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SOLUTIONS MANUAL

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FUNDAMENTALS OF APPLIED ELECTROMAGNETICS

Chapter 1

Section 1-3: Traveling Waves

Problem 1.1 A 4-kHz sound wave traveling in the x-direction in air was observed to have a differential pressure $p(x,t) = 5$ N/m^2 at $x = 0$ and $t = 25$ μs. If the reference phase of $p(x,t)$ is 42°, find a complete expression for $p(x,t)$. The velocity of sound in air is 330 m/s.

Solution: The general form is given by Eq. (1.17),

$$p(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right),$$

where it is given that $\phi_0 = 42°$. From Eq. (1.26), $T = 1/f = 1/(4\times10^3) = 0.25$ ms. From Eq. (1.27),

$$\lambda = \frac{u_p}{f} = \frac{330}{4\times10^3} = 82.5\times10^{-3} \text{ m}.$$

Also, since

$$p(x=0,\ t=25\ \mu\text{s}) = 5\ (\text{N/m}^2) = A\cos\left(\frac{2\pi\times25\times10^{-6}}{2.5\times10^{-4}} + 42°\frac{\pi \text{ rad}}{180°}\right)$$
$$= A\cos(1.36 \text{ rad}) = 0.208A,$$

it follows that $A = 5/0.208 = 24$ N/m^2. So, with t in (s) and x in (m),

$$p(x,t) = 24\cos\left(2\pi\times10^6\frac{t}{250} - 2\pi\times10^3\frac{x}{82.5} + 42°\right) \quad (\text{N/m}^2)$$
$$= 24\cos(8\pi\times10^3 t - 24.24\pi x + 42°) \quad (\text{N/m}^2).$$

Problem 1.2 For the pressure wave described in Example 1-1, plot
(a) $p(x,t)$ versus x at $t = 0$,
(b) $p(x,t)$ versus t at $x = 0$.
Be sure to use appropriate scales for x and t so that each of your plots covers at least two cycles.

Solution: Refer to Fig. P1.2(a) and Fig. P1.2(b).

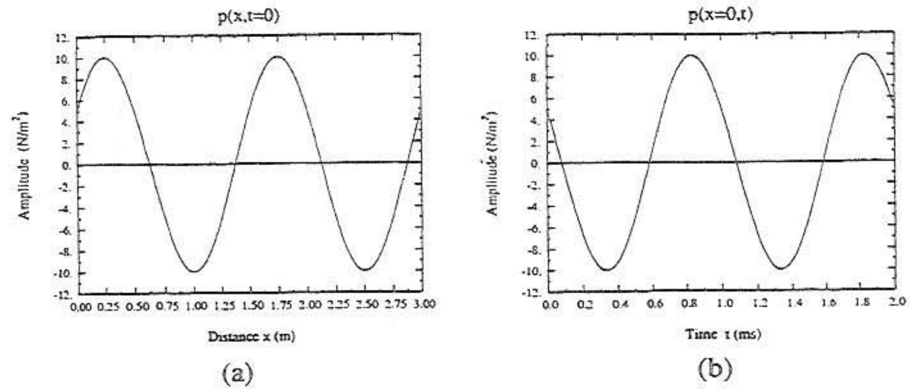

Figure P1.2: (a) Pressure wave as a function of distance at $t = 0$ and (b) pressure wave as a function of time at $x = 0$.

Problem 1.3 A harmonic wave traveling along a string is generated by an oscillator that completes 120 vibrations per minute. If it is observed that a given crest, or maximum, travels 250 cm in 10 s, what is the wavelength?

Solution:

$$f = \frac{120}{60} = 2 \text{ Hz}.$$
$$u_{\text{p}} = \frac{250 \text{ cm}}{10 \text{ s}} = 0.25 \text{ m/s}.$$
$$\lambda = \frac{u_{\text{p}}}{f} = \frac{0.25}{2} = 0.125 \text{ m} = 12.5 \text{ cm}.$$

Problem 1.4 Two waves, $y_1(t)$ and $y_2(t)$, have identical amplitudes and oscillate at the same frequency, but $y_2(t)$ leads $y_1(t)$ by a phase angle of 60°. If

$$y_1(t) = 4\cos(2\pi \times 10^3 t),$$

write down the expression appropriate for $y_2(t)$ and plot both functions over the time span from 0 to 2 ms.

Solution:

