Chapter Six:

Capacitance and Inductance

6.1 A 6-μF capacitor was charged to 12 V. Find the charge accumulated in the capacitor.

SOLUTION:

$Q = CV = (6\times10^{-6})(12)$ $\boxed{Q = 72\,\mu C}$

6.2 A capacitor has an accumulated charge of 600 μC with 5 V across it. What is the value of capacitance?

SOLUTION:

$$C = Q/V = \frac{600 \times 10^{-6}}{5}$$

$$\boxed{C = 120\mu F}$$

6.3 An uncharged 100-μF capacitor is charged by a constant current of 1 mA. Find the voltage across the capacitor after 4 s. CS

SOLUTION:

$v = \frac{1}{C}\int_0^T i\,dt$ $\quad C = 10^{-4}$ F $\quad i = 10^{-3}$ A $\quad T = 4$s

$v = \frac{iT}{C}$ $\quad \boxed{v = 40\text{V}}$

6.4 A 10-μF capacitor is charged by a constant current source, and its voltage is increased to 2 V in 5 s. Find the value of the constant current source.

SOLUTION:

$$v = \frac{1}{C}\int_0^5 i\,dt = 2 = \frac{i}{C}t\Big|_0^5 = \frac{5i}{C}$$

$$\boxed{i = 4\mu A}$$

6.5 A 50-μF capacitor initially charged to −12 V is charged by a constant current of 2.5 μA. Find the voltage across the capacitor after 3 min.

SOLUTION:

$v = v_0 + \frac{1}{C}\int_0^T i \, dt$ $\quad v_0 = -12V \quad T = 180s \quad i = 2.5\mu A$

$\boxed{v = -3V}$

6.6 The energy that is stored in a 25-μF capacitor is $w(t) = 12\sin^2 377t$ J. Find the current in the capacitor.

CS

SOLUTION:

$w(t) = \frac{1}{2} C v^2(t)$ $\quad v(t) = \sqrt{\frac{2w(t)}{C}}$

$i(t) = C\,dv/dt = \frac{d\sqrt{2C\,w(t)}}{dt}$ $\quad i(t) = \sqrt{24C}\,\omega \cos \omega t$

$$\boxed{i(t) = 9.23 \cos \omega t \text{ A} \quad \omega = 377 \text{ r/s}}$$

6.7 The voltage across a 150-μF capacitor is given by the expression $v(t) = 60 \sin 377t$ V. Find (a) the current in the capacitor and (b) the expression for the energy stored in the element.

SOLUTION:

a) $i = C\,dv/dt = 60 C \omega \cos \omega t$ $\boxed{i(t) = 3.39 \cos \omega t \text{ A}}$

b) $w(t) = \frac{1}{2} C v^2$ $\boxed{w(t) = 270 \sin^2 \omega t \text{ mJ}}$

6.8 The voltage across a 12-μF capacitor is shown in Fig. P6.8. Compute the waveform for the current in the capacitor.

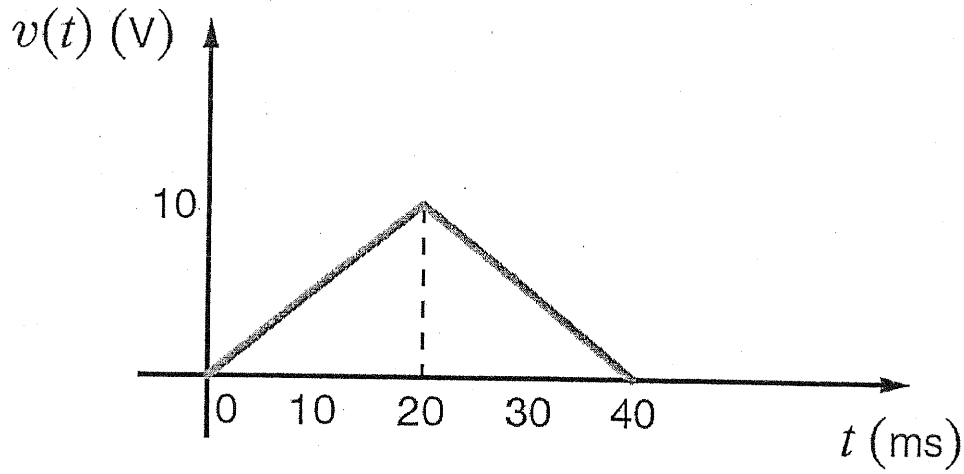


Figure P6.8

SOLUTION:

$t_1 = 20ms$ $\quad t_2 = 40ms$ $\quad i = c\, dv/dt$

$t < 0$ $\quad v(t) = 0\ V$ $\quad i = 0$

$0 \le t < t_1$ $\quad v(t) = 500t\ V$ $\quad i = 6mA$

$t_1 \le t < t_2$ $\quad v(t) = 20 - 500t\ V$ $\quad i = -6mA$

$t > t_2$ $\quad v(t) = 0$ $\quad i = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 6mA & 0 \le t < t_1 \\ -6mA & t_1 \le t < t_2 \\ 0 & t > t_2 \end{cases}$$

6.9 The current in a 100-μF capacitor is shown in Fig. P6.9. Determine the waveform for the voltage across the capacitor if it is initially uncharged. CS

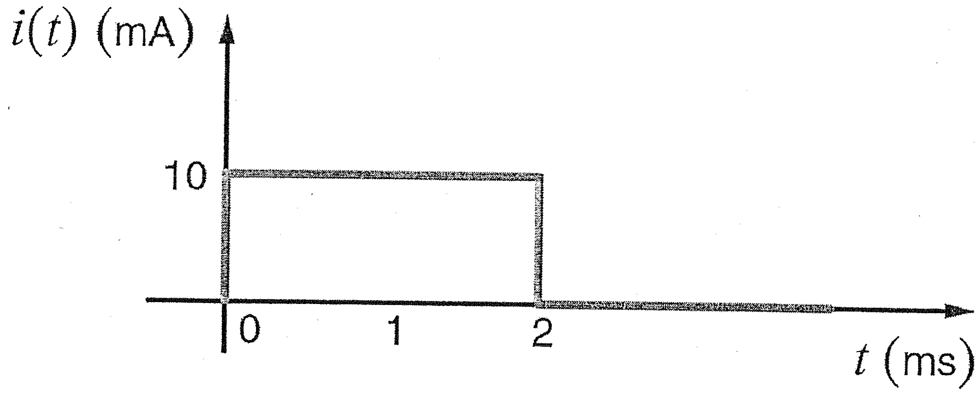


Figure P6.9

SOLUTION:

$t_1 = 2ms$ $\quad v = \frac{1}{C}\int i\,dt$

$t < 0$ $\quad i = 0$ $\quad v = 0$

$0 \le t < t_1$ $\quad i = 10mA$ $\quad v = 100t$ V

$t \ge t_1$ $\quad i = 0$ $\quad v = 0.2V$

$$v(t) = \begin{cases} 0 \text{ V} & t < 0 \\ 100t \text{ V} & 0 \le t < t_1 \\ 0.2 \text{ V} & t \ge t_1 \end{cases}$$

6.10 The voltage across a 50-μF capacitor is shown in Fig. P6.10. Compute the waveform for the current in the capacitor.

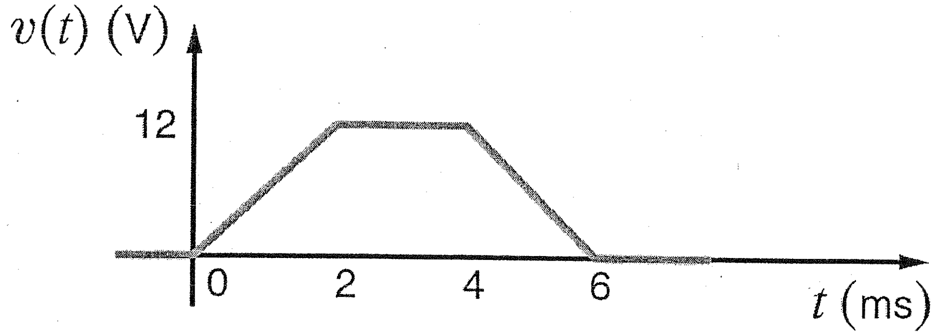


Figure P6.10

SOLUTION:

$t_1 = 2ms$ $t_2 = 4ms$ $t_3 = 6ms$ $i = C\,dv/dt$

$t < 0$	$v = 0$	$i = 0$
$0 \le t < t_1$	$v = 6000t$	$i = 300mA$
$t_1 \le t < t_2$	$v = 12$	$i = 0$
$t_2 \le t < t_3$	$v = 36 - 6000t$	$i = -300mA$
$t \ge t_3$	$v = 0$	$i = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 300mA & 0 \le t < t_1 \\ 0 & t_1 \le t < t_2 \\ -300mA & t_2 \le t < t_3 \\ 0 & t \ge t_3 \end{cases}$$

6.11 The voltage across a 25-μF capacitor is shown in Fig. P6.11. Determine the current waveform. PSV

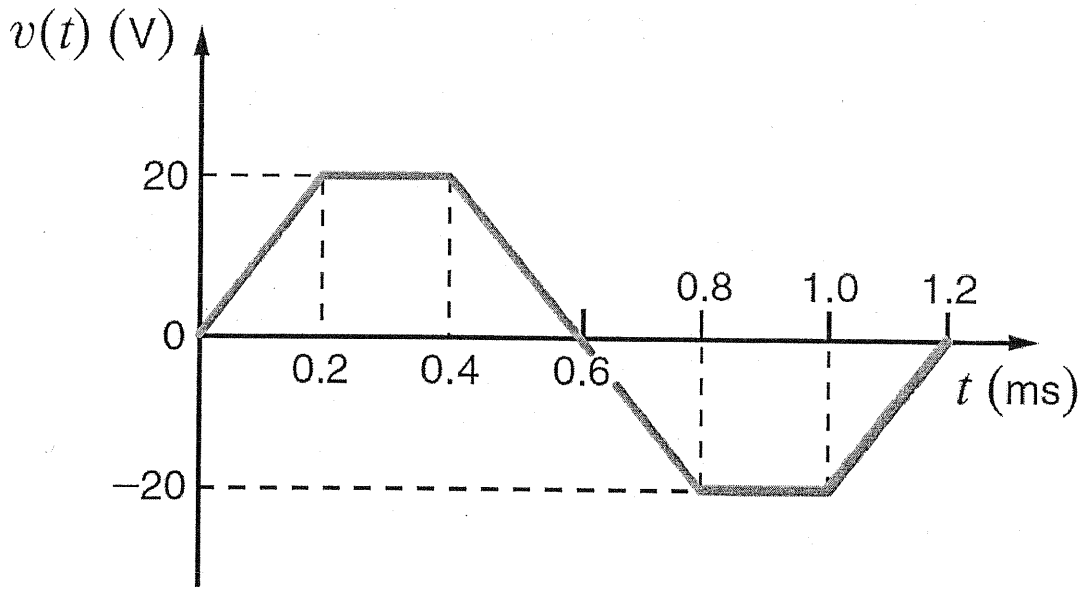


Figure P6.11

SOLUTION:

$t_1 = 0.2\,ms$ $t_2 = 0.4\,ms$ $t_3 = 0.8\,mA$ $t_4 = 1\,mA$ $t_5 = 1.2\,ms$ $i = C\,dv/dt$

$t < 0$	$v = 0$	$i = 0$
$0 \le t < t_1$	$v = 10^5 t$	$i = 2.5A$
$t_1 \le t < t_2$	$v = 20$	$i = 0$
$t_2 \le t < t_3$	$v = 60 - 10^5 t$	$i = -2.5A$
$t_3 \le t < t_4$	$v = 0$	$i = 0$
$t_4 \le t < t_5$	$v = -120 + 10^5 t$	$i = 2.5A$

$$i(t) = \begin{cases} 0 & t < 0 \\ 2.5 & 0 \le t < t_1 \\ 0 & t_1 \le t < t_2 \\ -2.5 & t_2 \le t < t_3 \\ 0 & t_3 \le t < t_4 \\ 2.5 & t_4 \le t < t_5 \\ 0 & t \ge t_5 \end{cases}$$

6.12 The voltage across a 2-F capacitor is given by the waveform in Fig. P6.12. Find the waveform for the current in the capacitor.

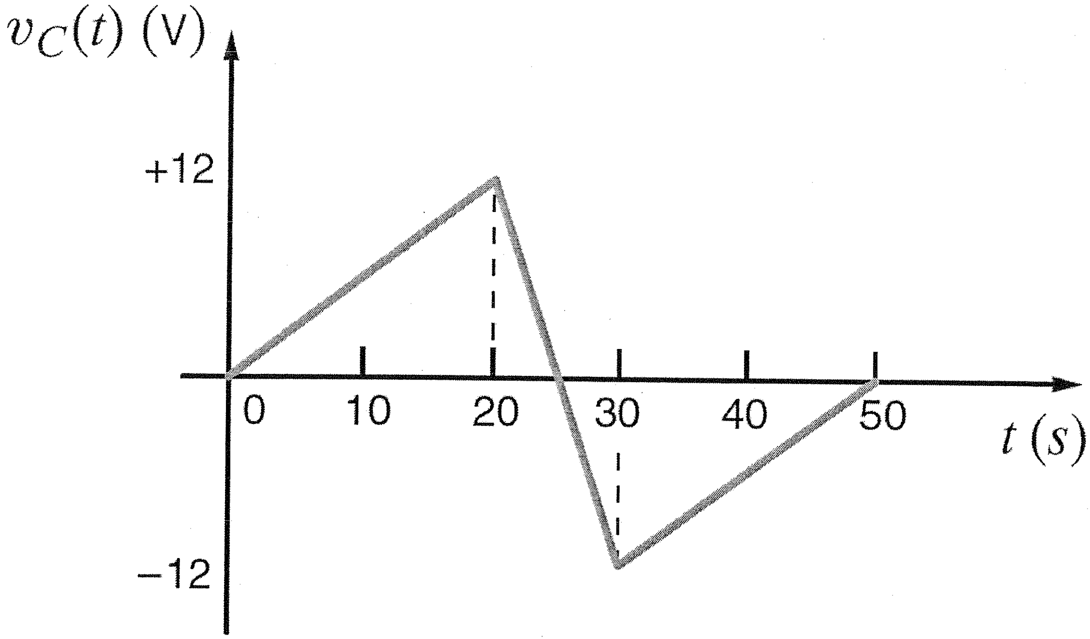


Figure P6.12

SOLUTION:

$t_1 = 20s \quad t_2 = 30s \quad t_3 = 50s \quad i = c\, dv/dt$

$t < 0$	$v = 0$	$i = 0$
$0 < t < t_1$	$v = 0.6t$ V	$i = 1.2$A
$t_1 < t < t_2$	$v = 60 - 2.4t$	$i = -4.8$A
$t_2 < t < t_3$	$v = -30 + 0.6t$	$i = 1.2$A
$t > t_3$	$v = 0$	$i = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 1.2\text{ A} & 0 < t < t_1 \\ -4.8\text{ A} & t_1 < t < t_2 \\ 1.2\text{ A} & t_2 < t < t_3 \\ 0 & t > t_3 \end{cases}$$

6.13 The voltage across a 2-μF capacitor is given by the waveform in Fig. P6.13. Compute the current waveform. CS

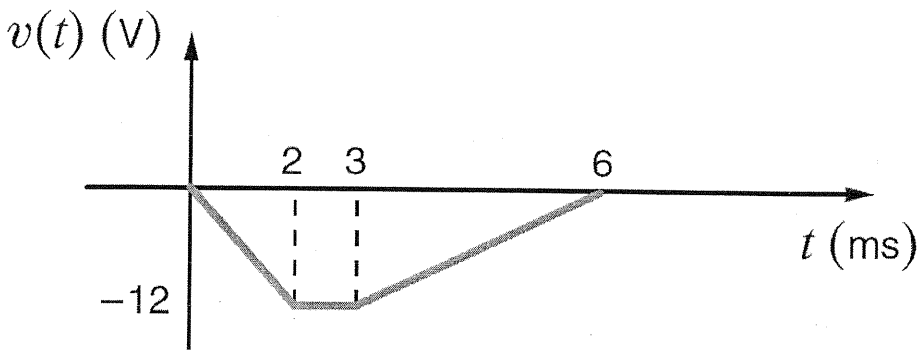


Figure P6.13

SOLUTION:

$t_1 = 2ms$ $\quad t_2 = 3ms$ $\quad t_3 = 6ms$ $\quad i = C\, dv/dt$

$t < 0$	$v = 0$	$i = 0$
$0 < t < t_1$	$v = -6000t$	$i = -12mA$
$t_1 < t < t_2$	$v = -12$	$i = 0$
$t_2 < t < t_3$	$v = -24 + 4000t$	$i = 8mA$
$t > t_3$	$v = 0$	$i = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ -12\ mA & 0 < t < t_1 \\ 0 & t_1 < t < t_2 \\ 8mA & t_2 < t < t_3 \\ 0 & t > t_3 \end{cases}$$

6.14 Draw the waveform for the current in a 24-μF capacitor when the capacitor voltage is as described in Fig. P6.14.

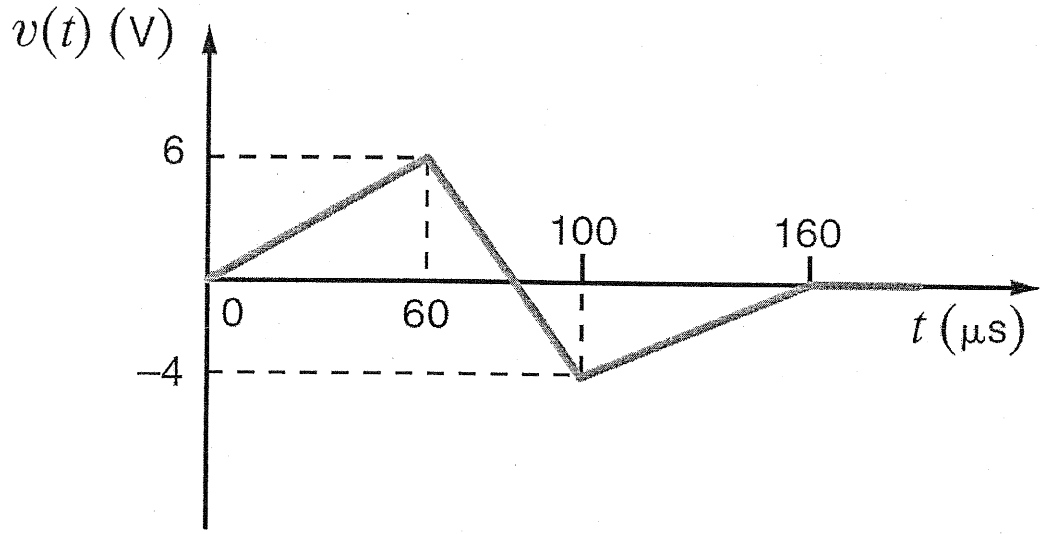


Figure P6.14

SOLUTION:

$t_1 = 60\mu s$ $\quad t_2 = 100\mu s$ $\quad t_3 = 160\mu s$ $\quad i = C\,dv/dt$

$t<0$	$v=0$	$i=0$
$0\le t<t_1$	$v=10^5 t$	$i=2.4A$
$t_1\le t<t_2$	$v=21-2.5\times10^5 t$	$i=-6A$
$t_2\le t<t_3$	$v=-\frac{32}{3}+\frac{10^6}{15}t$	$i=1.6A$
$t\ge t_3$	$v=0$	$i=0$

$$i(t) = \begin{cases} 0 & t<0 \\ 2.4A & 0\le t<t_1 \\ -2.4A & t_1\le t<t_2 \\ 1.6A & t_2\le t<t_3 \\ 0 & t\ge t_3 \end{cases}$$

6.15 Draw the waveform for the current in a 3-μF capacitor when the voltage across the capacitor is given in Fig. P6.15. CS

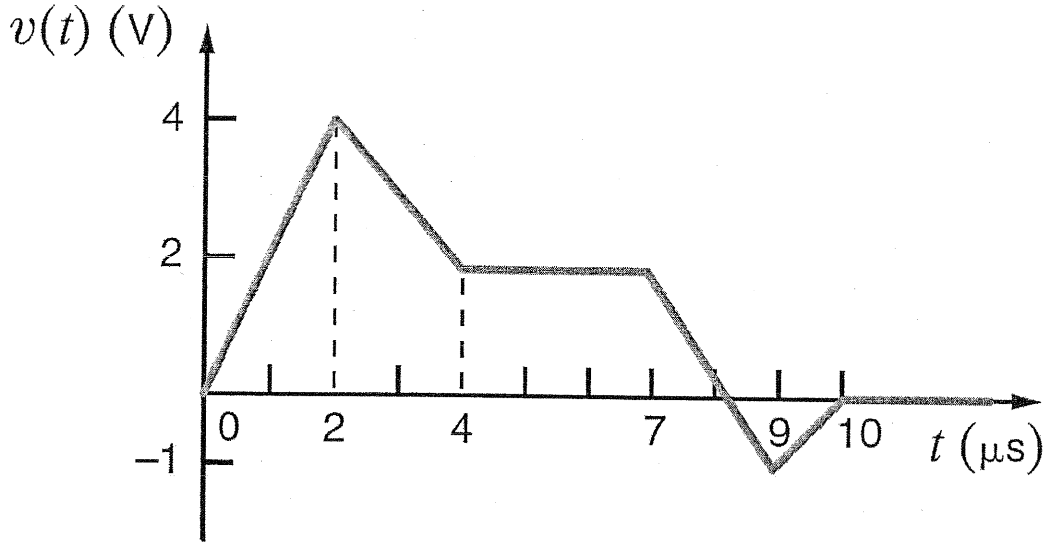


Figure P6.15

SOLUTION: $t_1 = 2\mu s$ $t_2 = 4\mu s$ $t_3 = 7\mu s$ $t_4 = 9\mu$ $t_5 = 10\mu s$

