Chapter Ten:

Magnetically Coupled Networks

10.1 Given the network in Fig. P10.1,

(a) find the equations for $v_a(t)$ and $v_b(t)$.

(b) find the equations for $v_c(t)$ and $v_d(t)$.

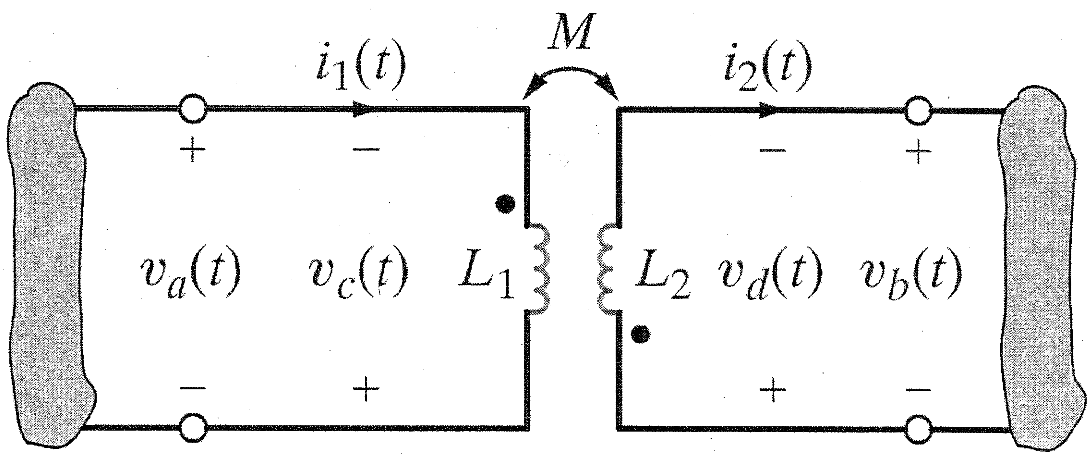


Figure P10.1

SOLUTION:

a) $v_a(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ $\quad v_b(t) = -M\frac{di_1}{dt} - L_2 \frac{di_2}{dt}$

b) $v_c(t) = -v_a(t) = -L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$

$v_d(t) = -v_b(t) = M\frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

10.2 Given the network in Fig. P10.2,

(a) write the equations for $v_a(t)$ and $v_b(t)$.

(b) write the equations for $v_c(t)$ and $v_d(t)$.

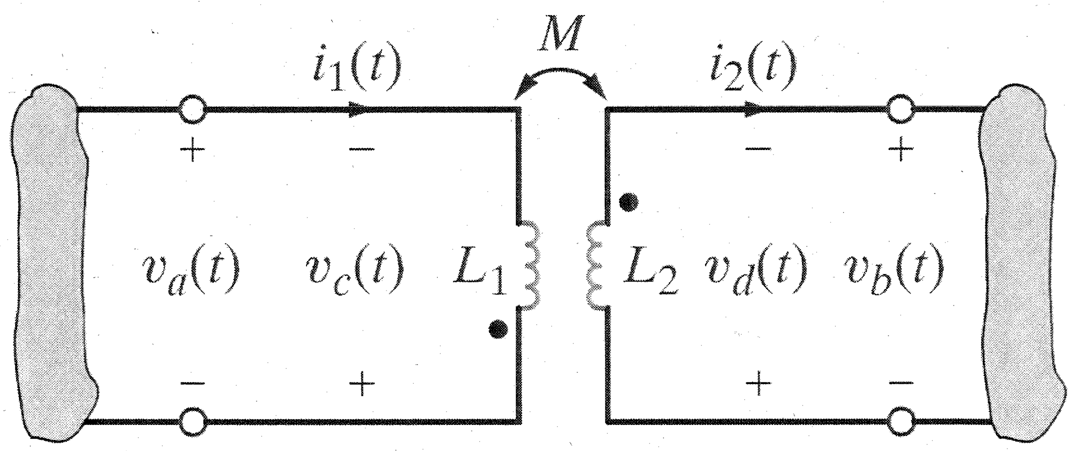


Figure P10.2

SOLUTION:

a) $v_a(t) = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$ $\qquad v_b(t) = -M\frac{di_1}{dt} - L_2\frac{di_2}{dt}$

b) $v_c(t) = -v_a(t) = -L_1 di_1/dt - M di_2/dt$

$v_d(t) = -v_b(t) = M\frac{di_1}{dt} + L_2 di_2/dt$

10.3 Given the network in Fig. P10.3, CS

(a) find the equations for $v_a(t)$ and $v_b(t)$.

(b) find the equations for $v_c(t)$ and $v_d(t)$.

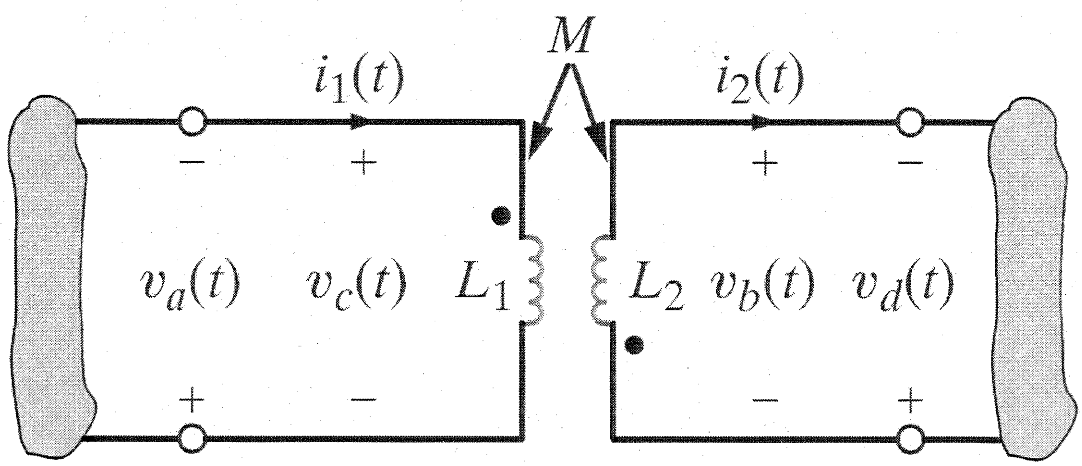


Figure P10.3

SOLUTION:

a) $v_a(t) = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$ $\qquad v_b(t) = -M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$

b) $v_c(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ $\qquad v_d(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

10.4 Given the network in Fig. P10.4,

(a) write the equations for $v_a(t)$ and $v_b(t)$.

(b) write the equations for $v_c(t)$ and $v_d(t)$.

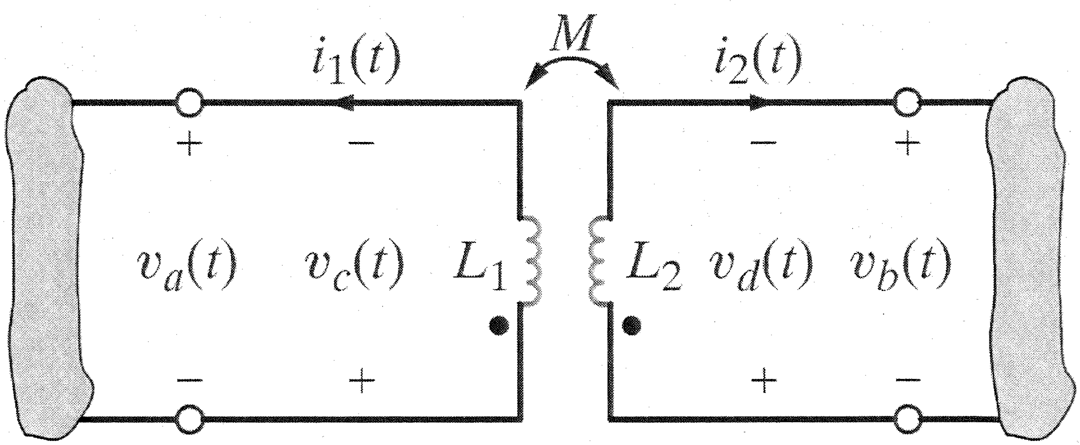


Figure P10.4

SOLUTION:

a) $v_a(t) = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$ $\quad$ $v_b(t) = -M\frac{di_1}{dt} - L_2\frac{di_2}{dt}$

b) $v_c(t) = L_1\, di_1/dt + M\, di_2/dt$ $\quad$ $v_d(t) = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$

10.5 Find the voltage gain $\mathbf{V}_o/\mathbf{V}_S$ of the network shown in Fig. P10.5. CS

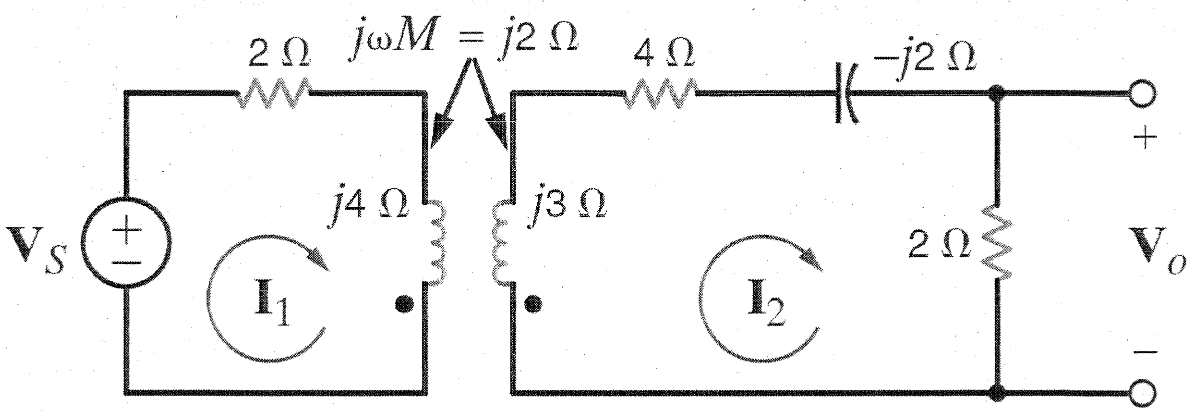


Figure P10.5

SOLUTION:

$V_S = I_1(2+j4) - j2I_2$ & $0 = -j2I_1 + I_2(6+j1)$

$V_o = 2I_2$ $\quad I_1 = \dfrac{V_S + j2I_2}{2+j4}$ $\quad 0 = \dfrac{-j2(V_S + j2I_2)}{2+j4} + I_2(6+j1)$

$I_2 = \dfrac{jV_S}{6+j13}$ $\quad V_o = \dfrac{j2V_S}{6+j13}$ $\quad \dfrac{V_o}{V_S} = \dfrac{j2}{6+j13}$

$$\boxed{\frac{V_o}{V_S} = 0.140\angle 24.8^\circ}$$

10.6 Find $\mathbf{V}_o$ in the network in Fig. P10.6.

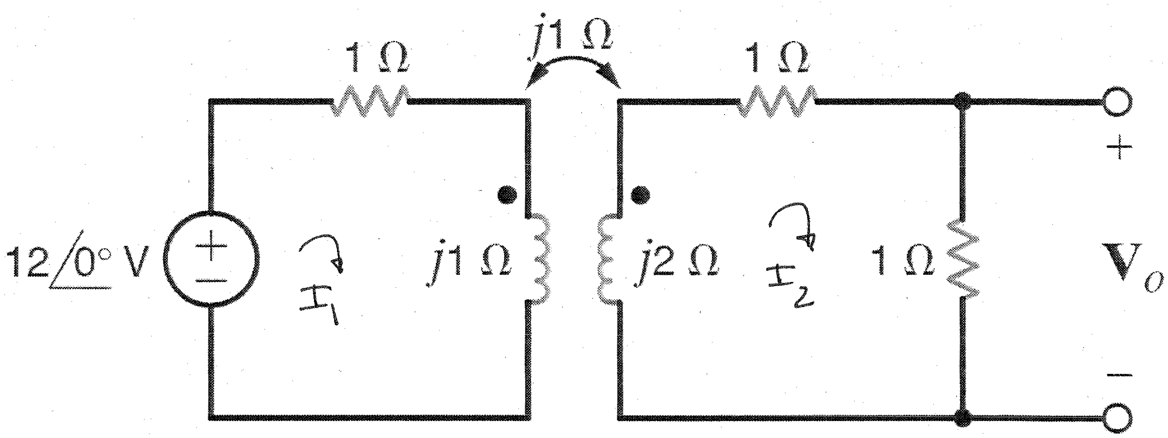


Figure P10.6

SOLUTION:

$12\angle 0 = I_1(1+j1) - j1\, I_2$ & $0 = -j1\, I_1 + I_2(2+j2)$ $V_o = 1(I_2)$

$$\begin{bmatrix} 1+j1 & -j1 \\ -j1 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow I_2 = 2.91 \angle 14.0^\circ \text{ A}$$

$$\boxed{V_o = 2.91 \angle 14.0^\circ \text{ V}}$$

10.7 Given the network in Fig. P10.7, find $\mathbf{V}_o$. PSV

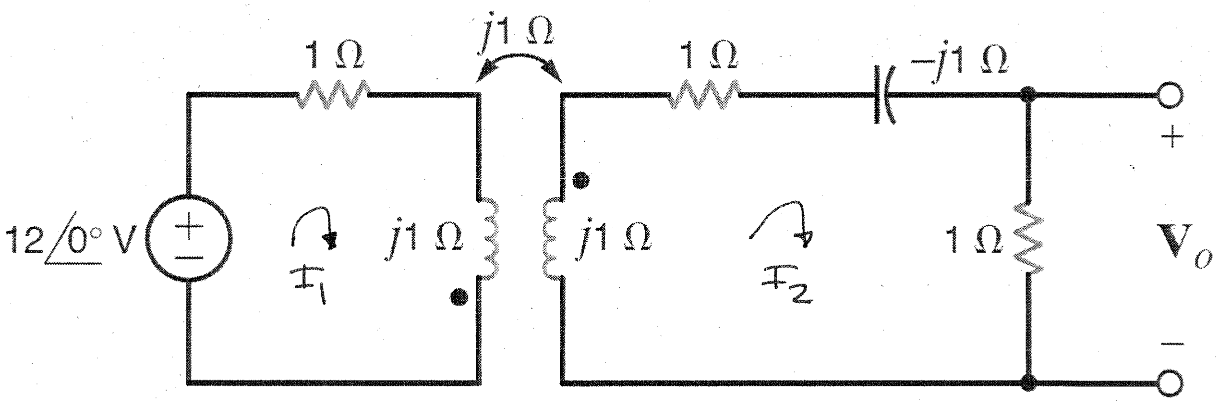


Figure P10.7

SOLUTION:

$12\angle 0^\circ = I_1(1+j1) + j1\, I_2$ & $0 = j1 I_1 + I_2(2)$ & $V_o = (1) I_2$

$$\begin{bmatrix} 1+j1 & j1 \\ j1 & 2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow I_2 = 3.32\angle -124^\circ \text{ A}$$

$$\boxed{V_o = 3.32\angle -124^\circ \text{ V}}$$

10.8 Find $\mathbf{V}_o$ in the circuit in Fig. P10.8. CS

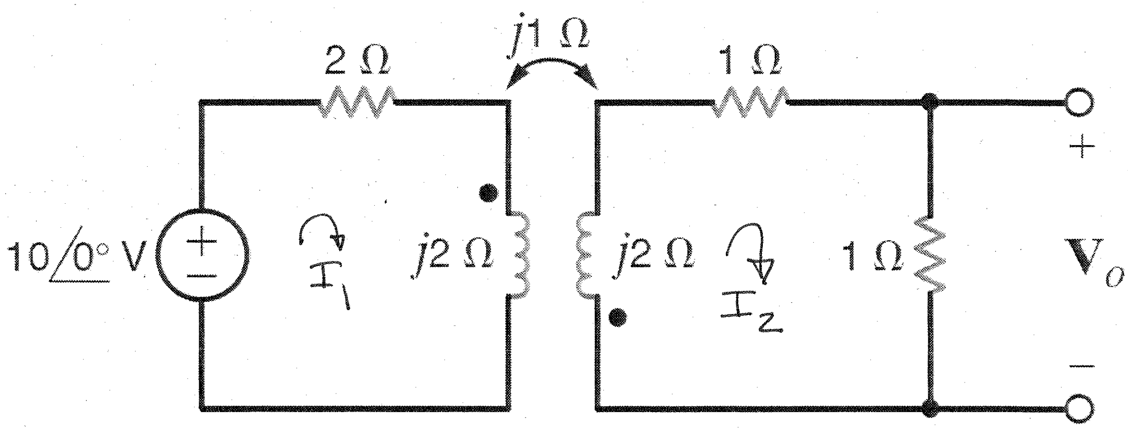


Figure P10.8

SOLUTION:

$10\angle 0^\circ = I_1(2+j2) + j1 I_2$ & $j1 I_1 + I_2(2+j2) = 0$ & $V_o = (1) I_2$

$$\begin{bmatrix} 2+j2 & j1 \\ j1 & 2+j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow I_2 = 1.24\angle -173^\circ \text{ A}$$

$$\boxed{V_o = 1.24\angle -173^\circ \text{ V}}$$

10.9 Find the voltage gain $\mathbf{V}_o/\mathbf{V}_S$ of the network shown in Fig. P10.9.

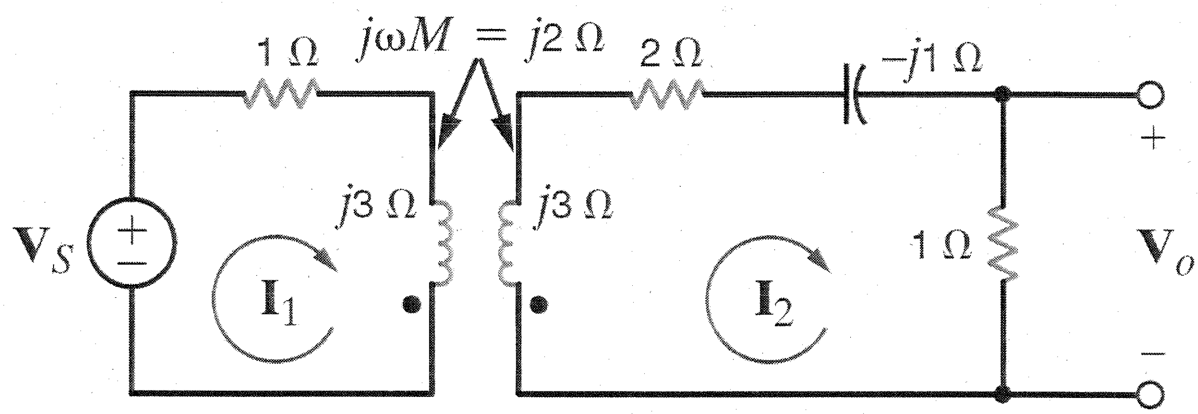


Figure P10.9

SOLUTION:

$V_o = (1)I_2$ & $V_S = I_1(1+j3) - j2I_2$ & $-j2I_1 + I_2(3+j2) = 0$

Let $V_S = 1\angle 0°$ V. Then $\frac{V_o}{V_S} = V_o$

$$\begin{bmatrix} 1+j3 & -j2 \\ -j2 & 3+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow I_2 = 0.181\angle 5.19° \text{ A}$$

$$\boxed{\frac{V_o}{V_S} = 0.181\angle 5.19°}$$

10.10 Find $\mathbf{V}_o$ in the network in Fig. P10.10.

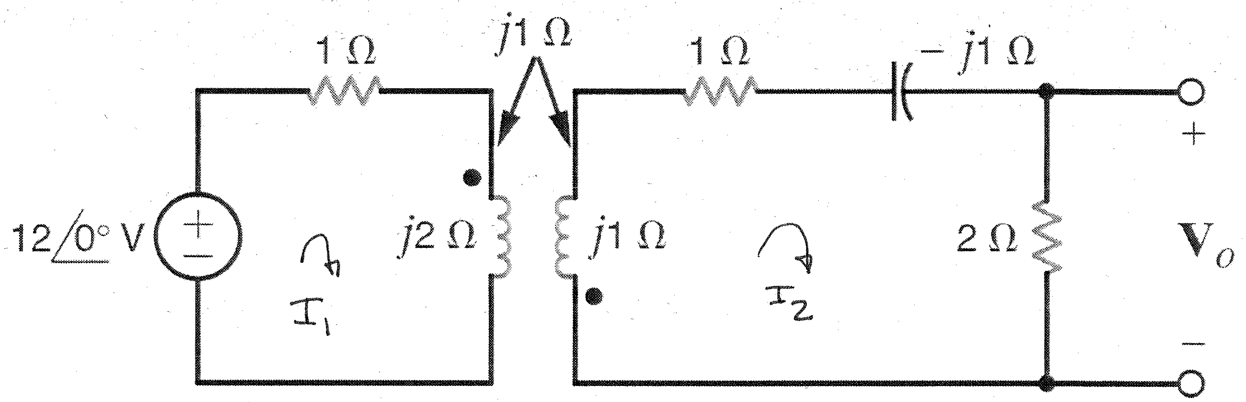


Figure P10.10

SOLUTION:

$12\angle 0^\circ = I_1(1+j2) + j1 I_2$ & $j1 I_1 + I_2(3) = 0$ & $V_o = 2I_2$

$$\begin{bmatrix} 1+j2 & j1 \\ j1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow I_2 = 1.66\angle -146^\circ \text{ A}$$

$$\boxed{V_o = 3.33\angle -146^\circ \text{ V}}$$

10.11 Find $\mathbf{V}_o$ in the network in Fig. P10.11.

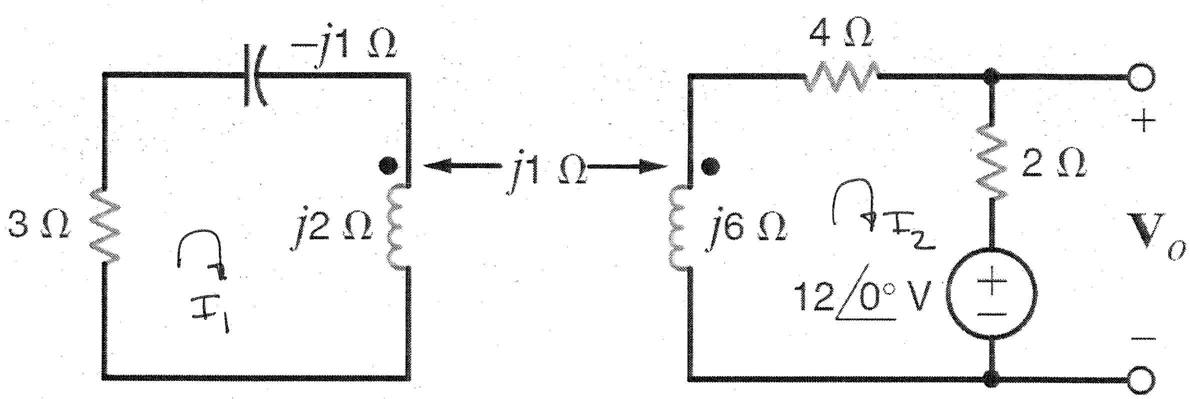


Figure P10.11

SOLUTION:

$I_1(3+j1) - jI_2 = 0$ & $-12\angle 0° = -jI_1 + I_2(6+j6)$ & $V_o = 12\angle 0 + 2I_2$

$$\begin{bmatrix} 3+j1 & -j1 \\ -j1 & 6+j6 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \end{bmatrix} \Rightarrow I_2 = 1.39\angle 137° \text{ A}$$

$$\boxed{V_o = 10.15\angle 10.8° \text{ V}}$$

10.12 Find $\mathbf{V}_o$ in the circuit in Fig. P10.12.

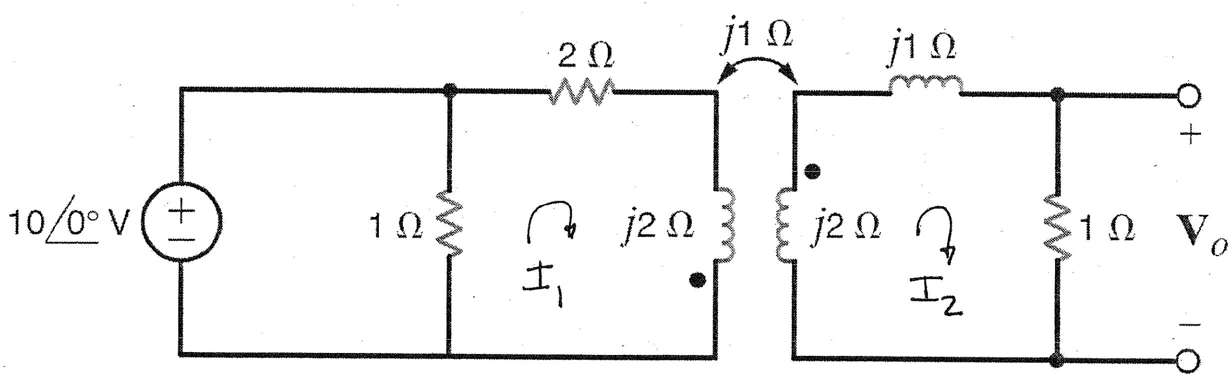


Figure P10.12

SOLUTION:

$10\angle 0^\circ = I_1(2+j2) + j1 I_2$ & $0 = j1 I_1 + I_2(1+j3)$ & $V_o = 1 I_2$

$$\begin{bmatrix} 2+j2 & j1 \\ j1 & 1+j3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow I_2 = 1.17\angle 159^\circ \text{ A}$$

$$\boxed{V_o = 1.17\angle 159^\circ \text{ V}}$$

10.13 Find $\mathbf{V}_o$ in the network in Fig. P10.13.

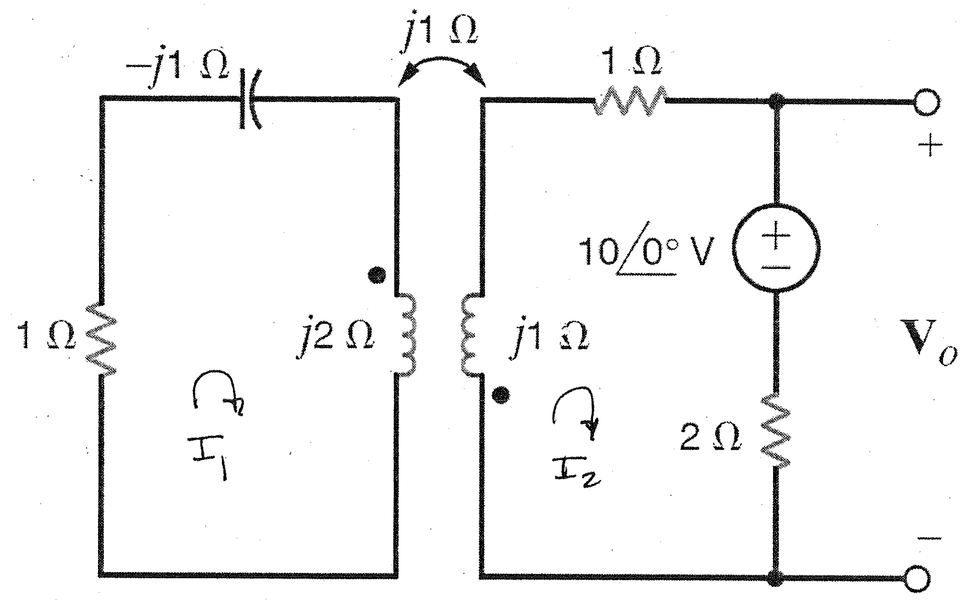


Figure P10.13

SOLUTION:

$I_1(1+j1) + jI_2 = 0$ & $-10\angle 0° = jI_1 + I_2(3+j1)$ & $V_o = 2I_2 + 10\angle 0$

$$\begin{bmatrix} 1+j1 & j1 \\ j1 & 3+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \end{bmatrix} \Rightarrow I_2 = 2.83\angle 172° \text{ A}$$

$$\boxed{V_o = 4.47\angle 10.3° \text{ V}}$$

10.14 Find $\mathbf{V}_o$ in the network in Fig. P10.14.

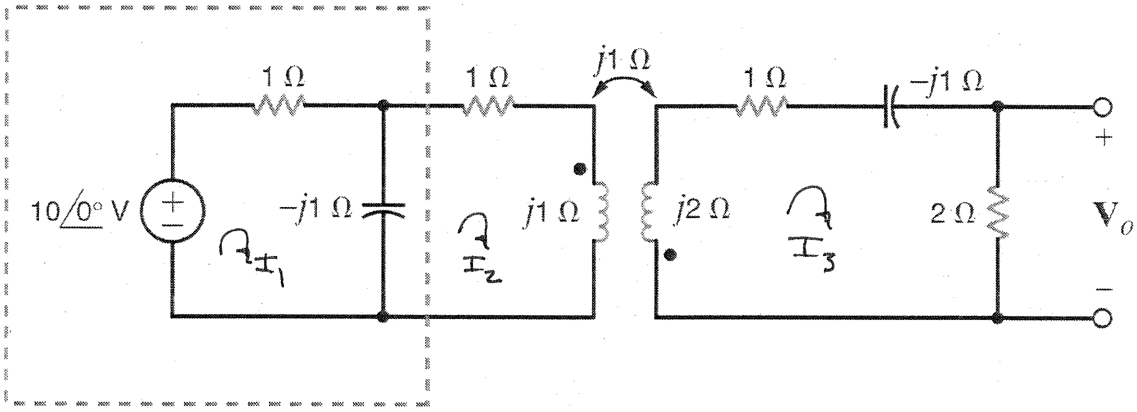


Figure P10.14

SOLUTION:

$10\angle 0 = I_1(1-j1) + j1\, I_2 \quad \& \quad j1 I_1 + I_2(1) + j I_3 = 0 \quad \& \quad j I_2 + I_3(3+j1) = 0$

$$\begin{bmatrix} 1-j1 & j1 & 0 \\ j1 & 1 & j1 \\ 0 & j1 & 3+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$V_o = 2 I_3$

$I_3 = 1.21 \angle -166^\circ$ A

$\boxed{V_o = 2.42 \angle -166^\circ \text{ V}}$

10.15 Find $\mathbf{V}_o$ in the network in Fig. P10.15.

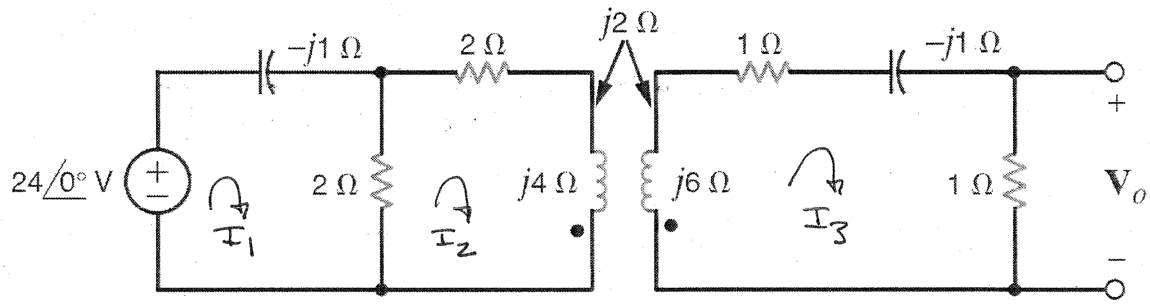


Figure P10.15

SOLUTION:

$24\angle 0° = I_1(2-j1) - 2I_2$ & $-2I_1 + I_2(4+j4) - j2I_3 = 0$ & $-j2I_2 + I_3(2+j5) = 0$

$$\begin{bmatrix} 2-j1 & -2 & 0 \\ -2 & 4+j4 & -j2 \\ 0 & -j2 & 2+j5 \end{bmatrix} \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$V_o = (1)I_3$

$I_3 = 2.17\angle 5.19°$ A

$\boxed{V_o = 2.17\angle 5.19°\ V}$

10.16 Find $\mathbf{V}_o$ in the circuit in Fig. P10.16. CS

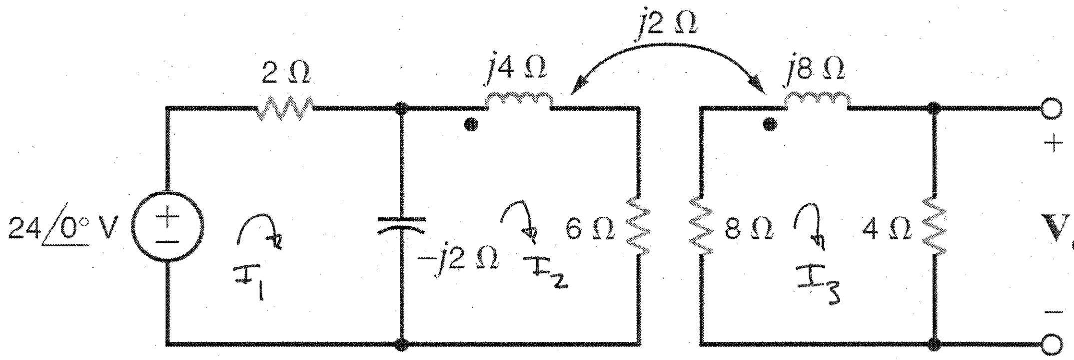


Figure P10.16

SOLUTION:

$24\angle 0^\circ = I_1(2-j2) + j2I_2$ & $j2I_1 + I_2(6+j2) + j2I_3 = 0$ & $j2I_2 + I_3(12+j8) = 0$

$$\begin{bmatrix} 2-j2 & j2 & 0 \\ j2 & 6+j2 & j2 \\ 0 & j2 & 12+j8 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$I_3 = 0.303\angle 170^\circ$ A

$V_o = 4 I_3$

$$\boxed{V_o = 1.21\angle 170^\circ \text{ V}}$$

10.17 Find $\mathbf{V}_o$ in the network in Fig. P10.17.

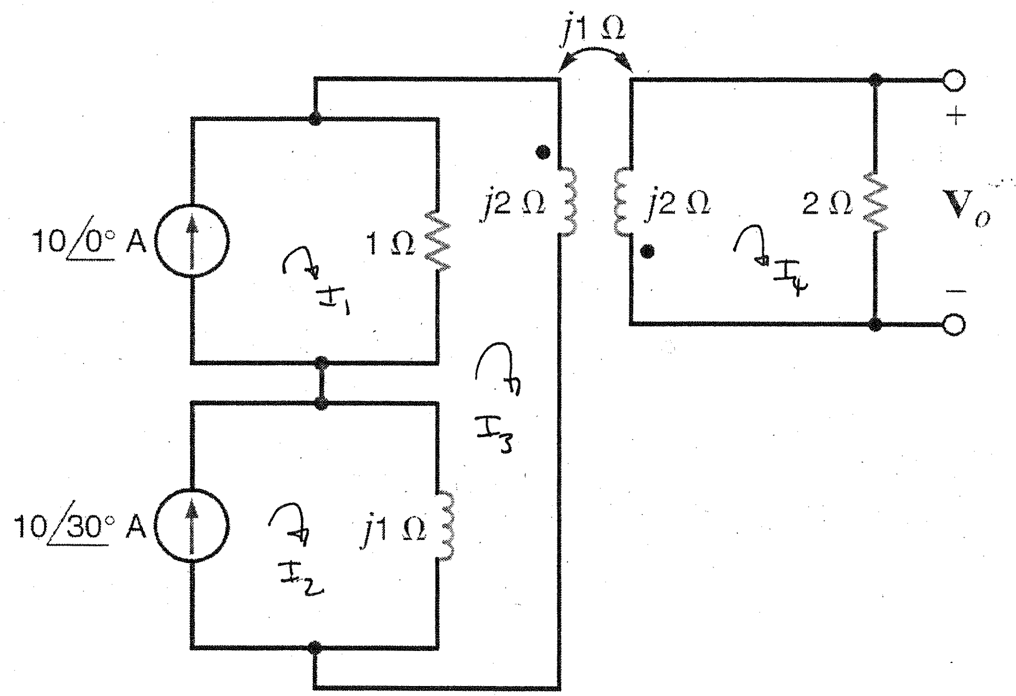


Figure P10.17

SOLUTION:

$I_1 = 10\angle 0°$ A $\quad I_2 = 10\angle 30°$A $\quad I_3(1+j3) - I_1 - jI_2 + jI_4 = 0$

yields, $\quad I_3(1+j3) + jI_4 = I_1 + jI_2 = 5 + j8.66$

and $\quad jI_3 + I_4(2+j2) = 0 \quad$ & $V_o = 2I_4$

$$\begin{bmatrix} 1+j3 & j1 \\ j1 & (2+j2) \end{bmatrix}\begin{bmatrix} I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 5+j8.66 \\ 0 \end{bmatrix} \Rightarrow \quad I_4 = 1.17\angle -141° \text{ A}$$

$$\boxed{V_o = 2.34\angle -141° \text{ V}}$$

10.18 Find $\mathbf{I}_o$ in the circuit in Fig. P10.18.

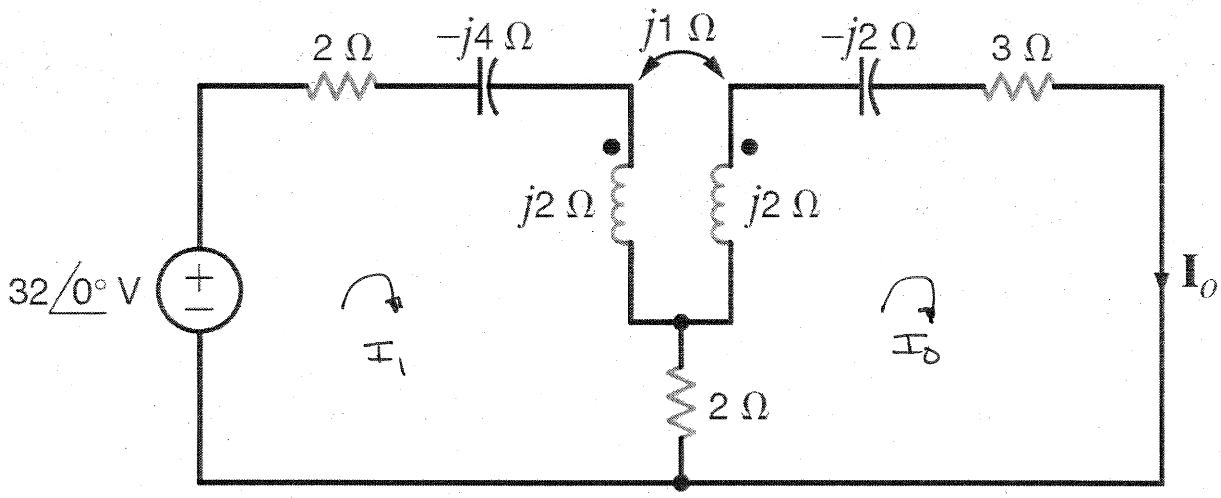


Figure P10.18

SOLUTION:

$$32\angle 0^\circ = I_1(2 - j4 + j2 + 2) - jI_o - 2I_o \Rightarrow 32\angle 0 = I_1(4 - j2) + I_o(-2 - j1)$$

$$0 = -I_1(j1) - 2I_1 + I_2(2 + j2 - j2 + 3) \Rightarrow 0 = I_1(-2 - j1) + I_2(5)$$

$$\begin{bmatrix} 4 - j2 & -2 - j1 \\ -2 - j1 & 5 \end{bmatrix}\begin{bmatrix} I_1 \\ I_o \end{bmatrix} = \begin{bmatrix} 32 \\ 0 \end{bmatrix} \Rightarrow \boxed{I_o = 3.25\angle 66.0^\circ \text{ A}}$$

10.19 Write the mesh equations for the network in Fig. P10.19.

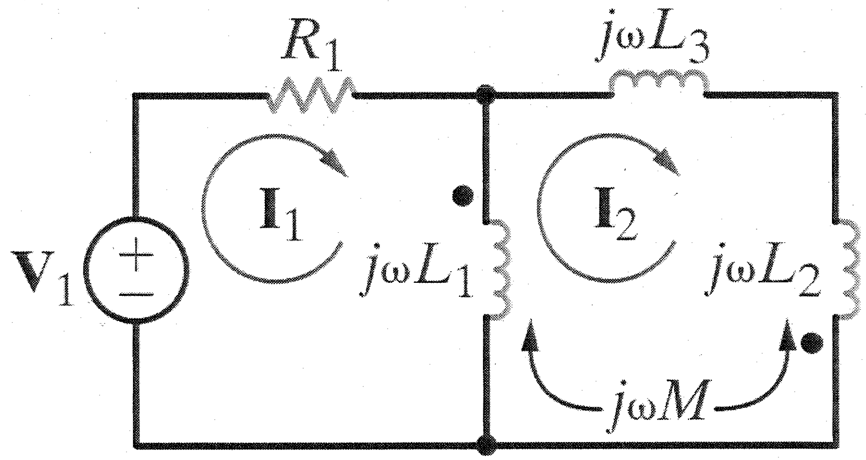


Figure P10.19

SOLUTION:

$V_1 = I_1 (R_1 + j\omega L_1) - j\omega M I_2$

$0 = -j\omega M I_1 + I_2 (j\omega)(L_1 + L_2 + L_3)$

10.20 Write the mesh equations for the network in Fig. P10.20.

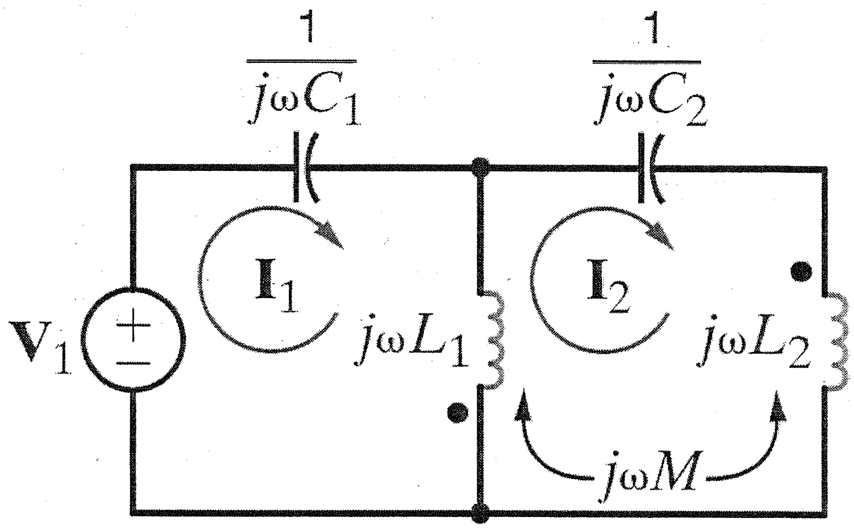


Figure P10.20

SOLUTION:

$$V_1 = I_1\left(j\omega L - j/\omega C_1\right) - j\omega M I_2 - j\omega L_1 I_2$$

$$0 = -j\omega M I_1 + I_2\left(j\omega(L_1+L_2) - j/\omega C_2\right) - j\omega L_1 I_1$$

10.21 Write the mesh equations for the network shown in Fig. P10.21. CS

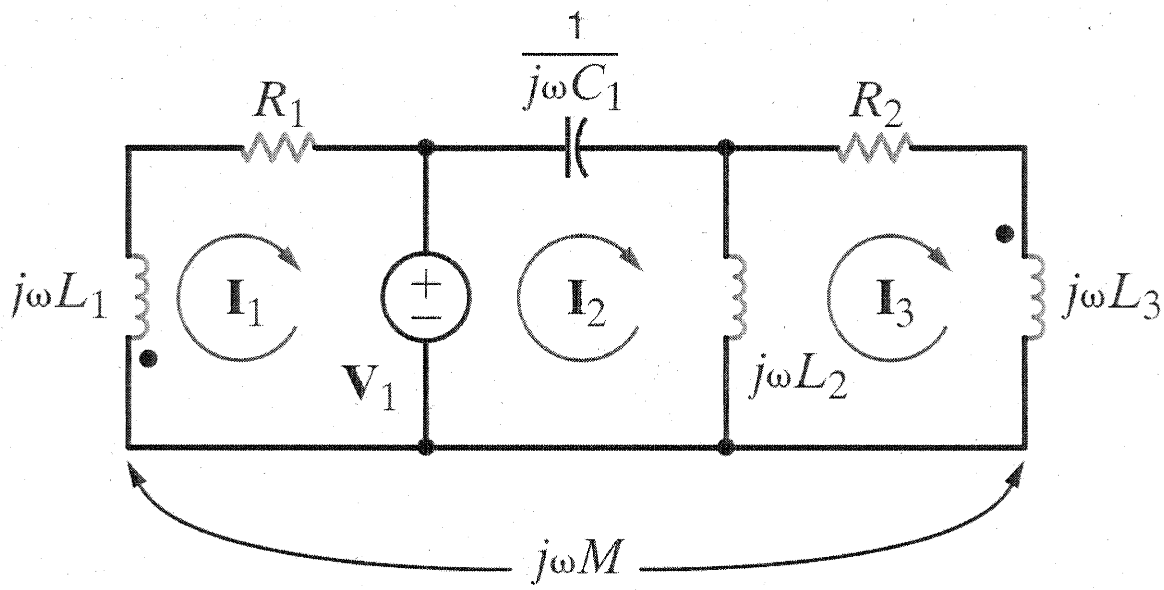


Figure P10.21

SOLUTION:

$$-V_1 = I_1(R_1 + j\omega L_1) + j\omega M I_3$$

$$V_1 = I_2(j\omega L_2 - j/\omega C_1) - j\omega L_2 I_3$$

$$0 = -j\omega L_2 I_2 + I_3(R_2 + j\omega(L_2 + L_3)) + j\omega M I_1$$

10.22 Write the mesh equations for the network shown in Fig. P10.22.

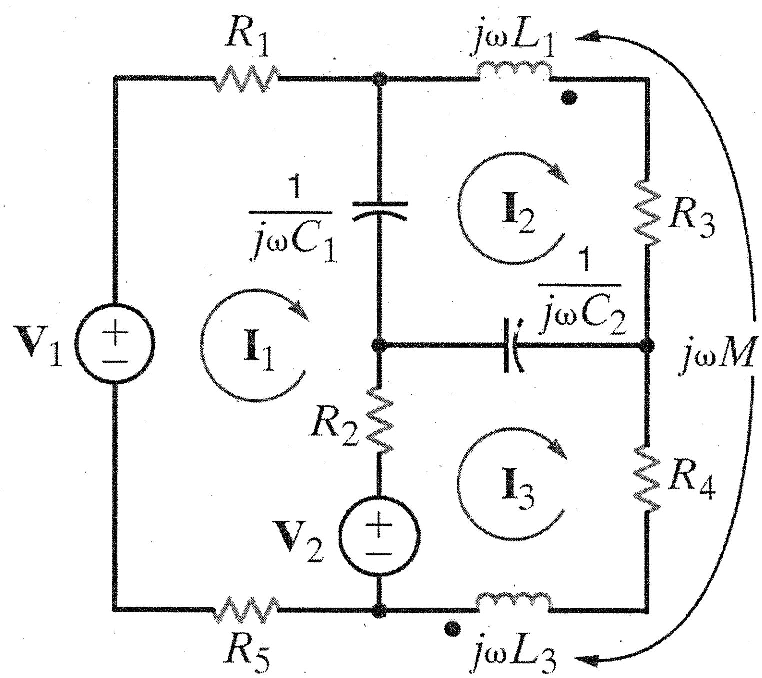


Figure P10.22

SOLUTION:

$$V_1 - V_2 = I_1\left(R_1 + R_2 + R_5 - j/\omega C_1\right) + I_2\left(+ j/\omega C_1\right) - R_2 I_3$$

$$0 = \frac{j}{\omega C_1} I_1 + I_2\left(R_3 + j\left[\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right]\right) + j\omega M I_3 + \frac{j}{\omega C_2} I_3$$

$$V_2 = -R_2 I_1 + I_2\left(j\omega M + \frac{j}{\omega C_2}\right) + I_3\left(R_2 + R_4 + j\omega L_3 - j/\omega C_2\right)$$

