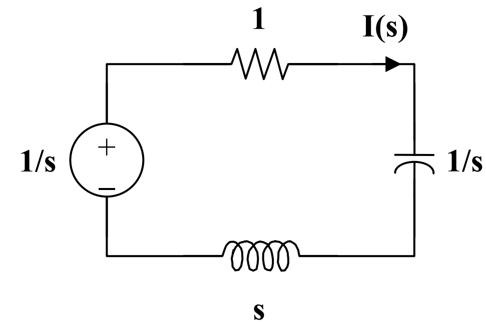
Chapter 16, Solution 1.

Consider the s-domain form of the circuit which is shown below.

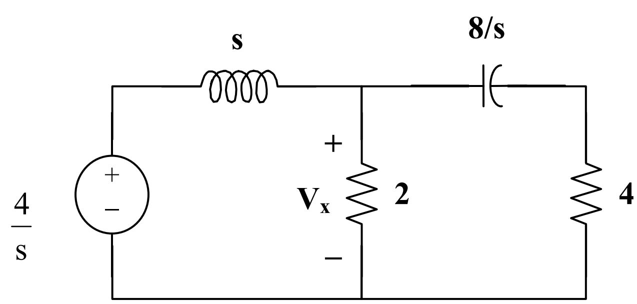


$$I(s) = \frac{1/s}{1+s+1/s} = \frac{1}{s^2+s+1} = \frac{1}{(s+1/2)^2+(\sqrt{3}/2)^2}$$

$$i(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

$$i(t) = \underline{\mathbf{1.155 e^{-0.5t} \sin(0.866t)\ A}}$$

Chapter 16, Solution 2.



$$\frac{V_x - \frac{4}{s}}{s} + \frac{V_x - 0}{2} + \frac{V_x - 0}{4 + \frac{8}{s}} = 0$$

$$V_x(4s+8) - \frac{(16s+32)}{s} + (2s^2+4s)V_x + s^2 V_x = 0$$

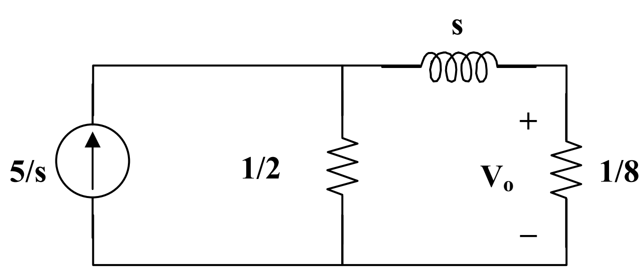
$$V_x(3s^2+8s+8) = \frac{16s+32}{s}$$

$$V_x = -16\frac{s+2}{s(3s^2+8s+8)} = -16\left(\frac{0.25}{s} + \frac{-0.125}{s + \frac{4}{3} + j\frac{\sqrt{8}}{3}} + \frac{-0.125}{s + \frac{4}{3} - j\frac{\sqrt{8}}{3}}\right)$$

$$v_x = \underline{(-4 + 2e^{-(1.3333 + j0.9428)t} + 2e^{-(1.3333 - j0.9428)t})u(t)\ \text{V}}$$

$$v_x = \underline{4u(t) - e^{-4t/3}\cos\left(\frac{2\sqrt{2}}{3}t\right) - \frac{6}{\sqrt{2}}e^{-4t/3}\sin\left(\frac{2\sqrt{2}}{3}t\right)\ \text{V}}$$

Chapter 16, Solution 3.



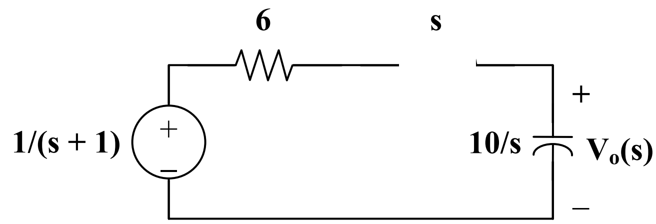
Current division leads to:

$$V_o = \frac{1}{8}\frac{5}{s}\left(\frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{8} + s}\right) = \frac{5}{10+16s} = \frac{5}{16(s+0.625)}$$

$$v_o(t) = \underline{0.3125\left(1 - e^{-0.625t}\right)u(t)\ \text{V}}$$

Chapter 16, Solution 4.

The s-domain form of the circuit is shown below.



Using voltage division,

$$V_o(s) = \frac{10/s}{s+6+10/s}\left(\frac{1}{s+1}\right) = \frac{10}{s^2+6s+10}\left(\frac{1}{s+1}\right)$$

$$V_o(s) = \frac{10}{(s+1)(s^2+6s+10)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+6s+10}$$

$$10 = A(s^2+6s+10) + B(s^2+s) + C(s+1)$$

Equating coefficients :

s^2 : $\quad 0 = A + B \longrightarrow B = -A$

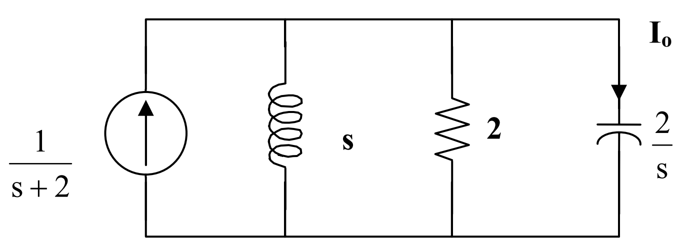
s^1: $\quad 0 = 6A + B + C = 5A + C \longrightarrow C = -5A$

s^0 : $\quad 10 = 10A + C = 5A \longrightarrow A = 2, B = -2, C = -10$

$$V_o(s) = \frac{2}{s+1} - \frac{2s+10}{s^2+6s+10} = \frac{2}{s+1} - \frac{2(s+3)}{(s+3)^2+1^2} - \frac{4}{(s+3)^2+1^2}$$

$$v_o(t) = \underline{\mathbf{2\,e^{-t} - 2\,e^{-3t}\cos(t) - 4\,e^{-3t}\sin(t)\ V}}$$

Chapter 16, Solution 5.



$$V = \frac{1}{s+2}\left(\frac{1}{\frac{1}{s}+\frac{1}{2}+\frac{s}{2}}\right) = \frac{1}{s+2}\left(\frac{2s}{s^2+s+2}\right) = \frac{2s}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$I_o = \frac{Vs}{2} = \frac{s^2}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

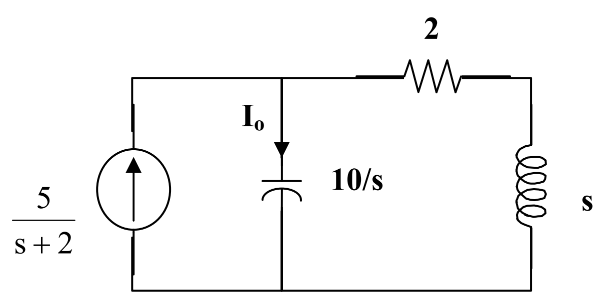
$$= \frac{1}{s+2} + \frac{\frac{(-0.5-j1.3229)^2}{(1.5-j1.3229)(-j2.646)}}{s+0.5+j1.3229} + \frac{\frac{(-0.5+j1.3229)^2}{(1.5+j1.3229)(+j2.646)}}{s+0.5-j1.3229}$$

$$\underline{i_o(t) = \left(e^{-2t} + 0.3779e^{-90^\circ}e^{-t/2}e^{-j1.3229t} + 0.3779e^{90^\circ}e^{-t/2}e^{j1.3229t}\right)u(t)\,A}$$

or

$$\underline{= \left(e^{-2t} - 0.7559\sin 1.3229t\right)u(t)\,A}$$

Chapter 16, Solution 6.



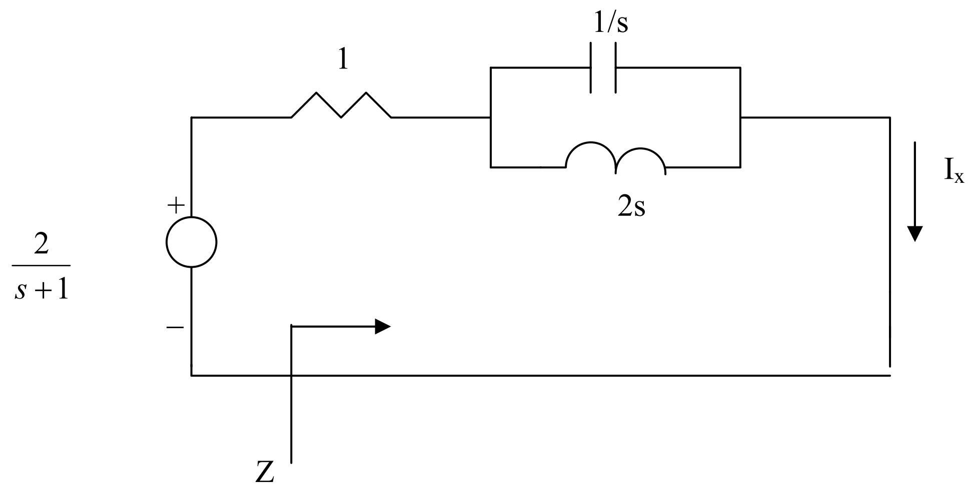
Use current division.

$$I_o = \frac{s+2}{s+2+\frac{10}{s}}\,\frac{5}{s+2} = \frac{5s}{s^2+2s+10} = \frac{5(s+1)}{(s+1)^2+3^2} - \frac{5}{(s+1)^2+3^2}$$

$$\underline{i_o(t) = 5e^{-t}\cos 3t - \frac{5}{3}e^{-t}\sin 3t}$$

Chapter 16, Solution 7.

The s-domain version of the circuit is shown below.



$$Z = 1 + \frac{1}{s} // 2s = 1 + \frac{\frac{1}{s}(2s)}{\frac{1}{s} + 2s} = 1 + \frac{2s}{1+2s^2} = \frac{2s^2 + 2s + 1}{1 + 2s^2}$$

$$I_x = \frac{V}{Z} = \frac{2}{s+1} x \frac{1+2s^2}{2s^2 + 2s + 1} = \frac{2s^2 + 1}{(s+1)(s^2 + s + 0.5)} = \frac{A}{(s+1)} + \frac{Bs + C}{(s^2 + s + 0.5)}$$

$$2s^2 + 1 = A(s^2 + s + 0.5) + B(s^2 + s) + C(s+1)$$

$$s^2: \qquad 2 = A + B$$

$$s: \qquad 0 = A + B + C = 2 + C \longrightarrow C = -2$$

$$\text{constant}: \qquad 1 = 0.5A + C \text{ or } 0.5A = 3 \longrightarrow A = 6, \ B = -4$$

$$I_x = \frac{6}{s+1} - \frac{4s+2}{(s+0.5)^2 + 0.75} = \frac{6}{s+1} - \frac{4(s+0.5)}{(s+0.5)^2 + 0.866^2}$$

$$\underline{i_x(t) = \left[6 - 4e^{-0.5t} \cos 0.866t\right] u(t) \, A}$$

Chapter 16, Solution 8.

(a) $$Z = \frac{1}{s} + 1//(1+2s) = \frac{1}{s} + \frac{(1+2s)}{2+2s} = \underline{\frac{s^2 + 1.5s + 1}{s(s+1)}}$$

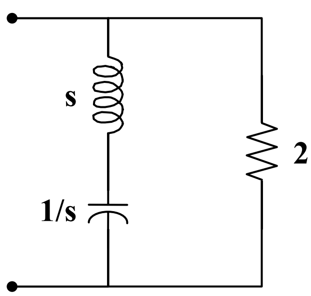
(b) $$\frac{1}{Z} = \frac{1}{2} + \frac{1}{s} + \frac{1}{1+\frac{1}{s}} = \frac{3s^2 + 3s + 2}{2s(s+1)}$$

$$Z = \underline{\frac{2s(s+1)}{3s^2 + 3s + 2}}$$

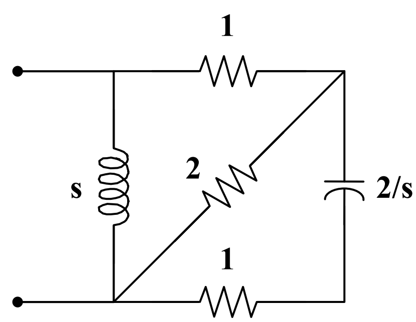
Chapter 16, Solution 9.

(a) The s-domain form of the circuit is shown in Fig. (a).

$$Z_{in} = 2 \parallel (s + 1/s) = \frac{2(s+1/s)}{2+s+1/s} = \underline{\mathbf{\frac{2(s^2+1)}{s^2+2s+1}}}$$



(a)



(b)

(b) The s-domain equivalent circuit is shown in Fig. (b).

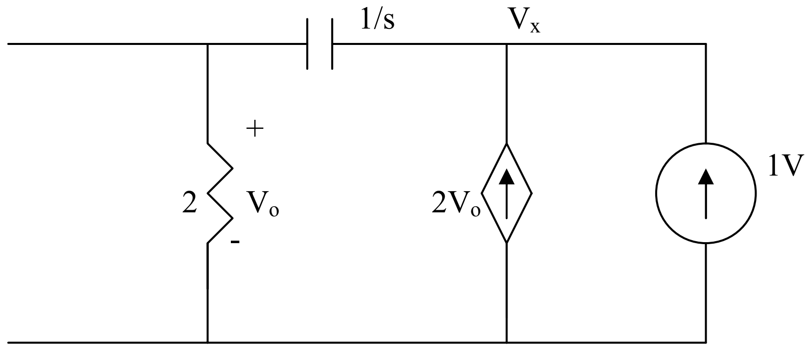
$$2 \parallel (1 + 2/s) = \frac{2(1+2/s)}{3+2/s} = \frac{2(s+2)}{3s+2}$$

$$1 + 2 \parallel (1 + 2/s) = \frac{5s+6}{3s+2}$$

$$Z_{in} = s \parallel \left(\frac{5s+6}{3s+2}\right) = \frac{s \cdot \left(\frac{5s+6}{3s+2}\right)}{s + \left(\frac{5s+6}{3s+2}\right)} = \underline{\mathbf{\frac{s(5s+6)}{3s^2+7s+6}}}$$

Chapter 16, Solution 10.

To find Z_{Th}, consider the circuit below.



Applying KCL gives

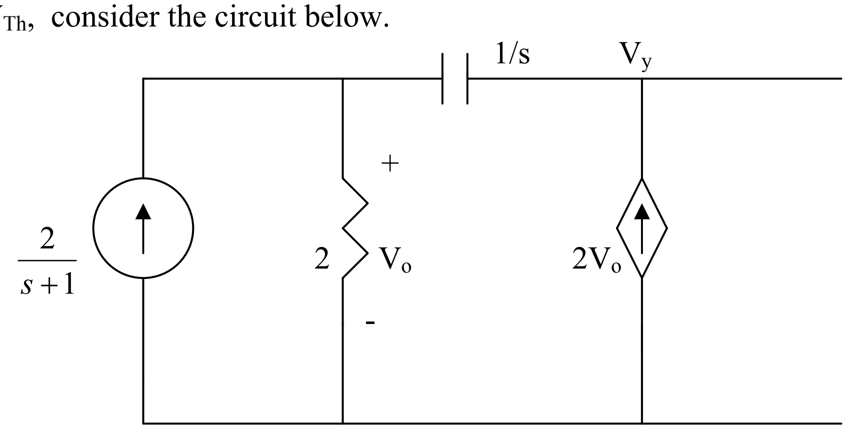
$$1 + 2V_o = \frac{V_x}{2 + 1/s}$$

But $V_o = \dfrac{2}{2 + 1/s} V_x$. Hence

$$1 + \frac{4V_x}{2 + 1/s} = \frac{V_x}{2 + 1/s} \longrightarrow V_x = -\frac{(2s+1)}{3s}$$

$$Z_{Th} = \frac{V_x}{1} = \underline{-\frac{(2s+1)}{3s}}$$

To find V_{Th}, consider the circuit below.