$$y_2(t) = 4\cos(2\pi \times 10^3 t + 60°).$$

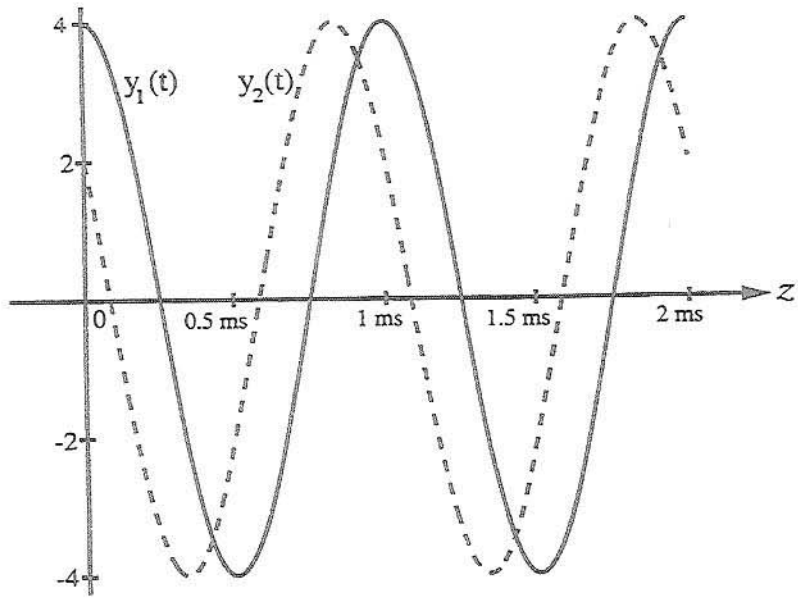

Figure P1.4: Plots of $y_1(t)$ and $y_2(t)$.

Problem 1.5 The height of an ocean wave is described by the function

$$y(x,t) = 1.5\sin(0.5t - 0.6x) \quad \text{(m)}.$$

Determine the phase velocity and the wavelength and then sketch $y(x,t)$ at $t = 2$ s over the range from $x = 0$ to $x = 2\lambda$.

Solution: The given wave may be rewritten as a cosine function:

$$y(x,t) = 1.5\cos(0.5t - 0.6x - \pi/2).$$

By comparison of this wave with Eq. (1.32),

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0),$$

we deduce that

$$\omega = 2\pi f = 0.5 \text{ rad/s}, \qquad \beta = \frac{2\pi}{\lambda} = 0.6 \text{ rad/m},$$

$$u_p = \frac{\omega}{\beta} = \frac{0.5}{0.6} = 0.83 \text{ m/s}, \qquad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.6} = 10.47 \text{ m}.$$

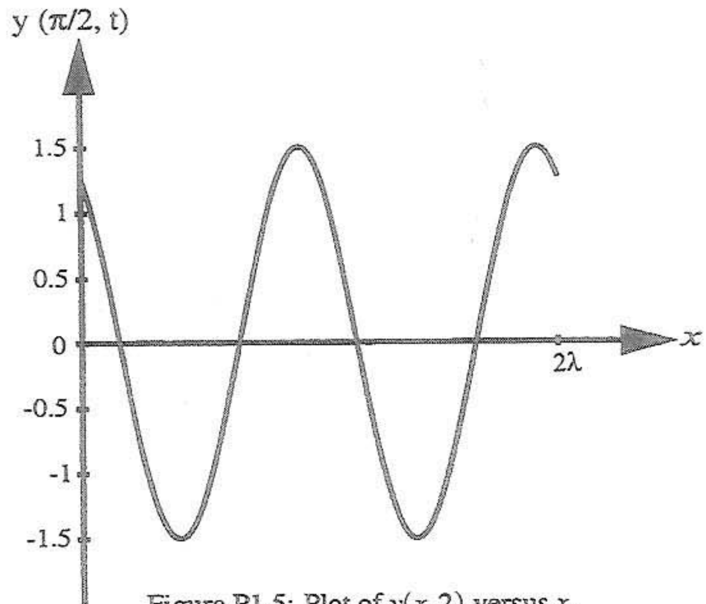

Figure P1.5: Plot of $y(x,2)$ versus x.

At $t = 2$ s, $y(x,2) = 1.5\sin(1-0.6x)$ (m), with the argument of the cosine function given in radians. Plot is shown in Fig. P1.5.

Problem 1.6 A wave traveling along a string in the $+x$-direction is given by

$$y_1(x,t) = A\cos(\omega t - \beta x),$$

where $x = 0$ is the end of the string, which is tied rigidly to a wall, as shown in Fig. 1-21 (P1.6). When wave $y_1(x,t)$ arrives at the wall, a reflected wave $y_2(x,t)$ is generated. Hence, at any location on the string, the vertical displacement y_s will be the sum of the incident and reflected waves:

$$y_s(x,t) = y_1(x,t) + y_2(x,t).$$

(a) Write down an expression for $y_2(x,t)$, keeping in mind its direction of travel and the fact that the end of the string cannot move.

(b) Generate plots of $y_1(x,t)$, $y_2(x,t)$ and $y_s(x,t)$ versus x over the range $-2\lambda \le x \le 0$ at $\omega t = \pi/4$ and at $\omega t = \pi/2$.

