Chapter 9

Sections 9-1 and 9-2: Short Dipole and Antenna Radiation Characteristics

Problem 9.1 A center-fed Hertzian dipole is excited by a current $I_0 = 10$ A. If the dipole is $\lambda/50$ in length, determine the maximum radiated power density at a distance of 1 km.

Solution: From Eq. (9.14), the maximum power density radiated by a Hertzian dipole is given by

$$S_0 = \frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2} = \frac{377 \times (2\pi/\lambda)^2 \times 10^2 \times (\lambda/50)^2}{32\pi^2 (10^3)^2}$$
$$= 1.9 \times 10^{-6} \text{ W/m}^2 = 1.9 \quad (\mu\text{W/m}^2).$$

Problem 9.2 A 1-m-long dipole is excited by a 1-MHz current with an amplitude of 12 A. What is the average power density radiated by the dipole at a distance of 5 km in a direction that is 30° from the dipole axis?

Solution: At 1 MHz, $\lambda = c/f = 3 \times 10^8 / 10^6 = 300$ m. Hence $l/\lambda = 1/300$, and therefore the antenna is a Hertzian dipole. From Eq. (9.12),

$$S(R,\theta) = \left(\frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2}\right) \sin^2\theta$$
$$= \frac{120\pi \times (2\pi/300)^2 \times 12^2 \times 1^2}{32\pi^2 \times (5 \times 10^3)^2} \sin^2 30° = 7.54 \times 10^{-10} \quad (\text{W/m}^2).$$

Problem 9.3 Determine the (a) direction of maximum radiation, (b) directivity, (c) beam solid angle, and (d) half-power beamwidth in the x–z plane for an antenna whose normalized radiation intensity is given by

$$F(\theta,\phi) = \begin{cases} 1, & \text{for } 0 \le \theta \le 60° \text{and } 0 \le \phi \le 2\pi, \\ 0, & \text{elsewhere.} \end{cases}$$

Suggestion: Sketch the pattern prior to calculating the desired quantities.

Solution: The direction of maximum radiation is a circular cone 120° wide centered around the $+\hat{\mathbf{z}}$-axis. From Eq. (9.23),

$$D = \frac{4\pi}{\iint_{4\pi} F \, d\Omega} = \frac{4\pi}{\int_0^{2\pi}\int_0^{60°} \sin\theta \, d\theta \, d\phi} = \frac{4\pi}{2\pi(-\cos\theta)|_{0°}^{60°}} = \frac{2}{-\frac{1}{2}+1} = 4 = 6 \text{ dB},$$

$$\Omega_p = \frac{4\pi \text{ sr}}{D} = \frac{4\pi \text{ sr}}{4} = \pi \quad \text{(sr)}.$$

The half power beamwidth is $\beta = 120°$.

Problem 9.4 Repeat Problem 9.3 for an antenna with

$$F(\theta,\phi) = \begin{cases} \sin^2\theta\cos^2\phi, & \text{for } 0 \le \theta \le \pi \text{ and } -\pi/2 \le \phi \le \pi/2, \\ 0, & \text{elsewhere.} \end{cases}$$

Solution: The direction of maximum radiation is the $+\hat{\mathbf{x}}$-axis (where $\theta = \pi/2$ and $\phi = 0$). From Eq. (9.23),

$$\begin{aligned} D &= \frac{4\pi}{\iint_{4\pi} F\,d\Omega} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2}\int_0^\pi \sin^2\theta\cos^2\phi\sin\theta\,d\theta\,d\phi} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2}\cos^2\phi\,d\phi\int_0^\pi \sin^3\theta\,d\theta} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2}\frac{1}{2}(1+\cos 2\phi)\,d\phi\int_{-1}^{1}(1-x^2)\,dx} \\ &= \frac{4\pi}{\frac{1}{2}\left(\phi+\frac{1}{2}\sin 2\phi\right)\Big|_{-\pi/2}^{\pi/2}(x-x^3/3)\Big|_{-1}^{1}} = \frac{4\pi}{\frac{1}{2}\pi(4/3)} = 6 = 7.8 \text{ dB}, \\ \Omega_p &= \frac{4\pi \text{ sr}}{D} = \frac{4\pi \text{ sr}}{6} = \frac{2}{3}\pi \quad \text{(sr)}. \end{aligned}$$

In the x-z plane, $\phi = 0$ and the half power beamwidth is 90°, since $\sin^2(45°) = \sin^2(135°) = \frac{1}{2}$.

Problem 9.5 A 2-m-long center-fed dipole antenna operates in the AM broadcast band at 1 MHz. The dipole is made of copper wire with a radius of 1 mm.

(a) Determine the radiation efficiency of the antenna.

(b) What is the antenna gain in dB?

(c) What antenna current is required so that the antenna would radiate 20 W, and how much power will the generator have to supply to the antenna?

Solution:

(a) Following Example 9-3, $\lambda = c/f = (3 \times 10^8 \text{ m/s})/(10^6 \text{ Hz}) = 300$ m. As $l/\lambda = (2 \text{ m})/(300 \text{ m}) = 6.7 \times 10^{-3}$, this antenna is a short (Hertzian) dipole. Thus, from respectively Eqs. (9.35), (9.32), and (9.31),

$$R_{\text{rad}} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2(6.7 \times 10^{-3})^2 = 35 \quad (\text{m}\Omega),$$

$$R_{\text{loss}} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2 \text{ m}}{2\pi(10^{-3} \text{ m})}\sqrt{\frac{\pi(10^6 \text{ Hz})(4\pi \times 10^{-7} \text{ H/m})}{5.8 \times 10^7 \text{ S/m}}} = 83 \quad (\text{m}\Omega),$$

$$\xi = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{35 \text{ m}\Omega}{35 \text{ m}\Omega + 83 \text{ m}\Omega} = 29.7\%.$$

(b) From Example 9-2, a Hertzian dipole has a directivity of 1.5. The gain, from Eq. (9.29), is $G = \xi D = 0.297 \times 1.5 = 0.44 = -3.5$ dB.

(c) From Eq. (9.30a),

$$I_0 = \sqrt{\frac{2P_{\text{rad}}}{R_{\text{rad}}}} = \sqrt{\frac{2(20 \text{ W})}{35 \text{ m}\Omega}} = 33.8 \text{ A}$$

and from Eq. (9.31),

$$P_t = \frac{P_{\text{rad}}}{\xi} = \frac{20 \text{ W}}{0.297} = 67.3 \text{ W}.$$

Problem 9.6 Repeat Problem 9.5 for a 20-cm-long antenna operating at 5 MHz.

Solution:

(a) At 5 MHz, $\lambda = c/f = 3 \times 10^8/(5 \times 10^6) = 60$ m. As $l/\lambda = 0.2/60 = 3.33 \times 10^{-3}$, the antenna length satisfies the condition of a short dipole. From Eqs. (9.35), (9.32), and (9.31),

$$R_{\text{rad}} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \times (3.33 \times 10^{-3})^2 = 8.76 \quad (\text{m}\Omega),$$

$$R_{\text{loss}} = \frac{l}{2\pi a}\sqrt{\frac{\pi f u_c}{\sigma_c}} = \frac{0.2}{2\pi \times 10^{-3}}\sqrt{\frac{\pi \times 5 \times 10^6 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}} = 18.57 \quad (\text{m}\Omega),$$

$$\xi = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{8.76}{8.76 + 18.57} = 0.32, \quad \text{or } 32\%.$$

(b) For Hertzian dipole, $D = 1.5$, and $G = \xi D = 0.32 \times 1.5 = 0.48 = -3.2$dB.

(c) From Eq. (9.30a),

$$I_0 = \sqrt{\frac{2P_{\text{rad}}}{R_{\text{rad}}}} = \sqrt{\frac{2 \times 20}{8.76 \times 10^{-3}}} = 67.6 \text{ A}.$$

Problem 9.7 An antenna with a pattern solid angle of 1.5 (sr) radiates 30 W of power. At a range of 1 km, what is the maximum power density radiated by the antenna?

Solution: From Eq. (9.23), $D = 4\pi/\Omega_p$, and from Eq. (9.24), $D = 4\pi R^2 S_{max}/P_{rad}$. Combining these two equations gives

$$S_{max} = \frac{P_{rad}}{\Omega_p R^2} = \frac{30}{1.5 \times (10^3)^2} = 2 \times 10^{-5} \quad (\text{W/m}^2).$$

Problem 9.8 An antenna with a radiation efficiency of 90% has a directivity of 6.7 dB. What is its gain in dB?

Solution: $D = 6.7$ dB corresponds to $D = 4.68$.

$$G = \xi D = 0.9 \times 4.68 = 4.21 = 6.24 \text{ dB}.$$

Alternatively,

$$G\,(\text{dB}) = \xi\,(\text{dB}) + D\,(\text{dB}) = 10\log 0.9 + 6.7 = -0.46 + 6.7 = 6.24 \text{ dB}.$$

Problem 9.9 The radiation pattern of a circular parabolic-reflector antenna consists of a circular major lobe with a half-power beamwidth of 2° and a few minor lobes. Ignoring the minor lobes, obtain an estimate for the antenna directivity in dB.