Applying KCL gives

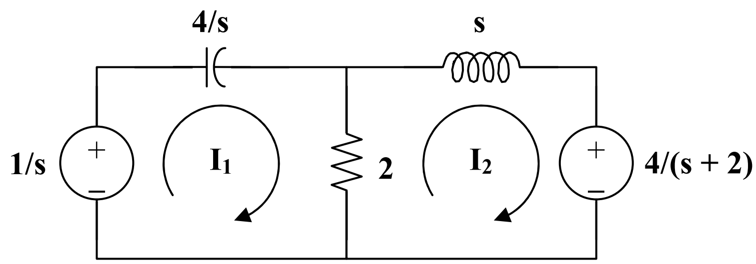
$$\frac{2}{s+1} + 2V_o = \frac{V_o}{2} \longrightarrow V_o = -\frac{4}{3(s+1)}$$

But $-V_y + 2V_o \bullet \frac{1}{s} + V_o = 0$

$$V_{Th} = V_y = V_o(1+\frac{2}{s}) = -\frac{4}{3(s+1)}\left(\frac{s+2}{s}\right) = \underline{\frac{-4(s+2)}{3s(s+1)}}$$

Chapter 16, Solution 11.

The s-domain form of the circuit is shown below.



Write the mesh equations.

$$\frac{1}{s} = \left(2+\frac{4}{s}\right)I_1 - 2I_2 \qquad (1)$$

$$\frac{-4}{s+2} = -2I_1 + (s+2)\,I_2 \qquad (2)$$

Put equations (1) and (2) into matrix form.

$$\begin{bmatrix} 1/s \\ -4/(s+2) \end{bmatrix} = \begin{bmatrix} 2+4/s & -2 \\ -2 & s+2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \frac{2}{s}(s^2+2s+4), \quad \Delta_1 = \frac{s^2-4s+4}{s\,(s+2)}, \qquad \Delta_2 = \frac{-6}{s}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1/2\cdot(s^2-4s+4)}{(s+2)(s^2+2s+4)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+4}$$

$$1/2\cdot(s^2-4s+4) = A\,(s^2+2s+4) + B\,(s^2+2s) + C\,(s+2)$$

Equating coefficients :

s^2: $\quad 1/2 = A + B$

s^1: $\quad -2 = 2A + 2B + C$

$s^0: \quad 2 = 4A + 2C$

Solving these equations leads to $\quad A = 2, \quad B = -3/2, \quad C = -3$

$$I_1 = \frac{2}{s+2} + \frac{-3/2\,s - 3}{(s+1)^2 + (\sqrt{3})^2}$$

$$I_1 = \frac{2}{s+2} + \frac{-3}{2} \cdot \frac{(s+1)}{(s+1)^2 + (\sqrt{3})^2} + \frac{-3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{(s+1)^2 + (\sqrt{3})^2}$$

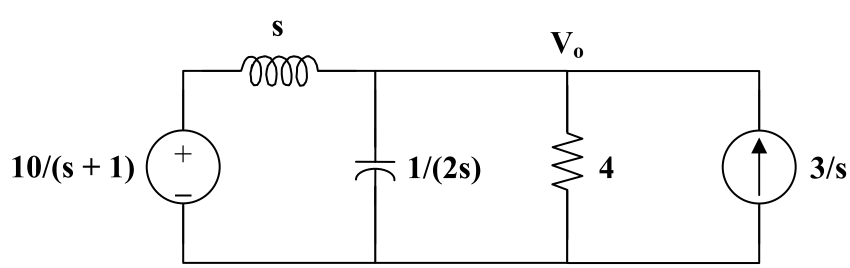
$$i_1(t) = \underline{\left[\mathbf{2\,e^{-2t} - 1.5\,e^{-t}\cos(1.732t) - 0.866\sin(1.732t)}\right]\mathbf{u(t)\ A}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-6}{s} \cdot \frac{s}{2(s^2 + 2s + 4)} = \frac{-3}{(s+1)^2 + (\sqrt{3})^2}$$

$$i_2(t) = \frac{-3}{\sqrt{3}} e^{-t} \sin(\sqrt{3}t) = \underline{\mathbf{-1.732\,e^{-t}\sin(1.732t)\,u(t)\ A}}$$

Chapter 16, Solution 12.

We apply nodal analysis to the s-domain form of the circuit below.



$$\frac{\frac{10}{s+1} - V_o}{s} + \frac{3}{s} = \frac{V_o}{4} + 2sV_o$$

$$(1 + 0.25s + s^2)\,V_o = \frac{10}{s+1} + 15 = \frac{10 + 15s + 15}{s+1}$$

$$V_o = \frac{15s + 25}{(s+1)(s^2 + 0.25s + 1)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + 0.25s + 1}$$

$$A = (s+1)V_o\Big|_{s=-1} = \frac{40}{7}$$

$$15s + 25 = A(s^2 + 0.25s + 1) + B(s^2 + s) + C(s+1)$$

Equating coefficients :

s^2: $\quad 0 = A + B \longrightarrow B = -A$

s^1: $\quad 15 = 0.25A + B + C = -0.75A + C$

s^0: $\quad 25 = A + C$

$A = 40/7, \qquad B = -40/7, \qquad C = 135/7$

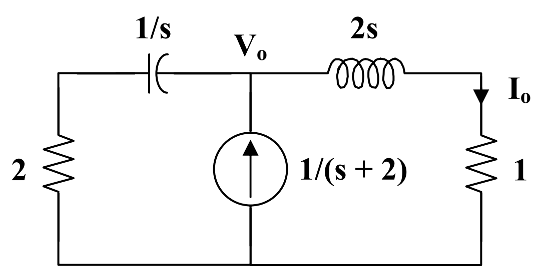
$$V_o = \frac{\frac{40}{7}}{s+1} + \frac{\frac{-40}{7}s + \frac{135}{7}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{40}{7}\frac{1}{s+1} - \frac{40}{7}\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} + \left(\frac{155}{7}\cdot\frac{2}{\sqrt{3}}\right)\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$v_o(t) = \frac{40}{7}e^{-t} - \frac{40}{7}e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{(155)(2)}{(7)(\sqrt{3})}e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$v_o(t) = \underline{\mathbf{5.714\,e^{-t} - 5.714\,e^{-t/2}\cos(0.866t) + 25.57\,e^{-t/2}\sin(0.866t)\ V}}$$

Chapter 16, Solution 13.

Consider the following circuit.



Applying KCL at node o,

$$\frac{1}{s+2} = \frac{V_o}{2s+1} + \frac{V_o}{2+1/s} = \frac{s+1}{2s+1}V_o$$

$$V_o = \frac{2s+1}{(s+1)(s+2)}$$

$$I_o = \frac{V_o}{2s+1} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

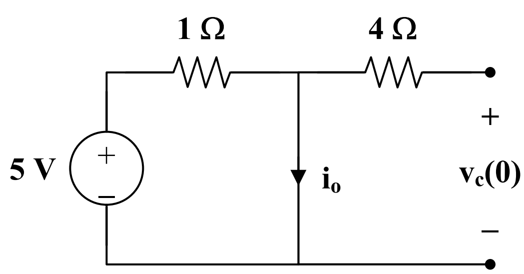
$A = 1, \qquad B = -1$

$$I_o = \frac{1}{s+1} - \frac{1}{s+2}$$

$$i_o(t) = \underline{\left(\mathbf{e^{-t} - e^{-2t}}\right)\mathbf{u(t)\ A}}$$

Chapter 16, Solution 14.

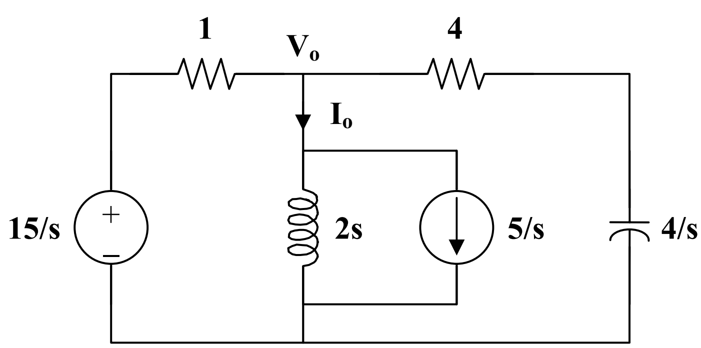
We first find the initial conditions from the circuit in Fig. (a).



(a)

$i_o(0^-) = 5\ A, \quad v_c(0^-) = 0\ V$

We now incorporate these conditions in the s-domain circuit as shown in Fig.(b).



(b)

At node o,

$$\frac{V_o - 15/s}{1} + \frac{V_o}{2s} + \frac{5}{s} + \frac{V_o - 0}{4 + 4/s} = 0$$

$$\frac{15}{s} - \frac{5}{s} = \left(1 + \frac{1}{2s} + \frac{s}{4(s+1)}\right) V_o$$

$$\frac{10}{s} = \frac{4s^2 + 4s + 2s + 2 + s^2}{4s(s+1)} V_o = \frac{5s^2 + 6s + 2}{4s(s+1)} V_o$$

$$V_o = \frac{40(s+1)}{5s^2 + 6s + 2}$$

$$I_o = \frac{V_o}{2s} + \frac{5}{s} = \frac{4(s+1)}{s(s^2 + 1.2s + 0.4)} + \frac{5}{s}$$

$$I_o = \frac{5}{s} + \frac{A}{s} + \frac{Bs + C}{s^2 + 1.2s + 0.4}$$

$$4(s+1) = A(s^2 + 1.2s + 0.4) + B s^s + C s$$

Equating coefficients :

s^0: $\quad 4 = 0.4A \longrightarrow A = 10$

s^1: $\quad 4 = 1.2A + C \longrightarrow C = -1.2A + 4 = -8$

s^2: $\quad 0 = A + B \longrightarrow B = -A = -10$

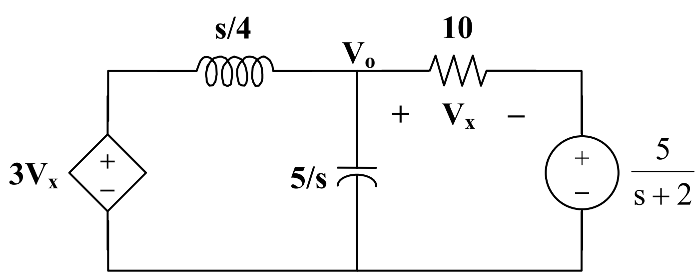
$$I_o = \frac{5}{s} + \frac{10}{s} - \frac{10s + 8}{s^2 + 1.2s + 0.4}$$

$$I_o = \frac{15}{s} - \frac{10(s+0.6)}{(s+0.6)^2 + 0.2^2} - \frac{10(0.2)}{(s+0.6)^2 + 0.2^2}$$

$$i_o(t) = \underline{\left[\mathbf{15 - 10\,e^{-0.6t}\left(\cos(0.2\,t) - \sin(0.2\,t)\right)}\right]\ \mathbf{u(t)\ A}}$$

Chapter 16, Solution 15.

First we need to transform the circuit into the s-domain.



$$\frac{V_o - 3V_x}{s/4} + \frac{V_o - 0}{5/s} + \frac{V_o - \frac{5}{s+2}}{10} = 0$$

$$40V_o - 120V_x + 2s^2V_o + sV_o - \frac{5s}{s+2} = 0 = (2s^2 + s + 40)V_o - 120V_x - \frac{5s}{s+2}$$

$$\text{But, } V_x = V_o - \frac{5}{s+2} \rightarrow V_o = V_x + \frac{5}{s+2}$$

We can now solve for V_x.

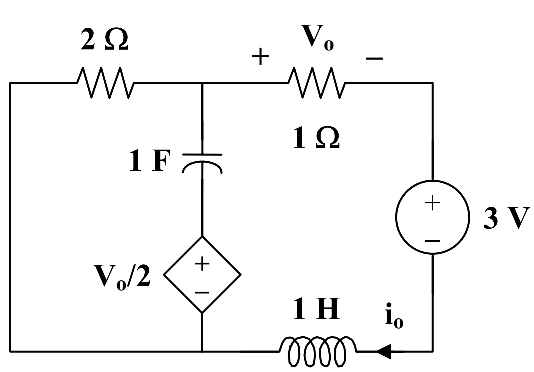
$$(2s^2 + s + 40)\left(V_x + \frac{5}{s+2}\right) - 120V_x - \frac{5s}{s+2} = 0$$

$$2(s^2 + 0.5s - 40)V_x = -10\frac{(s^2 + 20)}{s+2}$$

$$\underline{V_x = -5\frac{(s^2 + 20)}{(s+2)(s^2 + 0.5s - 40)}}$$

Chapter 16, Solution 16.

We first need to find the initial conditions. For $t < 0$, the circuit is shown in Fig. (a). To dc, the capacitor acts like an open circuit and the inductor acts like a short circuit.



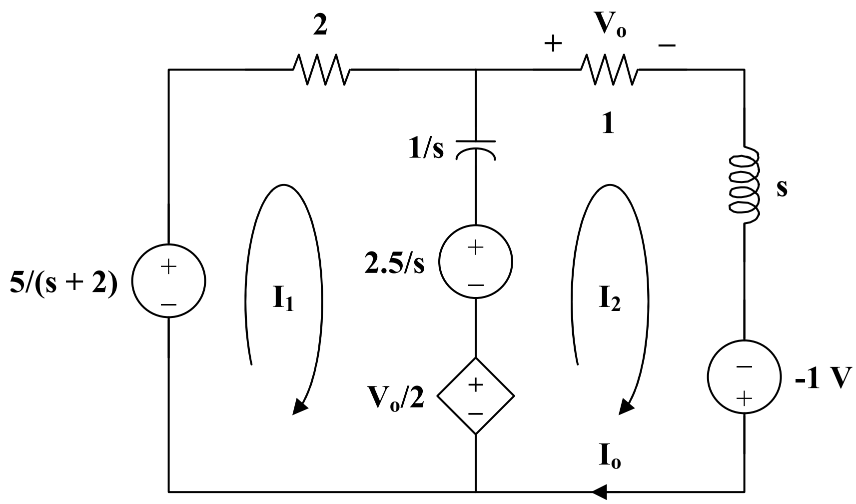
(a)

Hence,

$$i_L(0) = i_o = \frac{-3}{3} = -1\text{ A}, \qquad v_o = -1\text{ V}$$

$$v_c(0) = -(2)(-1) - \left(\frac{-1}{2}\right) = 2.5\text{ V}$$

We now incorporate the initial conditions for $t > 0$ as shown in Fig. (b).



