$$H(s) = \frac{2}{s^2}(1 - e^{-s}) - \frac{2}{s^2}e^{-3s} + \frac{2}{s^2}e^{-4s} = \underline{\mathbf{\frac{2}{s^2}(1 - e^{-s} - e^{-3s} + e^{-4s})}}$$

Chapter 15, Solution 19.

Since $\mathsf{L}[\delta(t)] = 1$ and $T = 2$, $F(s) = \underline{\mathbf{\frac{1}{1 - e^{-2s}}}}$

Chapter 15, Solution 20.

Let $$g_1(t) = \sin(\pi t), \quad 0 < t < 1$$
$$= \sin(\pi t)[u(t) - u(t-1)]$$
$$= \sin(\pi t)u(t) - \sin(\pi t)u(t-1)$$

Note that $\sin(\pi(t-1)) = \sin(\pi t - \pi) = -\sin(\pi t)$.

So, $g_1(t) = \sin(\pi t)u(t) + \sin(\pi(t-1))u(t-1)$

$$G_1(s) = \frac{\pi}{s^2 + \pi^2}(1 + e^{-s})$$

$$G(s) = \frac{G_1(s)}{1 - e^{-2s}} = \underline{\mathbf{\frac{\pi(1 + e^{-s})}{(s^2 + \pi^2)(1 - e^{-2s})}}}$$

Chapter 15, Solution 21.

$$T = 2\pi$$

Let $f_1(t) = \left(1 - \frac{t}{2\pi}\right)\left[u(t) - u(t-1)\right]$

$$f_1(t) = u(t) - \frac{t}{2\pi}u(t) + \frac{1}{2\pi}(t-1)u(t-1) - \left(1 - \frac{1}{2\pi}\right)u(t-1)$$

$$F_1(s) = \frac{1}{s} - \frac{1}{2\pi s^2} + \frac{e^{-s}}{2\pi s^2} + \left(-1 + \frac{1}{2\pi}\right)e^{-s}\cdot\frac{1}{s} = \frac{\left[2\pi + (-2\pi+1)e^{-s}\right]s + \left[-1 + e^{-s}\right]}{2\pi s^2}$$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}} = \underline{\boldsymbol{\frac{\left[2\pi + (-2\pi+1)e^{-s}\right]s + \left[-1 + e^{-s}\right]}{2\pi s^2(1 - e^{-2\pi s})}}}$$

Chapter 15, Solution 22.

(a) Let $g_1(t) = 2t, \quad 0 < t < 1$

$$= 2t\left[u(t) - u(t-1)\right]$$
$$= 2t\,u(t) - 2(t-1)u(t-1) + 2u(t-1)$$

$$G_1(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} + \frac{2}{s}e^{-s}$$

$$G(s) = \frac{G_1(s)}{1 - e^{-sT}}, \quad T = 1$$

$$G(s) = \underline{\boldsymbol{\frac{2(1 - e^{-s} + s e^{-s})}{s^2(1 - e^{-s})}}}$$

(b) Let $h = h_0 + u(t)$, where h_0 is the periodic triangular wave.

Let h_1 be h_0 within its first period, i.e.

$$h_1(t) = \begin{cases} 2t & 0 < t < 1 \\ 4 - 2t & 1 < t < 2 \end{cases}$$

$$h_1(t) = 2t\,u(t) - 2t\,u(t-1) + 4u(t-1) - 2t\,u(t-1) - 2(t-2)u(t-2)$$
$$h_1(t) = 2t\,u(t) - 4(t-1)u(t-1) - 2(t-2)u(t-2)$$

$$H_1(s) = \frac{2}{s^2} - \frac{4}{s^2}e^{-s} - \frac{2e^{-2s}}{s^2} = \frac{2}{s^2}(1-e^{-s})^2$$

$$H_0(s) = \frac{2}{s^2}\frac{(1-e^{-s})^2}{(1-e^{-2s})}$$

$$H(s) = \underline{\mathbf{\frac{1}{s} + \frac{2}{s^2}\frac{(1-e^{-s})^2}{(1-e^{-2s})}}}$$

Chapter 15, Solution 23.

(a) Let $f_1(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \end{cases}$

$$f_1(t) = \left[u(t) - u(t-1)\right] - \left[u(t-1) - u(t-2)\right]$$
$$f_1(t) = u(t) - 2u(t-1) + u(t-2)$$

$$F_1(s) = \frac{1}{s}(1 - 2e^{-s} + e^{-2s}) = \frac{1}{s}(1-e^{-s})^2$$

$$F(s) = \frac{F_1(s)}{(1-e^{-sT})}, \quad T = 2$$

$$F(s) = \underline{\mathbf{\frac{(1-e^{-s})^2}{s(1-e^{-2s})}}}$$

(b) Let $h_1(t) = t^2\left[u(t) - u(t-2)\right] = t^2 u(t) - t^2 u(t-2)$

$$h_1(t) = t^2 u(t) - (t-2)^2 u(t-2) - 4(t-2)u(t-2) - 4u(t-2)$$

$$H_1(s) = \frac{2}{s^3}(1-e^{-2s}) - \frac{4}{s^2}e^{-2s} - \frac{4}{s}e^{-2s}$$

$$H(s) = \frac{H_1(s)}{(1-e^{-Ts})}, \quad T = 2$$

$$H(s) = \underline{\mathbf{\frac{2(1-e^{-2s}) - 4s\,e^{-2s}(s+s^2)}{s^3(1-e^{-2s})}}}$$

Chapter 15, Solution 24.

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{10s^4 + s}{s^2 + 6s + 5}$$

$$= \lim_{s\to\infty} \frac{10 + \frac{1}{s^3}}{\frac{1}{s^2} + \frac{6}{s^3} + \frac{5}{s^4}} = \frac{10}{0} = \underline{\infty}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{10s^4 + s}{s^2 + 6s + 5} = \underline{\mathbf{0}}$$

(b) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^2 + s}{s^2 - 4s + 6} = \underline{\mathbf{1}}$$

The complex poles are not in the left-half plane.
$f(\infty)$ **does not exist**

(c) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{2s^3 + 7s}{(s+1)(s+2)(s^2 + 2s + 5)}$$

$$= \lim_{s\to\infty} \frac{\frac{2}{s} + \frac{7}{s^3}}{\left(1 + \frac{1}{s}\right)\left(1 + \frac{2}{s}\right)\left(1 + \frac{2}{s} + \frac{5}{s^2}\right)} = \frac{0}{1} = \underline{\mathbf{0}}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{2s^3 + 7s}{(s+1)(s+2)(s^2 + 2s + 5)} = \frac{0}{10} = \underline{\mathbf{0}}$$

Chapter 15, Solution 25.

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{(8)(s+1)(s+3)}{(s+2)(s+4)}$$

$$= \lim_{s\to\infty} \frac{(8)\left(1+\frac{1}{s}\right)\left(1+\frac{3}{s}\right)}{\left(1+\frac{2}{s}\right)\left(1+\frac{4}{s}\right)} = \underline{\mathbf{8}}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{(8)(1)(3)}{(2)(4)} = \underline{\mathbf{3}}$$

(b) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{6s(s-1)}{s^4 - 1}$$

$$f(0) = \lim_{s\to\infty} \frac{6\left(\frac{1}{s^2} - \frac{1}{s^4}\right)}{1 - \frac{1}{s^4}} = \frac{0}{1} = \underline{\mathbf{0}}$$

All poles are not in the left-half plane.
$f(\infty)$ **<u>does not exist</u>**

Chapter 15, Solution 26.

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^3 + 3s}{s^3 + 4s^2 + 6} = \underline{\mathbf{1}}$$

Two poles are not in the left-half plane.
$f(\infty)$ **<u>does not exist</u>**

(b) $f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^3 - 2s^2 + s}{(s-2)(s^2 + 2s + 4)}$

$$= \lim_{s\to\infty} \frac{1 - \frac{2}{s} + \frac{1}{s^2}}{\left(1 - \frac{2}{s}\right)\left(1 + \frac{2}{s} + \frac{4}{s^2}\right)} = \underline{\mathbf{1}}$$

One pole is not in the left-half plane.
$f(\infty)$ **does not exist**

Chapter 15, Solution 27.

(a) $f(t) = \underline{\mathbf{u(t) + 2e^{-t}}}$

(b) $G(s) = \frac{3(s+4) - 11}{s+4} = 3 - \frac{11}{s+4}$

$g(t) = \underline{\mathbf{3\delta(t) - 11e^{-4t}}}$

(c) $H(s) = \frac{4}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$

$A = 2$, $B = -2$

$H(s) = \frac{2}{s+1} - \frac{2}{s+3}$

$h(t) = \underline{\mathbf{2e^{-t} - 2e^{-3t}}}$

(d) $$J(s) = \frac{12}{(s+2)^2(s+4)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s+4}$$

$$B = \frac{12}{2} = 6, \quad C = \frac{12}{(-2)^2} = 3$$

$$12 = A(s+2)(s+4) + B(s+4) + C(s+2)^2$$

Equating coefficients :

s^2: $\quad 0 = A + C \longrightarrow A = -C = -3$

s^1: $\quad 0 = 6A + B + 4C = 2A + B \longrightarrow B = -2A = 6$

s^0: $\quad 12 = 8A + 4B + 4C = -24 + 24 + 12 = 12$

$$J(s) = \frac{-3}{s+2} + \frac{6}{(s+2)^2} + \frac{3}{s+4}$$

$$j(t) = \underline{\mathbf{3\,e^{-4t} - 3\,e^{-2t} + 6\,t\,e^{-2t}}}$$

Chapter 15, Solution 28.

(a)

$$F(s) = \frac{\frac{2(-2)}{2}}{s+3} + \frac{\frac{2(-4)}{-2}}{s+5} = \frac{-2}{s+3} + \frac{4}{s+5}$$

$$f(t) = \underline{(-2e^{-3t} + 4e^{-5t})ut(t)}$$

(b)

$$H(s) = \frac{3s+11}{(s+1)(s^2+2s+5)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+2s+5}$$

$$3s + 11 = A(s^2+2s+5) + (Bs+C)(s+1) = (A+B)s^2 + (2A+B+C)s + 5A + C$$

$$5A + C = 11;\ A = -B;\ -B + C = 3,\ B = C - 3 \rightarrow A = 2;\ B = -2;\ C = 1$$

$$H(s) = \frac{2}{s+1} + \frac{-2s+1}{s^2+2s+5} \rightarrow h(t) = \underline{\left(2e^{-t} - 2e^{-t}\cos 2t + 1.5e^{-t}\sin 2t\right)u(t)}$$

Chapter 15, Solution 29.

$$V(s) = \frac{2}{s} + \frac{As + B}{(s+2)^2 + 3^2};\ 2s^2 + 8s + 26 + As^2 + Bs = 2s + 26 \rightarrow A = -2 \text{ and } B = -6$$

$$V(s) = \frac{2}{s} - \frac{2(s+2)}{(s+2)^2 + 3^2} - \frac{2}{3}\frac{3}{(s+2)^2 + 3^2}$$

$$v(t) = \underline{2u(t) - 2e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t,\ \ t \ge 0}$$

Chapter 15, Solution 30.