Solution: A circular lobe means that $\beta_{xz} = \beta_{yz} = 2° = 0.035$ rad. Using Eq. (9.26), we have

$$D = \frac{4\pi}{\beta_{xz}\beta_{yz}} = \frac{4\pi}{(0.035)^2} = 1.03 \times 10^4.$$

In dB,

$$D(\text{dB}) = 10\log D = 10\log(1.03 \times 10^4) = 40.13 \text{ dB}.$$

Problem 9.10 The normalized radiation intensity of a certain antenna is given by

$$F(\theta) = \exp(-20\theta^2) \qquad \text{for } 0 \le \theta \le \pi,$$

where θ is in radians. Determine:

(a) the half-power beamwidth,
(b) the pattern solid angle,

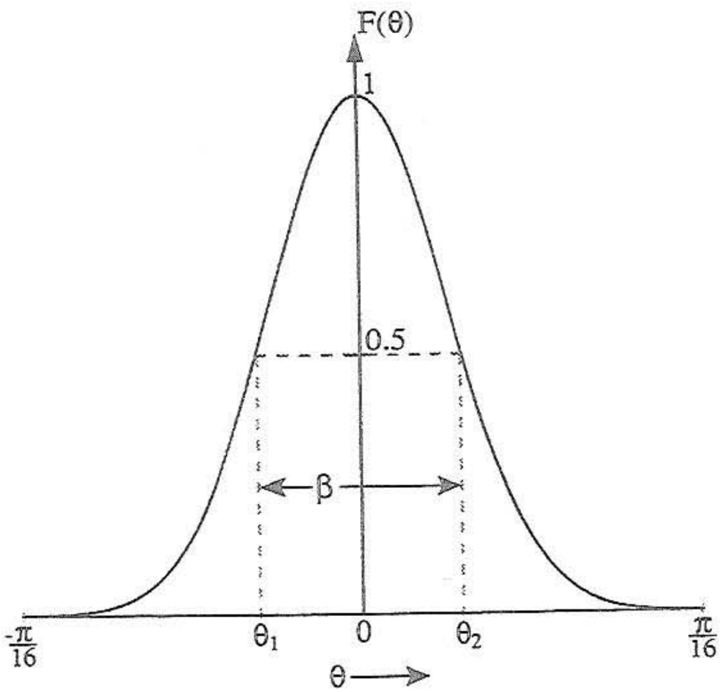

Figure P9.10: $F(\theta)$ versus θ.

(c) the antenna directivity.

Solution:

(a) Since $F(\theta)$ is independent of ϕ, the beam is symmetrical about $z = 0$. Upon setting $F(\theta) = 0.5$, we have

$$F(\theta) = \exp(-20\theta^2) = 0.5,$$
$$\ln[\exp(-20\theta^2)] = \ln(0.5),$$
$$20\theta^2 = -0.693,$$
$$\theta = \pm\left(\frac{0.693}{20}\right)^{1/2} = \pm 0.186 \text{ radians.}$$

Hence, $\beta = 2 \times 0.186 = 0.372$ radians $= 21.31°$.

(b) By Eq. (9.21),

$$\begin{aligned}\Omega_p &= \iint_{4\pi} F(\theta)\sin\theta\, d\theta\, d\phi \\ &= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \exp(-20\theta^2)\sin\theta\, d\theta\, d\phi \\ &= 2\pi\int_0^{\pi} \exp(-20\theta^2)\sin\theta\, d\theta.\end{aligned}$$

Numerical evaluation yields

$$\Omega_p = 0.156 \text{ sr}.$$

(c)

$$D = \frac{4\pi}{\Omega_p} = \frac{4\pi}{0.156} = 80.55.$$

Sections 9-3 and 9-4: Dipole Antennas

Problem 9.11 Repeat Problem 9.5 for a 1-m-long half-wave dipole that operates in the FM/TV broadcast band at 150 MHz.

Solution:

(a) Following Example 9-3,

$$\lambda = c/f = (3\times10^8 \text{ m/s})/(150\times10^6 \text{ Hz}) = 2 \text{ m}.$$

As $l/\lambda = (1 \text{ m})/(2 \text{ m}) = \frac{1}{2}$, this antenna is a half-wave dipole. Thus, from Eq. (9.48), (9.32), and (9.31),

$$R_{rad} = 73\ \Omega,$$

$$R_{loss} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{1 \text{ m}}{2\pi(10^{-3}\text{ m})}\sqrt{\frac{\pi(150\times10^6\text{ Hz})(4\pi\times10^{-7}\text{ H/m})}{5.8\times10^7\text{ S/m}}} = 0.5\ \Omega,$$

$$\xi = \frac{R_{rad}}{R_{rad}+R_{loss}} = \frac{73\ \Omega}{73\ \Omega + 0.5\ \Omega} = 99.3\%.$$

(b) From Eq. (9.47), a half-wave dipole has a directivity of 1.64. The gain, from Eq. (9.29), is $G = \xi D = 0.993\times1.64 = 1.63 = 2.1$ dB.

(c) From Eq. (9.30a),

$$I_0 = \sqrt{\frac{2P_{rad}}{R_{rad}}} = \sqrt{\frac{2(20\text{ W})}{73\ \Omega}} = 0.74 \text{ A},$$

and from Eq. (9.31),

$$P_t = \frac{P_{rad}}{\xi} = \frac{20 \text{ W}}{0.993} = 20.1 \text{ W}.$$

Problem 9.12 Assuming the loss resistance of a half-wave dipole antenna to be negligibly small and ignoring the reactance component of its antenna impedance, calculate the standing wave ratio on a 60-Ω transmission line connected to the dipole antenna.

Solution: According to Eq. (9.48), a half wave dipole has a radiation resistance of 73 Ω. To the transmission line, this behaves as a load, so the reflection coefficient is

$$\Gamma = \frac{R_{rad} - Z_0}{R_{rad} + Z_0} = \frac{73\ \Omega - 60\ \Omega}{73\ \Omega + 60\ \Omega} = 0.098,$$

and the standing wave ratio is

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.098}{1 - 0.098} = 1.22.$$

Problem 9.13 For the short dipole with length l such that $l \ll \lambda$, instead of treating the current $\widetilde{I}(z)$ as constant along the dipole, as was done in Section 9-1, a more realistic approximation that insures that the current goes to zero at the ends is to describe $\widetilde{I}(z)$ by the triangular function

$$\widetilde{I}(z) = \begin{cases} I_0(1 - 2z/l), & \text{for } 0 \le z \le l/2, \\ I_0(1 + 2z/l), & \text{for } -l/2 \le z \le 0, \end{cases}$$

as shown in Fig. 9-36 (P9.13). Use this current distribution to determine (a) the far-field $\widetilde{E}(R, \theta, \phi)$, (b) the power density $S(R, \theta, \phi)$, (c) the directivity D, and (d) the radiation resistance R_{rad}.

Solution:

(a) When the current along the dipole was assumed to be constant and equal to I_0, the vector potential was given by Eq. (9.3) as:

$$\widetilde{\mathbf{A}}(R) = \hat{\mathbf{z}} \frac{\mu_0}{4\pi} \left(\frac{e^{-jkR}}{R} \right) \int_{-l/2}^{l/2} I_0 \, dz.$$

If the triangular current function is assumed instead, then I_0 in the above expression should be replaced with the given expression. Hence,

$$\widetilde{\mathbf{A}} = \hat{\mathbf{z}} \frac{\mu_0}{4\pi} \left(\frac{e^{-jkR}}{R} \right) I_0 \left[\int_0^{l/2} \left(1 - \frac{2z}{l} \right) dz + \int_{-l/2}^{0} \left(1 + \frac{2z}{l} \right) dz \right] = \hat{\mathbf{z}} \frac{\mu_0 I_0 l}{8\pi} \left(\frac{e^{-jkR}}{R} \right),$$

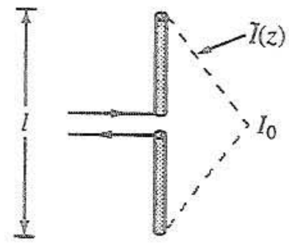

Figure P9.13: Triangular current distribution on a short dipole (Problem 9.13).

which is half that obtained for the constant-current case given by Eq. (9.3). Hence, the expression given by (9.9a) need only be modified by the factor of 1/2:

$$\tilde{\mathbf{E}} = \hat{\theta}\tilde{E}_\theta = \hat{\theta}\,\frac{jI_0lk\eta_0}{8\pi}\left(\frac{e^{-jkR}}{R}\right)\sin\theta.$$

(b) The corresponding power density is

$$S(R,\theta) = \frac{|\tilde{E}_\theta|^2}{2\eta_0} = \left(\frac{\eta_0k^2I_0^2l^2}{128\pi^2R^2}\right)\sin^2\theta.$$

(c) The power density is 4 times smaller than that for the constant current case, but the reduction is true for all directions. Hence, D remains unchanged at 1.5.

(d) Since $S(R,\theta)$ is 4 times smaller, the total radiated power P_{rad} is 4-times smaller. Consequently, $R_{\text{rad}} = 2P_{\text{rad}}/I_0^2$ is 4 times smaller than the expression given by Eq. (9.35); that is,

$$R_{\text{rad}} = 20\pi^2(l/\lambda)^2 \quad (\Omega).$$

Problem 9.14 For a dipole antenna of length $l = 3\lambda/2$, (a) determine the directions of maximum radiation, (b) obtain an expression for $S_{\max}$, and (c) generate a plot of the normalized radiation pattern $F(\theta)$. Compare your pattern with that shown in Fig. 9.17(c).

Solution:

(a) From Eq. (9.56), $S(\theta)$ for an arbitrary length dipole is given by

$$S(\theta) = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos\left(\frac{\pi l}{\lambda}\cos\theta\right) - \cos\left(\frac{\pi l}{\lambda}\right)}{\sin\theta}\right]^2.$$

For $l = 3\lambda/2$, $S(\theta)$ becomes

$$S(\theta) = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos\left(\frac{3\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2.$$

Solving for the directions of maximum radiation numerically yields two maximum directions of radiation given by

$$\theta_{\max_1} = 42.6^\circ, \qquad \theta_{\max_2} = 137.4^\circ.$$

(b) From the numerical results, it was found that $S(\theta) = 15I_0^2/(\pi R^2)(1.96)$ at $\theta_{\max}$. Thus,

$$S_{\max} = \frac{15I_0^2}{\pi R^2}(1.96).$$

(c) The normalized radiation pattern is given by Eq. (9.13) as

$$F(\theta) = \frac{S(\theta)}{S_{\max}}.$$

Using the expression for $S(\theta)$ from part (a) with the value of $S_{\max}$ found in part (b),

$$F(\theta) = \frac{1}{1.96}\left[\frac{\cos\left(\frac{3\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2.$$

The normalized radiation pattern is shown in Fig. P9.14, which is identical to that shown in Fig. 9.17(c).