(b)

For mesh 1,

$$\frac{-5}{s+2} + \left(2 + \frac{1}{s}\right)I_1 - \frac{1}{s}I_2 + \frac{2.5}{s} + \frac{V_o}{2} = 0$$

But, $\quad V_o = I_o = I_2$

$$\left(2 + \frac{1}{s}\right)I_1 + \left(\frac{1}{2} - \frac{1}{s}\right)I_2 = \frac{5}{s+2} - \frac{2.5}{s} \qquad (1)$$

For mesh 2,

$$\left(1 + s + \frac{1}{s}\right)I_2 - \frac{1}{s}I_1 + 1 - \frac{V_o}{2} - \frac{2.5}{s} = 0$$

$$-\frac{1}{s}I_1 + \left(\frac{1}{2} + s + \frac{1}{s}\right)I_2 = \frac{2.5}{s} - 1 \qquad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 2+\frac{1}{s} & \frac{1}{2}-\frac{1}{s} \\ \frac{-1}{s} & \frac{1}{2}+s+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{s+2}-\frac{2.5}{s} \\ \frac{2.5}{s}-1 \end{bmatrix}$$

$$\Delta = 2s + 2 + \frac{3}{s}, \qquad \Delta_2 = -2 + \frac{4}{s} + \frac{5}{s(s+2)}$$

$$I_o = I_2 = \frac{\Delta_2}{\Delta} = \frac{-2s^2 + 13}{(s+2)(2s^2+2s+3)} = \frac{A}{s+2} + \frac{Bs + C}{2s^2+2s+3}$$

$$-2s^2 + 13 = A(2s^2+2s+3) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $\quad -2 = 2A + B$

s^1: $\quad 0 = 2A + 2B + C$

s^0: $\quad 13 = 3A + 2C$

Solving these equations leads to

$A = 0.7143, \quad B = -3.429, \quad C = 5.429$

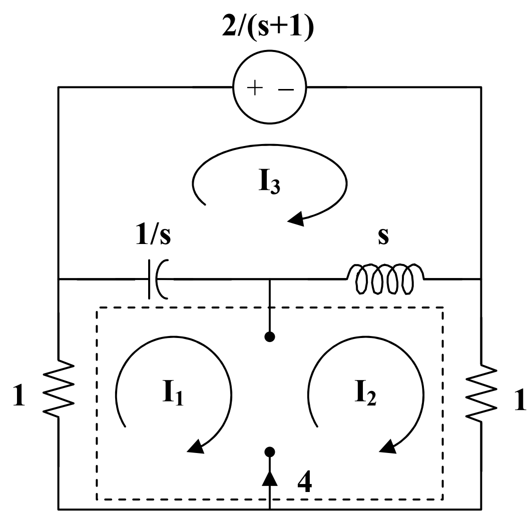
$$I_o = \frac{0.7143}{s+2} - \frac{3.429s - 5.429}{2s^2+2s+3} = \frac{0.7143}{s+2} - \frac{1.7145s - 2.714}{s^2+s+1.5}$$

$$I_o = \frac{0.7143}{s+2} - \frac{1.7145(s+0.5)}{(s+0.5)^2+1.25} + \frac{(3.194)(\sqrt{1.25})}{(s+0.5)^2+1.25}$$

$$i_o(t) = \underline{\left[\mathbf{0.7143e^{-2t} - 1.7145e^{-0.5t}\cos(1.25t) + 3.194e^{-0.5t}\sin(1.25t)}\right]\mathbf{u(t)\ A}}$$

Chapter 16, Solution 17.

We apply mesh analysis to the s-domain form of the circuit as shown below.



For mesh 3,

$$\frac{2}{s+1} + \left(s + \frac{1}{s}\right)I_3 - \frac{1}{s}I_1 - s\,I_2 = 0 \qquad (1)$$

For the supermesh,

$$\left(1 + \frac{1}{s}\right)I_1 + (1+s)\,I_2 - \left(\frac{1}{s} + s\right)I_3 = 0 \qquad (2)$$

But $\quad I_1 = I_2 - 4 \qquad (3)$

Substituting (3) into (1) and (2) leads to

$$\left(2 + s + \frac{1}{s}\right)I_2 - \left(s + \frac{1}{s}\right)I_3 = 4\left(1 + \frac{1}{s}\right) \qquad (4)$$

$$-\left(s + \frac{1}{s}\right)I_2 + \left(s + \frac{1}{s}\right)I_3 = \frac{-4}{s} - \frac{2}{s+1} \qquad (5)$$

Adding (4) and (5) gives

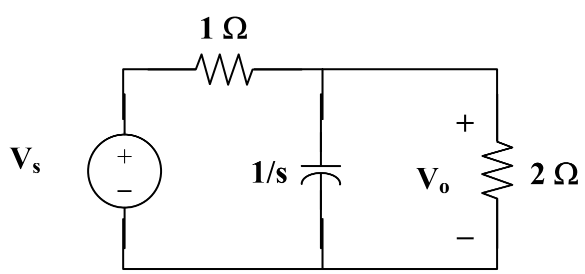
$$2\,I_2 = 4 - \frac{2}{s+1}$$

$$I_2 = 2 - \frac{1}{s+1}$$

$$i_o(t) = i_2(t) = \underline{\mathbf{(2 - e^{-t})u(t)\ A}}$$

Chapter 16, Solution 18.

$v_s(t) = 3u(t) - 3u(t-1)$ or $V_s = \dfrac{3}{s} - \dfrac{e^{-s}}{s} = \dfrac{3}{s}(1 - e^{-s})$



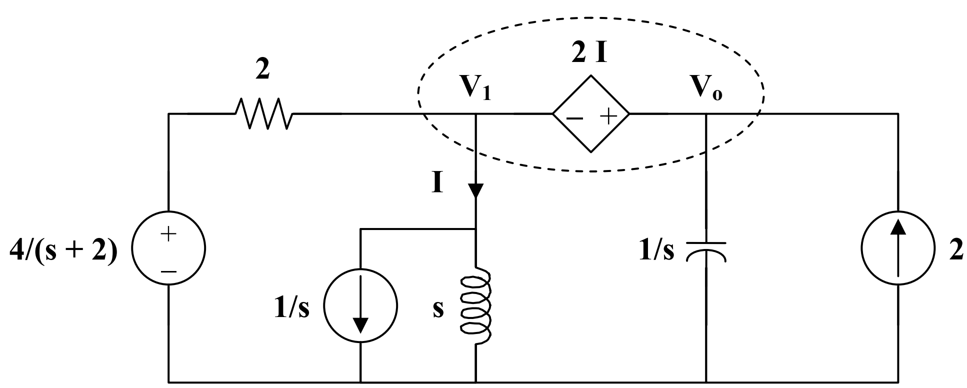
$$\frac{V_o - V_s}{1} + sV_o + \frac{V_o}{2} = 0 \rightarrow (s + 1.5)V_o = V_s$$

$$V_o = \frac{3}{s(s+1.5)}(1 - e^{-s}) = \left(\frac{2}{s} - \frac{2}{s+1.5}\right)(1 - e^{-s})$$

$$v_o(t) = \underline{[(2 - 2e^{-1.5t})u(t) - (2 - 2e^{-1.5(t-1)})u(t-1)]\,V}$$

Chapter 16, Solution 19.

We incorporate the initial conditions in the s-domain circuit as shown below.



At the supernode,

$$\frac{4/(s+2)-V_1}{2}+2=\frac{V_1}{s}+\frac{1}{s}+sV_o$$

$$\frac{2}{s+2}+2=\left(\frac{1}{2}+\frac{1}{s}\right)V_1+\frac{1}{s}+s\,V_o \tag{1}$$

But $V_o = V_1 + 2I$ and $I=\dfrac{V_1+1}{s}$

$$V_o = V_1+\frac{2(V_1+1)}{s} \longrightarrow V_1=\frac{V_o-2/s}{(s+2)/s}=\frac{s\,V_o-2}{s+2} \tag{2}$$

Substituting (2) into (1)

$$\frac{2}{s+2}+2-\frac{1}{s}=\left(\frac{2s+1}{s}\right)\left[\left(\frac{s}{s+2}\right)V_o-\frac{2}{s+2}\right]+s\,V_o$$

$$\frac{2}{s+2}+2-\frac{1}{s}+\frac{2(2s+1)}{s(s+2)}=\left[\left(\frac{2s+1}{s+2}\right)+s\right]V_o$$

$$\frac{2s^2+9s}{s(s+2)}=\frac{2s+9}{s+2}=\frac{s^2+4s+1}{s+2}V_o$$

$$V_o=\frac{2s+9}{s^2+4s+1}=\frac{A}{s+0.2679}+\frac{B}{s+3.732}$$

A = 2.443, B = -0.4434

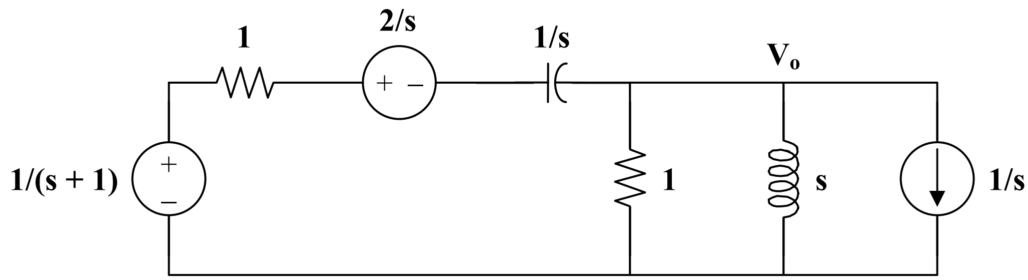
$$V_o=\frac{2.443}{s+0.2679}-\frac{0.4434}{s+3.732}$$

Therefore,

$v_o(t) = \underline{\mathbf{(2.443\,e^{-0.2679t} - 0.4434\,e^{-3.732t})\,u(t)\ V}}$

Chapter 16, Solution 20.

We incorporate the initial conditions and transform the current source to a voltage source as shown.



At the main non-reference node, KCL gives

$$\frac{1/(s+1) - 2/s - V_o}{1 + 1/s} = \frac{V_o}{1} + \frac{V_o}{s} + \frac{1}{s}$$

$$\frac{s}{s+1} - 2 - s\,V_o = (s+1)(s+1/s)\,V_o + \frac{s+1}{s}$$

$$\frac{s}{s+1} - \frac{s+1}{s} - 2 = (2s + 2 + 1/s)\,V_o$$

$$V_o = \frac{-2s^2 - 4s - 1}{(s+1)(2s^2 + 2s + 1)}$$

$$V_o = \frac{-s - 2s - 0.5}{(s+1)(s^2 + s + 0.5)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + s + 0.5}$$

$$A = (s+1)\,V_o\big|_{s=-1} = 1$$

$$-s^2 - 2s - 0.5 = A(s^2 + s + 0.5) + B(s^2 + s) + C(s+1)$$

Equating coefficients :

$$s^2: \quad -1 = A + B \longrightarrow B = -2$$

$$s^1: \quad -2 = A + B + C \longrightarrow C = -1$$

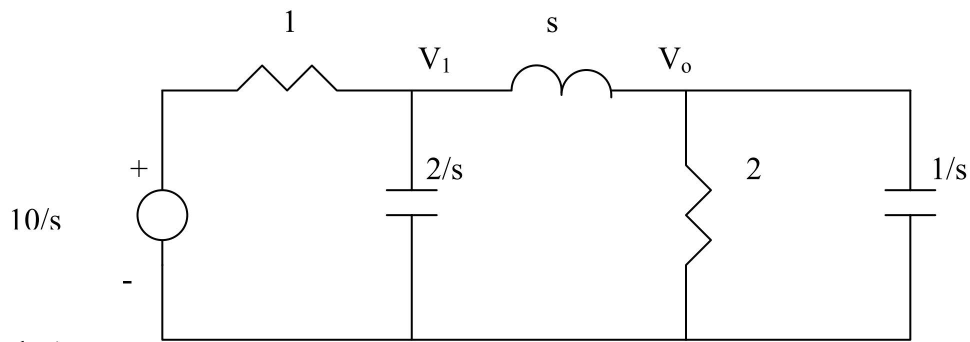
$$s^0: \quad -0.5 = 0.5A + C = 0.5 - 1 = -0.5$$

$$V_o = \frac{1}{s+1} - \frac{2s+1}{s^2 + s + 0.5} = \frac{1}{s+1} - \frac{2(s+0.5)}{(s+0.5)^2 + (0.5)^2}$$

$$v_o(t) = \underline{\left[e^{-t} - 2e^{-t/2}\cos(t/2)\right]u(t)\ \text{V}}$$

Chapter 16, Solution 21.

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{\frac{10}{s}-V_1}{1}=\frac{V_1-V_o}{s}+\frac{s}{2}V_o \qquad \longrightarrow \qquad 10=(s+1)V_1+(\frac{s^2}{2}-1)V_o \qquad (1)$$

At node 2,

$$\frac{V_1-V_o}{s}=\frac{V_o}{2}+sV_o \qquad \longrightarrow \qquad V_1=V_o(\frac{s}{2}+s^2+1) \qquad (2)$$

Substituting (2) into (1) gives

$$10=(s+1)(s^2+s/2+1)V_o+(\frac{s^2}{2}-1)V_o=s(s^2+2s+1.5)V_o$$

$$V_o=\frac{10}{s(s^2+2s+1.5)}=\frac{A}{s}+\frac{Bs+C}{s^2+2s+1.5}$$

$$10=A(s^2+2s+1.5)+Bs^2+Cs$$

$$s^2: \quad 0=A+B$$

$$s: \quad 0=2A+C$$

$$\text{constant}: \quad 10=1.5A \qquad \longrightarrow \qquad A=20/3,\ \text{B = -20/3},\ \text{C = -40/3}$$

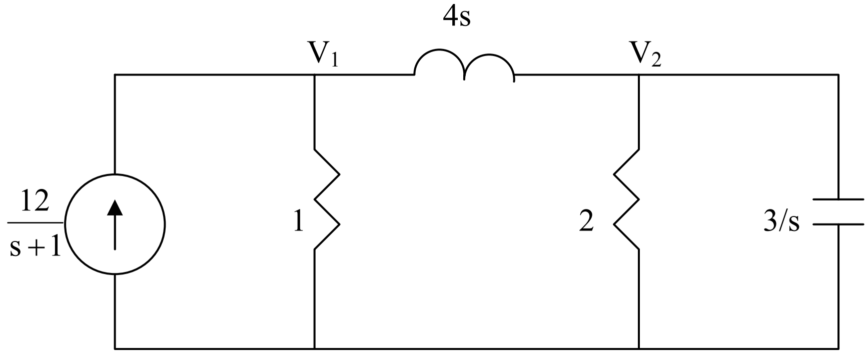
$$V_o=\frac{20}{3}\left[\frac{1}{s}-\frac{s+2}{s^2+2s+1.5}\right]=\frac{20}{3}\left[\frac{1}{s}-\frac{s+1}{(s+1)^2+0.7071^2}-1.414\frac{0.7071}{(s+1)^2+0.7071^2}\right]$$

Taking the inverse Laplace tranform finally yields

$$\underline{v_o(t)=\frac{20}{3}\left[1-e^{-t}\cos 0.7071t-1.414e^{-t}\sin 0.7071t\right]u(t)\,V}$$

Chapter 16, Solution 22.

