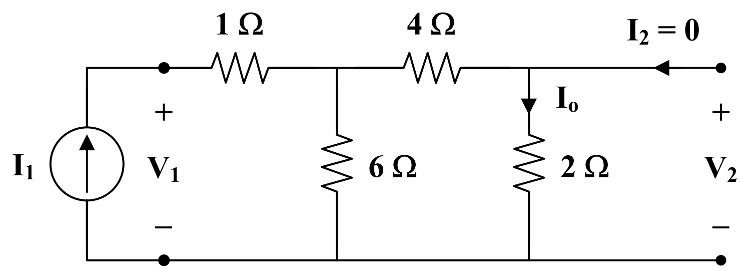
Chapter 19, Solution 1.

To get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



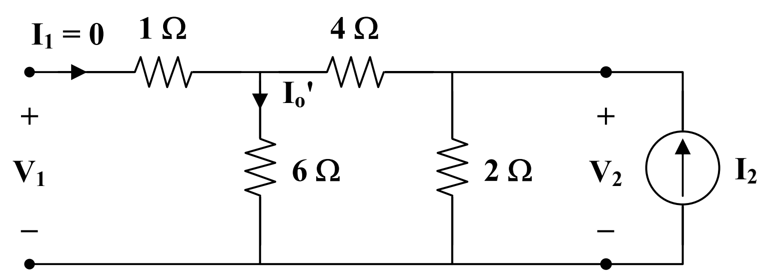
(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 1 + 6 \,||\, (4+2) = 4\,\Omega$$

$$\mathbf{I}_o = \frac{1}{2}\mathbf{I}_1, \qquad \mathbf{V}_2 = 2\mathbf{I}_o = \mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = 1\,\Omega$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 2 \,||\, (4+6) = 1.667\,\Omega$$

$$\mathbf{I}_o' = \frac{2}{2+10}\mathbf{I}_2 = \frac{1}{6}\mathbf{I}_2, \qquad \mathbf{V}_1 = 6\mathbf{I}_o' = \mathbf{I}_2$$

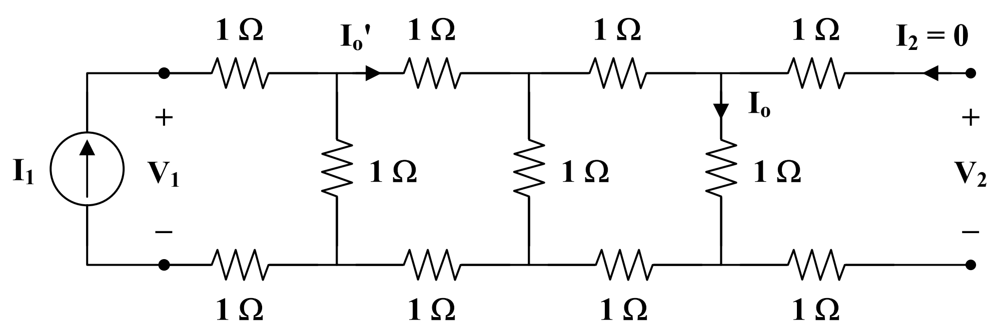
$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = 1\,\Omega$$

Hence,

$$[\mathbf{z}] = \underline{\begin{bmatrix} \mathbf{4} & \mathbf{1} \\ \mathbf{1} & \mathbf{1.667} \end{bmatrix} \Omega}$$

Chapter 19, Solution 2.

Consider the circuit in Fig. (a) to get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$.



(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2 + 1 \,||\, [\,2 + 1 \,||\, (2+1)\,]$$

$$\mathbf{z}_{11} = 2 + 1 \,||\left(2 + \frac{3}{4}\right) = 2 + \frac{(1)(11/4)}{1 + 11/4} = 2 + \frac{11}{15} = 2.733$$

$$\mathbf{I}_o = \frac{1}{1+3}\mathbf{I}_o{}' = \frac{1}{4}\mathbf{I}_o{}'$$

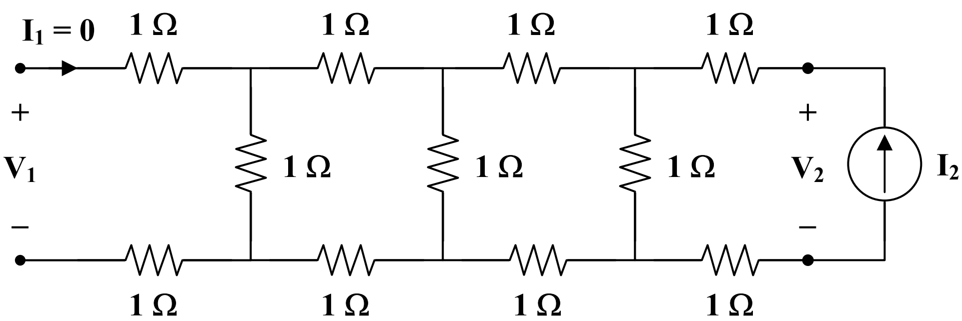
$$\mathbf{I}_o{}' = \frac{1}{1 + 11/4}\mathbf{I}_1 = \frac{4}{15}\mathbf{I}_1$$

$$\mathbf{I}_o = \frac{1}{4}\cdot\frac{4}{15}\mathbf{I}_1 = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_o = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{15} = \mathbf{z}_{12} = 0.06667$$

To get $\mathbf{z}_{22}$, consider the circuit in Fig. (b).



(b)

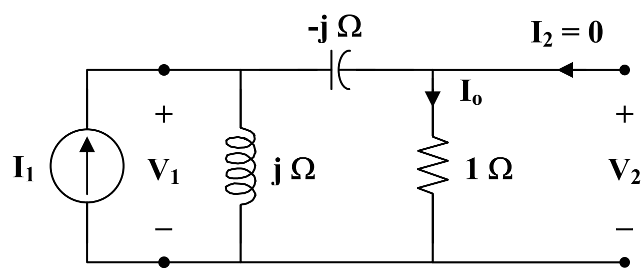
$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 2 + 1 \,\|\, (2 + 1 \,\|\, 3) = \mathbf{z}_{11} = 2.733$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{2.733} & \mathbf{0.06667} \\ \mathbf{0.06667} & \mathbf{2.733} \end{bmatrix} \Omega$$

Chapter 19, Solution 3.

(a) To find $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = j \,\|\, (1 - j) = \frac{j(1-j)}{j + 1 - j} = 1 + j$$

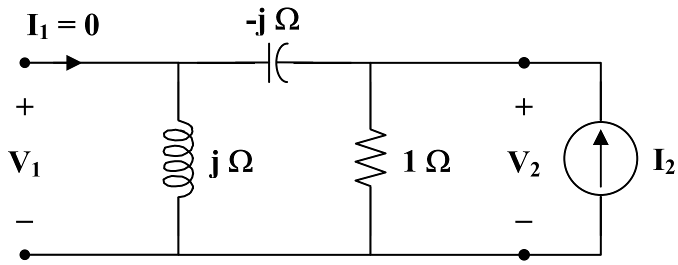
By current division,

$$\mathbf{I}_o = \frac{j}{j + 1 - j}\mathbf{I}_1 = j\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_o = j\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = j$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 1 \,\|\, (j - j) = 0$$

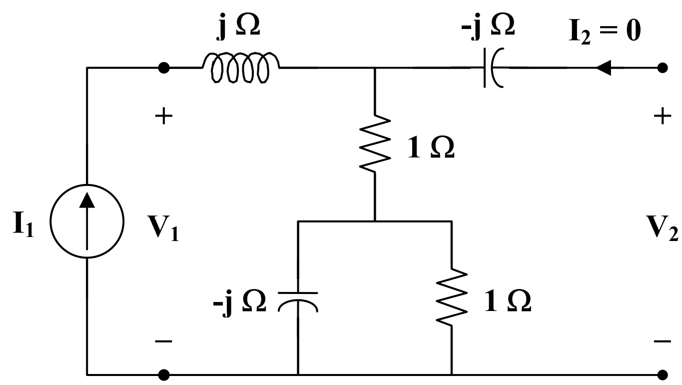
$$\mathbf{V}_1 = j\mathbf{I}_2$$

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = j$$

Thus,

$$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{1+j} & \mathbf{j} \\ \mathbf{j} & \mathbf{0} \end{bmatrix} \Omega}$$

(b) To find $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (c).



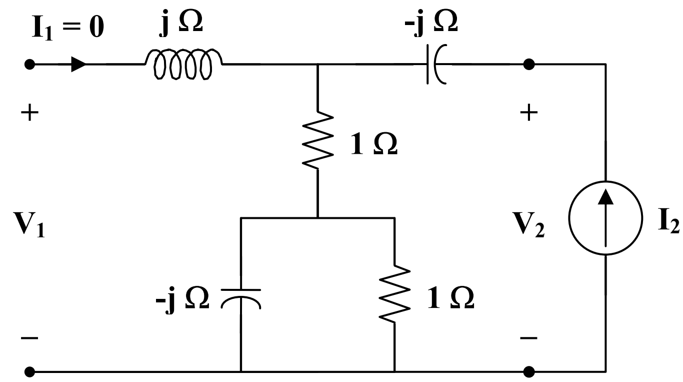
(c)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = j + 1 + 1 \parallel (-j) = 1 + j + \frac{-j}{1-j} = 1.5 + j0.5$$

$$\mathbf{V}_2 = (1.5 - j0.5)\,\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = 1.5 - j0.5$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (d).



(d)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = -j + 1 + 1 \parallel (-j) = 1.5 - j1.5$$

$$\mathbf{V}_1 = (1.5 - j0.5)\,\mathbf{I}_2$$

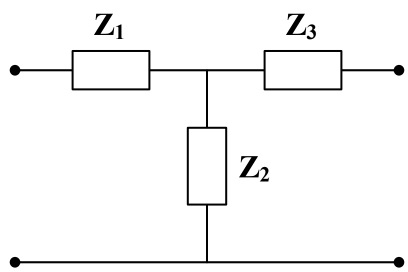
$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = 1.5 - j0.5$$

Thus,

$$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{1.5 + j0.5} & \mathbf{1.5 - j0.5} \\ \mathbf{1.5 - j0.5} & \mathbf{1.5 - j1.5} \end{bmatrix} \Omega}$$

Chapter 19, Solution 4.

Transform the Π network to a T network.



$$\mathbf{Z}_1 = \frac{(12)(j10)}{12 + j10 - j5} = \frac{j120}{12 + j5}$$

$$\mathbf{Z}_2 = \frac{-j60}{12 + j5}$$

$$\mathbf{Z}_3 = \frac{50}{12 + j5}$$

The z parameters are

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{Z}_2 = \frac{(-j60)(12 - j5)}{144 + 25} = -1.775 - j4.26$$

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{z}_{12} = \frac{(j120)(12 - j5)}{169} + \mathbf{z}_{12} = 1.775 + j4.26$$

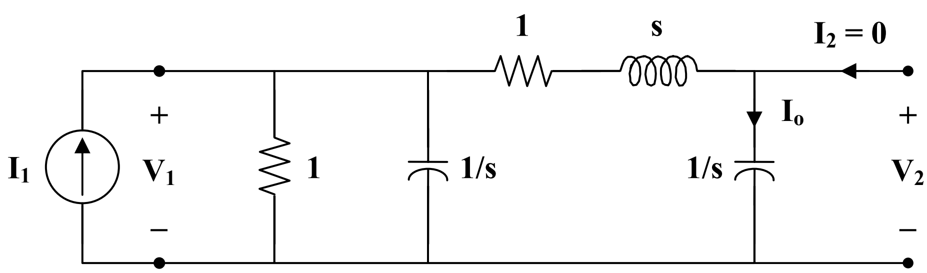
$$\mathbf{z}_{22} = \mathbf{Z}_3 + \mathbf{z}_{21} = \frac{(50)(12 - j5)}{169} + \mathbf{z}_{21} = 1.7758 - j5.739$$

Thus,

$$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{1.775 + j4.26} & \mathbf{-1.775 - j4.26} \\ \mathbf{-1.775 - j4.26} & \mathbf{1.775 - j5.739} \end{bmatrix} \Omega}$$

Chapter 19, Solution 5.

Consider the circuit in Fig. (a).



(a)

$$\mathbf{z}_{11} = 1 \,\|\, \frac{1}{s} \,\|\, \left(1 + s + \frac{1}{s}\right) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} \,\|\, \left(1 + s + \frac{1}{s}\right) = \frac{\left(\frac{1}{s+1}\right)\left(1 + s + \frac{1}{s}\right)}{\left(\frac{1}{s+1}\right) + 1 + s + \frac{1}{s}}$$

$$\mathbf{z}_{11} = \frac{s^2 + s + 1}{s^3 + 2s^2 + 3s + 1}$$

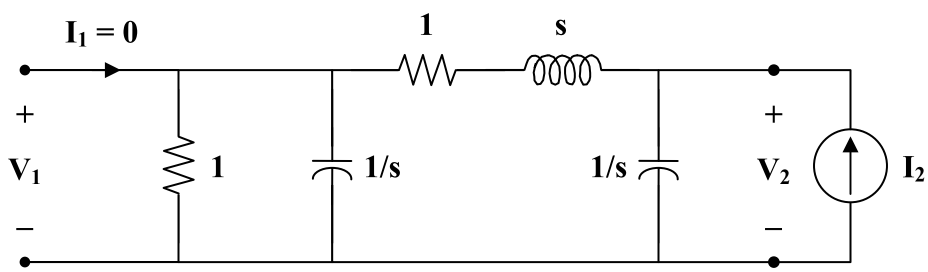
$$\mathbf{I}_o = \frac{1 \| \frac{1}{s}}{1 \| \frac{1}{s} + 1 + s + \frac{1}{s}} \mathbf{I}_1 = \frac{\frac{1}{s+1}}{\frac{1}{s+1} + 1 + s + \frac{1}{s}} \mathbf{I}_1 = \frac{\frac{s}{s+1}}{\frac{s}{s+1} + s^2 + s + 1} \mathbf{I}_1$$

$$\mathbf{I}_o = \frac{s}{s^3 + 2s^2 + 3s + 1} \mathbf{I}_1$$

$$\mathbf{V}_2 = \frac{1}{s} \mathbf{I}_o = \frac{\mathbf{I}_1}{s^3 + 2s^2 + 3s + 1}$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

Consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{s} \| \left(1 + s + 1 \| \frac{1}{s} \right) = \frac{1}{s} \| \left(1 + s + \frac{1}{s+1} \right)$$

$$\mathbf{z}_{22} = \frac{\left(\frac{1}{s}\right)\left(1 + s + \frac{1}{s+1}\right)}{\frac{1}{s} + 1 + s + \frac{1}{s+1}} = \frac{1 + s + \frac{1}{s+1}}{1 + s + s^2 + \frac{s}{s+1}}$$

$$\mathbf{z}_{22} = \frac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1}$$

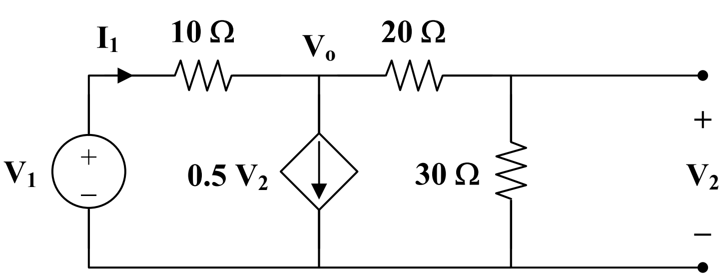
$$\mathbf{z}_{12} = \mathbf{z}_{21}$$

Hence,

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\mathbf{s}^2+\mathbf{s}+1}{\mathbf{s}^3+2\mathbf{s}^2+3\mathbf{s}+1} & \dfrac{1}{\mathbf{s}^3+2\mathbf{s}^2+3\mathbf{s}+1} \\ \dfrac{1}{\mathbf{s}^3+2\mathbf{s}^2+3\mathbf{s}+1} & \dfrac{\mathbf{s}^2+2\mathbf{s}+2}{\mathbf{s}^3+2\mathbf{s}^2+3\mathbf{s}+1} \end{bmatrix}$$

Chapter 19, Solution 6.

To find $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, connect a voltage source $\mathbf{V}_1$ to the input and leave the output open as in Fig. (a).



(a)

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{10} = 0.5\mathbf{V}_2 + \frac{\mathbf{V}_o}{50}, \qquad \text{where } \mathbf{V}_2 = \frac{30}{20+30}\mathbf{V}_o = \frac{3}{5}\mathbf{V}_o$$

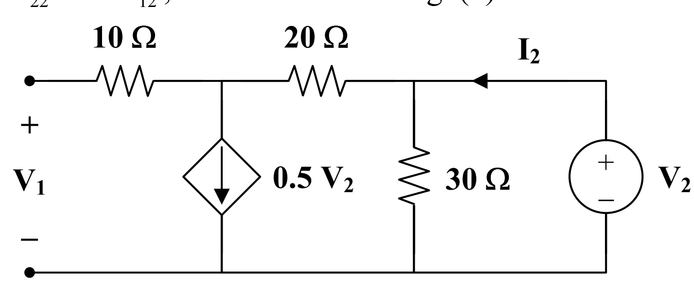
$$\mathbf{V}_1 = \mathbf{V}_o + 5\left(\frac{3}{5}\mathbf{V}_o\right) + \frac{\mathbf{V}_o}{5} = 4.2\,\mathbf{V}_o$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{10} = \frac{3.2}{10}\mathbf{V}_o = 0.32\,\mathbf{V}_o$$

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{4.2\,\mathbf{V}_o}{0.32\,\mathbf{V}_o} = 13.125\,\Omega$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{0.6\,\mathbf{V}_o}{0.32\,\mathbf{V}_o} = 1.875\,\Omega$$

To obtain $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, use the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = 0.5\mathbf{V}_2 + \frac{\mathbf{V}_2}{30} = 0.5333\mathbf{V}_2$$

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{0.5333} = 1.875\,\Omega$$

$$\mathbf{V}_1 = \mathbf{V}_2 - (20)(0.5\mathbf{V}_2) = \text{-}9\mathbf{V}_2$$

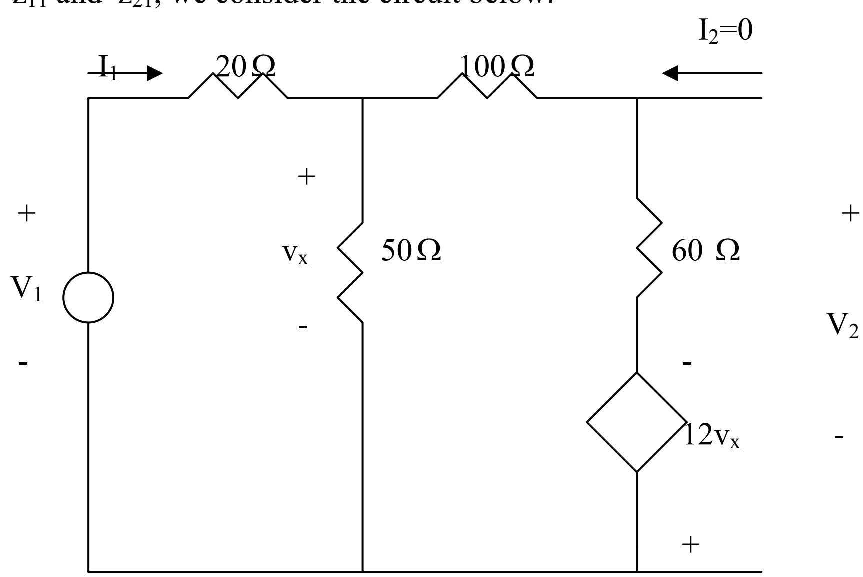
$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{\text{-}9\mathbf{V}_2}{0.5333\mathbf{V}_2} = \text{-}16.875\,\Omega$$

Thus,

$$\underline{[\mathbf{z}] = \begin{bmatrix} \mathbf{13.125} & \mathbf{-16.875} \\ \mathbf{1.875} & \mathbf{1.875} \end{bmatrix} \Omega}$$

Chapter 19, Solution 7.

To get z_{11} and z_{21}, we consider the circuit below.



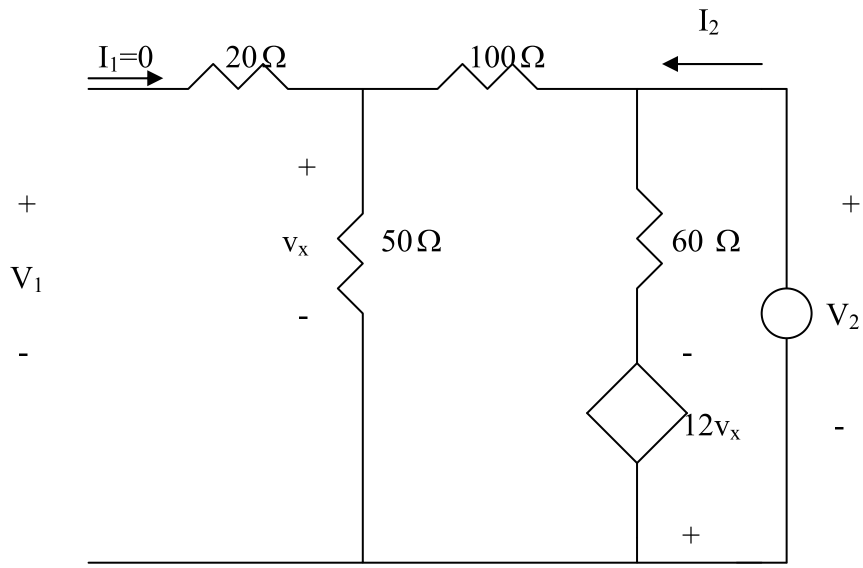
$$\frac{V_1 - V_x}{20} = \frac{V_x}{50} + \frac{V_x + 12V_x}{160} \longrightarrow V_x = \frac{40}{121}V_1$$

$$I_1 = \frac{V_1 - V_x}{20} = \frac{81}{121}(\frac{V_1}{20}) \longrightarrow z_{11} = \frac{V_1}{I_1} = 29.88$$

$$V_2 = 60\left(\frac{13V_x}{160}\right) - 12V_x = -\frac{57}{8}V_x = -\frac{57}{8}\left(\frac{40}{121}\right)V_1 = -\frac{57}{8}\left(\frac{40}{121}\right)\frac{20x121}{81}I_1$$

$$= -70.37I_1 \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = -70.37$$

To get z_{12} and z_{22}, we consider the circuit below.



$$V_x = \frac{50}{100+50}V_2 = \frac{1}{3}V_2, \qquad I_2 = \frac{V_2}{150} + \frac{V_2 + 12V_x}{60} = 0.09V_2$$

$$z_{22} = \frac{V_2}{I_2} = 1/0.09 = 11.11$$

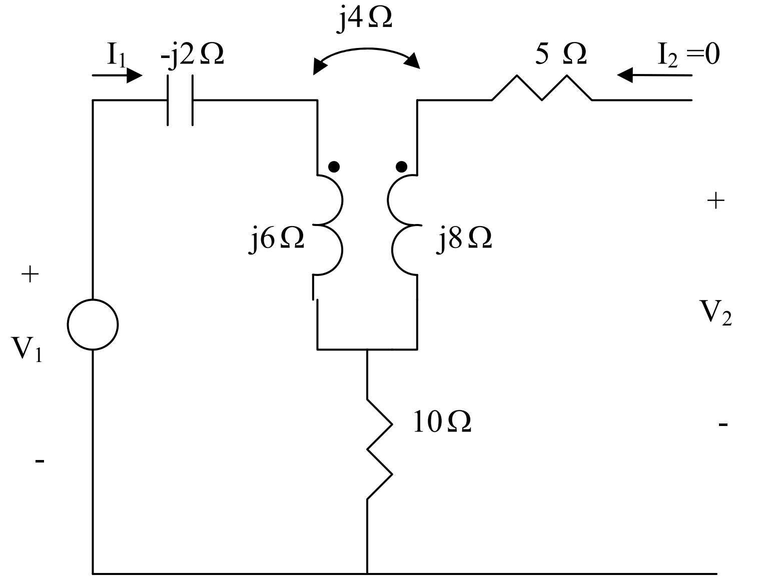
$$V_1 = V_x = \frac{1}{3}V_2 = \frac{11.11}{3}I_2 = 3.704I_2 \qquad \longrightarrow \qquad z_{12} = \frac{V_1}{I_2} = 3.704$$

Thus,

$$\underline{[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix}\Omega}$$

Chapter 19, Solution 8.

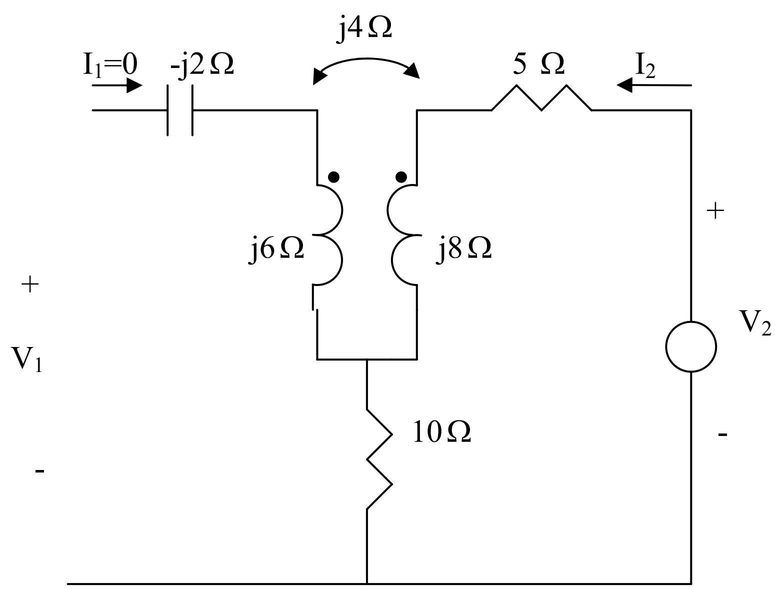
To get z_{11} and z_{21}, consider the circuit below.



$$V_1 = (10 - j2 + j6)I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 10 + j4$$

$$V_2 = -10I_1 - j4I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = -(10 + j4)$$

To get z_{22} and z_{12}, consider the circuit below.



$$V_2 = (5+10+j8)I_2 \longrightarrow z_{22} = \frac{V_2}{I_2} = 15 + j8$$

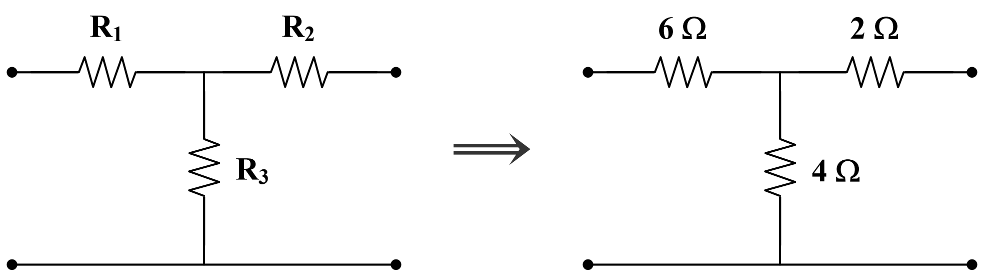
$$V_1 = -(10+j4)I_2 \longrightarrow z_{12} = \frac{V_1}{I_2} = -(10+j4)$$

Thus,

$$[z] = \underline{\begin{bmatrix} (10+j4) & -(10+j4) \\ -(10+j4) & (15+j8) \end{bmatrix} \Omega}$$

Chapter 19, Solution 9.

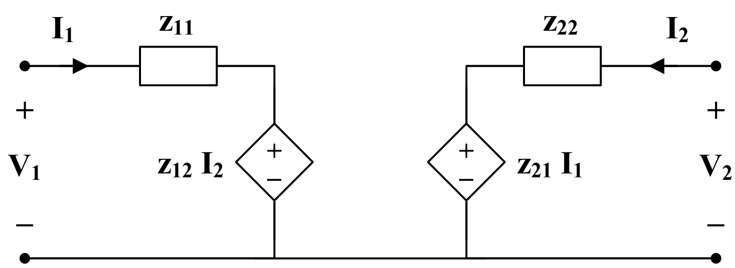
It is evident from Fig. 19.5 that **a T network is appropriate for realizing the z parameters**.



$$R_1 = \mathbf{z}_{11} - \mathbf{z}_{12} = 10 - 4 = \underline{\mathbf{6\ \Omega}}$$
$$R_2 = \mathbf{z}_{22} - \mathbf{z}_{12} = 6 - 4 = \underline{\mathbf{2\ \Omega}}$$
$$R_3 = \mathbf{z}_{12} = \mathbf{z}_{21} = \underline{\mathbf{4\ \Omega}}$$

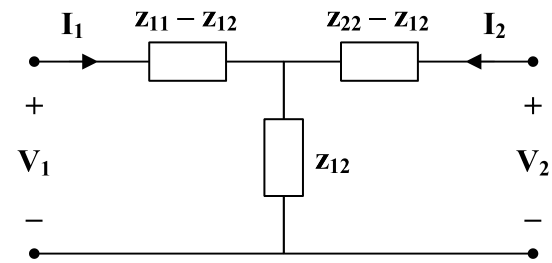
Chapter 19, Solution 10.

(a) This is a non-reciprocal circuit so that **the two-port looks like the one shown in Figs. (a) and (b)**.

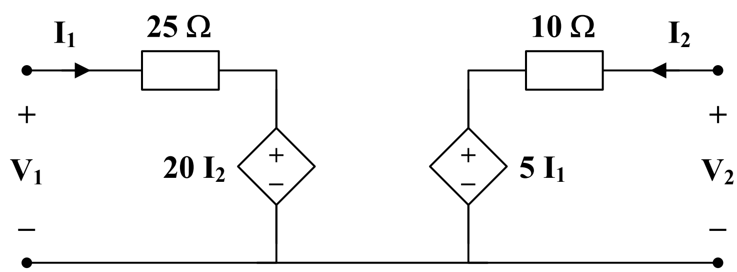


(a)

(b) This is a reciprocal network and **the two-port look like the one shown in Figs. (c) and (d)**.



(c)

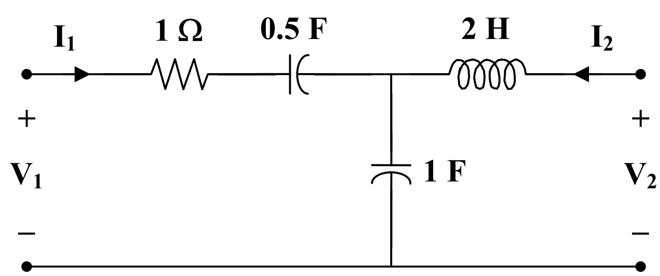


(b)

$$\mathbf{z}_{11} - \mathbf{z}_{12} = 1 + \frac{2}{s} = 1 + \frac{1}{0.5s}$$

$$\mathbf{z}_{22} - \mathbf{z}_{12} = 2s$$

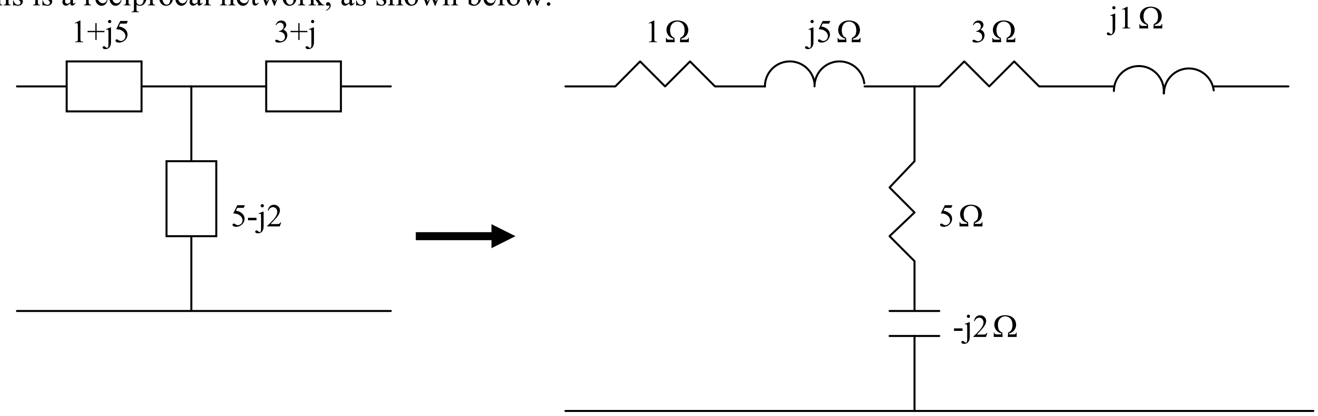
$$\mathbf{z}_{12} = \frac{1}{s}$$



(d)

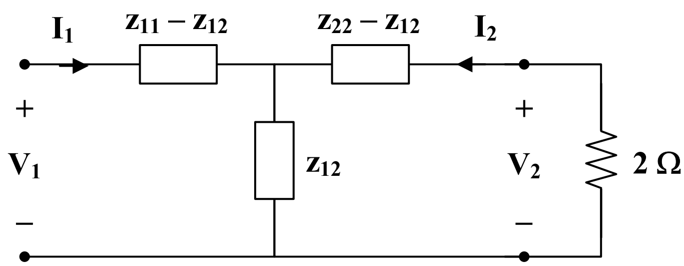
Chapter 19, Solution 11.

This is a reciprocal network, as shown below.

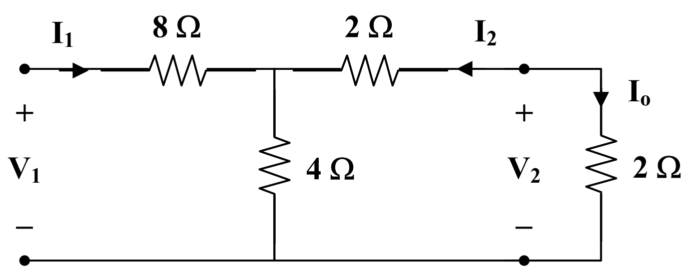


Chapter 19, Solution 12.

This is a reciprocal two-port so that it can be represented by the circuit in Figs. (a) and (b).



(a)



(b)

From Fig. (b),

$$\mathbf{V}_1 = (8 + 4 \,||\, 4)\mathbf{I}_1 = 10\mathbf{I}_1$$

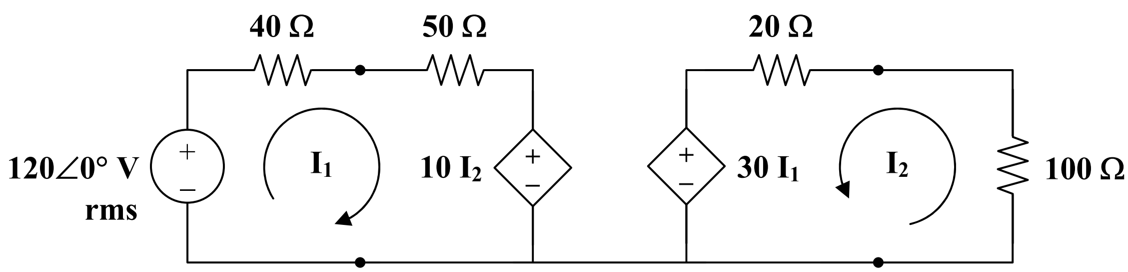
By current division,

$$\mathbf{I}_o = \frac{1}{2}\mathbf{I}_1, \qquad \mathbf{V}_2 = 2\mathbf{I}_o = \mathbf{I}_1$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{10\mathbf{I}_1} = \underline{\mathbf{0.1}}$$

Chapter 19, Solution 13.

This is a reciprocal two-port so that the circuit can be represented by the circuit below.



We apply mesh analysis.
For mesh 1,

$$-120 + 90\mathbf{I}_1 + 10\mathbf{I}_2 = 0 \longrightarrow 12 = 9\mathbf{I}_1 + \mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$30\mathbf{I}_1 + 120\mathbf{I}_2 = 0 \longrightarrow \mathbf{I}_1 = -4\mathbf{I}_2 \qquad (2)$$

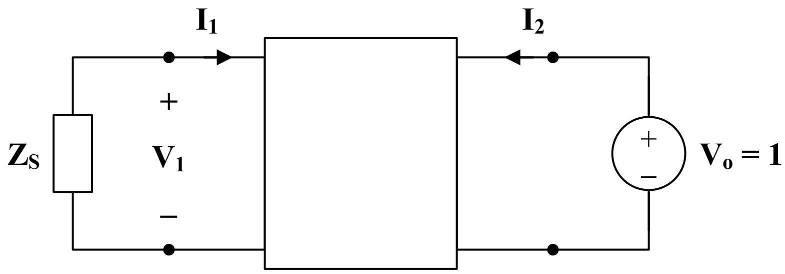
Substituting (2) into (1),

$$12 = -36\mathbf{I}_2 + \mathbf{I}_2 = -35\mathbf{I}_2 \longrightarrow \mathbf{I}_2 = \frac{-12}{35}$$

$$P = \frac{1}{2}|\mathbf{I}_2|^2 R = \frac{1}{2}\left(\frac{12}{35}\right)^2 (100) = \underline{\mathbf{5.877\ W}}$$

Chapter 19, Solution 14.