Solution:

(a) Since wave $y_2(x,t)$ was caused by wave $y_1(x,t)$, the two waves must have the same angular frequency ω, and since $y_2(x,t)$ is traveling on the same string as $y_1(x,t)$,

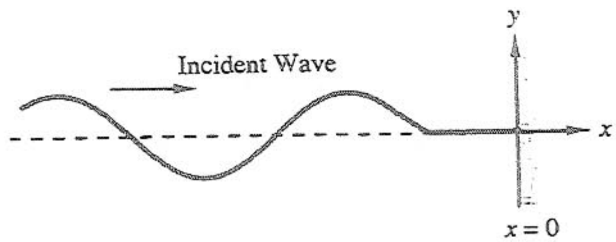

Figure P1.6: Wave on a string tied to a wall at $x = 0$ (Problem 1.6).

the two waves must have the same phase constant β. Hence, with its direction being in the negative x-direction, $y_2(x,t)$ is given by the general form

$$y_2(x,t) = B\cos(\omega t + \beta x + \phi_0), \quad (1)$$

where B and ϕ_0 are yet-to-be-determined constants. The total displacement is

$$y_s(x,t) = y_1(x,t) + y_2(x,t) = A\cos(\omega t - \beta x) + B\cos(\omega t + \beta x + \phi_0).$$

Since the string cannot move at $x = 0$, the point at which it is attached to the wall, $y_s(0,t) = 0$ for all t. Thus,

$$y_s(0,t) = A\cos\omega t + B\cos(\omega t + \phi_0) = 0. \quad (2)$$

(i) Easy Solution: The physics of the problem suggests that a possible solution for (2) is $B = -A$ and $\phi_0 = 0$, in which case we have

$$y_2(x,t) = -A\cos(\omega t + \beta x). \quad (3)$$

(ii) Rigorous Solution: By expanding the second term in (2), we have

$$A\cos\omega t + B(\cos\omega t\cos\phi_0 - \sin\omega t\sin\phi_0) = 0,$$

or

$$(A + B\cos\phi_0)\cos\omega t - (B\sin\phi_0)\sin\omega t = 0. \quad (4)$$

This equation has to be satisfied for all values of t. At $t = 0$, it gives

$$A + B\cos\phi_0 = 0, \quad (5)$$

and at $\omega t = \pi/2$, (4) gives

$$B \sin\phi_0 = 0. \tag{6}$$

Equations (5) and (6) can be satisfied simultaneously only if

$$A = B = 0 \tag{7}$$

or

$$A = -B \quad \text{and} \quad \phi_0 = 0. \tag{8}$$

Clearly (7) is not an acceptable solution because it means that $y_1(x,t) = 0$, which is contrary to the statement of the problem. The solution given by (8) leads to (3).

(b) At $\omega t = \pi/4$,

$$y_1(x,t) = A\cos(\pi/4 - \beta x) = A\cos\left(\frac{\pi}{4} - \frac{2\pi x}{\lambda}\right),$$

$$y_2(x,t) = -A\cos(\omega t + \beta x) = -A\cos\left(\frac{\pi}{4} + \frac{2\pi x}{\lambda}\right).$$

Plots of y_1, y_2, and y_3 are shown in Fig. P1.6(b).

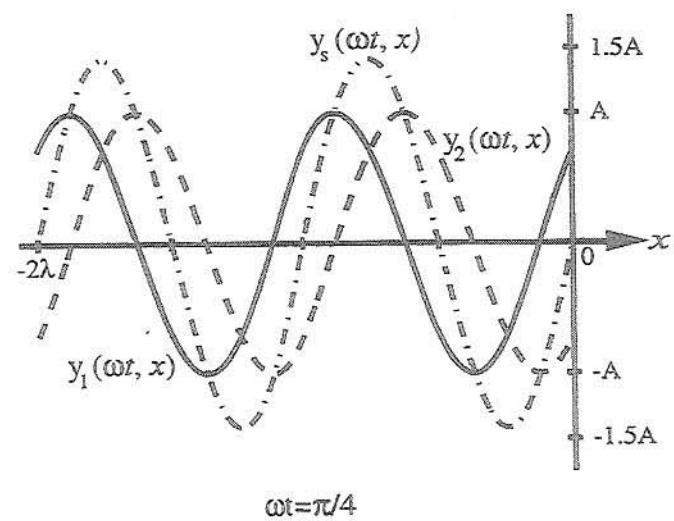

Figure P1.6: (b) Plots of y_1, y_2, and y_s versus x at $\omega t = \pi/4$.

At $\omega t = \pi/2$,

$$y_1(x,t) = A\cos(\pi/2 - \beta x) = A\sin\beta x = A\sin\frac{2\pi x}{\lambda},$$

$$y_2(x,t) = -A\cos(\pi/2+\beta x) = A\sin\beta x = A\sin\frac{2\pi x}{\lambda}.$$

Plots of y_1, y_2, and y_3 are shown in Fig. P1.6(c).

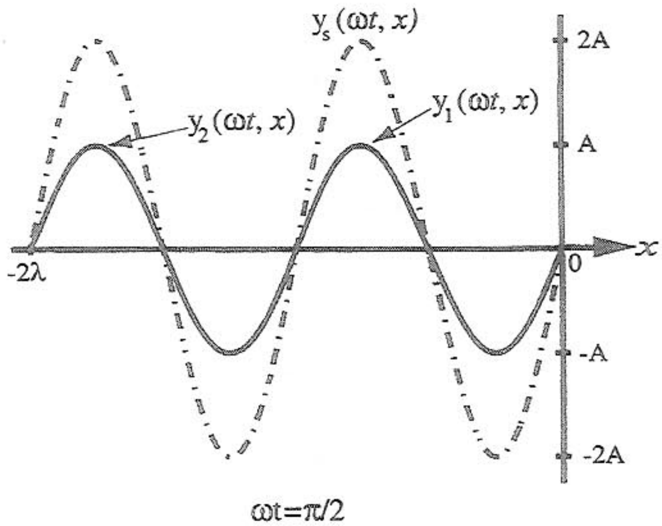

Figure P1.6: (c) Plots of y_1, y_2, and y_s versus x at $\omega t = \pi/2$.