(a) $$H_1(s) = \frac{2(s+2)+2}{(s+2)^2 + 3^2} = \frac{2(s+2)}{(s+2)^2 + 3^2} + \frac{2}{3}\frac{3}{(s+2)^2 + 3^2}$$

$$\underline{h_1(t) = 2e^{-2t}\cos 3t + \frac{2}{3}e^{-2t}\sin 3t}$$

(b) $$H_2(s) = \frac{s^2 + 4}{(s+1)^2(s^2 + 2s + 5)} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{Cs + D}{(s^2 + 2s + 5)}$$

$$s^2 + 4 = A(s+1)(s^2 + 2s + 5) + B(s^2 + 2s + 5) + Cs(s+1)^2 + D(s+1)^2$$

or

$$s^2 + 4 = A(s^3 + 3s^2 + 7s + 5) + B(s^2 + 2s + 5) + C(s^3 + 2s^2 + s) + D(s^2 + 2s + 1)$$

Equating coefficients:

s^3: $\quad 0 = A + C \longrightarrow C = -A$

s^2: $\quad 1 = 3A + B + 2C + D = A + B + D$

s: $\quad 0 = 7A + 2B + C + 2D = 6A + 2B + 2D = 4A + 2 \longrightarrow A = -1/2, C = 1/2$

constant: $\quad 4 = 5A + 5B + D = 4A + 4B + 1 \longrightarrow B = 5/4, D = 1/4$

$$H_2(s) = \frac{1}{4}\left[\frac{-2}{(s+1)} + \frac{5}{(s+1)^2} + \frac{2s \;\; +1}{(s^2+2s+5)}\right] = \frac{1}{4}\left[\frac{-2}{(s+1)} + \frac{5}{(s+1)^2} + \frac{2(s \;\; +1)-1}{(s+1)^2+2^2)}\right]$$

Hence,

$$\underline{h_2(t) = \frac{1}{4}\left(-2e^{-t} + 5te^{-t} + 2e^{-t}\cos 2t - 0.5e^{-t}\sin 2t\right)u(t)}$$

(c) $$H_3(s) = \frac{(s+2)e^{-s}}{(s \;\; +1)(s+3)} = e^{-s}\left[\frac{A}{(s \;\; +1)} + \frac{B}{(s+3)}\right] = \frac{1}{2}e^{-s}\left[\frac{1}{(s \;\; +1)} + \frac{1}{(s+3)}\right]$$

$$\underline{h_3(t) = \frac{1}{2}\left(e^{-(t-1)} + e^{-3(t-1)}\right)u(t-1)}$$

Chapter 15, Solution 31.

(a) $$F(s) = \frac{10s}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = F(s)\,(s+1)\Big|_{s=-1} = \frac{-10}{2} = -5$$

$$B = F(s)\,(s+2)\Big|_{s=-2} = \frac{-20}{-1} = 20$$

$$C = F(s)\,(s+3)\Big|_{s=-3} = \frac{-30}{2} = -15$$

$$F(s) = \frac{-5}{s+1} + \frac{20}{s+2} - \frac{15}{s+3}$$

$$f(t) = \underline{\mathbf{-5e^{-t} + 20e^{-2t} - 15e^{-3t}}}$$

(b) $$F(s) = \frac{2s^2+4s+1}{(s+1)(s+2)^3} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+2)^3}$$

$$A = F(s)\,(s+1)\Big|_{s=-1} = -1$$

$$D = F(s)\,(s+2)^3\Big|_{s=-2} = -1$$

$$2s^2 + 4s + 1 = A(s+2)(s^2+4s+4) + B(s+1)(s^2+4s+4)$$

$$+ C(s+1)(s+2) + D(s+1)$$

Equating coefficients :

s^3: $\quad 0 = A + B \longrightarrow B = -A = 1$

s^2: $\quad 2 = 6A + 5B + C = A + C \longrightarrow C = 2 - A = 3$

s^1: $\quad 4 = 12A + 8B + 3C + D = 4A + 3C + D$

$\quad 4 = 6 + A + D \longrightarrow D = -2 - A = -1$

s^0: $\quad 1 = 8A + 4B + 2C + D = 4A + 2C + D = -4 + 6 - 1 = 1$

$$F(s) = \frac{-1}{s+1} + \frac{1}{s+2} + \frac{3}{(s+2)^2} - \frac{1}{(s+2)^3}$$

$$f(t) = -e^{-t} + e^{-2t} + 3\,t\,e^{-2t} - \frac{t^2}{2}e^{-2t}$$

$$f(t) = \underline{\mathbf{-\,e^{-t}} + \left(\mathbf{1 + 3\,t - \frac{t^2}{2}}\right)\mathbf{e^{-2t}}}$$

(c) $$F(s) = \frac{s+1}{(s+2)(s^2+2s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+5}$$

$$A = F(s)\,(s+2)\Big|_{s=-2} = \frac{-1}{5}$$

$$s + 1 = A\,(s^2 + 2s + 5) + B\,(s^2 + 2s) + C\,(s+2)$$

Equating coefficients :

s^2: $\quad 0 = A + B \longrightarrow B = -A = \frac{1}{5}$

s^1: $\quad 1 = 2A + 2B + C = 0 + C \longrightarrow C = 1$

s^0: $\quad 1 = 5A + 2C = -1 + 2 = 1$

$$F(s) = \frac{-1/5}{s+2} + \frac{1/5 \cdot s + 1}{(s+1)^2 + 2^2} = \frac{-1/5}{s+2} + \frac{1/5\,(s+1)}{(s+1)^2 + 2^2} + \frac{4/5}{(s+1)^2 + 2^2}$$

$$f(t) = \underline{\mathbf{-\,0.2\,e^{-2t} + 0.2\,e^{-t}\cos(2t) + 0.4\,e^{-t}\sin(2t)}}$$

Chapter 15, Solution 32.

(a) $$F(s) = \frac{8(s+1)(s+3)}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$A = F(s)\,s\big|_{s=0} = \frac{(8)(3)}{(2)(4)} = 3$$

$$B = F(s)\,(s+2)\big|_{s=-2} = \frac{(8)(-1)}{(-4)} = 2$$

$$C = F(s)\,(s+4)\big|_{s=-4} = \frac{(8)(-1)(-3)}{(-4)(-2)} = 3$$

$$F(s) = \frac{3}{s} + \frac{2}{s+2} + \frac{3}{s+4}$$

$$f(t) = \underline{\mathbf{3\,u(t) + 2\,e^{-2t} + 3\,e^{-4t}}}$$

(b) $$F(s) = \frac{s^2 - 2s + 4}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$s^2 - 2s + 4 = A\,(s^2 + 4s + 4) + B\,(s^2 + 3s + 2) + C\,(s+1)$$

Equating coefficients :

s^2: $\quad 1 = A + B \longrightarrow B = 1 - A$

s^1: $\quad -2 = 4A + 3B + C = 3 + A + C$

s^0: $\quad 4 = 4A + 2B + C = -B - 2 \longrightarrow B = -6$

$A = 1 - B = 7 \qquad\qquad C = -5 - A = -12$

$$F(s) = \frac{7}{s+1} - \frac{6}{s+2} - \frac{12}{(s+2)^2}$$

$$f(t) = \underline{\mathbf{7\,e^{-t} - 6(1 + 2\,t)\,e^{-2t}}}$$

(c) $$F(s) = \frac{s^2 + 1}{(s+3)(s^2 + 4s + 5)} = \frac{A}{s+3} + \frac{Bs + C}{s^2 + 4s + 5}$$

$$s^2 + 1 = A\,(s^2 + 4s + 5) + B\,(s^2 + 3s) + C\,(s+3)$$

Equating coefficients :

$s^2: \quad 1 = A + B \longrightarrow B = 1 - A$

$s^1: \quad 0 = 4A + 3B + C = 3 + A + C \longrightarrow A + C = -3$

$s^0: \quad 1 = 5A + 3C = -9 + 2A \longrightarrow A = 5$

$B = 1 - A = -4 \qquad C = -A - 3 = -8$

$$F(s) = \frac{5}{s+3} - \frac{4s+8}{(s+2)^2+1} = \frac{5}{s+3} - \frac{4(s+2)}{(s+2)^2+1}$$

$f(t) = \underline{\mathbf{5e^{-3t} - 4e^{-2t}\cos(t)}}$

Chapter 15, Solution 33.

(a) $$F(s) = \frac{6(s-1)}{s^4-1} = \frac{6}{(s^2+1)(s+1)} = \frac{As+B}{s^2+1} + \frac{C}{s+1}$$

$$6 = A(s^2+s) + B(s+1) + C(s^2+1)$$

Equating coefficients :

$s^2: \quad 0 = A + C \longrightarrow A = -C$

$s^1: \quad 0 = A + B \longrightarrow B = -A = C$

$s^0: \quad 6 = B + C = 2B \longrightarrow B = 3$

$A = -3, \qquad B = 3, \qquad C = 3$

$$F(s) = \frac{3}{s+1} + \frac{-3s+3}{s^2+1} = \frac{3}{s+1} + \frac{-3s}{s^2+1} + \frac{3}{s^2+1}$$

$f(t) = \underline{\mathbf{3e^{-t} + 3\sin(t) - 3\cos(t)}}$

(b) $$F(s) = \frac{s\,e^{-\pi s}}{s^2+1}$$

$f(t) = \underline{\mathbf{\cos(t-\pi)\,u(t-\pi)}}$

(c) $$F(s) = \frac{8}{s(s+1)^3} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)^3}$$

$A = 8$, $\qquad D = -8$

$$8 = A(s^3 + 3s^2 + 3s + 1) + B(s^3 + 2s^2 + s) + C(s^2 + s) + Ds$$

Equating coefficients :

s^3: $\quad 0 = A + B \longrightarrow B = -A$

s^2: $\quad 0 = 3A + 2B + C = A + C \longrightarrow C = -A = B$

s^1: $\quad 0 = 3A + B + C + D = A + D \longrightarrow D = -A$

s^0: $\quad A = 8, \quad B = -8, \quad C = -8, \quad D = -8$

$$F(s) = \frac{8}{s} - \frac{8}{s+1} - \frac{8}{(s+1)^2} - \frac{8}{(s+1)^3}$$

$$f(t) = \underline{\mathbf{8\left[1 - e^{-t} - t\,e^{-t} - 0.5\,t^2\,e^{-t}\right]u(t)}}$$

Chapter 15, Solution 34.