To find $\mathbf{Z}_{Th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{z}_{11}\,\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\,\mathbf{I}_1 + \mathbf{z}_{22}\,\mathbf{I}_2 \qquad (2)$$

But

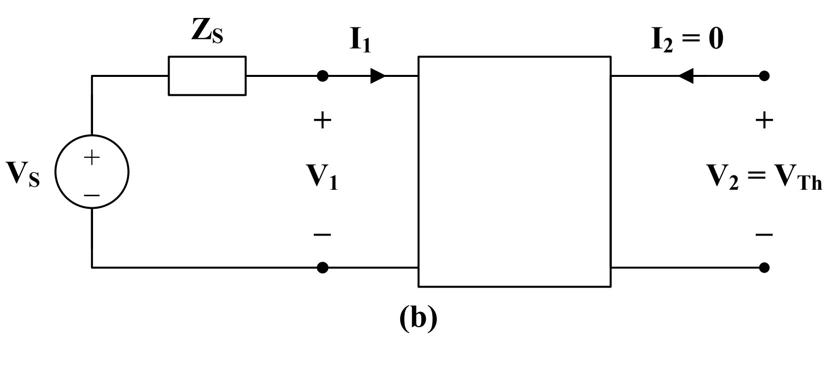
$$\mathbf{V}_2 = 1, \qquad \mathbf{V}_1 = -\mathbf{Z}_s\,\mathbf{I}_1$$

Hence, $0 = (\mathbf{z}_{11} + \mathbf{Z}_s)\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \longrightarrow \mathbf{I}_1 = \dfrac{-\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}\mathbf{I}_2$

$$1 = \left(\frac{-\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s} + \mathbf{z}_{22}\right)\mathbf{I}_2$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \underline{\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}}$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = 0, \qquad \mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\,\mathbf{Z}_s$$

Substituting these into (1) and (2),

$$\mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s = \mathbf{z}_{11} \mathbf{I}_1 \longrightarrow \mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 = \frac{\mathbf{z}_{21} \mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_{Th} = \mathbf{V}_2 = \underline{\frac{\mathbf{z}_{21} \mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}}$$

Chapter 19, Solution 15.

(a) From Prob. 18.12,

$$Z_{Th} = z_{22} - \frac{z_{12} z_{21}}{z_{11} + Z_s} = 120 - \frac{80x60}{40+10} = 24$$

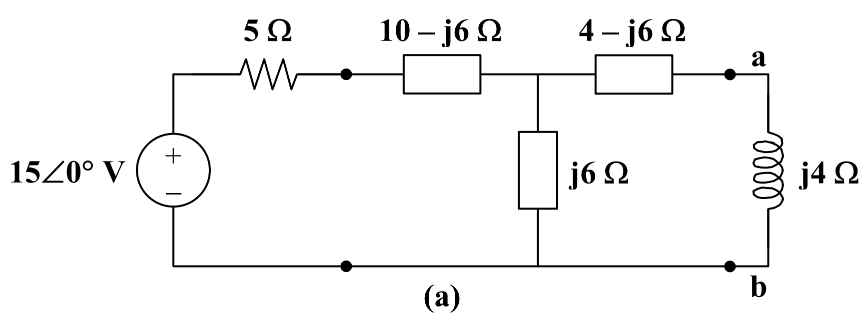
$$\underline{Z_L = Z_{Th} = 24\Omega}$$

(b) $V_{Th} = \dfrac{z_{21}}{z_{11} + Z_s} V_s = \dfrac{80}{40+10}(120) = 192$

$$P_{max} = \frac{V^2{}_{Th}}{8R_{Th}} = \frac{192^2}{8x24} = \underline{192\ W}$$

Chapter 19, Solution 16.

As a reciprocal two-port, the given circuit can be represented as shown in Fig. (a).



(a)

At terminals a-b,

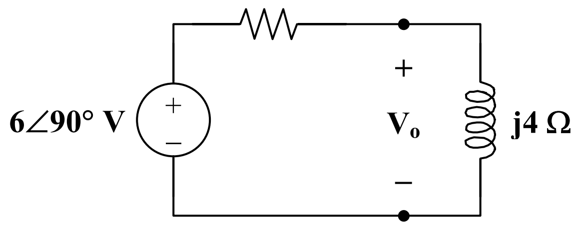
$$\mathbf{Z}_{Th} = (4 - j6) + j6 \,||\, (5 + 10 - j6)$$

$$\mathbf{Z}_{Th} = 4 - j6 + \frac{j6(15 - j6)}{15} = 4 - j6 + 2.4 + j6$$

$$\mathbf{Z}_{Th} = \underline{\mathbf{6.4\ \Omega}}$$

$$\mathbf{V}_{Th} = \frac{j6}{j6 + 5 + 10 - j6}(15\angle 0°) = j6 = \underline{\mathbf{6\angle 90°\ V}}$$

The Thevenin equivalent circuit is shown in Fig. (b).



(b)

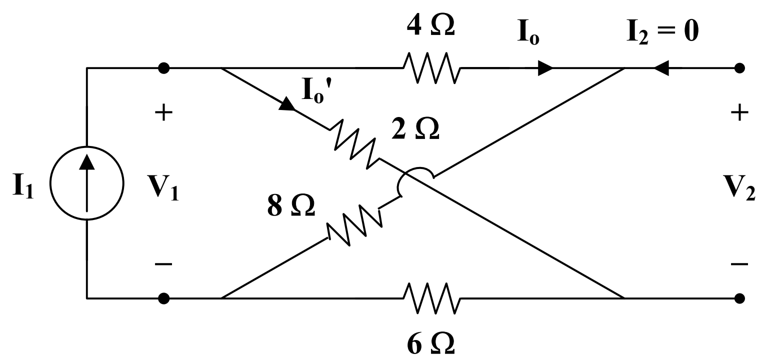
From this,

$$\mathbf{V}_o = \frac{j4}{6.4 + j4}(j6) = 3.18\angle 148°$$

$$v_o(t) = \underline{\mathbf{3.18\cos(2t + 148°)\ V}}$$

Chapter 19, Solution 17.

To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

In this case, the 4-Ω and 8-Ω resistors are in series, since the same current, $\mathbf{I}_o$, passes through them. Similarly, the 2-Ω and 6-Ω resistors are in series, since the same current, $\mathbf{I}_o^{'}$, passes through them.

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (4+8) \,||\, (2+6) = 12 \,||\, 8 = \frac{(12)(8)}{20} = 4.8\,\Omega$$

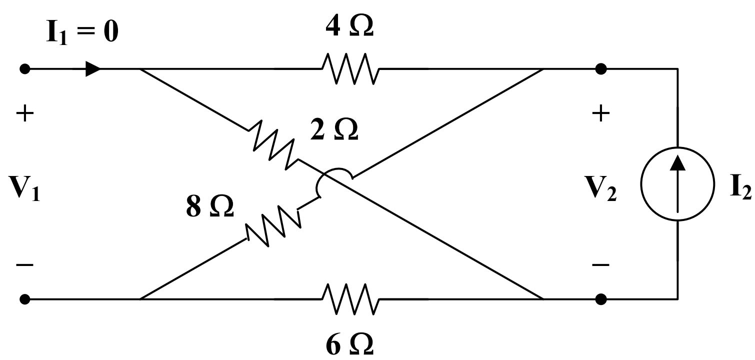
$$\mathbf{I}_o = \frac{8}{8+12}\mathbf{I}_1 = \frac{2}{5}\mathbf{I}_1 \qquad\qquad \mathbf{I}_o^{'} = \frac{3}{5}\mathbf{I}_1$$

But $\quad -\mathbf{V}_2 - 4\mathbf{I}_o + 2\mathbf{I}_o^{'} = 0$

$$\mathbf{V}_2 = -4\mathbf{I}_o + 2\mathbf{I}_o^{'} = \frac{-8}{5}\mathbf{I}_1 + \frac{6}{5}\mathbf{I}_1 = \frac{-2}{5}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{-2}{5} = -0.4\,\Omega$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = (4+2) \,||\, (8+6) = 6 \,||\, 14 = \frac{(6)(14)}{20} = 4.2\,\Omega$$

$$\mathbf{z}_{12} = \mathbf{z}_{21} = -0.4\,\Omega$$

Thus,

$$[\mathbf{z}] = \underline{\begin{bmatrix} \mathbf{4.8} & \mathbf{-0.4} \\ \mathbf{-0.4} & \mathbf{4.2} \end{bmatrix}\Omega}$$

We may take advantage of Table 18.1 to get [**y**] from [**z**].

$$\Delta_z = (4.8)(4.2) - (0.4)^2 = 20$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{4.2}{20} = 0.21 \qquad \mathbf{y}_{12} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{0.4}{20} = 0.02$$

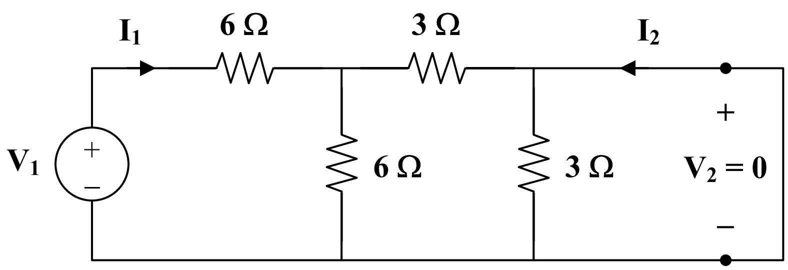
$$\mathbf{y}_{21} = \frac{-\mathbf{z}_{21}}{\Delta_z} = \frac{0.4}{20} = 0.02 \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} = \frac{4.8}{20} = 0.24$$

Thus,

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{0.21} & \mathbf{0.02} \\ \mathbf{0.02} & \mathbf{0.24} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Solution 18.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig.(a).



(a)

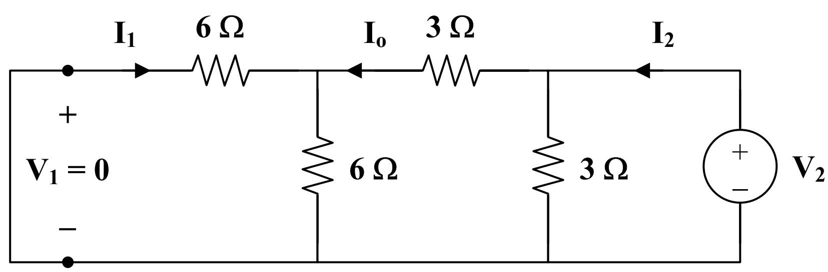
$$\mathbf{V}_1 = (6 + 6 \parallel 3)\mathbf{I}_1 = 8\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{8}$$

$$\mathbf{I}_2 = \frac{-6}{6+3}\mathbf{I}_1 = \frac{-2}{3}\frac{\mathbf{V}_1}{8} = \frac{-\mathbf{V}_1}{12}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{12}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{3 \,\|\, (3+6 \,\|\, 6)} = \frac{1}{3 \,\|\, 6} = \frac{1}{2}$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_o}{2}, \qquad \mathbf{I}_o = \frac{3}{3+6}\mathbf{I}_2 = \frac{1}{3}\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_2}{6} = \left(\frac{-1}{6}\right)\left(\frac{1}{2}\mathbf{V}_2\right) = \frac{-\mathbf{V}_2}{12}$$

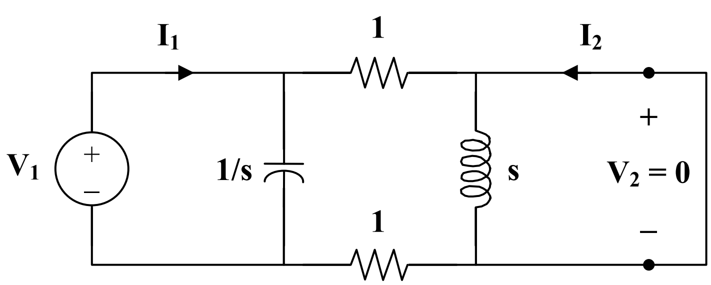
$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{12} = \mathbf{y}_{21}$$

Thus,

$$\underline{[\mathbf{y}] = \begin{bmatrix} \frac{1}{8} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{2} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Solution 19.

Consider the circuit in Fig.(a) for calculating $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$.



(a)

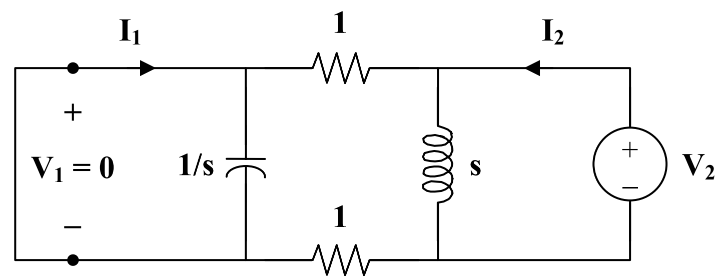
$$\mathbf{V}_1 = \left(\frac{1}{s} \,\|\, 2\right)\mathbf{I}_1 = \frac{2/s}{2+(1/s)}\mathbf{I}_1 = \frac{2}{2s+1}\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{2s+1}{2} = s + 0.5$$

$$\mathbf{I}_2 = \frac{(-1/s)}{(1/s)+2}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{2s+1} = \frac{-\mathbf{V}_1}{2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \text{-}0.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig.(b).



(b)

$$\mathbf{V}_2 = (s \,\|\, 2)\,\mathbf{I}_2 = \frac{2s}{s+2}\mathbf{I}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{s+2}{2s} = 0.5 + \frac{1}{s}$$

$$\mathbf{I}_1 = \frac{-s}{s+2}\mathbf{I}_2 = \frac{-s}{s+2}\cdot\frac{s+2}{2s}\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

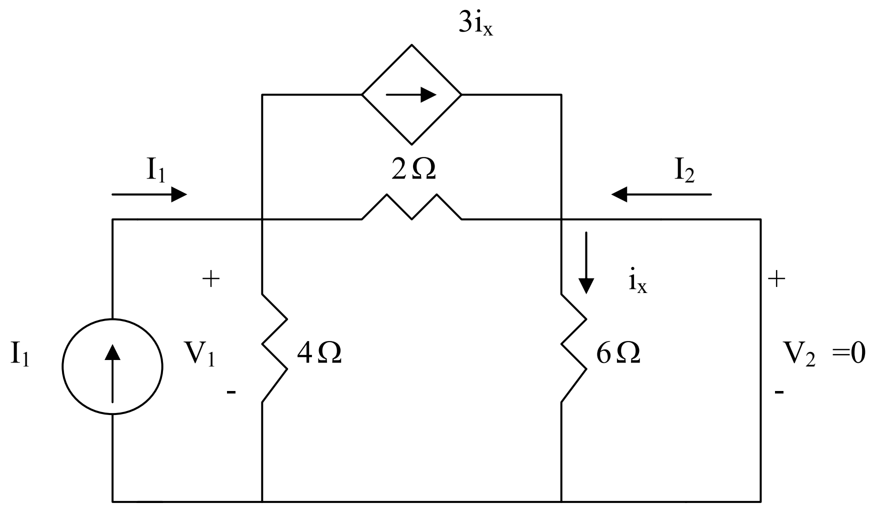
$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \text{-}0.5$$

Thus,

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{s+0.5} & \mathbf{-0.5} \\ \mathbf{-0.5} & \mathbf{0.5+1/s} \end{bmatrix}\mathbf{S}}$$

Chapter 19, Solution 20.

To get y_{11} and y_{21}, consider the circuit below.

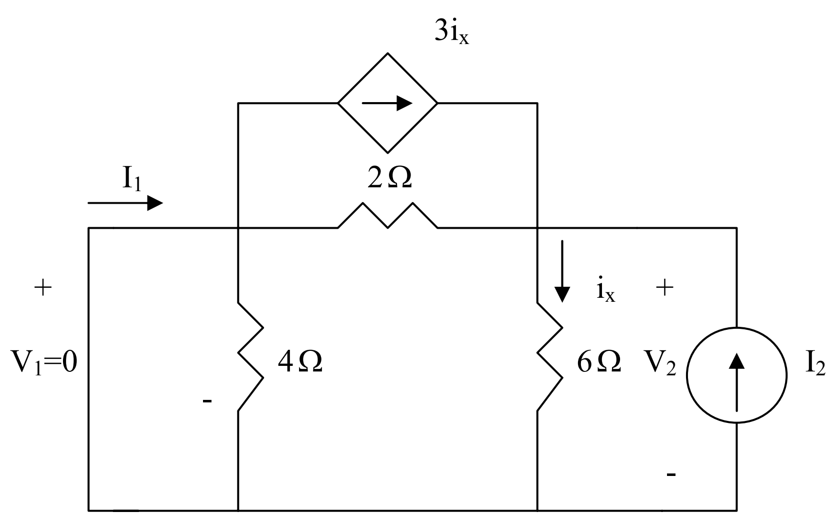


Since 6-ohm resistor is short-circuited, $i_x = 0$

$$V_1 = I_1(4//2) = \frac{8}{6}I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = 0.75$$

$$I_2 = -\frac{4}{4+2}I_1 = -\frac{2}{3}(\frac{6}{8}V_1) = -\frac{1}{2}V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = -0.5$$

To get y_{22} and y_{12}, consider the circuit below.



$$i_x = \frac{V_2}{6}, \quad I_2 = i_x - 3i_x + \frac{V_2}{2} = \frac{V_2}{6} \longrightarrow y_{22} = \frac{I_2}{V_2} = \frac{1}{6} = 0.1667$$

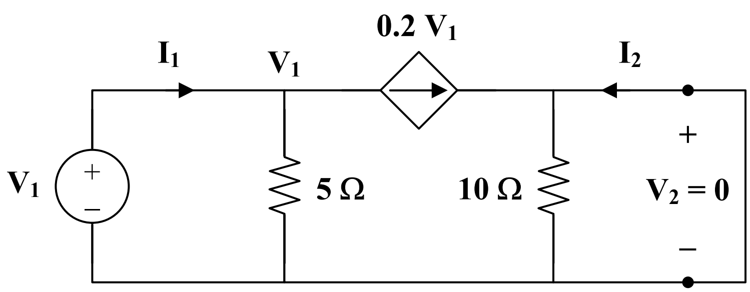
$$I_1 = 3i_x - \frac{V_2}{2} = 0 \longrightarrow y_{12} = \frac{I_1}{V_2} = 0$$

Thus,

$$\underline{[y] = \begin{bmatrix} 0.75 & 0 \\ -0.5 & 0.1667 \end{bmatrix} S}$$

Chapter 19, Solution 21.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, refer to Fig. (a).



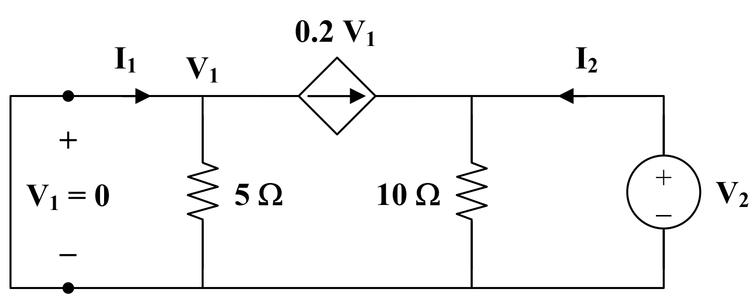
(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{5} + 0.2\mathbf{V}_1 = 0.4\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 0.4$$

$$\mathbf{I}_2 = -0.2\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.2$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (b).



(b)

Since $\mathbf{V}_1 = 0$, the dependent current source can be replaced with an open circuit.

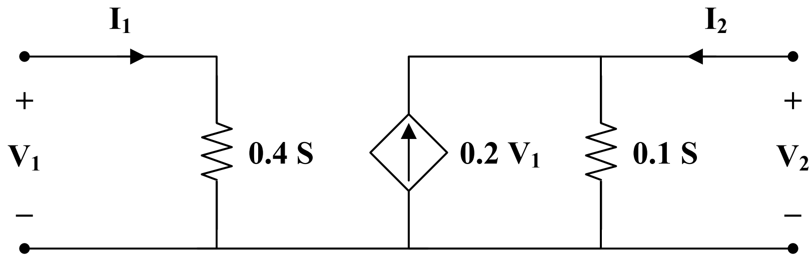
$$\mathbf{V}_2 = 10\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{10} = 0.1$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} 0.4 & 0 \\ -0.2 & 0.1 \end{bmatrix} \text{S}$$

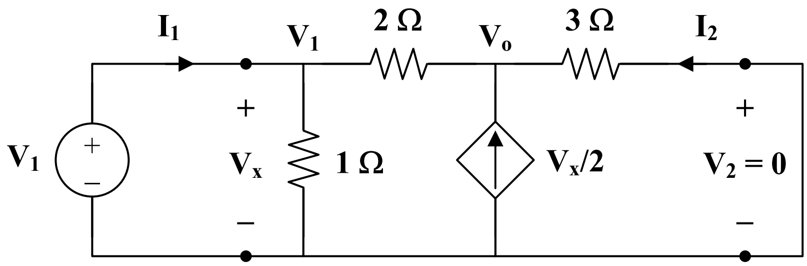
Consequently, **<u>the y parameter equivalent circuit is shown in Fig. (c)</u>**.



(c)

Chapter 19, Solution 22.

(a) To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ refer to the circuit in Fig. (a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{2} \longrightarrow \mathbf{I}_1 = 1.5\mathbf{V}_1 - 0.5\mathbf{V}_o \qquad (1)$$

At node 2,

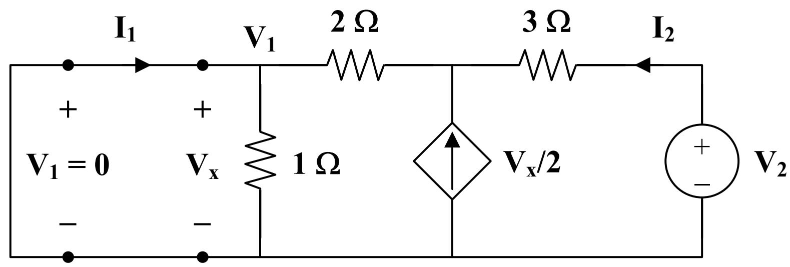
$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{2} + \frac{\mathbf{V}_1}{2} = \frac{\mathbf{V}_o}{3} \longrightarrow 1.2\mathbf{V}_1 = \mathbf{V}_o \qquad (2)$$

Substituting (2) into (1) gives,

$$\mathbf{I}_1 = 1.5\mathbf{V}_1 - 0.6\mathbf{V}_1 = 0.9\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 0.9$$

$$\mathbf{I}_2 = \frac{-\mathbf{V}_o}{3} = -0.4\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.4$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$ refer to the circuit in Fig. (b).



(b)

$\mathbf{V}_x = \mathbf{V}_1 = 0$ so that the dependent current source can be replaced by an open circuit.

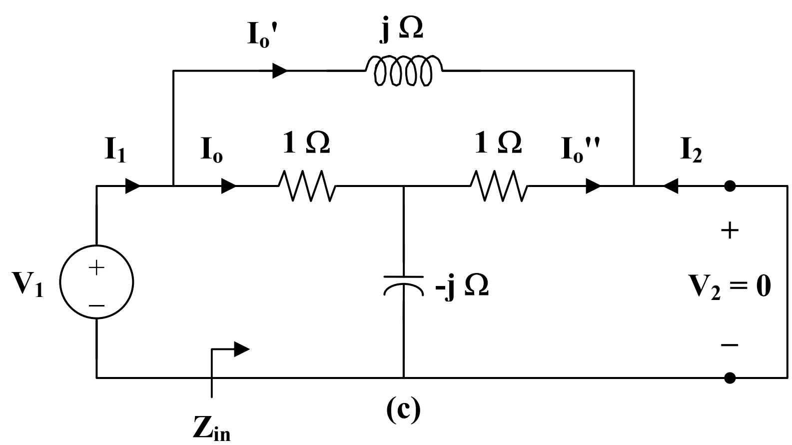
$$\mathbf{V}_2 = (3 + 2 + 0)\mathbf{I}_2 = 5\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2$$

$$\mathbf{I}_1 = -\mathbf{I}_2 = -0.2\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = -0.2$$

Thus,

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{0.9} & \mathbf{-0.2} \\ \mathbf{-0.4} & \mathbf{0.2} \end{bmatrix} \mathbf{S}}$$

(b) To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ refer to Fig. (c).



(c)

$$\mathbf{Z}_{in} = j \,||\, (1 + 1 \,||\, -j) = j \,||\, \left(1 + \frac{-j}{1-j}\right) = j \,||\, (1.5 - j0.5)$$

$$= \frac{j(1.5 - j0.5)}{1.5 + j0.5} = 0.6 + j0.8$$

$$\mathbf{V}_1 = \mathbf{Z}_{in}\,\mathbf{I}_1 \quad \longrightarrow \quad \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{\mathbf{Z}_{in}} = \frac{1}{0.6 + j0.8} = 0.6 - j0.8$$

$$\mathbf{I}_o = \frac{j}{1.5 + j0.5}\mathbf{I}_1, \qquad \mathbf{I}_o' = \frac{1.5 - j0.5}{1.5 + j0.5}\mathbf{I}_1$$

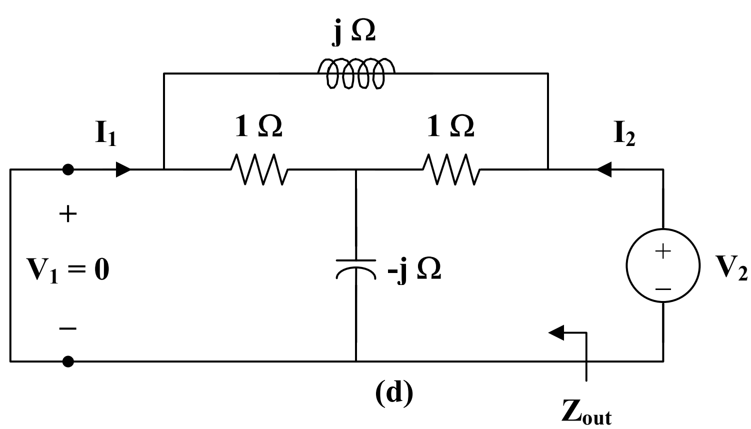
$$\mathbf{I}_o'' = \frac{-j}{1 - j}\mathbf{I}_o = \frac{\mathbf{I}_1}{(1 - j)(1.5 + j0.5)} = \frac{\mathbf{I}_1}{2 - j}$$

$$-\mathbf{I}_2 = \mathbf{I}_o' + \mathbf{I}_o'' = \frac{(1.5 - j0.5)^2}{2.5}\mathbf{I}_1 + \frac{2 + j}{5}\mathbf{I}_1 = (1.2 - j0.4)\,\mathbf{I}_1$$

$$-\mathbf{I}_2 = (1.2 - j0.4)(0.6 - j0.8)\,\mathbf{V}_1 = (0.4 - j1.2)\,\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.4 + j1.2 = \mathbf{y}_{12}$$

To get $\mathbf{y}_{22}$ refer to the circuit in Fig.(d).



(d)

$$\mathbf{Z}_{out} = j \,||\, (1 + 1 \,||\, -j) = 0.6 + j0.8$$

$$\mathbf{y}_{22} = \frac{1}{\mathbf{Z}_{out}} = 0.6 - j0.8$$

Thus,

$$\underline{[\mathbf{y}] = \begin{bmatrix} \mathbf{0.6 - j0.8} & \mathbf{-0.4 + j1.2} \\ \mathbf{-0.4 + j1.2} & \mathbf{0.6 - j0.8} \end{bmatrix}\mathbf{S}}$$

Chapter 19, Solution 23.

(a)

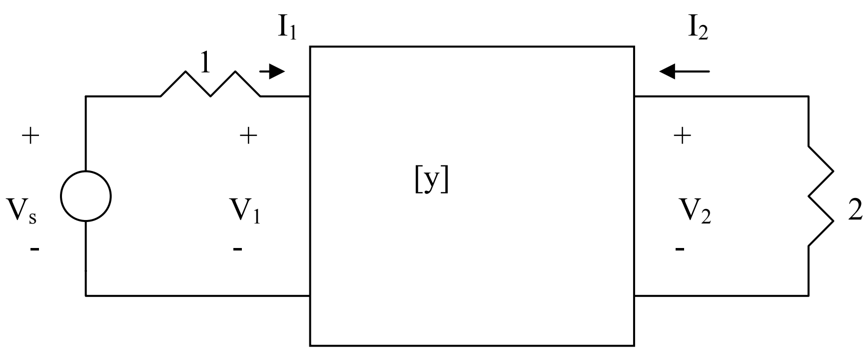
$$-y_{12} = 1//\frac{1}{s} = \frac{1}{s+1} \qquad \longrightarrow \qquad y_{12} = -\frac{1}{s+1}$$

$$y_{11} + y_{12} = 1 \qquad \longrightarrow \qquad y_{11} = 1 - y_{12} = 1 + \frac{1}{s+1} = \frac{s+2}{s+1}$$

$$y_{22} + y_{12} = s \qquad \longrightarrow \qquad y_{22} = s - y_{12} = s + \frac{1}{s+1} = \frac{s^2+s+1}{s+1}$$

$$\underline{[y] = \begin{bmatrix} \frac{s+2}{s+1} & \frac{-1}{s+1} \\ \frac{-1}{s+1} & \frac{s^2+s+1}{s+1} \end{bmatrix}}$$

(b) Consider the network below.



$$V_s = I_1 + V_1 \qquad (1)$$

$$V_2 = -2I_2 \qquad (2)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad (3)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad (4)$$

From (1) and (3)

$$V_s - V_1 = y_{11}V_1 + y_{12}V_2 \quad \longrightarrow \quad V_s = (1 + y_{11})V_1 + y_{12}V_2 \qquad (5)$$

From (2) and (4),

$$-0.5V_2 = y_{21}V_1 + y_{22}V_2 \quad \longrightarrow \quad V_1 = -\frac{1}{y_{21}}(0.5 + y_{22})V_2 \qquad (6)$$

Substituting (6) into (5),

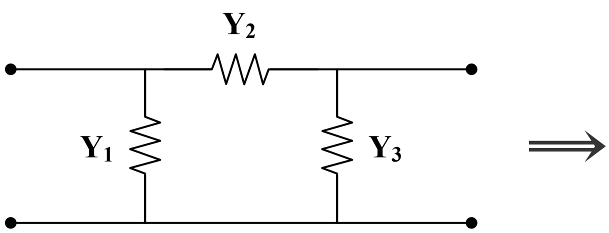
$$V_s = -\frac{(1 + y_{11})(0.5 + y_{22})}{y_{21}}V_2 + y_{12}V_2$$

$$= \frac{2}{s} \quad \longrightarrow \quad V_2 = \frac{2/s}{\left[y_{12} - \frac{1}{y_{21}}(1 + y_{11})(0.5 + y_{22}) \right]}$$

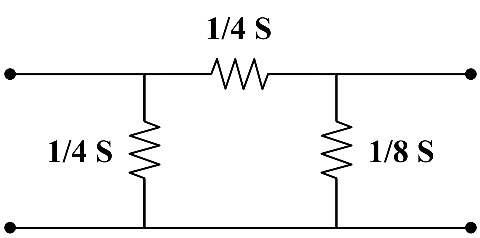
$$V_2 = \frac{2/s}{-\frac{1}{s+1} + (s+1)\left(\frac{2s+3}{s+1}\right)\left(\frac{1}{2} + \frac{s^2+s+1}{s+1}\right)} = \underline{\frac{2(s+1)}{s(2s^3 + 6s^2 + 7.5s + 3.5)}}$$

Chapter 19, Solution 24.

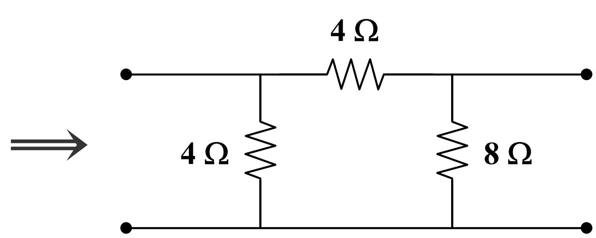
Since this is a reciprocal network, **<u>a Π network is appropriate, as shown below</u>**.



(a)



(b)



(c)

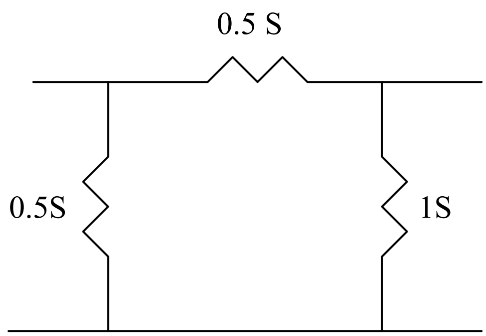
$$\mathbf{Y}_1 = \mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\,\text{S}, \qquad \mathbf{Z}_1 = \underline{\mathbf{4}\,\boldsymbol{\Omega}}$$

$$\mathbf{Y}_2 = -\mathbf{y}_{12} = \frac{1}{4}\,\text{S}, \qquad \mathbf{Z}_2 = \underline{\mathbf{4}\,\boldsymbol{\Omega}}$$

$$\mathbf{Y}_3 = \mathbf{y}_{22} + \mathbf{y}_{21} = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}\,\text{S}, \qquad \mathbf{Z}_3 = \underline{\mathbf{8}\,\boldsymbol{\Omega}}$$

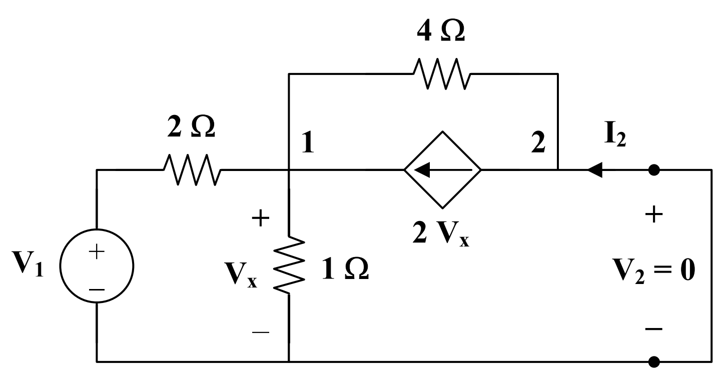
Chapter 19, Solution 25.

This is a reciprocal network and is shown below.



Chapter 19, Solution 26.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).



(a)

At node 1,

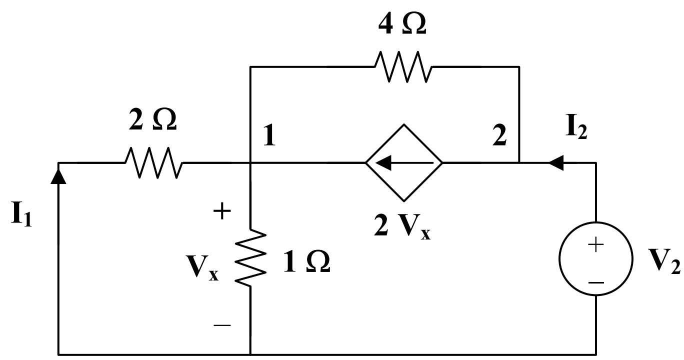
$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{2} + 2\mathbf{V}_x = \frac{\mathbf{V}_x}{1} + \frac{\mathbf{V}_x}{4} \longrightarrow 2\mathbf{V}_1 = -\mathbf{V}_x \tag{1}$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{2} = \frac{\mathbf{V}_1 + 2\mathbf{V}_1}{2} = 1.5\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 1.5$$

Also, $$\mathbf{I}_2 + \frac{\mathbf{V}_x}{4} = 2\mathbf{V}_x \longrightarrow \mathbf{I}_2 = 1.75\mathbf{V}_x = -3.5\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -3.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

At node 2,

$$\mathbf{I}_2 = 2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} \tag{2}$$

At node 1,

$$2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} = \frac{\mathbf{V}_x}{2} + \frac{\mathbf{V}_x}{1} = \frac{3}{2}\mathbf{V}_x \longrightarrow \mathbf{V}_2 = -\mathbf{V}_x \tag{3}$$

Substituting (3) into (2) gives

$$\mathbf{I}_2 = 2\mathbf{V}_x - \frac{1}{2}\mathbf{V}_x = 1.5\mathbf{V}_x = -1.5\mathbf{V}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = -1.5$$

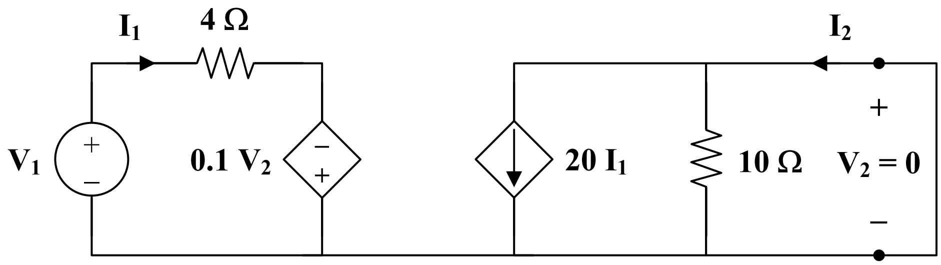
$$\mathbf{I}_1 = \frac{-\mathbf{V}_x}{2} = \frac{\mathbf{V}_2}{2} \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0.5$$

Thus,

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{1.5} & \mathbf{0.5} \\ \mathbf{-3.5} & \mathbf{-1.5} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Solution 27.

Consider the circuit in Fig. (a).