Problem 1.7 Two waves on a string are given by the following functions:

$$y_1(x,t) = 3\cos(20t - 30x) \quad \text{(cm)},$$
$$y_2(x,t) = -3\cos(20t + 30x) \quad \text{(cm)},$$

where x is in centimeters. The waves are said to interfere constructively when their superposition $|y_s| = |y_1 + y_2|$ is a maximum and they interfere destructively when $|y_s|$ is a minimum.

(a) What are the directions of propagation of waves $y_1(x,t)$ and $y_2(x,t)$?

(b) At $t = (\pi/50)$ s, at what location x do the two waves interfere constructively, and what is the corresponding value of $|y_s|$?

(c) At $t = (\pi/50)$ s, at what location x do the two waves interfere destructively, and what is the corresponding value of $|y_s|$?

Solution:

(a) $y_1(x,t)$ is traveling in positive x-direction. $y_2(x,t)$ is traveling in negative x-direction.

(b) At $t = (\pi/50)$ s, $y_s = y_1 + y_2 = 3[\cos(0.4\pi - 30x) - \cos(0.4\pi + 3x)]$. Using the formulas from Appendix C,

$$2\sin x \sin y = \cos(x-y) - (\cos x + y),$$

we have

$$y_s = 2\sin(0.4\pi)\sin 30x = 1.9\sin 30x.$$

Hence,

$$|y_s|_{\max} = 1.9$$

and it occurs when $\sin 30x = 1$, or $30x = \frac{\pi}{2} + 2n\pi$, or $x = \left(\frac{\pi}{60} + \frac{2n\pi}{30}\right)$ cm, where $n = 0, 1, 2, \ldots$.

(c) $|y_s|_{\min} = 0$ and it occurs when $30x = n\pi$, or $x = \frac{n\pi}{30}$ cm.

Problem 1.8 Give expressions for $y(x,t)$ for a sinusoidal wave traveling along a string in the negative x-direction, given that $y_{\max} = 20$ cm, $\lambda = 30$ cm, $f = 5$ Hz, and

(a) $y(x,0) = 0$ at $x = 0$,

(b) $y(x,0) = 0$ at $x = 7.5$ cm.

Solution: For a wave traveling in the negative x-direction, we use Eq. (1.17) with $\omega = 2\pi f = 10\pi$ (rad/s), $\beta = 2\pi/\lambda = 2\pi/0.3 = 20\pi/3$ (rad/s), $A = 20$ cm, and x assigned a positive sign:

$$y(x,t) = 20\cos\left(10\pi t + \frac{20\pi}{3}x + \phi_0\right) \quad \text{(cm)},$$

with x in meters.

(a) $y(0,0) = 0 = 20\cos\phi_0$. Hence, $\phi_0 = \pm\pi/2$, and

$$\begin{aligned} y(x,t) &= 20\cos\left(10\pi t + \frac{20\pi}{3}x \pm \frac{\pi}{2}\right) \\ &= \begin{cases} -20\sin\left(10\pi t + \frac{20\pi}{3}x\right) \text{ (cm)}, & \text{if } \phi_0 = \pi/2, \\ 20\sin\left(10\pi t + \frac{20\pi}{3}x\right) \text{ (cm)}, & \text{if } \phi_0 = -\pi/2. \end{cases} \end{aligned}$$

(b) At $x = 7.5$ cm $= 7.5 \times 10^{-2}$ m, $y = 0 = 20\cos(\pi/2 + \phi_0)$. Hence, $\phi_0 = 0$ or π, and

$$y(x,t) = \begin{cases} 20\cos\left(10\pi t + \frac{20\pi}{3}x\right) \text{ (cm)}, & \text{if } \phi_0 = 0, \\ -20\cos\left(10\pi t + \frac{20\pi}{3}x\right) \text{ (cm)}, & \text{if } \phi_0 = \pi. \end{cases}$$

Problem 1.9 An oscillator that generates a sinusoidal wave on a string completes 20 vibrations in 30 s. The wave peak is observed to travel a distance of 2.8 m along the string in 5 s. What is the wavelength?

Solution:

$$T = \frac{30}{20} = 1.5 \text{ s}, \qquad u_p = \frac{2.8}{5} = 0.56 \text{ m/s},$$
$$\lambda = u_p T = 0.56 \times 1.5 = 0.84 \text{ m}.$$

Problem 1.10 The vertical displacement of a string is given by the harmonic function:

$$y(x,t) = 5\cos(12\pi t - 20\pi x) \quad \text{(m)},$$

where x is the horizontal distance along the string in meters. Suppose a tiny particle were to be attached to the string at $x = 5$ cm, obtain an expression for the vertical velocity of the particle as a function of time.