(a) $$F(s) = 10 + \frac{s^2 + 4 - 3}{s^2 + 4} = 11 - \frac{3}{s^2 + 4}$$

$$f(t) = \underline{\mathbf{11\,\delta(t) - 1.5\sin(2t)}}$$

(b) $$G(s) = \frac{e^{-s} + 4\,e^{-2s}}{(s+2)(s+4)}$$

Let $$\frac{1}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$

$A = 1/2 \qquad B = 1/2$

$$G(s) = \frac{e^{-s}}{2}\left(\frac{1}{s+2} + \frac{1}{s+4}\right) + 2\,e^{-2s}\left(\frac{1}{s+2} + \frac{1}{s+4}\right)$$

$$g(t) = \underline{\mathbf{0.5\left[e^{-2(t-1)} - e^{-4(t-1)}\right]u(t-1) + 2\left[e^{-2(t-2)} - e^{-4(t-2)}\right]u(t-2)}}$$

(c) Let $\frac{s+1}{s(s+3)(s+4)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$

$A = 1/12, \qquad B = 2/3, \qquad C = -3/4$

$$H(s) = \left(\frac{1}{12} \cdot \frac{1}{s} + \frac{2/3}{s+3} - \frac{3/4}{s+4} \right) e^{-2s}$$

$$h(t) = \underline{\left(\mathbf{\frac{1}{12} + \frac{2}{3} e^{-3(t-2)} - \frac{3}{4} e^{-4(t-2)}} \right) \mathbf{u(t-2)}}$$

Chapter 15, Solution 35.

(a) Let $G(s) = \frac{s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$

$A = 2, \qquad B = -1$

$$G(s) = \frac{2}{s+1} - \frac{1}{s+2} \longrightarrow g(t) = 2e^{-t} - e^{-2t}$$

$$F(s) = e^{-6s}\,G(s) \longrightarrow f(t) = g(t-6)\,u(t-6)$$

$$f(t) = \underline{\left[\mathbf{2e^{-(t-6)} - e^{-2(t-6)}} \right] \mathbf{u(t-6)}}$$

(b) Let $G(s) = \frac{1}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4}$

$A = 1/3, \qquad B = -1/3$

$$G(s) = \frac{1}{3(s+1)} - \frac{1}{3(s+4)}$$

$$g(t) = \frac{1}{3}\left[e^{-t} - e^{-4t} \right]$$

$$F(s) = 4\,G(s) - e^{-2t}\,G(s)$$

$$f(t) = 4\,g(t)\,u(t) - g(t-2)\,u(t-2)$$

$$f(t) = \underline{\mathbf{\frac{4}{3}\left[e^{-t} - e^{-4t} \right] u(t) - \frac{1}{3}\left[e^{-(t-2)} - e^{-4(t-2)} \right] u(t-2)}}$$

(c) Let $G(s) = \dfrac{s}{(s+3)(s^2+4)} = \dfrac{A}{s+3} + \dfrac{Bs+C}{s^2+4}$

$A = -3/13$

$s = A(s^2+4) + B(s^2+3s) + C(s+3)$

Equating coefficients :

$s^2: \quad 0 = A + B \longrightarrow B = -A$

$s^1: \quad 1 = 3B + C$

$s^0: \quad 0 = 4A + 3C$

$A = -3/13, \quad B = 3/13, \quad C = 4/13$

$$13\,G(s) = \frac{-3}{s+3} + \frac{3s+4}{s^2+4}$$

$$13\,g(t) = -3e^{-3t} + 3\cos(2t) + 2\sin(2t)$$

$$F(s) = e^{-s}\,G(s)$$

$$f(t) = g(t-1)\,u(t-1)$$

$$f(t) = \underline{\boldsymbol{\frac{1}{13}\left[-3e^{-3(t-1)} + 3\cos(2(t-1)) + 2\sin(2(t-1))\right]u(t-1)}}$$

Chapter 15, Solution 36.

(a) $X(s) = \dfrac{1}{s^2(s+2)(s+3)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s+2} + \dfrac{D}{s+3}$

$B = 1/6, \quad C = 1/4, \quad D = -1/9$

$1 = A(s^3 + 5s^2 + 6s) + B(s^2 + 5s + 6) + C(s^3 + 3s^2) + D(s^3 + 2s^2)$

Equating coefficients :

$s^3: \quad 0 = A + C + D$

$s^2: \quad 0 = 5A + B + 3C + 2D = 3A + B + C$

$s^1: \quad 0 = 6A + 5B$

$s^0: \quad 1 = 6B \longrightarrow B = 1/6$

$$A = -5/6 \, B = -5/36$$

$$X(s) = \frac{-5/36}{s} + \frac{1/6}{s^2} + \frac{1/4}{s+2} - \frac{1/9}{s+3}$$

$$x(t) = \underline{\mathbf{\frac{-5}{36} u(t) + \frac{1}{6} t + \frac{1}{4} e^{-2t} - \frac{1}{9} e^{-3t}}}$$

(b) $$Y(s) = \frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$A = 1, \qquad C = -1$$

$$1 = A(s^2 + 2s + 1) + B(s^2 + s) + Cs$$

Equating coefficients :

s^2: $\quad 0 = A + B \longrightarrow B = -A$

s^1: $\quad 0 = 2A + B + C = A + C \longrightarrow C = -A$

s^0: $\quad 1 = A, \quad B = -1, \quad C = -1$

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$y(t) = \underline{\mathbf{u(t) - e^{-t} - t e^{-t}}}$$

(c) $$Z(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs + D}{s^2 + 6s + 10}$$

$$A = 1/10, \qquad B = -1/5$$

$$1 = A(s^3 + 7s^2 + 16s + 10) + B(s^3 + 6s^2 + 10s) + C(s^3 + s^2) + D(s^2 + s)$$

Equating coefficients :

s^3: $\quad 0 = A + B + C$

s^2: $\quad 0 = 7A + 6B + C + D = 6A + 5B + D$

s^1: $\quad 0 = 16A + 10B + D = 10A + 5B \longrightarrow B = -2A$

s^0: $\quad 1 = 10A \longrightarrow A = 1/10$

$$A = 1/10, \qquad B = -2A = -1/5, \qquad C = A = 1/10, \qquad D = 4A = \frac{4}{10}$$

$$10\,Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+4}{s^2+6s+10}$$

$$10\,Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+3}{(s+3)^2+1} + \frac{1}{(s+3)^2+1}$$

$$z(t) = \underline{\mathbf{0.1\left[\,1 - 2\,e^{-t} + e^{-3t}\cos(t) + e^{-3t}\sin(t)\right]\,u(t)}}$$

Chapter 15, Solution 37.

(a) Let $P(s) = \dfrac{12}{s\,(s^2+4)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+4}$

$$A = P(s)\,s\big|_{s=0} = 12/4 = 3$$

$$12 = A\,(s^2+4) + B\,s^2 + C\,s$$

Equating coefficients :

s^0: $12 = 4A \longrightarrow A = 3$

s^1: $0 = C$

s^2: $0 = A + B \longrightarrow B = -A = -3$

$$P(s) = \frac{3}{s} - \frac{3s}{s^2+4}$$

$$p(t) = 3\,u(t) - 3\cos(2t)$$

$$F(s) = e^{-2s}\,P(s)$$

$$f(t) = \underline{\mathbf{3\left[1 - \cos(2(t-2))\right]u(t-2)}}$$

(b) Let $G(s) = \dfrac{2s+1}{(s^2+1)(s^2+9)} = \dfrac{As+B}{s^2+1} + \dfrac{Cs+D}{s^2+9}$

$$2s+1 = A\,(s^3+9s) + B\,(s^2+9) + C\,(s^3+s) + D\,(s^2+1)$$

Equating coefficients :

s^3: $0 = A + C \longrightarrow C = -A$

$$s^2: \quad 0 = B + D \longrightarrow D = -B$$

$$s^1: \quad 2 = 9A + C = 8A \longrightarrow A = 2/8, \quad C = -2/8$$

$$s^0: \quad 1 = 9B + D = 8B \longrightarrow B = 1/8, \quad D = -1/8$$

$$G(s) = \frac{1}{8}\left(\frac{2s+1}{s^2+1}\right) - \frac{1}{8}\left(\frac{2s+1}{s^2+9}\right)$$

$$G(s) = \frac{1}{4} \cdot \frac{s}{s^2+1} + \frac{1}{8} \cdot \frac{1}{s^2+1} - \frac{1}{4} \cdot \frac{s}{s^2+9} - \frac{1}{8} \cdot \frac{1}{s^2+9}$$

$$g(t) = \underline{\mathbf{\frac{1}{4}\cos(t) + \frac{1}{8}\sin(t) - \frac{1}{4}\cos(3t) - \frac{1}{24}\sin(3t)}}$$

(c) Let $H(s) = \dfrac{9s^2}{s^2 + 4s + 13} = 9 - \dfrac{36s + 117}{s^2 + 4s + 13}$

$$H(s) = 9 - 36 \cdot \frac{s+2}{(s+2)^2 + 3^2} - 15 \cdot \frac{3}{(s+2)^2 + 3^2}$$

$$h(t) = \underline{\mathbf{9\delta(t) - 36e^{-2t}\cos(3t) - 15e^{-2t}\sin(3t)}}$$

Chapter 15, Solution 38.

(a) $F(s) = \dfrac{s^2 + 4s}{s^2 + 10s + 26} = \dfrac{s^2 + 10s + 26 - 6s - 26}{s^2 + 10s + 26}$

$$F(s) = 1 - \frac{6s + 26}{s^2 + 10s + 26}$$

$$F(s) = 1 - \frac{6(s+5)}{(s+5)^2 + 1^2} + \frac{4}{(s+5)^2 + 1^2}$$

$$f(t) = \underline{\mathbf{\delta(t) - 6e^{-t}\cos(5t) + 4e^{-t}\sin(5t)}}$$

(b) $$F(s) = \frac{5s^2 + 7s + 29}{s(s^2 + 4s + 29)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 29}$$

$$5s^2 + 7s + 29 = A(s^2 + 4s + 29) + Bs^2 + Cs$$

Equating coefficients :

s^0: $29 = 29A \longrightarrow A = 1$

s^1: $7 = 4A + C \longrightarrow C = 7 - 4A = 3$

s^2: $5 = A + B \longrightarrow B = 5 - A = 4$

$A = 1, \qquad B = 4, \qquad C = 3$

$$F(s) = \frac{1}{s} + \frac{4s + 3}{s^2 + 4s + 29} = \frac{1}{s} + \frac{4(s + 2)}{(s + 2)^2 + 5^2} - \frac{5}{(s + 2)^2 + 5^2}$$

$$f(t) = \underline{\mathbf{u(t) + 4e^{-2t}\cos(5t) - e^{-2t}\sin(5t)}}$$

Chapter 15, Solution 39.

