(b) At the point SWR: $S = 1.64$.

(c) Z_{in} is 0.350λ from the load, which is at 0.144λ on the wavelengths to generator scale. So point Z-IN is at $0.144\lambda + 0.350\lambda = 0.494\lambda$ on the WTG scale. At point Z-IN:

$$Z_{in} = z_{in}Z_0 = (0.61 - j0.022) \times 50\ \Omega = (30.5 - j1.09)\ \Omega.$$

(d) At the point on the SWR circle opposite Z-IN,

$$Y_{in} = \frac{y_{in}}{Z_0} = \frac{(1.64 + j0.06)}{50\ \Omega} = (32.7 + j1.17)\ \text{mS}.$$

(e) Traveling from the point Z-$LOAD$ in the direction of the generator (clockwise), the SWR circle crosses the $x_L = 0$ line first at the point SWR. To travel from Z-$LOAD$ to SWR one must travel $0.250\lambda - 0.144\lambda = 0.106\lambda$. (Readings are on the wavelengths to generator scale.) So the shortest line length would be 0.106λ.

(f) The voltage max occurs at point SWR. From the previous part, this occurs at $z = -0.106\lambda$.

Problem 2.39 A lossless 50-Ω transmission line is terminated in a short circuit. Use the Smith chart to find

(a) the input impedance at a distance 2.3λ from the load,

(b) the distance from the load at which the input admittance is $Y_{in} = -j0.04$ S.

Solution: Refer to Fig. P2.39.

(a) For a short, $z_{in} = 0 + j0$. This is point Z-$SHORT$ and is at 0.000λ on the WTG scale. Since a lossless line repeats every $\lambda/2$, traveling 2.3λ toward the generator is equivalent to traveling 0.3λ toward the generator. This point is at $A : Z$-IN, and

$$Z_{in} = z_{in}Z_0 = (0 - j3.08) \times 50\ \Omega = -j154\ \Omega.$$

(b) The admittance of a short is at point Y-$SHORT$ and is at 0.250λ on the WTG scale:

$$y_{in} = Y_{in}Z_0 = -j0.04\ \text{S} \times 50\ \Omega = -j2,$$

which is point $B : Y$-IN and is at 0.324λ on the WTG scale. Therefore, the line length is $0.324\lambda - 0.250\lambda = 0.074\lambda$. Any integer half wavelengths farther is also valid.

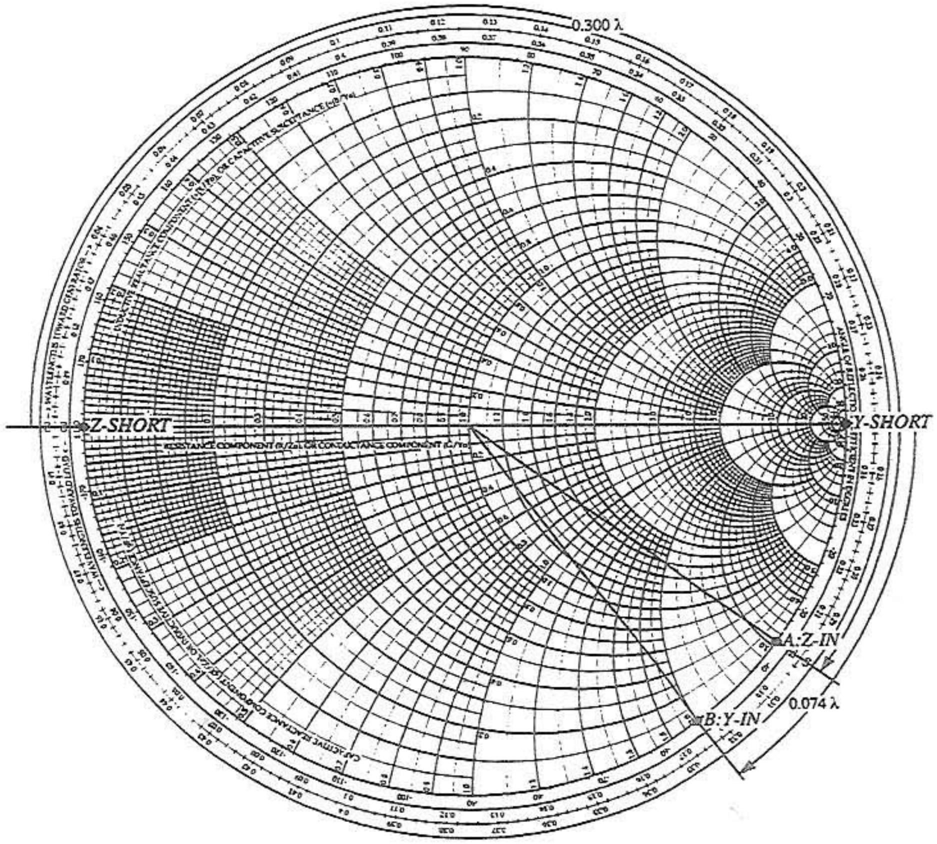

Figure P2.39: Solution of Problem 2.39.

Problem 2.40 Use the Smith chart to find y_L if $z_L = 1.5 - j0.7$.

Solution: Refer to Fig. P2.40. The point Z represents $1.5 - j0.7$. The reciprocal of point Z is at point Y, which is at $0.55 + j0.26$.

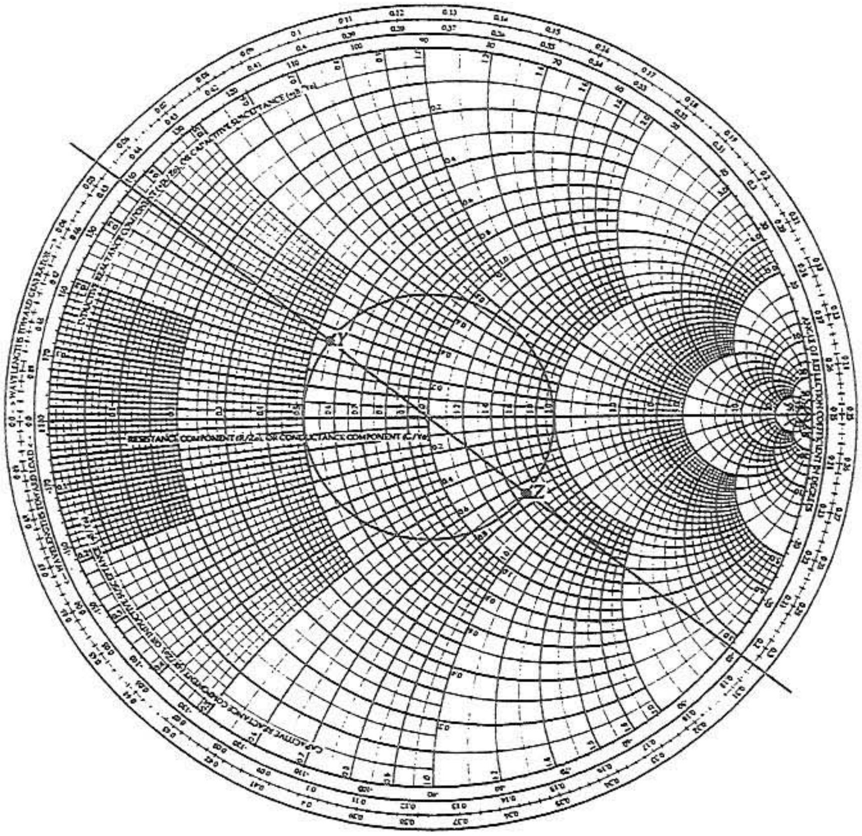

Figure P2.40: Solution of Problem 2.40.

Problem 2.41 A lossless 100-Ω transmission line $3\lambda/8$ in length is terminated in an unknown impedance. If the input impedance is $Z_{in} = -j2.5\ \Omega$,

(a) use the Smith chart to find Z_L.

(b) What length of open-circuit line could be used to replace Z_L?

Solution: Refer to Fig. P2.41. $z_{in} = Z_{in}/Z_0 = -j2.5\ \Omega/100\ \Omega = 0.0 - j0.025$ which is at point *Z-IN* and is at 0.004λ on the wavelengths to load scale.