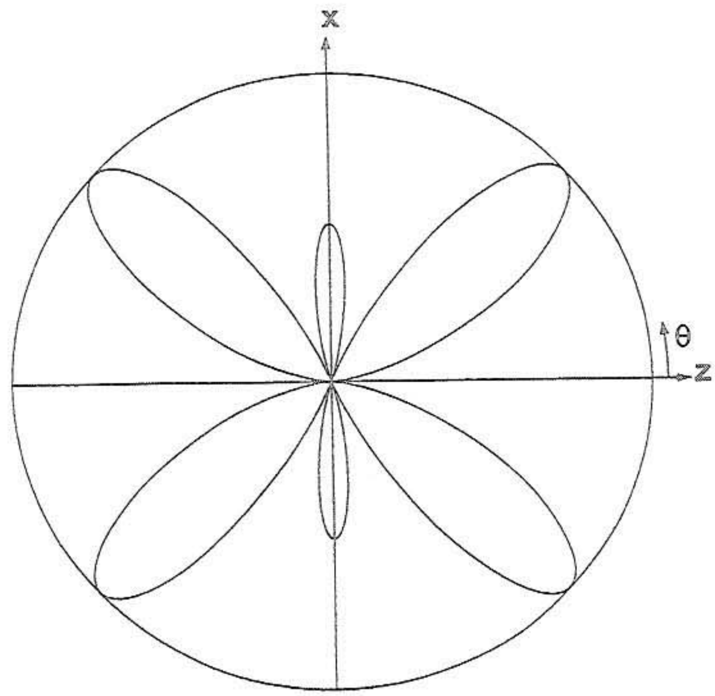

Figure P9.14: Radiation pattern of dipole of length $3\lambda/2$.

Problem 9.15 Repeat parts (a)–(c) of Problem 9.14 for a dipole of length $l = 3\lambda/4$.

Solution:

(a) For $l = 3\lambda/4$, Eq. (9.56) becomes

$$S(\theta) = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos\left(\frac{3\pi}{4}\cos\theta\right) - \cos\left(\frac{3\pi}{4}\right)}{\sin\theta}\right]^2 = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos\left(\frac{3\pi}{4}\cos\theta\right) + \frac{1}{\sqrt{2}}}{\sin\theta}\right]^2.$$

Solving for the directions of maximum radiation numerically yields

$$\theta_{\max_1} = 90^\circ, \qquad \theta_{\max_2} = 270^\circ.$$

(b) From the numerical results, it was found that $S(\theta) = 15I_0^2/(\pi R^2)(2.91)$ at $\theta_{\max}$.

Thus,

$$S_{\max} = \frac{15I_0^2}{\pi R^2}(2.91).$$

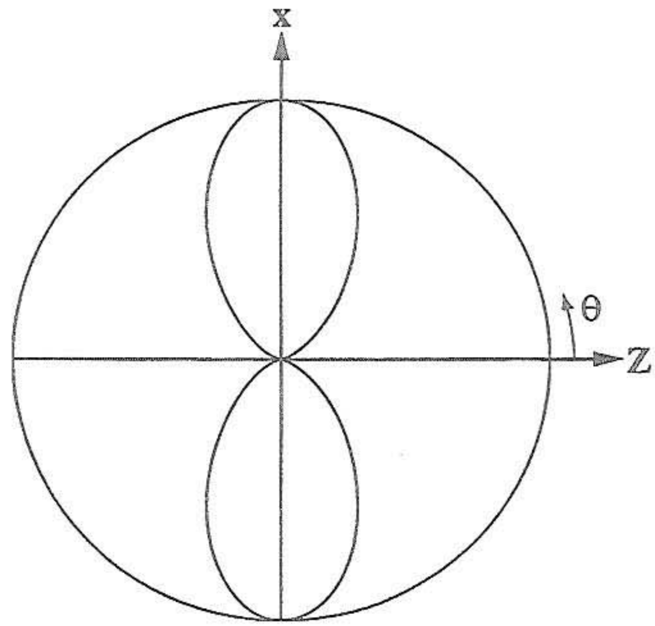

Figure P9.15: Radiation pattern of dipole of length $l = 3\lambda/4$.

(c) The normalized radiation pattern is given by Eq. (9.13) as

$$F(\theta) = \frac{S(\theta)}{S_{\max}}.$$

Using the expression for $S(\theta)$ from part (a) with the value of $S_{\max}$ found in part (b),

$$F(\theta) = \frac{1}{2.91}\left[\frac{\cos\left(\frac{3\pi}{4}\cos\theta\right) + \frac{1}{\sqrt{2}}}{\sin\theta}\right]^2.$$

The normalized radiation pattern is shown in Fig. P9.15.

Problem 9.16 Repeat parts (a)–(c) of Problem 9.14 for a dipole of length $l = \lambda$.

Solution: For $l = \lambda$, Eq. (9.56) becomes

$$S(\theta) = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos(\pi\cos\theta) - \cos(\pi)}{\sin\theta}\right]^2 = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta}\right]^2.$$

Solving for the directions of maximum radiation numerically yields

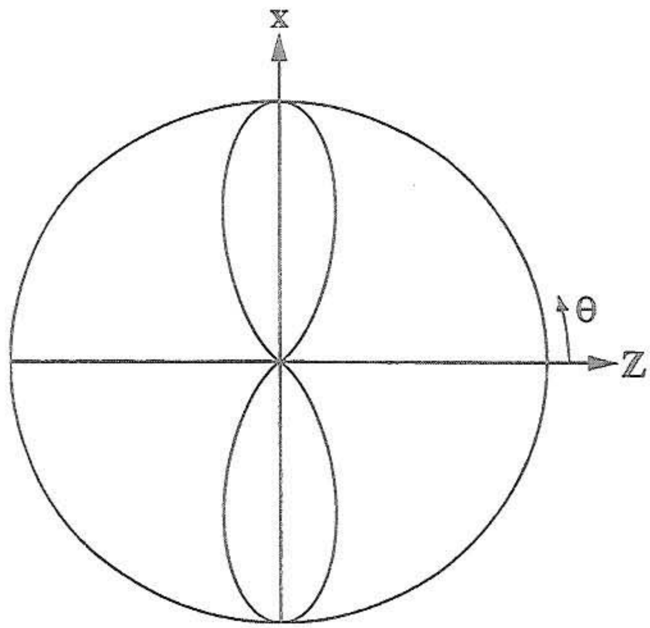

Figure P9.16: Radiation pattern of dipole of length $l = \lambda$.

$$\theta_{\max_1} = 90°, \qquad \theta_{\max_2} = 270°.$$

(b) From the numerical results, it was found that $S(\theta) = 15I_0^2/(\pi R^2)(4)$ at $\theta_{\max}$. Thus,

$$S_{\max} = \frac{60I_0^2}{\pi R^2}.$$

(c) The normalized radiation pattern is given by Eq. (9.13), as

$$F(\theta) = \frac{S(\theta)}{S_{\max}}.$$

Using the expression for $S(\theta)$ from part (a) with the value of $S_{\max}$ found in part (b),

$$F(\theta) = \frac{1}{4}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta}\right]^2.$$

The normalized radiation pattern is shown in Fig. P9.16.

Problem 9.17 A car antenna is a vertical monopole over a conducting surface. Repeat Problem 9.5 for a 1-m-long car antenna operating at 1 MHz. The antenna wire is made of aluminum with $\mu_c = \mu_0$ and $\sigma_c = 3.5 \times 10^7$ S/m, and its diameter is 1 cm.

Solution:

(a) Following Example 9-3, $\lambda = c/f = (3 \times 10^8 \text{ m/s})/(10^6 \text{ Hz}) = 300$ m. As $l/\lambda = 2 \times (1 \text{ m})/(300 \text{ m}) = 0.0067$, this antenna is a short (Hertzian) monopole. From Section 9-3.3, the radiation resistance of a monopole is half that for a corresponding dipole. Thus,

$$R_{\text{rad}} = \tfrac{1}{2}80\pi^2\left(\frac{l}{\lambda}\right)^2 = 40\pi^2(0.0067)^2 = 17.7 \quad (\text{m}\Omega),$$

$$R_{\text{loss}} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{1 \text{ m}}{\pi(10^{-2} \text{ m})}\sqrt{\frac{\pi(10^6 \text{ Hz})(4\pi \times 10^{-7} \text{ H/m})}{3.5 \times 10^7 \text{ S/m}}} = 10.7 \text{ m}\Omega,$$

$$\xi = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{17.7 \text{ m}\Omega}{17.7 \text{ m}\Omega + 10.7 \text{ m}\Omega} = 62\%.$$

(b) From Example 9-2, a Hertzian dipole has a directivity of 1.5. The gain, from Eq. (9.29), is $G = \xi D = 0.62 \times 1.5 = 0.93 = -0.3$ dB.

(c) From Eq. (9.30a),

$$I_0 = \sqrt{\frac{2P_{\text{rad}}}{R_{\text{rad}}}} = \sqrt{\frac{2(20 \text{ W})}{17.7 \text{ m}\Omega}} = 47.5 \text{ A},$$

and from Eq. (9.31),

$$P_t = \frac{P_{\text{rad}}}{\xi} = \frac{20 \text{ W}}{0.62} = 32.3 \text{ W}.$$

Sections 9-5 and 9-6: Effective Area and Friis Formula

Problem 9.18 Determine the effective area of a half-wave dipole antenna at 100 MHz, and compare it to its physical cross section if the wire diameter is 1 cm.

Solution: At $f = 100$ MHz, $\lambda = c/f = (3 \times 10^8 \text{ m/s})/(100 \times 10^6 \text{ Hz}) = 3$ m. From Eq. (9.47), a half wave dipole has a directivity of $D = 1.64$. From Eq. (9.64), $A_e = \lambda^2 D/4\pi = (3 \text{ m})^2 \times 1.64/4\pi = 1.17 \text{ m}^2$.

