Chapter 7

Section 7-2: Propagation in Lossless Media

Problem 7.1 The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{z}}\,50\cos(10^9 t - 5y) \quad \text{(mA/m)}.$$

Find (a) the direction of wave propagation, (b) the phase velocity, (c) the wavelength in the material, (d) the relative permittivity of the material, and (e) the electric field phasor.

Solution:

(a) Positive y-direction.

(b) $\omega = 10^9$ rad/s, $k = 5$ rad/m.

$$u_p = \frac{\omega}{k} = \frac{10^9}{5} = 2 \times 10^8 \text{ m/s}.$$

(c) $\lambda = 2\pi/k = 2\pi/5 = 1.26$ m.

(d) $\varepsilon_r = \left(\dfrac{c}{u_p}\right)^2 = \left(\dfrac{3 \times 10^8}{2 \times 10^8}\right)^2 = 2.25.$

(e) From Eq. (7.39b),

$$\widetilde{\mathbf{E}} = -\eta \hat{\mathbf{k}} \times \widetilde{\mathbf{H}},$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{120\pi}{1.5} = 251.33 \quad (\Omega),$$

$$\hat{\mathbf{k}} = \hat{\mathbf{y}}, \quad \text{and} \quad \widetilde{\mathbf{H}} = \hat{\mathbf{z}}\,50 e^{-j5y} \times 10^{-3} \quad \text{(A/m)}.$$

Hence,

$$\widetilde{\mathbf{E}} = -251.33\hat{\mathbf{y}} \times \hat{\mathbf{z}}\,50 e^{-j5y} \times 10^{-3} = -\hat{\mathbf{x}}\,12.57 e^{-j5y} \quad \text{(V/m)},$$

and

$$\mathbf{E}(y,t) = \mathfrak{Re}(\widetilde{\mathbf{E}} e^{j\omega t}) = -\hat{\mathbf{x}}\,12.57\cos(10^9 t - 5y) \quad \text{(V/m)}.$$

Problem 7.2 Write general expressions for the electric and magnetic fields of a 1-GHz sinusoidal plane wave traveling in the $+y$-direction in a lossless nonmagnetic medium with relative permittivity $\varepsilon_r = 9$. The electric field is polarized along the x-direction, its peak value is 3 V/m and its intensity is 2 V/m at $t = 0$ and $y = 2$ cm.

Solution: For $f = 1$ GHz, $\mu_r = 1$, and $\varepsilon_r = 9$,

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s},$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r} = \frac{2\pi f}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 10^9}{3 \times 10^8}\sqrt{9} = 20\pi \text{ rad/m},$$

$$\mathbf{E}(y,t) = \hat{\mathbf{x}}3\cos(2\pi \times 10^9 t - 20\pi y + \phi_0) \quad \text{(V/m)}.$$

At $t = 0$ and $y = 2$ cm, $E = 2$ V/m:

$$2 = 3\cos(-20\pi \times 2 \times 10^{-2} + \phi_0) = 3\cos(-0.4\pi + \phi_0).$$

Hence,

$$\phi_0 - 0.4\pi = \cos^{-1}\left(\frac{2}{3}\right) = 0.84 \text{ rad},$$

which gives

$$\phi_0 = 2.1 \text{ rad} = 120.19^\circ$$

and

$$\mathbf{E}(y,t) = \hat{\mathbf{x}}\,3\cos(2\pi \times 10^9 t - 20\pi y + 120.19^\circ) \quad \text{(V/m)}.$$

Problem 7.3 The electric field phasor of a uniform plane wave is given by $\widetilde{\mathbf{E}} = \hat{\mathbf{y}}\,10e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5×10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find (a) the wavelength, (b) the frequency f of the wave, (c) the relative permittivity of the medium, and (d) the magnetic field $\mathbf{H}(z,t)$.

Solution:

(a) From $\widetilde{\mathbf{E}} = \hat{\mathbf{y}}10e^{j0.2z}$ (V/m), we deduce that $k = 0.2$ rad/m. Hence,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2} = 10\pi = 31.42 \text{ m}.$$

(b)

$$f = \frac{u_p}{\lambda} = \frac{1.5 \times 10^8}{31.42} = 4.77 \times 10^6 \text{ Hz} = 4.77 \text{ MHz}.$$

(c) From

$$u_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}}, \qquad \varepsilon_r = \frac{1}{\mu_r}\left(\frac{c}{u_p}\right)^2 = \frac{1}{2.4}\left(\frac{3}{1.5}\right)^2 = 1.67.$$

(d)

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \simeq 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{2.4}{1.67}} = 451.94 \quad (\Omega),$$

$$\widetilde{\mathbf{H}} = \frac{1}{\eta}(-\hat{\mathbf{z}}) \times \widetilde{\mathbf{E}} = \frac{1}{\eta}(-\hat{\mathbf{z}}) \times \hat{\mathbf{y}}10e^{j0.2z} = \hat{\mathbf{x}}22.13e^{j0.2z} \quad \text{(mA/m)},$$

$$\mathbf{H}(z,t) = \hat{\mathbf{x}}22.13\cos(\omega t + 0.2z) \quad \text{(mA/m)},$$

with $\omega = 2\pi f = 9.54\pi \times 10^6$ rad/s.

Problem 7.4 The electric field of a plane wave propagating in a nonmagnetic material is given by

$$\mathbf{E} = [\hat{\mathbf{y}}3\sin(2\pi \times 10^7 t - 0.4\pi x) + \hat{\mathbf{z}}4\cos(2\pi \times 10^7 t - 0.4\pi x)] \quad \text{(V/m)}.$$

Determine (a) the wavelength, (b) ε_r, and (c) $\mathbf{H}$.

Solution:

(a) Since $k = 0.4\pi$,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.4\pi} = 5 \text{ m}.$$

(b)

$$u_p = \frac{\omega}{k} = \frac{2\pi \times 10^7}{0.4\pi} = 5 \times 10^7 \text{ m/s}.$$

But

$$u_p = \frac{c}{\sqrt{\varepsilon_r}}.$$

Hence,

$$\varepsilon_r = \left(\frac{c}{u_p}\right)^2 = \left(\frac{3 \times 10^8}{5 \times 10^7}\right)^2 = 36.$$

(c)

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E} = \frac{1}{\eta}\hat{\mathbf{x}} \times [\hat{\mathbf{y}}3\sin(2\pi \times 10^7 t - 0.4\pi x) + \hat{\mathbf{z}}4\cos(2\pi \times 10^7 t - 0.4\pi x)]$$

$$= \hat{\mathbf{z}}\frac{3}{\eta}\sin(2\pi \times 10^7 t - 0.4\pi x) - \hat{\mathbf{y}}\frac{4}{\eta}\cos(2\pi \times 10^7 t - 0.4\pi x) \quad \text{(A/m)},$$

with

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} \simeq \frac{120\pi}{6} = 20\pi = 62.83 \quad (\Omega).$$

Problem 7.5 A wave radiated by a source in air is incident upon a soil surface, whereupon a part of the wave is transmitted into the soil medium. If the wavelength of the wave is 30 cm in air and 15 cm in the soil medium, what is the soil's relative permittivity? Assume the soil to be a very low loss medium.