(a) Point *Z-LOAD* is 0.375λ toward the load from the end of the line. Thus, on the wavelength to load scale, it is at $0.004\lambda + 0.375\lambda = 0.379\lambda$.

$$Z_L = z_L Z_0 = (0 + j0.95) \times 100\ \Omega = j95\ \Omega.$$

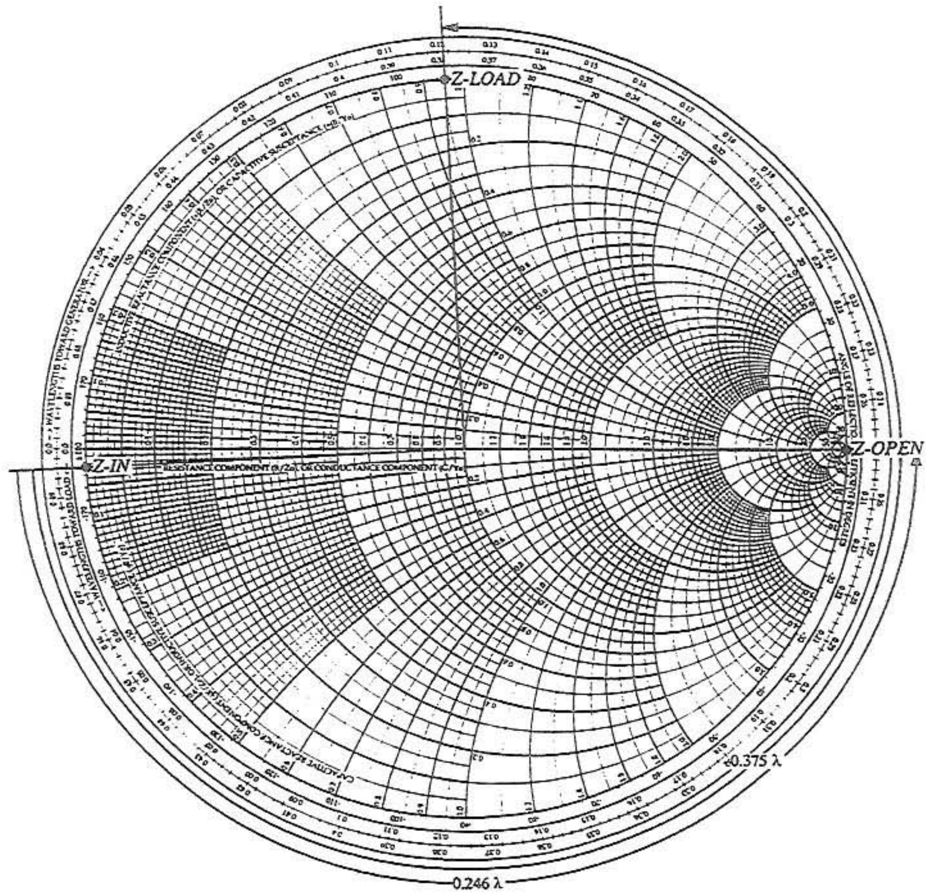

Figure P2.41: Solution of Problem 2.41.

(b) An open circuit is located at point *Z-OPEN*, which is at 0.250λ on the wavelength to load scale. Therefore, an open circuited line with $Z_{in} = -j0.025$ must have a length of $0.250\lambda - 0.004\lambda = 0.246\lambda$.

Problem 2.42 A 75-Ω lossless line is 0.6λ long. If $S = 1.8$ and $\theta_r = -60°$, use the Smith chart to find $|\Gamma|$, Z_L, and Z_{in}.

Solution: Refer to Fig. P2.42. The SWR circle must pass through $S = 1.8$ at point *SWR*. A circle of this radius has

$$|\Gamma| = \frac{S-1}{S+1} = 0.29.$$

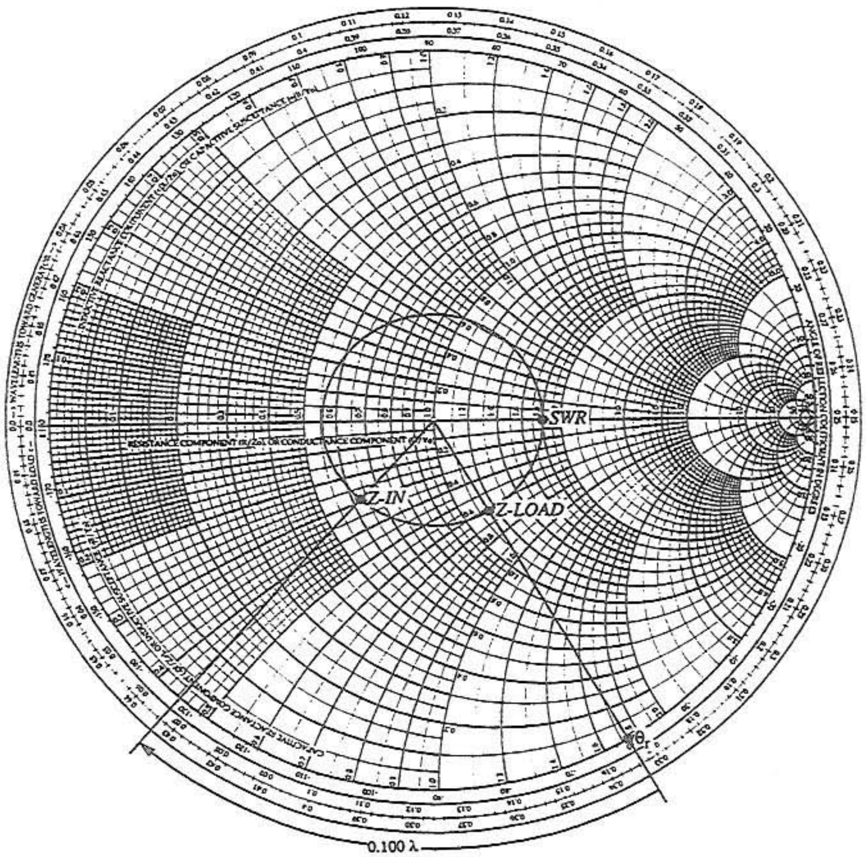

Figure P2.42: Solution of Problem 2.42.

The load must have a reflection coefficient with $\theta_r = -60°$. The angle of the reflection coefficient is read off that scale at the point θ_r. The intersection of the circle of constant $|\Gamma|$ and the line of constant θ_r is at the load, point *Z-LOAD*, which has a value $z_L = 1.15 - j0.62$. Thus,

$$Z_L = z_L Z_0 = (1.15 - j0.62) \times 75\ \Omega = (86.5 - j46.6)\ \Omega.$$

A 0.6λ line is equivalent to a 0.1λ line. On the WTG scale, *Z-LOAD* is at 0.333λ, so *Z-IN* is at $0.333\lambda + 0.100\lambda = 0.433\lambda$ and has a value

$$z_{in} = 0.63 - j0.29.$$

Therefore $Z_{\text{in}} = z_{\text{in}} Z_0 = (0.63 - j0.29) \times 75\ \Omega = (47.0 - j21.8)\ \Omega$.

Problem 2.43 Using a slotted line on a 50-Ω air-spaced lossless line, the following measurements were obtained: $S = 1.6$, $|\widetilde{V}|_{\max}$ occurred only at 10 cm and 24 cm from the load. Use the Smith chart to find Z_{L}.

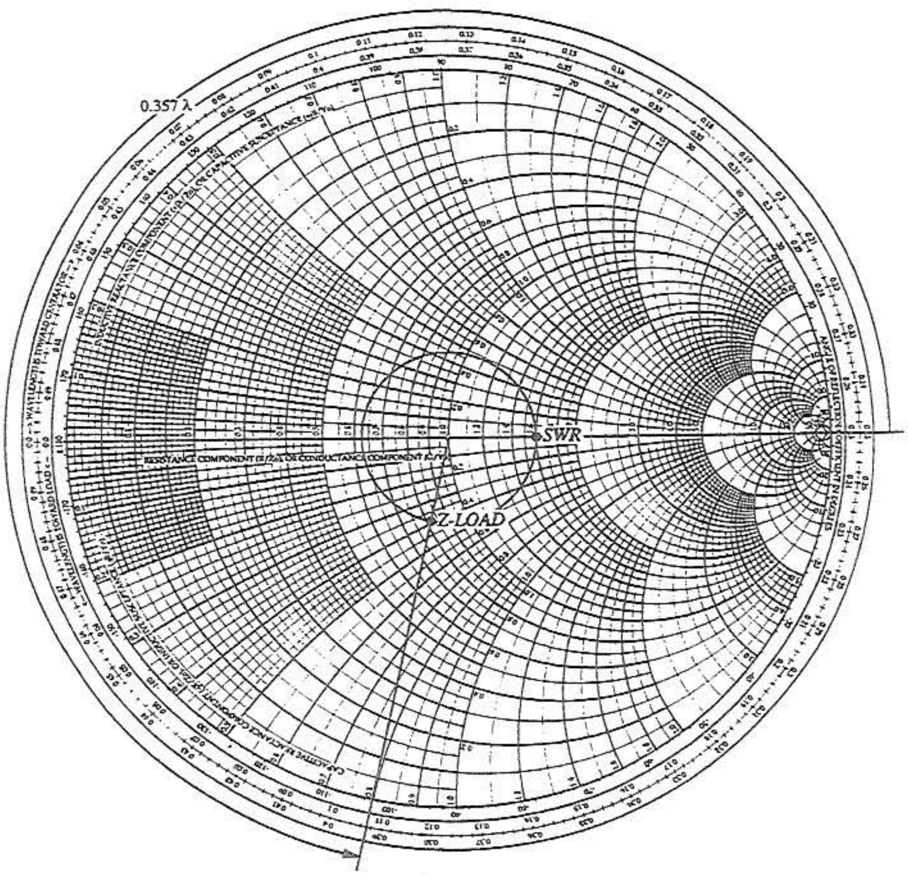

Figure P2.43: Solution of Problem 2.43.

Solution: Refer to Fig. P2.43. The point *SWR* denotes the fact that $S = 1.6$. This point is also the location of a voltage maximum. From the knowledge of the locations of adjacent maxima we can determine that $\lambda = 2(24\text{ cm} - 10\text{ cm}) = 28$ cm. Therefore, the load is $\frac{10\text{ cm}}{28\text{ cm}}\lambda = 0.357\lambda$ from the first voltage maximum, which is at 0.250λ on the WTL scale. Traveling this far on the SWR circle we find point *Z-LOAD* at $0.250\lambda + 0.357\lambda - 0.500\lambda = 0.107\lambda$ on the WTL scale, and here

$$z_L = 0.82 - j0.39.$$

Therefore $Z_L = z_L Z_0 = (0.82 - j0.39) \times 50\ \Omega = (41.0 - j19.5)\ \Omega$.