$i = c\, dv/dt$

$t < 0$ $dv/dt = 0$

$0 \le t < t_1$ $\Delta v/\Delta t = (4-0)/t_1 = 2\times10^6$

$t_1 \le t < t_2$ $\Delta v/\Delta t = (2-4)/(t_2-t_1) = -10^6$

$t_2 \le t < t_3$ $\Delta v/\Delta t = 0$

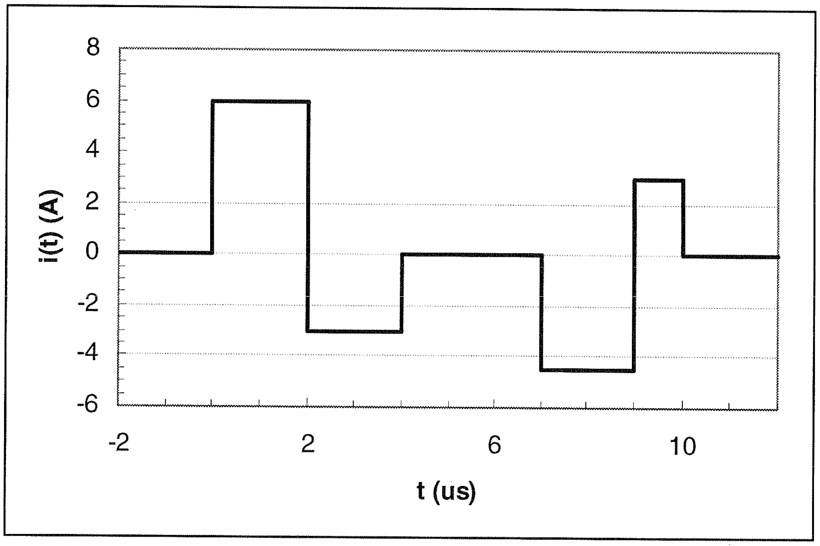
$t_3 \le t < t_4$ $\Delta v/\Delta t = (-1-2)/(t_4-t_3) = -1.5\times10^6$

$t_4 \le t < t_5$ $\Delta v/\Delta t = 1/(t_5-t_4) = 10^6$

$t \ge t_5$ $\Delta v/\Delta t = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 6A & 0 \le t < t_1 \\ -3A & t_1 \le t < t_2 \\ 0 & t_2 \le t < t_3 \\ -4.5A & t_3 \le t < t_4 \\ 3A & t_4 \le t < t_5 \\ 0 & t \ge t_5 \end{cases}$$

6.15



6.16 The voltage across a 10-μF capacitor is given by the waveform in Fig. P6.16. Plot the waveform for the capacitor current.

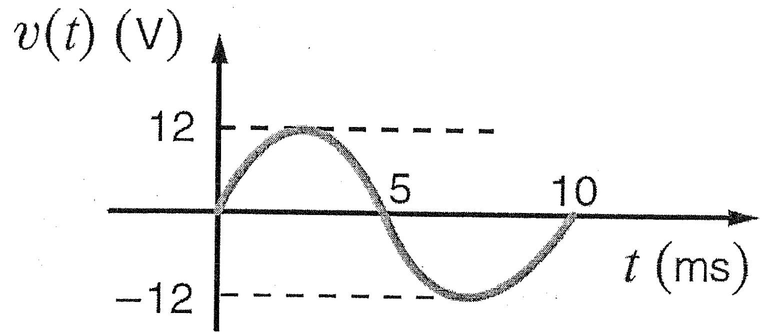


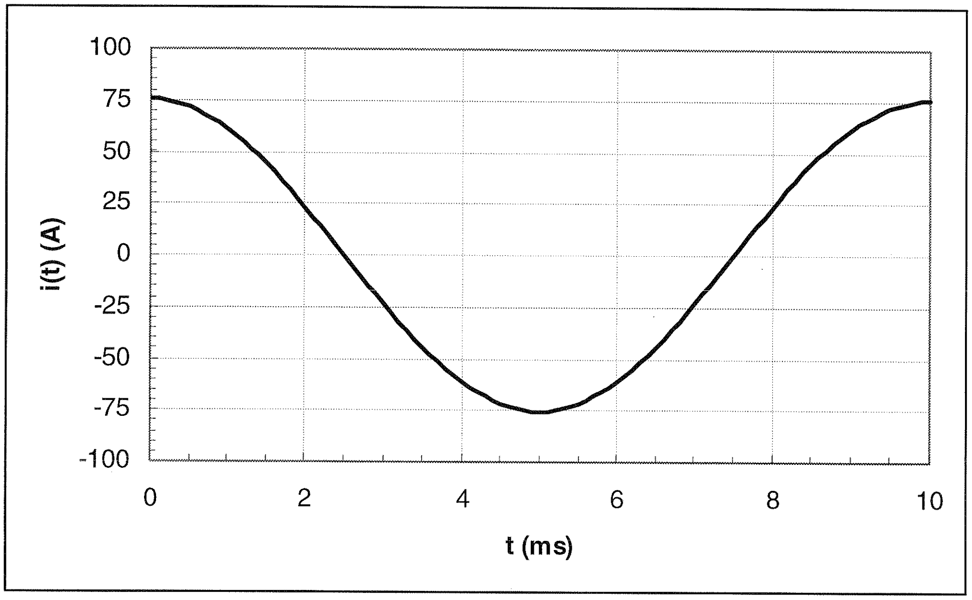
Figure P6.16

SOLUTION:

$i = C\,dv/dt$ $\quad v = 12 \sin \omega t$ $\quad \omega = \frac{2\pi}{T}$ $\quad T = 10\,ms \Rightarrow \omega = 200\pi$ r/s

$$\boxed{i = 75.4 \cos \omega t \text{ mA} \quad \omega = 200\pi \text{ r/s}}$$

6.16



6.17 The waveform for the current in a 50-μF capacitor is shown in Fig. P6.17. Determine the waveform for the capacitor voltage. PSV

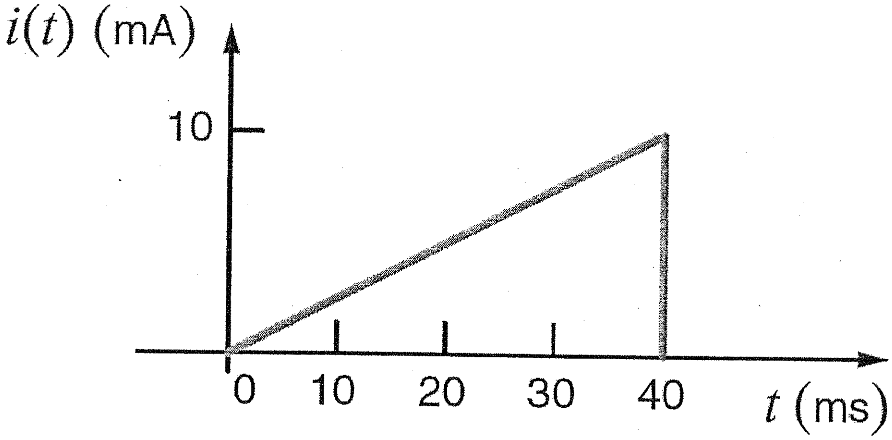


Figure P6.17

SOLUTION: $t_1 = 40\,ms$ $\quad v = \frac{1}{C}\int i\,dt + v_0$

$t < 0$ $\quad i(t) = 0$ $\quad v = 0$

$0 \le t < t_1$ $\quad i(t) = 0.25\,t$ $\quad v = 2500\,t^2$

$t \ge t_1$ $\quad i(t) = 0$ $\quad v = 2500\,(40\times10^{-3})^2 + 0 = 4V$

$$v(t) = \begin{cases} 0 & t < 0 \\ 2500\,t^2\ V & 0 \le t < t_1 \\ 4V & t \ge t_1 \end{cases}$$

6.18 The waveform for the current in a 100-μF initially uncharged capacitor is shown in Fig. P6.18. Determine the waveform for the capacitor's voltage.

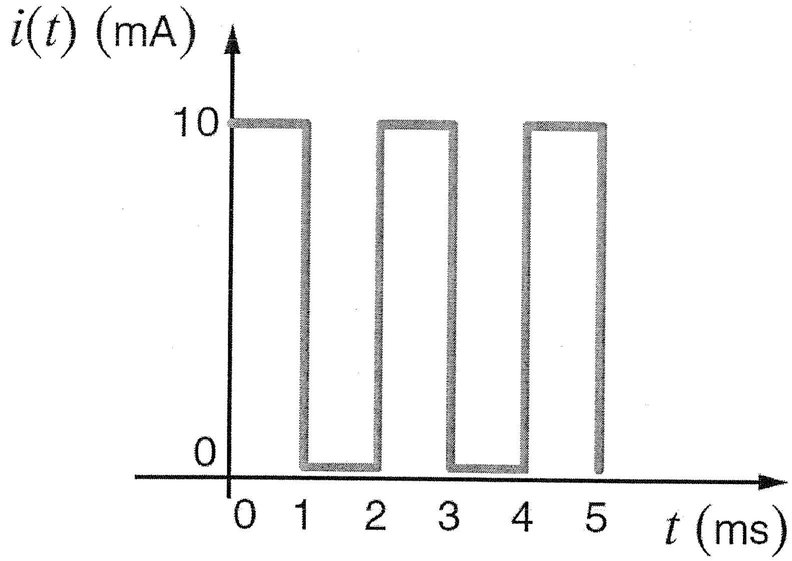


Figure P6.18

SOLUTION:

$v_c(t) = \frac{1}{C}\int i_c(t)\,dt = 10^4 \int i_c(t)\,dt$

for $i(t) = 10\,mA$, $v_c(t) = 100t + v_0$ (v_0 = initial voltage)

$$v_c(t) = \begin{cases} 0 \text{ V} & t < 0 \\ 100t \text{ V} & 0 \le t \le 1ms \\ 0.1 \text{ V} & 1ms \le t \le 2ms \\ -0.1 + 100t \text{ V} & 2ms \le t \le 3ms \\ 0.2 \text{ V} & 3ms \le t \le 4ms \\ -0.2 + 100t \text{ V} & 4ms \le t \le 5ms \\ 0.3\text{V} & t > 5m \end{cases}$$

6.19 The waveform for the current in a 50-μF initially uncharged capacitor is shown in Fig. P6.19. Determine the waveform for the capacitor's voltage.

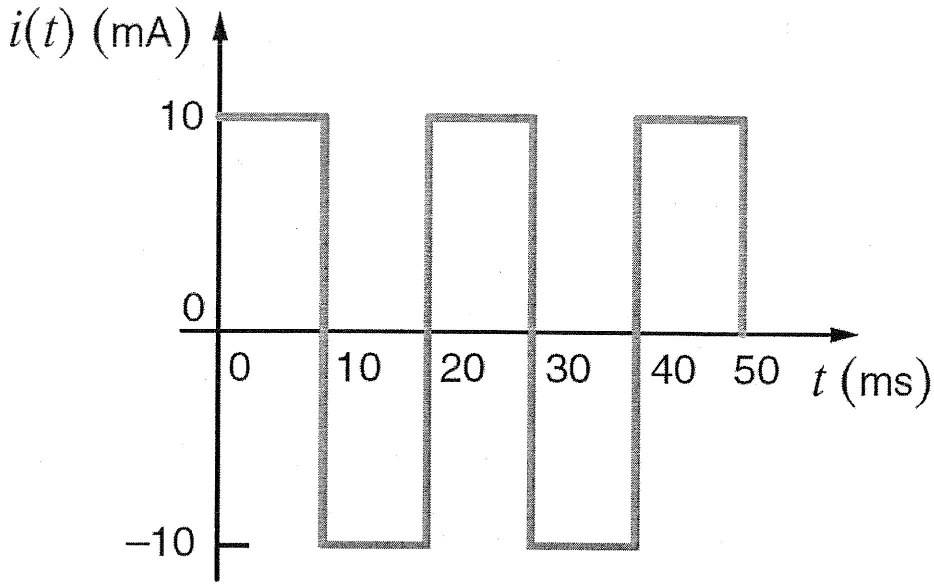


Figure P6.19

SOLUTION:

$v_c(t) = \frac{1}{C}\int i_c(t)\,dt$

for $i = 10mA$ $v_c(t) = 200t + V_o$ V_o = initial voltage

for $i = -10mA$ $v_c(t) = -200t + V_o$

$$v_c(t) = \begin{cases} 0 \text{ V} & t < 0 \\ 200t \text{ V} & 0 \le t \le 10ms \\ 4 - 200t \text{ V} & 10ms \le t \le 20ms \\ -4 + 200t \text{ V} & 20ms \le t \le 30ms \\ 8 - 200t \text{ V} & 30ms \le t \le 40ms \\ -8 + 200t \text{ V} & 40ms \le t \le 50ms \\ 0 \text{ V} & t > 50ms \end{cases}$$

6.20 If $v_C(t = 2\text{ s}) = 10$ V in the circuit in Fig. P6.20, find the energy stored in the capacitor and the power supplied by the source at $t = 6$ s.

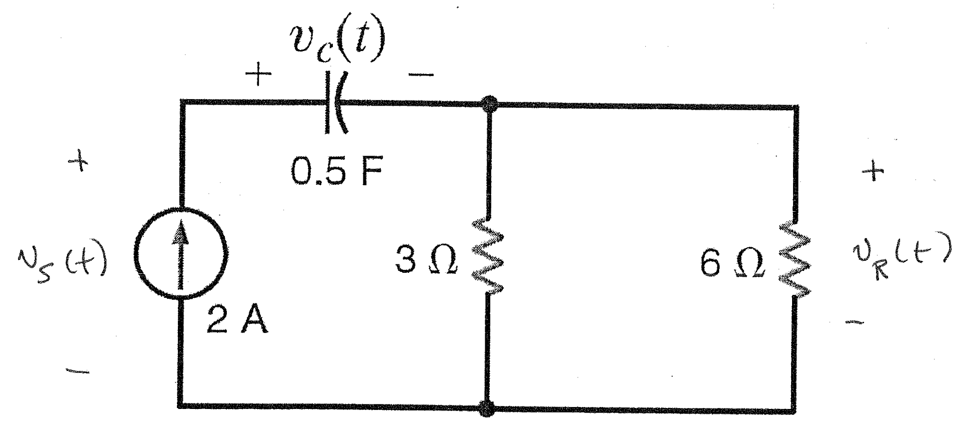


Figure P6.20

SOLUTION:

$$v_c(t_2) = \frac{1}{C}\int_{t_1}^{t_2} i_c(t)\,dt + V_o \qquad t_1 = 2s \quad t_2 = 6s \quad V_o = 10V$$

$$v_c(t_2) = 2(2)\,t\Big|_2^6 + 10 \qquad \boxed{v_c(t_2) = 26V}$$

$$w_c(t_2) = \frac{1}{2}Cv_c(t_2)^2 \Rightarrow \boxed{w_c(t_2) = 169J}$$

$$v_R(t_2) = i(t_2)\left[3 // 6\right] = 2(2) = 4\text{ V}$$

$$v_s(t_2) = v_c(t_2) + v_R(t_2) = 30V$$

$$P_s(t_2) = v_s(t_2)\,i_s(t_2) = 30(2) \Rightarrow \boxed{P_s(t_2) = 60W}$$

6.21 The current in an inductor changes from 0 to 200 mA in 4 ms and induces a voltage of 100 mV. What is the value of the inductor?

SOLUTION:

$v = L\, di/dt \Rightarrow L \frac{\Delta I}{\Delta t}$ $\Delta I = 200\text{mA}$ $\Delta t = 4\text{ms}$ $v = 0.1\text{V}$

$L = v\left(\frac{\Delta t}{\Delta I}\right)$ $\boxed{L = 2\text{mH}}$

6.22 The current in a 100-mH inductor is $i(t) = 2 \sin 377t$ A. Find (a) the voltage across the inductor and (b) the expression for the energy stored in the element. CS

SOLUTION:

a) $v = L\, di/dt = 0.1\ (2)(377) \cos 377t$

$$\boxed{v(t) = 75.4 \cos 377t \quad V}$$

b) $w(t) = \frac{1}{2} L\, i(t)^2$ $\boxed{w(t) = 0.2 \sin^2 377t \quad J}$

6.23 If the current $i(t) = 1.5t$ A flows through a 2-H inductor, find the energy stored at $t = 2s$.

SOLUTION:

$$w(t) = \frac{1}{2} L\, i(t)^2$$

$$w(2) = \left(\frac{1}{2}\right)(2)\left[1.5\,(2)\right]^2 \qquad \boxed{w(2) = 9\,J}$$

6.24 The current in a 25-mH inductor is given by the expressions

$$i(t) = 0 \qquad t < 0$$

$$i(t) = 10(1 - e^{-t})\text{mA} \qquad t > 0$$

Find (a) the voltage across the inductor and (b) the expression for the energy stored in it.

SOLUTION:

a) $v = L\, di/dt$

$$v = \begin{cases} 0 \text{ V} & t < 0 \\ 250e^{-t}\ \mu\text{V} & t > 0 \end{cases}$$

b) $w(t) = \frac{1}{2} L\, i(t)^2$

$$w(t) = \begin{cases} 0 \text{ J} & t < 0 \\ 1.25(1 - e^{-t})^2\ \mu\text{J} & t > 0 \end{cases}$$

6.25 Given the data in the previous problem, find the voltage across the inductor and the energy stored in it after 1 s. CS

SOLUTION:

$v(t) = 250e^{-t}\ \mu V \quad t > 0$

$w(t) = 1.25\,(1 - e^{-t})^2\ \mu J \quad t > 0$

$v(1) = 92.0\ \mu V$

$w(1) = 0.50\ \mu J$

6.26 The current in a 10-mH inductor is shown in Fig. P6.26. Determine the waveform for the voltage across the inductor.

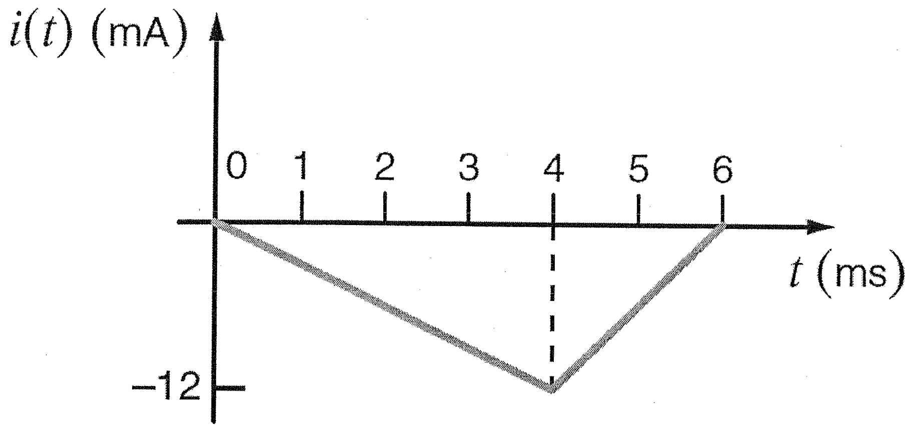


Figure P6.26

SOLUTION:

$v = L\,di/dt$

for $0 \le t \le 4\text{ms}$, $\quad di/dt = -\dfrac{12\times10^{-3}}{4\times10^{-3}} = -3\text{ A/s}$

for $4\text{ms} \le t \le 6\text{ms}$, $\quad di/dt = +12\times10^{-3}/2\times10^{-3} = +6\text{ A/s}$

$$v = \begin{cases} 0\text{ V} & t < 0 \\ -30\text{ mV} & 0 < t \le 4\text{ms} \\ +60\text{ mV} & 4\text{ms} < t \le 6\text{ms} \\ 0\text{ V} & t > 6\text{ms} \end{cases}$$

