Chapter Nine:

Steady-State Power Analysis

9.1 Determine the equations for the current and the instantaneous power in the network in Fig. P9.1. CS

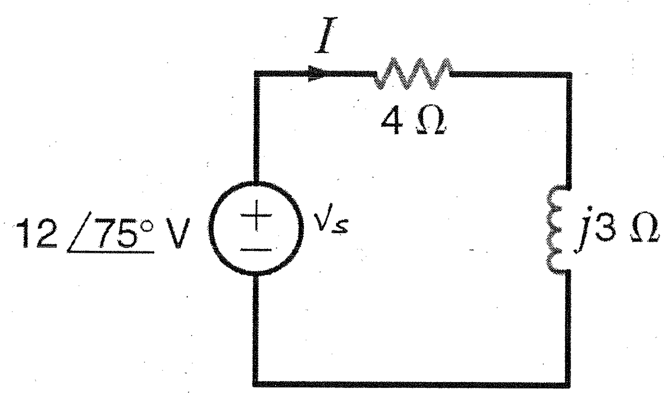


Figure P9.1

SOLUTION:

$$I = \frac{12\angle 75^\circ}{4+j3}$$

$$\boxed{\begin{aligned} I &= 2.4\angle 38.1^\circ \text{ A} \\ i(t) &= 2.4\cos(\omega t + 38.1^\circ) \text{ A} \end{aligned}}$$

$p(t) = i(t)\, v_s(t)$ $\qquad v_s(t) = 12\cos(\omega t + 75^\circ)$ V

$$p(t) = 28.8\left[\cos(\omega t + 38.1^\circ)\cos(\omega t + 75^\circ)\right]$$

$$\boxed{p(t) = 14.4\cos(2\omega t + 113.1^\circ) + 11.5 \quad \text{W}}$$

9.2 Determine the equations for the voltage and the instantaneous power in the network in Fig. P9.2.

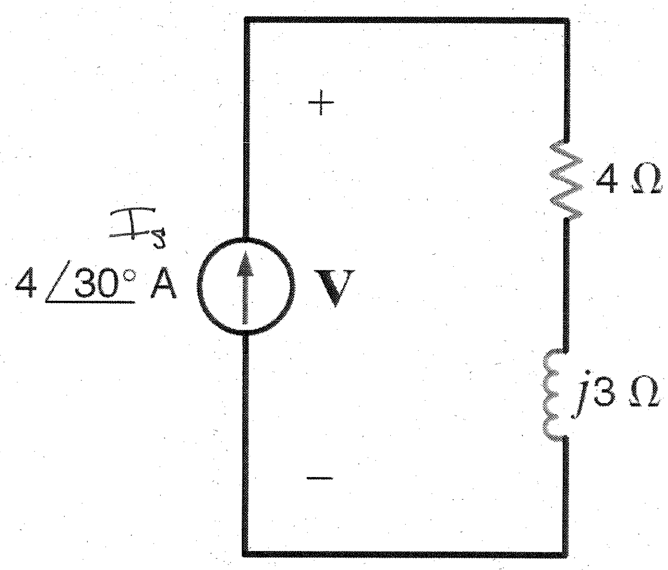


Figure P9.2

SOLUTION:

$V = 4\angle 30\,(4+j3) = 20\angle 66.9°\text{ V}$

$\boxed{\begin{array}{l} V = 20\angle 66.9°\text{ V} \\ v(t) = 20\cos(\omega t + 66.9°)\text{ V} \end{array}}$

$i_s(t) = 4\cos(\omega t + 30°)\text{ A}$

$p(t) = i_s(t)\,v(t) = 80\cos(\omega t + 30°)\cos(\omega t + 66.9°)\text{ W}$

$\boxed{p(t) = 40\cos(2\omega t + 96.9°) + 32.0\text{ W}}$

9.3 The voltage and current at the input of a circuit are given by the expressions

$$v(t) = 170\cos(\omega t + 30^\circ)\ \mathrm{V}$$
$$i(t) = 5\cos(\omega t + 45^\circ)\ \mathrm{A}$$

Determine the average power absorbed by the circuit.

SOLUTION:

$$P = \frac{V_M I_M}{2}\cos(\theta_v - \theta_i) = \frac{170(5)}{2}\left[\cos(30-45)\right]$$

$$\boxed{P = 411\ \mathrm{W}}$$

9.4 The voltage and current at the input of a network are given by the expressions

$$v(t) = 6\cos\omega t \text{ V}$$
$$i(t) = 4\sin\omega t \text{ A}$$

Determine the average power absorbed by the network.

SOLUTION:

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$i(t) = 4\cos(\omega t - 90°) \Rightarrow \theta_i = -90°$$

$$\boxed{P = 0\text{W}}$$

9.5 Find the average power absorbed by the resistor in the circuit shown in Fig. P9.5 if $v_1(t) = 10\cos(377t + 60°)$ V and $v_2(t) = 20\cos(377t + 120°)$ V.

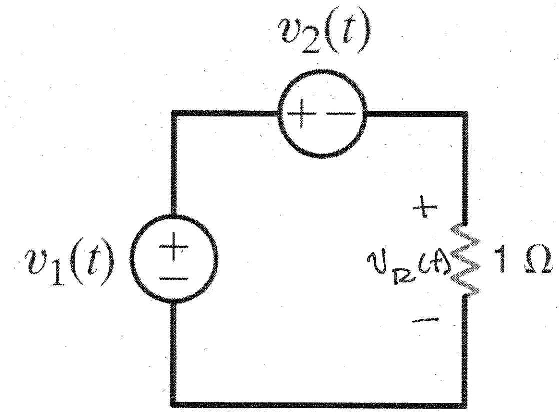


Figure P9.5

SOLUTION:

$V_1 = 10\angle 60°$ V $\qquad V_2 = 20\angle 120°$ V

$V_R = V_1 - V_2 = 17.3\angle -30°$ V

$P_R = \dfrac{V_m^2}{2R} \Rightarrow \boxed{P = 150\text{W}}$

9.6 Find the average power absorbed by the resistor in the circuit shown in Fig. P9.6. Let $i_1(t) = 4\cos(377t + 60°)$ A, $i_2(t) = 6\cos(754t + 10°)$ A, and $i_3(t) = 4\cos(377t - 30°)$ A.

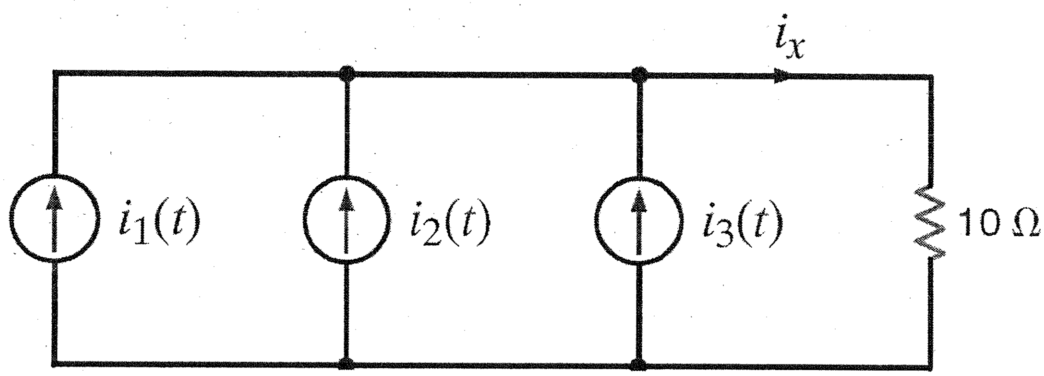


Figure P9.6

SOLUTION:

$$I_x = I_1 + I_2 + I_3 = 4\angle 60° + 6\angle 10° + 4\angle -30° = 11.64\angle 12.4°\ A$$

$$P = \frac{I_M^2}{2} R \qquad \boxed{P = 678\ W}$$

9.7 Compute the average power absorbed by each of the elements to the right of the dashed line in the circuit shown in Fig. P9.7.

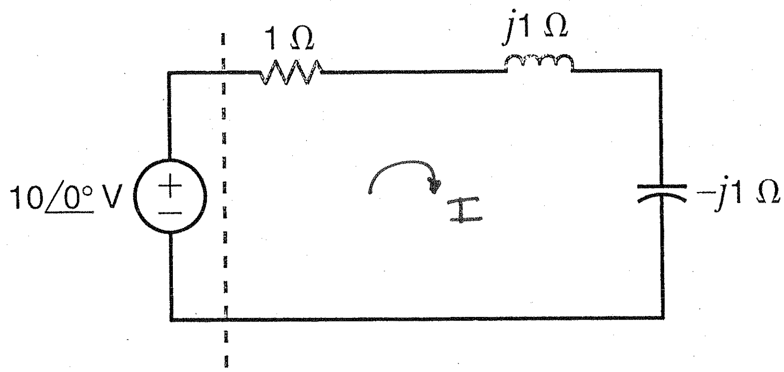


Figure P9.7

SOLUTION

The inductor & capacitor consume zero average power. As for the resistor,

$$P_R = \frac{|I|^2}{2} R$$

and $$I = \frac{10\angle 0^\circ}{1 + j1 - j1} = 10\angle 0^\circ \text{ A}$$

$$P_R = \frac{(10)^2}{2}(1) = 50\text{W}$$

$$\boxed{\begin{aligned} P_R &= 50\text{W} \\ P_L &= 0\text{W} \\ P_C &= 0\text{W} \end{aligned}}$$

9.8 Determine the average power supplied by each source in the network shown in Fig. P9.8. CS

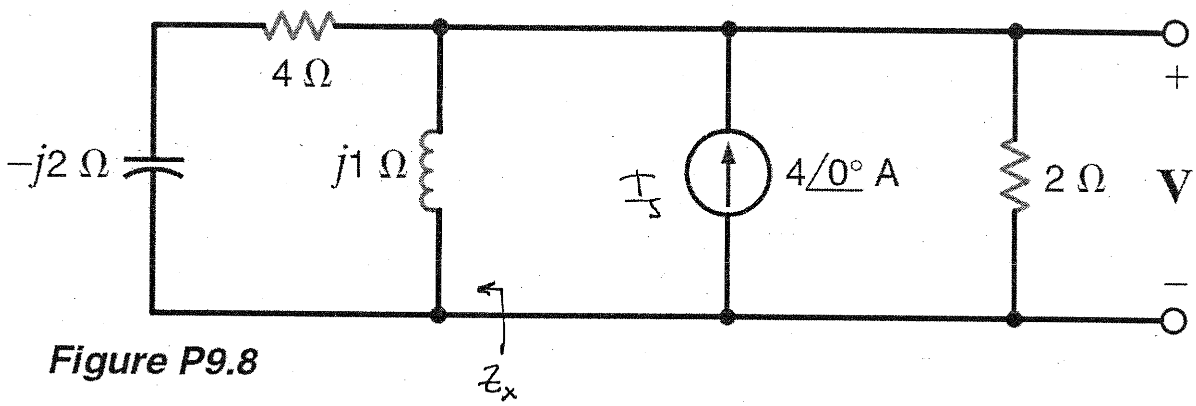
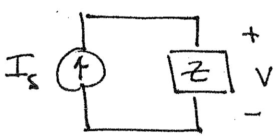


Figure P9.8

SOLUTION:



$$Z = \frac{2Z_x}{2+Z_x} \qquad Z_x = \frac{j1\,(4-j2)}{4-j1} = 1.09\,\angle 77.5^\circ\ \Omega$$

$$Z = 0.539 + j\,0.692\ \Omega = R_{eq} + j\,X_{eq}$$

Only the resistive part of Z consumes average power.

$$P = \frac{I_M^{\,2}}{2} R_{eq} \qquad P = \frac{(4)^2}{2}(0.539) \qquad \boxed{P = 4.31\text{ W}}$$

9.9 Find the average power absorbed by the network shown in Fig. P9.9.

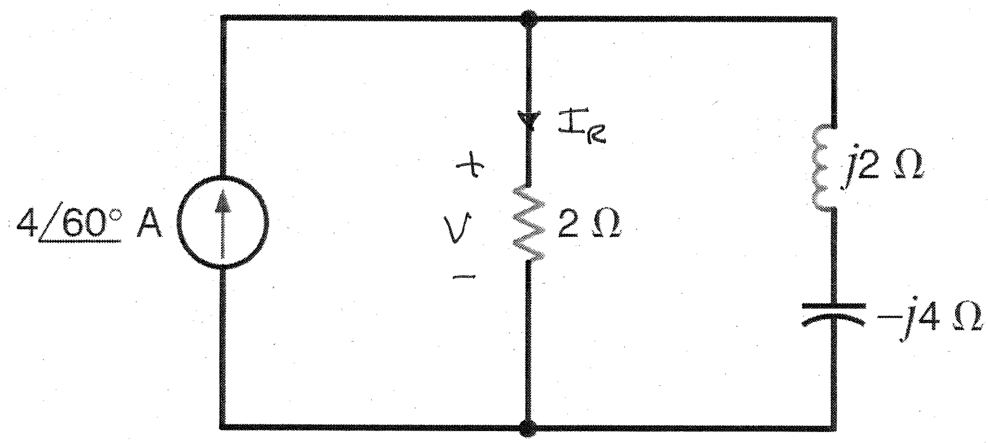


Figure P9.9

SOLUTION:

$$I_R = 4\angle 60° \left[\frac{-j2}{2-j2}\right] = 2.83\angle 15° \text{ A}$$

$$V = 2I_R = 5.66\angle 15° \text{ V}$$

$$P = \frac{V_m I_{RM}}{2} \qquad \boxed{P = 8 \text{ W}}$$

9.10 Given the network in Fig. P9.10, find the power supplied and the average power absorbed by each element. **PSV**

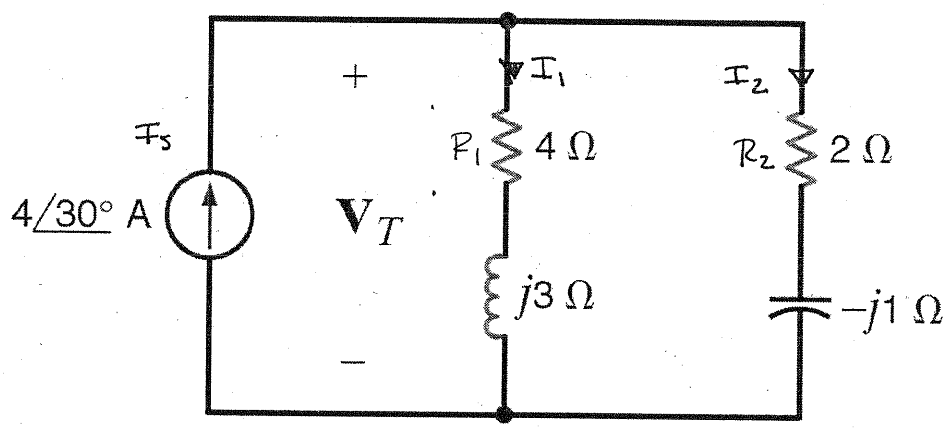


Figure P9.10

SOLUTION:

Let $Z_1 = 4+j3\,\Omega$ and $Z_2 = 2-j1\,\Omega$ ∴ $Z_T = \dfrac{Z_1 Z_2}{Z_1+Z_2} = 1.75 - j0.25\,\Omega$

$V_T = 4\angle 30°\, Z_T = 7.07\angle 21.9°$ V

for the source: $P_S(t) = \dfrac{4(7.07)}{2}\left[\cos(2\omega t + 68.13°) + \cos 8.13°\right]$

$$\boxed{P_S(t) = 14.14 \cos(2\omega t + 51.9°) + 14.0 \text{ W}}$$

$I_1 = V_T / Z_1 = 1.41\angle -15°$ A $\qquad I_2 = V_T/Z_2 = 3.16\angle 48.4°$ A

$P_{R1} = \dfrac{I_{1M}^2}{2} R_1 = 4.0$ W $\qquad P_{R2} = \dfrac{I_{2M}^2}{2} R_2 = 10$ W

$P_{R1} = 4.0$W	$P_L = 0$
$P_{R2} = 10$ W	$P_C = 0$

9.11 Given the network in Fig. P9.11, determine which elements are supplying power, which ones are absorbing power, and how much power is being supplied and absorbed.

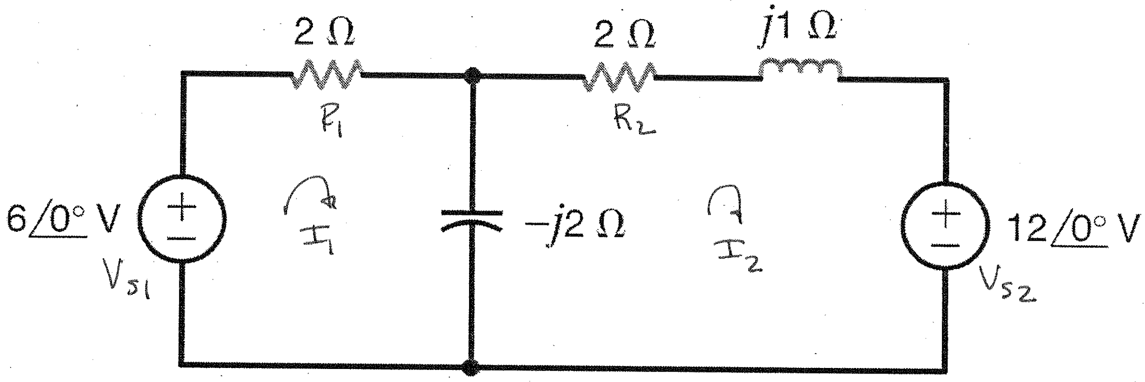


Figure P9.11

SOLUTION:

$6 = I_1(2-j2) + j2I_2$ & $-12 = j2I_1 + I_2(2-j1)$

$$\begin{bmatrix} 2-j2 & j2 \\ j2 & 2-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \end{bmatrix} \Rightarrow \begin{cases} I_1 = 2.55\angle 101^\circ \text{ A} \\ I_2 = 3.16\angle -161^\circ \text{ A} \end{cases}$$

$P_{R_1} = \dfrac{I_{1M}^2}{2} R_1 = 6.5\text{ W}$ absorbed $\quad P_L = 0$

$P_{R2} = \dfrac{I_{2M}^2}{2} R_2 = 10\text{ W}$ absorbed $\quad P_C = 0$

$P_{S1} = \dfrac{V_{S1M} I_{1M}}{2} \cos(\theta_{V_{S1}} - \theta_{I_1}) = -1.46\text{ W}$ supplied

So, V_{S1} absorbs 1.46W (V_{S1} & I_1 have active sign format)

$P_{S2} = \dfrac{V_{S2M} I_{2M}}{2} \cos(\theta_{V_{S2}} - \theta_{I_2}) = -17.96\text{ W}$

So, V_{S2} delivers 17.96W (V_{S2} & I_2 have passive sign format)

9.12 Given the network in Fig. P9.12, show that the power supplied by the sources is equal to the power absorbed by the passive elements.

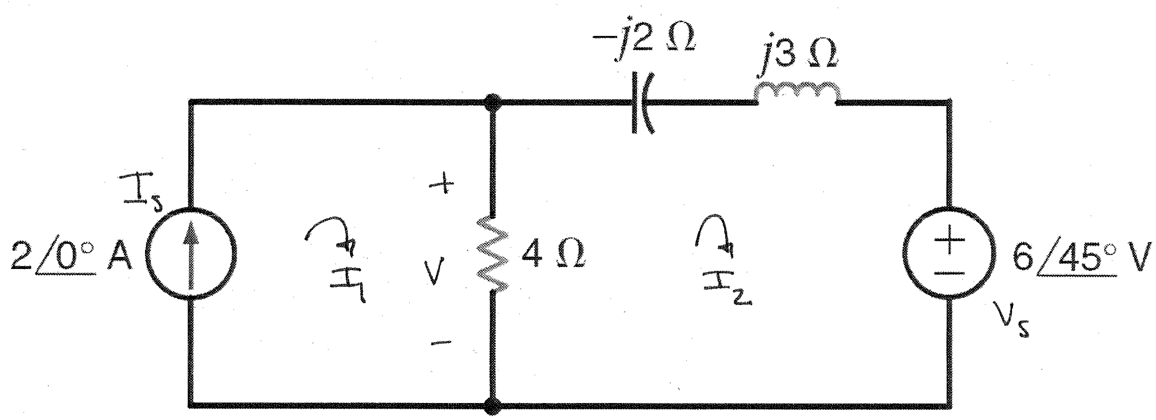


Figure P9.12

SOLUTION:

$I_1 = 2\angle 0°$ A $\qquad -4I_1 + I_2(4+j1) = -6\angle 45 \qquad \begin{bmatrix} 1 & 0 \\ -4 & 4+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -6\angle 45° \end{bmatrix}$

$I_2 = 1.37\angle -62.5°$ A $\qquad V = 4(I_1 - I_2) = 7.32\angle 41.8°$ V

I_s supplies $P_{I_s} = \dfrac{I_{sm} V_M}{2}\cos(\theta_V - \theta_{I_s}) \Rightarrow P_{I_s} = 5.46$ W

V_s absorbs $P_{V_s} = \dfrac{V_{sm} I_{2M}}{2}\cos(\theta_{V_s} - \theta_{I_2}) \Rightarrow P_{V_s} = -1.24$ W

So, V_s actually delivers 1.24W

$P_R = \dfrac{V_m^2}{2R} = 6.70$ W $\qquad P_L = P_c = 0$ W

$\left.\begin{array}{l}\text{Power supplied} = 5.46 + 1.24 = 6.70\text{W} \\ \text{Power absorbed} = P_R = 6.70\text{W}\end{array}\right\}$ balance!

9.13 Calculate the average power absorbed by the 1-Ω resistor in the network shown in Fig. P9.13.

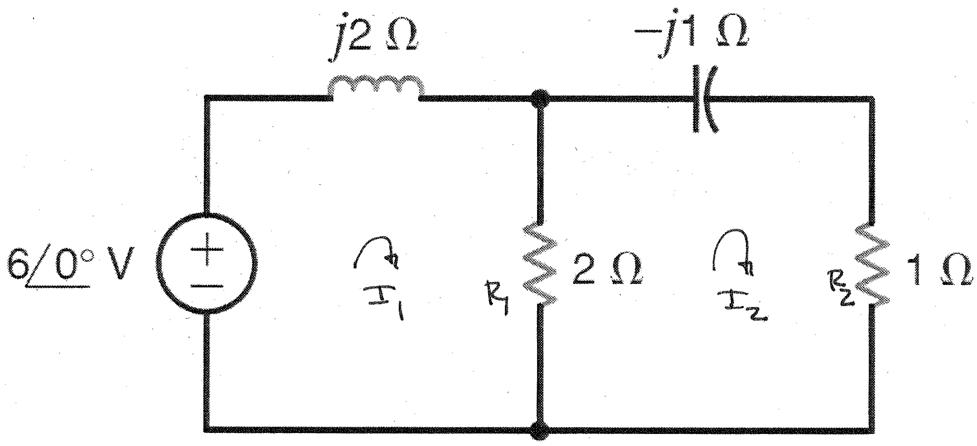


Figure P9.13

SOLUTION:

$6\angle 0 = I_1(2+j2) - 2I_2 \quad \& \quad -2I_1 + I_2(3-j1) = 0$

$$\begin{bmatrix} 2+j2 & -2 \\ -2 & 3-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \Rightarrow I_2 = 2.12\angle -45° \text{ A}$$

$$P_{1\Omega} = \frac{I_{2M}^2}{2}(R_2) \qquad \boxed{P_{1\Omega} = 2.25 \text{ W}}$$

9.14 Given the network in Fig. P9.14, find the average power supplied to the circuit. CS

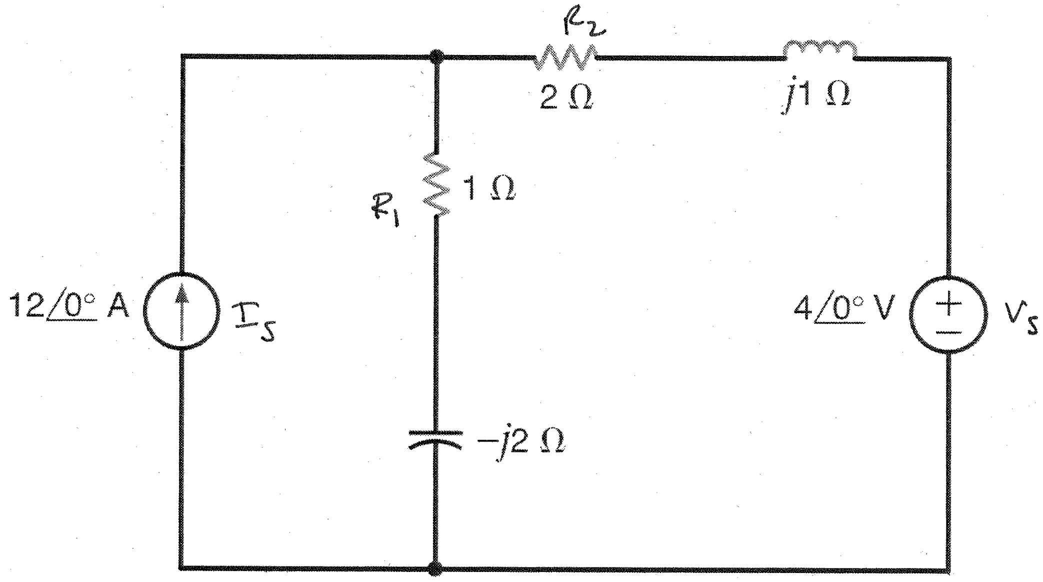
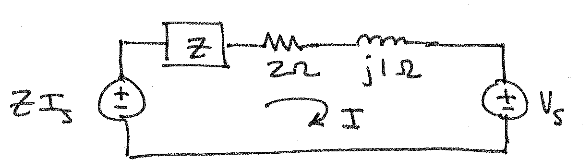


Figure P9.14

SOLUTION:

Let $Z = 1 - j2\Omega$ & use source transformation.