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{12}{s+1} = \frac{V_1}{1} + \frac{V_1 - V_2}{4s} \longrightarrow \frac{12}{s+1} = V_1\left(1 + \frac{1}{4s}\right) - \frac{V_2}{4s} \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{4s} = \frac{V_2}{2} + \frac{s}{3}V_2 \longrightarrow V_1 = V_2\left(\frac{4}{3}s^2 + 2s + 1\right) \qquad (2)$$

Substituting (2) into (1),

$$\frac{12}{s+1} = V_2\left[\left(\frac{4}{3}s^2 + 2s + 1\right)\left(1 + \frac{1}{4s}\right) - \frac{1}{4s}\right] = \left(\frac{4}{3}s^2 + \frac{7}{3}s + \frac{3}{2}\right)V_2$$

$$V_2 = \frac{9}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{A}{(s+1)} + \frac{Bs + C}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9 = A(s^2 + \frac{7}{4}s + \frac{9}{8}) + B(s^2 + s) + C(s+1)$$

Equating coefficients:

$$s^2: \qquad 0 = A + B$$

$$s: \qquad 0 = \frac{7}{4}A + B + C = \frac{3}{4}A + C \longrightarrow C = -\frac{3}{4}A$$

$$\text{constant}: \qquad 9 = \frac{9}{8}A + C = \frac{3}{8}A \longrightarrow A = 24,\ B = -24,\ C = -18$$

$$V_2 = \frac{24}{(s+1)} - \frac{24s+18}{(s^2+\frac{7}{4}s+\frac{9}{8})} = \frac{24}{(s+1)} - \frac{24(s+7/8)}{(s+\frac{7}{8})^2+\frac{23}{64}} + \frac{3}{(s+\frac{7}{8})^2+\frac{23}{64}}$$

Taking the inverse of this produces:

$$\underline{v_2(t) = \left[24e^{-t} - 24e^{-0.875t}\cos(0.5995t) + 5.004e^{-0.875t}\sin(0.5995t)\right]u(t)}$$

Similarly,

$$V_1 = \frac{9\left(\frac{4}{3}s^2+2s+1\right)}{(s+1)(s^2+\frac{7}{4}s+\frac{9}{8})} = \frac{D}{(s+1)} + \frac{Es+F}{(s^2+\frac{7}{4}s+\frac{9}{8})}$$

$$9\left(\frac{4}{3}s^2+2s+1\right) = D(s^2+\frac{7}{4}s+\frac{9}{8}) + E(s^2+s) + F(s+1)$$

Equating coefficients:

s^2 : $\quad 12 = D + E$

s : $\quad 18 = \frac{7}{4}D + E + F$ or $6 = \frac{3}{4}D + F \longrightarrow F = 6 - \frac{3}{4}D$

constant : $\quad 9 = \frac{9}{8}D + F$ or $3 = \frac{3}{8}D \longrightarrow D = 8,\ E = 4,\ F = 0$

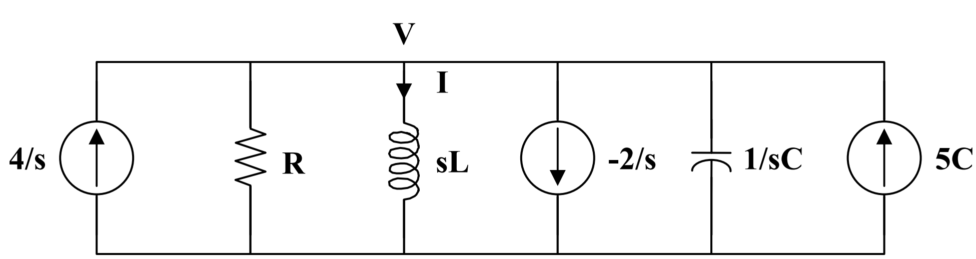
$$V_1 = \frac{8}{(s+1)} + \frac{4s}{(s^2+\frac{7}{4}s+\frac{9}{8})} = \frac{8}{(s+1)} + \frac{4(s+7/8)}{(s+\frac{7}{8})^2+\frac{23}{64}} - \frac{7/2}{(s+\frac{7}{8})^2+\frac{23}{64}}$$

Thus,

$$\underline{v_1(t) = \left[8e^{-t} + 4e^{-0.875t}\cos(0.5995t) - 5.838e^{-0.875t}\sin(0.5995t)\right]u(t)}$$

Chapter 16, Solution 23.

The s-domain form of the circuit with the initial conditions is shown below.



At the non-reference node,

$$\frac{4}{s}+\frac{2}{s}+5C=\frac{V}{R}+\frac{V}{sL}+sCV$$

$$\frac{6+5sC}{s}=\frac{CV}{s}\left(s^2+\frac{s}{RC}+\frac{1}{LC}\right)$$

$$V=\frac{5s+6/C}{s^2+s/RC+1/LC}$$

But $\quad \frac{1}{RC}=\frac{1}{10/80}=8, \qquad \frac{1}{LC}=\frac{1}{4/80}=20$

$$V=\frac{5s+480}{s^2+8s+20}=\frac{5(s+4)}{(s+4)^2+2^2}+\frac{(230)(2)}{(s+4)^2+2^2}$$

$$v(t)=\underline{\mathbf{5e^{-4t}\cos(2t)+230e^{-4t}\sin(2t)\ V}}$$

$$I=\frac{V}{sL}=\frac{5s+480}{4s(s^2+8s+20)}$$

$$I=\frac{1.25s+120}{s(s^2+8s+20)}=\frac{A}{s}+\frac{Bs+C}{s^2+8s+20}$$

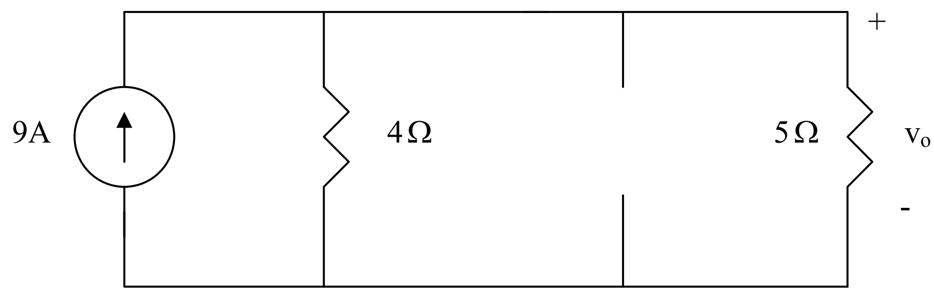
$A=6, \qquad B=-6, \qquad C=-46.75$

$$I=\frac{6}{s}-\frac{6s+46.75}{s^2+8s+20}=\frac{6}{s}-\frac{6(s+4)}{(s+4)^2+2^2}-\frac{(11.375)(2)}{(s+4)^2+2^2}$$

$$i(t)=\underline{\mathbf{6u(t)-6e^{-4t}\cos(2t)-11.375e^{-4t}\sin(2t),\quad t>0}}$$

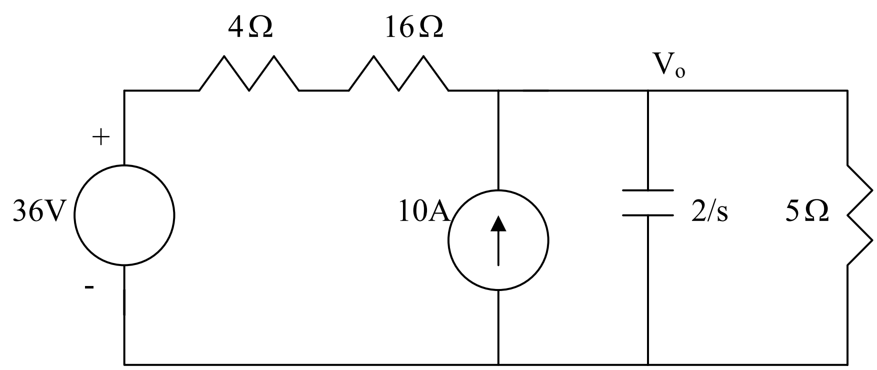
Chapter 16, Solution 24.

At $t = 0^-$, the circuit is equivalent to that shown below.



$$v_o(0) = 5x\frac{4}{4+5}(9) = 20$$

For $t > 0$, we have the Laplace transform of the circuit as shown below after transforming the current source to a voltage source.



Applying KCL gives

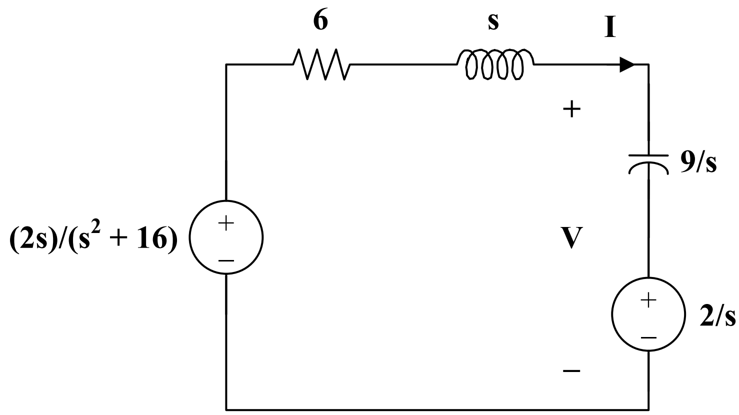
$$\frac{36 - V_o}{20} + 10 = \frac{sV_o}{2} + \frac{V_o}{5} \quad \longrightarrow \quad V_o = \frac{3.6 + 20s}{s(s+0.5)} = \frac{A}{s} + \frac{B}{s+0.5}, \qquad A = 7.2, B = -12.8$$

Thus,

$$v_o(t) = \underline{\left[7.2 - 12.8e^{-0.5t}\right]u(t)}$$

Chapter 16, Solution 25.

For $t > 0$, the circuit in the s-domain is shown below.



Applying KVL,

$$\frac{-2s}{s^2+16} + \left(6 + s + \frac{9}{s}\right)I + \frac{2}{s} = 0$$

$$I = \frac{4s^2 + 32}{(s^2 + 6s + 9)(s^2 + 16)}$$

$$V = \frac{9}{s}I + \frac{2}{s} = \frac{2}{s} + \frac{36s^2 + 288}{s(s+3)^2(s^2+16)}$$

$$= \frac{2}{s} + \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+3)^2} + \frac{Ds + E}{s^2 + 16}$$

$$\begin{aligned} 36s^2 + 288 = {} & A(s^4 + 6s^3 + 25s^2 + 96s + 144) + B(s^4 + 3s^3 + 16s^2 + 48s) \\ & + C(s^3 + 16s) + D(s^4 + 6s^3 + 9s^2) + E(s^3 + 6s^2 + 9s) \end{aligned}$$

Equating coefficients :

s^0 :	$288 = 144A$	(1)
s^1:	$0 = 96A + 48B + 16C + 9E$	(2)
s^2 :	$36 = 25A + 16B + 9D + 6E$	(3)
s^3 :	$0 = 6A + 3B + C + 6D + E$	(4)
s^4 :	$0 = A + B + D$	(5)

Solving equations (1), (2), (3), (4) and (5) gives

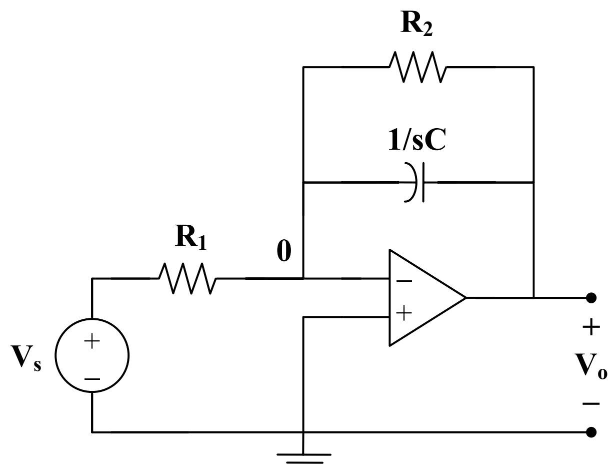
$A = 2$, $B = -1.7984$, $C = -8.16$, $D = -0.2016$, $E = 2.765$

$$V(s) = \frac{4}{s} - \frac{1.7984}{s+3} - \frac{8.16}{(s+3)^2} - \frac{0.2016\,s}{s^2+16} + \frac{(0.6912)(4)}{s^2+16}$$

$$v(t) = \underline{\mathbf{4\,u(t) - 1.7984\,e^{-3t} - 8.16\,t\,e^{-3t} - 0.2016\cos(4t) + 0.6912\sin(4t)\ V}}$$

Chapter 16, Solution 26.

Consider the op-amp circuit below.



At node 0,

$$\frac{V_s - 0}{R_1} = \frac{0 - V_o}{R_2} + (0 - V_o)\,sC$$

$$V_s = R_1\left(\frac{1}{R_2} + sC\right)\left(-V_o\right)$$

$$\frac{V_o}{V_s} = \frac{-1}{sR_1C + R_1/R_2}$$

But $\quad \frac{R_1}{R_2} = \frac{20}{10} = 2\,, \qquad R_1C = (20\times10^3)(50\times10^{-6}) = 1$

So, $\quad \frac{V_o}{V_s} = \frac{-1}{s+2}$

$$V_s = 3\,e^{-5t} \longrightarrow V_s = 3/(s+5)$$

$$V_o = \frac{-3}{(s+2)(s+5)}$$

$$-V_o = \frac{3}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

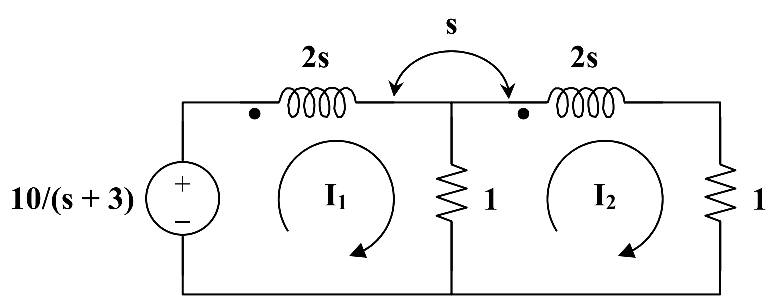
$$A = 1, \qquad B = -1$$

$$V_o = \frac{1}{s+5} - \frac{1}{s+2}$$

$$v_o(t) = \underline{\left(\mathbf{e^{-5t} - e^{-2t}}\right)\mathbf{u(t)}}$$

Chapter 16, Solution 27.

Consider the following circuit.



For mesh 1,

$$\frac{10}{s+3} = (1+2s)\,I_1 - I_2 - s\,I_2$$

$$\frac{10}{s+3} = (1+2s)\,I_1 - (1+s)\,I_2 \qquad (1)$$

For mesh 2,

$$0 = (2+2s)\,I_2 - I_1 - s\,I_1$$
$$0 = -(1+s)\,I_1 + 2\,(s+1)\,I_2 \qquad (2)$$

(1) and (2) in matrix form,

$$\begin{bmatrix} 10/(s+3) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+1 & -(s+1) \\ -(s+1) & 2\,(s+1) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 3s^2 + 4s + 1$$

$$\Delta_1 = \frac{20(s+1)}{s+3}$$

$$\Delta_2 = \frac{10(s+1)}{s+3}$$

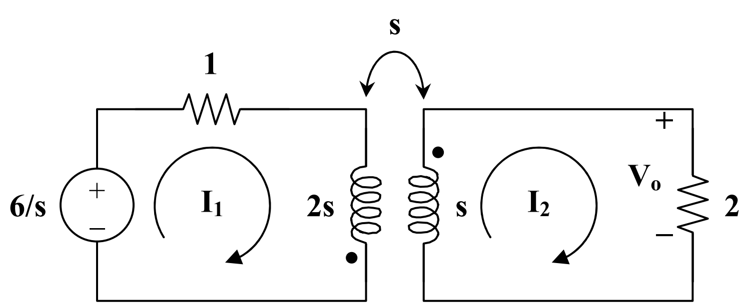
Thus

$$I_1 = \frac{\Delta_1}{\Delta} = \underline{\frac{\mathbf{20(s+1)}}{\mathbf{(s+3)(3s^2+4s+1)}}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \underline{\frac{\mathbf{10(s+1)}}{\mathbf{(s+3)(3s^2+4s+1)}}} = \frac{I_1}{2}$$

Chapter 16, Solution 28.

Consider the circuit shown below.