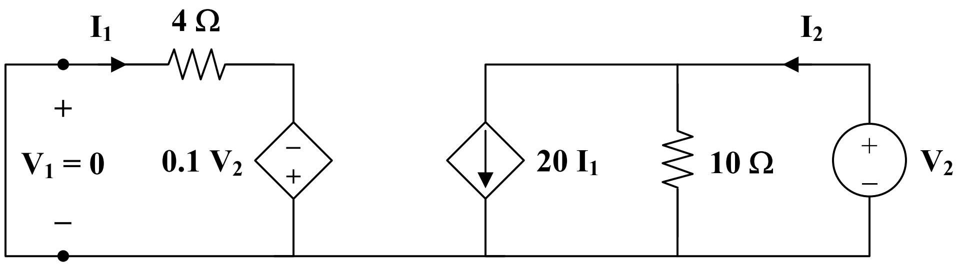


(a)

$$\mathbf{V}_1 = 4\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{4\mathbf{I}_1} = 0.25$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 = 5\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = 5$$

Consider the circuit in Fig. (b).



(b)

$$4\mathbf{I}_1 = 0.1\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{0.1}{4} = 0.025$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 + \frac{\mathbf{V}_2}{10} = 0.5\mathbf{V}_2 + 0.1\mathbf{V}_2 = 0.6\mathbf{V}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = 0.6$$

Thus,

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{0.25} & \mathbf{0.025} \\ \mathbf{5} & \mathbf{0.6} \end{bmatrix} \mathbf{S}}$$

Alternatively, from the given circuit,

$$\mathbf{V}_1 = 4\mathbf{I}_1 - 0.1\mathbf{V}_2$$
$$\mathbf{I}_2 = 20\mathbf{I}_1 + 0.1\mathbf{V}_2$$

Comparing these with the equations for the h parameters show that

$$\mathbf{h}_{11} = 4, \qquad \mathbf{h}_{12} = \text{-}0.1, \qquad \mathbf{h}_{21} = 20, \qquad \mathbf{h}_{22} = 0.1$$

Using Table 18.1,

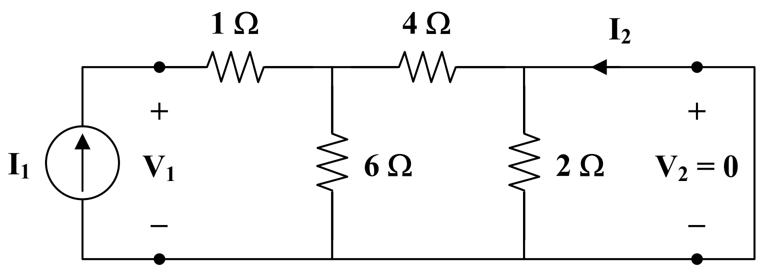
$$\mathbf{y}_{11} = \frac{1}{\mathbf{h}_{11}} = \frac{1}{4} = 0.25, \qquad \mathbf{y}_{12} = \frac{\text{-}\mathbf{h}_{12}}{\mathbf{h}_{11}} = \frac{0.1}{4} = 0.025$$

$$\mathbf{y}_{21} = \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} = \frac{20}{4} = 5, \qquad \mathbf{y}_{22} = \frac{\Delta_h}{\mathbf{h}_{11}} = \frac{0.4 + 2}{4} = 0.6$$

as above.

Chapter 19, Solution 28.

We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by considering the circuit in Fig.(a).



(a)

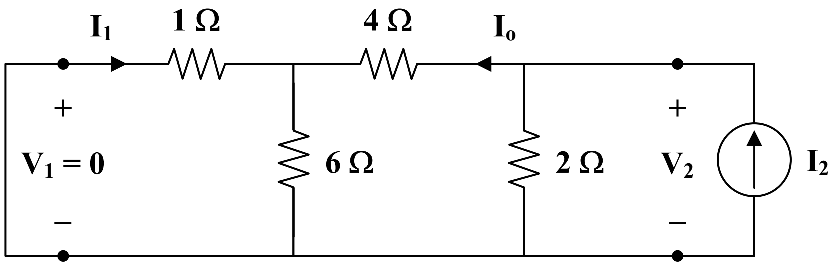
$$\mathbf{Z}_{in} = 1 + 6 \,||\, 4 = 3.4$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{\mathbf{Z}_{in}} = 0.2941$$

$$\mathbf{I}_2 = \frac{\text{-}6}{10}\mathbf{I}_1 = \left(\frac{\text{-}6}{10}\right)\left(\frac{\mathbf{V}_1}{3.4}\right) = \frac{\text{-}6}{34}\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-6}{34} = -0.1765$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

$$\frac{1}{\mathbf{y}_{22}} = 2 \,||\, (4 + 6 \,||\, 1) = 2 \,||\left(4 + \frac{6}{7}\right) = \frac{(2)(34/7)}{2 + (34/7)} = \frac{34}{24} = \frac{V_2}{I_2}$$

$$\mathbf{y}_{22} = \frac{24}{34} = 0.7059$$

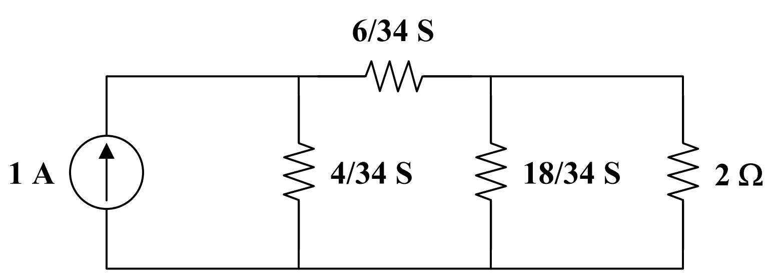
$$\mathbf{I}_1 = \frac{-6}{7}\mathbf{I}_o \qquad \mathbf{I}_o = \frac{2}{2 + (34/7)}\mathbf{I}_2 = \frac{14}{48}\mathbf{I}_2 = \frac{7}{34}\mathbf{V}_2$$

$$\mathbf{I}_1 = \frac{-6}{34}\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-6}{34} = -0.1765$$

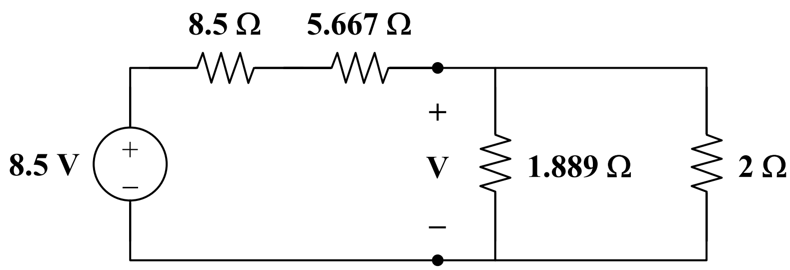
Thus,

$$[\mathbf{y}] = \begin{bmatrix} 0.2941 & -0.1765 \\ -0.1765 & 0.7059 \end{bmatrix} \mathrm{S}$$

The equivalent circuit is shown in Fig. (c). After transforming the current source to a voltage source, we have the circuit in Fig. (d).



(c)



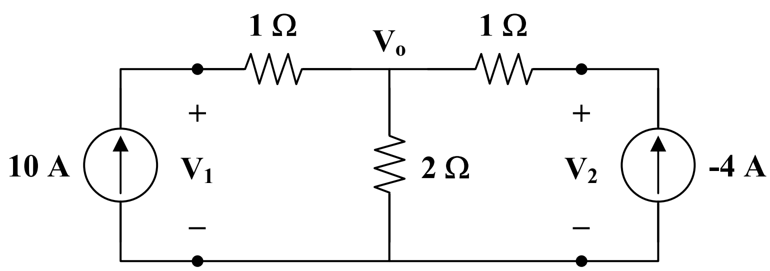
(d)

$$V = \frac{(2 \| 1.889)(8.5)}{2 \| 1.889 + 8.5 + 5.667} = \frac{(0.9714)(8.5)}{0.9714 + 14.167} = 0.5454$$

$$P = \frac{V^2}{R} = \frac{(0.5454)^2}{2} = \underline{\mathbf{0.1487\ W}}$$

Chapter 19, Solution 29.

(a) Transforming the Δ subnetwork to Y gives the circuit in Fig. (a).



(a)

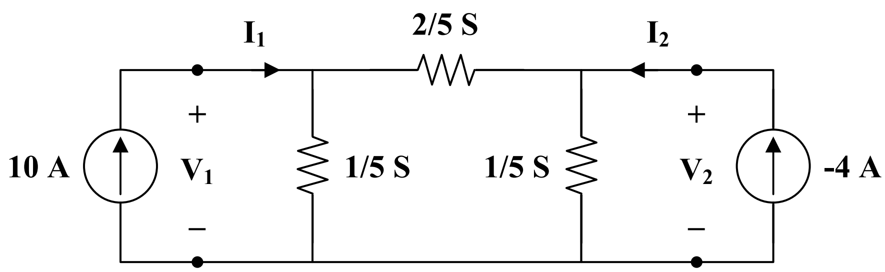
It is easy to get the z parameters

$$\mathbf{z}_{12} = \mathbf{z}_{21} = 2, \quad \mathbf{z}_{11} = 1 + 2 = 3, \quad \mathbf{z}_{22} = 3$$

$$\Delta_z = \mathbf{z}_{11}\,\mathbf{z}_{22} - \mathbf{z}_{12}\,\mathbf{z}_{21} = 9 - 4 = 5$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{3}{5} = \mathbf{y}_{22}, \qquad \mathbf{y}_{12} = \mathbf{y}_{21} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{-2}{5}$$

Thus, the equivalent circuit is as shown in Fig. (b).



(b)

$$\mathbf{I}_1 = 10 = \frac{3}{5}\mathbf{V}_1 - \frac{2}{5}\mathbf{V}_2 \quad \longrightarrow \quad 50 = 3\mathbf{V}_1 - 2\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \text{-}4 = \frac{\text{-}2}{5}\mathbf{V}_1 + \frac{3}{5}\mathbf{V}_2 \quad \longrightarrow \quad \text{-}20 = \text{-}2\mathbf{V}_1 + 3\mathbf{V}_2$$

$$10 = \mathbf{V}_1 - 1.5\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$50 = 30 + 4.5\mathbf{V}_2 - 2\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{V}_2 = \underline{\mathbf{8\ V}}$$

$$\mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 = \underline{\mathbf{22\ V}}$$

(b) For direct circuit analysis, consider the circuit in Fig. (a).

For the main non-reference node,

$$10 - 4 = \frac{\mathbf{V}_o}{2} \quad \longrightarrow \quad \mathbf{V}_o = 12$$

$$10 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \quad \longrightarrow \quad \mathbf{V}_1 = 10 + \mathbf{V}_o = \underline{\mathbf{22\ V}}$$

$$\text{-}4 = \frac{\mathbf{V}_2 - \mathbf{V}_o}{1} \quad \longrightarrow \quad \mathbf{V}_2 = \mathbf{V}_o - 4 = \underline{\mathbf{8\ V}}$$

Chapter 19, Solution 30.

(a) Convert to z parameters; then, convert to h parameters using Table 18.1.

$$\mathbf{z}_{11} = \mathbf{z}_{12} = \mathbf{z}_{21} = 60\,\Omega, \qquad \mathbf{z}_{22} = 100\,\Omega$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 6000 - 3600 = 2400$$

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}} = \frac{2400}{100} = 24, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} = \frac{60}{100} = 0.6$$

$$\mathbf{h}_{21} = \frac{\text{-}\mathbf{z}_{21}}{\mathbf{z}_{22}} = \text{-}0.6, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} = 0.01$$

Thus,

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{24\,\Omega} & \mathbf{0.6} \\ \mathbf{-0.6} & \mathbf{0.01\,S} \end{bmatrix}}$$

(b) Similarly,

$$\mathbf{z}_{11} = 30\,\Omega \qquad \mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 20\,\Omega$$

$$\Delta_z = 600 - 400 = 200$$

$$\mathbf{h}_{11} = \frac{200}{20} = 10 \qquad \mathbf{h}_{12} = \frac{20}{20} = 1$$

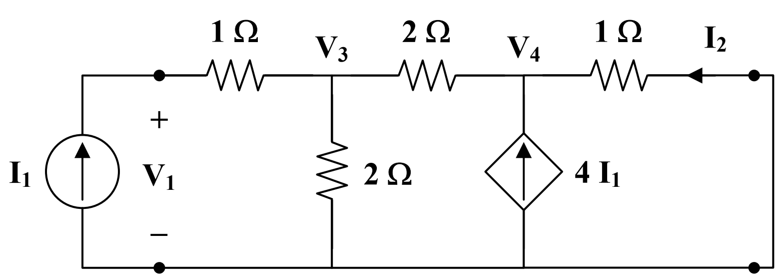
$$\mathbf{h}_{21} = -1 \qquad \mathbf{h}_{22} = \frac{1}{20} = 0.05$$

Thus,

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0.05\,S} \end{bmatrix}}$$

Chapter 19, Solution 31.

We get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by considering the circuit in Fig. (a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_3}{2} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{2} \longrightarrow 2\mathbf{I}_1 = 2\mathbf{V}_3 - \mathbf{V}_4 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_3 - \mathbf{V}_4}{2} + 4\mathbf{I}_1 = \frac{\mathbf{V}_4}{1}$$

$$8\mathbf{I}_1 = -\mathbf{V}_3 + 3\mathbf{V}_4 \longrightarrow 16\mathbf{I}_1 = -2\mathbf{V}_3 + 6\mathbf{V}_4 \qquad (2)$$

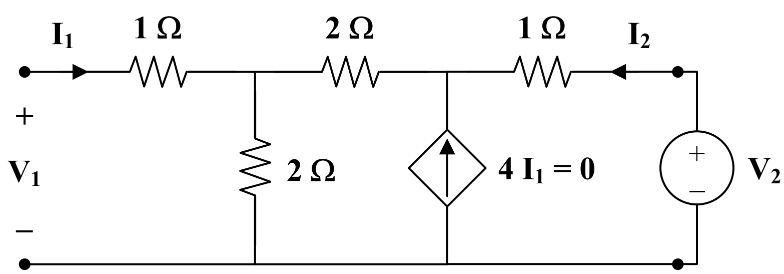
Adding (1) and (2),

$$18\mathbf{I}_1 = 5\mathbf{V}_4 \longrightarrow \mathbf{V}_4 = 3.6\mathbf{I}_1$$
$$\mathbf{V}_3 = 3\mathbf{V}_4 - 8\mathbf{I}_1 = 2.8\mathbf{I}_1$$
$$\mathbf{V}_1 = \mathbf{V}_3 + \mathbf{I}_1 = 3.8\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 3.8\,\Omega$$

$$\mathbf{I}_2 = \frac{\text{-}\,\mathbf{V}_4}{1} = \text{-}3.6\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \text{-}3.6$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b). The dependent current source can be replaced by an open circuit since $4\mathbf{I}_1 = 0$.



(b)

$$\mathbf{V}_1 = \frac{2}{2+2+1}\mathbf{V}_2 = \frac{2}{5}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 0.4$$

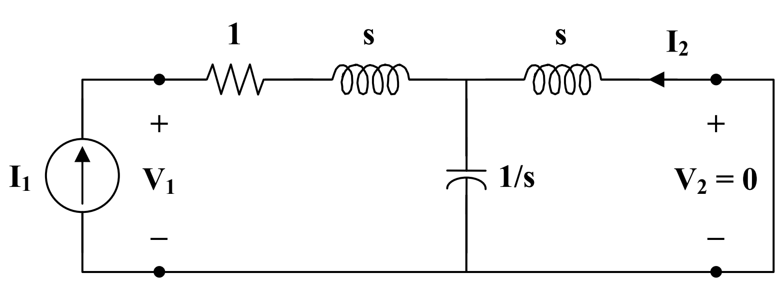
$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2+2+1} = \frac{\mathbf{V}_2}{5} \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2\text{ S}$$

Thus,

$$\underline{[\mathbf{h}] = \begin{bmatrix} \mathbf{38\,\Omega} & \mathbf{0.4} \\ \mathbf{-3.6} & \mathbf{0.2\ S} \end{bmatrix}}$$

Chapter 19, Solution 32.

(a) We obtain $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by referring to the circuit in Fig. (a).



(a)

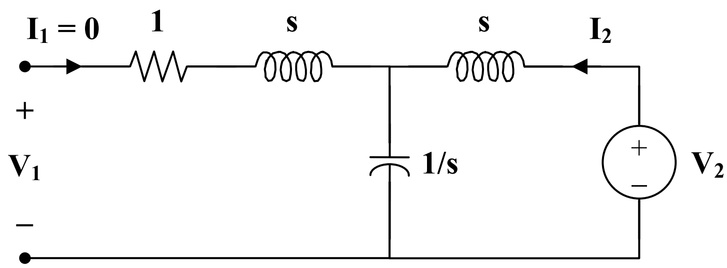
$$\mathbf{V}_1 = \left(1 + s + s \,\|\, \frac{1}{s}\right)\mathbf{I}_1 = \left(1 + s + \frac{s}{s^2+1}\right)\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = s + 1 + \frac{s}{s^2+1}$$

By current division,

$$\mathbf{I}_2 = \frac{-1/s}{s+1/s}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{s+1} \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{s^2+1}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

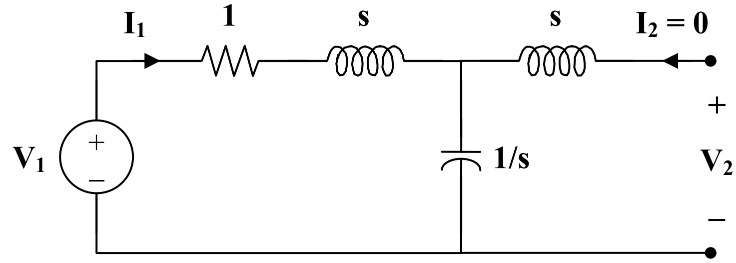
$$\mathbf{V}_1 = \frac{1/s}{s+1/s}\mathbf{V}_2 = \frac{\mathbf{V}_2}{s^2+1} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{s^2+1}$$

$$\mathbf{V}_2 = \left(s + \frac{1}{s}\right)\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{s+1/s} = \frac{s}{s^2+1}$$

Thus,

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{s+1+\frac{s}{s^2+1}} & \mathbf{\frac{1}{s^2+1}} \\ \mathbf{\frac{-1}{s^2+1}} & \mathbf{\frac{s}{s^2+1}} \end{bmatrix}}$$

(b) To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).

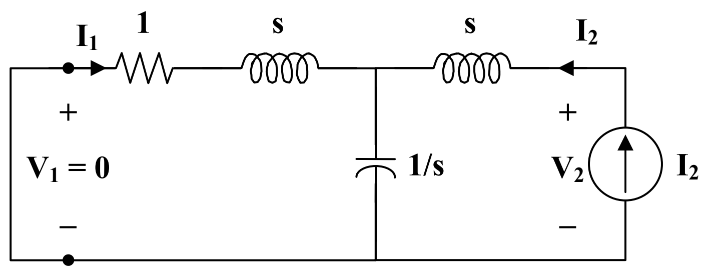


(c)

$$\mathbf{V}_1 = \left(1 + s + \frac{1}{s}\right)\mathbf{I}_1 \longrightarrow \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{1+s+1/s} = \frac{s}{s^2+s+1}$$

$$\mathbf{V}_2 = \frac{1/s}{1+s+1/s}\mathbf{V}_1 = \frac{\mathbf{V}_1}{s^2+s+1} \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s^2+s+1}$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$\mathbf{V}_2 = \left(s + \frac{1}{s} \parallel (s+1)\right)\mathbf{I}_2 = \left(s + \frac{(s+1)/s}{1+s+1/s}\right)\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = s + \frac{s+1}{s^2+s+1}$$

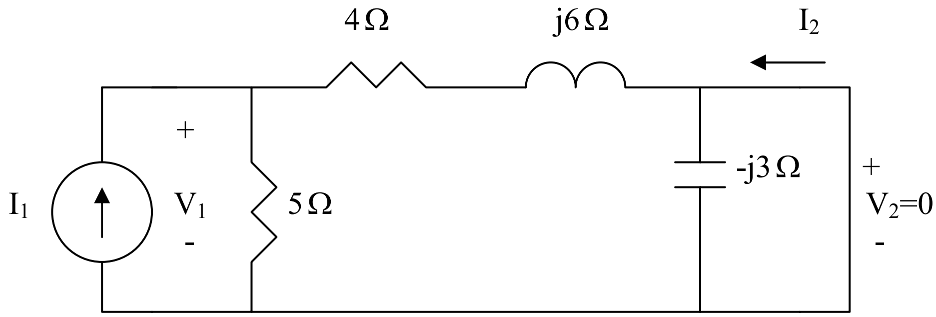
$$\mathbf{I}_1 = \frac{-1/s}{1+s+1/s}\mathbf{I}_2 = \frac{-\mathbf{I}_2}{s^2+s+1} \longrightarrow \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-1}{s^2+s+1}$$

Thus,

$$[\mathbf{g}] = \underline{\begin{bmatrix} \dfrac{\mathbf{s}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} & \dfrac{-\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} \\ \dfrac{\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} & \mathbf{s}+\dfrac{\mathbf{s}+\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} \end{bmatrix}}$$

Chapter 19, Solution 33.

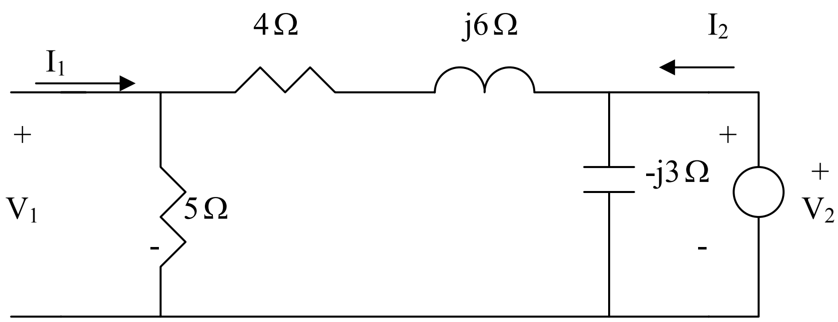
To get h_{11} and h_{21}, consider the circuit below.



$$V_1 = 5//(4+j6)I_1 = \frac{5(4+j6)I_1}{9+j6} \qquad h_{11} = \frac{V_1}{I_1} = 3.0769 + j1.2821$$

Also, $I_2 = -\dfrac{5}{9+j6}I_1 \quad \longrightarrow \quad h_{21} = \dfrac{I_2}{I_1} = -0.3846 + j0.2564$

To get h_{22} and h_{12}, consider the circuit below.



$$V_1 = \frac{5}{9+j6}V_2 \quad \longrightarrow \quad h_{12} = \frac{V_1}{V_2} = \frac{5}{9+j6} = 0.3846 - j0.2564$$

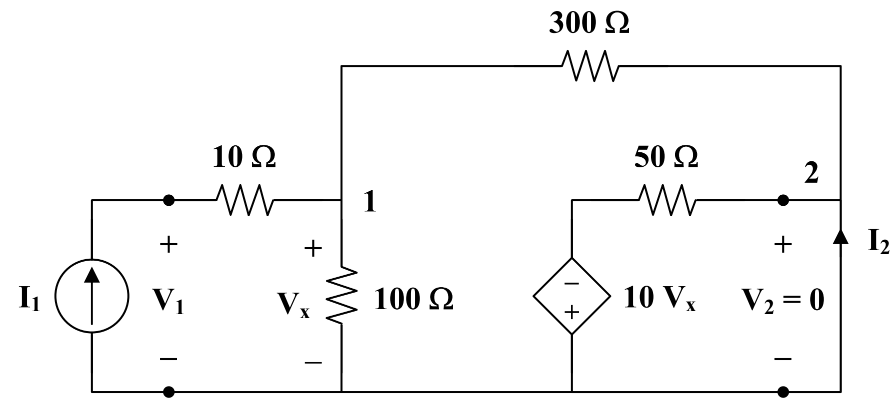
$$V_2 = -j3//(9+j6)I_2 \quad \longrightarrow \quad h_{22} = \frac{I_2}{V_2} = \frac{1}{-j3//(9+j6)} = \frac{9+j3}{-j3(9+j6)}$$
$$= 0.0769 + j0.2821$$

Thus,

$$[h] = \begin{bmatrix} 3.0769 + j1.2821 & 0.3846 - j0.2564 \\ -0.3846 + j0.2564 & 0.0769 + j0.2821 \end{bmatrix}$$

Chapter 19, Solution 34.

Refer to Fig. (a) to get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$.



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x - 0}{300} \longrightarrow 300\mathbf{I}_1 = 4\mathbf{V}_x \qquad (1)$$

$$\mathbf{V}_x = \frac{300}{4}\mathbf{I}_1 = 75\mathbf{I}_1$$

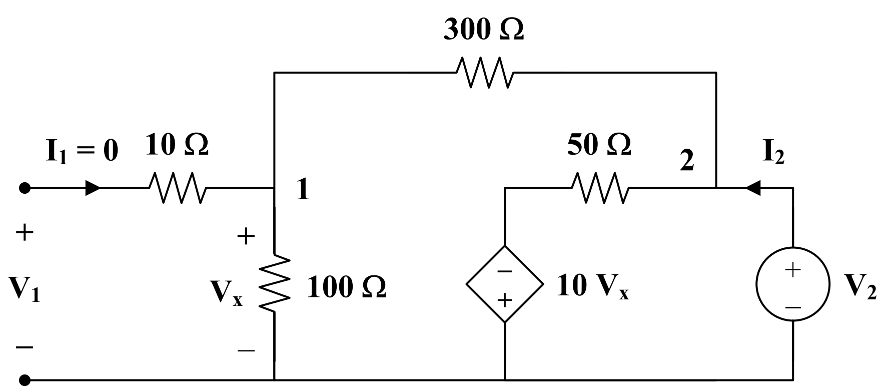
But $\qquad \mathbf{V}_1 = 10\mathbf{I}_1 + \mathbf{V}_x = 85\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \dfrac{\mathbf{V}_1}{\mathbf{I}_1} = 85\,\Omega$

At node 2,

$$\mathbf{I}_2 = \frac{0 + 10\mathbf{V}_x}{50} - \frac{\mathbf{V}_x}{300} = \frac{\mathbf{V}_x}{5} - \frac{\mathbf{V}_x}{300} = \frac{75}{5}\mathbf{I}_1 - \frac{75}{300}\mathbf{I}_1 = 14.75\mathbf{I}_1$$

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = 14.75$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

At node 2,

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{400} + \frac{\mathbf{V}_2 + 10\,\mathbf{V}_x}{50} \longrightarrow 400\,\mathbf{I}_2 = 9\,\mathbf{V}_2 + 80\,\mathbf{V}_x$$

But $\quad \mathbf{V}_x = \dfrac{100}{400}\mathbf{V}_2 = \dfrac{\mathbf{V}_2}{4}$

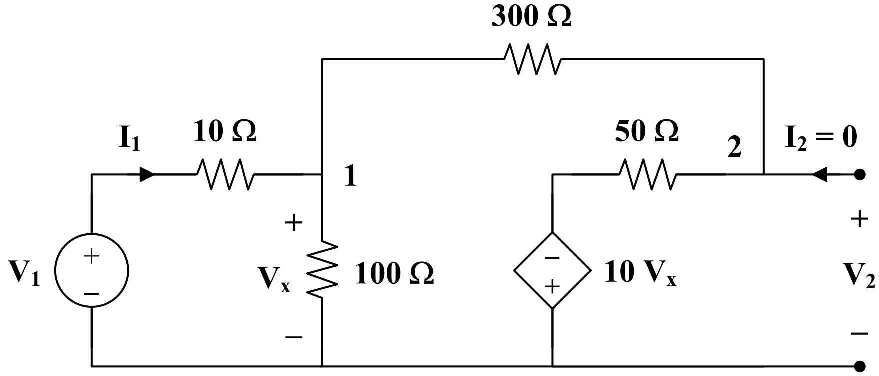
Hence, $\quad 400\,\mathbf{I}_2 = 9\,\mathbf{V}_2 + 20\,\mathbf{V}_2 = 29\,\mathbf{V}_2$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{29}{400} = 0.0725\text{ S}$$

$$\mathbf{V}_1 = \mathbf{V}_x = \frac{\mathbf{V}_2}{4} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{4} = 0.25$$

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{85\,\Omega} & \mathbf{0.25} \\ \mathbf{14.75} & \mathbf{0.0725\text{ S}} \end{bmatrix}}$$

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).



(c)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x + 10\,\mathbf{V}_x}{350} \longrightarrow 350\,\mathbf{I}_1 = 14.5\,\mathbf{V}_x \qquad (2)$$

But $\quad \mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_x}{10} \longrightarrow 10\,\mathbf{I}_1 = \mathbf{V}_1 - \mathbf{V}_x$

or $\quad \mathbf{V}_x = \mathbf{V}_1 - 10\,\mathbf{I}_1 \qquad (3)$

Substituting (3) into (2) gives

$$350\mathbf{I}_1 = 14.5\mathbf{V}_1 - 145\mathbf{I}_1 \longrightarrow 495\mathbf{I}_1 = 14.5\mathbf{V}_1$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{14.5}{495} = 0.02929 \text{ S}$$

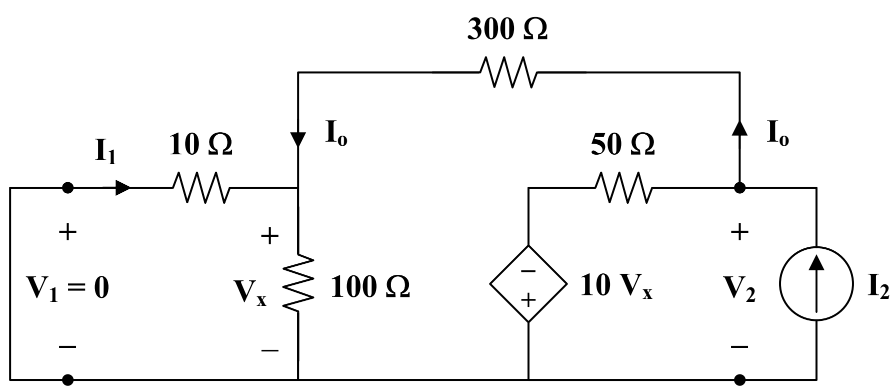
At node 2,

$$\mathbf{V}_2 = (50)\left(\frac{11}{350}\mathbf{V}_x\right) - 10\mathbf{V}_x = \text{-}8.4286\mathbf{V}_x$$

$$= \text{-}8.4286\mathbf{V}_1 + 84.286\mathbf{I}_1 = \text{-}8.4286\mathbf{V}_1 + (84.286)\left(\frac{14.5}{495}\right)\mathbf{V}_1$$

$$\mathbf{V}_2 = \text{-}5.96\mathbf{V}_1 \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \text{-}5.96$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$10 \,\|\, 100 = 9.091$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} + \frac{\mathbf{V}_2}{300 + 9.091}$$

$$309.091\mathbf{I}_2 = 7.1818\mathbf{V}_2 + 61.818\mathbf{V}_x \tag{4}$$

But $$\mathbf{V}_x = \frac{9.091}{309.091}\mathbf{V}_2 = 0.02941\mathbf{V}_2 \tag{5}$$

Substituting (5) into (4) gives

$$309.091\mathbf{I}_2 = 9\mathbf{V}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 34.34\ \Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{309.091} = \frac{34.34\,\mathbf{I}_2}{309.091}$$

$$\mathbf{I}_1 = \frac{-100}{110}\mathbf{I}_o = \frac{-34.34\,\mathbf{I}_2}{(1.1)(309.091)}$$

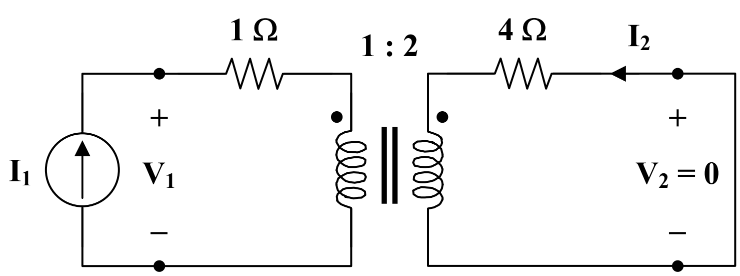
$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -0.101$$

Thus,

$$[\mathbf{g}] = \underline{\begin{bmatrix} \mathbf{0.02929\ S} & \mathbf{-0.101} \\ \mathbf{-5.96} & \mathbf{34.34\ \Omega} \end{bmatrix}}$$

Chapter 19, Solution 35.

To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ consider the circuit in Fig. (a).



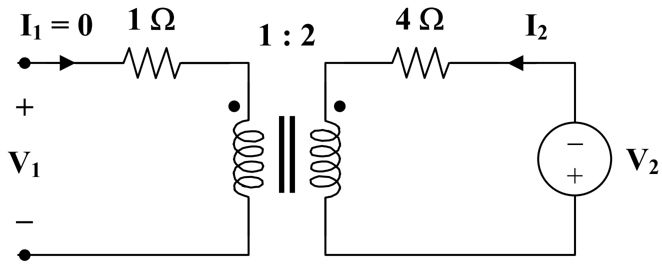
(a)

$$Z_R = \frac{4}{n^2} = \frac{4}{4} = 1$$

$$\mathbf{V}_1 = (1+1)\mathbf{I}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2\ \Omega$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-N_2}{N_1} = -2 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{2} = -0.5$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

Since $\mathbf{I}_1 = 0$, $\mathbf{I}_2 = 0$.
Hence, $\mathbf{h}_{22} = 0$.

At the terminals of the transformer, we have $\mathbf{V}_1$ and $\mathbf{V}_2$ which are related as

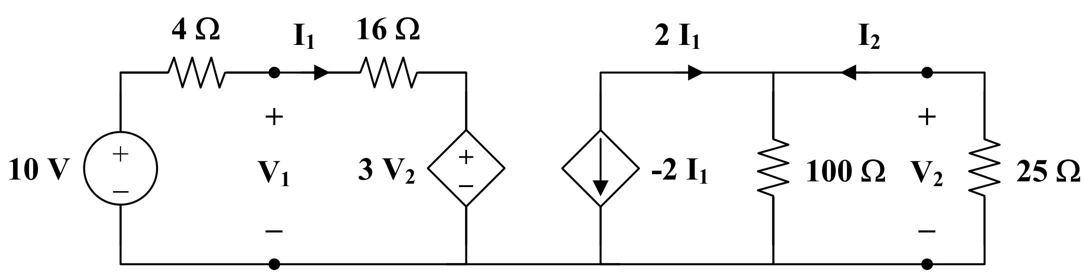
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n = 2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{2} = 0.5$$

Thus,

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{2\,\Omega} & \mathbf{0.5} \\ \mathbf{-0.5} & \mathbf{0} \end{bmatrix}}$$

Chapter 19, Solution 36.

We replace the two-port by its equivalent circuit as shown below.



$$100 \,||\, 25 = 20\,\Omega$$

$$\mathbf{V}_2 = (20)(2\mathbf{I}_1) = 40\mathbf{I}_1 \qquad (1)$$

$$-10 + 20\mathbf{I}_1 + 3\mathbf{V}_2 = 0$$
$$10 = 20\mathbf{I}_1 + (3)(40\mathbf{I}_1) = 140\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{1}{14}, \qquad \mathbf{V}_2 = \frac{40}{14}$$

$$\mathbf{V}_1 = 16\mathbf{I}_1 + 3\mathbf{V}_2 = \frac{136}{14}$$

$$\mathbf{I}_2 = \left(\frac{100}{125}\right)(2\mathbf{I}_1) = \frac{-8}{70}$$

(a) $$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{40}{136} = \underline{\mathbf{0.2941}}$$

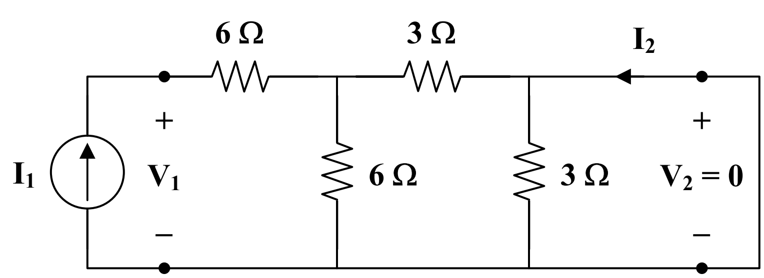
(b) $$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \underline{\mathbf{-1.6}}$$

(c) $$\frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{136} = \underline{\mathbf{7.353 \times 10^{-3}\ S}}$$

(d) $$\frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40}{1} = \underline{\mathbf{40\ \Omega}}$$

Chapter 19, Solution 37.

(a) We first obtain the h parameters. To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ refer to Fig. (a).



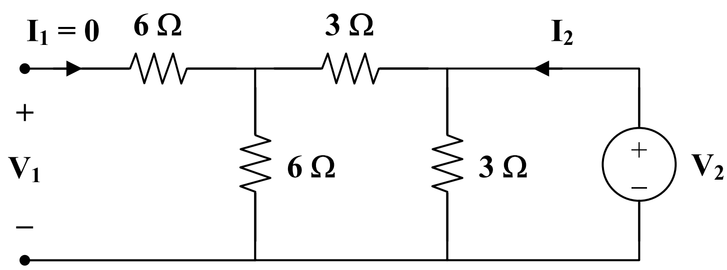
(a)

$$3 \,||\, 6 = 2$$

$$\mathbf{V}_1 = (6+2)\mathbf{I}_1 = 8\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 8\,\Omega$$

$$\mathbf{I}_2 = \frac{-6}{3+6}\mathbf{I}_1 = \frac{-2}{3}\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-2}{3}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b).