6.27 The current in a 50-mH inductor is given in Fig. P6.27. Sketch the inductor voltage. CS

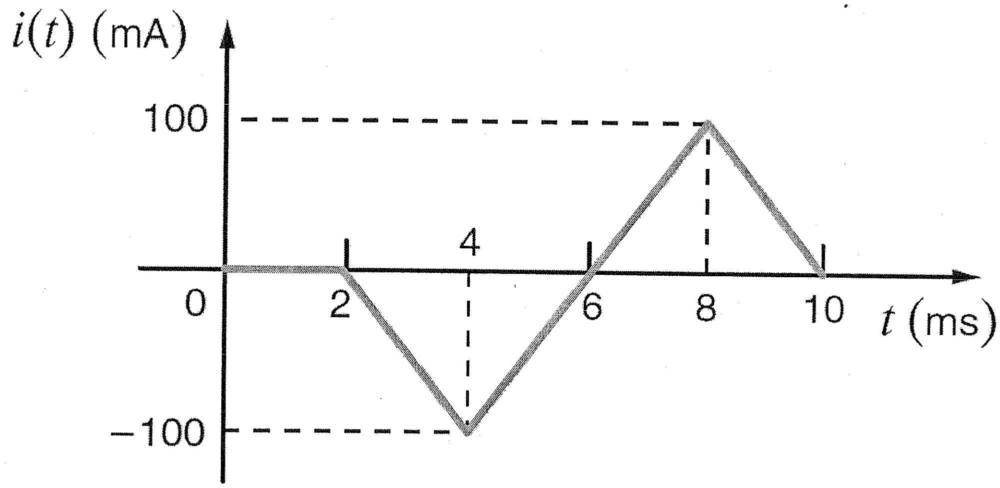
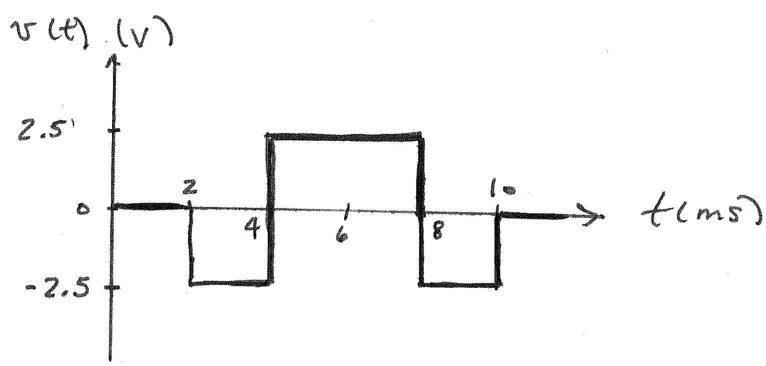


Figure P6.27

SOLUTION:

$v = L\,di/dt$

$$di/dt = \begin{cases} -50 \text{ A/s} & t \text{ between } 2 \text{ \& } 4\text{ms and } 8 \text{ to } 10\text{ms} \\ +50 \text{ A/s} & t \text{ between } 4\text{ms to } 8\text{ ms.} \end{cases}$$



6.28 The current in a 50-mH inductor is shown in Fig. P6.28. Find the voltage across the inductor. **PSV**

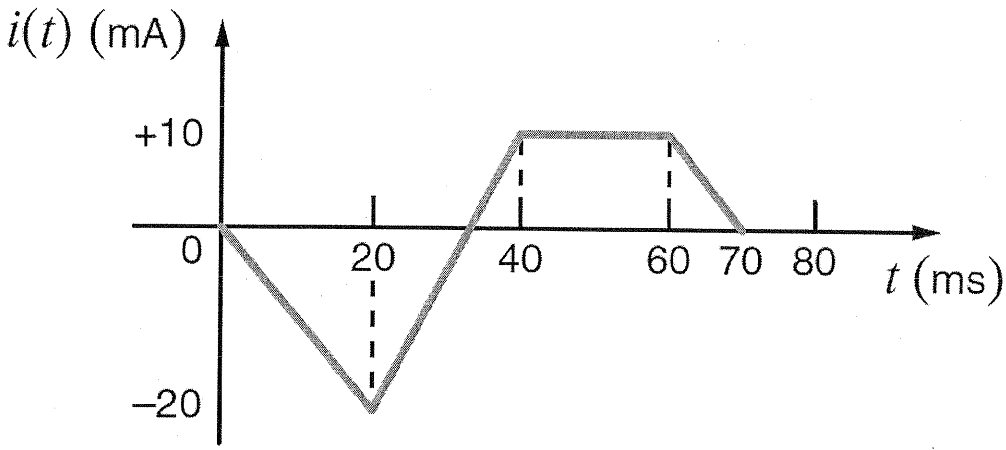


Figure P6.28

SOLUTION:

$v = L\,di/dt$

$$di/dt = \begin{cases} -1\ A/s & t \text{ between } 0 \text{ to } 20ms \\ 1.5\ A/s & t \text{ betwen } 20ms \text{ to } 40ms \\ -1\ A/s & t \text{ betwen } 60ms \text{ to } 70ms \\ 0 & \text{otherwise} \end{cases}$$

$$v(t) = \begin{cases} 0\ V & t < 0 \\ -50\ mV & 0 < t \le 20ms \\ +75\ mV & 20ms \le t \le 40ms \\ 0\ V & 40ms < t \le 60ms \\ -50\ mV & 60ms < t \le 70ms \\ 0V & t > 70ms \end{cases}$$

6.29 Draw the waveform for the voltage across a 24-mH inductor when the inductor current is given by the waveform shown in Fig. P6.29.

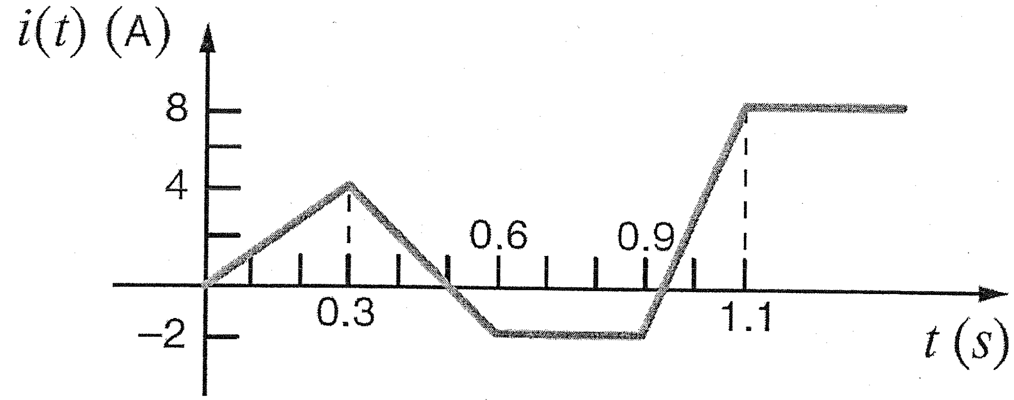


Figure P6.29

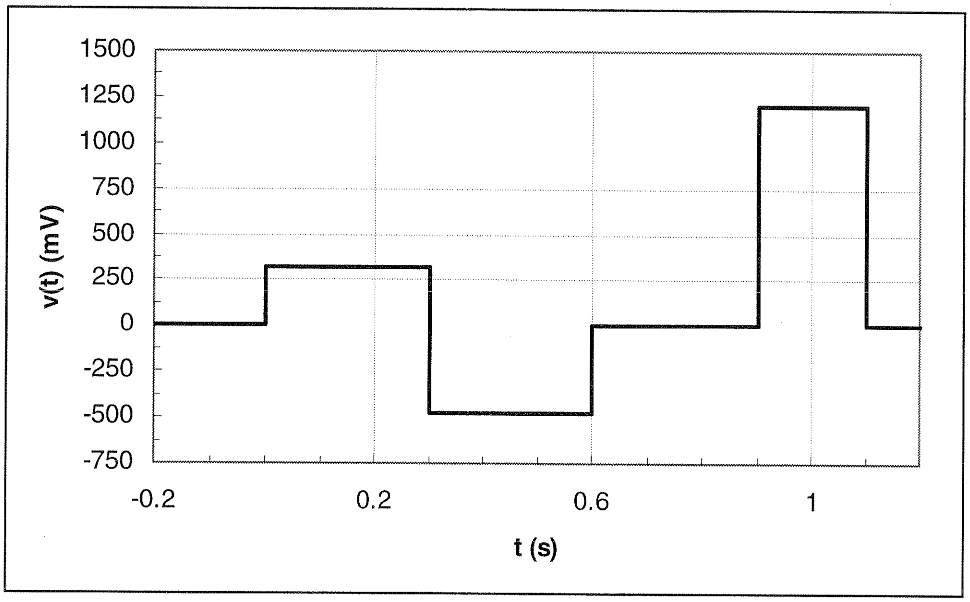
SOLUTION:

$v = L\,di/dt$

$$di/dt = \begin{cases} 40/3 \text{ A/s} & 0 \le t \le 0.3\text{ s} \\ -20 \text{ A/s} & 0.3 < t \le 0.6\text{ s} \\ 50 \text{ A/s} & 0.9 < t \le 1.1\text{ s} \\ 0 & \text{otherwise} \end{cases}$$

$$v(t) = \begin{cases} 0 & t \le 0 \\ 320\text{ mV} & 0 < t \le 0.3\text{ s} \\ -480\text{ mV} & 0.3 < t \le 0.6\text{ s} \\ 0 & 0.6 < t \le 0.9\text{ s} \\ 1200\text{ mV} & 0.9 < t \le 1.1\text{ s} \\ 0 & t > 1.1\text{ s} \end{cases}$$

6.29



6.30 The current in a 24-mH inductor is given by the waveform in Fig. P6.30. Find the waveform for the voltage across the inductor.

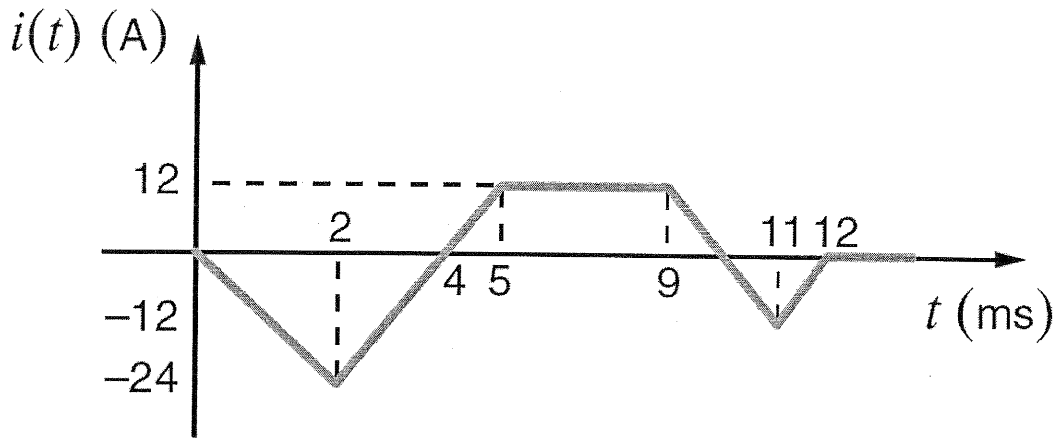


Figure P6.30

SOLUTION:

$v = L\, di/dt$

$$\frac{di}{dt} = \begin{cases} -12 \times 10^3 \text{ A/s} & 0 \le t \le 2\text{ms} \\ +12 \times 10^3 \text{ A/s} & 2\text{ms} < t \le 5\text{ms} \\ -12 \times 10^3 \text{ A/s} & 9\text{ms} \le t \le 11\text{ms} \\ +12 \times 10^3 \text{ A/s} & 11\text{ms} < t \le 12\text{ms} \end{cases}$$

$$v(t) = \begin{cases} 0 \text{ V} & t \le 0 \\ -288 \text{ V} & 0 < t \le 2\text{ms} \\ +288 \text{ V} & 2\text{ms} < t \le 5\text{ms} \\ 0 \text{ V} & 5\text{ms} < t \le 9\text{ms} \\ -288 \text{ V} & 9\text{ms} < t \le 11\text{ms} \\ +288 \text{ V} & 11\text{ms} < t \le 12\text{ms} \\ 0 \text{ V} & t > 12\text{ms} \end{cases}$$

6.31 The current in a 4-mH inductor is given by the waveform in Fig. P6.31. Plot the voltage across the inductor.

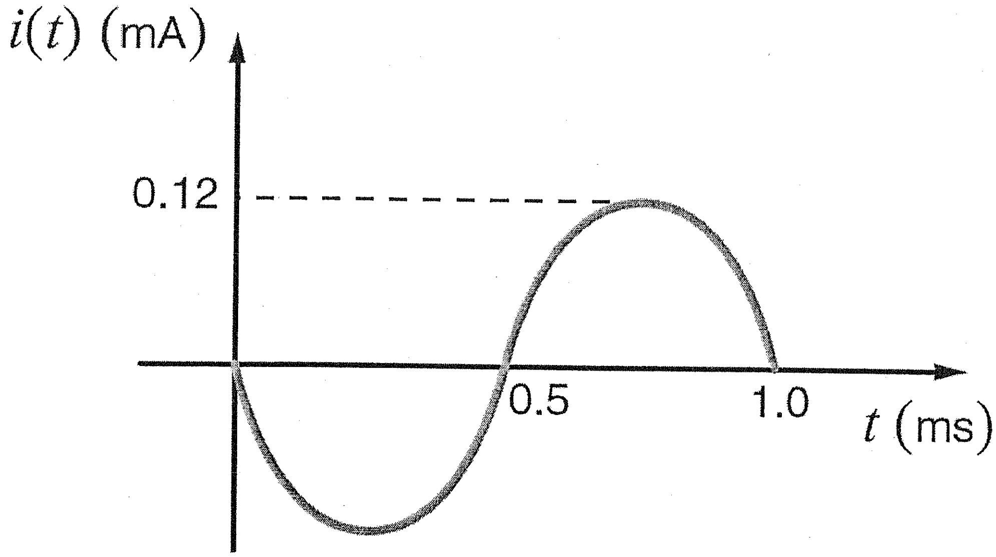
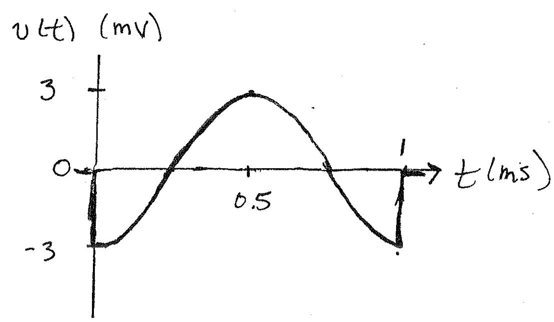


Figure P6.31

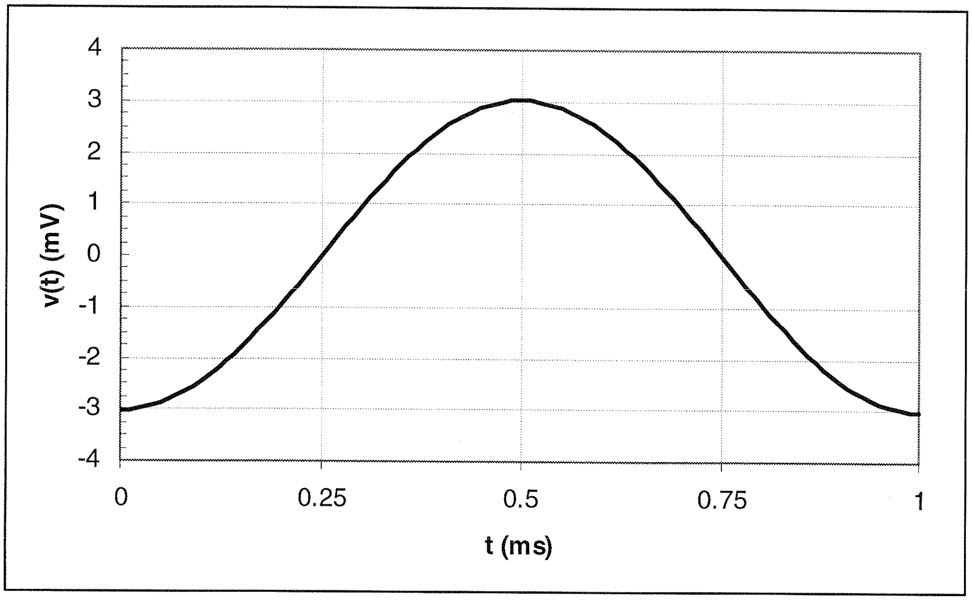
SOLUTION:

$i(t) = -120 \sin(\omega t)\ \mu A$ $\qquad \omega = \dfrac{2\pi}{T} \qquad T = 1\,ms \Rightarrow \omega = 2000\pi\ r/s$

$i(t) = -120 \sin(2000\pi t)\ \mu A \qquad v(t) = L\dfrac{di}{dt} = -3.02 \cos(2000\pi t)\ mV$



6.31



6.32 The voltage across a 2-H inductor is given by the waveform shown in Fig. P6.32. Find the waveform for the current in the inductor.

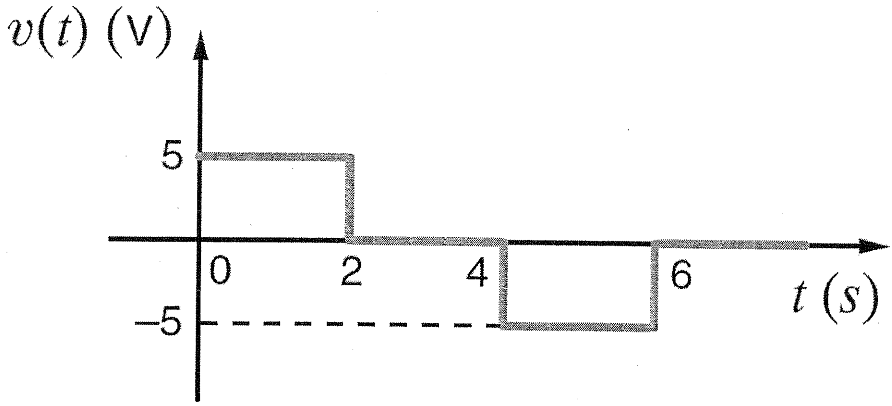


Figure P6.32

SOLUTION:

$i = \frac{1}{L}\int v(t)\,dt$

Since $v(t)$ is a constant across each time span, we can write

$i = \frac{V}{L}t + I_o$ V = constant voltage, I_o = initial current.

$$i(t) = \begin{cases} 0 & t<0 \\ 2.5t \text{ A} & 0 \le t \le 2s \\ 5 \text{ A} & 2 \le t \le 4s \\ 15 - 2.5t \text{ A} & 4 < t \le 6s \\ 0 \text{ A} & t \ge 6s \end{cases}$$

6.33 The waveform for the voltage across a 20-mH inductor is shown in Fig. P6.33. Compute the waveform for the inductor current. $v(t) = 0, t < 0$. CS

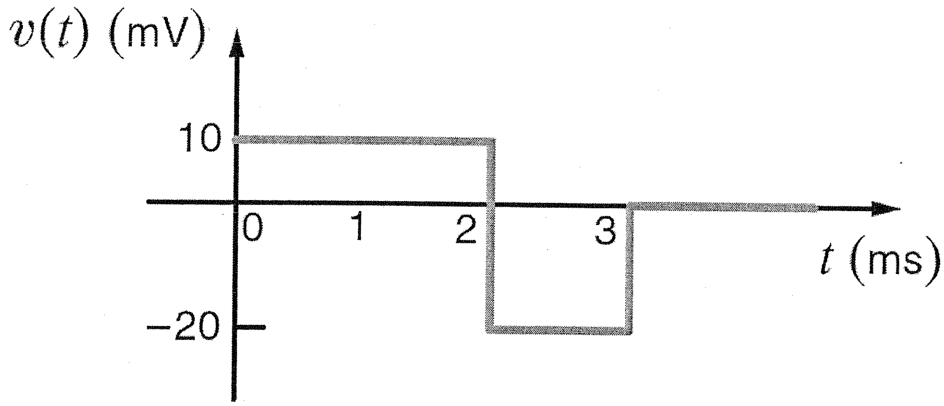


Figure P6.33

SOLUTION:

$i = \frac{1}{L}\int v(t)\,dt$

Since $v(t)$ is constant across time spans,

$i(t) = \frac{V}{L}t + I_0$ V = constant voltage
I_0 = initial current

$$i(t) = \begin{cases} 0 \text{ A} & t < 0 \\ t/2 \text{ A} & 0 \le t \le 2ms \\ 3\times10^{-3} - t \text{ A} & 2ms \le t \le 3ms \\ 0 \text{ A} & t \ge 3ms \end{cases}$$

6.34 The voltage across a 4-H inductor is given by the waveform shown in Fig. P6.34. Find the waveform for the current in the inductor. $v(t) = 0, t < 0$. **PSV**

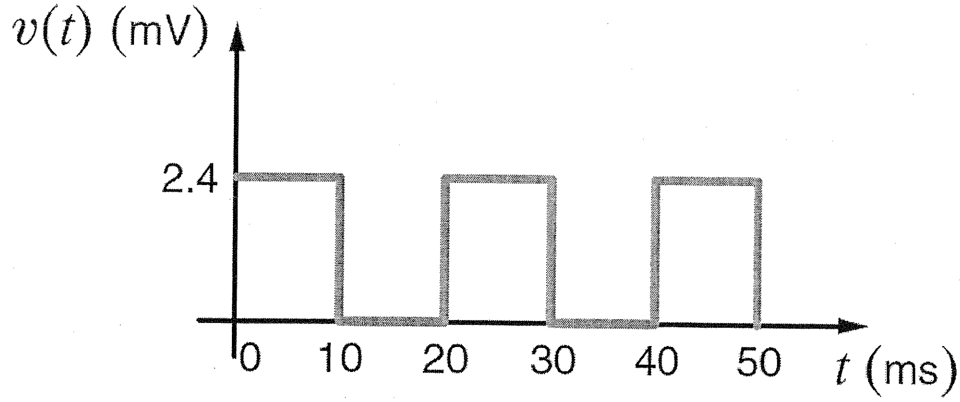


Figure P6.34

SOLUTION:

$i(t) = \frac{1}{L}\int v(t)\,dt$ $\qquad$ $v(t)$ is constant across time spans, so

$$i(t) = \frac{V}{L}t + I_0$$

V = constant voltage
I_0 = initial current

$$i(t) = \begin{cases} 0 & t < 0 \\ 600t\ \mu A & 0 \le t \le 10ms \\ 6\ \mu A & 10ms \le t \le 20ms \\ -6 + 600t\ \mu A & 20ms \le t \le 30ms \\ 12\ \mu A & 30ms \le t \le 40ms \\ -12 + 600t\ \mu A & 40ms \le t \le 50ms \\ 18\ \mu A & t \ge 50ms \end{cases}$$

6.35 The voltage across a 24-mH inductor is shown in Fig. P6.35. Determine the waveform for the inductor current. $v(t) = 0, t < 0$.