For mesh 1,

$$\frac{6}{s} = (1+2s)\,I_1 + s\,I_2 \tag{1}$$

For mesh 2,

$$0 = s\,I_1 + (2+s)\,I_2$$

$$I_1 = -\left(1+\frac{2}{s}\right)I_2 \tag{2}$$

Substituting (2) into (1) gives

$$\frac{6}{s} = -(1+2s)\left(1+\frac{2}{s}\right)I_2 + s\,I_2 = \frac{-(s^2+5s+2)}{s}\,I_2$$

or $\qquad I_2 = \dfrac{-6}{s^2+5s+2}$

$$V_o = 2I_2 = \frac{-12}{s^2 + 5s + 2} = \frac{-12}{(s + 0.438)(s + 4.561)}$$

Since the roots of $s^2 + 5s + 2 = 0$ are -0.438 and -4.561,

$$V_o = \frac{A}{s + 0.438} + \frac{B}{s + 4.561}$$

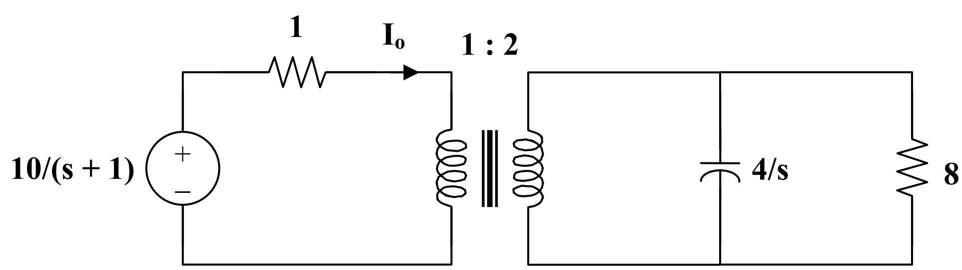
$$A = \frac{-12}{4.123} = -2.91, \qquad B = \frac{-12}{-4.123} = 2.91$$

$$V_o(s) = \frac{-2.91}{s + 0.438} + \frac{2.91}{s + 4.561}$$

$$v_o(t) = \underline{\mathbf{2.91\left[e^{-4.561t} - e^{0.438t}\right] u(t)\ V}}$$

Chapter 16, Solution 29.

Consider the following circuit.



Let $\quad Z_L = 8 \| \frac{4}{s} = \frac{(8)(4/s)}{8 + 4/s} = \frac{8}{2s + 1}$

When this is reflected to the primary side,

$$Z_{in} = 1 + \frac{Z_L}{n^2}, \quad n = 2$$

$$Z_{in} = 1 + \frac{2}{2s + 1} = \frac{2s + 3}{2s + 1}$$

$$I_o = \frac{10}{s + 1} \cdot \frac{1}{Z_{in}} = \frac{10}{s + 1} \cdot \frac{2s + 1}{2s + 3}$$

$$I_o = \frac{10s+5}{(s+1)(s+1.5)} = \frac{A}{s+1} + \frac{B}{s+1.5}$$

$$A = -10, \qquad B = 20$$

$$I_o(s) = \frac{-10}{s+1} + \frac{20}{s+1.5}$$

$$i_o(t) = \underline{\mathbf{10\left[2\,e^{-1.5t} - e^{-t}\right]u(t)\ A}}$$

Chapter 16, Solution 30.

$$Y(s) = H(s)\,X(s), \qquad X(s) = \frac{4}{s+1/3} = \frac{12}{3s+1}$$

$$Y(s) = \frac{12\,s^2}{(3s+1)^2} = \frac{4}{3} - \frac{8s+4/3}{(3s+1)^2}$$

$$Y(s) = \frac{4}{3} - \frac{8}{9}\cdot\frac{s}{(s+1/3)^2} - \frac{4}{27}\cdot\frac{1}{(s+1/3)^2}$$

Let $G(s) = \frac{-8}{9}\cdot\frac{s}{(s+1/3)^2}$

Using the time differentiation property,

$$g(t) = \frac{-8}{9}\cdot\frac{d}{dt}(t\,e^{-t/3}) = \frac{-8}{9}\left(\frac{-1}{3}t\,e^{-t/3} + e^{-t/3}\right)$$

$$g(t) = \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3}$$

Hence,

$$y(t) = \frac{4}{3}u(t) + \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3} - \frac{4}{27}t\,e^{-t/3}$$

$$y(t) = \underline{\mathbf{\frac{4}{3}u(t) - \frac{8}{9}e^{-t/3} + \frac{4}{27}t\,e^{-t/3}}}$$

Chapter 16, Solution 31.

$$x(t) = u(t) \longrightarrow X(s) = \frac{1}{s}$$

$$y(t) = 10\cos(2t) \longrightarrow Y(s) = \frac{10s}{s^2 + 4}$$

$$H(s) = \frac{Y(s)}{X(s)} = \underline{\mathbf{\frac{10s^2}{s^2 + 4}}}$$

Chapter 16, Solution 32.

(a) $$Y(s) = H(s)\,X(s)$$

$$= \frac{s+3}{s^2 + 4s + 5} \cdot \frac{1}{s}$$

$$= \frac{s+3}{s\,(s^2 + 4s + 5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 5}$$

$$s + 3 = A\,(s^2 + 4s + 5) + Bs^2 + Cs$$

Equating coefficients :

s^0 : $3 = 5A \longrightarrow A = 3/5$

s^1: $1 = 4A + C \longrightarrow C = 1 - 4A = -7/5$

s^2 : $0 = A + B \longrightarrow B = -A = -3/5$

$$Y(s) = \frac{3/5}{s} - \frac{1}{5} \cdot \frac{3s + 7}{s^2 + 4s + 5}$$

$$Y(s) = \frac{0.6}{s} - \frac{1}{5} \cdot \frac{3\,(s + 2) + 1}{(s + 2)^2 + 1}$$

$$y(t) = \underline{\mathbf{\left[0.6 - 0.6\,e^{-2t}\cos(t) - 0.2\,e^{-2t}\sin(t)\right]u(t)}}$$

(b) $x(t) = 6\,t\,e^{-2t} \longrightarrow X(s) = \dfrac{6}{(s+2)^2}$

$$Y(s) = H(s)\,X(s) = \frac{s+3}{s^2+4s+5} \cdot \frac{6}{(s+2)^2}$$

$$Y(s) = \frac{6(s+3)}{(s+2)^2(s^2+4s+5)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+4s+5}$$

Equating coefficients :

s^3: $0 = A + C \longrightarrow C = -A$ (1)

s^2: $0 = 6A + B + 4C + D = 2A + B + D$ (2)

s^1: $6 = 13A + 4B + 4C + 4D = 9A + 4B + 4D$ (3)

s^0: $18 = 10A + 5B + 4D = 2A + B$ (4)

Solving (1), (2), (3), and (4) gives

$A = 6$, $B = 6$, $C = -6$, $D = -18$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6s+18}{(s+2)^2+1}$$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6(s+2)}{(s+2)^2+1} - \frac{6}{(s+2)^2+1}$$

$$y(t) = \underline{\mathbf{\left[6e^{-2t} + 6\,t\,e^{-2t} - 6e^{-2t}\cos(t) - 6e^{-2t}\sin(t)\right]u(t)}}$$

Chapter 16, Solution 33.

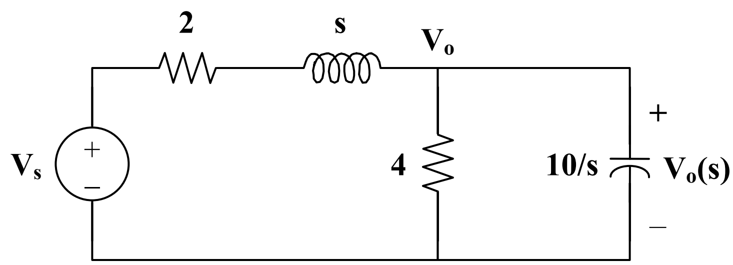
$H(s) = \dfrac{Y(s)}{X(s)}$, $X(s) = \dfrac{1}{s}$

$$Y(s) = \frac{4}{s} + \frac{1}{2(s+3)} - \frac{2s}{(s+2)^2+16} - \frac{(3)(4)}{(s+2)^2+16}$$

$$H(s) = s\,Y(s) = \underline{\mathbf{4 + \frac{s}{2(s+3)} - \frac{2s^2}{s^2+4s+20} - \frac{12s}{s^2+4s+20}}}$$

Chapter 16, Solution 34.

Consider the following circuit.



Using nodal analysis,

$$\frac{V_s - V_o}{s+2} = \frac{V_o}{4} + \frac{V_o}{10/s}$$

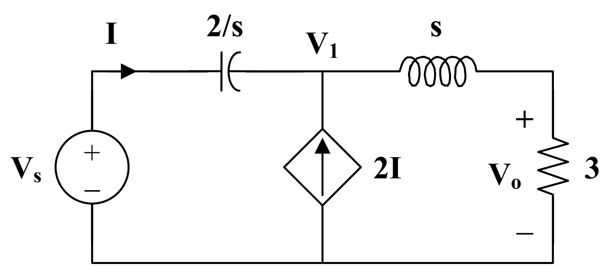
$$V_s = (s+2)\left(\frac{1}{s+2} + \frac{1}{4} + \frac{s}{10}\right)V_o = \left(1 + \frac{1}{4}(s+2) + \frac{1}{10}(s^2+2s)\right)V_o$$

$$V_s = \frac{1}{20}\left(2s^2 + 9s + 30\right)V_o$$

$$\frac{V_o}{V_s} = \underline{\frac{\mathbf{20}}{\mathbf{2s^2 + 9s + 30}}}$$

Chapter 16, Solution 35.

Consider the following circuit.



At node 1,

$$2I + I = \frac{V_1}{s+3}, \qquad \text{where} \quad I = \frac{V_s - V_1}{2/s}$$

$$3 \cdot \frac{V_s - V_1}{2/s} = \frac{V_1}{s+3}$$

$$\frac{V_1}{s+3} = \frac{3s}{2} V_s - \frac{3s}{2} V_1$$

$$\left(\frac{1}{s+3} + \frac{3s}{2}\right) V_1 = \frac{3s}{2} V_s$$

$$V_1 = \frac{3s\,(s+3)}{3s^2 + 9s + 2} V_s$$

$$V_o = \frac{3}{s+3} V_1 = \frac{9s}{3s^2 + 9s + 2} V_s$$

$$H(s) = \frac{V_o}{V_s} = \underline{\boldsymbol{\frac{9s}{3s^2 + 9s + 2}}}$$

Chapter 16, Solution 36.

From the previous problem,

$$3\,I = \frac{V_1}{s+3} = \frac{3s}{3s^2 + 9s + 2} V_s$$

$$I = \frac{s}{3s^2 + 9s + 2} V_s$$

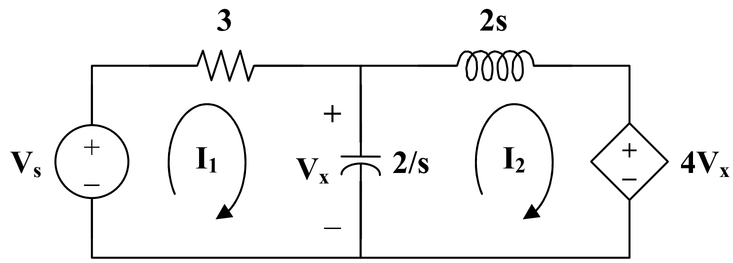
But $\quad V_s = \dfrac{3s^2 + 9s + 2}{9s} V_o$

$$I = \frac{s}{3s^2 + 9s + 2} \cdot \frac{3s^2 + 9s + 2}{9s} V_o = \frac{V_o}{9}$$

$$H(s) = \frac{V_o}{I} = \underline{\mathbf{9}}$$

Chapter 16, Solution 37.

(a) Consider the circuit shown below.



For loop 1,

$$V_s = \left(3+\frac{2}{s}\right)I_1 - \frac{2}{s}I_2 \qquad (1)$$

For loop 2,

$$4V_x + \left(2s+\frac{2}{s}\right)I_2 - \frac{2}{s}I_1 = 0$$

But, $V_x = (I_1 - I_2)\left(\frac{2}{s}\right)$

So, $\frac{8}{s}(I_1 - I_2) + \left(2s+\frac{2}{s}\right)I_2 - \frac{2}{s}I_1 = 0$

$$0 = \frac{-6}{s}I_1 + \left(\frac{6}{s} - 2s\right)I_2 \qquad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 3+2/s & -2/s \\ -6/s & 6/s-2s \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \left(3+\frac{2}{s}\right)\left(\frac{6}{s}-2s\right) - \left(\frac{6}{s}\right)\left(\frac{2}{s}\right)$$

$$\Delta = \frac{18}{s} - 6s - 4$$

$$\Delta_1 = \left(\frac{6}{s}-2s\right)V_s, \qquad \Delta_2 = \frac{6}{s}V_s$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(6/s - 2s)}{18/s - 4 - 6s} V_s$$

$$\frac{I_1}{V_s} = \frac{3/s - s}{9/s - 2 - 3} = \underline{\frac{\mathbf{s^2 - 3}}{\mathbf{3s^2 + 2s - 9}}}$$

(b) $$I_2 = \frac{\Delta_2}{\Delta}$$

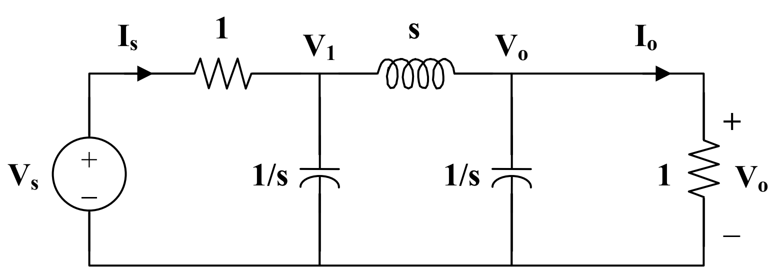
$$V_x = \frac{2}{s}(I_1 - I_2) = \frac{2}{s}\left(\frac{\Delta_1 - \Delta_2}{\Delta}\right)$$

$$V_x = \frac{2/s\, V_s\, (6/s - 2s - 6/s)}{\Delta} = \frac{-4V_s}{\Delta}$$

$$\frac{I_2}{V_x} = \frac{6/s\, V_s}{-4V_s} = \underline{\frac{\mathbf{-3}}{\mathbf{2s}}}$$

Chapter 16, Solution 38.

(a) Consider the following circuit.



At node 1,

$$\frac{V_s - V_1}{1} = s V_1 + \frac{V_1 - V_o}{s}$$

$$V_s = \left(1 + s + \frac{1}{s}\right) V_1 - \frac{1}{s} V_o \qquad (1)$$

At node o,

$$\frac{V_1 - V_o}{s} = s\,V_o + V_o = (s+1)\,V_o$$

$$V_1 = (s^2 + s + 1)\,V_o \qquad (2)$$

Substituting (2) into (1)

$$V_s = (s + 1 + 1/s)(s^2 + s + 1)V_o - 1/s\,V_o$$
$$V_s = (s^3 + 2s^2 + 3s + 2)V_o$$

$$H_1(s) = \frac{V_o}{V_s} = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 2}}}$$

(b) $\quad I_s = V_s - V_1 = (s^3 + 2s^2 + 3s + 2)V_o - (s^2 + s + 1)V_o$

$$I_s = (s^3 + s^2 + 2s + 1)V_o$$

$$H_2(s) = \frac{V_o}{I_s} = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + s^2 + 2s + 1}}}$$

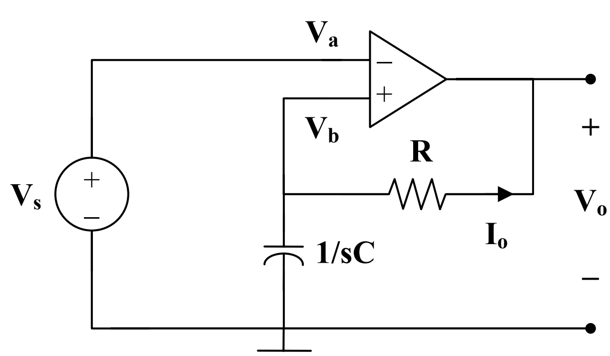
(c) $\quad I_o = \frac{V_o}{1}$

$$H_3(s) = \frac{I_o}{I_s} = \frac{V_o}{I_s} = H_2(s) = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + s^2 + 2s + 1}}}$$

(d). $\quad H_4(s) = \frac{I_o}{V_s} = \frac{V_o}{V_s} = H_1(s) = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 2}}}$

Chapter 16, Solution 39.