(b)

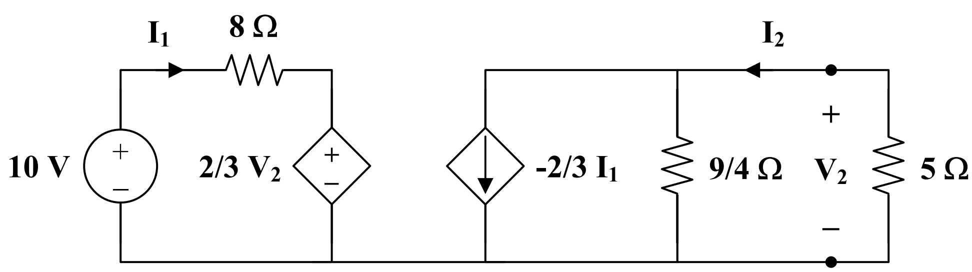
$$3 \,||\, 9 = \frac{9}{4}$$

$$\mathbf{V}_2 = \frac{9}{4}\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{4}{9}$$

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

$$[\mathbf{h}] = \begin{bmatrix} 8\,\Omega & \frac{2}{3} \\ \frac{-2}{3} & \frac{4}{9}\,\mathrm{S} \end{bmatrix}$$

The equivalent circuit of the given circuit is shown in Fig. (c).



(c)

$$8\mathbf{I}_1 + \frac{2}{3}\mathbf{V}_2 = 10 \qquad (1)$$

$$\mathbf{V}_2 = \frac{2}{3}\mathbf{I}_1\left(5 \,||\, \frac{9}{4}\right) = \frac{2}{3}\mathbf{I}_1\left(\frac{45}{29}\right) = \frac{30}{29}\mathbf{I}_1$$

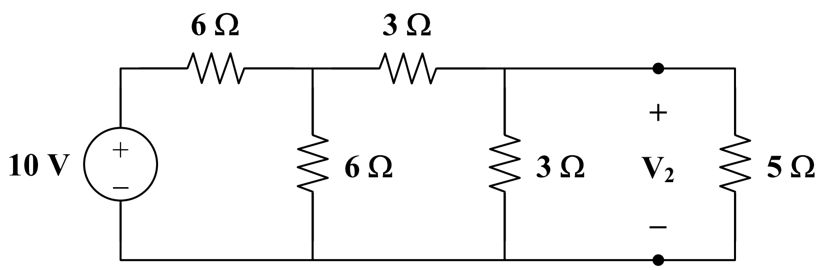
$$\mathbf{I}_1 = \frac{29}{30}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$(8)\left(\frac{29}{30}\right)\mathbf{V}_2 + \frac{2}{3}\mathbf{V}_2 = 10$$

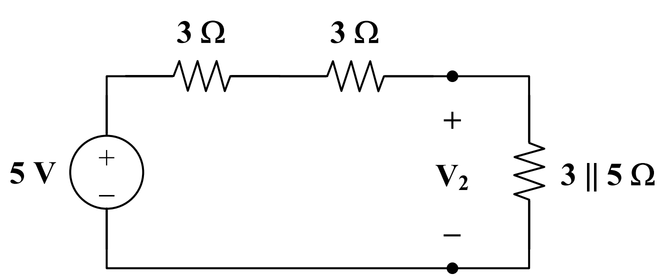
$$\mathbf{V}_2 = \frac{300}{252} = \underline{\mathbf{1.19\ V}}$$

(b) By direct analysis, refer to Fig.(d).



(d)

Transform the 10-V voltage source to a $\frac{10}{6}$-A current source. Since $6 \,||\, 6 = 3\ \Omega$, we combine the two 6-Ω resistors in parallel and transform the current source back to $\frac{10}{6} \times 3 = 5\ \text{V}$ voltage source shown in Fig. (e).



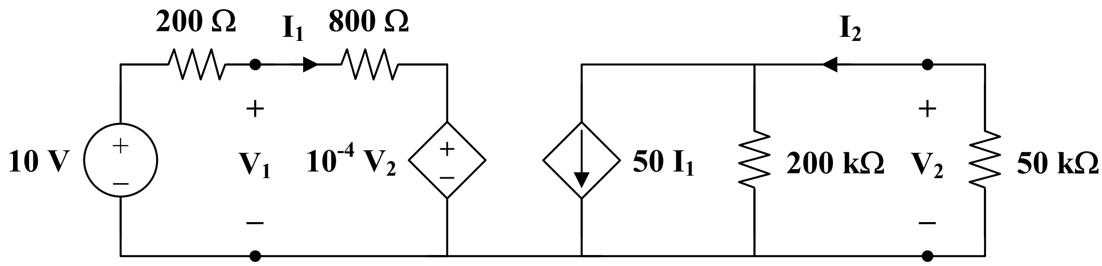
(e)

$$3 \,||\, 5 = \frac{(3)(5)}{8} = \frac{15}{8}$$

$$\mathbf{V}_2 = \frac{15/8}{6+15/8}(5) = \frac{75}{63} = \underline{\mathbf{1.19\ V}}$$

Chapter 19, Solution 38.

We replace the two-port by its equivalent circuit as shown below.



$$\mathbf{Z}_{in} = \frac{\mathbf{V}_s}{\mathbf{I}_1}, \qquad\qquad 200 \,||\, 50 = 40\ \text{k}\Omega$$

$$\mathbf{V}_2 = \text{-}50\mathbf{I}_1(40\times10^3) = (\text{-}2\times10^6)\mathbf{I}_1$$

For the left loop,

$$\frac{\mathbf{V}_s - 10^{-4}\,\mathbf{V}_2}{1000} = \mathbf{I}_1$$

$$\mathbf{V}_s - 10^{-4}(\text{-}2\times10^6\,\mathbf{I}_1) = 1000\mathbf{I}_1$$
$$\mathbf{V}_s = 1000\mathbf{I}_1 - 200\mathbf{I}_1 = 800\mathbf{I}_1$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_s}{\mathbf{I}_1} = \underline{\mathbf{800\ \Omega}}$$

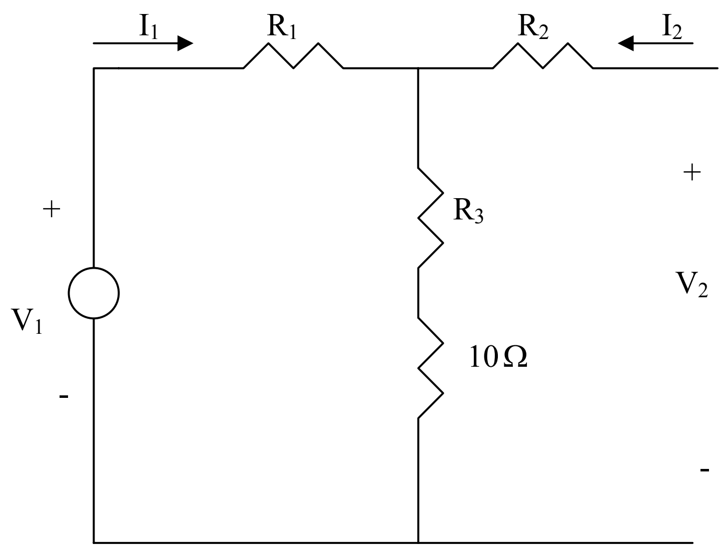
Alternatively,

$$\mathbf{Z}_{in} = \mathbf{Z}_s + \mathbf{h}_{11} - \frac{\mathbf{h}_{12}\,\mathbf{h}_{21}\,\mathbf{Z}_L}{1+\mathbf{h}_{22}\,\mathbf{Z}_L}$$

$$\mathbf{Z}_{in} = 200 + 800 - \frac{(10^{-4})(50)(50\times10^3)}{1+(0.5\times10^{-5})(50\times10^3)} = \underline{\mathbf{800\ \Omega}}$$

Chapter 19, Solution 39.

To get g_{11} and g_{21}, consider the circuit below which is partly obtained by converting the delta to wye subnetwork.

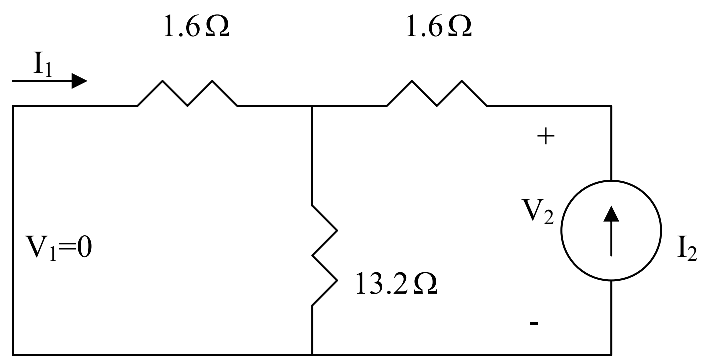


$$R_1 = \frac{4x8}{8+8+4} = 1.6 = R_2, \qquad R_3 = \frac{8x8}{20} = 3.2$$

$$V_2 = \frac{13.2}{13.2+1.6} V_1 = 0.8919 V_1 \quad \longrightarrow \quad g_{21} = \frac{V_2}{V_1} = 0.8919$$

$$V_1 = I_1(1.6+3.2+10) = 14.8 I_1 \quad \longrightarrow \quad g_{11} = \frac{I_1}{V_1} = \frac{1}{14.8} = 0.06757$$

To get g_{22} and g_{12}, consider the circuit below.



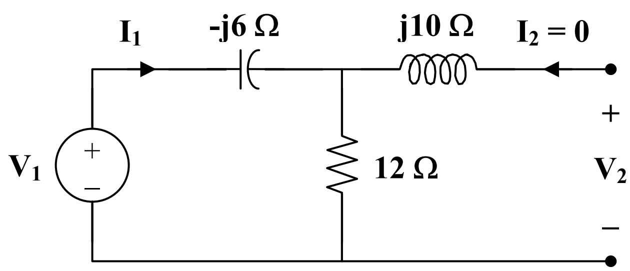
$$I_1 = -\frac{13.2}{13.2+1.6}I_2 = -0.8919 I_2 \longrightarrow g_{12} = \frac{I_1}{I_2} = -0.8919$$

$$V_2 = I_2(1.6 + 13.2//1.6) = 3.027 I_2 \longrightarrow g_{22} = \frac{V_2}{I_2} = 3.027$$

$$[g] = \underline{\begin{bmatrix} 0.06757 & -0.8919 \\ 0.8919 & 3.027 \end{bmatrix}}$$

Chapter 19, Solution 40.

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, consider the circuit in Fig. (a).

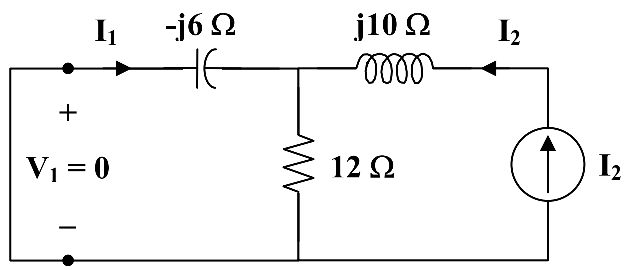


(a)

$$\mathbf{V}_1 = (12 - j6)\mathbf{I}_1 \longrightarrow \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{12 - j6} = 0.0667 + j0.0333 \text{ S}$$

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{12\mathbf{I}_1}{(12 - j6)\mathbf{I}_1} = \frac{2}{2 - j} = 0.8 + j0.4$$

To get $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_1 = \frac{-12}{12 - j6}\mathbf{I}_2 \quad \longrightarrow \quad \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-12}{12 - j6} = -\mathbf{g}_{21} = -0.8 - j0.4$$

$$\mathbf{V}_2 = (j10 + 12 \,||\, -j6)\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = j10 + \frac{(12)(-j6)}{12 - j6} = 2.4 + j5.2\,\Omega$$

$$[\mathbf{g}] = \underline{\begin{bmatrix} \mathbf{0.0667 + j0.0333\,S} & \mathbf{-0.8 - j0.4} \\ \mathbf{0.8 + j0.4} & \mathbf{2.4 + j5.2\,\Omega} \end{bmatrix}}$$

Chapter 19, Solution 41.

For the g parameters

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \qquad (2)$$

But $\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s$ and

$$\mathbf{V}_2 = -\mathbf{I}_2\mathbf{Z}_L = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

$$0 = \mathbf{g}_{21}\mathbf{V}_1 + (\mathbf{g}_{22} + \mathbf{Z}_L)\mathbf{I}_2$$

or

$$\mathbf{V}_1 = \frac{-(\mathbf{g}_{22} + \mathbf{Z}_L)}{\mathbf{g}_{21}}\mathbf{I}_2$$

Substituting this into (1),

$$\mathbf{I}_1 = \frac{(\mathbf{g}_{22}\mathbf{g}_{11} + \mathbf{Z}_L\mathbf{g}_{11} - \mathbf{g}_{21}\mathbf{g}_{12})}{-\mathbf{g}_{21}}\mathbf{I}_2$$

or

$$\underline{\frac{\mathbf{I_2}}{\mathbf{I_1}} = \frac{\mathbf{-g_{21}}}{\mathbf{g_{11}Z_L + \Delta_g}}}$$

Also,

$$\mathbf{V}_2 = \mathbf{g}_{21}(\mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s) + \mathbf{g}_{22}\mathbf{I}_2$$

$$= \mathbf{g}_{21}\mathbf{V}_s - \mathbf{g}_{21}\mathbf{Z}_s\mathbf{I}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

$$= \mathbf{g}_{21}\mathbf{V}_s + \mathbf{Z}_s(\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g)\mathbf{I}_2 + \mathbf{g}_{22}\mathbf{I}_2$$

But

$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{\mathbf{Z}_L}$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\,\mathbf{V}_s - [\mathbf{g}_{11}\,\mathbf{Z}_s\,\mathbf{Z}_L + \Delta_g\,\mathbf{Z}_s + \mathbf{g}_{22}]\left[\frac{\mathbf{V}_2}{\mathbf{Z}_L}\right]$$

$$\frac{\mathbf{V}_2[\mathbf{Z}_L + \mathbf{g}_{11}\,\mathbf{Z}_s\,\mathbf{Z}_L + \Delta_g\,\mathbf{Z}_s + \mathbf{g}_{22}]}{\mathbf{Z}_L} = \mathbf{g}_{21}\,\mathbf{V}_s$$

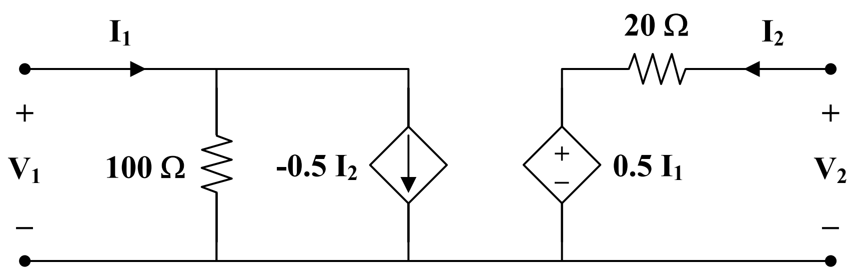
$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\,\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\,\mathbf{Z}_s\,\mathbf{Z}_L + \Delta_g\,\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\,\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\,\mathbf{Z}_s\,\mathbf{Z}_L + \mathbf{g}_{11}\,\mathbf{g}_{22}\,\mathbf{Z}_s - \mathbf{g}_{21}\,\mathbf{g}_{12}\,\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\underline{\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\,\mathbf{Z}_L}{(1+\mathbf{g}_{11}\,\mathbf{Z}_s)(\mathbf{g}_{22}+\mathbf{Z}_L) - \mathbf{g}_{12}\,\mathbf{g}_{21}\,\mathbf{Z}_s}}$$

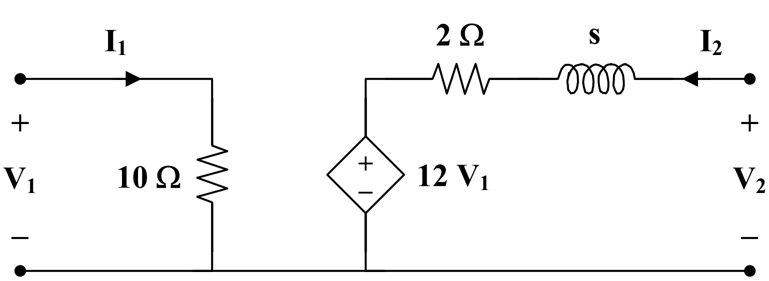
Chapter 19, Solution 42.

(a) **<u>The network is shown in Fig. (a)</u>.**



(a)

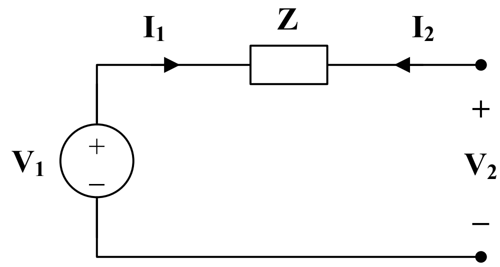
(b) **<u>The network is shown in Fig. (b)</u>.**



(b)

Chapter 19, Solution 43.

(a) To find **A** and **C**, consider the network in Fig. (a).

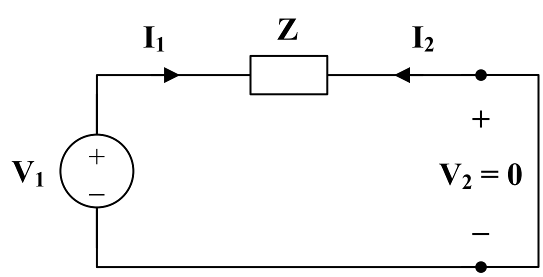


(a)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{I}_1 = 0 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

To get **B** and **D**, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_1 = \mathbf{Z}\mathbf{I}_1, \qquad \mathbf{I}_2 = \text{-}\mathbf{I}_1$$

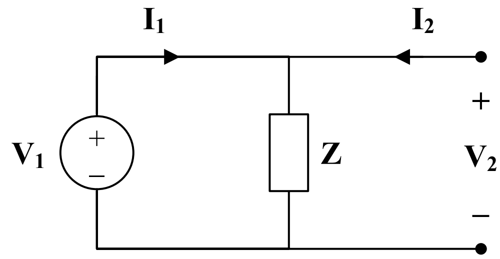
$$\mathbf{B} = \frac{\text{-}\mathbf{V}_1}{\mathbf{I}_2} = \frac{\text{-}\mathbf{Z}\mathbf{I}_1}{\text{-}\mathbf{I}_1} = \mathbf{Z}$$

$$\mathbf{D} = \frac{\text{-}\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Hence,

$$\underline{[\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{Z} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}}$$

(b) To find $\mathbf{A}$ and $\mathbf{C}$, consider the circuit in Fig. (c).

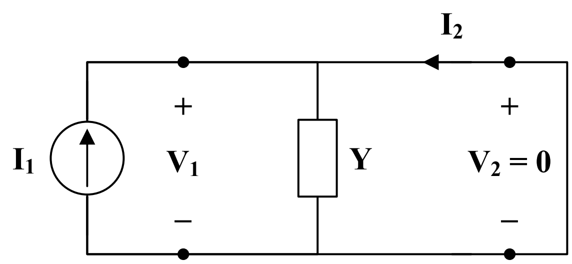


(c)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{V}_1 = \mathbf{Z}\mathbf{I}_1 = \mathbf{V}_2 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{\mathbf{Z}} = \mathbf{Y}$$

To get $\mathbf{B}$ and $\mathbf{D}$, refer to the circuit in Fig.(d).



(d)

$$\mathbf{V}_1 = \mathbf{V}_2 = 0 \qquad \mathbf{I}_2 = -\mathbf{I}_1$$

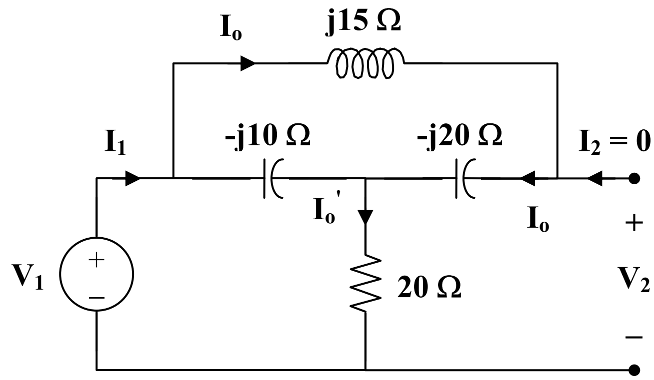
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = 0, \qquad \mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Thus,

$$\underline{[\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{Y} & \mathbf{1} \end{bmatrix}}$$

Chapter 19, Solution 44.

To determine $\mathbf{A}$ and $\mathbf{C}$, consider the circuit in Fig.(a).



(a)

$$\mathbf{V}_1 = [\,20 + (\text{-j}10) \,\|\, (\text{j}15 - \text{j}20)\,]\mathbf{I}_1$$

$$\mathbf{V}_1 = \left[20 + \frac{(\text{-j}10)(\text{-j}5)}{\text{-}\,\text{j}15}\right]\mathbf{I}_1 = \left[20 - \text{j}\frac{10}{3}\right]\mathbf{I}_1$$

$$\mathbf{I}_o{}' = \mathbf{I}_1$$

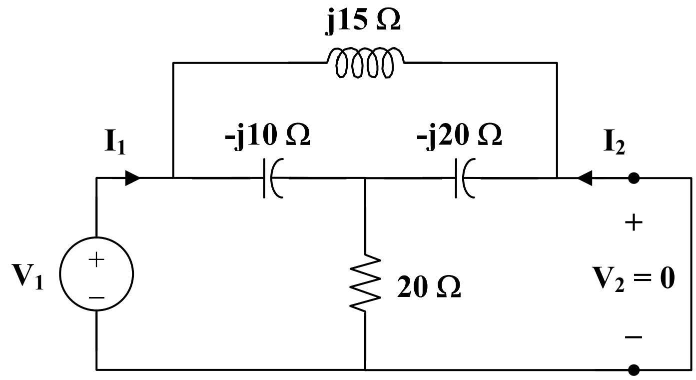
$$\mathbf{I}_o = \left(\frac{\text{-}\,\text{j}10}{\text{-}\,\text{j}10 - \text{j}5}\right)\mathbf{I}_1 = \left(\frac{2}{3}\right)\mathbf{I}_1$$

$$\mathbf{V}_2 = (\text{-j}20)\mathbf{I}_o + 20\mathbf{I}_o{}' = -\text{j}\frac{40}{3}\text{I}_1 + 20\text{I}_1 = \left(20 - \text{j}\frac{40}{3}\right)\text{I}_1$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{(20 - \text{j}10/3)\mathbf{I}_1}{\left(20 - \text{j}\frac{40}{3}\right)\text{I}_1} = 0.7692 + \text{j}0.3461$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{20 - \text{j}\frac{40}{3}} = 0.03461 + \text{j}0.023$$

To find $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



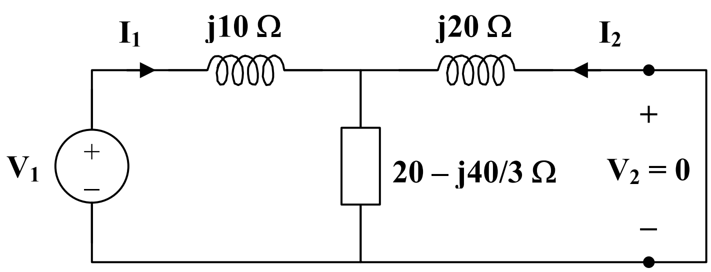
(b)

We may transform the Δ subnetwork to a T as shown in Fig. (c).

$$\mathbf{Z}_1 = \frac{(j15)(-j10)}{j15 - j10 - j20} = j10$$

$$\mathbf{Z}_2 = \frac{(-j10)(-j20)}{-j15} = -j\frac{40}{3}$$

$$\mathbf{Z}_3 = \frac{(j15)(-j20)}{-j15} = j20$$



(c)

$$-\mathbf{I}_2 = \frac{20 - j40/3}{20 - j40/3 + j20}\mathbf{I}_1 = \frac{3 - j2}{3 + j}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{3 + j}{3 - j2} = 0.5385 + j0.6923$$

$$\mathbf{V}_1 = \left[j10 + \frac{(j20)(20 - j40/3)}{20 - j40/3 + j20} \right]\mathbf{I}_1$$

$$\mathbf{V}_1 = [\,j10 + 2(9 + j7)\,]\mathbf{I}_1 = j\mathbf{I}_1(24 - j18)$$

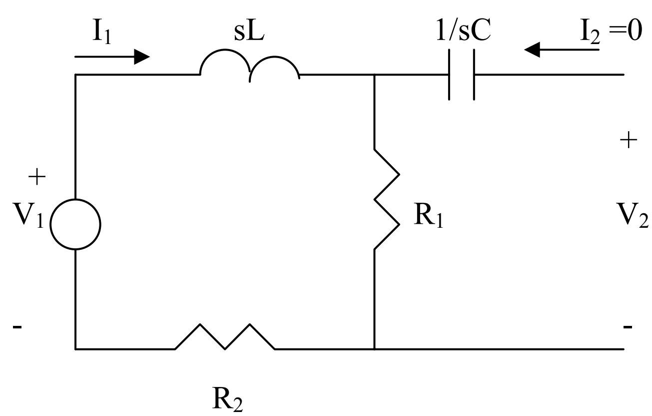
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-j\mathbf{I}_1(24 - j18)}{\dfrac{-(3-j2)}{3+j}\mathbf{I}_1} = \frac{6}{13}(-15 + j55)$$

$$\mathbf{B} = -6.923 + j25.385\,\Omega$$

$$[\mathbf{T}] = \underline{\begin{bmatrix} \mathbf{0.7692 + j0.3461} & \mathbf{-6.923 + j25.385\,\Omega} \\ \mathbf{0.03461 + j0.023\,S} & \mathbf{0.5385 + j0.6923} \end{bmatrix}}$$

Chapter 19, Solution 45.

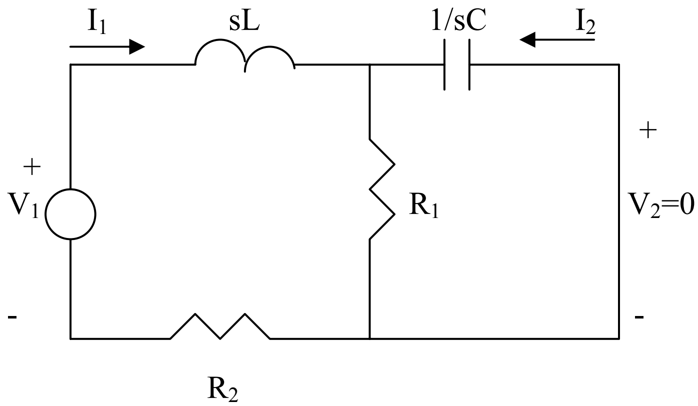
To obtain A and C, consider the circuit below.



$$V_2 = \frac{R_1}{R_1 + R_2 + sL}V_1 \longrightarrow A = \frac{V_1}{V_2} = \underline{\frac{R_1 + R_2 + sL}{R_1}}$$

$$V_2 = I_1 R_1 \longrightarrow \underline{C = \frac{I_1}{V_2} = \frac{1}{R_1}}$$

To obtain B and D, consider the circuit below.



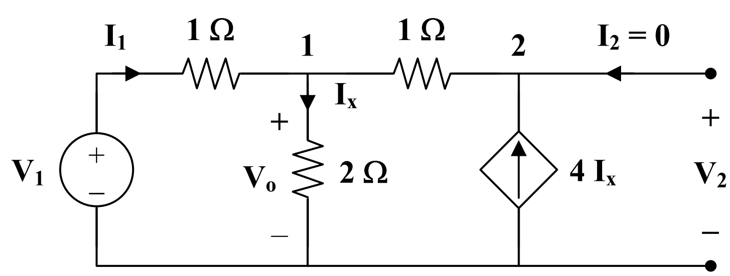
$$I_2 = -\frac{R_1}{R_1 + \frac{1}{sC}} I_1 = -\frac{sR_1C}{1+sR_1C} I_1 \longrightarrow \quad \underline{D = -\frac{I_1}{I_2} = \frac{1+sR_1C}{sR_1C}}$$

$$V_1 = \left(R_2 + sL + \frac{\frac{R_1}{sC}}{R_1 + \frac{1}{sC}} \right) I_1 = -\frac{\left[(1+sR_1C)(R_2+sL)+R_1\right]}{1+sR_1C} \frac{(1+sR_1C)}{sR_{1C}} I_2$$

$$B = -\frac{V_1}{I_2} = \underline{\frac{1}{sR_1C}\left[R_1 + (1+sR_1C)(R_2+sL)\right]}$$

Chapter 19, Solution 46.

To get $\mathbf{A}$ and $\mathbf{C}$, refer to the circuit in Fig.(a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{1} \longrightarrow \quad 2\mathbf{I}_1 = 3\mathbf{V}_o - 2\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_o - \mathbf{V}_2}{1} = 4\mathbf{I}_x = \frac{4\mathbf{V}_o}{2} = 2\mathbf{V}_o \longrightarrow \mathbf{V}_o = \text{-}\mathbf{V}_2 \qquad (2)$$

From (1) and (2),

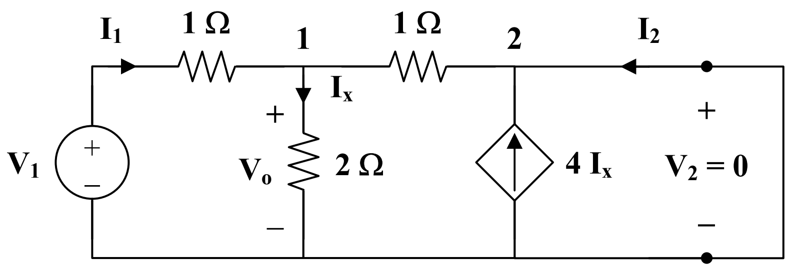
$$2\mathbf{I}_1 = \text{-}5\mathbf{V}_2 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\text{-}5}{2} = \text{-}2.5\text{ S}$$

But $\quad \mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_o}{1} = \mathbf{V}_1 + \mathbf{V}_2$

$$\text{-}2.5\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_2 \longrightarrow \mathbf{V}_1 = \text{-}3.5\mathbf{V}_2$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \text{-}3.5$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o}{1} \longrightarrow 2\mathbf{I}_1 = 3\mathbf{V}_o \qquad (3)$$

At node 2,

$$\mathbf{I}_2 + \frac{\mathbf{V}_o}{1} + 4\mathbf{I}_x = 0$$

$$-\mathbf{I}_2 = \mathbf{V}_o + 2\mathbf{V}_o = 0 \longrightarrow \mathbf{I}_2 = \text{-}3\mathbf{V}_o \qquad (4)$$

Adding (3) and (4),

$$2\mathbf{I}_1 + \mathbf{I}_2 = 0 \longrightarrow \mathbf{I}_1 = \text{-}0.5\mathbf{I}_2 \qquad (5)$$

$$\mathbf{D} = \frac{\text{-}\mathbf{I}_1}{\mathbf{I}_2} = 0.5$$

But $\quad \mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = \mathbf{I}_1 + \mathbf{V}_o \qquad (6)$

Substituting (5) and (4) into (6),

$$\mathbf{V}_1 = \frac{-1}{2}\mathbf{I}_2 + \frac{-1}{3}\mathbf{I}_2 = \frac{-5}{6}\mathbf{I}_2$$

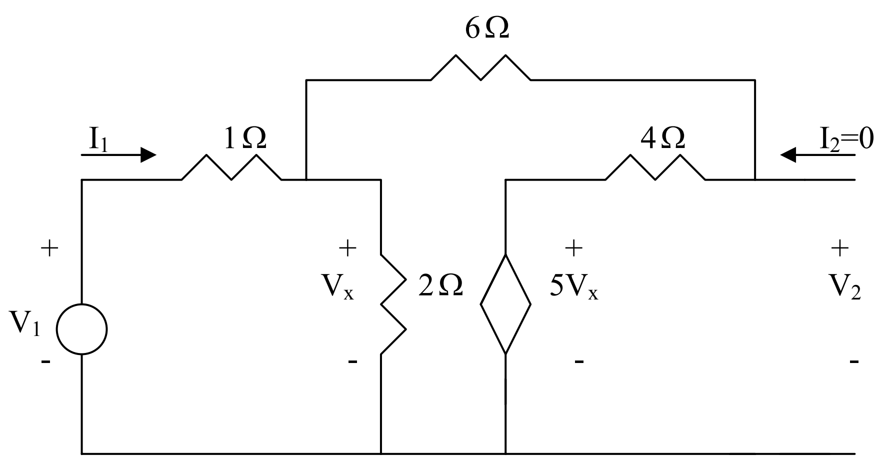
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{5}{6} = 0.8333\,\Omega$$

Thus,

$$[\mathbf{T}] = \underline{\begin{bmatrix} \mathbf{-3.5} & \mathbf{0.8333\,\Omega} \\ \mathbf{-2.5\,S} & \mathbf{-0.5} \end{bmatrix}}$$

Chapter 19, Solution 47.

To get A and C, consider the circuit below.



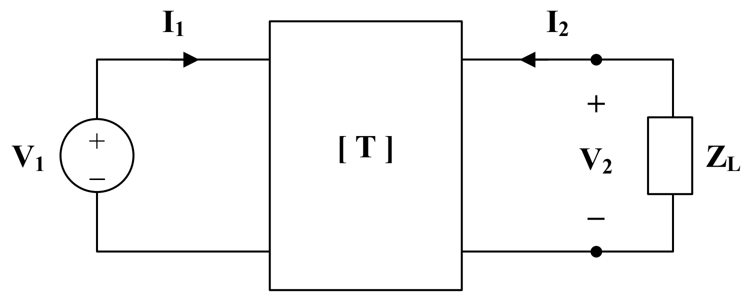
$$\frac{V_1 - V_x}{1} = \frac{V_x}{2} + \frac{V_x - 5V_x}{10} \longrightarrow V_1 = 1.1V_x$$

$$V_2 = 4(-0.4V_x) + 5V_x = 3.4V_x \longrightarrow A = \frac{V_1}{V_2} = 1.1/3.4 = 0.3235$$

$$I_1 = \frac{V_1 - V_x}{1} = 1.1V_x - V_x = 0.1V_x \longrightarrow C = \frac{I_1}{V_2} = 0.1/3.4 = 0.02941$$

Chapter 19, Solution 48.

(a) Refer to the circuit below.



$$\mathbf{V}_1 = 4\mathbf{V}_2 - 30\mathbf{I}_2 \qquad (1)$$

$$\mathbf{I}_1 = 0.1\mathbf{V}_2 - \mathbf{I}_2 \qquad (2)$$

When the output terminals are shorted, $\mathbf{V}_2 = 0$.
So, (1) and (2) become

$$\mathbf{V}_1 = \text{-}30\mathbf{I}_2 \qquad \text{and} \qquad \mathbf{I}_1 = \text{-}\mathbf{I}_2$$

Hence,

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{30\,\Omega}}$$

(b) When the output terminals are open-circuited, $\mathbf{I}_2 = 0$.
So, (1) and (2) become

$$\mathbf{V}_1 = 4\mathbf{V}_2$$

$$\mathbf{I}_1 = 0.1\mathbf{V}_2 \qquad \text{or} \qquad \mathbf{V}_2 = 10\mathbf{I}_1$$

$$\mathbf{V}_1 = 40\mathbf{I}_1$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{40\,\Omega}}$$

(c) When the output port is terminated by a 10-Ω load, $V_2 = \text{-}10I_2$.
So, (1) and (2) become

$$\mathbf{V}_1 = \text{-}40\mathbf{I}_2 - 30\mathbf{I}_2 = \text{-}70\mathbf{I}_2$$

$$\mathbf{I}_1 = \text{-}\mathbf{I}_2 - \mathbf{I}_2 = \text{-}2\mathbf{I}_2$$

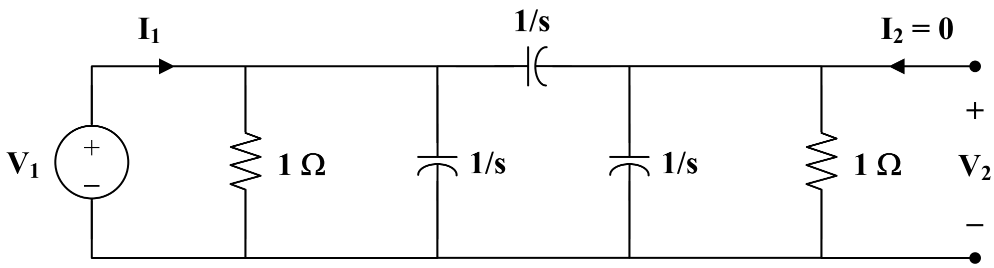
$$\mathbf{V}_1 = 35\mathbf{I}_1$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{35\,\Omega}}$$

Alternatively, we may use $\mathbf{Z}_{in} = \dfrac{\mathbf{A}\,\mathbf{Z}_L + \mathbf{B}}{\mathbf{C}\,\mathbf{Z}_L + \mathbf{D}}$

Chapter 19, Solution 49.

To get $\mathbf{A}$ and $\mathbf{C}$, refer to the circuit in Fig.(a).