10.23 Write the mesh equations for the network in Fig. P10.23.

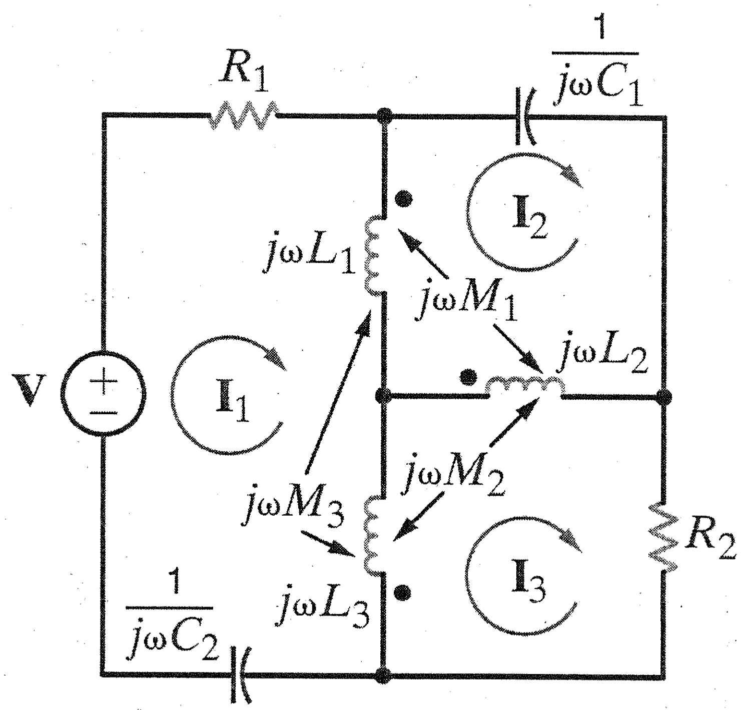


Figure P10.23

SOLUTION:

$$V = I_1\left[R_1 + j\left(\omega L_1 + \omega L_3 - 1/\omega C_2\right)\right] - j\omega L_1 I_2 - j\omega L_3 I_3 \qquad \text{(eq. 1)}$$
$$+ j\omega M_3(I_3 - I_1) + j\omega M_3(I_2 - I_1) + j\omega M_1(I_3 - I_2) + j\omega M_2(I_2 - I_3)$$

$$0 = I_2\left[j\left(\omega L_1 + \omega L_2 - 1/\omega C_1\right)\right] - j\omega L_1 I_1 - j\omega L_2 I_3 \qquad \text{(eq 2)}$$
$$+ j\omega M_1(I_2 - I_1) + j\omega M_1(I_2 - I_3) + j\omega M_2(I_1 - I_3) + j\omega M_3(I_1 - I_3)$$

$$0 = I_3\left[R_2 + j\omega(L_2 + L_3)\right] - j\omega L_2 I_2 - j\omega L_3 I_1 \qquad \text{(eq 3)}$$
$$+ j\omega M_1(I_1 - I_2) + j\omega M_2(I_3 - I_1) + j\omega M_3(I_1 - I_2) + j\omega M_2(I_3 - I_2)$$

$$V = I_1\left[R_1 + j\left(\omega L_1 + \omega L_3 - 2\omega M_3 - \frac{1}{\omega C_2}\right)\right] + I_2\left[j\omega\left(M_3 + M_2 - M_1 - L_1\right)\right]$$
$$+ I_3\left[M_1 - M_2 + M_3 - L_3\right]j\omega$$

$$0 = I_1\left[j\omega\left(M_2 + M_3 - M_1 - L_1\right)\right] + I_2\left[j\left(\omega\left(L_1 + L_2 + 2M_1\right) - 1/\omega C_1\right)\right]$$
$$+ I_3\left[j\omega\left(-M_1 - M_2 - M_3 - L_2\right)\right]$$

$$0 = I_1\left[j\omega\left(M_1 - M_2 + M_3 - L_3\right)\right] + I_2\left[j\omega\left(-M_1 - M_2 - M_3 - L_2\right)\right]$$
$$+ I_3\left[R_2 + j\omega\left(L_2 + L_3 + 2M_2\right)\right]$$

10.24 Find $\mathbf{V}_o$ in the network in Fig. P10.24.

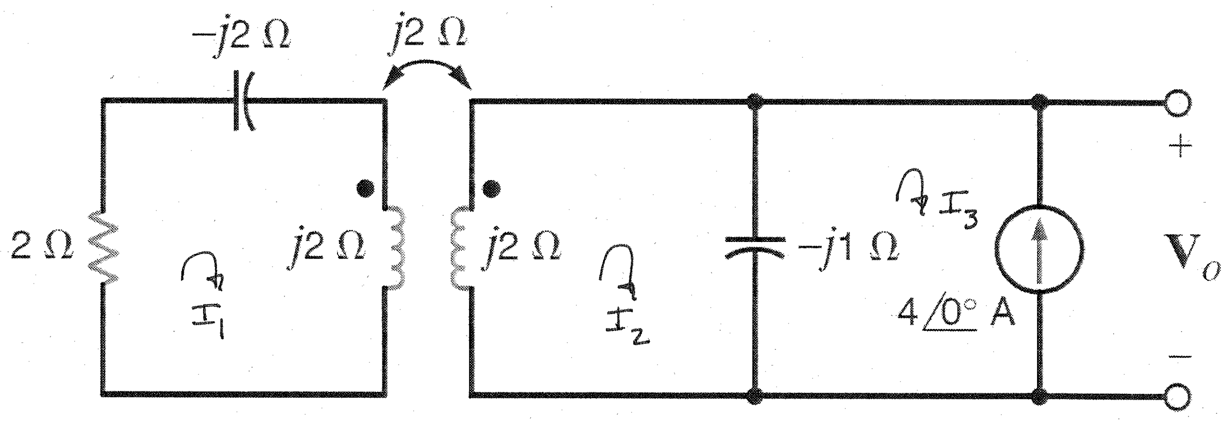


Figure P10.24

SOLUTION:

$0 = I_1(2) - j2\,I_2$ & $0 = -j2 I_1 + I_2(j1) + j1\,I_3$ & $I_3 = -4\angle 0°$ A

$$\begin{bmatrix} 2 & -j2 & 0 \\ -j2 & j1 & j1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix}$$

$V_o = -j1(I_2 - I_3)$

$I_2 = 0.8 + j1.6$ A

$I_3 = -4 + j0$ A

$$\boxed{V_o = 5.06\angle -71.6° \text{ V}}$$

10.25 Find $\mathbf{V}_o$ in the network in Fig. P10.25. CS

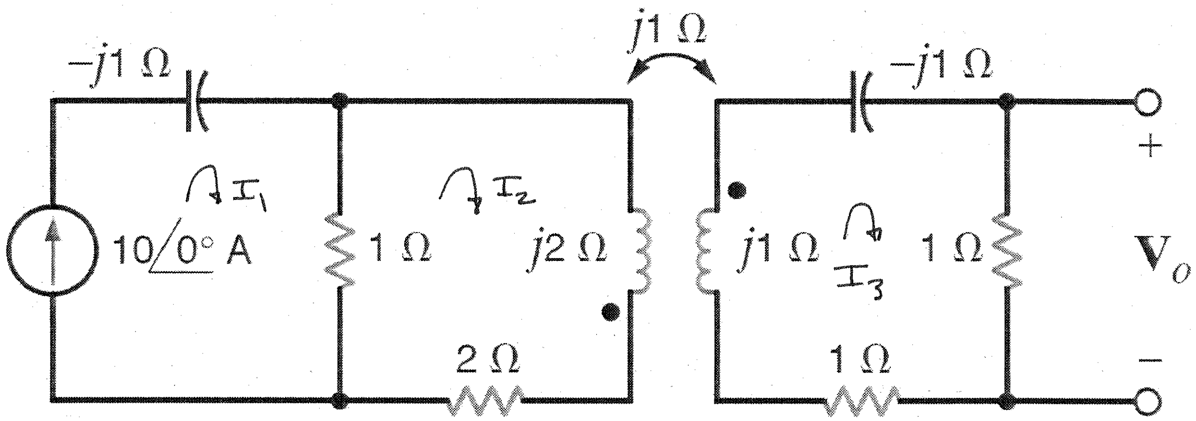


Figure P10.25

SOLUTION:

$I_1 = 10\angle 0°\,\text{A}$ & $I_2(3+j2) - I_1 + jI_3 = 0$ & $0 = jI_2 + I_3\,[2]$

$$\begin{bmatrix} -1 & 3+j2 & j1 \\ 0 & j1 & 2 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \rightarrow$$

$V_o = (1)I_3$

$I_3 = 1.24\angle -120°\ \text{A}$

$\boxed{V_o = 1.24\angle -120°\ \text{V}}$

10.26 Find $\mathbf{V}_o$ in the network in Fig. P10.26.

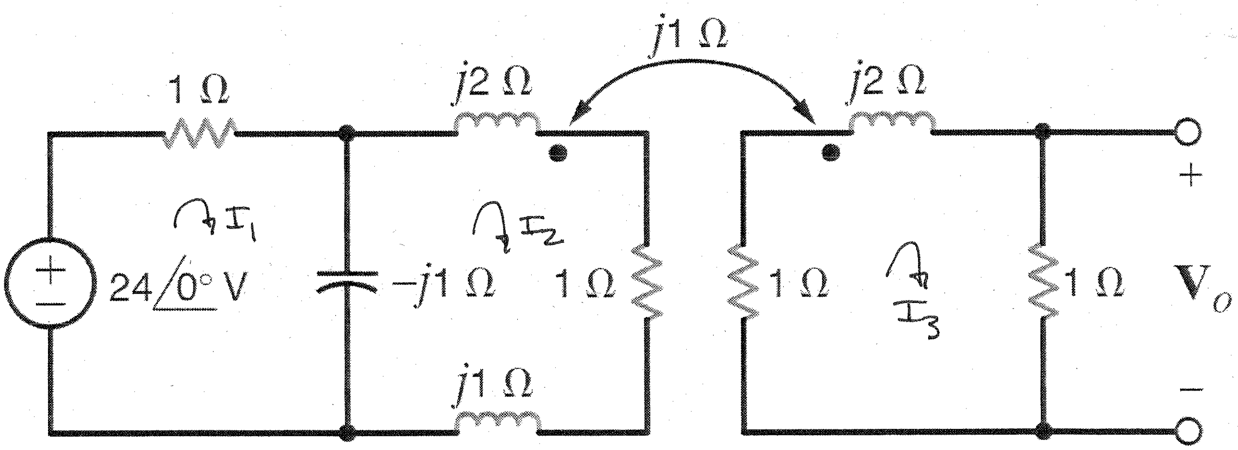


Figure P10.26

SOLUTION:

$24\angle 0^\circ = I_1(1-j1) + jI_2$ & $jI_1 + I_2(1+j2) - j1 I_3 = 0$ $V_o = (1) I_3$

$0 = -jI_2 + I_3(2+j2)$

$$\begin{bmatrix} 1-j1 & j1 & 0 \\ j1 & 1+j2 & -j1 \\ 0 & -j1 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$I_3 = 2.10 \angle -52.1^\circ$ A

$$\boxed{V_o = 2.10 \angle -52.1^\circ \text{ V}}$$

10.27 Find $\mathbf{V}_o$ in the network in Fig. P10.27. PSV

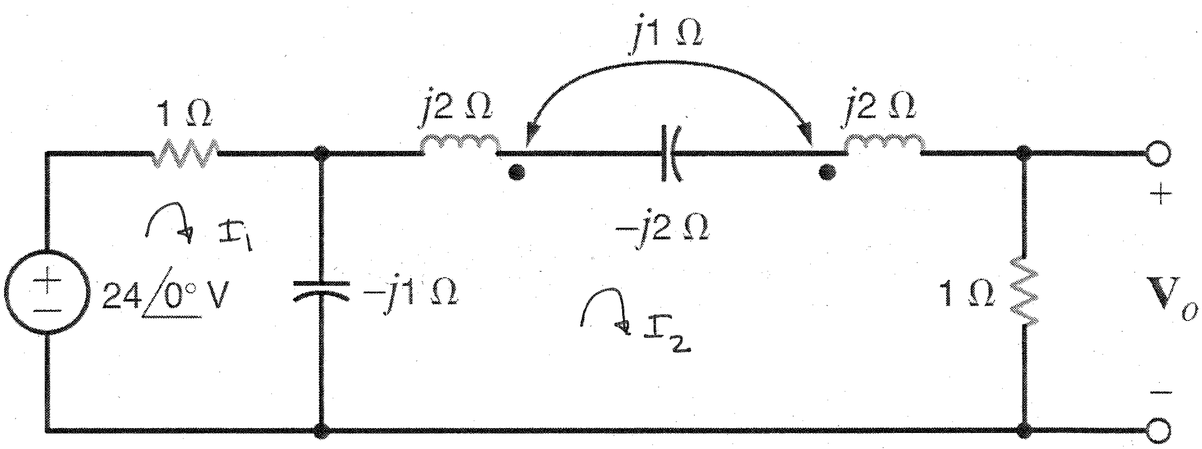


Figure P10.27

SOLUTION:

$$24\angle 0^\circ = I_1(1-j1) + j1\,I_2 \quad \& \quad V_o = (1)\,I_2$$

$$0 = I_2(1+j1) + j1\,I_1 - j1\,I_2 - j1\,I_2 \Rightarrow j\,I_1 + I_2(1-j1) = 0$$

$$\begin{bmatrix} 1-j1 & j1 \\ j1 & 1-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \end{bmatrix} \qquad V_o = (1)\,I_2$$

$$I_2 = 10.7\angle -26.6^\circ \text{ A}$$

$$\boxed{V_o = 10.7\angle -26.6^\circ \text{ V}}$$

10.28 Find $\mathbf{V}_o$ in the network in Fig. P10.28. CS

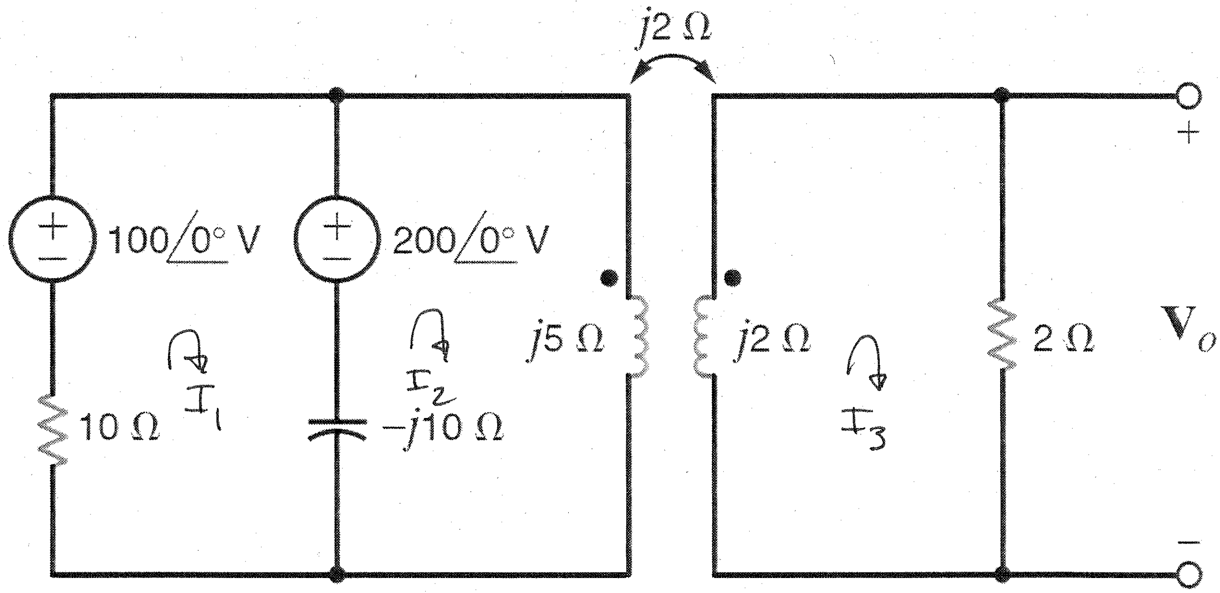


Figure P10.28

SOLUTION:

$100\angle 0 - 200\angle 0 = I_1(10-j10) + j10 I_2 = -100\angle 0$

$200\angle 0 = j10 I_1 + I_2(-j5) - j2 I_3$

$0 = -j2 I_2 + I_3(2+j2)$

$V_o = 2 I_3$

$$\begin{bmatrix} 10-j10 & j10 & 0 \\ j10 & -j5 & -j2 \\ 0 & -j2 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -100 \\ 200 \\ 0 \end{bmatrix}$$

$I_3 = 18.4\angle 72.9^\circ \text{ A}$

$\boxed{V_o = 36.8\angle 72.9^\circ \text{ V}}$

10.29 Find $\mathbf{V}_o$ in the network in Fig. P10.29.

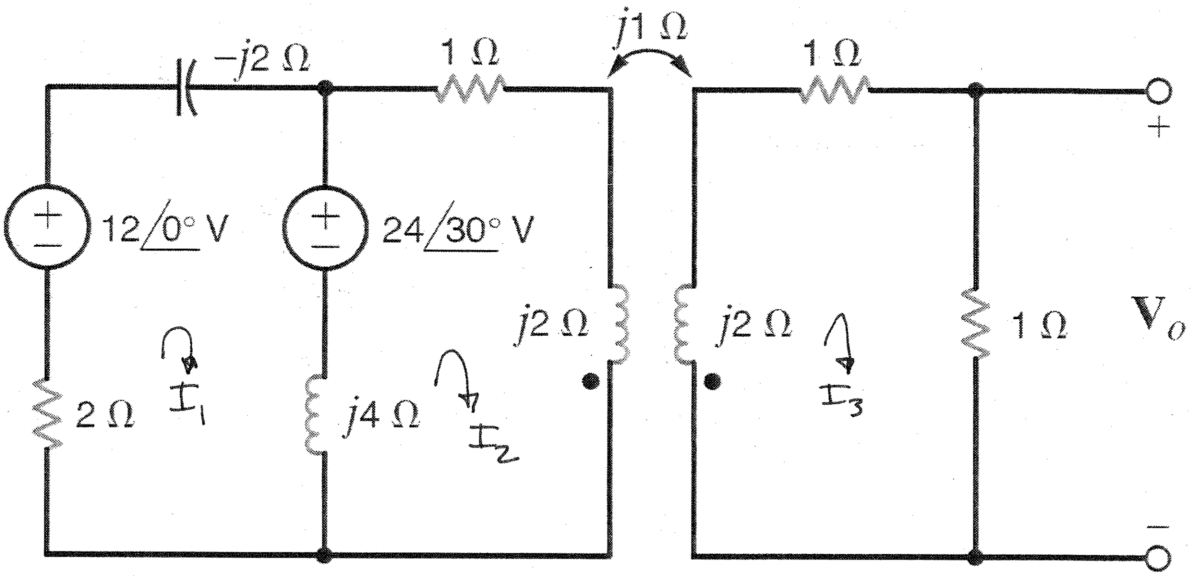


Figure P10.29

SOLUTION:

$$12\angle 0^\circ - 24\angle 30^\circ = I_1(2+j2) - j4I_2 = -8.78 - j12$$

$$24\angle 30^\circ = -j4I_1 + I_2(1+j6) - jI_3 \qquad I_3(1) = V_o$$

$$0 = -jI_2 + I_3(2+j2)$$

$$\begin{bmatrix} 2+j2 & -j4 & 0 \\ -j4 & 1+j6 & -j1 \\ 0 & -j1 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -8.78-j12 \\ 20.8+j12 \\ 0 \end{bmatrix}$$

$$I_3 = 1.63\angle 6.49^\circ \text{ A} \qquad \boxed{V_o = 1.63\angle 6.49^\circ \text{ V}}$$

10.30 Find $\mathbf{V}_o$ in the network in Fig. P10.30.

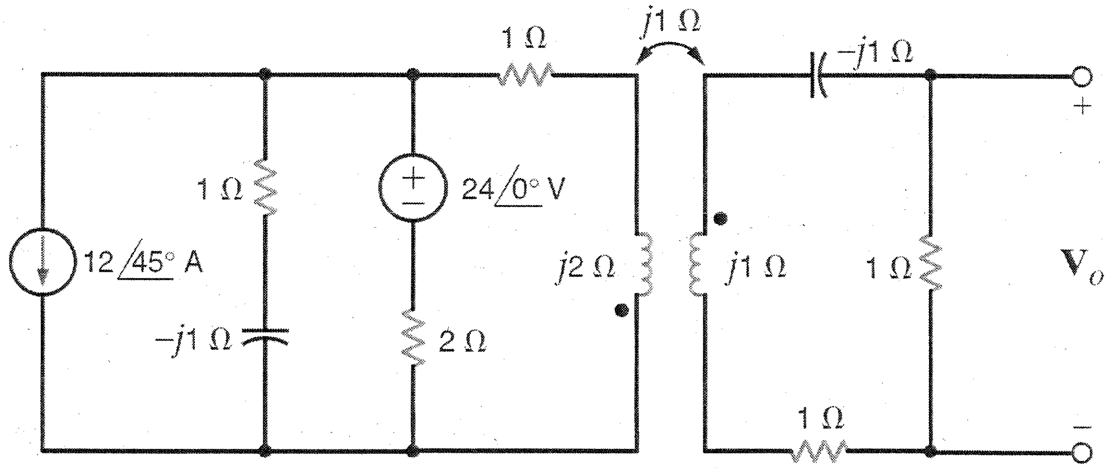
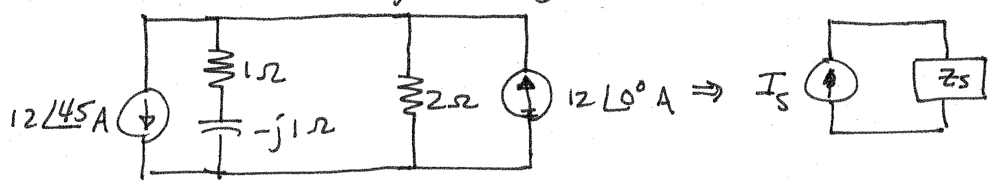


Figure P10.30

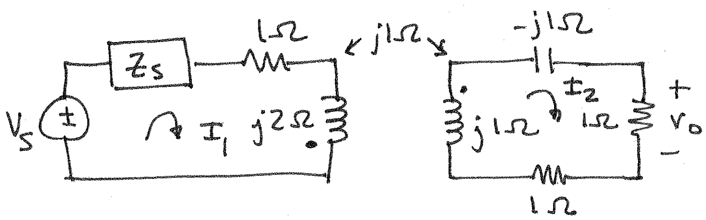
SOLUTION: Simplify through source transformation.