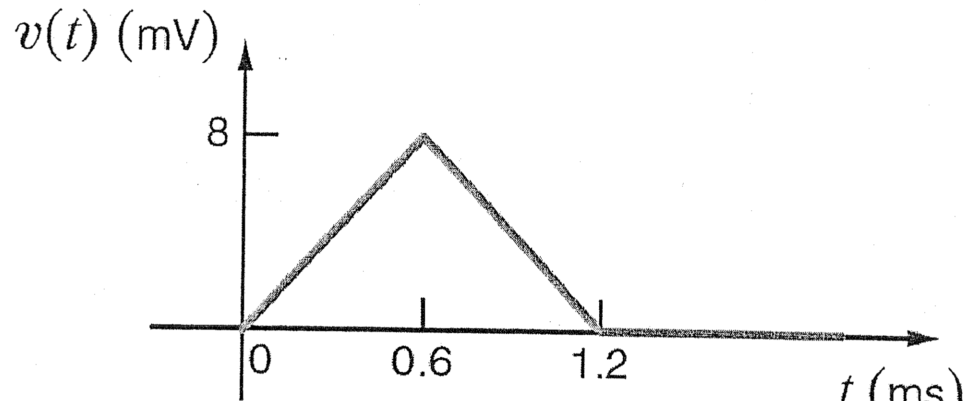


Figure P6.35

SOLUTION:

For $0 \le t \le 0.6ms$ $\quad v(t) = (40/3)t$ V $\quad i(t) = \frac{1}{L}\int v\,dt = \frac{20}{3L}t^2 = 278t^2$ A

For $0.6m \le t \le 1.2ms$ $\quad v(t) = 16\times10^{-3} - \left(\frac{40}{3}\right)t$

$$i(t) = \frac{1}{L}\int v\,dt = \frac{1}{L}\left[16\times10^{-3}t - \frac{20}{3}t^2 + K\right]$$

$$i(t_1) = 100\mu A = \left[16\times10^{-3}t_1 - \frac{20}{3}t_1^2 + K\right]/L \qquad t_1 = 0.6\,ms$$

$$K = -200\mu A$$

$$i(t) = \begin{cases} 0 \text{ A} & t < 0 \\ 278t^2 \text{ A} & 0 \le t \le 0.6ms \\ 0.667t - 278t^2 - 200\times10^{-6} \text{ A} & 0.6ms \le t \le 1.2ms \\ 200\mu A & t \ge 1.2ms \end{cases}$$

6.36 Find the possible capacitance range of the following capacitors.

(a) $0.068\ \mu F$ with a tolerance of 10%.

(b) 120 pF with a tolerance of 20%.

(c) $39\ \mu F$ with a tolerance of 20%.

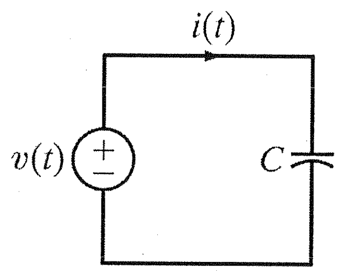
SOLUTION:

a) $C = 6.8 nF \pm 10\%$ $\qquad 6.12 nF \le C \le 7.48 nF$

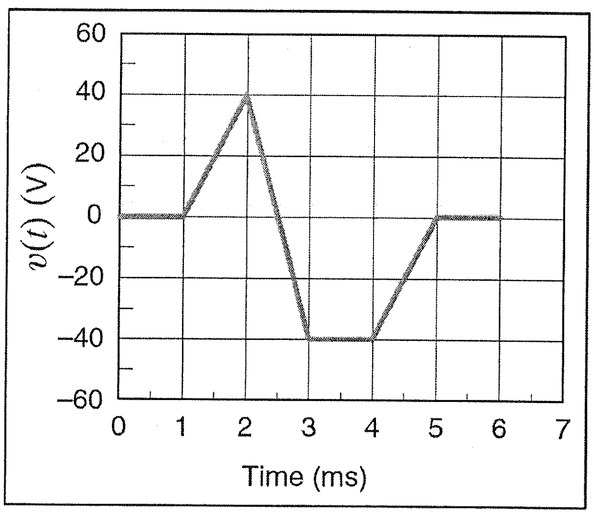
b) $C = 120 pF \pm 20\%$ $\qquad 96 pF \le C \le 144 pF$

c) $C = 39 \mu F \pm 20\%$ $\qquad 31.2 \mu F \le C \le 46.8 \mu F$

6.37 The capacitor in Fig. P6.37a is 51 nF with a tolerance of 10%. Given the voltage waveform in Fig. P6.37b, graph the current $i(t)$ for the minimum and maximum capacitor values.



(a)



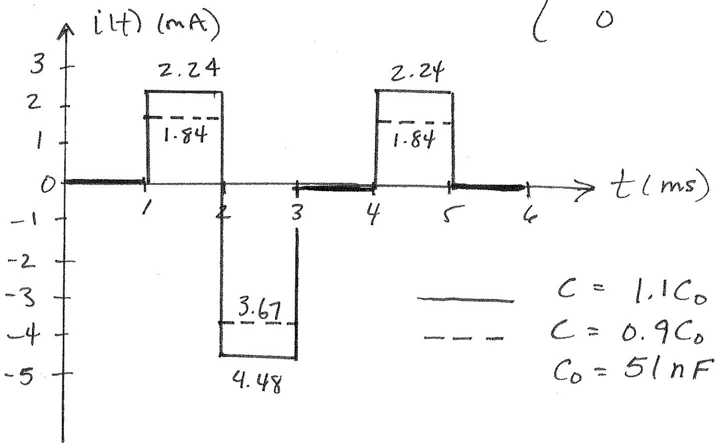
(b)

Figure P6.37

SOLUTION:

$i(t) = C\, dv/dt$

$$dv/dt = \begin{cases} 4\times10^4 \text{ V/s} & t = 1 \text{ to } 2 \text{ ms and } 4\text{–}5\text{ms} \\ -8\times10^4 \text{ V/s} & t = 2 \text{ to } 3\text{ms} \\ 0 & \text{otherwise} \end{cases}$$



6.38 Find the possible inductance range of the following inductors. CS

(a) 10 mH with a tolerance of 10%.

(b) 2.0 nH with a tolerance of 5%.

(c) 68 μH with a tolerance of 10%.

SOLUTION:

a) $L = 10mH \pm 10\%$ $\quad 9mH \le L \le 11mH$

b) $L = 2nH \pm 5\%$ $\quad 1.9nH \le L \le 2.1nH$

c) $L = 68\mu H \pm 10\%$ $\quad 61.2\mu H \le L \le 74.8\mu H$

6.39 The inductor in Fig. P6.39a is 330 μH with a tolerance of 5%. Given the current waveform in Fig. P6.39b, graph the voltage $v(t)$ for the minimum and maximum inductor values.

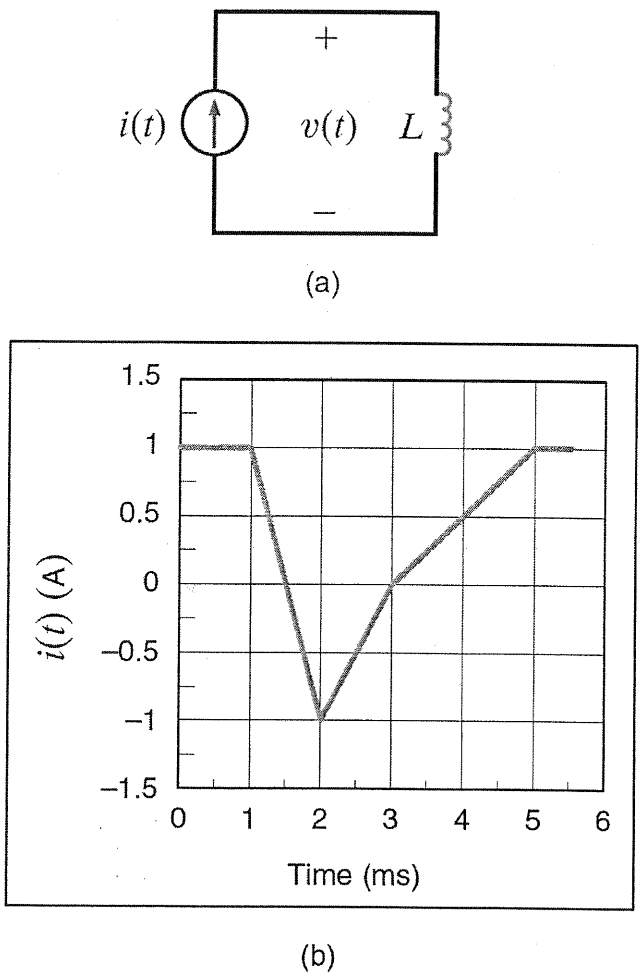
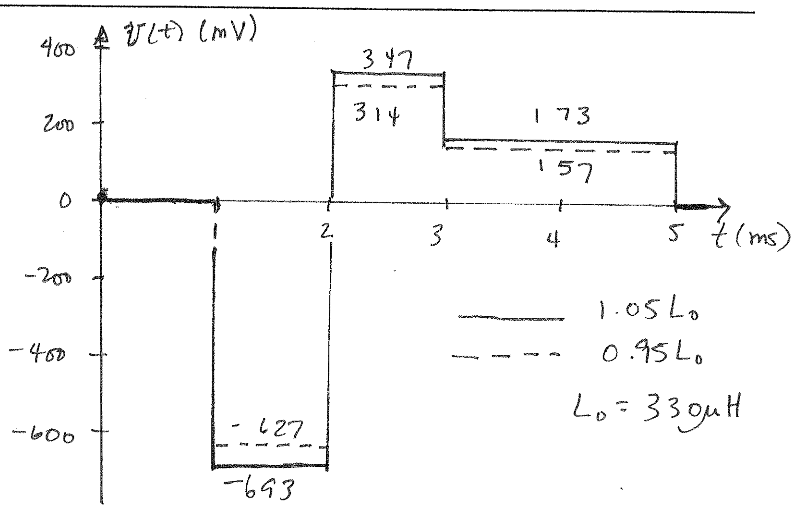


Figure P6.39

SOLUTION: $v = L\, di/dt$

$$\frac{di}{dt} = \begin{cases} -2000 \text{ A/s} & 1 \le t \le 2 \text{ ms} \\ 1000 \text{ A/s} & 2 \le t \le 3 \text{ ms} \\ 500 \text{ A/s} & 3 \le t \le 5 \text{ ms} \\ 0 & \text{otherwise} \end{cases}$$



6.40 The inductor in Fig. P6.40a is 4.7 μH with a tolerance of 20%. Given the current waveform in Fig. 6.40b, graph the voltage $v(t)$ for the minimum and maximum inductor values.

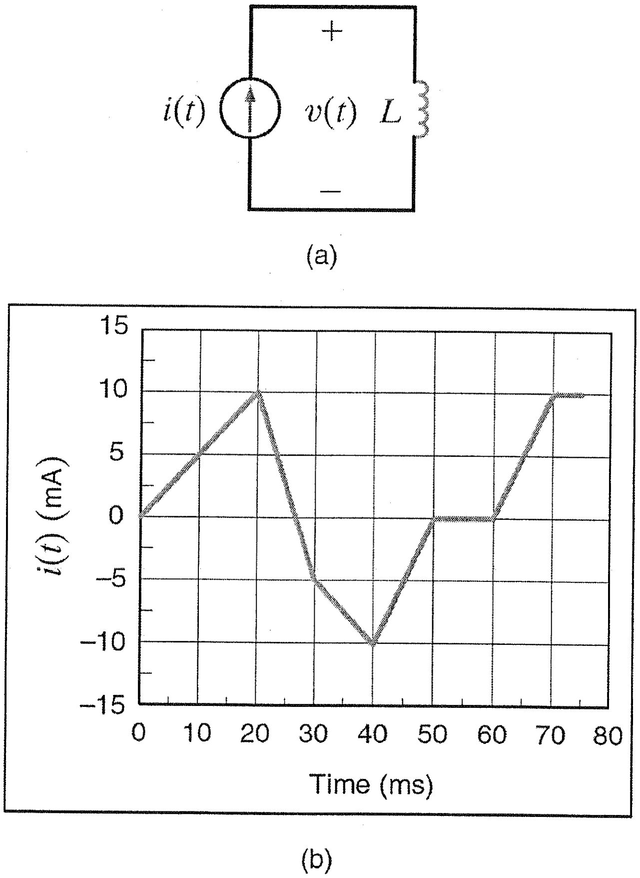
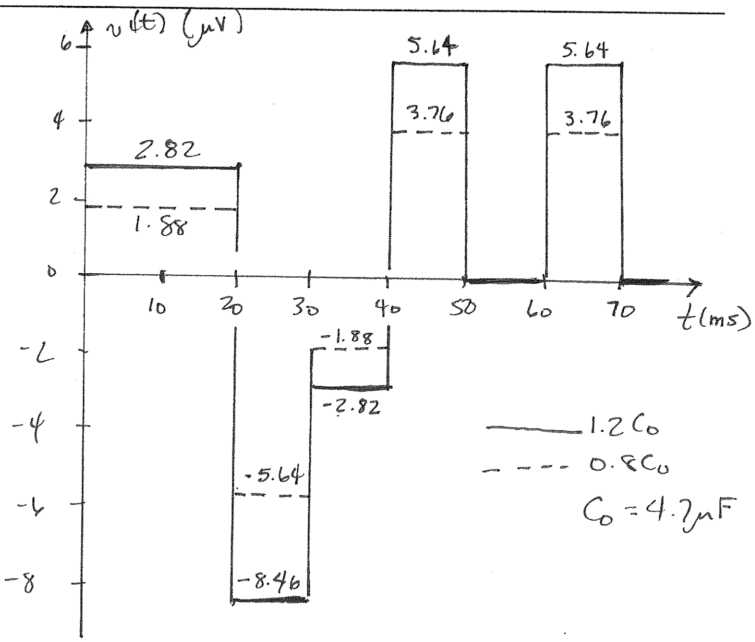


Figure P6.40

SOLUTION: $v = L\, di/dt$

$$\frac{di(t)}{dt} = \begin{cases} 1/2 \text{ A/s} & 0 \le t < 20\text{ms} \\ -1.5 \text{ "} & 20 \le t \le 30\text{ms} \\ -0.5 \text{ "} & 30 \le t \le 40\text{ms} \\ 1 \text{ "} & 40 \le t \le 50\text{ms} \\ 1 \text{ "} & 60 \le t \le 70\text{ms} \\ 0 \text{ "} & \text{otherwise} \end{cases}$$



6.41 If the total energy stored in the circuit in Fig. P6.41 is 80 mJ, what is the value of L?

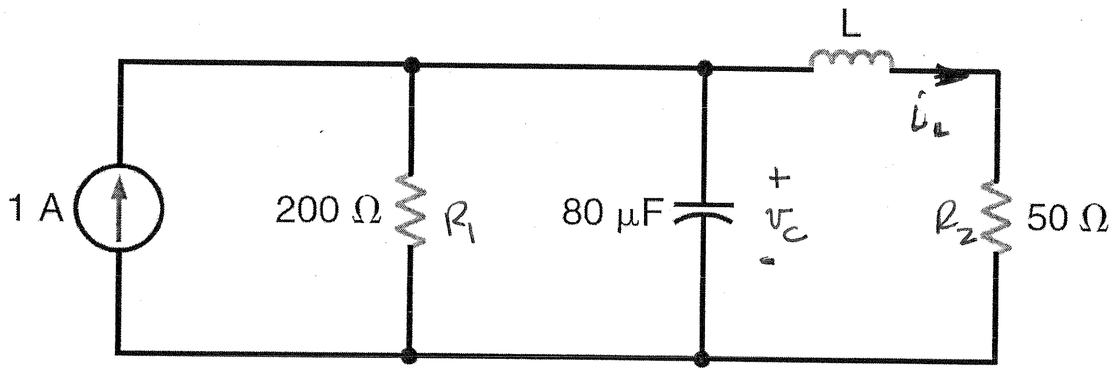


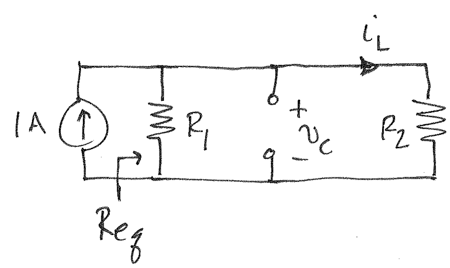
Figure P6.41

SOLUTION:

Since source (1A) is constant, v_C and i_L are also constants.

$i_C = C\frac{dv_C}{dt} = 0$ and $v_L = L\frac{di_L}{dt} = 0$

New circuit,



$R_{eq} = R_1 \parallel R_2 = 40\Omega$

$v_C = (1A)\,R_{eq} = 40V$

$w_C = \frac{1}{2}Cv_C^2 = 64mJ$

$w_{TOTAL} = 80mJ = w_C + w_L$

$w_L = \frac{1}{2}Li_L^2 = 16mJ$

$i_L = \frac{(1A)\,R_1}{R_1+R_2} = 0.8A$

$\boxed{L = 50mH}$

6.42 Find the value of C if the energy stored in the capacitor in Fig. P6.42 equals the energy stored in the inductor.

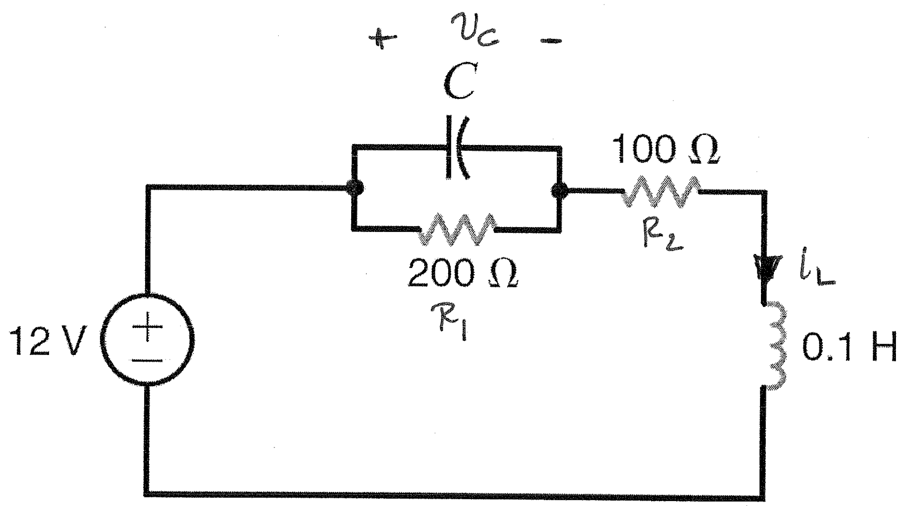


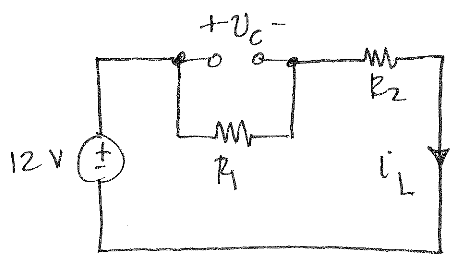
Figure P6.42

SOLUTION:

Since voltage source is constant, v_C and i_L are constant

$i_C = C\, dv_C/dt = 0$ & $v_L = L\, di_L/dt = 0$

<u>New circuit</u>



$$i_L = \frac{12}{R_1 + R_2} = 40\text{mA}$$

$$v_C = \frac{12\, R_1}{R_1 + R_2} = 8\text{V}$$

$$w_C = \frac{1}{2} C v_C^2 = w_L = \frac{1}{2} L i_L^2 \Rightarrow C = L\left(\frac{i_L}{v_C}\right)^2$$

$$\boxed{C = 2.5\mu\text{F}}$$

6.43 Given the network in Fig. P6.43, find the power dissipated in the 3-Ω resistor and the energy stored in the capacitor. **PSV**

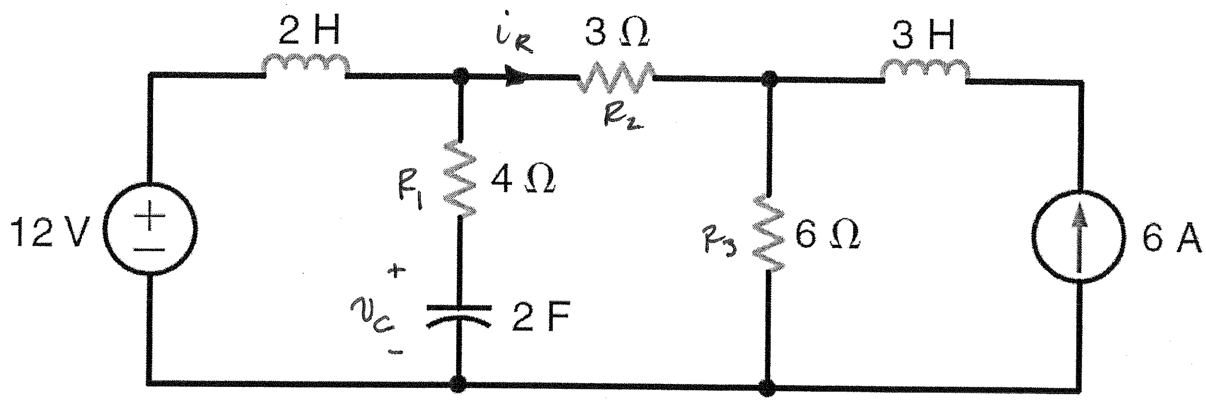
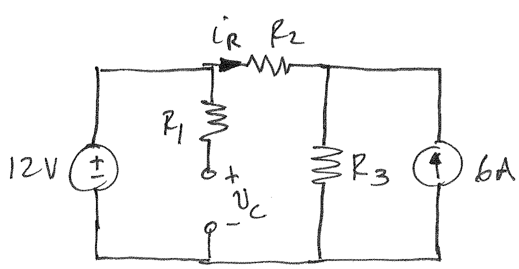


Figure P6.43

SOLUTION: Since all sources are constant, all voltages and currents are constant.