Solution: From $\lambda = \lambda_0/\sqrt{\varepsilon_r}$,

$$\varepsilon_r = \left(\frac{\lambda_0}{\lambda}\right)^2 = \left(\frac{30}{15}\right)^2 = 4.$$

Problem 7.6 The electric field of a plane wave propagating in a lossless, nonmagnetic, dielectric material with $\varepsilon_r = 2.56$ is given by

$$\mathbf{E} = \hat{\mathbf{y}}\, 20\cos(8\pi \times 10^9 t - kz) \quad \text{(V/m)}.$$

Determine:

(a) f, u_p, λ, k, and η, and

(b) the magnetic field $\mathbf{H}$.

Solution:

(a)

$$\omega = 2\pi f = 8\pi \times 10^9 \text{ rad/s},$$
$$f = 4 \times 10^9 \text{ Hz} = 4 \text{ GHz},$$
$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.56}} = 1.875 \times 10^8 \text{ m/s},$$
$$\lambda = \frac{u_p}{f} = \frac{1.875 \times 10^8}{4 \times 10^9} = 4.69 \text{ cm},$$
$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4.69 \times 10^{-2}} = 134.04 \text{ rad/m},$$
$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.56}} = \frac{377}{1.6} = 235.62\ \Omega.$$

(b)

$$\begin{aligned}\mathbf{H} &= -\hat{\mathbf{x}}\frac{20}{\eta}\cos(8\pi\times 10^9 t - kz)\\ &= -\hat{\mathbf{x}}\frac{20}{235.62}\cos(8\pi\times 10^9 t - 134.04z)\\ &= -\hat{\mathbf{x}}\,8.49\times 10^{-2}\cos(8\pi\times 10^9 t - 134.04z) \quad \text{(A/m)}.\end{aligned}$$

Section 7-3: Wave Polarization

Problem 7.7 An RHC-polarized wave with a modulus of 2 (V/m) is traveling in free space in the negative z-direction. Write down the expression for the wave's electric field vector, given that the wavelength is 6 cm.

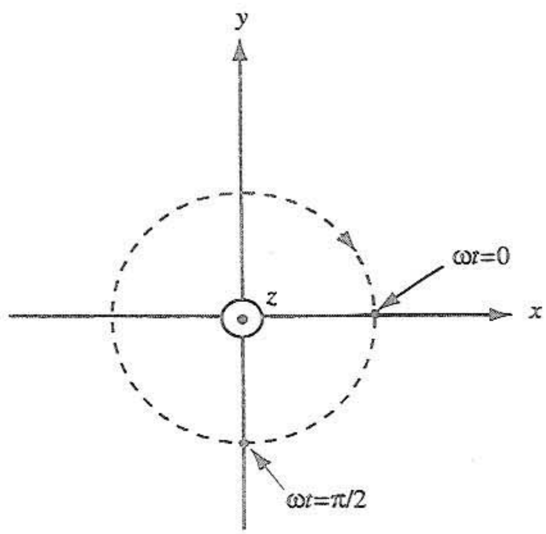

Figure P7.7: Locus of **E** versus time.

Solution: For an RHC wave traveling in $-\hat{\mathbf{z}}$, let us try the following:

$$\mathbf{E} = \hat{\mathbf{x}}a\cos(\omega t + kz) + \hat{\mathbf{y}}a\sin(\omega t + kz).$$

Modulus $|E| = \sqrt{a^2 + a^2} = a\sqrt{2} = 2$ (V/m). Hence,

$$a = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

Next, we need to check the sign of the $\hat{\mathbf{y}}$-component relative to that of the $\hat{\mathbf{x}}$-component. We do this by examining the locus of $\mathbf{E}$ versus t at $z = 0$: Since the wave is traveling along $-\hat{\mathbf{z}}$, when the thumb of the right hand is along $-\hat{\mathbf{z}}$ (into the page), the other four fingers point in the direction shown (clockwise as seen from above). Hence, we should reverse the sign of the $\hat{\mathbf{y}}$-component:

$$\mathbf{E} = \hat{\mathbf{x}}\sqrt{2}\cos(\omega t + kz) - \hat{\mathbf{y}}\sqrt{2}\sin(\omega t + kz) \quad \text{(V/m)}$$

with

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6\times 10^{-2}} = 104.72 \quad \text{(rad/m)},$$

and

$$\omega = kc = \frac{2\pi}{\lambda}\times 3\times 10^{8} = \pi\times 10^{10} \quad \text{(rad/s)}.$$

Problem 7.8 For a wave characterized by the electric field

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}a_x\cos(\omega t - kz) + \hat{\mathbf{y}}a_y\cos(\omega t - kz + \delta),$$

identify the polarization state, determine the polarization angles (γ, χ), and sketch the locus of $\mathbf{E}(0,t)$ for each of the following cases:

(a) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 0$,

(b) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 180°$,

(c) $a_x = 3$ V/m, $a_y = 3$ V/m, and $\delta = 45°$,

(d) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = -135°$.

Solution:

$$\psi_0 = \tan^{-1}(a_y/a_x), \quad \text{[Eq. (7.60)]},$$
$$\tan 2\gamma = (\tan 2\psi_0)\cos\delta \quad \text{[Eq. (7.59a)]},$$
$$\sin 2\chi = (\sin 2\psi_0)\sin\delta \quad \text{[Eq. (7.59b)]}.$$

Case	a_x	a_y	δ	ψ_0	γ	χ	Polarization State
(a)	3	4	0	53.13°	53.13°	0	Linear
(b)	3	4	180°	53.13°	−53.13°	0	Linear
(c)	3	3	45°	45°	45°	22.5°	Left elliptical
(d)	3	4	−135°	53.13°	−56.2°	−21.37°	Right elliptical

(a) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}4\cos(\omega t - kz)$.
(b) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) - \hat{\mathbf{y}}4\cos(\omega t - kz)$.
(c) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}3\cos(\omega t - kz + 45°)$.
(d) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}4\cos(\omega t - kz - 135°)$.