$I_s = 12\angle 0 - 12\angle 45 = 9.18\angle -67.5°$ A

$Z_s = 2(1-j1)/(3-j1) = 0.8 - j0.4\,\Omega = 0.894\angle -26.6°\,\Omega$

$V_s = I_s Z_s = 8.21\angle -94.1°$ V



$V_s = I_1[Z_s + 1 + j2] + jI_2$

$0 = jI_1 + I_2(2)$

$V_o = (1)I_2$

$$\begin{bmatrix} 1.8+j1.6 & j1 \\ j1 & 2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 8.21\angle -94.1° \\ 0 \end{bmatrix} \Rightarrow I_2 = 1.47\angle 141°\text{ A}$$

$$\boxed{V_o = 1.47\angle 141°\text{ V}}$$

10.31 Find $\mathbf{V}_o$ in the network in Fig. P10.31. CS

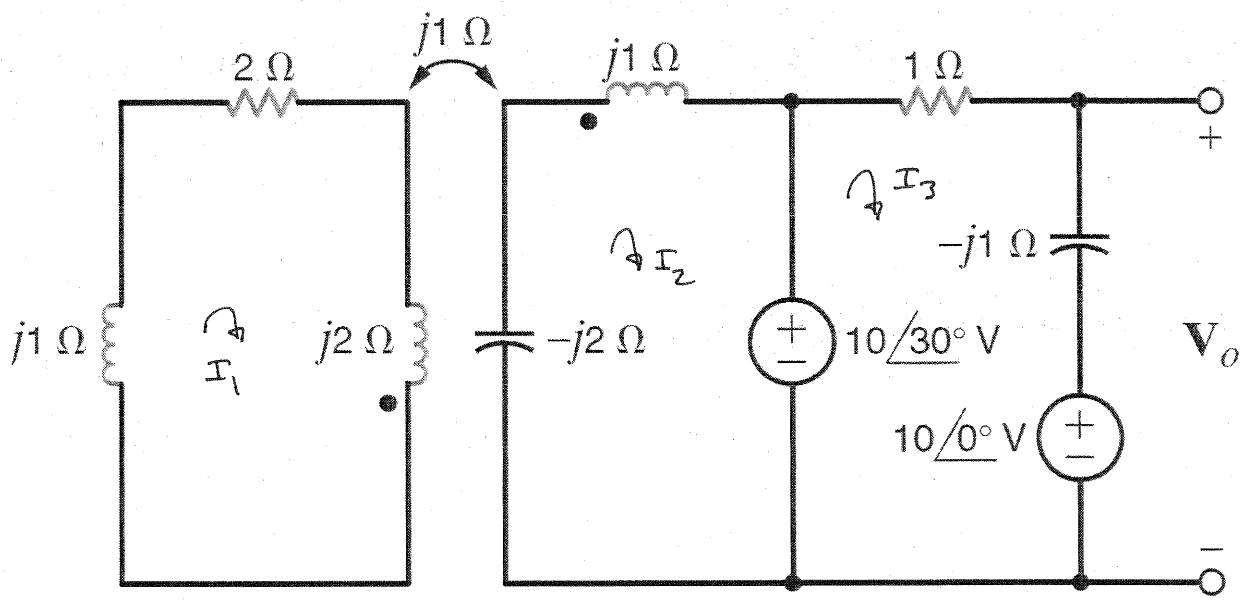


Figure P10.31

SOLUTION:

$I_1(2+j3) - jI_2 = 0$ & $-jI_1 + I_2(-j1) = -10\angle 30°$ & $V_o = -jI_3 + 10\angle 0°$

$10\angle 30 - 10\angle 0 = I_3(1-j1)$

$$\begin{bmatrix} 2+j3 & -j1 & 0 \\ -j1 & -j1 & 0 \\ 0 & 0 & 1-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -10\angle 30° \\ 10\angle 30 - 10\angle 0 \end{bmatrix}$$

$I_3 = 3.66\angle 150°$ A

$$\boxed{V_o = 12.2\angle 15.0° \text{ V}}$$

10.32 Find $\mathbf{I}_o$ in the circuit in Fig. P10.32.

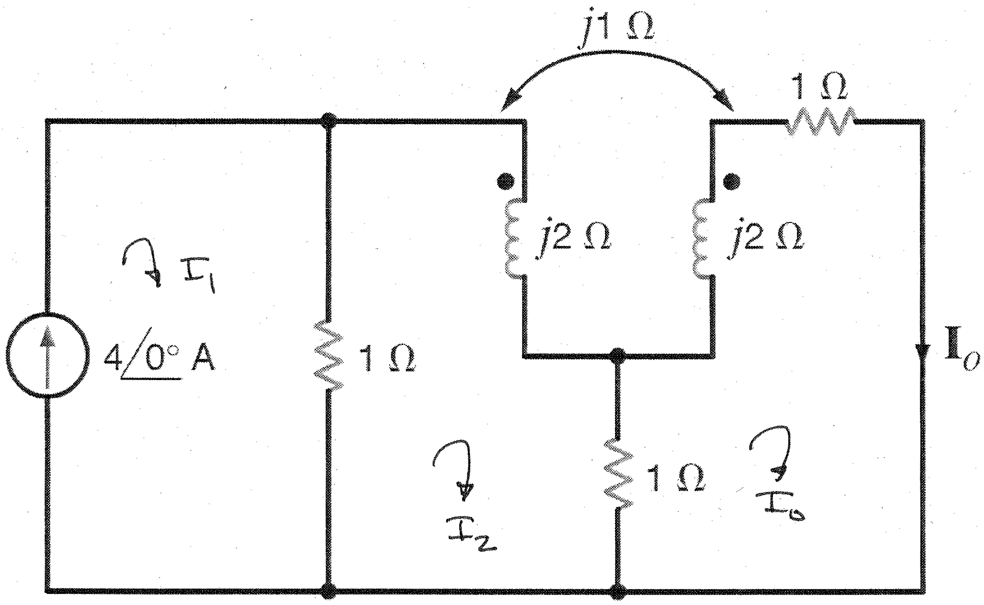


Figure P10.32

SOLUTION:

$I_1 = 4\angle 0°$ & $-I_1 + I_2(2+j2) - jI_o - I_o = 0$

and $0 = I_2(-1-j1) + I_o(2+j2)$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 2+j2 & -1-j1 \\ 0 & -1-j1 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{I_o = 0.667 - j0.667 \text{ A}}$

$\boxed{I_o = 0.943 \angle -45° \text{ A}}$

10.33 Find $\mathbf{I}_o$ in the circuit in Fig. P10.33.

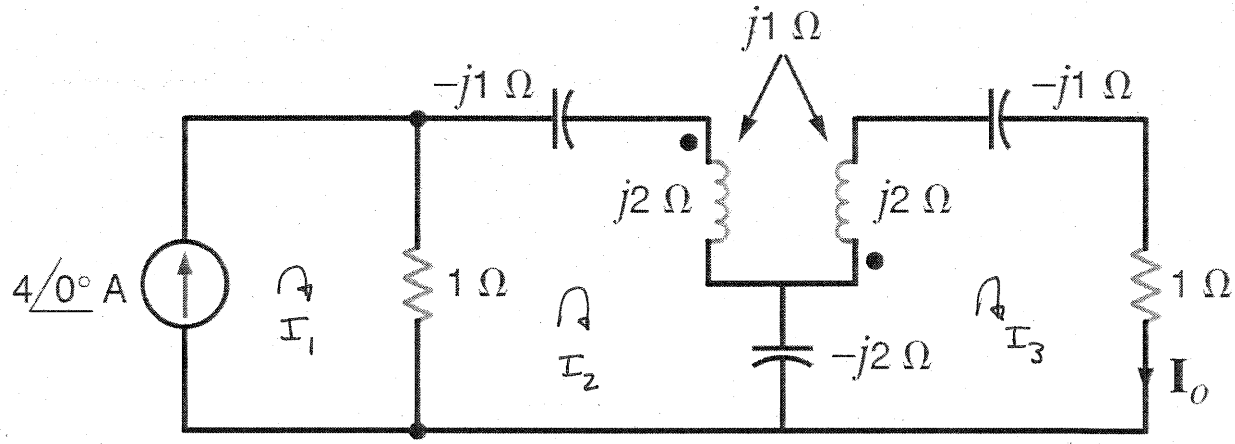


Figure P10.33

SOLUTION:

$I_1 = 4\angle 0°\,A$ & $0 = -I_1 + I_2(1-j1) + I_3(j1+j2)$ $\qquad I_3 = I_o$

and, $\quad 0 = I_2(j2+j1) + I_3(1-j1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1-j1 & j3 \\ 0 & j3 & 1-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{I_3 = I_o = 1.30\angle -72.5°\,A}$$

10.34 Find $\mathbf{V}_o$ in the network in Fig. P10.34.

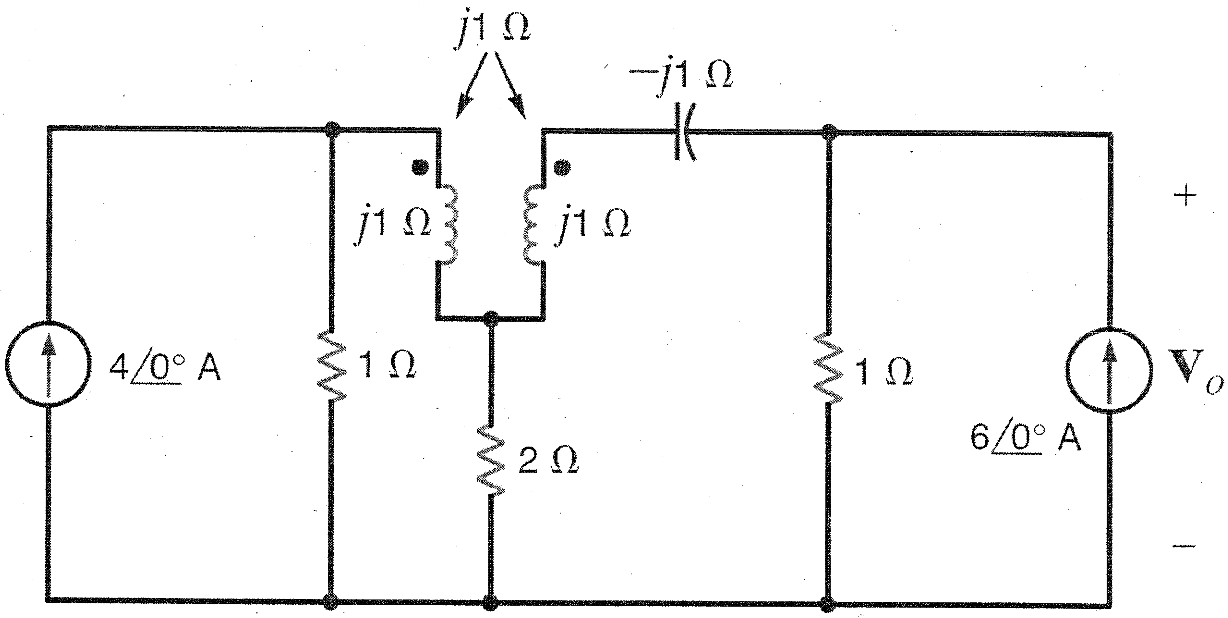
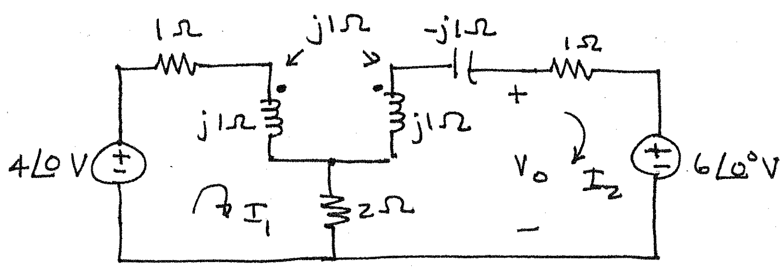


Figure P10.34

SOLUTION: Use source transformations



$$V_o = I_2(1) + 6\angle 0°$$

$$4\angle 0 = I_1(3+j1) - I_2(2+j1) \quad \& \quad -6\angle 0 = -I_1(2+j1) + I_2(3)$$

$$\begin{bmatrix} 3+j1 & -2-j1 \\ -2-j1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix} \Rightarrow I_2 = 1.53\angle -176° \text{ A}$$

$$\boxed{V_o = 4.47\angle -1.51° \text{ V}}$$

10.35 Find $\mathbf{V}_o$ in the circuit in Fig. P10.35. PSV

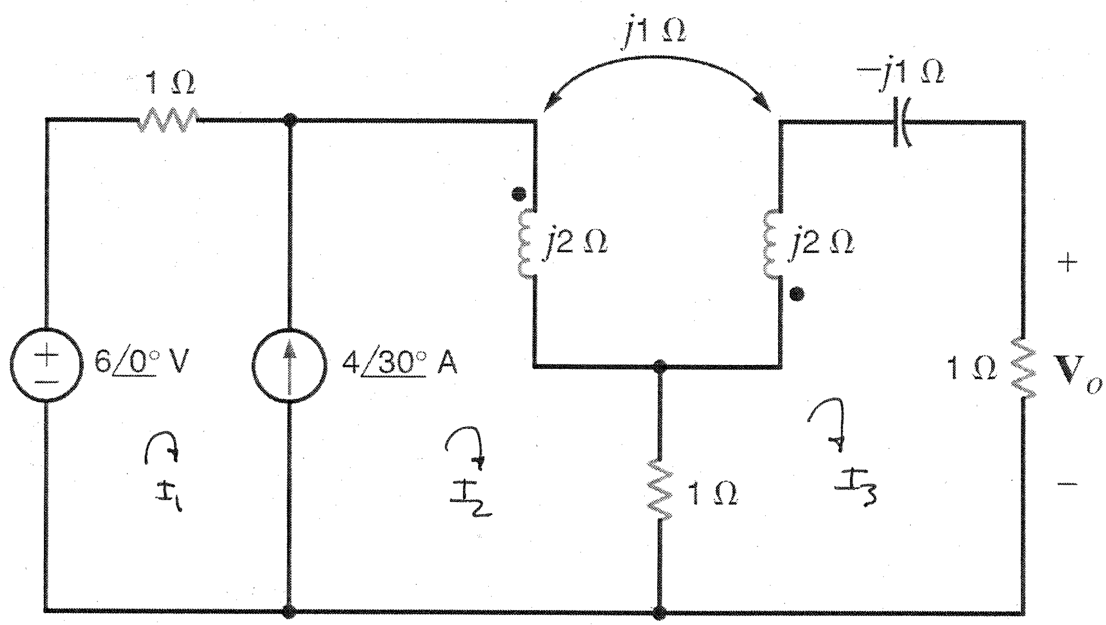


Figure P10.35

SOLUTION:

$6\angle 0 = I_1 + I_2(1+j2) + I_3(-1+j1)$ & $I_2(-1+j1) + I_3(2+j1) = 0$

$I_2 - I_1 = 4\angle 30°$

$V_o = (1)\, I_3$

$$\begin{bmatrix} 1 & 1+j2 & -1+j1 \\ 0 & -1+j1 & 2+j1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4\angle 30° \end{bmatrix} \Rightarrow I_3 = 1.66\angle -109° \text{ A}$$

$$\boxed{V_o = 1.66\angle -109° \text{ V}}$$

10.36 Find $\mathbf{V}_o$ in the network in Fig. P10.36. CS

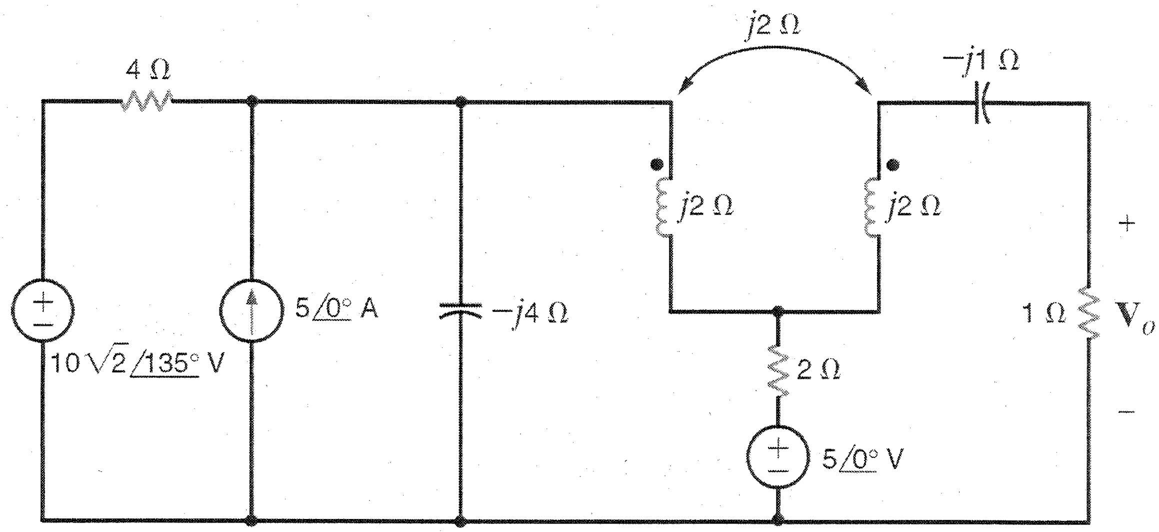
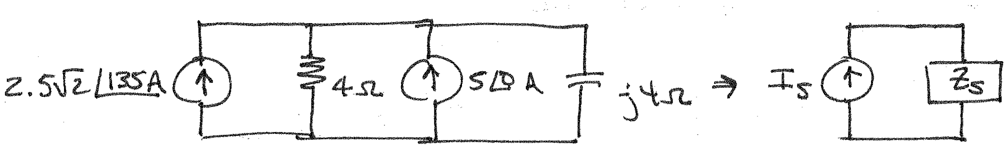


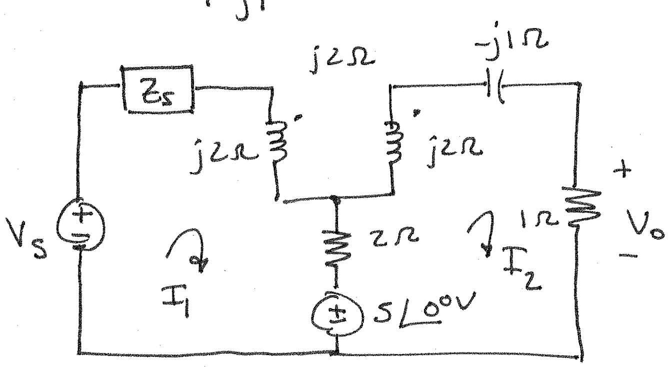
Figure P10.36

SOLUTION: Using source transformation



$I_s = 5\angle 0 + 2.5\sqrt{2}\angle 135 = 5 - 2.5 + j2.5 = 2.5 + j2.5\,A$

$Z_s = \frac{4(-j4)}{4-j4} = 2 - j2\,\Omega \qquad V_s = I_s Z_s = 10\angle 0\ V$



$V_s = I_1[Z_s + j2 + 2] + I_2[-2-j2] + 5\angle 0$

$5\angle 0^\circ = I_1[-2-j2] + I_2[3+j1]$

$$\begin{bmatrix} 4 & -2-j2 \\ -2-j2 & 3+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$I_2 = 2.5\angle 36.9^\circ\ A$

$V_o = (1)I_2 \qquad \boxed{V_o = 2.5\angle 36.9^\circ\ V}$

10.37 Determine the impedance seen by the source in the network shown in Fig. P10.37.

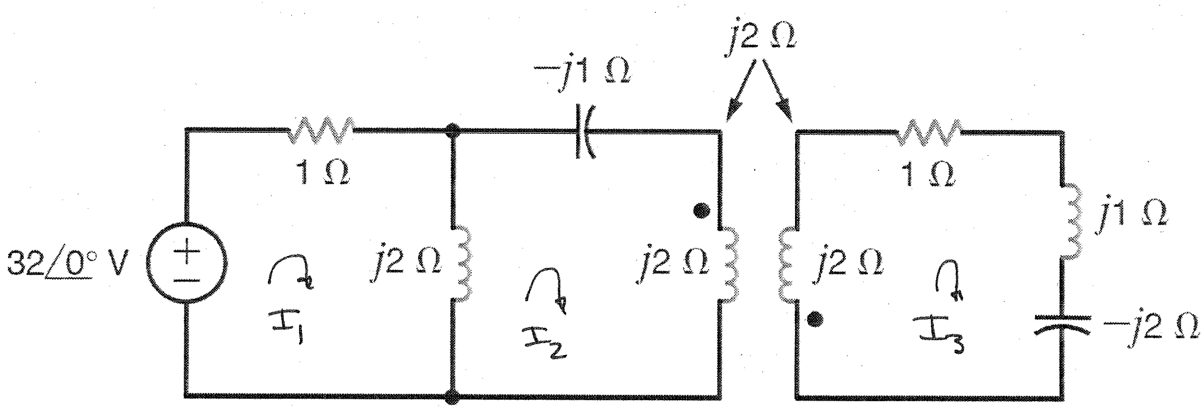


Figure P10.37

SOLUTION:

$32\angle 0 = I_1(1+j2) - j2I_2$ & $0 = -j2I_1 + I_2[j3] + j2I_3$

and $j2I_2 + I_3(1+j1) = 0$

$$Z_s = \frac{32\angle 0}{I_1}$$

$$\begin{bmatrix} 1+j2 & -j2 & 0 \\ -j2 & j3 & j2 \\ 0 & j2 & 1+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 32 \\ 0 \\ 0 \end{bmatrix} \Rightarrow I_1 = 11.2\angle -24.8^\circ \text{ A}$$

$$\boxed{Z_S = 2.6 + j1.2\,\Omega}$$

10.38 Determine the impedance seen by the source in the network shown in Fig. P10.38.

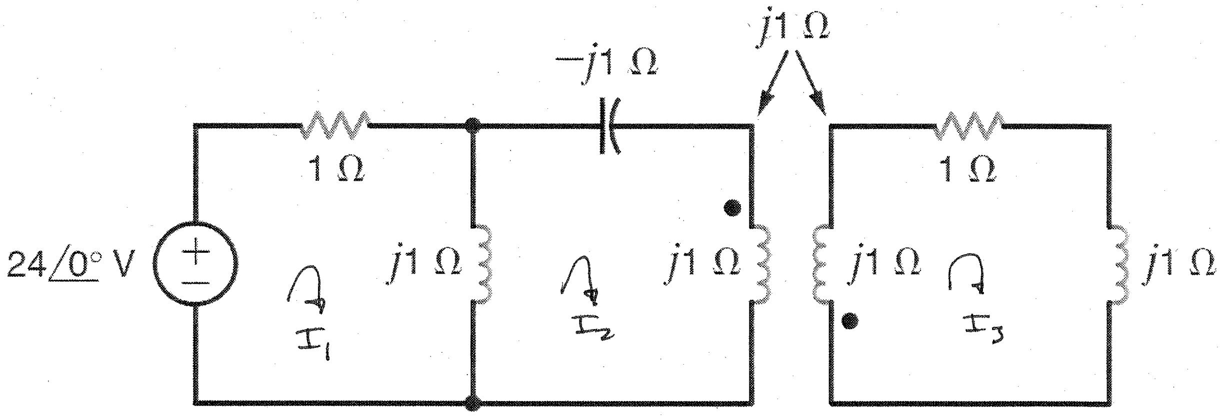


Figure P10.38

SOLUTION:

$24\angle 0 = I_1(1+j1) - jI_2$ & $0 = -jI_1 + I_2(j1) + j1I_3$

and $j1I_2 + I_3(1+j2) = 0$ $Z_S = 24\angle 0 / I_1$

$$\begin{bmatrix} 1+j1 & -j1 & 0 \\ -j1 & j1 & j1 \\ 0 & j1 & 1+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix} \Rightarrow I_1 = 14.4 + j4.8\ \text{A}$$

$$\boxed{Z_S = 1.5 - j0.5\ \Omega}$$

10.39 Determine the input impedance $\mathbf{Z}_{in}$ in the network in Fig. P10.39.

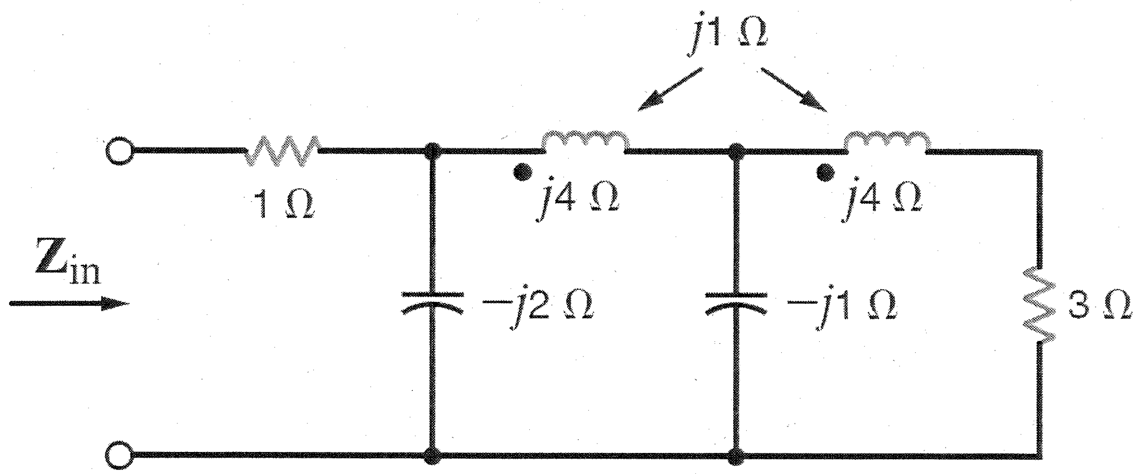
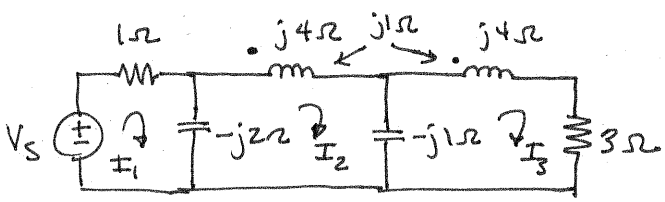


Figure P10.39

SOLUTION:



$Z_{IN} = V_S / I_1$

let $V_S = 1\angle 0^\circ$ V, so that

$Z_{IN} = 1/I_1$

$$\left.\begin{aligned} V_S &= I_1(1-j2) + j2I_2 \\ 0 &= j2I_1 + I_2(j1) + j1\,I_3 + j1I_3 \\ 0 &= jI_2 + jI_2 + I_3(3+j3) \end{aligned}\right\}$$

$$\begin{bmatrix} 1-j2 & j2 & 0 \\ j2 & j1 & j2 \\ 0 & j2 & 3+j3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 0.137\angle 37.2^\circ$

$$\boxed{Z_{IN} = 5.8 - j4.4\,\Omega}$$

10.40 Determine the input impedance $\mathbf{Z}_{\text{in}}$ of the circuit in Fig. P10.40. CS

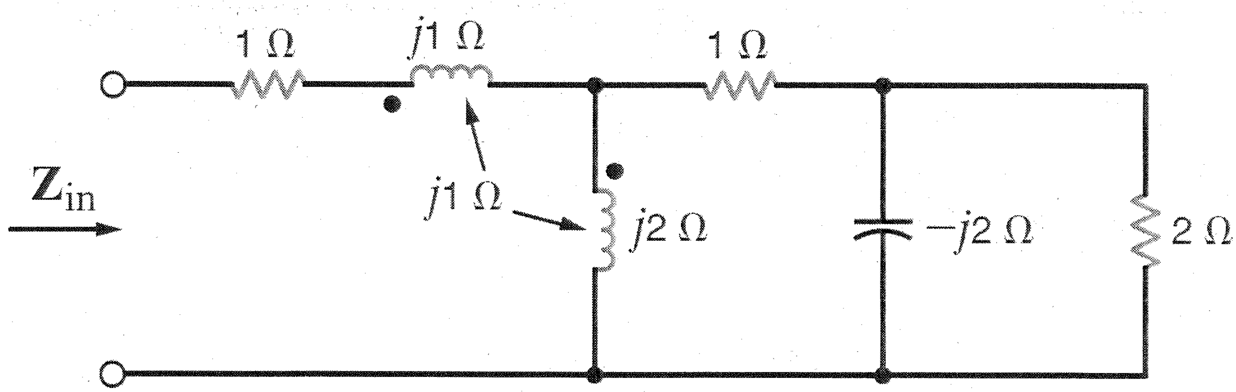
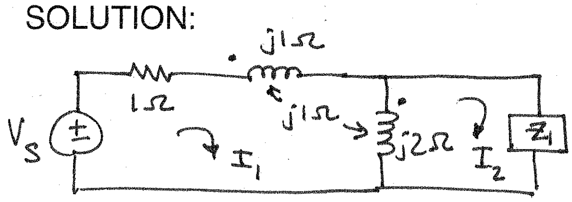


Figure P10.40

SOLUTION:



$Z_{IN} = V_S / I_1$ let $V_S = 1\angle 0^\circ$ V

so that $Z_{IN} = 1/I_1$

$$Z_1 = 1 + \frac{(2)(-j2)}{2-j2} = 2 - j1\,\Omega$$

$V_S = I_1(1+j3) - j2I_2 + jI_1 + j(I_1 - I_2)$

$$\left.\begin{aligned} V_S &= I_1(1+j5) + I_2(-j3) \\ 0 &= -j2I_1 + I_2(Z_1 + j2) - j1 I_1 \end{aligned}\right\} \begin{bmatrix} 1+j5 & -j3 \\ -j3 & 2+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$I_1 = 0.1785\angle -34.8^\circ$ A

$$\boxed{Z_{IN} = 4.6 + j\,3.2\,\Omega}$$

10.41 Given the network shown in Fig. P10.41, determine the value of the capacitor C that will cause the impedance seen by the $24\underline{/0^\circ}$ V voltage source to be purely resistive. $f = 60$ Hz.

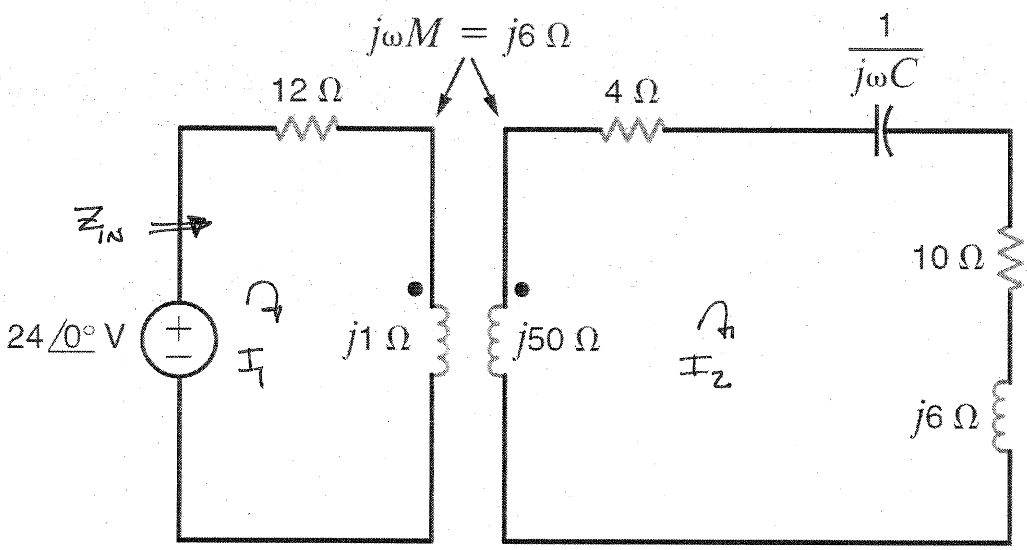


Figure P10.41

SOLUTION: for $Z_{IN} = R_{IN} + j0$, $\angle I_1 = 0°$. $X = 56 - 1/\omega C$

$24\angle 0 = I_1(12+j1) - j6 I_2$ & $-j6 I_1 + I_2[14 + jX] = 0$

$24\angle 0 = I_1\left[12 + j1 + \frac{36}{14+jX}\right]$ ⇐ $I_2 = j6 I_1 / (14+jX)$

$$\frac{24\angle 0}{I_1} = Z_{IN} = R + j0 = 12 + j1 + \frac{36}{14+jX} = 12 + j1 + \frac{36(14) - j36X}{14^2 + X^2}$$

$$j1 - \frac{j36X}{14^2+X^2} = 0 \Rightarrow X^2 - 36X + 14^2 = 0 \Rightarrow X = \begin{cases} 29.3 \\ 6.69 \end{cases}$$

$$\frac{1}{\omega C} = 56 - X = \begin{Bmatrix} 49.3 \\ 26.7 \end{Bmatrix} \Rightarrow \boxed{C = \begin{Bmatrix} 53.8\,\mu F \\ 99\,\mu F \end{Bmatrix} \text{ either will work!}}$$

10.42 Analyze the network in Fig. P10.42 and determine whether a value of X_C can be found such that the output voltage is equal to twice the input voltage.

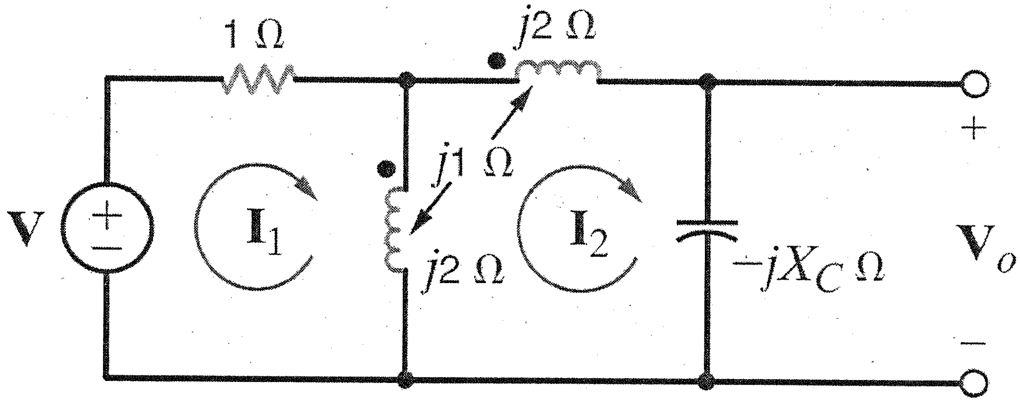


Figure P10.42

SOLUTION:

$V = I_1(1+j2) - j2I_2 + jI_2 \Rightarrow V = I_1(1+j2) - jI_2$

$0 = -j2I_1 + I_2(j(4-X_c)) - jI_2 + j(I_1 - I_2) \Rightarrow 0 = -jI_1 + jI_2(2-X_c)$

or, $I_1 = I_2(2-X_c)$

now, $V = I_2[(1+j2)(2-X_c) - j1] \Rightarrow V = I_2[2-X_c + j(3-2X_c)]$

But, $V_o = -jX_c I_2$

So, $\dfrac{V_o}{V} = \dfrac{-jX_c}{2-X+j(3-2X_c)} = 2+j0$

requires $2-X_c = 0 \Rightarrow X = 2$ and $\dfrac{V_o}{V} = \dfrac{-j2}{0+j(3-4)} = 2+j0$ ✓

$\boxed{X_c = 2\Omega}$

10.43 Two coils in a network are positioned such that there is 100% coupling between them. If the inductance of one coil is 10 mH and the mutual inductance is 6 mH, compute the inductance of the other coil. CS

SOLUTION:

$$k = 1 = \frac{M}{\sqrt{4 L_2}} = \frac{6 \times 10^{-3}}{\sqrt{10^{-2} L_2}} \qquad \boxed{L_2 = 3.6 \text{ mH}}$$

10.44 The currents in the network in Fig. P10.44 are known to be $i_1(t) = 10\cos(377t - 30°)$ mA and $i_2(t) = 20\cos(377t - 45°)$ mA. The inductances are $L_1 = 2$ H, $L_2 = 2$ H, and $k = 0.8$. Determine $v_1(t)$ and $v_2(t)$.

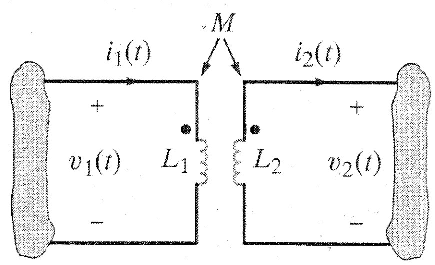
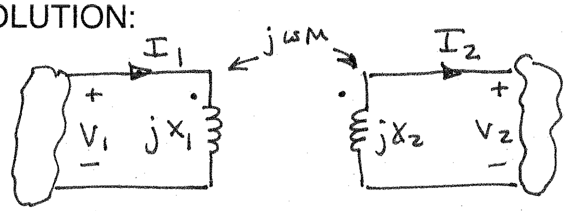


Figure P10.44

SOLUTION:



$X_1 = \omega L_1 = 754\,\Omega$

$X_2 = \omega L_2 = 754\,\Omega$

$M = k\sqrt{L_1L_2} = 1.6\text{H} \qquad \omega M = 603\,\Omega$

$V_1 = I_1(jX_1) - j\omega M I_2$

$-V_2 = -j\omega M I_1 + I_2 jX_2$

$I_1 = 10\angle{-30}$ mA

$I_2 = 20\angle{-45}$ mA

$V_1 = 5.16\angle{-157°}$ V

$V_2 = 9.39\angle{-145°}$ V

$$\boxed{\begin{aligned} v_1(t) &= 5.16\cos(377t - 157°)\text{ V} \\ v_2(t) &= 9.39\cos(377t - 145°)\text{ V}\end{aligned}}$$

10.45 Determine the energy stored in the coupled inductors in the circuit in P10.44 at $t = 1$ ms.

SOLUTION:

$$w(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) - M i_1(t) i_2(t)$$

$i_1(t) = 10 \cos(377t - 30°)$ mA $\quad i_2(t) = 20 \cos(377t - 45°)$ mA

$L_1 = 2H \quad L_2 = 2H \quad k = 0.8 \Rightarrow M = 1.6H$

let $t_1 = 1ms$

$i_1(t_1) = 9.89$ mA $\quad i_2(t_1) = 18.4$ mA

$$\boxed{w(t_1) = 145 \mu J}$$

10.46 The currents in the magnetically-coupled inductors shown in Fig. P10.46 are known to be $i_1(t) = 8\cos(377t - 20°)$ mA and $i_2(t) = 4\cos(377t - 50°)$ mA. The inductor values are $L_1 = 2$ H, $L_2 = 1$ H, and $k = 0.6$. Determine $v_1(t)$ and $v_2(t)$.

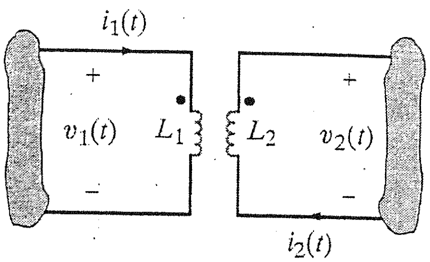


Figure P10.46

SOLUTION: $M = k\sqrt{L_1L_2} \Rightarrow M = 0.849\ H$

$$v_1(t) = L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = \frac{-2(8)(377)}{1000}\sin(377t - 20°) + \frac{M(4)(377)}{1000}\sin(377t - 50°)$$

$M = k\sqrt{L_1L_2} \Rightarrow M = 0.849\ H$ $\quad I_1 = 8\angle{-20°}\ mA \quad I_2 = 4\angle{-50°}\ mA$

$V_1 = j\omega L_1 I_1 - j\omega M I_2$ $\quad \omega = 377\ r/s \quad \omega M = 320\ \Omega$

$-V_2 = -j\omega M I_1 + j\omega L_2 I_2$

$V_1 = 4.96\angle 77.4°\ V \qquad V_2 = 5.12\angle 70°\ V$

$$\boxed{\begin{aligned} v_1(t) &= 4.96\cos(377t + 77.4°)\ V \\ v_2(t) &= 1.46\cos(377t + 101°)\ V \end{aligned}}$$

10.47 Determine the energy stored in the coupled inductors in Problem 10.46 at $t = 1$ ms. CS

SOLUTION: $w(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)$

$i_1(t) = 8 \cos(377t - 20°)$ mA $\quad i_2(t) = 4\cos(377t - 50°)$ mA

$L_1 = 2H \quad L_2 = 1H \quad k = 0.6 \Rightarrow M = 0.849\ H$

at $t_1 = 1ms$, $\quad i_1(t_1) = 8.00$ mA $\quad i_2(t_1) = 3.52$ mA

$\boxed{w(t_1) = 94.1 \mu J}$

10.48 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.48.

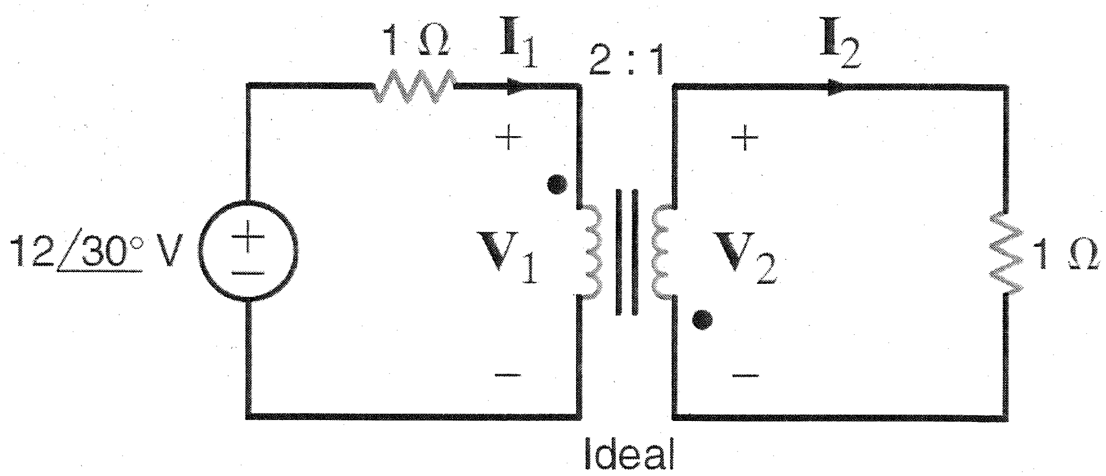


Figure P10.48

SOLUTION:

$12\angle 30 = (1)I_1 + V_1$ & $V_2 = (1)I_2$ & $-V_1 = 2V_2$ & $2I_1 = -I_2$

$12\angle 30 = I_1 - 2V_2 = I_1 - 2I_2 = I_1 + 4I_1 = 5I_1$

$I_1 = \frac{12\angle 30°}{5} = \boxed{2.4\angle 30°\ A = I_1}$

$\boxed{I_2 = 4.8\angle -150°\ A}$

$\boxed{V_2 = 4.8\angle -150°\ V}$ $\boxed{V_1 = 9.6\angle 30°\ V}$

10.49 Find all currents and voltages in the network in Fig. P10.49.

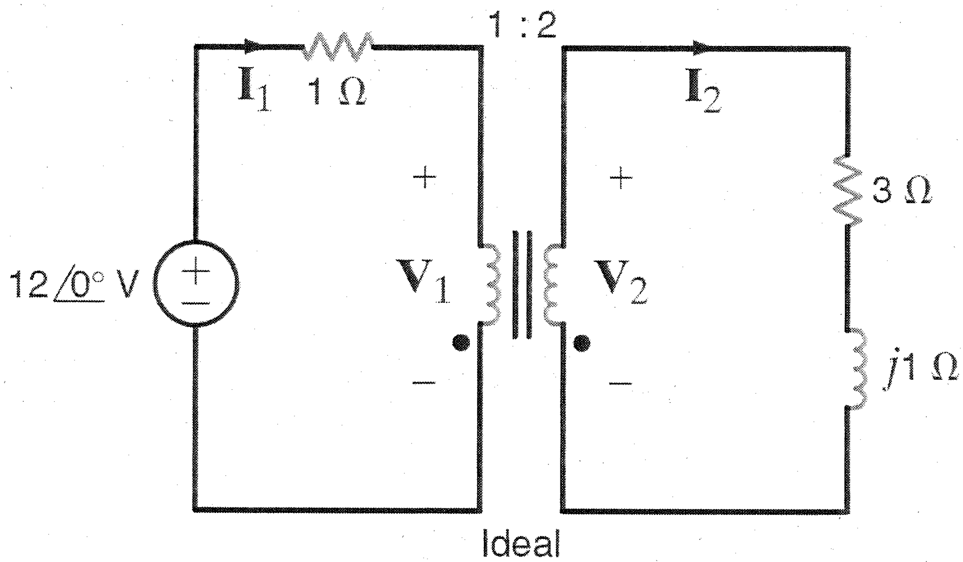
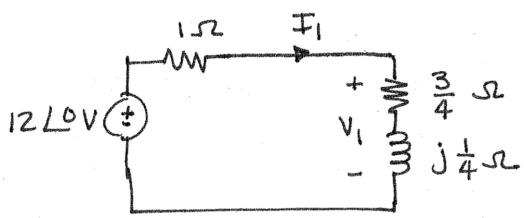


Figure P10.49

SOLUTION: $n = 2$ $n^2 = 4$

$12\angle 0 = (1)I_1 + V_1$ & $V_2 = I_2(3+j1)$ & $V_2 = 2V_1$ & $I_1 = 2I_2$



$I_1 = \dfrac{12\angle 0}{1.75 + j0.25}$

$V_1 = I_1(0.75 + j0.25)$

$\boxed{\begin{array}{ll} I_1 = 6.79\angle -8.13^\circ \text{ A} & I_2 = 3.39\angle -8.13^\circ \text{ A} \\ V_1 = 5.37\angle 10.3^\circ \text{ V} & V_2 = 10.74\angle 10.3^\circ \text{V} \end{array}}$

10.50 Determine $\mathbf{V}_o$ in the circuit in Fig. P10.50. **PSV**

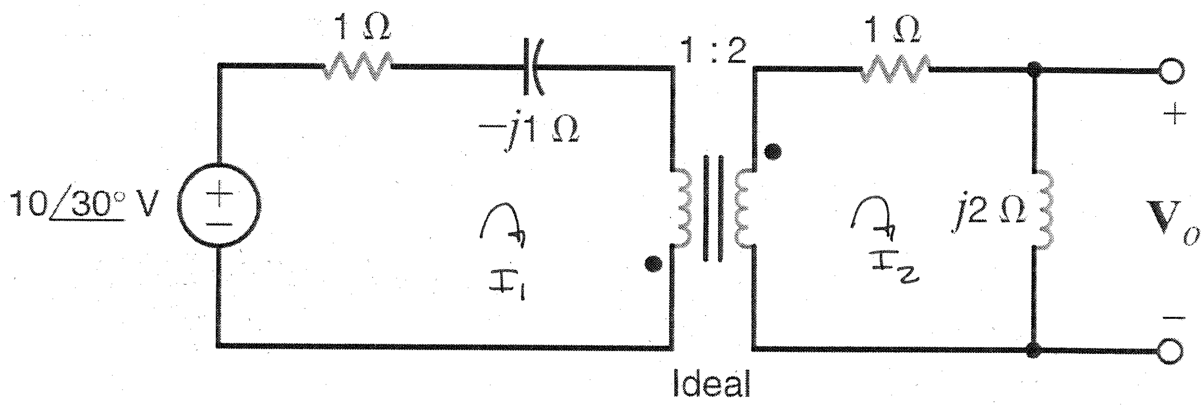
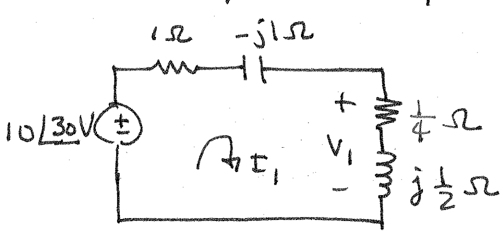