Problem 2.44 At an operating frequency of 5 GHz, a 50-Ω lossless coaxial line with insulating material having a relative permittivity $\varepsilon_r = 2.25$ is terminated in an antenna with an impedance $Z_L = 75\ \Omega$. Use the Smith chart to find Z_{in}. The line length is 30 cm.

Solution: To use the Smith chart the line length must be converted into wavelengths. Since $\beta = 2\pi/\lambda$ and $u_p = \omega/\beta$,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi u_p}{\omega} = \frac{c}{\sqrt{\varepsilon_r} f} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25} \times (5 \times 10^9 \text{ Hz})} = 0.04 \text{ m}.$$

Hence, $l = \frac{0.30 \text{ m}}{0.04 \text{ m}} \lambda = 7.5\lambda$. Since this is an integral number of half wavelengths,

$$Z_{in} = Z_L = 75\ \Omega.$$

Section 2-10: Impedance Matching

Problem 2.45 A 50-Ω lossless line 0.6λ long is terminated in a load with $Z_L = (50 + j25)\ \Omega$. At 0.3λ from the load, a resistor with resistance $R = 30\ \Omega$ is connected as shown in Fig. 2-43 (P2.45(a)). Use the Smith chart to find Z_{in}.

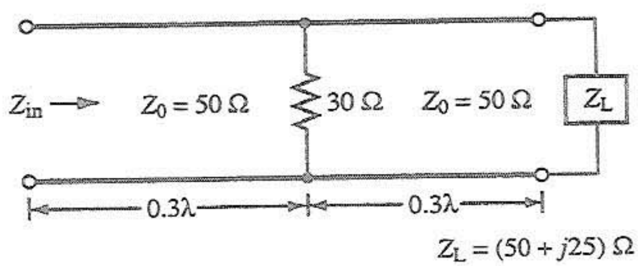

Figure P2.45: (a) Circuit for Problem 2.45.

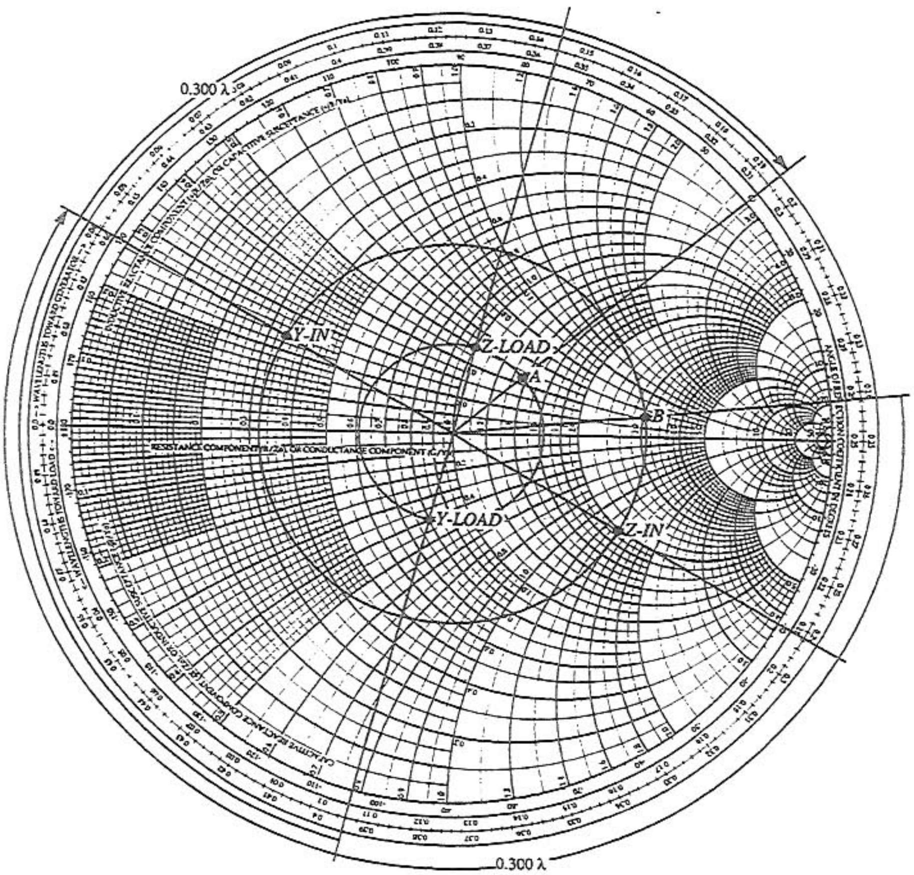

Figure P2.45: (b) Solution of Problem 2.45.

Solution: Refer to Fig. P2.45(b). Since the 30-Ω resistor is in parallel with the input impedance at that point, it is advantageous to convert all quantities to admittances.

$$z_L = \frac{Z_L}{Z_0} = \frac{(50 + j25)\ \Omega}{50\ \Omega} = 1 + j0.5$$

and is located at point *Z-LOAD*. The corresponding normalized load admittance is at point *Y-LOAD*, which is at 0.394λ on the WTG scale. The input admittance of the load only at the shunt conductor is at $0.394\lambda + 0.300\lambda - 0.500\lambda = 0.194\lambda$ and is denoted by point *A*. It has a value of

$$y_{\text{in}A} = 1.37 + j0.45.$$

The shunt conductance has a normalized conductance

$$g = \frac{50\ \Omega}{30\ \Omega} = 1.67.$$

The normalized admittance of the shunt conductance in parallel with the input admittance of the load is the sum of their admittances:

$$y_{\text{in}B} = g + y_{\text{in}A} = 1.67 + 1.37 + j0.45 = 3.04 + j0.45$$

and is located at point B. On the WTG scale, point B is at 0.242λ. The input admittance of the entire circuit is at $0.242\lambda + 0.300\lambda - 0.500\lambda = 0.042\lambda$ and is denoted by point *Y-IN*. The corresponding normalized input impedance is at *Z-IN* and has a value of

$$z_{\text{in}} = 1.9 - j1.4.$$

Thus,

$$Z_{\text{in}} = z_{\text{in}} Z_0 = (1.9 - j1.4) \times 50\ \Omega = (95 - j70)\ \Omega.$$

Problem 2.46 A 50-Ω lossless line is to be matched to an antenna with

$$Z_L = (75 - j20)\ \Omega$$

using a shorted stub. Use the Smith chart to determine the stub length and the distance between the antenna and the stub.

Solution: Refer to Fig. P2.46(a) and Fig. P2.46(b), which represent two different solutions.

$$z_L = \frac{Z_L}{Z_0} = \frac{(75 - j20)\ \Omega}{50\ \Omega} = 1.5 - j0.4$$

and is located at point *Z-LOAD* in both figures. Since it is advantageous to work in admittance coordinates, y_L is plotted as point *Y-LOAD* in both figures. *Y-LOAD* is at 0.041λ on the WTG scale.

For the first solution in Fig. P2.46(a), point *Y-LOAD-IN-*1 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN-*1 is at 0.145λ on the WTG scale, so the stub should be located at $0.145\lambda - 0.041\lambda = 0.104\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN-*1, $b = 0.52$, so a stub with an input admittance of $y_{\text{stub}} = 0 - j0.52$ is required. This point is *Y-STUB-IN-*1 and is at 0.423λ on the WTG scale. The short circuit admittance

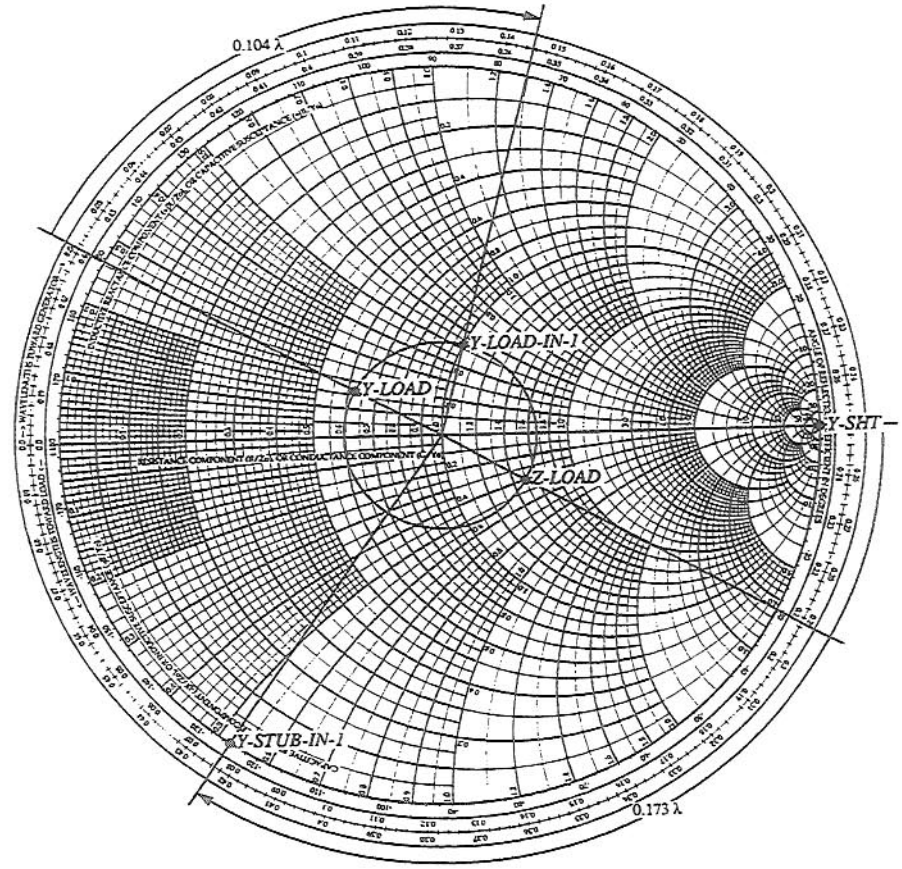

Figure P2.46: (a) First solution to Problem 2.46.