Solution:

$$y(x,t) = 5\cos(12\pi t - 20\pi x) \quad \text{(m)}.$$

$$\begin{aligned} u(0.05,t) &= \left.\frac{dy(x,t)}{dt}\right|_{x=0.05} \\ &= 60\pi\sin(12\pi t - 20\pi x)|_{x=0.05} \\ &= 60\pi\sin(12\pi t - \pi) \\ &= -60\pi\sin(12\pi t) \quad \text{(m/s)}. \end{aligned}$$

Problem 1.11 Given two waves characterized by

$$y_1(t) = 6\cos\omega t,$$
$$y_2(t) = 6\sin(\omega t + 30°),$$

does $y_2(t)$ lead or lag $y_1(t)$, and by what phase angle?

Solution: We need to express $y_2(t)$ in terms of a cosine function:

$$\begin{aligned} y_2(t) &= 6\sin(\omega t + 30°) \\ &= 6\cos\left(\frac{\pi}{2} - \omega t - 30°\right) = 6\cos(60° - \omega t) = 6\cos(\omega t - 60°). \end{aligned}$$

Hence, $y_2(t)$ lags $y_1(t)$ by 60°.

Problem 1.12 The voltage of an electromagnetic wave traveling on a transmission line is given by $v(z,t) = 3e^{-\alpha z}\sin(2\pi \times 10^9 t - 10\pi z)$ (V), where z is the distance in meters from the generator.

(a) Find the frequency, wavelength, and phase velocity of the wave.

(b) At $z = 2$ m, the amplitude of the wave was measured to be 1 V. Find α.

Solution:

(a) This equation is similar to that of Eq. (1.28) with $\omega = 2\pi \times 10^9$ rad/s and $\beta = 10\pi$ rad/m. From Eq. (1.29a), $f = \omega/2\pi = 10^9$ Hz $= 1$ GHz; from Eq. (1.29b), $\lambda = 2\pi/\beta = 0.2$ m. From Eq. (1.30),

$$u_p = \omega/\beta = 2 \times 10^8 \text{ m/s}.$$

(b) Using just the amplitude of the wave,

$$1 = 3e^{-\alpha 2}, \qquad \alpha = \frac{-1}{2\text{ m}} \ln\left(\frac{1}{3}\right) = 0.55 \text{ Np/m}.$$

Problem 1.13 A certain electromagnetic wave traveling in sea water was observed to have an amplitude of 19.025 (V/m) at a depth of 10 m and an amplitude of 12.13 (V/m) at a depth of 100 m. What is the attenuation constant of sea water?

Solution: The amplitude has the form $Ae^{\alpha z}$. At $z = 10$ m,

$$Ae^{-10\alpha} = 19.025$$

and at $z = 100$ m,

$$Ae^{-100\alpha} = 12.13.$$

The ratio gives

$$\frac{e^{-10\alpha}}{e^{-100\alpha}} = \frac{19.025}{12.13} = 1.568$$

or

$$e^{-10\alpha} = 1.568e^{-100\alpha}.$$

Taking the natural log of both sides gives

$$\ln(e^{-10\alpha}) = \ln(1.568e^{-100\alpha}),$$
$$-10\alpha = \ln(1.568) - 100\alpha,$$
$$90\alpha = \ln(1.568) = 0.45.$$

Hence,

$$\alpha = \frac{0.45}{90} = 5 \times 10^{-3} \quad \text{(Np/m)}.$$

Section 1-5: Complex Numbers

Problem 1.14 Evaluate each of the following complex numbers and express the result in rectangular form:

(a) $z_1 = 3e^{j\pi/4}$,
(b) $z_2 = \sqrt{3}\, e^{j3\pi/4}$,
(c) $z_3 = 2\, e^{-j\pi/2}$,
(d) $z_4 = j^3$,
(e) $z_5 = j^{-4}$,
(f) $z_6 = (1-j)^3$,
(g) $z_7 = (1-j)^{1/2}$.

Solution: (Note: In the following solutions, numbers are expressed to only two decimal places, but the final answers are found using a calculator with 10 decimal places.)

(a) $z_1 = 3e^{j\pi/4} = 3(\cos\pi/4 + j\sin\pi/4) = 2.12 + j2.12 = 2.12(1+j)$.

(b)

$$z_2 = \sqrt{3}\,e^{j3\pi/4} = \sqrt{3}\left[\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right] = -1.22 + j1.22 = 1.22(-1+j).$$

(c) $z_3 = 2e^{-j\pi/2} = 2[\cos(-\pi/2) + j\sin(-\pi/2)] = -j2$.

(d) $z_4 = j^3 = j \cdot j^2 = -j$, or

$$z_4 = j^3 = (e^{j\pi/2})^3 = e^{j3\pi/2} = \cos(3\pi/2) + j\sin(3\pi/2) = -j.$$

(e) $z_5 = j^{-4} = (e^{j\pi/2})^{-4} = e^{-j2\pi} = 1$.