$ZI_S - V_S = I(Z + 2 + j1)$

$I = 8\angle -53.1^\circ$ A

$P_{R_1} = \frac{I_M^2}{2} R_1 = 32W$

$P_{R_2} = \frac{I_M^2}{2} R_2 = 64W$

$P_L = P_C = 0$

$P_{\text{supplied to circuit}} = P_{R_1} + P_{R_2} = 96W$

9.15 Given $v_S(t) = 100 \cos 100t$ volts, find the average power supplied by the source and the current $i_2(t)$ in the network in Fig. P9.15.

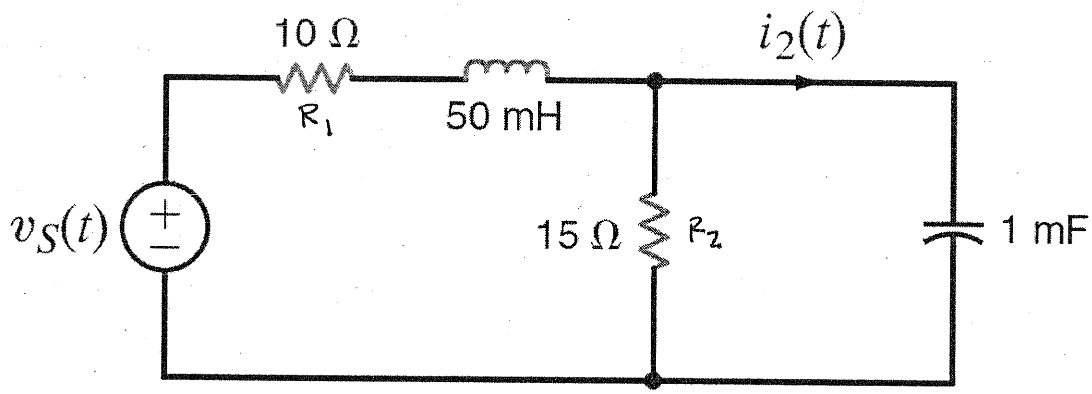
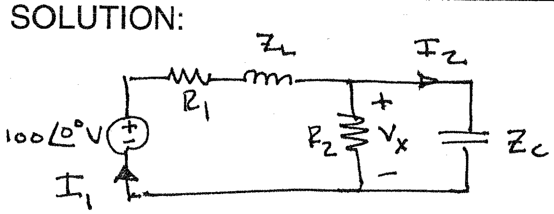


Figure P9.15

SOLUTION:



$Z_L = j5\,\Omega \qquad Z_C = -j10\,\Omega$

Let $Z_x = \dfrac{R_2 Z_C}{R_2 + Z_C} = \dfrac{-j30}{3 - j2}$

$V_x = \dfrac{100\angle 0 \; Z_x}{R_1 + Z_L + Z_x} \Rightarrow V_x = 56.4 \angle -48.8^\circ \text{ V}$

$I_2 = V_x / Z_C = 5.64 \angle 41.2^\circ \text{ A}$ $\qquad \boxed{i_2(t) = 5.64 \cos(100t + 41.2^\circ) \text{ A}}$

Let $Z_y = R_1 + Z_L + Z_x = 14.7 \angle -7.49^\circ\ \Omega$

$I_1 = V_S / Z_y = 6.78 \angle 7.49^\circ \text{ A}$

$P_{V_S} = \dfrac{V_{SM} I_{IM}}{2} \cos(0 - 7.49^\circ)$ $\qquad \boxed{P_{V_S} = 336 \text{ W}}$

9.16 If $i_g(t) = 0.5 \cos 2000t$ A, find the average power absorbed by each element in the circuit in Fig. P9.16.

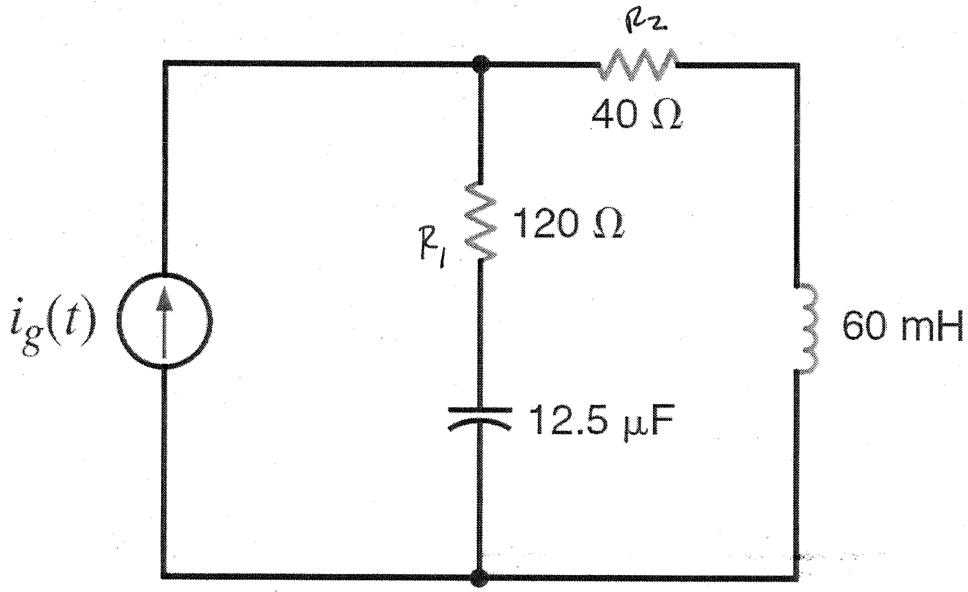
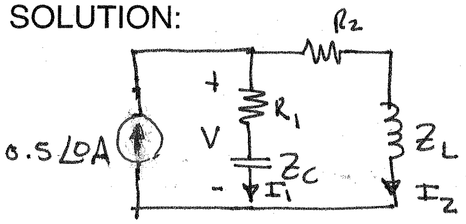


Figure P9.16

SOLUTION:



$Z_L = j120\,\Omega \qquad Z_C = -j40\,\Omega$

Let $Z_1 = R_1 + Z_C = 120 - j40\,\Omega$

$Z_2 = R_2 + Z_L = 40 + j120\,\Omega$

$Z_3 = Z_1 Z_2/(Z_1 + Z_2) = 80 + j40\,\Omega$

$V = (0.5\angle 0°) Z_3$

$V = 44.7\angle 26.6°$ V

$I_1 = V/Z_1 = 0.354\angle 45°$ A $\qquad I_2 = V/Z_2 = 0.354\angle -45°$ A

$P_{R1} = \dfrac{I_{1m}^2}{2} R_1 = 7.5\,W \qquad P_{R2} = \dfrac{I_{2m}^2}{2} R_2 = 2.5\,W$

$P_{I_s} = -\dfrac{I_{sm} V_m}{2} \cos(0 - 26.6°) = -10.0\,W$

$P_L = P_C = 0\,W$

9.17 Calculate the average power absorbed by the 1-Ω resistor in the network shown in Fig. P9.17.

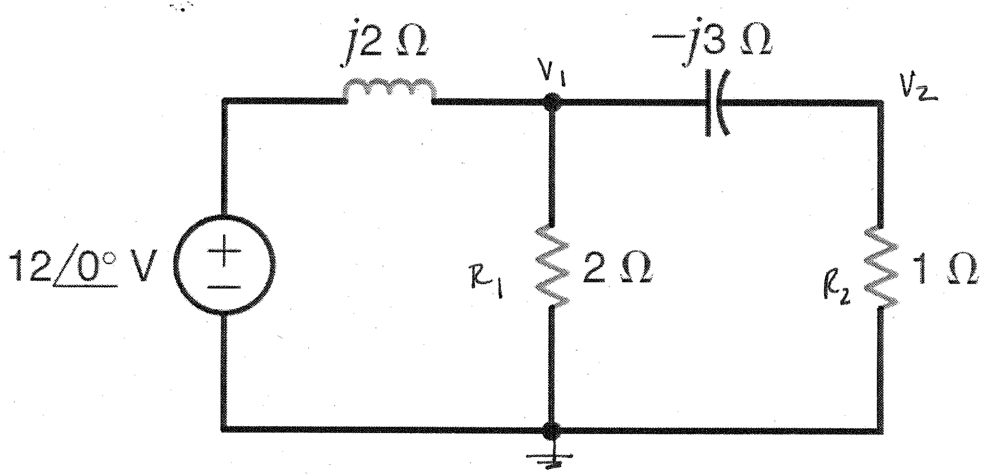


Figure P9.17

SOLUTION:

$$\frac{V_1 - 12}{j2} + \frac{V_1}{2} + \frac{V_1}{1-j3} = 0 \Rightarrow V_1 = 3 - j9 \text{ V} = 9.49\angle -71.6^\circ \text{ V}$$

$$V_2 = \frac{V_1(1)}{1-j3} = 3\angle 0^\circ \text{ V}$$

$$P_{1\Omega} = \frac{V_{2m}^2}{2R_2}$$

$$\boxed{P_{1\Omega} = 4.5\text{W}}$$

9.18 Find the average power supplied and/or absorbed by each element in Fig. P9.18.

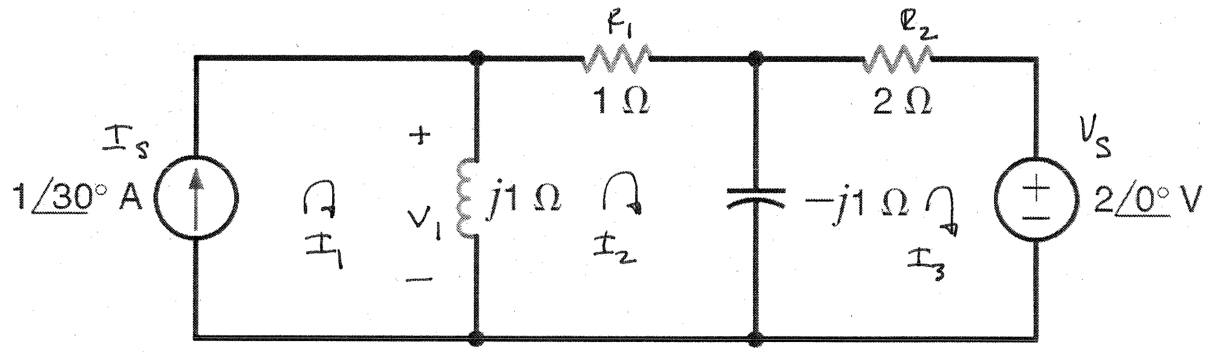


Figure P9.18

SOLUTION:

$I_1 = 1\angle 30°$ A $\quad -j1 I_1 + I_2(1) + j1 I_3 = 0 \quad j1 I_2 + I_3(2-j1) = -2\angle 0°$

$$\begin{bmatrix} 1 & 0 & 0 \\ -j1 & 1 & j1 \\ 0 & j1 & 2-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1\angle 30° \\ 0 \\ -2\angle 0° \end{bmatrix} \Rightarrow \begin{matrix} I_1 = 1.0\angle 30° \text{ A} \\ I_2 = 1.34\angle 110° \text{ A} \\ I_3 = 0.392\angle 174° \text{ A} \end{matrix}$$

$P_L = P_C = 0 \quad P_{R_1} = \frac{I_{2m}^2}{2} R_1 = 0.90\text{W} \quad P_{R_2} = \frac{I_{3m}^2}{2} R_2 = 0.154\text{W}$

$V_1 = j1(I_1 - I_2) \Rightarrow V_1 = 1.53\angle 60.4°$ V

$P_{I_S} = \frac{V_{1m} I_{Sm}}{2} \cos(60.4 - 30) = 0.660$ W supplied

$P_{V_S} = -\frac{V_{Sm} I_{3m}}{2} \cos(0 - 174°) = 0.394\text{W}$ supplied

power supplied = power absorbed

9.19 Determine the average power absorbed by the 4-Ω in the network shown in Fig. P9.19. **PSV**

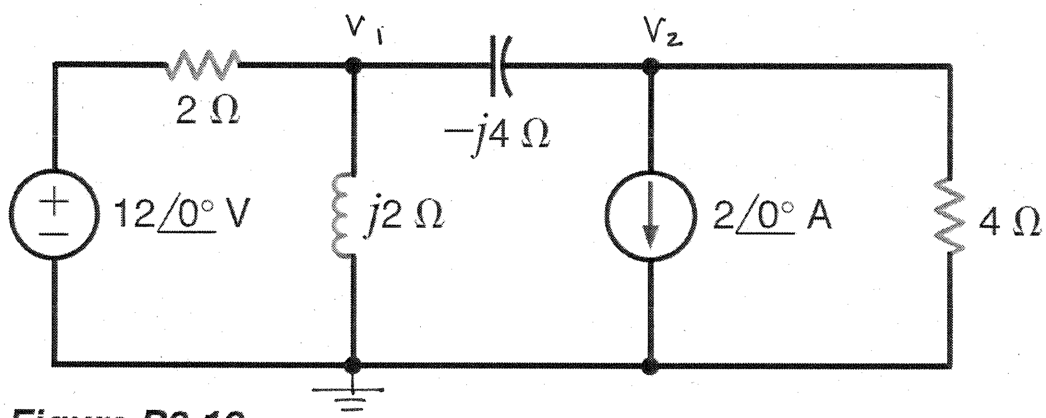


Figure P9.19

SOLUTION:

$$\frac{V_1 - 12\angle 0}{2} + \frac{V_1}{j2} + \frac{V_1 - V_2}{-j4} = 0 \Rightarrow V_1(2-j1) - jV_2 = 24$$

$$\frac{V_2 - V_1}{-j4} + 2\angle 0^\circ + \frac{V_2}{4} = 0 \Rightarrow -jV_1 + V_2(1+j1) = -8$$

$$\begin{bmatrix} 2-j1 & -j1 \\ -j1 & 1+j1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 24 \\ -8 \end{bmatrix} \qquad V_2 = 8.68\angle 102.6^\circ \text{ V}$$

$$P_{4\Omega} = \frac{V_{2M}^2}{2(4)} \qquad \boxed{P_{4\Omega} = 9.42 \text{ W}}$$

9.20 Given the network in Fig. P9.20, find the total average power supplied and the average power absorbed in the 4-Ω resistor.

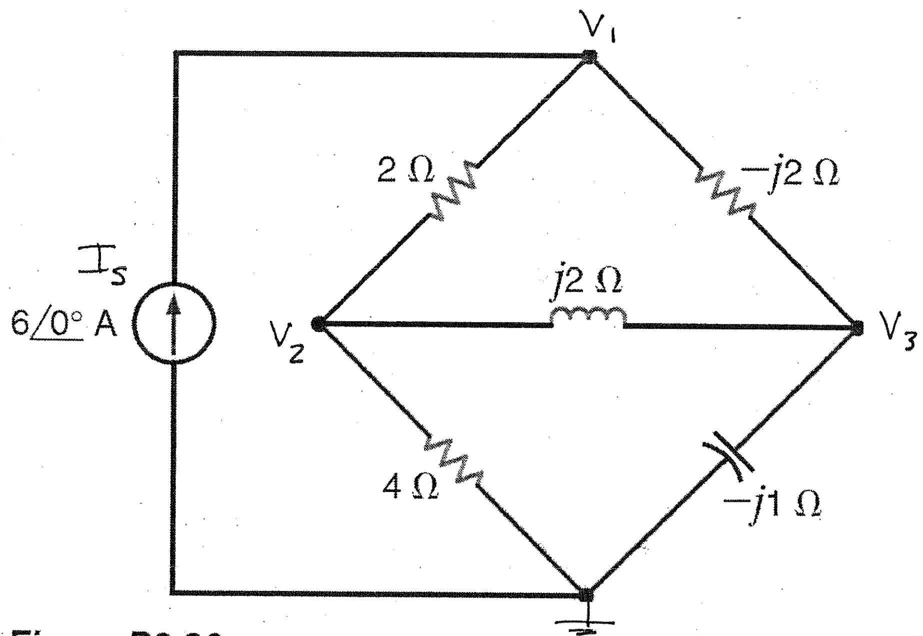


Figure P9.20

SOLUTION:

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{-j2} = 6\angle 0° \Rightarrow V_1(1+j1) - V_2 - jV_3 = 12$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{j2} + \frac{V_2}{4} = 0 \Rightarrow -2V_1 + V_2(3 - j2) + j2V_3 = 0$$

$$\frac{V_3 - V_1}{-j2} + \frac{V_3 - V_2}{j2} + \frac{V_3}{-j1} = 0 \Rightarrow -V_1 + V_2 + 2V_3 = 0$$

$$\begin{bmatrix} 1+j1 & -1 & -j1 \\ -2 & 3-j2 & j2 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} V_1 = 16.7\angle -54.4 \text{ V} \\ V_2 = 8.82\angle -36.0° \text{ V} \end{matrix}$$

$$P_{I_s} = \frac{I_{sm} V_{1m}}{2} \cos(-51.1 - 0) \Rightarrow \boxed{P_{I_s} = 29.2 \text{ W}}$$

$$P_{4\Omega} = \frac{V_{2m}^2}{2(4)} \Rightarrow \boxed{P_{4\Omega} = 9.73 \text{ W}}$$

9.21 Determine the average power supplied to the network in Fig. P9.21.

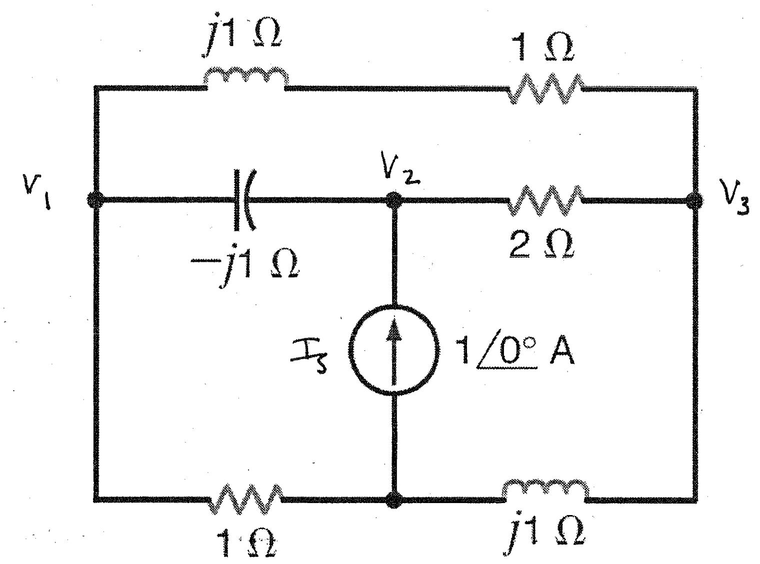


Figure P9.21

SOLUTION:

$$\frac{V_1 - V_2}{-j1} + \frac{V_1 - V_3}{1+j1} + \frac{V_1}{1} = 0 \Rightarrow \quad V_1(3+j1) - j2V_2 + V_3(-1+j1) = 0$$

$$\frac{V_2 - V_1}{-j1} + \frac{V_2 - V_3}{2} = 1\angle 0° \Rightarrow \quad -j2V_1 + V_2(1+j2) - V_3 = 2\angle 0$$

$$\frac{V_1}{1} + \frac{V_3}{j1} = 1\angle 0 \Rightarrow \quad V_1 - jV_3 = 1\angle 0$$

$$\begin{bmatrix} 3+j1 & -j2 & -1+j1 \\ -j2 & 1+j2 & -1 \\ 1 & 0 & -j1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \quad V_2 = 1.01\angle -17.7° \text{ V}$$

$$P_{I_s} = \frac{I_{sm} V_{2m}}{2} \cos(-40.4 - 0) \qquad \boxed{P_{I_s} = 481 \text{ mW}}$$

9.22 Find the average power absorbed by the 2-Ω resistor in the circuit shown in Fig. P9.22.

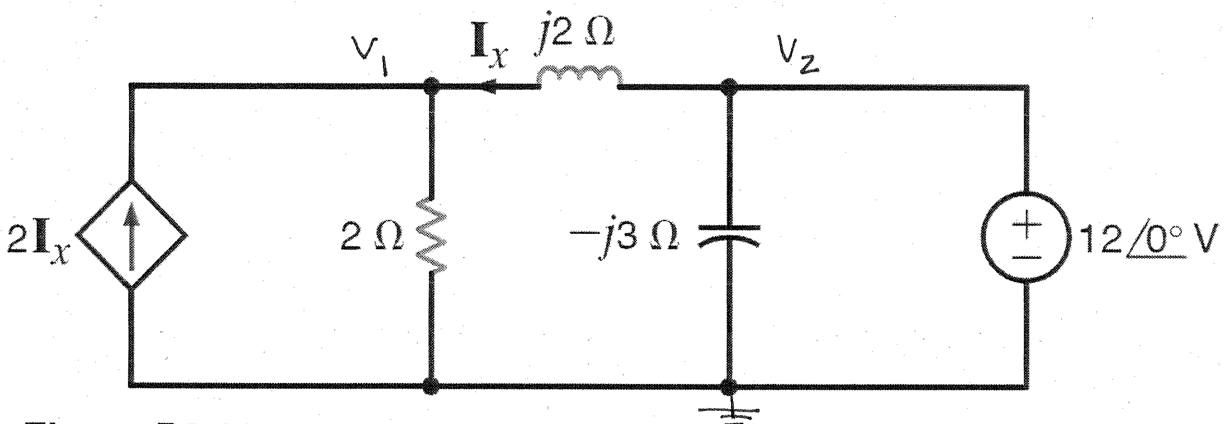


Figure P9.22

SOLUTION:

$$I_x + 2I_x = \frac{V_1}{2} \qquad I_x = \frac{V_2 - V_1}{j2} \quad \Rightarrow \quad V_1(3+j1) - 3V_2 = 0$$

$$V_2 = 12\angle 0° \quad \Rightarrow \quad V_1 = 11.4\angle -18.4°\ \text{V}$$

$$P_{2\Omega} = \frac{V_{1m}^2}{2(2)} \qquad \boxed{P_{2\Omega} = 32.5\ \text{W}}$$

9.23 Determine the average power absorbed by a 2-Ω resistor connected at the output terminals of the network shown in Fig. P9.23.