The physical cross section is: $A_p = ld = \frac{1}{2}\lambda d = \frac{1}{2}(3\text{ m})(10^{-2}\text{ m}) = 0.015\text{ m}^2$. Hence, $A_e/A_p = 78$.

Problem 9.19 A 3-GHz line-of-sight microwave communication link consists of two lossless parabolic dish antennas, each 1 m in diameter. If the receive antenna requires 1 nW of receive power for good reception and the distance between the antennas is 40 km, how much power should be transmitted?

Solution: At $f = 3$ GHz, $\lambda = c/f = (3\times10^8\text{ m/s})/(3\times10^9\text{ Hz}) = 0.10$ m. Solving the Friis transmission formula (Eq. (9.75)) for the transmitted power:

$$P_t = P_{rec}\frac{\lambda^2R^2}{\xi_t\xi_r A_t A_r}$$
$$= 10^{-9}\frac{(0.100\text{ m})^2(40\times10^3\text{ m})^2}{1\times1\times(\frac{\pi}{4}(1\text{ m})^2)(\frac{\pi}{4}(1\text{ m})^2)} = 25.9\times10^{-3}\text{ W} = 25.9\text{ mW}.$$

Problem 9.20 A half-wave dipole TV broadcast antenna transmits 1 kW at 50 MHz. What is the power received by a home television antenna with 13-dB gain if located at a distance of 30 km?

Solution: At $f = 50$ MHz, $\lambda = c/f = (3\times10^8\text{ m/s})/(50\times10^6\text{ Hz}) = 6$ m, for which a half wave dipole, or larger antenna, is very reasonable to construct. Assuming the TV transmitter to have a vertical half wave dipole, its gain in the direction of the home would be $G_t = 1.64$. The home antenna has a gain of $G_r = 13\text{ dB} = 20$. From the Friis transmission formula (Eq. (9.75)):

$$P_{rec} = P_t\frac{\lambda^2G_rG_t}{(4\pi)^2R^2} = 10^3\frac{(6\text{ m})^2\times1.64\times20}{(4\pi)^2(30\times10^3\text{ m})^2} = 8.3\times10^{-6}\text{ W} = 8.3\ \mu\text{W}.$$

Problem 9.21 A 150-MHz communication link consists of two vertical half-wave dipole antennas separated by 2 km. The antennas are lossless, the signal occupies a bandwidth of 3 MHz, the system noise temperature of the receiver is 600 K, and the desired signal-to-noise ratio is 20 dB. What transmitter power is required?

Solution: From Eq. (9.77), the receiver noise power is

$$P_n = KT_{sys}B = 1.38\times10^{-23}\times600\times3\times10^6 = 2.48\times10^{-14}\text{ W}.$$

For a signal to noise ratio $S_n = 20\text{ dB} = 100$, the received power must be at least

$$P_{rec} = S_nP_n = 100(2.48\times10^{-14}\text{ W}) = 2.48\times10^{-12}\text{ W}.$$

Since the two antennas are half-wave dipoles, Eq. (9.47) states $D_t = D_r = 1.64$, and since the antennas are both lossless, $G_t = D_t$ and $G_r = D_r$. Since the operating frequency is $f = 150$ MHz, $\lambda = c/f = (3\times10^8 \text{ m/s})/(150\times10^6 \text{ Hz}) = 2$ m. Solving the Friis transmission formula (Eq. (9.75)) for the transmitted power:

$$P_t = P_{rec}\frac{(4\pi)^2R^2}{\lambda^2 G_r G_t} = 2.48\times10^{-12}\frac{(4\pi)^2(2\times10^3 \text{ m})^2}{(2 \text{ m})^2(1.64)(1.64)} = 0.15 \quad \text{(mW)}.$$

Problem 9.22 Consider the communication system shown in Fig. 9-37 (P9.22), with all components properly matched. If $P_t = 10$ W and $f = 6$ GHz:

(a) what is the power density at the receiving antenna (assuming proper alignment of antennas)?

(b) What is the received power?

(c) If $T_{sys} = 1{,}000$ K and the receiver bandwidth is 10 MHz, what is the signal to noise ratio in dB?

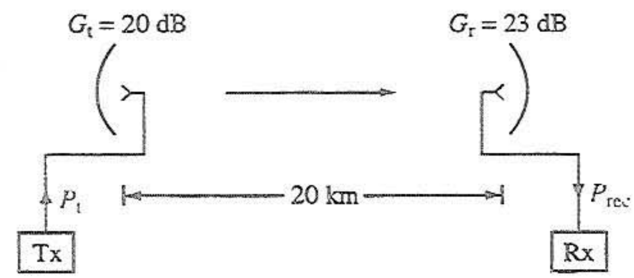

Figure P9.22: Communication system of Problem 9.22.

Solution:

(a) $G_t = 20$ dB $= 100$, $G_r = 23$ dB $= 200$, and $\lambda = c/f = 5$ cm. From Eq. (9.72),

$$S_r = G_t\frac{P_t}{4\pi R^2} = \frac{10^2\times10}{4\pi\times(2\times10^4)^2} = 2\times10^{-7} \quad (\text{W/m}^2).$$

(b)

$$P_{rec} = P_t G_t G_r\left(\frac{\lambda}{4\pi R}\right)^2 = 10\times100\times200\times\left(\frac{5\times10^{-2}}{4\pi\times2\times10^4}\right)^2 = 7.92\times10^{-9} \text{ W}.$$

(c)

$$P_n = KT_{sys}B = 1.38\times10^{-23}\times10^3\times10^7 = 1.38\times10^{-13} \text{ W},$$

$$S_n = \frac{P_{rec}}{P_n} = \frac{7.92 \times 10^{-9}}{1.38 \times 10^{-13}} = 5.74 \times 10^4 = 47.6 \text{ dB}.$$

Sections 9-7 and 9-8: Radiation by Apertures

Problem 9.23 A uniformly illuminated aperture is of length $l_x = 20\lambda$. Determine the beamwidth between first nulls in the x–z plane.

Solution: The radiation intensity of a uniformly illuminated antenna is given by Eq. (9.90):

$$F(\theta) = \text{sinc}^2(\pi l_x \sin\theta/\lambda) = \text{sinc}^2(\pi\gamma),$$

with

$$\gamma = l_x \sin\theta/\lambda.$$

For $l_x = 20\lambda$,

$$\gamma = 20\sin\theta.$$

The first zero of the sinc function occurs when $\gamma = \pm 1$, as shown in Fig. 9-23. Hence,

$$1 = 20\sin\theta,$$

or

$$\theta = \sin^{-1}\left(\frac{1}{20}\right) = 2.87^\circ,$$

and

$$\beta_{null} = 2\theta = 5.73^\circ.$$

Problem 9.24 The 10-dB beamwidth is the beam size between the angles at which $F(\theta)$ is 10 dB below its peak value. Determine the 10-dB beamwidth in the x–z plane for a uniformly illuminated aperture with length $l_x = 10\lambda$.

Solution: For a uniformly illuminated antenna of length $l_x = 10\lambda$ Eq. (9.90) gives

$$F(\theta) = \text{sinc}^2(\pi l_x \sin\theta/\lambda) = \text{sinc}^2(10\pi\sin\theta).$$

The peak value of $F(\theta)$ is 1, and the 10-dB level below the peak corresponds to when $F(\theta) = 0.1$ (because $10\log 0.1 = -10$ dB). Hence, we set $F(\theta) = 0.1$ and solve for θ:

$$0.1 = \text{sinc}^2(10\pi\sin\theta).$$

From tabulated values of the sinc function, it follows that the solution of this equation is

$$10\pi \sin\theta = 2.319$$

or

$$\theta \approx 4.23°.$$

Hence, the 10-dB beamwidth is

$$\beta \approx 2\theta = 8.46°.$$

Problem 9.25 A uniformly illuminated rectangular aperture situated in the x–y plane is 3 m high (along x) and 1 m wide (along y). If $f = 10$ GHz, determine

(a) the beamwidths of the radiation pattern in the elevation plane (x–z plane) and the azimuth plane (y–z plane), and

(b) the antenna directivity D in dB.

Solution: From Eqs. (9.94a), (9.94b), and (9.96),

$$\beta_{xz} = 0.88\frac{\lambda}{l_x} = \frac{0.88 \times 3 \times 10^{-2}}{3} = 8.8 \times 10^{-3} \text{ rad} = 0.50°,$$

$$\beta_{yz} = 0.88\frac{\lambda}{l_y} = \frac{0.88 \times 3 \times 10^{-2}}{1} = 2.64 \times 10^{-2} \text{ rad} = 1.51°,$$

$$D = \frac{4\pi}{\beta_{xz}\beta_{yz}} = \frac{4\pi}{(8.8 \times 10^{-3})(2.64 \times 10^{-2})} = 5.41 \times 10^4 = 47.3 \text{ dB}.$$

Problem 9.26 An antenna with a circular aperture has a circular beam with a beamwidth of 1.5°at 20 GHz.

(a) What is the antenna directivity in dB?

(b) If the antenna area is doubled, what would be the new directivity and new beamwidth?

(c) If the aperture is kept the same as in (a), but the frequency is doubled to 40 GHz, what would the directivity and beamwidth become then?

Solution:

(a) From Eq. (9.96),

$$D \simeq \frac{4\pi}{\beta^2} = \frac{4\pi}{(1.5° \times \pi/180°)^2} = 1.83 \times 10^4 = 42.6 \text{ dB}.$$

(b) If area is doubled, it means the diameter is increased by $\sqrt{2}$, and therefore the beamwidth decreases by $\sqrt{2}$ to

$$\beta = \frac{1.5^\circ}{\sqrt{2}} = 1.1^\circ.$$

The directivity increases by a factor of 2, or 3 dB, to $D = 42.6 + 3 = 45.6$ dB.