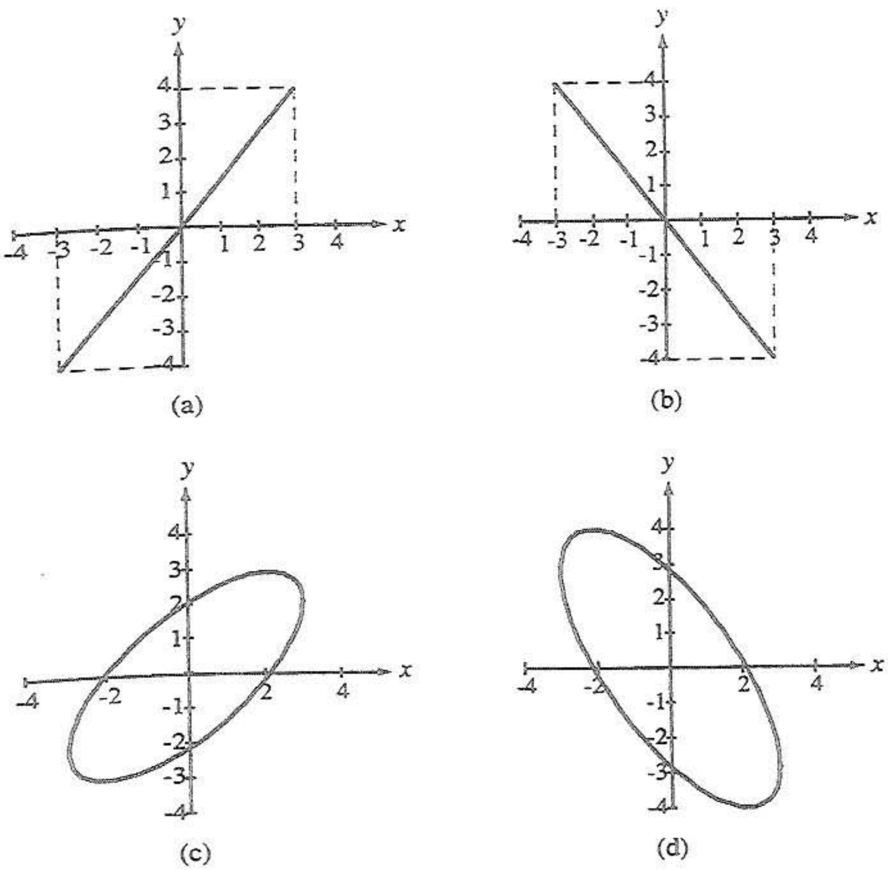

Figure P7.8: Plots of the locus of $\mathbf{E}(0,t)$.

Problem 7.9 The electric field of a uniform plane wave propagating in free space is given by $\widetilde{\mathbf{E}} = (\hat{\mathbf{x}} + j\hat{\mathbf{y}})20e^{-j\pi z/6}$ (V/m). Specify the modulus and direction of the electric field intensity at the $z = 0$ plane at $t = 0$, 5 and 10 ns.

Solution:

$$\begin{aligned}
\mathbf{E}(z,t) &= \Re e[\widetilde{\mathbf{E}} e^{j\omega t}] \\
&= \Re e[(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) 20 e^{-j\pi z/6} e^{j\omega t}] \\
&= \Re e[(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{j\pi/2}) 20 e^{-j\pi z/6} e^{j\omega t}] \\
&= \hat{\mathbf{x}} 20 \cos(\omega t - \pi z/6) + \hat{\mathbf{y}} 20 \cos(\omega t - \pi z/6 + \pi/2) \\
&= \hat{\mathbf{x}} 20 \cos(\omega t - \pi z/6) - \hat{\mathbf{y}} 20 \sin(\omega t - \pi z/6) \quad \text{(V/m)},
\end{aligned}$$

$$|\mathbf{E}| = \left[E_x^2 + E_y^2\right]^{1/2} = 20 \quad \text{(V/m)},$$

$$\psi = \tan^{-1}\left(\frac{E_y}{E_x}\right) = -(\omega t - \pi z/6).$$

From

$$f = \frac{c}{\lambda} = \frac{kc}{2\pi} = \frac{\pi/6 \times 3 \times 10^8}{2\pi} = 2.5 \times 10^7 \text{ Hz},$$

$$\omega = 2\pi f = 5\pi \times 10^7 \text{ rad/s}.$$

At $z = 0$,

$$\psi = -\omega t = -5\pi \times 10^7 t = \begin{cases} 0 & \text{at } t = 0, \\ -0.25\pi = -45^\circ & \text{at } t = 5 \text{ ns}, \\ -0.5\pi = -90^\circ & \text{at } t = 10 \text{ ns}. \end{cases}$$

Therefore, the wave is LHC polarized.

Problem 7.10 A linearly polarized plane wave of the form $\widetilde{\mathbf{E}} = \hat{\mathbf{x}} a_x e^{-jkz}$ can be expressed as the sum of an RHC polarized wave with magnitude a_R and an LHC polarized wave with magnitude a_L. Prove this statement by finding expressions for a_R and a_L in terms of a_x.

Solution:

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}} a_x e^{-jkz},$$

$$\text{RHC wave:} \quad \widetilde{\mathbf{E}}_R = a_R(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{-j\pi/2}) e^{-jkz} = a_R(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{-jkz},$$

$$\text{LHC wave:} \quad \widetilde{\mathbf{E}}_L = a_L(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{j\pi/2}) e^{-jkz} = a_L(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{-jkz},$$

$$\widetilde{\mathbf{E}} = \widetilde{\mathbf{E}}_R + \widetilde{\mathbf{E}}_L,$$

$$\hat{\mathbf{x}} a_x = a_R(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) + a_L(\hat{\mathbf{x}} + j\hat{\mathbf{y}}).$$

By equating real and imaginary parts, $a_x = a_R + a_L$, $0 = -a_R + a_L$, or $a_L = a_x/2$, $a_R = a_x/2$.

Problem 7.11 The electric field of an elliptically polarized plane wave is given by

$$\mathbf{E}(z,t) = [-\hat{\mathbf{x}}10\sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}}20\cos(\omega t - kz)] \quad \text{(V/m)}.$$

Determine (a) the polarization angles (γ, χ) and (b) the direction of rotation.

Solution:
(a)

$$\begin{aligned}\mathbf{E}(z,t) &= [-\hat{\mathbf{x}}10\sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}}20\cos(\omega t - kz)] \\ &= [\hat{\mathbf{x}}10\cos(\omega t - kz + 30^\circ) + \hat{\mathbf{y}}20\cos(\omega t - kz)] \quad \text{(V/m)}.\end{aligned}$$

Phasor form:

$$\widetilde{\mathbf{E}} = (\hat{\mathbf{x}}10e^{j30^\circ} + \hat{\mathbf{y}}20)e^{-jkz}.$$

Since δ is defined as the phase of E_y relative to that of E_x,

$$\begin{aligned}\delta &= -30^\circ, \\ \psi_0 &= \tan^{-1}\left(\frac{20}{10}\right) = 63.44^\circ, \\ \tan 2\gamma &= (\tan 2\psi_0)\cos\delta = -1.15 \quad \text{or} \quad \gamma = 65.5^\circ, \\ \sin 2\chi &= (\sin 2\psi_0)\sin\delta = -0.40 \quad \text{or} \quad \chi = -11.79^\circ.\end{aligned}$$

(b) Since $\chi < 0$, the wave is right-hand elliptically polarized.