(a)

$$1 \,||\, \frac{1}{s} = \frac{1/s}{1 + 1/s} = \frac{1}{s+1}$$

$$\mathbf{V}_2 = \frac{1 \,||\, 1/s}{1/s + 1 \,||\, 1/s}\mathbf{V}_1$$

$$\mathbf{A} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\dfrac{1}{s+1}}{\dfrac{1}{s} + \dfrac{1}{s+1}} = \frac{s}{2s+1}$$

$$\mathbf{V}_1 = \mathbf{I}_1\left(\frac{1}{s+1}\right) \,||\, \left(\frac{1}{s} + \frac{1}{s+1}\right) = \mathbf{I}_1\left(\frac{1}{s+1}\right) \,||\, \left(\frac{2s+1}{s(s+1)}\right)$$

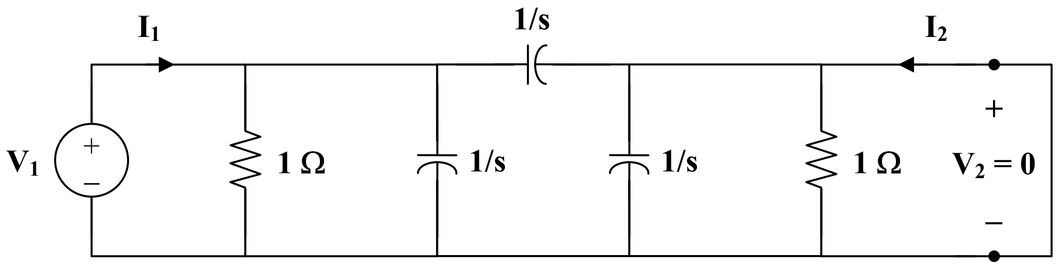
$$\frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\left(\dfrac{1}{s+1}\right)\cdot\left(\dfrac{2s+1}{s(s+1)}\right)}{\dfrac{1}{s+1} + \dfrac{2s+1}{s(s+1)}} = \frac{2s+1}{(s+1)(3s+1)}$$

But $\qquad \mathbf{V}_1 = \mathbf{V}_2 \cdot \dfrac{2s+1}{s}$

Hence, $$\frac{\mathbf{V}_2}{\mathbf{I}_1}\cdot\frac{2s+1}{s} = \frac{2s+1}{(s+1)(3s+1)}$$

$$\mathbf{C} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{(s+1)(3s+1)}{s}$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_1 = \mathbf{I}_1\left(1 \,\|\, \frac{1}{s} \,\|\, \frac{1}{s}\right) = \mathbf{I}_1\left(1 \,\|\, \frac{1}{2s}\right) = \frac{\mathbf{I}_1}{2s+1}$$

$$\mathbf{I}_2 = \frac{\frac{-1}{s+1}\mathbf{I}_1}{\frac{1}{s+1}+\frac{1}{s}} = \frac{-s}{2s+1}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{2s+1}{s} = 2+\frac{1}{s}$$

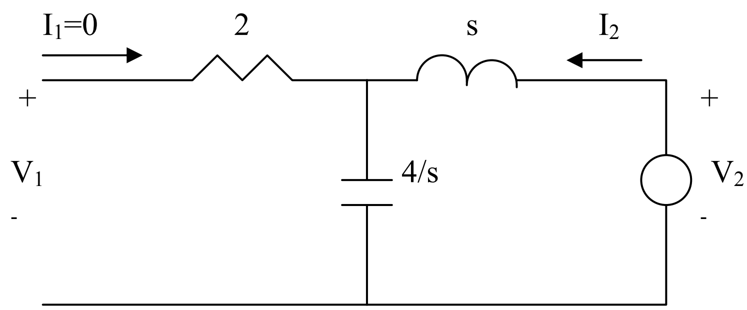
$$\mathbf{V}_1 = \left(\frac{1}{2s+1}\right)\left(\frac{2s+1}{-s}\right)\mathbf{I}_2 = \frac{\mathbf{I}_2}{-s} \longrightarrow \mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{1}{s}$$

Thus,

$$\underline{[\mathbf{T}] = \begin{bmatrix} \dfrac{\mathbf{2}}{\mathbf{2s+1}} & \dfrac{\mathbf{1}}{\mathbf{s}} \\ \dfrac{\mathbf{(s+1)(3s+1)}}{\mathbf{s}} & \mathbf{2+\dfrac{1}{s}} \end{bmatrix}}$$

Chapter 19, Solution 50.

To get a and c, consider the circuit below.

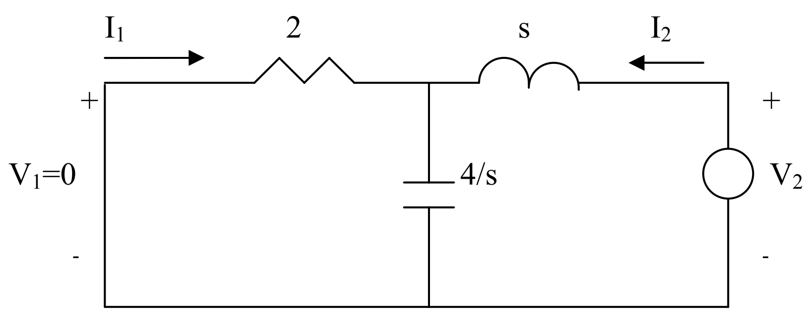


$$V_1 = \frac{4/s}{s+4/s} V_2 = \frac{4}{s^2+4} V_2 \quad \longrightarrow \quad a = V_2 \big/ V_1 = 1 + 0.25s^2$$

$$V_2 = (s+4/s) I_2 \ \text{ or}$$

$$I_2 = \frac{V_2}{s+4/s} = \frac{(1+0.25s^2)V_1}{s+4/s} \quad \longrightarrow \quad c = \frac{I_2}{V_1} = \frac{s+0.25s^3}{s^2+4}$$

To get b and d, consider the circuit below.



$$I_1 = \frac{-4/s}{2+4/s} I_2 = -\frac{2I_2}{s+2} \quad \longrightarrow \quad d = -\frac{I_2}{I_1} = 1 + 0.5s$$

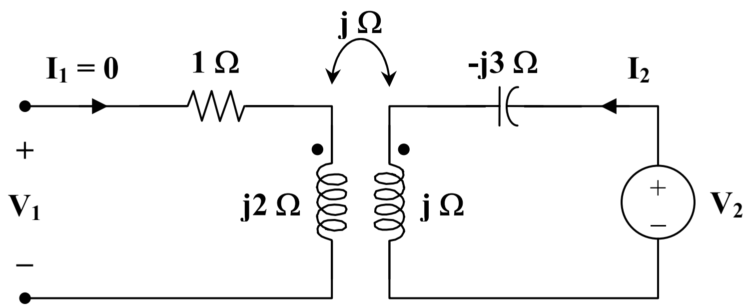
$$V_2 = (s + 2 // \frac{4}{s}) I_2 = \frac{(s^2+2s+4)}{s+2} I_2$$

$$= -\frac{(s^2+2s+4)}{s+2} \frac{(s+2)}{2} I_1 \quad \longrightarrow \quad b = -\frac{V_2}{I_1} = 0.5s^2 + s + 2$$

$$[t] = \begin{bmatrix} 0.25s^2+1 & 0.5s^2+s+2 \\ \dfrac{0.25s^2+s}{s^2+4} & 0.5s+1 \end{bmatrix}$$

Chapter 19, Solution 51.

To get **a** and **c**, consider the circuit in Fig. (a).



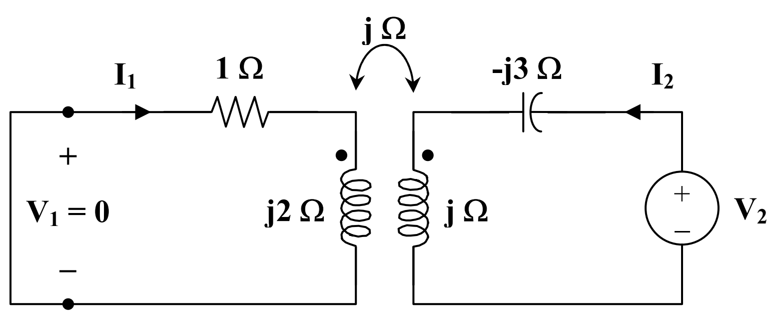
(a)

$$\mathbf{V}_2 = \mathbf{I}_2\,(\mathrm{j} - \mathrm{j}3) = \text{-j}2\,\mathbf{I}_2$$
$$\mathbf{V}_1 = \text{-j}\mathbf{I}_2$$

$$\mathbf{a} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-\,\mathrm{j}2\,\mathbf{I}_2}{-\,\mathrm{j}\mathbf{I}_2} = 2$$

$$\mathbf{c} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1}{-\,\mathrm{j}} = \mathrm{j}$$

To get **b** and **d**, consider the circuit in Fig. (b).



(b)

For mesh 1,

$$0 = (1 + \mathrm{j}2)\,\mathbf{I}_1 - \mathrm{j}\,\mathbf{I}_2$$

or
$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1 + \mathrm{j}2}{\mathrm{j}} = 2 - \mathrm{j}$$

$$\mathbf{d} = \frac{-\mathbf{I}_2}{\mathbf{I}_1} = -2 + j$$

For mesh 2,

$$\mathbf{V}_2 = \mathbf{I}_2(j - j3) - j\mathbf{I}_1$$
$$\mathbf{V}_2 = \mathbf{I}_1(2 - j)(-j2) - j\mathbf{I}_1 = (-2 - j5)\mathbf{I}_1$$

$$\mathbf{b} = \frac{-\mathbf{V}_2}{\mathbf{I}_1} = 2 + j5$$

Thus,

$$[\mathbf{t}] = \underline{\begin{bmatrix} \mathbf{2} & \mathbf{2 + j5} \\ \mathbf{j} & \mathbf{-2 + j} \end{bmatrix}}$$

Chapter 19, Solution 52.

It is easy to find the z parameters and then transform these to h parameters and T parameters.

$$[\mathbf{z}] = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

$$\Delta_z = (R_1 + R_2)(R_2 + R_3) - R_2^2$$
$$= R_1R_2 + R_2R_3 + R_3R_1$$

(a) $$[\mathbf{h}] = \begin{bmatrix} \frac{\Delta_z}{\mathbf{z}_{22}} & \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \frac{1}{\mathbf{z}_{22}} \end{bmatrix} = \begin{bmatrix} \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2 + R_3} & \frac{R_2}{R_2 + R_3} \\ \frac{-R_2}{R_2 + R_3} & \frac{1}{R_2 + R_3} \end{bmatrix}$$

Thus,

$$\underline{\mathbf{h}_{11} = \mathbf{R}_1 + \frac{\mathbf{R}_2\mathbf{R}_3}{\mathbf{R}_2 + \mathbf{R}_3}}, \quad \underline{\mathbf{h}_{12} = \frac{\mathbf{R}_2}{\mathbf{R}_2 + \mathbf{R}_3} = -\mathbf{h}_{21}}, \quad \underline{\mathbf{h}_{22} = \frac{1}{\mathbf{R}_2 + \mathbf{R}_3}}$$

as required.

(b) $$[\mathbf{T}] = \begin{bmatrix} \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \frac{\Delta_z}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \frac{R_1 + R_2}{R_2} & \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ \frac{1}{R_2} & \frac{R_2 + R_3}{R_2} \end{bmatrix}$$

Hence,

$$\underline{\mathbf{A} = 1 + \frac{\mathbf{R_1}}{\mathbf{R_2}}}, \quad \underline{\mathbf{B} = \mathbf{R_3} + \frac{\mathbf{R_1}}{\mathbf{R_2}}(\mathbf{R_2} + \mathbf{R_3})}, \quad \underline{\mathbf{C} = \frac{1}{\mathbf{R_2}}}, \quad \underline{\mathbf{D} = 1 + \frac{\mathbf{R_3}}{\mathbf{R_2}}}$$

as required.

Chapter 19, Solution 53.

For the z parameters,

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{12}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

For **ABCD** parameters,

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \qquad (3)$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \qquad (4)$$

From (4),

$$\mathbf{V}_2 = \frac{\mathbf{I}_1}{\mathbf{C}} + \frac{\mathbf{D}}{\mathbf{C}}\mathbf{I}_2 \qquad (5)$$

Comparing (2) and (5),

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}}, \qquad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}}$$

Substituting (5) into (3),

$$\mathbf{V}_1 = \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \left(\frac{\mathbf{AD}}{\mathbf{C}} - \mathbf{B}\right)\mathbf{I}_2$$

$$= \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}}\mathbf{I}_2 \qquad (6)$$

Comparing (6) and (1),

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} \qquad \mathbf{z}_{12} = \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} = \frac{\Delta_T}{\mathbf{C}}$$

Thus,

$$[Z] = \underline{\begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta_T}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}}$$

Chapter 19, Solution 54.

For the y parameters

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad (2)$$

From (2),

$$\mathbf{V}_1 = \frac{\mathbf{I}_2}{\mathbf{y}_{21}} - \frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2$$

or
$$\mathbf{V}_1 = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{12}}\mathbf{V}_2 + \frac{1}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (1) gives

$$\mathbf{I}_1 = \frac{-\mathbf{y}_{11}\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2 + \mathbf{y}_{12}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2$$

or
$$\mathbf{I}_1 = \frac{-\Delta_y}{\mathbf{y}_{21}}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

clearly shows that

$$\underline{\mathbf{A} = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{B} = \frac{-\mathbf{1}}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{C} = \frac{-\Delta_y}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{D} = \frac{-\mathbf{y}_{11}}{\mathbf{y}_{21}}}$$

as required.

Chapter 19, Solution 55.

For the z parameters

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

From (1),

$$\mathbf{I}_1 = \frac{1}{\mathbf{z}_{11}}\mathbf{V}_1 - \frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (2) gives

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \left(\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11}}\right)\mathbf{I}_2$$

or
$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \frac{\Delta_z}{\mathbf{z}_{11}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{I}_1 = \mathbf{g}_{11}\,\mathbf{V}_1 + \mathbf{g}_{12}\,\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{g}_{21}\,\mathbf{V}_1 + \mathbf{g}_{22}\,\mathbf{I}_2$$

indicates that

$$\underline{\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{12} = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}}$$

as required.

Chapter 19, Solution 56.

(a) $\Delta_y = (2+j)(3-j) + j4 = 7 + j5$

$$[z] = \begin{bmatrix} y_{22}/\Delta_y & -y_{12}/\Delta_y \\ -y_{21}/\Delta_y & y_{11}/\Delta_y \end{bmatrix} = \underline{\begin{bmatrix} 0.2162 - j0.2973 & -0.2703 - j0.3784 \\ 0.0946 - j0.0676 & 0.2568 - j0.0405 \end{bmatrix}}\Omega$$

(b) $$[h] = \begin{bmatrix} 1/y_{11} & -y_{12}/y_{11} \\ y_{21}/y_{11} & \Delta_y/y_{11} \end{bmatrix} = \underline{\begin{bmatrix} 0.4 - j0.2 & -0.8 - j1.6 \\ -0.4 + j0.2 & 3.8 + j0.6 \end{bmatrix}}$$

(c) $$[t] = \begin{bmatrix} -y_{11}/y_{12} & -1/y_{12} \\ -\Delta_y/y_{12} & -y_{22}/y_{12} \end{bmatrix} = \underline{\begin{bmatrix} -0.25 + j0.5 & j0.25 \\ -1.25 + j1.75 & 0.25 + j0.75 \end{bmatrix}}$$

Chapter 19, Solution 57.

$\Delta_T = (3)(7) - (20)(1) = 1$

$$[\mathbf{z}] = \begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta_T}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{7} \end{bmatrix}\Omega}$$

$$[\mathbf{y}] = \begin{bmatrix} \frac{\mathbf{D}}{\mathbf{B}} & \frac{-\Delta_T}{\mathbf{B}} \\ \frac{-1}{\mathbf{B}} & \frac{\mathbf{A}}{\mathbf{B}} \end{bmatrix} = \underline{\begin{bmatrix} \frac{7}{20} & \frac{-1}{20} \\ \frac{-1}{20} & \frac{3}{20} \end{bmatrix} \mathbf{S}}$$

$$[\mathbf{h}] = \begin{bmatrix} \frac{\mathbf{B}}{\mathbf{D}} & \frac{\Delta_T}{\mathbf{D}} \\ \frac{-1}{\mathbf{D}} & \frac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \underline{\begin{bmatrix} \frac{20}{7}\Omega & \frac{1}{7} \\ \frac{-1}{7} & \frac{1}{7}\mathbf{S} \end{bmatrix}}$$

$$[\mathbf{g}] = \begin{bmatrix} \frac{\mathbf{C}}{\mathbf{A}} & \frac{-\Delta_T}{\mathbf{A}} \\ \frac{1}{\mathbf{A}} & \frac{\mathbf{B}}{\mathbf{A}} \end{bmatrix} = \underline{\begin{bmatrix} \frac{1}{3}\mathbf{S} & \frac{-1}{3} \\ \frac{1}{3} & \frac{20}{3}\Omega \end{bmatrix}}$$

$$[\mathbf{t}] = \begin{bmatrix} \frac{\mathbf{D}}{\Delta_T} & \frac{\mathbf{B}}{\Delta_T} \\ \frac{\mathbf{C}}{\Delta_T} & \frac{\mathbf{A}}{\Delta_T} \end{bmatrix} = \underline{\begin{bmatrix} 7 & 20\,\Omega \\ 1\,\mathbf{S} & 3 \end{bmatrix}}$$

Chapter 19, Solution 58.

The given set of equations is for the h parameters.

$$[\mathbf{h}] = \begin{bmatrix} 1\,\Omega & 2 \\ -2 & 0.4\,\mathrm{S} \end{bmatrix} \qquad \Delta_h = (1)(0.4) - (2)(-2) = 4.4$$

(a)
$$[\mathbf{y}] = \begin{bmatrix} \frac{1}{\mathbf{h}_{11}} & \frac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} \\ \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \frac{\Delta_h}{\mathbf{h}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} 1 & -2 \\ -2 & 4.4 \end{bmatrix} \mathbf{S}}$$

(b)
$$[\mathbf{T}] = \begin{bmatrix} \frac{-\Delta_h}{\mathbf{h}_{21}} & \frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \frac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} 2.2 & 0.5\,\Omega \\ 0.2\,\mathbf{S} & 0.5 \end{bmatrix}}$$

Chapter 19, Solution 59.

$$\Delta_g = (0.06)(2) - (-0.4)(2) = 0.12 + 0.08 = 0.2$$

(a) $$[\mathbf{z}] = \begin{bmatrix} \dfrac{1}{\mathbf{g}_{11}} & \dfrac{-\mathbf{g}_{12}}{\mathbf{g}_{11}} \\ \dfrac{\mathbf{g}_{21}}{\mathbf{g}_{11}} & \dfrac{\Delta_g}{\mathbf{g}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{16.667} & \mathbf{6.667} \\ \mathbf{3.333} & \mathbf{3.333} \end{bmatrix} \Omega}$$

(b) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{\Delta_g}{\mathbf{g}_{22}} & \dfrac{\mathbf{g}_{12}}{\mathbf{g}_{22}} \\ \dfrac{-\mathbf{g}_{21}}{\mathbf{g}_{22}} & \dfrac{1}{\mathbf{g}_{22}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0.1} & \mathbf{-0.2} \\ \mathbf{-0.1} & \mathbf{0.5} \end{bmatrix} \mathbf{S}}$$

(c) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{g}_{22}}{\Delta_g} & \dfrac{-\mathbf{g}_{12}}{\Delta_g} \\ \dfrac{-\mathbf{g}_{21}}{\Delta_g} & \dfrac{\mathbf{g}_{11}}{\Delta_g} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{2} \\ \mathbf{-1} & \mathbf{0.3\ S} \end{bmatrix}}$$

(d) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{1}{\mathbf{g}_{21}} & \dfrac{\mathbf{g}_{22}}{\mathbf{g}_{21}} \\ \dfrac{\mathbf{g}_{11}}{\mathbf{g}_{21}} & \dfrac{\Delta_g}{\mathbf{g}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{5} & \mathbf{10\,\Omega} \\ \mathbf{0.3\ S} & \mathbf{1} \end{bmatrix}}$$

Chapter 19, Solution 60.

$$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = 0.3 - 0.02 = 0.28$$

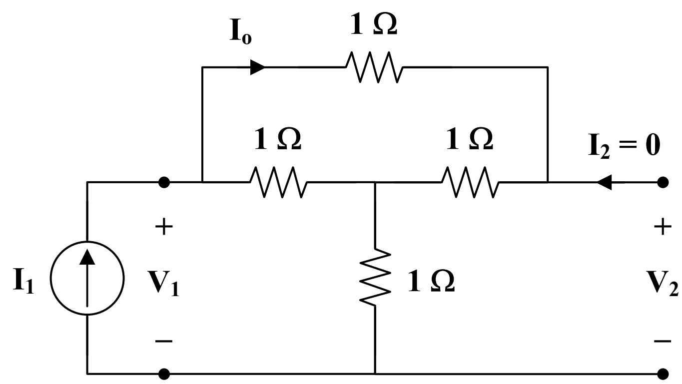
(a) $$[\mathbf{z}] = \begin{bmatrix} \dfrac{\mathbf{y}_{22}}{\Delta_y} & \dfrac{-\mathbf{y}_{12}}{\Delta_y} \\ \dfrac{-\mathbf{y}_{21}}{\Delta_y} & \dfrac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.786} & \mathbf{0.7143} \\ \mathbf{0.3571} & \mathbf{2.143} \end{bmatrix} \Omega}$$

(b) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.667\,\Omega} & \mathbf{0.3333} \\ \mathbf{-0.1667} & \mathbf{0.4667\ S} \end{bmatrix}}$$

(c) $$[\mathbf{t}] = \begin{bmatrix} \dfrac{-\mathbf{y}_{11}}{\mathbf{y}_{12}} & \dfrac{-1}{\mathbf{y}_{12}} \\ \dfrac{-\Delta_y}{\mathbf{y}_{12}} & \dfrac{-\mathbf{y}_{22}}{\mathbf{y}_{12}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{3} & \mathbf{5\,\Omega} \\ \mathbf{1.4\ S} & \mathbf{2.5} \end{bmatrix}}$$

Chapter 19, Solution 61.

(a) To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{I}_1[1 + 1 \,||\, (1+1)] = \mathbf{I}_1\left(1 + \frac{2}{3}\right) = \frac{5}{3}\mathbf{I}_1$$

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{5}{3}$$

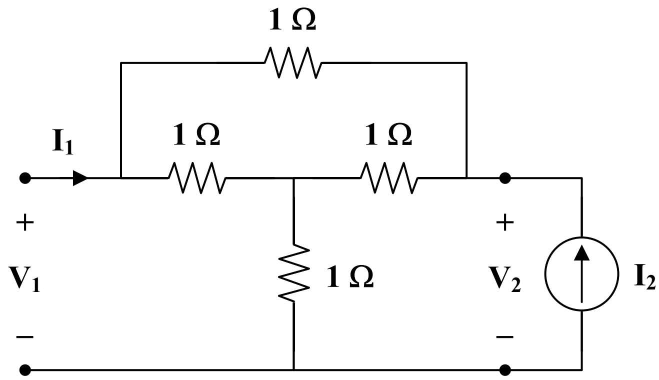
$$\mathbf{I}_o = \frac{1}{1+2}\mathbf{I}_1 = \frac{1}{3}\mathbf{I}_1$$

$$-\mathbf{V}_2 + \mathbf{I}_o + \mathbf{I}_1 = 0$$

$$\mathbf{V}_2 = \frac{1}{3}\mathbf{I}_1 + \mathbf{I}_1 = \frac{4}{3}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{4}{3}$$

To obtain $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

Due to symmetry, this is similar to the circuit in Fig. (a).

$$\mathbf{z}_{22} = \mathbf{z}_{11} = \frac{5}{3}, \qquad \mathbf{z}_{21} = \mathbf{z}_{12} = \frac{4}{3}$$

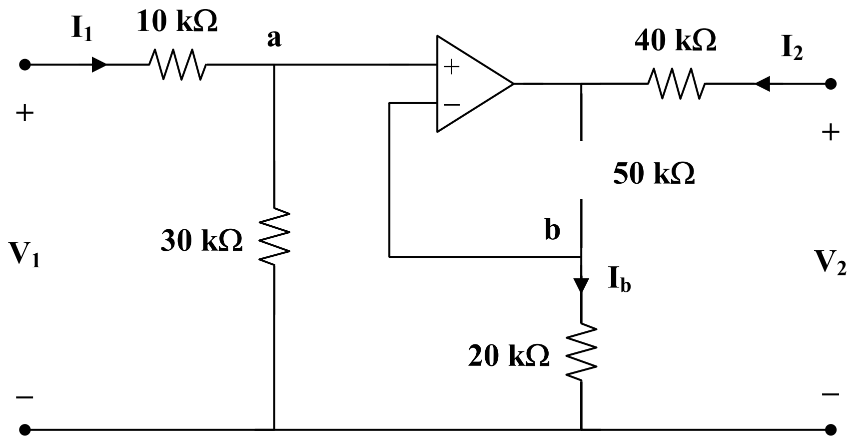
$$\underline{[\mathbf{z}] = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix} \Omega}$$

(b) $$[\mathbf{h}] = \begin{bmatrix} \frac{\Delta_z}{\mathbf{z}_{22}} & \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \frac{1}{\mathbf{z}_{22}} \end{bmatrix} = \underline{\begin{bmatrix} \frac{3}{5}\Omega & \frac{4}{5} \\ \frac{-4}{5} & \frac{3}{5}\mathrm{S} \end{bmatrix}}$$

(c) $$[\mathbf{T}] = \begin{bmatrix} \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \frac{\Delta_z}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \frac{5}{4} & \frac{3}{4}\Omega \\ \frac{3}{4}\mathrm{S} & \frac{5}{4} \end{bmatrix}}$$

Chapter 19, Solution 62.

Consider the circuit shown below.



Since no current enters the input terminals of the op amp,

$$\mathbf{V}_1 = (10+30)\times 10^3\,\mathbf{I}_1 \tag{1}$$

But $$\mathbf{V}_a = \mathbf{V}_b = \frac{30}{40}\mathbf{V}_1 = \frac{3}{4}\mathbf{V}_1$$

$$\mathbf{I}_b = \frac{\mathbf{V}_b}{20\times 10^3} = \frac{3}{80\times 10^3}\mathbf{V}_1$$

which is the same current that flows through the 50-kΩ resistor.

Thus, $$\mathbf{V}_2 = 40\times 10^3\,\mathbf{I}_2 + (50+20)\times 10^3\,\mathbf{I}_b$$

$$\mathbf{V}_2 = 40\times 10^3\,\mathbf{I}_2 + 70\times 10^3 \cdot \frac{3}{80\times 10^3}\mathbf{V}_1$$

$$\mathbf{V}_2 = \frac{21}{8}\mathbf{V}_1 + 40\times 10^3\,\mathbf{I}_2$$

$$\mathbf{V}_2 = 105\times 10^3\,\mathbf{I}_1 + 40\times 10^3\,\mathbf{I}_2 \tag{2}$$

From (1) and (2),

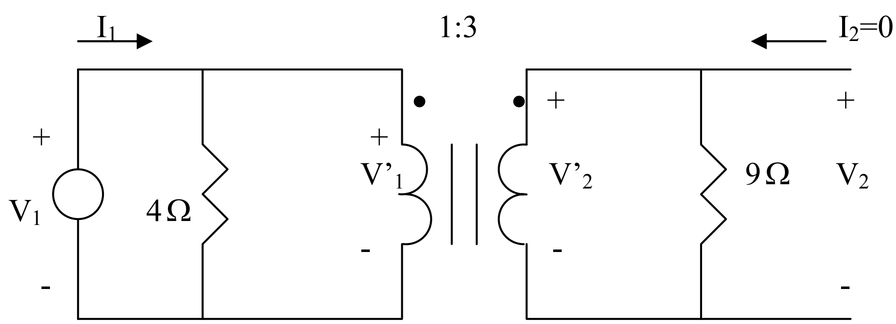
$$[\mathbf{z}] = \begin{bmatrix} \mathbf{40} & \mathbf{0} \\ \mathbf{105} & \mathbf{40} \end{bmatrix} \mathbf{k\Omega}$$

$$\Delta_z = \mathbf{z}_{11}\,\mathbf{z}_{22} - \mathbf{z}_{12}\,\mathbf{z}_{21} = 16\times 10^8$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0.381} & \mathbf{15.24\ k\Omega} \\ \mathbf{9.52\ \mu S} & \mathbf{0.381} \end{bmatrix}}$$

Chapter 19, Solution 63.

To get z_{11} and z_{21}, consider the circuit below.

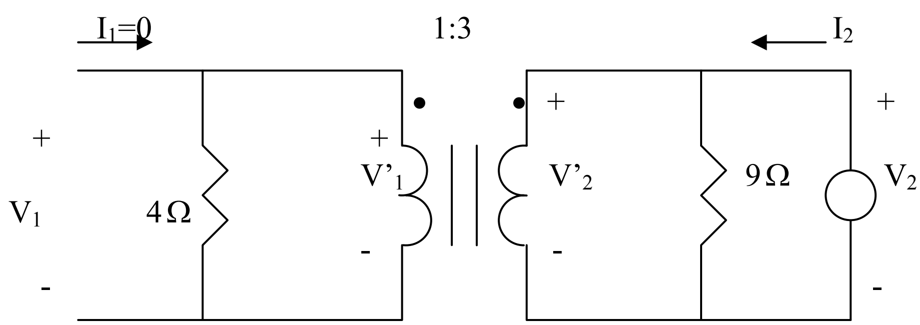


$$Z_R = \frac{9}{n^2} = 1, \qquad n = 3$$

$$V_1 = (4 // Z_R) I_1 = \frac{4}{5} I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 0.8$$

$$V_2 = V_2' = nV_1' = nV_1 = 3(4/5)I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = 2.4$$

To get z_{21} and z_{22}, consider the circuit below.



$$Z_R' = n^2(4) = 36, \qquad n = 3$$

$$V_2 = (9 // Z_R')I_2 = \frac{9x36}{45} I_2 \quad \longrightarrow \quad z_{22} = \frac{V_2}{I_2} = 7.2$$

$$V_1 = \frac{V_2}{n} = \frac{V_2}{3} = 2.4 I_2 \quad \longrightarrow \quad z_{21} = \frac{V_1}{I_2} = 2.4$$

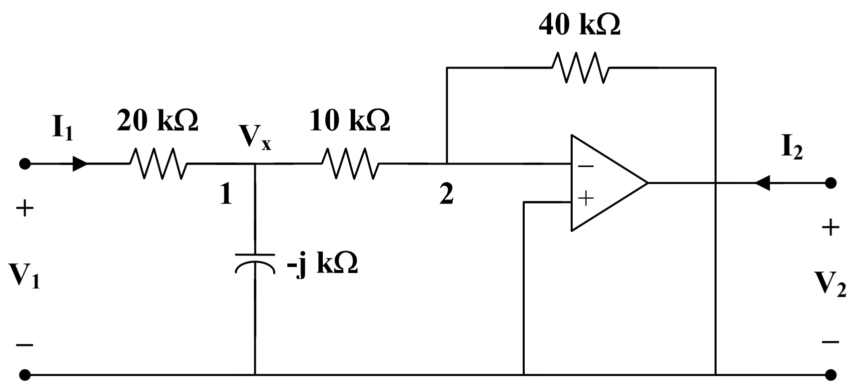
Thus,

$$\underline{[z] = \begin{bmatrix} 0.8 & 2.4 \\ 2.4 & 7.2 \end{bmatrix} \Omega}$$

Chapter 19, Solution 64.

$$1\ \mu\text{F} \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{-j}{(10^3)(10^{-6})} = -j\ \text{k}\Omega$$

Consider the op amp circuit below.



At node 1,

$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{20} = \frac{\mathbf{V}_x}{-j} + \frac{\mathbf{V}_x - 0}{10}$$

$$\mathbf{V}_1 = (3 + j20)\,\mathbf{V}_x \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_x - 0}{10} = \frac{0 - \mathbf{V}_2}{40} \quad \longrightarrow \quad \mathbf{V}_x = \frac{-1}{4}\mathbf{V}_2 \qquad (2)$$

But $\quad \mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_x}{20 \times 10^3} \qquad (3)$

Substituting (2) into (3) gives

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 + 0.25\,\mathbf{V}_2}{20\times 10^3} = 50\times 10^{-6}\,\mathbf{V}_1 + 12.5\times 10^{-6}\,\mathbf{V}_2 \qquad (4)$$

Substituting (2) into (1) yields

$$\mathbf{V}_1 = \frac{-1}{4}(3 + j20)\,\mathbf{V}_2$$

or

$$0 = \mathbf{V}_1 + (0.75 + j5)\,\mathbf{V}_2 \qquad (5)$$

Comparing (4) and (5) with the following equations

$$\mathbf{I}_1 = \mathbf{y}_{11}\,\mathbf{V}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2$$

indicates that $\mathbf{I}_2 = 0$ and that

$$[\mathbf{y}] = \underline{\begin{bmatrix} \mathbf{50\times 10^{-6}} & \mathbf{12.5\times 10^{-6}} \\ \mathbf{1} & \mathbf{0.75 + j5} \end{bmatrix}\mathbf{S}}$$

$$\Delta_y = (77.5 + j25. - 12.5)\times 10^{-6} = (65 + j250)\times 10^{-6}$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{2\times 10^4\ \Omega} & \mathbf{-0.25} \\ \mathbf{2\times 10^4} & \mathbf{1.3 + j5\ S} \end{bmatrix}}$$

Chapter 19, Solution 65.

The network consists of two two-ports in series. It is better to work with z parameters and then convert to y parameters.

For N_a, $$[\mathbf{z}_a] = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

For N_b, $$[\mathbf{z}_b] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 6 & 3 \\ 3 & 3 \end{bmatrix}$$

$$\Delta_z = 18 - 9 = 9$$

$$[\mathbf{y}] = \begin{bmatrix} \dfrac{\mathbf{z}_{22}}{\Delta_z} & \dfrac{-\mathbf{z}_{12}}{\Delta_z} \\ \dfrac{-\mathbf{z}_{21}}{\Delta_z} & \dfrac{\mathbf{z}_{11}}{\Delta_z} \end{bmatrix} = \underline{\begin{bmatrix} \dfrac{\mathbf{1}}{\mathbf{3}} & \dfrac{\mathbf{-1}}{\mathbf{3}} \\ \dfrac{\mathbf{-1}}{\mathbf{3}} & \dfrac{\mathbf{2}}{\mathbf{3}} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Solution 66.

Since we have two two-ports in series, it is better to convert the given y parameters to z parameters.

$$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = (2\times10^{-3})(10\times10^{-3}) - 0 = 20\times10^{-6}$$

$$[\mathbf{z}_a] = \begin{bmatrix} \dfrac{\mathbf{y}_{22}}{\Delta_y} & \dfrac{-\mathbf{y}_{12}}{\Delta_y} \\ \dfrac{-\mathbf{y}_{21}}{\Delta_y} & \dfrac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} 500\,\Omega & 0 \\ 0 & 100\,\Omega \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

i.e.
$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

or
$$\mathbf{V}_1 = 600\mathbf{I}_1 + 100\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = 100\mathbf{I}_1 + 200\mathbf{I}_2 \qquad (2)$$

But, at the input port,
$$\mathbf{V}_s = \mathbf{V}_1 + 60\mathbf{I}_1 \qquad (3)$$

and at the output port,
$$\mathbf{V}_2 = \mathbf{V}_o = -300\mathbf{I}_2 \qquad (4)$$

From (2) and (4),
$$100\mathbf{I}_1 + 200\mathbf{I}_2 = -300\mathbf{I}_2$$
$$\mathbf{I}_1 = -5\mathbf{I}_2 \qquad (5)$$

Substituting (1) and (5) into (3),
$$\mathbf{V}_s = 600\mathbf{I}_1 + 100\mathbf{I}_2 + 60\mathbf{I}_1$$

$$= (660)(\text{-}5)\mathbf{I}_2 + 100\mathbf{I}_2$$
$$= \text{-}3200\mathbf{I}_2 \qquad (6)$$

From (4) and (6),

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{\text{-}300\mathbf{I}_2}{\text{-}3200\mathbf{I}_2} = \underline{\mathbf{0.09375}}$$

Chapter 19, Solution 67.

The y parameters for the upper network is

$$[\mathbf{y}] = \begin{bmatrix} 2 & \text{-}1 \\ \text{-}1 & 2 \end{bmatrix}, \qquad \Delta_y = 4 - 1 = 3$$

$$[\mathbf{z}_a] = \begin{bmatrix} \dfrac{\mathbf{y}_{22}}{\Delta_y} & \dfrac{-\mathbf{y}_{12}}{\Delta_y} \\ \dfrac{-\mathbf{y}_{21}}{\Delta_y} & \dfrac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} \dfrac{2}{3} & \dfrac{1}{3} \\ \dfrac{1}{3} & \dfrac{2}{3} \end{bmatrix}$$

$$[\mathbf{z}_b] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 5/3 & 4/3 \\ 4/3 & 5/3 \end{bmatrix}$$

$$\Delta_z = \frac{25}{9} - \frac{16}{9} = 1$$

$$[\mathbf{T}] = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.25} & \mathbf{0.75\,\Omega} \\ \mathbf{0.75\,S} & \mathbf{1.25} \end{bmatrix}}$$

Chapter 19, Solution 68.