$v_L = L\, di_L/dt = 0$ & $i_c = C\, dv_c/dt = 0$

New Circuit



$P_{R2} = R_2 i_R^2$ $W_C = \frac{1}{2} C v_C^2$

By superposition:

$i_R = 12\left[\frac{1}{R_2+R_3}\right] - 6\left[\frac{R_3}{R_2+R_3}\right] = -\frac{8}{3}\,A$

$v_C = 12\,V$

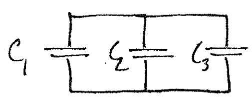
$\boxed{P_{R2} = 21.33\,W}$ $\boxed{W_C = 144\,J}$

6.44 What values of capacitance can be obtained by interconnecting a 2-μF capacitor, a 4-μF capacitor, and an 8-μF capacitor?

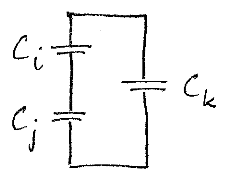
SOLUTION:

$C_1 = 2\mu F \qquad C_2 = 4\mu F \qquad C_3 = 8\mu F$

There are 4 configurations

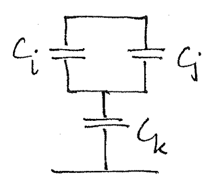


$$C_{eq} = C_1 + C_2 + C_3 = 14\mu F$$



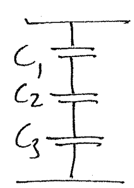
3 possibilities

$$C_{eq} = C_k + \frac{C_i C_j}{C_i + C_j} \qquad C_{eq} = \begin{cases} 9.33\mu F \\ 5.60\mu F \\ 4.67\mu F \end{cases}$$



3 possibilities

$$C_{eq} = \frac{(C_i + C_j)\, C_k}{C_i + C_j + C_k} \qquad C_{eq} = \begin{cases} 3.43\mu F \\ 2.86\mu F \\ 1.71\mu F \end{cases}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C_{eq} = 1.14\mu F$$

Possible values:

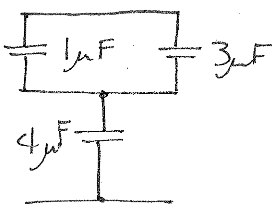
1.14μF	1.75μF
3.43μF	3.75μF
5.20μF	5.60μF
9.33μF	14μF

6.45 Given a 1-, 3-, and 4-μF capacitor, can they be interconnected to obtain an equivalent 2-μF capacitor?

CS

SOLUTION:

Yes!

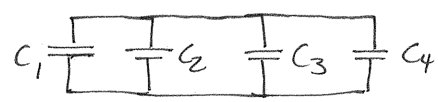


$$C_{eq} = \frac{(10^{-6} + 3\times10^{-6})\,4\times10^{-6}}{(10^{-6} + 3\times10^{-6}) + 4\times10^{-6}} = 2\mu F$$

6.46 Given four 6-μF capacitors, find the maximum value and minimum value that can be obtained by interconnecting the capacitors in series/parallel combinations.

SOLUTION:

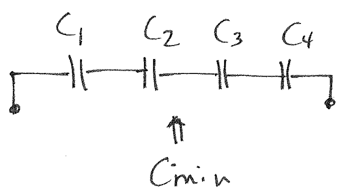
maximum C_{eq}:



all $C = 6\mu F$

$$\boxed{C_{max} = 4C = 24\mu F}$$

minimum C_{eq}:



$$\boxed{C_{min} = \frac{C}{4} = 1.5\mu F}$$

6.47 The two capacitors in Fig. P6.47 were charged and then connected as shown. Determine the equivalent capacitance, the initial voltage at the terminals, and the total energy stored in the network.

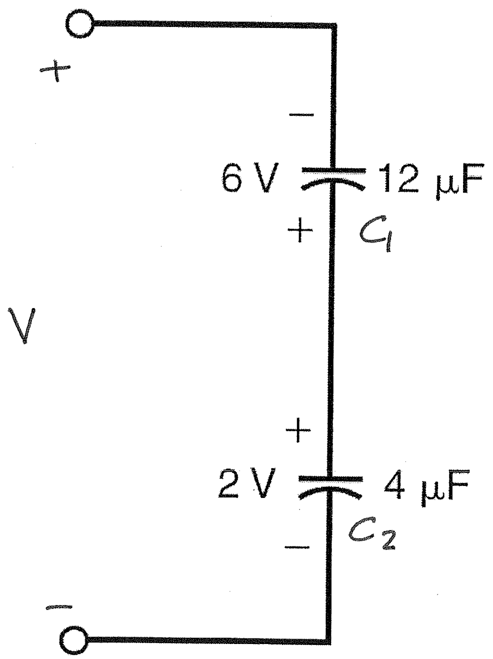


Figure P6.47

SOLUTION:

$V = -6 + 2 = -4V$ $\boxed{v = -4V}$

$w = \frac{1}{2} C_1 v_1^2 + \frac{1}{2} C_2 v_2^2$ $v_1 = 6V$ $v_2 = 2V$

$\boxed{w = 224\mu J}$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$ $\boxed{C_{eq} = 3\mu F}$

6.48 The two capacitors shown in Fig. P6.48 have been connected for some time and have reached their present values. Find V_o.

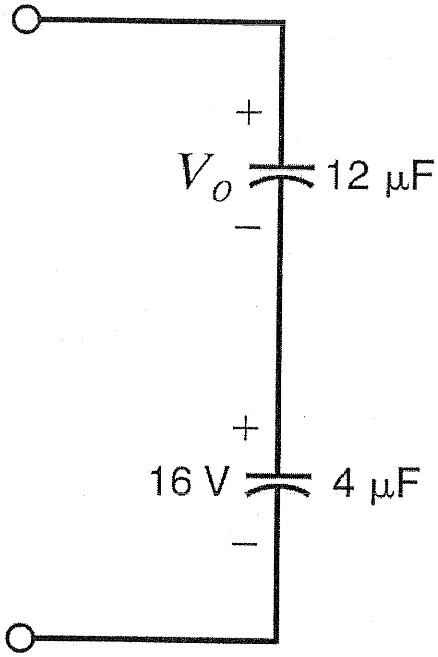


Figure P6.48

SOLUTION: Identical charge on each capacitor.

$$Q = CV = (4\times10^{-6})(16) = (12\times10^{-6})V_o \Rightarrow \boxed{V_o = 5.33\,V}$$

6.49 The three capacitors shown in Fig. P6.49 have been connected for some time and have reached their present values. Find V_1 and V_2. CS

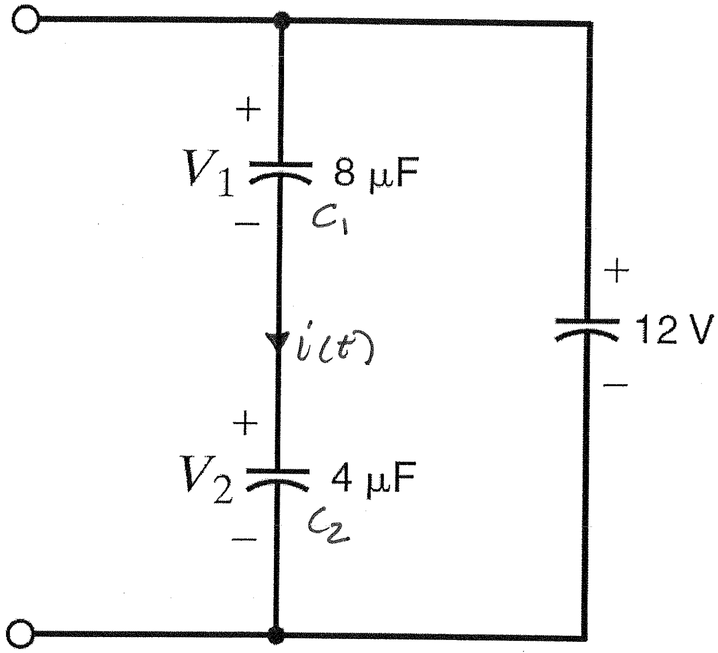


Figure P6.49

SOLUTION:

$v_1 = \frac{1}{C_1}\int i\,dt \qquad v_2 = \frac{1}{C}\int i\,dt$

$\frac{v_1}{v_2} = \frac{C_2}{C_1} = \frac{1}{2} \quad \& \quad v_1 + v_2 = 12V$

$\boxed{v_2 = 8V} \qquad \boxed{v_1 = 4V}$

6.50 Select the value of C to produce the desired total capacitance of $C_T = 10\ \mu F$ in the circuit in Fig. P6.50.

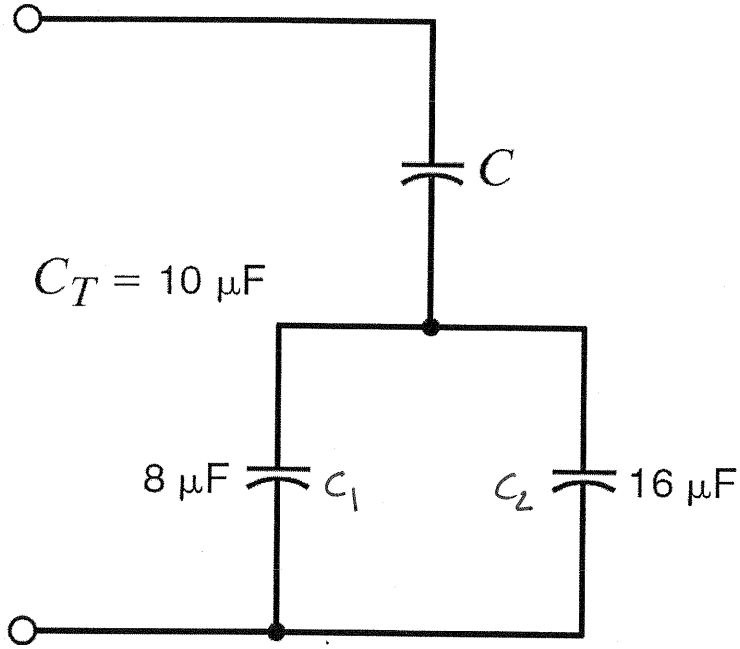


Figure P6.50

SOLUTION:

$$C_T = \frac{C\,(C_1 + C_2)}{C + C_1 + C_2} = 10\,\mu F$$

$$\boxed{C = 17.14\,\mu F}$$

6.51 Select the value of C to produce the desired total capacitance of $C_T = 1\ \mu\text{F}$ in the circuit in Fig. P6.51.

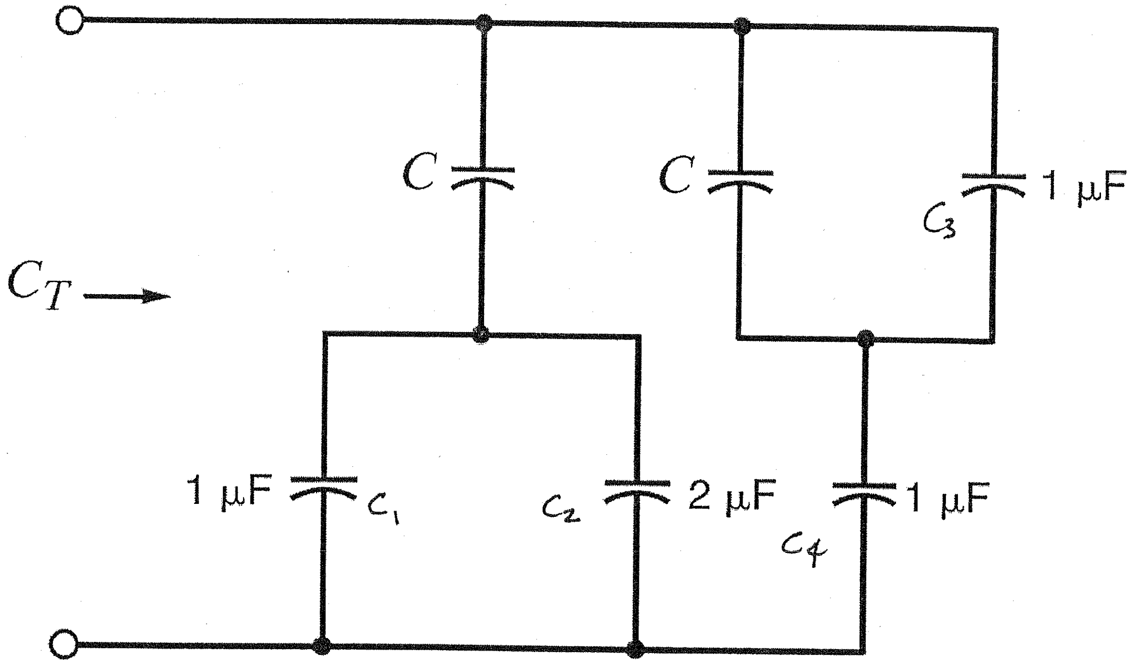


Figure P6.51

SOLUTION:

$$C_T = \frac{(C_1 + C_2)C}{C_1 + C_2 + C} + \frac{(C_3 + C)C_4}{C + C_3 + C_4} = 1\,\mu\text{F}$$

$$\frac{3C}{3+C} + \frac{(1+C)}{2+C} = 1 \quad \text{where } C \text{ is in } \mu\text{F}$$

$$\boxed{C = 468\ \text{nF}}$$

6.52 Find C_T in the network in Fig. P6.52 if (a) the switch is open and (b) the switch is closed.

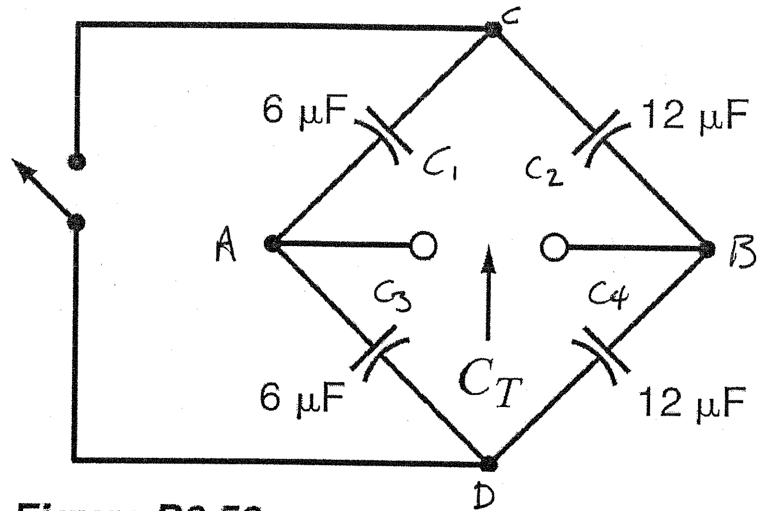
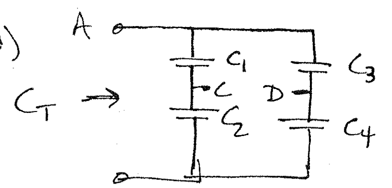


Figure P6.52

SOLUTION:

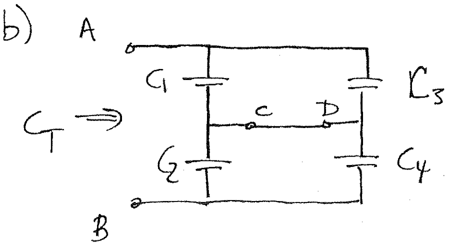
a)



$$C_T = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

$$C_T = 8\mu F$$

b)



$$C_T = \frac{(C_1 + C_3)(C_2 + C_4)}{C_1 + C_3 + C_2 + C_4}$$

$$\boxed{C_T = 8\mu F}$$

6.53 Find the equivalent capacitance at terminals A-B in Fig. P6.53. PSV

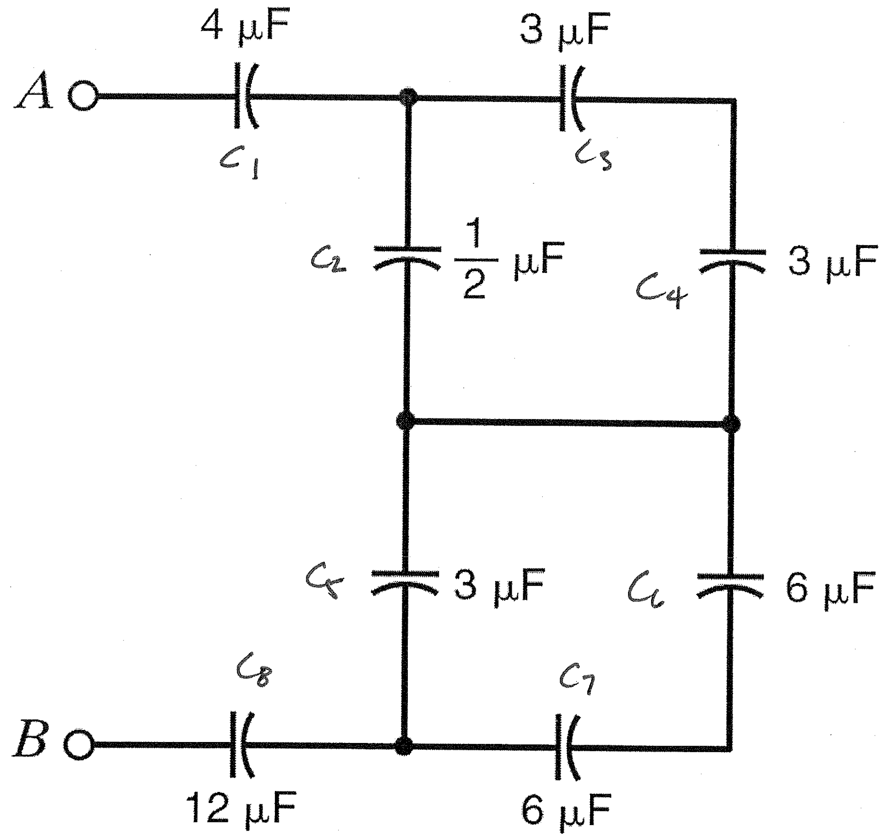


Figure P6.53

SOLUTION:

$C_A = C_2 + \frac{C_3 C_4}{C_3 + C_4} = 2\mu F$ $\qquad$ $C_B = C_5 + \frac{C_6 C_7}{C_6 + C_7} = 6\mu F$

$\frac{1}{C_{AB}} = \frac{1}{C_1} + \frac{1}{C_A} + \frac{1}{C_B} + \frac{1}{C_8}$ $\qquad$ $\boxed{C_{AB} = 1\mu F}$

6.54 Determine the total capacitance of the network in Fig. P6.54. **CS**

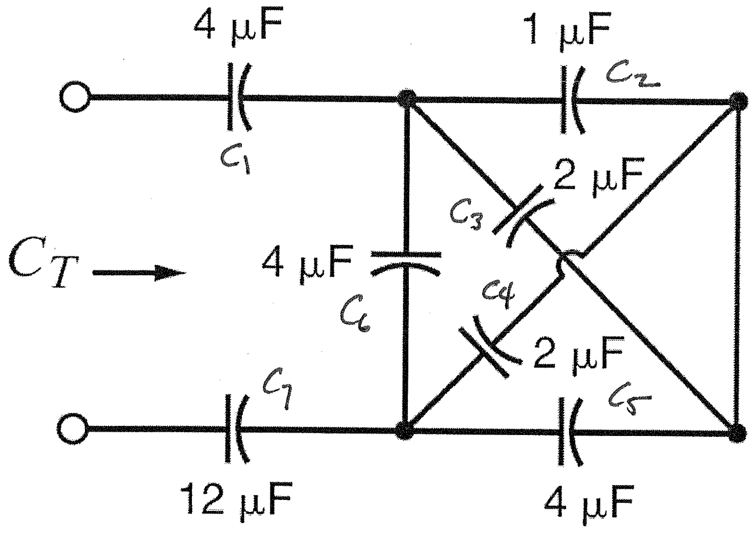
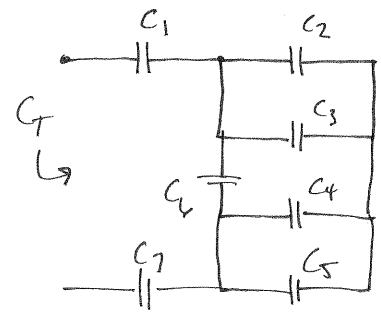