is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.423\lambda - 0.250\lambda = 0.173\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.46(b), point *Y-LOAD-IN-2* represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN-2* is at 0.355λ on the WTG scale, so the stub should be located at $0.355\lambda - 0.041\lambda = 0.314\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN-2*, $b = -0.52$, so a stub with an input admittance of $y_{\text{stub}} = 0 + j0.52$ is required. This point is *Y-STUB-IN-2* and is at 0.077λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.077\lambda - 0.250\lambda + 0.500\lambda = 0.327\lambda$ long (or some multiple of a half wavelength

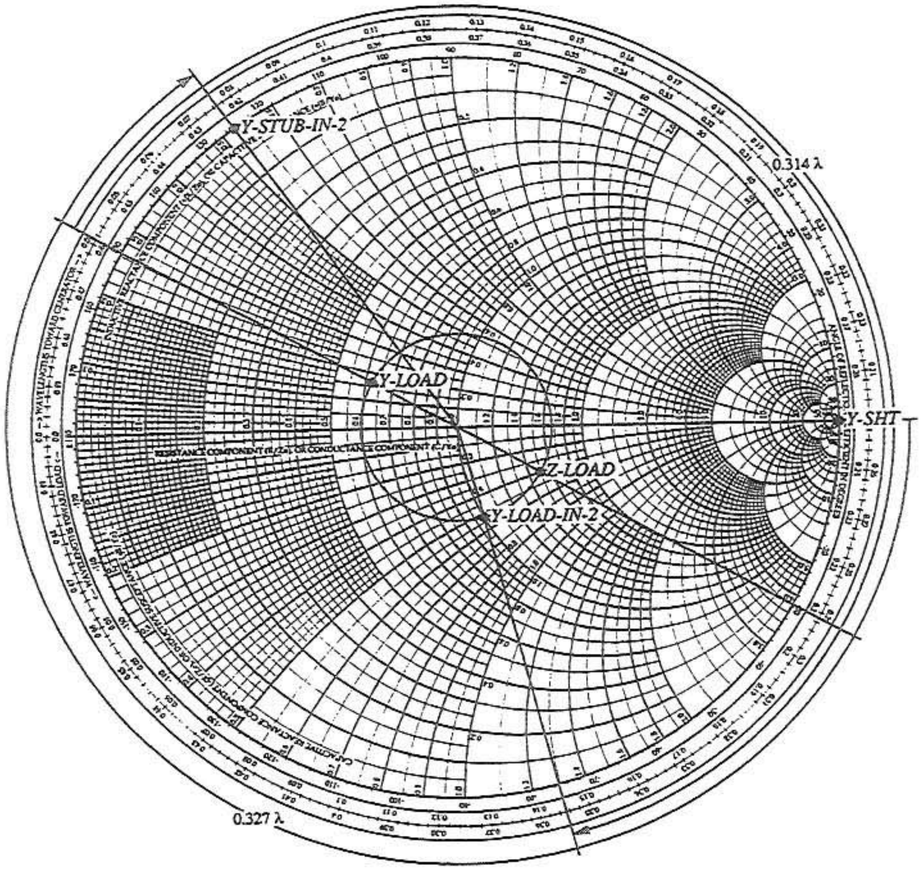

Figure P2.46: (b) Second solution to Problem 2.46.

longer).

Problem 2.47 Repeat Problem 2.46 for a load with $Z_L = (100 + j50)\ \Omega$.

Solution: Refer to Fig. P2.47(a) and Fig. P2.47(b), which represent two different solutions.

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j50\ \Omega}{50\ \Omega} = 2 + j1$$

and is located at point *Z-LOAD* in both figures. Since it is advantageous to work in admittance coordinates, y_L is plotted as point *Y-LOAD* in both figures. *Y-LOAD* is at 0.463λ on the WTG scale.

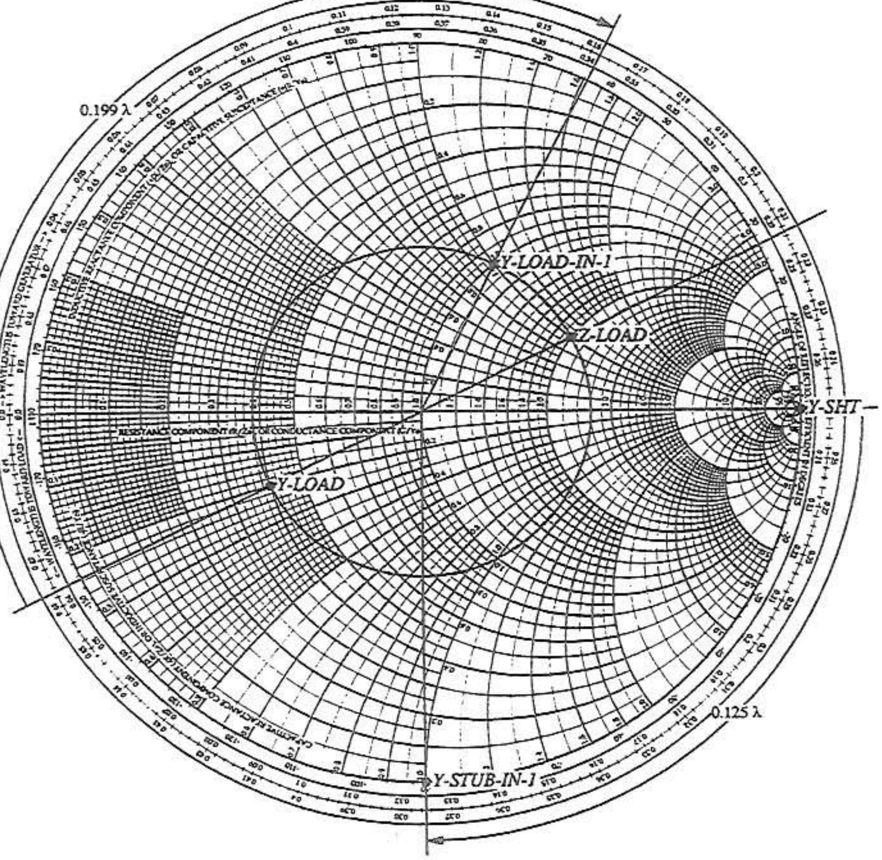

Figure P2.47: (a) First solution to Problem 2.47.

For the first solution in Fig. P2.47(a), point *Y-LOAD-IN*-1 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-1 is at 0.162λ on the WTG scale, so the stub should be located at $0.162\lambda - 0.463\lambda + 0.500\lambda = 0.199\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-1, $b = 1$, so a stub with an input admittance of $y_{\text{stub}} = 0 - j1$ is required. This point is *Y-STUB-IN*-1 and is at 0.375λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.375\lambda - 0.250\lambda = 0.125\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.47(b), point *Y-LOAD-IN*-2 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-2 is at 0.338λ on the

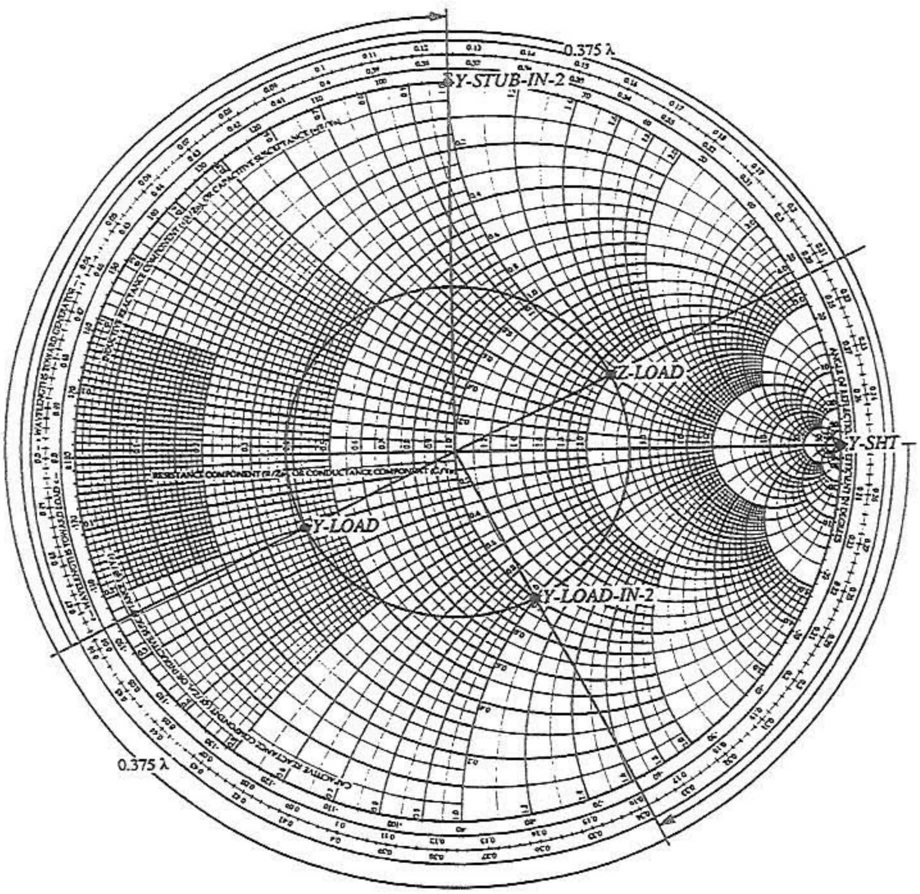

Figure P2.47: (b) Second solution to Problem 2.47.