For the upper network N_a, $[\mathbf{y}_a] = \begin{bmatrix} 4 & \text{-}2 \\ \text{-}2 & 4 \end{bmatrix}$

and for the lower network N_b, $[\mathbf{y}_b] = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

For the overall network,

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

$$\Delta_y = 36 - 9 = 27$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{\dfrac{1}{6}}\,\Omega & \mathbf{\dfrac{-1}{2}} \\ \mathbf{\dfrac{-1}{2}} & \mathbf{\dfrac{9}{2}\,S} \end{bmatrix}}$$

Chapter 19, Solution 69.

We first determine the y parameters for the upper network N_a.
To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).

$$n = \frac{1}{2}, \qquad\qquad \mathbf{Z}_R = \frac{1/s}{n^2} = \frac{4}{s}$$

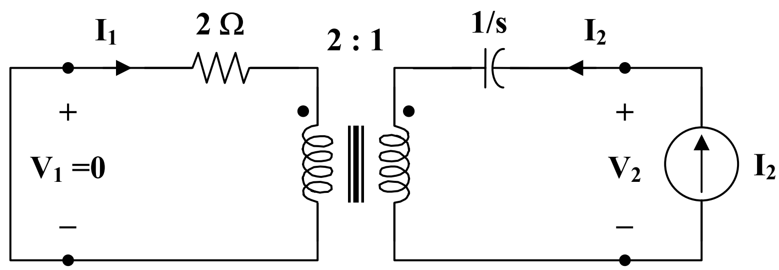
$$\mathbf{V}_1 = (2 + \mathbf{Z}_R)\mathbf{I}_1 = \left(2 + \frac{4}{s}\right)\mathbf{I}_1 = \left(\frac{2s+4}{s}\right)\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s}{2(s+2)}$$

$$\mathbf{I}_2 = \frac{-\mathbf{I}_1}{n} = -2\mathbf{I}_1 = \frac{-s\,\mathbf{V}_1}{s+2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-s}{s+2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{Z}_R{}' = (n^2)(2) = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

$$\mathbf{V}_2 = \left(\frac{1}{s} + \mathbf{Z}_R{}'\right)\mathbf{I}_2 = \left(\frac{1}{s} + \frac{1}{2}\right)\mathbf{I}_2 = \left(\frac{s+2}{2s}\right)\mathbf{I}_2$$

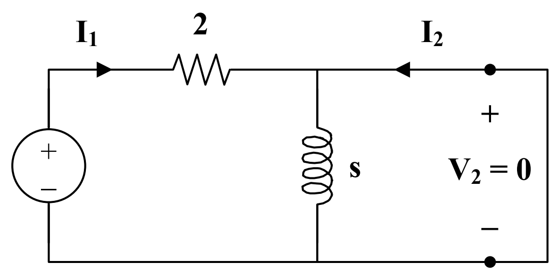
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{2s}{s+2}$$

$$\mathbf{I}_1 = -n\,\mathbf{I}_2 = \left(\frac{-1}{2}\right)\left(\frac{2s}{s+2}\right)\mathbf{V}_2 = \left(\frac{-s}{s+2}\right)\mathbf{V}_2$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-s}{s+2}$$

$$[\mathbf{y}_a] = \begin{bmatrix} \dfrac{s}{2(s+2)} & \dfrac{-s}{s+2} \\ \dfrac{-s}{s+2} & \dfrac{2s}{s+2} \end{bmatrix}$$

For the lower network N_b, we obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by referring to the network in Fig. (c).

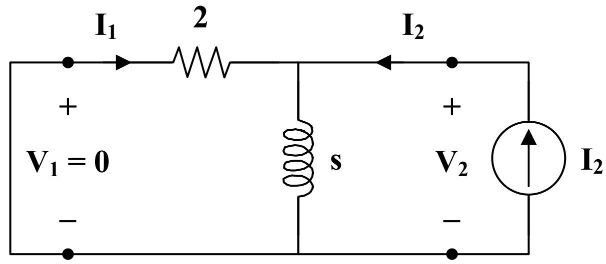


(c)

$$\mathbf{V}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{2}$$

$$\mathbf{I}_2 = -\mathbf{I}_1 = \frac{-\mathbf{V}_1}{2} \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (d).



(d)

$$\mathbf{V}_2 = (s \parallel 2)\mathbf{I}_2 = \frac{2s}{s+2}\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{s+2}{2s}$$

$$\mathbf{I}_1 = -\mathbf{I}_2 \cdot \frac{-s}{s+2} = \left(\frac{-s}{s+2}\right)\left(\frac{s+2}{2s}\right)\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{2}$$

$$[\mathbf{y}_b] = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & (s+2)/2s \end{bmatrix}$$

$$\underline{[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} \dfrac{\mathbf{s+1}}{\mathbf{s+2}} & \dfrac{\mathbf{-(3s+2)}}{\mathbf{2(s+2)}} \\ \dfrac{\mathbf{-(3s+2)}}{\mathbf{2(s+2)}} & \dfrac{\mathbf{5s^2+4s+4}}{\mathbf{2s(s+2)}} \end{bmatrix}}$$

Chapter 19, Solution 70.

We may obtain the g parameters from the given z parameters.

$$[\mathbf{z}_a] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix}, \qquad \Delta_{z_a} = 250 - 100 = 150$$

$$[\mathbf{z}_b] = \begin{bmatrix} 50 & 25 \\ 25 & 30 \end{bmatrix}, \qquad \Delta_{z_b} = 1500 - 625 = 875$$

$$[\mathbf{g}] = \begin{bmatrix} \frac{1}{z_{11}} & \frac{-z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}} \end{bmatrix}$$

$$[\mathbf{g}_a] = \begin{bmatrix} 0.04 & -0.8 \\ 0.2 & 6 \end{bmatrix}, \qquad [\mathbf{g}_b] = \begin{bmatrix} 0.02 & -0.5 \\ 0.5 & 17.5 \end{bmatrix}$$

$$[\mathbf{g}] = [\mathbf{g}_a] + [\mathbf{g}_b] = \underline{\begin{bmatrix} \mathbf{0.06\ S} & \mathbf{-1.3} \\ \mathbf{0.7} & \mathbf{23.5\ \Omega} \end{bmatrix}}$$

Chapter 19, Solution 71.

This is a parallel-series connection of two two-ports. We need to add their g parameters together and obtain z parameters from there.

For the transformer,

$$V_1 = \frac{1}{2} V_2, \quad I_1 = -2I_2$$

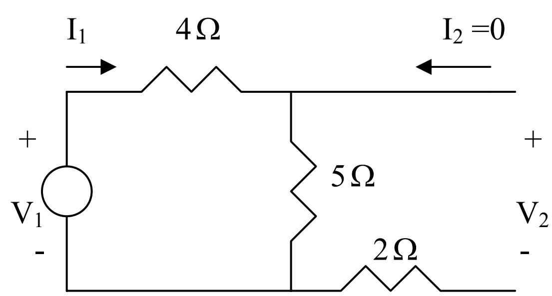
Comparing this with

$$V_1 = AV_2 - BI_2, \qquad I_1 = CV_2 - DI_2$$

shows that

$$[T_{b1}] = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

To get A and C for T_{b2} , consider the circuit below.

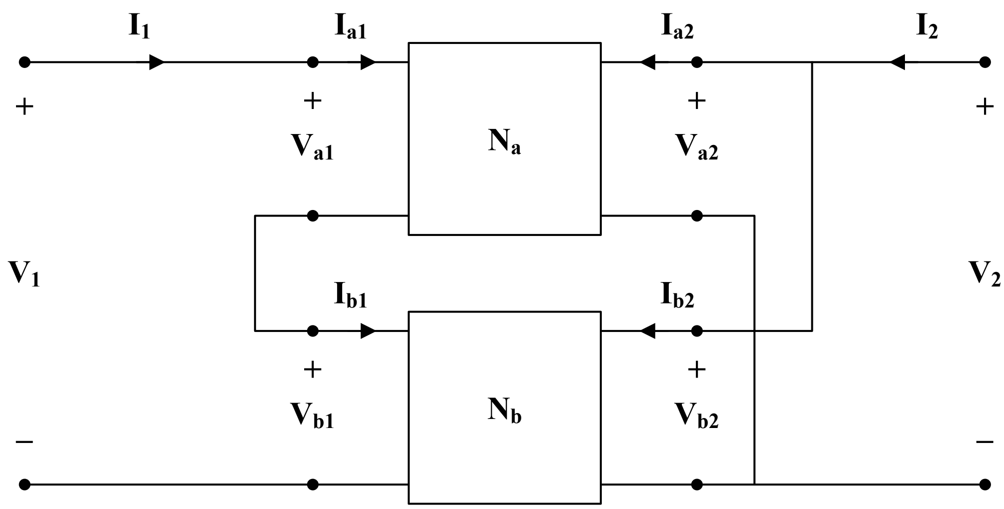


$V_1 = 9I_1, \qquad V_2 = 5I_1$

$$A = \frac{V_1}{V_2} = 9/5 = 1.8, \qquad C = \frac{I_1}{V_2} = 1/5 = 0.2$$

Chapter 19, Solution 72.

Consider the network shown below.



$$\mathbf{V}_{a1} = 25\mathbf{I}_{a1} + 4\mathbf{V}_{a2} \qquad (1)$$
$$\mathbf{I}_{a2} = -4\mathbf{I}_{a1} + \mathbf{V}_{a2} \qquad (2)$$
$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_{b2} \qquad (3)$$
$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_{b2} \qquad (4)$$
$$\mathbf{V}_1 = \mathbf{V}_{a1} + \mathbf{V}_{b1}$$
$$\mathbf{V}_2 = \mathbf{V}_{a2} = \mathbf{V}_{b2}$$
$$\mathbf{I}_2 = \mathbf{I}_{a2} + \mathbf{I}_{b2}$$
$$\mathbf{I}_1 = \mathbf{I}_{a1}$$

Now, rewrite (1) to (4) in terms of $\mathbf{I}_1$ and $\mathbf{V}_2$

$$\mathbf{V}_{a1} = 25\mathbf{I}_1 + 4\mathbf{V}_2 \qquad (5)$$
$$\mathbf{I}_{a2} = -4\mathbf{I}_1 + \mathbf{V}_2 \qquad (6)$$
$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_2 \qquad (7)$$
$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_2 \qquad (8)$$

Adding (5) and (7),

$$\mathbf{V}_1 = 25\mathbf{I}_1 + 16\mathbf{I}_{b1} + 5\mathbf{V}_2 \tag{9}$$

Adding (6) and (8),

$$\mathbf{I}_2 = -4\mathbf{I}_1 - \mathbf{I}_{b1} + 1.5\mathbf{V}_2 \tag{10}$$

$$\mathbf{I}_{b1} = \mathbf{I}_{a1} = \mathbf{I}_1 \tag{11}$$

Because the two networks $\mathrm{N_a}$ and $\mathrm{N_b}$ are independent,

$$\mathbf{I}_2 = -5\mathbf{I}_1 + 1.5\mathbf{V}_2$$

or

$$\mathbf{V}_2 = 3.333\mathbf{I}_1 + 0.6667\mathbf{I}_2 \tag{12}$$

Substituting (11) and (12) into (9),

$$\mathbf{V}_1 = 41\mathbf{I}_1 + \frac{25}{1.5}\mathbf{I}_1 + \frac{5}{1.5}\mathbf{I}_2$$

$$\mathbf{V}_1 = 57.67\mathbf{I}_1 + 3.333\mathbf{I}_2 \tag{13}$$

Comparing (12) and (13) with the following equations

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

indicates that

$$[\mathbf{z}] = \underline{\begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix}\Omega}$$

Alternatively,

$$[\mathbf{h}_a] = \begin{bmatrix} 25 & 4 \\ -4 & 1 \end{bmatrix}, \qquad [\mathbf{h}_b] = \begin{bmatrix} 16 & 1 \\ -1 & 0.5 \end{bmatrix}$$

$$[\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b] = \begin{bmatrix} 41 & 5 \\ -5 & 1.5 \end{bmatrix} \qquad \Delta_h = 61.5 + 25 = 86.5$$

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{22}} & \dfrac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \dfrac{-\mathbf{h}_{21}}{\mathbf{h}_{22}} & \dfrac{1}{\mathbf{h}_{22}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix}\Omega}$$

as obtained previously.

Chapter 19, Solution 73.

From Example 18.14 and the cascade two-ports,

$$[\mathbf{T}_a] = [\mathbf{T}_b] = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b] = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{7} & \mathbf{12\,\Omega} \\ \mathbf{4\,S} & \mathbf{7} \end{bmatrix}}$$

When the output is short-circuited, $\mathbf{V}_2 = 0$ and by definition

$$\mathbf{V}_1 = -\mathbf{B}\mathbf{I}_2, \qquad \mathbf{I}_1 = -\mathbf{D}\mathbf{I}_2$$

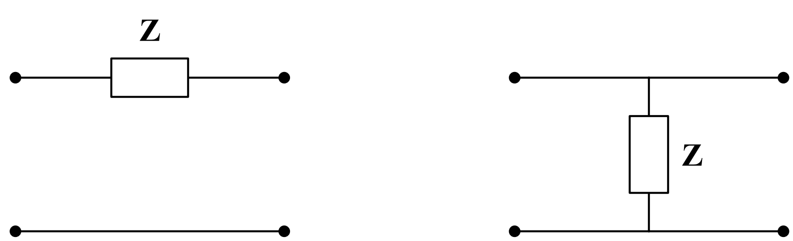
Hence,

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{B}}{\mathbf{D}} = \underline{\frac{\mathbf{12}}{\mathbf{7}}\,\Omega}$$

Chapter 19, Solution 74.

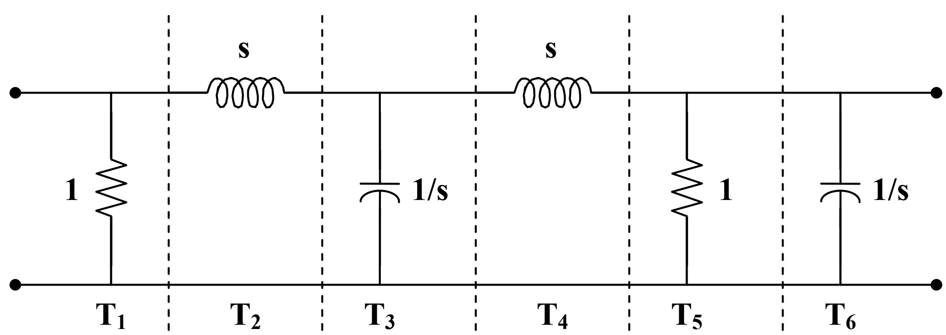
From Prob. 18.35, the transmission parameters for the circuit in Figs. (a) and (b) are

$$[\mathbf{T}_a] = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix}, \qquad [\mathbf{T}_b] = \begin{bmatrix} 1 & 0 \\ 1/\mathbf{Z} & 1 \end{bmatrix}$$



(a) (b)

We partition the given circuit into six subcircuits similar to those in Figs. (a) and (b) as shown in Fig. (c) and obtain $[\mathbf{T}]$ for each.



$$[\mathbf{T}_1]=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix},\qquad [\mathbf{T}_2]=\begin{bmatrix}1 & s\\ 0 & 1\end{bmatrix},\qquad [\mathbf{T}_3]=\begin{bmatrix}1 & 0\\ s & 1\end{bmatrix}$$

$$[\mathbf{T}_4]=[\mathbf{T}_2],\qquad [\mathbf{T}_5]=[\mathbf{T}_1],\qquad [\mathbf{T}_6]=[\mathbf{T}_3]$$

$$[\mathbf{T}]=[\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4][\mathbf{T}_5][\mathbf{T}_6]=[\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s & 1\end{bmatrix}$$

$$=[\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix}1 & 0\\ s+1 & 1\end{bmatrix}=[\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3]\begin{bmatrix}1 & s\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s+1 & 1\end{bmatrix}$$

$$=[\mathbf{T}_1][\mathbf{T}_2]\begin{bmatrix}1 & 0\\ s & 1\end{bmatrix}\begin{bmatrix}s^2+s+1 & s\\ s+1 & 1\end{bmatrix}$$

$$=[\mathbf{T}_1]\begin{bmatrix}1 & s\\ 0 & 1\end{bmatrix}\begin{bmatrix}s^2+s+1 & s\\ s^3+s^2+2s+1 & s^2+1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}s^4+s^3+3s^2+2s+1 & s^3+2s\\ s^3+s^2+2s+1 & s^2+1\end{bmatrix}$$

$$[\mathbf{T}]=\underline{\begin{bmatrix}\mathbf{s^4+s^3+3s^2+2s+1} & \mathbf{s^3+2s}\\ \mathbf{s^4+2s^3+4s^2+4s+2} & \mathbf{s^3+s^2+2s+1}\end{bmatrix}}$$

Note that $\mathbf{AB}-\mathbf{CD}=1$ as expected.

Chapter 19, Solution 75.

(a) We convert $[z_a]$ and $[z_b]$ to T-parameters. For N_a, $\Delta_z = 40-24=16$.

$$[T_a]=\begin{bmatrix}z_{11}/z_{21} & \Delta_z/z_{21}\\ 1/z_{21} & z_{22}/z_{21}\end{bmatrix}=\begin{bmatrix}2 & 4\\ 0.25 & 1.25\end{bmatrix}$$

For N_b, $\Delta_y = 80+8=88$.

$$[T_b]=\begin{bmatrix}-y_{22}/y_{21} & -1/y_{21}\\ -\Delta_y/y_{21} & -y_{11}/y_{21}\end{bmatrix}=\begin{bmatrix}-5 & -0.5\\ -44 & -4\end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} -186 & -17 \\ -56.25 & -5.125 \end{bmatrix}$$

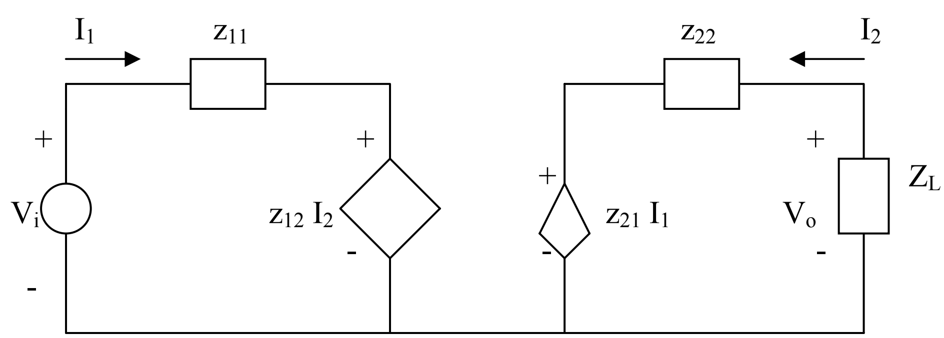
We convert this to y-parameters. $\Delta_T = AD - BC = -3.$

$$[y] = \begin{bmatrix} D/B & -\Delta_T/B \\ -1/B & A/B \end{bmatrix} = \underline{\begin{bmatrix} 0.3015 & -0.1765 \\ 0.0588 & 10.94 \end{bmatrix}}$$

(b) The equivalent z-parameters are

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 3.3067 & 0.0533 \\ -0.0178 & 0.0911 \end{bmatrix}$$

Consider the equivalent circuit below.



$$V_i = z_{11}I_1 + z_{12}I_2 \qquad (1)$$

$$V_o = z_{21}I_1 + z_{22}I_2 \qquad (2)$$

But $V_o = -I_2Z_L \quad \longrightarrow \quad I_2 = -V_o/Z_L$ (3)

From (2) and (3) ,

$$V_o = z_{21}I_1 - z_{22}\frac{V_o}{Z_L} \quad \longrightarrow \quad I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{Z_L z_{21}}\right) \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$\frac{V_i}{V_o} = \left(\frac{z_{11}}{z_{21}} + \frac{z_{11}z_{22}}{z_{21}Z_L}\right) - \frac{z_{12}}{Z_L} = -194.3 \quad \longrightarrow \quad \underline{\frac{V_{o.}}{V_i} = -0.0051}$$

Chapter 19, Solution 76.

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 3.849, \qquad z_{21} = V_2 = 1.122$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

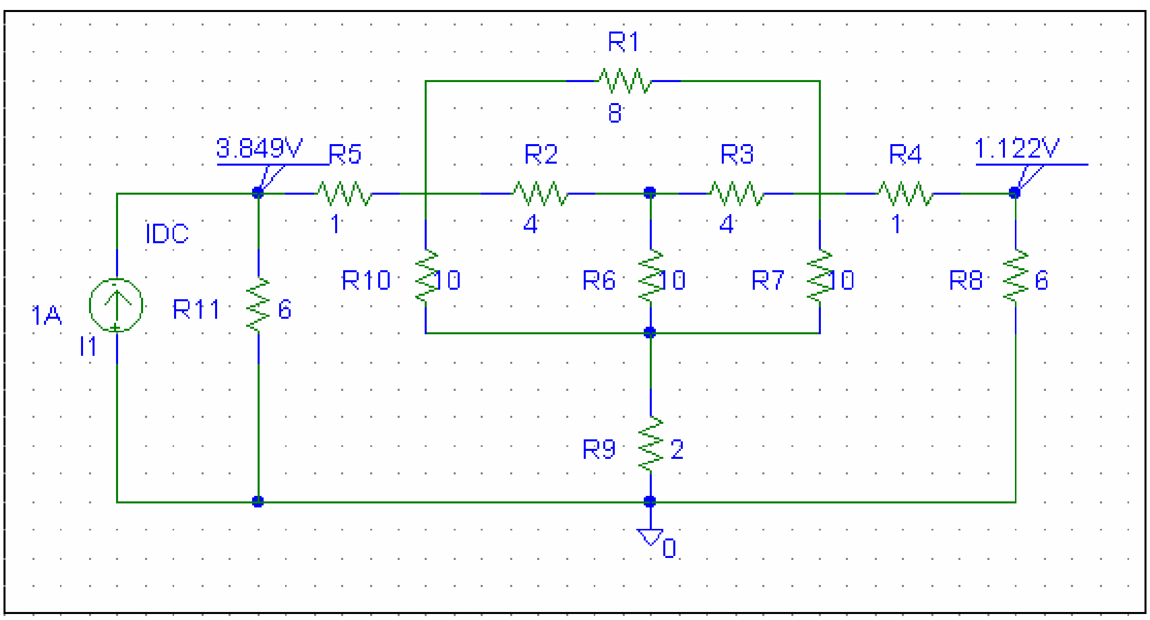
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

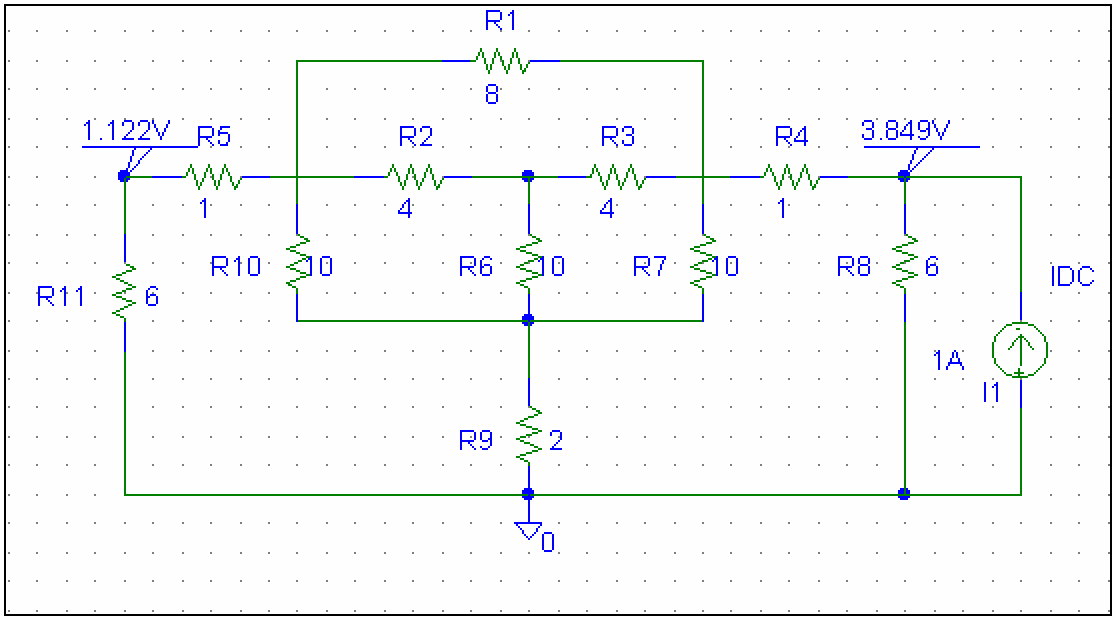
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 1.122, \qquad z_{22} = V_2 = 3.849$$

Thus,

$$\underline{[z] = \begin{bmatrix} 3.949 & 1.122 \\ 1.122 & 3.849 \end{bmatrix} \Omega}$$





Chapter 19, Solution 77.

We follow Example 19.15 except that this is an AC circuit.

(a) We set $V_2 = 0$ and $I_1 = 1$ A. The schematic is shown below. In the AC Sweep Box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

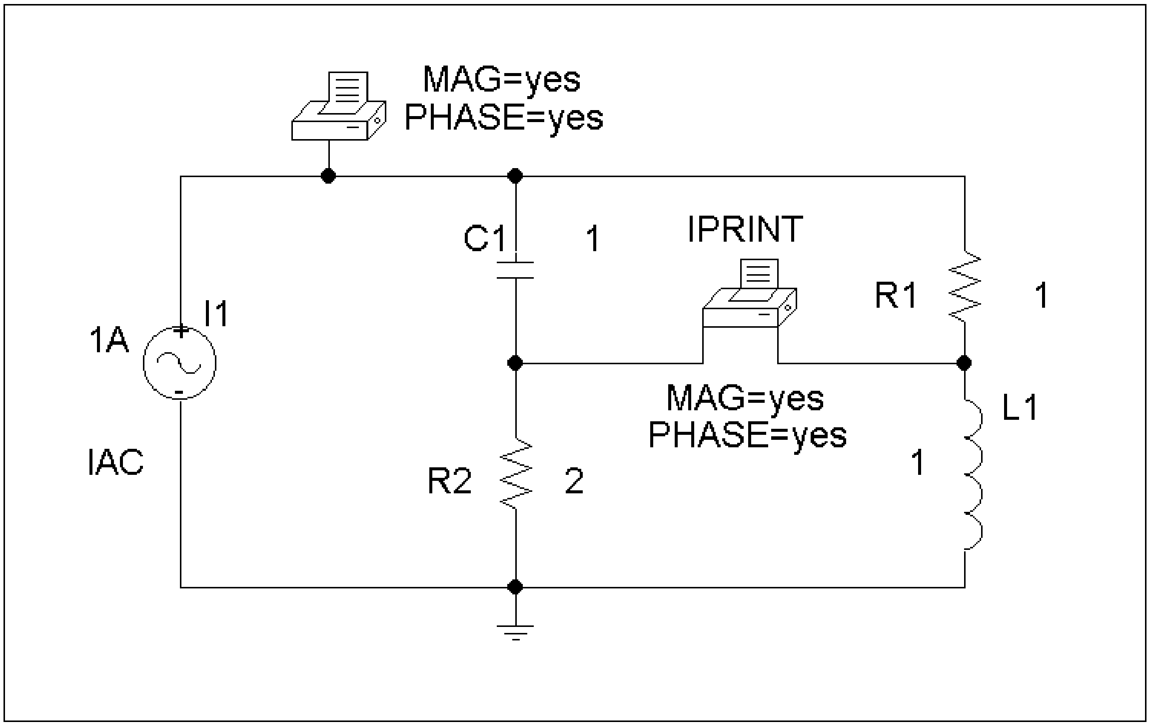
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	3.163 E–.01	–1.616 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	9.488 E–01	–1.616 E+02

From this we obtain

$$h_{11} = V_1/1 = 0.9488\angle{-161.6°}$$

$$h_{21} = I_2/1 = 0.3163\angle{-161.6°}.$$



(b) In this case, we set $I_1 = 0$ and $V_2 = 1V$. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	3.163 E–.01	1.842 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	9.488 E–01	–1.616 E+02

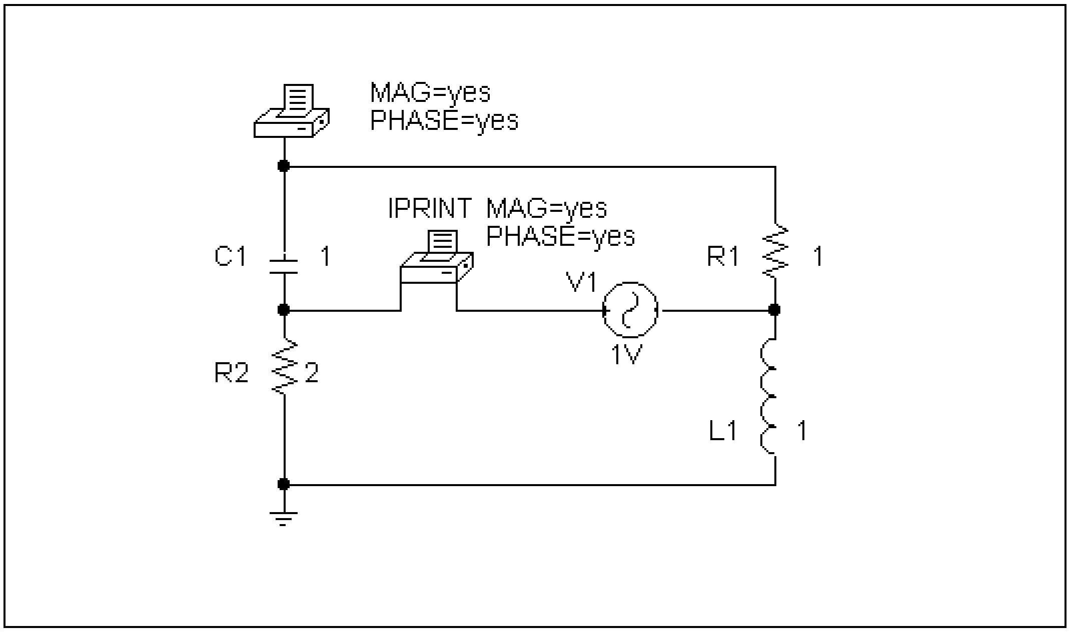
From this,

$$h_{12} = V_1/1 = 0.3163\angle 18.42°$$

$$h_{21} = I_2/1 = 0.9488\angle -161.6°.$$

Thus,

$$[h] = \underline{\begin{bmatrix} \mathbf{0.9488\angle -161.6°} & \mathbf{0.3163\angle 18.42°} \\ \mathbf{0.3163\angle -161.6°} & \mathbf{0.9488\angle -161.6°} \end{bmatrix}}$$



Chapter 19, Solution 78

For h_{11} and h_{21}, short-circuit the output port and let $I_1 = 1A$. $f = \omega / 2\pi = 0.6366$. The schematic is shown below. When it is saved and run, the output file contains the following:

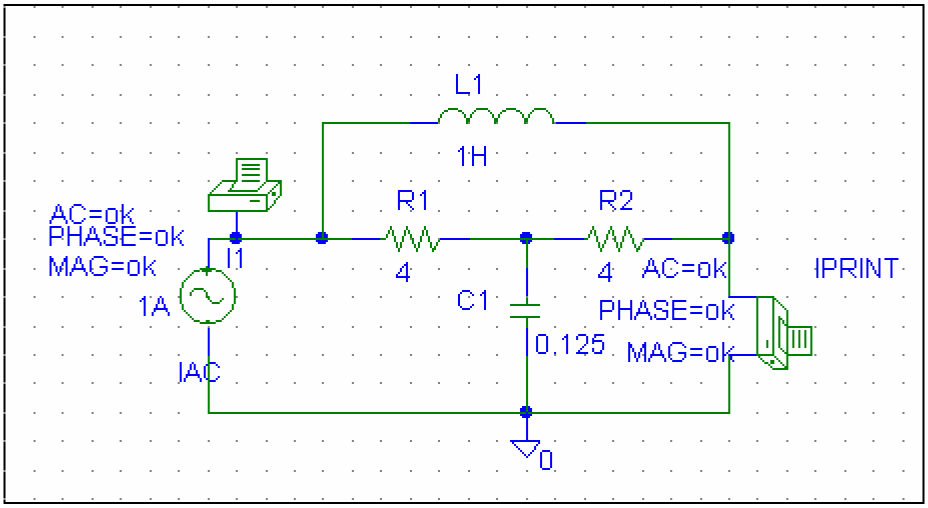
```
FREQ        IM(V_PRINT1)IP(V_PRINT1)

   6.366E-01   1.202E+00   1.463E+02

  FREQ        VM($N_0003) VP($N_0003)

   6.366E-01   3.771E+00  -1.350E+02
```

From the output file, we obtain

$$I_2 = 1.202\angle 146.3^o, \qquad V_1 = 3.771\angle -135^o$$

so that

$$h_{11} = \frac{V_1}{1} = 3.771\angle -135^o, \qquad h_{21} = \frac{I_2}{1} = 1.202\angle 146.3^o$$



For h_{12} and h_{22}, open-circuit the input port and let $V_2 = 1V$. The schematic is shown below. When it is saved and run, the output file includes:

```
FREQ          VM($N_0003) VP($N_0003)
 6.366E-01    1.202E+00   -3.369E+01
FREQ          IM(V_PRINT1)IP(V_PRINT1)
 6.366E-01    3.727E-01   -1.534E+02
```

From the output file, we obtain

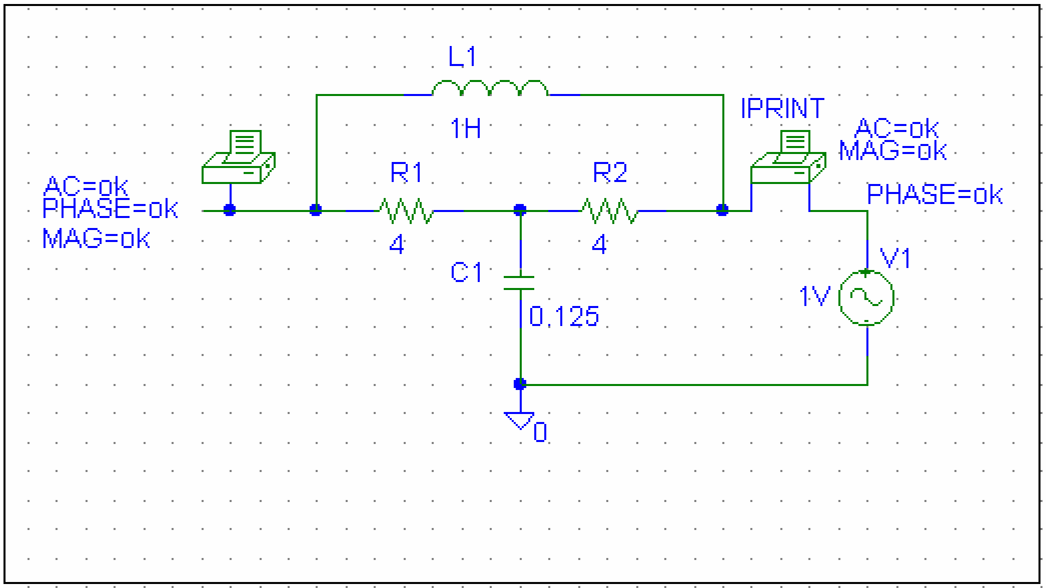
$$I_2 = 0.3727\angle -153.4^o, \qquad V_1 = 1.202\angle -33.69^o$$

so that

$$h_{12} = \frac{V_1}{1} = 1.202\angle -33.69^o, \qquad h_{22} = \frac{I_2}{1} = 0.3727\angle -153.4^o$$

Thus,

$$\underline{[h] = \begin{bmatrix} 3.771\angle -135^o & 1.202\angle -33.69^o \\ 1.202\angle 146.3 & 0.3727\angle -153.4^o \end{bmatrix}}$$



Chapter 19, Solution 79

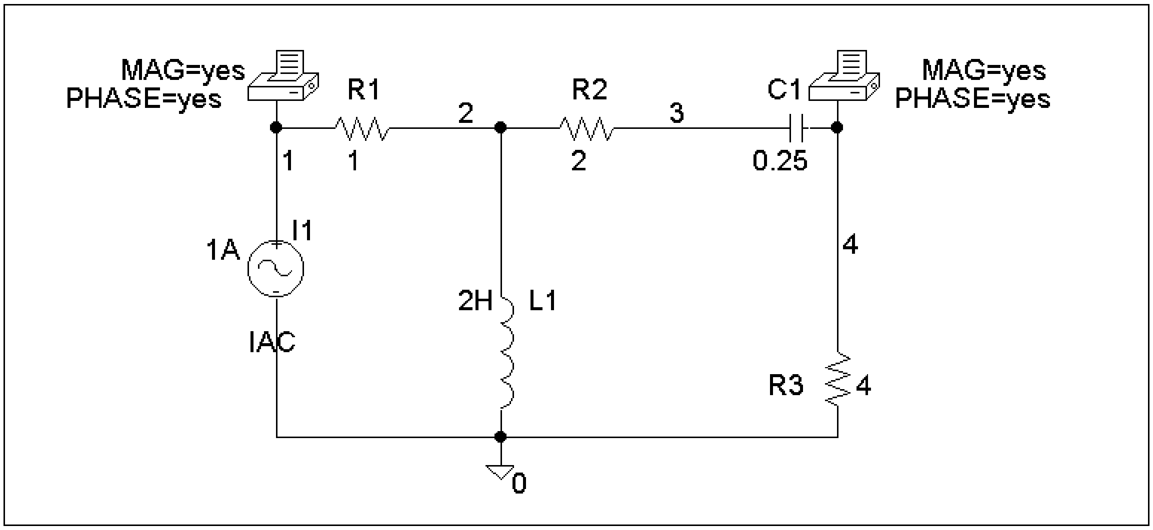
We follow Example 19.16.
(a) We set $I_1 = 1$ A and open-circuit the output-port so that $I_2 = 0$. The schematic is shown below with two VPRINT1s to measure V_1 and V_2. In the AC Sweep box, we enter Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

