Chapter Eight:

AC Steady-State Analysis

8.1 Given $i(t) = 5\cos(400t - 120°)$ A, determine the period of the current and the frequency in Hertz. CS

SOLUTION:

$\omega = 400$ r/s $\quad f = \frac{\omega}{2\pi} = \frac{200}{\pi} = 63.7$ Hz $\quad T = \frac{1}{f} = 15.7$ ms

$$\boxed{f = 63.7\text{Hz} \qquad T = 15.7 \text{ ms}}$$

8.2 Determine the relative phase relationship of the two waves.

$$v_1(t) = 10\cos(377t - 30°)\text{ V}$$
$$v_2(t) = 10\cos(377t + 90°)\text{ V}$$

SOLUTION:

$\theta_2 = 90°$ $\theta_1 = -30°$ $\theta_2 - \theta_1 = 120°$

$\boxed{v_2 \text{ leads } v_1 \text{ by } 120°}$

8.3 Given the following voltage and current

$$i(t) = 5\sin(377t - 20^\circ)\ \text{V}$$
$$v(t) = 10\cos(377t + 30^\circ)\ \text{V}$$

determine the phase relationship between $i(t)$ and $v(t)$.

SOLUTION:

$i(t) = 5\cos(377t - 20 - 90) = 5\cos(377t - 110)$

$\theta_v = 30^\circ \qquad \theta_i = -110^\circ \qquad \theta_v - \theta_i = 140^\circ$

$\boxed{v(t) \text{ leads } i(t) \text{ by } 140^\circ}$

8.4 Determine the phase angles by which $v_1(t)$ leads $i_1(t)$ and $v_1(t)$ leads $i_2(t)$, where

$$v_1(t) = 4\sin(377t + 25°)\text{ V}$$
$$i_1(t) = 0.05\cos(377t - 20°)\text{ A}$$
$$i_2(t) = -0.1\sin(377t + 45°)\text{ A}$$

SOLUTION:

$i_1(t) = 0.05 \sin(\omega t - 20 + 90) = 0.05 \sin(\omega t + 70°)$

$\theta_{v_1} - \theta_{i_1} = -45°$ $\boxed{v_1 \text{ lead } i_1 \text{ by } -45°}$

$i_2(t) = 0.1 \sin(\omega t - 135°)$

$\theta_{v_1} - \theta_{U_2} = 160°$ $\boxed{v_1 \text{ lead } i_2 \text{ by } 160°}$

8.5 Calculate the current in the resistor in Fig. P8.5 if the voltage input is

(a) $v_1(t) = 10\cos(377t + 180°)$ V.

(b) $v_2(t) = 12\sin(377t + 45°)$ V.

Give the answers in both the time and frequency domains.

CS

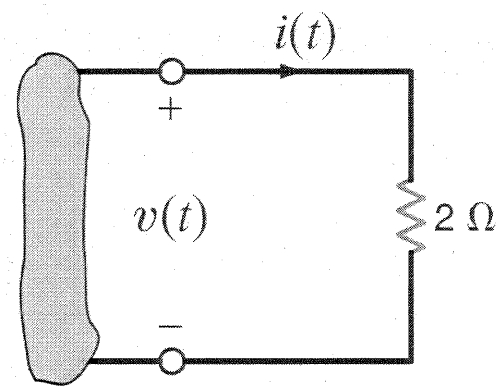


Figure P8.5

SOLUTION:

a) $i = v/R$ $\quad i(t) = 5\cos(377t + 180°)$ A $\quad I = 5\angle 180°$ A

b) $i(t) = 6\sin(377t + 45°) = 6\cos(377t - 45°)$ A $\quad I = 6\angle -45°$ A

8.6 Calculate the current in the inductor shown in Fig. P8.6 if the voltage input is

(a) $v_1(t) = 10\cos(377t + 45°)$ V

(b) $v_2(t) = 5\sin(377t - 90°)$ V

Give the answers in both the time and frequency domains.

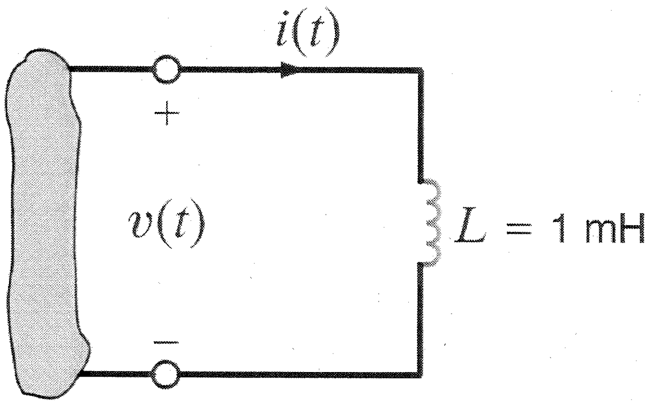
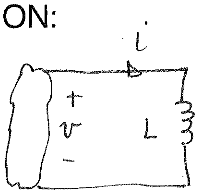


Figure P8.6

SOLUTION:

a)



$i = \frac{1}{L}\int v\,dt = \frac{10}{377L}\sin(377t + 45°)$

$$\boxed{\begin{array}{l} i(t) = 26.5\cos(377t - 45°)\text{ A} \\ I = 26.5\angle -45°\text{ A} \end{array}}$$

b) $i = \frac{5}{377L}\left(-\cos(377t - 90°)\right)$

$$\boxed{\begin{array}{l} i(t) = 13.3\cos(377t + 90)\text{ A} \\ I = 13.3\angle 90°\text{ A} \end{array}}$$

8.7 Calculate the current in the capacitor shown in Fig. P8.7 if the voltage input is

(a) $v_1(t) = 10\cos(377t - 30°)$ V

(b) $v_2(t) = 5\sin(377t + 60°)$ V

Give the answers in both the time and frequency domains.

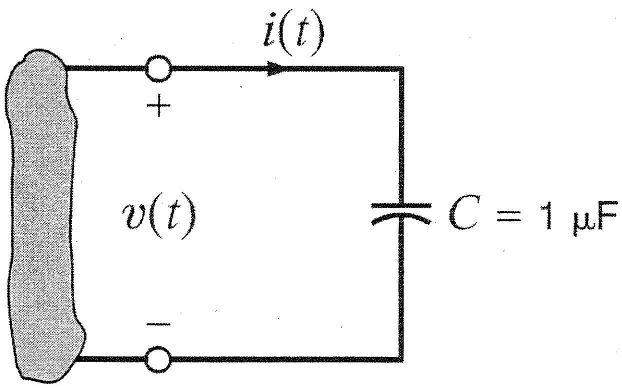


Figure P8.7

SOLUTION:

a) $i = C\dfrac{dv}{dt} = 10^{-6}(10)(377)\left[-\sin(377t - 30°)\right]$

$i(t) = 3.77\sin(\omega t + 150°)$ mV

$$\boxed{\begin{aligned} i(t) &= 3.77\cos(377t + 60°)\text{ mA} \\ I &= 3.77\angle 60°\text{ mA}\end{aligned}}$$

b) $i = (10^{-6})(5)(377)\cos(\omega t + 60°)$

$$\boxed{\begin{aligned} i(t) &= 1.89\cos(377t + 60°)\text{ mA} \\ I &= 1.89\angle 60°\text{ mA}\end{aligned}}$$

8.8 Find the frequency-domain impedance, **Z**, as shown in Fig. P8.8. CS

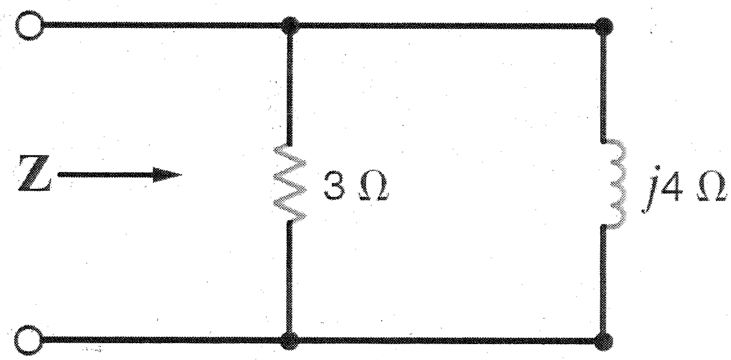


Figure P8.8

SOLUTION:

$$Z = \frac{3(j4)}{3+j4} = \frac{j12}{3+j4} = \frac{12\angle 90^\circ}{5\angle 53.1^\circ} = 2.4\angle 36.9^\circ\ \Omega$$

$$\boxed{Z = 2.4\angle 36.9^\circ\ \Omega}$$

8.9 Find the impedance, **Z**, shown in Fig. P8.9 at a frequency of 60 Hz.

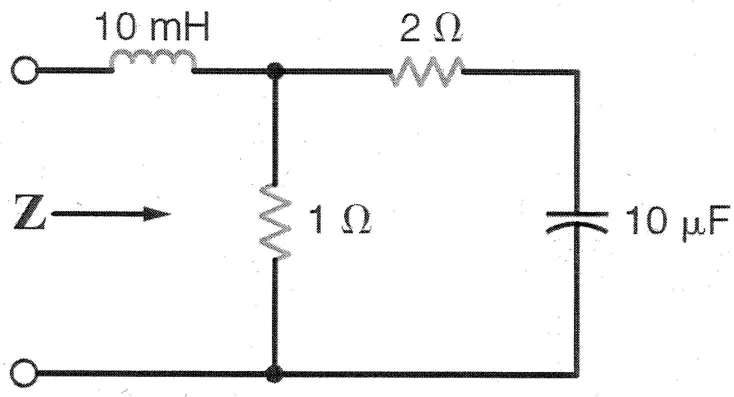
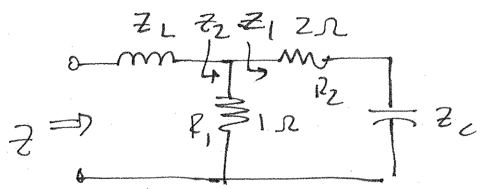


Figure P8.9

SOLUTION: $\omega = 2\pi f = 377$



$Z_L = j\omega L = j\,3.77\,\Omega$

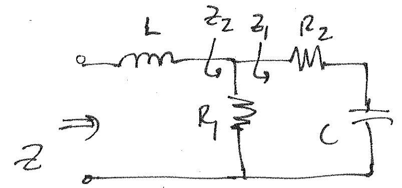
$Z_C = \dfrac{1}{j\omega C} = -j\,265\,\Omega$

$Z_1 = 2 - j265\ \Omega$ $\quad Z_2 = \dfrac{R_1 Z_1}{R_1 + Z_1}$ $\quad Z = Z_L + Z_2$

$\boxed{Z = 1.00 + j\,3.77\,\Omega}$

8.10 Find the impedance, **Z**, shown in Fig. P8.9 at a frequency of 400 Hz.

SOLUTION:



$L = 10mH \quad C = 10\mu F \quad R_1 = 1\Omega \quad R_2 = 2\Omega$

$Z_L = j\omega L = j25.1\Omega$

$Z_C = \frac{1}{j\omega C} = -j39.8\Omega$

$Z_1 = R_2 + Z_C$

$Z_1 = 2 - j39.8\Omega$

$Z_2 = \frac{R_1 Z_1}{R_1 + Z_1} = 1.00 - j0.02\Omega$

$Z = Z_L + Z_2$

$\boxed{Z = 25.10 \angle 87.7° \ \Omega}$

8.11 In the network in Fig. P8.11, find $\mathbf{Z}(j\omega)$ at a frequency of 60 Hz. CS

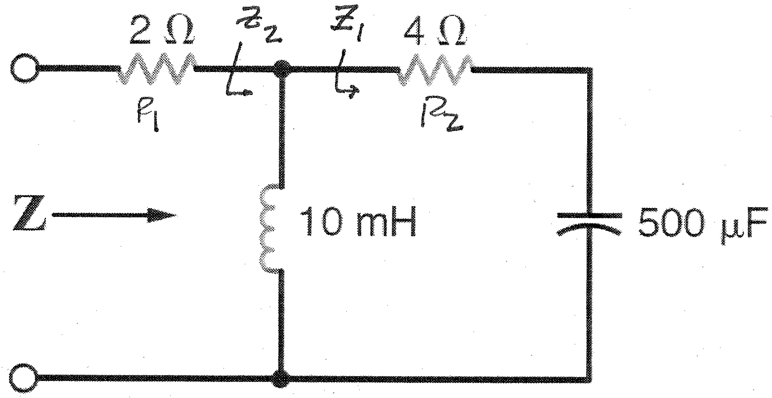


Figure P8.11

SOLUTION: $\omega = 377\ r/s$

$Z_1 = R_2 + jZ_c$ $\qquad Z_c = \frac{1}{j\omega C} = -j\,5.31\,\Omega$

$Z_2 = \frac{Z_L Z_1}{Z_L + Z_1}$ $\qquad Z_L = j\,3.77\,\Omega$

$Z = R_1 + Z_2$ $\qquad \boxed{Z = 7.11\angle 44.3^\circ\ \Omega}$

8.12 Find the frequency-domain impedance, **Z**, shown in Fig. P8.12.

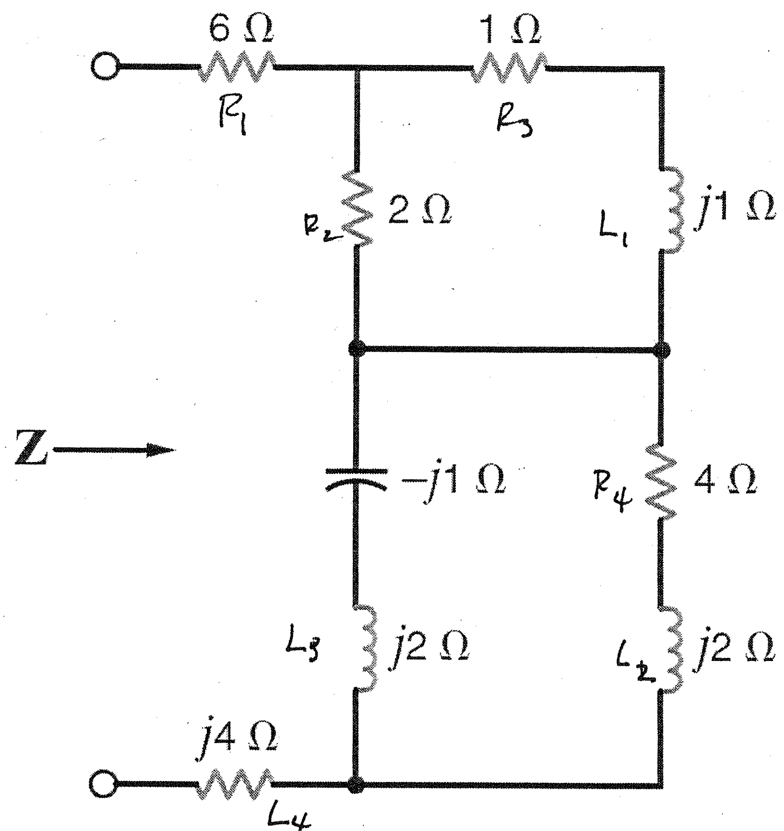
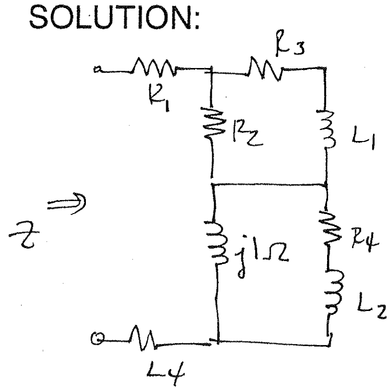


Figure P8.12

SOLUTION:



$Z_1 = R_3 + Z_{L1} = 1 + j1\ \Omega$

$Z_2 = R_4 + Z_{L2} = 4 + j2$

$Z_3 = \dfrac{Z_1 R_2}{Z_1 + R_2} = 0.8 + j0.4$

$Z_4 = \dfrac{j1\,(Z_2)}{j1 + Z_2} = 0.16 + j0.88$

$Z = R_1 + Z_3 + Z_4 + Z_{L4}$

$\boxed{Z = 8.74\ \angle 37.2^\circ\ \Omega}$

8.13 Find the frequency-domain impedance, **Z**, shown in Fig. P8.13. **PSV**

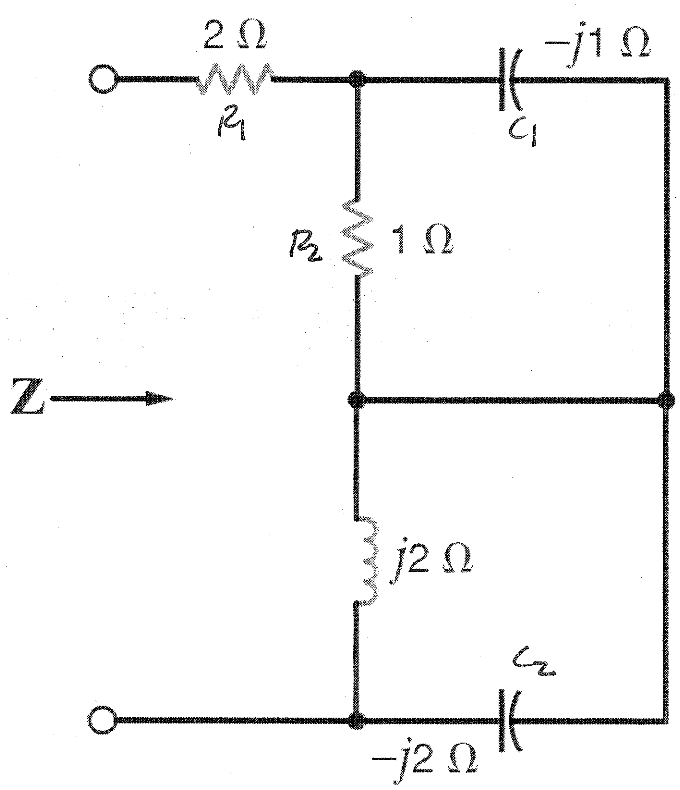


Figure P8.13

SOLUTION:

$$Z_1 = \frac{R_2 Z_{C1}}{R_2 + Z_{C1}} = 0.5 - j0.5 \qquad Z_2 = \frac{Z_L Z_{C2}}{Z_L + Z_{C2}} = \infty$$

$$Z = R_1 + Z_1 + Z_2 = \infty \qquad \boxed{Z = \infty}$$

8.14 Find **Z** in the network in Fig. P8.14. CS

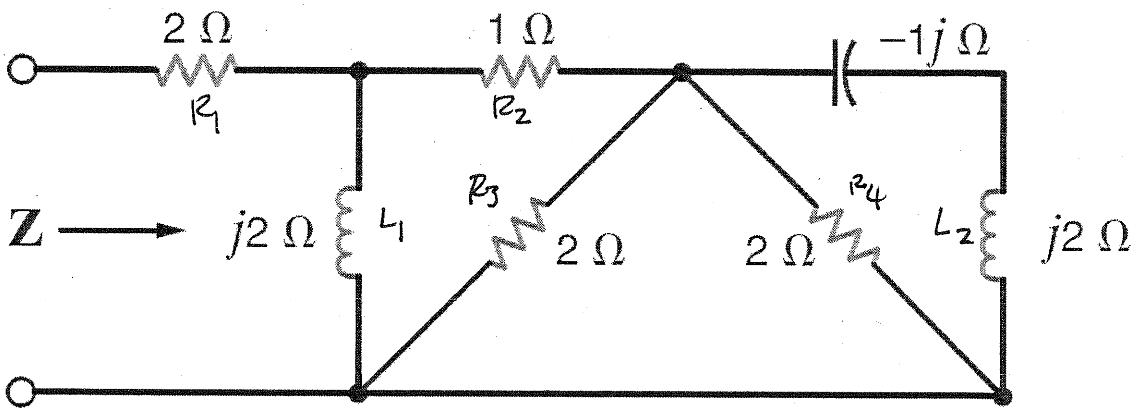


Figure P8.14

SOLUTION:

$Z_1 = Z_{L2} + Z_C = j1\,\Omega$ $\qquad Z_2 = \dfrac{R_4 Z_1}{Z_1 + R_4} = 0.4 + j0.8\,\Omega$

$Z_3 = \dfrac{R_3 Z_2}{R_3 + Z_2} = 0.5 + j0.5\,\Omega$ $\qquad Z_4 = R_2 + Z_3 = 1.5 + j0.5\,\Omega$

$Z_5 = \dfrac{Z_{L1} Z_4}{Z_{L1} + Z_4} = 0.706 + j\,0.824\,\Omega$ $\qquad Z = R_1 + Z_5$

$Z = 2.706 + j0.824$ $\qquad \boxed{Z = 2.83\,\angle 16.9^\circ\,\Omega}$

8.15 Find **Z** in the network in Fig. P8.15.

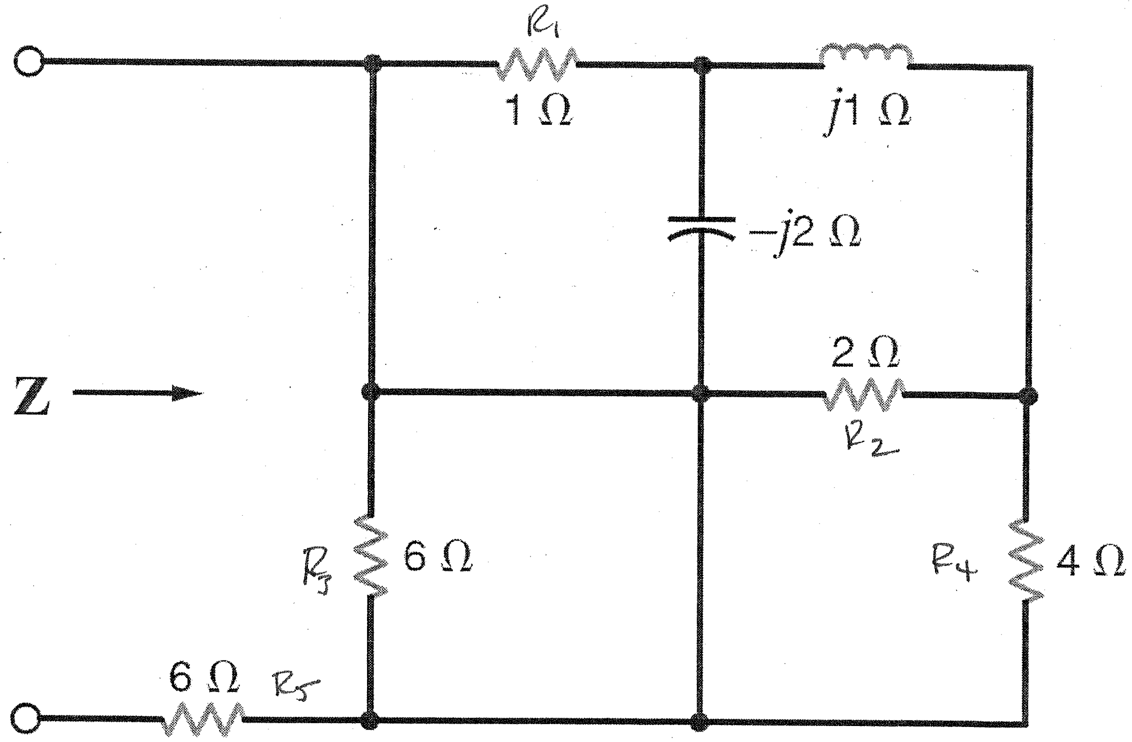
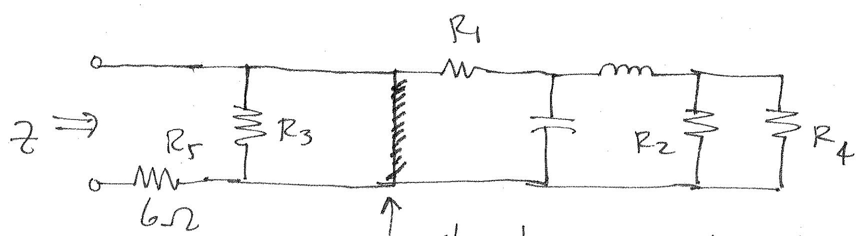


Figure P8.15

SOLUTION:

Redraw:



shorts everything but R_5!

$\boxed{Z = 6\,\Omega}$

8.16 Draw the frequency-domain circuit and calculate $i(t)$ for the circuit shown in Fig. P8.16 if $v_S(t) = 2\cos(377t)$ V.

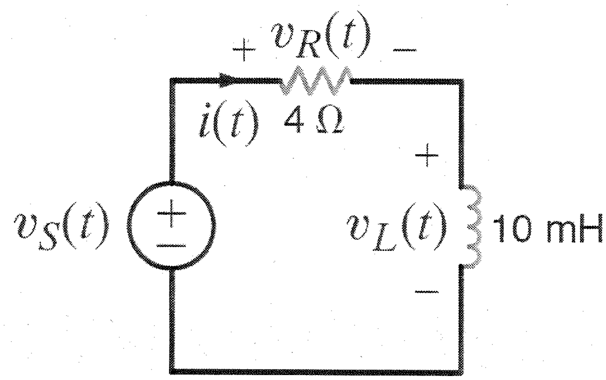
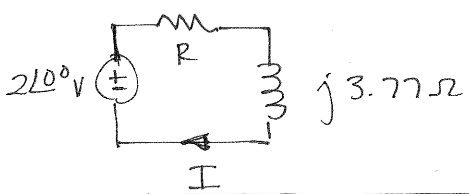


Figure P8.16

SOLUTION:



$$I = \frac{2\angle 0^\circ}{4+j3.77} =$$

$$\boxed{I = 0.36\angle -43.3^\circ \text{ A}}$$

$$\boxed{i(t) = 0.36\cos(377t - 43.3^\circ) \text{ A}}$$

8.17 Draw the frequency-domain circuit and calculate $v(t)$ for the circuit shown in Fig. P8.17 if $i_S(t) = 10\cos(377t + 30°)$ A.

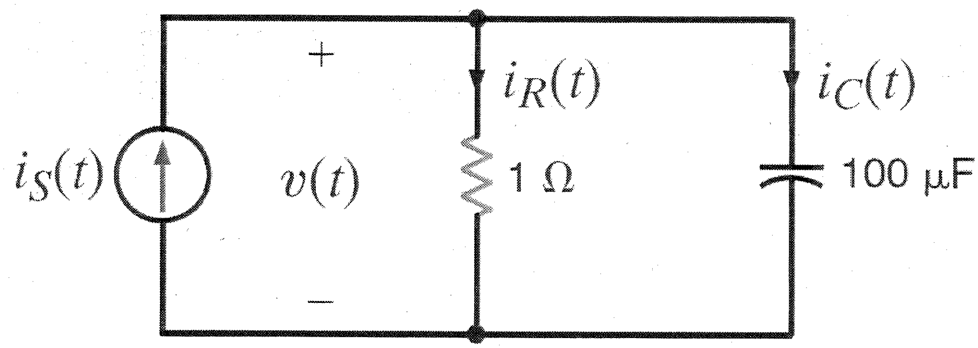
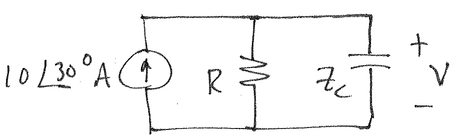


Figure P8.17

SOLUTION:



$Z_C = \frac{1}{j\omega C} = -j26.5\,\Omega$

$V = \frac{10\angle 30° \,(R Z_C)}{R + Z_C}$

$\boxed{V = 9.99\ \angle 27.8°\ V}$

$\boxed{v(t) = 9.99\cos(377t + 27.8°)\ V}$

8.18 Draw the frequency-domain circuit and calculate $v(t)$ for the circuit shown in Fig. P8.18 if $i_S(t) = 20\cos(377t + 120°)$ A. CS

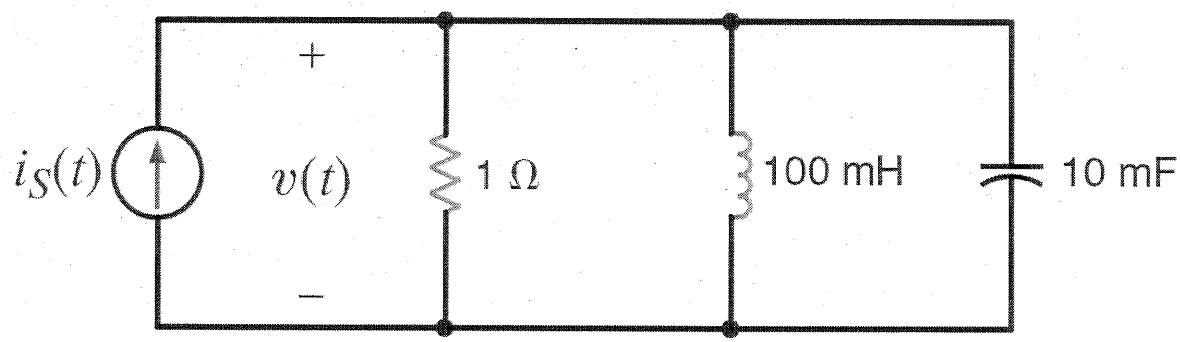
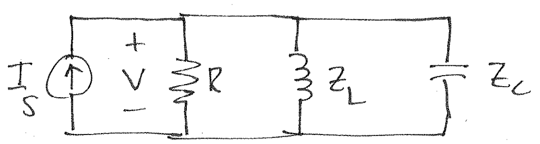


Figure P8.18

SOLUTION:



$Z_C = -j0.265\ \Omega$

$Z_L = j\,37.7\ \Omega$

$Z_{eq} = R /\!/ Z_L /\!/ Z_C = 0.258\ \angle -74.9°\ \Omega$

$I_S = 20\ \angle 120°$ A

$V = I_S Z_{eq}$

$\boxed{V = 5.16\ \angle 45.1°\ \text{V}}$

$\boxed{v(t) = 5.16\cos(377t + 45.1°)\ \text{V}}$

8.19 Draw the frequency-domain circuit and calculate $v(t)$ for the circuit shown in Fig. P8.19 if $i_S(t) = 2\cos(1000t + 120°)$ A.

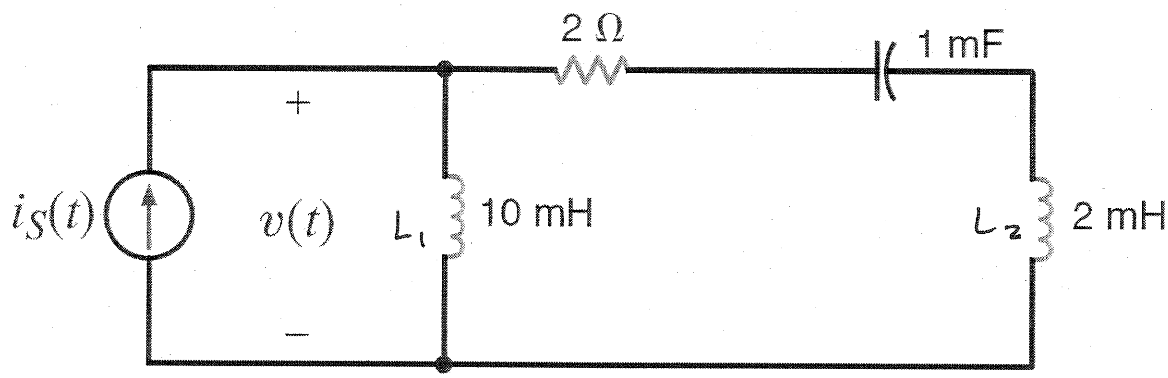
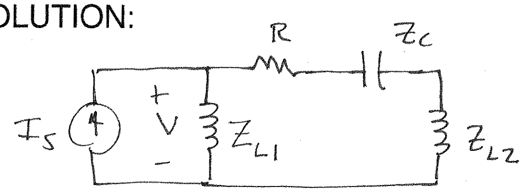


Figure P8.19

SOLUTION:



$I_S = 2\angle 120°$ A

$Z_1 = R + Z_C + Z_{L2} = 2 + j1\ \Omega$

$Z_{L1} = j10\Omega \quad Z_{L2} = j2\Omega \quad Z_C = -j1\Omega$

$Z_2 = \dfrac{Z_{L1} Z_1}{Z_{L1} + Z_1}$

$Z_2 = 1.6 + j1.2\Omega$

$V = I_S Z_2$

$\boxed{V = 4\angle 156.9°\ V}$

$\boxed{v(t) = 4\cos(1000t + 156.9°)\ V}$

8.20 Draw the frequency-domain network and calculate $v_o(t)$ in the circuit shown in Fig. P8.20 if $v_S(t)$ is $4\sin(500t + 45°)$ V and $i_S(t)$ is $1\cos(500t + 45°)$ A. Also, use a phasor diagram to determine $v_1(t)$.

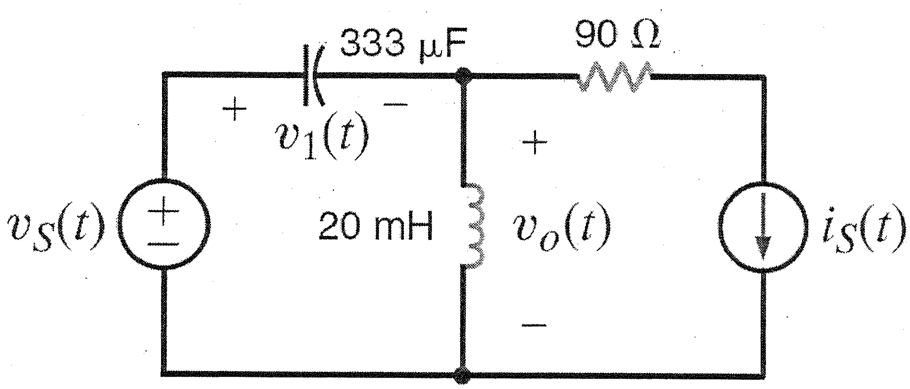
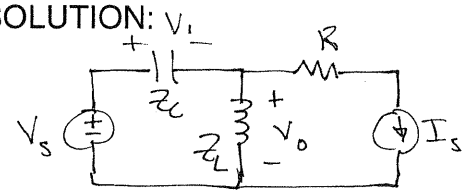


Figure P8.20

SOLUTION:



$V_S = 4\angle{-45°}$ V $\quad I_S = 1\angle 45°$ A

$Z_C = \frac{1}{j\omega C} = -j6\,\Omega \quad Z_L = j\omega L = j10\,\Omega$

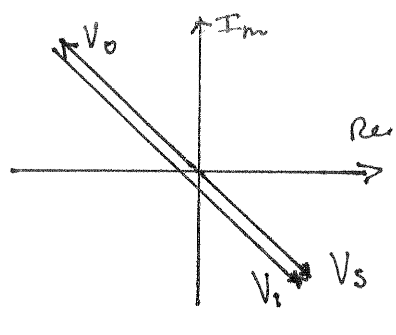
Superposition: $V_o = V_S\left[\frac{Z_L}{Z_L + Z_C}\right] - I_S\left[\frac{Z_L Z_C}{Z_L + Z_C}\right]$

$V_o = 5\angle 135°$ V

$$\boxed{v_o(t) = 5\cos(500t + 135°)\ \text{V}}$$

$V_1 = V_S - V_o = 4\angle{-45°} - 5\angle 135 = 9\angle{-45°}$

$$\boxed{v_1(t) = 9\cos(500t - 45°)\ \text{V}}$$



8.21 Draw the frequency-domain network and calculate $v_o(t)$ in the circuit shown in Fig. P8.21 if $i_1(t)$ is $200\cos(10^5 t + 60°)$ mA, $i_2(t)$ is $100\sin(10^5 t + 90°)$ mA, and $v_S(t) = 10\sin(10^5 t)$ V. Also, use a phasor diagram to determine $v_C(t)$.