WTG scale, so the stub should be located at $0.338\lambda - 0.463\lambda + 0.500\lambda = 0.375\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN-2*, $b = -1$, so a stub with an input admittance of $y_{\text{stub}} = 0 + j1$ is required. This point is *Y-STUB-IN-2* and is at 0.125λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.125\lambda - 0.250\lambda + 0.500\lambda = 0.375\lambda$ long (or some multiple of a half wavelength longer).

Problem 2.48 Use the Smith chart to find Z_{in} of the feed line shown in Fig. 2-44 (P2.48(a)). All lines are lossless with $Z_0 = 50\ \Omega$.

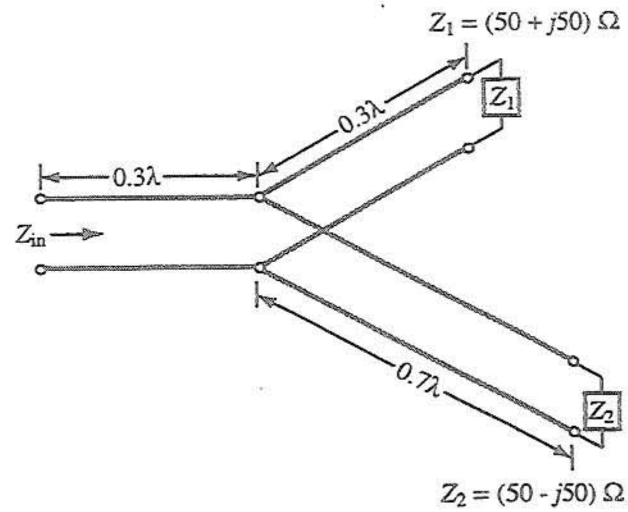

Figure P2.48: (a) Circuit of Problem 2.48.

Solution: Refer to Fig. P2.48(b).

$$z_1 = \frac{Z_1}{Z_0} = \frac{50 + j50\ \Omega}{50\ \Omega} = 1 + j1$$

and is at point *Z-LOAD*-1.

$$z_2 = \frac{Z_2}{Z_0} = \frac{50 - j50\ \Omega}{50\ \Omega} = 1 - j1$$

and is at point *Z-LOAD*-2. Since at the junction the lines are in parallel, it is advantageous to solve the problem using admittances. y_1 is point *Y-LOAD*-1, which is at 0.412λ on the WTG scale. y_2 is point *Y-LOAD*-2, which is at 0.088λ on the WTG scale. Traveling 0.300λ from *Y-LOAD*-1 toward the generator one obtains the input admittance for the upper feed line, point *Y-IN*-1, with a value of $1.97 + j1.02$. Since traveling 0.700λ is equivalent to traveling 0.200λ on any transmission line, the input admittance for the lower line feed is found at point *Y-IN*-2, which has a value of $1.97 - j1.02$. The admittance of the two lines together is the sum of their admittances: $1.97 + j1.02 + 1.97 - j1.02 = 3.94 + j0$ and is denoted *Y-JUNCT*. 0.300λ from *Y-JUNCT* toward the generator is the input admittance of the entire feed line, point *Y-IN*, from which *Z-IN* is found.

$$Z_{in} = z_{in} Z_0 = (1.65 - j1.79) \times 50\ \Omega = (82.5 - j89.5)\ \Omega.$$

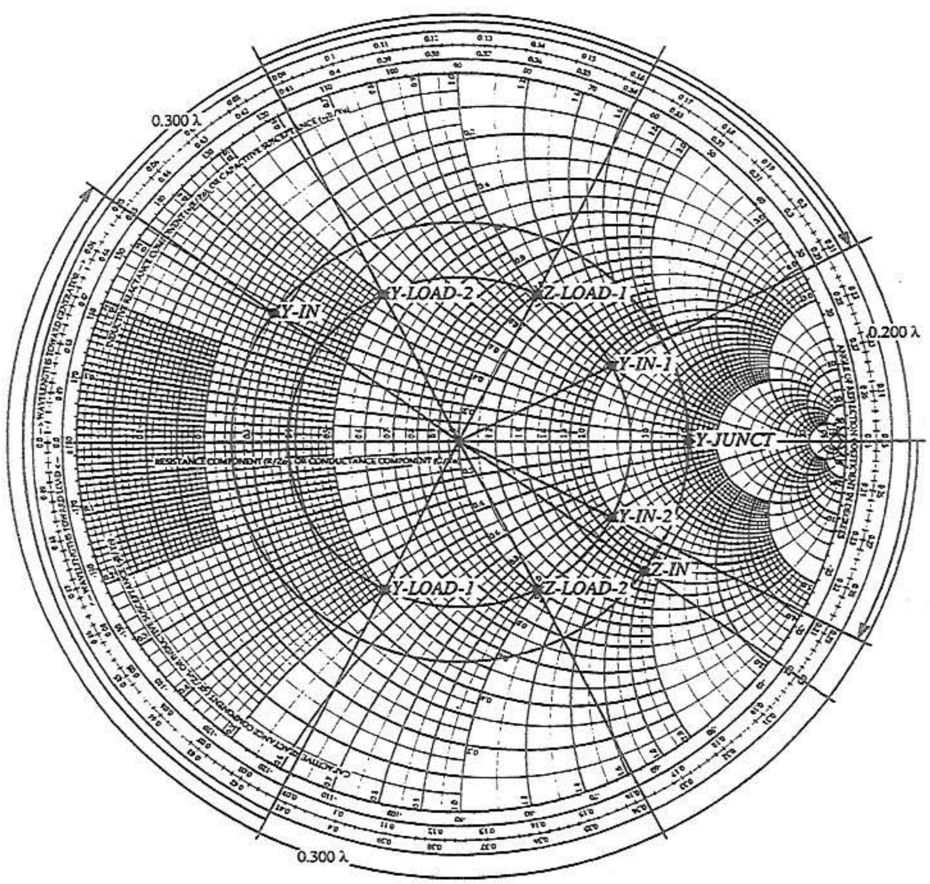

Figure P2.48: (b) Solution of Problem 2.48.

Problem 2.49 Repeat Problem 2.48 for the case where all three transmission lines are $\lambda/4$ in length.

Solution: Since the transmission lines are in parallel, it is advantageous to express loads in terms of admittances. In the upper branch, which is a quarter wave line,

$$Y_{1\,\text{in}} = \frac{Y_0^2}{Y_1} = \frac{Z_1}{Z_0^2},$$

and similarly for the lower branch,

$$Y_{2\,\text{in}} = \frac{Y_0^2}{Y_2} = \frac{Z_2}{Z_0^2}.$$

Thus, the total load at the junction is

$$Y_{\text{JCT}} = Y_{1\,\text{in}} + Y_{2\,\text{in}} = \frac{Z_1 + Z_2}{Z_0^2}.$$

Therefore, since the common transmission line is also quarter-wave,

$$Z_{\text{in}} = Z_0^2/Z_{\text{JCT}} = Z_0^2 Y_{\text{JCT}} = Z_1 + Z_2 = (50 + j50)\ \Omega + (50 - j50)\ \Omega = 100\ \Omega.$$

Section 2-11: Transients on Transmission Lines

Problem 2.50 Generate a bounce diagram for the voltage $V(z,t)$ for a 1-m long lossless line characterized by $Z_0 = 50\ \Omega$ and $u_p = 2c/3$ (where c is the velocity of light) if the line is fed by a step voltage applied at $t = 0$ by a generator circuit with $V_g = 60$ V and $R_g = 100\ \Omega$. The line is terminated in a load $Z_L = 25\ \Omega$. Use the bounce diagram to plot $V(t)$ at a point midway along the length of the line from $t = 0$ to $t = 25$ ns.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3},$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = \frac{-1}{3}.$$

From Eq. (2.124b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20\ \text{V}.$$

Also,

$$T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5\ \text{ns}.$$

The bounce diagram is shown in Fig. P2.50(a) and the plot of $V(t)$ in Fig. P2.50(b).

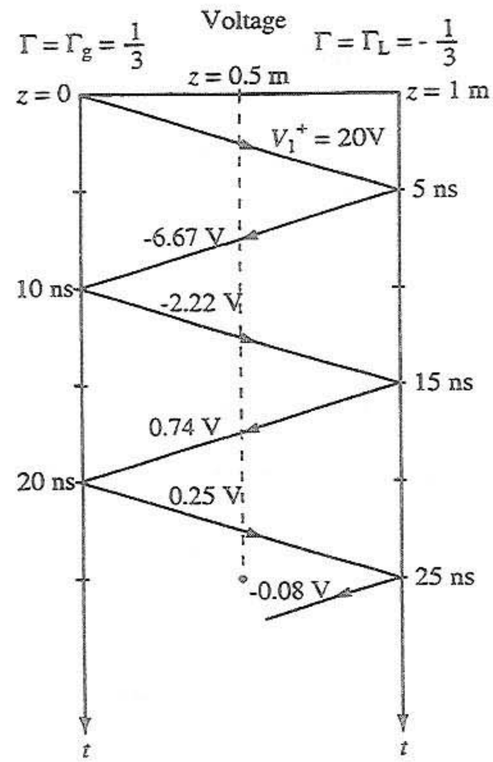

Figure P2.50: (a) Bounce diagram for Problem 2.50.