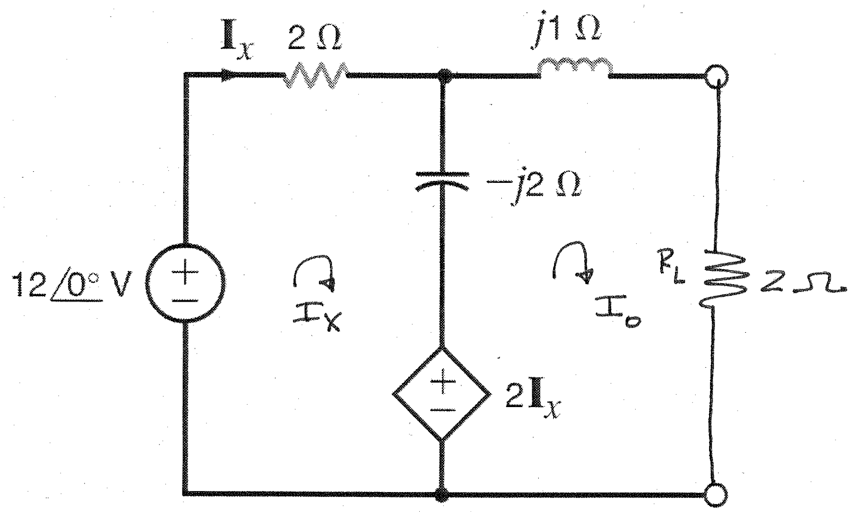


Figure P9.23

SOLUTION:

$12 = I_x(2-j2) + 2I_x + j2I_o \Rightarrow \quad 12\angle 0 = I_x(4-j2) + j2I_o$

$2I_x = j2I_x + I_o(2-j1) \Rightarrow \quad 0 = I_x(-2+j2) + I_o(2-j1)$

$$\begin{bmatrix} 4-j2 & j2 \\ -2+j2 & 2-j1 \end{bmatrix}\begin{bmatrix} I_x \\ I_o \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$$

$I_o = 3.15 \angle -23.2°$ A

$P_{R_L} = \frac{I_{oM}^2}{2} R_L$

$$\boxed{P_{R_L} = 9.92 \text{ W}}$$

9.24 Determine the average power absorbed by the 2-kΩ output resistor in Fig. P9.24.

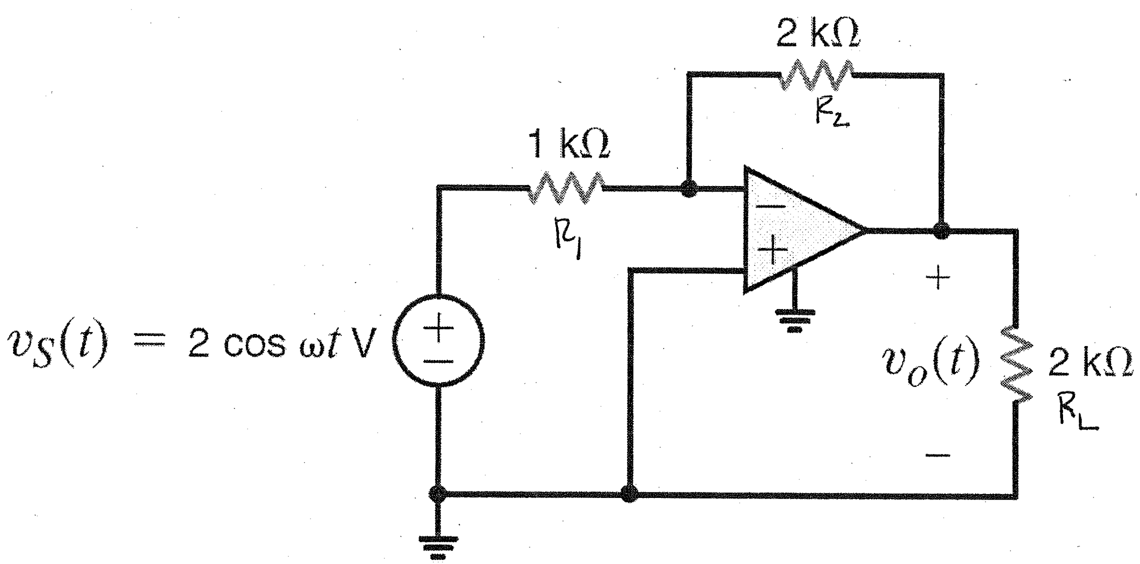


Figure P9.24

SOLUTION:

$$\frac{v_o(t)}{v_s(t)} = -\frac{R_2}{R_1} = -2 \qquad v_o(t) = -4\cos(\omega t) = 4\cos(\omega t + 180°)\text{ V}$$

$$V_o = 4\angle 180°\text{ V} \qquad P_{R_L} = \frac{V_{om}^2}{2R_L} \qquad \boxed{P_{R_L} = 4\text{ mW}}$$

9.25 Determine the average power absorbed by the 4-kΩ resistor in Fig. P9.25.

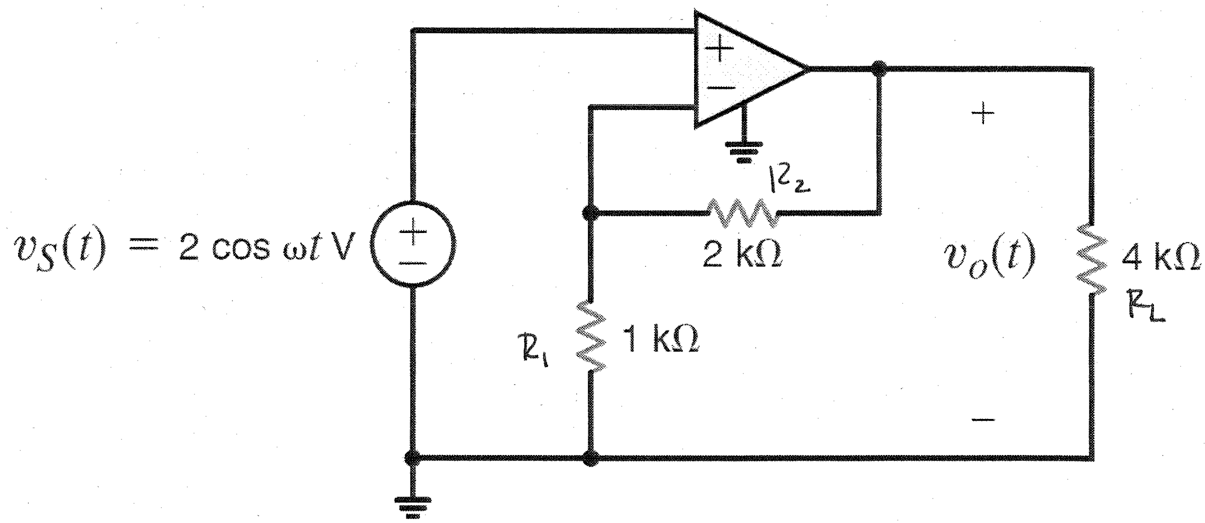


Figure P9.25

SOLUTION:

$V_S = 2\angle 0° \text{ V}$ $\quad V_o = V_S\left(1 + R_2/R_1\right) = 6\angle 0° \text{ V}$

$P_{R_L} = \dfrac{V_{om}^2}{2R_L}$ $\quad \boxed{P_{R_L} = 4.5 \text{ mW}}$

9.26 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.26.

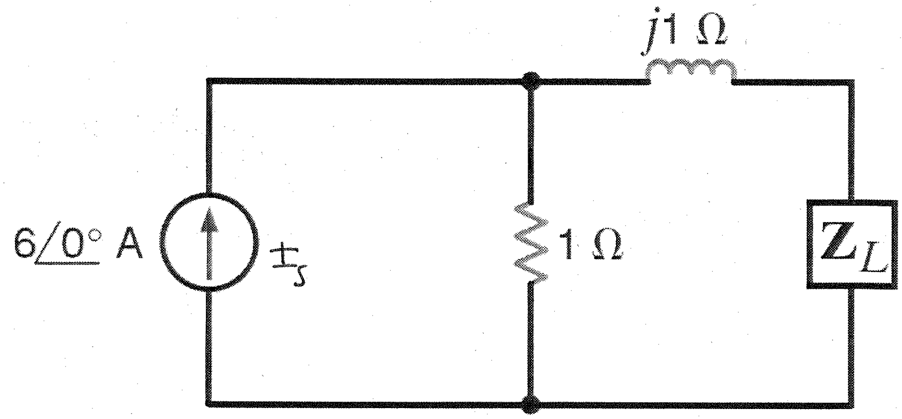
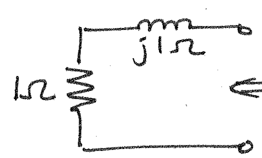


Figure P9.26

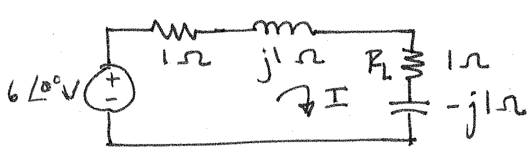
SOLUTION:



$\Leftarrow Z_{TH} = 1 + j1\,\Omega$

for max. power transfer, $Z_L = Z_{TH}^* = 1 - j1$

$\boxed{Z_L = 1 - j1\,\Omega}$



$$I = \frac{6\angle 0}{2} = 3\angle 0°$$

$$P_{MAX} = \frac{I_M^2}{2} R_L$$

$$\boxed{P_{MAX} = 4.5\,W}$$

9.27 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.27. CS

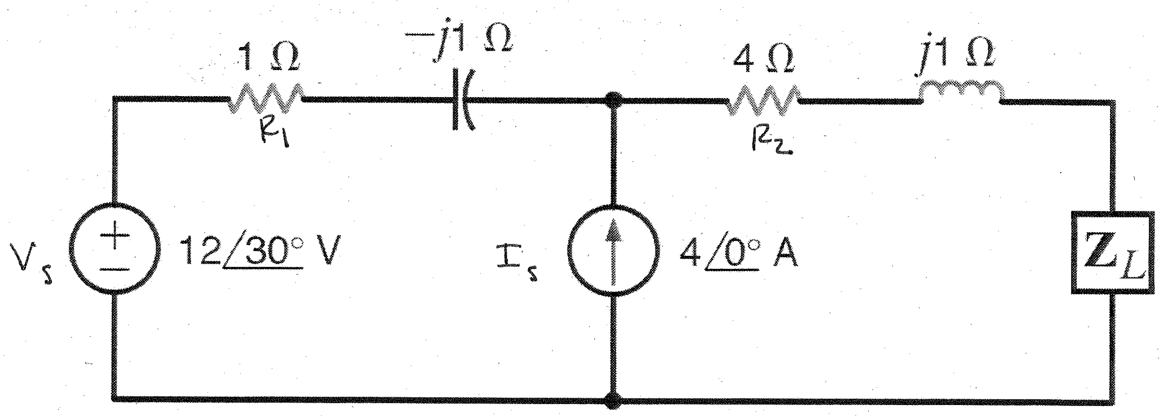
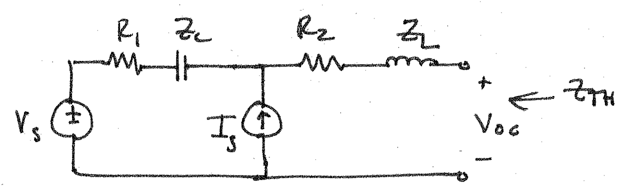


Figure P9.27

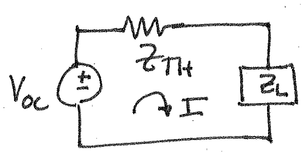
SOLUTION:

Thevenin eq



Superposition: $V_{OC} = V_S + I_S(R_1 + Z_C) \Rightarrow V_{OC} = 14.5\angle 7.91°\ V$

$Z_{TH} = R_1 + Z_C + R_2 + Z_L = 5\,\Omega$



Z_L is purely resistive, so for max. power transfer,

$R_L = |Z_{TH}| = 5\,\Omega$

$I = \dfrac{V_{OC}}{Z_{TH} + Z_L} = 1.45\angle 7.91°\ A$ $\qquad P_L = \frac{1}{2} I_M^2 R_L$

$\boxed{P_L = 5.26\,W}$

9.28 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig. P9.28.

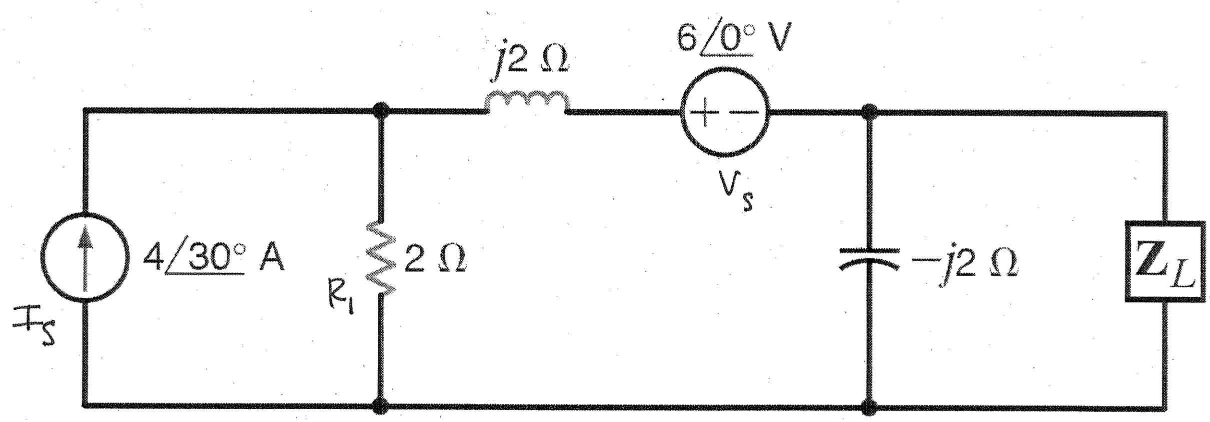
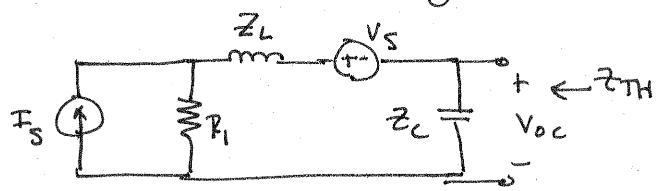


Figure P9.28

SOLUTION: Thevenin eq.:



$$Z_{TH} = \frac{(R_1 + Z_L)\,Z_C}{R_1 + Z_L + Z_C}$$

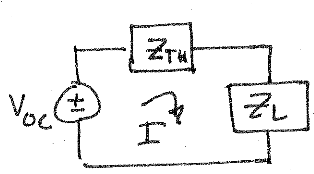
$$Z_{TH} = 2 - j2\ \Omega$$

Superposition:

$$V_{oc} = \frac{I_S R_1 Z_C}{R_1 + Z_L + Z_C} - \frac{V_S Z_C}{R_1 + Z_L + Z_C} = 4.10\angle{-13.1^\circ}\ \text{V}$$

for max power transfer, $Z_L = Z_{TH}^* = 2 + j2\,\Omega$

$$\boxed{Z_L = 2 + j2\,\Omega}$$



$$P_L = \frac{1}{2} I_M^2 R_L$$

$$I = \frac{V_{oc}}{Z_{TH} + Z_L} = 1.03\angle{-13.1^\circ}\ \text{A}$$

$$\boxed{P_L = 1.05\ \text{W}}$$

9.29 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig. P9.29.

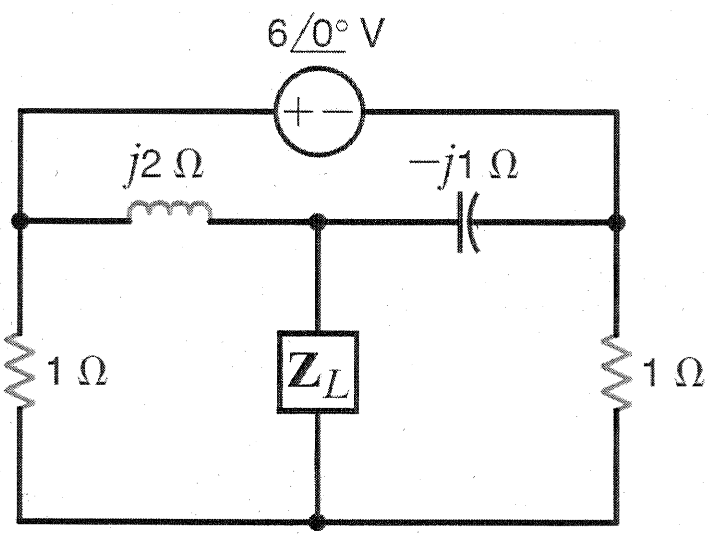
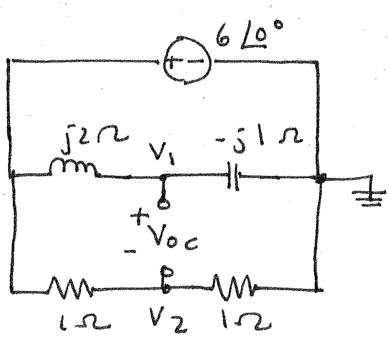


Figure P9.29

SOLUTION: Thevenin eq.



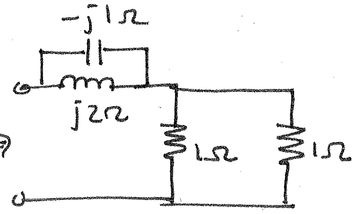
$$V_1 = 6\angle 0 \left[\frac{-j1}{j2-j1}\right] = -6\angle 0^\circ \text{ V}$$

$$V_2 = 6\angle 0 \left[\frac{1}{1+1}\right] = 3\angle 0^\circ$$

$$V_{oc} = V_1 - V_2 = -9\angle 0^\circ = 9\angle 180^\circ \text{ V}$$

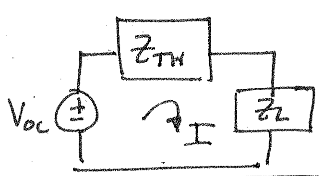
Z_{TH} redraw ⇒

Z_{TH} ⇒



$$Z_{TH} = \frac{j2(-j1)}{j2-j1} + \frac{1(1)}{1+1}$$

$$Z_{TH} = \frac{1}{2} - j2\,\Omega$$



$$P_L = \frac{I_m^2}{2} R_L \qquad I = \frac{V_{oc}}{Z_{TH}+Z_L} = 9\angle 180^\circ \text{ A}$$

$$\boxed{Z_L = Z_{TH}^* = \frac{1}{2} + j2\,\Omega}$$

$$\boxed{P_L = 20.25 \text{ W}}$$

9.30 Repeat Problem 9.29 for the network in Fig. P9.30.

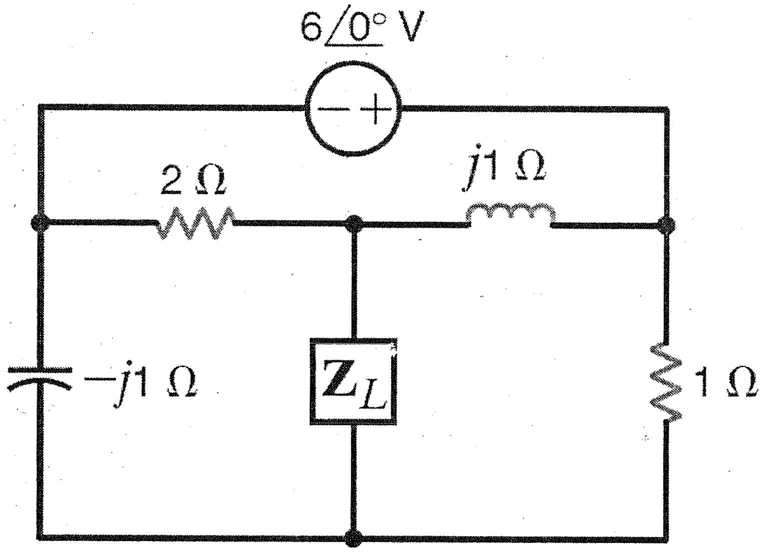
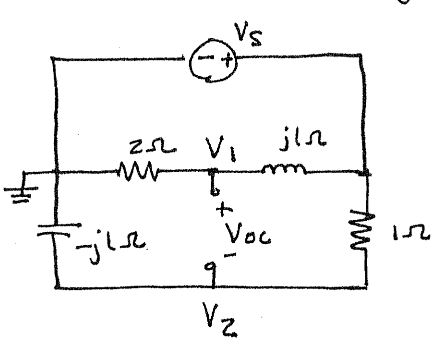


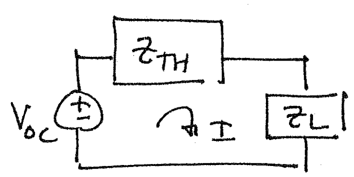
Figure P9.30

SOLUTION: Thevenin eq.

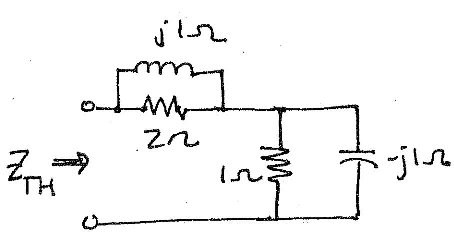


$$V_1 = V_S\left[\frac{2}{2+j1}\right] \qquad V_2 = V_S\left[\frac{-j1}{1-j1}\right]$$

$$V_{oc} = V_1 - V_2 = 1.90\,\angle 18.4^\circ \text{ V}$$



Redraw for Z_{TH}



$$I = \frac{V_{oc}}{Z_{TH}+Z_L} = 1.06\,\angle 18.4^\circ \text{ A}$$

$$P_L = \frac{I_M^2}{2} R_L$$

$$\boxed{P_L = 0.501\text{ W}}$$

$$Z_{TH} = \frac{2(j1)}{2+j1} + \frac{1(-j1)}{1-j1}$$

$$Z_{TH} = 0.9 + j0.3\ \Omega$$

$$\boxed{Z_L = Z_{TH}^* = 0.9 + j0.3\ \Omega}$$

9.31 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.31. **PSV**

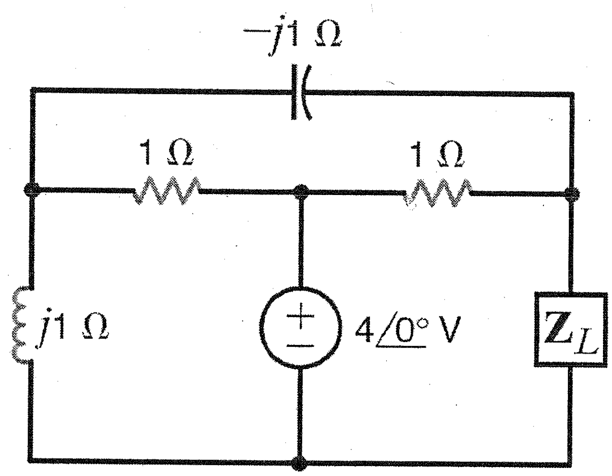
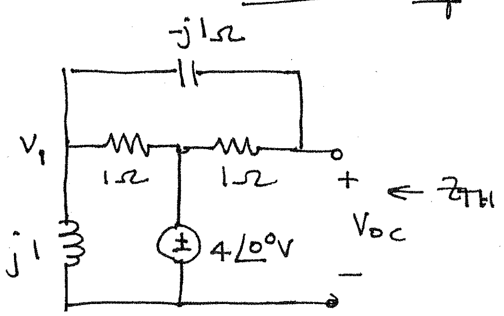
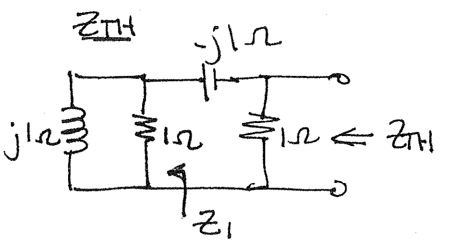


Figure P9.31

SOLUTION: Thevenin eq.





$$z_1 = 1(j1)/(1+j) = j1/(1+j1)\ \Omega$$

$$Z_{TH} = 1[z_1 - j1]/\{1 + z_1 - j1\}$$

$$Z_{TH} = 0.4 - j0.2\ \Omega$$

$$\frac{V_1 - V_{oc}}{-j1} + \frac{V_1}{j1} + \frac{V_1 - 4}{1} = 0$$

$$V_1 = 4 + (j1)V_{oc}$$

$$\frac{V_1 - V_{oc}}{-j1} = \frac{V_{oc} - 4}{1}$$

$$-j1 V_1 + V_{oc}(1 + j1) = 4$$

yields $V_{oc} = 2.4 + j0.8\ V$

$$Z_L = Z_{TH}^* = 0.4 + j0.2\ \Omega$$

$$P_L = \frac{1}{2} I_m^2 R_L$$

$$I = V_{oc}/(Z_{TH} + Z_L) = 3 + j1\ A$$

$$\boxed{P_L = 2\ W}$$

9.32 In the network in Fig. P9.32, find $\mathbf{Z}_L$ for maximum average power transfer and the maximum average power transferred. CS

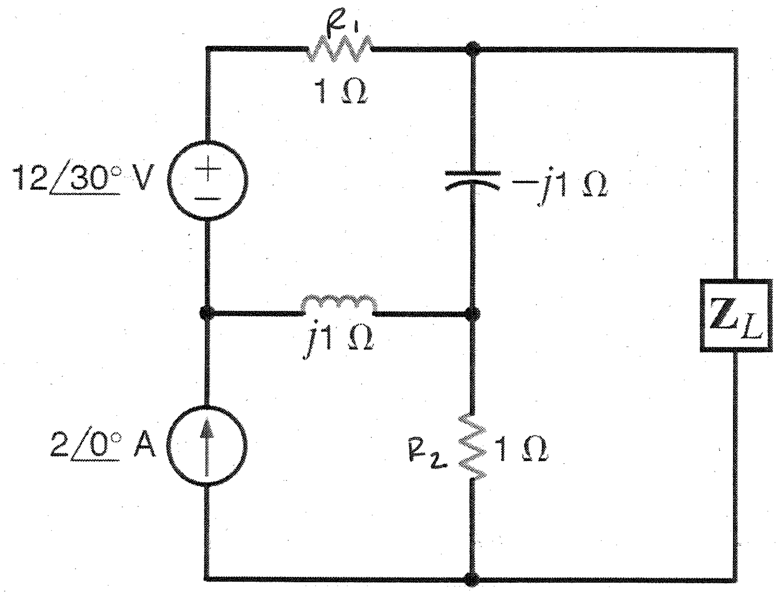
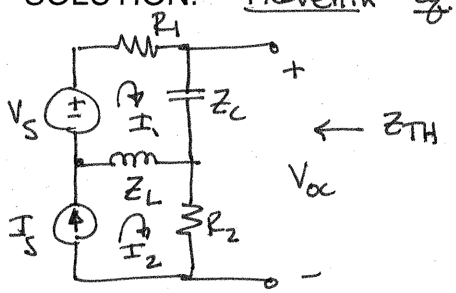


Figure P9.32

SOLUTION: Thevenin eq.