(a) $$F(s) = \frac{2s^3 + 4s^2 + 1}{(s^2 + 2s + 17)(s^2 + 4s + 20)} = \frac{As + B}{s^2 + 2s + 17} + \frac{Cs + D}{s^2 + 4s + 20}$$

$$s^3 + 4s^2 + 1 = A(s^3 + 4s^2 + 20s) + B(s^2 + 4s + 20)$$
$$+ C(s^3 + 2s^2 + 17s) + D(s^2 + 2s + 17)$$

Equating coefficients :

s^3: $2 = A + C$

s^2: $4 = 4A + B + 2C + D$

s^1: $0 = 20A + 4B + 17C + 2D$

s^0: $1 = 20B + 17D$

Solving these equations (Matlab works well with 4 unknowns),

$A = -1.6, \qquad B = -17.8, \qquad C = 3.6, \qquad D = 21$

$$F(s) = \frac{-1.6s - 17.8}{s^2 + 2s + 17} + \frac{3.6s + 21}{s^2 + 4s + 20}$$

$$F(s) = \frac{(-1.6)(s + 1)}{(s + 1)^2 + 4^2} + \frac{(-4.05)(4)}{(s + 1)^2 + 4^2} + \frac{(3.6)(s + 2)}{(s + 2)^2 + 4^2} + \frac{(3.45)(4)}{(s + 2)^2 + 4^2}$$

$$f(t) = \underline{\mathbf{-1.6e^{-t}\cos(4t) - 4.05e^{-t}\sin(4t) + 3.6e^{-2t}\cos(4t) + 3.45e^{-2t}\sin(4t)}}$$

(b) $$F(s) = \frac{s^2+4}{(s^2+9)(s^2+6s+3)} = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2+6s+3}$$

$$s^2+4 = A(s^3+6s^2+3s) + B(s^2+6s+3) + C(s^3+9s) + D(s^2+9)$$

Equating coefficients :

s^3: $0 = A + C \longrightarrow C = -A$

s^2: $1 = 6A + B + D$

s^1: $0 = 3A + 6B + 9C = 6B + 6C \longrightarrow B = -C = A$

s^0: $4 = 3B + 9D$

Solving these equations,

$A = 1/12$, $B = 1/12$, $C = -1/12$, $D = 5/12$

$$12F(s) = \frac{s+1}{s^2+9} + \frac{-s+5}{s^2+6s+3}$$

$$s^2+6s+3 = 0 \longrightarrow \frac{-6 \pm \sqrt{36-12}}{2} = -0.551, -5.449$$

Let $$G(s) = \frac{-s+5}{s^2+6s+3} = \frac{E}{s+0.551} + \frac{F}{s+5.449}$$

$$E = \frac{-s+5}{s+5.449}\bigg|_{s=-0.551} = 1.133$$

$$F = \frac{-s+5}{s+0.551}\bigg|_{s=-5.449} = -2.133$$

$$G(s) = \frac{1.133}{s+0.551} - \frac{2.133}{s+5.449}$$

$$12F(s) = \frac{s}{s^2+3^2} + \frac{1}{3}\cdot\frac{3}{s^2+3^2} + \frac{1.133}{s+0.551} - \frac{2.133}{s+5.449}$$

$$f(t) = \underline{\mathbf{0.08333\cos(3t) + 0.02778\sin(3t) + 0.0944\,e^{-0.551t} - 0.1778\,e^{-5.449t}}}$$

Chapter 15, Solution 40.

Let $H(s) = \left[\dfrac{4s^2+7s+13}{(s+2)(s^2+2s+5)}\right] = \dfrac{A}{s+2} + \dfrac{Bs+C}{s^2+2s+5}$

$$4s^2+7s+13 = A(s^2+2s+5) + B(s^2+2s) + C(s+2)$$

Equating coefficients gives:

$s^2:\quad 4 = A + B$

$s:\quad 7 = 2A + 2B + C \longrightarrow C = -1$

$\text{constant}:\quad 13 = 5A + 2C \longrightarrow 5A = 15 \text{ or } A = 3,\ B = 1$

$$H(s) = \frac{3}{s+2} + \frac{s-1}{s^2+2s+5} = \frac{3}{s+2} + \frac{(s+1)-2}{(s+1)^2+2^2}$$

Hence,

$$h(t) = 3e^{-2t} + e^{-t}\cos 2t - e^{-t}\sin 2t = 3e^{-2t} + e^{-t}(A\cos\alpha\cos 2t - A\sin\alpha\sin 2t)$$

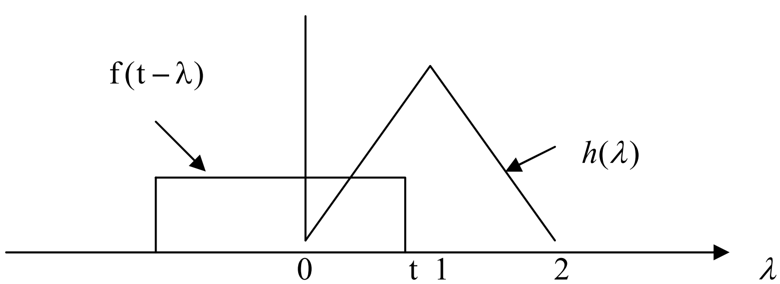
where $A\cos\alpha = 1, \quad A\sin\alpha = 1 \longrightarrow A = \sqrt{2}, \quad \alpha = 45^o$

Thus,

$$h(t) = \underline{\left[\sqrt{2}e^{-t}\cos(2t+45^o) + 3e^{-2t}\right]u(t)}$$

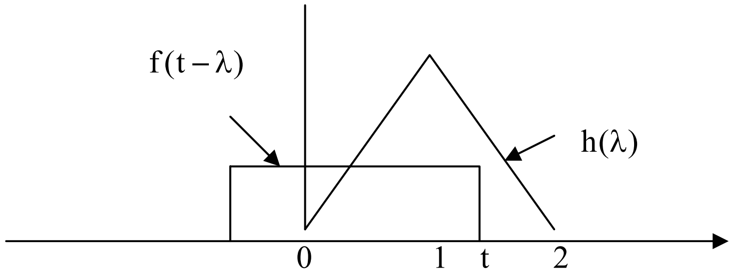
Chapter 15, Solution 41.

Let $y(t) = f(t)*h(t)$. For $0 < t < 1$,



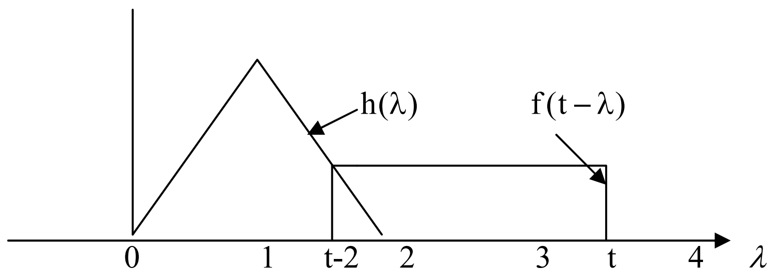
$$y(t) = \int_0^t (1)4\lambda d\lambda = 2\lambda^2 \Big|_0^t = 2t^2$$

For 1 <t<3,



$$y(t)=\int_0^1 (1)4\lambda d\lambda+\int_1^t (1)(8-4\lambda)d\lambda = 2\lambda^2 \Big|_0^t + (8\lambda-2\lambda^2)\Big|_1^t = 8t-2t^2-4$$

For $3<t<4$



$$y(t)=\int_{t-2}^{2}(8-4\lambda)\lambda d\lambda = 8\lambda-2\lambda^2\Big|_{t-2}^{2} = 32-16t+2t^2$$

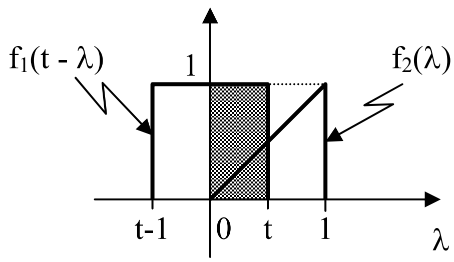
Thus,

$$y(t)=\begin{cases} 2t^2, & 0<t<1 \\ 8t-2t^2-4, & 1<t<3 \\ 32-16t+2t^2, & 3<t<4 \\ 0, & \text{otherwise} \end{cases}$$

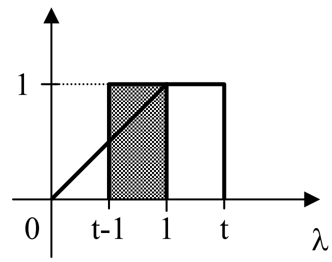
Chapter 15, Solution 42.

(a) For $0 < t < 1$, $f_1(t - \lambda)$ and $f_2(\lambda)$ overlap from 0 to t, as shown in Fig. (a).

$$y(t) = f_1(t) * f_2(t) = \int_0^t (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^t = \frac{t^2}{2}$$



(a)



(b)

For $1 < t < 2$, $f_1(t - \lambda)$ and $f_2(\lambda)$ overlap as shown in Fig. (b).

$$y(t) = \int_{t-1}^1 (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_{t-1}^1 = t - \frac{t^2}{2}$$

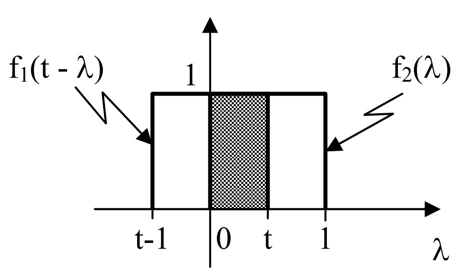
For $t > 2$, there is no overlap.