Problem 7.12 Compare the polarization states of each of the following pairs of plane waves:

(a) wave 1: $\mathbf{E}_1 = \hat{\mathbf{x}}2\cos(\omega t - kz) + \hat{\mathbf{y}}2\sin(\omega t - kz)$,
wave 2: $\mathbf{E}_2 = \hat{\mathbf{x}}2\cos(\omega t + kz) + \hat{\mathbf{y}}2\sin(\omega t + kz)$,

(b) wave 1: $\mathbf{E}_1 = \hat{\mathbf{x}}2\cos(\omega t - kz) - \hat{\mathbf{y}}2\sin(\omega t - kz)$,
wave 2: $\mathbf{E}_2 = \hat{\mathbf{x}}2\cos(\omega t + kz) - \hat{\mathbf{y}}2\sin(\omega t + kz)$.

Solution:
(a)

$$\begin{aligned}\mathbf{E}_1 &= \hat{\mathbf{x}}2\cos(\omega t - kz) + \hat{\mathbf{y}}2\sin(\omega t - kz) \\ &= \hat{\mathbf{x}}2\cos(\omega t - kz) + \hat{\mathbf{y}}2\cos(\omega t - kz - \pi/2), \\ \widetilde{\mathbf{E}}_1 &= \hat{\mathbf{x}}2e^{-jkz} + \hat{\mathbf{y}}2e^{-jkz}e^{-j\pi/2},\end{aligned}$$

$$\psi_0 = \tan^{-1}\left(\frac{ay}{ax}\right) = \tan^{-1} 1 = 45^\circ,$$
$$\delta = -\pi/2.$$

Hence, wave 1 is RHC.

Similarly,

$$\widetilde{\mathbf{E}}_2 = \hat{\mathbf{x}}2e^{jkz} + \hat{\mathbf{y}}2e^{jkz}e^{-j\pi/2}.$$

Wave 2 has the same magnitude and phases as wave 1 except that its direction is along $-\hat{\mathbf{z}}$ instead of $+\hat{\mathbf{z}}$. Hence, the locus of rotation of $\mathbf{E}$ will match the left hand instead of the right hand. Thus, wave 2 is LHC.

(b)

$$\mathbf{E}_1 = \hat{\mathbf{x}}2\cos(\omega t - kz) - \hat{\mathbf{y}}2\sin(\omega t - kz),$$
$$\widetilde{\mathbf{E}}_1 = \hat{\mathbf{x}}2e^{-jkz} + \hat{\mathbf{y}}2e^{-jkz}e^{j\pi/2}.$$

Wave 1 is LHC.

$$\widetilde{\mathbf{E}}_2 = \hat{\mathbf{x}}2e^{jkz} + \hat{\mathbf{y}}2e^{jkz}e^{j\pi/2}.$$

Reversal of direction of propagation (relative to wave 1) makes wave 2 RHC.

Problem 7.13 Plot the locus of $\mathbf{E}(0,t)$ for a plane wave with

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\sin(\omega t + kz) + \hat{\mathbf{y}}2\cos(\omega t + kz).$$

Determine the polarization state from your plot.

Solution:

$$\mathbf{E} = \hat{\mathbf{x}}\sin(\omega t + kz) + \hat{\mathbf{y}}2\cos(\omega t + kz).$$

Wave direction is $-\hat{\mathbf{z}}$. At $z = 0$,

$$\mathbf{E} = \hat{\mathbf{x}}\sin\omega t + \hat{\mathbf{y}}2\cos\omega t.$$

Tip of $\mathbf{E}$ rotates in accordance with right hand (with thumb pointing along $-\hat{\mathbf{z}}$). Hence, wave state is RHE.

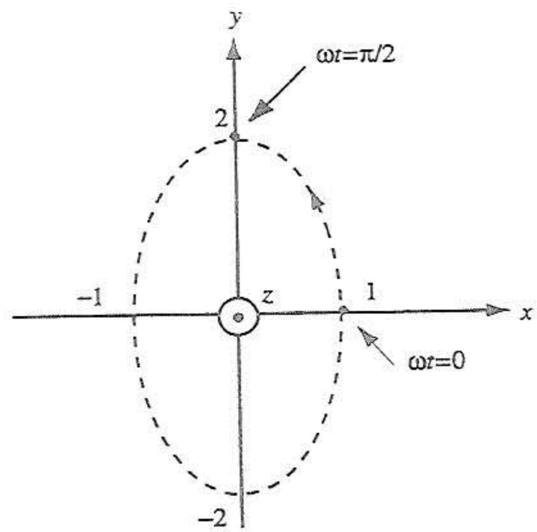

Figure P7.13: Locus of E versus time.

Sections 7-4: Propagation in a Lossy Medium

Problem 7.14 For each of the following combination of parameters, determine if the material is a low-loss dielectric, a quasi-conductor, or a good conductor, and then calculate α, β, λ, u_p, and η_c:

(a) glass with $\mu_r = 1$, $\varepsilon_r = 5$, and $\sigma = 10^{-12}$ S/m at 10 GHz,

(b) animal tissue with $\mu_r = 1$, $\varepsilon_r = 12$, and $\sigma = 0.3$ S/m at 100 MHz,

(c) wood with $\mu_r = 1$, $\varepsilon_r = 3$, and $\sigma = 10^{-4}$ S/m at 1 kHz.

Solution: Using equations given in Table 7-1:

	Case (a)	Case (b)	Case (c)
$\sigma/\omega\varepsilon$	3.6×10^{-13}	4.5	600
Type	low-loss dielectric	quasi-conductor	good conductor
α	8.42×10^{-11} Np/m	9.75 Np/m	6.3×10^{-4} Np/m
β	468.3 rad/m	12.16 rad/m	6.3×10^{-4} rad/m
λ	1.34 cm	51.69 cm	10 km
u_p	1.34×10^{8} m/s	0.52×10^{8} m/s	0.1×10^{8} m/s
η_c	$\simeq 168.5\ \Omega$	$39.54 + j31.72\ \Omega$	$6.28(1+j)\ \Omega$

Problem 7.15 Dry soil is characterized by $\varepsilon_r = 2.5$, $\mu_r = 1$, and $\sigma = 10^{-4}$ (S/m). At each of the following frequencies, determine if dry soil may be considered a good conductor, a quasi-conductor, or a low-loss dielectric, and then calculate α, β, λ, μ_p, and η_c:

(a) 60 Hz,
(b) 1 kHz,
(c) 1 MHz,
(d) 1 GHz.

Solution: $\varepsilon_r = 2.5$, $\mu_r = 1$, $\sigma = 10^{-4}$ S/m.