Figure P10.50

SOLUTION: $n = 2 \quad n^2 = 4$

Use impedance reflection,



$I_2 = -I_1/2 \qquad V_0 = j2\, I_2$

$$I_1 = \frac{10\angle 30^\circ}{1.25 - j0.5} = 7.43\angle 51.8^\circ \text{ A}$$

$I_2 = 3.72\angle -128^\circ$ A

$$\boxed{V_o = 7.43\angle -38.2^\circ \text{ V}}$$

10.51 Determine $\mathbf{V}_o$ in the circuit in Fig. P10.51.

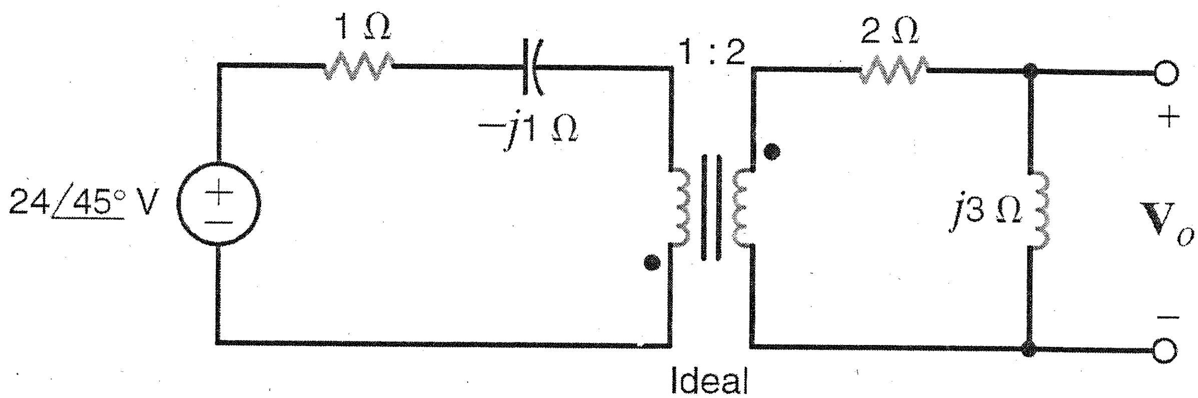
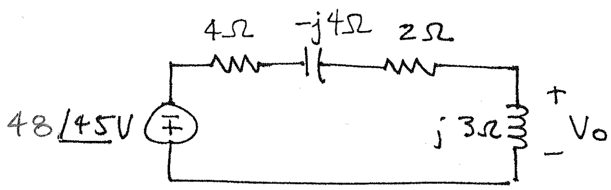


Figure P10.51

SOLUTION: $n = 2 \quad n^2 = 4$ Using reflection through transformer



$$V_o = -\frac{48\angle 45 \; (j3)}{6 - j4 + j3} = -\frac{48\angle 45 \; (j3)}{6 - j1}$$

$$\boxed{V_o = 23.7 \angle 35.5^\circ \text{ V}}$$

10.52 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.52. CS

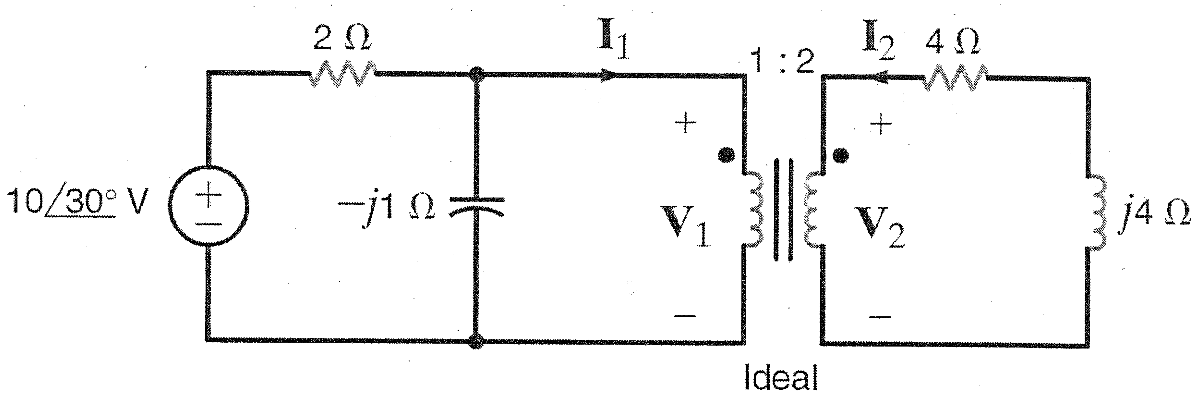
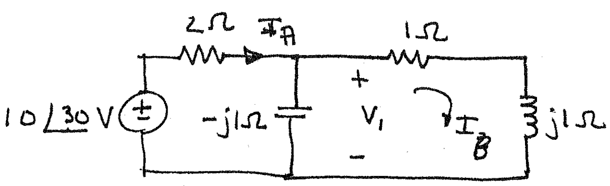


Figure P10.52

SOLUTION: $n = 2 \quad n^2 = 4$



$$10\angle 30 = I_A(2 - j1) + j I_B$$

$$0 = j I_A + I_B(1)$$

$$I_A = j I_B$$

$$10\angle 30 = I_B(1 + j2 + j1) = I_B(1 + j3)$$

$$I_B = \boxed{I_1 = 3.16\angle -41.6^\circ \text{ A}}$$

$$V_1 = I_B(1 + j1)$$

$$\boxed{V_1 = 4.47\angle 3.4^\circ \text{ V}}$$

$$\boxed{V_2 = nV_1 = 8.93\angle 3.4^\circ \text{ V}}$$

$$\boxed{I_2 = -I_1/n = 1.58\angle 138^\circ \text{ A}}$$

10.53 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.53.

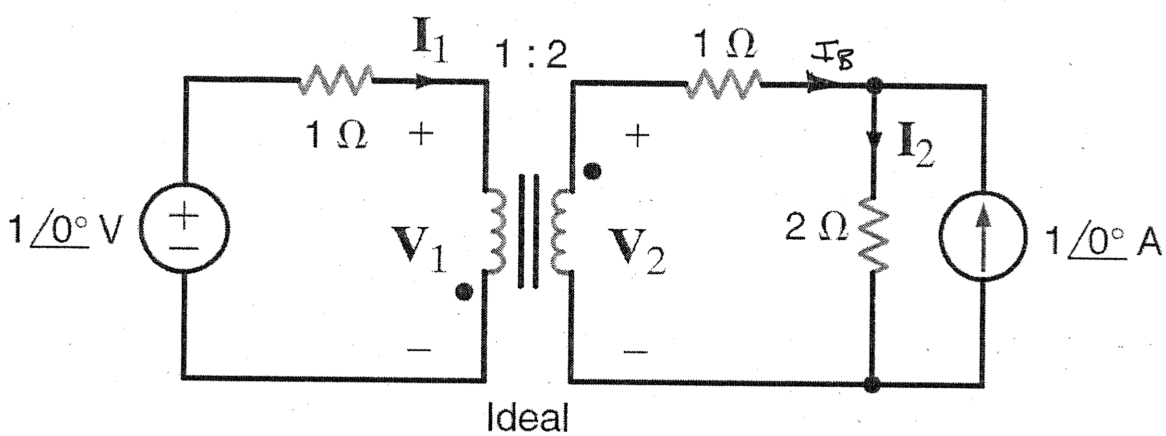
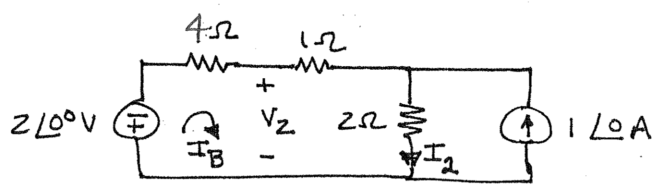


Figure P10.53

SOLUTION: $n = 2 = -\dfrac{V_2}{V_1} = -\dfrac{I_1}{I_B}$



$-2\angle 0 = I_B(4+1+2) + 2(1\angle 0)$

$I_B = 0.571\angle 180°$ A

$I_2 = I_B + 1\angle 0$

$\boxed{I_2 = 0.429\angle 0°\text{ A}}$

$V_2 = 2I_2 + I_B(1)$

$\boxed{V_2 = 0.287\angle 0°\text{ V}}$

$I_1 = -nI_B$

$\boxed{I_1 = 1.14\angle 0°\text{ A}}$

$V_1 = -V_2/n$

$\boxed{V_1 = 0.143\angle 180°\text{ V}}$

10.54 Find **I** in the network in Fig. P10.54.

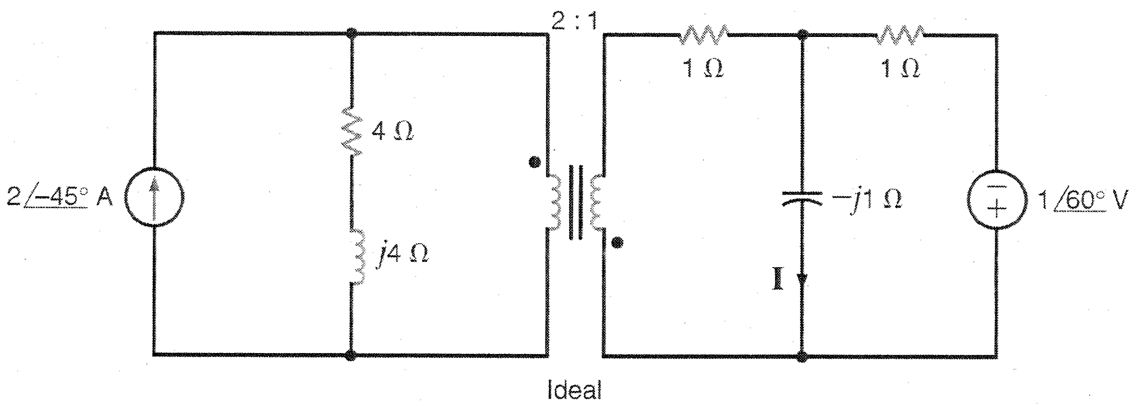
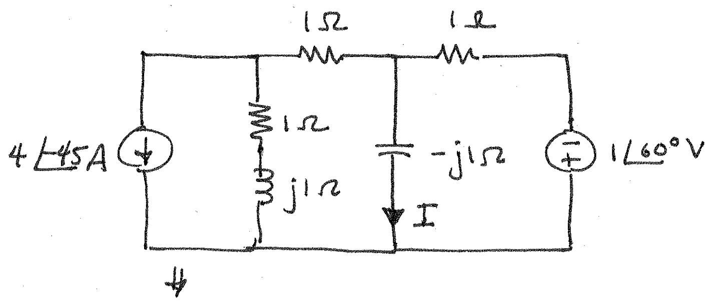
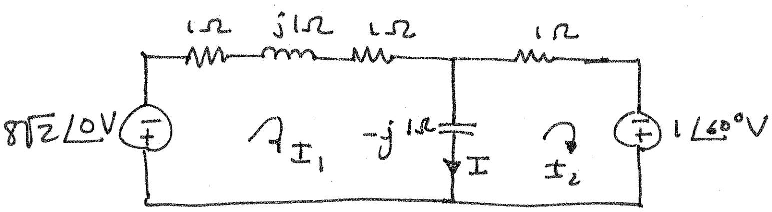


Figure P10.54

SOLUTION: $n = 1/2$





$$-8\sqrt{2}\angle 0 = I_1(2) + j1 I_2$$
$$1\angle 60 = j1 I_1 + (1-j1) I_2$$

$$\begin{bmatrix} 2 & j1 \\ j1 & 1-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -8\sqrt{2}\angle 0 \\ 1\angle 60 \end{bmatrix}$$

$I_1 = 4.17\angle 168^\circ\ A$ $\qquad I_2 = 3.63\angle 119^\circ\ A$

$I = I_1 - I_2$ $\qquad \boxed{I = 3.23\angle -135^\circ\ A}$

10.55 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.55.

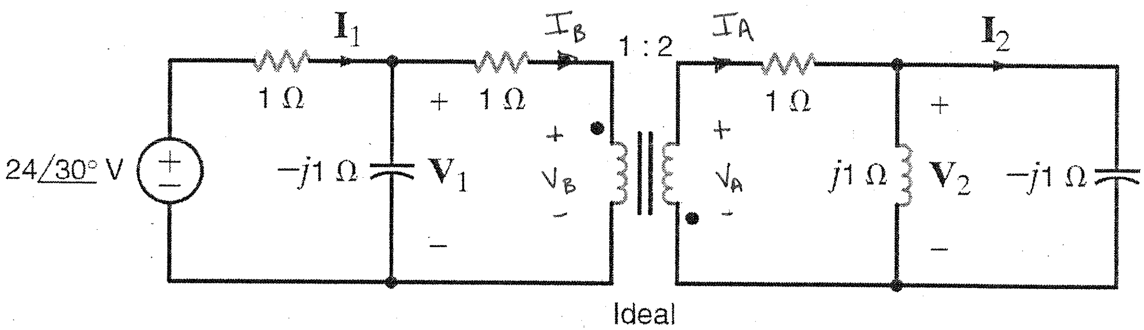
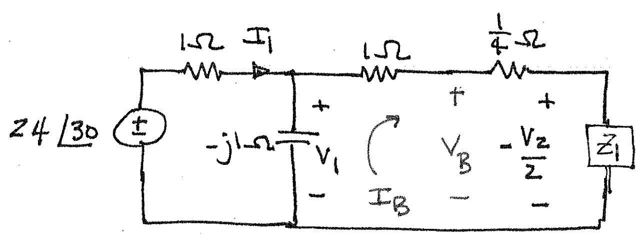


Figure P10.55

SOLUTION: $n = 2$



$$Z_1 = \frac{(j1)(-j1)}{j1 - j1} = \infty$$

$$I_2 = \frac{V_2}{-j1}$$

$$24\angle 30 = I_1(1 - j1) + j\,I_B \quad \& \quad I_B = 0$$

$$V_1 = (I_1 - I_B)(-j1) = 10\sqrt{2}\angle -15 \qquad \boxed{V_1 = 17.0\angle -15^\circ \text{ V}}$$

$$V_B = -\frac{V_2}{2} = V_1 \Rightarrow \qquad \boxed{V_2 = 34.0\angle 165^\circ \text{ V}} \qquad \boxed{I_1 = 17.0\angle 75^\circ \text{ A}}$$

$$I_2 = \frac{V_2}{-j1} \Rightarrow \qquad \boxed{I_2 = 34.0\angle -105^\circ \text{ A}}$$

10.56 Find the current **I** in the network in Fig. P10.56.

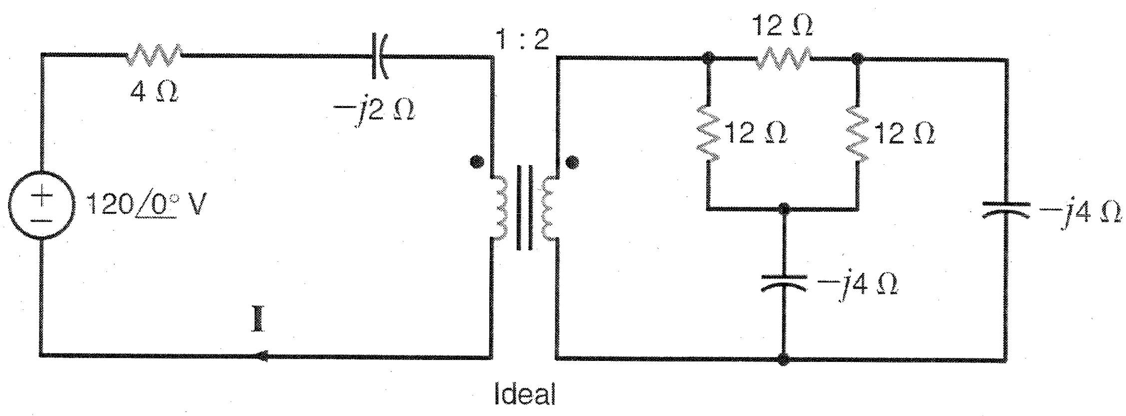
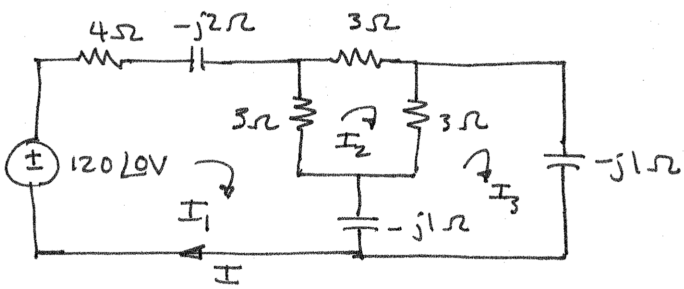


Figure P10.56

SOLUTION: $n = 2$



$$120\angle 0 = I_1(7-j3) - 3I_2 + jI_3$$

$$0 = -3I_1 + 9I_2 - 3I_3$$

$$0 = jI_1 - 3I_2 + I_3(3-j2)$$

$$\begin{bmatrix} 7-j3 & -3 & j1 \\ -3 & 9 & -3 \\ j1 & -3 & 3-j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$$

$I = I_1$

$I_1 = 19.9\angle 24.4°$ A

$$\boxed{I_o = 19.9\angle 24.4° \text{ A}}$$

10.57 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.57.

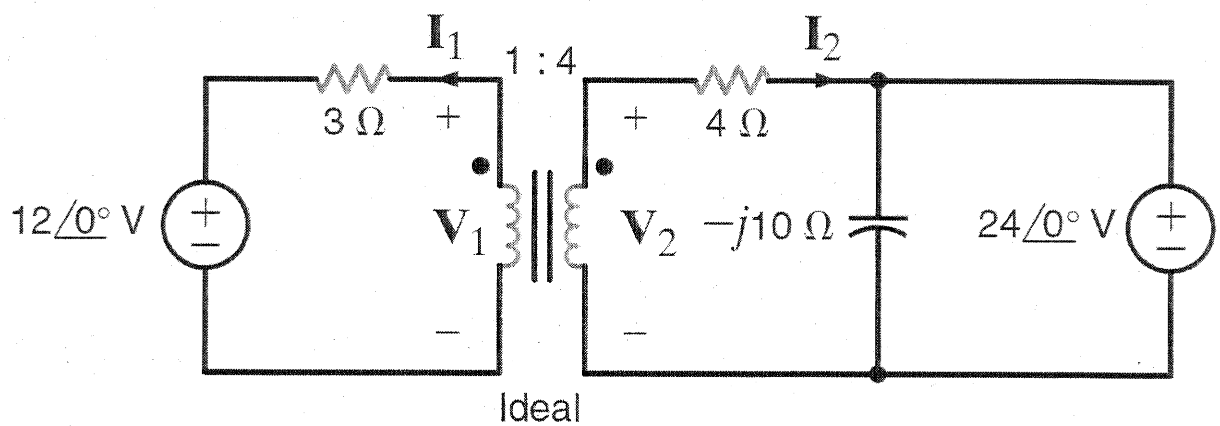
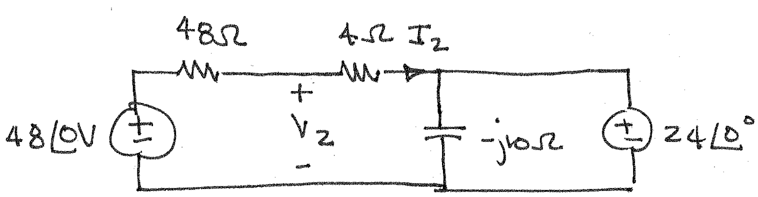


Figure P10.57

SOLUTION:



$n = 4$

$$I_2 = \frac{48\angle 0 - 24\angle 0}{48+4} = \frac{24\angle 0}{52} = 0.46\angle 0°\ \text{A}$$

$$V_2 = 48\angle 0 - 48 I_2 \Rightarrow V_2 = 25.9\angle 0°\ \text{V}$$

$$V_1 = V_2 / n = 6.48\angle 0°\ \text{V}$$

$$I_1 = -I_2 n = -1.84\angle 0°\ \text{A}$$

$I_1 = -1.84\angle 0°$ A	$V_1 = 6.48\angle 0°$ V
$I_2 = 0.46\angle 0°$ A	$V_2 = 25.9\angle 0°$ V

10.58 Find $\mathbf{V}_o$ in the network in Fig. P10.58.

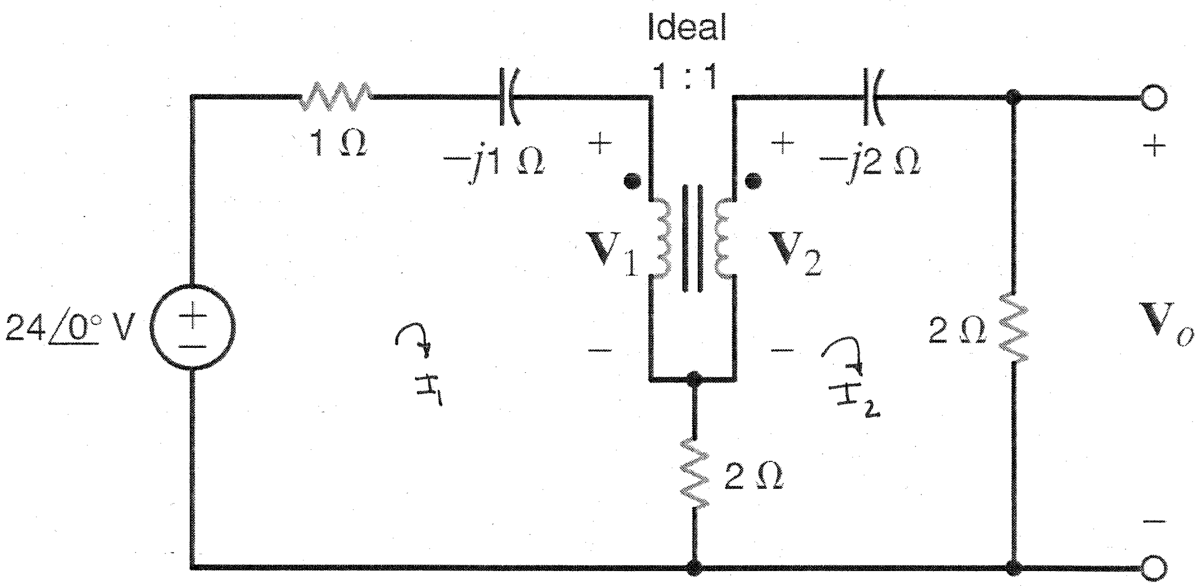


Figure P10.58

SOLUTION: $n = 1$ $\quad I_1 = I_2 \quad V_1 = V_2$

$$\left.\begin{aligned} 24\angle 0^\circ &= I_1(3-j1) - 2I_2 + V_1 \\ V_2 &= -2I_1 + I_2(4-j2) \end{aligned}\right\} \Rightarrow \begin{aligned} 24\angle 0 &= I_1(1-j1) + V_1 \\ V_1 &= I_1(2-j2) \end{aligned}$$

$$24\angle 0 = I_1(1-j1) + I_1(2-j2) = I_1(3-j3) \Rightarrow I_1 = \frac{24\angle 0}{3-j3} = 5.66\angle 45^\circ\text{ A}$$

$$I_2 = I_1 = 5.66\angle 45^\circ\text{ A}$$

$$V_o = 2I_2 \qquad \boxed{V_o = 11.32\angle 45^\circ\text{ V}}$$

10.59 Find the current **I** in the network in Fig. P10.59. CS

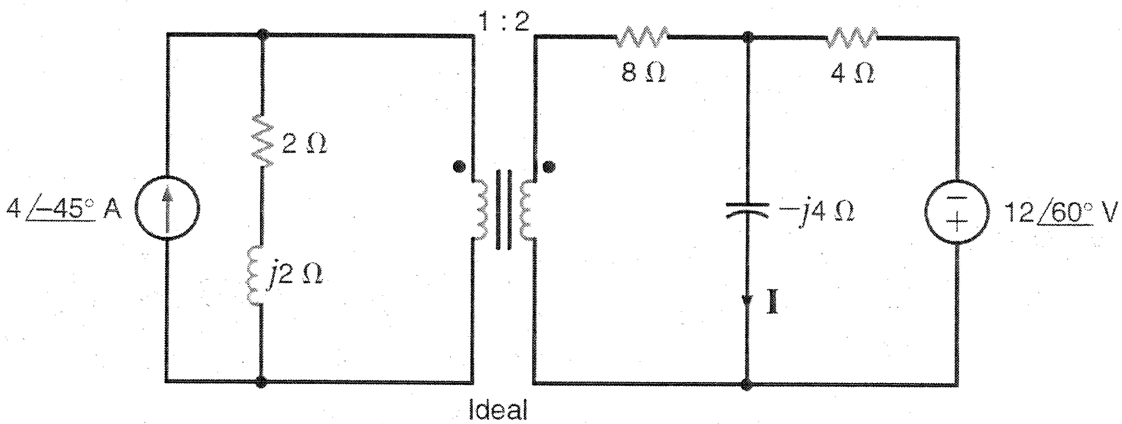
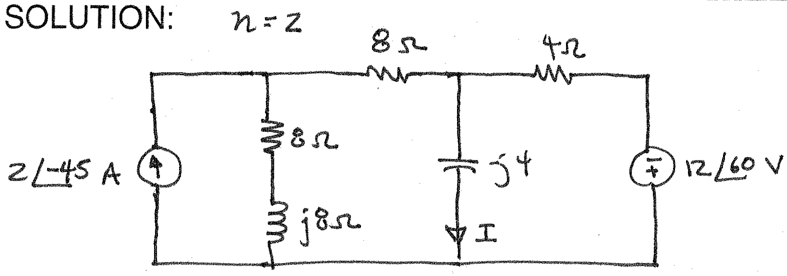
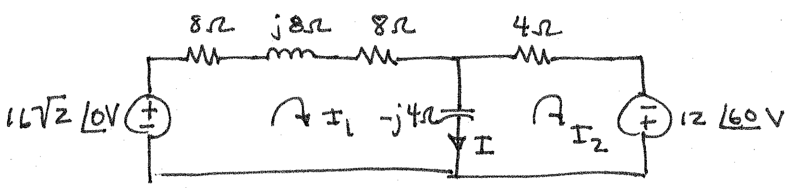