Therefore,

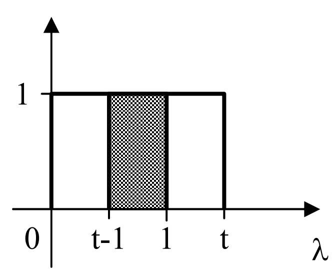
$$y(t) = \begin{cases} \mathbf{t^2/2}, & \mathbf{0 < t < 1} \\ \mathbf{t - t^2/2}, & \mathbf{1 < t < 2} \\ \mathbf{0}, & \textbf{otherwise} \end{cases}$$

(b) For $0 < t < 1$, the two functions overlap as shown in Fig. (c).

$$y(t) = f_1(t) * f_2(t) = \int_0^t (1)(1)\, d\lambda = t$$



(c)



(d)

For $1 < t < 2$, the functions overlap as shown in Fig. (d).

$$y(t) = \int_{t-1}^{1} (1)(1)\, d\lambda = \lambda \Big|_{t-1}^{1} = 2 - t$$

For $t > 2$, there is no overlap.

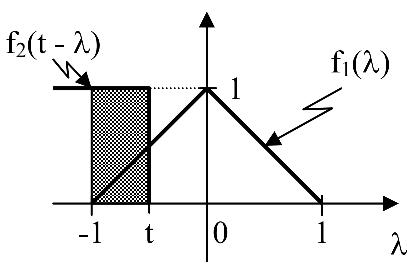
Therefore,

$$y(t) = \begin{cases} \mathbf{t}, & \mathbf{0 < t < 1} \\ \mathbf{2 - t}, & \mathbf{1 < t < 2} \\ \mathbf{0}, & \textbf{otherwise} \end{cases}$$

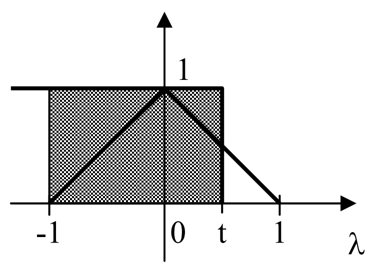
(c) For $t < -1$, there is no overlap. For $-1 < t < 0$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = f_1(t) * f_2(t) = \int_{-1}^{t} (1)(\lambda + 1)\, d\lambda = \left(\frac{\lambda^2}{2} + \lambda \right) \Big|_{-1}^{t}$$

$$y(t) = \frac{1}{2}(t^2 + 2t + 1) = \frac{1}{2}(t+1)^2$$



(e)



(f)

For $0 < t < 1$, the functions overlap as shown in Fig. (f).

$$y(t) = \int_{-1}^{0} (1)(\lambda + 1)\, d\lambda + \int_{0}^{t} (1)(1 - \lambda)\, d\lambda$$

$$y(t) = \left(\frac{\lambda^2}{2} + \lambda \right) \Big|_{-1}^{0} + \left(\lambda - \frac{\lambda^2}{2} \right) \Big|_{0}^{t}$$

$$y(t) = \frac{1}{2}(1 + 2t - t^2)$$

For $t > 1$, the two functions overlap.

$$y(t) = \int_{-1}^{0} (1)(\lambda+1)\, d\lambda + \int_{0}^{1} (1)(1-\lambda)\, d\lambda$$

$$y(t) = \frac{1}{2} + \left(\lambda - \frac{\lambda^2}{2}\right)\Bigg|_0^1 = \frac{1}{2} + 1 - \frac{1}{2} = 1$$

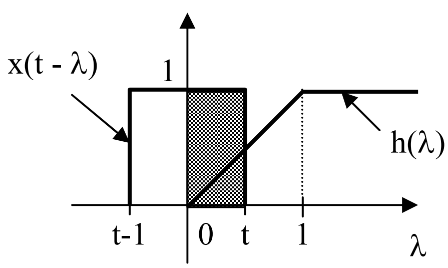
Therefore,

$$y(t) = \begin{cases} \mathbf{0}, & \mathbf{t < -1} \\ \mathbf{0.5(t^2 + 2t + 1)}, & \mathbf{-1 < t < 0} \\ \mathbf{0.5(-t^2 + 2t + 1)}, & \mathbf{0 < t < 1} \\ \mathbf{1}, & \mathbf{t > 1} \end{cases}$$

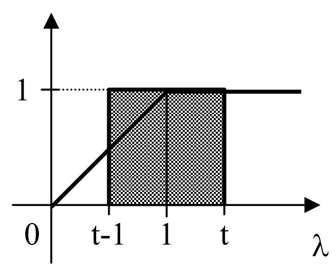
Chapter 15, Solution 43.

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^t = \frac{t^2}{2}$$



(a)



(b)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

$$y(t) = \int_{t-1}^{1} (1)(\lambda)\, d\lambda + \int_1^t (1)(1)\, d\lambda = \frac{\lambda^2}{2}\Big|_{t-1}^{1} + \lambda\Big|_1^t = \frac{-1}{2}t^2 + 2t - 1$$

For $t > 2$, there is a complete overlap so that

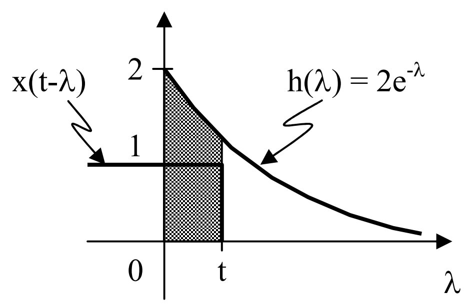
$$y(t) = \int_{t-1}^{t} (1)(1)\, d\lambda = \lambda\Big|_{t-1}^{t} = t - (t-1) = 1$$

Therefore,

$$y(t) = \begin{cases} \mathbf{t^2/2,} & \mathbf{0 < t < 1} \\ \mathbf{-(t^2/2) + 2t - 1,} & \mathbf{1 < t < 2} \\ \mathbf{1,} & \mathbf{t > 2} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

(b) For $t > 0$, the two functions overlap as shown in Fig. (c).

$$y(t) = x(t) * h(t) = \int_0^t (1)\, 2\, e^{-\lambda}\, d\lambda = -2\, e^{-\lambda}\Big|_0^t$$



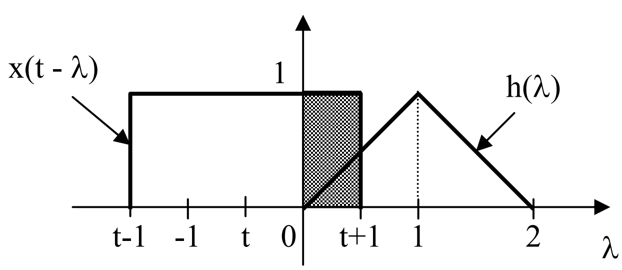
(c)

Therefore,

$$y(t) = \underline{\mathbf{2\,(1 - e^{-t}), \quad t > 0}}$$

(c) For $-1 < t < 0$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (d).

$$y(t) = x(t) * h(t) = \int_0^{t+1} (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^{t+1} = \frac{1}{2}(t+1)^2$$

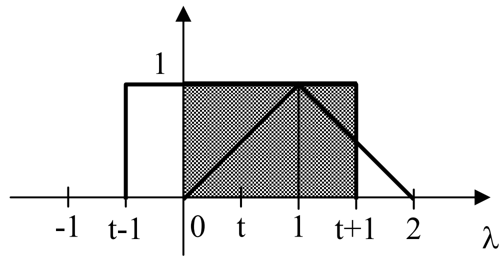


(d)

For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_0^1 (1)(\lambda)\, d\lambda + \int_1^{t+1} (1)(2-\lambda)\, d\lambda$$

$$y(t) = \frac{\lambda^2}{2}\Big|_0^1 + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_1^{t+1} = \frac{-1}{2}t^2 + t + \frac{1}{2}$$

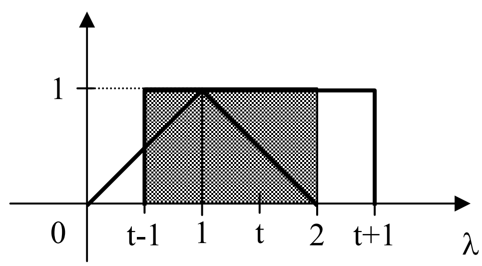


(e)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^{1} (1)(\lambda)\, d\lambda + \int_{1}^{2} (1)(2-\lambda)\, d\lambda$$

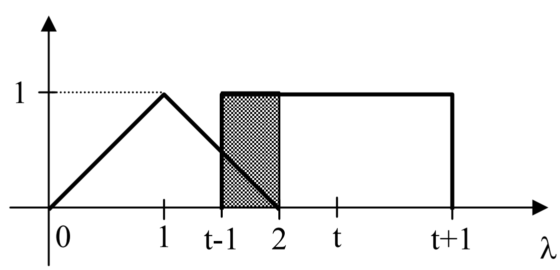
$$y(t) = \frac{\lambda^2}{2}\Big|_{t-1}^1 + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_1^{2} = \frac{-1}{2}t^2 + t + \frac{1}{2}$$



(f)

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (g).

$$y(t) = \int_{t-1}^{2} (1)(2-\lambda)\, d\lambda = \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_{t-1}^{2} = \frac{9}{2} - 3t + \frac{1}{2}t^2$$



(g)

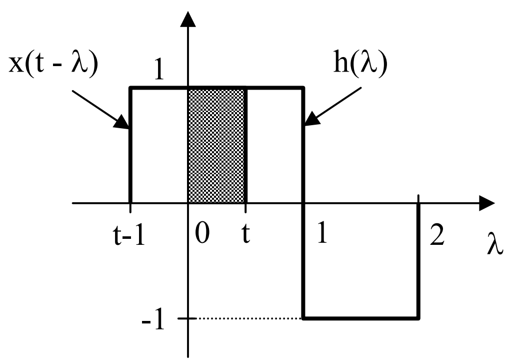
Therefore,

$$y(t) = \begin{cases} (t^2/2) + t + 1/2, & -1 < t < 0 \\ -(t^2/2) + t + 1/2, & 0 < t < 2 \\ (t^2/2) - 3t + 9/2, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$$

Chapter 15, Solution 44.