(c) If f is doubled, λ becomes half as long, which means that the diameter to wavelength ratio is twice as large. Consequently, the beamwidth is half as wide:

$$\beta = \frac{1.5^\circ}{2} = 0.75^\circ,$$

and D is four times as large, or 6 dB greater, $D = 42.6 + 6 = 48.6$ dB.

Problem 9.27 A 94-GHz automobile collision-avoidance radar uses a rectangular-aperture antenna placed above the car's bumper. If the antenna is 1 m in length and 10 cm in height,

(a) what are its elevation and azimuth beamwidths?

(b) what is the horizontal extent of the beam at a distance of 300 m?

Solution:

(a) At 94 GHz, $\lambda = 3 \times 10^8/(94 \times 10^9) = 3.2$ mm. The elevation beamwidth is $\beta_e = \lambda/0.1\text{ m} = 3.2 \times 10^{-2}$ rad $= 1.8^\circ$. The azimuth beamwidth is $\beta_a = \lambda/1\text{ m} = 3.2 \times 10^{-3}$ rad $= 0.18^\circ$.

(b) At a distance of 300 m, the horizontal extent of the beam is

$$\Delta y = \beta_a R = 3.2 \times 10^{-3} \times 300 = 0.96 \text{ m}.$$

Problem 9.28 A microwave telescope consisting of a very sensitive receiver connected to a 100-m parabolic-dish antenna is used to measure the energy radiated by astronomical objects at 10 GHz. If the antenna beam is directed toward the moon and the moon extends over a planar angle of 0.5° from Earth, what fraction of the moon's cross section will be occupied by the beam?

Solution:

$$\beta_{\text{ant}} = \frac{\lambda}{d} = \frac{3 \times 10^{-2}}{100} = 3 \times 10^{-4} \text{ rad}.$$

For the moon, $\beta_{\text{moon}} = 0.5^\circ \times \pi/180^\circ = 8.73 \times 10^{-3}$ rad. Fraction of the moon's cross section occupied by the beam is

$$\left(\frac{\beta_{\text{ant}}}{\beta_{\text{moon}}}\right)^2 = \left(\frac{3 \times 10^{-4}}{8.73 \times 10^{-3}}\right)^2 = 1.2 \times 10^{-3}, \quad \text{or } 0.12\%.$$

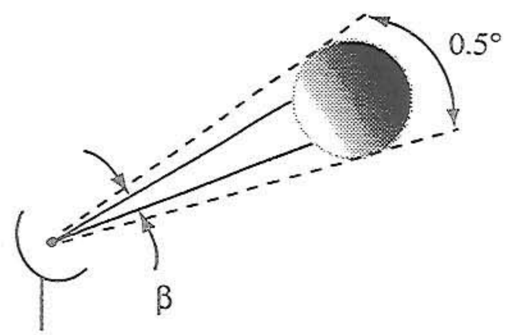

Figure P9.28: Antenna beam viewing the moon.

Sections 9-9 to 9-11: Antenna Arrays

Problem 9.29 A two-element array consisting of two isotropic antennas separated by a distance d along the z-axis is placed in a coordinate system whose z-axis points eastward and whose x-axis points toward the zenith. If a_0 and a_1 are the amplitudes of the excitations of the antennas at $z = 0$ and at $z = d$ respectively, and if δ is the phase of the excitation of the antenna at $z = d$ relative to that of the other antenna, find the array factor and plot the pattern in the x–z plane for

(a) $a_0 = a_1 = 1,\ \delta = \pi/4$, and $d = \lambda/2$,
(b) $a_0 = 1,\ a_1 = 2,\ \delta = 0$, and $d = \lambda$,
(c) $a_0 = a_1 = 1,\ \delta = -\pi/2$, and $d = \lambda/2$,
(d) $a_0 = a_1,\ a_1 = 2,\ \delta = \pi/4$, and $d = \lambda/2$, and
(e) $a_0 = a_1,\ a_1 = 2,\ \delta = \pi/2$, and $d = \lambda/4$.

Solution:
(a) Employing Eq. (9.110),

$$\begin{aligned}
F_a(\theta) &= \left|\sum_{i=0}^{1} a_i e^{j\psi_i} e^{jikd\cos\theta}\right|^2 \\
&= |1 + e^{j((2\pi/\lambda)(\lambda/2)\cos\theta + \pi/4)}|^2 \\
&= |1 + e^{j(\pi\cos\theta + \pi/4)}|^2 = 4\cos^2\left(\frac{\pi}{8}(4\cos\theta + 1)\right).
\end{aligned}$$

A plot of this array factor pattern is shown in Fig. P9.29(a).

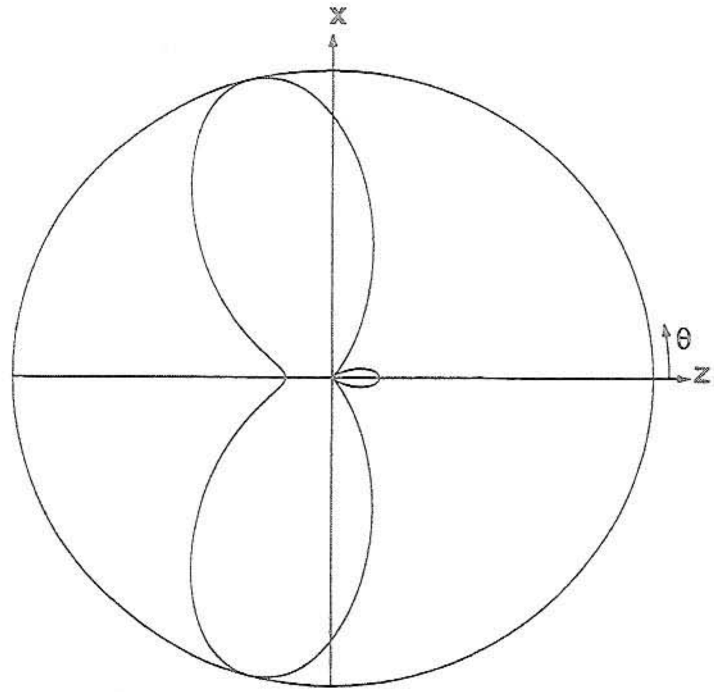

Figure P9.29: (a) Array factor in the elevation plane for Problem 9.29(a).

(b) Employing Eq. (9.110),

$$F_a(\theta) = \left| \sum_{i=0}^{1} a_i e^{j\psi_i} e^{jikd\cos\theta} \right|^2$$
$$= |1 + 2e^{j((2\pi/\lambda)\lambda\cos\theta + 0)}|^2 = |1 + 2e^{j2\pi\cos\theta}|^2 = 5 + 4\cos(2\pi\cos\theta).$$

A plot of this array factor pattern is shown in Fig. P9.29(b).

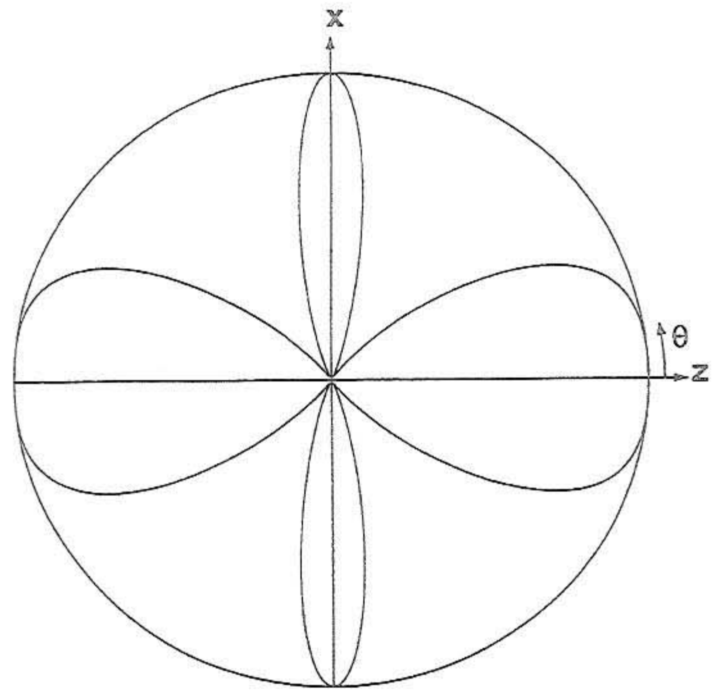

Figure P9.29: (b) Array factor in the elevation plane for Problem 9.29(b).

(c) Employing Eq. (9.110), and setting $a_0 = a_1 = 1,\ \psi = 0,\ \psi_1 = \delta = -\pi/2$ and $d = \lambda/2$, we have

$$\begin{aligned} F_a(\theta) &= \left| \sum_{i=0}^{1} a_i e^{j\psi_i} e^{jikd\cos\theta} \right|^2 \\ &= \left| 1 + e^{-j\pi/2} e^{j(2\pi/\lambda)(\lambda/2)\cos\theta} \right|^2 \\ &= \left| 1 + e^{j(\pi\cos\theta - \pi/2)} \right|^2 \\ &= 4\cos^2\left(\frac{\pi}{2}\cos\theta - \frac{\pi}{4}\right). \end{aligned}$$

A plot of the array factor is shown in Fig. P9.29(c).