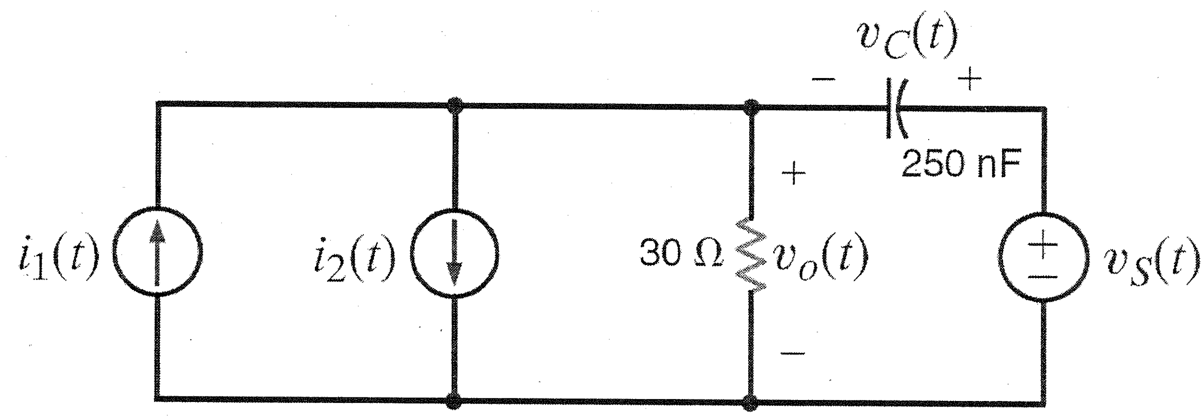
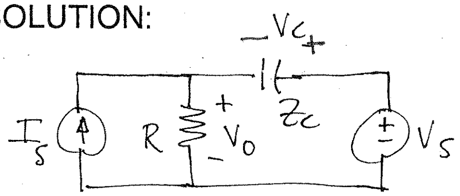


Figure P8.21

SOLUTION:



$I_1 = 200 \angle 60°$ mA $\quad I_2 = 100\angle 0°$ mA

$I_S = I_1 - I_2 = 173 \angle 90°$ mA

$V_S = 10 \angle -90°$ V

$Z_C = \frac{1}{j\omega C} = -j\,40\,\Omega$

Superposition:

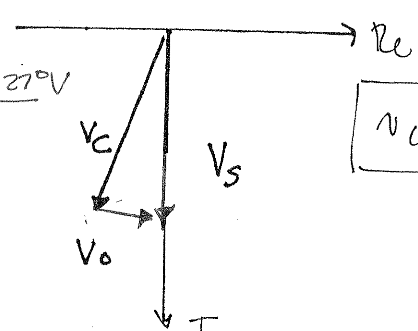
$$V_o = \frac{I_S R Z_C}{R + Z_C} + \frac{V_S R}{R + Z_C}$$

$V_o = 7.30 \angle -2.16$ V

$$\boxed{v_o(t) = 7.30\cos(10^5 t - 2.16°)\text{ V}}$$

$V_C = V_S - V_o$

$V_C = 12.2 \angle -127°$ V



$$\boxed{v_C(t) = 12.2\cos(10^5 t - 127°)\text{ V}}$$

8.22 The impedance of the network in Fig. P8.22 is found to be purely real at $f = 400$ Hz. What is the value of C?

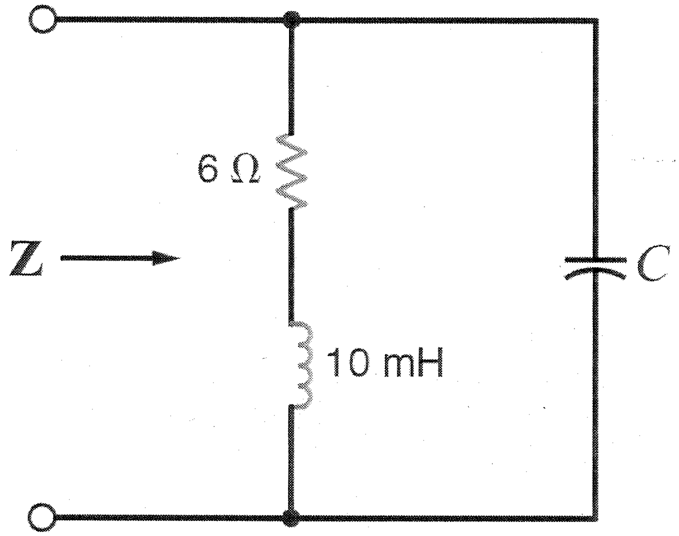


Figure P8.22

SOLUTION:

$$Z = \frac{(6 + j\omega L)(-j/\omega C)}{6 + j(\omega L - 1/\omega C)} = R_{eq}$$

Requires angles of numerator = angle of denominator

$$\frac{-\frac{6}{\omega C}}{L/C} = \frac{\omega L - 1/\omega C}{6} \Rightarrow \frac{-36}{\omega L} = \omega L - \frac{1}{\omega C}$$

$$C = \frac{L}{36 + \omega^2 L^2}$$

$$\boxed{C = 15.0\mu F}$$

8.23 In the circuit shown in Fig. P8.23, determine the value of the inductance such that the current is in phase with the source voltage. **PSV**

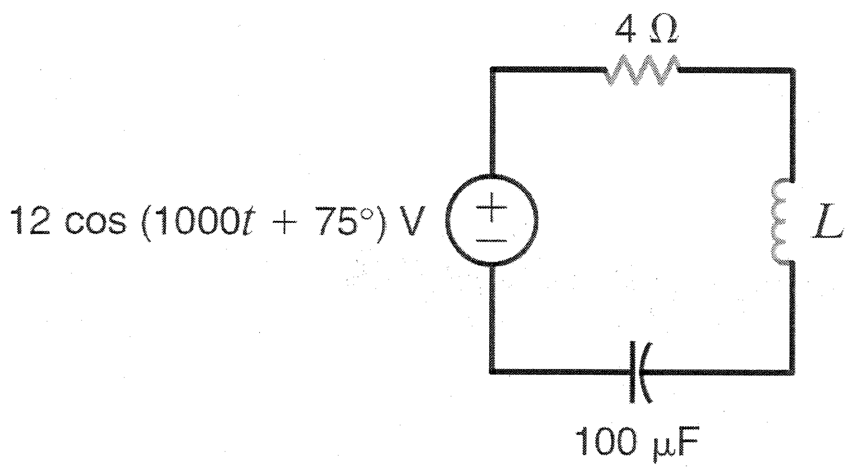
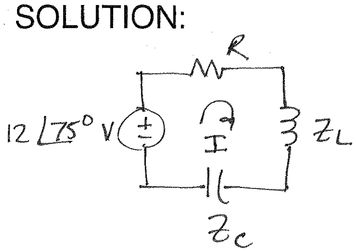


Figure P8.23

SOLUTION:



If V_s & I are in phase, then

$$\frac{V_s}{I} = R_{eq} + j0 = R + Z_L + Z_C$$

or $Z_L + Z_C = 0$

$Z_L = j\,1000L \qquad Z_C = -j10\,\Omega \qquad \Rightarrow Z_L = j10\,\Omega$

$$\boxed{L = 10\,mH}$$

8.24 The impedance of the box in Fig. P8.24 is $5 + j4\ \Omega$ at 1000 rad/s. What is the impedance at 1300 rad/s?

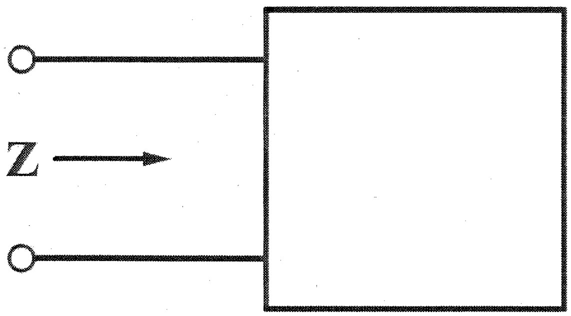


Figure P8.24

SOLUTION:

$Z = 5 + j4\,\Omega = R_{eq} + j\omega L_{eq}$ @ $\omega = 1000$ r/s $\Rightarrow L_{eq} = 4$ mH

at $\omega = 1300$ r/s $\quad Z = R_{eq} + j(1300)(0.004)$

$$\boxed{Z = 5 + j5.2\,\Omega}$$

8.25 The admittance of the box in Fig. P8.25 is $0.1 + j0.2$ S at 500 rad/s. What is the impedance at 300 rad/s?

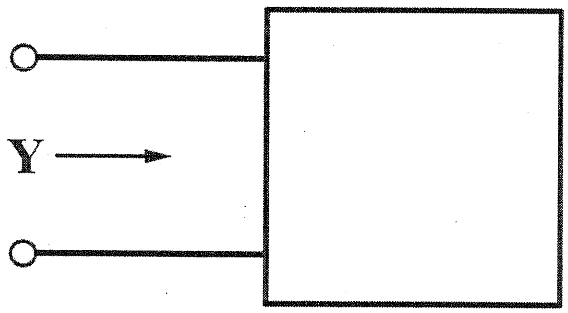


Figure P8.25

SOLUTION:

$$Y = 0.1 + j0.2 \ S = \frac{1}{R_{eq}} + \frac{1}{\frac{1}{j\omega C_{eq}}} \qquad \omega C_{eq} = 0.2 \quad @ \ 500 \ r/s$$

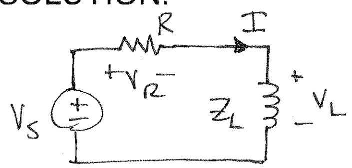
$$C_{eq} = 400 \mu F$$

at 300 r/s $\quad \boxed{Y = 0.1 + j\, 0.12 \ S}$

$$\boxed{Z = 1/Y = 4.10 - j4.92 \ \Omega}$$

8.26 The voltages $v_R(t)$ and $v_L(t)$ in the circuit shown in Fig. P8.16 can be drawn as phasors in a phasor diagram. Use a phasor diagram to show that $v_R(t) + v_L(t) = v_S(t)$.

SOLUTION:

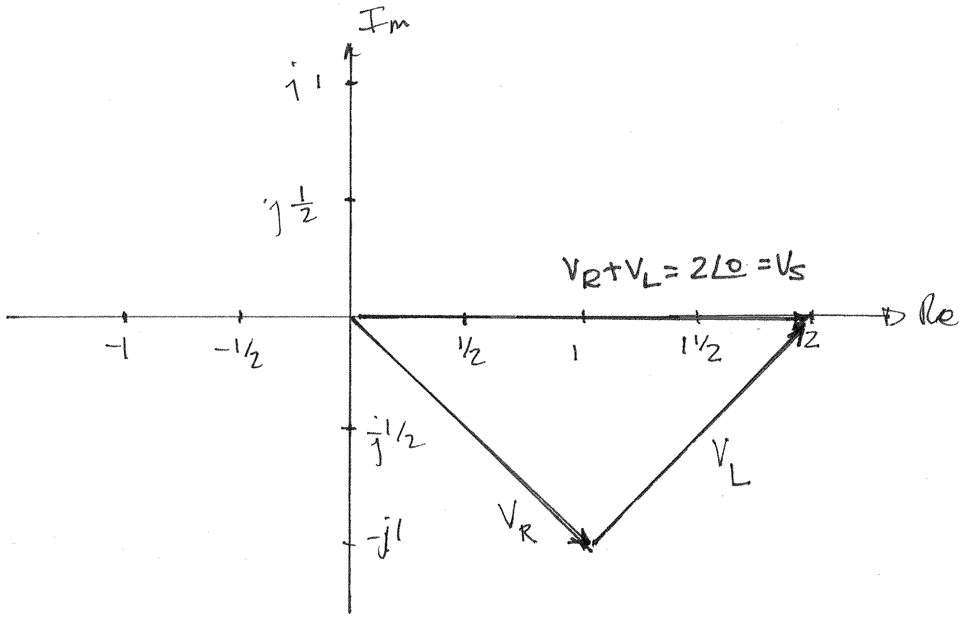


$R = 4\,\Omega \qquad L = 10\,\text{mH} \qquad \omega = 377\ \text{r/s}$

$Z_L = j\,3.77\,\Omega \qquad V_S = 2\angle 0^\circ\ \text{V}$

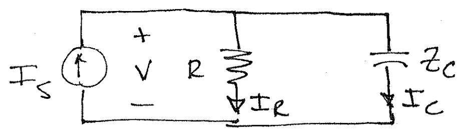
$$I = \frac{V_S}{R + j\omega L} = 0.36\angle -43.3^\circ\ \text{A}$$

$V_R = RI = 1.46\angle -43.3^\circ\ \text{V} \qquad V_L = Z_L I = 1.37\angle 46.7^\circ\ \text{V}$



8.27 The currents $i_R(t)$ and $i_C(t)$ in the circuit shown in Fig. P8.17 can be drawn as phasors in a phasor diagram. Use the diagram to show that $i_R(t) + i_C(t) = i_S(t)$.

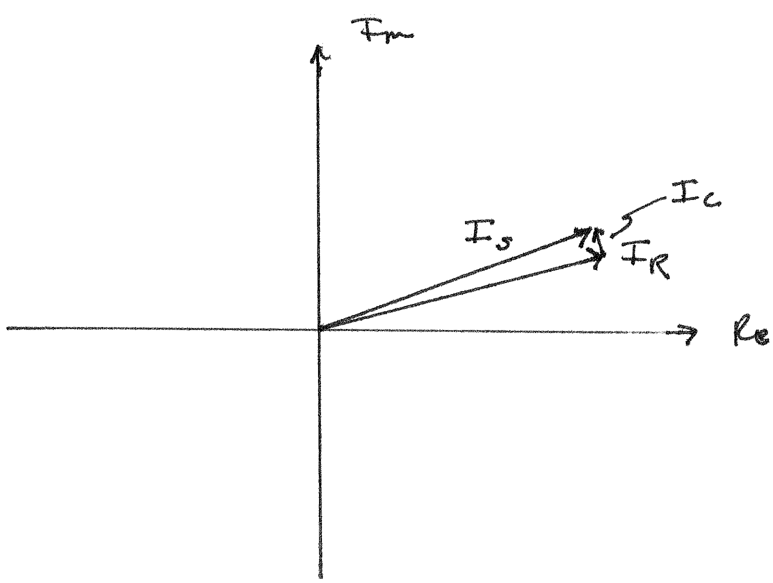
SOLUTION:



$I_S = 10\angle 30^\circ$ A $\quad R = 1\,\Omega \quad C = 100\,\mu F$

$\omega = 377 \quad Z_C = -j26.5\,\Omega$

$$I_R = \frac{I_S Z_C}{R + Z_C} = 9.993\angle 27.8^\circ \text{ A} \qquad I_C = \frac{I_S R}{R + Z_C} = 0.377\angle 118^\circ \text{ A}$$



8.28 The currents $i_R(t)$ and $i_C(t)$ in the circuit shown in Fig. P8.28 can be drawn as phasors in a phasor diagram. Use the diagram to show that $i_R(t) + i_C(t) = i_S(t)$.
CS

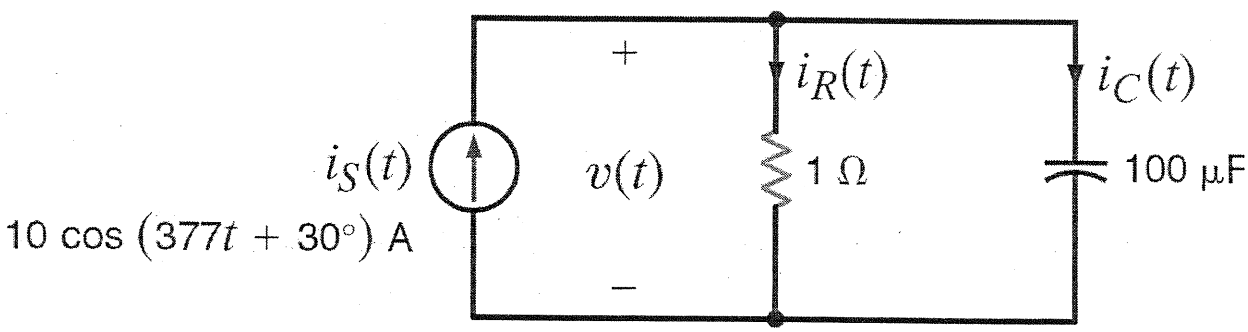
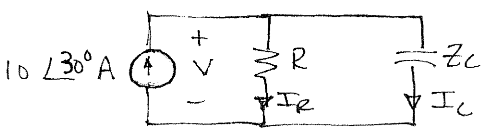


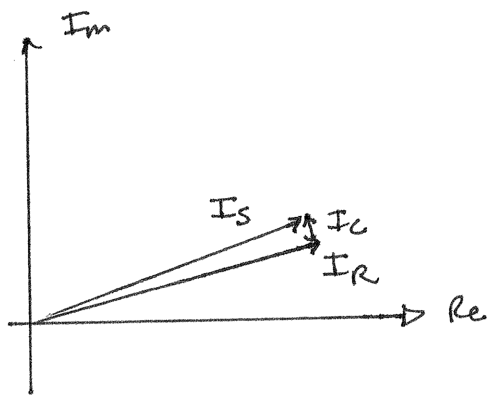
Figure P8.28

SOLUTION:



$$Z_C = \frac{1}{j\omega C} = -j26.5\,\Omega$$

$$I_R = \frac{I_S Z_C}{Z_C + R} = 9.993\ \angle 27.8^\circ\ \text{A} \qquad I_C = \frac{I_S R}{R + Z_C} = 0.377\ \angle 118^\circ\ \text{A}$$



8.29 Draw the frequency-domain network and calculate $v_o(t)$ in the circuit shown in Fig. P8.29 if $i_S(t)$ is $300 \sin(10^4 t - 45°)$ mA. Also, using a phasor diagram, show that $i_1(t) + i_2(t) = i_S(t)$. CS

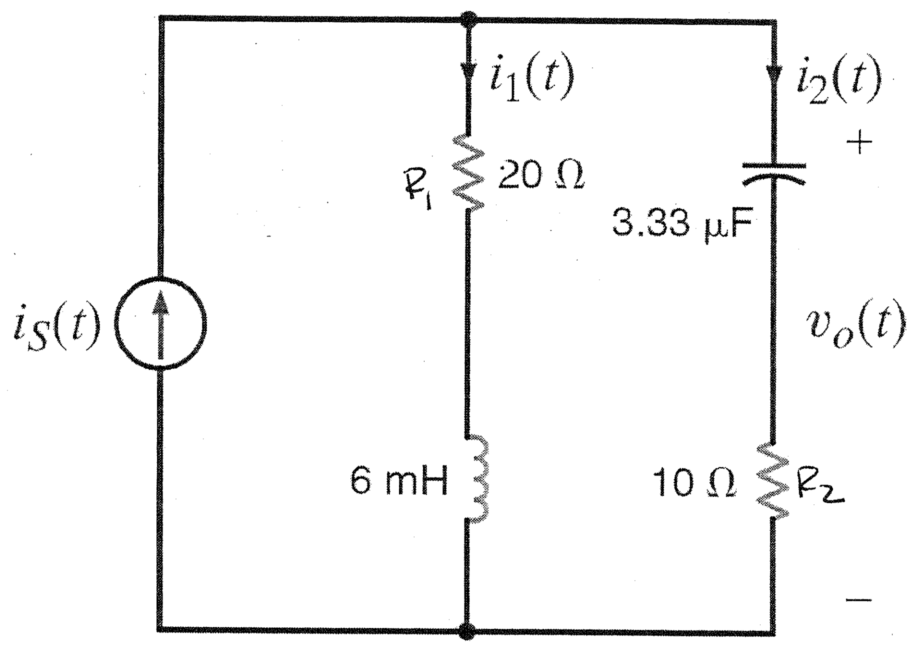
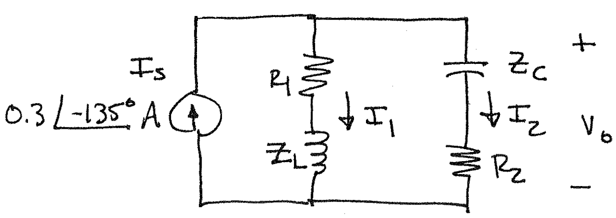


Figure P8.29

SOLUTION:



$Z_L = j60\ \Omega \qquad Z_C = -j30$

$Z_1 = R_1 + Z_L = 20 + j60\ \Omega$

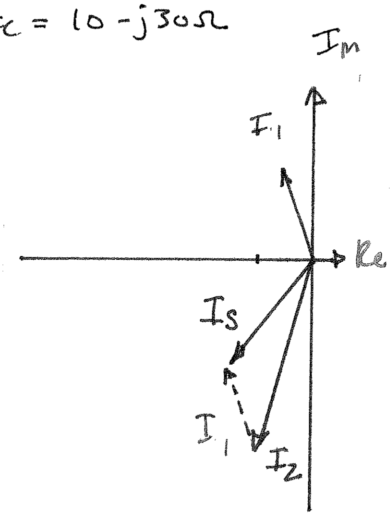
$Z_2 = R_2 + Z_C = 10 - j30\ \Omega$

$I_1 = I_S Z_2/(Z_1+Z_2) = 0.224\angle 108.4°$ A

$I_2 = I_S Z_1/(Z_1+Z_2) = 0.448\angle{-108.4°}$ A

$V_o = I_2 Z_2 = 14.1\angle 180$ V

$$\boxed{v_o(t) = 14.1 \cos(10^4 t + 180°)\,\text{V}}$$



8.30 Find the value of C in the circuit shown in Fig. P8.30 so that $\mathbf{Z}$ is purely resistive at the frequency of 60 Hz.

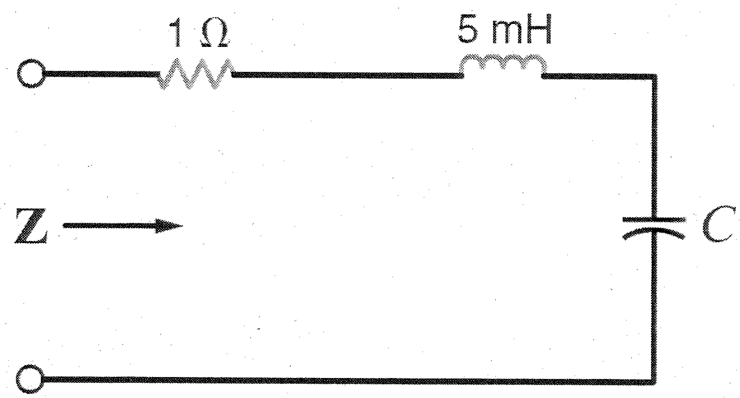


Figure P8.30

SOLUTION:

$Z = R + Z_L + Z_C = R_{eq}$ requires $\omega L = \frac{1}{\omega C}$

$C = \frac{1}{\omega^2 L}$ $\boxed{C = 1.41\, mF}$

8.31 Find the frequency at which the circuit shown in Fig. P8.31 is purely resistive.

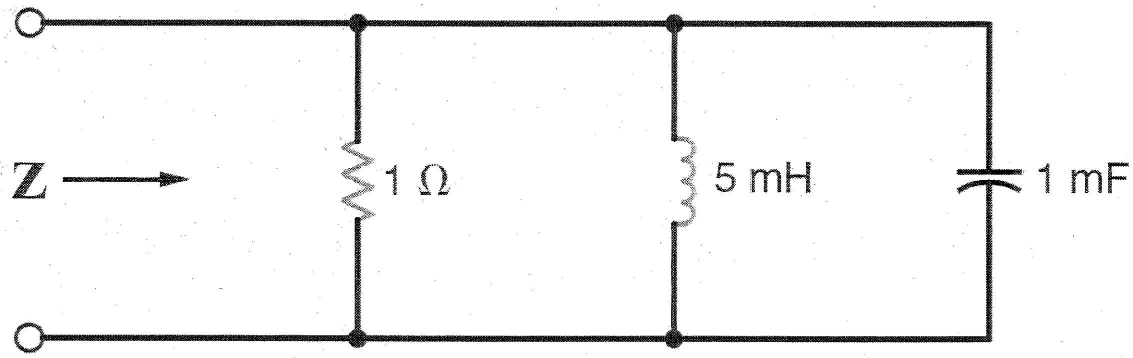


Figure P8.31

SOLUTION:

$Z = R_{eq}$ $\quad Y = \frac{1}{R_{eq}} = G_{eq} = \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}$

Requires $\frac{1}{\omega L} = \omega C \Rightarrow \omega = \frac{1}{\sqrt{LC}}$ $\quad \omega = 447.2\ r/s$

$f = \frac{\omega}{2\pi}$ $\quad \boxed{f = 71.2\ Hz}$

8.32 In the circuit shown in Fig. P8.32, determine the frequency at which $i(t)$ is in phase with $v_S(t)$.

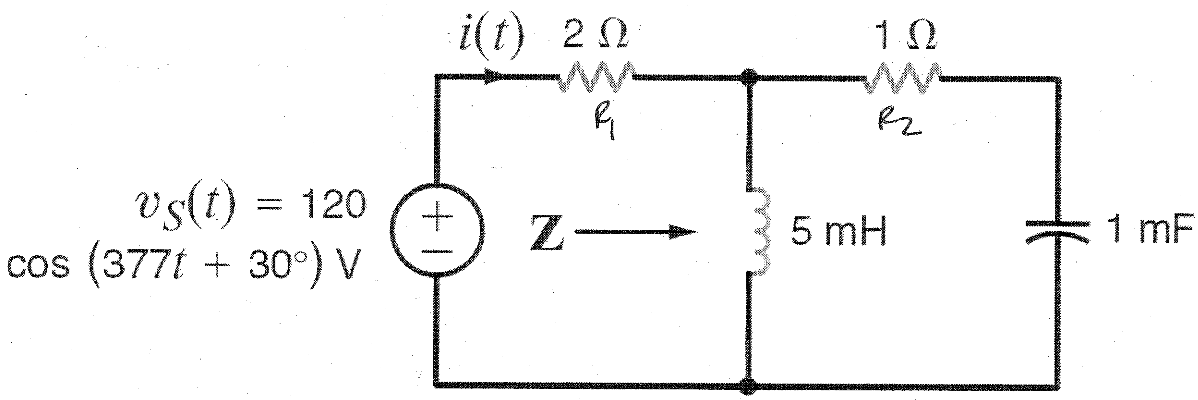


Figure P8.32

SOLUTION: for $i(t)$ and $V_S(t)$ to be in phase, Z must be real.

$$Z_1 = R_2 + Z_C = 1 - \frac{j}{\omega C} \qquad Z_2 = \frac{Z_L Z_1}{Z_L + Z_1} \text{ must be real!}$$

$$Z_2 = \frac{L/C + j\omega L}{1 + j\left(\omega L - \frac{1}{\omega C}\right)} = R_{eq} \Rightarrow \frac{\omega L}{L/C} = \frac{\omega L - 1/\omega C}{1}$$

$$(\omega C)^2 = \omega^2 LC - 1 \qquad \omega^2 (LC - C^2) = 1 \qquad \omega = \frac{1}{\sqrt{LC - C^2}}$$

$$\boxed{\omega = 500 \text{ r/s}}$$

8.33 In the circuit shown in Fig. P8.33, determine the value of the inductance such that $v(t)$ is in phase with $i_S(t)$.

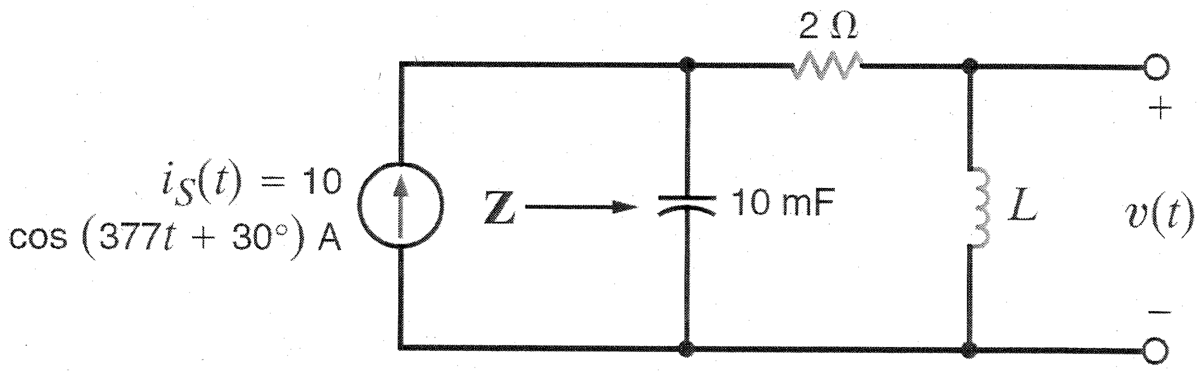
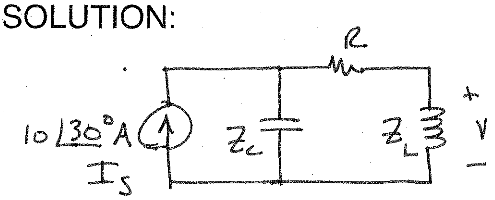


Figure P8.33

SOLUTION:



$$V = \frac{I_S Z_C Z_L}{R + Z_L + Z_C}$$

$$\frac{V}{I_S} = \frac{Z_C Z_L}{R + Z_L + Z_C} = \frac{L/C}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = R_{eq} + j0$$

Requires $\omega L = \frac{1}{\omega C} \Rightarrow L = \frac{1}{\omega^2 C}$ $\boxed{L = 0.704 \text{ mH}}$

8.34 Find the current **I** shown in Fig. P8.34.

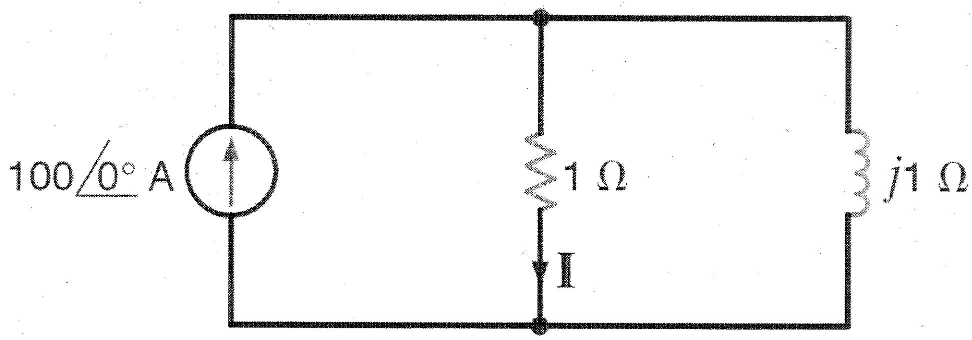


Figure P8.34

SOLUTION:

$$I = 100\angle 0^\circ \left(\frac{j1}{1+j1}\right)$$

$$\boxed{I = 70.7\angle 45^\circ \text{ A}}$$

8.35 Find the voltage $\mathbf{V}$ shown in Fig. P8.35.

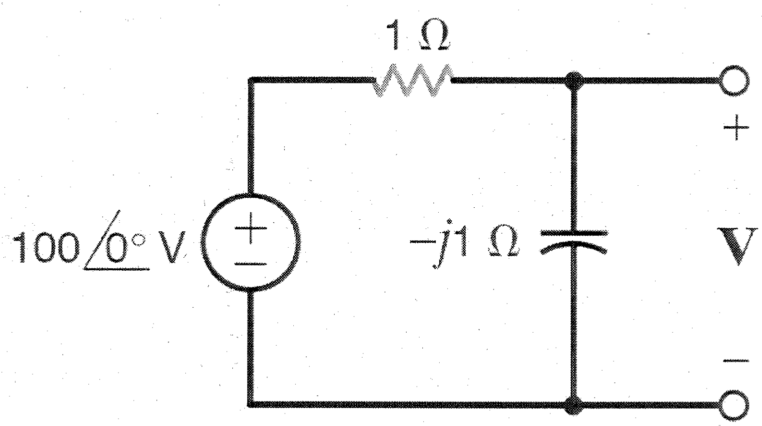


Figure P8.35

SOLUTION:

$$V = 100\angle 0^\circ \left(\frac{-j1}{1-j1}\right)$$

$$\boxed{V = 70.7\angle -45^\circ \text{ V}}$$

8.36 Find the frequency-domain voltage $\mathbf{V}_o$, as shown in Fig. P8.36. CS

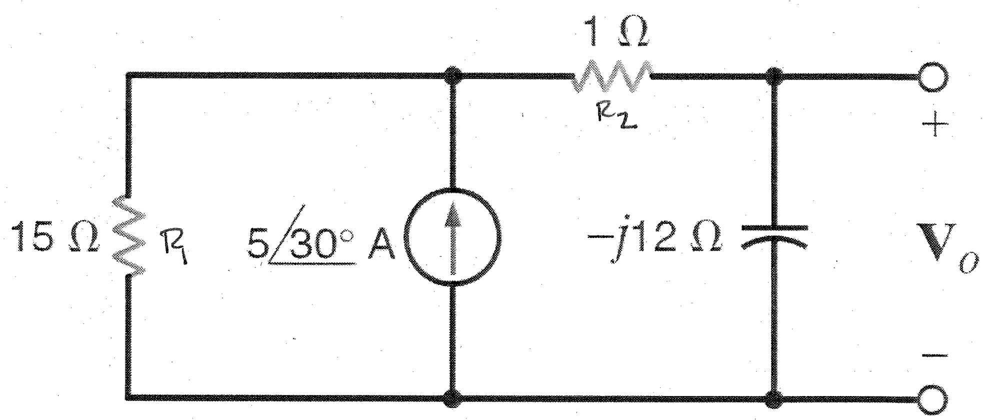
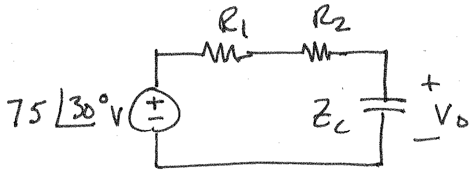


Figure P8.36

SOLUTION:

Source transformation:



$$V_o = \frac{75\angle 30\ Z_C}{Z_C + R_1 + R_2}$$

$$\boxed{V_o = 45\angle -23.1^\circ \text{ V}}$$

8.37 Find the voltage, $\mathbf{V}_o$, shown in Fig. P8.37.

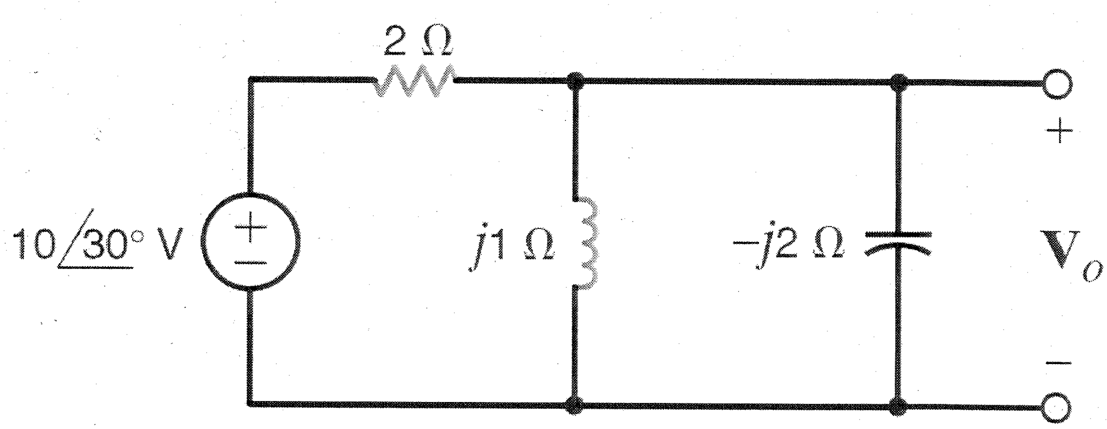
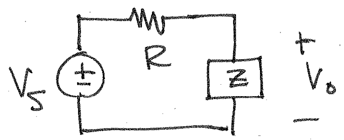


Figure P8.37

SOLUTION:



$$Z = \frac{Z_L Z_C}{Z_L + Z_C} = j2\,\Omega$$

$$V_o = \frac{V_s Z}{R + Z}$$

$$\boxed{V_o = 7.07\angle 75^\circ\ \text{V}}$$

8.38 Given the network in Fig. P8.38, determine the value of $\mathbf{V}_o$ if $\mathbf{V}_S = 24\,\underline{/0^\circ}$ V.