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(1)\, d\lambda = t$$



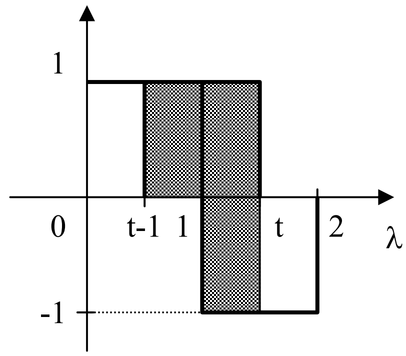
(a)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

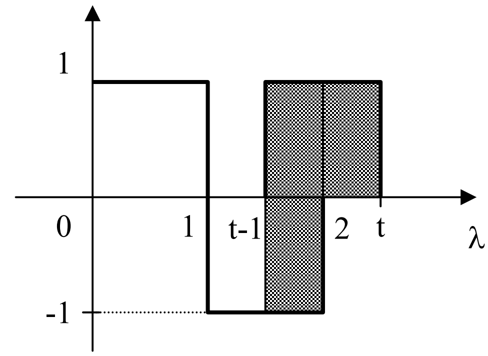
$$y(t) = \int_{t-1}^{1} (1)(1)\, d\lambda + \int_1^t (-1)(1)\, d\lambda = \lambda\Big|_{t-1}^{1} - \lambda\Big|_1^t = 3 - 2t$$

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (c).

$$y(t) = \int_{t-1}^{2} (1)(-1)\, d\lambda = -\lambda\Big|_{t-1}^{2} = t - 3$$



(b)



(c)

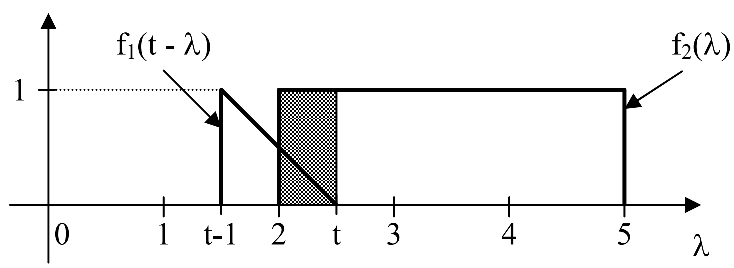
Therefore,

$$y(t) = \begin{cases} \mathbf{t}, & \mathbf{0 < t < 1} \\ \mathbf{3 - 2t}, & \mathbf{1 < t < 2} \\ \mathbf{t - 3}, & \mathbf{2 < t < 3} \\ \mathbf{0}, & \mathbf{otherwise} \end{cases}$$

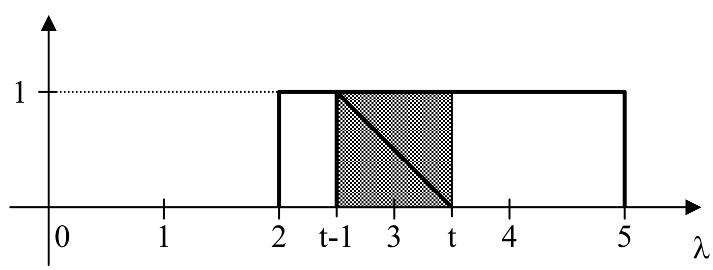
(b) For $t < 2$, there is no overlap. For $2 < t < 3$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap, as shown in Fig. (d).

$$y(t) = f_1(t) * f_2(t) = \int_2^t (1)(t-\lambda)\, d\lambda$$

$$= \left(\lambda t - \frac{\lambda^2}{2}\right)\Bigg|_2^t = \frac{t^2}{2} - 2t + 2$$



(d)



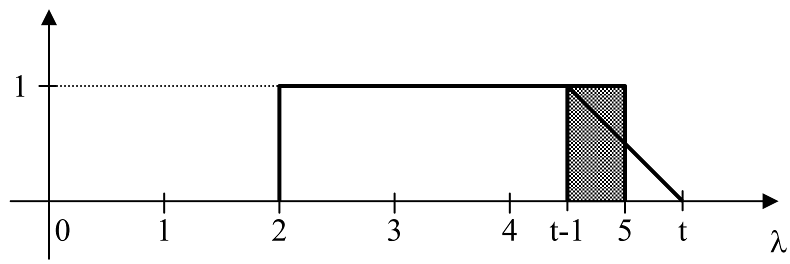
(e)

For $3 < t < 5$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_{t-1}^{t} (1)(t-\lambda)\, d\lambda = \left(\lambda t - \frac{\lambda^2}{2}\right)\Bigg|_{t-1}^{t} = \frac{1}{2}$$

For $5 < t < 6$, the functions overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^{5} (1)(t-\lambda)\, d\lambda = \left(\lambda t - \frac{\lambda^2}{2}\right)\Bigg|_{t-1}^{5} = \frac{-1}{2}t^2 + 5t - 12$$



(f)

Therefore,

$$y(t) = \begin{cases} \mathbf{(t^2/2) - 2t + 2,} & \mathbf{2 < t < 3} \\ \mathbf{1/2,} & \mathbf{3 < t < 5} \\ \mathbf{-(t^2/2) + 5t - 12,} & \mathbf{5 < t < 6} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

Chapter 15, Solution 45.

(a) $f(t) * \delta(t) = \int_0^t f(\lambda)\delta(t-\lambda)\, d\lambda = f(\lambda)\big|_{\lambda=t}$

$$\mathbf{f(t) * \delta(t) = f(t)}$$

(b) $f(t) * u(t) = \int_0^t f(\lambda) u(t-\lambda)\, d\lambda$

Since $u(t-\lambda) = \begin{cases} 1 & \lambda < t \\ 0 & \lambda > t \end{cases}$

$$\mathbf{f(t) * u(t) = \int_0^t f(\lambda)\, d\lambda}$$

Alternatively,

$$\mathsf{L}\{f(t) * u(t)\} = \frac{F(s)}{s}$$

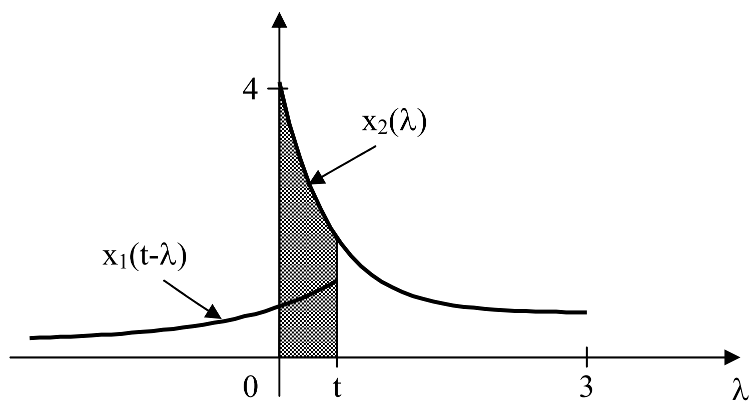
$$\mathsf{L}^{-1}\left\{\frac{F(s)}{s}\right\} = f(t) * u(t) = \int_o^t f(\lambda)\, d\lambda$$

Chapter 15, Solution 46.

(a) Let $y(t) = x_1(t) * x_2(t) = \int_0^t x_2(t)\, x_1(t-\lambda)\, d\lambda$

For $0 < t < 3$, $x_1(t-\lambda)$ and $x_2(\lambda)$ overlap as shown in Fig. (a).

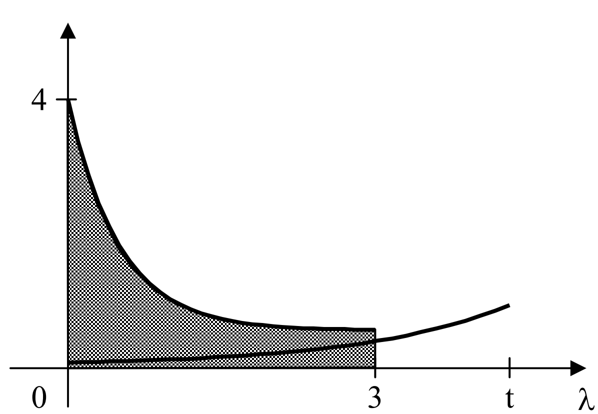
$$y(t) = \int_0^t 4\, e^{-2\lambda}\, e^{-(t-\lambda)}\, d\lambda = 4\, e^{-t} \int_0^t e^{-\lambda}\, d\lambda = 4\,(e^{-t} - e^{-2t})$$



(a)

For $t > 3$, the two functions overlap as shown in Fig. (b).

$$y(t) = \int_0^3 4\, e^{-2\lambda}\, e^{-(t-\lambda)}\, d\lambda = 4\, e^{-t}\left(-\, e^{-\lambda}\right)\Big|_0^3 = 4\, e^{-t}\,(1 - e^{-3})$$



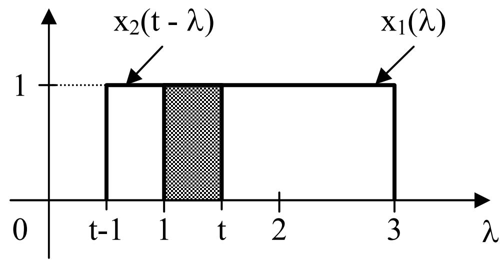
(b)

Therefore,

$$y(t) = \begin{cases} \mathbf{4(e^{-t} - e^{-2t}),} & \mathbf{0 < t < 3} \\ \mathbf{4e^{-t}(1 - e^{-3}),} & \mathbf{t > 3} \end{cases}$$

(b) For $1 < t < 2$, $x_1(\lambda)$ and $x_2(t-\lambda)$ overlap as shown in Fig. (c).

$$y(t) = x_1(t) * x_2(t) = \int_1^t (1)(1)\, d\lambda = \lambda\Big|_1^t = t - 1$$



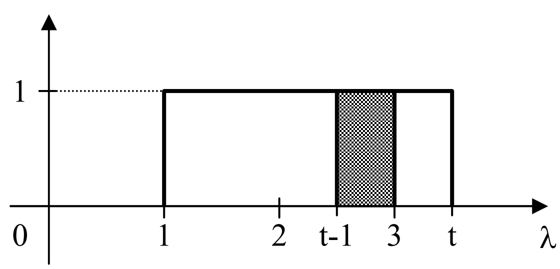
(c)

For $2 < t < 3$, the two functions overlap completely.

$$y(t) = \int_{t-1}^t (1)(1)\, d\lambda = \lambda\Big|_{t-1}^t = t - (t-1) = 1$$

For $3 < t < 4$, the two functions overlap as shown in Fig. (d).

$$y(t) = \int_{t-1}^3 (1)(1)\, d\lambda = \lambda\Big|_{t-1}^3 = 4 - t$$



(d)