(f)

$$\begin{aligned} z_6 = (1-j)^3 = (\sqrt{2}e^{-j\pi/4})^3 &= (\sqrt{2})^3 e^{-j3\pi/4} \\ &= (\sqrt{2})^3[\cos(3\pi/4) - j\sin(3\pi/4)] \\ &= -2 - j2 = -2(1+j). \end{aligned}$$

(g)

$$\begin{aligned} z_7 = (1-j)^{1/2} = (\sqrt{2}e^{-j\pi/4})^{1/2} = \pm 2^{1/4}e^{-j\pi/8} &= \pm 1.19(0.92 - j0.38) \\ &= \pm(1.10 - j0.45). \end{aligned}$$

Problem 1.15 Complex numbers z_1 and z_2 are given by

$$z_1 = 3 - j2,$$
$$z_2 = -4 + j2.$$

(a) Express z_1 and z_2 in polar form.
(b) Find $|z_1|$ by applying Eq. (1.41) and again by applying Eq. (1.43).
(c) Determine the product $z_1 z_2$ in polar form.
(d) Determine the ratio z_1/z_2 in polar form.
(e) Determine z_1^3 in polar form.

Solution:
(a) Using Eq. (1.41),

$$z_1 = 3 - j2 = 3.6e^{-j33.7^\circ},$$
$$z_2 = -4 + j2 = 4.5e^{j153.4^\circ}.$$

(b) By Eq. (1.41) and Eq. (1.43), respectively,

$$|z_1| = |3 - j2| = \sqrt{3^2 + (-2)^2} = \sqrt{13} = 3.60,$$
$$|z_1| = \sqrt{(3 - j2)(3 + j2)} = \sqrt{13} = 3.60.$$

(c) By applying Eq. (1.47b) to the results of part (a),

$$z_1 z_2 = 3.6e^{-j33.7^\circ} \times 4.5e^{j153.4^\circ} = 16.2e^{j119.7^\circ}.$$

(d) By applying Eq. (1.48b) to the results of part (a),

$$\frac{z_1}{z_2} = \frac{3.6e^{-j33.7^\circ}}{4.5e^{j153.4^\circ}} = 0.80e^{-j187.1^\circ}.$$

(e) By applying Eq. (1.49) to the results of part (a),

$$z_1^3 = \left(3.6e^{-j33.7^\circ}\right)^3 = (3.6)^3 e^{-j3\times 33.7^\circ} = 46.66e^{-j101.1^\circ}.$$

Problem 1.16 If $z = -2 + j3$, determine the following quantities in polar form:
(a) $1/z$,
(b) z^3,
(c) $|z|^2$,
(d) $\Im\mathfrak{m}\{z\}$,
(e) $\Im\mathfrak{m}\{z^*\}$.

Solution: (Note: In the following solutions, numbers are expressed to only two decimal places, but the final answers are found using a calculator with 10 decimal places.)

(a)

$$\frac{1}{z} = \frac{1}{-2+j3} = (-2+j3)^{-1} = (3.61\,e^{j123.7^\circ})^{-1} = (3.61)^{-1}e^{-j123.7^\circ} = 0.28\,e^{-j123.7^\circ}.$$

(b) $z^3 = (-2+j3)^3 = (3.61\,e^{j123.7^\circ})^3 = (3.61)^3 e^{j371.1^\circ} = 46.87\,e^{j11.07^\circ}$.

(c) $|z|^2 = z \cdot z^* = (-2+j3)(-2-j3) = 4+9 = 13$.

(d) $\mathfrak{Im}\{z\} = \mathfrak{Im}\{-2+j3\} = 3$.

(e) $\mathfrak{Im}\{z^*\} = \mathfrak{Im}\{-2-j3\} = -3 = 3e^{j\pi}$.

Problem 1.17 Find complex numbers $t = z_1 + z_2$ and $s = z_1 - z_2$, both in polar form, for each of the following pairs:

(a) $z_1 = 2+j3,\ z_2 = 1-j2$,

(b) $z_1 = 2,\ z_2 = -j2$,

(c) $z_1 = 3\angle 30^\circ,\ z_2 = 3\angle{-30^\circ}$,

(d) $z_1 = 3\angle 30^\circ,\ z_2 = 3\angle{-150^\circ}$.

Solution:

(a)

$$t = z_1 + z_2 = (2+j3) + (1-j2) = 3+j1 = 3.16\,e^{j18.43^\circ},$$
$$s = z_1 - z_2 = (2+j3) - (1-j2) = 1+j5 = 5.10\,e^{j78.69^\circ}.$$

(b)

$$t = z_1 + z_2 = 2-j2 = 2.83\,e^{-j45^\circ},$$
$$s = z_1 - z_2 = 2+j2 = 2.83\,e^{j45^\circ}.$$

(c)

$$t = z_1 + z_2 = 3\angle 30^\circ + 3\angle{-30^\circ}$$
$$= 3e^{j30^\circ} + 3e^{-j30^\circ} = (2.6+j1.5) + (2.6-j1.5) = 5.2,$$
$$s = z_1 - z_2 = 3e^{j30^\circ} - 3e^{-j30^\circ} = (2.6+j1.5) - (2.6-j1.5) = j3 = 3e^{j90^\circ}.$$