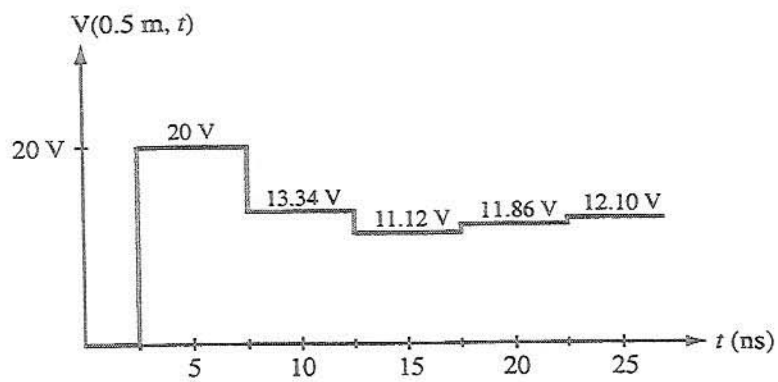

Figure P2.50: (b) Time response of voltage.

Problem 2.51 Repeat Problem 2.50 for the current I on the line.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3},$$
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-1}{3}.$$

From Eq. (2.124a),

$$I_1^+ = \frac{V_g}{R_g + Z_0} = \frac{60}{100 + 50} = 0.4 \text{ A}.$$

The bounce diagram is shown in Fig. P2.51(a) and $I(t)$ in Fig. P2.51(b).

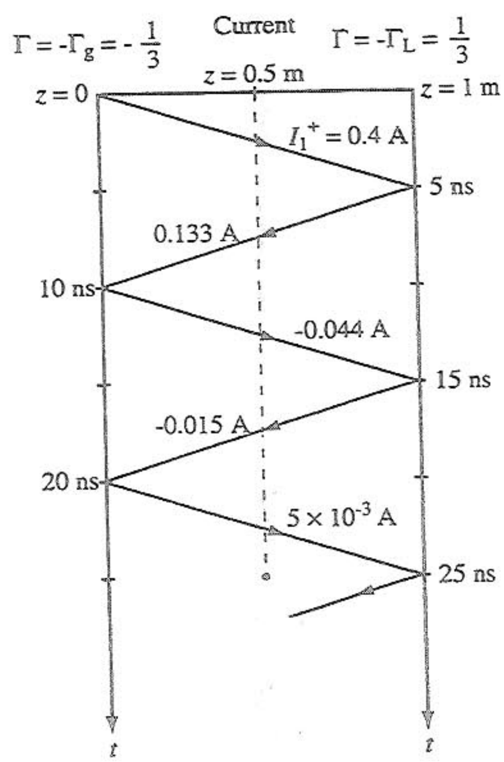

Figure P2.51: (a) Bounce diagram for Problem 2.51.

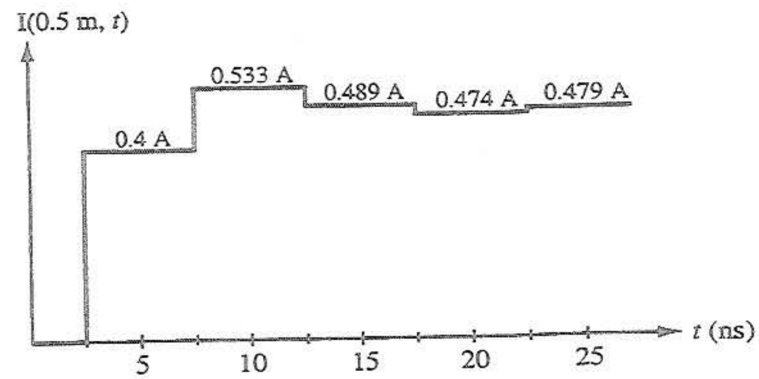

Figure P2.51: (b) Time response of current.

Problem 2.52 In response to a step voltage, the voltage waveform shown in Fig. 2-45 (P2.52) was observed at the sending end of a lossless transmission line with $R_g = 50\ \Omega$, $Z_0 = 50\ \Omega$, and $\varepsilon_r = 2.25$. Determine (a) the generator voltage, (b) the length of the line, and (c) the load impedance.

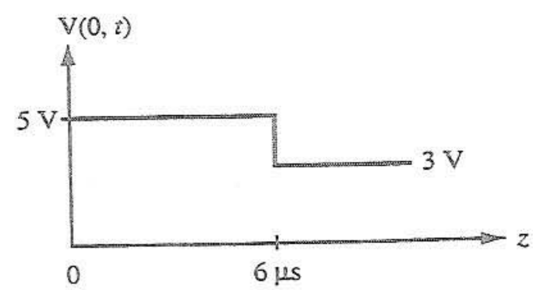

Figure P2.52: Observed voltage at sending end.

Solution:

(a) From the figure, $V_1^+ = 5$ V. Applying Eq. (2.124b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{V_g Z_0}{Z_0 + Z_0} = \frac{V_g}{2},$$

which gives $V_g = 2V_1^+ = 10$ V.

(b) $u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8$ m/s. The first change in the waveform occurs at $\Delta t = 6\ \mu$s. But $\Delta t = 2l/u_p$. Hence,

$$l = \frac{\Delta t \mu_p}{2} = \frac{6\times10^{-6}}{2} \times 2\times10^8 = 600 \text{ m}.$$

(c) The change in level from 5 V down to 3 V means that $V_1^- = -2$ V. But

$$V_1^- = \Gamma_L V_1^+, \qquad \text{or } \Gamma_L = \frac{V_1^-}{V_1^+} = \frac{-2}{5} = -0.4.$$

From

$$Z_L = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) = 50\left(\frac{1-0.4}{1+0.4}\right) = 21.43\ \Omega.$$

Problem 2.53 In response to a step voltage, the voltage waveform shown in Fig. 2.46 (P2.53) was observed at the sending end of a shorted line with $Z_0 = 50\ \Omega$ and $\varepsilon_r = 4$. Determine V_g, R_g, and the line length.

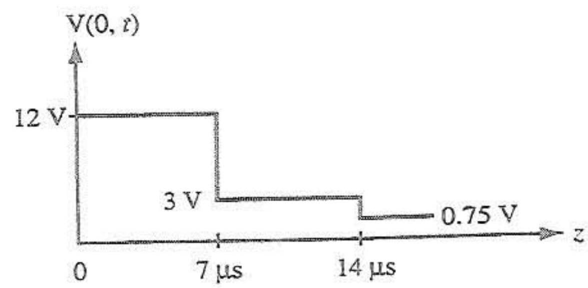

Figure P2.53: Observed voltage at sending end.

Solution:

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8}{\sqrt{4}} = 1.5\times10^8 \text{ m/s},$$

$$7\ \mu\text{s} = 7\times10^{-6} \text{ s} = \frac{2l}{u_p} = \frac{2l}{1.5\times10^8}.$$

Hence, $l = 525$ m.

From the voltage waveform, $V_1^+ = 12$ V. At $t = 7\mu$s, the voltage at the sending end is

$$V(z=0, t=7\mu\text{s}) = V_1^+ + \Gamma_L V_1^+ + \Gamma_g \Gamma_L V_1^+ = -\Gamma_g V_1^+ \qquad \text{(because } \Gamma_L = -1\text{)}.$$

Hence, 3 V$= -\Gamma_g \times 12$ V, or $\Gamma_g = -0.25$. From Eq. (2.128),

$$R_g = Z_0 \left(\frac{1+\Gamma_g}{1-\Gamma_g} \right) = 50 \left(\frac{1-0.25}{1+0.25} \right) = 30\ \Omega.$$

Also,

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}, \qquad \text{or} \quad 12 = \frac{V_g \times 50}{30+50},$$

which gives $V_g = 19.2$ V.

Problem 2.54 Suppose the voltage waveform shown in Fig. 2-45 was observed at the sending end of a 50-Ω transmission line in response to a step voltage introduced by a generator with $V_g = 15$ V and an unknown series resistance R_g. The line is 1 km in length, its velocity of propagation is 1×10^8 m/s, and it is terminated in a load $Z_L = 100\ \Omega$.

(a) Determine R_g.

(b) Explain why the drop in level of $V(0,t)$ at $t = 6\ \mu$s cannot be due to reflection from the load.

(c) Determine the shunt resistance R_f and the location of the fault responsible for the observed waveform.

Solution:

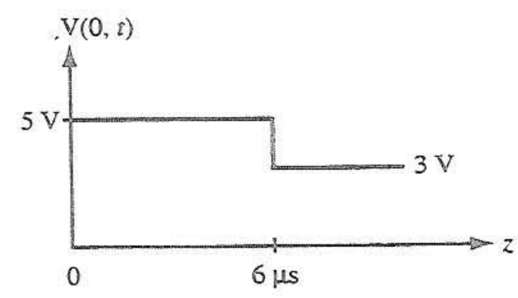

Figure P2.54: Observed voltage at sending end.

(a)

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}.$$

From Fig. 2-45, $V_1^+ = 5$ V. Hence,

$$5 = \frac{15 \times 50}{R_g + 50},$$

which gives $R_g = 100\ \Omega$.

(b) Roundtrip time delay of pulse return from the load is

$$2T = \frac{2l}{u_p} = \frac{2 \times 10^3}{1 \times 10^8} = 20\ \mu\text{s},$$

which is much longer than 6 μs, the instance at which $V(0,t)$ drops in level.