Consider the circuit below.



Since no current enters the op amp, I_o flows through both R and C.

$$V_o = -I_o\left(R + \frac{1}{sC}\right)$$

$$V_a = V_b = V_s = \frac{-I_o}{sC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{R + 1/sC}{1/sC} = \underline{\mathbf{sRC + 1}}$$

Chapter 16, Solution 40.

(a) $$H(s) = \frac{V_o}{V_s} = \frac{R}{R + sL} = \frac{R/L}{s + R/L}$$

$$h(t) = \underline{\frac{\mathbf{R}}{\mathbf{L}}\mathbf{e}^{\mathbf{-Rt/L}}\ \mathbf{u(t)}}$$

(b) $$v_s(t) = u(t) \longrightarrow V_s(s) = 1/s$$

$$V_o = \frac{R/L}{s + R/L}V_s = \frac{R/L}{s(s + R/L)} = \frac{A}{s} + \frac{B}{s + R/L}$$

$$A = 1, \qquad B = -1$$

$$V_o = \frac{1}{s} - \frac{1}{s + R/L}$$

$$v_o(t) = u(t) - e^{-Rt/L}\,u(t) = \underline{\mathbf{(1 - e^{-Rt/L})\,u(t)}}$$

Chapter 16, Solution 41.

$$Y(s) = H(s)\,X(s)$$

$$h(t) = 2e^{-t}\,u(t) \longrightarrow H(s) = \frac{2}{s + 1}$$

$$v_i(t) = 5u(t) \quad \longrightarrow \quad V_i(s) = X(s) = 5/s$$

$$Y(s) = \frac{10}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$A = 10$, $\qquad B = -10$

$$Y(s) = \frac{10}{s} - \frac{10}{s+1}$$

$$y(t) = \underline{\mathbf{10(1 - e^{-t})u(t)}}$$

Chapter 16, Solution 42.

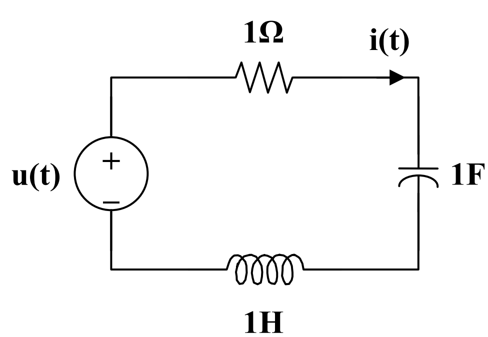
$$2s\,Y(s) + Y(s) = X(s)$$

$$(2s+1)\,Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{2s+1} = \frac{1}{2(s+1/2)}$$

$$h(t) = \underline{\mathbf{0.5e^{-t/2}\,u(t)}}$$

Chapter 16, Solution 43.



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KVL we get:

$$-u(t) + i + v_C + i' = 0; \;\; i = v_C'$$

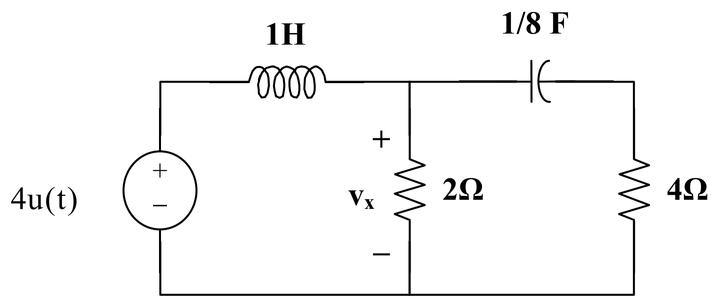
Thus,

$$v_C' = i$$
$$i' = -v_C - i + u(t)$$

Finally we get,

$$\begin{bmatrix} v_C' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} v_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t);\ i(t) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} v_C \\ i \end{bmatrix} + [0]u(t)$$

Chapter 16, Solution 44.



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KCL we get:

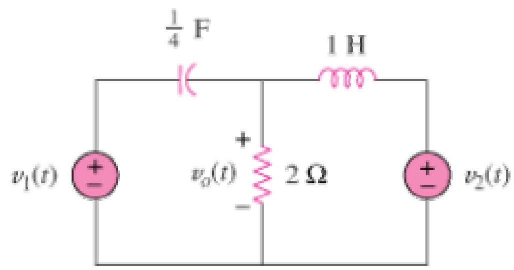
$$-i_L + \frac{v_x}{2} + \frac{v_C'}{8} = 0;\ \text{or}\ v_C' = 8i_L - 4v_x$$
$$i_L' = 4u(t) - v_x$$
$$v_x = v_C + 4\frac{v_C'}{8} = v_C + \frac{v_C'}{2} = v_C + 4i_L - 2v_x;\ \text{or}\ v_x = 0.3333v_C + 1.3333i_L$$

$$v_C' = 8i_L - 1.3333v_C - 5.333i_L = -1.3333v_C + 2.666i_L$$
$$i_L' = 4u(t) - 0.3333v_C - 1.3333i_L$$

Now we can write the state equations.

$$\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \begin{bmatrix} -1.3333 & 2.666 \\ -0.3333 & -1.3333 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix}u(t);\ v_x = \begin{bmatrix} 0.3333 \\ 1.3333 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

Chapter 16, Solution 45.



First select the inductor current i_L (current flowing left to right) and the capacitor voltage v_C (voltage positive on the left and negative on the right) to be the state variables.

Applying KCL we get:

$$-\frac{v_C'}{4}+\frac{v_o}{2}+i_L=0 \text{ or } v_C'=4i_L+2v_o$$

$$i_L'=v_o-v_2$$

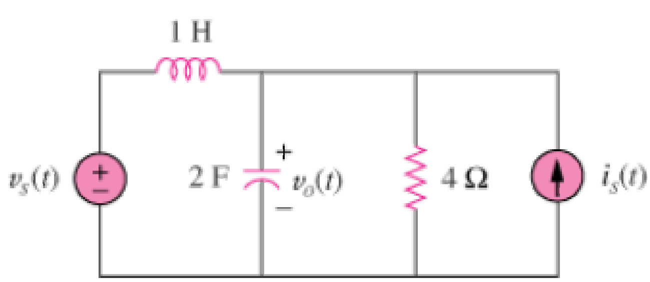
$$v_o=-v_C+v_1$$

$$v_C'=4i_L-2v_C+2v_1$$

$$i_L'=-v_C+v_1-v_2$$

$$\begin{bmatrix} i_L' \\ v_C' \end{bmatrix}=\underline{\begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix}+\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}};\ v_o(t)=\underline{\begin{bmatrix} 0 & -1 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix}+\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}}$$

Chapter 16, Solution 46.



First select the inductor current i_L (left to right) and the capacitor voltage v_C to be the state variables.

Letting $v_o = v_C$ and applying KCL we get:

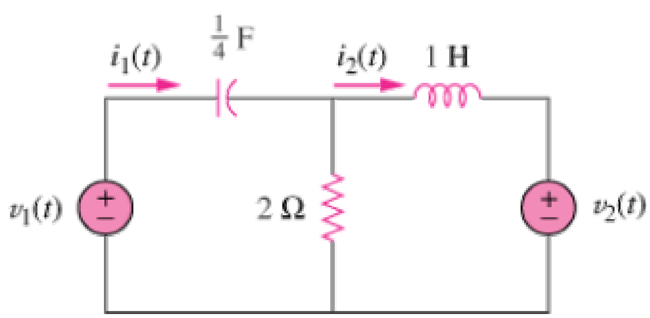
$$-i_L + v_C' + \frac{v_C}{4} - i_s = 0 \text{ or } v_C' = -0.25v_C + i_L + i_s$$

$$i_L' = -v_C + v_s$$

Thus,

$$\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \begin{bmatrix} -0.25 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} v_s \\ i_s \end{bmatrix};\ v_o(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_s \\ i_s \end{bmatrix}$$

Chapter 16, Solution 47.



First select the inductor current i_L (left to right) and the capacitor voltage v_C (+ on the left) to be the state variables.

Letting $i_1 = \frac{v_C'}{4}$ and $i_2 = i_L$ and applying KVL we get:

Loop 1:

$$-v_1 + v_C + 2\left(\frac{v_C'}{4} - i_L\right) = 0 \text{ or } v_C' = 4i_L - 2v_C + 2v_1$$

Loop 2:

$$2\left(i_L - \frac{v_C'}{4}\right) + i_L' + v_2 = 0 \text{ or}$$

$$i_L' = -2i_L + \frac{4i_L - 2v_C + 2v_1}{2} - v_2 = -v_C + v_1 - v_2$$

$$i_1 = \frac{4i_L - 2v_C + 2v_1}{4} = i_L - 0.5v_C + 0.5v_1$$

$$\begin{bmatrix} i_L{}' \\ v_C{}' \end{bmatrix} = \underline{\begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}};\ \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = \underline{\begin{bmatrix} 1 & -0.5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}}$$

Chapter 16, Solution 48.

Let $x_1 = y(t)$. Thus, $x_1' = y' = x_2$ and $x_2' = y'' = -3x_1 - 4x_2 + z(t)$

This gives our state equations.

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z(t)};\ y(t) = \underline{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] z(t)}$$

Chapter 16, Solution 49.

Let $x_1 = y(t)$ and $x_2 = x_1' - z = y' - z$ or $y' = x_2 + z$

Thus,

$$x_2' = y'' - z' = -6x_1 - 5(x_2 + z) + z' + 2z - z' = -6x_1 - 5x_2 - 3z$$

This now leads to our state equations,

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} z(t)};\ y(t) = \underline{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] z(t)}$$

Chapter 16, Solution 50.

Let $x_1 = y(t)$, $x_2 = x_1'$, and $x_3 = x_2'$.
Thus,

$$x_3'' = -6x_1 - 11x_2 - 6x_3 + z(t)$$

We can now write our state equations.

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z(t)};\ y(t) = \underline{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0] z(t)}$$

Chapter 16, Solution 51.

We transform the state equations into the s-domain and solve using Laplace transforms.

$$sX(s) - x(0) = AX(s) + B\left(\frac{1}{s}\right)$$

Assume the initial conditions are zero.

$$(sI - A)X(s) = B\left(\frac{1}{s}\right)$$

$$X(s) = \begin{bmatrix} s+4 & -4 \\ 2 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 2 \end{bmatrix} \left(\frac{1}{s}\right) = \frac{1}{s^2+4s+8} \begin{bmatrix} s & 4 \\ 2 & s+4 \end{bmatrix} \begin{bmatrix} 0 \\ (2/s) \end{bmatrix}$$

$$Y(s) = X_1(s) = \frac{8}{s(s^2+4s+8)} = \frac{1}{s} + \frac{-s-4}{s^2+4s+8}$$

$$= \frac{1}{s} + \frac{-s-4}{(s+2)^2+2^2} = \frac{1}{s} + \frac{-(s+2)}{(s+2)^2+2^2} + \frac{-2}{(s+2)^2+2^2}$$

y(t) = $\underline{\left(1 - e^{-2t}(\cos 2t + \sin 2t)\right)u(t)}$

Chapter 16, Solution 52.

Assume that the initial conditions are zero. Using Laplace transforms we get,

$$X(s) = \begin{bmatrix} s+2 & 1 \\ -2 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1/s \\ 2/s \end{bmatrix} = \frac{1}{s^2+6s+10} \begin{bmatrix} s+4 & -1 \\ 2 & s+2 \end{bmatrix} \begin{bmatrix} 3/s \\ 4/s \end{bmatrix}$$

$$X_1 = \frac{3s+8}{s((s+3)^2+1^2)} = \frac{0.8}{s} + \frac{-0.8s-1.8}{(s+3)^2+1^2}$$

$$= \frac{0.8}{s} - 0.8\frac{s+3}{(s+3)^2+1^2} + .6\frac{1}{(s+3)^2+1^2}$$

$$x_1(t) = (0.8 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)$$

$$X_2 = \frac{4s+14}{s((s+3)^2+1^2} = \frac{1.4}{s} + \frac{-1.4s-4.4}{(s+3)^2+1^2}$$

$$= \frac{1.4}{s} - 1.4\frac{s+3}{(s+3)^2+1^2} - 0.2\frac{1}{(s+3)^2+1^2}$$

$$x_2(t) = (1.4 - 1.4e^{-3t}\cos t - 0.2e^{-3t}\sin t)u(t)$$

$$y_1(t) = -2x_1(t) - 2x_2(t) + 2u(t)$$
$$= \underline{(-2.4 + 4.4e^{-3t}\cos t - 0.8e^{-3t}\sin t)u(t)}$$

$$y_2(t) = x_1(t) - 2u(t) = \underline{(-1.2 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)}$$

Chapter 16, Solution 53.

If V_o is the voltage across R, applying KCL at the non-reference node gives

$$I_s = \frac{V_o}{R} + sC\,V_o + \frac{V_o}{sL} = \left(\frac{1}{R} + sC + \frac{1}{sL}\right)V_o$$

$$V_o = \frac{I_s}{\frac{1}{R} + sC + \frac{1}{sL}} = \frac{sRL\,I_s}{sL + R + s^2RLC}$$

$$I_o = \frac{V_o}{R} = \frac{sL\,I_s}{s^2RLC + sL + R}$$

$$H(s) = \frac{I_o}{I_s} = \frac{sL}{s^2RLC + sL + R} = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

The roots

$$s_{1,2} = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

both lie in the left half plane since R, L, and C are positive quantities.

Thus, <u>**the circuit is stable**</u>.

Chapter 16, Solution 54.

(a) $H_1(s) = \dfrac{3}{s+1}$, $\qquad H_2(s) = \dfrac{1}{s+4}$

$$H(s) = H_1(s)H_2(s) = \frac{3}{(s+1)(s+4)}$$

$$h(t) = \mathcal{L}^{-1}\left[H(s)\right] = \mathcal{L}^{-1}\left[\frac{A}{s+1} + \frac{B}{s+4}\right]$$

$A = 1$, $\qquad B = -1$

$h(t) = \underline{\mathbf{(e^{-t} - e^{-4t})u(t)}}$

(b) Since the poles of H(s) all lie in the left half s-plane, **the system is stable**.

Chapter 16, Solution 55.

Let V_{o1} be the voltage at the output of the first op amp.

$\dfrac{V_{o1}}{V_s} = \dfrac{-1/sC}{R} = \dfrac{-1}{sRC}$, $\qquad \dfrac{V_o}{V_{o1}} = \dfrac{-1}{sRC}$

$$H(s) = \frac{V_o}{V_s} = \frac{1}{s^2R^2C^2}$$

$$h(t) = \frac{t}{R^2C^2}$$

$\lim\limits_{t\to\infty} h(t) = \infty$, i.e. the output is unbounded.

Hence, **the circuit is unstable**.