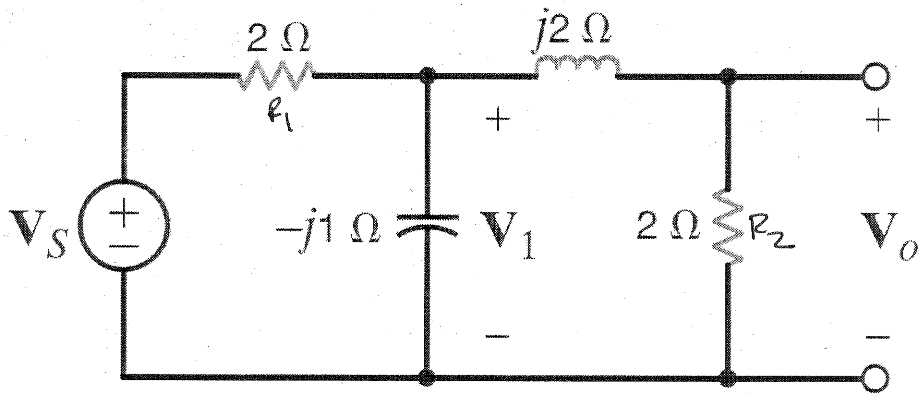


Figure P8.38

SOLUTION:

$Z_1 = R_2 + Z_L = 2 + j2\,\Omega \qquad Z_2 = \dfrac{Z_1 Z_C}{Z_1 + Z_C} = \dfrac{2 - j2}{2 + j1}$

$V_1 = V_S \dfrac{Z_2}{Z_2 + R_1} = 8 - j8\ \text{V} \qquad V_o = \dfrac{V_1 R_2}{R_2 + Z_L} = \dfrac{8 - j8}{1 + j1} = 8\,\angle{-90^\circ}\ \text{V}$

$$\boxed{V_o = 8\,\angle{-90^\circ}\ \text{V}}$$

8.39 Find $\mathbf{V}_S$ in the network in Fig. P8.39, if $\mathbf{V}_1 = 4\angle 0°$ V.

CS

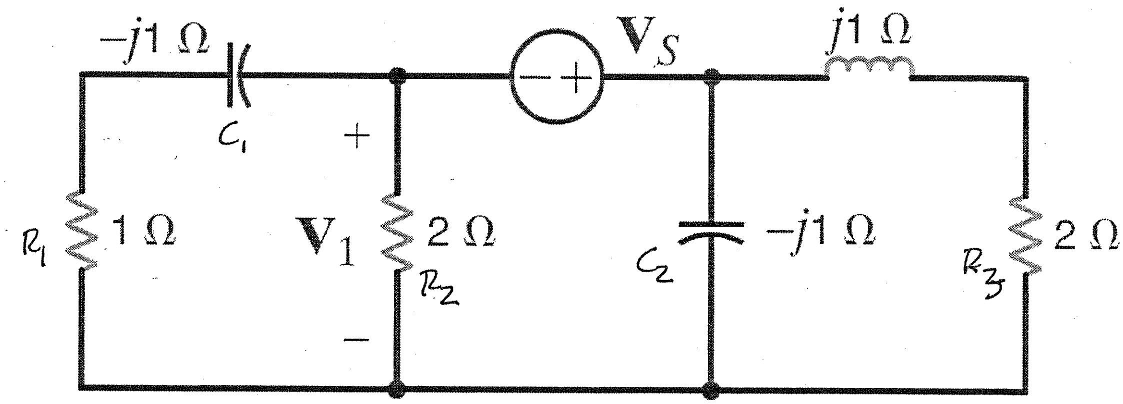
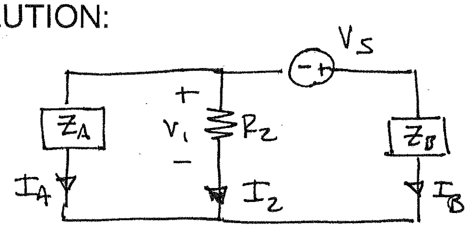


Figure P8.39

SOLUTION:



$Z_A = 1 - j1\,\Omega$

$Z_B = \dfrac{-j1(2+j1)}{2} = \dfrac{1-j2}{2}\,\Omega$

$I_2 = V_1/R_2 = 2\angle 0°\,A$ $\quad I_A = \dfrac{V_1}{Z_A} = 2 + j2\,A$ $\quad -I_B = I_A + I_2 = 4 + j2\,A$

$V_S = I_B Z_B - V_1 = -8 + j3\ V$

$$\boxed{V_S = 8.54\angle +159.4°\ V}$$

8.40 Find $\mathbf{V}_o$ in the network in Fig. P8.40. **PSV**

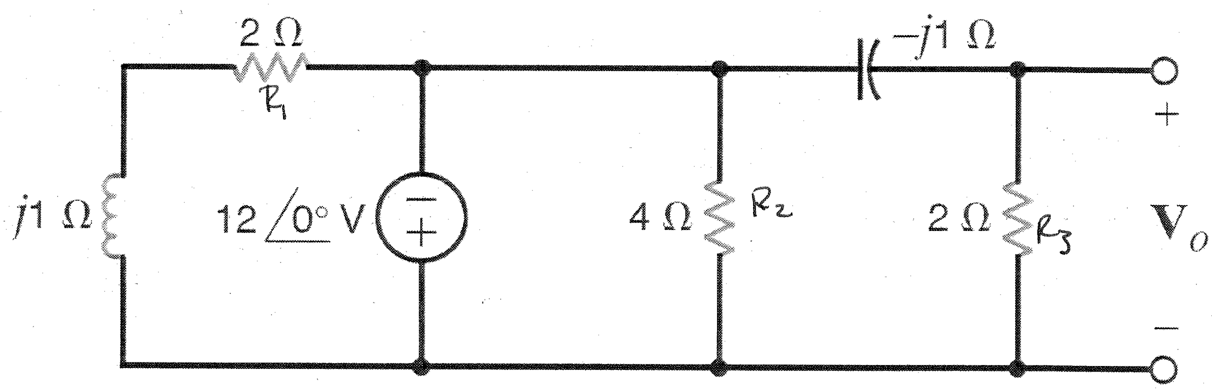
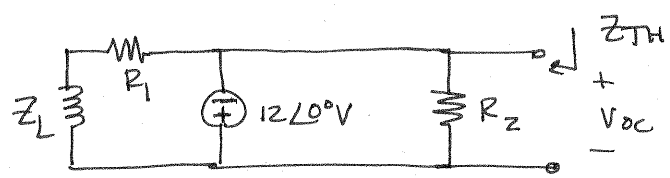


Figure P8.40

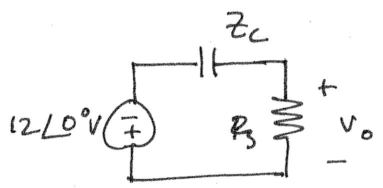
SOLUTION:

Use Thevenin's Theorem:



$V_{oc} = 12 \angle 180° \text{ V}$

$Z_{TH} = 0 \ \Omega$



$$V_o = -12 \angle 0° \left(\frac{R_3}{R_3 + Z_C} \right)$$

$$\boxed{V_o = 10.7 \angle -153.4° \text{ V}}$$

8.41 If $\mathbf{V}_1 = 4\angle 0°$ V, find $\mathbf{I}_o$ in Fig. P8.41. CS

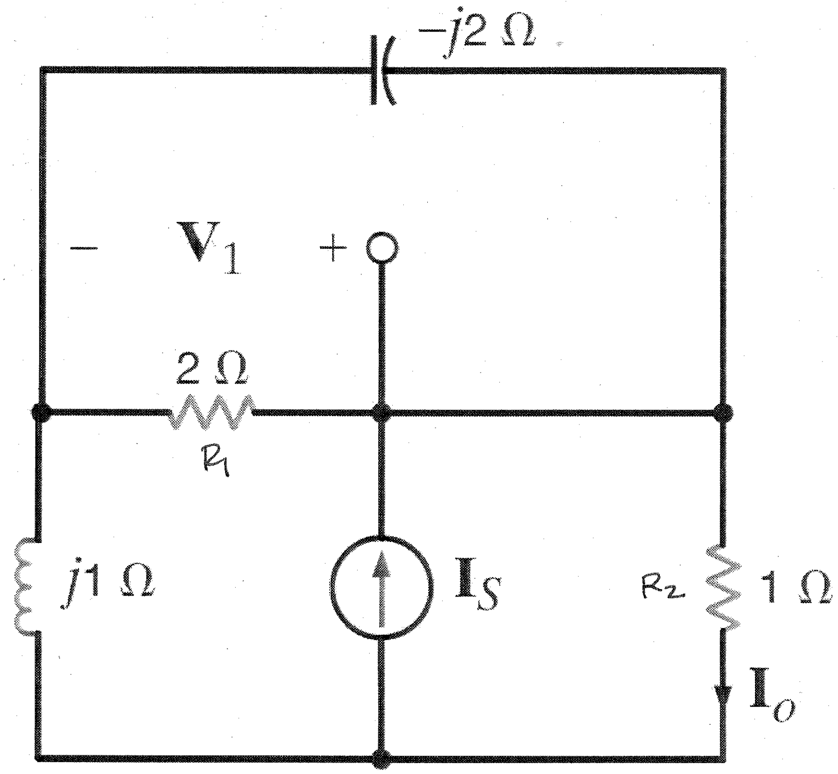
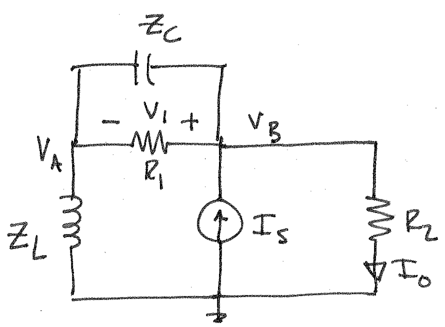


Figure P8.41

SOLUTION:



Nodal: $\dfrac{V_1}{R_1} + \dfrac{V_1}{Z_C} = \dfrac{V_A}{Z_L} \Rightarrow V_A = -2 + j2\ \text{V}$

$V_B = V_A + V_1 = V_A + 4 = 2 + j2\ \text{V}$

$I_o = \dfrac{V_B}{R_2} \Rightarrow \boxed{I_o = 2.83\angle 45°\ \text{A}}$

8.42 In the network in Fig. P8.42 $\mathbf{I}_o = 4\underline{/0^\circ}$ A find $\mathbf{I}_x$.

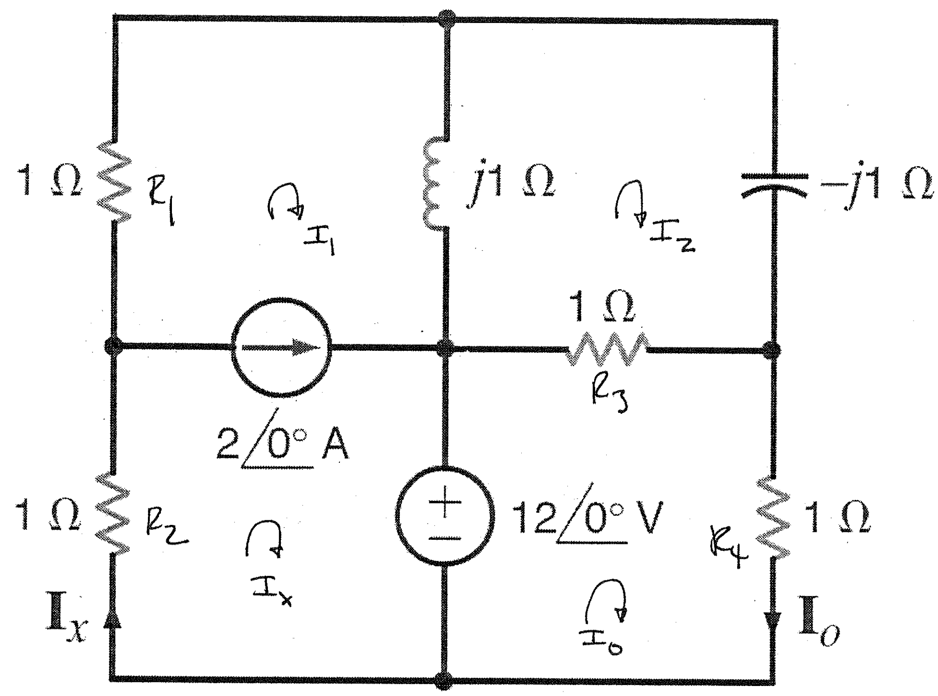


Figure P8.42

SOLUTION:

$I_x - I_1 = 2\angle 0$ $\Downarrow$ $I_1 = I_x - 2$

$12 = 2I_o - I_2$ $\Downarrow$ $I_2 = -4A$

$I_x + I_1 - j1\,I_2 + I_o = 0$ $\Downarrow$ $2I_x - 2 + j4 + 4 = 0$

$$\boxed{I_x = -1 - j2 \text{ A} = 2.24\angle{-116.6^\circ} \text{ A}}$$

8.43 If $\mathbf{I}_o = 4\angle 0^\circ$ A in the circuit in Fig. P8.43, find $\mathbf{I}_x$.

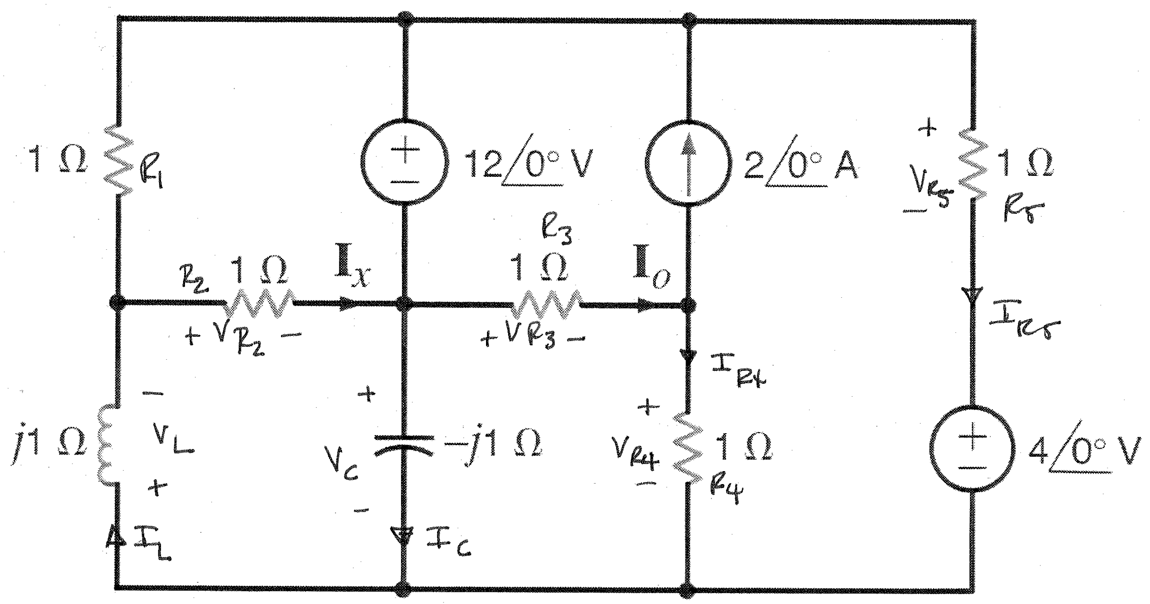


Figure P8.43

SOLUTION:

$V_{R3} = I_o R_3 = 4\angle 0^\circ V$ $\quad I_{R4} = I_o - 2\angle 0^\circ = 2\angle 0^\circ A$ $\quad V_C = V_{R3} + V_{R4} = 6\angle 0^\circ V$

$I_C = V_C / _{-j1} = j6\,A$ $\quad V_C + 12 = V_{R5} + 4 \Rightarrow V_{R5} = 14V$

$I_{R5} = V_{R5}/R_5 = 14A$ $\quad I_L = I_C + I_{R4} + I_{R5} = 16 + j6\,A$

$V_L = I_L(j1) = -6 + j16V$ $\quad V_L + V_{R2} + V_C = 0 \Rightarrow V_{R2} = -j16V$

$I_x = \frac{V_{R2}}{R_2} = -j16A$ $\quad \boxed{I_x = -j16\,A = 16\angle -90^\circ A}$

8.44 If $\mathbf{I}_o = 4\underline{/0^\circ}$ A in the network in Fig. P8.44, find $\mathbf{I}_x$.

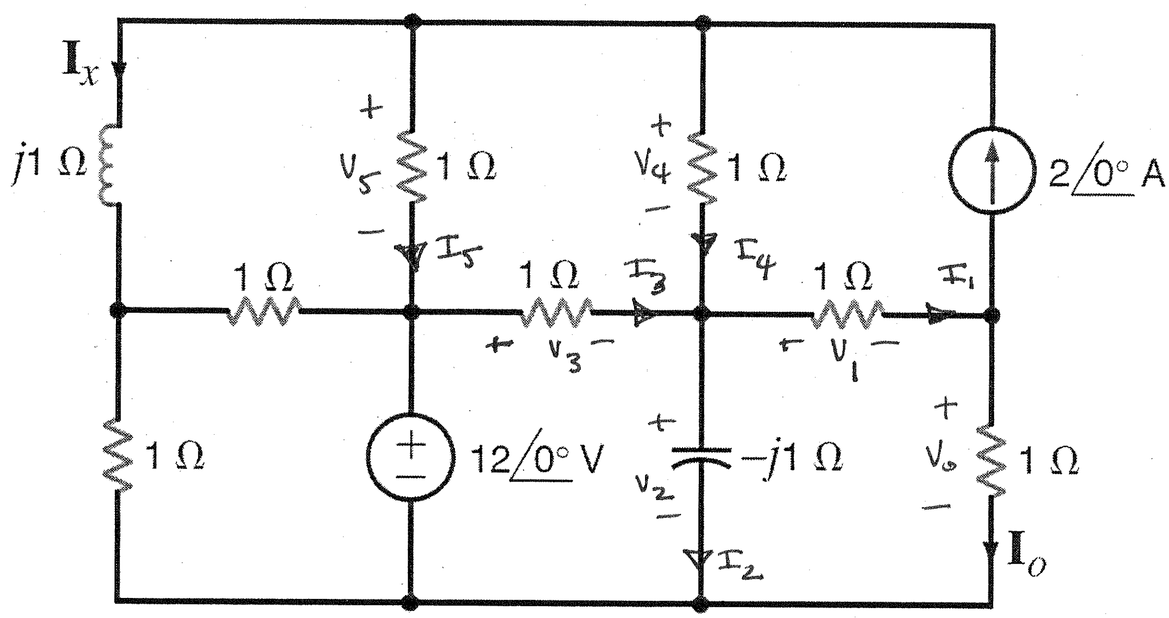


Figure P8.44

SOLUTION: $V_o = (1)I_o = 4\angle 0^\circ V$

$I_1 = 2 + I_o = 6\angle 0^\circ A$

$V_1 = 1 I_1 = 6\angle 0^\circ V$

$V_2 = V_1 + V_o = 10\angle 0^\circ V$

$I_2 = V_2 / -j1 = j10 A$

$V_3 = 12\angle 0 - V_2 = 2\angle 0 V$

$I_3 = V_3 / 1 = 2\angle 0 A$

$I_4 = I_1 + I_2 - I_3 = 4 + j10 A$

$V_4 = (1) I_4 = 4 + j10 A$

$V_5 = V_4 - V_3 = 2 + j10 V$

$I_5 = V_5 / 1 = 2 + j10 A$

$I_x = 2 - I_4 - I_5$

$\boxed{I_x = -4 - j20 A}$

8.45 In the network in Fig. P8.45, $\mathbf{V}_o$ is known to be $4\,\underline{/45°}$ V. Find $\mathbf{Z}$.

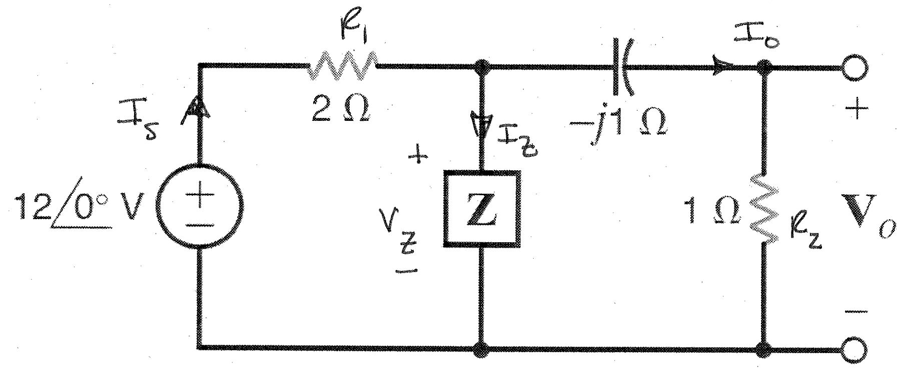


Figure P8.45

SOLUTION:

$$I_o = \frac{V_o}{R_2} = 4\,\angle 45°\text{ A} \qquad V_Z = I_o(R_2 - j1) = 4\sqrt{2}\,\angle 0°\text{ V}$$

$$I_S = \frac{12\angle 0° - V_Z}{R_1} = 3.17\angle 0°\text{ A} \qquad I_Z = I_S - I_o = 2.85\,\angle -83.1°\text{ A}$$

$$\boxed{Z = \frac{V_Z}{I_Z} = 1.98\,\angle 83.1°\ \Omega}$$

8.46 In the network in Fig. P8.46, $\mathbf{V}_1 = 2\underline{/45^\circ}$ V. Find $\mathbf{Z}$.

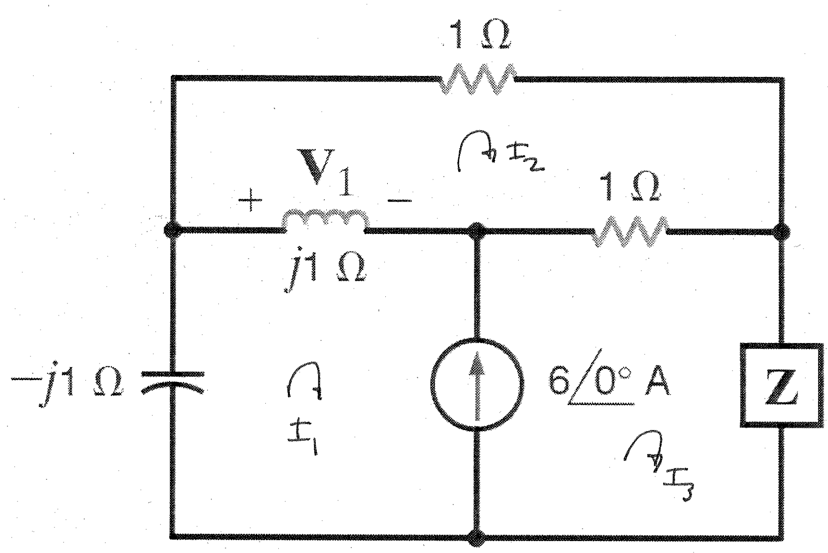


Figure P8.46

SOLUTION:

① $I_3 - I_1 = 6\underline{/0^\circ}\,A$ ② $I_3(1+Z) - I_2(1+j1) = 0$

③ $I_2(2+j1) - j1\,I_1 - I_3 = 0$ ④ $\dfrac{V_1}{j1} = 2\underline{/-45^\circ} = I_1 - I_2$

Solve for Z, yields

$$\boxed{\begin{gathered} Z = -0.508 + j\,0.586\ \Omega \\ Z = 0.776\underline{/130.9^\circ}\ \Omega \end{gathered}}$$

8.47 Find $\mathbf{V}_o$ in the circuit in Fig. P8.47.

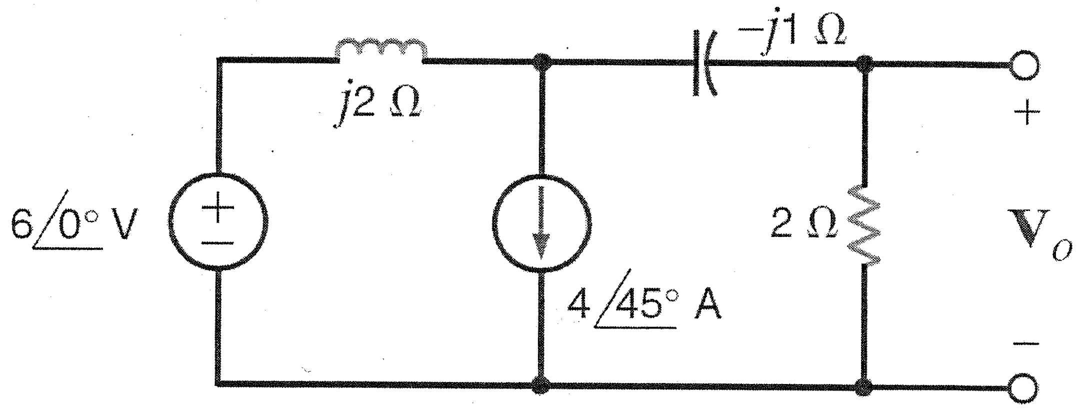
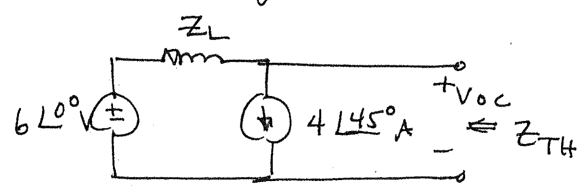


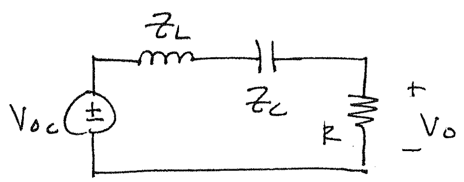
Figure P8.47

SOLUTION:

Thevenin eq.:



$Z_{TH} = Z_L = j2$



Superposition

$V_{oc} = 6\angle 0^\circ - 4\angle 45^\circ\,(j2)$

$V_{oc} = 13.0\angle -25.9^\circ$ V

$$V_o = \frac{V_{oc}\,(2)}{2 + j2 - j1}$$

$$\boxed{V_o = 11.6\angle -52.5^\circ\ \text{V}}$$

8.48 Using nodal analysis, find $\mathbf{I}_o$ in the circuit in Fig. P8.48.
CS

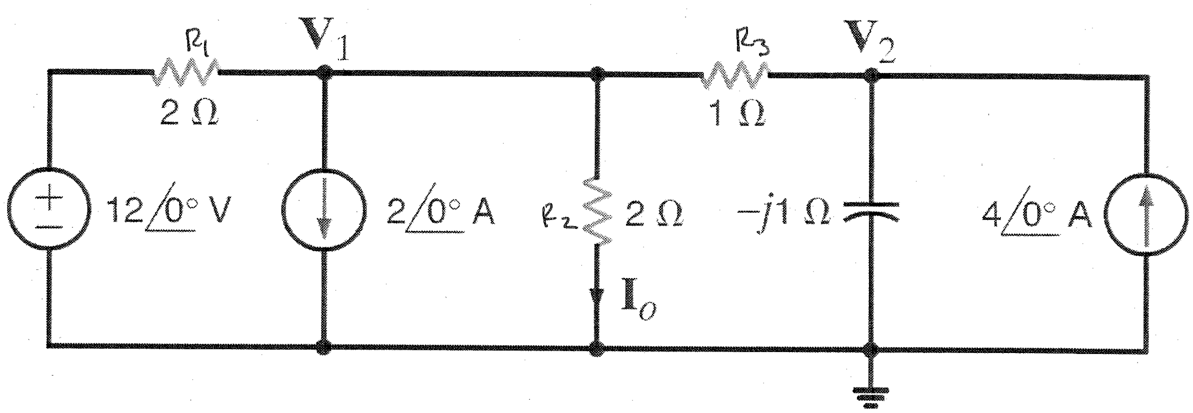


Figure P8.48

SOLUTION:

$$\frac{V_1-12}{2} + 2 + \frac{V_1}{2} + \frac{V_1-V_2}{1} = 0 \Rightarrow 2V_1 - V_2 = 4$$

$$\frac{V_2-V_1}{1} + \frac{V_2}{-j1} = 4 \Rightarrow -V_1 + V_2(1+j1) = 4$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 1+j1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$I_o = \frac{V_1}{2} \qquad \boxed{I_o = 2\angle -36.9^\circ A} \Leftarrow V_1 = 4\angle -36.9^\circ V$$

8.49 Determine $\mathbf{V}_o$ in the circuit in Fig. P8.49.

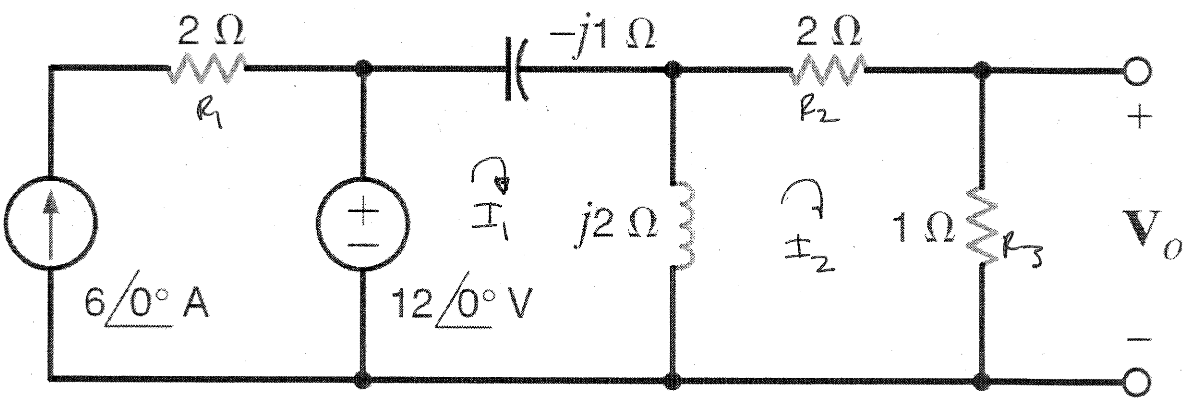


Figure P8.49

SOLUTION:

① $12 = I_1(j1) - j2 I_2$ ② $-j2 I_1 + I_2(3+j2) = 0$

$$\begin{bmatrix} j1 & -j2 \\ -j2 & 3+j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow I_2 = 6.66\angle 33.67° \text{ A}$$

$V_o = R_3 I_2$ $\boxed{V_o = 6.66\angle 33.67° \text{ V}}$

8.50 Using nodal analysis, find $\mathbf{I}_o$ in the circuit in Fig. P8.50.

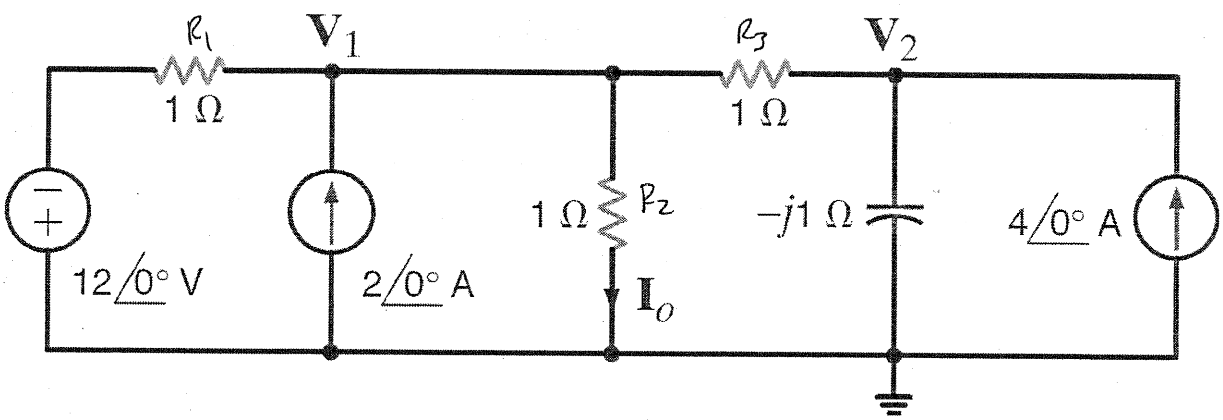


Figure P8.50

SOLUTION:

① $$\frac{V_1+12}{1} + \frac{V_1}{1} + \frac{V_1 - V_2}{1} = 2\angle 0^\circ \qquad I_o = \frac{V_1}{R_2}$$

or, $$3V_1 - V_2 = -10$$

② $$\frac{V_2 - V_1}{1} + \frac{V_2}{-j1} = 4 \Rightarrow -V_1 + V_2(1+j1) = 4$$

$$\begin{bmatrix} 3 & -1 \\ -1 & 1+j1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -10 \\ 4 \end{bmatrix} \rightarrow V_1 = 3.23\angle -177^\circ \text{ V}$$

$$\boxed{I_o = 3.23\angle -177^\circ \text{ A}}$$

8.51 Use nodal analysis to find $\mathbf{I}_o$ in the circuit in Fig. P8.57.

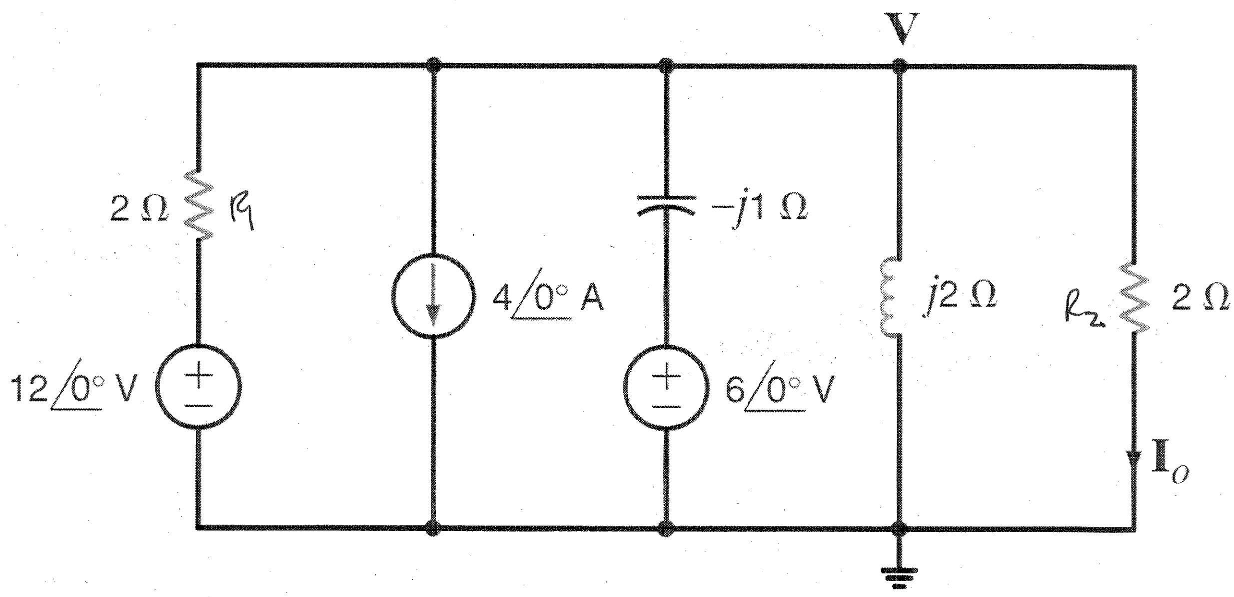


Figure P8.51

SOLUTION:

$$\frac{V-12}{2} + 4 + \frac{V-6}{-j1} + \frac{V}{j2} + \frac{V}{2} = 0 \qquad \frac{V}{R_2} = I_o$$

$$V\left(1 + j\tfrac{1}{2}\right) = 2 + j6 \qquad V = \frac{4 + j12}{2 + j1} = 5.66\angle 45^\circ \text{ V}$$

$$\boxed{I_o = 2.83\angle 45^\circ \text{ A}}$$

8.52 Find $\mathbf{V}_o$ in the network in Fig. P8.52. CS

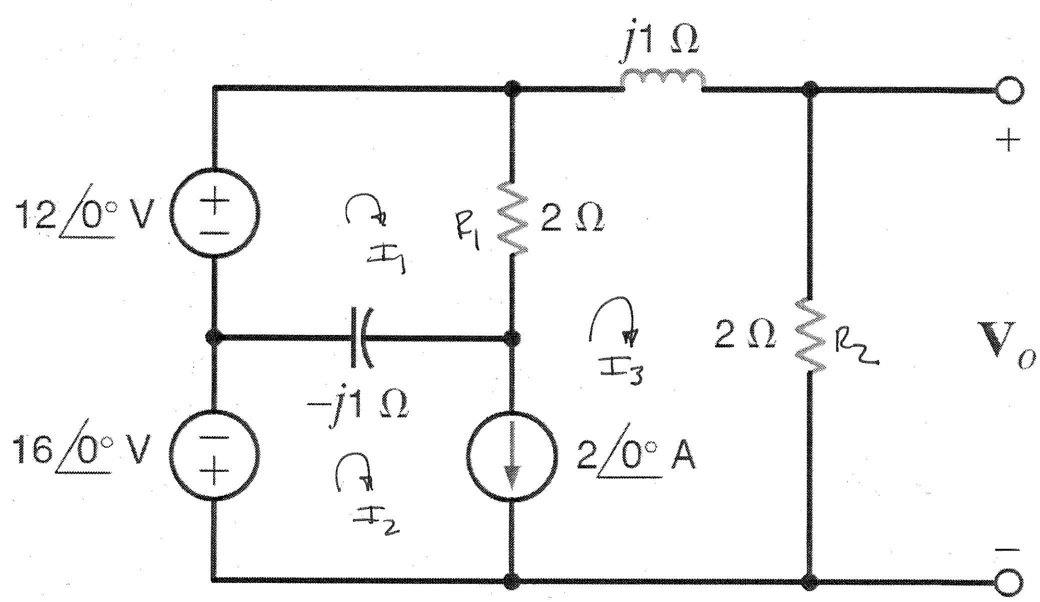


Figure P8.52

SOLUTION:

① $12 = I_1(2-j1) + j1 I_2 - 2I_3$

② $-16+12 = I_3(2+j1) = -4 \Rightarrow I_3 = \frac{-4}{2+j1} = 1.79\angle 153.4° A$

$V_o = R_2 I_3$ $\boxed{V_o = 3.58\angle 153.4°\ V}$

8.53 Find $\mathbf{V}_o$ in the network in Fig. P8.53 using nodal analysis. PSV

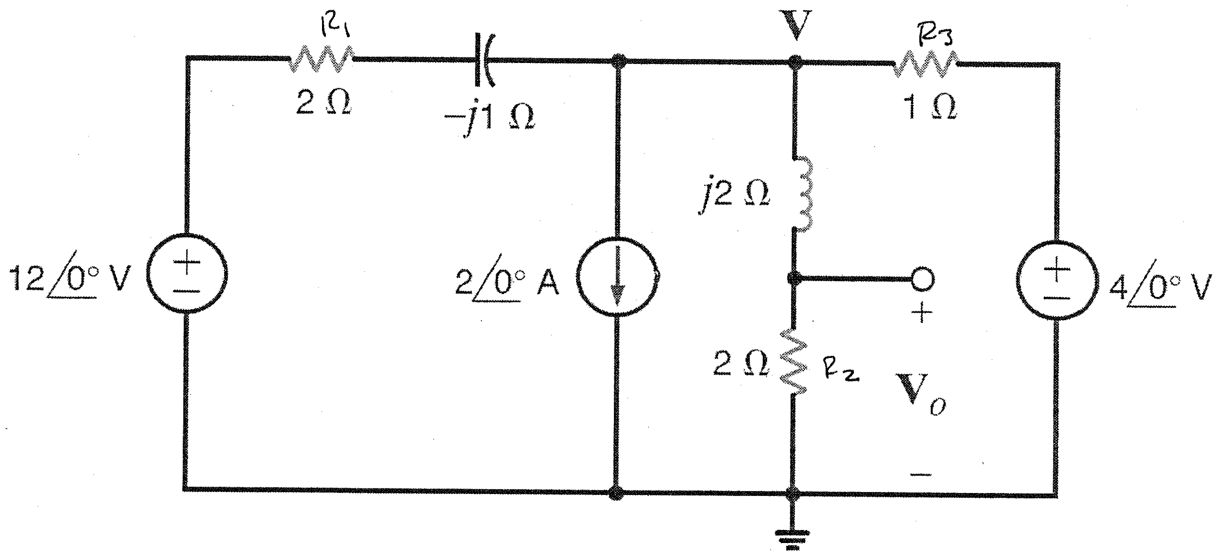


Figure P8.53

SOLUTION:

$$\frac{V-12}{2-j1} + 2 + \frac{V}{2+j2} + \frac{V-4}{1} = 0 \qquad V_o = \frac{V(2)}{2+j2}$$

$$\boxed{V_o = 3.09 \angle -23.8^\circ \text{ V}}$$

8.54 Find $\mathbf{I}_o$ in the circuit in Fig. P8.54 using nodal analysis.

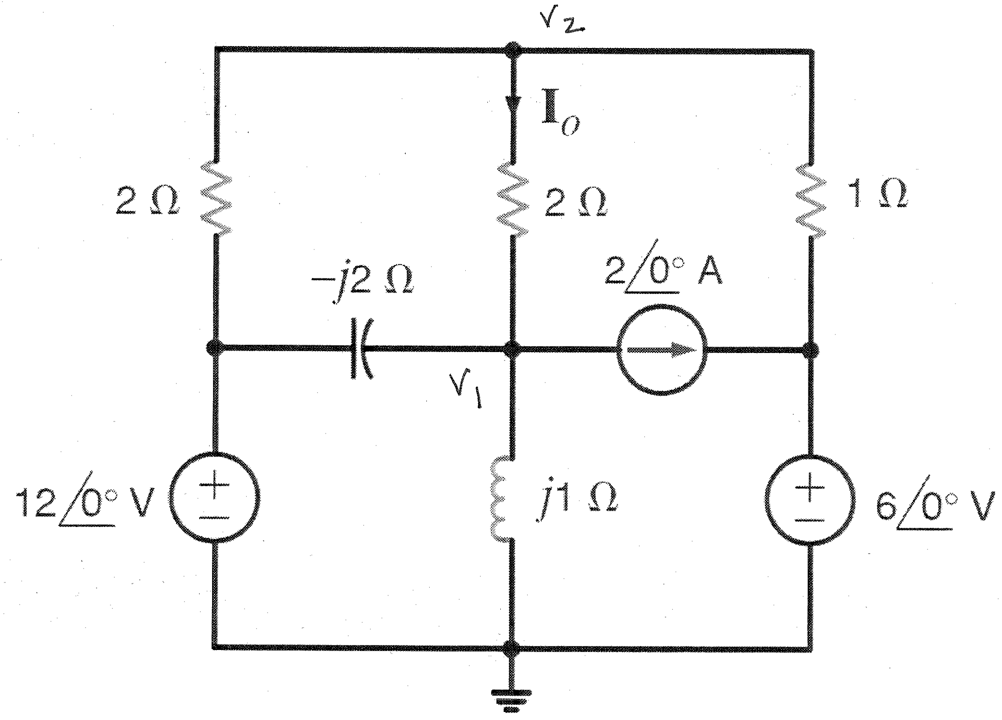


Figure P8.54

SOLUTION:

@ V_1: $\dfrac{V_1-12}{-j2}+\dfrac{V_1}{j1}+\dfrac{V_1-V_2}{2}=-2 \Rightarrow V_1(1-j1)-V_2=-4+j12$