$V_S = I_1(1 - j1 + j1) - j1 I_2 = 12\angle 30°$

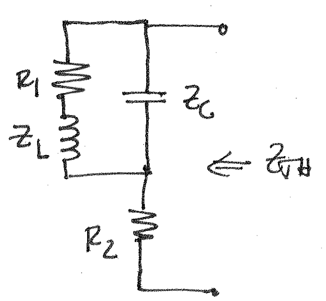
$I_1 - j1 I_2 = 12\angle 30°$

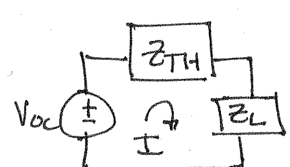
$I_2 = 2\angle 0°$

yields $I_1 = 13.1\angle 37.6°\text{A}$

$V_{oc} = I_1(-j1) + I_2(1) = 13.1\angle -52.4°\text{ V}$

Z_{TH}





$I = \dfrac{V_{oc}}{Z_{TH} + Z_L} = 3.28\angle -52.4°\text{A}$

$\boxed{Z_L = Z_{TH}^* = 2 + j1\,\Omega}$

$P_L = \frac{1}{2} I_M^2 R_L$

$\boxed{P_L = 10.7\text{ W}}$

$Z_1 = R_1 + Z_L = 1 + j1\,\Omega$

$Z_{TH} = R_2 + Z_c // Z_1 = 2 - j1\,\Omega$

9.33 Repeat Problem 9.32 for the network in Fig. P9.33.

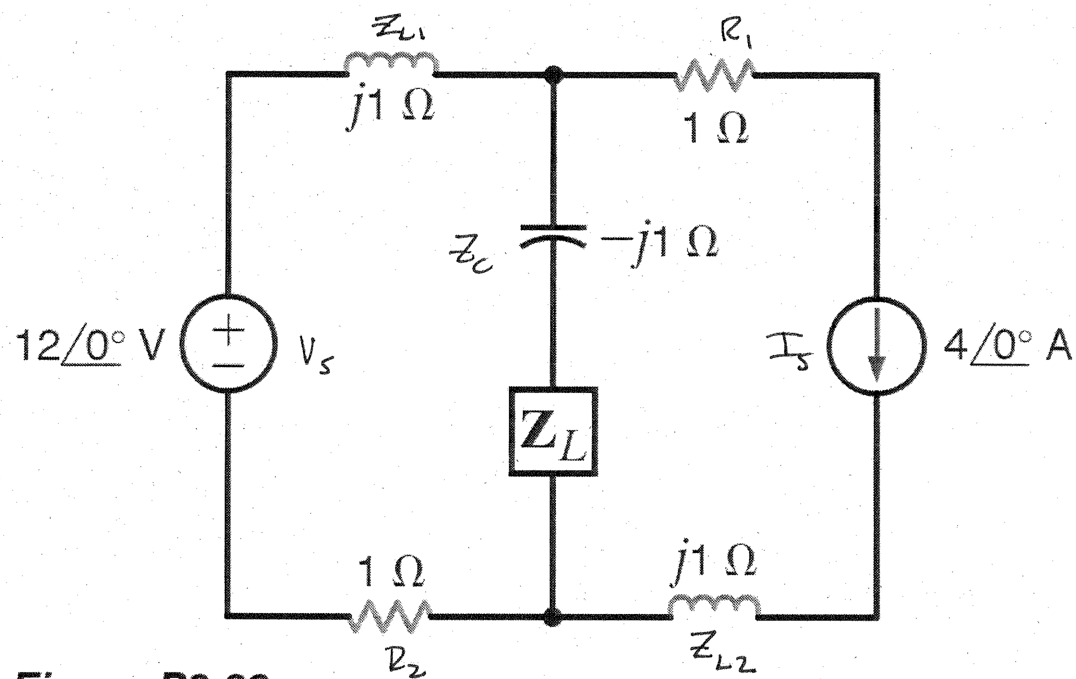
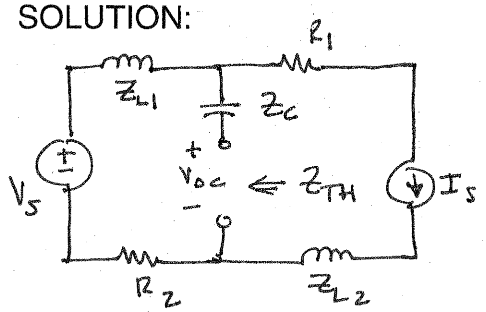


Figure P9.33

SOLUTION:



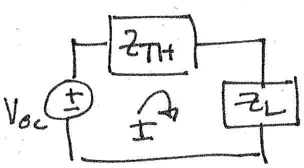
$$V_S = Z_{L1} I_S + V_{oc} + R_2 I_S$$

$$\therefore V_{oc} = 12\angle 0 - j1\,(4\angle 0) - 1\,(4\angle 0)$$

$$V_{oc} = 8 - j4 \text{ V}$$

Let $Z_1 = R_2 + Z_{L1} = 1 + j1\,\Omega$

$$Z_{TH} = Z_C + Z_1 = 1\Omega$$



$$I = \frac{V_{oc}}{Z_{TH} + Z_L} = \frac{8 - j4}{2} = 4 - j2 = 4.47\angle -26.6° \text{ A}$$

$$\boxed{Z_L = Z_{TH}^* = 1\Omega}$$

$$P_L = \frac{1}{2} I_M^2 R_L$$

$$\boxed{P_L = 10.0\text{W}}$$

9.34 Repeat Problem 9.32 for the network in Fig. P9.34. CS

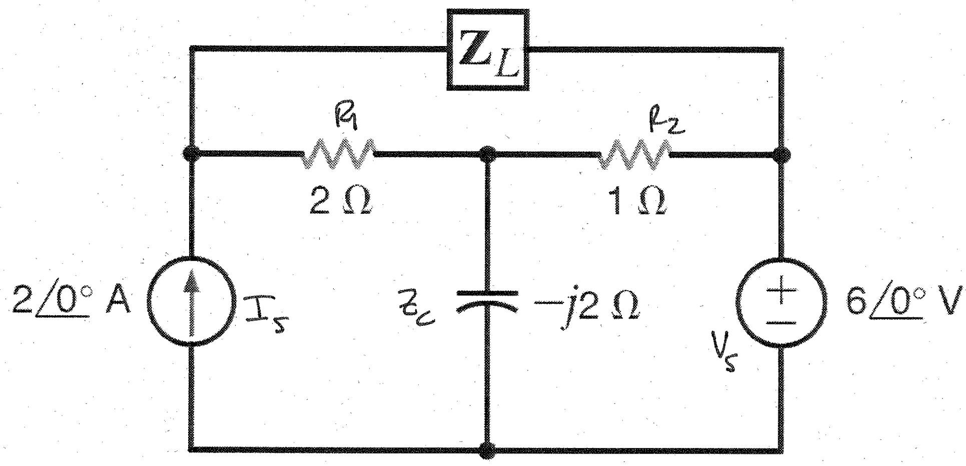
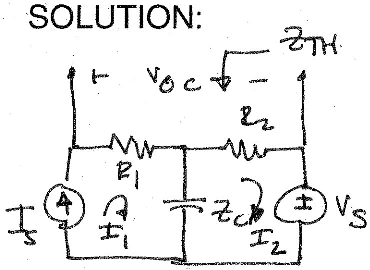


Figure P9.34

SOLUTION:

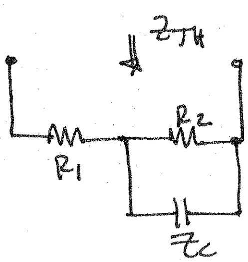


$I_1 = 2\angle 0°$ A

$j2I_1 + I_2(1-j2) = -6\angle 0°$

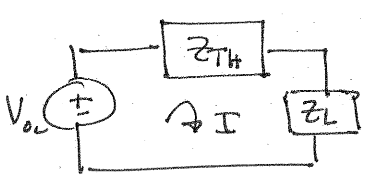
$I_2 = \dfrac{-6-j4}{1-j2} = 0.4 - j3.2$ A

$V_{oc} = I_1R_1 + I_2R_2 = 4.4 - j3.2$ V



$Z_{TH} = R_1 + \dfrac{(R_2Z_c)}{R_2+Z_c} = 2.8 - j0.4\,\Omega$

$Z_L = Z_{TH}^*$ $\boxed{Z_L = 2.8 + j0.4\,\Omega}$



$I = \dfrac{V_{oc}}{Z_{TH}+Z_L} = 0.972\angle -36.0°$ A

$P_L = \frac{1}{2} I_m^2 R_L$ $\boxed{P_L = 1.32\text{W}}$

9.35 Repeat Problem 9.32 for the network in Fig. P9.35.

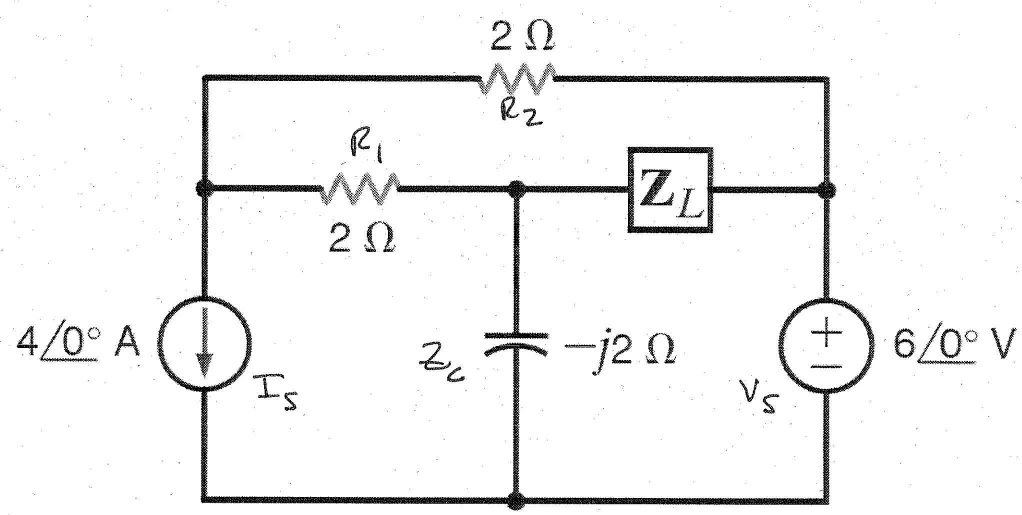
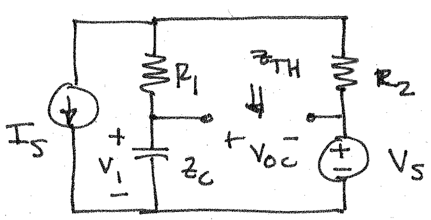
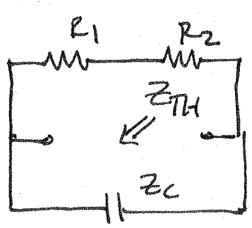
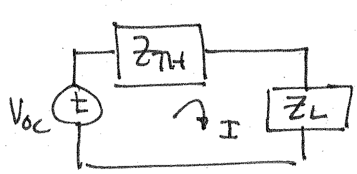


Figure P9.35

SOLUTION:







$V_{OC} = V_1 - V_S = V_1 - 6\angle 0°$

Use superposition to find V_1

$$V_1 = V_S\left\{\frac{Z_C}{R_1+R_2+Z_C}\right\} - \frac{I_S R_2 Z_C}{R_1+R_2+Z_C}$$

$V_1 = -0.4 + j0.8\ \text{V}$

$V_{OC} = -6.4 + j0.8\ \text{V}$

$Z_{TH} = Z_C(R_1+R_2)/[R_1+R_2+Z_C]$

$Z_{TH} = 0.8 - j1.6\ \Omega$

$\boxed{Z_L = Z_{TH}^* = 0.8 + j1.6\ \Omega}$

$$I = \frac{V_{OC}}{Z_{TH}+Z_L} = 4.03\angle 173°\ \text{A}$$

$P_L = \frac{1}{2} I_M^2 R_L$ $\boxed{P_L = 6.50\text{W}}$

9.36 Repeat Problem 9.32 for the network in Fig. P9.36.

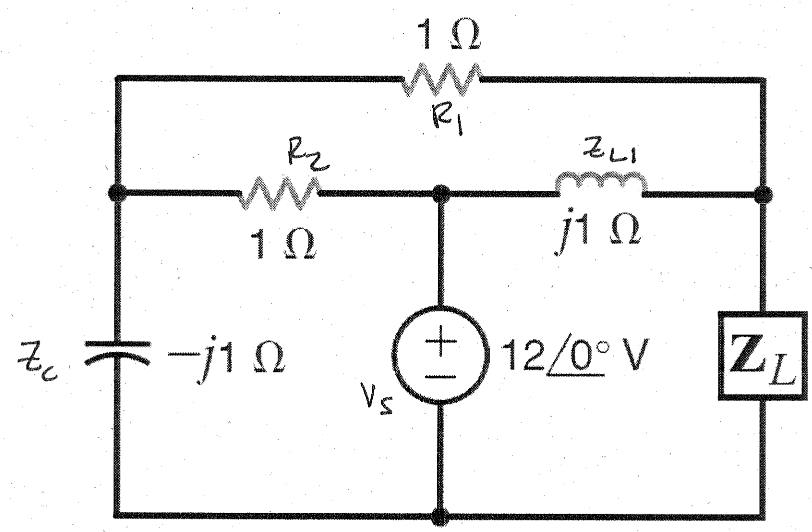
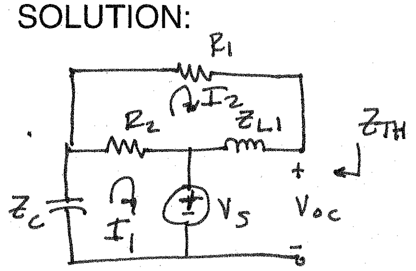


Figure P9.36

SOLUTION:

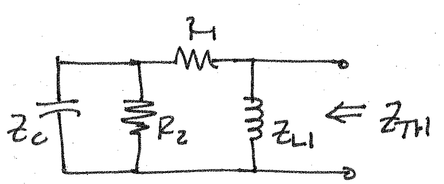


$$I_1(1-j1) - I_2 = -12\angle 0$$

$$-I_1 + I_2(2+j1) = 0$$

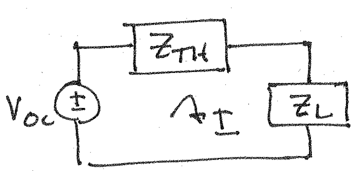
$$\begin{bmatrix} 1-j1 & -1 \\ -1 & 2+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}$$

$$I_2 = -4.8 - j2.4\,A$$



$$V_{oc} = I_2(j1) + V_s = 14.4 - j4.8\,V$$

Let $Z_1 = \dfrac{R_2 Z_c}{R_2 + Z_c} = 0.5 - j0.5$ $\qquad Z_{TH} = \dfrac{Z_{L1}[R_1 + Z_1]}{Z_{L1} + R_1 + Z_1} = 0.6 + j0.8\,\Omega$



$$\boxed{Z_L = Z_{TH}^* = 0.6 - j0.8\,\Omega}$$

$$I = \frac{V_{oc}}{Z_{TH} + Z_L} = 12.6\angle -18.4°\,A$$

$$P_L = \frac{1}{2} I_m^2 R_L \qquad \boxed{P_L = 47.6\,W}$$

9.37 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig. P9.37. CS

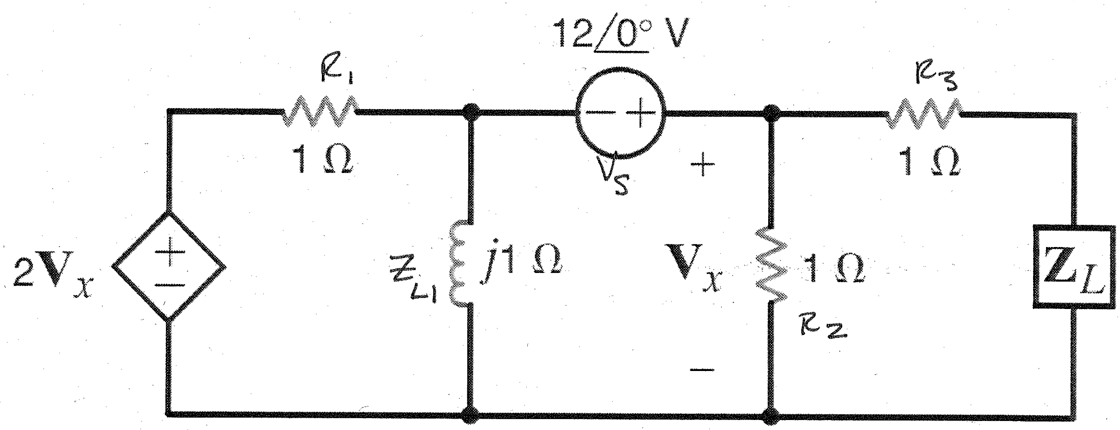
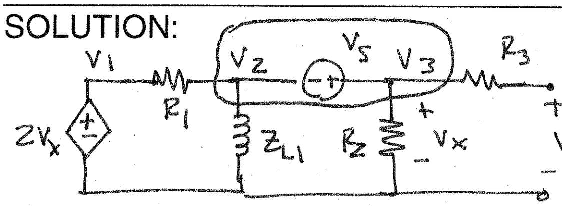


Figure P9.37

SOLUTION:



$V_1 = 2V_x \qquad V_x = V_3 = V_{oc}$

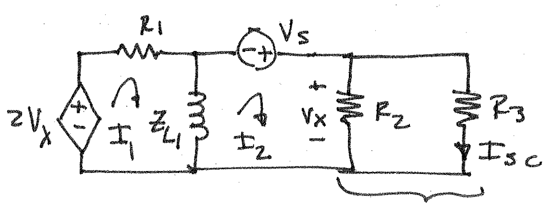
$\hookrightarrow V_1 = 2V_{oc}$

$V_3 - V_2 = V_s = 12\angle 0$

@ supernode: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2}{Z_{L1}} + \dfrac{V_3}{R_2} = 0$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 1 \\ -1 & 1-j1 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix}$$

$V_3 = V_{oc} = 12 + j12\ \text{V}$



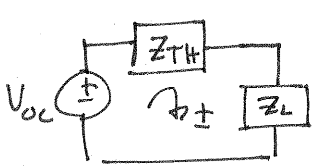
$I_{sc} = I_2/2$

$V_x = I_2/2$

$2V_x = I_1(1+j1) - jI_2 \Rightarrow I_1 = I_2$

$V_s = \dfrac{I_2}{2} + I_2(j1) - I_1(j1) = 12\angle 0°$

$I_2 = 24\angle 0°\ \text{A} \qquad I_{sc} = 12\angle 0°\ \text{A}$



$Z_{TH} = V_{oc}/I_{sc}$

$Z_{TH} = 1 + j1\ \Omega \qquad \boxed{Z_L = Z_{TH}^* = 1 - j1\ \Omega}$

$I = V_{oc}/(Z_{TH} + Z_L) = 8.49\angle 45°\ \text{A}$

$P_L = \frac{1}{2} I_m^2 R_L \qquad \boxed{P_L = 36\ \text{W}}$

9.38 Find the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.38.

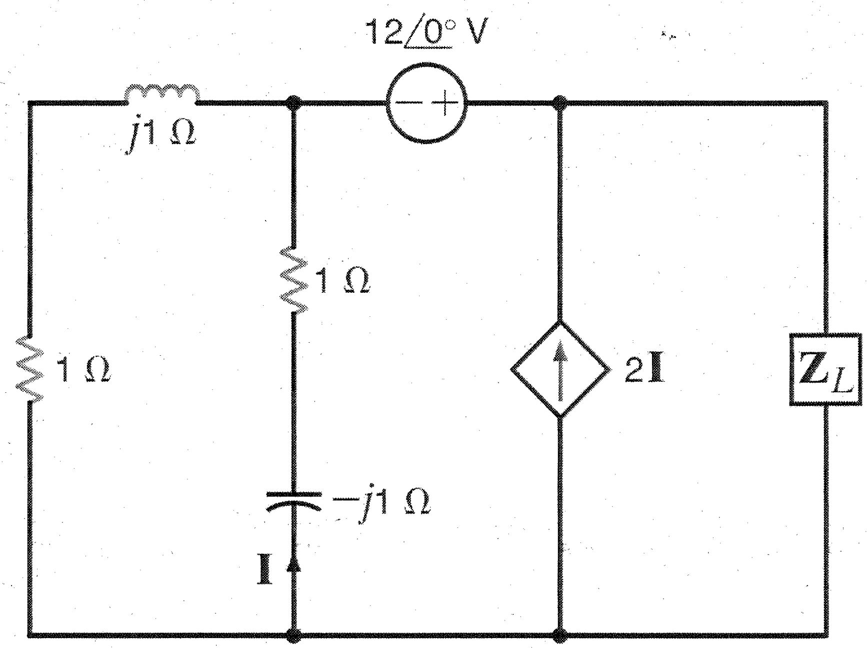
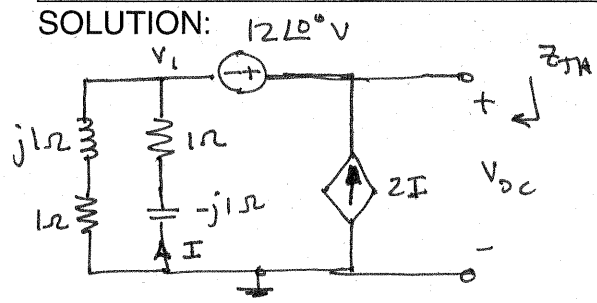


Figure P9.38

SOLUTION:

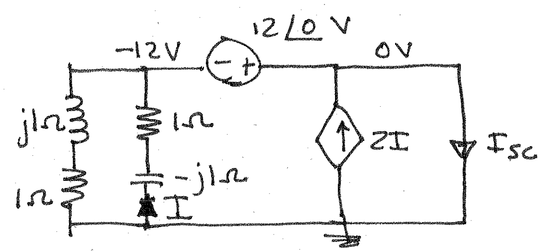


$V_{oc} - V_1 = 12\angle 0 \Rightarrow V_1 = V_{oc} - 12$

$\frac{V_1}{1+j1} + \frac{V_1}{1-j1} = 2I \qquad I = \frac{-V_1}{1-j1}$

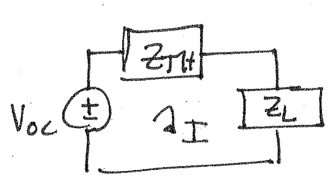
yields $V_1 = 0 \Rightarrow I = 0$

$V_{oc} = 12\angle 0°\ V$



$I_{sc} = 2I + I + \frac{12}{1+j1}$ & $I = \frac{12}{1-j1}$

$I_{sc} = 24 + j12\ A$



$Z_{TH} = V_{oc}/I_{sc} = 0.4 - j0.2\ \Omega$ $\boxed{Z_L = Z_{TH}^* = 0.4 + j0.2\ \Omega}$

$I = V_{oc}/(Z_L + Z_{TH}) = 15\angle 0°\ A$

$P_L = \frac{1}{2} I_M^2 R_L$ $\boxed{P_L = 45\ W}$

9.39 Repeat Problem 9.38 for the network in Fig. P9.39.
PSV

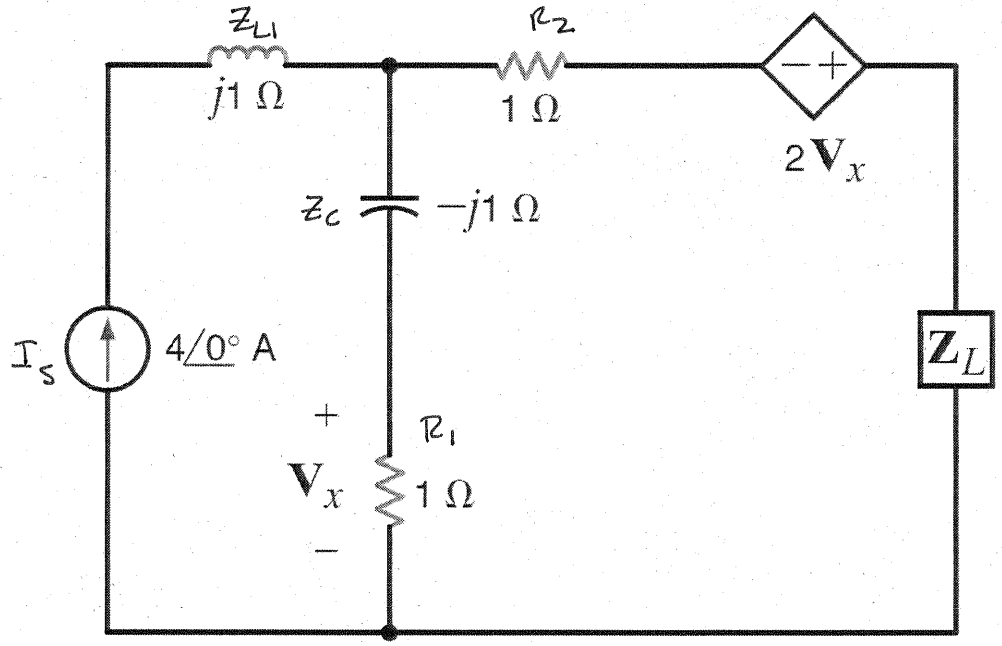
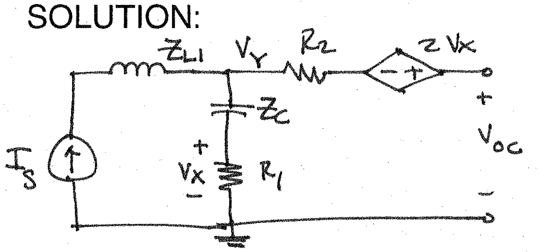


Figure P9.39

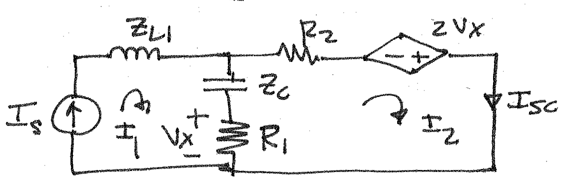
SOLUTION:



$V_Y = I_S\,(R_1 + Z_C) = 4 - j4$ V

$V_X = I_S R_1 = 4\angle 0$ V

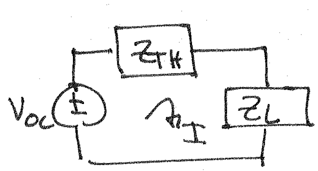
$V_{OC} = V_Y + 2V_X = 12 - j4$ V



$I_1 = 4\angle 0°$ $\quad I_2 = I_{SC}$ $\quad V_X = R_1(I_1 - I_2)$

$2V_X = I_2(2 - j1) - I_1(1 - j1)$

yields $\quad I_{SC} = 3.07\angle -4.40°$ A



$Z_{TH} = V_{OC}/I_{SC} = 4 - j1\ \Omega$ $\quad \boxed{Z_L = Z_{TH}^* = 4 + j1\ \Omega}$

$I = V_{OC} / (Z_{TH} + Z_L) = 1.58\angle -18.4°$ A

$P_L = \frac{1}{2} I_M^2 R_L$ $\quad \boxed{P_L = 5.00\ \text{W}}$

9.40 Find the value of $\mathbf{Z}_L$ in the circuit in Fig. P9.40 for maximum average power transfer.

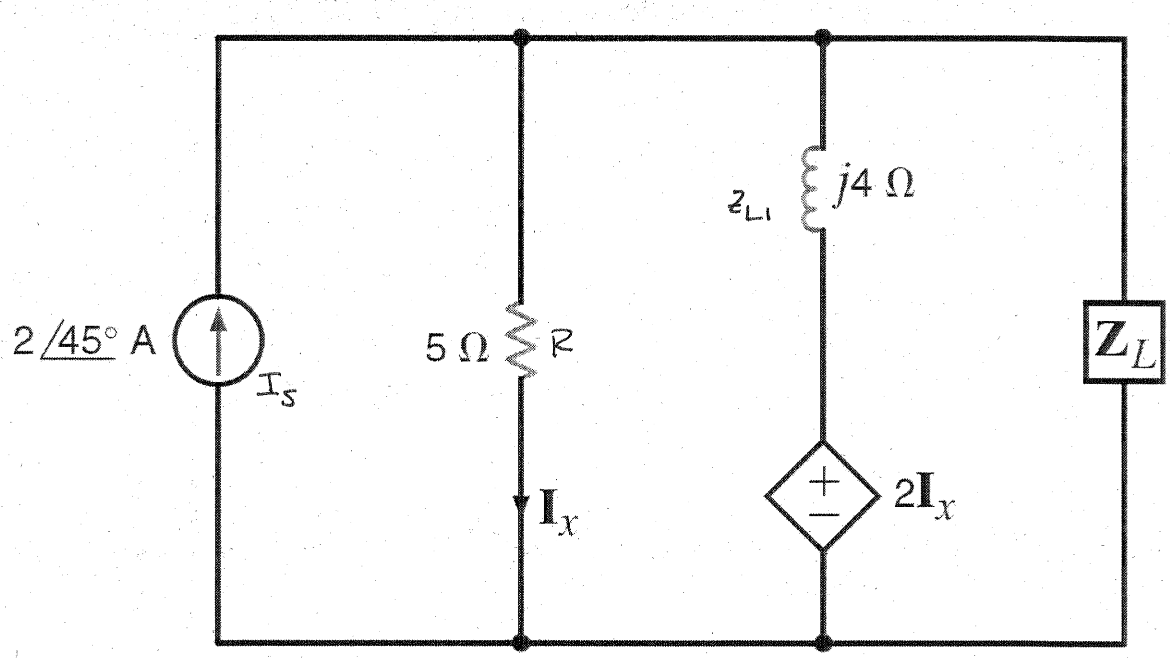
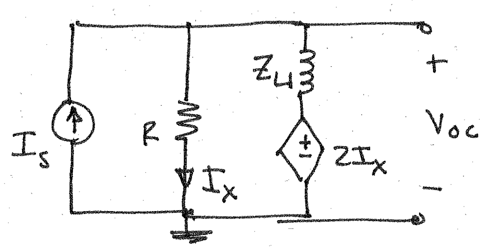


Figure P9.40

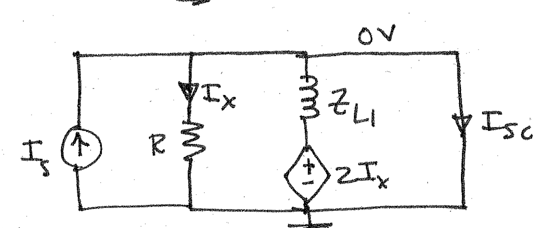
SOLUTION:



$$I_s = \frac{V_{oc}}{R} + \frac{V_{oc} - 2I_x}{Z_{L1}} \qquad I_x = \frac{V_{oc}}{R}$$

$$2\angle 45 = \frac{V_{oc}}{5} + \frac{V_{oc} - 0.4V_{oc}}{j4}$$

$$V_{oc} = 7.98\angle 81.9°\ \text{V}$$



$$I_x = 0\text{A} \Rightarrow I_{sc} = I_s = 2\angle 45°\text{A}$$

$$Z_{TH} = \frac{V_{oc}}{I_{sc}} = 3.2 + j2.4\,\Omega$$

$$\boxed{Z_L = Z_{TH}^* = 3.2 - j2.4\,\Omega}$$

9.41 Compute the rms value of the voltage given by the expression $v(t) = 10 + 20\cos(377t + 30°)$ V.

SOLUTION:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \qquad v^2(t) = 100 + 400\cos(\omega t+\theta) + 400\cos^2(\omega t+\theta)$$

$$\frac{1}{T}\int_0^T 100\,dt = \frac{1}{T}(100t)\Big|_0^T = 100$$

$$\frac{1}{T}\int_0^T 400\cos(\omega t+\theta)\,dt = 0$$

$$\frac{1}{T}\int_0^T 400\cos^2(\omega t+\theta)\,dt = \frac{400}{2} = 200$$

$$V_{rms} = \sqrt{100 + 0 + 200} \qquad \boxed{V_{rms} = 17.3\text{ V}}$$

9.42 Compute the rms value of the voltage given by the waveform shown in Fig. P9.42.

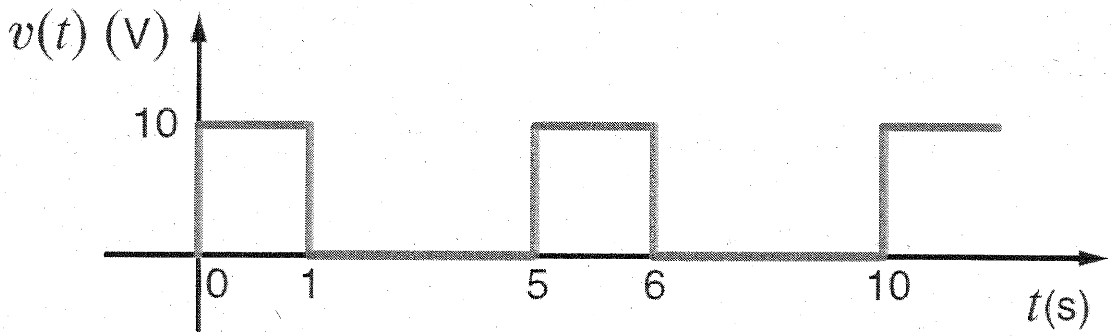


Figure P9.42

SOLUTION:

$T = 5s \qquad v^2(t) = \begin{cases} 100 & t < 1 \\ 0 & 1 \le t < 5 \end{cases}$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^1 100\,dt} = \sqrt{\frac{1}{5}(100t)\Big|_0^1} = \sqrt{20}$$

$$\boxed{V_{rms} = 4.47\text{ V}}$$

9.43 Calculate the rms value of the waveform shown in Fig. P9.43.

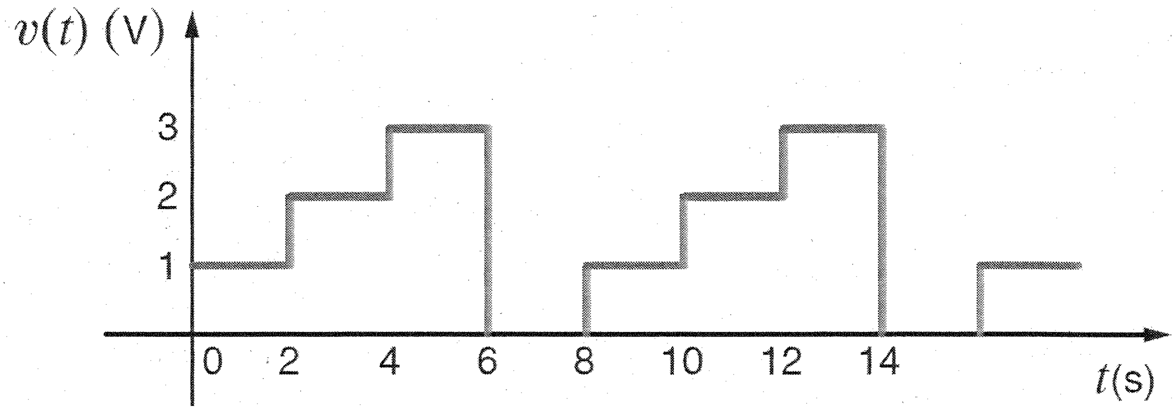


Figure P9.43

SOLUTION:

$T = 8\text{ s}$

$$v^2(t) = \begin{cases} 1 & 0 < t < 2 \\ 4 & 2 < t < 4 \\ 9 & 4 < t < 6 \\ 0 & 6 < t < 8 \end{cases}$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \left[\frac{1}{8}\left\{(1)(2) + (4)(2) + 9(2)\right\}\right]^{1/2}$$

$$\boxed{V_{Rms} = 1.87\text{ V}}$$

9.44 Calculate the rms value of the waveform in Fig. P9.44.

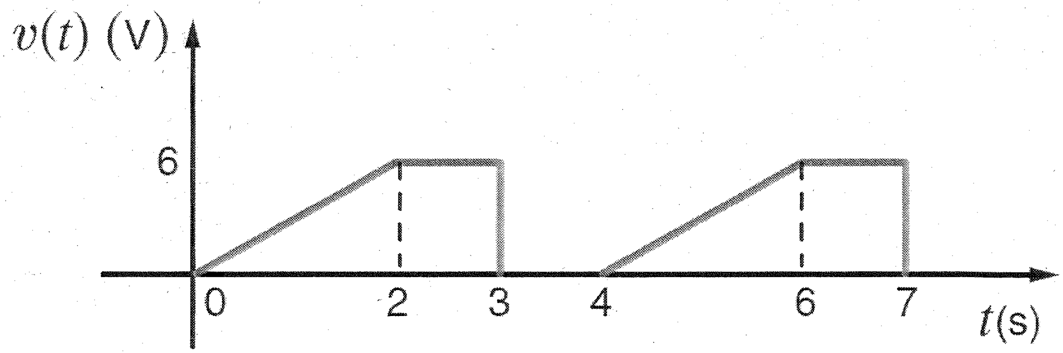


Figure P9.44

SOLUTION:

$T = 4\,s$

$$v(t) = \begin{cases} 3t & 0<t<2 \\ 6 & 2<t<3 \\ 0 & 3<t<4 \end{cases} \qquad v^2(t) = \begin{cases} 9t^2 & 0<t<2 \\ 36 & 2<t<3 \\ 0 & 3<t<4 \end{cases}$$

$$V_{rms} = \left[\frac{1}{4}\left\{ \frac{9t^3}{3}\Big|_0^2 + 36t\Big|_2^3 \right\}\right]^{1/2} = \left\{\frac{1}{4}\left[24 + 36\right]\right\}^{1/2}$$

$$\boxed{V_{rms} = 3.87\text{ V}}$$

9.45 Calculate the rms value of the waveform in Fig. P9.45.

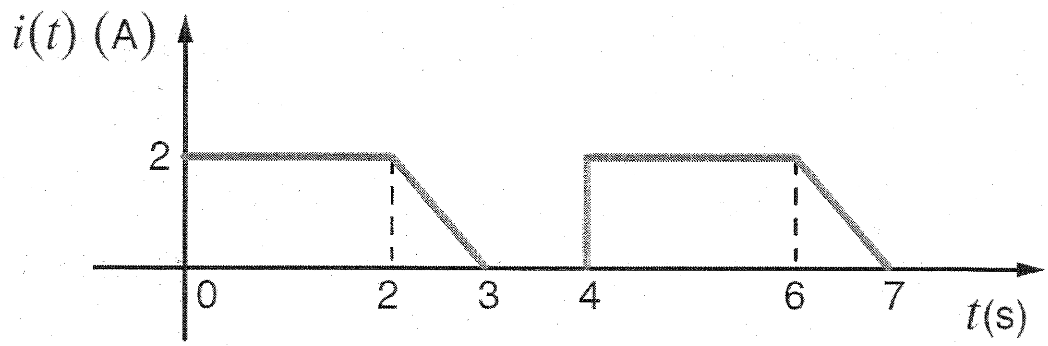


Figure P9.45

SOLUTION:

$T = 4s$ $\quad i(t) = \begin{cases} 2 & 0<t<2 \\ 6-2t & 2<t<3 \\ 0 & 3<t<4 \end{cases} \Rightarrow i^2(t) = \begin{cases} 4 \\ 36-24t+4t^2 \\ 0 \end{cases}$

$$I_{rms} = \left\{ \frac{1}{4}\left[4t\Big|_0^2 + \left(36t\Big|_2^3 - \frac{24t^2}{2}\Big|_2^3 + \frac{4t^3}{3}\Big|_2^3 \right)\right]\right\}^{1/2}$$

$$I_{RMS} = \left\{ \frac{1}{4}\left[8 + 36 - 60 + 25.33 \right]\right\}^{1/2}$$

$$\boxed{I_{rms} = 1.53\ A}$$

9.46 Calculate the rms value of the waveform in Fig. P9.46.

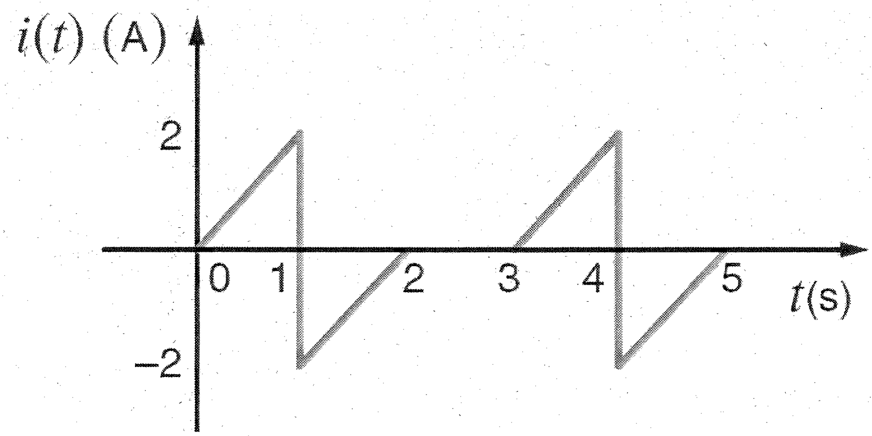


Figure P9.46

SOLUTION:

$T = 3s \qquad i(t) = \begin{cases} 2t & 0<t<1 \\ -4+2t & 1<t<2 \\ 0 & 2<t<3 \end{cases} \qquad i^2(t) = \begin{cases} 4t^2 \\ 16-16t+4t^2 \\ 0 \end{cases}$

$$I_{rms} = \left\{\frac{1}{T}\int_0^T i^2(t)\,dt\right\}^{1/2} = \left\{\frac{1}{3}\left[\frac{4t^3}{3}\Big|_0^1 + 16t\Big|_1^2 - 8t^2\Big|_1^2 + \frac{4}{3}t^3\Big|_1^2\right]\right\}^{1/2}$$

$$I_{rms} = \left\{\frac{1}{3}\left[\frac{4}{3} + 16 - 8(4-1) + \frac{4}{3}(8-1)\right]\right\}^{1/2}$$

$$\boxed{I_{rms} = 0.94\ A}$$

9.47 Calculate the rms value of the waveform shown in Fig. P9.47.

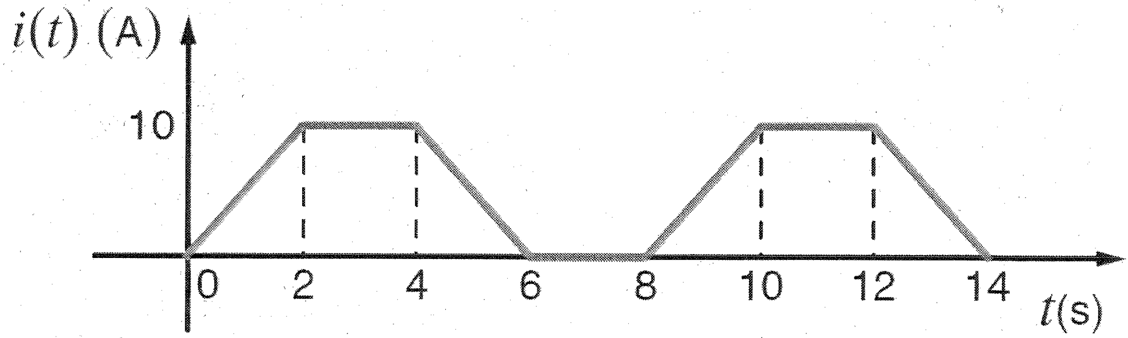


Figure P9.47

SOLUTION:

$i(t)$ consists of 3 parts – 2 identical triangles and a rectangle.