(c) $V_1^+ = 5$ V. The new level of 3 V is equal to V_1^+ plus V_1^- due to shunt resistance:

$$V_1^+ + V_1^- = 5 + 5\Gamma = 3 \quad \text{(V)},$$

or $\Gamma = -\frac{2}{5} = -0.4$. But

$$\Gamma = \frac{R_f - Z_0}{R_f + Z_0} = -0.4,$$

which gives $R_f = 21.43\ \Omega$.

Problem 2.55 A generator circuit with $V_g = 200$ V and $R_g = 25\ \Omega$ was used to excite a 75-Ω lossless line with a rectangular pulse of duration $\tau = 0.4\ \mu$s. The line is 200 m long, its $u_p = 2 \times 10^8$ m/s, and it is terminated in a load $Z_L = 125\ \Omega$.

(a) Synthesize the voltage pulse exciting the line as the sum of two step functions, $V_{g_1}(t)$ and $V_{g_2}(t)$.

(b) For each voltage step function, generate a bounce diagram for the voltage on the line.

(c) Use the bounce diagrams to plot the total voltage at the sending end of the line.

Solution:

(a) pulse length $= 0.4\ \mu$s.

$$V_g(t) = V_{g_1}(t) + V_{g_2}(t),$$

with

$$V_{g_1}(t) = 200\,U(t) \quad \text{(V)},$$
$$V_{g_2}(t) = -200\,U(t - 0.4\ \mu\text{s}) \quad \text{(V)}.$$

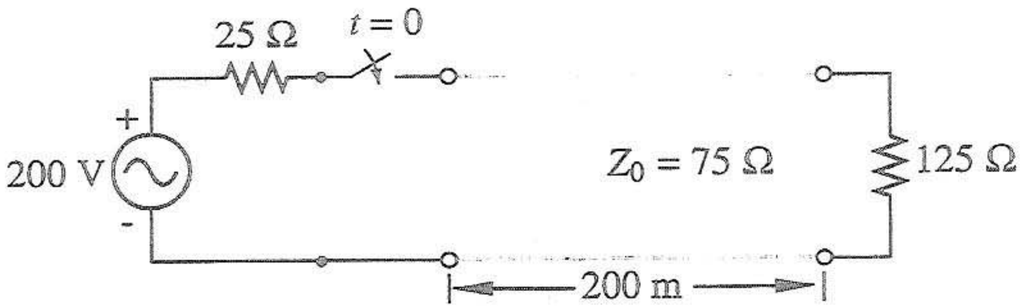

Figure P2.55: (a) Circuit for Problem 2.55.

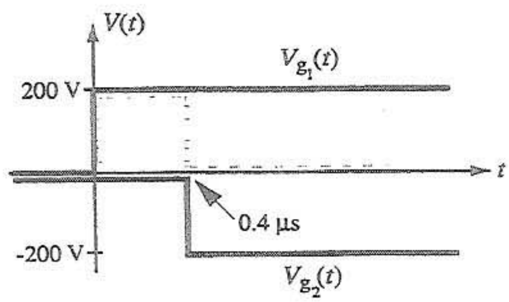

Figure P2.55: (b) Solution of part (a).

(b)

$$T = \frac{l}{u_p} = \frac{200}{2 \times 10^8} = 1\ \mu\text{s}.$$

We will divide the problem into two parts, one for $V_{g_1}(t)$ and another for $V_{g_2}(t)$ and then we will use superposition to determine the solution for the sum. The solution for $V_{g_2}(t)$ will mimic the solution for $V_{g_1}(t)$, except for a reversal in sign and a delay by 0.4 μs.

For $V_{g_1}(t) = 200\,U(t)$:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{25-75}{25+75} = -0.5,$$
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{125-75}{125+75} = 0.25,$$

$$V_1^+ = \frac{V_1 Z_0}{R_g + Z_0} = \frac{200 \times 75}{25 + 75} = 150 \text{ V},$$
$$V_\infty = \frac{V_g Z_L}{R_g + Z_L} = \frac{200 \times 125}{25 + 125} = 166.67 \text{ V}.$$

(i) $V_1(0,t)$ at sending end due to $V_{g_1}(t)$:

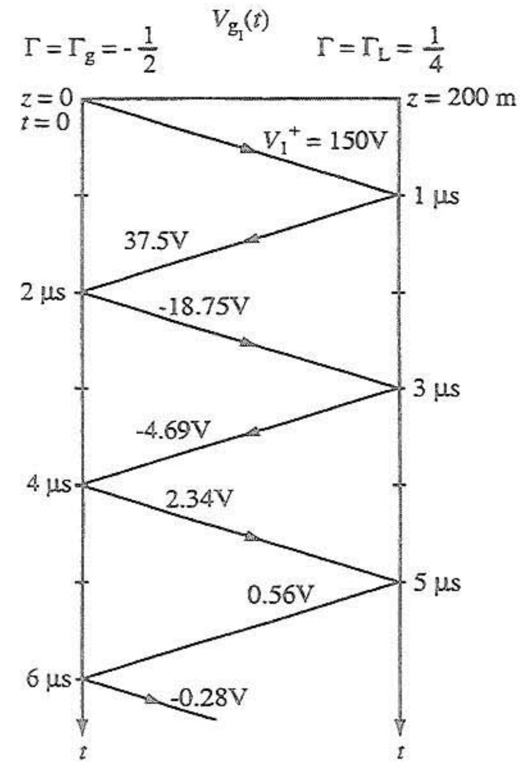

Figure P2.55: (c) Bounce diagram for voltage in reaction to $V_{g_1}(t)$.

(ii) $V_2(0,t)$ at sending end due to $V_{g_2}(t)$:

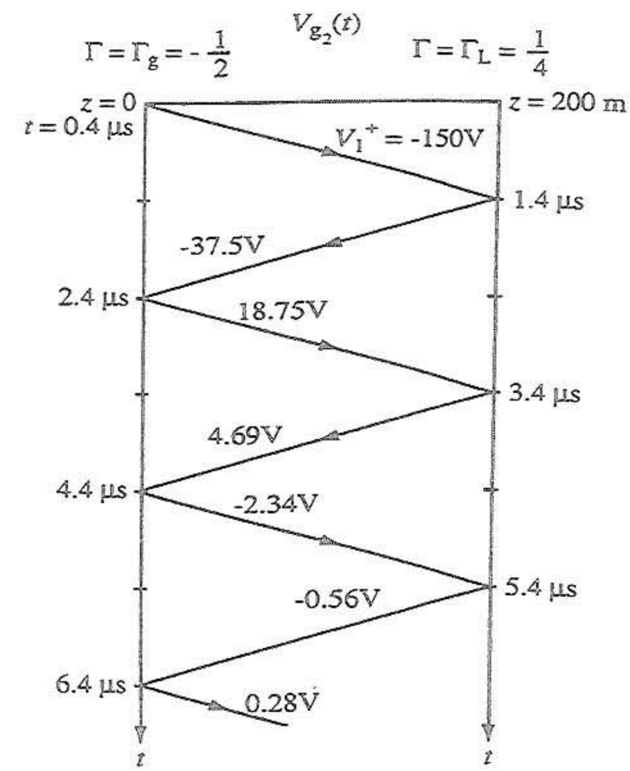

Figure P2.55: (d) Bounce diagram for voltage in reaction to $V_{g_2}(t)$.

(b)
(i) $V_1(0,t)$ at sending end due to $V_{g_1}(t)$:

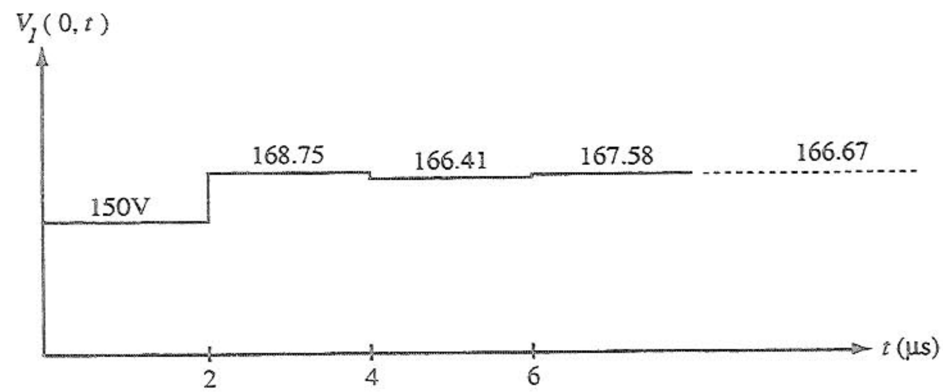

Figure P2.55: (e) $V_1(0,t)$.

(ii) $V_2(0,t)$ at sending end:

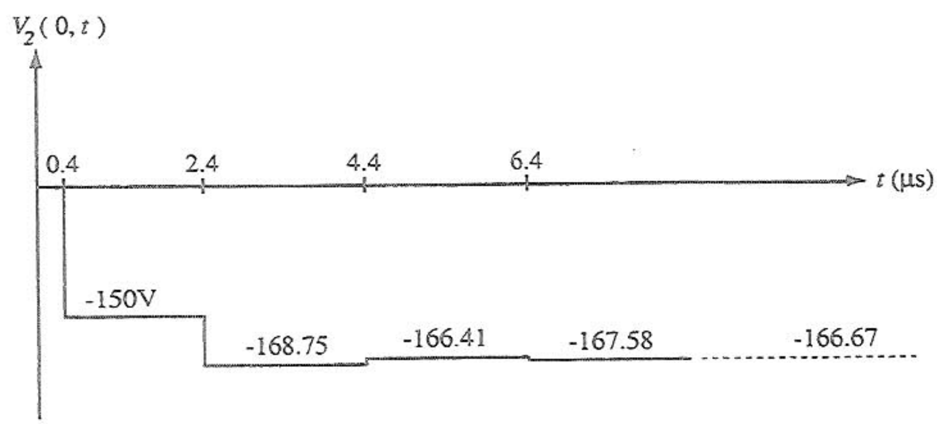

Figure P2.55: (f) $V_2(0,t)$.