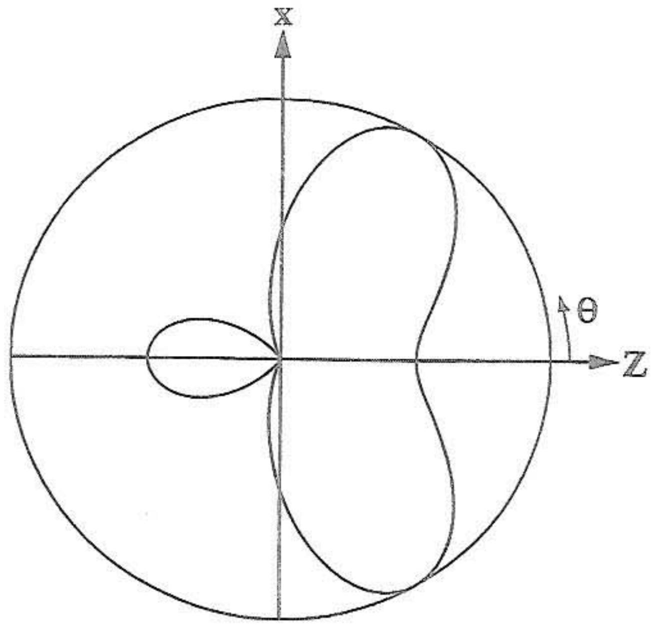

Figure P9.29: (c) Array factor in the elevation plane for Problem 9.29(c).

(d) Employing Eq. (9.110), and setting $a_0 = 1,\ a_1 = 2,\ \psi_0 = 0,\ \psi_1 = \delta = \pi/4$, and $d = \lambda/2$, we have

$$F_a(\theta) = \left| \sum_{i=0}^{1} a_i e^{j\psi_i} e^{jikd\cos\theta} \right|^2$$
$$= \left| 1 + 2e^{j\pi/4} e^{j(2\pi/\lambda)(\lambda/2)\cos\theta} \right|^2$$
$$= \left| 1 + 2e^{j(\pi\cos\theta + \pi/4)} \right|^2$$
$$= 5 + 4\cos\left(\pi\cos\theta + \frac{\pi}{4}\right).$$

A plot of the array factor is shown in Fig. P9.29(d).

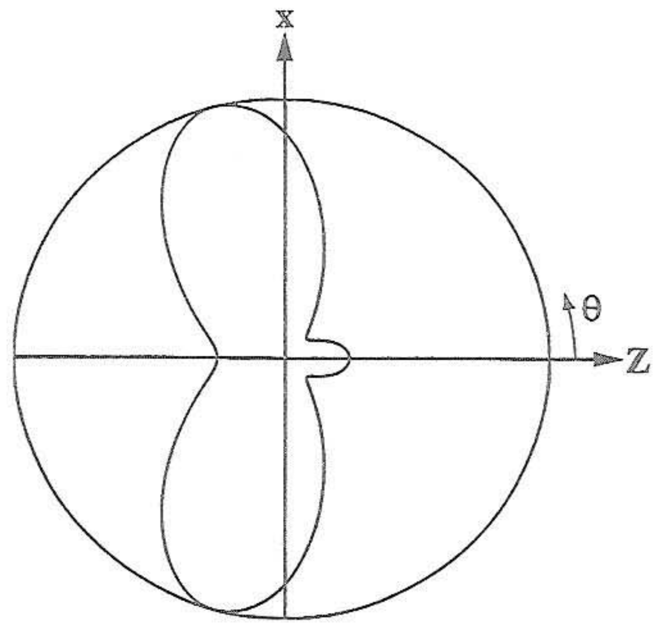

Figure P9.29: (d) Array factor in the elevation plane for Problem 9.29(d).

(e) Employing Eq. (9.110), and setting $a_0 = 1,\ a_1 = 2,\ \psi_0 = 0,\ \psi_1 = \delta = \pi/2$, and $d = \lambda/4$, we have

$$\begin{aligned} F_{\text{a}}(\theta) &= \left|\sum_{i=0}^{1} a_i e^{j\psi_i} e^{jikd\cos\theta}\right|^2 \\ &= \left|1 + 2e^{j\pi/2} e^{j(2\pi/\lambda)(\lambda/4)\cos\theta}\right|^2 \\ &= \left|1 + 2e^{j(\pi\cos\theta+\pi)/2}\right|^2 \\ &= 5 + 4\cos\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right) = 5 - 4\sin\left(\frac{\pi}{2}\cos\theta\right). \end{aligned}$$

A plot of the array factor is shown in Fig. P9.29(e).

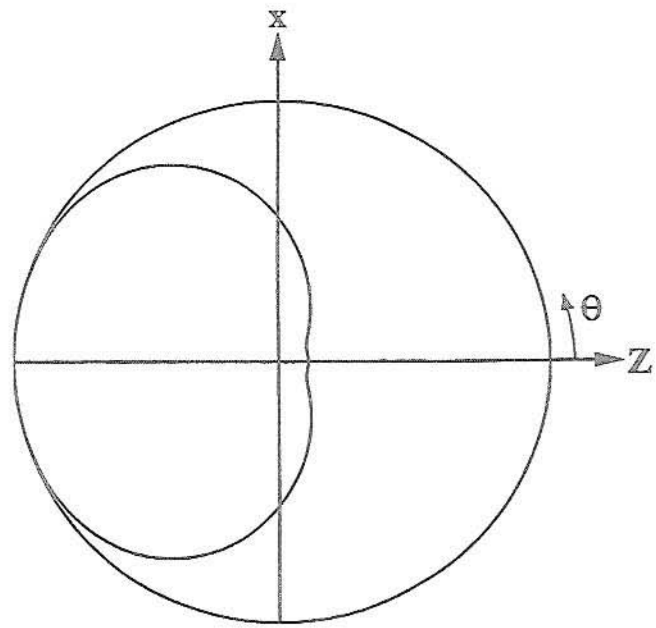

Figure P9.29: (e) Array factor in the elevation plane for Problem 9.29(e).

Problem 9.30 If the antennas in part (a) of Problem 9.29 are parallel vertical Hertzian dipoles with axes along the x-direction, determine the normalized radiation intensity in the x-z plane and plot it.

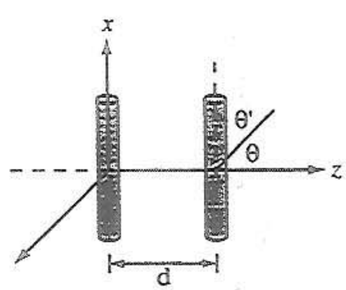

Figure P9.30: (a) Two vertical dipoles of Problem 9.30.

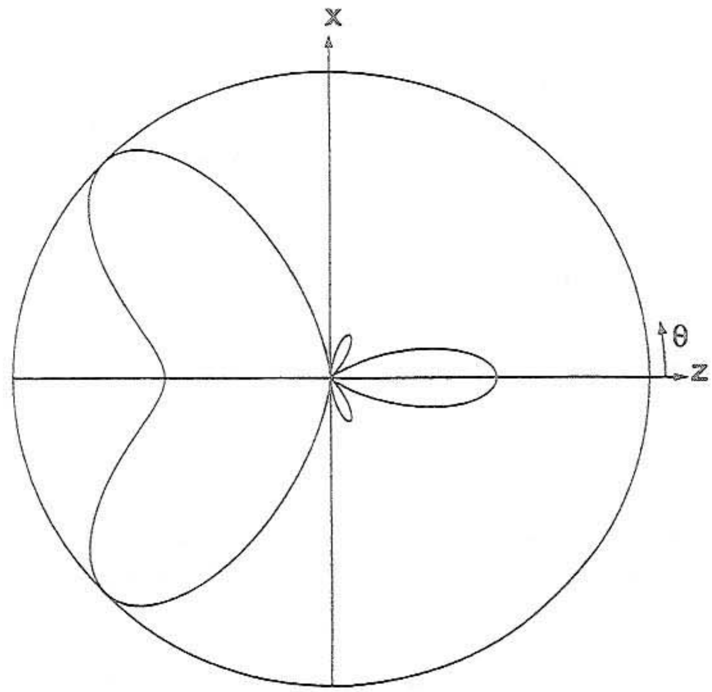

Figure P9.30: (b) Pattern factor in the elevation plane of the array in Problem 9.30(a).

Solution: The power density radiated by a Hertzian dipole is given from Eq. (9.12) by $S_e(\theta') = S_0 \sin^2\theta'$, where θ' is the angle measured from the dipole axis, which in the present case is the x-axis (Fig. P9.30).

Hence, $\theta' = \pi/2 - \theta$ and $S_e(\theta) = S_0 \sin^2(\frac{1}{2}\pi - \theta) = S_0 \cos^2\theta$. Then, from Eq. (9.108), the total power density is the product of the element pattern and the array factor. From part (a) of the previous problem:

$$S(\theta) = S_e(\theta)F_a(\theta) = 4S_0 \cos^2\theta \cos^2\left(\frac{\pi}{8}(4\cos\theta + 1)\right).$$

This function has a maximum value of $3.52S_0$ and it occurs at $\theta_{max} = \pm 135.5°$. The maximum must be found by trial and error. A plot of the normalized array antenna pattern is shown in Fig. P9.30.

Problem 9.31 Consider the two-element dipole array of Fig. 9.29(a). If the two dipoles are excited with identical feeding coefficients ($a_0 = a_1 = 1$ and $\psi_0 = \psi_1 = 0$), choose (d/λ) such that the array factor has a maximum at $\theta = 45°$.

Solution: With $a_0 = a_1 = 1$ and $\psi_0 = \psi_1 = 0$,

$$F_a(\theta) = |1 + e^{j(2\pi d/\lambda)\cos\theta}|^2 = 4\cos^2\left(\frac{\pi d}{\lambda}\cos\theta\right).$$

$F_a(\theta)$ is a maximum when the argument of the cosine function is zero or a multiple of π. Hence, for a maximum at $\theta = 45°$,

$$\frac{\pi d}{\lambda}\cos 45° = n\pi, \qquad n = 0, 1, 2, \ldots.$$

The first value of n, namely $n = 0$, does not provide a useful solution because it requires d to be zero, which means that the two elements are at the same location. While this gives a maximum at $\theta = 45°$, it also gives the same maximum at all angles θ in the y-z plane because the two-element array will have become a single element with an azimuthally symmetric pattern. The value $n = 1$ leads to

$$\frac{d}{\lambda} = \frac{1}{\cos 45°} = 1.414.$$

Problem 9.32 Choose (d/λ) so that the array pattern of the array of Problem 9.31 has a null, rather than a maximum, at $\theta = 45°$.