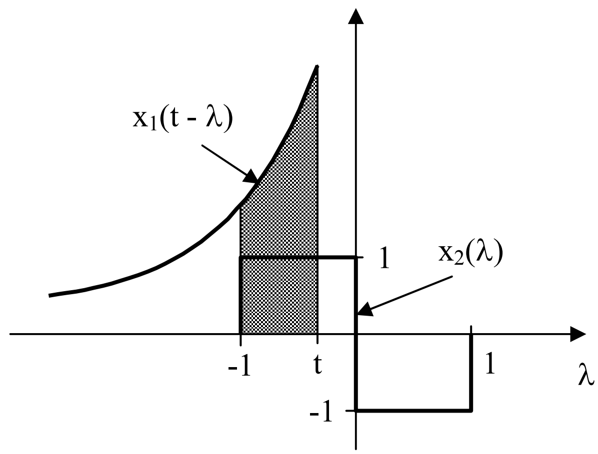
Therefore,

$$y(t) = \begin{cases} \mathbf{t - 1,} & \mathbf{1 < t < 2} \\ \mathbf{1,} & \mathbf{2 < t < 3} \\ \mathbf{4 - t,} & \mathbf{3 < t < 4} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

(c) For $-1 < t < 0$, $x_1(t-\lambda)$ and $x_2(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = x_1(t) * x_2(t) = \int_{-1}^t (1)\, 4\, e^{-(t-\lambda)}\, d\lambda$$

$$y(t) = 4e^{-t}\int_{-1}^{t} e^{\lambda}\, d\lambda = 4\left[1 - e^{-(t+1)}\right]$$



(e)

For $0 < t < 1$,

$$y(t) = \int_{-1}^{0} (1)\,4e^{-(t-\lambda)}\, d\lambda + \int_{0}^{t} (-1)\,4e^{-(t-\lambda)}\, d\lambda$$

$$y(t) = 4e^{-t}\, e^{\lambda}\Big|_{-1}^{0} - 4e^{-t}\, e^{\lambda}\Big|_{0}^{t} = 8e^{-t} - 4e^{-(t+1)} - 4$$

For $t > 1$, the two functions overlap completely.

$$y(t) = \int_{-1}^{0} (1)\,4e^{-(t-\lambda)}\, d\lambda + \int_{0}^{1} (-1)\,4e^{-(t-\lambda)}\, d\lambda$$

$$y(t) = 4e^{-t}\, e^{\lambda}\Big|_{-1}^{0} - 4e^{-t}\, e^{\lambda}\Big|_{0}^{1} = 8e^{-t} - 4e^{-(t+1)} - 4e^{-(t-1)}$$

Therefore,

$$y(t) = \begin{cases} \mathbf{4\left[1 - e^{-(t+1)}\right],} & \mathbf{-1 < t < 0} \\ \mathbf{8e^{-t} - 4e^{-(t+1)} - 4,} & \mathbf{0 < t < 1} \\ \mathbf{8e^{-t} - 4e^{-(t+1)} - 4e^{-(t-1)},} & \mathbf{t > 1} \end{cases}$$

Chapter 15, Solution 47.

$$f_1(t) = f_2(t) = \cos(t)$$

$$L^{-1}\left[F_1(s)F_2(s)\right] = \int_{0}^{t} \cos(\lambda)\cos(t - \lambda)\, d\lambda$$

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A + B) + \cos(A - B)\right]$$

$$\mathcal{L}^{-1}\left[F_1(s)F_2(s)\right] = \frac{1}{2}\int_0^t [\cos(t) + \cos(t - 2\lambda)]\, d\lambda$$

$$\mathcal{L}^{-1}\left[F_1(s)F_2(s)\right] = \frac{1}{2}\cos(t)\cdot\lambda\Big|_0^t + \frac{1}{2}\cdot\frac{\sin(t-2\lambda)}{-2}\Big|_0^t$$

$$\mathcal{L}^{-1}\left[F_1(s)F_2(s)\right] = \underline{\mathbf{0.5\,t\cos(t) + 0.5\sin(t)}}$$

Chapter 15, Solution 48.

(a) Let $G(s) = \dfrac{2}{s^2 + 2s + 5} = \dfrac{2}{(s+1)^2 + 2^2}$

$$g(t) = e^{-t}\sin(2t)$$

$$F(s) = G(s)\,G(s)$$

$$f(t) = \mathcal{L}^{-1}\left[G(s)\,G(s)\right] = \int_0^t g(\lambda)\,g(t-\lambda)\, d\lambda$$

$$f(t) = \int_0^t e^{-\lambda}\sin(2\lambda)\,e^{-(t-\lambda)}\sin(2(t-\lambda))\, d\lambda$$

$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$f(t) = \frac{1}{2}e^{-t}\int_0^t e^{-\lambda}\left[\cos(2t) - \cos(2(t-2\lambda))\right] d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\, d\lambda - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\cos(2t-4\lambda)\, d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\cdot\frac{e^{-2\lambda}}{-2}\Big|_0^t - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\left[\cos(2t)\cos(4\lambda) + \sin(2t)\sin(4\lambda)\right] d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)\,(-e^{-2t}+1) - \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\cos(4\lambda)\, d\lambda$$

$$-\frac{e^{-t}}{2}\sin(2t)\int_0^t e^{-2\lambda}\sin(4\lambda)\, d\lambda$$

$$f(t) = \frac{1}{4} e^{-t} \cos(2t)\,(1 - e^{-2t})$$

$$- \frac{e^{-t}}{2} \cos(2t) \left[\frac{e^{-2\lambda}}{4+16} \left(-2\cos(4\lambda) - 4\sin(4\lambda) \right) \right]_0^t$$

$$- \frac{e^{-t}}{2} \sin(2t) \left[\frac{e^{-2\lambda}}{4+16} \left(-2\sin(4\lambda) + 4\cos(4\lambda) \right) \right]_0^t$$

$$f(t) = \underline{\mathbf{\frac{e^{-t}}{2} \cos(2t) - \frac{e^{-3t}}{4} \cos(2t) - \frac{e^{-t}}{20} \cos(2t) + \frac{e^{-3t}}{20} \cos(2t)\cos(4t)}}$$

$$\underline{\mathbf{+ \frac{e^{-3t}}{10} \cos(2t)\sin(4t) + \frac{e^{-t}}{10} \sin(2t)}}$$

$$\underline{\mathbf{+ \frac{e^{-t}}{20} \sin(2t)\sin(4t) - \frac{e^{-t}}{10} \sin(2t)\cos(4t)}}$$

(b) Let $X(s) = \frac{2}{s+1}$, $\qquad Y(s) = \frac{s}{s+4}$

$x(t) = 2\,e^{-t}\,u(t)$, $\qquad y(t) = \cos(2t)\,u(t)$

$$F(s) = X(s)\,Y(s)$$

$$f(t) = L^{-1}\left[X(s)\,Y(s) \right] = \int_0^\infty y(\lambda)\,x(t-\lambda)\,d\lambda$$

$$f(t) = \int_0^t \cos(2\lambda) \cdot 2\,e^{-(t-\lambda)}\,d\lambda$$

$$f(t) = 2\,e^{-t} \cdot \frac{e^{\lambda}}{1+4} \left(\cos(2\lambda) + 2\sin(2\lambda) \right) \Big|_0^t$$

$$f(t) = \frac{2}{5} e^{-t} \left[e^{t} \left(\cos(2t) + 2\sin(2t) - 1 \right) \right]$$

$$f(t) = \underline{\mathbf{\frac{2}{5} \cos(2t) + \frac{4}{5} \sin(2t) - \frac{2}{5} e^{-t}}}$$

Chapter 15, Solution 49.

Let x(t) = u(t) – u(t-1) and y(t) = h(t)*x(t).

$$y(t) = \mathsf{L}^{-1}[H(s)X(s)] = \mathsf{L}^{-1}\left[\frac{4}{s+2}(\frac{1}{s} - \frac{e^{-s}}{s})\right] = \mathsf{L}^{-1}\left[\frac{4(1-e^{-s})}{s(s+2)}\right]$$

But

$$\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} = \frac{1}{2}\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$Y(s) = 2\left[\frac{1}{s} - \frac{1}{s+2} - \frac{e^{-s}}{s} + \frac{e^{-s}}{s+2}\right]$$

$$y(t) = \underline{2[1 - e^{-2t}]u(t) - 4[1 - e^{-2(t-1)}]u(t-1)}$$

Chapter 15, Solution 50.

Take the Laplace transform of each term.

$$\left[s^2\,V(s) - s\,v(0) - v'(0)\right] + 2\left[s\,V(s) - v(0)\right] + 10\,V(s) = \frac{3s}{s^2+4}$$

$$s^2\,V(s) - s + 2 + 2s\,V(s) - 2 + 10\,V(s) = \frac{3s}{s^2+4}$$

$$(s^2 + 2s + 10)\,V(s) = s + \frac{3s}{s^2+4} = \frac{s^3 + 7s}{s^2+4}$$

$$V(s) = \frac{s^3 + 7s}{(s^2+4)(s^2+2s+10)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+2s+10}$$

$$s^3 + 7s = A\,(s^3 + 2s^2 + 10s) + B\,(s^2 + 2s + 10) + C\,(s^3 + 4s) + D\,(s^2 + 4)$$

Equating coefficients :

s^3: $\quad 1 = A + C \longrightarrow C = 1 - A$

s^2: $\quad 0 = 2A + B + D$

s^1: $\quad 7 = 10A + 2B + 4C = 6A + 2B + 4$

$s^0: \quad 0 = 10B + 4D \longrightarrow D = \text{-}2.5B$

Solving these equations yields

$$A = \frac{9}{26}, \qquad B = \frac{12}{26}, \qquad C = \frac{17}{26}, \qquad D = \frac{-30}{26}$$

$$V(s) = \frac{1}{26}\left[\frac{9s+12}{s^2+4} + \frac{17s-30}{s^2+2s+10}\right]$$

$$V(s) = \frac{1}{26}\left[\frac{9s}{s^2+4} + 6\cdot\frac{2}{s^2+4} + 17\cdot\frac{s+1}{(s+1)^2+3^2} - \frac{47}{(s+1)^2+3^2}\right]$$

$$v(t) = \underline{\mathbf{\frac{9}{26}\cos(2t) + \frac{6}{26}\sin(2t) + \frac{17}{26}e^{-t}\cos(3t) - \frac{47}{78}e^{-t}\sin(3t)}}$$

Chapter 15, Solution 51.

Taking the Laplace transform of the differential equation yields

$$\left[s^2V(s) - sv(0) - v'(0)\right] + 5[sV(s) - v(0)]] + 6V(s) = \frac{10}{s+1}$$

or $\left(s^2 + 5s + 6\right)V(s) - 2s - 4 - 10 = \dfrac{10}{s+1} \longrightarrow V(s) = \dfrac{2s^2 + 16s + 24}{(s+1)(s+2)(s+3)}$

Let $V(s) = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3}, \quad A = 5, \quad B = 0, \quad C = -3$

Hence,

$$v(t) = \underline{(5e^{-t} - 3e^{-3t})u(t)}$$

Chapter 15, Solution 52.