FREQ	VM(1)	VP(1)
3.183 E–01	4.669 E+00	–1.367 E+02
FREQ	VM(4)	VP(4)
3.183 E–01	2.530 E+00	–1.084 E+02

From this,

$$z_{11} = V_1/I_1 = 4.669\angle{-136.7°}/1 = 4.669\angle{-136.7°}$$

$$z_{21} = V_2/I_1 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108.4°}.$$



(b) In this case, we let I_2 = 1 A and open-circuit the input port. The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

FREQ	VM(1)	VP(1)
3.183 E–01	2.530 E+00	–1.084 E+02

FREQ	VM(2)	VP(2)
3.183 E–01	1.789 E+00	–1.534 E+02

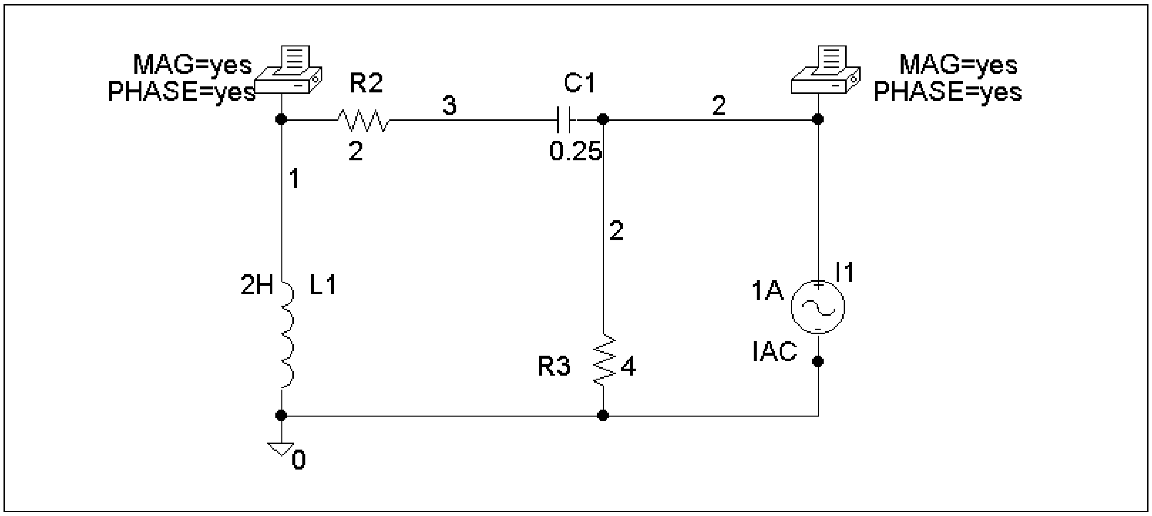
From this,

$$z_{12} = V_1/I_2 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108..4°}$$

$$z_{22} = V_2/I_2 = 1.789\angle{-153.4°}/1 = 1.789\angle{-153.4°}.$$

Thus,

$$[z] = \underline{\begin{bmatrix} \mathbf{4.669\angle -136.7°} & \mathbf{2.53\angle -108.4°} \\ \mathbf{2.53\angle -108.4°} & \mathbf{1.789\angle -153.4°} \end{bmatrix}}$$



Chapter 19, Solution 80

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 29.88, \qquad z_{21} = V_2 = -70.37$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

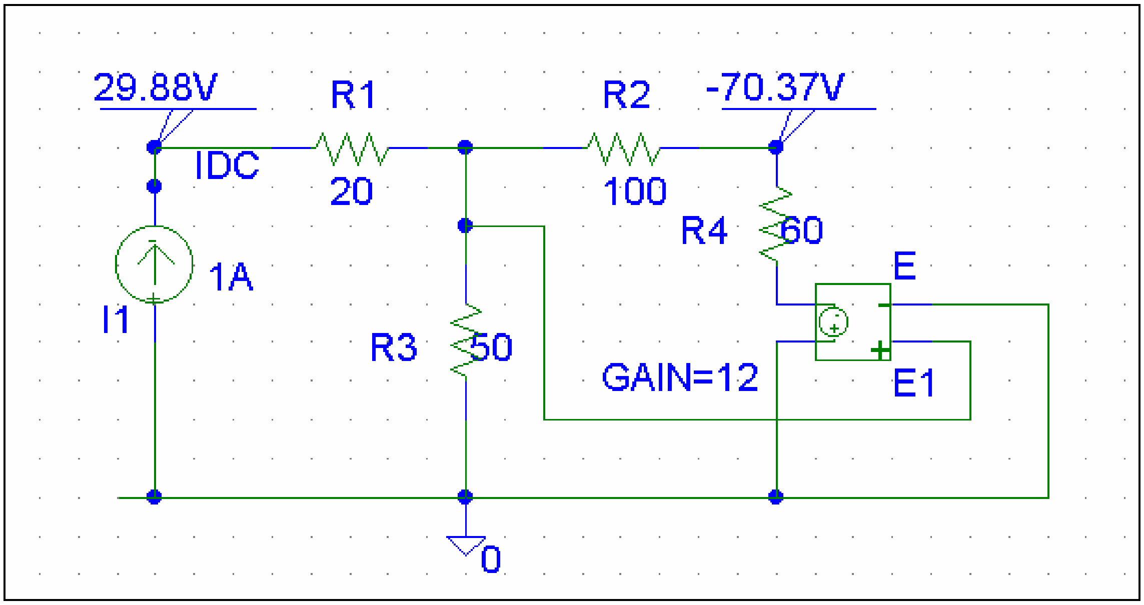
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

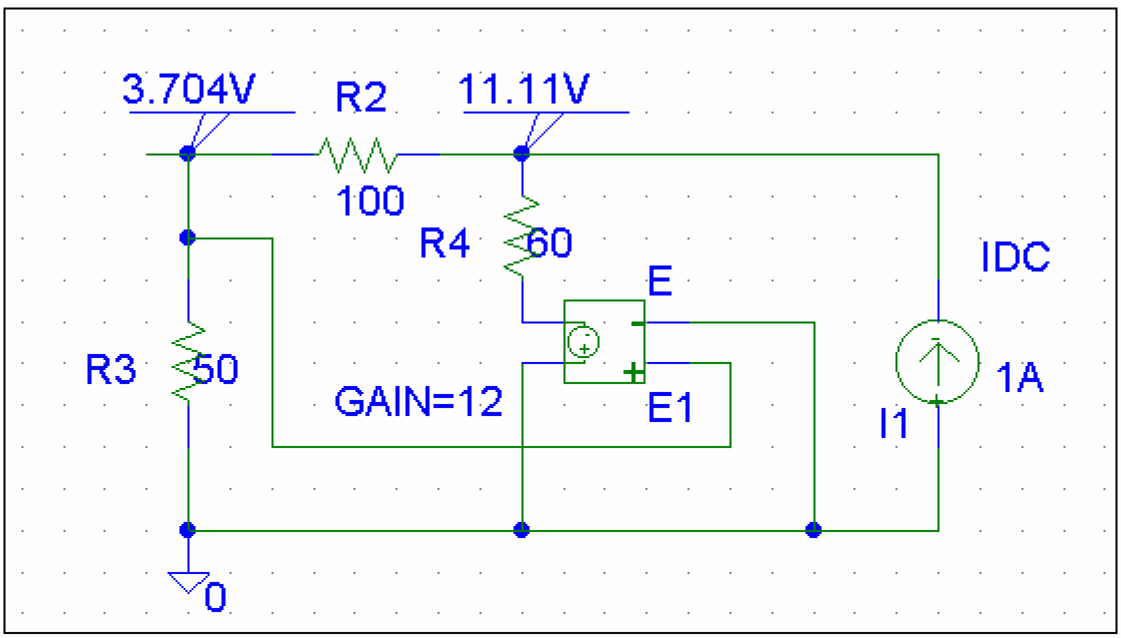
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 3.704, \qquad z_{22} = V_2 = 11.11$$

Thus,

$$\underline{[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix}\Omega}$$

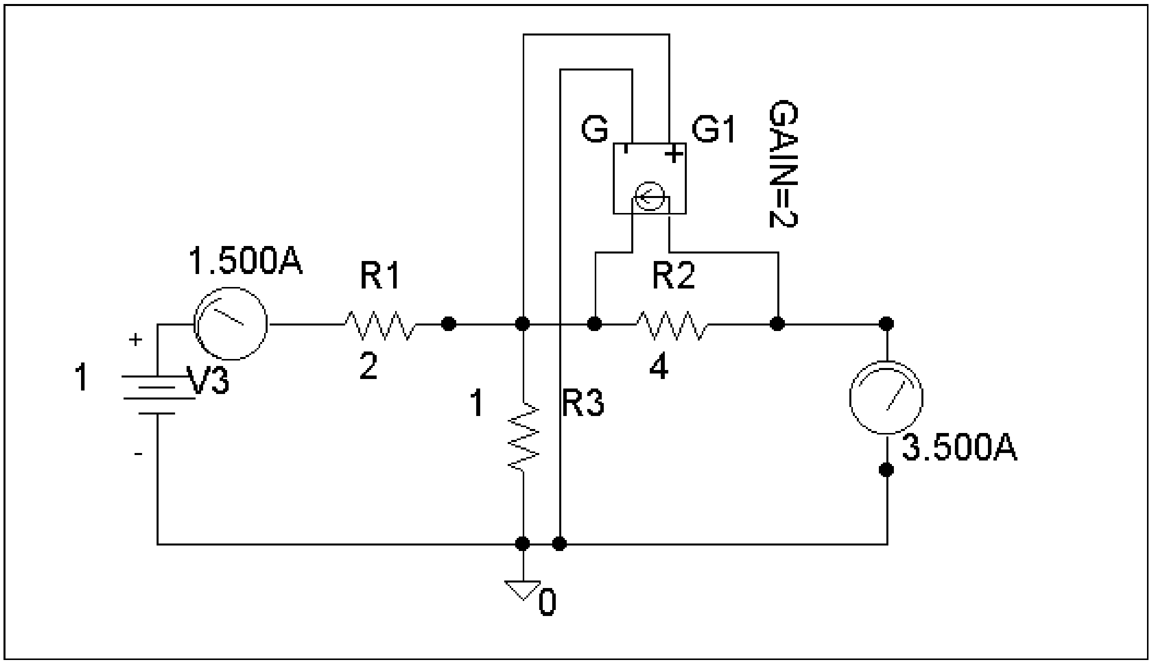




Chapter 19, Solution 81

(a) We set $V_1 = 1$ and short circuit the output port. The schematic is shown below. After simulation we obtain

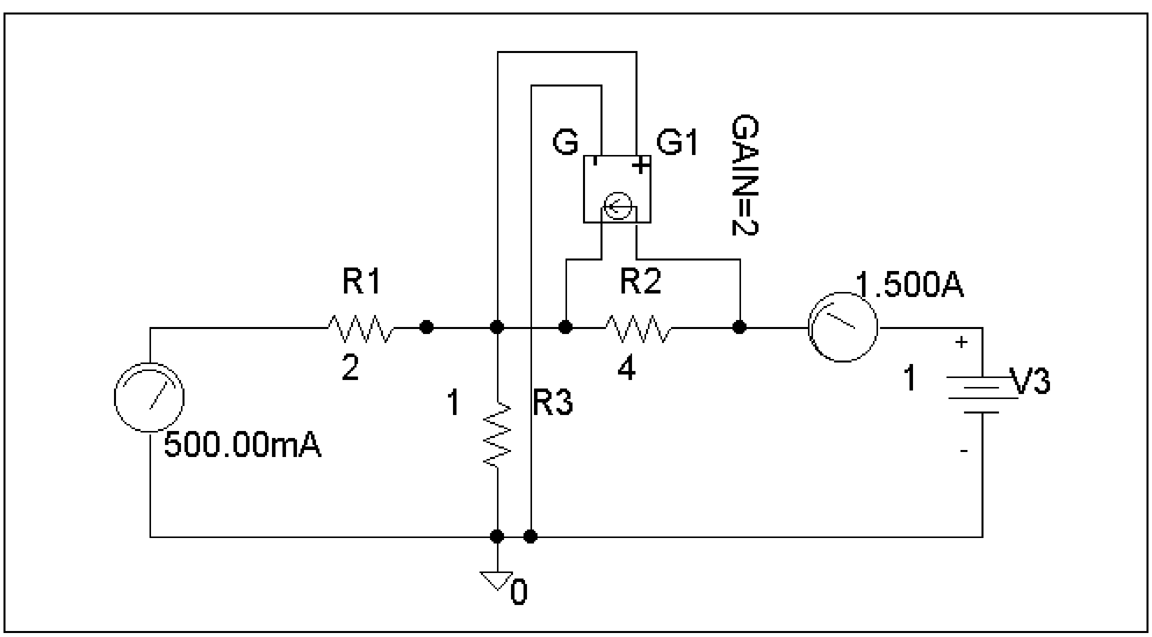
$$y_{11} = I_1 = 1.5, \ \ y_{21} = I_2 = 3.5$$



(b) We set $V_2 = 1$ and short-circuit the input port. The schematic is shown below. Upon simulating the circuit, we obtain

$$y_{12} = I_1 = -0.5, \; y_{22} = I_2 = 1.5$$

$$[Y] = \underline{\begin{bmatrix} \mathbf{1.5} & \mathbf{-0.5} \\ \mathbf{3.5} & \mathbf{1.5} \end{bmatrix}}$$

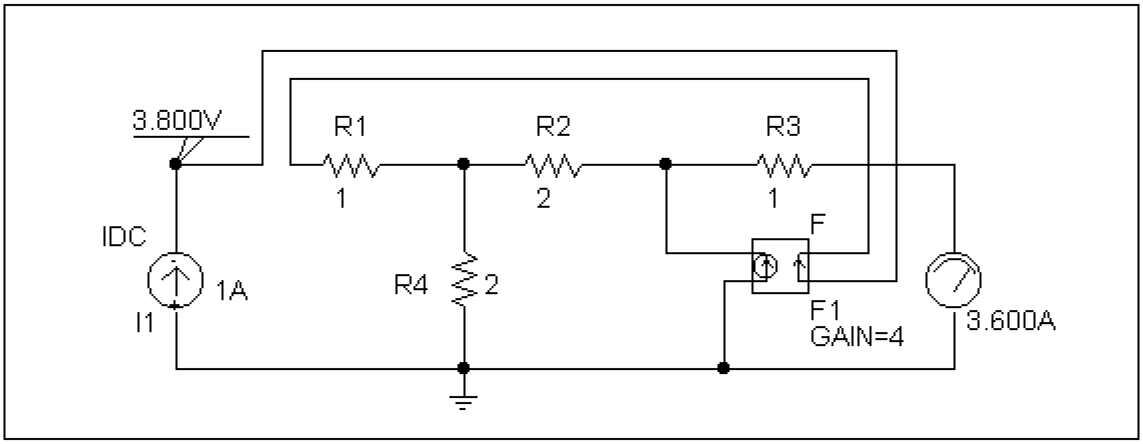


Chapter 19, Solution 82

We follow Example 19.15.

(a) Set $V_2 = 0$ and $I_1 = 1A$. The schematic is shown below. After simulation, we obtain

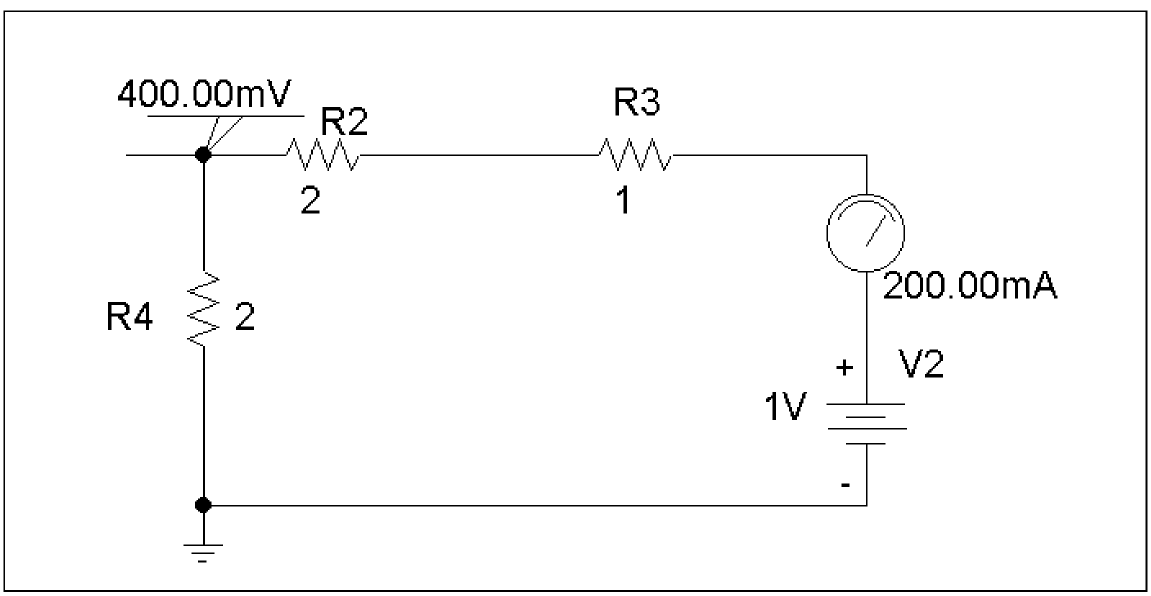
$$h_{11} = V_1/1 = 3.8, \quad h_{21} = I_2/1 = 3.6$$



(b) Set $V_1 = 1$ V and $I_1 = 0$. The schematic is shown below. After simulation, we obtain

$$h_{12} = V_1/1 = 0.4, \quad h_{22} = I_2/1 = 0.25$$

Hence, $$[h] = \underline{\begin{bmatrix} \mathbf{3.8} & \mathbf{0.4} \\ \mathbf{3.6} & \mathbf{0.25} \end{bmatrix}}$$



Chapter 19, Solution 83

To get A and C, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 11$ and $V_2 = 34$.

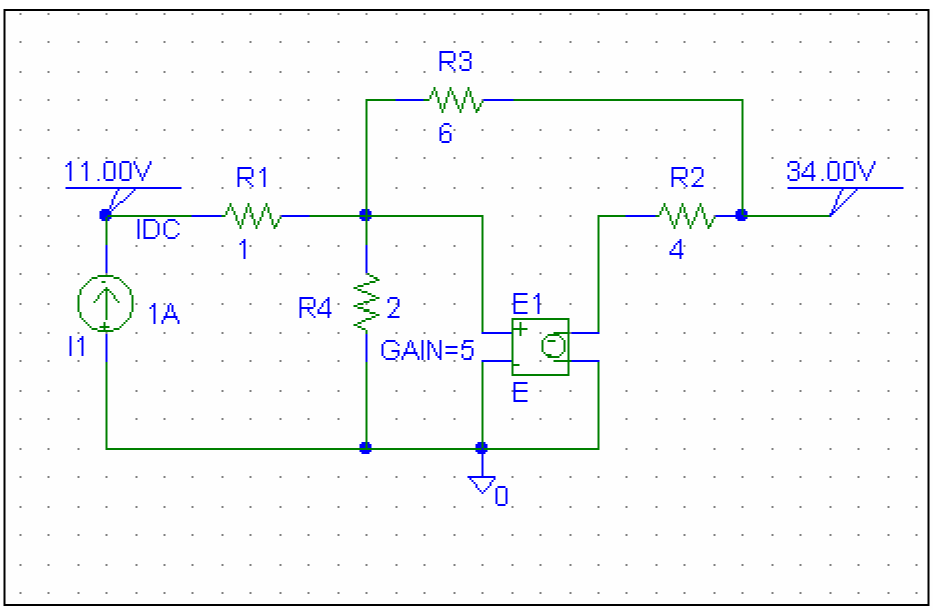
$$A = \frac{V_1}{V_2} = 0.3235, \qquad C = \frac{I_1}{V_2} = \frac{1}{34} = 0.02941$$

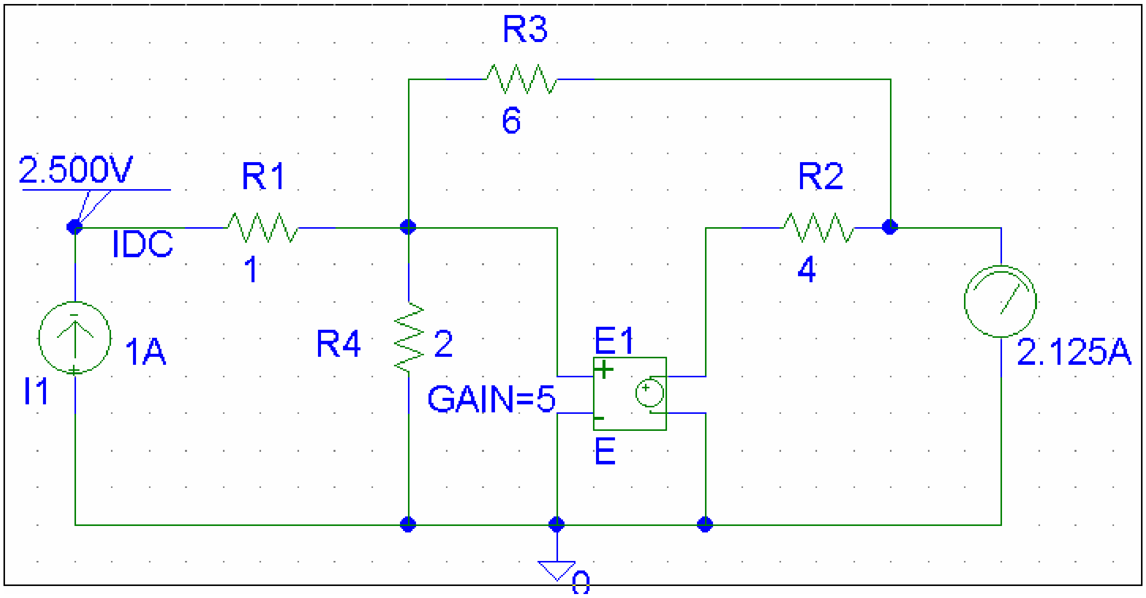
Similarly, to get B and D, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 2.5$ and $I_2 = -2.125$.

$$B = -\frac{V_1}{I_2} = \frac{2.5}{2.125} = 1.1765, \qquad D = -\frac{I_1}{I_2} = \frac{1}{2.125} = 0.4706$$

Thus,

$$\underline{[T] = \begin{bmatrix} 0.3235 & 1.1765 \\ 0.02941 & 0.4706 \end{bmatrix}}$$

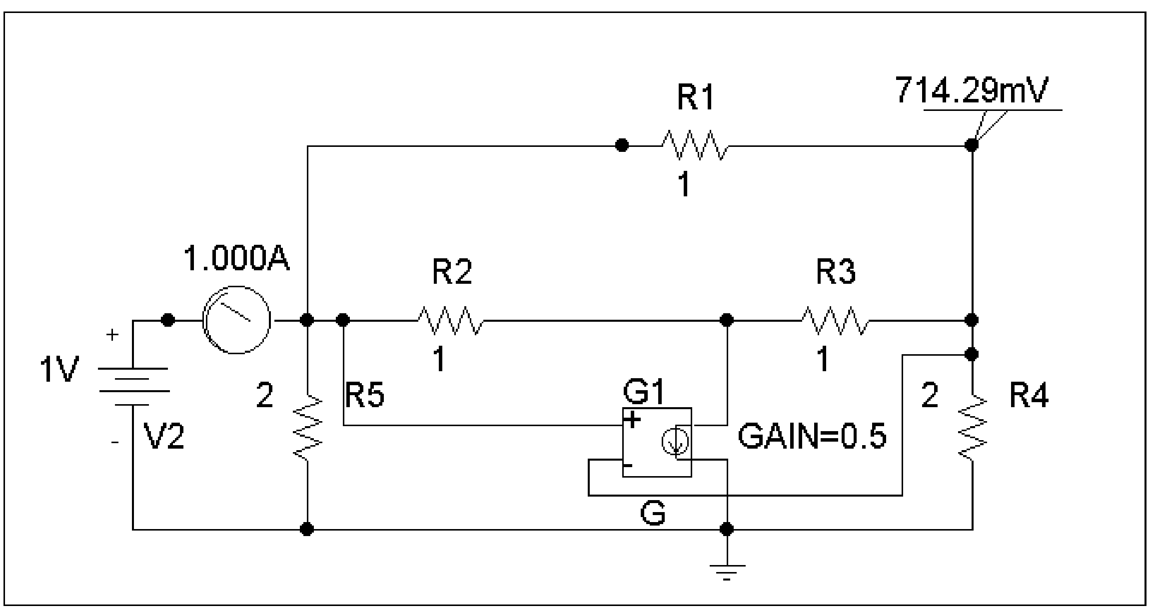




Chapter 19, Solution 84

(a) Since $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$, we open-circuit the output port and let $V_1 = 1$ V. The schematic is as shown below. After simulation, we obtain

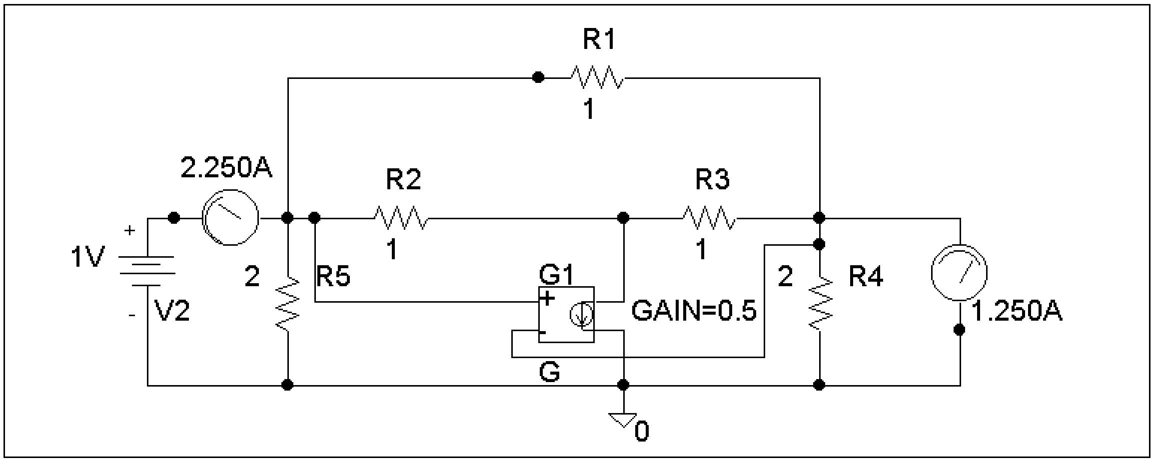
$$A = 1/V_2 = 1/0.7143 = 1.4$$
$$C = I_2/V_2 = 1.0/0.7143 = 1.4$$



(b) To get B and D, we short-circuit the output port and let $V_1 = 1$. The schematic is shown below. After simulating the circuit, we obtain

$$B = -V_1/I_2 = -1/1.25 = -0.8$$
$$D = -I_1/I_2 = -2.25/1.25 = -1.8$$

Thus
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.4} & \mathbf{-0.8} \\ \mathbf{1.4} & \mathbf{-1.8} \end{bmatrix}}$$



Chapter 19, Solution 85

(a) Since $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$, we let $V_1 = 1$ V and open-circuit the output port. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

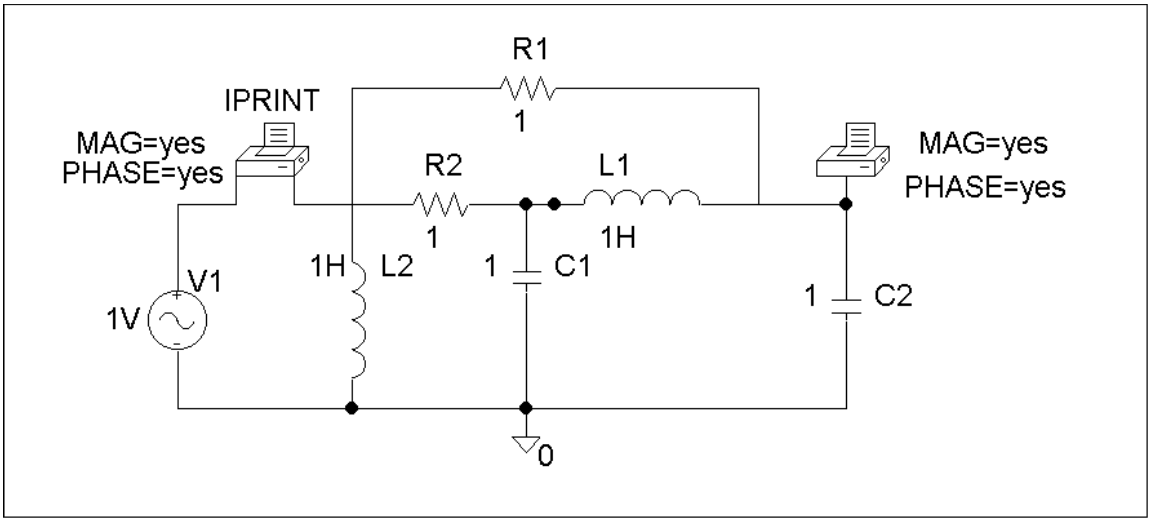
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	6.325 E–01	1.843 E+01

FREQ	VM($N_0002)	VP($N_0002)
1.592 E–01	6.325 E–01	–7.159 E+01

From this, we obtain

$$A = \frac{1}{V_2} = \frac{1}{0.6325\angle -71.59^\circ} = 1.581\angle 71.59^\circ$$

$$C = \frac{I_1}{V_2} = \frac{0.6325\angle 18.43°}{0.6325\angle -71.59°} = 1\angle 90° \;=\; j$$



(b) Similarly, since $B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$ and $D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$, we let $V_1 = 1$ V and short-circuit the output port. The schematic is shown below. Again, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. After simulation, we get an output file which includes the following results:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	5.661 E–04	8.997 E+01

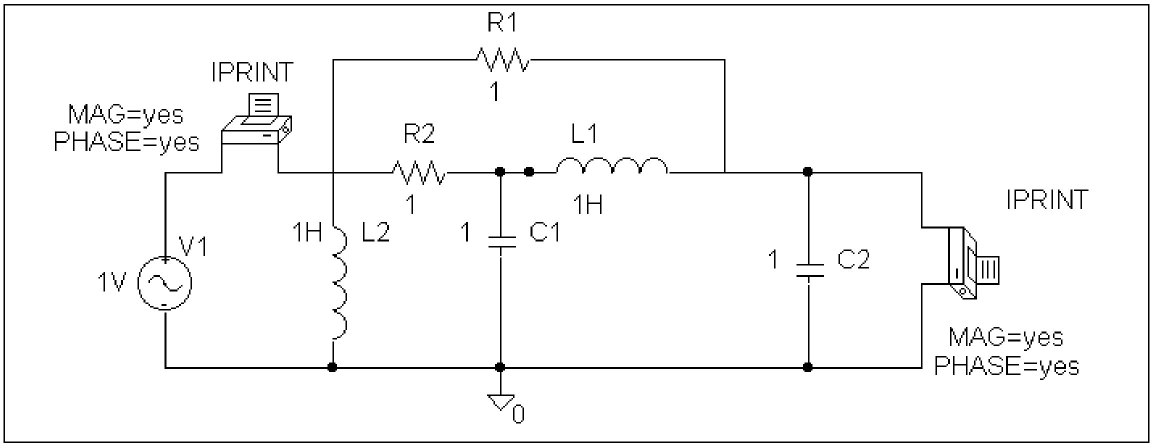
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	9.997 E–01	–9.003 E+01

From this,

$$B = -\frac{1}{I_2} = -\frac{1}{0.9997\angle -90°} = -1\angle 90° = -j$$

$$D = -\frac{I_1}{I_2} = -\frac{5.661\text{x}10^{-4}\angle 89.97°}{0.9997\angle -90°} \;=\; 5.561\text{x}10^{-4}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.581\angle 71.59°} & \mathbf{-j} \\ \mathbf{j} & \mathbf{5.661x10^{-4}} \end{bmatrix}}$$

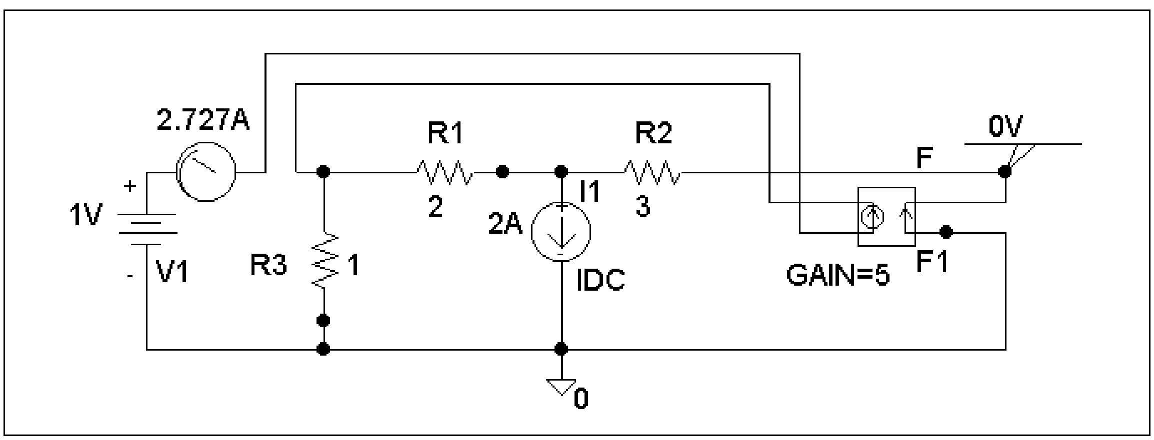


Chapter 19, Solution 86

(a) By definition, $g_{11} = \left.\dfrac{I_1}{V_1}\right|_{I_2=0}$, $g_{21} = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$.