Solution: With $a_0 = a_1 = 1$ and $\psi_0 = \psi_1 = 0$,

$$F_a(\theta) = |1 + e^{j(2\pi d/\lambda)\cos\theta}|^2 = 4\cos^2\left(\frac{\pi d}{\lambda}\cos\theta\right).$$

$F_a(\theta)$ is equal to zero when the argument of the cosine function is $[(\pi/2) + n\pi]$. Hence, for a null at $\theta = 45°$,

$$\frac{\pi d}{\lambda}\cos 45° = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, \ldots.$$

For $n = 0$,

$$\frac{d}{\lambda} = \frac{1}{2\cos 45°} = 0.707.$$

Problem 9.33 Find and plot the normalized array factor and determine the half-power beamwidth for a five-element linear array excited with equal phase and a uniform amplitude distribution. The interelement spacing is $3\lambda/4$.

Solution: Using Eq. (9.121),

$$F_{an}(\theta) = \frac{\sin^2[(N\pi d/\lambda)\cos\theta]}{N^2\sin^2[(\pi d/\lambda)\cos\theta]} = \frac{\sin^2[(15\pi/4)\cos\theta]}{25\sin^2[(3\pi/4)\cos\theta]}$$

and this pattern is shown in Fig. P9.33. The peak values of the pattern occur at $\theta = \pm 90°$. From numerical values of the pattern, the angles at which $F_{an}(\theta) = 0.5$ are approximately 6.75°on either side of the peaks. Hence, $\beta \simeq 13.5°$.

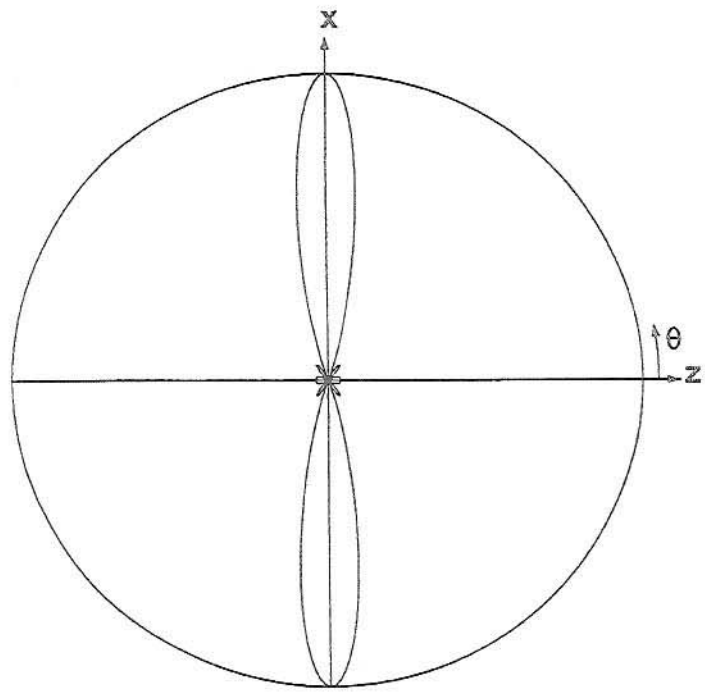

Figure P9.33: Normalized array pattern of a 5-element array with uniform amplitude distribution in Problem 9.33.

Problem 9.34 A three-element linear array of isotropic sources aligned along the z-axis has an interelement spacing of $\lambda/4$ Fig. 9-38 (P9.34). The amplitude excitation of the center element is twice that of the bottom and top elements and the phases are $-\pi/2$ for the bottom element and $\pi/2$ for the top element, relative to that of the center element. Determine the array factor and plot it in the elevation plane.

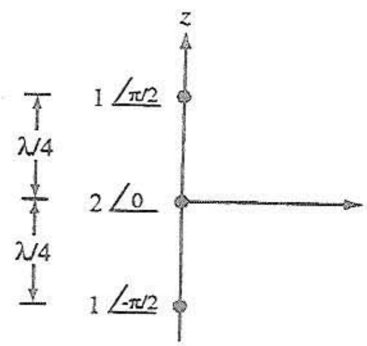

Figure P9.34: (a) Three-element array of Problem 9.34.

Solution: From Eq. (9.110),

$$\begin{aligned}
F_a(\theta) &= \left|\sum_{i=0}^{2} a_i e^{j\psi_i} e^{jikd\cos\theta}\right|^2 \\
&= \left|a_0 e^{j\psi_0} + a_1 e^{j\psi_1} e^{jkd\cos\theta} + a_2 e^{j\psi_2} e^{j2kd\cos\theta}\right|^2 \\
&= \left|e^{j(\psi_1-\pi/2)} + 2e^{j\psi_1} e^{j(2\pi/\lambda)(\lambda/4)\cos\theta} + e^{j(\psi_1+\pi/2)} e^{j2(2\pi/\lambda)(\lambda/4)\cos\theta}\right|^2 \\
&= \left|e^{j\psi_1} e^{j(\pi/2)\cos\theta}\right|^2 \left|e^{-j\pi/2} e^{-j(\pi/2)\cos\theta} + 2 + e^{j\pi/2} e^{j(\pi/2)\cos\theta}\right|^2 \\
&= 4\left(1+\cos\left(\tfrac{1}{2}\pi(1+\cos\theta)\right)\right)^2, \\
F_{an}(\theta) &= \tfrac{1}{4}\left(1+\cos\left(\tfrac{1}{2}\pi(1+\cos\theta)\right)\right)^2.
\end{aligned}$$

This normalized array factor is shown in Fig. P9.34(b).

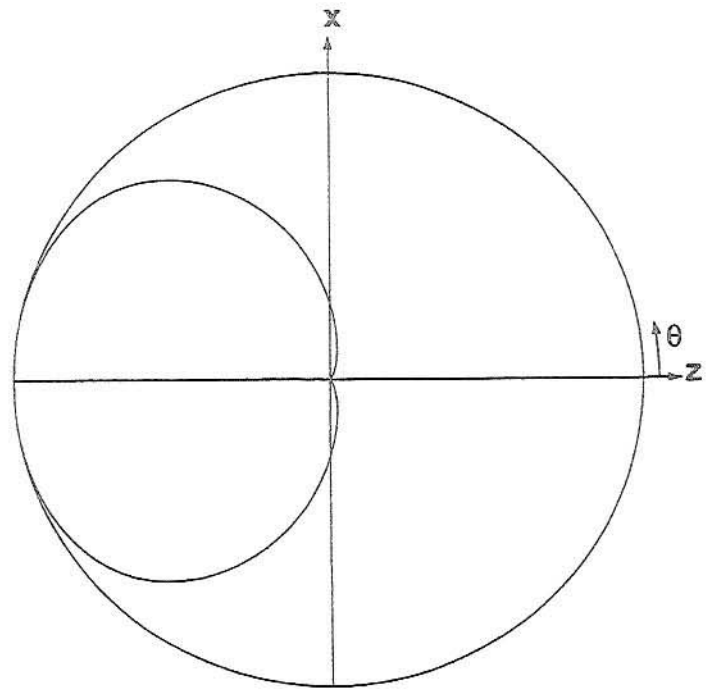

Figure P9.34: (b) Normalized array pattern of the 3-element array of Problem 9.34.

Problem 9.35 An eight-element linear array with $\lambda/2$ spacing is excited with equal amplitudes. To steer the main beam to a direction 60° below the broadside direction, what should be the incremental phase delay between adjacent elements? Also, give the expression for the array factor and plot the pattern.

Solution: Since broadside corresponds to $\theta = 90^\circ$, 60° below broadside is $\theta_0 = 150^\circ$. From Eq. (9.125),

$$\delta = kd\cos\theta_0 = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos 150^\circ = -2.72\,(\text{rad}) = -155.9^\circ.$$

Combining Eq. (9.126) with (9.127) gives

$$F_{\text{an}}(\theta) = \frac{\sin^2\left(\frac{1}{2}Nkd(\cos\theta - \cos\theta_0)\right)}{N^2\sin^2\left(\frac{1}{2}kd(\cos\theta - \cos\theta_0)\right)} = \frac{\sin^2\left(4\pi(\cos\theta + \frac{1}{2}\sqrt{3})\right)}{64\sin^2\left(\frac{1}{2}\pi(\cos\theta + \frac{1}{2}\sqrt{3})\right)}.$$

The pattern is shown in Fig. P9.35.

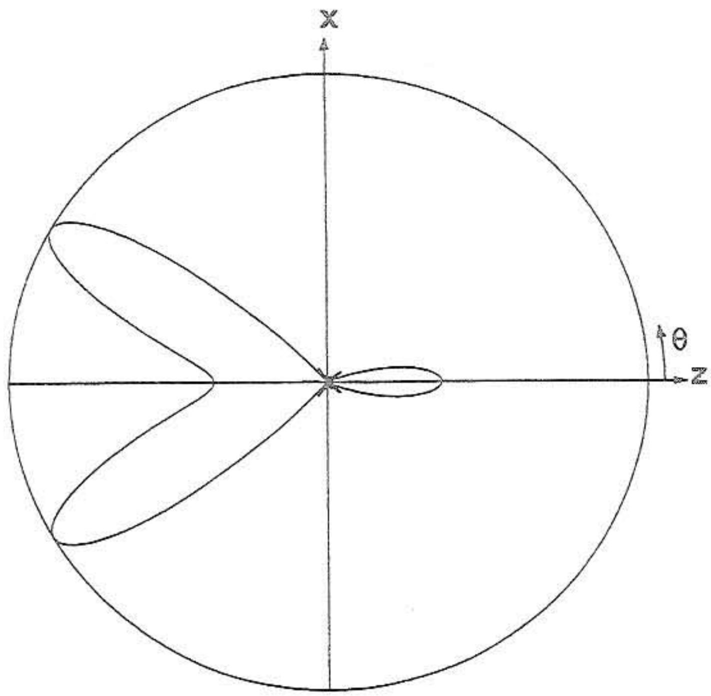

Figure P9.35: Pattern of the array of Problem 9.35.