(iii) Net voltage $V(0,t) = V_1(0,t) + V_2(0,t)$:

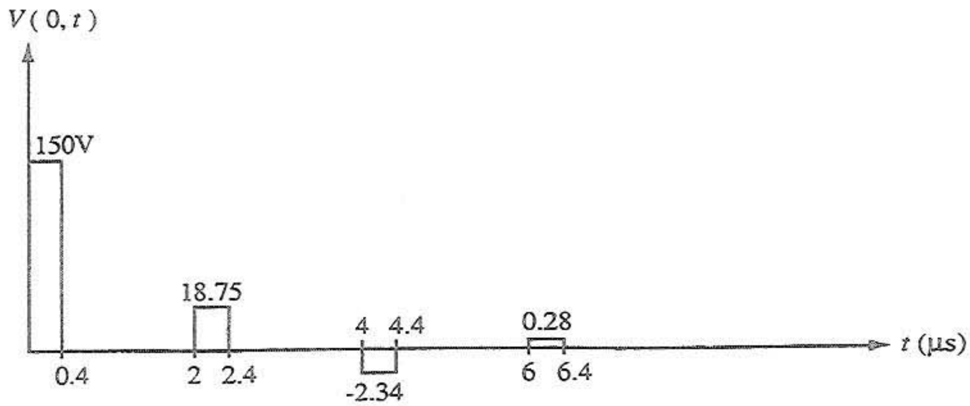

Figure P2.55: (g) Net voltage $V(0,t)$.

Problem 2.56 For the circuit of Problem 2.55, generate a bounce diagram for the current and plot its time history at the middle of the line.

Solution: Using the values for Γ_g and Γ_L calculated in Problem 2.55, we reverse their signs when using them to construct a bounce diagram for the current.

$$I_1^+ = \frac{V_1^+}{Z_0} = \frac{150}{75} = 2 \text{ A},$$
$$I_2^+ = \frac{V_2^+}{Z_0} = \frac{-150}{75} = -2 \text{ A},$$
$$I_\infty^+ = \frac{V_\infty}{Z_L} = 1.33 \text{ A}.$$

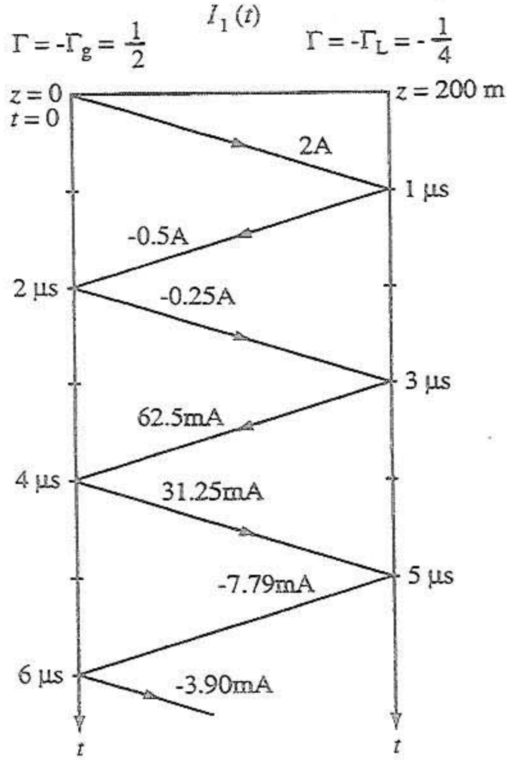

Figure P2.56: (a) Bounce diagram for $I_1(t)$ in reaction to $V_{g_1}(t)$.

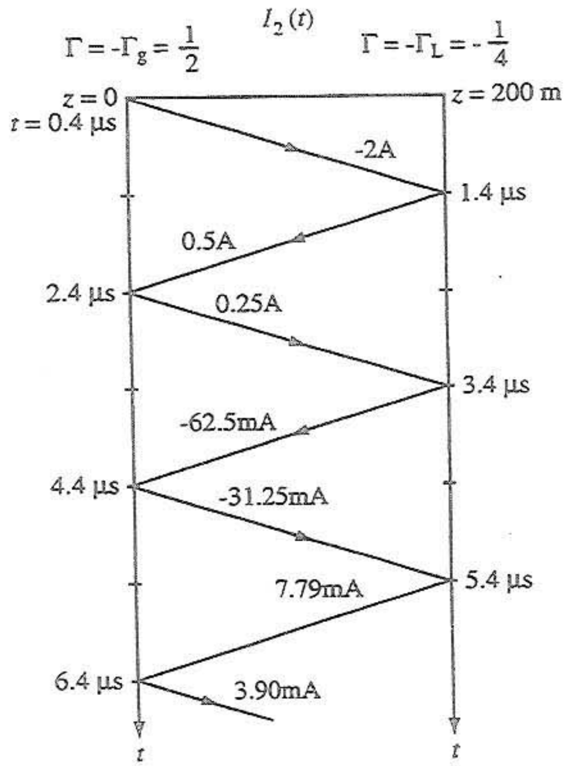

Figure P2.56: (b) Bounce diagram for current $I_2(t)$ in reaction to $V_{g_2}(t)$.

(i) $I_1(l/2,t)$ due to $V_{g_1}(t)$:

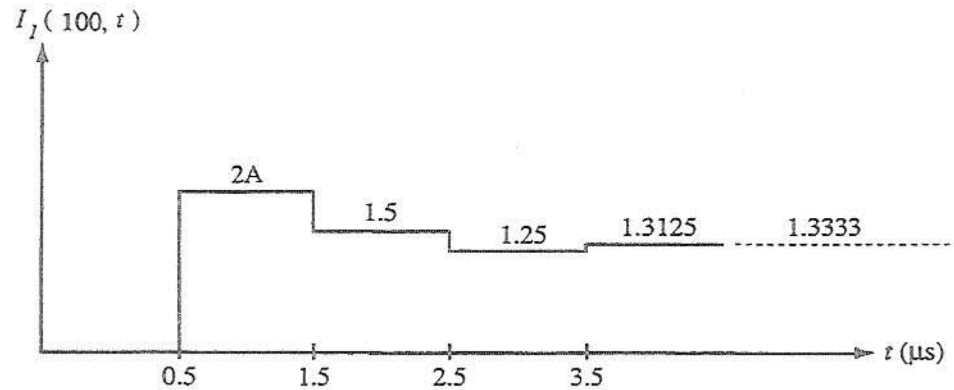

Figure P2.56: (c) $I_1(l/2,t)$.

(ii) $I_2(l/2,t)$ due to $V_{g_2}(t)$:

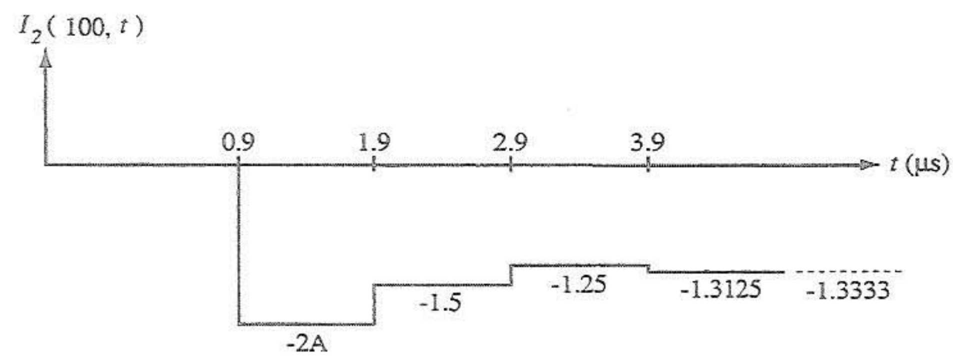

Figure P2.56: (d) $I_2(l/2,t)$.

(iii) Net current $I(l/2,t) = I_1(l/2,t) + I_2(l/2,t)$:

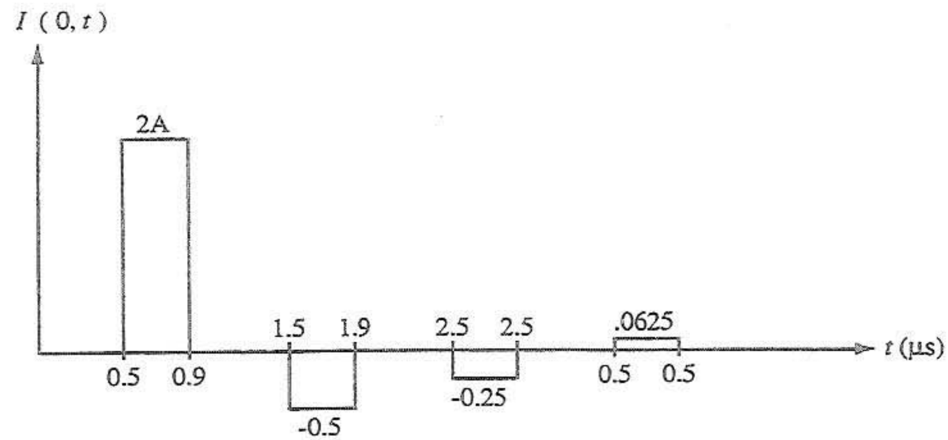

Figure P2.56: (e) Total $I(l/2,t)$.