Figure P6.54

SOLUTION:

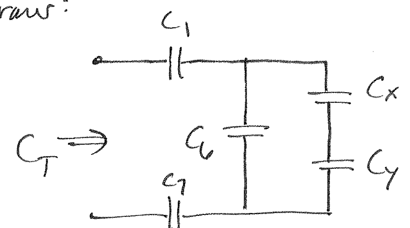
Redraw:



$C_x = C_2 + C_3 = 3\mu F$

$C_y = C_4 + C_5 = 6\mu F$

Redraw:



$$C_z = C_6 + \frac{C_x C_y}{C_x + C_y} = 6\mu F$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_z} + \frac{1}{C_7}$$

$$\boxed{C_T = 2\mu F}$$

6.55 Find the total capacitance C_T shown in the network in Fig. P6.55.

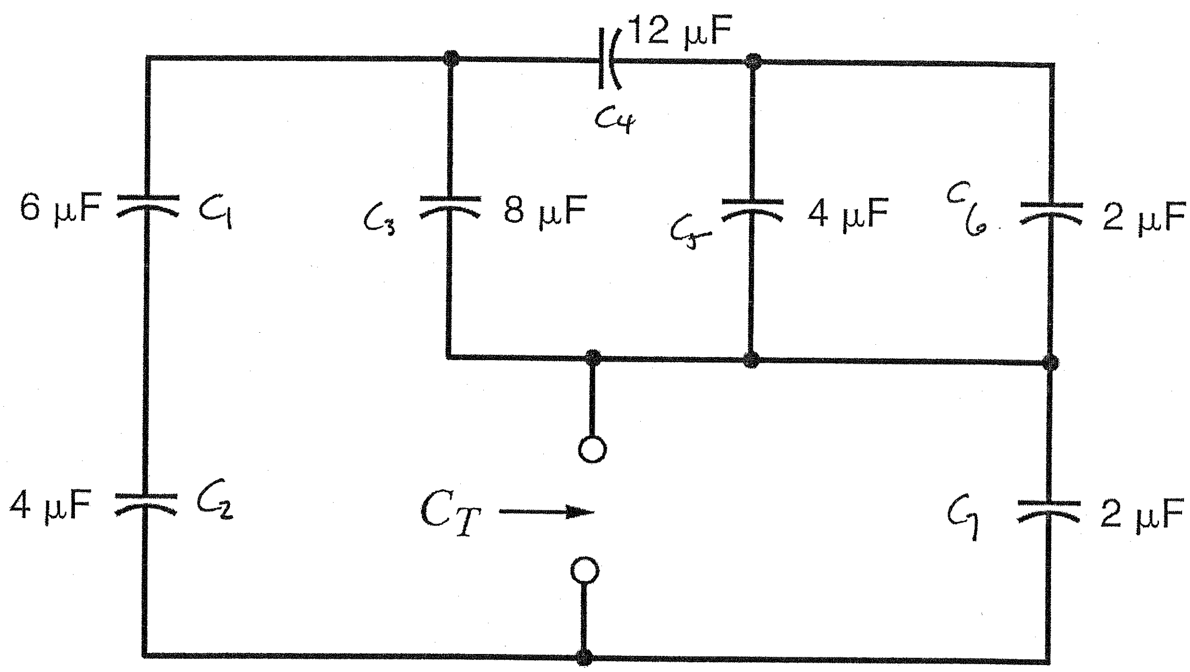
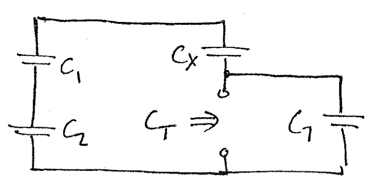


Figure P6.55

SOLUTION:



$$C_x = C_3 + \left[\frac{C_4 (C_5 + C_6)}{C_4 + C_5 + C_6} \right] = 12\mu F$$

$$\frac{1}{C_y} = \frac{1}{C_x} + \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_y = 2\mu F$$

$$C_T = C_7 + C_y = 4\ \mu F$$

$$\boxed{C_T = 4\mu F}$$

6.56 Find the total capacitance C_T of the network in Fig. P6.56.

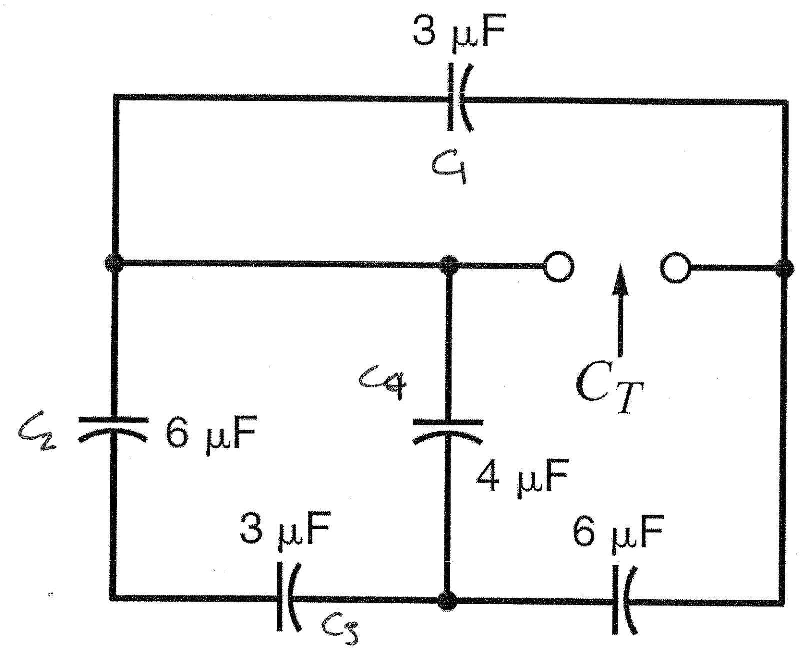
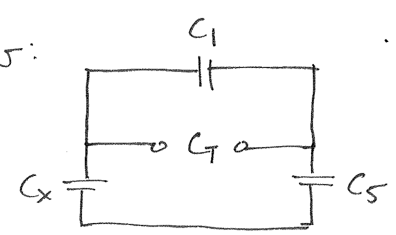


Figure P6.56

SOLUTION:

Redraw:



$$C_x = C_4 + \frac{C_2 C_3}{C_2 + C_3} = 6\mu F$$

$$C_T = C_1 + \frac{C_x C_5}{C_x + C_5}$$

$$\boxed{C_T = 6\mu F}$$

6.57 Find the total capacitance C_T of the network in Fig. P6.57.

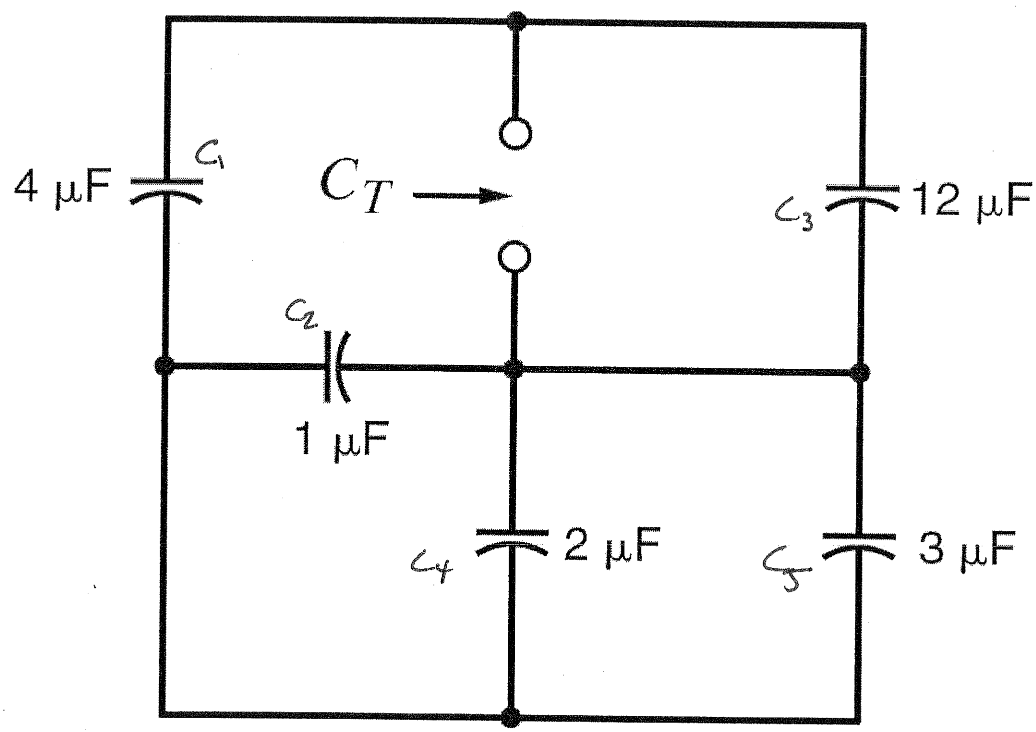
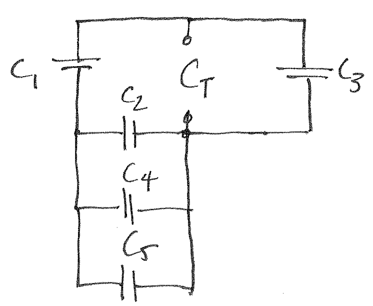


Figure P6.57

SOLUTION:

Redraw:



$$C_X = C_2 + C_4 + C_5 = 6\mu F$$

$$C_T = C_3 + \frac{C_1 C_X}{C_1 + C_X}$$

$$\boxed{C_T = 14.4\mu F}$$

6.58 In the network in Fig. P6.58, find the capacitance C_T if (a) the switch is open and (b) the switch is closed.

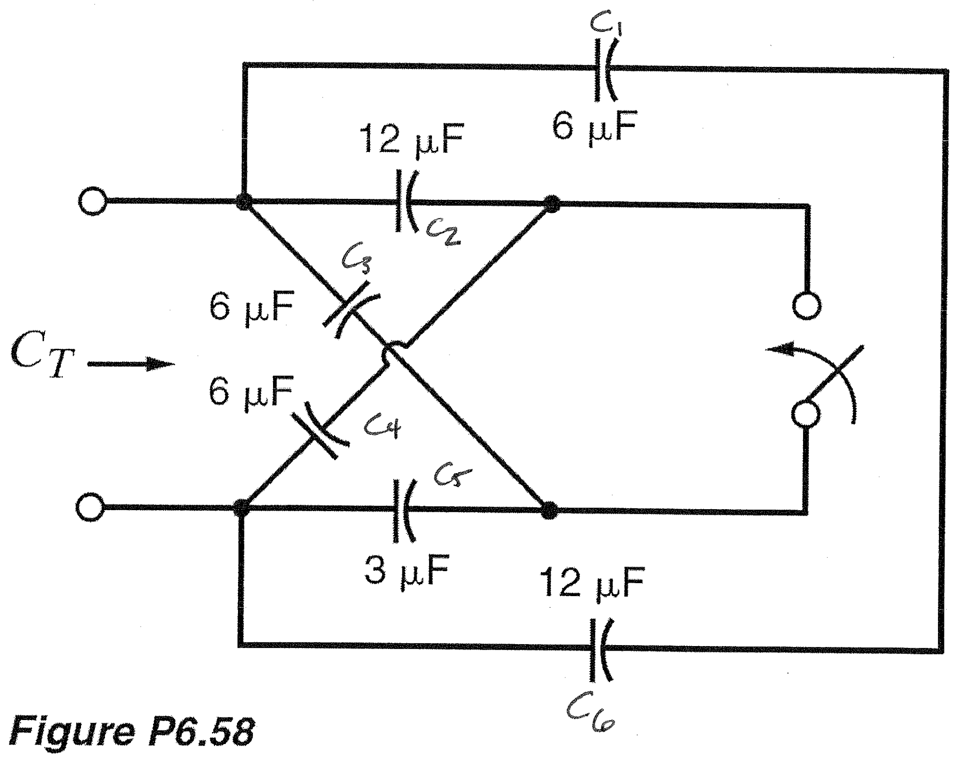
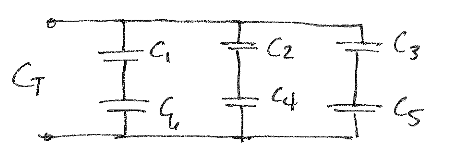


Figure P6.58

SOLUTION:

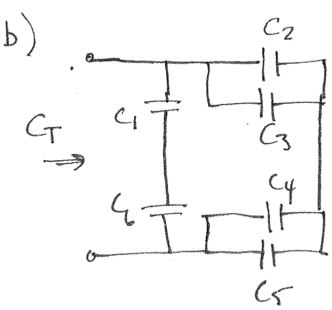
a)



$$C_T = \frac{C_1 C_6}{C_1 + C_6} + \frac{C_2 C_4}{C_2 + C_4} + \frac{C_3 C_5}{C_3 + C_5}$$

$$\boxed{C_T = 10\mu F}$$

b)



$$C_x = \frac{(C_2 + C_3)(C_4 + C_5)}{C_2 + C_3 + C_4 + C_5} = 6\mu F$$

$$C_T = C_x + \frac{C_1 C_6}{C_1 + C_6}$$

$$\boxed{C_T = 10\mu F}$$

6.59 Compute the equivalent capacitance of the network in Fig. P6.59 if all the capacitors are 6 μF.

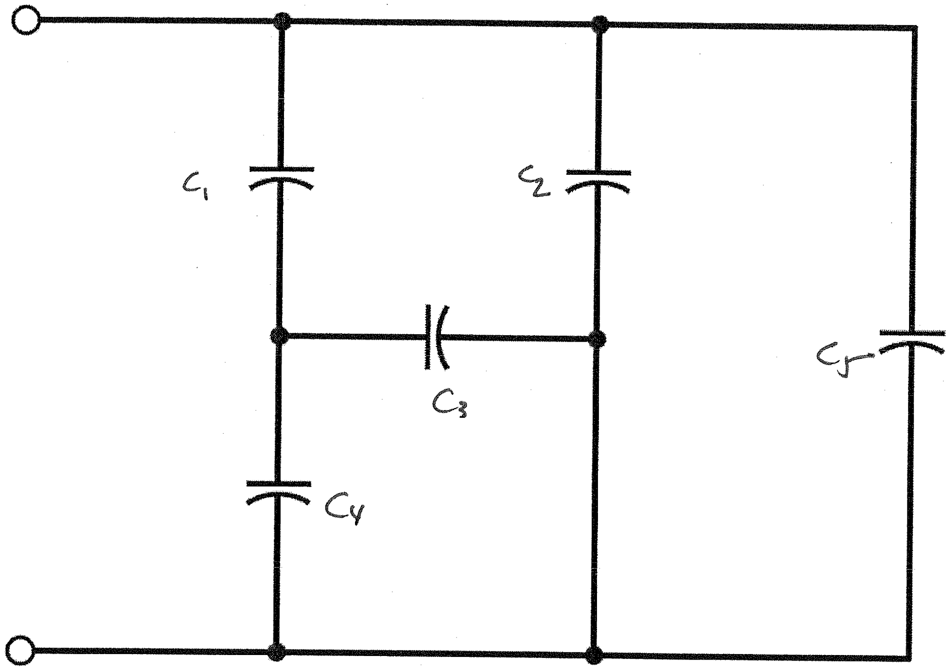
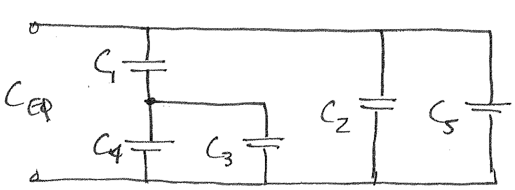


Figure P6.59

SOLUTION:

Redraw



$$C_X = \frac{(C_3 + C_4)\, C_1}{C_3 + C_4 + C_1} = 4\mu F$$

$$C_T = C_X + C_2 + C_5 \qquad \boxed{C_T = 16\mu F}$$

6.60 If all the capacitors in Fig. P6.60 are 6 μF, find C_{eq}. CS

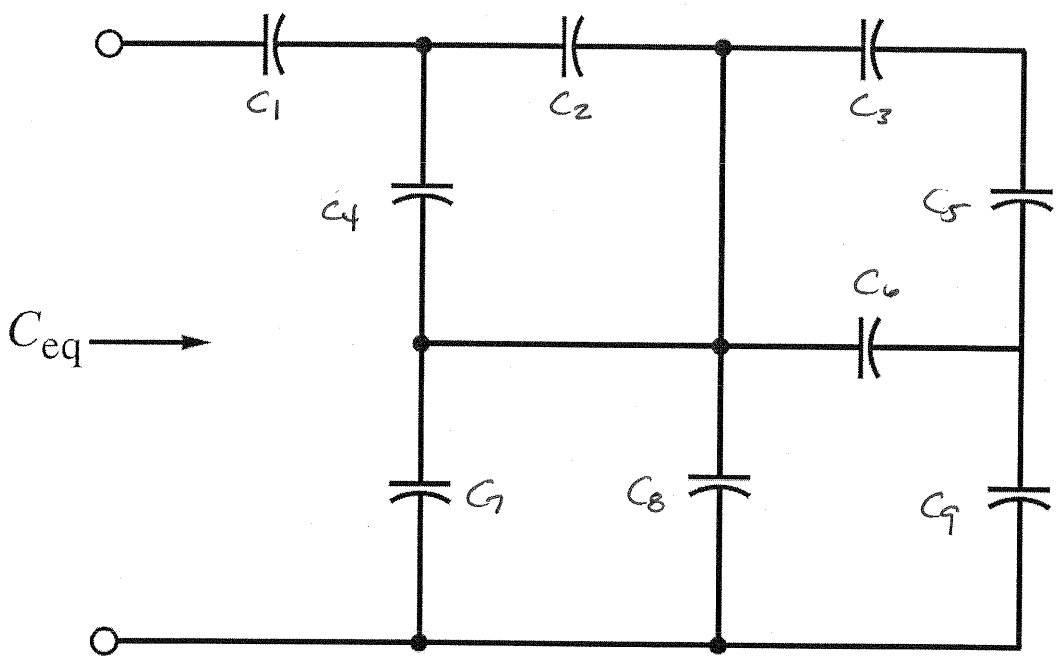
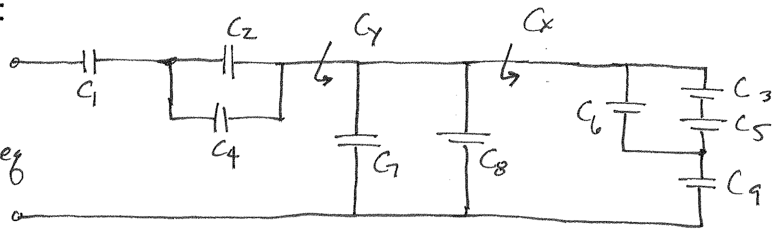


Figure P6.60

SOLUTION:

Redraw:



$$C_X = \frac{\left(\frac{C_3 C_5}{C_3+C_5}+C_6\right)C_9}{\frac{C_3 C_5}{C_3+C_5}+C_6+C_9} = 3.6\mu F$$

$$C_Y = C_7 + C_8 + C_X = 15.6\mu F$$

$$C_Z = C_2 + C_4 = 12\mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_Z} + \frac{1}{C_Y}$$

$$\boxed{C_{eq} = 3.18\mu F}$$

6.61 Given the capacitors in Fig. P6.61, are $C_1 = 2.0\ \mu F$ with a tolerance of 2% and $C_2 = 2.0\ \mu F$ with a tolerance of 20%, find the following.

(a) The nominal value of C_{eq}.

(b) The minimum and maximum possible values of C_{eq}.

(c) The percent errors of the minimum and maximum values.