For the triangle: $i_1(t) = 5t$ and $i_1^2(t) = 25t^2$

$$\int_0^2 i_1^2(t)\,dt = \frac{25t^3}{3}\Big|_0^2 = 66.67$$ (same for 2nd triangle

For the rectangle $i_2(t) = 10$ and $i_2^2(t) = 100$

$$\int_2^4 i_2^2(t)\,dt = 100t\Big|_2^4 = 200$$

$$I_{rms} = \left\{\frac{1}{T}\left[2(66.67) + 200\right]\right\}^{1/2} \qquad T = 8s$$

$$\boxed{I_{rms} = 6.45\ A}$$

9.48 Calculate the rms value of the waveform shown in Fig. P9.48. CS

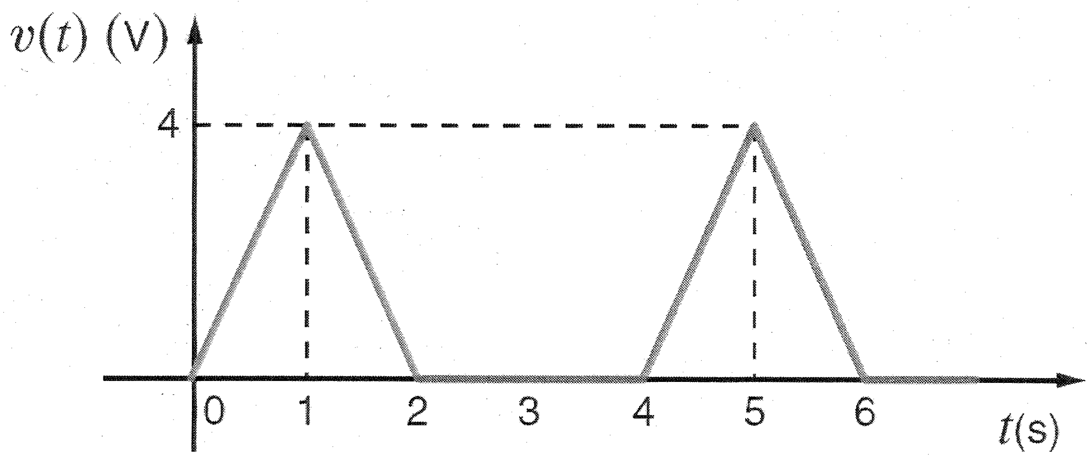


Figure P9.48

SOLUTION:

$v(t)$ consists of 2 identical triangles:

1st triangle: $v_1(t) = 4t$ and $v_1^2(t) = 16t^2$

$$\int_0^1 v_1^2(t)\,dt = \frac{16t^3}{3}\Big|_0^1 = \frac{16}{3}$$ (same for 2nd triangle)

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \left\{\frac{1}{T}\left[2\left(16/3\right)\right]\right\}^{1/2} \qquad T = 4s$$

$$\boxed{V_{rms} = 1.63 \text{ V}}$$

9.49 The current waveform in Fig. P9.49 is flowing through a 5-Ω resistor. Find the average power absorbed by the resistor. **PSV**

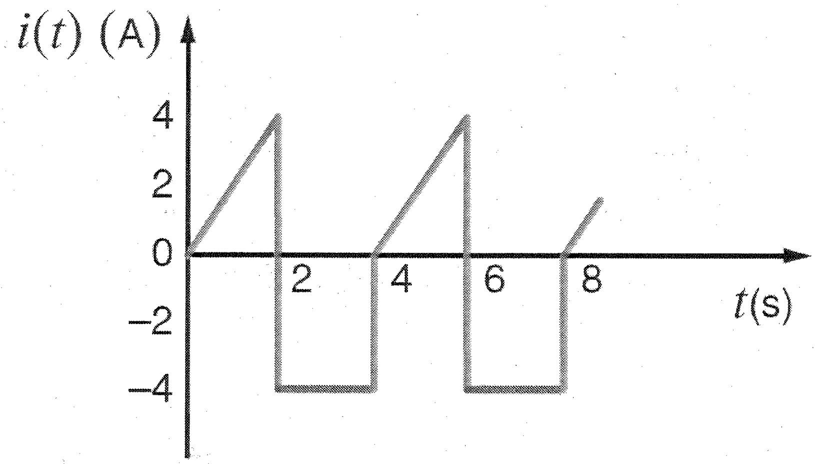


Figure P9.49

SOLUTION: $T = 4s$

$i(t)$ consists of a triangle and a rectangle

Triangle: $i_1(t) = 2t$ and $i_1^2(t) = 4t^2$

$$\int_0^2 i_1^2(t)\,dt = \frac{4}{3}t^3\Big|_0^2 = 10.67$$

Rectangle: $i_2(t) = -4$ and $i_2^2(t) = 16$

$$\int_2^4 16\,dt = 16t\Big|_2^4 = 32$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = \left\{\frac{1}{4}\left[10.67 + 32\right]\right\}^{1/2}$$

$$\boxed{I_{rms} = 3.27\text{ A}}$$

9.50 Calculate the rms value of the waveform shown in Fig. P9.50. CS

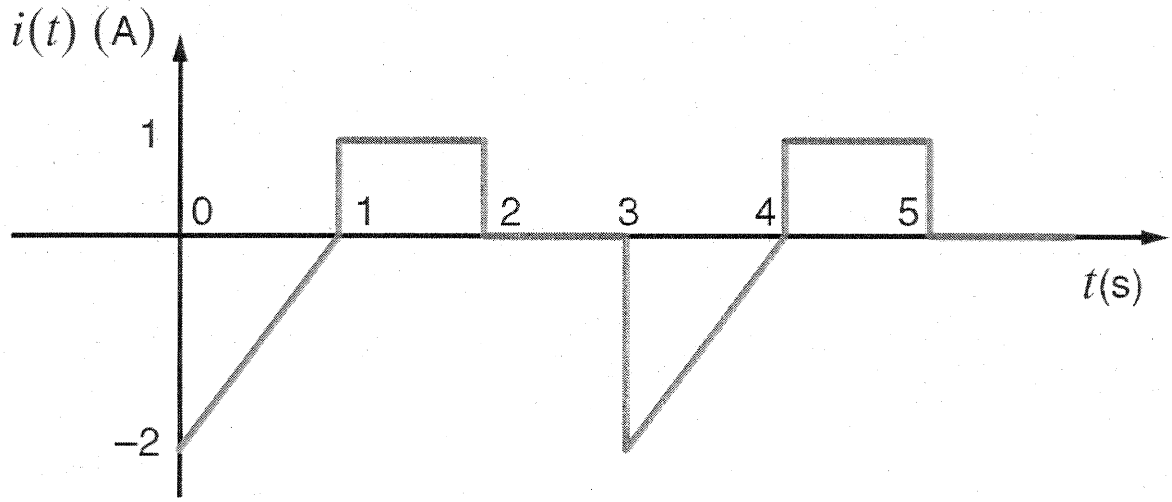


Figure P9.50

SOLUTION: $T = 3\text{s}$

$i(t)$ consists of a triangle of 1 second duration and a rectangle of 1 second duration.

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

Triangle: $i_1(t) = 2t - 2$ and $i_1^2(t) = 4t^2 - 8t + 4$

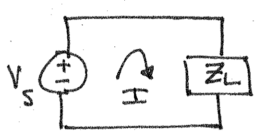
$$\int_0^1 i_1^2(t)dt = \frac{4t^3}{3}\Big|_0^1 - 4t^2\Big|_0^1 + 4t\Big|_0^1 = \frac{4}{3}$$

Rectangle: $i_2(t) = 1$ and $i_2^2(t) = 1$ $\int_1^2 i_2^2(t)\,dt = 1$

$$I_{RMS} = \left\{\frac{1}{3}\left[4/3 + 1\right]\right\}^{1/2}$$ $\boxed{I_{RMS} = 0.882\text{ A}}$

9.51 A plant consumes 20 kW of power from a 240-V rms line. If the load power factor is 0.9, what is the angle by which the load voltage leads the load current? What is the load current phasor if the line voltage has a phasor of 240 $\angle 0^\circ$ V rms?

SOLUTION:



$V_S = 240 \angle 0^\circ \; V_{rms}$

$P_L = 20 \text{ kW}$ pf = 0.9 (lagging implied)

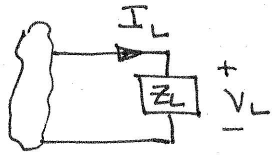
$\theta = \cos^{-1}(pf) = \cos^{-1}(0.9) \Rightarrow \boxed{\theta = 25.8^\circ}$

$P_L = |V_S||I|\, pf \Rightarrow |I| = \dfrac{P_L}{|V_S|\, pf} = \dfrac{2 \times 10^4}{240\,(.9)} = 92.6 A$

$$\boxed{I = 92.6 \angle -25.8^\circ \; A_{rms}}$$

9.52 A plant consumes 100 kW of power at 0.9 pf lagging. If the load current is 200 A rms, find the load voltage.

SOLUTION:



$|I_L| = 200\,A_{rms}$ $\quad$ $pf = 0.9$ lag

$P_L = 100\,kW$ $\quad$ $\theta_{V_L} - \theta_{I_L} = \cos^{-1}(pf) = 25.8°$

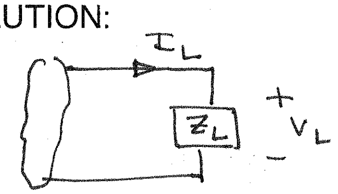
$$P_L = |V_L||I_L|\,pf \implies |V_L| = \frac{P_L}{|I_L|\,pf} = 556\,V_{rms}$$

assuming $I_L = 200\angle 0°\,A_{rms}$,

$$\boxed{V_L = 556\angle 25.8°\,V_{rms}}$$

9.53 A plant draws 250 A rms from a 240-V rms line to supply a load with 50 kW. What is the power factor of the load?

SOLUTION:



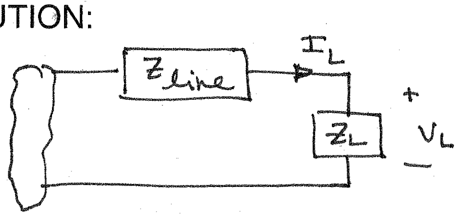
$$P_L = |V_L|\,|I_L|\,pf$$

$$pf = \frac{P_L}{|V_L|\,|I_L|}$$

$$\boxed{pf = 0.833}$$

9.54 The power company supplies 80 kW to an industrial load. The load draws 220 A rms from the transmission line. If the load voltage is 440 V rms and the load power factor is 0.8 lagging, find the losses in the transmission line. CS

SOLUTION:



assume $\theta_{V_L} = 0°$

$V_L = 440\angle 0°\ V_{rms}$

$I_L = 220\angle -\theta\ A_{rms}$

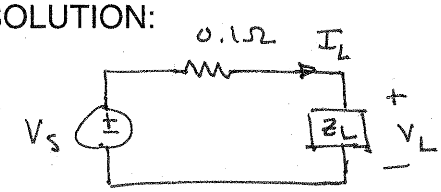
$P_L = |V_L||I_L|\,pf = 77440\ W$

$\theta = \cos^{-1}(0.8) = 36.9°$

$P_{line} = P_S - P_L = 80000 - 77440$ $\boxed{P_{line} = 2560\ W}$

9.55 An industrial load that consumes 40 kW is supplied by the power company through a transmission line with 0.1 Ω resistance, with 44 kW. If the voltage at the load is 240 V rms, find the power factor at the load.

SOLUTION:



$P_L = 4\times10^4$ W $\quad |V_L| = 240V_{rms}$

$P_L = |V_L||I_L|\,pf \qquad P_S = 44kW$

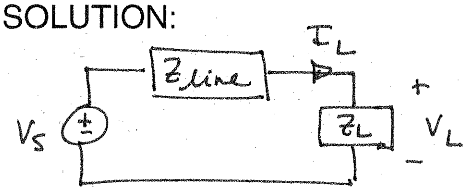
$P_{line} = P_S - P_L = 4kW = |I_L|^2(0.1)$

$|I_L| = 200A_{rms}$

$pf = \dfrac{P_L}{|V_L||I_L|} \qquad \boxed{pf = 0.83}$

9.56 The power company supplies 40 kW to an industrial load. The load draws 200 A rms from the transmission line. If the load voltage is 240 V rms and the load power factor is 0.8 lagging, find the losses in the transmission line.

SOLUTION:



$P_S = 40 \text{ kW}$

$|V_L| = 240 \text{ V}_{rms}$ $\quad |I_L| = 200 \text{ A}_{rms}$

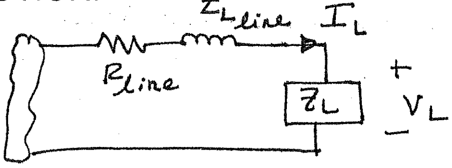
$P_L = |V_L||I_L| pf = 38.4 \text{ kW}$

$$\boxed{P_{line} = P_S - P_L = 1600 \text{ W}}$$

9.57 A transmission line with impedance $0.08 + j0.25\ \Omega$ is used to deliver power to a load. The load is inductive and the load voltage is $220\angle 0^\circ$ V rms at 60 Hz. If the load requires 12 kW and the real power loss in the line is 560 W, determine the power factor angle of the load.

CS

SOLUTION:



$R_{line} = 0.08\ \Omega \quad Z_{L_{line}} = j0.25\ \Omega$

$V_L = 220\angle 0^\circ\ V_{rms}$

$P_{line} = |I_L|^2 R_{line} = 560 \Rightarrow |I_L| = 83.7\ A$

$P_L = |V_L||I_L|\ pf \Rightarrow pf = \dfrac{12\times10^3}{(220)(83.7)} = 0.65$

Since load is inductive, pf is lagging.

$\boxed{pf = 0.65 \text{ lagging}}$ $\boxed{\theta_{Z_L} = \tan^{-1}(pf) = 49.4^\circ}$

9.58 Determine the real power, the reactive power, the complex power, and the power factor for a load having the following characteristics.

(a) $\mathbf{I} = 2\,\underline{/40^\circ}$ A rms, $\mathbf{V} = 450\,\underline{/70^\circ}$ V rms.

(b) $\mathbf{I} = 1.5\,\underline{/-20^\circ}$ A rms, $\mathbf{Z} = 5000\,\underline{/15^\circ}$ Ω.

(c) $\mathbf{V} = 200\,\underline{/+35^\circ}$ V rms, $\mathbf{Z} = 1500\,\underline{/-15^\circ}$ Ω.

SOLUTION:

a) $P = |I||V| \cos(\theta_V - \theta_I) = (2)(450)\cos(70-40)$

$\boxed{P = 779\ \text{W}}$

$Q = |V||I| \sin(\theta_V - \theta_I) \Rightarrow \boxed{Q = 450\ \text{VARs}}$

$S = VI^* = (450\,\underline{/70})(2\,\underline{/-40^\circ})$ $\boxed{S = 900\,\underline{/30^\circ}\ \text{VA}}$

$pf = \cos^{-1}(30^\circ) \Rightarrow \boxed{pf = 0.866\ \text{lag}}$

b) $V = IZ = 7500\,\underline{/-5^\circ}\ V_{rms}$ $P = (1.5)(7500)\cos(15) \Rightarrow \boxed{P = 10.9\text{kW}}$

$Q = (1.5)(7500)\sin(15) \Rightarrow \boxed{Q = 2912\ \text{VARs}}$

$S = VI^* \rightarrow \boxed{S = 11250\,\underline{/15^\circ}\ \text{VA}}$ $\boxed{pf = 0.966\ \text{lagging}}$

c) $I = V/Z = 0.133\,\underline{/50^\circ}$ $S = VI^* \Rightarrow \boxed{S = 26.7\,\underline{/-15^\circ}\ \text{VA}}$

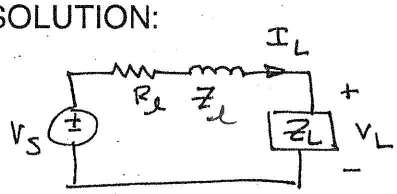
$P = |S|\cos(\theta_V - \theta_I) = 26.7\cos(-15^\circ)$ $\boxed{P = 25.8\ \text{W}}$

$Q = |S|\sin(\theta_V - \theta_I) \Rightarrow \boxed{Q = -6.91\ \text{VARs}}$

$pf = \cos^{-1}(-15^\circ) \Rightarrow \boxed{pf = 0.966\ \text{leading}}$

9.59 An industrial load operates at 30 kW, 0.8 pf lagging. The load voltage is $240\,\underline{/0^\circ}$ V rms. The real and reactive power losses in the transmission-line feeder are 1.8 kW and 2.4 kvar, respectively. Find the impedance of the trasmission line and the input voltage to the line. **PSV**

SOLUTION:



$$|I_L| = \frac{P_L}{|V_L|\,pf} = \frac{3\times10^4}{240(0.8)} = 156\ A_{rms}$$

$$\theta_{Z_L} = \cos^{-1}(pf) = 36.9^\circ$$

$$I_L = 156\,\underline{/-36.9^\circ}\ A_{rms}$$

$$|I_L|^2 R_\ell = 1800$$

$$R_\ell = 74\,m\Omega$$

$$|I_L|^2 Z_\ell = j2400$$

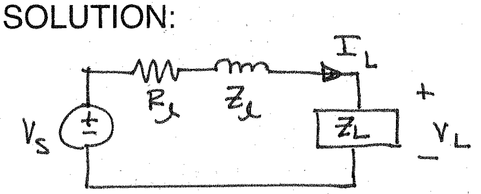
$$Z_\ell = j98\,m\Omega$$

$$\boxed{Z_{line} = 74 + j98\ m\Omega}$$

$$V_S = Z_{line} I_L + V_L \Rightarrow \boxed{V_S = 259\,\underline{/1.17^\circ}\ V_{rms}}$$

9.60 A transmission line with impedance $0.1 + j0.2\ \Omega$ is used to deliver power to a load. The load is capacitive and the load voltage is $240\,\underline{/0^\circ}$ V rms at 60 Hz. If the load requires 15 kW and the real power loss in the line is 660 W, determine the input voltage to the line.

SOLUTION:



$P_\ell = R_\ell |I_L|^2 \Rightarrow |I_L| = 81.2$ Arms

$P_L = 1500 = |V_L||I_L|\,pf_L \Rightarrow pf_L = 0.77$

Since load is capacitive, pf = 0.77 leading

$\theta_{Z_L} = -\cos^{-1}(pf) = -39.7^\circ = \theta_{V_L} - \theta_{I_L}$

$\theta_{I_L} = 39.7^\circ$

$I_L = 81.2\,\underline{/39.7^\circ}$ Arms

$V_S = I_L(R_\ell + Z_\ell) + V_L$

$= 81.2\,\underline{/39.7}\,(0.1 + j0.2) + 240\,\underline{/0^\circ}$

$$\boxed{V_S = 236\,\underline{/4.29^\circ}\ V_{rms}}$$

9.61 Find the real and reactive power absorbed by each element in the circuit in Fig. P9.61.

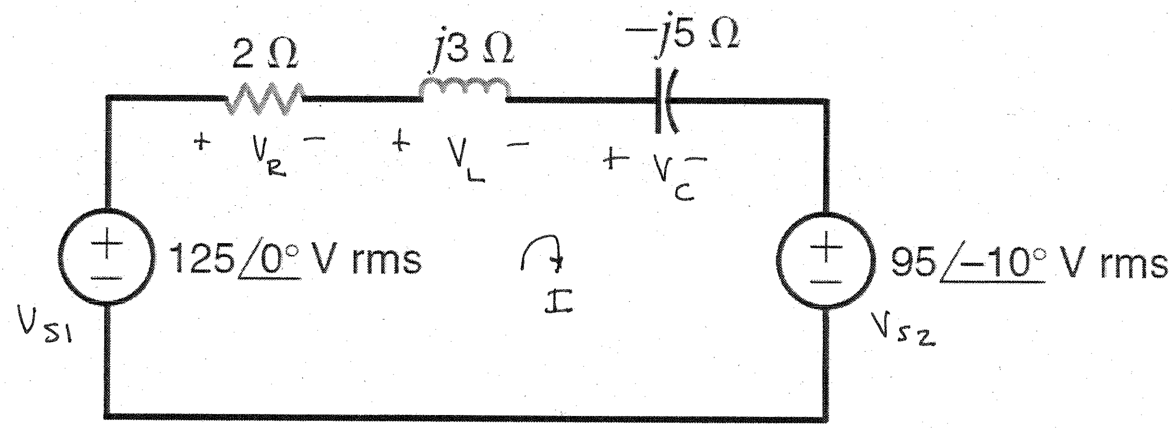


Figure P9.61

SOLUTION:

$$I = \frac{V_{S1} - V_{S2}}{2 + j3 - j5} = \frac{35.5\angle{+27.7°}}{2 - j2} = 12.5\angle{72.7°}\ A_{rms}$$

for V_{S1}: $S = -V_{S1} I^* = -(125\angle 0)(12.5\angle{-72.7°})$

$S_{V_{S1}} = 1563\angle 107.3$ VA

$\boxed{P_{VS_1} = -463\ W \qquad Q_{VS_1} = 1492\ VAR}$

for V_{S2}: $S_{V_{S2}} = I^* V_{S2} = 1188\angle{-82.7°}$ VA

$\boxed{P_{VS2} = 150\ W \qquad Q_{VS2} = -1178\ VAR}$

$P_R = |I|^2 R$ $\boxed{P_R = 313\ W}$ $\boxed{Q_R = 0}$

$Q_L = |I|^2(3)$ $\boxed{Q_L = 469\ VAR \quad P_L = 0}$

$Q_C = |I|^2(-5)$ $\boxed{Q_C = -781\ VAR \quad P_C = 0}$

9.62 Given the network in Fig. P9.62, determine the input voltage $\mathbf{V}_S$.

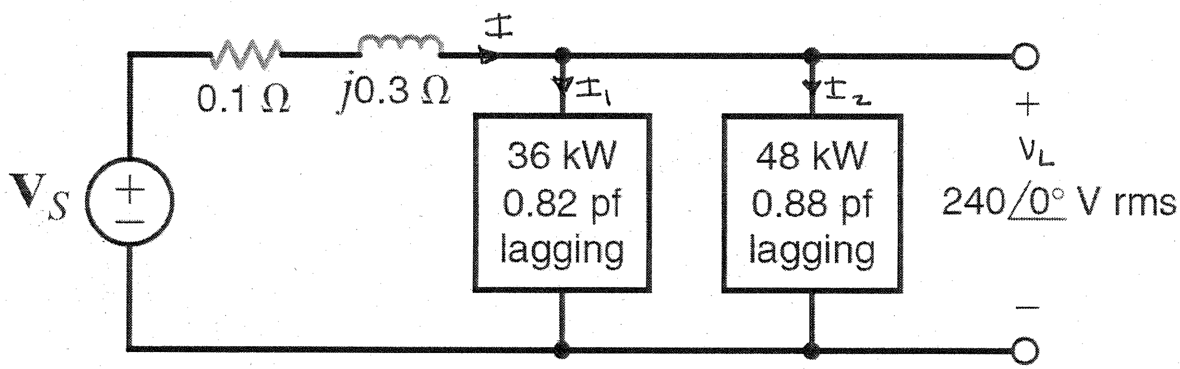


Figure P9.62

SOLUTION:

Load 1: $P_{L1} = |V_L||I_1|(pf_1) \Rightarrow |I_1| = \dfrac{P_{L1}}{|V_L|\, pf_1} = 183$ Arms

$\angle I_1 = \theta_{V_L} - \cos^{-1}(pf_1) = -34.9°$

$I_1 = 183 \angle -34.9°$ Arms

Load 2: $|I_2| = \dfrac{P_{L2}}{|V_L|\, pf_2} = 227$ Arms $\quad \theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -28.4°$

$I_2 = 227 \angle -28.4°$ Arms

$I = I_1 + I_2 = 409 \angle -31.3°$ Arms

$V_s = I(0.1 + j0.3) + V_L$

$$\boxed{V_s = 349 \angle 13.9° \text{ Vrms}}$$

9.63 Given the network in Fig. P9.63, compute the input source voltage and the input power factor.

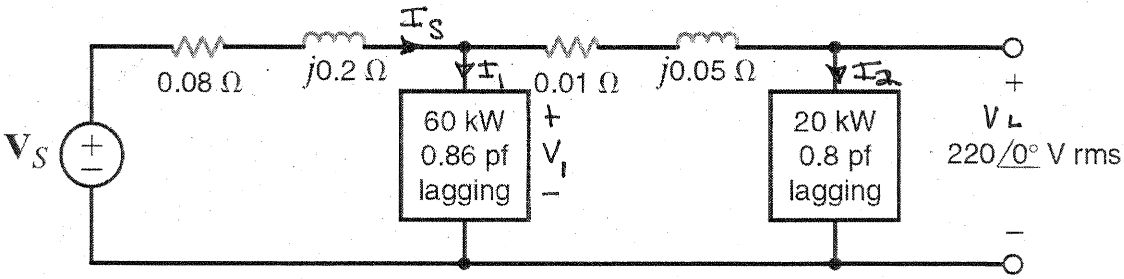


Figure P9.63

SOLUTION:

Load 2: $|I_2| = \frac{P_{L2}}{|V_L|(pf_2)} = \frac{20\times10^3}{220\ (0.8)} = 114$ Arms

$\theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -36.9°$

$I_2 = 114\angle{-36.9°}$ Arms

Load 1: $|I_1| = \frac{P_{L1}}{|V_1|(pf_1)}$ $\quad \theta_{I_2} = \theta_{V_1} - \cos^{-1}(pf_1)$

$V_1 = (0.01 + j0.05)I_2 + V_L = 224\angle{1.00°}$ Vrms

$I_1 = 311\angle{-29.7°}$ Arms

$I_S = I_1 + I_2 = 424\angle{-31.6°}$ Arms

$V_S = (0.08 + j0.2)\, I_S + V_1$

$$\boxed{V_S = 303\angle{11.1°}\ \text{Vrms}}$$

$pf_S = \cos(\theta_{V_S} - \theta_{I_S}) = \cos(11.1 - (-31.6))$

$$\boxed{pf_S = 0.73\ \text{lagging}}$$

9.64 Use Kirchhoff's laws to compute the source voltage of the network shown in Fig. P9.64.

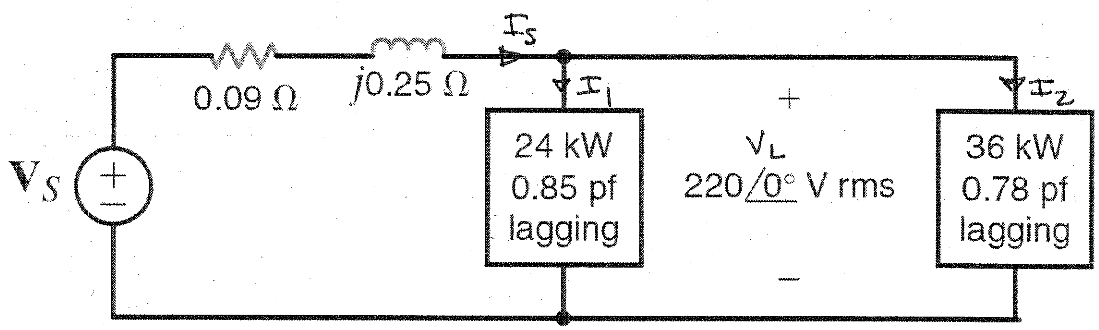


Figure P9.64

SOLUTION:

Load 2:
$$|I_2| = \frac{P_{L2}}{|V_L|(pf_2)} = \frac{36\times10^3}{220\,(0.78)} = 210\,A\,rms$$
$$\theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -38.7°$$
$$I_2 = 210\,\angle -38.7°\ Arms$$

Load 1:
$$|I_1| = \frac{P_{L1}}{|V_L|(pf_1)} = \frac{24\times10^3}{220\,(0.85)} = 128\,A\,rms$$
$$\theta_{I_1} = \theta_{V_L} - \cos^{-1}(pf_1) = -31.8°$$
$$I_1 = 128\,\angle -31.8°\ Arms$$

$$I_s = I_1 + I_2 = 337\,\angle -36.1°\ Arms$$

$$V_s = (0.09 + j0.25)\,I_s + V_L$$

$$\boxed{V_s = 298\,\angle 9.70°\ V_{rms}}$$

9.65 Given the network in Fig. P9.65, determine the input voltage $\mathbf{V}_S$. **PSV**

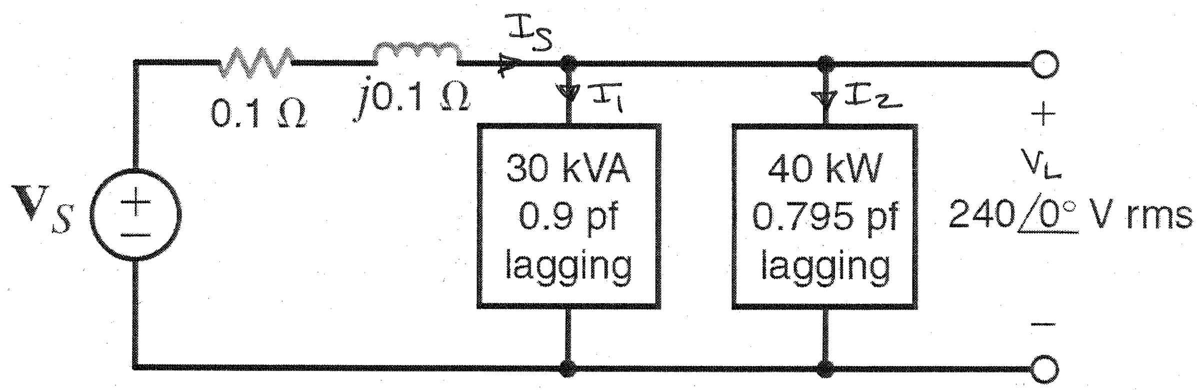


Figure P9.65

SOLUTION:

Load1: $\left\{ \begin{array}{l} |I_1| = \dfrac{P_{L1}}{|V_L|(pf_1)} = \dfrac{30\,000}{240(0.9)} = 139 \text{ Arms} \\ \theta_{I_1} = \theta_{V_L} - \cos^{-1}(0.9) = -25.8° \end{array} \right\} \quad I_1 = 139\underline{/-25.8°} \text{ Arms}$

Load2: $\left\{ \begin{array}{l} |I_2| = \dfrac{P_{L2}}{|V_L|(pf_2)} = 210 \text{ Arms} \\ \theta_{I_2} = \theta_{V_L} - \cos^{-1}(0.795) = -37.3° \end{array} \right\} \quad I_2 = 210\underline{/-37.3°} \text{ Arms}$

$I_s = I_1 + I_2 = 347\underline{/-32.7°} \text{ Arms}$

$V_s = (0.1 + j0.1) I_s + V_L$

$$\boxed{V_s = 288\underline{/2.08°} \text{ Vrms}}$$

9.66 Find the input source voltage and the power factor of the source for the network shown in Fig. P9.66. CS

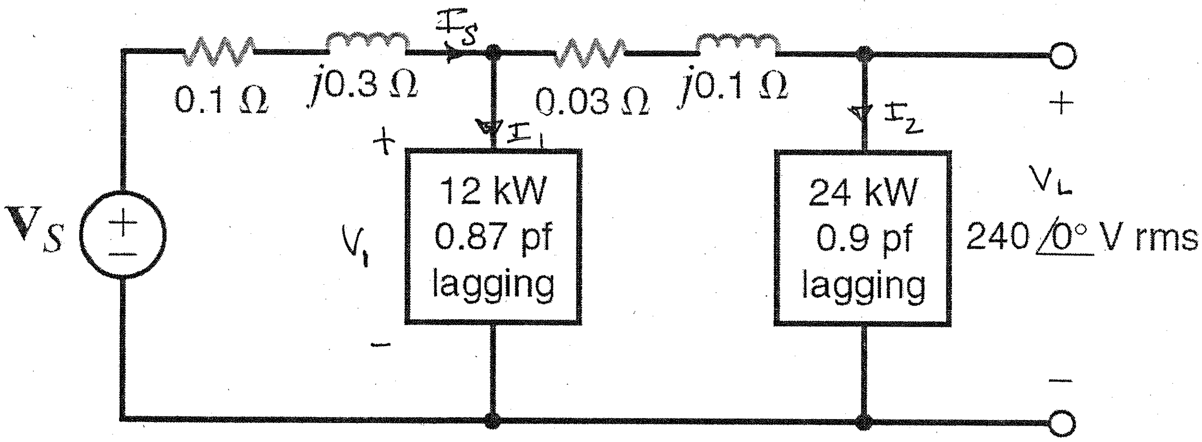


Figure P9.66

SOLUTION:

Load 2: $\left\{ |I_2| = \dfrac{P_{L2}}{|V_L|(pf_2)} = \dfrac{24000}{240\,(0.9)} = 111\ A_{rms};\quad \theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -25.8^\circ \right\}$ $I_2 = 111\,\underline{/-25.8^\circ}\ A_{rms}$

$$V_1 = (0.03 + j0.1)I_2 + V_L = 249\,\underline{/1.97^\circ}\ V_{rms}$$

Load 1: $\left\{ |I_1| = \dfrac{P_{L1}}{|V_1|(pf_1)} = \dfrac{12000}{249\,(0.87)} = 55.4\ A_{rms};\quad \theta_{I_1} = \theta_{V_1} - \cos^{-1}(pf_1) = -27.6^\circ \right\}$ $I_1 = 55.4\,\underline{/-27.6^\circ}\ A_{rms}$

$$I_S = I_1 + I_2 = 166\,\underline{/-26.4^\circ}\ A_{rms}$$

$$V_S = (0.1 + j0.3)\,I_S + V_1 \qquad \boxed{V_S = 289\,\underline{/9.14^\circ}\ V_{rms}}$$

$$pf_s = \cos(\theta_{V_S} - \theta_{I_S}) = \cos(35.54^\circ) = 0.814$$

$$\boxed{pf_s = 0.814\ \text{lagging}}$$

9.67 Given the network in Fig. P9.67, find the complex power supplied by the source, the power factor of the source, and the voltage $v_S(t)$. The frequency is 60 Hz.

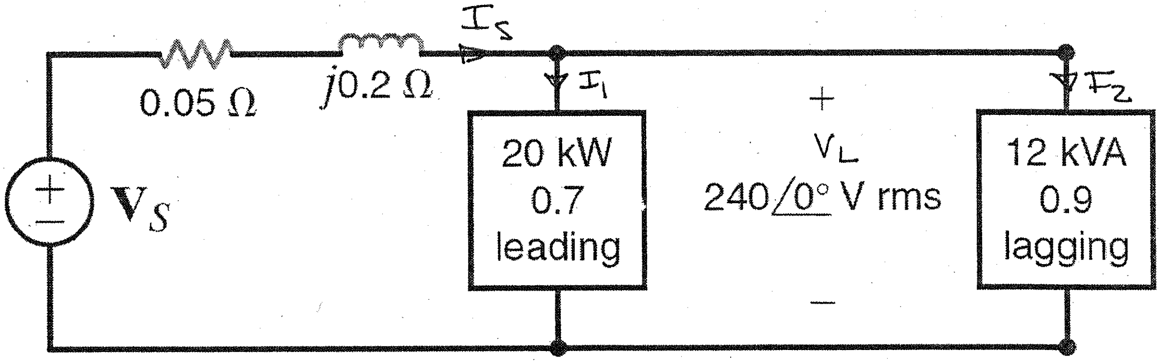


Figure P9.67

SOLUTION:

Load 2: $|I_2| = \dfrac{P_{L2}}{|V_L|(pf_2)} = \dfrac{12000}{240(0.9)} = 55.6 \text{ Arms}$

$\theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -25.8°$

$I_2 = 55.6\angle -25.8° \text{ Arms}$

Load 1: $|I_1| = \dfrac{P_{L1}}{|V_L|(pf_1)} = \dfrac{20000}{240(0.7)} = 119 \text{ Arms}$

$\theta_{I_1} = \theta_{V_L} + \cos^{-1}(pf_1) = +45.6°$

$I_1 = 119\angle +45.6° \text{ Arms}$

$I_S = I_1 + I_2 = 147\angle 24.5° \text{ Arms}$

$V_S = (0.05 + j0.2)I_S + V_L = 236\angle 7.22° \text{ Vrms}$

$S_{V_S} = V_S I_S^*$ $\quad pf_S = \cos(\theta_{S_S}) = \cos(43.8)$ $\quad \boxed{pf_S = 0.955 \text{ leading}}$

$\boxed{S_{V_S} = 34.6\angle -17.3° \text{ kVA}}$

$v_S(t) = 334\cos(377t + 7.22°) \text{ V}$

9.68 Given the circuit in Fig. P9.68, find the complex power supplied by the source and the source power factor. If $f = 60$ Hz, find $v_s(t)$.

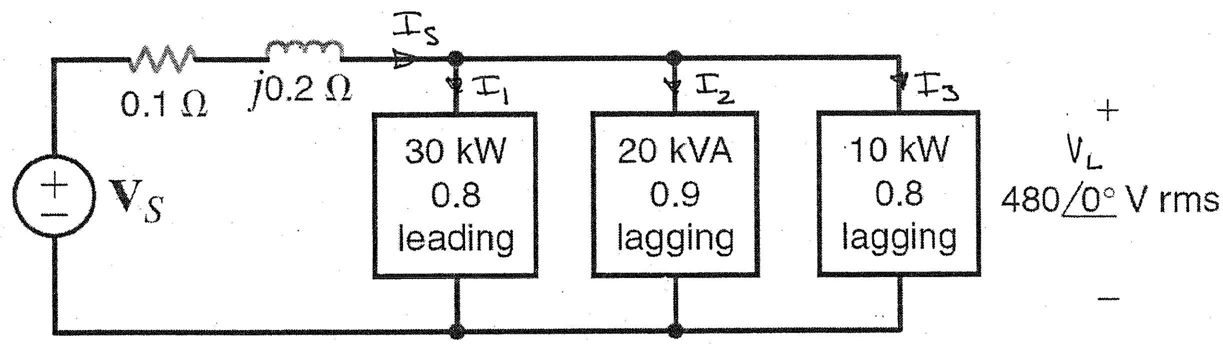


Figure P9.68

SOLUTION:

Load 3: $\left\{ \begin{array}{l} |I_3| = \dfrac{P_{L3}}{|V_L|(pf_3)} = \dfrac{10000}{480(0.8)} = 26.0 \text{ Arms} \\ \theta_{I_3} = \theta_{V_L} - \cos^{-1}(pf_3) = -36.8^\circ \end{array} \right\} I_3 = 26.0\angle{-36.8^\circ} \text{ Arms}$

Load 2: $\left\{ \begin{array}{l} |I_2| = \dfrac{20000}{480(0.9)} = 46.3 \text{ Arms} \\ \theta_{I_2} = 0 - \cos^{-1}(0.9) = -25.8^\circ \end{array} \right\} I_2 = 46.3\angle{-25.8^\circ} \text{ Arms}$

Load 1: $\left\{ \begin{array}{l} |I_1| = \dfrac{30000}{480(0.8)} = 78.1 \text{ Arms} \\ \theta_{I_3} = \theta_{V_L} + \cos^{-1}(pf_1) = 36.9^\circ \end{array} \right\} I_3 = 78.1\angle{36.9^\circ} \text{ Arms}$

$I_s = I_1 + I_2 + I_3 = 125\angle{5.25^\circ} \text{ Arms}$

$V_s = (0.1 + j0.2)I_s + V_L = 491\angle{3.06^\circ}$

$S_s = V_s I_s^* = 61.4\angle{-2.19^\circ} \text{ kVA}$

$\boxed{S_s = 61.4\angle{-2.19^\circ} \text{ kVA}}$

$pf_s = \cos(\theta_{V_s} - \theta_{I_s}) = 0.999$

$\boxed{pf_s = 0.999 \text{ leading}}$

since $\theta_V - \theta_I < 0$, leading

$v_s(t) = \sqrt{2}\,|V_s| \cos(2\pi f t + 3.06^\circ)$ V

$\boxed{v_s = 694\cos(377t + 3.06^\circ) \text{ V}}$

9.69 In the circuit in Fig. P9.69, the complex power supplied by source $\mathbf{V}_1$ is $2000\,\underline{/-30^\circ}$ VA. If $\mathbf{V}_1 = 200\,\underline{/10^\circ}$ V rms, find $\mathbf{V}_2$.

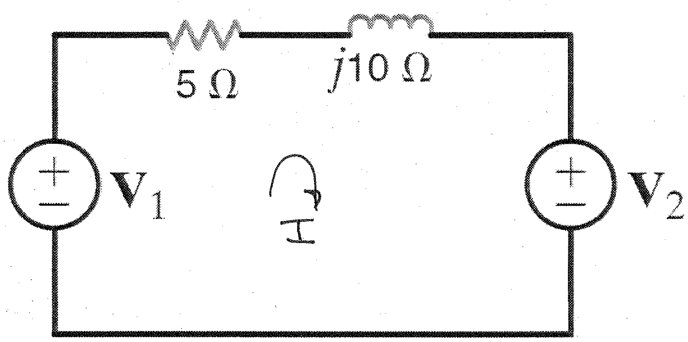


Figure P9.69

SOLUTION:

$$S_1 = V_1 I^* \Rightarrow 2000\,\underline{/-30} = 200\,\underline{/10}\; I^*$$

$$I^* = 10\,\underline{/-40} \qquad I = 10\,\underline{/40^\circ} \text{ Arms}$$

$$V_1 = I(5 + j10) + V_2 \Rightarrow V_2 = V_1 - (5 + j10)\, I$$

$$\boxed{V_2 = 235\,\underline{/-18.4^\circ} \text{ Vrms}}$$

9.70 For the network in Fig. P9.70, the complex power absorbed by the source on the right is $0 + j1582.5$ VA. Find the value of R and the unknown element and its value if $f = 60$ Hz. (If the element is a capacitor, give its capacitance; if the element is an inductor, give its inductance.)

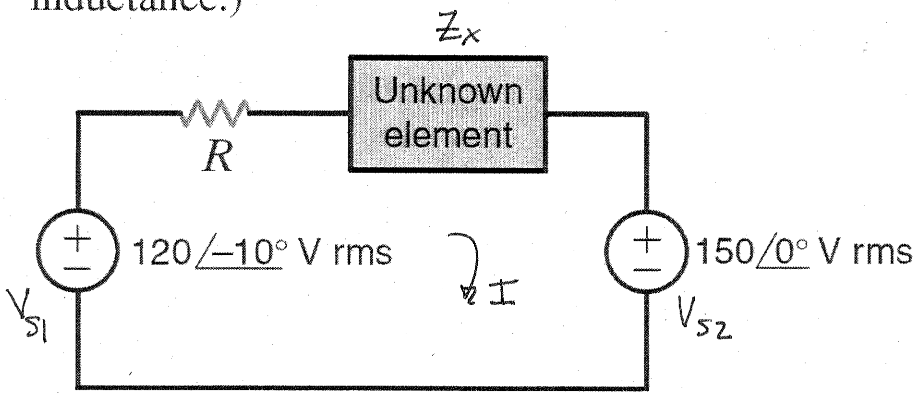


Figure P9.70

SOLUTION:

$$S_2 = j1582.5 \text{ VA} = V_{S2} I^* = 150 \angle 0 \, I^*$$

$$I^* = \frac{1582.5 \angle 90}{150 \angle 0^\circ} = 10.6 \angle 90^\circ \Rightarrow I = 10.6 \angle -90^\circ \text{ Arms}$$

$$\frac{V_{S1} - V_{S2}}{I} = R + Z_X = 1.97 - j3.00 \, \Omega$$

$$\boxed{R = 1.97 \Omega} \qquad Z_X = -j3 \Omega = -j/\omega C_X \qquad \omega = 377 \text{ r/s}$$

$$\boxed{C_X = 884 \mu F}$$

9.71 A plant consumes 60 kW at a power factor of 0.75 lagging from a 240-V rms 60-Hz line. Determine the value of the capacitor that when placed in parallel with the load will change the load power factor to 0.9 lagging.

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{60000}{0.75} = 80\text{kVA} \qquad \theta_{old} = \cos^{-1}(0.75) = 41.4^\circ$$

$$S_{old} = 80\,\angle 41.4^\circ\ \text{kVA} \qquad Q_{old} = |S_{old}|\sin\theta_{old} = 52.9\ \text{kVARs}$$

$$|S_{new}| = \frac{P_L}{pf_{new}} = \frac{60000}{0.9} = 66.7\ \text{kVA} \qquad \theta_{new} = 25.8^\circ$$

$$S_{new} = 66.7\,\angle 25.8^\circ\ \text{kVA} \qquad Q_{new} = 29.1\ \text{kVARs}$$

$$Q_C + Q_{old} = Q_{new} \Rightarrow -\omega C|V_L|^2 = (29.1 - 52.9)\times 10^3$$

$$|V_L| = 240\ V_{rms} \qquad \omega = 377\ r/s$$

$$\boxed{C = 1.1\,\text{mF}}$$

9.72 A particular load has a pf of 0.8 lagging. The power delivered to the load is 40 kW from a 270-V rms 60-Hz line. What value of capacitance placed in parallel with the load will raise the pf to 0.9 lagging? **PSV**

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{40000}{0.8} = 50\,kVA \qquad \theta_{old} = \cos^{-1}(0.8) = 36.9^\circ$$

$$Q_{old} = |S_{old}| \sin\theta_{old} = 30\,kVAR$$

$$|S_{new}| = \frac{40000}{0.9} = 44.4\,kVA \qquad \theta_{new} = \cos^{-1}(0.9) = 25.8^\circ$$

$$Q_{new} = (44.4\times10^3)\sin(\theta_{new}) = 19.4\,kVAR$$

$$Q_C + Q_{old} = Q_{new} \Rightarrow -\omega C|V_L|^2 = (19.4 - 30)\times10^3$$

$$\omega = 377\,r/s \qquad |V_L| = 270\,V_{rms}$$

$$\boxed{C = 386\,\mu F}$$

9.73 An industrial load is supplied through a transmission line that has a line impedance of $0.1 + j0.2\ \Omega$. The 60-Hz line voltage at the load is $480\angle 0^\circ$ V rms. The load consumes 124 kW at 0.75 pf lagging. What value of capacitance when placed in parallel with the load will change the power factor to 0.9 lagging? CS

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{124\times10^3}{0.75} = 165.3\text{ kVA} \qquad \theta_{old} = \cos^{-1}(pf_{old}) = 41.4^\circ$$

$$S_{old} = 165.3\angle 41.4^\circ\text{ kVA} \qquad Q_{old} = \text{Im}(S_{old}) = 109\text{ kVAR}$$

$$|S_{new}| = \frac{124\times10^3}{0.9} = 137.8\text{ kVA} \qquad \theta_{new} = \cos^{-1}(0.9) = 25.8^\circ$$

$$S_{new} = 137.8\angle 25.8^\circ\text{ kVA} \qquad Q_{new} = \text{Im}(S_{new}) = 60.1\text{ kVAR}$$

$$Q_C = Q_{new} - Q_{old} \Rightarrow -\omega C|V_L|^2 = (60.1 - 109)\times10^3$$

$\omega = 377$ r/s $\qquad |V_L| = 480$ Vrms

$$\boxed{C = 563\ \mu F}$$

9.74 The 60-Hz line voltage for a 60-kW, 0.76-pf lagging industrial load is 240 $\angle 0°$ V rms. Find the value of capacitance that when placed in parallel with the load will raise the power factor to 0.9 lagging.

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{60000}{0.76} = 78.9\ kVA$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = \cos^{-1}(0.76) = 40.5°$$

$$S_{old} = 78.9 \angle 40.5°\ kVA$$

$$Q_{old} = 51.3\ kVAR$$

$$|S_{new}| = \frac{60000}{0.9} = 66.6\ kVA$$

$$\theta_{new} = \cos^{-1}(0.9) = 25.8°$$

$$S_{new} = 66.6 \angle 25.8°\ kVA$$

$$Q_{new} = 29.1\ kVAR$$

$$Q_C = Q_{new} - Q_{old} \Rightarrow -\omega C |V_L|^2 = (29.1 - 51.3) \times 10^3$$

$\omega = 377\ r/s$ $\quad |V_L| = 240\text{V rms}$

$$\boxed{C = 1.02\ mF}$$

9.75 An industrial load consumes 44 kW at 0.82 pf lagging from a 240 $\angle 0°$-V-rms 60-Hz line. A bank of capacitors totaling 600 μF is available. If these capacitors are placed in parallel with the load, what is the new power factor of the total load?

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{44\,000}{0.82} = 53.7\text{ kVA}$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = 34.9°$$

$$S_{old} = 53.7\angle 34.9°\text{ kVA}$$

$$Q_{old} = \text{Im}\{S_{old}\}$$

$$Q_{old} = 30.7\text{ kVAR}$$

$$Q_C = -\omega C|V_L|^2 = -377(600\times10^{-6})(240)^2 = -13.0\text{ kVAR}$$

$$Q_C = Q_{new} - Q_{old} \Rightarrow Q_{new} = 17.7\text{ kVAR}$$

$$\theta_{new} = \tan^{-1}\left(\frac{Q_{new}}{P_L}\right) = 21.9 \quad \text{lagging since } \theta > 0°$$

$$pf_{new} = \cos\theta_{new} \qquad \boxed{pf_{new} = 0.928 \text{ lagging}}$$

9.76 A particular load has a pf of 0.8 lagging. The power delivered to the load is 40 kW from a 220-V rms 60-Hz line. What value of capacitance placed in parallel with the load will raise the pf to 0.9 lagging? CS

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{40000}{0.8} = 50\text{ kVA}$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = 36.9^\circ$$

$$S_{old} = 50\angle 36.9^\circ \text{ kVA}$$

$$Q_{old} = \text{Im}\{S_{old}\}$$

$$Q_{old} = 30\text{ kVAR}$$

$$|S_{new}| = \frac{40000}{0.9} = 44.4\text{ kVA}$$

$$\theta_{new} = \cos^{-1}(0.9) = 25.8^\circ$$

$$S_{new} = 44.4\angle 25.8^\circ\text{ kVA}$$

$$Q_{new} = 19.4\text{ kVAR}$$

$$Q_C = -\omega C|V_L|^2 = Q_{new} - Q_{old} = -10.6\text{ kVAR}$$

$$\omega = 377\text{ r/s} \qquad |V_L| = 220\text{ V}_{rms}$$

$$\boxed{C = 581\,\mu F}$$

9.77 A single-phase three-wire 60-Hz circuit serves three loads, as shown in Fig. P9.77. Determine $\mathbf{I}_{aA}$, $\mathbf{I}_{nN}$, $\mathbf{I}_c$, and the energy use over a 24-hour period in kilowatt-hours.