(d)

$$t = z_1 + z_2 = 3\angle 30^\circ + 3\angle{-150^\circ} = (2.6+j1.5) + (-2.6-j1.5) = 0,$$
$$s = z_1 - z_2 = (2.6+j1.5) - (-2.6-j1.5) = 5.2+j3 = 6e^{j30^\circ}.$$

Problem 1.18 Complex numbers z_1 and z_2 are given by

$$z_1 = 5\angle{-60^\circ},$$
$$z_2 = 2\angle 45^\circ.$$

(a) Determine the product $z_1 z_2$ in polar form.
(b) Determine the product $z_1 z_2^*$ in polar form.
(c) Determine the ratio z_1/z_2 in polar form.
(d) Determine the ratio z_1^*/z_2^* in polar form.
(e) Determine $\sqrt{z_1}$ in polar form.

Solution:

(a) $z_1 z_2 = 5e^{-j60^\circ} \times 2e^{j45^\circ} = 10e^{-j15^\circ}$.

(b) $z_1 z_2^* = 5e^{-j60^\circ} \times 2e^{-j45^\circ} = 10e^{-j105^\circ}$.

(c) $\dfrac{z_1}{z_2} = \dfrac{5e^{-j60^\circ}}{2e^{j45^\circ}} = 2.5^{-j105^\circ}$.

(d) $\dfrac{z_1^*}{z_2^*} = \left(\dfrac{z_1}{z_2}\right)^* = 2.5^{j105^\circ}$.

(e) $\sqrt{z_1} = \sqrt{5e^{-j60^\circ}} = \pm\sqrt{5}\,e^{-j30^\circ}$.

Problem 1.19 If $z = 3 - j4$, find the value of $\ln(z)$.

Solution:

$$|z| = +\sqrt{3^2+4^2} = 5, \quad \theta = \tan^{-1}\left(\frac{-4}{3}\right) = -53.1^\circ,$$

$$z = |z|e^{j\theta} = 5e^{-j53.1^\circ},$$

$$\begin{aligned}\ln(z) &= \ln(5e^{-j53.1^\circ}) \\ &= \ln(5) + \ln(e^{-j53.1^\circ}) \\ &= 1.61 - j53.1^\circ = 1.61 - j\frac{53.1^\circ\pi}{180^\circ} = 1.61 - j0.93.\end{aligned}$$

Problem 1.20 If $z = 3 - j4$, find the value of e^z.

Solution:

$$e^z = e^{3-j4} = e^3 \cdot e^{-j4} = e^3(\cos 4 - j\sin 4),$$

$$e^3 = 20.09, \quad \text{and} \quad 4 \text{ rad} = \frac{4}{\pi} \times 180^\circ = 229.18^\circ.$$

Hence, $e^z = 20.08(\cos 229.18^\circ - j\sin 229.18^\circ) = -13.13 + j15.20$.

Section 1-6: Phasors

Problem 1.21 A voltage source given by $v_s(t) = 10\cos(2\pi \times 10^3 t - 30°)$ (V) is connected to a series RC load as shown in Fig. 1-19. If $R = 1$ MΩ and $C = 100$ pF, obtain an expression for $v_c(t)$, the voltage across the capacitor.

Solution: In the phasor domain, the circuit is a voltage divider, and

$$\widetilde{V}_c = \widetilde{V}_s \frac{1/j\omega C}{R + 1/j\omega C} = \frac{\widetilde{V}_s}{(1 + j\omega RC)}.$$

Now $\widetilde{V}_s = 10e^{-j30°}$ V with $\omega = 2\pi \times 10^3$ rad/s, so

$$\begin{aligned}\widetilde{V}_c &= \frac{10e^{-j30°}\text{ V}}{1 + j((2\pi \times 10^3\text{ rad/s}) \times (10^6\ \Omega) \times (100 \times 10^{-12}\text{ F}))} \\ &= \frac{10e^{-j30°}\text{ V}}{1 + j\pi/5} = 8.5e^{-j62.1°}\text{ V}.\end{aligned}$$

Converting back to an instantaneous value,

$$v_c(t) = \Re e \widetilde{V}_c e^{j\omega t} = \Re e 8.5 e^{j(\omega t - 62.1°)}\text{ V} = 8.5\cos(2\pi \times 10^3 t - 62.1°)\text{ V},$$

where t is expressed in seconds.

Problem 1.22 Find the phasors of the following time functions:
(a) $v(t) = 3\cos(\omega t - \pi/4)$ (V),
(b) $v(t) = 12\sin(\omega t + \pi/4)$ (V),
(c) $i(x,t) = 4e^{-3x}\sin(\omega t - \pi/6)$ (A),
(d) $i(t) = -2\cos(\omega t + 3\pi/4)$ (A),
(e) $i(t) = 2\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$ (A).