$f \rightarrow$	60 Hz	1 kHz	1 MHz	1 GHz
$\dfrac{\varepsilon''}{\varepsilon'} = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{\sigma}{2\pi f \varepsilon_r \varepsilon_0}$	1.2×10^4	720	0.72	7.2×10^{-4}
Type of medium	Good conductor	Good conductor	Quasi-conductor	Low-loss dielectric
α (Np/m)	1.54×10^{-4}	6.28×10^{-4}	1.13×10^{-2}	1.19×10^{-2}
β (rad/m)	1.54×10^{-4}	6.28×10^{-4}	3.49×10^{-2}	33.14
λ (m)	4.08×10^4	10^4	180	0.19
u_p (m/s)	2.45×10^6	10^7	1.8×10^8	1.9×10^8
η_c (Ω)	$1.54(1+j)$	$6.28(1+j)$	$204.28 + j65.89$	238.27

Problem 7.16 In a medium characterized by $\varepsilon_r = 9$, $\mu_r = 1$, and $\sigma = 0.1$ S/m, determine the phase angle by which the magnetic field leads the electric field at 100 MHz.

Solution: The phase angle by which the magnetic field leads the electric field is $-\theta_\eta$ where θ_η is the phase angle of η_c.

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.1 \times 36\pi}{2\pi \times 10^8 \times 10^{-9} \times 9} = 2.$$

Hence, quasi-conductor.

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} = \frac{120\pi}{\sqrt{\varepsilon_r}}\left(1 - j\frac{\sigma}{\omega\varepsilon_0\varepsilon_r}\right)^{-1/2}$$
$$= 125.67(1 - j2)^{-1/2} = 71.49 + j44.18 = 84.04\angle 31.72^\circ.$$

Therefore $\theta_\eta = 31.72°$.

Since $\mathbf{H} = (1/\eta_c)\hat{\mathbf{k}} \times \mathbf{E}$, $\mathbf{H}$ leads $\mathbf{E}$ by $-\theta_\eta$, or by $-31.72°$. In other words, $\mathbf{H}$ lags $\mathbf{E}$ by 31.72°.

Problem 7.17 Generate a plot for the skin depth δ_s versus frequency for seawater for the range from 1 kHz to 10 GHz (use log-log scales). The constitutive parameters of seawater are $\mu_r = 1$, $\varepsilon_r = 80$ and $\sigma = 4$ S/m.

Solution:

$$\delta_s = \frac{1}{\alpha} = \frac{1}{\omega}\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right]^{-1/2},$$

$$\omega = 2\pi f,$$

$$\mu\varepsilon' = \mu_0\varepsilon_0\varepsilon_r = \frac{\varepsilon_r}{c^2} = \frac{80}{c^2} = \frac{80}{(3\times 10^8)^2},$$

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{\omega\varepsilon_0\varepsilon_r} = \frac{4\times 36\pi}{2\pi f \times 10^{-9}\times 80} = \frac{72}{80f}\times 10^9.$$

See Fig. P7.17 for plot of δ_s versus frequency.

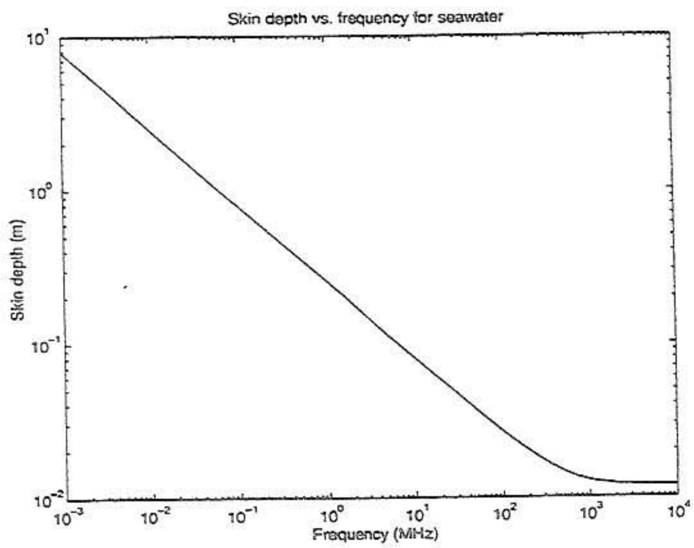

Figure P7.17: Skin depth versus frequency for seawater.

Problem 7.18 Ignoring reflection at the air-soil boundary, if the amplitude of a 2-GHz incident wave is 10 V/m at the surface of a wet soil medium, at what depth will it be down to 1 mV/m? Wet soil is characterized by $\mu_r = 1$, $\varepsilon_r = 16$, and $\sigma = 5\times10^{-4}$ S/m.

Solution:

$$E(z) = E_0 e^{-\alpha z} = 10e^{-\alpha z},$$

$$\frac{\sigma}{\omega\varepsilon} = \frac{5\times10^{-4}\times36\pi}{2\pi\times2\times10^9\times10^{-9}\times16} = 2.8\times10^{-4}.$$

Hence, medium is a low-loss dielectric.

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2}\cdot\frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{5\times10^{-4}\times120\pi}{2\times\sqrt{16}} = 0.024 \quad \text{(Np/m)},$$

$$10^{-3} = 10e^{-0.024z}, \quad \ln 10^{-4} = -0.024z,$$

$$z = 383.76 \text{ m}.$$

Problem 7.19 The skin depth of a certain nonmagnetic conducting material is 2 μm at 5 GHz. Determine the phase velocity in the material.

Solution: For a good conductor, $\alpha = \beta$, and for any material $\delta_s = 1/\alpha$. Hence,

$$u_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = 2\pi f\delta_s = 2\pi\times5\times10^9\times2\times10^{-6} = 6.28\times10^4 \quad \text{(m/s)}.$$

Problem 7.20 Based on wave attenuation and reflection measurements conducted at 1 MHz, it was determined that the intrinsic impedance of a certain medium is $28.1\angle 45^\circ$ (Ω) and the skin depth is 5 m. Determine (a) the conductivity of the material, (b) the wavelength in the medium, and (c) the phase velocity.

Solution:

(a) Since the phase angle of η_c is 45°, the material is a good conductor. Hence,

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = 28.1e^{j45^\circ} = 28.1\cos45^\circ + j28.1\sin45^\circ,$$

or

$$\frac{\alpha}{\sigma} = 28.1\cos45^\circ = 19.87.$$

Since $\alpha = 1/\delta_s = 1/5 = 0.2$ Np/m,

$$\sigma = \frac{\alpha}{19.87} = \frac{0.2}{19.87} = 0.01 \text{ S/m}.$$

(b) Since $\alpha = \beta$ for a good conductor, and $\alpha = 0.2$, it follows that $\beta = 0.2$. Therefore,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.2} = 10\pi = 31.4 \text{ m}.$$

(c) $u_p = f\lambda = 10^6 \times 31.4 = 3.14 \times 10^7$ m/s.