Take the Laplace transform of each term.

$$\left[s^2\, I(s) - s\, i(0) - i'(0) \right] + 3\left[s\, I(s) - i(0) \right] + 2\, I(s) + 1 = 0$$

$$(s^2 + 3s + 2)\, I(s) - s - 3 - 3 + 1 = 0$$

$$I(s) = \frac{s+5}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = 4, \qquad B = -3$$

$$I(s) = \frac{4}{s+1} - \frac{3}{s+2}$$

$$i(t) = \underline{\mathbf{(4\,e^{-t} - 3\,e^{-2t})\,u(t)}}$$

Chapter 15, Solution 53.

Take the Laplace transform of each term.

$$\left[s^2\, Y(s) - s\, y(0) - y'(0) \right] + 5\left[s\, Y(s) - y(0) \right] + 6\, V(s) = \frac{s}{s^2 + 4}$$

$$(s^2 + 5s + 6)\, Y(s) - s - 4 - 5 = \frac{s}{s^2 + 4}$$

$$(s^2 + 5s + 6)\, Y(s) = s + 9 + \frac{s}{s^2 + 4} = \frac{s + (s+9)(s^2 + 4)}{s^2 + 4}$$

$$Y(s) = \frac{s^3 + 9s^2 + 5s + 36}{(s+2)(s+3)(s^2 + 4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{Cs + D}{s^2 + 4}$$

$$A = (s+2)\, Y(s)\Big|_{s=-2} = \frac{27}{4}, \qquad B = (s+3)\, Y(s)\Big|_{s=-3} = \frac{-75}{13}$$

When $s = 0$,

$$\frac{36}{(2)(3)(4)} = \frac{A}{2} + \frac{B}{3} + \frac{D}{4} \longrightarrow D = \frac{5}{26}$$

When $s = 1$,

$$\frac{46+5}{(12)(5)} = \frac{A}{3} + \frac{B}{4} + \frac{C}{5} + \frac{D}{5} \longrightarrow C = \frac{1}{52}$$

Thus, $Y(s) = \dfrac{27/4}{s+2} - \dfrac{75/13}{s+3} + \dfrac{1/52 \cdot s + 5/26}{s^2+4}$

$$y(t) = \underline{\mathbf{\frac{27}{4} e^{-2t} - \frac{75}{13} e^{-3t} + \frac{1}{52}\cos(2t) + \frac{5}{52}\sin(2t)}}$$

Chapter 15, Solution 54.

Taking the Laplace transform of the differential equation gives

$$\left[s^2 V(s) - s\,v(0) - v'(0) \right] + 3\left[s\,V(s) - v(0) \right] + 2\,V(s) = \frac{5}{s+3}$$

$$(s^2 + 3s + 2)\,V(s) = \frac{5}{s+3} - 1 = \frac{2-s}{s+3}$$

$$V(s) = \frac{2-s}{(s+3)(s^2+3s+2)} = \frac{2-s}{(s+1)(s+2)(s+3)}$$

$$V(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$A = 3/2$, $\quad B = -4$, $\quad C = 5/2$

$$V(s) = \frac{3/2}{s+1} - \frac{4}{s+2} + \frac{5/2}{s+3}$$

$$v(t) = \underline{\mathbf{(1.5 e^{-t} - 4 e^{-2t} + 2.5 e^{-3t})\,u(t)}}$$

Chapter 15, Solution 55.

Take the Laplace transform of each term.

$$\left[s^3\,Y(s) - s^2\,y(0) - s\,y'(0) - y''(0)\right] + 6\left[s^2\,Y(s) - s\,y(0) - y'(0)\right]$$

$$+8\left[s\,Y(s) - y(0)\right] = \frac{s+1}{(s+1)^2 + 2^2}$$

Setting the initial conditions to zero gives

$$(s^3 + 6s^2 + 8s)\,Y(s) = \frac{s+1}{s^2 + 2s + 5}$$

$$Y(s) = \frac{(s+1)}{s\,(s+2)(s+4)(s^2+2s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4} + \frac{Ds+E}{s^2+2s+5}$$

$$A = \frac{1}{40}, \qquad B = \frac{1}{20}, \qquad C = \frac{-3}{104}, \qquad D = \frac{-3}{65}, \qquad E = \frac{-7}{65}$$

$$Y(s) = \frac{1}{40}\cdot\frac{1}{s} + \frac{1}{20}\cdot\frac{1}{s+2} - \frac{3}{104}\cdot\frac{1}{s+4} - \frac{1}{65}\cdot\frac{3s+7}{(s+1)^2+2^2}$$

$$Y(s) = \frac{1}{40}\cdot\frac{1}{s} + \frac{1}{20}\cdot\frac{1}{s+2} - \frac{3}{104}\cdot\frac{1}{s+4} - \frac{1}{65}\cdot\frac{3(s+1)}{(s+1)^2+2^2} - \frac{1}{65}\cdot\frac{4}{(s+1)^2+2^2}$$

$$y(t) = \underline{\mathbf{\frac{1}{40}u(t) + \frac{1}{20}e^{-2t} - \frac{3}{104}e^{-4t} - \frac{3}{65}e^{-t}\cos(2t) - \frac{2}{65}e^{-t}\sin(2t)}}$$

Chapter 15, Solution 56.

Taking the Laplace transform of each term we get:

$$4\left[s\,V(s) - v(0)\right] + \frac{12}{s}V(s) = 0$$

$$\left[4s + \frac{12}{s}\right]V(s) = 8$$

$$V(s) = \frac{8s}{4s^2 + 12} = \frac{2s}{s^2 + 3}$$

$$v(t) = \underline{\mathbf{2\cos\left(\sqrt{3}t\right)}}$$

Chapter 15, Solution 57.

Take the Laplace transform of each term.

$$\left[s\,Y(s) - y(0)\right] + \frac{9}{s}Y(s) = \frac{s}{s^2+4}$$

$$\left(\frac{s^2+9}{s}\right)Y(s) = 1 + \frac{s}{s^2+4} = \frac{s^2+s+4}{s^2+4}$$

$$Y(s) = \frac{s^3+s^2+4s}{(s^2+4)(s^2+9)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+9}$$

$$s^3+s^2+4s = A\,(s^3+9s) + B\,(s^2+9) + C\,(s^3+4s) + D\,(s^2+4)$$

Equating coefficients :

s^0 : $0 = 9B + 4D$

s^1 : $4 = 9A + 4C$

s^2 : $1 = B + D$

s^3 : $1 = A + C$

Solving these equations gives

$A = 0$, $B = -4/5$, $C = 1$, $D = 9/5$

$$Y(s) = \frac{-4/5}{s^2+4} + \frac{s+9/5}{s^2+9} = \frac{-4/5}{s^2+4} + \frac{s}{s^2+9} + \frac{9/5}{s^2+9}$$

$$y(t) = \underline{\mathbf{-0.4\sin(2t) + \cos(3t) + 0.6\sin(3t)}}$$

Chapter 15, Solution 58.

We take the Laplace transform of each term and obtain

$$6V(s) + [sV(s) - v(0)] + \frac{10}{s}V(s) = e^{-2s} \longrightarrow V(s) = \frac{se^{-2s}}{s^2+6s+10}$$

$$V(s) = \frac{(s+3)e^{-2s} - 3e^{-2s}}{(s+3)^2+1}$$

Hence,

$$v(t) = \underline{\left[e^{-3(t-2)} \cos(t-2) - 3e^{-3(t-2)} \sin(t-2) \right] u(t-2)}$$

Chapter 15, Solution 59.

Take the Laplace transform of each term of the integrodifferential equation.

$$\left[s\,Y(s) - y(0) \right] + 4\,Y(s) + \frac{3}{s} Y(s) = \frac{6}{s+2}$$

$$(s^2 + 4s + 3)\,Y(s) = s\left(\frac{6}{s+2} - 1 \right)$$

$$Y(s) = \frac{s\,(4-s)}{(s+2)(s^2+4s+3)} = \frac{(4-s)\,s}{(s+1)(s+2)(s+3)}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$A = 2.5$, $\quad B = 6$, $\quad C = -10.5$

$$Y(s) = \frac{2.5}{s+1} + \frac{6}{s+2} - \frac{10.5}{s+3}$$

$$y(t) = \underline{\mathbf{2.5\,e^{-t} + 6\,e^{-2t} - 10.5\,e^{-3t}}}$$

Chapter 15, Solution 60.

Take the Laplace transform of each term of the integrodifferential equation.

$$2\left[s\,X(s) - x(0) \right] + 5\,X(s) + \frac{3}{s} X(s) + \frac{4}{s} = \frac{4}{s^2+16}$$

$$(2s^2 + 5s + 3)\,X(s) = 2s - 4 + \frac{4s}{s^2+16} = \frac{2s^3 - 4s^2 + 36s - 64}{s^2+16}$$

$$X(s) = \frac{2s^3 - 4s^2 + 36s - 64}{(2s^2+5s+3)(s^2+16)} = \frac{s^3 - 2s^2 + 18s - 32}{(s+1)(s+1.5)(s^2+16)}$$

$$X(s) = \frac{A}{s+1} + \frac{B}{s+1.5} + \frac{Cs+D}{s^2+16}$$

$$A = (s+1)\,X(s)\Big|_{s=-1} = -6.235$$

$$B = (s+1.5)\,X(s)\Big|_{s=-1.5} = 7.329$$

When $s = 0$,

$$\frac{-32}{(1.5)(16)} = A + \frac{B}{1.5} + \frac{D}{16} \quad \longrightarrow \quad D = 0.2579$$

$$\begin{aligned} s^3 - 2s^2 + 18s - 32 &= A\,(s^3 + 1.5s^2 + 16s + 24) + B\,(s^3 + s^2 + 16s + 16) \\ &\quad + C\,(s^3 + 2.5s^2 + 1.5s) + D\,(s^2 + 2.5s + 1.5) \end{aligned}$$

Equating coefficients of the s^3 terms,

$$1 = A + B + C \quad \longrightarrow \quad C = -0.0935$$

$$X(s) = \frac{-6.235}{s+1} + \frac{7.329}{s+1.5} + \frac{-0.0935s + 0.2579}{s^2+16}$$

$$x(t) = \underline{\mathbf{-6.235\,e^{-t} + 7.329\,e^{-1.5t} - 0.0935\cos(4t) + 0.0645\sin(4t)}}$$