@ V_2: $\dfrac{V_2-12}{2}+\dfrac{V_2-V_1}{2}+\dfrac{V_2-6}{1}=0 \Rightarrow -\dfrac{V_1}{2}+2V_2=12$

$$\begin{bmatrix} 1-j1 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} -4+j12 \\ 24 \end{bmatrix} \qquad I_o=\frac{V_2-V_1}{2}$$

$V_1=-6.72+j7.04\ V$
$V_2=4.32+j1.76V$

$$\boxed{I_o=6.12\angle{-25.5^\circ}\ A}$$

8.55 Use the supernode technique to find $\mathbf{I}_o$ in the circuit in Fig. P8.55.

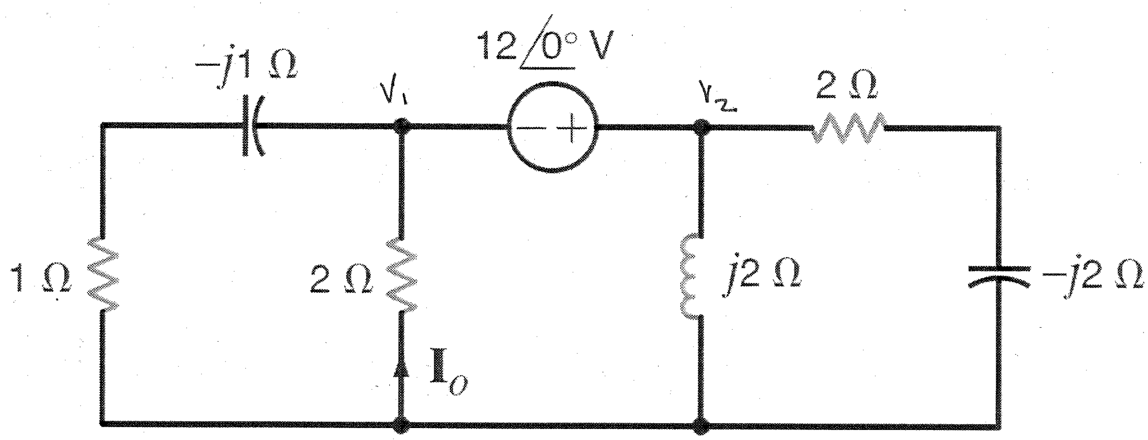


Figure P8.55

SOLUTION:

① $$\frac{V_1}{1-j1} + \frac{V_1}{2} + \frac{V_2}{j2} + \frac{V_2}{2-j2} = 0 \Rightarrow V_1[8+j4] + V_2[2-j2] = 0$$

② $$V_2 - V_1 = 12\angle 0^\circ$$

$$\begin{bmatrix} 8+j4 & 2-j2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 12\angle 0^\circ \end{bmatrix} \Rightarrow V_1 = 3.33\angle 123.7^\circ \text{ V}$$

$$I_o = -\frac{V_1}{2}$$

$$\boxed{I_o = 1.67\angle -56.3^\circ \text{ A}}$$

8.56 Find $\mathbf{I}_o$ in the network in Fig. P8.56.

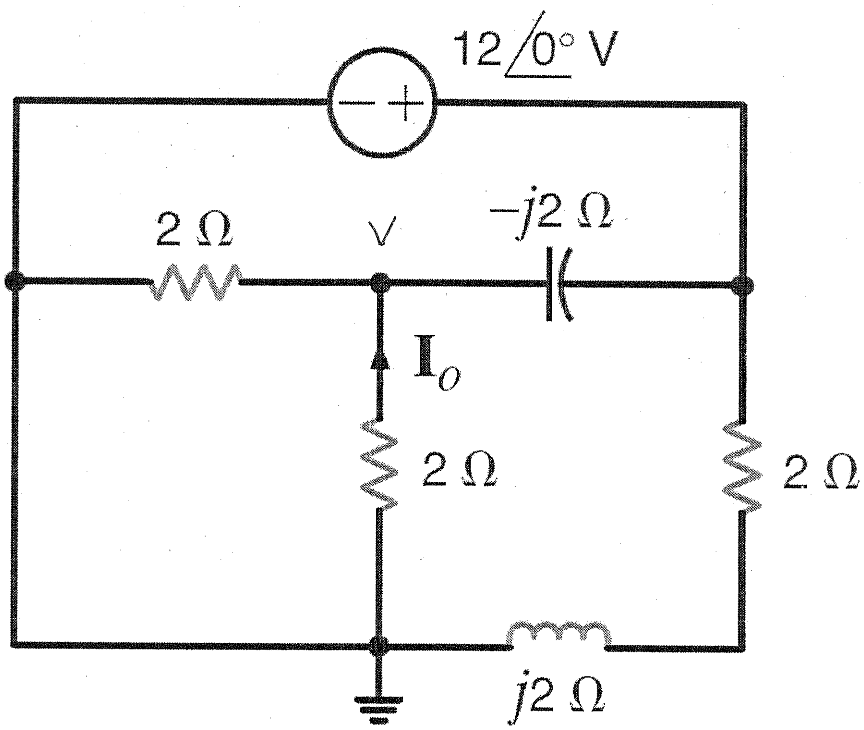


Figure P8.56

SOLUTION:

$$\frac{V}{2} + \frac{V}{2} + \frac{V - 12\angle 0}{-j2} = 0 \Rightarrow V\left(1 + j/2\right) = +j6 \Rightarrow V = 5.37\angle 63.4^\circ \text{ V}$$

$I_o = -V/2$

$$\boxed{I_o = 2.69\angle -167^\circ \text{ A}}$$

8.57 Find $\mathbf{V}_o$ in the network in Fig. P8.57.

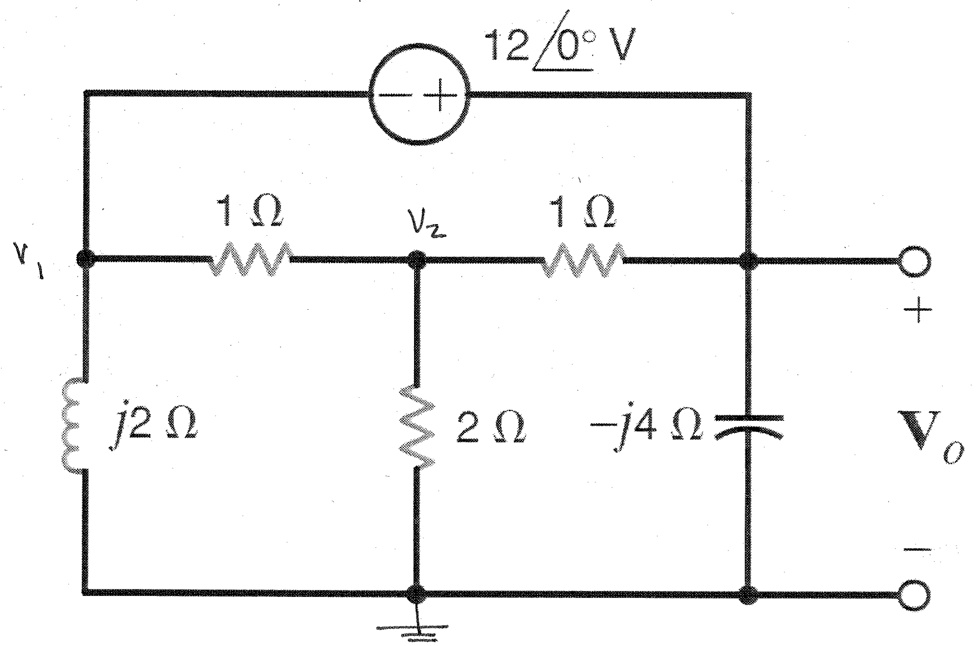


Figure P8.57

SOLUTION:

Nodal: @ V_2: $\frac{V_2 - V_1}{1} + \frac{V_2 - V_o}{1} + \frac{V_2}{2} = 0 \Rightarrow -V_1 + 2.5V_2 - V_o = 0$

@ GND: $\frac{V_1}{j2} + \frac{V_2}{2} + \frac{V_o}{-j4} = 0 \Rightarrow 2V_1 + j2V_2 - V_o = 0$

and, $V_o - V_1 = 12\angle 0°\ \text{V}$

$$\begin{bmatrix} -1 & 2.5 & -1 \\ 2 & j2 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow \boxed{V_o = 13.7\angle -36.2°\ \text{V}}$$

8.58 Use nodal analysis to find $\mathbf{V}_o$ in the circuit in Fig. P8.58.

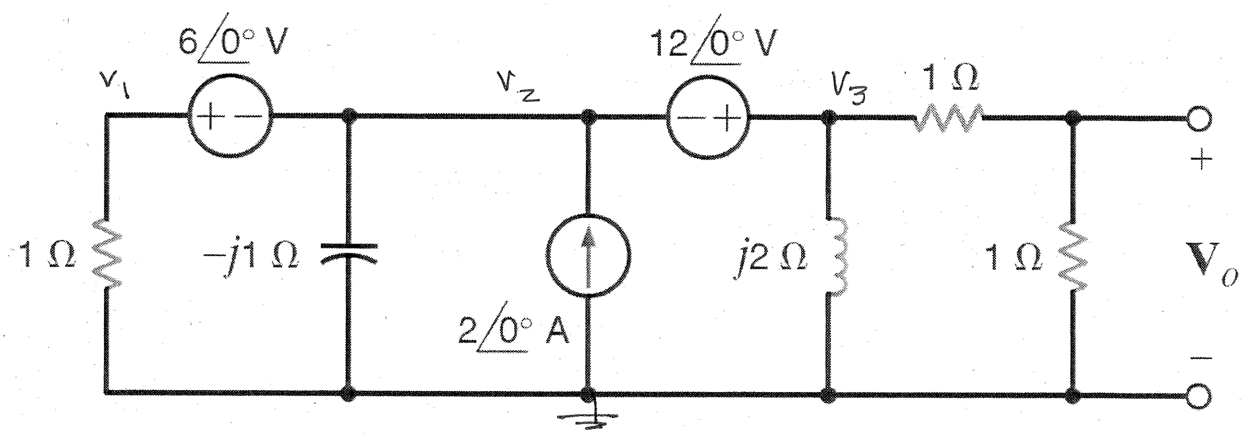


Figure P8.58

SOLUTION:

$V_1 - V_2 = 6\angle 0^\circ$ $\quad V_3 - V_2 = 12\angle 0^\circ$ $\quad V_3 - V_o = V_o \Rightarrow V_3 = 2V_o$

at GND NODE: $\frac{V_1}{1} + \frac{V_2}{-j1} + \frac{V_3}{j2} + \frac{V_o}{1} = 2\angle 0^\circ \Rightarrow V_1 + jV_2 - \frac{jV_3}{2} + V_o = 2$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 \\ 1 & j1 & -j/2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_o \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 0 \\ 2 \end{bmatrix} \Rightarrow \boxed{V_o = 4.56\angle 37.9^\circ \text{ V}}$$

8.59 The low-frequency equivalent circuit for a common-emitter transistor amplifier is shown in Fig. P8.59. Compute the voltage gain $\mathbf{V}_o/\mathbf{V}_S$.

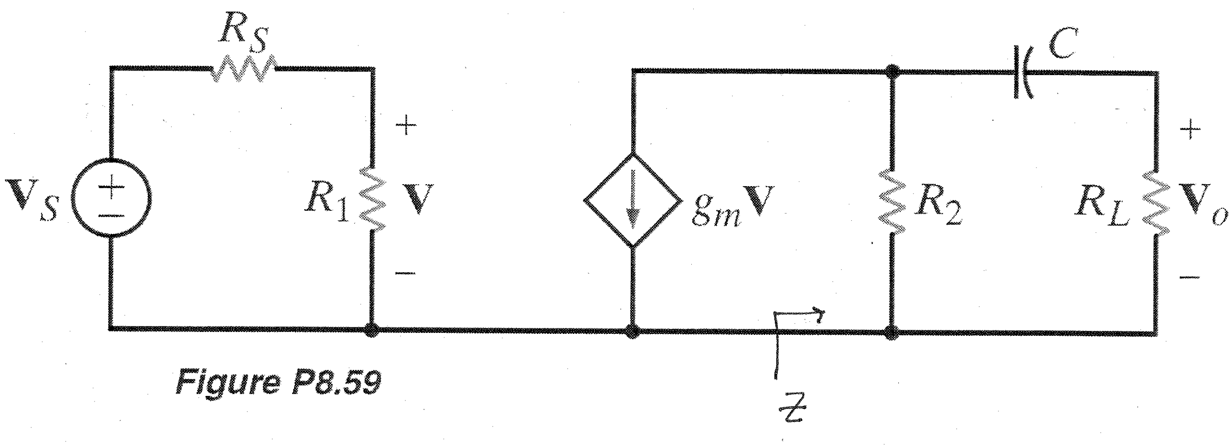


Figure P8.59

SOLUTION:

$$\frac{V}{V_S} = \frac{R_1}{R_1 + R_S} \qquad V_o = -g_m V Z$$

$$Z = \frac{R_2(R_L + Z_C)}{R_2 + R_L + Z_C} \qquad \frac{V_o}{V_s} = -g_m\left(\frac{R_1 R_2}{R_1 + R_S}\right)\left(\frac{j\omega C R_L + 1}{j\omega C(R_L + R_2) + 1}\right)$$

$$\boxed{\frac{V_o}{V_S} = -g_m R_2\left(\frac{R_1}{R_1 + R_S}\right)\left[\frac{j\omega C R_L + 1}{j\omega C(R_L + R_2) + 1}\right]}$$

8.60 Use nodal analysis to find $\mathbf{V}_o$ in the circuit in Fig. P8.60.
PSV

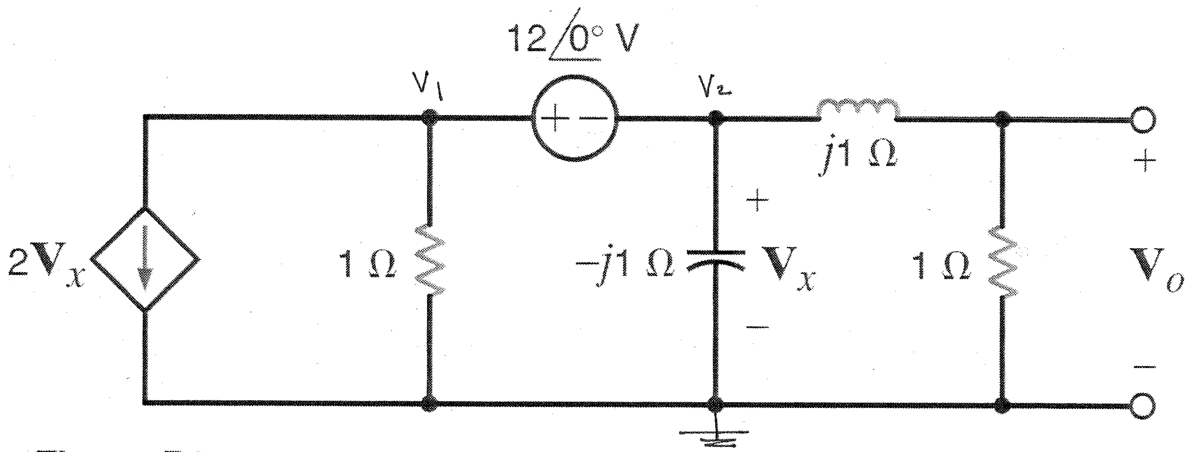


Figure P8.60

SOLUTION:

$V_1 - V_2 = 12\angle 0°\ V$

@ V_o: $\frac{V_2 - V_o}{j1} = \frac{V_o}{1} \Rightarrow V_2 - V_o(1 + j1) = 0$

@ GND: $2V_x + \frac{V_1}{1} + \frac{V_2}{-j1} + \frac{V_o}{1} = 0$ where $V_x = V_2$

so

$$V_1 + V_2(2 + j1) + V_o = 0$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1-j1 \\ 1 & 2+j1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{V_o = 2.4\angle 126.9°\ V}$$

8.61 Find the voltage across the inductor in the circuit shown in Fig. P8.61 using nodal analysis.

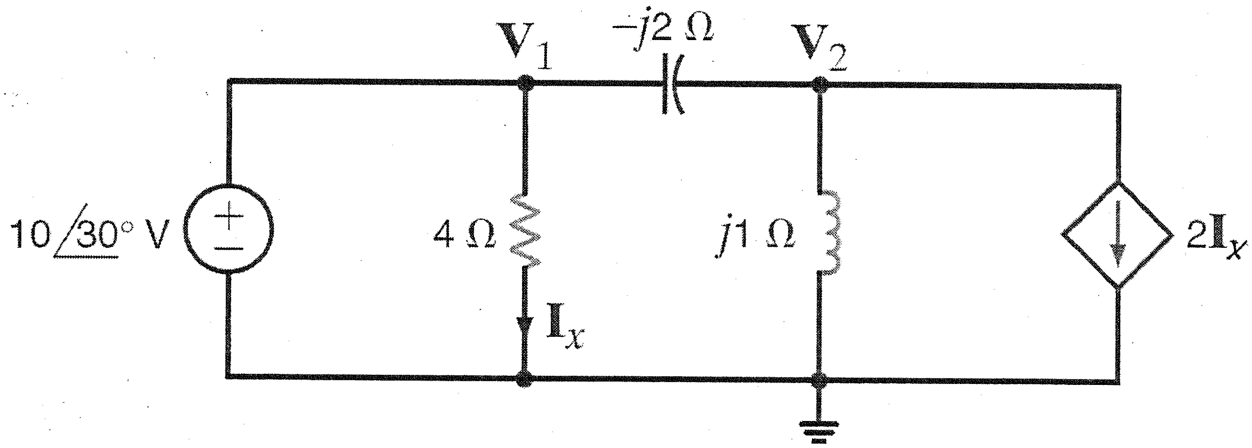


Figure P8.61

SOLUTION:

$V_1 = 10\angle 30^\circ$ & $\dfrac{-V_1 + V_2}{-j2} + \dfrac{V_2}{j1} + 2I_x = 0$ where $I_x = \dfrac{V_1}{4}$

yields; $V_1(1+j1) + V_2 = 0$

$$\begin{bmatrix} 1 & 0 \\ 1+j1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 30^\circ \\ 0 \end{bmatrix} \Rightarrow \boxed{V_2 = 14.1\angle -105^\circ \text{ V}}$$

8.62 Use nodal analysis to find $\mathbf{I}_o$ in the circuit in Fig. P8.62.

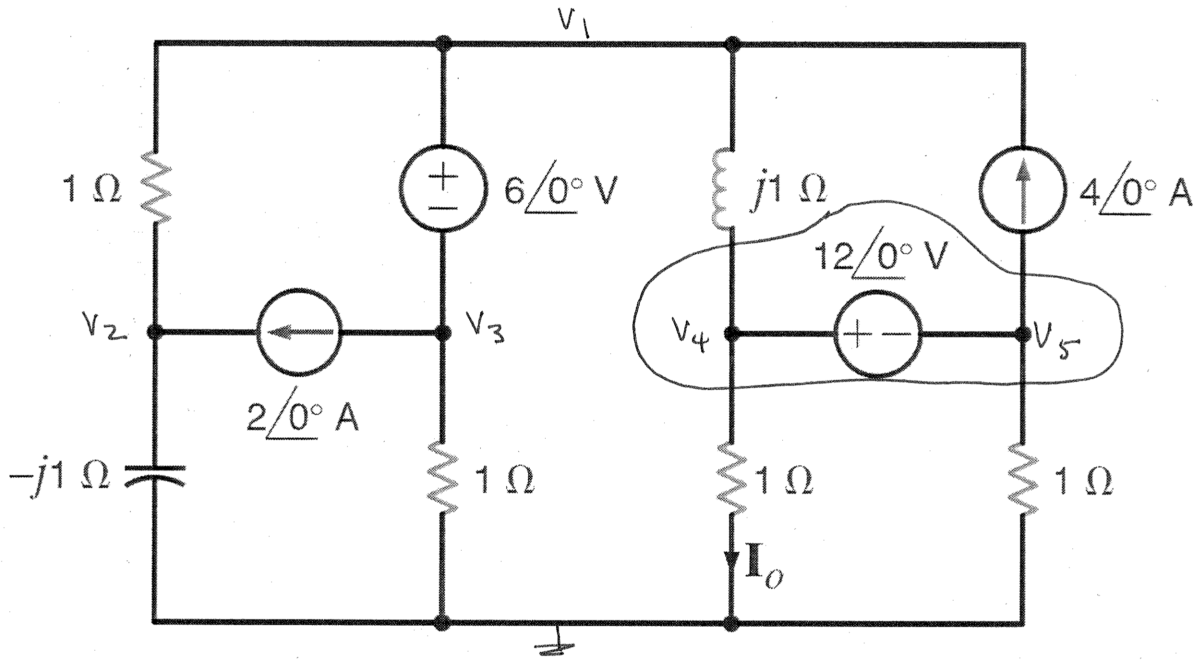


Figure P8.62

SOLUTION:

$V_1 - V_3 = 6\angle 0°$ $\quad V_4 - V_5 = 12\angle 0°$ $\quad I_o = V_4/1 = V_4$

@ V_2: $\quad \dfrac{V_2 - V_1}{1} + \dfrac{V_2}{-j1} = 2 \Rightarrow -V_1 + V_2(1+j1) = 2$

@ GND: $\quad \dfrac{V_2}{-j1} + \dfrac{V_3}{1} + \dfrac{V_4}{1} + \dfrac{V_5}{1} = 0 \Rightarrow jV_2 + V_3 + V_4 + V_5 = 0$

@ super node: $\quad \dfrac{V_4 - V_1}{j1} + \dfrac{V_4}{1} + \dfrac{V_5}{1} = -4 \Rightarrow jV_1 + V_4(1-j1) + V_5 = -4$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 1+j1 & 0 & 0 & 0 \\ 0 & j1 & 1 & 1 & 1 \\ j1 & 0 & 0 & 1-j1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 2 \\ 0 \\ -4 \end{bmatrix} \Rightarrow$$

$V_4 = 3.96\angle -14.2°$ V

$$\boxed{I_o = 3.96\angle -14.2 \text{ A}}$$

8.63 Use mesh analysis to find $\mathbf{V}_o$ in the circuit shown in Fig. P8.63.

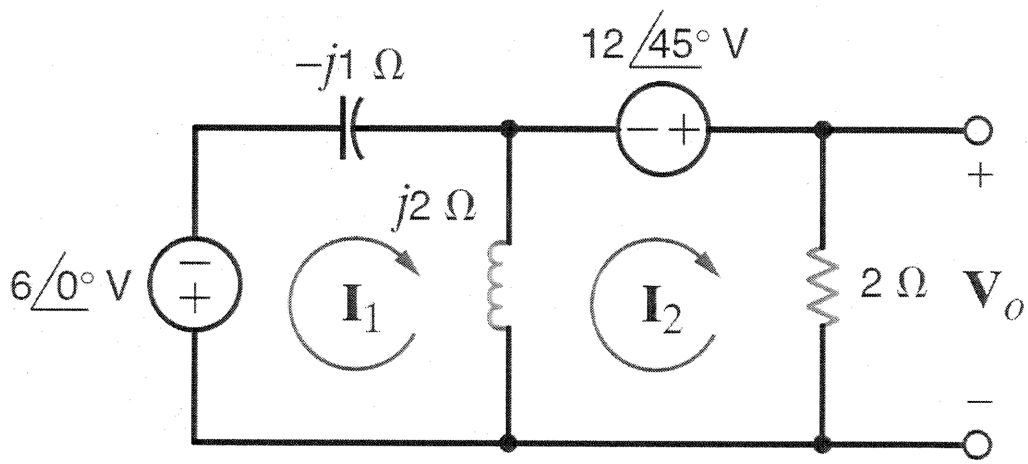


Figure P8.63

SOLUTION:

$-6\angle 0^\circ = I_1(j1) - I_2(j2) \Rightarrow \quad jI_1 - j2I_2 = -6\angle 0^\circ$

$12\angle 45^\circ = I_2(2+j2) - j2I_1 \Rightarrow \quad -j2I_1 + (2+j2)I_2 = 12\angle 45^\circ$

$$\begin{bmatrix} j1 & -j2 \\ -j2 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -6\angle 0 \\ 12\angle 45^\circ \end{bmatrix} \Rightarrow \quad I_2 = 3.25\angle 157.5^\circ \text{ A}$$

$V_o = 2I_2 \quad \boxed{V_o = 6.50\angle 157.5^\circ \text{ V}}$

8.64 Use mesh analysis to find $\mathbf{V}_o$ in the circuit shown in Fig. P8.64.

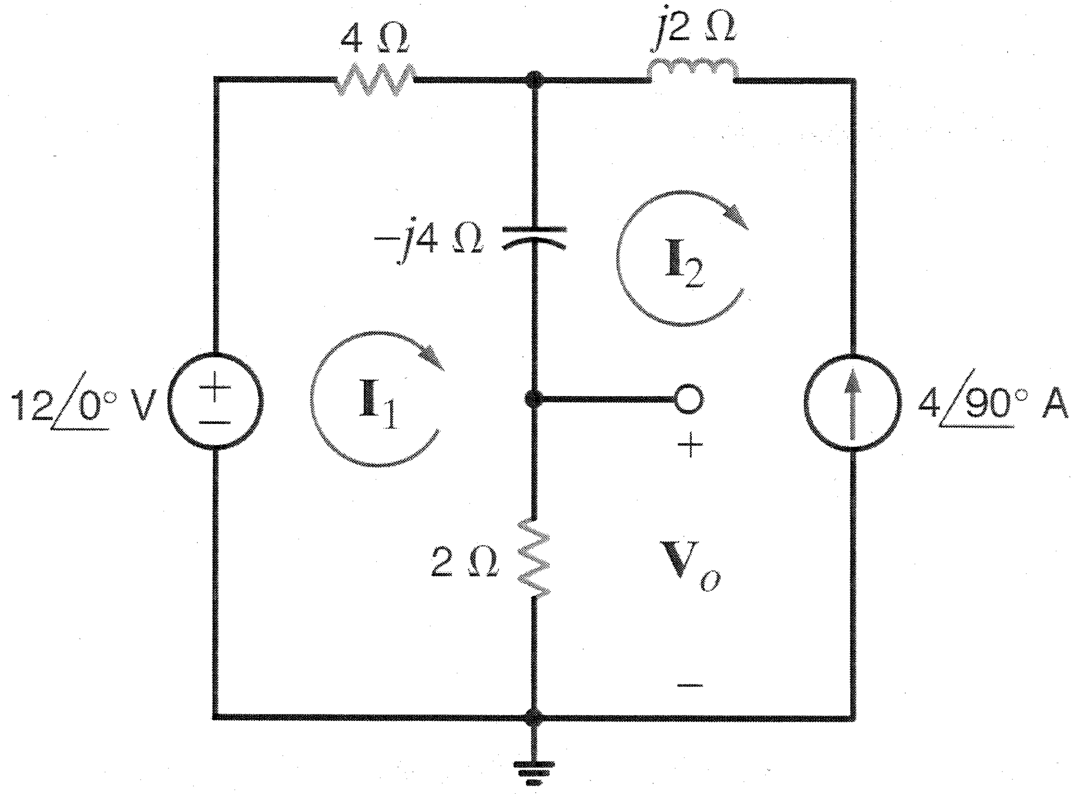


Figure P8.64

SOLUTION:

$12\angle 0 = I_1(6-j4) - I_2(2-j4) \Rightarrow I_1(6-j4) + I_2(-2+j4) = 12+j0$

$I_2 = -4\angle 90^\circ = -j4\text{ A} \qquad V_o = 2(I_1 - I_2)$

$$\begin{bmatrix} 6-j4 & -2+j4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -j4 \end{bmatrix} \Rightarrow \begin{cases} I_1 = 0.154 - j1.23\text{ A} \\ I_2 = -j4\text{ A} \end{cases}$$

$$\boxed{V_o = 5.55\angle 86.8^\circ\text{ V}}$$

8.65 Find $\mathbf{V}_o$ in the circuit in Fig. P8.65 using mesh analysis.

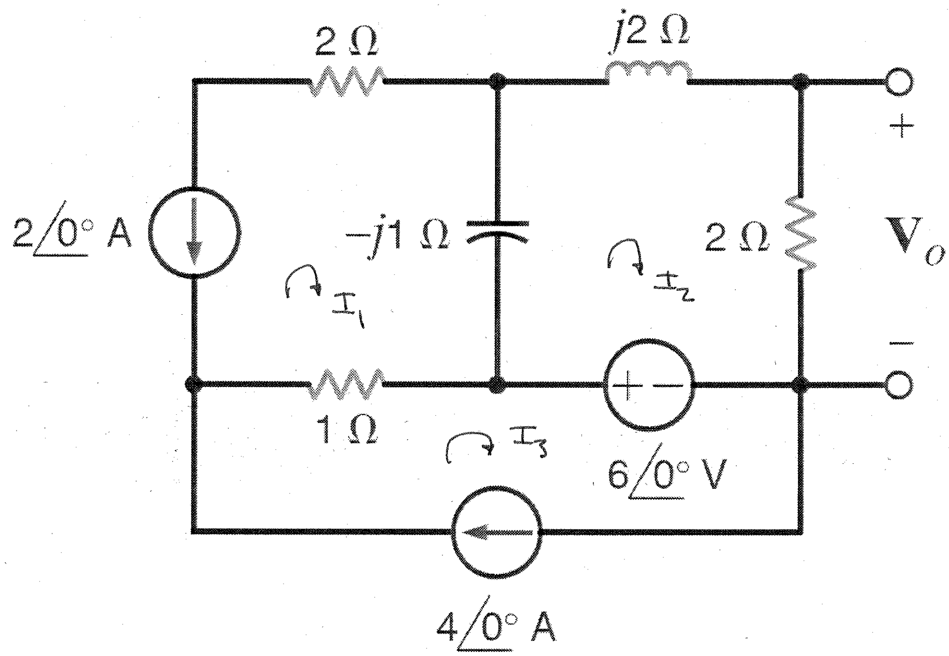


Figure P8.65

SOLUTION:

$I_1 = -2\angle 0° \text{ A}$ $\quad I_3 = 4\angle 0° \text{ A}$ $\quad 6\angle 0 = jI_1 + I_2(2 + j1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ j1 & 2+j1 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -2+j0 \\ 4+j0 \\ 6+j0 \end{bmatrix} \Rightarrow I_2 = 2.83\angle -8.13° \text{ A}$$

$V_o = 2I_2$ $\quad \boxed{V_o = 5.66\angle -8.13° \text{ V}}$

8.66 Use mesh analysis to find $\mathbf{V}_o$ in the circuit in Fig. P8.66.

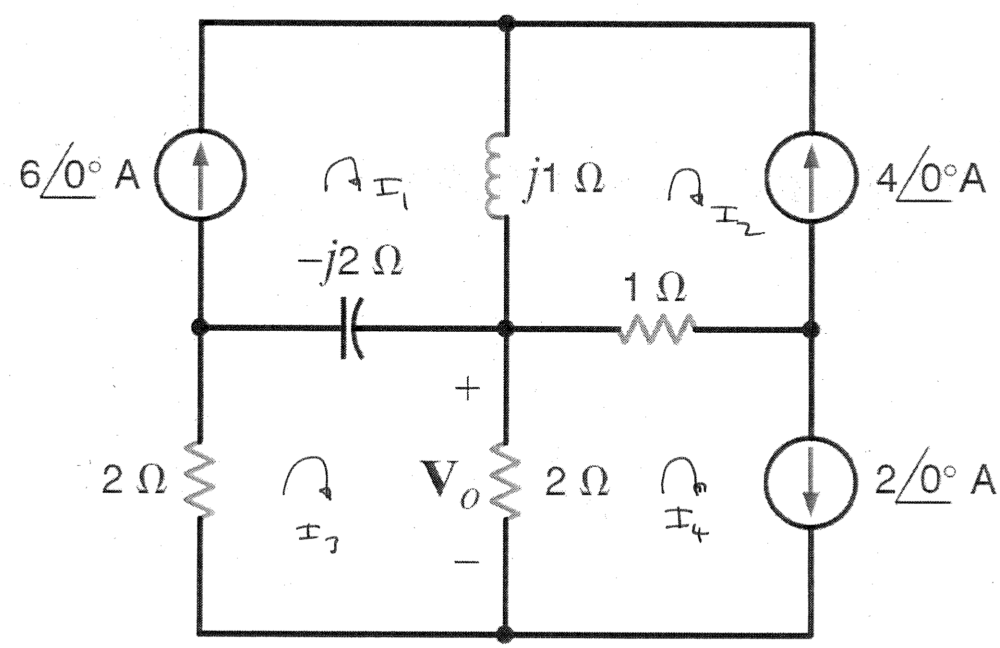


Figure P8.66

SOLUTION:

$I_1 = 6\angle 0° \text{ A}$ $\quad I_2 = -4\angle 0° \text{ A}$ $\quad I_4 = 2\angle 0° \text{ A}$

$j2 I_1 + I_3(4 - j2) - 2I_4 = 0$ $\quad V_o = 2(I_3 - I_4)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ j2 & 0 & 4-j2 & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 2 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} I_3 = 2 - j2 \text{ A} \\ I_4 = 2 \text{ A} \end{array}$$

$$\boxed{V_o = 4\angle -90° \text{ V}}$$

8.67 Using loop analysis and MATLAB, find $\mathbf{I}_o$ in the network in Fig. P8.67. CS

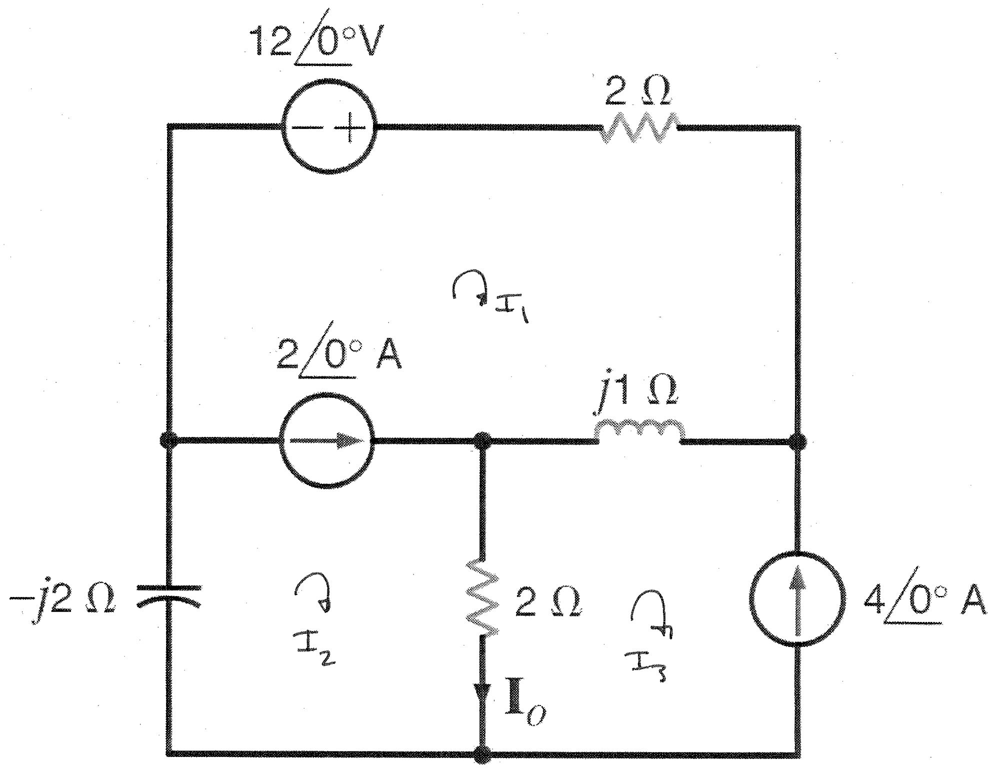


Figure P8.67

SOLUTION:

$I_2 - I_1 = 2\angle 0°\ A \qquad I_3 = -4\angle 0°\ A \qquad 12 = I_1(2+j1) + I_2(2-j2) - I_3(2+j1)$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 2+j1 & 2-j2 & -2-j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 12 \end{bmatrix}$$

MATLAB

```
» Z=[-1 1 0;0 0 1;2+1i 2-2i -2-1i];
» V=[2;-4;12];
» I=inv(z)*v

I =   0     + 0i
      2.0 + 0i
     -4.0
```

$I_o = I_2 - I_3 \qquad \boxed{I_o = 6\angle 0°\ A}$

8.68 Find $\mathbf{V}_o$ in the network in Fig. P8.68.

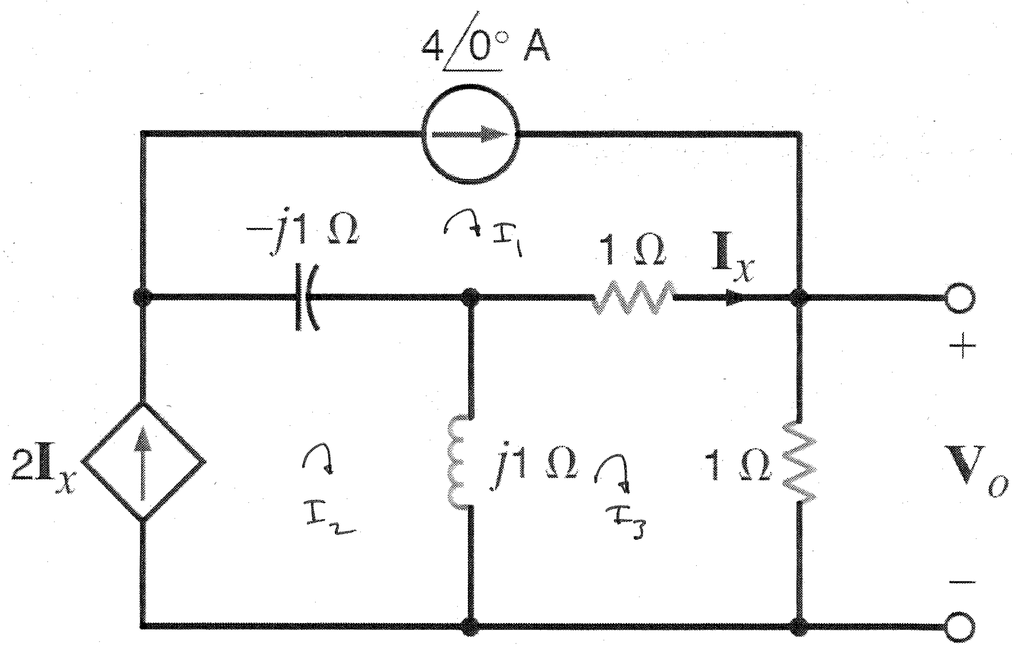


Figure P8.68

SOLUTION:

$I_1 = 4\angle 0° \text{ A} \qquad I_2 = 2I_x = 2I_3 - 2I_1 \Rightarrow +2I_1 + I_2 - 2I_3 = 0$

$-I_1 - jI_2 + I_3(2+j1) = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ -1 & -j1 & 2+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4+j0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow I_3 = 4\angle -36.9° \text{ A}$$

$V_o = (1)I_3 \qquad \boxed{V_o = 4\angle -36.9° \text{ V}}$

8.69 Find $\mathbf{V}_o$ in the network in Fig. P8.69. PSV

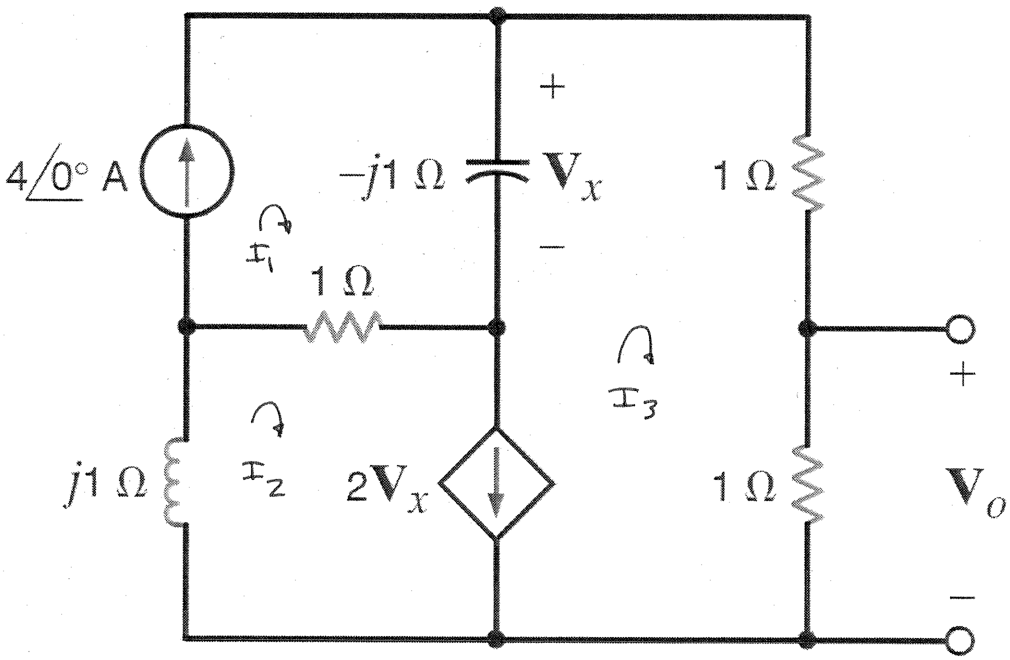


Figure P8.69

SOLUTION:

$I_1 = 4\angle 0°$ A $\quad I_2 - I_3 = 2V_x = 2(-j1)(I_1 - I_3) \Rightarrow j2I_1 + I_2 + I_3(-1-j2) = 0$

and, $\quad jI_2 + (I_2 - I_1)1 - j1(I_3 - I_1) + 2I_3 = 0 \quad$ & $V_o = I_3(1)$

or, $\quad I_1(-1+j1) + I_2(1+j1) + I_3(2-j1) = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ j2 & 1 & -1-j2 \\ -1+j1 & 1+j1 & 2-j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} \Rightarrow I_3 = 2.53\angle 71.6°\ A$$

$$\boxed{V_o = 2.53\angle 71.6°\ V}$$

8.70 Use loop analysis to find $\mathbf{I}_o$ in the network in Fig. P8.70.

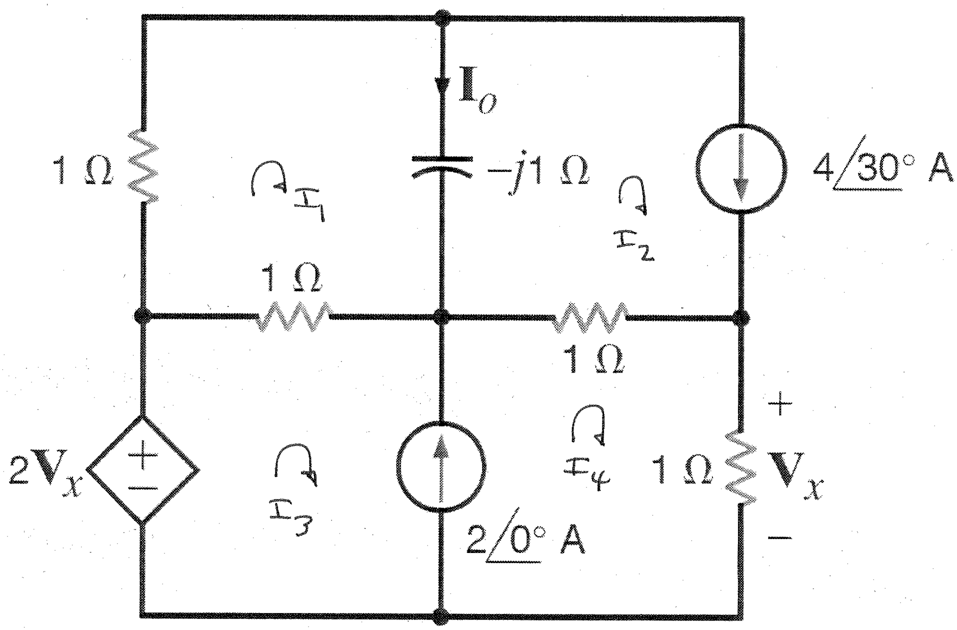


Figure P8.70

SOLUTION: $I_o = I_1 - I_2$

$I_2 = 4\angle 30°\ A$ $\quad I_4 - I_3 = 2\angle 0\ A$ $\quad I_1(2-j1) + jI_2 - I_3 = 0$

$$\left.\begin{array}{l} 2V_x = -I_1 - I_2 + I_3 + 2I_4 \\ \text{and} \quad V_x = (1)\,I_4 \end{array}\right\} \quad -I_1 - I_2 + I_3 = 0$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 2-j1 & j1 & -1 & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 4\angle 30 \\ 2\angle 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} I_1 = 4\angle 30°\ A \\ I_2 = 4\angle 30°\ A \end{array}$$

$$\boxed{I_o = 0\ A}$$

8.71 Use superposition to find $\mathbf{V}_o$ in the network in Fig. P8.71.

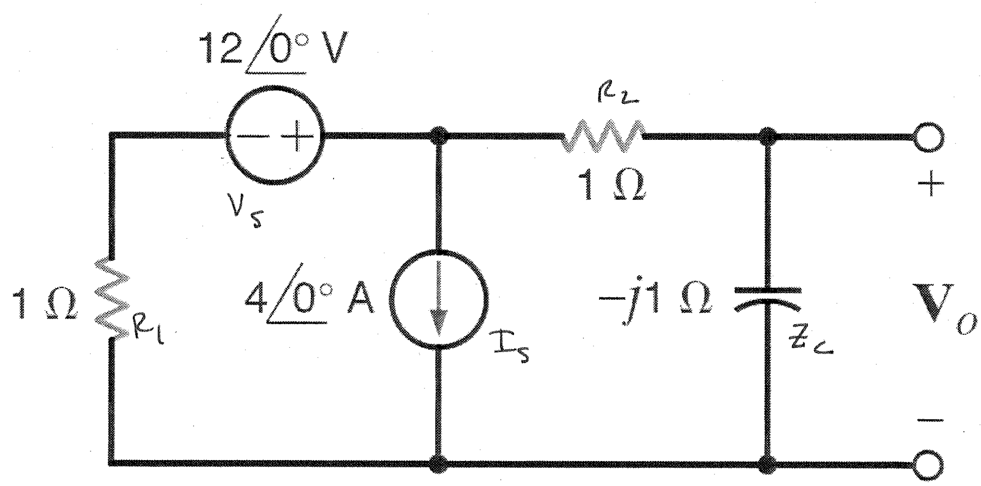
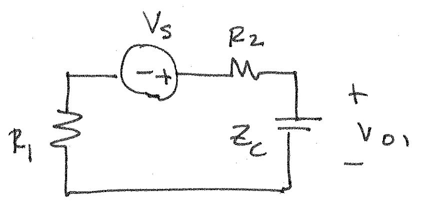
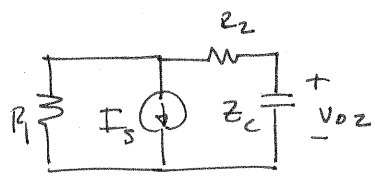


Figure P8.71

SOLUTION:



$$V_{o1} = \frac{V_s Z_C}{R_1 + R_2 + Z_C} \qquad V_{o1} = 2.4 - j4.8\ \text{V}$$



$$V_{o2} = \frac{-I_S R_1}{R_1 + R_2 + Z_C} Z_C \qquad V_{o2} = -0.8 + j1.6\ \text{V}$$

$$V_o = V_{o1} + V_{o2}$$

$$\boxed{V_o = 3.58 \angle -63.4^\circ\ \text{V}}$$

8.72 Using superposition, find $\mathbf{V}_o$ in the circuit in Fig. P8.72.

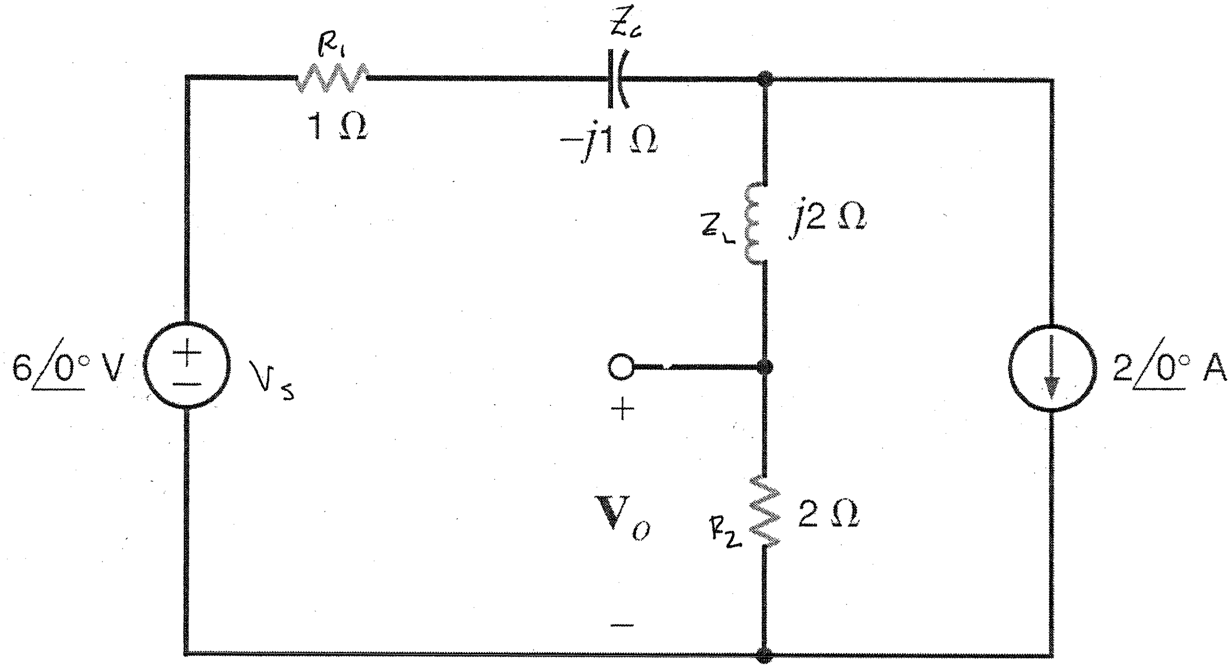
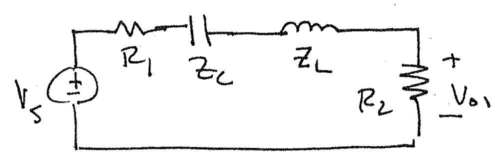


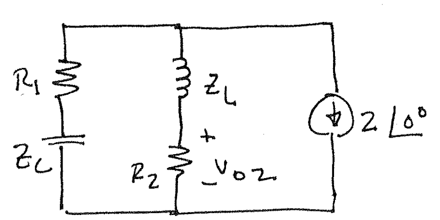
Figure P8.72

SOLUTION:



$$V_{o1} = \frac{V_S R_2}{R_1 + R_2 + Z_L + Z_C}$$

$$V_{o1} = 3.79 \angle -18.4^\circ \text{ V}$$



$$V_{o2} = \frac{-2\,(R_1 + Z_C)}{R_1 + Z_C + R_2 + Z_L} R_2$$

$$V_{o2} = 1.79 \angle 116.6^\circ \text{ V}$$

$$V_o = V_{o1} + V_{o2}$$

$$\boxed{V_o = 2.83 \angle 8.13 \text{ V}}$$

8.73 Find $\mathbf{V}_o$ in the network in Fig. P8.73 using superposition.

CS

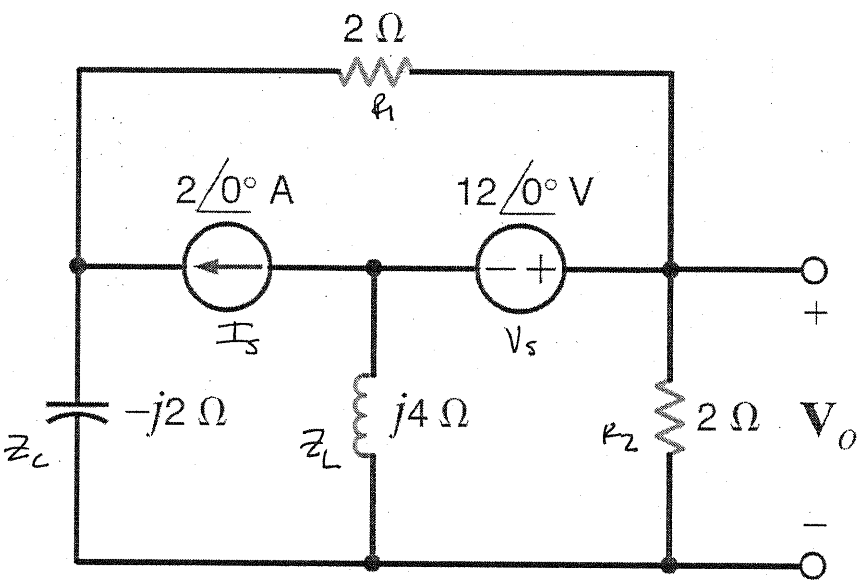
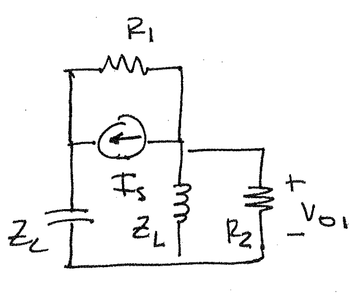


Figure P8.73

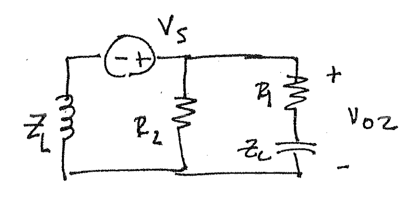
SOLUTION:



$$Z_A = \frac{Z_L R_2}{Z_L + R_2} = \frac{4}{2-j1}$$

$$Z_B = Z_C + Z_A = \frac{2-j2}{2-j1}$$

$$V_{O1} = \frac{I_S R_1}{R_1 + Z_B}\left(\frac{-Z_L R_2}{Z_L + R_2}\right) = -1.33 - j\,1.33 \text{ V}$$



$$Z_x = R_1 + X_C$$

$$Z_y = R_2 Z_x / (R_2 + Z_x)$$

$$V_{O2} = V_s Z_y / (Z_y + Z_L) = 0 - j4$$

$$V_o = V_{o1} + V_{o2}$$

$$\boxed{V_o = 5.50\angle -104^\circ \text{ V}}$$

8.74 Use both superposition and MATLAB to determine $\mathbf{V}_o$ in the circuit in Fig. P8.74.

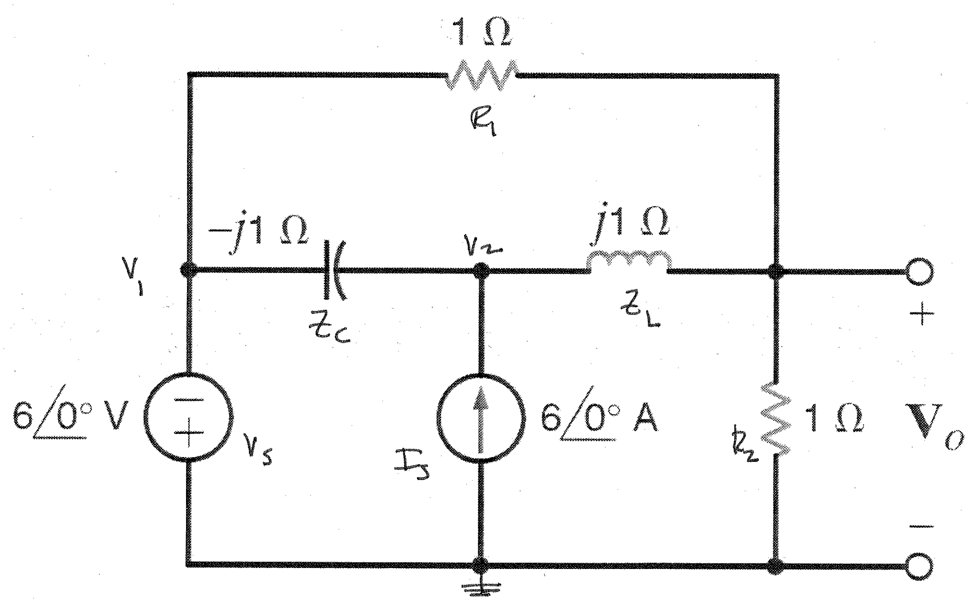
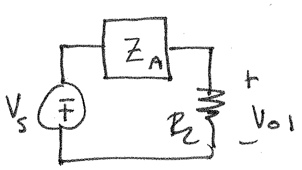


Figure P8.74

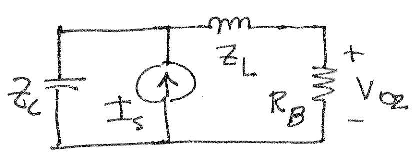
SOLUTION:

Superposition



$Z_A = R_1 \parallel (Z_L + Z_C)$
$= 0$

$V_{o1} = -V_S = -6\angle 0^\circ V$



$R_B = R_1 \parallel R_2 = 1/2\,\Omega$

$V_{o2} = I_S Z_C R_B / (R_B + Z_L + Z_C) = 6\angle -90^\circ V$

MATLAB

$V_1 = -6\angle 0^\circ V$; $\dfrac{V_2 - V_1}{Z_C} + \dfrac{V_2 - V_o}{Z_L} = I_S$

$\dfrac{V_o - V_1}{R_1} + \dfrac{V_o - V_2}{Z_L} + \dfrac{V_o}{R_2} = 0$

$$\begin{bmatrix} -j1 & 0 & j1 \\ -1 & j1 & 2-j1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -6 \end{bmatrix}$$

```
» y=[-1i 0 1i;-1 1i 2-1i;1 0 0];
»  i=[6;0;-6];
» v=inv(y)*i

v =   -6.0000
       6.0000 -12.0000i
      -6.0000 - 6.0000i
```

$$\boxed{V_o = 8.49\angle -135^\circ\ V}$$

8.75 Find $\mathbf{V}_o$ in the network in Fig. P8.75 using superposition.
PSV

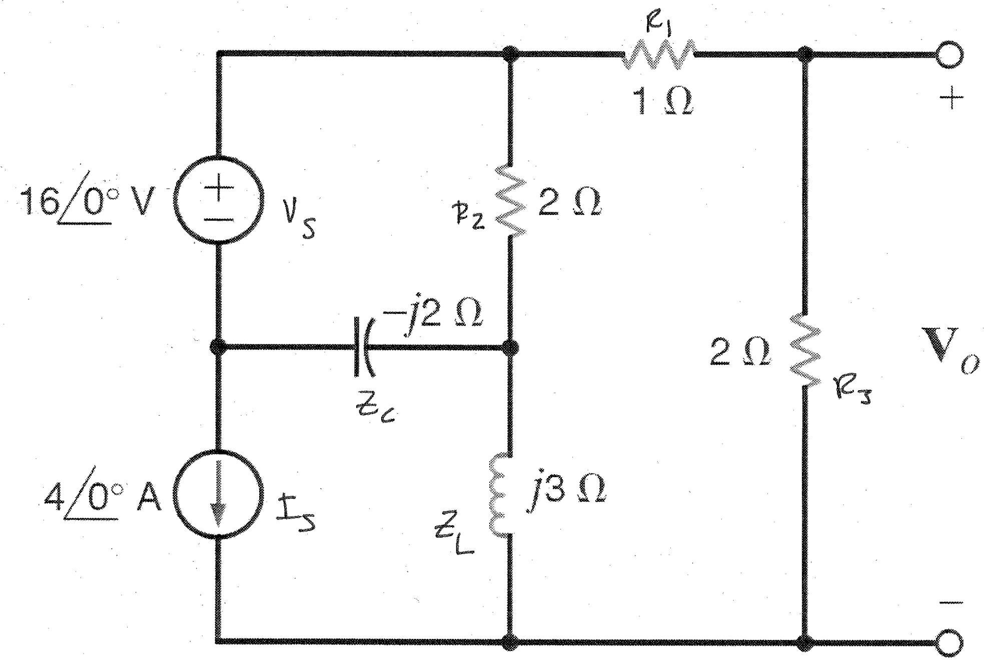
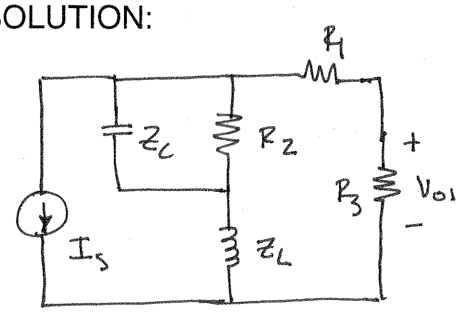


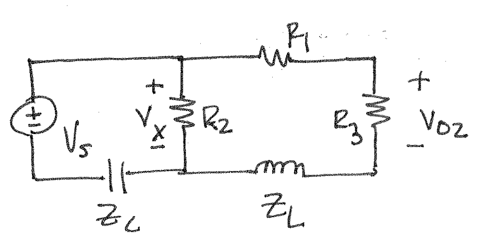
Figure P8.75

SOLUTION:



$Z_A = R_2 \,//\, Z_C \qquad Z_B = Z_L + Z_A$

$V_{o1} = \dfrac{-I_S Z_B}{Z_B + R_1 + R_3} R_3 = -3.2 - j2.4\text{V}$



$Z_X = R_1 + R_3 + Z_L \qquad Z_Y = R_2 \,//\, Z_X$

$V_X = \dfrac{V_S Z_Y}{Z_C + Z_Y} \qquad V_{o2} = \dfrac{V_X R_3}{R_1 + R_3 + Z_L}$

$V_{o2} = 4.8 + j1.6\text{V}$

$$\boxed{V_o = V_{o1} + V_{o2} = 1.79 \angle 26.6^\circ \text{ V}}$$

8.76 Use source exchange to determine $\mathbf{V}_o$ in the network in Fig. P8.76.

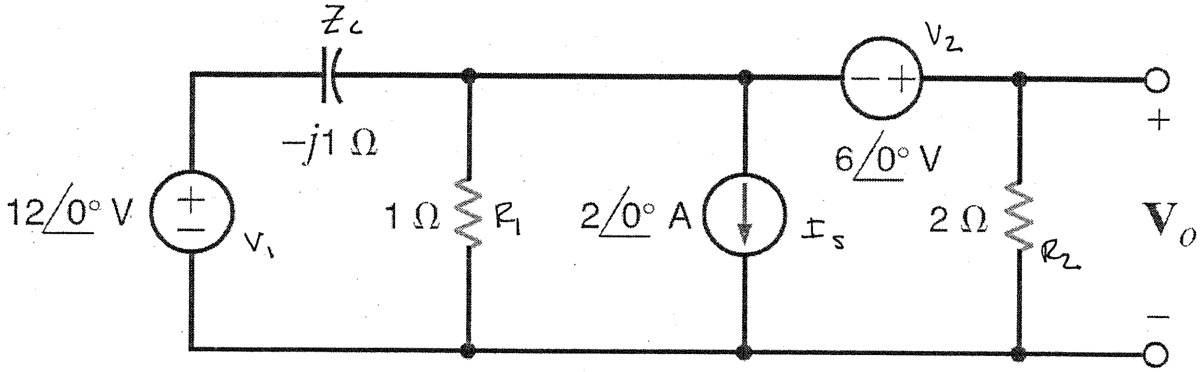
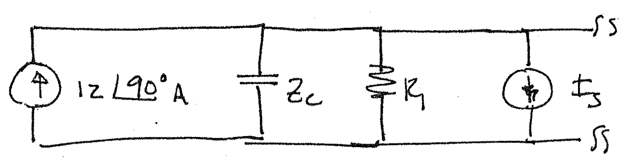
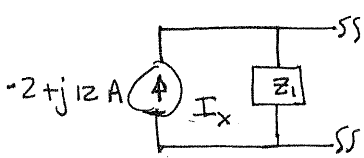


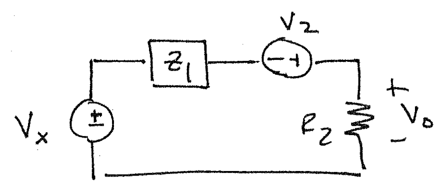
Figure P8.76

SOLUTION:





$\Rightarrow$



$Z_1 = R_1 // Z_c$

$V_x = I_x Z_1$

$$V_o = \frac{(V_x + V_2) R_2}{R_2 + Z_1}$$

$$\boxed{V_o = 10.2 \angle 43.8^\circ \text{ V}}$$

8.77 Use source exchange to find the current $\mathbf{I}_o$ in the network in Fig. P8.77.

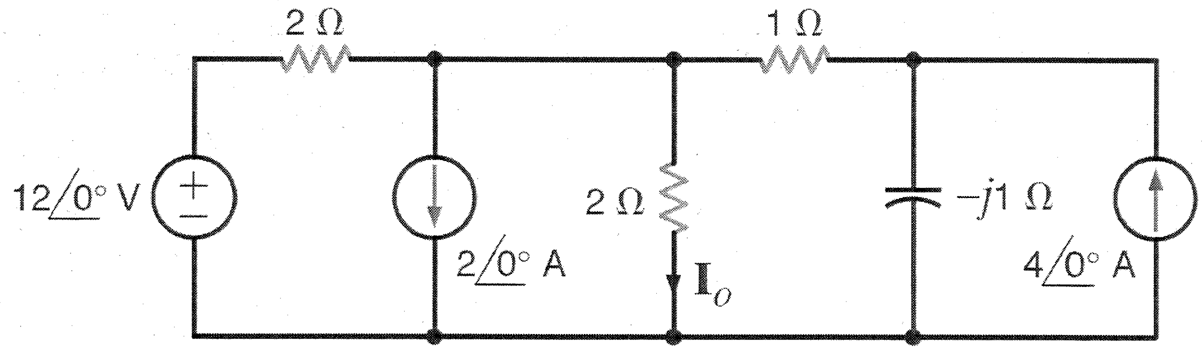
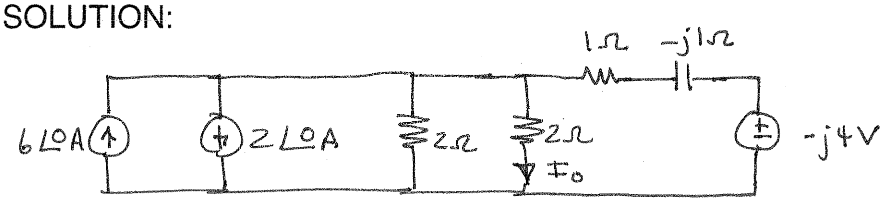
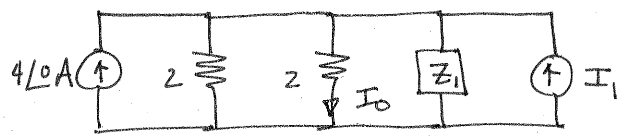


Figure P8.77

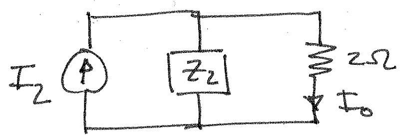
SOLUTION:





$Z_1 = 1 - j1\ \Omega$

$I_1 = -j4/Z_1$



$I_2 = 4\angle 0 + I_1$

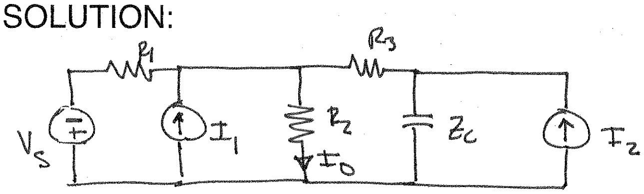
$Z_2 = 2Z_1/(2+Z_1)$

$$I_o = \frac{I_2 Z_2}{Z_2 + 2}$$

$$\boxed{I_o = 2\angle -36.9^\circ\ \text{A}}$$

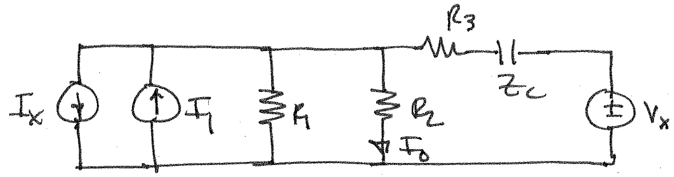
8.78 Use source transformation to determine $\mathbf{I}_o$ in the network in Fig. P8.50.

SOLUTION:



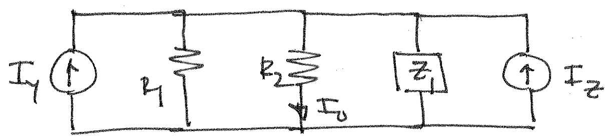
$R_1 = R_2 = R_3 = 1\Omega \quad Z_C = -j1\Omega$

$V_s = 12\angle 0° V \quad I_1 = 2\angle 0 A \quad I_2 = 4\angle 0 A$



$I_x = V_s / R_1 = 12\angle 0° A$

$V_x = I_2 Z_C = -j4 \ V$

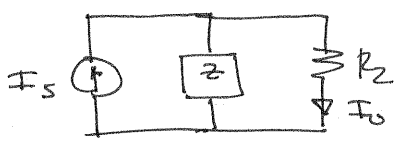


$I_y = I_1 - I_x = -10A$

$I_z = \dfrac{V_x}{Z_1} = 2 - j2 \ A$

$Z_1 = R_3 - Z_C = 1 - j1\Omega$

$I_s = I_y + I_z = -8 - j2 A$



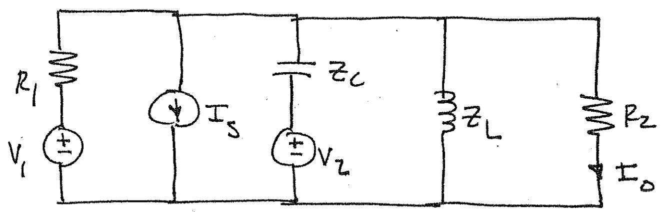
$Z = \dfrac{Z_1 R_1}{Z_1 + R_1} = \dfrac{1-j1}{2-j1}$

$I_o = \dfrac{I_s Z}{Z + R_2}$

$\boxed{I_o = 3.23\angle -177.3° \ A}$

8.79 Use source transformation to determine $\mathbf{I}_o$ in the network in Fig. P8.51.

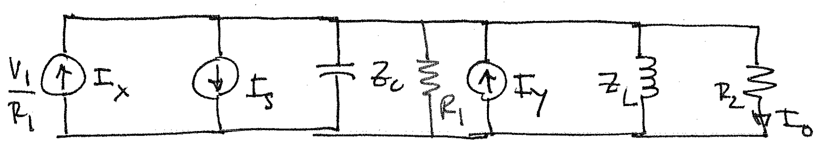
SOLUTION:



$R_1 = R_2 = 2\Omega \qquad Z_C = -j1\,\Omega$

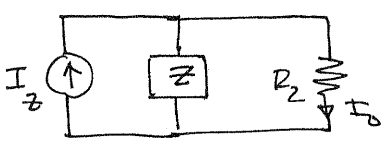
$Z_L = j2\Omega \qquad I_S = 4\angle 0°\,A$

$V_1 = 12\angle 0°\,V \qquad V_2 = 6\angle 0°\,V$



$I_x = V_1/R_1 = 6\angle 0°A$

$I_y = V_2/Z_C = 6\angle 90°A$



$I_Z = I_x + I_y - I_S = 2 + j6\,A$

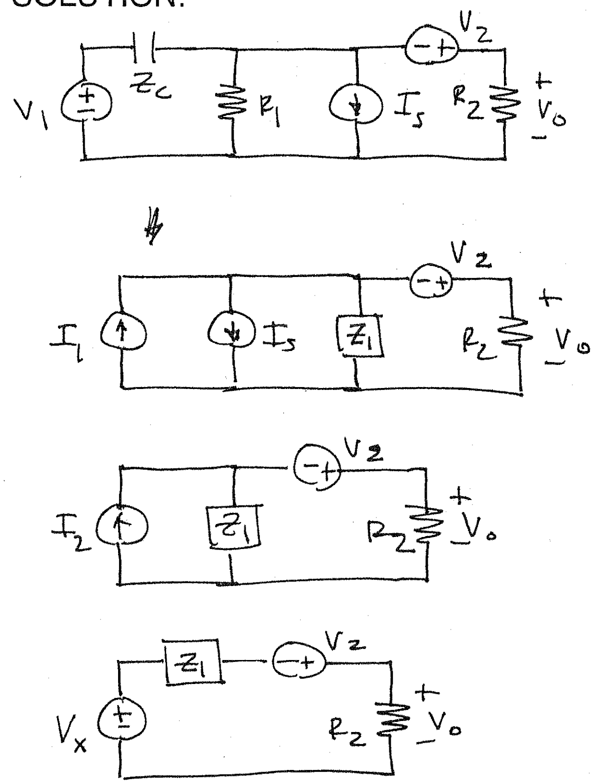
$Z = \left[\frac{1}{R_1} + \frac{1}{Z_C} + \frac{1}{Z_L}\right]^{-1} = 1 - j1\,\Omega$

$I_o = \frac{I_Z Z}{Z + R_2}$

$\boxed{I_o = 2.83\angle 45°\,A}$

8.80 Use source transformation to determine $\mathbf{V}_o$ in the network in Fig. P8.76.

SOLUTION:



$Z_C = -j1\,\Omega \quad R_1 = 1\,\Omega \quad R_2 = 2\,\Omega$

$V_1 = 12\angle 0°\,V \quad I_s = 2\angle 0°\,A$

$V_2 = 6\angle 0°\,V$

$I_1 = V_1 / Z_C = j12\,A$

$Z_1 = \dfrac{R_1 Z_C}{R_1 + Z_C} = 0.5 - j0.5\,\Omega$

$I_2 = I_1 - I_s = -2 + j12\,A$

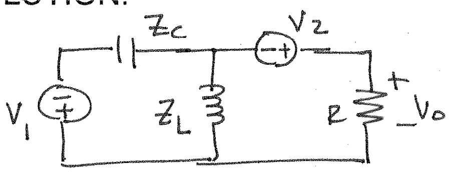
$V_x = I_2 Z_1 = 5 + j7\,V$

$V_o = \dfrac{(V_x + V_2)R_2}{R_2 + Z_1}$

$\boxed{V_o = 10.2\angle 43.8°\,V}$

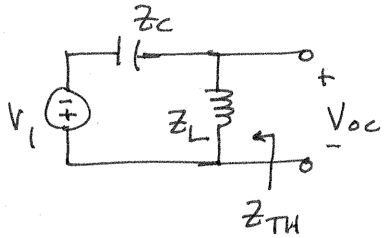
8.81 Using Thévenin's theorem, find $\mathbf{V}_o$ in the network in Fig. P8.63.