Problem 7.21 The electric field of a plane wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{z}}25e^{-30x}\cos(2\pi \times 10^9 t - 40x) \quad \text{(V/m)}.$$

Obtain the corresponding expression for $\mathbf{H}$.

Solution: From the given expression for $\mathbf{E}$,

$$\omega = 2\pi \times 10^9 \quad \text{(rad/s)},$$
$$\alpha = 30 \quad \text{(Np/m)},$$
$$\beta = 40 \quad \text{(rad/m)}.$$

From (7.65a) and (7.65b),

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon' = -\omega^2\mu_0\varepsilon_0\varepsilon'_r = -\frac{\omega^2}{c^2}\varepsilon'_r,$$
$$2\alpha\beta = \omega^2\mu\varepsilon'' = \frac{\omega^2}{c^2}\varepsilon''_r.$$

Using the above values for ω, α, and β, we obtain the following:

$$\varepsilon'_r = 1.6,$$
$$\varepsilon''_r = 5.47.$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$$
$$= \frac{\varepsilon_0}{\sqrt{\varepsilon'_r}}\left(1 - j\frac{\varepsilon''_r}{\varepsilon'_r}\right)^{-1/2} = \frac{377}{\sqrt{1.6}}\left(1 - j\frac{5.47}{1.6}\right)^{-1/2} = 157.9\,e^{j36.85^\circ} \quad (\Omega).$$

$$\widetilde{\mathbf{E}} = \hat{\mathbf{z}}25e^{-30x}e^{-j40x},$$

$$\widetilde{\mathbf{H}} = \frac{1}{\eta_c}\hat{\mathbf{k}} \times \widetilde{\mathbf{E}} = \frac{1}{157.9\,e^{j36.85°}}\hat{\mathbf{x}} \times \hat{\mathbf{z}}25e^{-30x}e^{-j40x} = -\hat{\mathbf{y}}0.16\,e^{-30x}e^{-40x}e^{-j36.85°},$$

$$\mathbf{H} = \Re e\{\widetilde{\mathbf{H}}e^{j\omega t}\} = -\hat{\mathbf{y}}0.16\,e^{-30x}\cos(2\pi \times 10^9 t - 40x - 36.85°) \quad \text{(A/m)}.$$

Section 7-5: Current Flow in Conductors

Problem 7.22 In a nonmagnetic, lossy, dielectric medium, a 300-MHz plane wave is characterized by the magnetic field phasor

$$\widetilde{\mathbf{H}} = (\hat{\mathbf{x}} - j4\hat{\mathbf{z}})e^{-2y}e^{-j9y} \quad \text{(A/m)}.$$

Obtain time-domain expressions for the electric and magnetic field vectors.

Solution:

$$\widetilde{\mathbf{E}} = -\eta_c\hat{\mathbf{k}} \times \widetilde{\mathbf{H}}.$$

To find η_c, we need ε' and ε''. From the given expression for $\widetilde{\mathbf{H}}$,

$$\alpha = 2 \quad \text{(Np/m)},$$
$$\beta = 9 \quad \text{(rad/m)}.$$

Also, we are given than $f = 300$ MHz $= 3 \times 10^8$ Hz. From (7.65a),

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon',$$

$$4 - 81 = -(2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \varepsilon_r' \times \frac{10^{-9}}{36\pi},$$

whose solution gives

$$\varepsilon_r' = 1.95.$$

Similarly, from (7.65b),

$$2\alpha\beta = \omega^2\mu\varepsilon'',$$

$$2 \times 2 \times 9 = (2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \varepsilon_r'' \times \frac{10^{-9}}{36\pi},$$

which gives

$$\varepsilon_r'' = 0.91.$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$$
$$= \frac{\eta_0}{\sqrt{\varepsilon'_r}}\left(1 - j\frac{0.91}{1.95}\right)^{-1/2} = \frac{377}{\sqrt{1.95}}(0.93 + j0.21) = 256.9\,e^{j12.6^\circ}.$$

Hence,

$$\begin{aligned}
\widetilde{\mathbf{E}} &= -256.9\,e^{j12.6^\circ}\,\hat{\mathbf{y}} \times (\hat{\mathbf{x}} - j4\hat{\mathbf{z}})e^{-2y}e^{-j9y} \\
&= (\hat{\mathbf{x}}\,j4 + \hat{\mathbf{z}})\,256.9\,e^{-2y}e^{-j9y}e^{j12.6^\circ} \\
&= (\hat{\mathbf{x}}4e^{j\pi/2} + \hat{\mathbf{z}})\,256.9\,e^{-2y}e^{-j9y}e^{j12.6^\circ}, \\
\mathbf{E} &= \mathfrak{Re}\{\widetilde{\mathbf{E}}e^{j\omega t}\} \\
&= \hat{\mathbf{x}}\,1.03 \times 10^3 e^{-2y}\cos(\omega t - 9y + 102.6^\circ) \\
&\quad + \hat{\mathbf{z}}\,256.9\,e^{-2y}\cos(\omega t - 9y + 12.6^\circ) \quad \text{(V/m)}, \\
\mathbf{H} &= \mathfrak{Re}\{\widetilde{\mathbf{H}}e^{j\omega t}\} \\
&= \mathfrak{Re}\{(\hat{\mathbf{x}} + j4\hat{\mathbf{z}})e^{-2y}e^{-j9y}e^{j\omega t}\} \\
&= \hat{\mathbf{x}}\,e^{-2y}\cos(\omega t - 9y) + \hat{\mathbf{z}}\,4e^{-2y}\sin(\omega t - 9y) \quad \text{(A/m)}.
\end{aligned}$$

Problem 7.23 A rectangular copper block is 30 cm in height (along z). In response to a wave incident upon the block from above, a current is induced in the block in the positive x-direction. Determine the ratio of the a-c resistance of the block to its d-c resistance at 1 kHz. The relevant properties of copper are given in Appendix B.

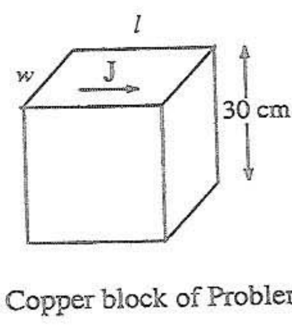

Figure P7.23: Copper block of Problem 7.23.

Solution:

$$\text{d-c resistance } R_{dc} = \frac{l}{\sigma A} = \frac{l}{0.3\,\sigma w},$$
$$\text{a-c resistance } R_{ac} = \frac{l}{\sigma w \delta_s}.$$

$$\frac{R_{ac}}{R_{dc}} = \frac{0.3}{\delta_s} = 0.3\sqrt{\pi f \mu \sigma} = 0.3[\pi \times 10^3 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7]^{1/2} = 143.55.$$

Problem 7.24 The inner and outer conductors of a coaxial cable have radii of 0.5 cm and 1 cm, respectively. The conductors are made of copper with $\varepsilon_r = 1$, $\mu_r = 1$ and $\sigma = 5.8 \times 10^7$ S/m, and the outer conductor is 0.1 cm thick. At 10 MHz:

(a) Are the conductors thick enough to be considered infinitely thick so far as the flow of current through them is concerned?