We let $V_1 = 1$ V and open-circuit the output port. The schematic is shown below. After simulation, we obtain

$$g_{11} = I_1 = 2.7$$
$$g_{21} = V_2 = 0.0$$



(b) Similarly,

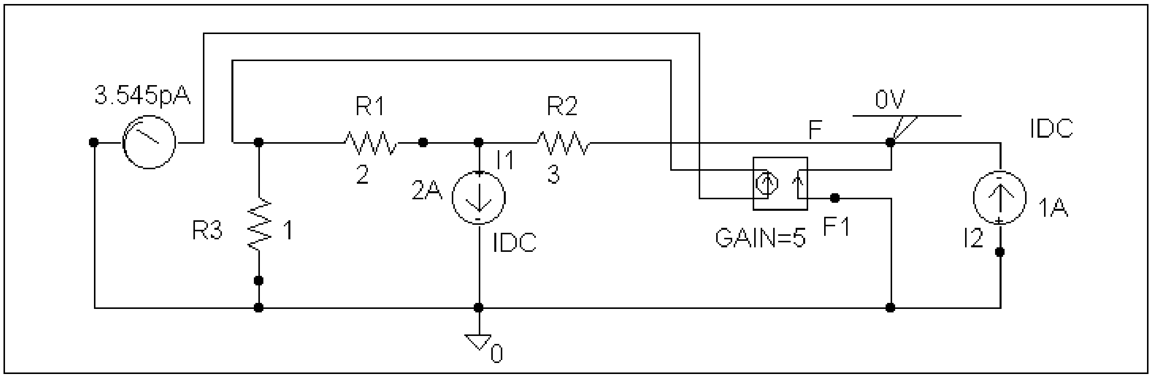
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}, \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

We let $I_2 = 1$ A and short-circuit the input port. The schematic is shown below. After simulation,

$$g_{12} = I_1 = 0$$

$$g_{22} = V_2 = 0$$

Thus $$[g] = \underline{\begin{bmatrix} \mathbf{2.727S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}$$



Chapter 19, Solution 87

(a) Since $$a = \left.\frac{V_2}{V_1}\right|_{I_1=0} \text{ and } c = \left.\frac{I_2}{V_1}\right|_{I_1=0},$$

we open-circuit the input port and let $V_2 = 1$ V. The schematic is shown below. In the AC Sweep box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

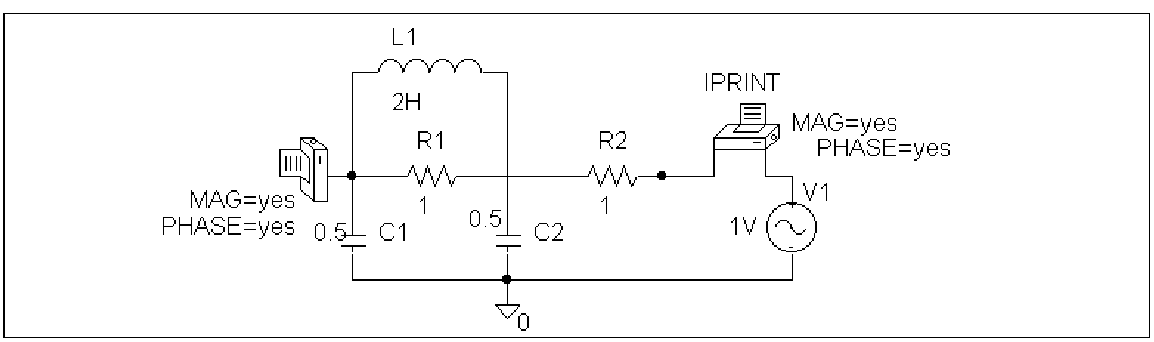
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	5.000 E–01	1.800 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	5.664 E–04	8.997 E+01

From this,

$$a = \frac{1}{5.664\text{x}10^{-4}\angle 89.97^\circ} = 1765\angle -89.97^\circ$$

$$c = \frac{0.5\angle 180^\circ}{5.664\text{x}10^{-4}\angle 89.97^\circ} = -882.28\angle -89.97^\circ$$



(b) Similarly,

$$b = -\left.\frac{V_2}{I_1}\right|_{V_1=0} \quad \text{and} \quad d = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

We short-circuit the input port and let $V_2 = 1$ V. The schematic is shown below. After simulation, we obtain an output file which includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	5.000 E–01	1.800 E+02

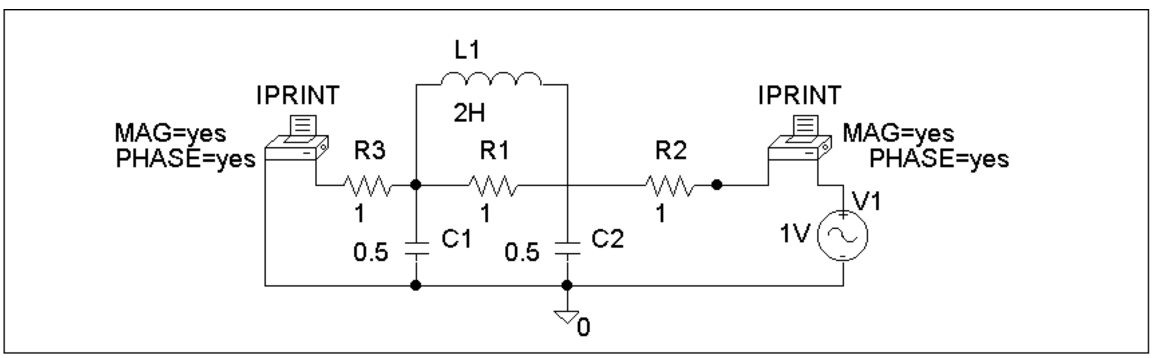
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	5.664 E–04	–9.010 E+01

From this, we get

$$b = -\frac{1}{5.664\text{x}10^{-4}\angle-90.1°} = -j1765$$

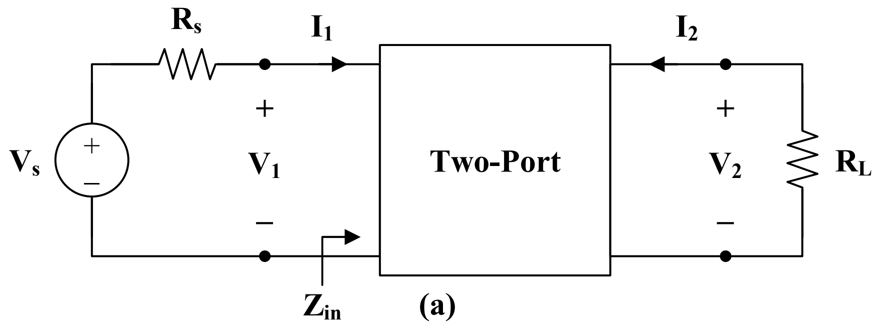
$$d = -\frac{0.5\angle 180°}{5.664\text{x}10^{-4}\angle-90.1°} = j888.28$$

Thus $$[t] = \underline{\begin{bmatrix} \mathbf{-j1765} & \mathbf{-j1765} \\ \mathbf{j888.2} & \mathbf{j888.2} \end{bmatrix}}$$



Chapter 19, Solution 88

To get Z_{in}, consider the network in Fig. (a).



(a)

$$\mathbf{I}_1 = \mathbf{y}_{11}\,\mathbf{V}_1 + \mathbf{y}_{12}\,\mathbf{V}_2 \qquad (1)$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2 \qquad (2)$$

But $$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{R_L} = \mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{-\mathbf{y}_{21}\,\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L} \qquad (3)$$

Substituting (3) into (1) yields

$$\mathbf{I}_1 = \mathbf{y}_{11}\,\mathbf{V}_1 + \mathbf{y}_{12}\cdot\left(\frac{-\mathbf{y}_{21}\,\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L}\right), \qquad \mathbf{Y}_L = \frac{1}{R_L}$$

$$\mathbf{I}_1 = \left(\frac{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)\mathbf{V}_1, \qquad \Delta_y = \mathbf{y}_{11}\,\mathbf{y}_{22} - \mathbf{y}_{12}\,\mathbf{y}_{21}$$

or $$Z_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}}$$

$$A_i = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2}{\mathbf{I}_1} = \mathbf{y}_{21}\,Z_{in} + \left(\frac{\mathbf{y}_{22}}{\mathbf{I}_1}\right)\left(\frac{-\mathbf{y}_{21}\,\mathbf{V}_1}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

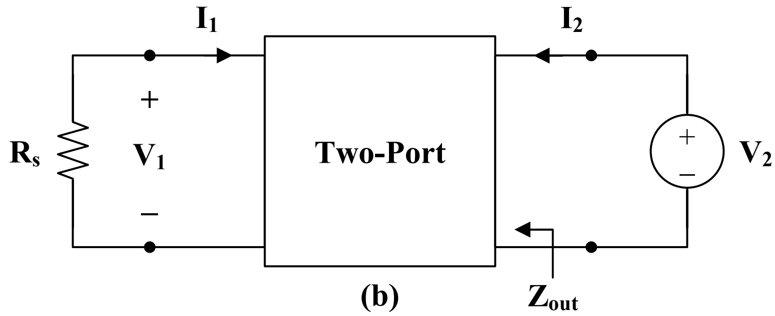
$$= \mathbf{y}_{21}\,Z_{in} - \frac{\mathbf{y}_{22}\,\mathbf{y}_{21}\,Z_{in}}{\mathbf{y}_{22} + \mathbf{Y}_L} = \left(\frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\,\mathbf{Y}_L}\right)\left(\mathbf{y}_{21} - \frac{\mathbf{y}_{22}\,\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

$$A_i = \underline{\frac{\mathbf{y}_{21}\,\mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\,\mathbf{Y}_L}}$$

From (3),

$$A_v = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \underline{\frac{-\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}}$$

To get Z_{out}, consider the circuit in Fig. (b).



(b)

$$Z_{out} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2} \tag{4}$$

But $\mathbf{V}_1 = -R_s\,\mathbf{I}_1$

Substituting this into (1) yields

$$\mathbf{I}_1 = -\mathbf{y}_{11}\,R_s\,\mathbf{I}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$

$$(1 + \mathbf{y}_{11}\,R_s)\,\mathbf{I}_1 = \mathbf{y}_{12}\,\mathbf{V}_2$$

$$\mathbf{I}_1 = \frac{\mathbf{y}_{12}\,\mathbf{V}_2}{1 + \mathbf{y}_{11}\,R_s} = \frac{-\mathbf{V}_1}{R_s}$$

or

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{-\mathbf{y}_{12}\,R_s}{1 + \mathbf{y}_{11}\,R_s}$$

Substituting this into (4) gives

$$Z_{out} = \frac{1}{\mathbf{y}_{22} - \dfrac{\mathbf{y}_{12}\,\mathbf{y}_{21}\,R_s}{1 + \mathbf{y}_{11}\,R_s}}$$

$$= \frac{1 + \mathbf{y}_{11}\,R_s}{\mathbf{y}_{22} + \mathbf{y}_{11}\,\mathbf{y}_{22}\,R_s - \mathbf{y}_{21}\,\mathbf{y}_{22}\,R_s}$$

$$\underline{Z_{out} = \frac{\mathbf{y}_{11} + \mathbf{Y}_s}{\Delta_y + \mathbf{y}_{22}\,\mathbf{Y}_s}}$$

Chapter 19, Solution 89

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + (2640 \times 16 \times 10^{-6} - 2.6 \times 10^{-4} \times 72) \cdot 10^5}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + 1824} = \underline{\mathbf{-1613}}$$

$$\text{dc gain} = 20 \log \left| A_v \right| = 20 \log (1613) = \underline{\mathbf{64.15}}$$

Chapter 19, Solution 90

(a) $$Z_{in} = h_{ie} - \frac{h_{re} h_{fe} R_L}{1 + h_{oe} R_L}$$

$$1500 = 2000 - \frac{10^{-4} \times 120 R_L}{1 + 20 \times 10^{-6} R_L}$$

$$500 = \frac{12 \times 10^{-3}}{1 + 2 \times 10^{-5} R_L}$$

$$500 + 10^{-2} R_L = 12 \times 10^{-3} R_L$$
$$500 \times 10^2 = 0.2 R_L$$
$$R_L = \underline{\mathbf{250\ k\Omega}}$$

(b) $$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-120 \times 250 \times 10^3}{2000 + (2000 \times 20 \times 10^{-6} - 120 \times 10^{-4}) \times 250 \times 10^3}$$

$$A_v = \frac{-30 \times 10^6}{2 \times 10^3 + 7 \times 10^3} = \underline{-3333}$$

$$A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{120}{1 + 20 \times 10^{-6} \times 250 \times 10^3} = \underline{\mathbf{20}}$$

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}} = \frac{600 + 2000}{(600 + 2000) \times 20 \times 10^{-6} - 10^{-4} \times 120}$$

$$Z_{out} = \frac{2600}{40} \text{ k}\Omega = \underline{\mathbf{65\ k\Omega}}$$

(c) $$A_v = \frac{\mathbf{V}_c}{\mathbf{V}_b} = \frac{\mathbf{V}_c}{\mathbf{V}_s} \longrightarrow \mathbf{V}_c = A_v \mathbf{V}_s = -3333 \times 4 \times 10^{-3} = \underline{\mathbf{-13.33\ V}}$$

Chapter 19, Solution 91

$$R_s = 1.2 \text{ k}\Omega, \qquad R_L = 4 \text{ k}\Omega$$

(a) $$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-80 \times 4 \times 10^3}{1200 + (1200 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80) \times 4 \times 10^3}$$

$$A_v = \frac{-32000}{1248} = \underline{\mathbf{-25.64}}$$

(b) $$A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{80}{1 + 20 \times 10^{-6} \times 4 \times 10^3} = \underline{\mathbf{74.074}}$$

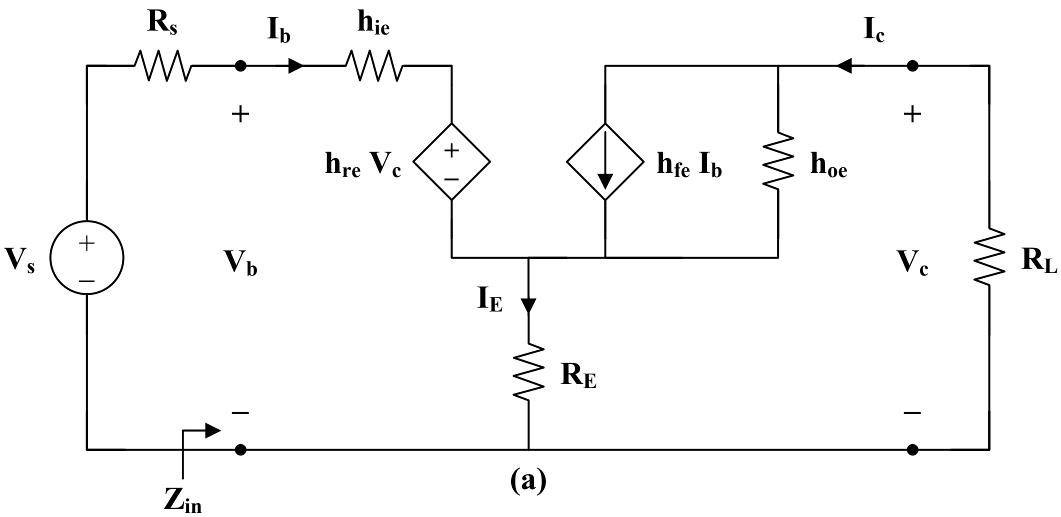
(c) $$Z_{in} = h_{ie} - h_{re} A_i$$
$$Z_{in} = 1200 - 1.5 \times 10^{-4} \times 74.074 \cong \underline{\mathbf{1.2\ k\Omega}}$$

(d) $$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{1200 + 1200}{2400 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80} = \frac{2400}{0.0468} = \underline{\mathbf{51.282\ k\Omega}}$$

Chapter 19, Solution 92

Due to the resistor $R_E = 240\,\Omega$, we cannot use the formulas in section 18.9.1. We will need to derive our own. Consider the circuit in Fig. (a).



(a)

$$\mathbf{I}_E = \mathbf{I}_b + \mathbf{I}_c \tag{1}$$

$$\mathbf{V}_b = h_{ie}\,\mathbf{I}_b + h_{re}\,\mathbf{V}_c + (\mathbf{I}_b + \mathbf{I}_c)R_E \tag{2}$$

$$\mathbf{I}_c = h_{fe}\,\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \tag{3}$$

But
$$\mathbf{V}_c = -\mathbf{I}_c R_L \tag{4}$$

Substituting (4) into (3),

$$\mathbf{I}_c = h_{fe}\,\mathbf{I}_b - \frac{R_L}{R_E + 1/h_{oe}}\mathbf{I}_c$$

or
$$A_i = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L} \tag{5}$$

$$A_i = \frac{100(1 + 240\text{x}30\text{x}10^{-6})}{1 + 30 \times 10^{-6}(4{,}000 + 240)}$$

$$A_i = \underline{\mathbf{79.18}}$$

From (3) and (5),

$$\mathbf{I}_c = \frac{h_{fe}(1+R_E)h_{oe}}{1+h_{oe}(R_L+R_E)}\mathbf{I}_b = h_{fe}\,\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \qquad (6)$$

Substituting (4) and (6) into (2),

$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + h_{re}\,\mathbf{V}_c + \mathbf{I}_c\,R_E$$

$$\mathbf{V}_b = \frac{\mathbf{V}_c\,(h_{ie}+R_E)}{\left(R_E+\dfrac{1}{h_{oe}}\right)\left[\dfrac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} - h_{fe}\right]} + h_{re}\,\mathbf{V}_c - \frac{\mathbf{V}_c}{R_L}R_E$$

$$\frac{1}{A_v} = \frac{\mathbf{V}_b}{\mathbf{V}_c} = \frac{(h_{ie}+R_E)}{\left(R_E+\dfrac{1}{h_{oe}}\right)\left[\dfrac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L} \qquad (7)$$

$$\frac{1}{A_v} = \frac{(4000+240)}{\left(240+\dfrac{1}{30\text{x}10^{-6}}\right)\left[\dfrac{100(1+240\text{x}30\text{x}10^{-6})}{1+30\times10^{-6}\times4240} - 100\right]} + 10^{-4} - \frac{240}{4000}$$

$$\frac{1}{A_v} = -6.06\text{x}10^{-3} + 10^{-4} - 0.06 = -0.066$$

$A_v =$ $\underline{\mathbf{-15.15}}$

From (5),

$$\mathbf{I}_c = \frac{h_{fe}}{1+h_{oe}R_L}\mathbf{I}_b$$

We substitute this with (4) into (2) to get

$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + (R_E - h_{re}R_L)\mathbf{I}_c$$

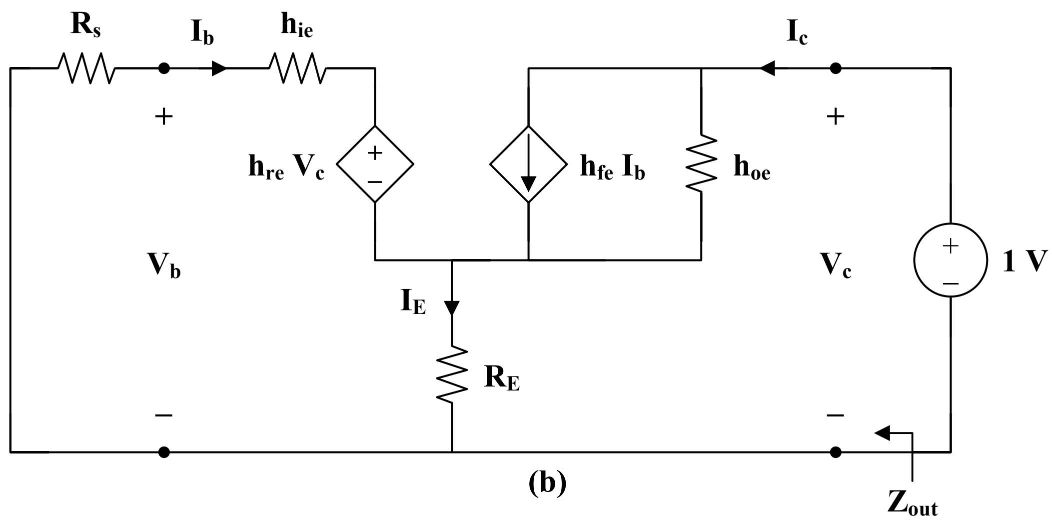
$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + (R_E - h_{re}R_L)\left(\frac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)}\mathbf{I}_b\right)$$

$$Z_{in} = \frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re}R_L)(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} \qquad (8)$$

$$Z_{in} = 4000 + 240 + \frac{(100)(240\times10^{-4}\times4\times10^{3})(1+240\text{x}30\text{x}10^{-6})}{1+30\times10^{-6}\times4240}$$

$Z_{in} =$ $\underline{\mathbf{12.818\ k\Omega}}$

To obtain Z_{out}, which is the same as the Thevenin impedance at the output, we introduce a 1-V source as shown in Fig. (b).



(b)

From the input loop,

$$\mathbf{I}_b(R_s + h_{ie}) + h_{re}\mathbf{V}_c + R_E(\mathbf{I}_b + \mathbf{I}_c) = 0$$

But $\quad \mathbf{V}_c = 1$

So,

$$\mathbf{I}_b(R_s + h_{ie} + R_E) + h_{re} + R_E\mathbf{I}_c = 0 \tag{9}$$

From the output loop,

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{R_E + \dfrac{1}{h_{oe}}} + h_{fe}\mathbf{I}_b = \frac{h_{oe}}{R_E h_{oe} + 1} + h_{fe}\mathbf{I}_b$$

or

$$\mathbf{I}_b = \frac{\mathbf{I}_c}{h_{fe}} - \frac{h_{oe}/h_{fe}}{1 + R_E h_{oe}} \tag{10}$$

Substituting (10) into (9) gives

$$(R_s + R_E + h_{ie})\left(\frac{\mathbf{I}_c}{h_{fe}}\right) + h_{re} + R_E\mathbf{I}_c - \frac{(R_s + R_E + h_{ie})\left(h_{oe}/h_{fe}\right)}{1 + R_E h_{oe}} = 0$$

$$\frac{R_s + R_E + h_{ie}}{h_{fe}}\mathbf{I}_c + R_E\mathbf{I}_c = \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\left(\frac{h_{oe}}{h_{fe}}\right) - h_{re}$$

$$\mathbf{I}_c = \frac{(h_{oe}/h_{fe})\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] - h_{re}}{R_E + (R_s + R_E + h_{ie})/h_{fe}}$$

$$Z_{out} = \frac{1}{\mathbf{I}_c} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{240 \times 100 + (1200 + 240 + 4000)}{\left[\dfrac{1200 + 240 + 4000}{1 + 240x30x10^{-6}}\right] \times 30 \times 10^{-6} - 10^{-4} \times 100}$$

$$Z_{out} = \frac{24000 + 5440}{0.152} = \underline{\mathbf{193.7\ k\Omega}}$$

Chapter 19, Solution 93

We apply the same formulas derived in the previous problem.

$$\frac{1}{A_v} = \frac{(h_{ie} + R_E)}{\left(R_E + \dfrac{1}{h_{oe}}\right)\left[\dfrac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L}$$

$$\frac{1}{A_v} = \frac{(2000 + 200)}{(200 + 10^5)\left[\dfrac{150(1 + 0.002)}{1 + 0.04} - 150\right]} + 2.5 \times 10^{-4} - \frac{200}{3800}$$

$$\frac{1}{A_v} = -0.004 + 2.5 \times 10^{-4} - 0.05263 = -0.05638$$

$$A_v = \underline{\mathbf{-17.74}}$$

$$A_i = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} = \frac{150(1 + 200x10^{-5})}{1 + 10^{-5} \times (200 + 3800)} = \underline{\mathbf{144.5}}$$

$$Z_{in} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re} R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

$$Z_{in} = 2000 + 200 + \frac{(150)(200 - 2.5\times10^{-4}\times3.8\times10^{3})(1.002)}{1.04}$$

$$Z_{in} = 2200 + 28966$$

$$Z_{in} = \underline{\mathbf{31.17\ k\Omega}}$$

$$Z_{out} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{200\times150 + 1000 + 200 + 2000}{\left[\dfrac{3200\times10^{-5}}{1.002}\right] - 2.5\times10^{-4}\times150} = \frac{33200}{-0.0055}$$

$$Z_{out} = \underline{\mathbf{-6.148\ M\Omega}}$$

Chapter 19, Solution 94

We first obtain the **ABCD** parameters.

Given $[\mathbf{h}] = \begin{bmatrix} 200 & 0 \\ 100 & 10^{-6} \end{bmatrix}$, $\Delta_h = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21} = 2\times10^{-4}$

$$[\mathbf{T}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}$$

The overall **ABCD** parameters for the amplifier are

$$[\mathbf{T}] = \begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}\begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \cong \begin{bmatrix} 2\times10^{-8} & 2\times10^{-2} \\ 10^{-10} & 10^{-4} \end{bmatrix}$$

$$\Delta_T = 2\times10^{-12} - 2\times10^{-12} = 0$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}} \\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} 200 & 0 \\ -10^{-4} & 10^{-6} \end{bmatrix}$$

Thus, $\quad h_{ie} = 200, \quad h_{re} = 0, \quad h_{fe} = -10^{-4}, \quad h_{oe} = 10^{-6}$

$$A_v = \frac{(10^4)(4\times10^3)}{200 + (2\times10^{-4} - 0)\times4\times10^3} = \underline{\mathbf{2\times10^5}}$$

$$Z_{in} = h_{ie} - \frac{h_{re}\,h_{fe}\,R_L}{1 + h_{oe}\,R_L} = 200 - 0 = \underline{\mathbf{200\ \Omega}}$$

Chapter 19, Solution 95

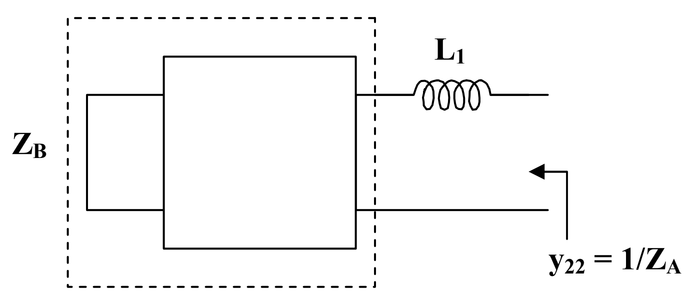
Let $\mathbf{Z}_A = \dfrac{1}{\mathbf{y}_{22}} = \dfrac{s^4 + 10s^2 + 8}{s^3 + 5s}$

Using long division,

$$\mathbf{Z}_A = s + \frac{5s^2 + 8}{s^3 + 5s} = s\,L_1 + \mathbf{Z}_B$$

i.e. $\quad L_1 = 1\text{ H} \quad$ and $\quad \mathbf{Z}_B = \dfrac{5s^2 + 8}{s^3 + 5s}$

as shown in Fig (a).



(a)

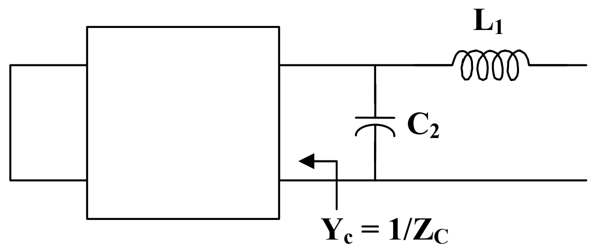
$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{s^3 + 5s}{5s^2 + 8}$$

Using long division,

$$\mathbf{Y}_B = 0.2s + \frac{3.4s}{5s^2 + 8} = sC_2 + \mathbf{Y}_C$$

where $\quad C_2 = 0.2\text{ F} \quad$ and $\quad \mathbf{Y}_C = \dfrac{3.4s}{5s^2 + 8}$

as shown in Fig. (b).



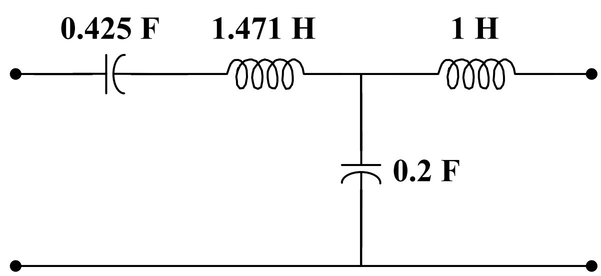
(b)

$$\mathbf{Z}_C = \frac{1}{\mathbf{Y}_C} = \frac{5s^2 + 8}{3.4s} = \frac{5s}{3.4} + \frac{8}{3.4s} = sL_3 + \frac{1}{sC_4}$$

i.e. an inductor in series with a capacitor

$$L_3 = \frac{5}{3.4} = 1.471\text{ H} \quad \text{and} \quad C_4 = \frac{3.4}{8} = 0.425\text{ F}$$

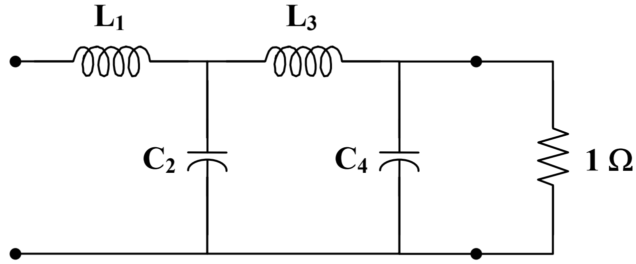
Thus, **<u>the LC network is shown in Fig. (c)</u>**.



(c)

Chapter 19, Solution 96

This is a fourth order network which can be realized with the network shown in Fig. (a).



(a)

$$\Delta(s) = (s^4 + 3.414s^2 + 1) + (2.613s^3 + 2.613s)$$

$$H(s) = \frac{\dfrac{1}{2.613s^3 + 2.613s}}{1 + \dfrac{s^4 + 3.414s^2 + 1}{2.613s^3 + 2.613s}}$$

which indicates that

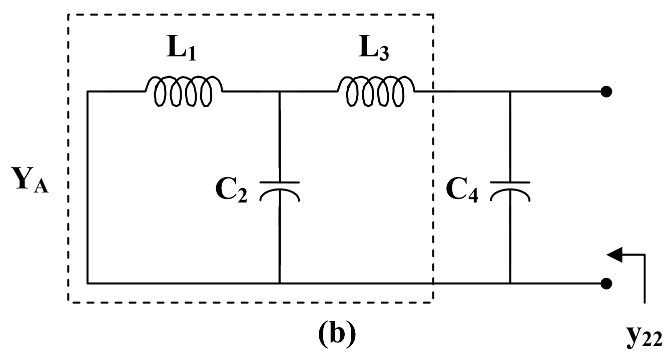
$$\mathbf{y}_{21} = \frac{-1}{2.613s^3 + 2.613s}$$
$$\mathbf{y}_{22} = \frac{s^4 + 3.414s + 1}{2.613s^3 + 2.613s}$$

We seek to realize $\mathbf{y}_{22}$. By long division,

$$\mathbf{y}_{22} = 0.383s + \frac{2.414s^2 + 1}{2.613s^3 + 2.613s} = sC_4 + \mathbf{Y}_A$$

i.e. $C_4 = 0.383$ F and $\mathbf{Y}_A = \dfrac{2.414s^2 + 1}{2.613s^3 + 2.613s}$

as shown in Fig. (b).



(b)

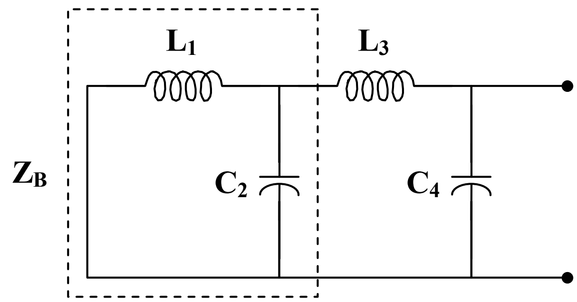
$$\mathbf{Z}_A = \frac{1}{\mathbf{Y}_A} = \frac{2.613s^3 + 2.613s}{2.414s^2 + 1}$$

By long division,

$$\mathbf{Z}_A = 1.082s + \frac{1.531s}{2.414s^2 + 1} = sL_3 + \mathbf{Z}_B$$

i.e. $L_3 = 1.082$ H and $\mathbf{Z}_B = \dfrac{1.531s}{2.414s^2 + 1}$

as shown in Fig.(c).

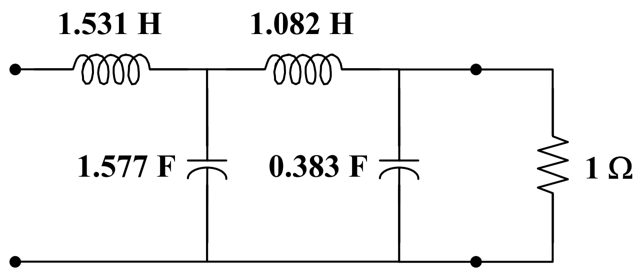


(c)

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = 1.577s + \frac{1}{1.531s} = sC_2 + \frac{1}{sL_1}$$

i.e. $C_2 = 1.577\text{ F}$ and $L_1 = 1.531\text{ H}$

Thus, **the network is shown in Fig. (d)**.



(d)

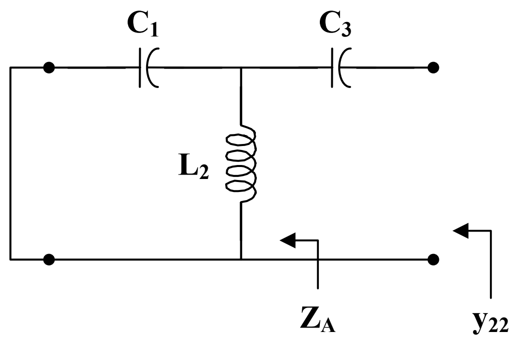
Chapter 19, Solution 97

$$H(s) = \frac{s^3}{(s^3 + 12s) + (6s^2 + 24)} = \frac{\dfrac{s^3}{s^3 + 12s}}{1 + \dfrac{6s^2 + 24}{s^3 + 12s}}$$

Hence,

$$\mathbf{y}_{22} = \frac{6s^2 + 24}{s^3 + 12s} = \frac{1}{sC_3} + \mathbf{Z}_A \qquad (1)$$

where $\mathbf{Z}_A$ is shown in the figure below.



We now obtain C_3 and $\mathbf{Z}_A$ using partial fraction expansion.

Let $$\frac{6s^2+24}{s(s^2+12)} = \frac{A}{s} + \frac{Bs+C}{s^2+12}$$

$$6s^2+24 = A(s^2+12) + Bs^2 + Cs$$

Equating coefficients :

s^0 : $24 = 12A \longrightarrow A = 2$

s^1 : $0 = C$

s^2 : $6 = A + B \longrightarrow B = 4$

Thus,

$$\frac{6s^2+24}{s(s^2+12)} = \frac{2}{s} + \frac{4s}{s^2+12} \qquad (2)$$

Comparing (1) and (2),

$$C_3 = \frac{1}{A} = \frac{1}{2}\ \text{F}$$

$$\frac{1}{\mathbf{Z}_A} = \frac{s^2+12}{4s} = \frac{1}{4}s + \frac{3}{s} \qquad (3)$$

But $$\frac{1}{\mathbf{Z}_A} = sC_1 + \frac{1}{sL_2} \qquad (4)$$

Comparing (3) and (4),

$$C_1 = \frac{1}{4}\ \text{F} \quad \text{and} \quad L_2 = \frac{1}{3}\ \text{H}$$

Therefore,

$C_1 = \underline{\mathbf{0.25\ F}}$, $L_2 = \underline{\mathbf{0.3333\ H}}$, $C_3 = \underline{\mathbf{0.5\ F}}$

Chapter 19, Solution 98

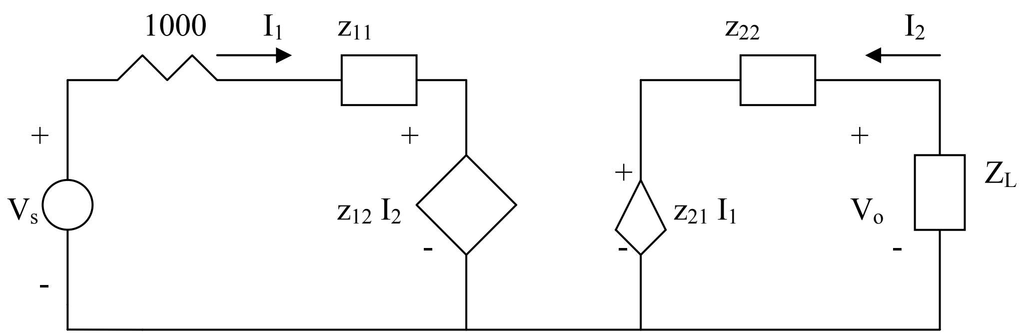
$$\Delta_h = 1 - 0.8 = 0.2$$

$$[T_a] = [T_b] = \begin{bmatrix} -\Delta_h / h_{21} & -h_{11}/h_{21} \\ -h_{22}/h_{21} & -1/h_{21} \end{bmatrix} = \begin{bmatrix} -0.001 & -10 \\ -2.5x10^{-6} & -0.005 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} 2.6x10^{-5} & 0.06 \\ 1.5x10^{-8} & 5x10^{-5} \end{bmatrix}$$

We now convert this to z-parameters

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 1.733x10^3 & 0.0267 \\ 6.667x10^7 & 3.33x10^3 \end{bmatrix}$$



$$V_s = (1000 + z_{11})I_1 + z_{12}I_2 \tag{1}$$

$$V_o = z_{22}I_2 + z_{21}I_1 \tag{2}$$

But $V_o = -I_2 Z_L \quad \longrightarrow \quad I_2 = -V_o / Z_L$ (3)

Substituting (3) into (2) gives

$$I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{z_{21}Z_L}\right) \tag{4}$$

We substitute (3) and (4) into (1)

$$V_s = (1000 + z_{11})\left(\frac{1}{z_{11}} + \frac{z_{22}}{z_{21}Z_L}\right)V_o - \frac{z_{12}}{Z_L}V_o$$

$$= 7.653x10^{-4} - 2.136x10^{-5} = \underline{744\mu V}$$

Chapter 19, Solution 99

$$\mathbf{Z}_{ab} = \mathbf{Z}_1 + \mathbf{Z}_3 = \mathbf{Z}_c \parallel (\mathbf{Z}_b + \mathbf{Z}_a)$$

$$\mathbf{Z}_1 + \mathbf{Z}_3 = \frac{\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (1)$$

$$\mathbf{Z}_{cd} = \mathbf{Z}_2 + \mathbf{Z}_3 = \mathbf{Z}_a \parallel (\mathbf{Z}_b + \mathbf{Z}_c)$$

$$\mathbf{Z}_2 + \mathbf{Z}_3 = \frac{\mathbf{Z}_a(\mathbf{Z}_b + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (2)$$

$$\mathbf{Z}_{ac} = \mathbf{Z}_1 + \mathbf{Z}_2 = \mathbf{Z}_b \parallel (\mathbf{Z}_a + \mathbf{Z}_c)$$

$$\mathbf{Z}_1 + \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_a + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (3)$$

Subtracting (2) from (1),

$$\mathbf{Z}_1 - \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_c - \mathbf{Z}_a)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (4)$$

Adding (3) and (4),

$$\mathbf{Z}_1 = \underline{\frac{\mathbf{Z_b Z_c}}{\mathbf{Z_a + Z_b + Z_c}}} \qquad (5)$$

Subtracting (5) from (3),

$$\mathbf{Z}_2 = \underline{\frac{\mathbf{Z_a Z_b}}{\mathbf{Z_a + Z_b + Z_c}}} \qquad (6)$$

Subtracting (5) from (1),

$$\mathbf{Z}_3 = \underline{\frac{\mathbf{Z_c Z_a}}{\mathbf{Z_a + Z_b + Z_c}}} \qquad (7)$$

Using (5) to (7)

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)}{(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)^2}$$

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (8)$$

Dividing (8) by each of (5), (6), and (7),

$$\underline{\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}}$$

$$\underline{\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}}$$

$$\underline{\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}}$$

as required. Note that the formulas above are not exactly the same as those in Chapter 9 because the locations of $\mathbf{Z}_b$ and $\mathbf{Z}_c$ are interchanged in Fig. 18.122.