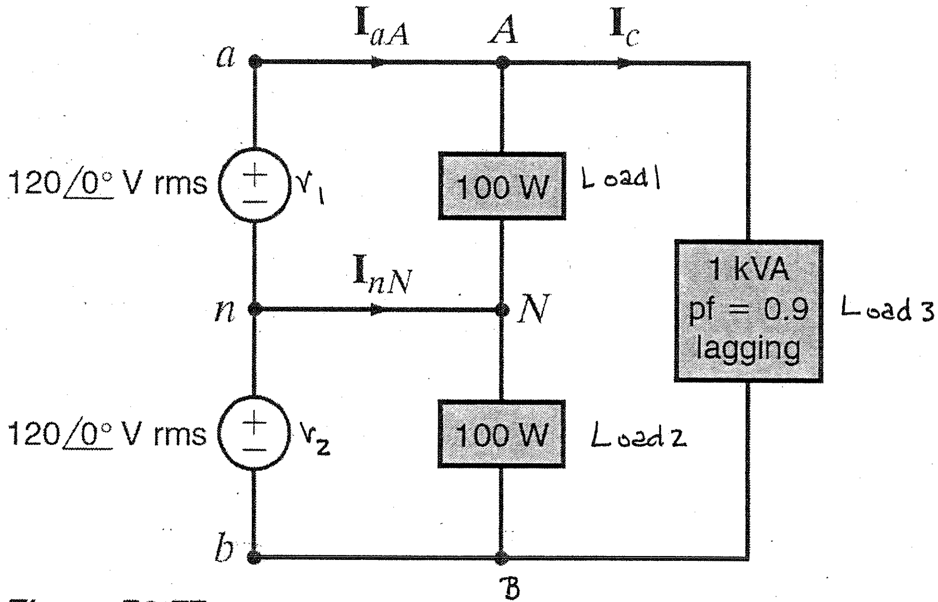


Figure P9.77

SOLUTION:

$S_1 = S_2 = 100 \angle 0° \text{ VA}$

$|S_3| = 1 \text{ kVA}$, $\theta_3 = \cos^{-1}(0.9) = 25.8°$ $\Big\}$ $S_3 = 1000 \angle 25.8 \text{ VA}$

$I_{AN}^* = \dfrac{S_1}{V_1} = 0.833 \angle 0° \text{ Arms}$ $\qquad I_{AN} = 0.833 \angle 0° \text{ Arms}$

$I_{NB}^* = S_2/V_2 = 0.833 \angle 0° \text{ Arms}$ $\qquad I_{NB} = 0.833 \angle 0° \text{ Arms}$

$I_c^* = S_3/(V_1+V_2) = \dfrac{1000\angle 25.8°}{240 \angle 0} = 4.17 \angle 25.8° \Rightarrow \boxed{I_c = 4.17 \angle -25.8° \text{ Arms}}$

$I_{aA} = I_{AN} + I_c \Rightarrow \boxed{I_{aA} = 4.93 \angle -21.6° \text{ Arms}}$

$I_{nN} = I_{NB} - I_{AN} \Rightarrow \boxed{I_{nN} = 0}$

Total power $= P = P_1 + P_2 + P_3 = 100 + 100 + 900 = 1100 \text{ W}$

There are 24 hours/day $\qquad \boxed{W = P(24) = 26.4 \text{ kW-hrs}}$

9.78 A 5.1-kW household range is designed to operate on a 240-V rms sinusoidal voltage, as shown in Fig. P9.78a. However, the electrician has mistakenly connected the range to 120 V rms, as shown in Fig. P9.78b. What is the effect of this error?

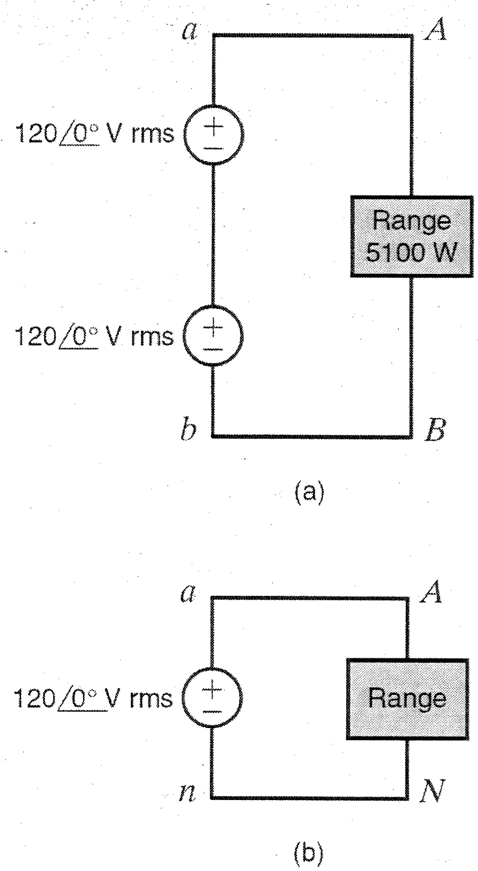


Figure P9.78

SOLUTION: As designed,

$$|I_{range}| = \frac{P_{range}}{|V_{ab}|} = \frac{5100}{|240\angle 0°|} = 21.25 \text{ A}_{rms}$$

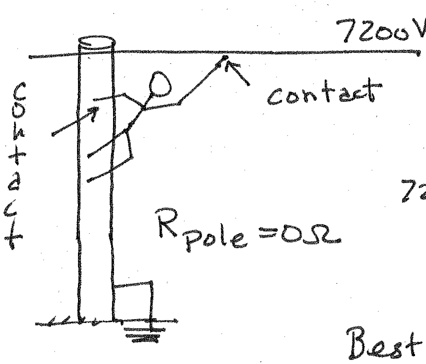
As connected

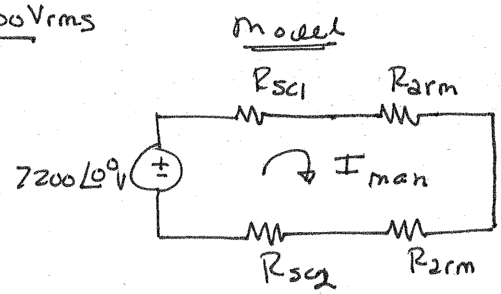
$$|I_{range}| = \frac{5100}{|120\angle 0°|} = 42.5 \text{ A}_{rms}$$

The current drawn by the range is doubled to offset the reduction in voltage by half.

9.79 A man and his son are flying a kite. The kite becomes entangled in a 7200-V rms power line close to a power pole. The man crawls up the pole to remove the kite. While trying to remove the kite, the man accidentally touches the 7200-V rms line. Assuming the power pole is well grounded, what is the potential current through the man's body? CS

SOLUTION:





Current flows from one hand, through both arms, to other hand & down the pole

Best case: skin is dry & $R_{sc} = 15k\Omega$, $R_{arm} = 100\Omega$

$$|I_{man}| = \frac{7200}{R_{sc1} + 2R_{arm} + R_{sc2}} = 238\,mA$$

Worst case: skin is wet & $R_{sc} = 150\Omega$, $R_{arm} = 100\Omega$

$$|I_{man}| = \frac{7200}{2(150) + 2(100)} = 14.4\,A_{rms}$$

$$|I_{man}| = \begin{cases} 238\,mA & \text{dry skin} \\ 14.4\,A & \text{wet skin} \end{cases}$$

9.80 A number of 120-V rms household fixtures are to be used to provide lighting for a large room. The total lighting load is 8 kW. The National Electric Code requires that no circuit breaker be larger than 20 A rms with a 25% safety margin. Determine the number of identical branch circuits needed for this requirement.

SOLUTION:

$$|I_{TOTAL}| = \frac{P}{|V|} = \frac{8000}{120} = 66.7 \text{ A rms}$$

$$|I_{BREAKER}| \leq 0.75(20) = 15 \text{ A rms} \quad \text{(25\% safety margin)}$$

$$\text{\# of branches} = \frac{|I_{TOTAL}|}{|I_{BREAKER}|} = 4.44$$

$$\boxed{\text{Use 5 branches!}}$$

9.81 To test a light socket, a woman, while standing on cushions that insulate her from the ground, sticks her finger into the socket, as shown in Fig. P9.81. The tip of her finger makes contact with one side of the line, and the side of her finger makes contact with the other side of the line. Assuming that any portion of a limb has a resistance of 95 Ω, is there any current in the body? Is there any current in the vicinity of the heart?

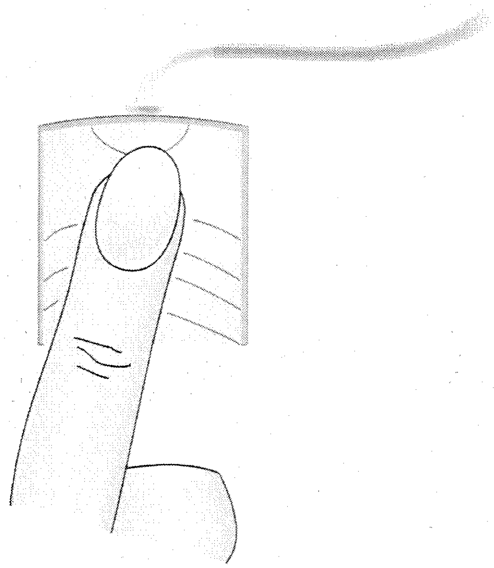
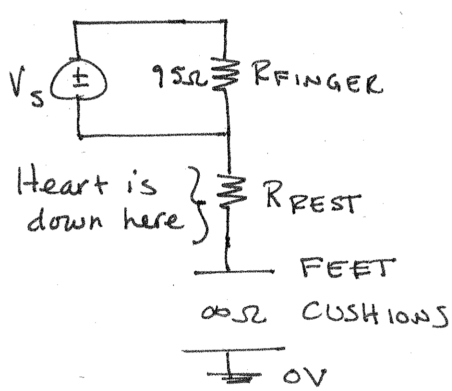


Figure P9.81

SOLUTION:

Model



$R_{FINGER} = 95\Omega$ $|V_S| = 120V$

R_{REST} models rest of body.

cushions have infinite resistance.

So, while $\frac{120}{95} = 1.26$ Arms flows through the fingertip (ouch), there is no current near the heart.

9.82 An inexperienced mechanic is installing a 12-V battery in a car. The negative terminal has been connected. He is currently tightening the bolts on the positive terminal. With a tight grip on the wrench, he turns it so that the gold ring on his finger makes contact with the frame of the car. This situation is modeled in Fig. P9.82, where we assume that the resistance of the wrench is negligible and the resistance of the contact is as follows:

$$R_1 = R_{\text{bolt to wrench}} = 0.012\ \Omega$$
$$R_2 = R_{\text{wrench to ring}} = 0.012\ \Omega$$
$$R_3 = R_{\text{ring}} = 0.012\ \Omega$$
$$R_4 = R_{\text{ring to frame}} = 0.012\ \Omega$$

What power is quickly dissipated in the gold ring, and what is the impact of this power dissipation? CS

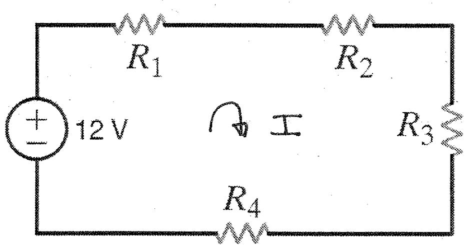


Figure P9.82

SOLUTION:

$$I = \frac{12}{R_1 + R_2 + R_3 + R_4} = \frac{12}{0.048} = 250\text{A}$$

$$P_{ring} = I^2 R_3 \Rightarrow \boxed{P_{ring} = 750\text{ W}}$$

The ring will get EXTREMELY hot, burning our mechanic!

9.83 A 5-kW load operates at 60 Hz, 240 V rms and has a power factor of 0.866 lagging. We wish to create a power factor of at least 0.975 lagging using a single capacitor. Can this requirement be met using a single capacitor from Table 6.1?

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{5000}{0.866} = 5.77 \text{ kVA}$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = 30°$$

$$S_{old} = 5.77 \angle 30° \text{ kVA}$$

$$Q_{old} = 2.88 \text{ kVAR}$$

$$|S_{new}| = \frac{5000}{0.975} = 5.13 \text{ kVA}$$

$$\theta_{new} = \cos^{-1}(0.975) = 12.8°$$

$$S_{new} = 5.13 \angle 12.8° \text{ kVA}$$

$$Q_{new} = 1.14 \text{ kVAR}$$

$$Q_C = -\omega C |V_L|^2 = Q_{new} - Q_{old} = -1.74 \text{ kVAR}$$

$$\omega = 377 \text{ r/s} \quad |V_L| = 240 \text{ V}$$

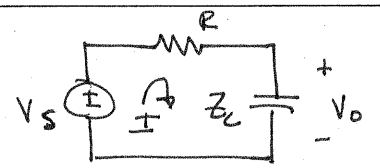
$$\boxed{C = 80.1 \mu F}$$

$\boxed{\text{Easily, just use an } 81 \mu F \text{ capacitor}}$

9.84 Use an *RC* combination to design a circuit that will reduce a 120-V rms line voltage to a voltage between 75 and 80 V rms while dissipating less than 30 W.

SOLUTION:

This circuit should do



assume $f = 60\text{Hz}$

$|V_o| = 77.5 \text{ Vrms}$

$|V_s| = 120 \text{ Vrms}$

$$\left|\frac{V_o}{V_s}\right| = \frac{77.5}{120} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$$

or,

$$\left(\frac{77.5}{120}\right)^2 = \frac{1}{1 + (\omega RC)^2} \Rightarrow 1 + (\omega RC)^2 = \left(\frac{120}{77.5}\right)^2 \qquad (1)$$

Also,

$$|I|^2 R < 30 \Rightarrow |I| = \frac{|V_s|}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

or,

$$\frac{|V_s|^2 (\omega C)^2 R}{1 + (\omega RC)^2} < 30 \qquad (2)$$

Substitute (1) into (2) yields

$$\frac{|V_s|^2 (\omega C)^2 R}{|V_s|^2 / |V_o|^2} = |V_o|^2 (\omega C)^2 R < 30$$

Arbitrarily choose $\boxed{C = 10\mu F}$

yields $R < 351\Omega$

Quite arbitrarily choose $\boxed{R = 300\Omega}$

9FE-1 An industrial load consumes 120 kW at 0.707 pf lagging and is connected to a 480 $\underline{/0^\circ}$ V rms 60-Hz line. Determine the value of the capacitor that, when connected in parallel with the load, will raise the power factor to 0.95 lagging. CS

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{120\times10^3}{0.707} = 170\text{ kVA}$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = 45^\circ$$

$$S_{old} = 170\angle 45^\circ\text{ kVA}$$

$$Q_{old} = 120\text{ kVAR}$$

$$|S_{new}| = 120\times10^3/0.95 = 126\text{ kVA}$$

$$\theta_{new} = \cos^{-1}(0.95) = 18.2^\circ$$

$$S_{new} = 126\angle 18.2^\circ\text{ kVA}$$

$$Q_{new} = 39.3\text{ kVAR}$$

$$Q_C = -\omega C|V_L|^2 = Q_{new} - Q_{old} = -80.7\text{ kVAR}$$

$\omega = 377$ r/s $\qquad |V_L| = 480$ Vrms

$$\boxed{C = 929\ \mu F}$$

9FE-2 Determine the average and rms values of the following waveform.

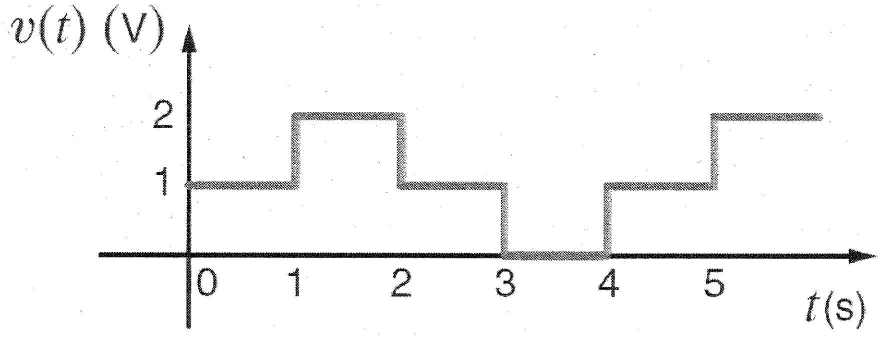


Figure 9PFE-2

SOLUTION:

Average: $\overline{v(t)} = \frac{1}{T}\int_0^T v(t)\,dt \qquad T = 4\,s$

$$\left.\begin{aligned}&\int_0^1 v(t)\,dt = \int_0^1 dt = 1\\&\int_1^2 v(t)\,dt = \int_1^2 2\,dt = 2\\&\int_2^3 v(t)\,dt = 1\end{aligned}\right\} \quad \overline{v(t)} = \frac{1}{T}\left[1+2+1\right] = 3/4$$

$$\boxed{\overline{v(t)} = 0.75\,V}$$

rms: $V_{rms} = \left\{\frac{1}{T}\int_0^T v^2(t)\,dt\right\}^{1/2}$

for $0<t<1$, $v^2(t) = 1^2 = 1$ & $\int_0^1 v^2 dt = \int_0^1 dt = 1$

for $1<t<2$, $v^2(t) = 2^2 = 4$ & $\int_2^3 v^2 dt = \int_2^3 4\,dt = 4$

for $2<t<3$ same contribution as $0<t<1$, $\int_2^3 v^2 dt = 1$

$$V_{rms} = \left\{\frac{1}{4}\left[1+4+1\right]\right\}^{1/2} \qquad \boxed{V_{rms} = 1.22\,V}$$

9FE-3 Find the impedance $\mathbf{Z}_L$ in the network in Fig. 9PFE-3 for maximum average power transfer. CS

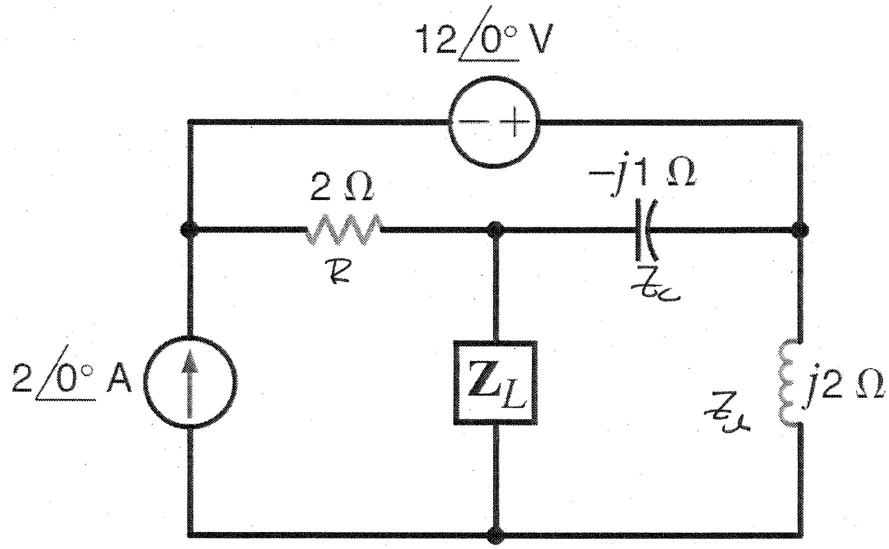
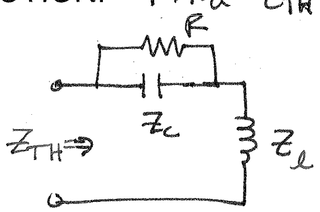


Figure 9PFE-3

SOLUTION: Find Z_{TH}



$$Z_{TH} = Z_l + \frac{R Z_c}{R + Z_c} = j2 + \frac{2(-j1)}{2 - j1}$$

$$Z_{TH} = 0.4 + j\,1.2\,\Omega$$

$$Z_L = Z_{TH}^{*} \qquad \boxed{Z_L = 0.4 - j\,1.2\,\Omega}$$

9FE-4 An rms-reading voltmeter is connected to the output of the op-amp shown in Fig. 9PFE-4. Determine the meter reading.

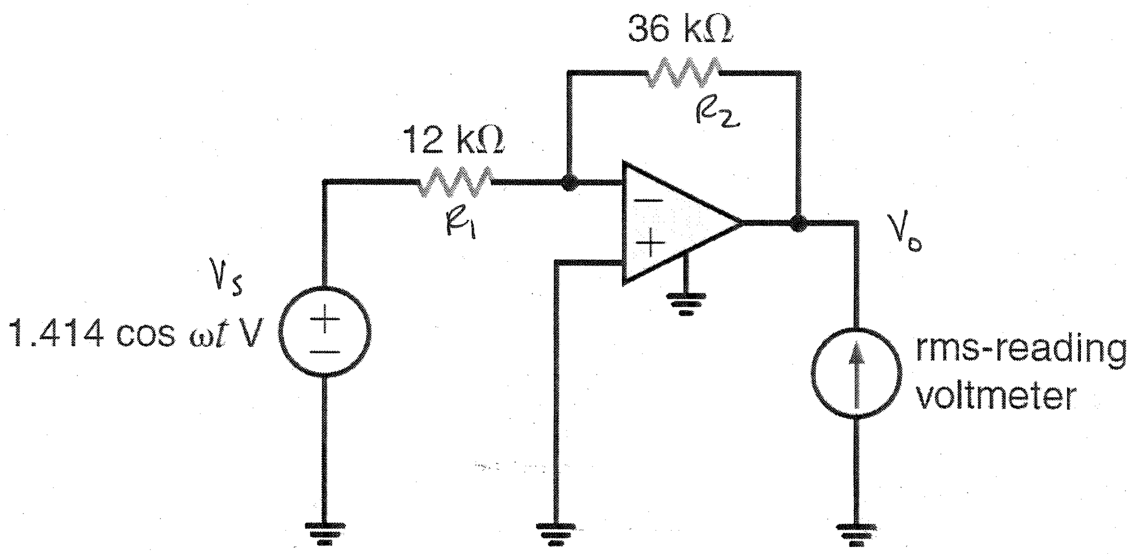


Figure 9PFE-4

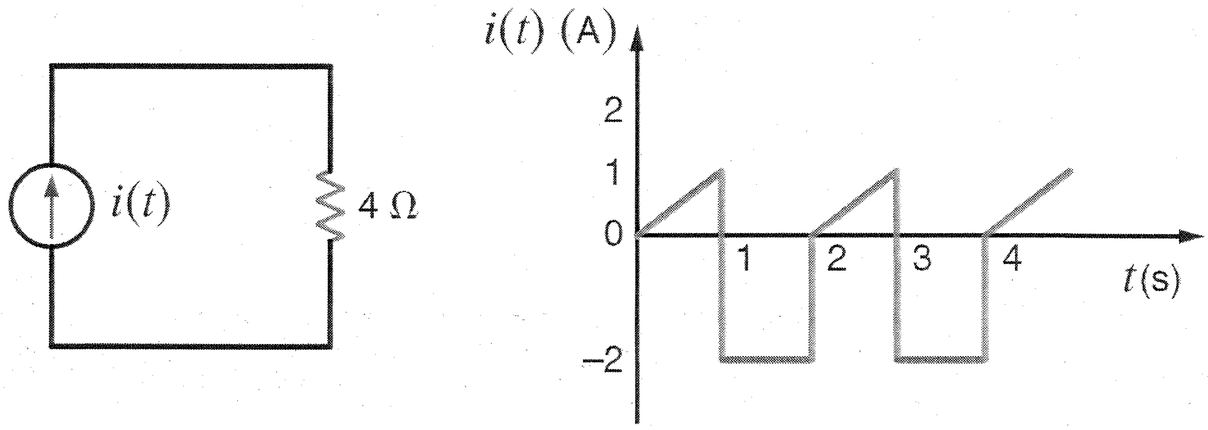
SOLUTION:

$$V_o = -\frac{R_2}{R_1} V_s \qquad V_s = 1.414 \angle 0 \text{ V} = 1 \angle 0^\circ \text{ Vrms}$$

$$V_o = -3V_s = -3 \angle 0^\circ \text{ Vrms}$$

$$\boxed{\text{meter reads } 3\text{ V}}$$

9FE-5 Determine the average power delivered to the resistor in Fig. 9PFE-5a if the current waveform is shown in Fig. 9PFE-5b. CS



(a) (b)

Figure 9PFE-5

SOLUTION:

$P = I_{rms}^2 R$ $\quad T = 2\,s$ $\quad I_{rms} = \left\{ \frac{1}{T} \int_0^T i^2(t)\,dt \right\}^{1/2}$

for $0 < t \leq 1$, $i(t) = t$ & $i^2(t) = t^2$ $\quad \int_0^1 t^2 dt = \frac{t^3}{3}\Big|_0^1 = 1/3$

for $1 < t \leq 2$, $i(t) = -2$ & $i^2(t) = 4$ $\quad \int_1^2 4\,dt = 4$

$$I_{rms} = \left\{ \frac{1}{2} \left[1/3 + 4 \right] \right\}^{1/2} = 1.47\,A$$

$P = (1.47)^2 (4)$ $\quad \boxed{P = 8.67\,W}$