(b) Determine the surface resistance R_s.

(c) Determine the a-c resistance per unit length of the cable.

Solution:

(a) From Eqs. (7.72) and (7.77b),

$$\delta_s = [\pi f \mu \sigma]^{-1/2} = [\pi \times 10^7 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7]^{-1/2} = 0.021 \text{ mm}.$$

Hence,

$$\frac{d}{\delta_s} = \frac{0.1 \text{ cm}}{0.021 \text{ mm}} \approx 50.$$

Hence, conductor is plenty thick.

(b) From Eq. (7.92a),

$$R_s = \frac{1}{\sigma \delta_s} = \frac{1}{5.8 \times 10^7 \times 2.1 \times 10^{-5}} = 8.2 \times 10^{-4}\ \Omega.$$

(c) From Eq. (7.96),

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{8.2 \times 10^{-4}}{2\pi}\left(\frac{1}{5 \times 10^{-3}} + \frac{1}{10^{-2}}\right) = 0.039 \quad (\Omega/\text{m}).$$

Section 7-6: EM Power Density

Problem 7.25 The magnetic field of a plane wave traveling in air is given by $\mathbf{H} = \hat{\mathbf{x}}25\sin(2\pi\times 10^7 t - ky)$ (mA/m). Determine the average power density carried by the wave.

Solution:

$$\mathbf{H} = \hat{\mathbf{x}}25\sin(2\pi\times 10^7 t - ky) \quad \text{(mA/m)},$$
$$\mathbf{E} = -\eta_0\hat{\mathbf{y}}\times\mathbf{H} = \hat{\mathbf{z}}\eta_0 25\sin(2\pi\times 10^7 t - ky) \quad \text{(mV/m)},$$
$$\mathbf{S}_{\text{av}} = (\hat{\mathbf{z}}\times\hat{\mathbf{x}})\frac{\eta_0(25)^2}{2}\times 10^{-6} = \hat{\mathbf{y}}\frac{120\pi}{2}(25)^2\times 10^{-6} = \hat{\mathbf{y}}0.12 \quad (\text{W/m}^2).$$

Problem 7.26 A wave traveling in a nonmagnetic medium with $\varepsilon_r = 9$ is characterized by an electric field given by

$$\mathbf{E} = [\hat{\mathbf{y}}3\cos(\pi\times 10^7 t + kx) - \hat{\mathbf{z}}2\cos(\pi\times 10^7 t + kx)] \quad \text{(V/m)}.$$

Determine the direction of wave travel and the average power density carried by the wave.

Solution:

$$\eta \simeq \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi \quad (\Omega).$$

The wave is traveling in the negative x-direction.

$$\mathbf{S}_{\text{av}} = -\hat{\mathbf{x}}\frac{[3^2+2^2]}{2\eta} = -\hat{\mathbf{x}}\frac{13}{2\times 40\pi} = -\hat{\mathbf{x}}0.05 \quad (\text{W/m}^2).$$

Problem 7.27 The electric-field phasor of a uniform plane wave traveling downward in water is given by

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}}10e^{-0.2z}e^{-j0.2z} \quad \text{(V/m)},$$

where $\hat{\mathbf{z}}$ is the downward direction and $z = 0$ is the water surface. If $\sigma = 4$ S/m,

(a) obtain an expression for the average power density,

(b) determine the attenuation rate, and

(c) determine the depth at which the power density has been reduced by 40 dB.

Solution:

(a) Since $\alpha = \beta = 0.2$, the medium is a good conductor.

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{0.2}{4} = (1+j)0.05 = 0.0707e^{j45^\circ} \quad (\Omega).$$

From Eq. (7.109),

$$\mathbf{S}_{av} = \hat{\mathbf{z}}\frac{|E_0|^2}{2|\eta_c|}e^{-2\alpha z}\cos\theta_\eta = \hat{\mathbf{z}}\frac{100}{2\times 0.0707}e^{-0.4z}\cos 45^\circ = \hat{\mathbf{z}}500e^{-0.4z} \quad (\mathrm{W/m^2}).$$

(b) $A = -8.68\alpha z = -8.68 \times 0.2z = -1.74z$ (dB).

(c) 40 dB is equivalent to 10^{-4}. Hence,

$$10^{-4} = e^{-2\alpha z} = e^{-0.4z}, \qquad \ln(10^{-4}) = -0.4z,$$

or $z = 23.03$ m.

Problem 7.28 The amplitudes of an elliptically polarized plane wave traveling in a lossless, nonmagnetic medium with $\varepsilon_r = 4$ are $H_{y0} = 6$ (mA/m) and $H_{z0} = 8$ (mA/m). Determine the average power flowing through an aperture in the y-z plane if its area is 20 m^2.

Solution:

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{4}} = 60\pi = 188.5\ \Omega,$$

$$\mathbf{S}_{av} = \hat{\mathbf{x}}\frac{\eta}{2}[H_{y0}^2 + H_{z0}^2] = \hat{\mathbf{x}}\frac{188.5}{2}[36+64]\times 10^{-6} = 9.43 \quad (\mathrm{mW/m^2}),$$

$$P = S_{av}A = 9.43\times 10^{-3}\times 20 = 0.19\ \mathrm{W}.$$

Problem 7.29 A wave traveling in a lossless, nonmagnetic medium has an electric field amplitude of 24.56 V/m and an average power density of 4 W/m^2. Determine the phase velocity of the wave.

Solution:

$$S_{av} = \frac{|E_0|^2}{2\eta}, \qquad \eta = \frac{|E_0|^2}{2S_{av}},$$

or

$$\eta = \frac{(24.56)^2}{2\times 4} = 75.4\ \Omega.$$

But

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{\varepsilon_r}}, \qquad \varepsilon_r = \left(\frac{377}{75.4}\right)^2 = 25.$$

Hence,

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8}{5} = 6\times10^7 \text{ m/s}.$$

Problem 7.30 At microwave frequencies, the power density considered safe for human exposure is 1 (mW/cm^2). A radar radiates a wave with an electric field amplitude E that decays with distance as $E(R) = (3{,}000/R)$ (V/m), where R is the distance in meters. What is the radius of the unsafe region?

Solution:

$$S_{av} = \frac{|E(R)|^2}{2\eta_0}, \qquad 1 \text{ (mW/cm}^2) = 10^{-3} \text{ W/cm}^2 = 10 \text{ W/m}^2,$$

$$10 = \left(\frac{3\times10^3}{R}\right)^2 \times \frac{1}{2\times120\pi} = \frac{1.2\times10^4}{R^2},$$

$$R = \left(\frac{1.2\times10^4}{10}\right)^{1/2} = 34.64 \text{ m}.$$

Problem 7.31 Consider the imaginary rectangular box shown in Fig. 7-19 (P7.31).