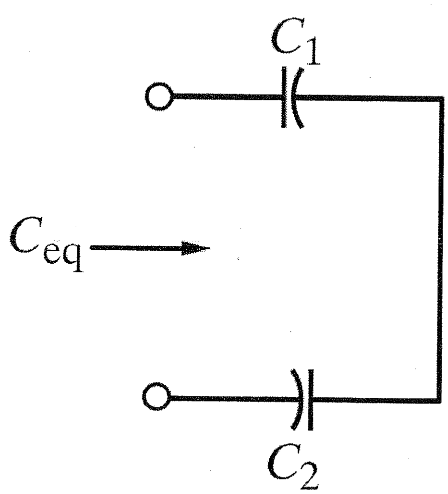


Figure P6.61

SOLUTION:

a) $C_{eq} = C_1 C_2 / (C_1 + C_2)$ $\boxed{C_{eq} = 1 \mu F}$

b) $C_{max} = C_{1max} C_{2max} / (C_{1max} + C_{2max})$ $C_{1max} = 2.04 \mu F$ $C_{2max} = 2.4 \mu F$

$\boxed{C_{max} = 1.103 \mu F}$

$C_{1min} = 1.96 \mu F$ $C_{2min} = 1.6 \mu F$ $\boxed{C_{min} = 0.881 \mu F}$

c) $+$ percent error $= \dfrac{C_{max} - C_{eq}}{C_{eq}} = 10.3\%$

$\boxed{\text{errors} = \begin{cases} +10.3\% \\ -11.9\% \end{cases}}$

$-$ percent error $= \dfrac{C_{min} - C_{eq}}{C_{eq}} = -11.9\%$

6.62 The capacitor values for the network in Fig. P6.62 are $C_1 = 0.1\ \mu F$ with a tolerance of 10%, $C_2 = 0.33\ \mu F$ with a tolerance of 20%, and $C_3 = 1\ \mu F$ with a tolerance of 10%. Find the following.

(a) The nominal value of C_{eq}.

(b) The minimum and maximum possible values of C_{eq}.

(c) The percent errors of the minimum and maximum values.

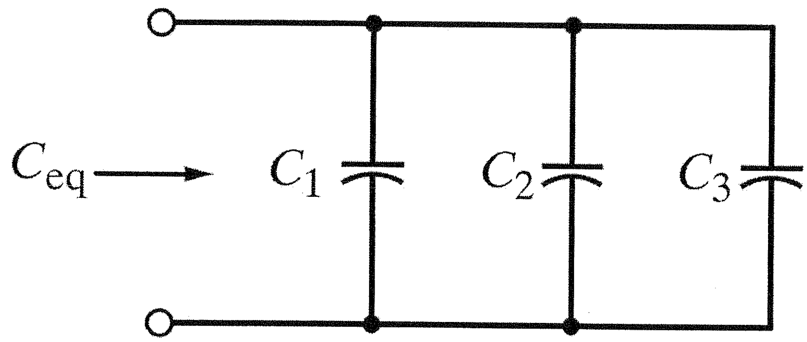


Figure P6.62

SOLUTION:

a) $C_{eq} = C_1 + C_2 + C_3 = 1.43\mu F$ $\boxed{C_{eq} = 1.43\mu F}$

b) $C_{max} = C_{1max} + C_{2max} + C_{3max}$ $C_{1max} = 0.11\mu F$ $C_{2max} = 0.396\mu F$ $C_{3max} = 1.1\mu F$

$\boxed{C_{max} = 1.606\mu F}$

$C_{1min} = 0.09\mu F$ $C_{2min} = 0.264\mu F$ $C_{3min} = 0.9\mu F$ $\boxed{C_{min} = 1.254\mu F}$

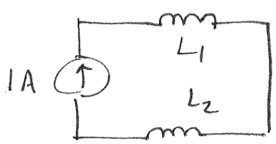
c) $+\%\ error = \dfrac{C_{max} - C_{eq}}{C_{eq}} = 12.3\%$

$\boxed{\%error = \pm 12.3\%}$

$-\%\ error = \dfrac{C_{min} - C_{eq}}{C_{eq}} = -12.3\%$

6.63 A 20-mH inductor and a 12-mH inductor are connected in series with a 1-A current source. Find (a) the equivalent inductance and (b) the total energy stored.

SOLUTION:



$L_1 = 20\text{mH}$ $\qquad L_2 = 12\text{mH}$

a) $L_{eq} = L_1 + L_2$ $\qquad \boxed{L_{eq} = 32\text{mH}}$

b) $W_{TOTAL} = W_1 + W_2 = \frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2$ $\qquad I = 1\text{A}$

$\boxed{W_{TOTAL} = 16\text{mJ}}$

6.64 Two inductors are connected in parallel, as shown in Fig. P6.64. Find i.

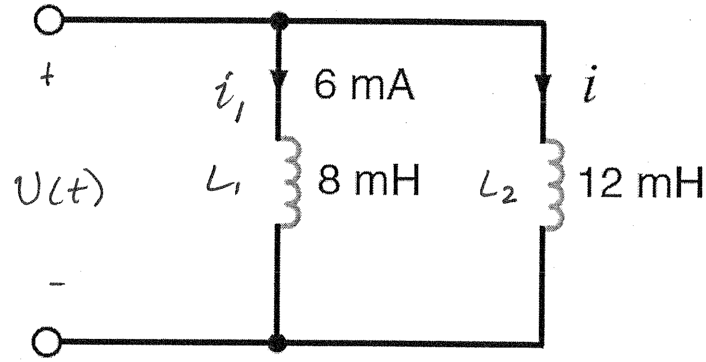


Figure P6.64

SOLUTION:

For L_1 $\quad i_1 = \frac{1}{L_1}\int v(t)\,dt$

for L_2 $\quad i = \frac{1}{L_2}\int v(t)\,dt$

$$\frac{i}{i_1} = \frac{L_1}{L_2}$$

$i = i_1\left(L_1/L_2\right)$ $\quad \boxed{i = 4\text{mA}}$

6.65 Find the value of L in the network in Fig. P6.65 so that the total inductance L_T will be 2 mH.

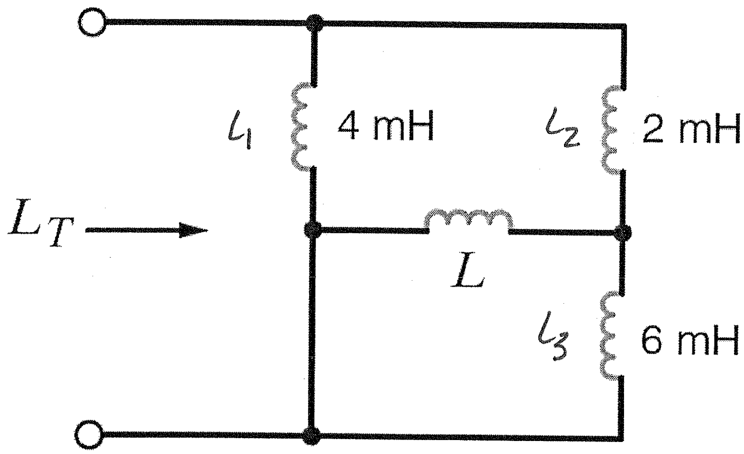
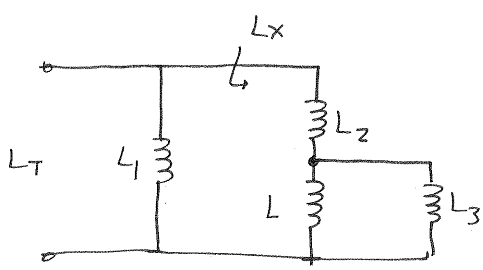


Figure P6.65

SOLUTION:

Redraw:



$L_x = L_2 + \frac{L L_3}{L + L_3}$

$L_T = \frac{L_1 L_x}{L_1 + L_x} = 2\,mH$

$L_x = \frac{L_1 L_T}{L_1 - L_T}$ $\quad L_x = 4\,mH$ $\quad \frac{L L_3}{L + L_3} = 2\,mH$

$\boxed{L = 3\,mH}$

6.66 Determine the inductance at terminals *A-B* in the network in Fig. P6.66. **PSV**

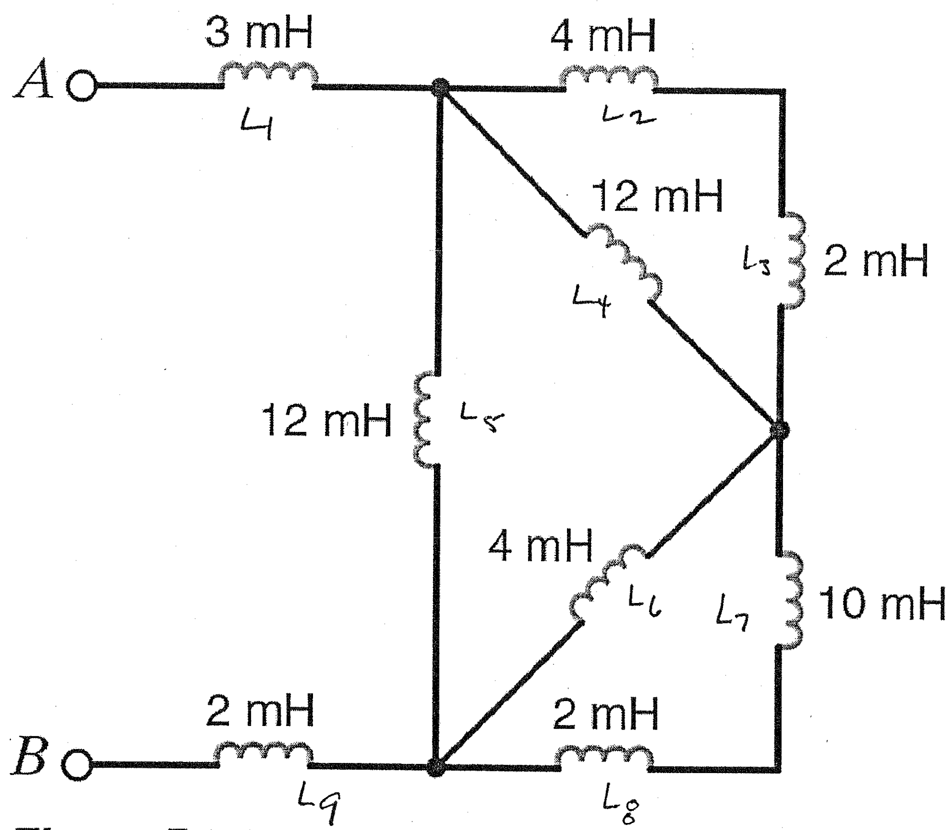
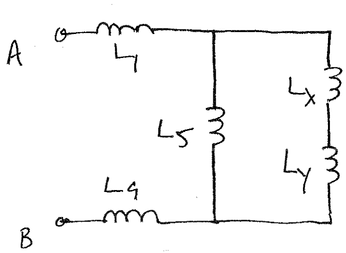


Figure P6.66

SOLUTION:

$$L_x = \frac{L_4(L_2+L_3)}{L_4+L_2+L_3} = 4\,mH \qquad L_y = \frac{L_6(L_7+L_8)}{L_6+L_7+L_8} = 3\,mH$$



$$L_z = \frac{L_5(L_x+L_y)}{L_5+L_x+L_y} = 4.42\,mH$$

$$L_T = L_1 + L_z + L_9 \qquad \boxed{L_T = 9.42\,mH}$$

6.67 Determine the inductance at terminals *A*-*B* in the network in Fig P6.67. CS

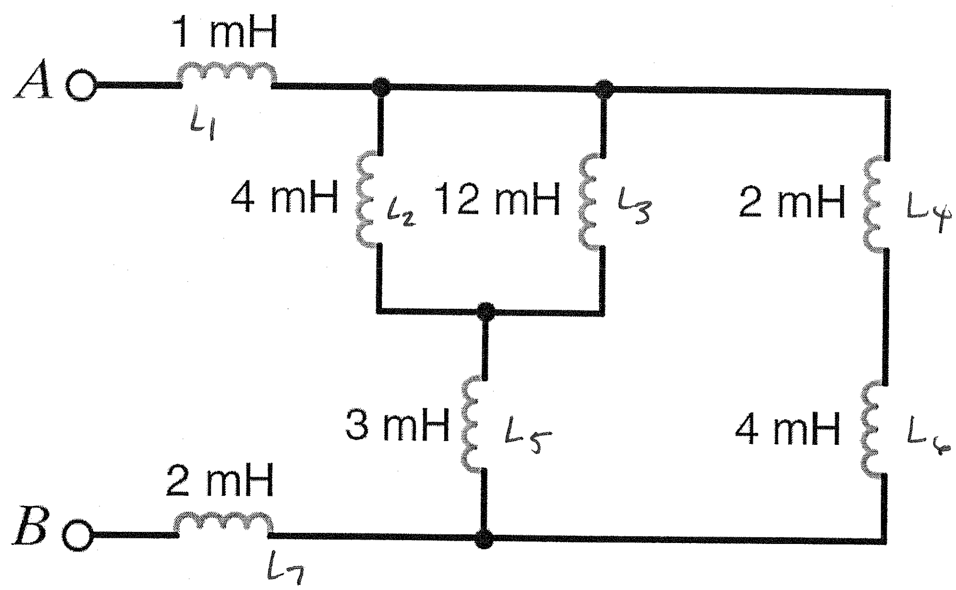
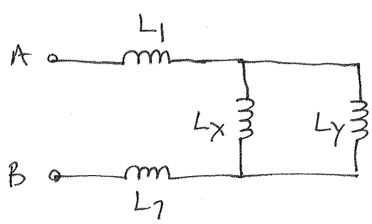


Figure P6.67

SOLUTION:



$$L_X = L_5 + \frac{L_2 L_3}{L_2 + L_3} = 6\,mH$$

$$L_Y = L_4 + L_6 = 6\,mH$$

$$L_{AB} = L_1 + L_7 + \frac{L_X L_Y}{L_X + L_Y}$$

$$\boxed{L_{AB} = 6\,mH}$$

6.68 Given the network shown in Fig. P6.68, find (a) the equivalent inductance at terminals *A-B* with terminals *C-D* short circuited, and (b) the equivalent inductance at terminals *C-D* with terminals *A-B* open circuited.

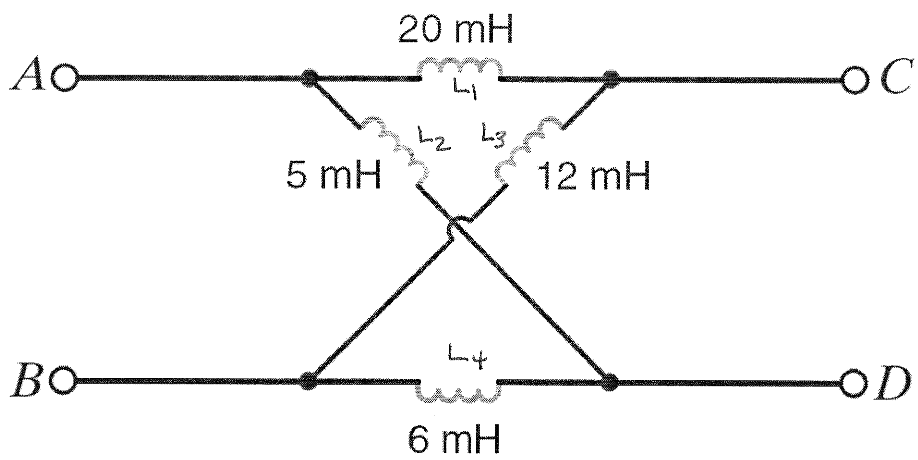
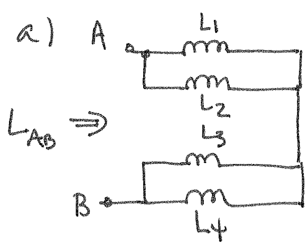


Figure P6.68

SOLUTION:

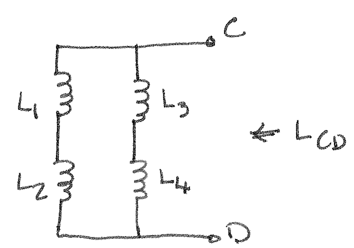
a)



$$L_{AB} = \frac{L_1 L_2}{L_1 + L_2} + \frac{L_3 L_4}{L_3 + L_4}$$

$$\boxed{L_{AB} = 8\text{ mH}}$$

b)



$$L_{CD} = \frac{(L_1 + L_2)(L_3 + L_4)}{L_1 + L_2 + L_3 + L_4}$$

$$\boxed{L_{CD} = 10.47\text{ mH}}$$

6.69 Find the total inductance at the terminals of the network in Fig. P6.69.

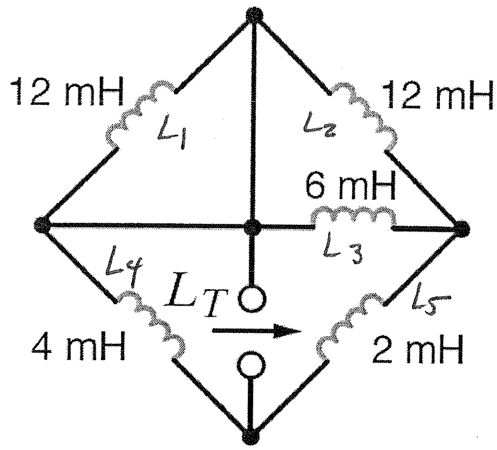
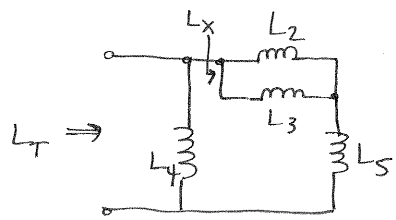


Figure P6.69

SOLUTION:

Redraw



$$L_x = \frac{L_2 L_3}{L_2 + L_3} + L_5 = 6\,mH$$

$$L_T = \frac{L_4 L_x}{L_4 + L_x}$$

$$\boxed{L_T = 2.4\,mH}$$

6.70 Compute the equivalent inductance of the network in Fig. P6.70 if all inductors are 12 mH.

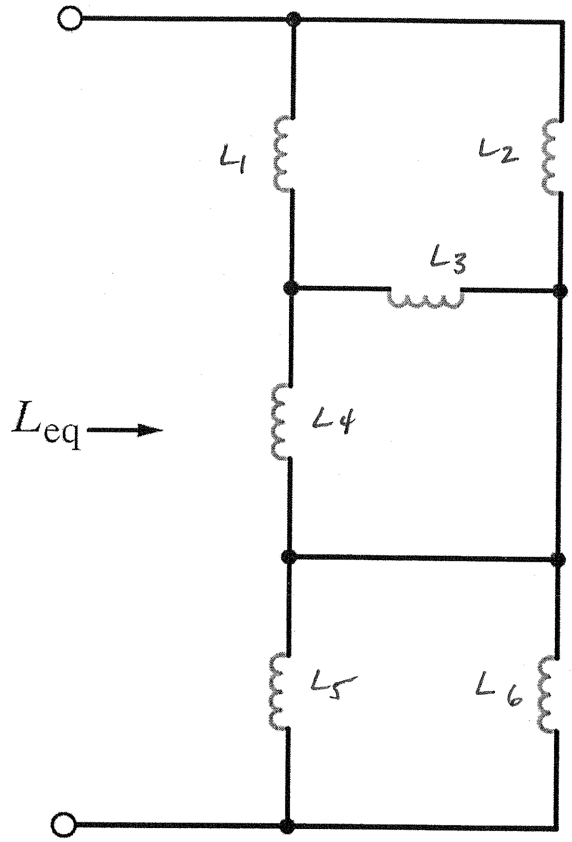
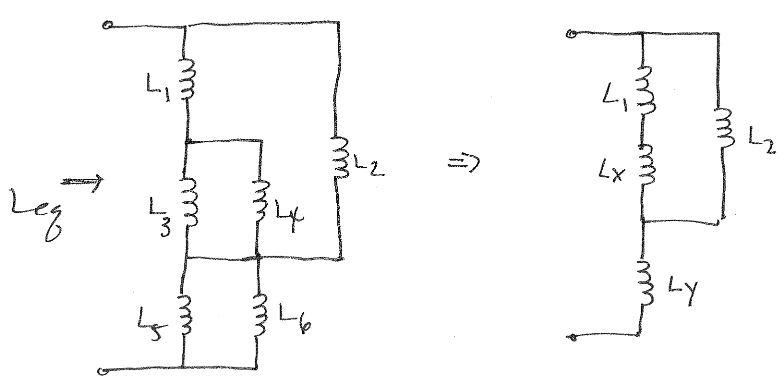


Figure P6.70

SOLUTION:

Redraw:



$$L_x = \frac{L_3 L_y}{L_3 + L_y} = 6\,mH$$

$$L_y = L_5 // L_6 = 6\,mH$$

$$L_z = \frac{(L_1 + L_x)\,L_2}{L_1 + L_x + L_2} = 7.2\,mH$$

$$L_{eq} = L_z + L_y$$

$$\boxed{L_{eq} = 13.2\,mH}$$

6.71 Find L_T in the network in Fig. P6.71 (a) with the switch open and (b) with the switch closed. All inductors are 12 mH.