SOLUTION:



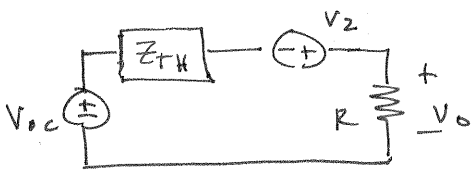
$V_1 = 6\angle 0°\ V \qquad V_2 = 12\angle 45°\ V$

$Z_C = -j1\ \Omega \qquad Z_L = j2\ \Omega \qquad R = 2\ \Omega$



$$V_{oc} = \frac{-V_1 Z_L}{Z_L + Z_C} = -12\angle 0°\ V$$

$$Z_{TH} = \frac{Z_L Z_C}{Z_L + Z_C} = -j2\ \Omega$$

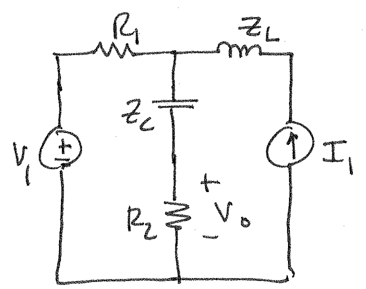


$$V_o = \frac{(V_{oc} + V_2)R}{R + Z_{TH}}$$

$$\boxed{V_o = 6.50\angle 157.5°\ V}$$

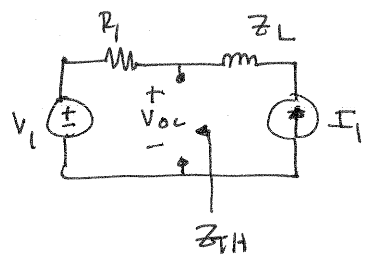
8.82 Use Thévenin's theorem to find $\mathbf{V}_o$ in the circuit in Fig. P8.64. CS

SOLUTION:



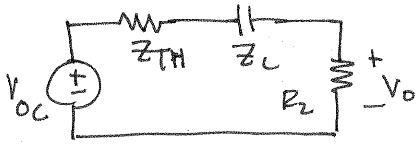
$R_1 = 4\Omega \quad R_2 = 2\Omega \quad Z_L = j2\Omega \quad Z_C = -j4\Omega$

$V_1 = 12\angle 0°\,V \quad I_1 = 4\angle 90°\,A$



$V_{oc} = V_1 + I_1 R_1 = 12 + j16\,V$

$Z_{TH} = R_1 = 4\Omega$

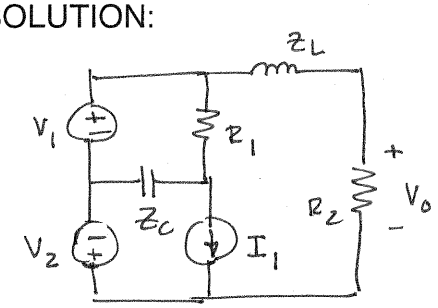


$$V_o = \frac{V_{oc} R_2}{R_2 + Z_C + Z_{TH}}$$

$$\boxed{V_o = 5.55\angle 86.8°\ V}$$

8.83 Solve Problem 8.52 using Thévenin's theorem.

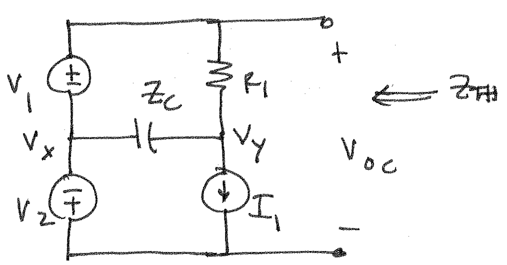
SOLUTION:



$V_1 = 12\angle 0°\ V \qquad V_2 = 16\angle 0°\ V$

$I_1 = 2\angle 0°\ A \qquad R_1 = R_2 = 2\ \Omega$

$Z_L = j1\ \Omega \qquad Z_C = -j1\ \Omega$

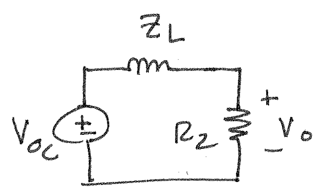


$V_{oc} - V_x = V_1 \qquad V_x = -V_2$

$$\frac{V_{oc} - V_y}{R_1} = \frac{V_y - V_x}{Z_C} + I_1$$

$V_{oc} = V_1 - V_2 = -4\angle 0°\ V$

$Z_{TH} = 0\ \Omega$ ($V_1 - V_2$ path shorts)



$$V_o = \frac{V_{oc} R_2}{R_2 + Z_L}$$

$$\boxed{V_o = 3.58\angle 153.4°\ V}$$

8.84 Apply Thévenin's theorem twice to find $\mathbf{V}_o$ in the circuit in Fig. P8.84.

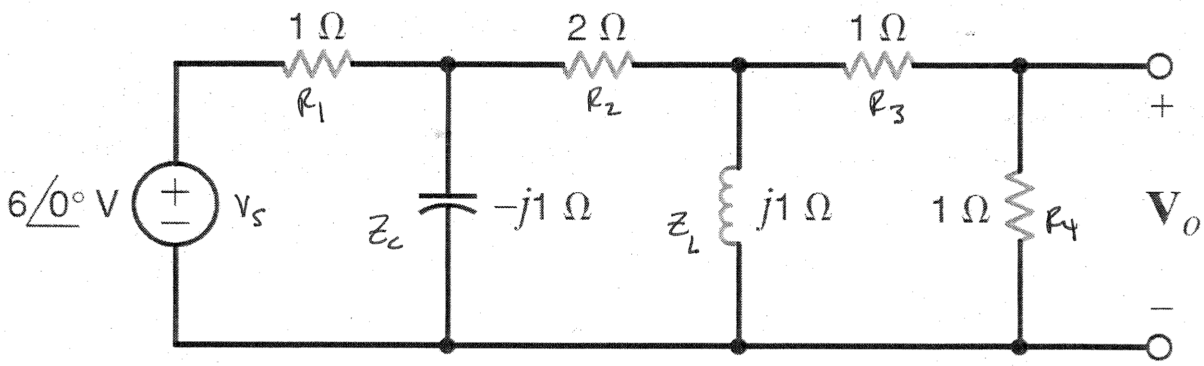
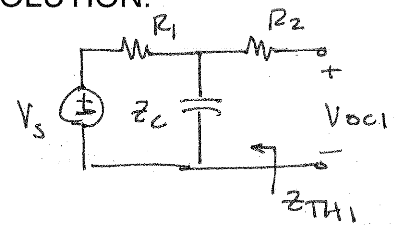


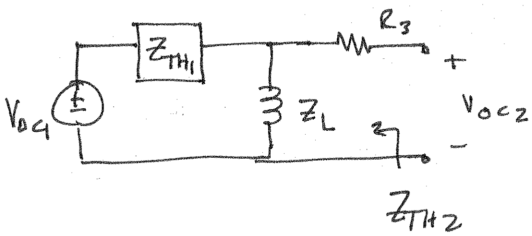
Figure P8.84

SOLUTION:



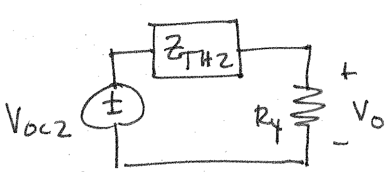
$$V_{oc1} = \frac{V_S Z_C}{Z_C + R_1} = 3 - j3 \text{ V}$$

$$Z_{TH1} = R_2 + \frac{R_1 Z_C}{R_1 + Z_C} = \frac{2-j3}{1-j1} \ \Omega$$



$$V_{oc2} = \frac{V_{oc1} Z_L}{Z_L + Z_{TH1}} = 1.66 \angle 33.7^\circ \text{ V}$$

$$Z_{TH2} = R_3 + \frac{Z_L Z_{TH1}}{Z_L + Z_{TH1}} = 1.66 \angle 33.7^\circ \ \Omega$$

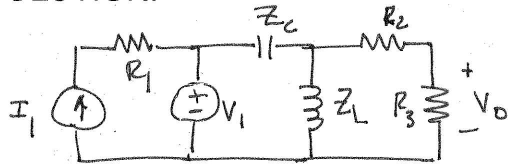


$$V_o = \frac{V_{oc2} R_4}{R_4 + Z_{TH2}}$$

$$\boxed{V_o = 0.651 \angle 12.5^\circ \text{ V}}$$

8.85 Solve Problem 8.49 using Thévenin's theorem.

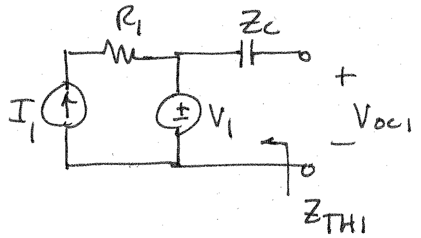
SOLUTION:



$R_1 = R_2 = 2\Omega \quad R_3 = 1\Omega$

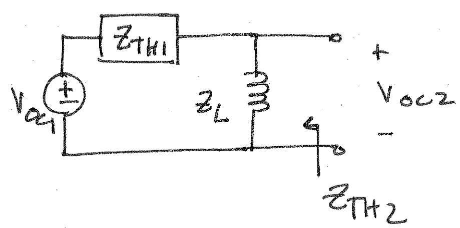
$Z_L = j1\Omega \quad Z_C = -j1\Omega$

$I_1 = 6\angle 0^\circ A \quad V_1 = 12\angle 0^\circ V$



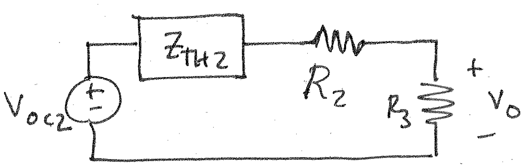
$V_{oc1} = V_1 = 12\angle 0^\circ V$

$Z_{TH1} = Z_C = -j1\Omega$



$$V_{oc2} = \frac{V_{oc1} Z_L}{Z_L + Z_C} = 24\angle 0^\circ V$$

$$Z_{TH2} = \frac{Z_L Z_{TH1}}{Z_L + Z_{TH1}} = -j2\Omega$$



$$V_o = \frac{V_{oc2} R_3}{R_2 + R_3 + Z_{TH2}}$$

$$\boxed{V_o = 6.66\angle 33.67^\circ V}$$

8.86 Use Thévenin's theorem to find $\mathbf{V}_o$ in the network in Fig. P8.86.

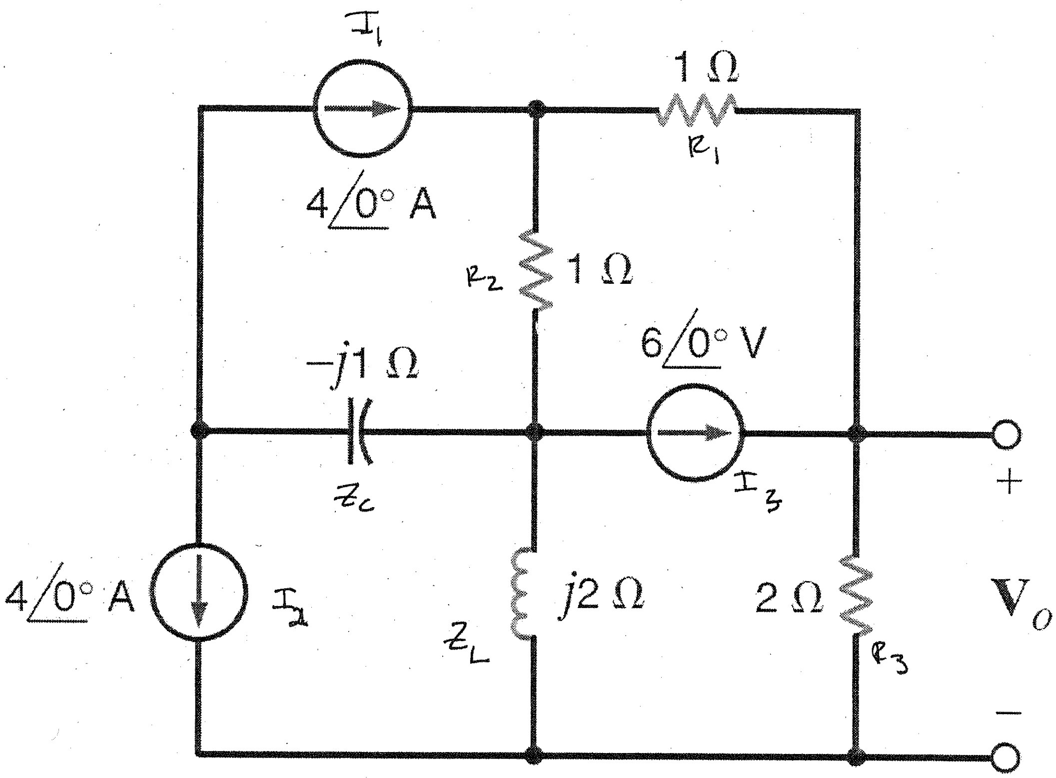
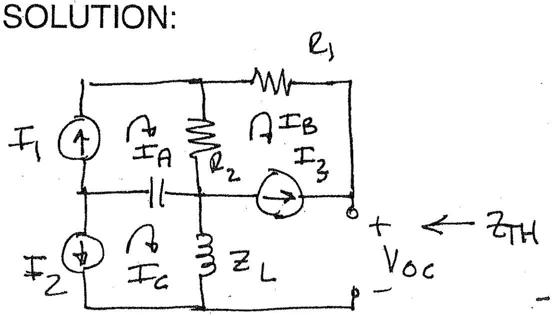


Figure P8.86

SOLUTION:



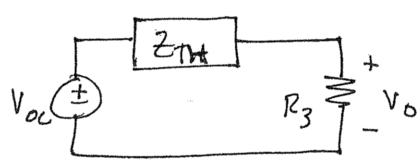
$$I_A = I_1 = 4\angle 0° \text{ A}$$

$$I_B = -I_3 = -6\angle 0° \text{ A}$$

$$I_C = -I_2 = -4\angle 0° \text{ A}$$

$$-Z_L I_C + R_2(I_B - I_A) + R_1 I_B + V_{OC} = 0$$

$$Z_{TH} = R_1 + R_2 + Z_L = 2 + j2\ \Omega \qquad V_{OC} = 16 - j8 \text{ V}$$



$$V_o = \frac{V_{OC} R_3}{R_3 + Z_{TH}}$$

$$\boxed{V_o = 4\angle -53.1° \text{ V}}$$

8.87 Find $\mathbf{V}_o$ in the network in Fig. P8.87 using Thévenin's theorem.

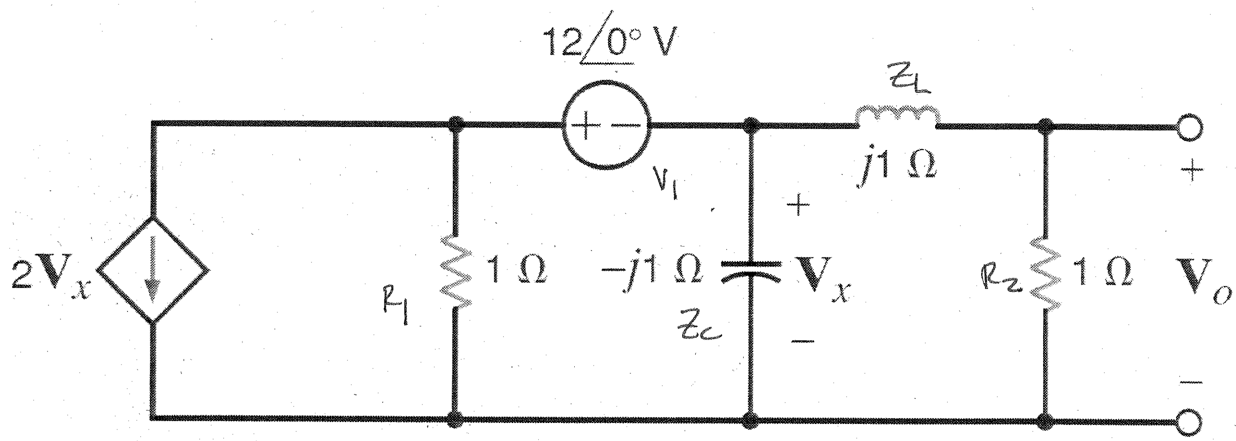
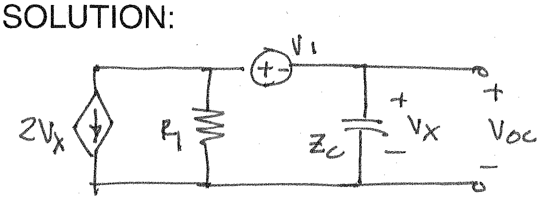


Figure P8.87

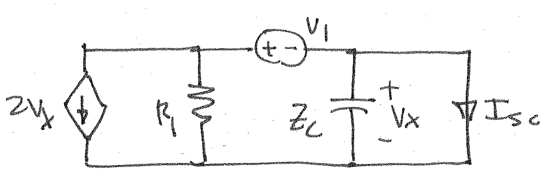
SOLUTION:



$V_x = V_{oc}$

$$2V_{oc} + \frac{V_{oc}+V_1}{R_1} + \frac{V_{oc}}{Z_C} = 0$$

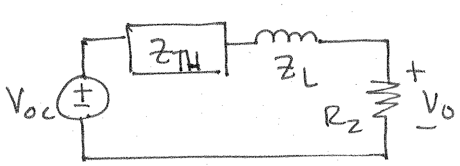
$V_{oc} = -3.6 + j1.2\ \text{V}$



$V_x = 0$

$\dfrac{V_1}{R_1} + I_{sc} = 0 \qquad I_{sc} = -12\angle 0°\ \text{A}$

$Z_{TH} = V_{oc}/I_{sc} = 0.3 - j0.1\ \Omega$



$$V_o = \frac{V_{oc}\,R_2}{R_2 + Z_L + Z_{TH}}$$

$$\boxed{V_o = 2.4\angle 127°\ \text{V}}$$

8.88 Find the Thévenin's equivalent for the network in Fig. P8.88 at the terminals *A*-*B*. CS

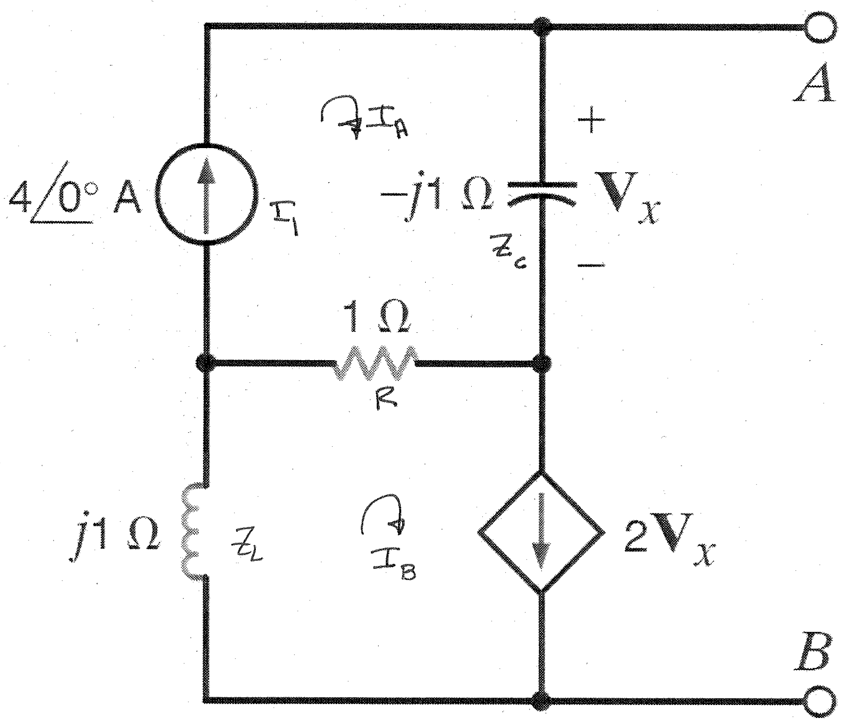
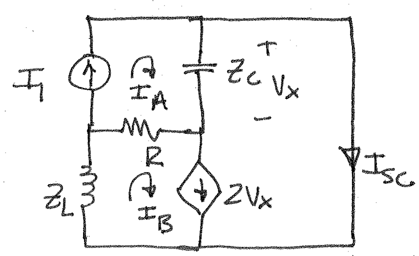


Figure P8.88

SOLUTION:



$I_A = I_1 = 4\angle 0° A$

$V_x = Z_C(I_1 - I_{sc})$

$I_B - I_{sc} = 2V_x$

$Z_C(I_{sc} - I_A) + Z_L I_B + R(I_B - I_A) = 0$

$I_{sc} = 2.53\ \angle 18.4°\ A$

$Z_{TH} = V_{oc}/I_{sc} = 2.24\ \angle 117°\ \Omega$

$V_{AB} = V_{oc}$

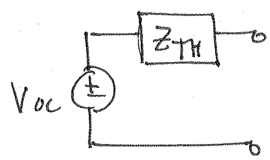
$I_A = 4\ \angle 0°\ A$

$V_x = Z_C I_A = -j4\ V$

$I_B = 2V_x = -j8\ A$

$Z_L I_B + R(I_B - I_A) - Z_C I_A + V_{oc} = 0$

$V_{oc} = -4 + j4\ V$



8.89 Given the network in Fig. P8.89, find the Thévenin's equivalent of the network at the terminals *A*-*B*.

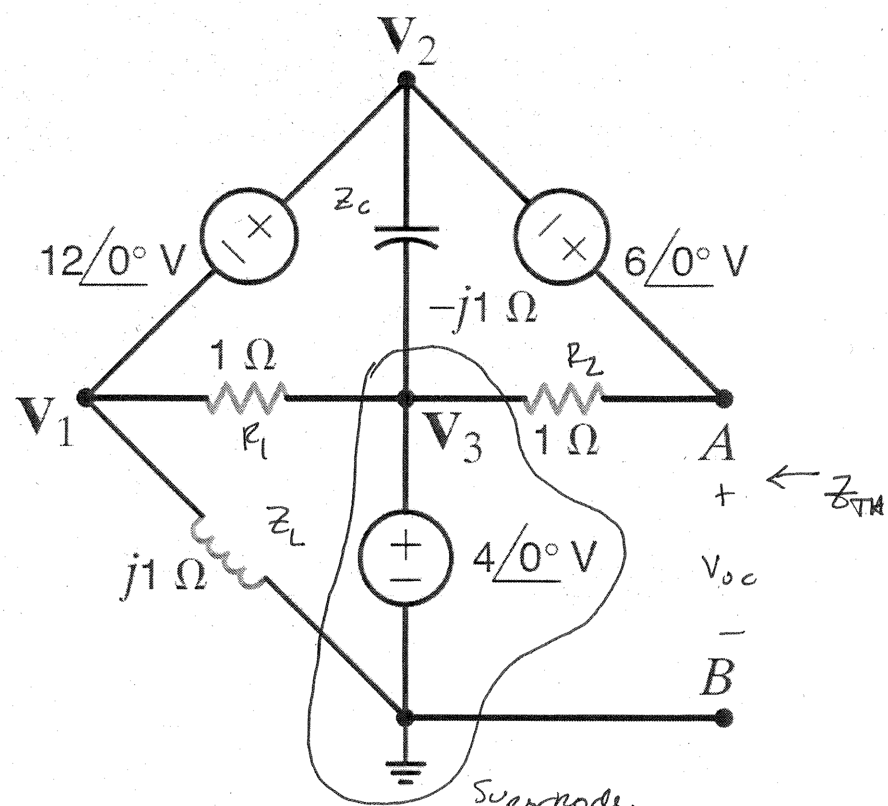


Figure P8.89

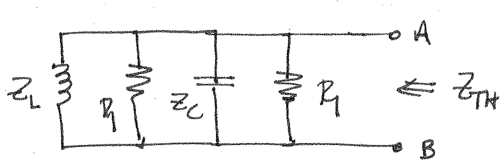
SOLUTION:

$V_2 - V_1 = 12\angle 0° \text{ V} \qquad V_{oc} - V_2 = 6\angle 0° \text{ V} \qquad V_3 = 4\angle 0° \text{ V}$

Supernode: $\dfrac{V_3 - V_2}{Z_C} + \dfrac{V_3 - V_1}{R_1} + \dfrac{-V_1}{Z_L} + \dfrac{V_3 - V_{oc}}{R_2} = 0$

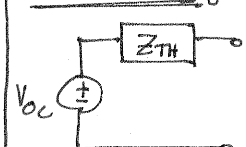
$V_{oc} = 13.6\angle -17.1° \text{ V}$

Z_{TH}



$\dfrac{1}{Z_{TH}} = \dfrac{1}{Z_C} + \dfrac{1}{R_1} + \dfrac{1}{Z_C} + \dfrac{1}{R_1} \Rightarrow Z_{TH} = 0.5\,\Omega$

Thevenin Eq



$V_{oc} = 13.6\angle -17.1° \text{ V}$

$Z_{TH} = 0.5\,\Omega$

8.90 Find $\mathbf{V}_x$ in the circuit in Fig. P8.90 using Norton's theorem.

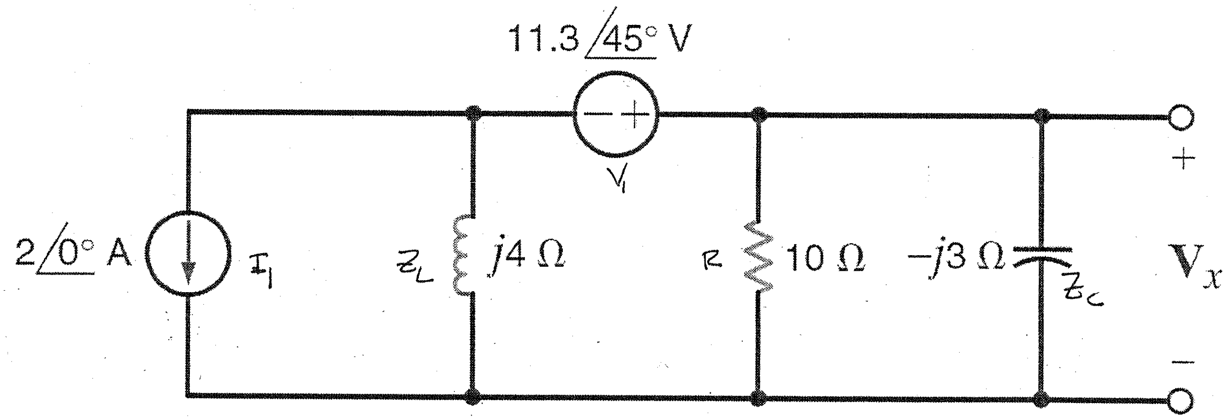
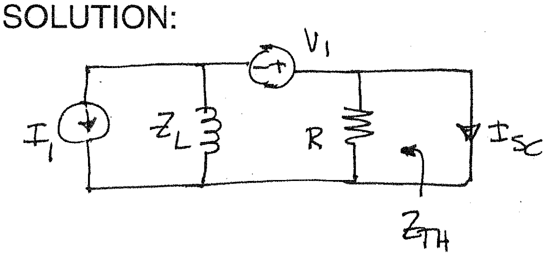


Figure P8.90

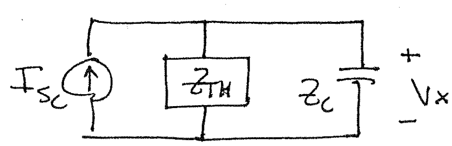
SOLUTION:



Superposition:

$$I_{SC} = -I_1 + \frac{V_1}{Z_L} = -j2\ \text{A}$$

$$Z_{TH} = \frac{R Z_L}{R + Z_L} = 5.90 - j11.8\ \Omega$$



$$V_x = I_{SC} Z \qquad Z = \frac{Z_{TH} Z_C}{Z_{TH} + Z_C}$$

$$\boxed{V_x = 15.4\,\angle -129.8^\circ\ \text{V}}$$

8.91 Find $\mathbf{I}_o$ in the network in Fig. P8.91 using Norton's theorem.

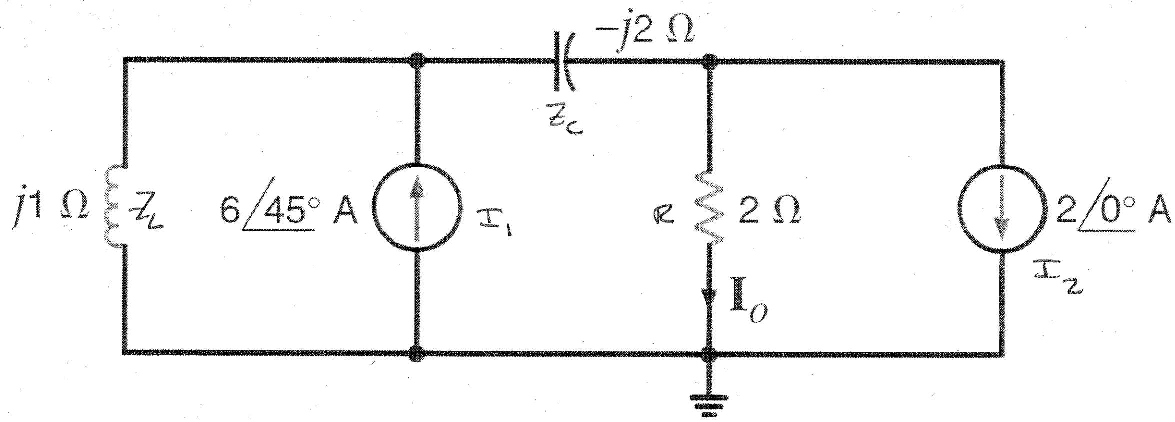
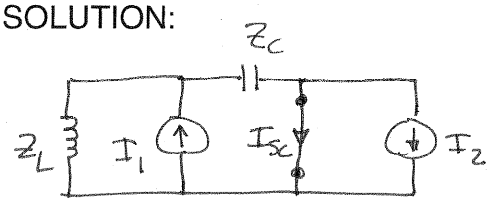


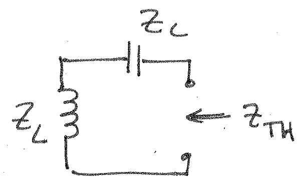
Figure P8.91

SOLUTION:

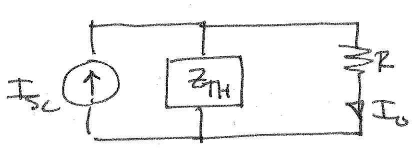


Superposition:

$$I_{SC} = -I_2 + \frac{I_1 Z_L}{Z_L + Z_C}$$



$$Z_{TH} = Z_L + Z_C = -j1\,\Omega$$



$$I_o = \frac{I_{SC} Z_{TH}}{R + Z_{TH}}$$

$$\boxed{I_o = 3.38 \angle 151^\circ \text{ A}}$$

8.92 Apply both Norton's theorem and MATLAB to find $\mathbf{V}_o$ in the network in Fig. P8.92.

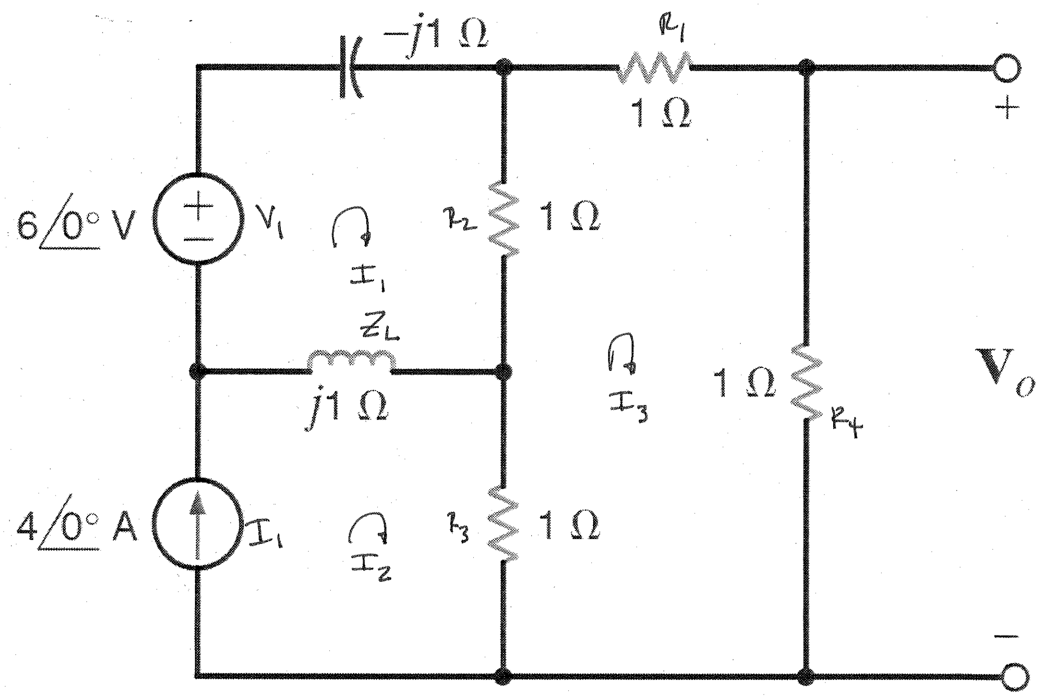


Figure P8.92

SOLUTION:

MATLAB:

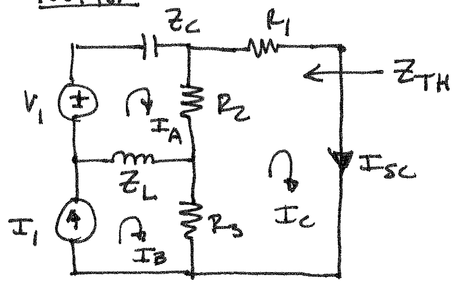
$$\begin{cases} 6 = I_1(1) - j1\, I_2 - I_3 \\ I_2 = 4\angle 0^\circ \\ 0 = -I_1 - I_2 + I_3(4) \end{cases} \Rightarrow \begin{bmatrix} 1 & -j1 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$V_o = R_4 I_3$

```
» z=[1 -1i -1;0 1 0;-1 -1 4];
» v=[6;4;0];
» i=inv(z)*v

i = 9.3333 + 5.3333i
    4.0000
    3.3333 + 1.3333i
```

$$\boxed{V_o = 3.59\angle 21.8^\circ \text{ V}}$$

Norton



① $V_1 = I_A\,(R_2 + Z_L + Z_C) - Z_L I_B - R_2 I_C$

or, $V_1 = I_A - j\,I_B - I_C = 6\angle 0°$

② $I_B = I_1 = 4\angle 0°\,A$

③ $-R_2 I_A - R_3 I_B + I_C\,(R_1 + R_2 + R_3) = 0$

or $-I_A - I_B + 3I_C = 0$

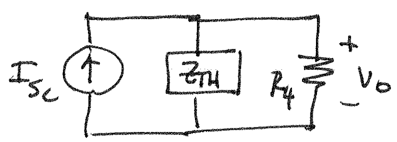
$I_{sc} = I_C$

yields $I_C = I_{sc} = 5 + j2\ A$

$Z_{TH} = R_1 + Z + R_3$

$$Z = \frac{R_2\,(Z_C + Z_L)}{R_2 + Z_C + Z_L} = 0$$

$Z_{TH} = 2\,\Omega$



$$V_o = \frac{I_{sc}\,R_4 Z_{TH}}{R_4 + Z_{TH}}$$

$$\boxed{V_o = 3.59\angle 21.8°\ V}$$

8.93 Find $\mathbf{V}_o$ using Norton's theorem for the circuit in Fig. P8.93. CS

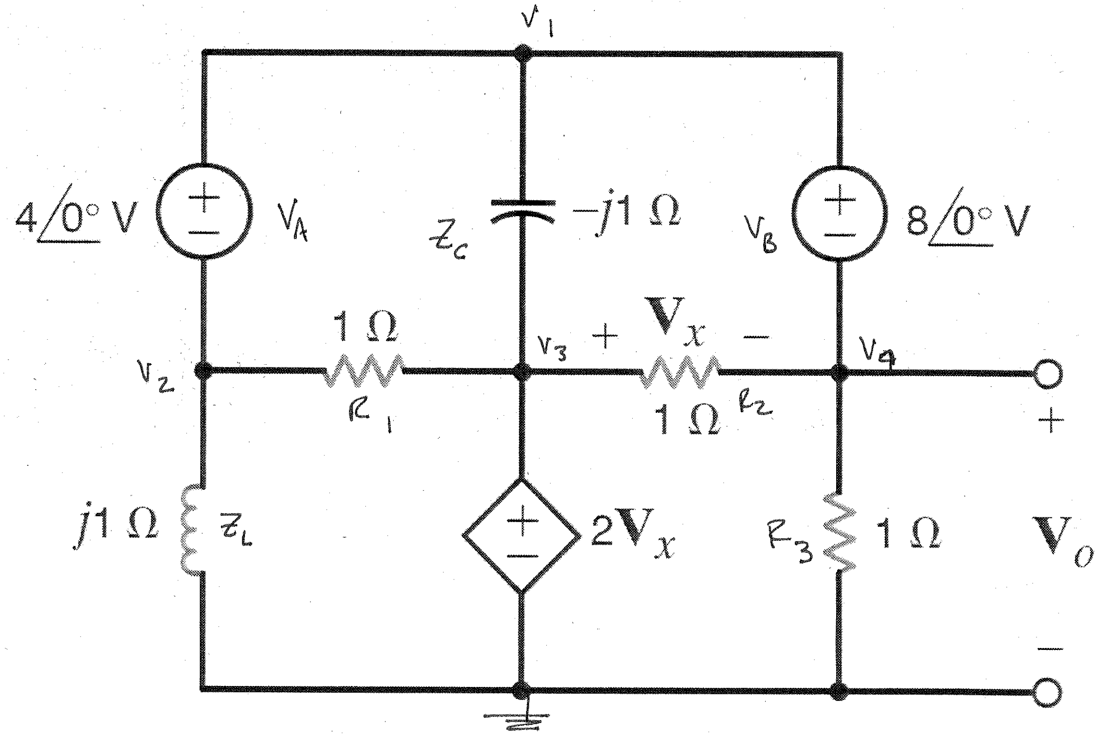
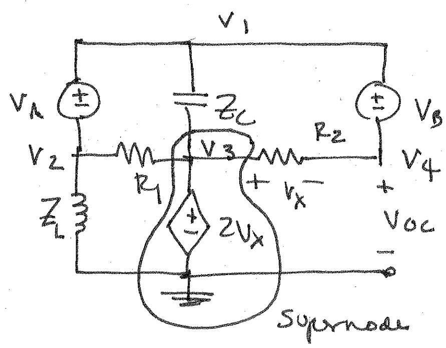


Figure P8.93

SOLUTION:



$V_{oc} = V_4$

$V_1 - V_2 = 4\angle 0° \text{ V} \qquad V_1 - V_4 = 8\angle 0° \text{ V}$

yield $V_1 = 8 + V_4$ & $V_2 = 4 + V_4$

$2V_x = V_3 = 2(V_3 - V_4) \Rightarrow V_3 = 2V_4$

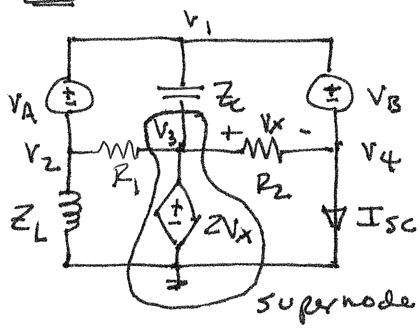
Supernode:

$$\frac{V_3 - V_2}{R_1} + \frac{V_3 - V_1}{Z_C} + \frac{V_3 - V_4}{R_2} - \frac{V_2}{Z_L} = 0$$

or, $V_3(2+j1) + V_2(-1+j1) - j1V_1 - V_4 = 0$

yields $V_{oc} = V_4 = 2\angle 0° \text{V}$

$\underline{I_{sc}}$



$V_4 = 0$

$V_3 - V_4 = V_x \quad \& \quad V_3 = 2V_x \Rightarrow V_3 = 0$

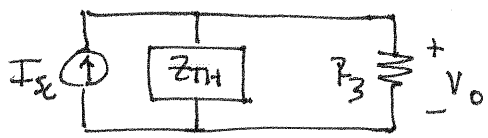
$V_1 = V_B = 8\angle 0° \text{ V}$

$V_1 - V_2 = V_A = 4\angle 0° \Rightarrow V_2 = 4\angle 0° \text{ V}$

at supernode: $\dfrac{V_3 - V_1}{Z_C} + \dfrac{V_3 - V_4}{R_2} + \dfrac{V_3 - V_2}{R_1} + \dfrac{0 - V_2}{Z_L} = I_{sc}$

yields $I_{sc} = -4 - j4 \text{ A}$

$Z_{TH} = \dfrac{V_{oc}}{I_{sc}} = 0.354\angle -135° \; \Omega$



$V_o = \dfrac{I_{sc}\left[Z_{TH} R_3\right]}{Z_{TH} + R_3}$

$\boxed{V_o = 4\angle 90° \text{ V}}$

8.94 Use Norton's theorem to find $\mathbf{V}_o$ in the network in Fig. P8.94. **PSV**

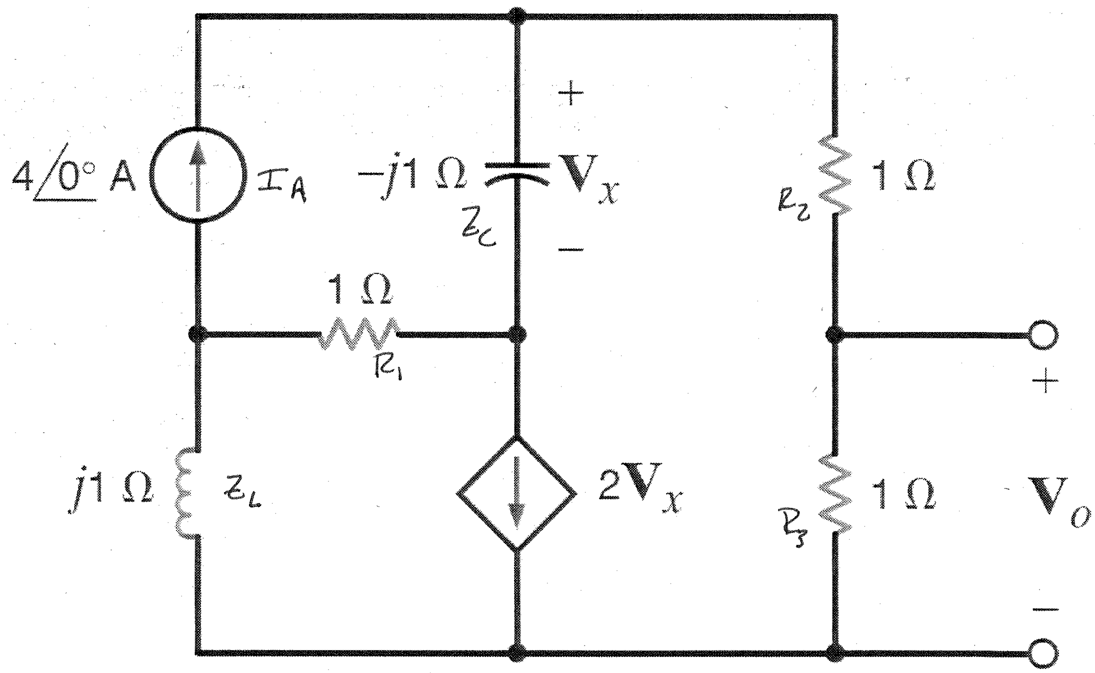
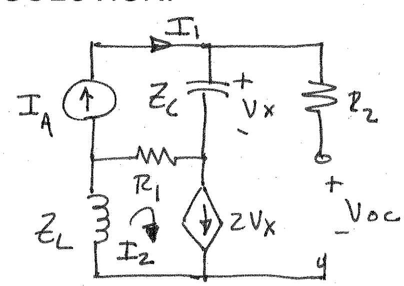


Figure P8.94

SOLUTION:

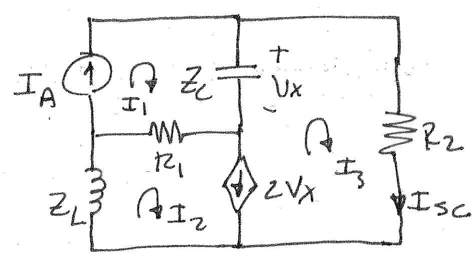


$I_1 = I_A = 4\angle 0°\,A$

$I_2 = 2V_x = 2Z_C I_1 = -j8\,A$

$Z_L I_2 + R_1(I_2 - I_1) - Z_C I_1 + V_{oc} = 0$

yields $V_{oc} = -4 + j4\,V$



$I_{sc} = I_3$

$I_1 = 4\angle 0°$ $\quad 2V_x = I_2 - I_3 = 2Z_C(I_1 - I_3)$

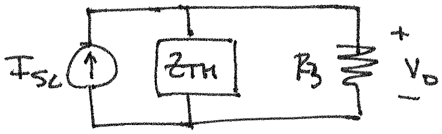
or, $-I_2 + I_3(1 + j2) = j8$

$I_2(Z_L + R_1) + I_3(Z_C + R_2) - I_1(R_1 + Z_C) = 0$

or $-I_1(1 - j1) + I_2(1 + j1) + I_3(1 - j1) = 0$

yields $I_{sc} = 2 + j2\,A$

$$Z_{TH} = \frac{V_{OC}}{I_{SC}} = j2\,\Omega$$



$$V_o = I_{SC}\left[\frac{R_3 Z_{TH}}{R_3 + Z_{TH}}\right]$$

$$\boxed{V_o = 2.53 \angle 71.6^\circ \text{ V}}$$

8.95 Use MATLAB to find the node voltages in the network in Fig. P8.95.

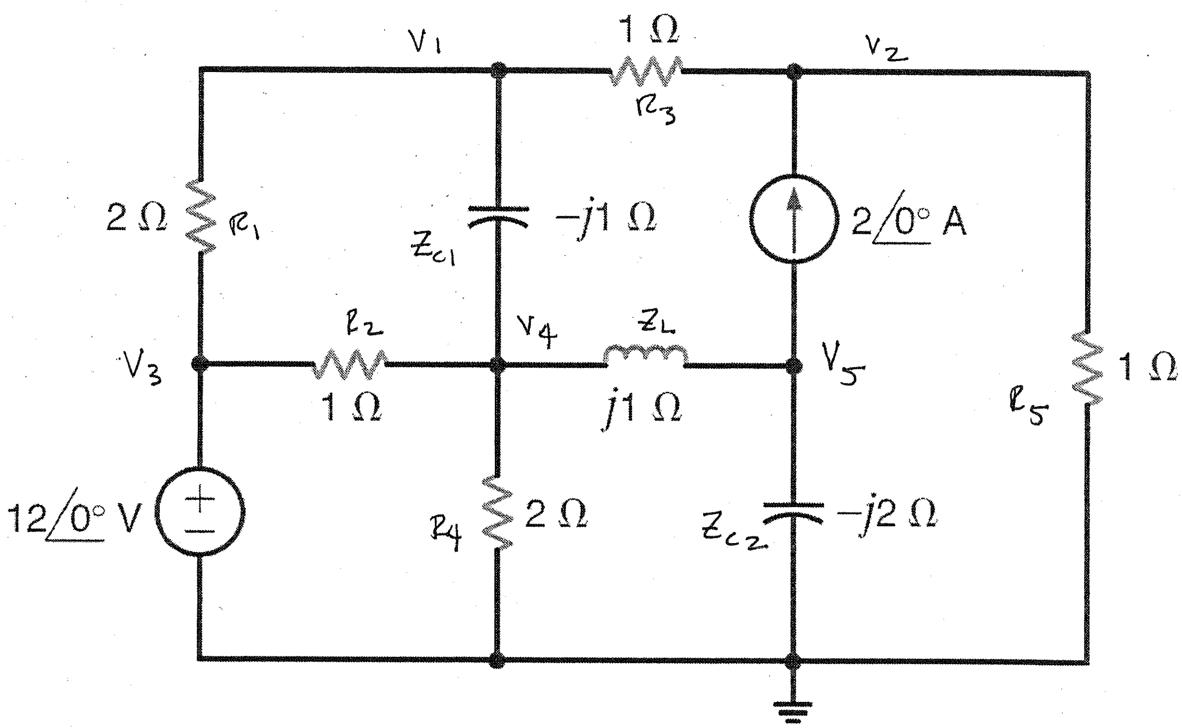


Figure P8.95

SOLUTION:

$$\frac{V_1 - V_3}{R_1} + \frac{V_1 - V_2}{R_3} + \frac{V_1 - V_4}{Z_{c1}} = 0 \Rightarrow V_1[3 + j2] - 2V_2 - V_3 - j2V_4 = 0$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_5} = 2\angle 0 \Rightarrow -V_1 + 2V_2 = 2\angle 0^\circ$$

$$\frac{V_4 - V_1}{Z_{c1}} + \frac{V_4 - V_5}{Z_L} + \frac{V_4}{R_4} + \frac{V_4 - V_3}{R_2} = 0 \Rightarrow -j2V_1 - 2V_3 + 3V_4 + j2V_5 = 0$$

$$\frac{V_5 - V_4}{Z_L} + \frac{V_5}{Z_{c2}} = -2\angle 0^\circ \rightarrow -j2V_4 + jV_5 = 4$$

$$V_3 = 12\angle 0^\circ$$

$$\begin{bmatrix} 3+j2 & -2 & -1 & -j2 & 0 \\ -1 & 2 & 0 & 0 & 0 \\ -j2 & 0 & -2 & 3 & j2 \\ 0 & 0 & 0 & -j2 & j1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 4 \\ 12 \end{bmatrix}$$

```
» y=[3+2i -2 -1 -2i 0;-1 2 0 0 0;-2i 0 -2 3 2i;0 0 0 -2i 1i;0 0 1 0 0];

» i=[0;2;0;4;12];

» v=inv(y)*i

v =

   6.5800 - 2.0600i
   4.2900 - 1.0300i
  12.0000 + 0.0000i
   4.5200 - 1.6400i
   9.0400 - 7.2800i
```

8.96 Use MATLAB to find $\mathbf{I}_o$ in the network in Fig. P8.96.

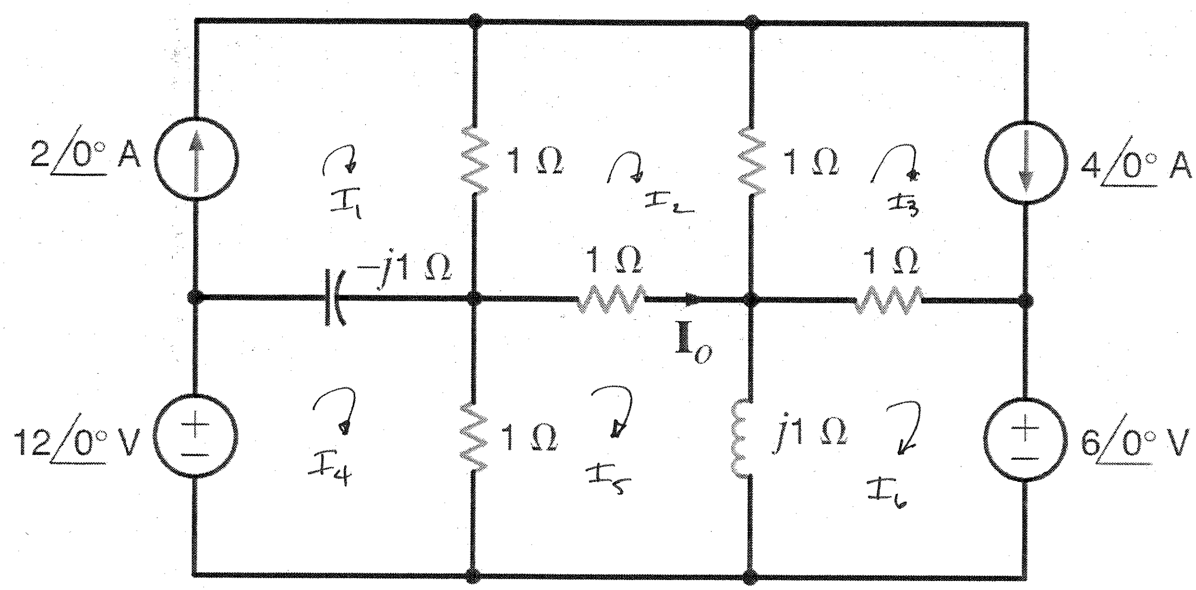


Figure P8.96

SOLUTION:

$I_1 = 2\angle 0° A$ $\quad I_3 = 4\angle 0° A$ $\quad -I_1 + 3I_2 - I_3 - I_5 = 0$

$12 = j1 I_1 + I_4(1-j1) - I_5 \quad -I_2 - I_4 + I_5(2+j1) - j1 I_6 = 0$

$-I_3 - j1 I_5 + I_6(1+j1) = -6$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ j1 & 0 & 0 & 1-j1 & -1 & 0 \\ 0 & -1 & 0 & -1 & 2+j1 & -j1 \\ 0 & 0 & -1 & 0 & -j1 & 1+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 12 \\ 0 \\ -6 \end{bmatrix}$$

$I_o = I_5 - I_2$

```
» z=[1 0 0 0 0 0;0 0 1 0 0 0;-1 3 -1 0 -1 0;
  1i 0 0 1-1i -1 0;0 -1 0 -1 2+1i -1i;0 0 -1 0 -1i 1+1i];
» v=[2;4;0;12;0;-6];
» i=inv(z)*v

i = 2.0
    3.6 + 0.8i
    4.0
    8.2 + 8.6i
    4.8 + 2.4i
    0.2 + 4.6i
```

$I_5 = 4.8 + j2.4 A$

$I_2 = 3.6 + j0.8 A$

$\boxed{I_o = 2\angle 53.1° A}$

8.97 Find $\mathbf{V}_o$ in the circuit in Fig. P8.97 using MATLAB.

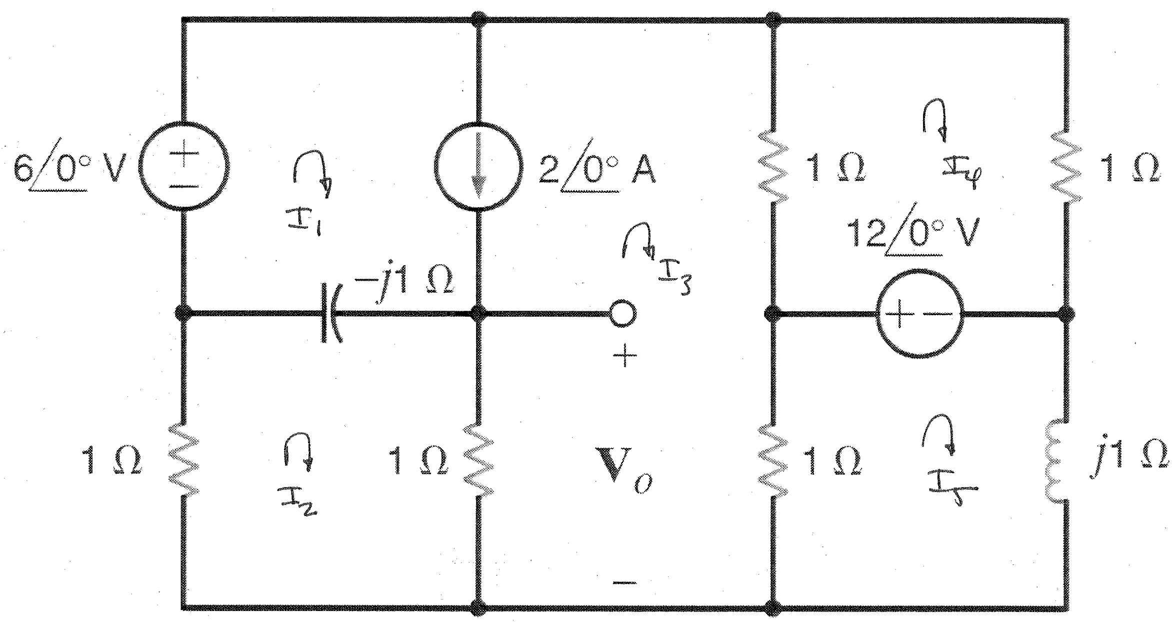


Figure P8.97

SOLUTION:

$I_1 - I_3 = 2\angle 0$ $\qquad j1 I_1 + I_2(2-j1) - I_3 = 0$ $\qquad 12\angle 0° = -I_3 + I_4(2)$

$-12 = -I_3 + (1+j1) I_5$ $\qquad 6 = I_4 + j1\, I_5 + I_2$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ j1 & 2-j1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 1+j1 \\ 0 & 1 & 0 & 1 & j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 12 \\ -12 \\ 6 \end{bmatrix}$$

$V_o = 1\,[I_2 - I_3]$

```
» z=[1 0 -1 0 0;1i 2-1i -1 0 0;0 0 -1 2 0;0 0 -1 0 1+1i;0 1 0 1 1i];
» v=[2;0;12;-12;6];
» i=inv(z)*v

i = 6.1509 + 3.4717i
    3.5849 + 0.4528i
    4.1509 + 3.4717i
    8.0755 + 1.7358i
   -2.1887 + 5.6604i
```

$I_2 = 3.59 + j0.45 A$

$I_3 = 4.15 + j3.42 A$

$\boxed{V_o = 3.07 \angle -101° \text{ V}}$

8.98 Determine $\mathbf{V}_o$ in the network in Fig. P8.98 using MATLAB.

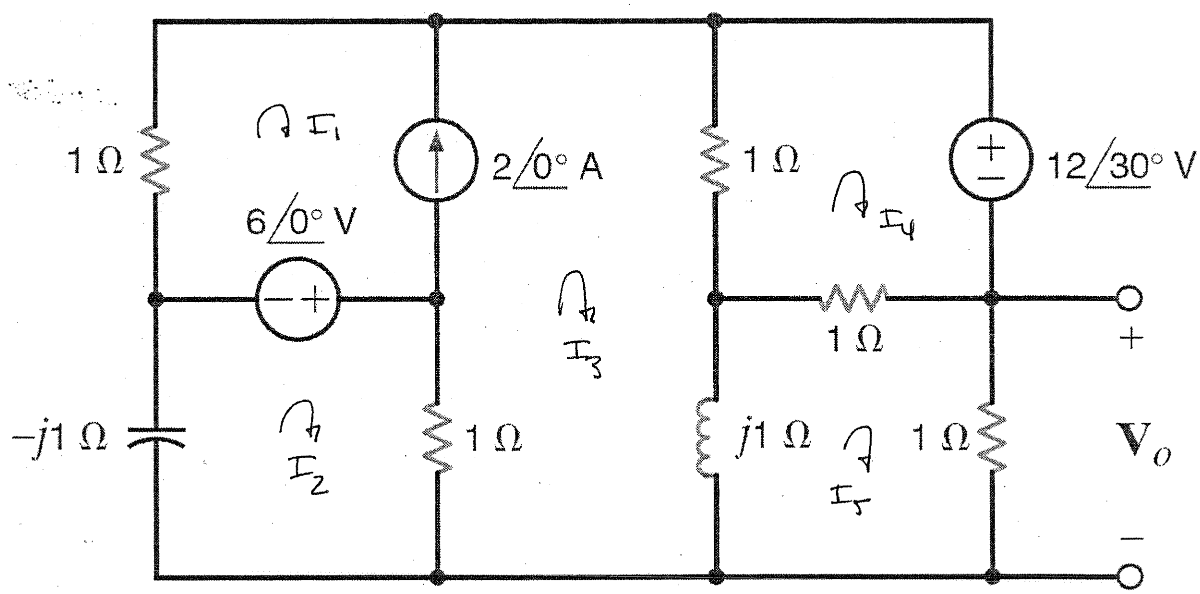


Figure P8.98

SOLUTION:

$I_3 - I_1 = 2\angle 0^\circ \qquad I_2(1-j1) - I_3 = 6\angle 0 \qquad -I_3 + 2I_4 - I_5 = -12\angle 30$

$-j1I_3 - I_4 + I_5(2+j1) = 0 \qquad I_1 - jI_2 + I_5 = -12\angle 30^\circ$

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & 1-j1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -j1 & -1 & 2+j1 \\ 1 & -j1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -12\angle 30 \\ 0 \\ -12\angle 30 \end{bmatrix}$$

$V_o = 1(I_5)$

```
» z=[-1 0 1 0 0;0 1-1i -1 0 0;0 0 -1 2 -1;0 0 -1i -1 2+1i;1 -1i 0 0 1];

» v=[2;6;-10.4-6.0i;0;-10.4-6.0i];

» i=inv(z)*v

i =

  -5.7798 - 1.9339i
   2.0771 + 0.1431i
  -3.7798 - 1.9339i
  -9.4716 - 4.9615i
  -4.7633 - 1.9890i
```

$I_5 = -4.76 - j1.99$ A

$V_o = (1)I_5$

$\boxed{V_o = 5.16 \angle -157^\circ \text{ V}}$

8.99 Find $\mathbf{I}_o$ in the network in Fig. P8.99 using MATLAB.

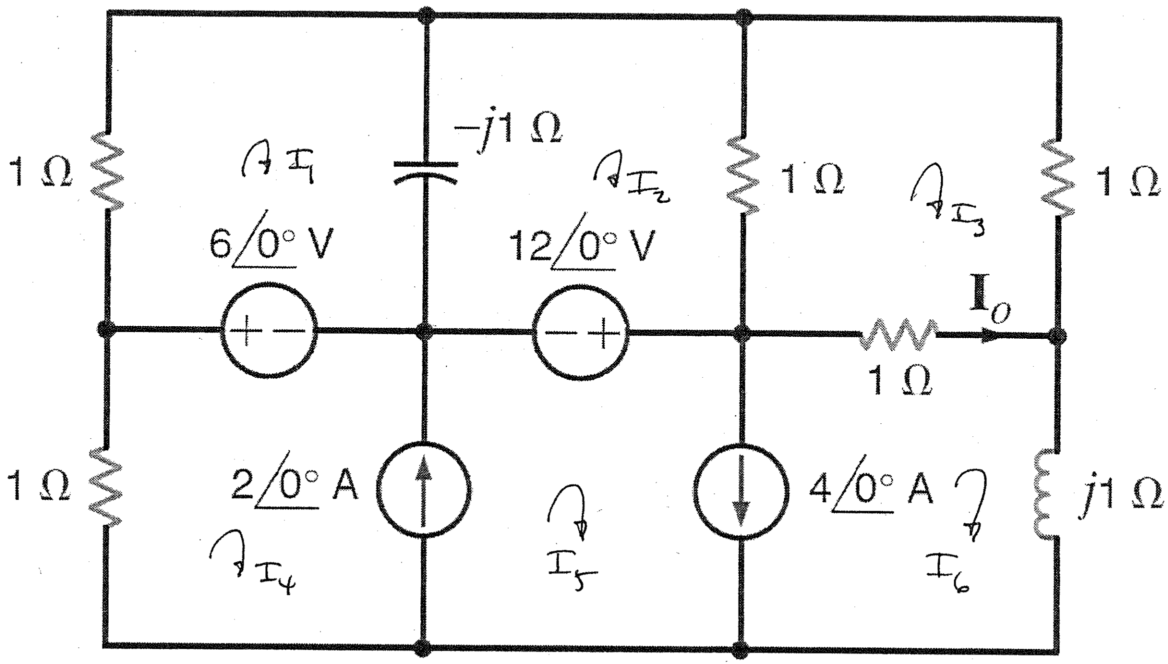


Figure P8.99

SOLUTION:

$$(1 - j1)\, I_1 + j I_2 = 6 \angle 0 \qquad j I_1 + I_2 (1 - j1) - I_3 = -12 \angle 0°$$

$$-I_2 + I_3 (3) - I_6 = 0 \qquad I_5 - I_4 = 2\angle 0° \qquad I_5 - I_6 = 4 \angle 0°$$

$$I_1 + I_3 + j I_6 + I_4 = 0$$

$$\begin{bmatrix} 1-j1 & j1 & 0 & 0 & 0 & 0 \\ j1 & 1-j1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 1 & 1 & 0 & j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ 0 \\ 2 \\ 4 \\ 0 \end{bmatrix}$$

```
» z=[1-1i 1i 0 0 0 0;1i 1-1i -1 0 0 0;
     0 -1 3 0 0 -1;0 0 0 -1 1 0;
     0 0 0 0 1 -1;1 0 1 1 0 1i];

» v=[6;-12;0;2;4;0];

» i=inv(z)*v

i =

  -2.2800 + 3.0400i
  -5.3200 - 5.2400i
  -1.6000 - 2.2000i
   2.5200 - 1.3600i
   4.5200 - 1.3600i
   0.5200 - 1.3600i
```

$$I_o = I_6 - I_3$$

$$I_6 = 0.52 - j1.36 \qquad I_3 = -1.6 - j2.2\text{A}$$

$$\boxed{I_o = 2.28 \angle 21.62°\ \text{A}}$$

8.100 Use MATLAB to determine $\mathbf{I}_o$ in the network in Fig. P8.100.

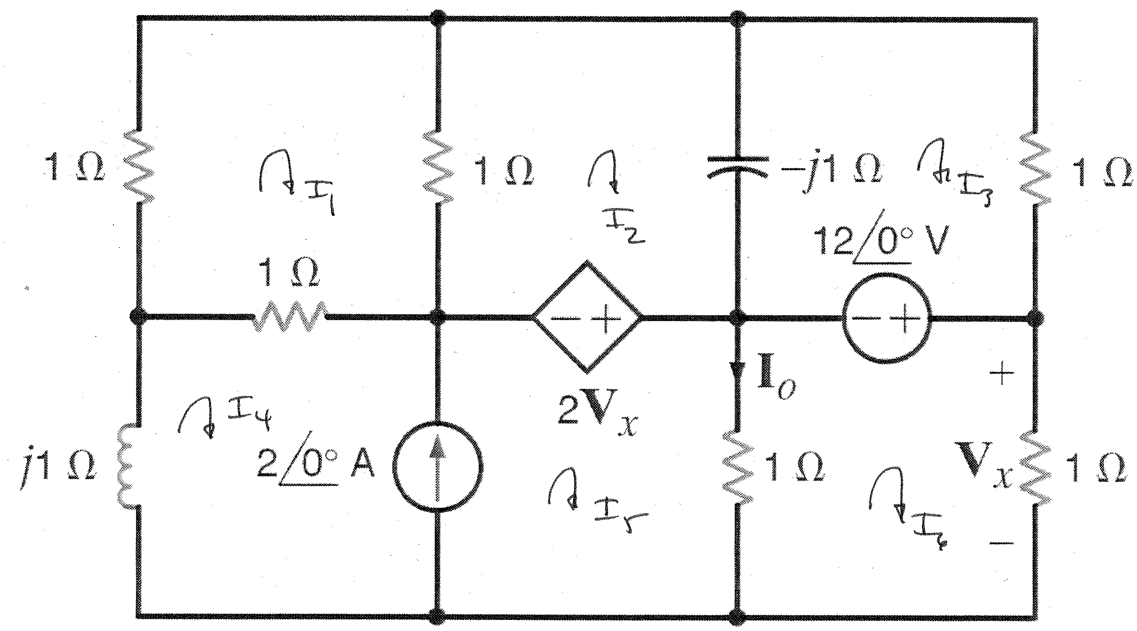


Figure P8.100

SOLUTION:

$3I_1 - I_2 - I_4 = 0 \qquad -I_1 + I_2(1-j1) + j1\, I_3 = -2V_x \qquad V_x = I_6\,[1]$

$j1 I_2 + I_3(1-j1) = -12\angle 0 \qquad I_5 - I_4 = 2\angle 0^\circ \qquad -I_5 + I_6(2) = 12\angle 0^\circ$

$I_1 + I_3 + jI_4 + I_6 = 0$

$$\begin{bmatrix} 3 & -1 & 0 & -1 & 0 & 0 \\ -1 & 1-j1 & j1 & 0 & 0 & 2 \\ 0 & j1 & 1-j1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 \\ 1 & 0 & 1 & j1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -12 \\ 2 \\ 12 \\ 0 \end{bmatrix}$$

```
» z=[3 -1 0 -1 0 0;-1 1-1i 1i 0 0 2;
     0 1i 1-1i 0 0 0;0 0 0 -1 1 0;
     0 0 0 0 -1 2;1 0 1 1i 0 1];
» v=[0;0;-12;2;12;0];
» i=inv(z)*v

i =
   -5.7379 - 2.8552i
  -20.2345 + 3.1862i
  -14.5241 + 5.7103i
    3.0207 -11.7517i
    5.0207 -11.7517i
    8.5103 - 5.8759i
```

$I_o = I_5 - I_6$

$$\boxed{I_o = 6.83\angle -121^\circ\ \text{A}}$$

8.101 Find $\mathbf{I}_o$ in the circuit in Fig. P8.101 using MATLAB.

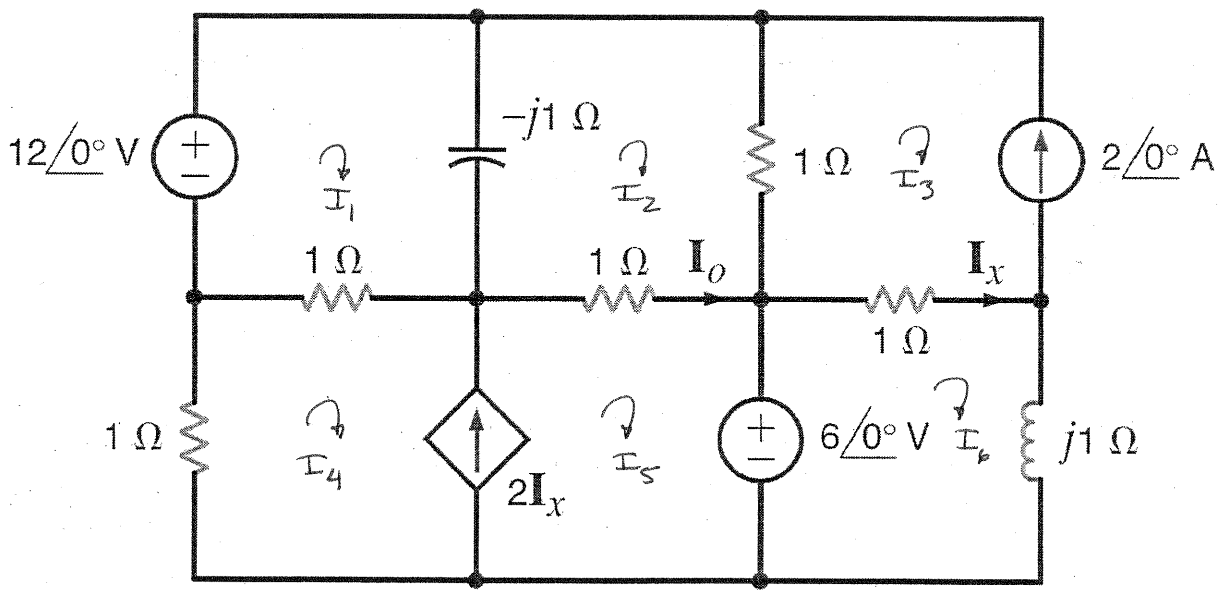


Figure P8.101

SOLUTION: $I_3 = -2\angle 0°\ A$

$I_5 - I_4 = 2I_x = 2(I_6 - I_3) \Rightarrow 2I_3 - I_4 + I_5 - 2I_6 = 0$

$12\angle 0° = I_1(1-j1) + I_2 - I_4$

$I_1(j1) + I_2(2-j1) - I_3 - I_5 = 0$

$6\angle 0° = -I_3 + I_6(1+j1)$

$-I_1 - I_2 + I_4(2) + I_5 = -6\angle 0$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & -2 \\ 1-j1 & j1 & 0 & -1 & 0 & 0 \\ j1 & 2-j1 & -1 & 0 & -1 & 0 \\ -1 & -1 & 0 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 12+j0 \\ 0 \\ -6+j0 \\ 6+j0 \end{bmatrix}$$

$I_o = I_5 - I_2 = 2.12 - j0.471$

$$\boxed{I_o = 2.17\angle -12.5°\ A}$$

8.102 Use MATLAB to find $\mathbf{I}_o$ in the network in Fig. P8.102.

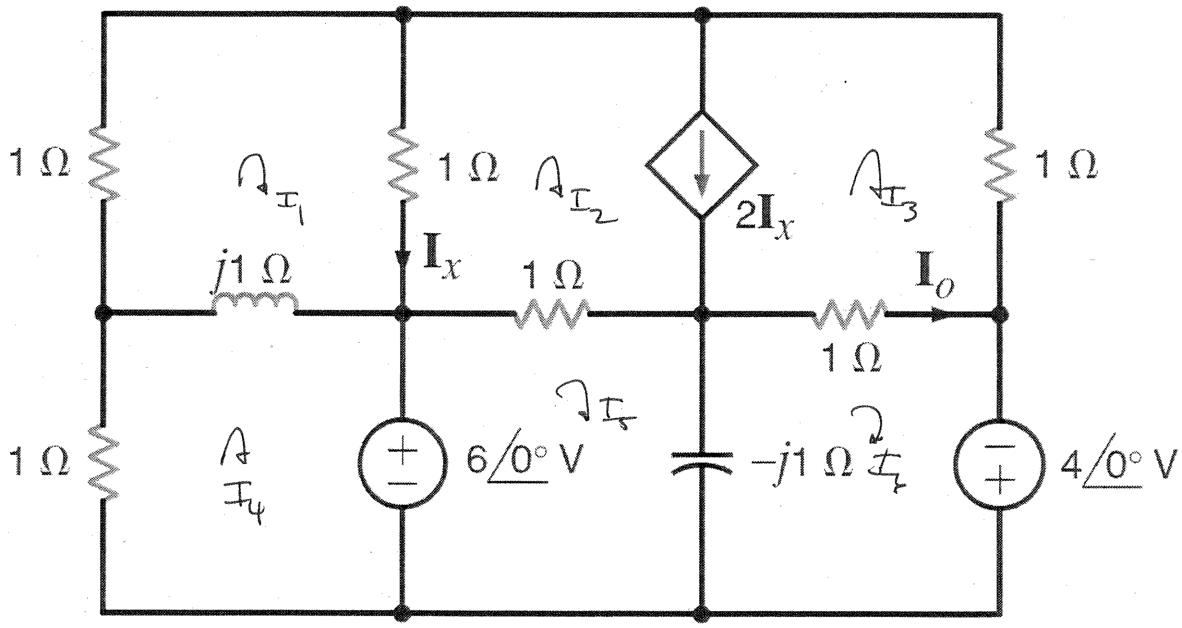


Figure P8.102

SOLUTION:

$I_1(2+j1) - I_2 - j1 I_4 = 0 \qquad I_2 - I_3 = 2I_x = 2(I_1 - I_2) \Rightarrow -2I_1 + 3I_2 - I_3 = 0$

$-j1 I_1 + I_4(1+j1) = -6\angle 0° \qquad -I_2 + I_5(1-j1) + j1 I_6 = 6\angle 0°$

$-I_3 + j I_5 + I_6(1-j1) = 4\angle 0 \qquad I_1 + I_3 + I_4 = 4\angle 0$

$$\begin{bmatrix} 2+j1 & -1 & 0 & -j1 & 0 & 0 \\ -2 & 3 & -1 & 0 & 0 & 0 \\ -j1 & 0 & 0 & 1+j1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1-j1 & j1 \\ 0 & 0 & -1 & 0 & j1 & 1-j1 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -6 \\ 6 \\ 4 \\ 4 \end{bmatrix}$$

```
» z=[2+1i -1 0 -1i 0 0;-2 3 -1 0 0 0;
     -1i 0 0 1+1i 0 0;0 -1 0 0 1-1i 1i;
     0 0 -1 0 1i 1-1i;1 0 1 1 0 0];
» v=[0;0;-6;6;4;4];
» i=inv(z)*v

i =
  -0.7170 - 1.5094i
   1.9623 - 1.1321i
   7.3208 - 0.3774i
  -2.6038 + 1.8868i
   9.4566 - 1.5019i
   9.8264 - 0.0075i

» io=i(6)-i(3)
io = 2.51 + 0.400i
```

$I_o = I_6 - I_3$

$\boxed{I_o = 2.53\angle 8.40°}$

8.103 Use both a nodal analysis and a loop analysis, each in conjunction with MATLAB, to find $\mathbf{I}_o$ in the network in Fig. P8.103.