Chapter 16, Solution 56.

$$sL \,||\, \frac{1}{sC} = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1 + s^2LC}$$

$$\frac{V_2}{V_1} = \frac{\frac{sL}{1+s^2LC}}{R + \frac{sL}{1+s^2LC}} = \frac{sL}{s^2RLC + sL + R}$$

$$\frac{V_2}{V_1} = \frac{s \cdot \frac{1}{RC}}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$$

Comparing this with the given transfer function,

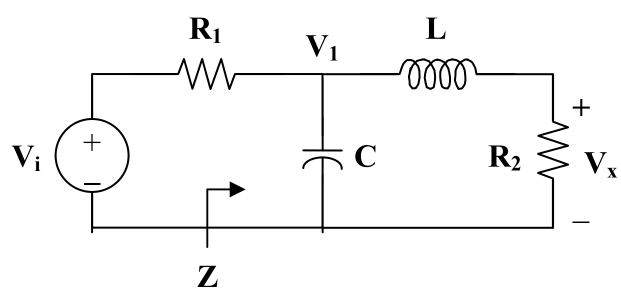
$$2 = \frac{1}{RC}, \qquad 6 = \frac{1}{LC}$$

If $R = 1\,k\Omega$, $\quad C = \frac{1}{2R} = \underline{\mathbf{500\ \mu F}}$

$$L = \frac{1}{6C} = \underline{\mathbf{333.3\ H}}$$

Chapter 16, Solution 57.

The circuit in the s-domain is shown below.



$$Z = \frac{1}{sC} \,||\, (R_2 + sL) = \frac{(1/sC)\cdot(R_2 + sL)}{R_2 + sL + 1/sC} = \frac{R_2 + sL}{1 + s^2LC + sR_2C}$$

$$V_1 = \frac{Z}{R_1 + Z} V_i$$

$$V_o = \frac{R_2}{R_2 + sL} V_1 = \frac{R_2}{R_2 + sL} \cdot \frac{Z}{R_1 + Z} V_i$$

$$\frac{V_o}{V_i} = \frac{R_2}{R_2 + sL} \cdot \frac{Z}{R_1 + Z} = \frac{R_2}{R_2 + sL} \cdot \frac{\dfrac{R_2 + sL}{1 + s^2 LC + sR_2 C}}{R_1 + \dfrac{R_2 + sL}{1 + s^2 LC + sR_2 C}}$$

$$\frac{V_o}{V_i} = \frac{R_2}{s^2 R_1 LC + sR_1 R_2 C + R_1 + R_2 + sL}$$

$$\frac{V_o}{V_i} = \frac{\dfrac{R_2}{R_1 LC}}{s^2 + s\left(\dfrac{R_2}{L} + \dfrac{1}{R_1 C}\right) + \dfrac{R_1 + R_2}{R_1 LC}}$$

Comparing this with the given transfer function,

$$5 = \frac{R_2}{R_1 LC} \qquad 6 = \frac{R_2}{L} + \frac{1}{R_1 C} \qquad 25 = \frac{R_1 + R_2}{R_1 LC}$$

Since $R_1 = 4\,\Omega$ and $R_2 = 1\,\Omega$,

$$5 = \frac{1}{4LC} \longrightarrow LC = \frac{1}{20} \tag{1}$$

$$6 = \frac{1}{L} + \frac{1}{4C} \tag{2}$$

$$25 = \frac{5}{4LC} \longrightarrow LC = \frac{1}{20}$$

Substituting (1) into (2),

$$6 = 20C + \frac{1}{4C} \longrightarrow 80C^2 - 24C + 1 = 0$$

Thus, $C = \dfrac{1}{4},\ \dfrac{1}{20}$

When $C = \frac{1}{4}$, $\qquad L = \frac{1}{20C} = \frac{1}{5}$.

When $C = \frac{1}{20}$, $\qquad L = \frac{1}{20C} = 1$.

Therefore, there are two possible solutions.
C = **0.25 F** L = **0.2 H** or C = **0.05 F** L = **1 H**

Chapter 16, Solution 58.

We apply KCL at the noninverting terminal at the op amp.

$$(V_s - 0)\,Y_3 = (0 - V_o)(Y_1 - Y_2)$$

$$Y_3\,V_s = -(Y_1 + Y_2)V_o$$

$$\frac{V_o}{V_s} = \frac{-Y_3}{Y_1 + Y_2}$$

Let $\quad Y_1 = sC_1, \qquad Y_2 = 1/R_1, \qquad Y_3 = sC_2$

$$\frac{V_o}{V_s} = \frac{-sC_2}{sC_1 + 1/R_1} = \frac{-sC_2/C_1}{s + 1/R_1C_1}$$

Comparing this with the given transfer function,

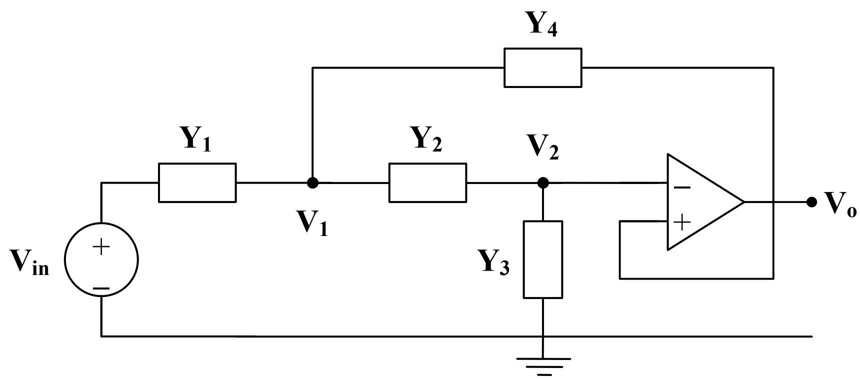
$$\frac{C_2}{C_1} = 1, \qquad \frac{1}{R_1C_1} = 10$$

If $R_1 = 1\ \text{k}\Omega$,

$$C_1 = C_2 = \frac{1}{10^4} = \underline{\mathbf{100\ \mu F}}$$

Chapter 16, Solution 59.

Consider the circuit shown below. We notice that $V_3 = V_o$ and $V_2 = V_3 = V_o$.



At node 1,

$$(V_{in} - V_1)Y_1 = (V_1 - V_o)Y_2 + (V_1 - V_o)Y_4$$

$$V_{in}\,Y_1 = V_1(Y_1 + Y_2 + Y_4) - V_o(Y_2 + Y_4) \qquad (1)$$

At node 2,

$$(V_1 - V_o)Y_2 = (V_o - 0)Y_3$$

$$V_1\,Y_2 = (Y_2 + Y_3)V_o$$

$$V_1 = \frac{Y_2 + Y_3}{Y_2}V_o \qquad (2)$$

Substituting (2) into (1),

$$V_{in}\,Y_1 = \frac{Y_2 + Y_3}{Y_2}\cdot(Y_1 + Y_2 + Y_4)V_o - V_o(Y_2 + Y_4)$$

$$V_{in}Y_1Y_2 = V_o(Y_1Y_2 + Y_2^2 + Y_2Y_4 + Y_1Y_3 + Y_2Y_3 + Y_3Y_4 - Y_2^2 - Y_2Y_4)$$

$$\frac{V_o}{V_{in}} = \frac{Y_1Y_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3 + Y_3Y_4}$$

Y_1 and Y_2 must be resistive, while Y_3 and Y_4 must be capacitive.

Let $\quad Y_1 = \dfrac{1}{R_1}, \qquad Y_2 = \dfrac{1}{R_2}, \qquad Y_3 = sC_1, \qquad Y_4 = sC_2$

$$\frac{V_o}{V_{in}} = \frac{\frac{1}{R_1R_2}}{\frac{1}{R_1R_2} + \frac{sC_1}{R_1} + \frac{sC_1}{R_2} + s^2C_1C_2}$$

$$\frac{V_o}{V_{in}} = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + s \cdot \left(\frac{R_1 + R_2}{R_1R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}}$$

Choose $R_1 = 1\ \text{k}\Omega$, then

$$\frac{1}{R_1R_2C_1C_2} = 10^6 \qquad \text{and} \qquad \frac{R_1 + R_2}{R_1R_2C_2} = 100$$

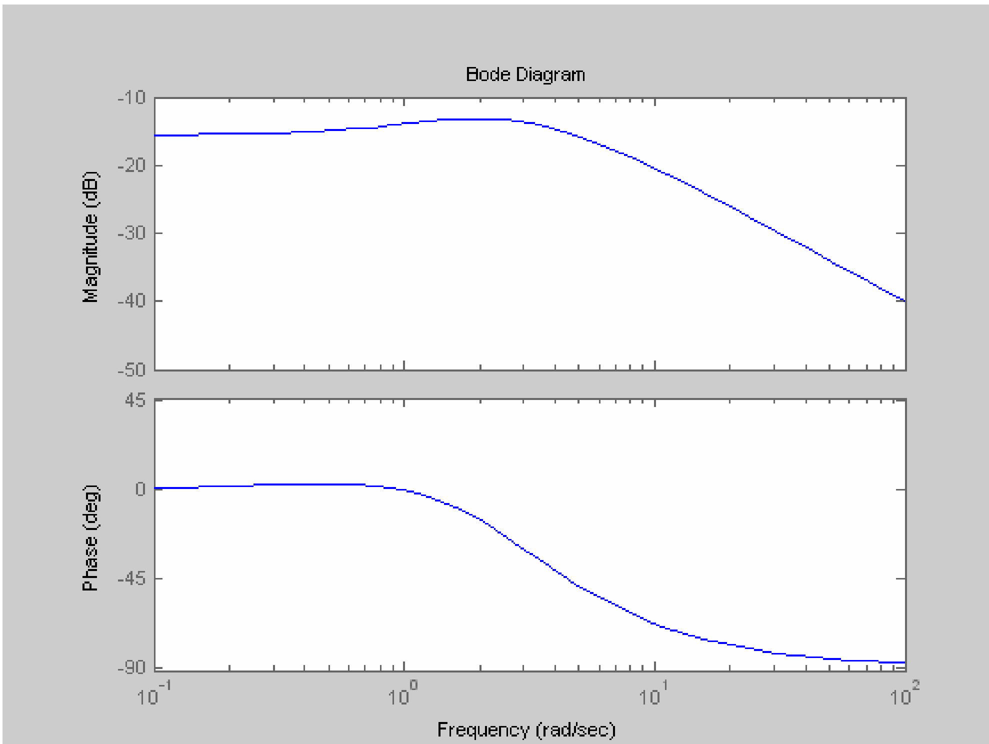
We have three equations and four unknowns. Thus, there is a family of solutions. One such solution is

$R_2 = \underline{\mathbf{1\ k\Omega}}$, $C_1 = \underline{\mathbf{50\,nF}}$, $C_2 = \underline{\mathbf{20\ \mu F}}$

Chapter 16, Solution 60.

With the following MATLAB codes, the Bode plots are generated as shown below.

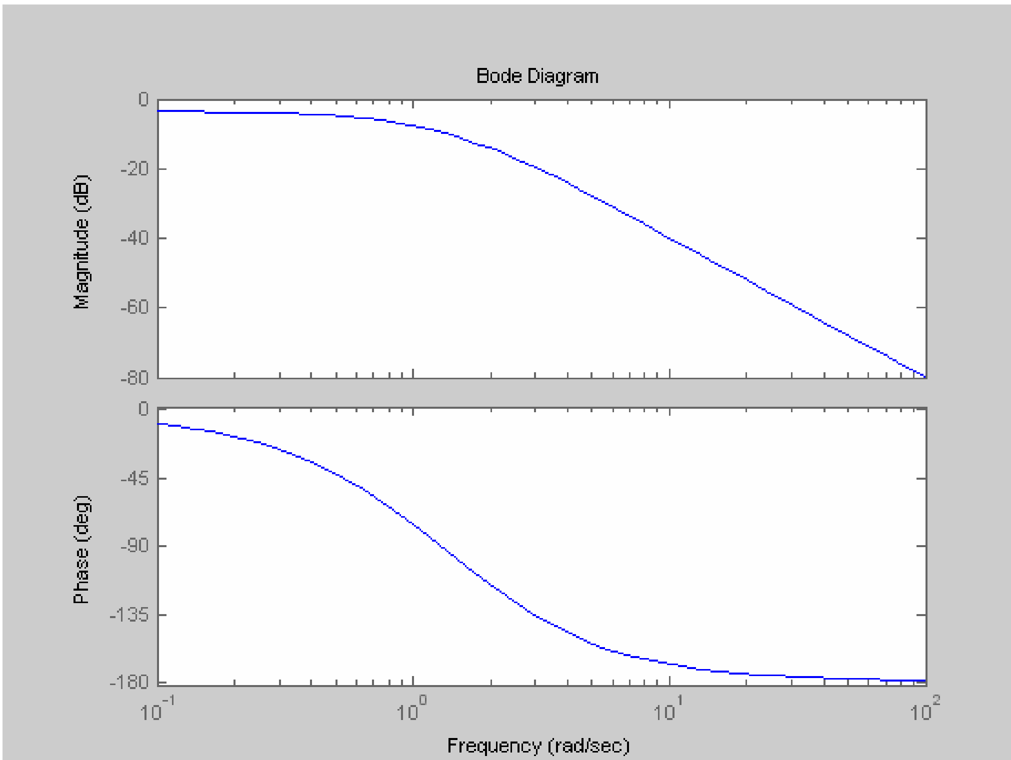
```
num=[1 1];
den= [1 5 6];
bode(num,den);
```



Chapter 16, Solution 61.

We use the following codes to obtain the Bode plots below.

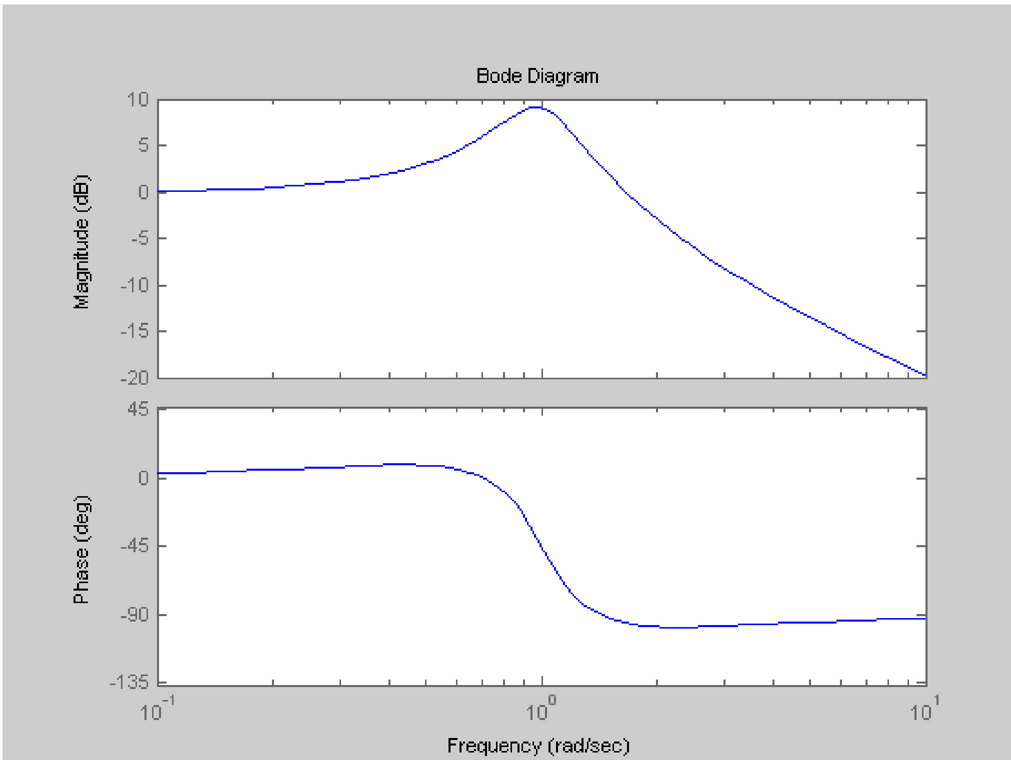
```
num=[1 4];
den= [1 6  11  6];
bode(num,den);
```



Chapter 16, Solution 62.