Figure P10.59

SOLUTION: $n = 2$





$I = I_1 - I_2$

$$\left.\begin{aligned} 16\sqrt{2}\angle 0 &= I_1(16+j4) + j4 I_2 \\ 12\angle 60 &= j4 I_1 + I_2(4-j4) \end{aligned}\right\} \quad \begin{bmatrix} 16+j4 & j4 \\ j4 & 4-j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 16\sqrt{2} \\ 12\angle 60 \end{bmatrix}$$

$I_1 = 1.63\angle -14.3^\circ \text{ A} \qquad I_2 = 1.06\angle 88.0^\circ \text{ A}$

$$\boxed{I = 2.12\angle -43.5^\circ \text{ A}}$$

10.60 Find the voltage $\mathbf{V}_o$ in the network in Fig. P10.60. PSV

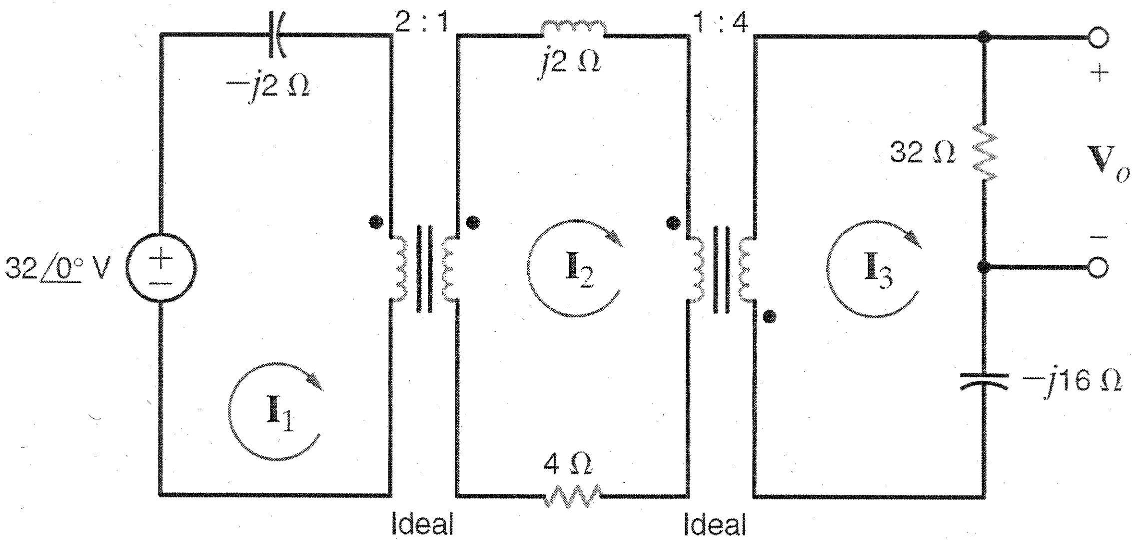
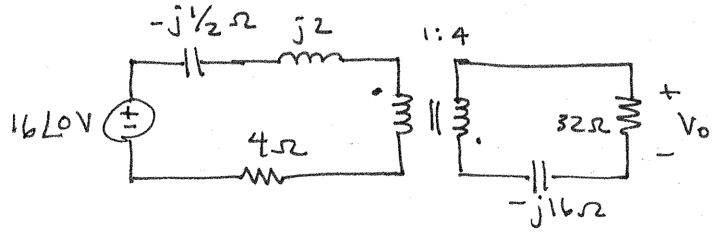
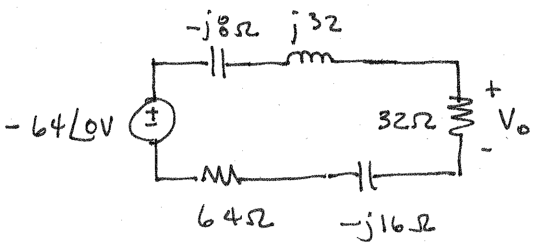


Figure P10.60

SOLUTION: $n_1 = 1/2$ $n_2 = 4$





$$V_o = \frac{-64\angle 0\ (32)}{32+64+j32-j16-j8}$$

$$\boxed{V_o = 21.3\ \angle 175°\ \text{V}}$$

10.61 Find $\mathbf{V}_o$ in the circuit in Fig. P10.61.

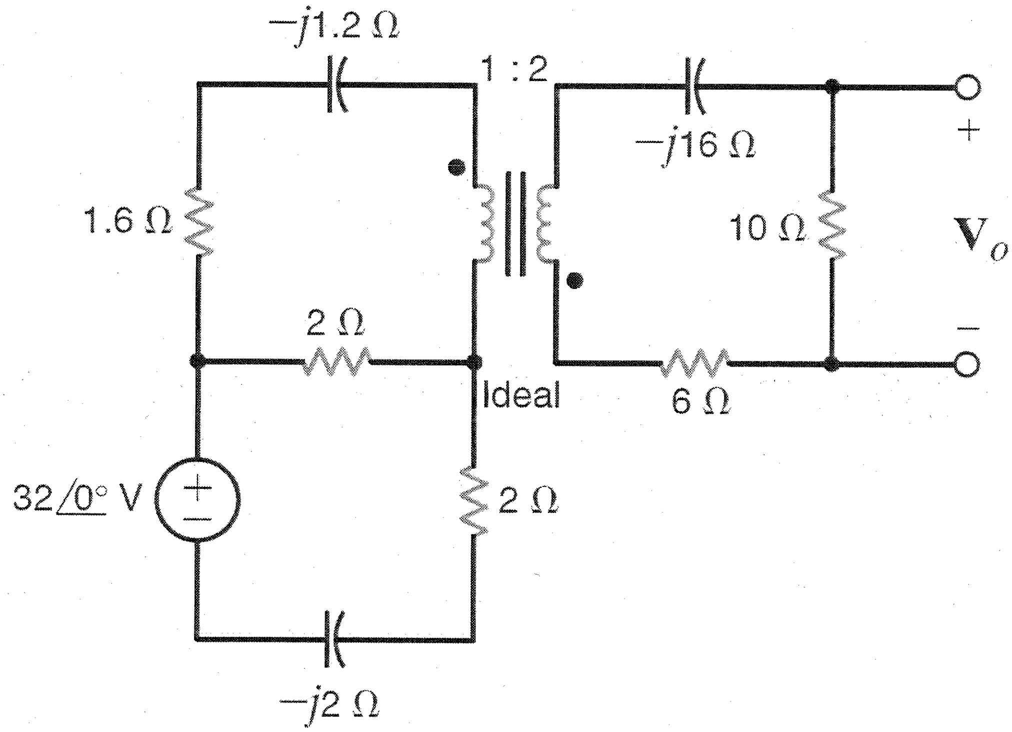
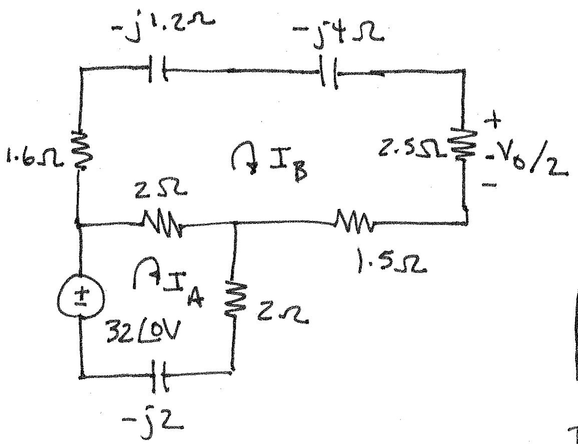


Figure P10.61

SOLUTION: $n = 2$



$$32\angle 0 = I_A(4-j2) - 2I_B$$

$$0 = -2I_A + I_B(7.6 - j5.2)$$

$$\frac{V_o}{2} = -I_B(2.5) \Rightarrow V_o = -5I_B$$

$$\begin{bmatrix} 4-j2 & -2 \\ -2 & 7.6-j5.2 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 32 \\ 0 \end{bmatrix}$$

$$I_B = 1.62\angle 66.0^\circ \text{ A}$$

$$\boxed{V_o = 8.12\angle -114^\circ \text{ V}}$$

10.62 Find $\mathbf{V}_o$ in the network in Fig. P10.62.

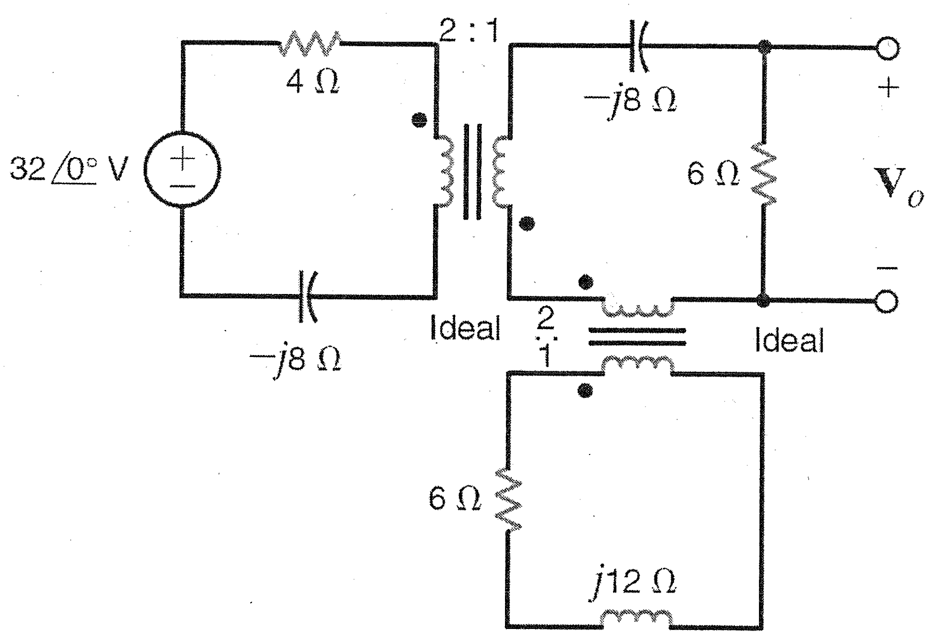
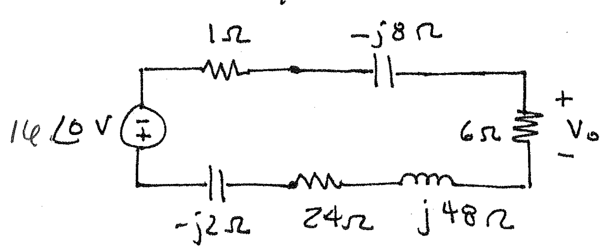


Figure P10.62

SOLUTION: $n_1 = 1/2$ $\quad n_2 = 2$



$$V_o = \frac{-16\angle 0\ (6)}{1+6+24+j(48-8-2)}$$

$$\boxed{V_o = 1.96\ \angle 129^\circ\ V}$$

10.63 Find $\mathbf{V}_o$ in the circuit in Fig. P10.63. CS

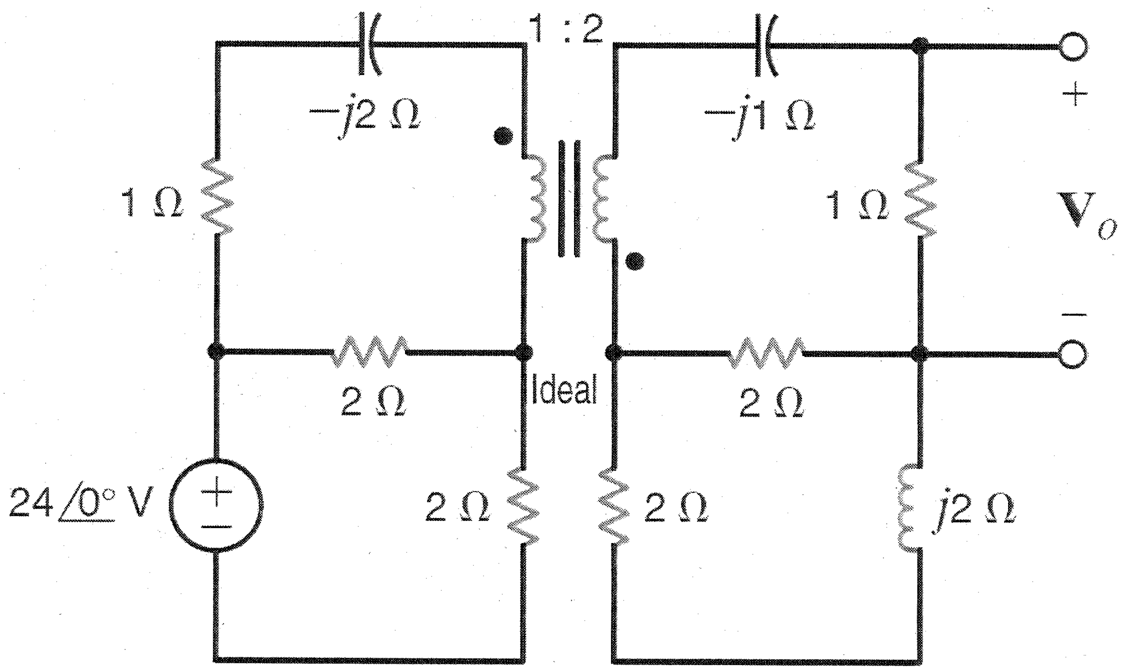
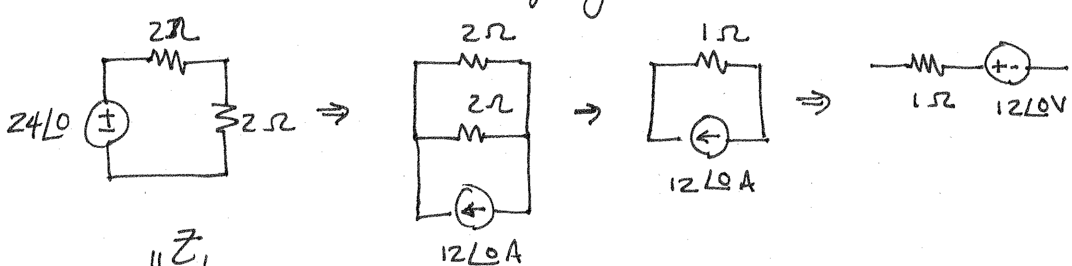
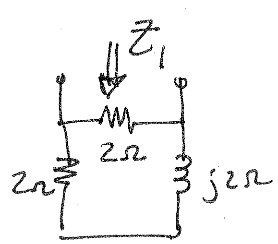


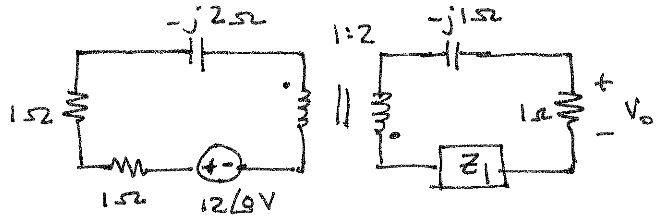
Figure P10.63

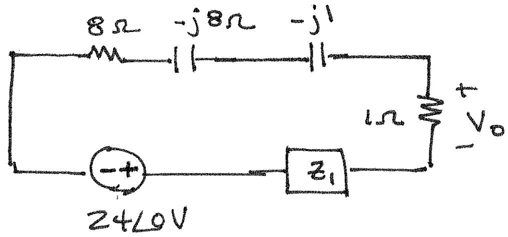
SOLUTION: First, we simplify,





$$Z_1 = \frac{2(2+j2)}{4+j2} = 1.2 + j0.4\,\Omega$$





$$V_o = \frac{-24\angle 0\,(1)}{8+1-j(8+1)+Z_1}$$

$$\boxed{V_o = 1.80\angle{-140^\circ}\ \text{V}}$$

10.64 Determine the input impedance seen by the source in the circuit in Fig. P10.64.

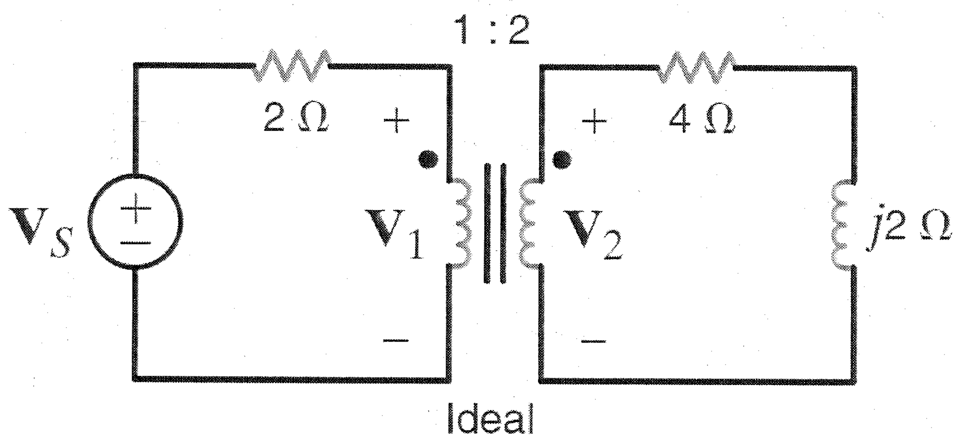
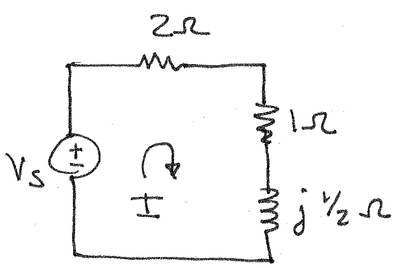


Figure P10.64

SOLUTION:



$Z_{IN} = V_S / I = 3 + j0.5\,\Omega$

$\boxed{Z_{IN} = 3 + j0.5\,\Omega}$

10.65 Determine the input impedance seen by the source in the circuit in Fig. P10.65. CS

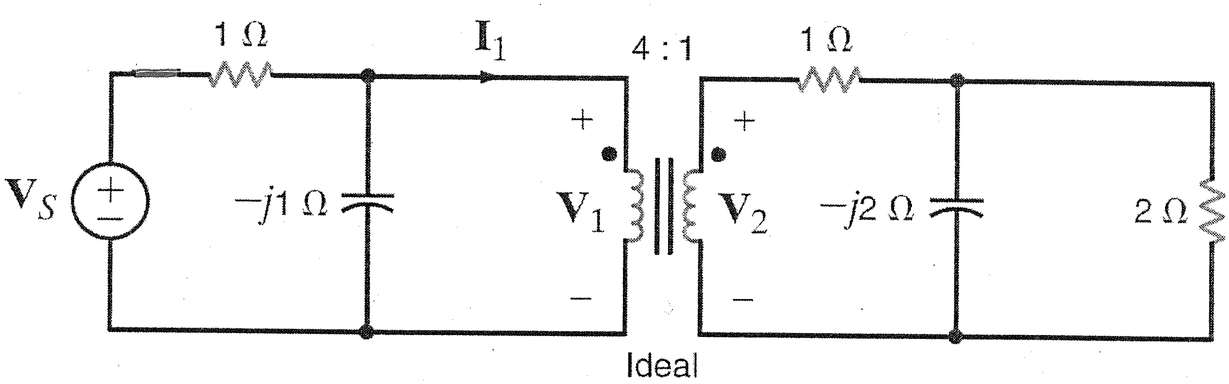
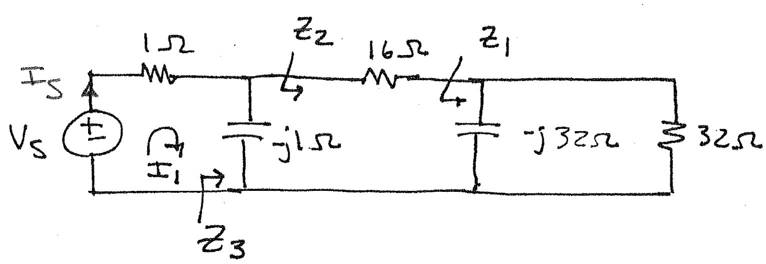