Problem 9.36 A linear array arranged along the z-axis consists of 12 equally spaced elements with $d = \lambda/2$. Choose an appropriate incremental phase delay δ so as to steer the main beam to a direction 30° above the broadside direction. Provide an expression for the array factor of the steered antenna and plot the pattern. From the pattern, estimate the beamwidth.

Solution: Since broadside corresponds to $\theta = 90°$, 30° above broadside is $\theta_0 = 60°$. From Eq. (9.125),

$$\delta = kd\cos\theta_0 = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos 60° = 1.57\ (\text{rad}) = 90°.$$

Combining Eq. (9.126) with (9.127) gives

$$F_{\text{an}}(\theta) = \frac{\sin^2\left(\frac{1}{2}12kd(\cos\theta - \cos\theta_0)\right)}{144\sin^2\left(\frac{1}{2}kd(\cos\theta - \cos\theta_0)\right)} = \frac{\sin^2\left(6\pi(\cos\theta - 0.5)\right)}{144\sin^2\left(\frac{\pi}{2}(\cos\theta - 0.5)\right)}.$$

The pattern is shown in Fig. P9.36. The beamwidth is $\approx 10°$.

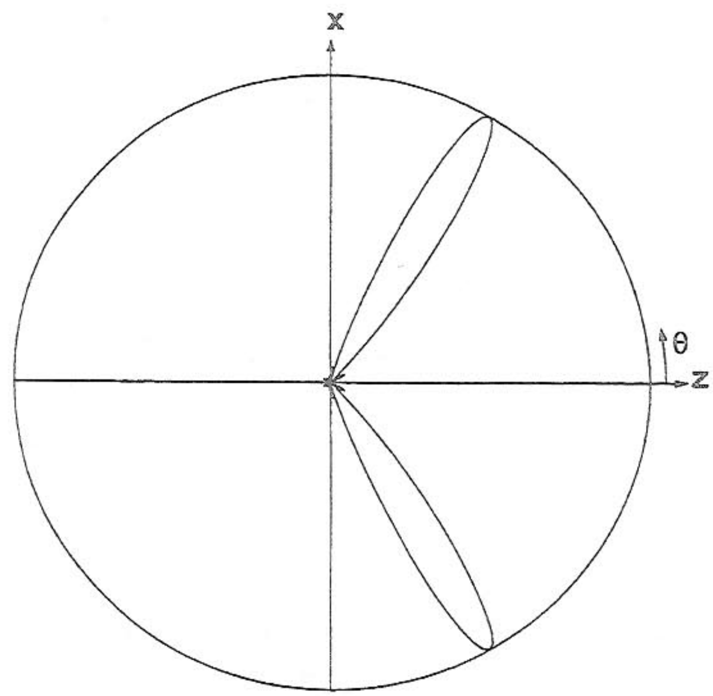

Figure P9.36: Array pattern of Problem 9.36.

Chapter 10

Sections 10-1 to 10-4: Satellite Communication Systems

Problem 10.1 A remote sensing satellite is in circular orbit around the earth at an altitude of 1,000 km above the earth's surface. What is its orbital period?

Solution: The orbit's radius is $R_0 = R_e + h = 6,378 + 1000 = 7378$ km. Rewriting Eq. (10.6) for T:

$$T = \left(\frac{4\pi^2 R_0^3}{GMe}\right)^{1/2} = \left[\frac{4\pi^2 \times (7.378 \times 10^6)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}\right]^{1/2}$$
$$= 6304.8439 \text{ s} = 105.08 \text{ minutes.}$$

Problem 10.2 A transponder with a bandwidth of 500 MHz uses polarization diversity. If the bandwidth allocated to transmit a single telephone channel is 4 kHz, how many telephone channels can be carried by the transponder?

Solution: Number of telephone channels $= \dfrac{2 \times 500 \text{ MHz}}{4 \text{ kHz}} = \dfrac{2 \times 5 \times 10^8}{4 \times 10^3} = 2.5 \times 10^5$ channels.

Problem 10.3 Repeat Problem 10.2 for TV channels, each requiring a bandwidth of 6 MHz.

Solution: Number of telephone channels $= \dfrac{2 \times 5 \times 10^8}{6 \times 10^6} = 166.67 \simeq 166$ channels. We need to round down becasue we cannot have a partial channel.

Problem 10.4 A geostationary satellite is at a distance of 40,000 km from a ground receiving station. The satellite transmitting antenna is a circular aperture with a 1-m diameter and the ground station uses a parabolic dish antenna with an effective diameter of 30 cm. If the satellite transmits 1 kW of power at 12 GHz and the ground receiver is characterized by a system noise temperature of 1,000 K, what would be the signal-to-noise ratio of a received TV signal with a bandwidth of 6 MHz? The antennas and the atmosphere may be assumed lossless.

Solution: We are given

$$R = 4 \times 10^7 \text{ m}, \quad d_t = 1 \text{ m}, \quad d_r = 0.3 \text{ m}, \quad P_t = 10^3 \text{ W},$$
$$f = 12 \text{ GHz}, \quad T_{sys} = 1,000 \text{ K}, \quad B = 6 \text{ MHz}.$$

At $f = 12$ GHz, $\lambda = c/f = 3\times10^8/12\times10^9 = 2.5\times10^{-2}$ m. With $\xi_t = \xi_r = 1$,

$$G_t = D_t = \frac{4\pi A_t}{\lambda^2} = \frac{4\pi(\pi d_t^2/4)}{\lambda^2} = \frac{4\pi\times\pi\times1}{4\times(2.5\times10^{-2})^2} = 15,791.37,$$

$$G_r = D_r = \frac{4\pi A_r}{\lambda^2} = \frac{4\pi(\pi d_r^2/4)}{\lambda^2} = \frac{4\pi\times\pi(0.3)^2}{4\times(2.5\times10^{-2})^2} = 1421.22.$$

Applying Eq. (10.11) with $\Upsilon(\theta) = 1$ gives:

$$S_n = \frac{P_t G_t G_r}{KT_{sys}B}\left(\frac{\lambda}{4\pi R}\right)^2 = \frac{10^3\times15,791.37\times1421.22}{1.38\times10^{-23}\times10^3\times6\times10^6}\left(\frac{2.5\times10^{-2}}{4\pi\times4\times10^7}\right)^2 = 670.49.$$

Sections 10-5 to 10-8: Radar Sensors

Problem 10.5 A collision avoidance automotive radar is designed to detect the presence of vehicles up to a range of 1 km. What is the maximum usable PRF?

Solution: From Eq. (10.14),

$$f_p = \frac{c}{2R_u} = \frac{3\times10^8}{2\times10^3} = 1.5\times10^5 \text{ Hz}.$$

Problem 10.6 A 10-GHz weather radar uses a 30-cm-diameter lossless antenna. At a distance of 1 km, what are the dimensions of the volume resolvable by the radar if the pulse length is 1 μs?

Solution: Resolvable volume has dimensions $\Delta x, \Delta y$, and ΔR.

$$\Delta x = \Delta y = \beta R = \frac{\lambda}{d}R = \frac{3\times10^{-2}}{0.3}\times10^3 = 100 \text{ m},$$

$$\Delta R = \frac{c\tau}{2} = \frac{3\times10^8}{2}\times10^{-6} = 150 \text{ m}.$$

Problem 10.7 A radar system is characterized by the following parameters: $P_t = 1$ kW, $\tau = 0.1$ μs, $G = 30$ dB, $\lambda = 3$ cm, and $T_{sys} = 1,500$ K. The radar cross section of a car is typically 10 m^2. How far can the car be and remain detectable by the radar with a minimum signal-to-noise ratio of 13 dB?

Solution: $S_{min} = 13$ dB means $S_{min} = 20$. $G = 30$ dB means $G = 1000$. Hence, by Eq. (10.27),

$$R_{max} = \left[\frac{P_t \tau G^2 \lambda^2 \sigma_t}{(4\pi)^3 K T_{sys} S_{min}}\right]^{1/4}$$
$$= \left[\frac{10^3 \times 10^{-7} \times 10^6 \times (3 \times 10^{-2})^2 \times 10}{(4\pi)^3 \times 1.38 \times 10^{-23} \times 1.5 \times 10^3 \times 20}\right]^{1/4} = 5753.12 \text{ m} = 5.75 \text{ km}.$$

Problem 10.8 A 3-cm-wavelength radar is located at the origin of an x–y coordinate system. A car located at $x = 100$ m and $y = 200$ m is heading east (x-direction) at a speed of 120 km/hr. What is the Doppler frequency measured by the radar?

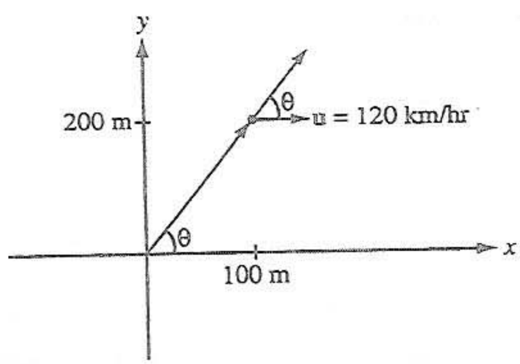

Figure P10.8: Geometry of Problem 10.8.

Solution:

$$\theta = \tan^{-1}\left(\frac{200}{100}\right) = 63.43°,$$
$$u = 120 \text{ km/hr} = \frac{1.2 \times 10^5}{3600} = 33.33 \text{ m/s},$$
$$f_d = \frac{-2u}{\lambda}\cos\theta = \frac{-2 \times 33.33}{3 \times 10^{-2}}\cos 63.43° = -993.88 \text{ Hz}.$$