The following codes are used to obtain the Bode plots below.

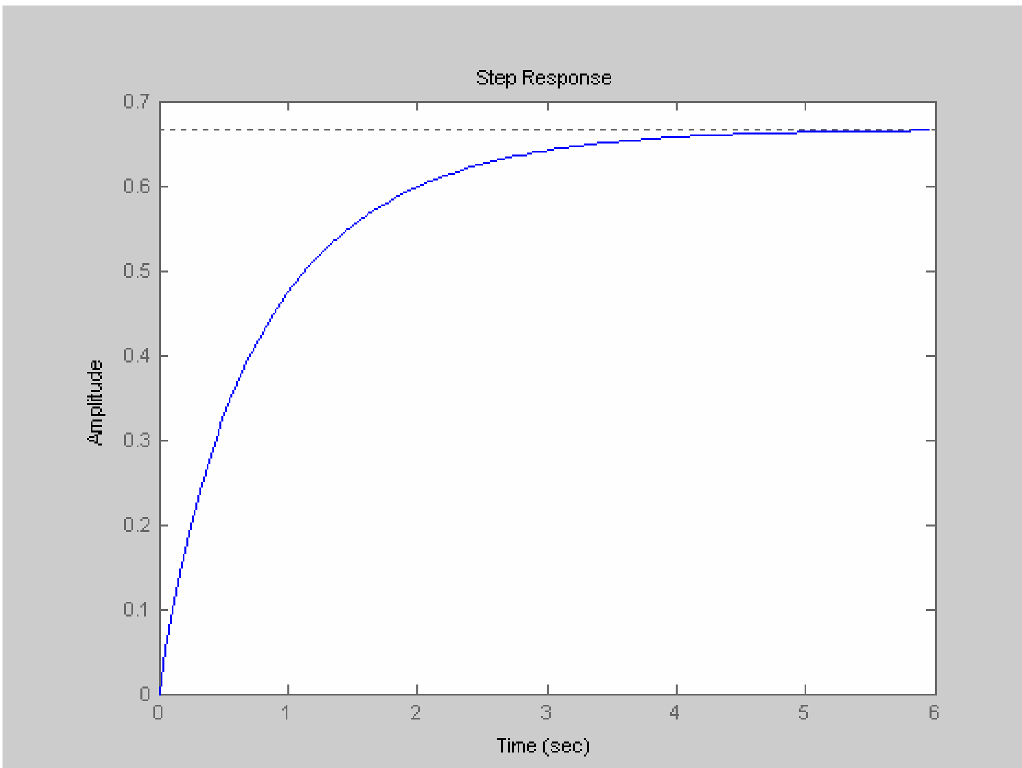
```
num=[1 1];
den= [1 0.5  1];
bode(num,den);
```



Chapter 16, Solution 63.

We use the following commands to obtain the unit step as shown below.

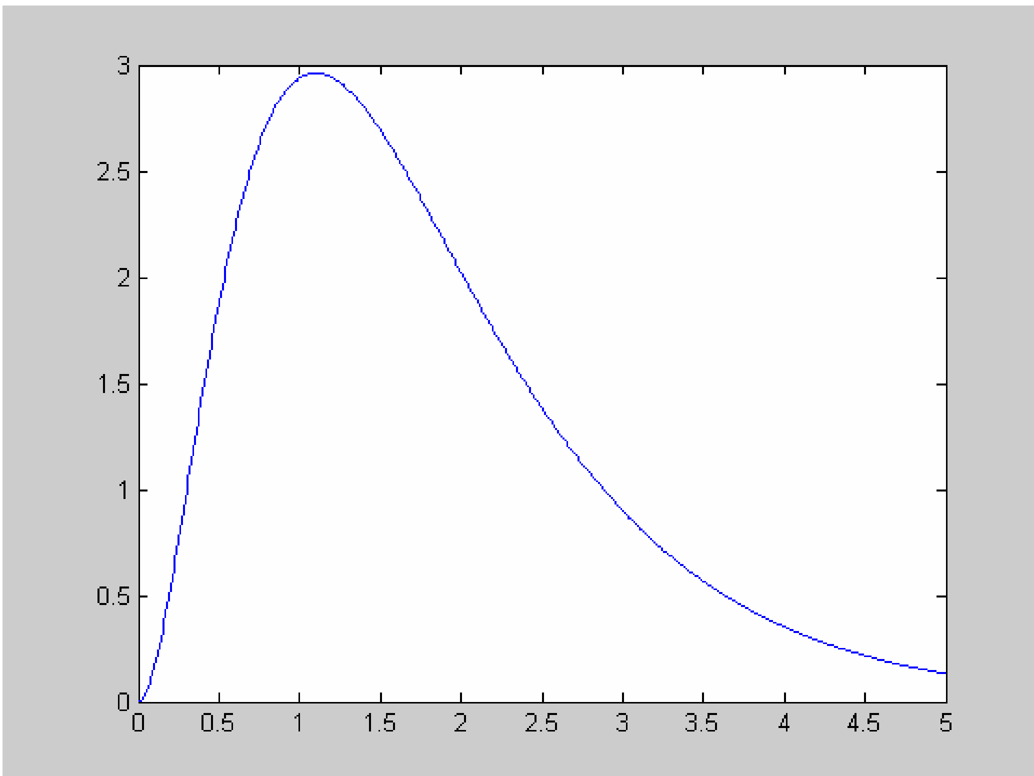
```
num=[1 2];
den= [1  4  3];
step(num,den);
```



Chapter 16, Solution 64.

With the following commands, we obtain the response as shown below.

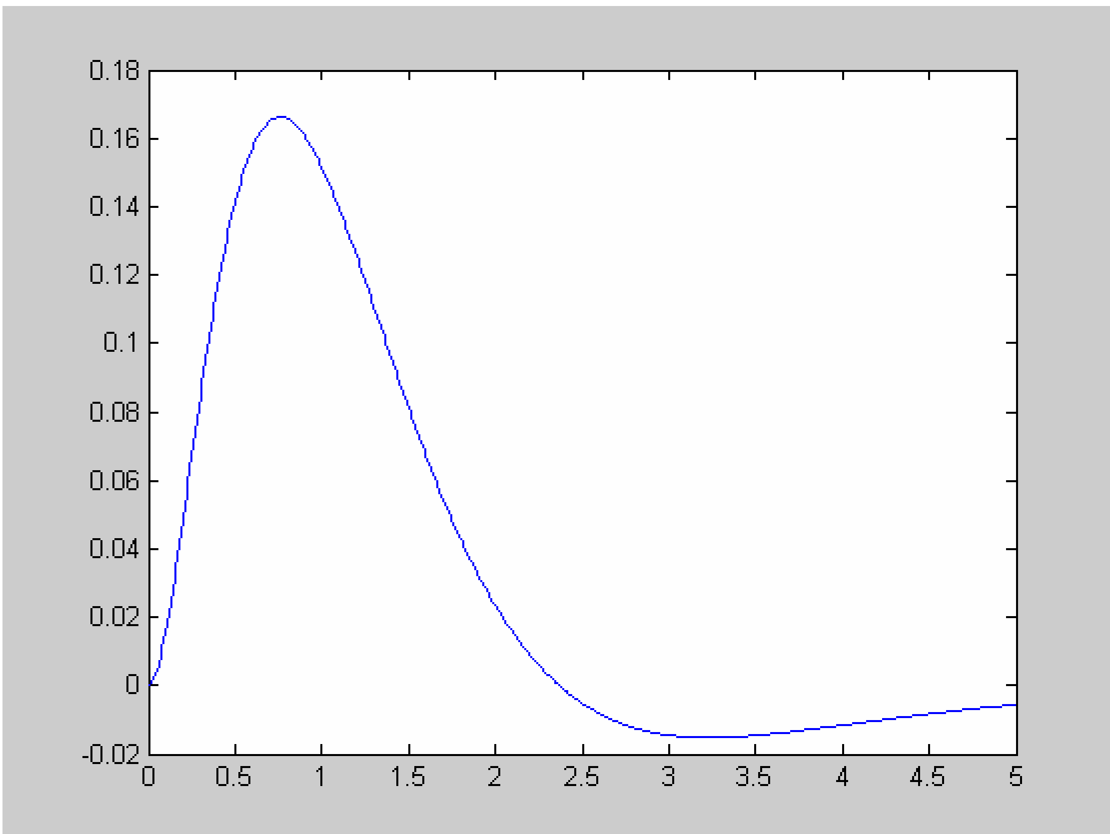
```
t=0:0.01:5;
x=10*exp(-t);
num=4;
den= [1  5  6];
y=lsim(num,den,x,t);
plot(t,y)
```



Chapter 16, Solution 65.

We obtain the response below using the following commands.

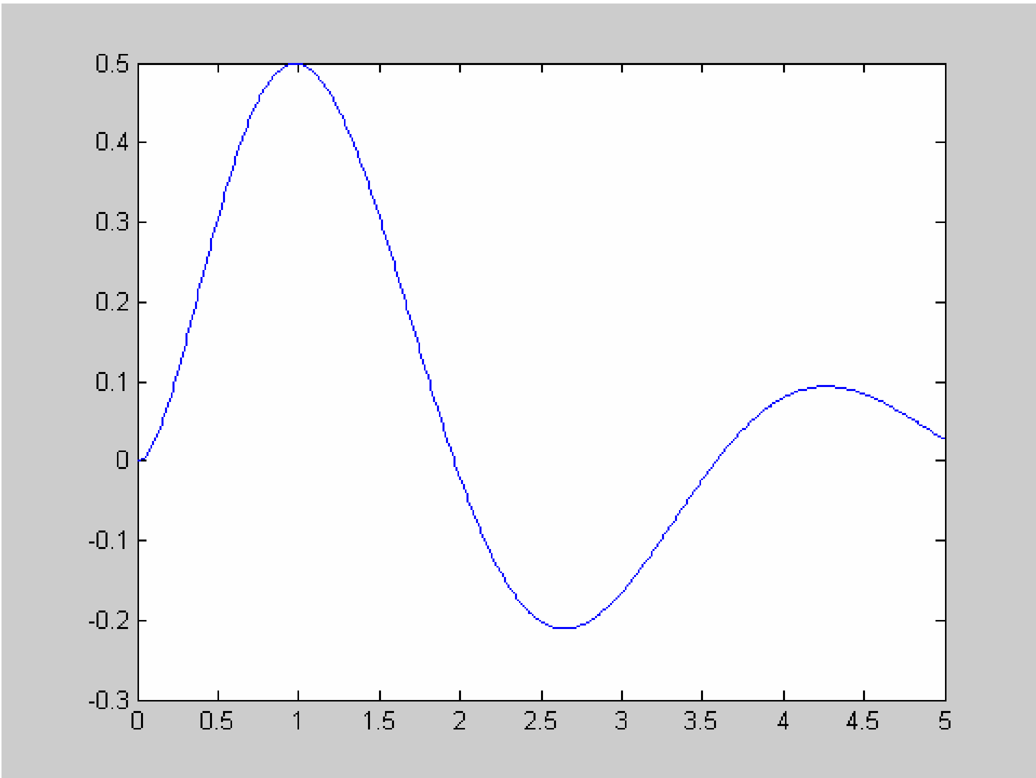
```
t=0:0.01:5;
x=1 + 3*exp(-2*t);
num=[1 0];
den= [1  6  11  6];
y=lsim(num,den,x,t);
plot(t,y)
```



Chapter 16, Solution 66.

We obtain the response below using the following MATLAB commands.

```
t=0:0.01:5;
x=5*exp(-3*t);
num=1;
den= [1  1  4];
y=lsim(num,den,x,t);
plot(t,y)
```



Chapter 16, Solution 67.

Using the result of Practice Problem 16.14,

$$\frac{V_o}{V_i} = \frac{-Y_1Y_2}{Y_2Y_3 + Y_4(Y_1 + Y_2 + Y_3)}$$

When $Y_1 = sC_1, \qquad C_1 = 0.5\ \mu F$

$$Y_2 = \frac{1}{R_1}, \qquad R_1 = 10\ k\Omega$$

$$Y_3 = Y_2, \qquad Y_4 = sC_2, \qquad C_2 = 1\ \mu F$$

$$\frac{V_o}{V_i} = \frac{-sC_1/R_1}{1/R_1^2 + sC_2(sC_1 + 2/R_1)} = \frac{-sC_1R_1}{1 + sC_2R_1(2 + sC_1R_1)}$$

$$\frac{V_o}{V_i} = \frac{-sC_1R_1}{s^2C_1C_2R_1^2 + s \cdot 2C_2R_1 + 1}$$

$$\frac{V_o}{V_i} = \frac{-s(0.5\times10^{-6})(10\times10^3)}{s^2(0.5\times10^{-6})(1\times10^{-6})(10\times10^3)^2 + s(2)(1\times10^{-6})(10\times10^3) + 1}$$

$$\frac{V_o}{V_i} = \frac{-100s}{s^2 + 400s + 2\times10^4}$$

Therefore,

$a = \underline{\mathbf{-100}}, \qquad b = \underline{\mathbf{400}}, \qquad c = \underline{\mathbf{2\times10^4}}$

Chapter 16, Solution 68.

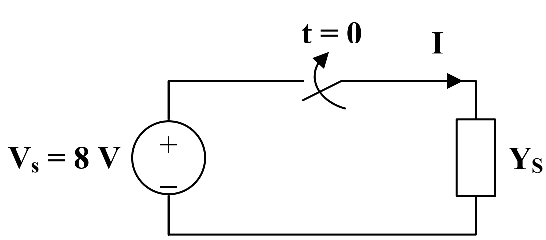
(a) Let $Y(s) = \dfrac{K(s+1)}{s+3}$

$$Y(\infty) = \lim_{s\to\infty} \frac{K(s+1)}{s+3} = \lim_{s\to\infty} \frac{K(1+1/s)}{1+3/s} = K$$

i.e. $0.25 = K$.

Hence, $Y(s) = \underline{\dfrac{\mathbf{s+1}}{\mathbf{4(s+3)}}}$

(b) Consider the circuit shown below.



$$V_s = 8u(t) \longrightarrow V_s = 8/s$$

$$I = \frac{V_s}{Z} = Y(s)V_s(s) = \frac{8}{4s} \cdot \frac{s+1}{s+3} = \frac{2(s+1)}{s(s+3)}$$

$$I = \frac{A}{s} + \frac{B}{s+3}$$

$$A = 2/3, \qquad B = -4/3$$

$$i(t) = \underline{\mathbf{\frac{1}{3}\left[2 - 4e^{-3t}\right] u(t)\ A}}$$

Chapter 16, Solution 69.

The gyrator is equivalent to two cascaded inverting amplifiers. Let V_1 be the voltage at the output of the first op amp.

$$V_1 = \frac{-R}{R} V_i = -V_i$$

$$V_o = \frac{-1/sC}{R} V_1 = \frac{1}{sCR} V_i$$

$$I_o = \frac{V_o}{R} = \frac{V_o}{sR^2C}$$

$$\frac{V_o}{I_o} = sR^2C$$

$$\underline{\mathbf{\frac{V_o}{I_o} = sL, \quad when\ L = R^2C}}$$