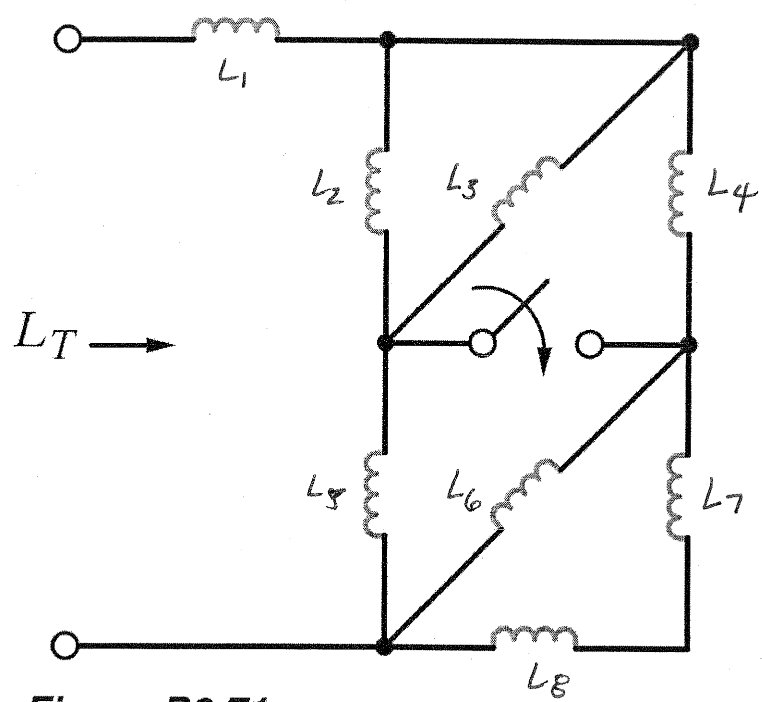
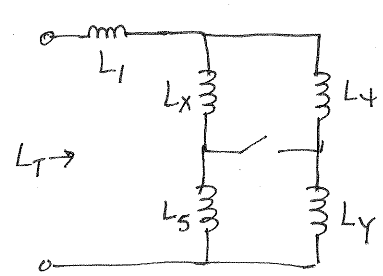


Figure P6.71

SOLUTION:



$$L_X = \frac{L_2 L_3}{L_2 + L_3} = 6\,mH$$

$$L_Y = \frac{L_6 (L_7 + L_8)}{L_6 + L_7 + L_8} = 8\,mH$$

a) $$L_T = L_1 + \frac{(L_X + L_5)(L_4 + L_Y)}{L_X + L_5 + L_4 + L_Y}$$ $\boxed{L_T = 21.47\,mH}$

b) $$L_T = L_1 + \frac{(L_X L_4)}{L_X + L_4} + \frac{L_5 L_Y}{L_5 + L_Y}$$ $\boxed{L_T = 20.8\,mH}$

6.72 For the network in Fig. P6.72, $v_S(t) = 120 \cos 377t$ V. Find $v_o(t)$.

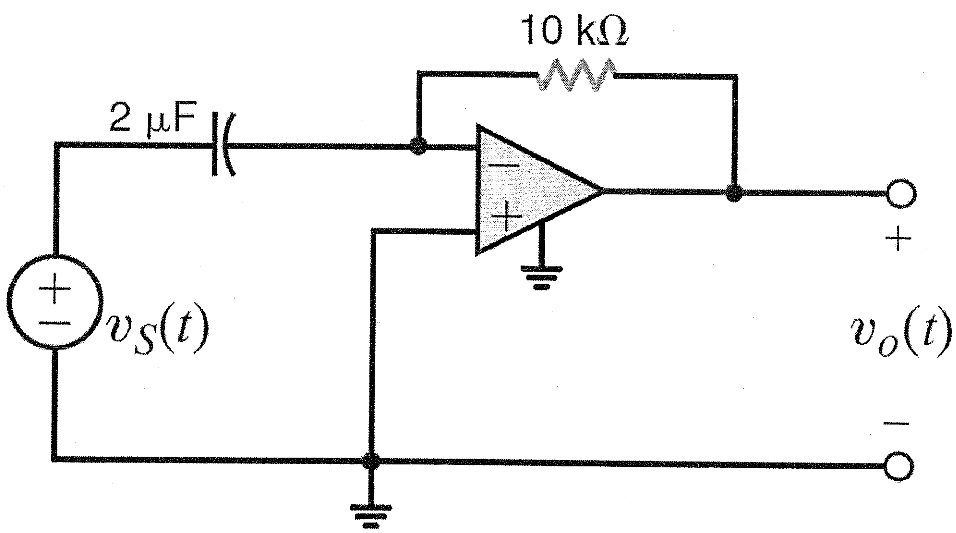


Figure P6.72

SOLUTION:

Differentiator: $v_o = -RC \dfrac{dv_s(t)}{dt}$

$$\boxed{v_o(t) = 904.8 \sin(377t) \text{ V}}$$

6.73 For the network in Fig. P6.73, $v_S(t) = 115 \sin 377t$ V. Find $v_o(t)$.

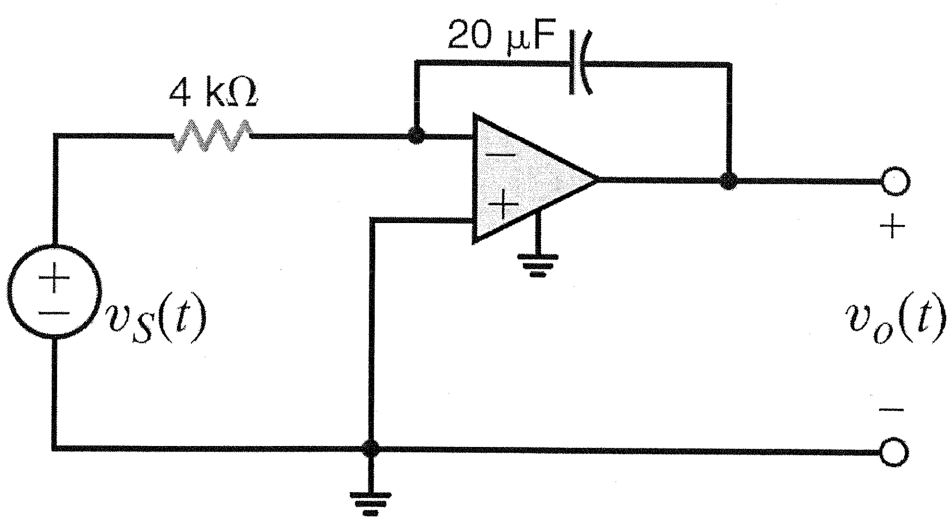


Figure P6.73

SOLUTION:

Integrator: $v_o(t) = \frac{-1}{RC} \int v_S(t)\,dt$

$$\boxed{v_o(t) = 3.81 \cos(377t) \text{ V}}$$

6.74 For the network in Fig. P6.74, $v_{S_1}(t) = 80 \cos 377t$ V and $v_{S_2}(t) = 40 \cos 377t$ V. Find $v_o(t)$.

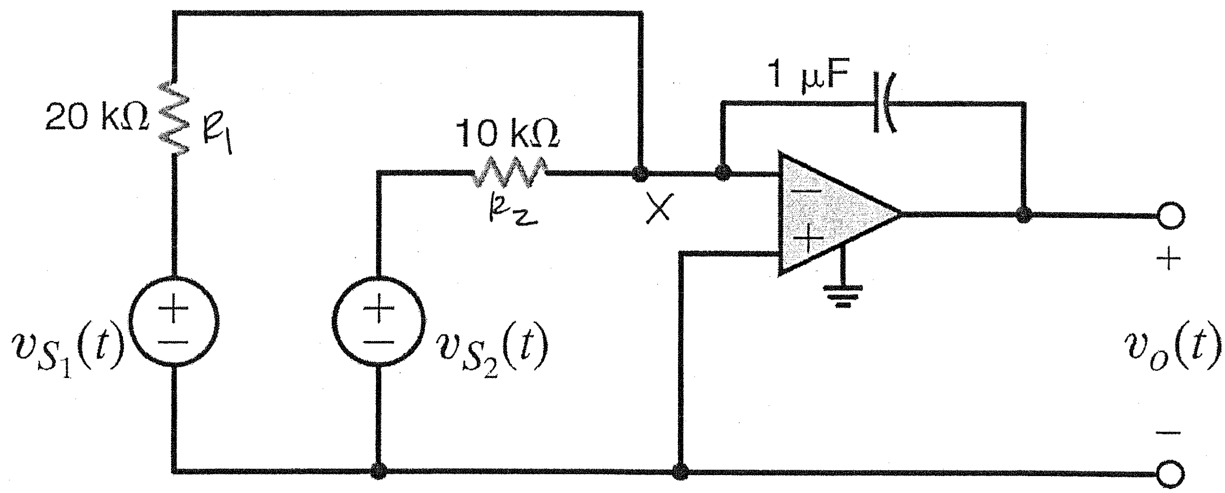


Figure P6.74

SOLUTION: Nodal analysis at node X!

$$\frac{v_{S1}}{R_1} + \frac{v_{S2}}{R_2} = -C\frac{dv_o}{dt} \qquad v_o(t) = -\frac{1}{C}\int \left(\frac{v_{S1}}{R_1} + \frac{v_{S2}}{R_2}\right) dt$$

$$v_o(t) = -\frac{8000}{\omega}\sin(\omega t) \qquad \boxed{v_o(t) = -21.2 \sin(377t) \text{ V}}$$

6.75 For the network in Fig. P6.75, choose C such that

$$v_o = -10 \int v_S \, dt$$

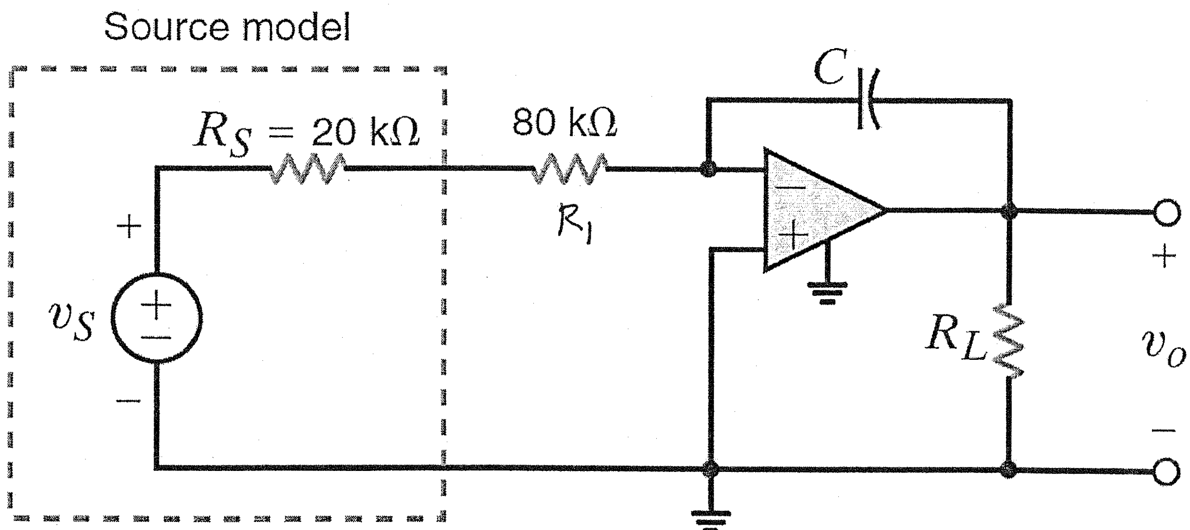


Figure P6.75

SOLUTION:

$$V_o = \frac{-1}{R_{eq} C} \int V_s \, dt \qquad R_{eq} = R_S + R_1 = 100\,k\Omega$$

Need $R_{eq} C = 0.1$ $\boxed{C = 1.0\mu F}$

6.76 An integrator is required that has the following performance

$$v_o(t) = 10^6 \int v_s \, dt.$$

where the capacitor values must be greater than 10 nF and the resistor values must be greater than 10 kΩ.

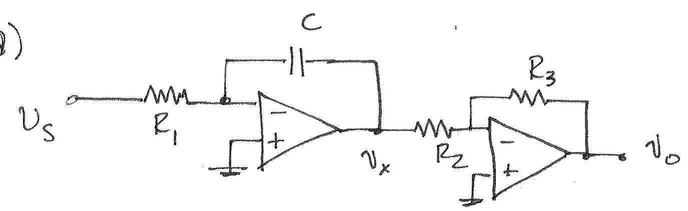
(a) Design the integrator.

(b) If ±10-V supplies are used, what are the maximum and minimum values of v_o?

(c) Suppose $V_S = 1$ V. What is the rate of change of v_o?

SOLUTION:

a)



$v_x = -\frac{1}{R_1 C}\int v_s\,dt$ $\qquad v_o = -\frac{R_3}{R_2} v_x$ $\qquad v_o = \frac{R_3}{C R_1 R_2}\int v_s\,dt$

Arbitrarily select: $C = 20\,nF$, $R_1 = 20\,k\Omega$ $R_2 = 20\,k\Omega$ $\Rightarrow$ $\boxed{R_3 = 8\,M\Omega}$

b) v_o cannot exceed the supplies and so is limited to ±10V

$\boxed{v_{o\,max} = 10V \qquad v_{o\,min} = -10V}$

c) $v_s = 1V$ $\qquad v_o(t) = 10^6 \int dt = 10^6 t$

$\boxed{\frac{dv_o}{dt} = 10^6 \ V/s}$

6.77 The circuit shown in Fig. P6.77 is known as a "Deboo" integrator.

(a) Express the output voltage in terms of the input voltage and circuit parameters.

(b) How is the Deboo integrator's performance different from that of a standard integrator?

(c) What kind of application would justify the use of this device?

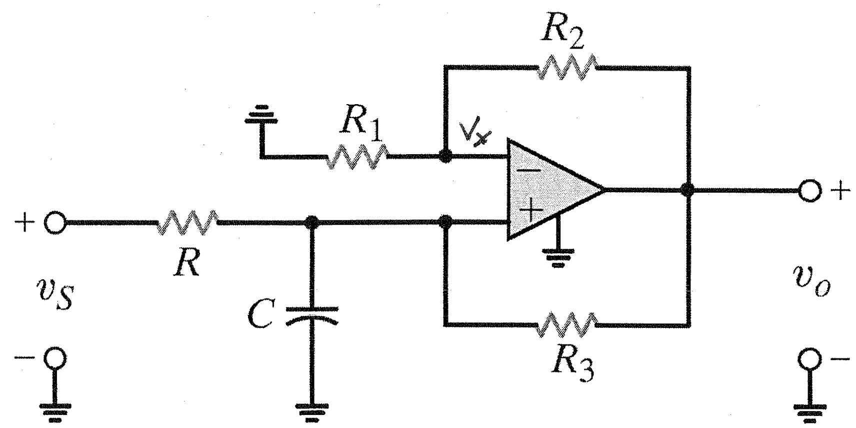


Figure P6.77

SOLUTION:

a) $v_x = v_o \left(\frac{R_1}{R_1+R_2}\right) = \alpha v_o$

$$\frac{v_S - v_x}{R} = C\frac{dv_x}{dt} + \frac{v_x - v_o}{R_3} \Rightarrow v_S = \frac{R_1 RC}{R_1+R_2}\frac{dv_o}{dt} + \left[\frac{R_1(R+R_3)}{R_3(R_1+R_2)} - \frac{R}{R_3}\right]v_o$$

$$v_o = \left(\frac{R_1+R_2}{R_1}\right)\frac{1}{RC}\int\left\{v_s(t) + \left[\frac{R}{R_3} - \frac{R_1(R+R_3)}{R_3(R_1+R_2)}\right]v_o\right\}dt$$

b) Consider the case $\frac{R_1}{R_1+R_2} = \frac{R}{R+R_3} \Rightarrow v_o = \frac{R+R_3}{R^2 C}\int v_s\,dt$

Major difference is that the integration is positive

c) Where positive integration is needed!

6.78 A driverless automobile is under development. One critical issue is braking, particularly at red lights. It is decided that the braking effort should depend on distance to the light (if you're close, you better stop now) and speed (if you're going fast, you'll need more brakes). The resulting design equation is

$$\text{braking effort} = K_1\left[\frac{dx(t)}{dt}\right] + K_2x(t)$$

where x, the distance from the vehicle to the intersection, is measured by a sensor whose output is proportional to x, $v_{sense} = \alpha x$. Use superposition to show that the circuit in Fig. P6.78 can produce the braking effort signal.

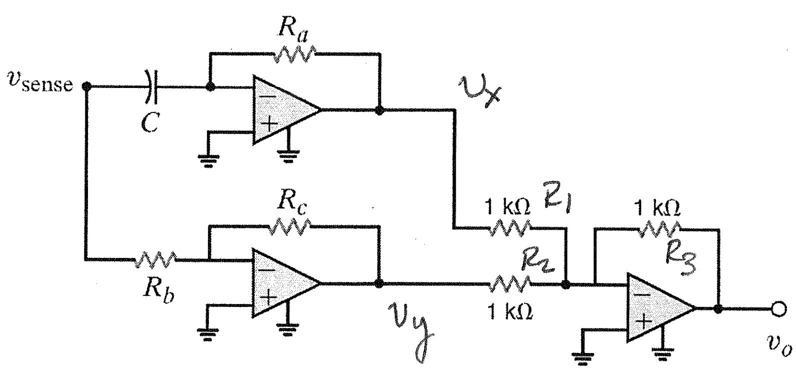


Figure P6.78

SOLUTION:

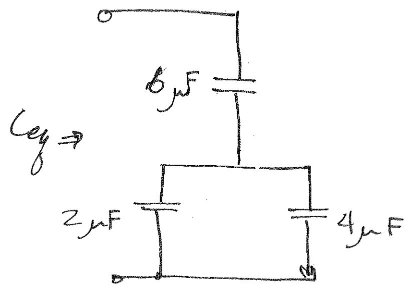
$$v_x = -R_a C \frac{d v_{sense}}{dt} \qquad v_y = -\frac{R_c}{R_b} v_{sense} \qquad v_o = -\frac{R_3}{R_1} v_x - \frac{R_3}{R_2} v_y$$

$$\boxed{v_o = \frac{\alpha R_3 R_a C}{R_1}\frac{dx}{dt} + \frac{R_3}{R_2}\frac{R_c}{R_b}\alpha x \qquad K_1 = \frac{\alpha R_3 R_a C}{R_1} \qquad K_2 = \frac{R_3 R_c}{R_2 R_b}}$$

6FE-1 Given three capacitors with values 2 μF, 4 μF, and 6 μF, can the capacitors be interconnected so that the combination is an equivalent 3 μF? CS

SOLUTION:

Yes!



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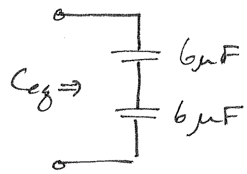

$$C_{eq} = \frac{(6\times10^{-6})(6\times10^{-6})}{6\times10^{-6} + 6\times10^{-6}} = 3\mu F$$

6FE-2 The current pulse shown in Fig. 5PFE-2 is applied to a 1-μF capacitor. Determine the charge on the capacitor and the energy stored.

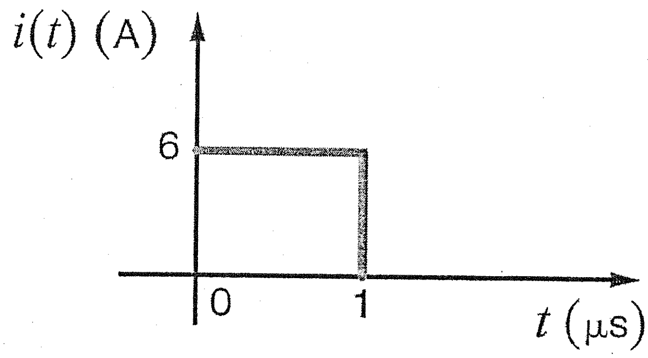

Figure 5PFE-2

SOLUTION:

$$q(t) = \int i \, dt$$

$$\boxed{q(t) = \begin{cases} 0 \quad C & t < 0 \\ 6t \quad C & 0 < t \le 1\mu s \\ 6 \quad \mu C & t > 1\mu s \end{cases}}$$

$$w(t) = \frac{1}{2} C v^2(t) \qquad v(t) = q(t)/C = \begin{cases} 0 \quad V & t \le 0 \\ 6 \times 10^6 t \quad V & 0 < t \le 1\mu s \\ 6 \quad V & t > 1\mu s \end{cases}$$

$$\boxed{w(t) = \begin{cases} 0 \quad J & t \le 0 \\ 18 \times 10^6 t^2 \quad J & 0 < t \le 1\mu s \\ 18 \quad \mu J & t > 1\mu s \end{cases}}$$

6FE-3 The two capacitors shown in Fig. 5PFE-3 have been connected for some time and have reached their present values. Determine the energy stored in the unknown capacitor C_x. CS

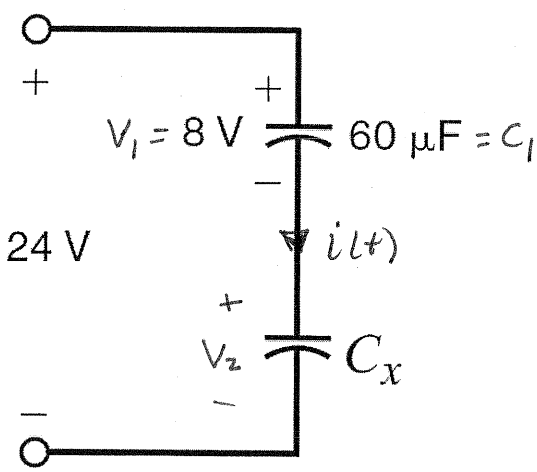

Figure 5PFE-3

SOLUTION:

$v_1 + v_2 = 24$

$v_2 = 16V$

$v_1 = \frac{1}{C_1}\int i(t)\,dt$

$v_2 = \frac{1}{C_x}\int i(t)\,dt$

$\frac{v_1}{v_2} = \frac{C_x}{C_1} = \frac{1}{2}$

$\boxed{C_x = 30\mu F}$