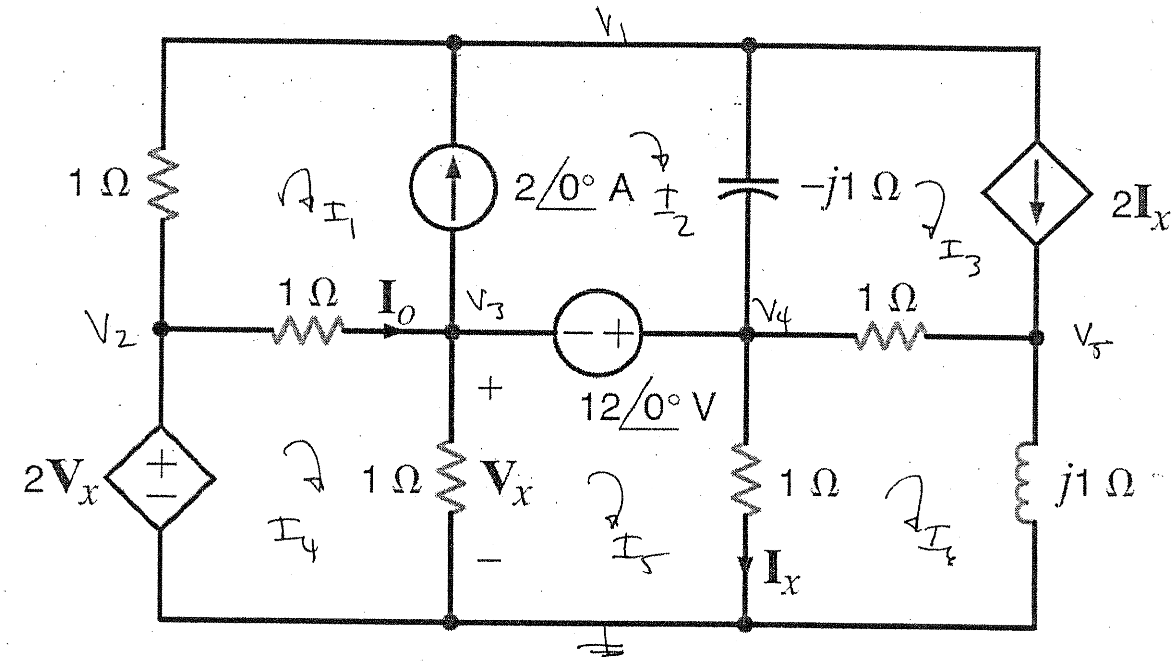


Figure P8.103

SOLUTION:

Loop: $I_2 - I_1 = 2\angle 0°$ $\quad I_3 = 2I_x = 2[I_5 - I_6] \Rightarrow I_3 - 2I_5 + 2I_6 = 0$

$-I_1 + 2I_4 - I_5 = 2V_x = 2[I_4 - I_5](1) \Rightarrow \quad -I_1 + I_5 = 0$

$-I_4 + 2I_5 - I_6 = 12\angle 0$ $\quad -I_3 - I_5 + (2+j1)I_6 = 0$

$2I_1 - jI_2 + jI_3 - I_4 = -12\angle 0$

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2+j1 \\ 2 & -j1 & j1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 12 \\ 0 \\ -12 \end{bmatrix}$$

$$\boxed{I_o = I_4 - I_1 = 17.7\angle -137°\ \text{A}}$$

Nodal: $V_2 = 2V_x = 2V_3 \Rightarrow V_2 - 2V_3 = 0 \qquad V_4 - V_3 = 12\angle 0^\circ$

$$\frac{V_1-V_2}{1} + \frac{V_1-V_4}{-j1} + 2I_x = 2\angle 0^\circ \qquad I_x = \frac{V_4}{1} \Rightarrow \quad V_1(1+j1) - V_2 + V_4(2-j1) = 2\angle 0^\circ$$

$$\frac{V_5-V_4}{1} + \frac{V_5}{j1} - 2I_x = 0 \Rightarrow -3V_4 + V_5(1-j1) = 0$$

$$\frac{V_3-V_2}{1} + \frac{V_3}{1} + \frac{V_4-V_1}{-j1} + \frac{V_4-V_5}{1} + \frac{V_4}{1} = -2\angle 0^\circ$$

$$\to -jV_1 - V_2 + 2V_3 + V_4(2+j1) - V_5 = -2\angle 0^\circ$$

$$\begin{bmatrix} 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 1+j1 & -1 & 0 & 2-j1 & 0 \\ 0 & 0 & 0 & -3 & 1-j1 \\ -j1 & -1 & 2 & 2+j1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 2 \\ 0 \\ -2 \end{bmatrix}$$

$I_o = (V_2 - V_3)/1$

$$\boxed{I_o = 17.7\angle -137^\circ \text{ A}}$$

MATLAB for LOOP

```
» z=[-1 1 0 0 0 0;0 0 1 0 -2 2;-1 0 0 0 1 0;
    0 0 0 -1 2 -1;0 0 -1 0 -1 2+1i;2 -1i 1i -1 0 0];

» v=[2;0;0;12;0;-12];

» i=inv(z)*v;

» io=i(4)-i(1)

io = -13.0 -12.0i
```

MATLAB for NODAL

```
» y=[0 1 -2 0 0;0 0 -1 1 0;1+1i -1 0 2-1i 0;
    0 0 0 -3 1-1i;-1i -1 2 2+1i -1];

» i=[0;12;2;0;-2];

» v=inv(y)*i;

» io=v(2)-v(3)

io = -13.0 -12.0i
```

8.104 Use Thévenin's theorem, in conjunction with MATLAB, to determine $\mathbf{I}_o$ in the network in Fig. P8.104.

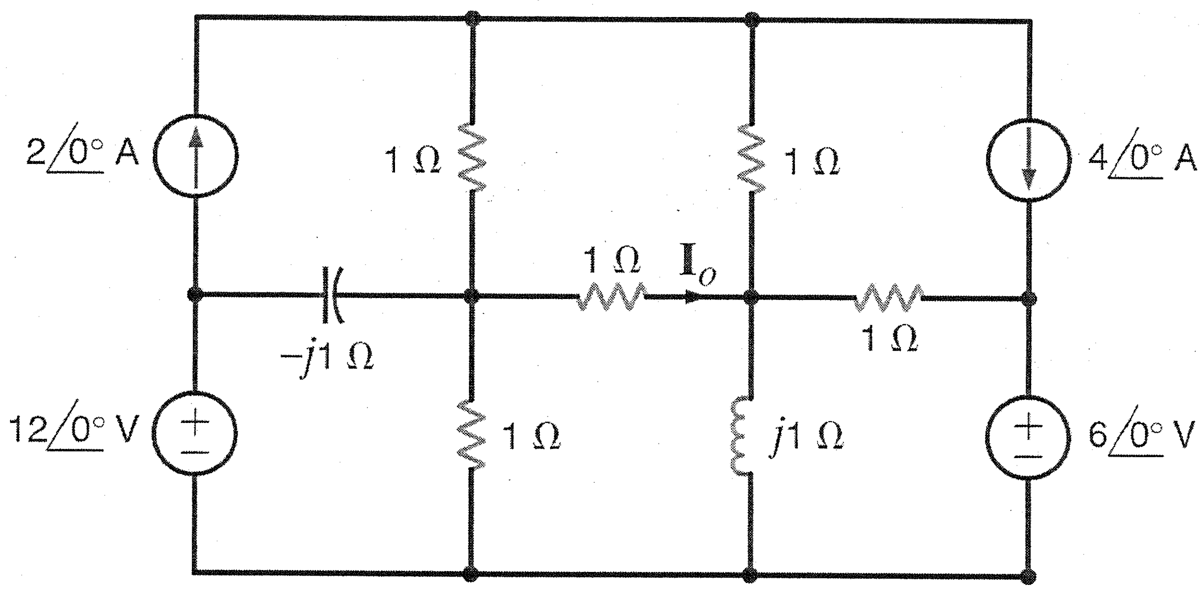
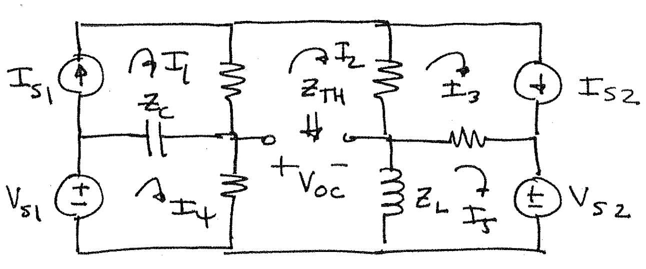


Figure P8.104

SOLUTION:



$Z_c = -j1\ \Omega \qquad Z_L = j1\ \Omega$

all $R = 1\ \Omega$

$2\angle 0° = I_1 \qquad I_3 = 4\angle 0°\ A \qquad j1 I_1 + I_4(1-j1) - I_2 = 12\angle 0°$

$0 = -I_1 - I_3 - I_4 - jI_5 + I_2(3+j1) \qquad -I_3 - jI_2 + I_5(1+j1) = -6\angle 0°$

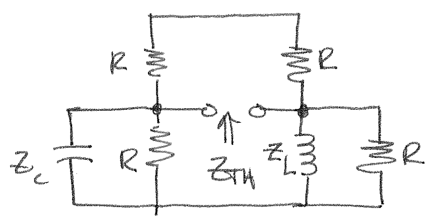
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ j1 & -1 & 0 & 1-j1 & 0 \\ 0 & -j1 & -1 & 0 & 1+j1 \\ -1 & 3+j1 & -1 & -1 & -j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 12 \\ -6 \\ 0 \end{bmatrix}$$

$(I_2 - I_4)1 + V_{oc} + j1(I_2 - I_5) = 0$

$V_{oc} = I_4 - I_2 + j1(I_5 - I_2)$

```
» z=[1 0 0 0 0;0 0 1 0 0;1i -1 0 1-1i 0;0 -1i -1 0 1+1i;-1 3+1i -1 -1 -1i];

» v=[2;4;12;-6;0];

» i=inv(z)*v

i =
    2.0000
    4.0000 + 1.3333i
    4.0000
    8.3333 + 7.6667i
    0.3333 + 3.6667i

» voc=i(4)-i(2)+1i*(i(5)-i(2))

voc = 2.0000 + 2.6667i
```

$V_{oc} = 3.33 \angle 53.1^\circ \text{ V}$

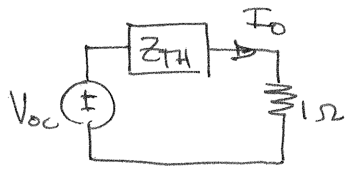


$$Z_{TH} = 2R \,//\, \left[(Z_C // R) + (Z_L // R) \right]$$

$$= 2 \,//\, \left[\tfrac{1}{2} - j\tfrac{1}{2} + \tfrac{1}{2} + j\tfrac{1}{2} \right]$$

$$= 2 // 1 = 2/3 \ \Omega$$

$$I_o = \frac{V_{oc}}{Z_{TH} + 1}$$



$$\boxed{I_o = 2 \angle 53.1^\circ \text{ A}}$$

8.105 Using the PSPICE *Schematics* editor, draw the circuit in Fig. P8.105. At what frequency are the magnitudes of $i_C(t)$ and $i_L(t)$ equal?

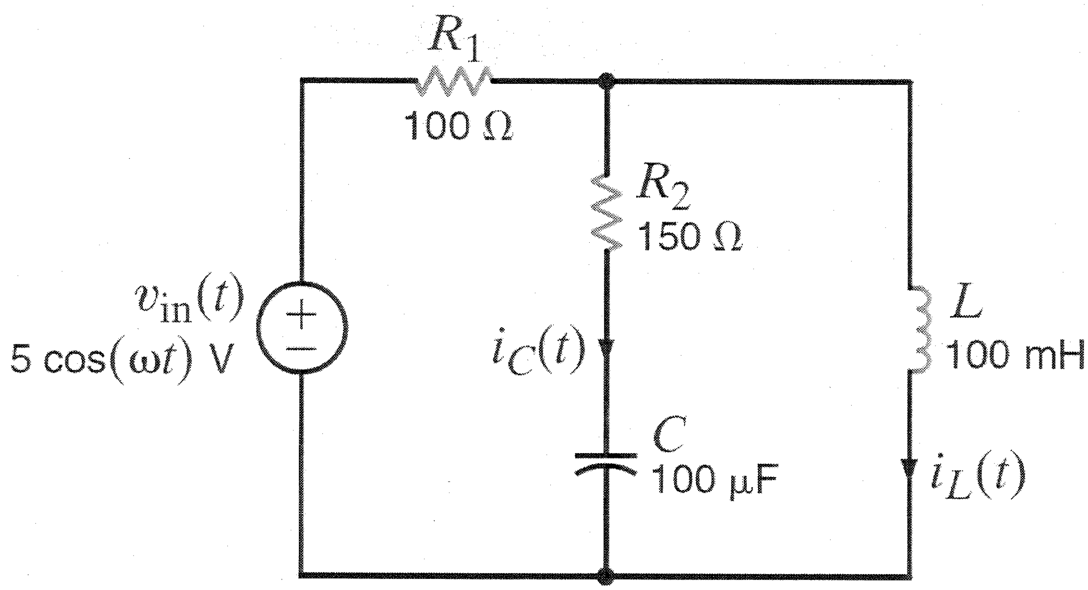
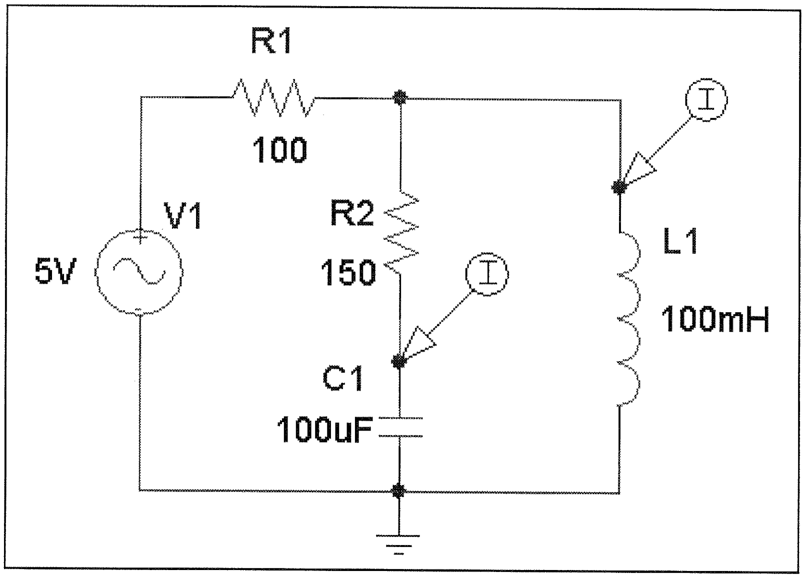
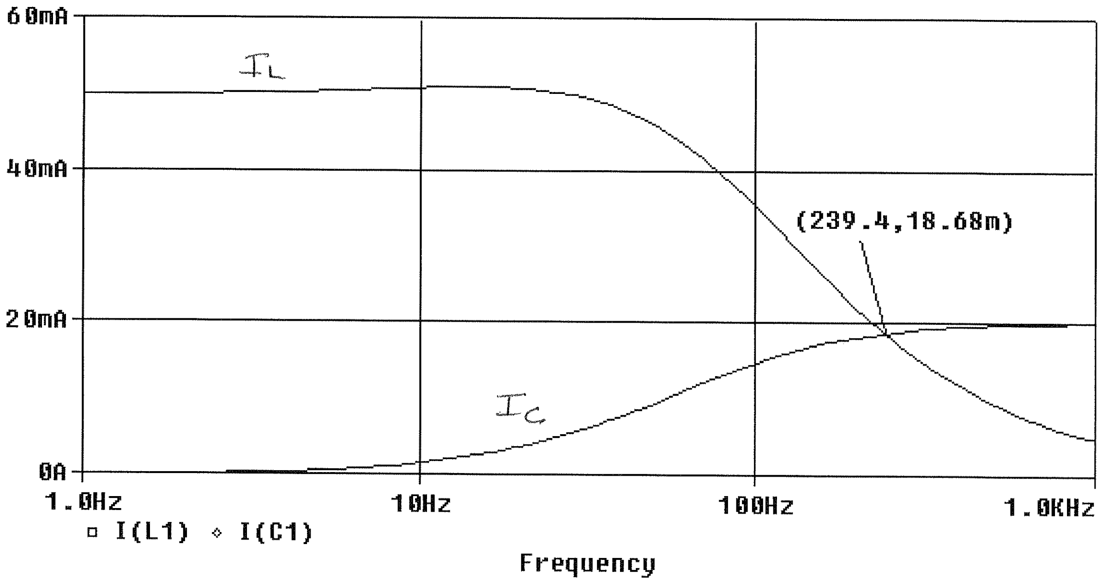


Figure P8.105

SOLUTION: $\boxed{f = 239.4\,Hz}$


R1
100
V1
5V
R2
150
C1
100uF
L1
100mH


60mA
40mA
20mA
0A
I_L
I_C
(239.4,18.68m)
1.0Hz
10Hz
100Hz
1.0KHz
□ I(L1) ◇ I(C1)
Frequency

8.106 Using the PSPICE *Schematics* editor, draw the circuit in Fig. P8.106. At what frequency are the phases of $i_1(t)$ and $v_x(t)$ equal?

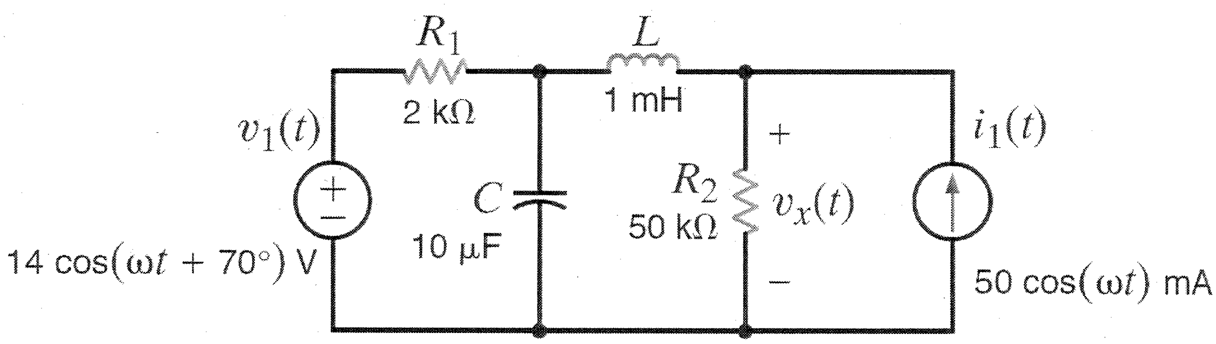
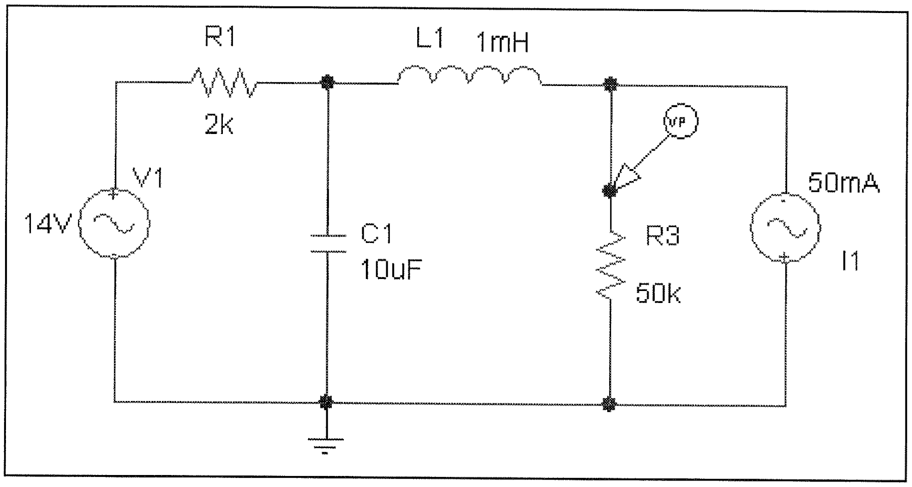
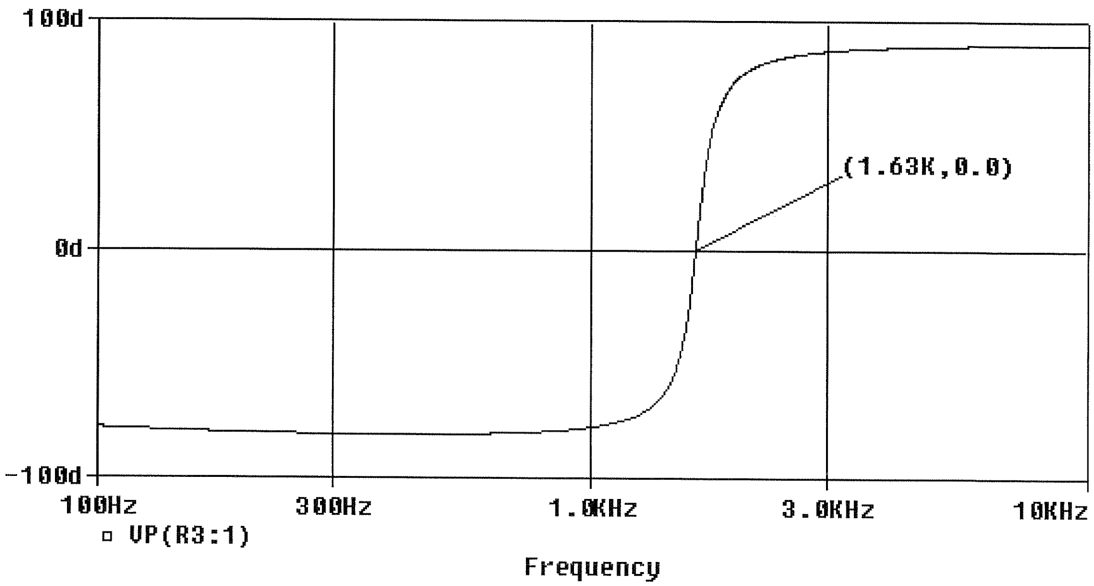


Figure P8.106

SOLUTION: Phase of $i_1(t) = 0°$ find frequency where phase of $v(t)$ is also $0°$.

$$\boxed{f = 1.63\,\text{kHz}}$$

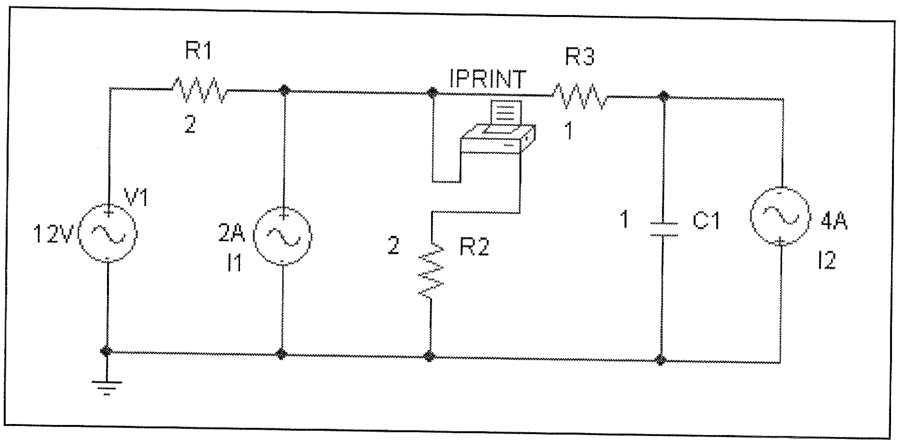

R1
2k
L1 1mH
V1
14V
C1
10uF
R3
50k
50mA
I1
VP


100d
0d
-100d
(1.63K,0.0)
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
VP(R3:1)
Frequency

8.107 Solve Problem 8.48 using PSPICE.

SOLUTION:

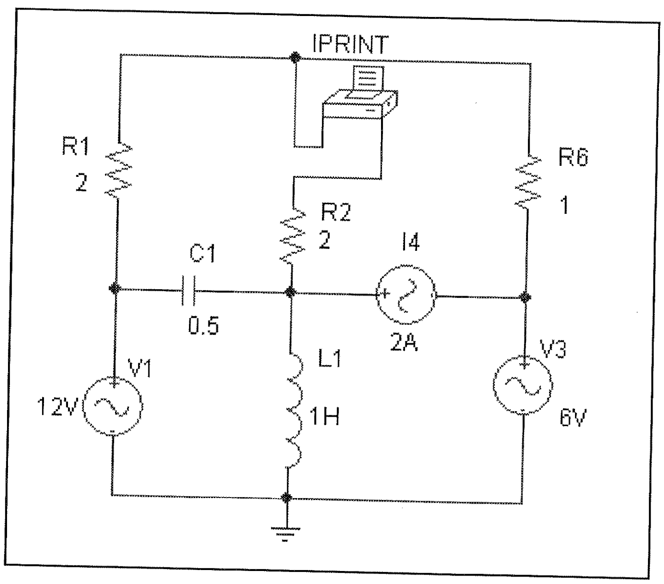
f (Hz)	Imag (A)	Iphase (°)
0.159	2.0	-36.87



8.108 Solve Problem 8.54 using PSPICE.

SOLUTION:

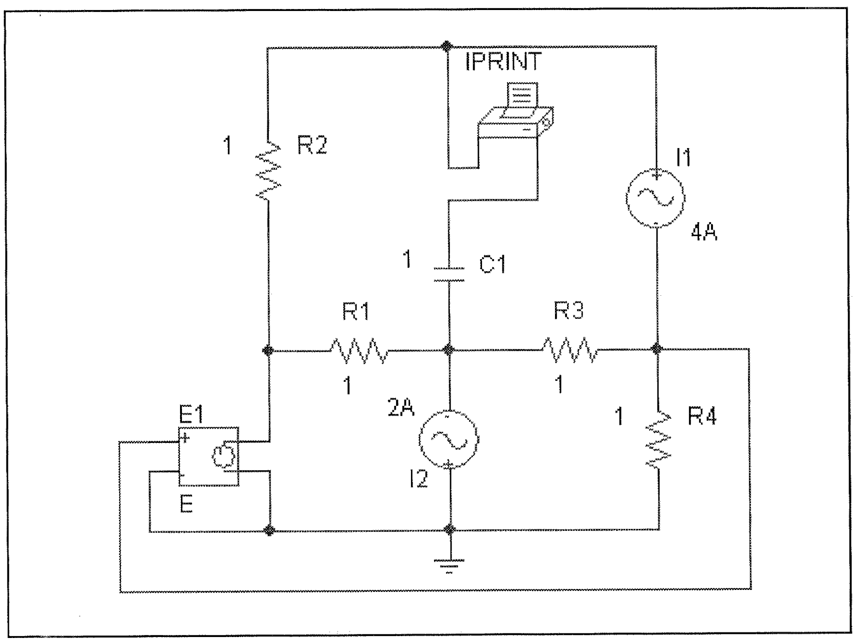
f(Hz)	Imag (A)	Iphase (°)
0.159	6.11	-25.5



8.109 Solve Problem 8.70 using PSPICE.

SOLUTION:

f (Hz)	Imag (A)	Iphase (°)
0.159	1.776E-15	1.800E+02



8.110 Physical inductors are essentially coils of "long" pieces of wire, usually copper with a very thin enamel coating for insulation. Since copper has some resistivity, inductors have some resistance. In most inductors, the coils touch each other. As we learned earlier, conductors in close proximity have capacitance between them. Since the enamel insulation is so thin, the capacitance in an inductor is larger than you would expect for coils of plastic coated wire. Thus, one practical electrical model of a specific inductor is shown in Fig. 8.110. Develop an equation for the inductor's impedance and determine the frequency at which the impedance is real.

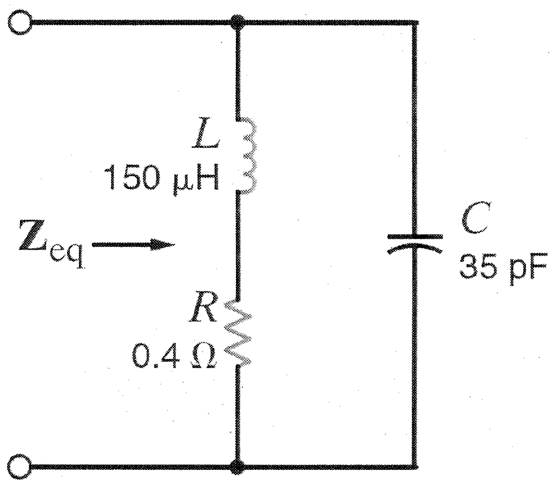


Figure P8.110

SOLUTION: $Z_{eq} = Z_C(R+Z_L)/(R+Z_L+Z_C) = (R+j\omega L)/[1-\omega^2 LC + j\omega CR]$

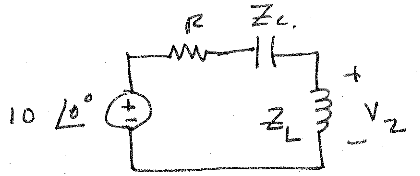
$Z_{eq} = R_{eq} + j0$ requires numerator & denominator have same angle.

$\frac{\omega L}{R} = \frac{\omega CR}{1-\omega^2 LC} \Rightarrow L - \omega^2 L^2 C = R^2 C \Rightarrow \omega^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$

$\omega = 13.88$ Mr/s $\boxed{f = 2.21 \text{ MHz}}$

8.111 We have available a sinusoidal voltage, $v_1(t) = 10\cos\left[2\pi(10^3 t)\right]$ V. We are asked to design a circuit that will produce a second voltage that has the same magnitude as $v_1(t)$ but leads it by 60°. A good starting point for the design is an *RC* voltage divider.

SOLUTION:



$$\frac{V_2}{10\angle 0} = 1\angle 60° = \frac{Z_L}{R + Z_L + Z_C}$$

$$1\angle 60° = \frac{j\omega L}{R + j\omega L - j/\omega C} = \frac{\omega L \angle 90°}{Z_D \angle \theta_D}$$

$$90° - \theta_D = 60° \Rightarrow \theta_D = 30° = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

$$\frac{\omega L - 1/\omega C}{R} = 0.577$$

Arbitrarily select $R = 100\Omega$ yields $\omega L - \frac{1}{\omega C} = 57.7\Omega$

Also, $\frac{(\omega L)^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 1 \Rightarrow (\omega L)^2 = 13323$

$\omega = 2000\pi$ r/s $\Rightarrow L = 18.4$ mH & $C = 2.76\,\mu F$

$$\boxed{R = 100\,\Omega \qquad L = 18.4\text{mH} \qquad C = 2.76\,\mu F}$$

8FE-1 Find $\mathbf{V}_o$ in the network in Fig. 8PFE-1. CS

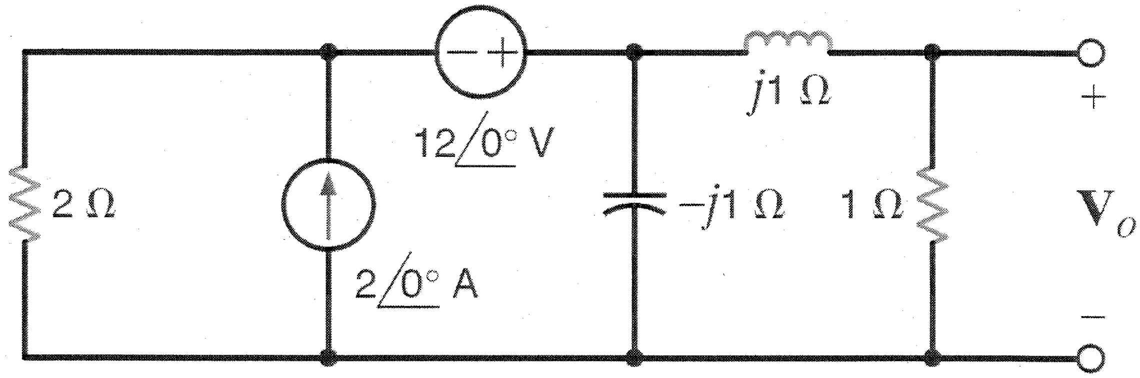
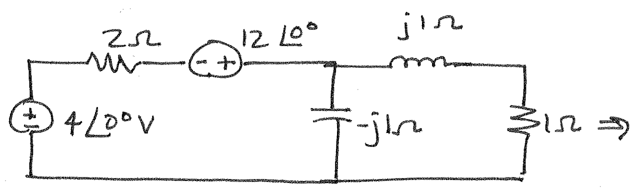
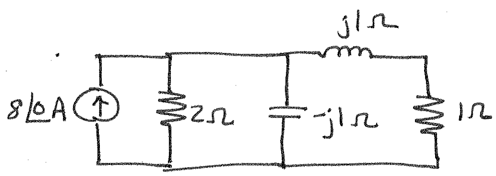


Figure 8PFE-1

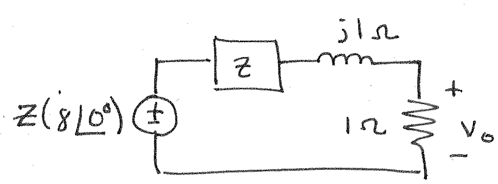
SOLUTION:

Use source transformation:





$$Z = 2(-j1)/(2-j1) = -j2/(2-j1)$$



$$V_o = \frac{Z(8\angle 0^\circ)(1)}{1+j1+Z}$$

$$\boxed{V_o = 5.06\angle -71.6^\circ \text{ V}}$$

8FE-2 Find $\mathbf{V}_o$ in the circuit in Fig. 8PFE-2.

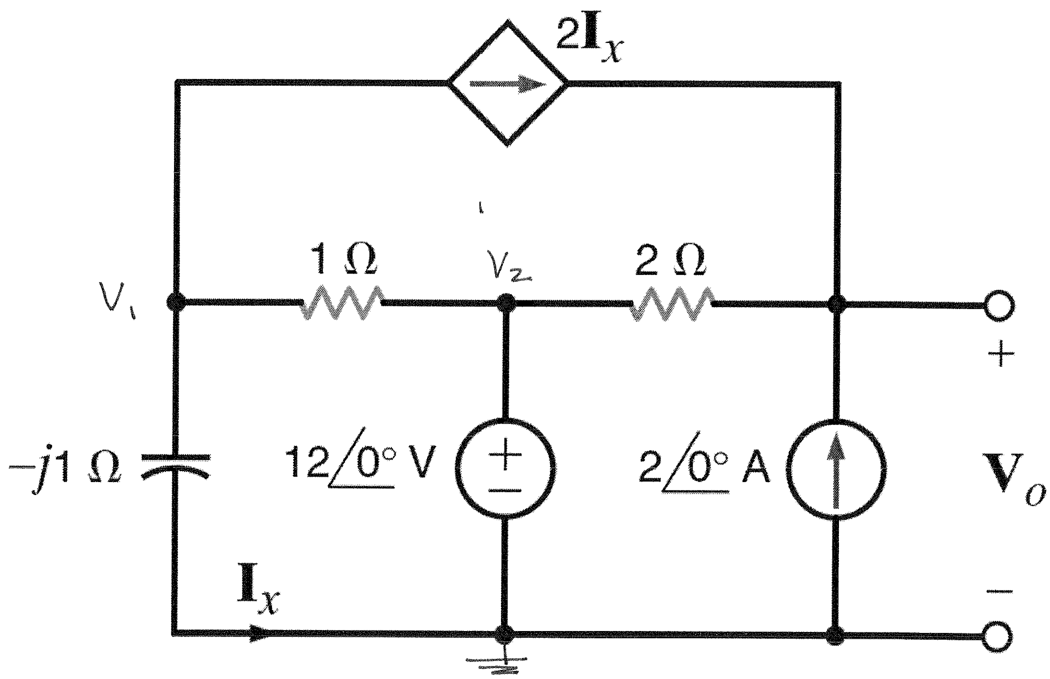


Figure 8PFE-2

SOLUTION:

$V_2 = 12\angle 0°\ V$ $\qquad \frac{V_1 - V_2}{1} + \frac{V_1}{-j1} + 2I_x = 0 \qquad I_x = \frac{V_1}{-j1} \Rightarrow V_1(1+j3) - V_2 = 0$

$\frac{V_2 - V_0}{2} + 2I_x + 2\angle 0 = 0 \Rightarrow V_2 - V_0 + j4 I_1 = -4$

$$\begin{bmatrix} 1+j3 & -1 & 0 \\ j4 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 12 \end{bmatrix} \qquad \boxed{V_0 = 30.8\angle 8.97°\ V}$$

8FE-3 Find $\mathbf{V}_o$ in the network in Fig. 8PFE-3. CS

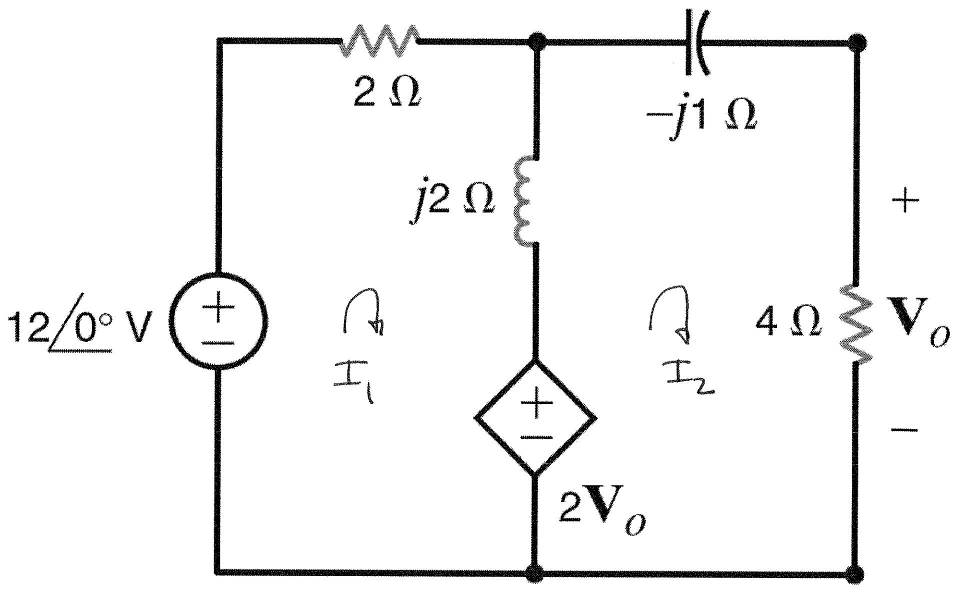


Figure 8PFE-3

SOLUTION:

$12\angle 0 = (2+j2)I_1 - j2I_2 + 2V_o \qquad V_o = 4I_2$

$12\angle 0^\circ = I_1(2+j2) + I_2(8-j2)$

$12\angle 0 = 2I_1 + I_2(4-j1) \qquad V_o = 4I_2$

$$\begin{bmatrix} 2+j2 & 8-j2 \\ 2 & 4-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$$

$I_2 = 2.06\angle -31.0^\circ$ A

$$\boxed{V_o = 8.23\angle -31.0^\circ \text{ V}}$$

8FE-4 Determine the midband (where the coupling capacitors can be ignored) gain of the single-stage transistor amplifier shown in Fig. 8PFE-4.

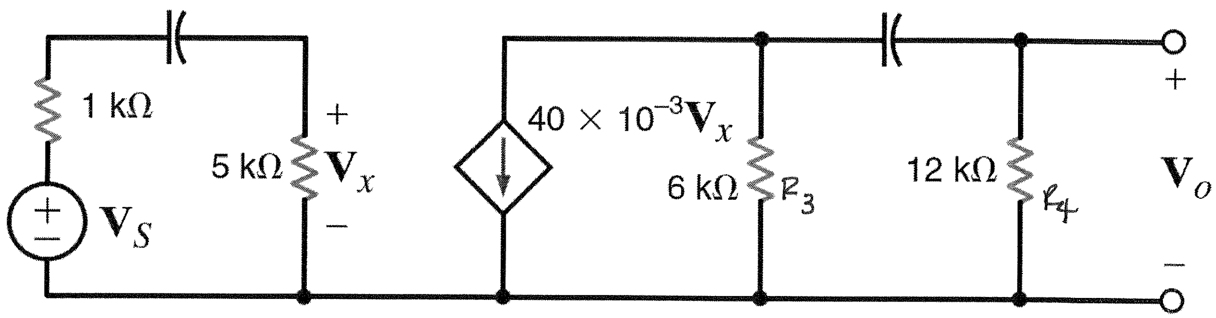


Figure 8PFE-4

SOLUTION:

at midband Z_C is small!

$$\frac{V_x}{V_S} = \frac{5000}{5000+1000} = 5/_6$$

$$V_o = -40\times10^{-3}\,V_x \left\{\frac{R_3 R_4}{R_3+R_4}\right\} \Rightarrow \frac{V_o}{V_x} = -40\times10^{-3}\,(4000) = -160$$

$$\frac{V_o}{V_S} = \frac{V_x}{V_S}\,\frac{V_o}{V_x} \qquad \boxed{\frac{V_o}{V_S} = -133}$$