Solution:
(a) $\widetilde{V} = 3e^{-j\pi/4}$ V.
(b) $v(t) = 12\sin(\omega t + \pi/4) = 12\cos(\pi/2 - (\omega t + \pi/4)) = 12\cos(\omega t - \pi/4)$ V, $\widetilde{V} = 12e^{-j\pi/4}$ V.
(c)

$$\begin{aligned}i(t) &= 4e^{-3x}\sin(\omega t - \pi/6)\text{ A} = 4e^{-3x}\cos(\pi/2 - (\omega t - \pi/6))\text{ A} \\ &= 4e^{-3x}\cos(\omega t - 2\pi/3)\text{ A}, \\ \widetilde{I} &= 4e^{-3x}e^{-j2\pi/3}\text{ A}.\end{aligned}$$

(d)

$$\begin{aligned} i(t) &= -2\cos(\omega t + 3\pi/4), \\ \widetilde{I} &= -2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A}. \end{aligned}$$

(e)

$$\begin{aligned} i(t) &= 2\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6) \\ &= 2\cos[\pi/2 - (\omega t + \pi/3)] + 3\cos(\omega t - \pi/6) \\ &= 2\cos(-\omega t + \pi/6) + 3\cos(\omega t - \pi/6) \\ &= 2\cos(\omega t - \pi/6) + 3\cos(\omega t - \pi/6) = 5\cos(\omega t - \pi/6), \\ \widetilde{I} &= 5e^{-j\pi/6} \text{ A}. \end{aligned}$$

Problem 1.23 Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) $\widetilde{V} = -3e^{j\pi/3}$ (V),
(b) $\widetilde{V} = j6e^{j\pi/4}$ (V),
(c) $\widetilde{I} = (3 + j4)$ (A),
(d) $\widetilde{I} = -3 + j2$ (A),
(e) $\widetilde{I} = j$ (A),
(f) $\widetilde{I} = 2e^{j3\pi/4}$ (A).

Solution:

(a)

$$\begin{aligned} \widetilde{V} &= -3e^{j\pi/3} \text{ V} = 3e^{j(\pi/3-\pi)} \text{ V} = 3e^{-j2\pi/3} \text{ V}, \\ v(t) &= 3\cos(\omega t - 2\pi/3) \text{ V}. \end{aligned}$$

(b)

$$\begin{aligned} \widetilde{V} &= j6e^{j\pi/4} \text{ V} = 6e^{j(\pi/4+\pi/2)} \text{ V} = 6e^{j3\pi/4} \text{ V}, \\ v(t) &= 6\cos(\omega t + 3\pi/4) \text{ V}. \end{aligned}$$

(c)

$$\begin{aligned} \widetilde{I} &= (3 + j4) \text{ A} = 5e^{j53.1^\circ} \text{ A}, \\ i(t) &= 5\cos(\omega t + 53.1^\circ) \text{ A}. \end{aligned}$$

(d)

$$\begin{aligned} \widetilde{I} &= -3 + j2 = 3.61\, e^{j146.31^\circ}, \\ i(t) &= \Re\mathfrak{e}\{3.61\, e^{j146.31^\circ} e^{j\omega t}\} = 3.61 \cos(\omega t + 146.31^\circ) \text{ A}. \end{aligned}$$

(e)

$$\widetilde{I} = j = e^{j\pi/2},$$
$$i(t) = \mathfrak{Re}\{e^{j\pi/2}e^{j\omega t}\} = \cos(\omega t + \pi/2) = -\sin\omega t \text{ A}.$$

(f)

$$\widetilde{I} = 2e^{j3\pi/4},$$
$$i(t) = \mathfrak{Re}\{2e^{j3\pi/4}e^{j\omega t}\} = 2\cos(\omega t + 3\pi/4) \text{ A}.$$

Problem 1.24 A series RLC circuit is connected to a generator with a voltage $v_s(t) = V_0\cos(\omega t + \pi/3)$ (V).

(a) Write down the voltage loop equation in terms of the current $i(t)$, R, L, C, and $v_s(t)$.

(b) Obtain the corresponding phasor-domain equation.

(c) Solve the equation to obtain an expression for the phasor current $\widetilde{I}$.

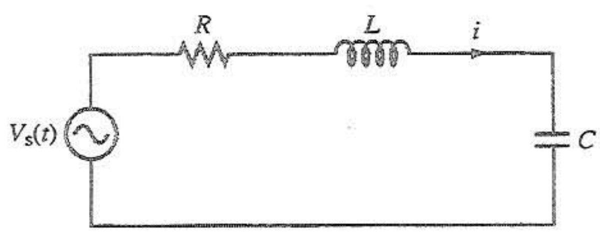

Figure P1.24: RLC circuit.

Solution:

(a) $v_s(t) = Ri + L\dfrac{di}{dt} + \dfrac{1}{C}\displaystyle\int i\,dt.$

(b) In phasor domain: $\widetilde{V}_s = R\widetilde{I} + j\omega L\widetilde{I} + \dfrac{\widetilde{I}}{j\omega C}.$

(c) $\widetilde{I} = \dfrac{\widetilde{V}_s}{R + j(\omega L - 1/\omega C)} = \dfrac{V_0e^{j\pi/3}}{R + j(\omega L - 1/\omega C)} = \dfrac{\omega CV_0e^{j\pi/3}}{\omega RC + j(\omega^2LC - 1)}.$