(a) Determine the net power flux $P(t)$ entering the box due to a plane wave in air given by

$$\mathbf{E} = \hat{\mathbf{x}}E_0\cos(\omega t - ky) \quad \text{(V/m)}.$$

(b) Determine the net time-average power entering the box.

Solution:

(a)

$$\mathbf{E} = \hat{\mathbf{x}}E_0\cos(\omega t - ky),$$

$$\mathbf{H} = -\hat{\mathbf{z}}\frac{E_0}{\eta_0}\cos(\omega t - ky).$$

$$\mathbf{S}(t) = \mathbf{E}\times\mathbf{H} = \hat{\mathbf{y}}\frac{E_0^2}{\eta_0}\cos^2(\omega t - ky),$$

$$P(t) = S(t)A|_{y=0} - S(t)A|_{y=b} = \frac{E_0^2}{\eta_0}ac[\cos^2\omega t - \cos^2(\omega t - kb)].$$

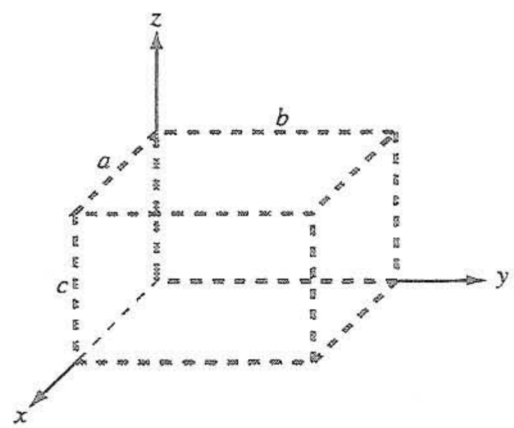

Figure P7.31: Imaginary rectangular box of Problems 7.31 and 7.32.

(b)

$$P_{av} = \frac{1}{T}\int_0^T P(t)\,dt.$$

where $T = 2\pi/\omega$.

$$P_{av} = \frac{E_0^2 ac}{\eta_0}\left\{\frac{\omega}{2\pi}\int_0^{2\pi/\omega}[\cos^2\omega t - \cos^2(\omega t - kb)]\,dt\right\} = 0.$$

Net average energy entering the box is zero, which is as expected since the box is in a lossless medium (air).

Problem 7.32 Repeat Problem 7.31 for a wave traveling in a lossy medium in which

$$\mathbf{E} = \hat{\mathbf{x}}100e^{-30y}\cos(2\pi\times10^9 t - 40y) \quad \text{(V/m)},$$
$$\mathbf{H} = -\hat{\mathbf{z}}0.64e^{-30y}\cos(2\pi\times10^9 t - 40y - 36.85^\circ) \quad \text{(A/m)}.$$

The box has dimensions $A = 1$ cm, $b = 2$ cm, and $c = 0.5$ cm.

Solution:

(a)

$$\begin{aligned}\mathbf{S}(t) &= \mathbf{E}\times\mathbf{H}\\ &= \hat{\mathbf{x}}100e^{-30y}\cos(2\pi\times10^9 t - 40y)\\ &\quad\times(-\hat{\mathbf{z}}0.64)e^{-30y}\cos(2\pi\times10^9 t - 40y - 36.85^\circ)\\ &= \hat{\mathbf{y}}64e^{-60y}\cos(2\pi\times10^9 t - 40y)\cos(2\pi\times10^9 t - 40y - 36.85^\circ).\end{aligned}$$

Using the identity $\cos\theta\cos\phi = \frac{1}{2}[\cos(\theta+\phi)+\cos(\theta-\phi)]$,

$$S(t) = \frac{64}{2} e^{-60y}[\cos(4\pi\times 10^9 t - 80y - 36.85^\circ) + \cos 36.85^\circ],$$
$$P(t) = S(t)A|_{y=0} - S(t)A|_{y=b}$$
$$= 32ac\{[\cos(4\pi\times 10^9 t - 36.85^\circ) + \cos 36.85^\circ]$$
$$- e^{-60b}[\cos(4\pi\times 10^9 t - 80y - 36.85^\circ) + \cos 36.85^\circ]\}.$$

(b)

$$P_{av} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} P(t)\,dt.$$

The average of $\cos(\omega t+\theta)$ over a period T is equal to zero, regardless of the value of θ. Hence,

$$P_{av} = 32ac(1-e^{-60b})\cos 36.85^\circ.$$

With $a = 1$ cm, $b = 2$ cm, and $c = 0.5$ cm,

$$P_{av} = 8.95\times 10^{-4} \quad \text{(W)}.$$

This is the average power absorbed by the lossy material in the box.

Problem 7.33 Given a wave with

$$\mathbf{E} = \hat{\mathbf{x}}E_0\cos(\omega t - kz),$$

calculate:

(a) the time-average electric energy density

$$(w_e)_{av} = \frac{1}{T}\int_0^T w_e\,dt = \frac{1}{2T}\int_0^T \varepsilon E^2\,dt,$$

(b) the time-average magnetic energy density

$$(w_m)_{av} = \frac{1}{T}\int_0^T w_m\,dt = \frac{1}{2T}\int_0^T \mu H^2\,dt,$$

and

(c) show that $(w_e)_{av} = (w_m)_{av}$.

Solution:

(a)

$$(w_e)_{av} = \frac{1}{2T}\int_0^T \varepsilon E_0^2\cos^2(\omega t - kz)\,dt.$$

With $T = \frac{2\pi}{\omega}$,

$$
\begin{aligned}
(w_e)_{av} &= \frac{\omega\varepsilon E_0^2}{4\pi} \int_0^{2\pi/\omega} \cos^2(\omega t - kz)\,dt \\
&= \frac{\varepsilon E_0^2}{4\pi} \int_0^{2\pi} \cos^2(\omega t - kz)\,d(\omega t) \\
&= \frac{\varepsilon E_0^2}{4}.
\end{aligned}
$$

(b)

$$
\mathbf{H} = \hat{\mathbf{y}} \frac{E_0}{\eta} \cos(\omega t - kz).
$$

$$
\begin{aligned}
(w_m)_{av} &= \frac{1}{2T} \int_0^T \mu H^2\,dt \\
&= \frac{1}{2T} \int_0^T \mu \frac{E_0^2}{\eta^2} \cos^2(\omega t - kz)\,dt \\
&= \frac{\mu E_0^2}{4\eta^2}.
\end{aligned}
$$

(c)

$$
(w_m)_{av} = \frac{\mu E_0^2}{4\eta^2} = \frac{\mu E_0^2}{4\left(\frac{\mu}{\varepsilon}\right)} = \frac{\varepsilon E_0^2}{4} = (w_e)_{av}.
$$