Figure P10.65

SOLUTION:



$$Z_{IN} = V_S / I_S$$

$$Z_1 = \frac{32(-j32)}{32-j32} = 16 - j16\,\Omega \qquad Z_2 = 16 + Z_1 = 32 - j16\,\Omega$$

$$Z_3 = \frac{-j1\,(Z_2)}{-j1 + Z_2} = 0.987\angle -88.6^\circ\,\Omega \qquad Z_{IN} = 1 + Z_3$$

$$\boxed{Z_{IN} = 1.42\angle -43.9\,\Omega}$$

10.66 Determine the input impedance seen by the source in the network shown in Fig. P10.66.

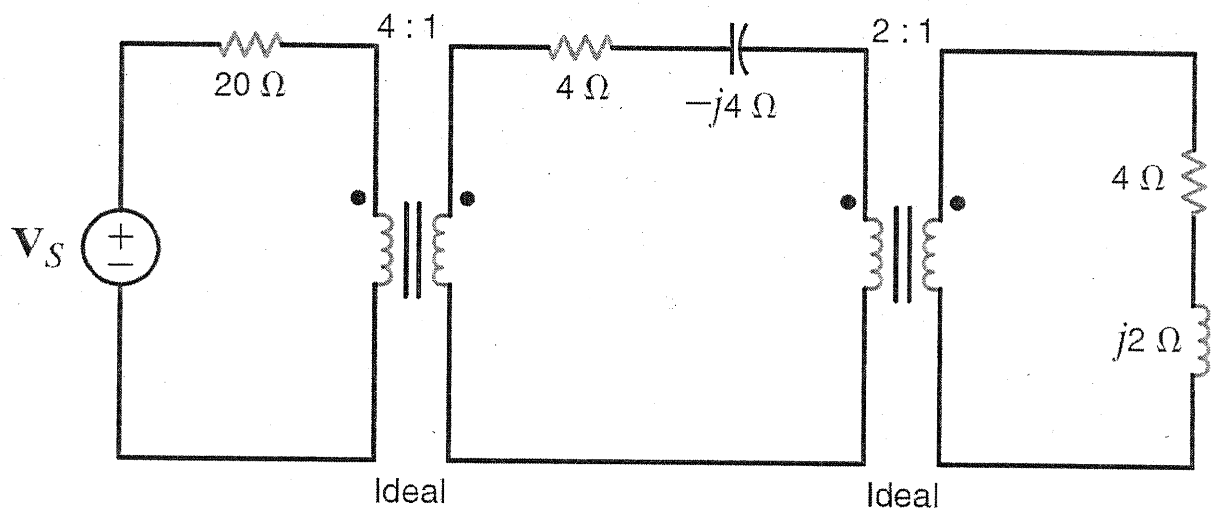
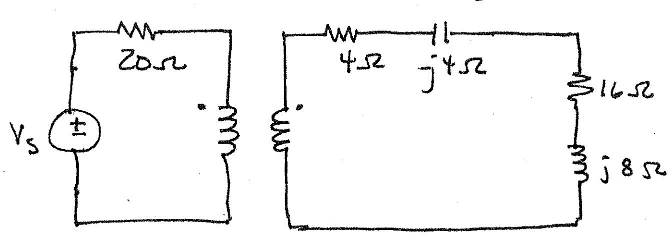
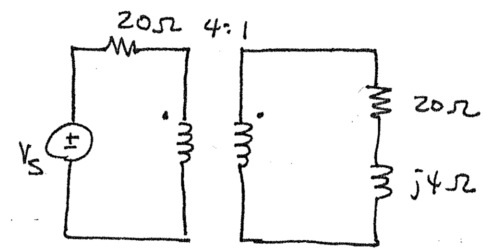


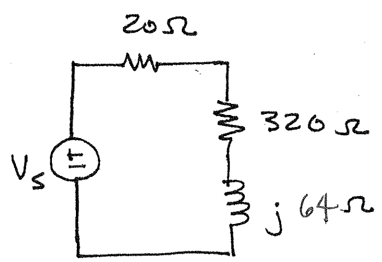
Figure P10.66

SOLUTION: $n_1 = 1/4$ $n_2 = 1/2$



⇒





$$\boxed{Z_{IN} = 340 + j\,64\ \Omega}$$

10.67 Determine the input impedance seen by the source in the network shown in Fig. P10.67.

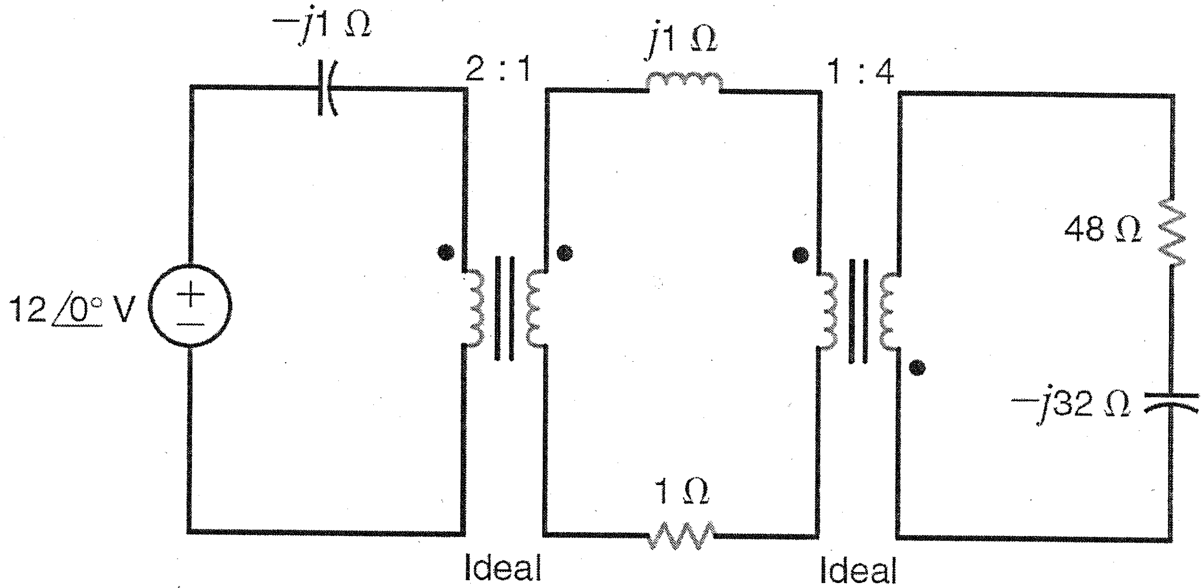
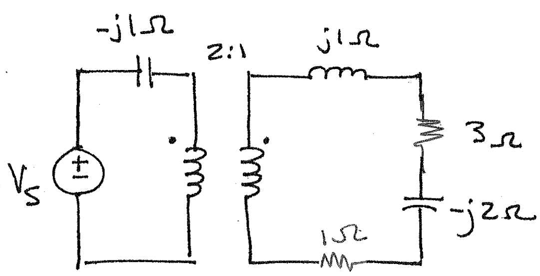
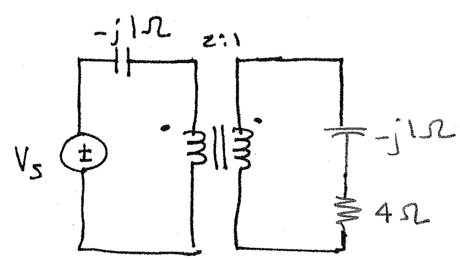


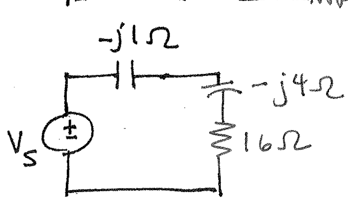
Figure P10.67

SOLUTION:



⇒





$Z_{IN} = 16 - j5\,\Omega$ $\boxed{Z_{IN} = 16 - j5\,\Omega}$

10.68 The output stage of an amplifier in an old radio is to be matched to the impedance of a speaker, as shown in Fig. P10.68. If the impedance of the speaker is 8 Ω and the amplifier requires a load impedance of 3.2 kΩ, determine the turns ratio of the ideal transformer.

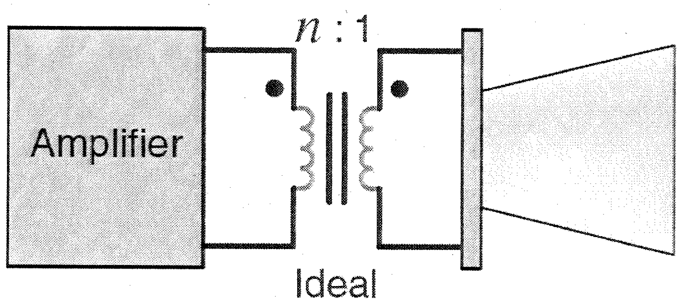


Figure P10.68

SOLUTION:

$Z_{SPEAKER} = 8\Omega \qquad Z_{SPEAKER}\, n^2 = 3200$

$n^2 = \frac{3200}{8} = 400 \qquad \boxed{n = 20}$

10.69 Determine $\mathbf{V}_S$ in the circuit in Fig. P10.69.

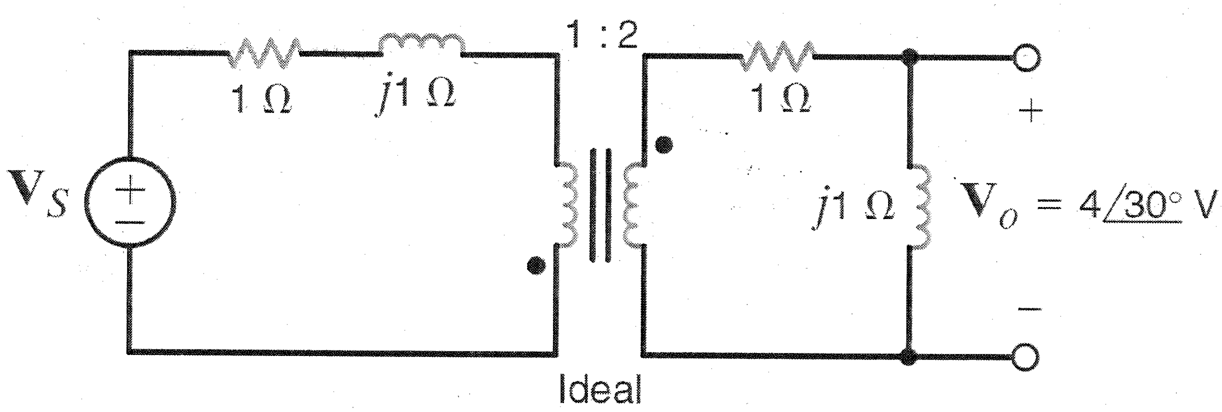
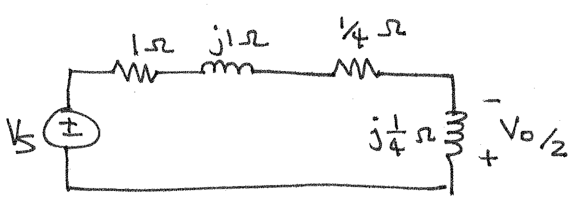


Figure P10.69

SOLUTION:



$$-\frac{V_o/2}{V_s} = \frac{j\,1/4}{1+\frac{1}{4}+j(1+1/4)} = 0.1 + j0.1$$

$$V_s = \frac{-2\angle 30}{0.1 + j0.1}$$

$$\boxed{V_s = 14.14\angle 165^\circ\ V}$$

10.70 Determine $\mathbf{I}_S$ in the circuit in Fig. P10.70. **PSV**

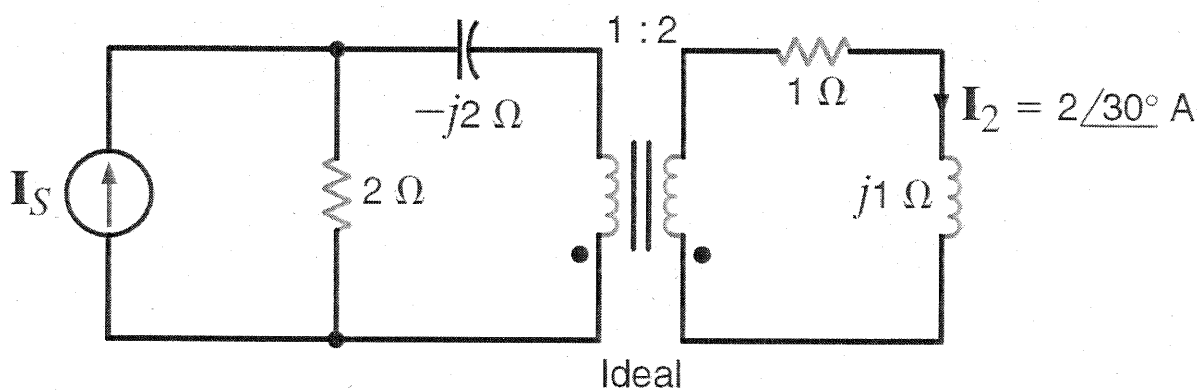
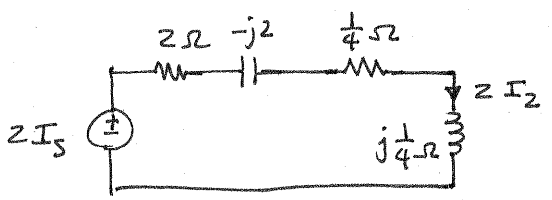


Figure P10.70

SOLUTION:



$$2I_2 = \frac{2I_s}{2.25 - j1.75} = 4\angle 30°$$

$$I_s = 2\angle 30° \,(2.25 - j1.75)$$

$$\boxed{I_s = 5.70\angle -7.87° \text{ A}}$$

10.71 Given that $\mathbf{V}_o = 48\,\underline{/30^\circ}$ V in the circuit shown in Fig. P10.71, determine $\mathbf{V}_S$. CS

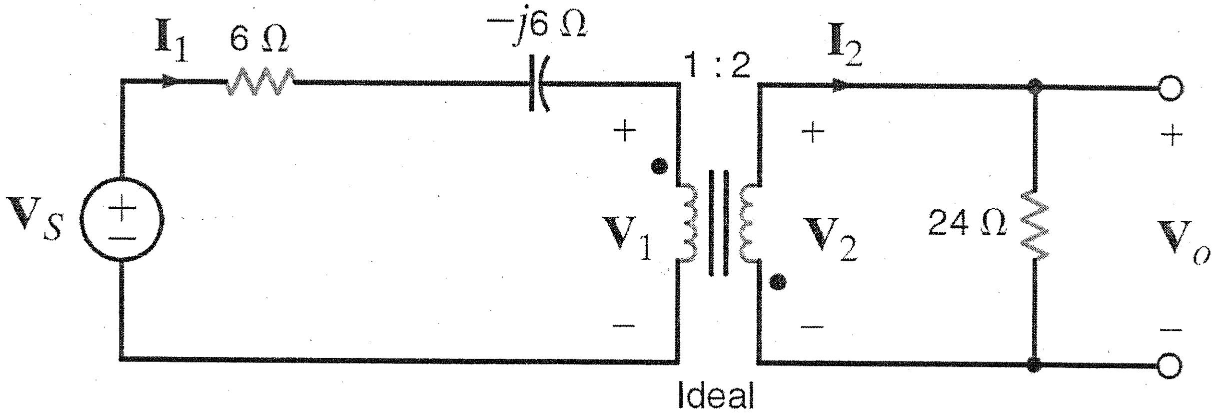


Figure P10.71

SOLUTION:

$V_2 = V_o = 48\,\underline{/30^\circ}$ $\quad V_1 = -\frac{V_2}{2} = -24\underline{/30^\circ} = 24\,\underline{/-150^\circ}\text{ V}$

$I_2 = V_o/24 = 2\,\underline{/30^\circ}\text{ A}$ $\quad I_1 = -2\,I_2 = -4\,\underline{/30^\circ}\text{ A} = 4\,\underline{/-150^\circ}\text{ A}$

$V_s = I_1\,(6 - j6) + V_1$

$$\boxed{V_s = 53.7\,\underline{/-177^\circ}\text{ V}}$$

10.72 In the circuit in Fig. P10.72, if $\mathbf{I}_x = 4\underline{/30°}$ A, find $\mathbf{V}_o$.

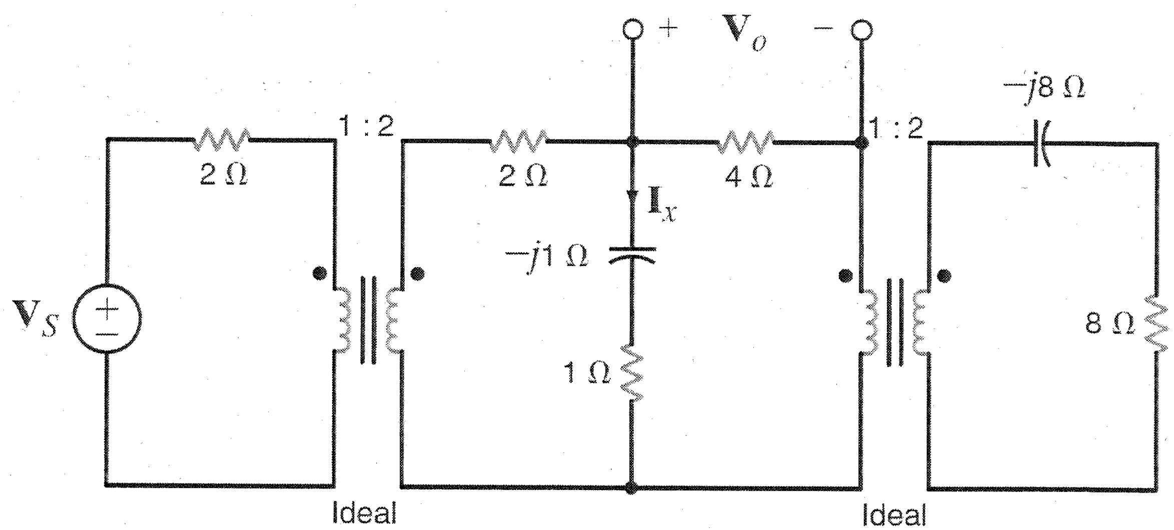
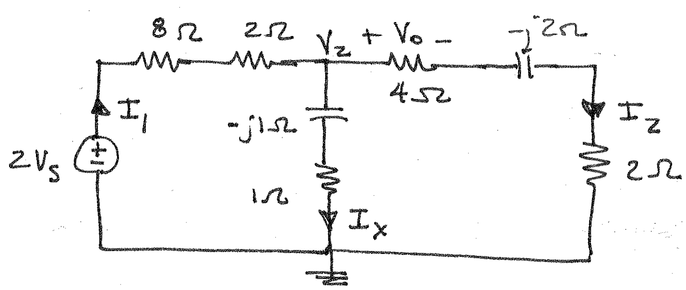


Figure P10.72

SOLUTION:



$V_2 = I_x(1-j1) = 4\sqrt{2}\,\angle{-15°}\,V$

$I_2 = \dfrac{V_2}{6-j2} = 0.894\,\underline{/3.43°}\,A$

$\boxed{V_o = 4I_2 = 3.58\,\underline{/3.43°}\,V}$

10.73 In the network in Fig. P10.73, if $\mathbf{I}_1 = 6\angle 0°$ A, find $\mathbf{V}_S$.

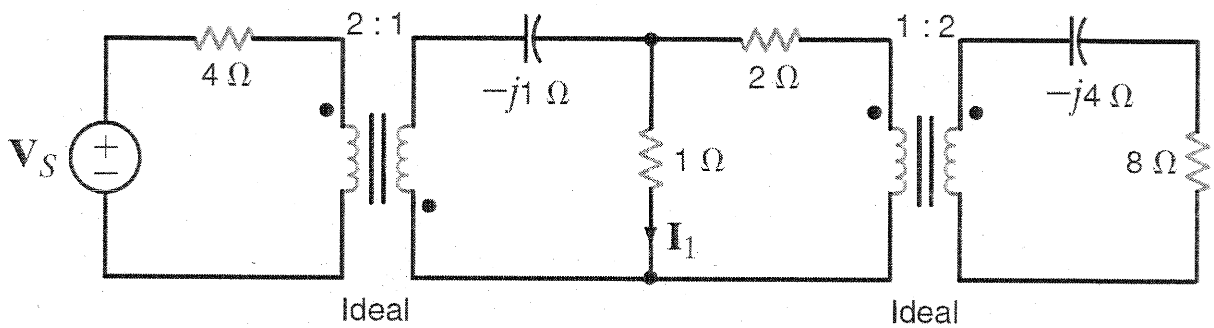
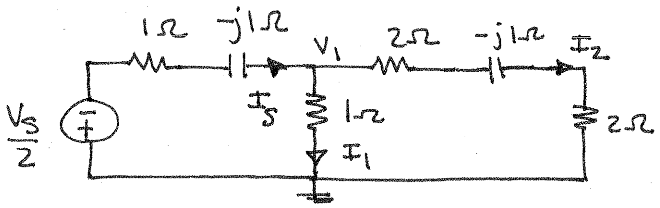


Figure P10.73

SOLUTION:



$V_1 = (1) I_1 = 6\angle 0°$ V

$I_2 = V_1 / (4 - j1) = 1.46\angle 14.0°$ A

$I_S = I_2 + I_1 = 7.42\angle 2.73°$ A

$-\frac{V_S}{2} = I_S(1 - j1) + V_1 = 15.5\angle -27.1°$ V

$\boxed{V_S = 30.9\angle 153° \text{ V}}$

10.74 For maximum power transfer, we desire to match the impedance of the inverting amplifier stage in Fig. P10.74 to the 50-Ω equivalent resistance of the ac input source. However, standard op-amps perform best when the resistances around them are at least a few hundred ohms. The gain of the op-amp circuit should be −10. Design the complete circuit by selecting resistors no smaller than 1 kΩ and specifying the turns ratio of the ideal transformer to satisfy both the gain and impedance matching requirements.

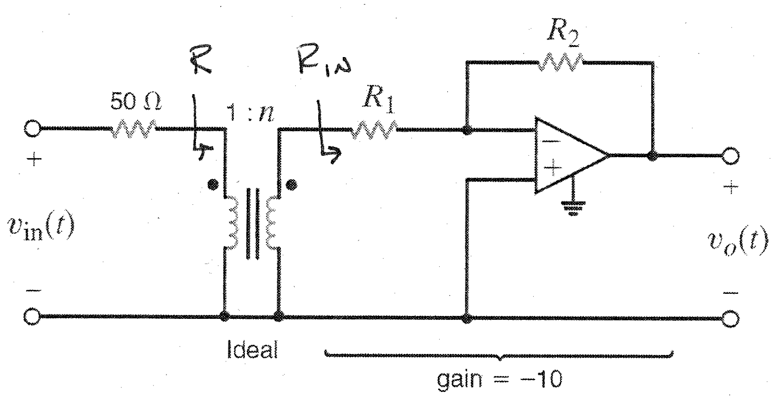


Figure P10.74

SOLUTION:

gain = $-10 = -R_2/R_1$ arbitrarily select $R_1 = 5\,k\Omega$

yields $R_2 = 50\,k\Omega$

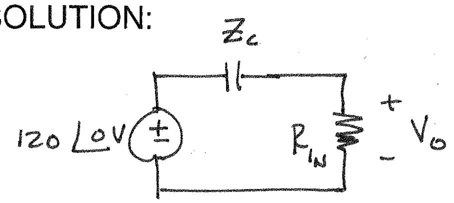
for op amp circuits $R_{IN} = R_1 = 5\,k\Omega$

$R = R_{IN}/n^2 \Rightarrow n^2 = 5000/50 = 100 \Rightarrow n = 10$

$$\boxed{R_1 = 5\,k\Omega \quad R_2 = 50\,k\Omega \quad n = 10}$$

10.75 Digital clocks often divide a 60-Hz frequency signal to obtain a 1-second, 1-minute, or 1-hour signal. A convenient source of this 60-Hz signal is the power line. However, 120 volts is too high to be used by the low-power electronics. Instead, a 3-V, 60-Hz signal is needed. If a resistive voltage divider is used to drop the voltage from 120 V to 3 V, the heat generated will be unacceptable. In addition, it is costly to use a transformer in this application. Digital clocks are consumer items and must be very inexpensive to be a competitive product. The problem then is to design a circuit that will produce between 2.5 V and 3 V at 60 Hz from the 120-V ac power line without dissipating any heat or the use of a transformer. The design will interface with a circuit that has an input resistance of 1200 ohms.

SOLUTION:



$|V_o| = 3\,V \qquad R_{IN} = 1.2\,k\Omega$

$$V_o = \frac{120\angle 0\, R_{IN}}{R_{IN} + Z_C} \Rightarrow |V_o| = 3 = \frac{120\,(1200)}{\sqrt{1200^2 + Z_C^2}}$$

$$Z_C^2 = 2.30 \times 10^9 \Rightarrow Z_C = -j47.98\,k\Omega$$

$$Z_C = -j/\omega C = -j47.98\,k\Omega \qquad \boxed{C = 55.3\,nF}$$

SOLUTION:

10FE-1 In the network in Fig. 10PFE-1, find the impedance seen by the source. CS

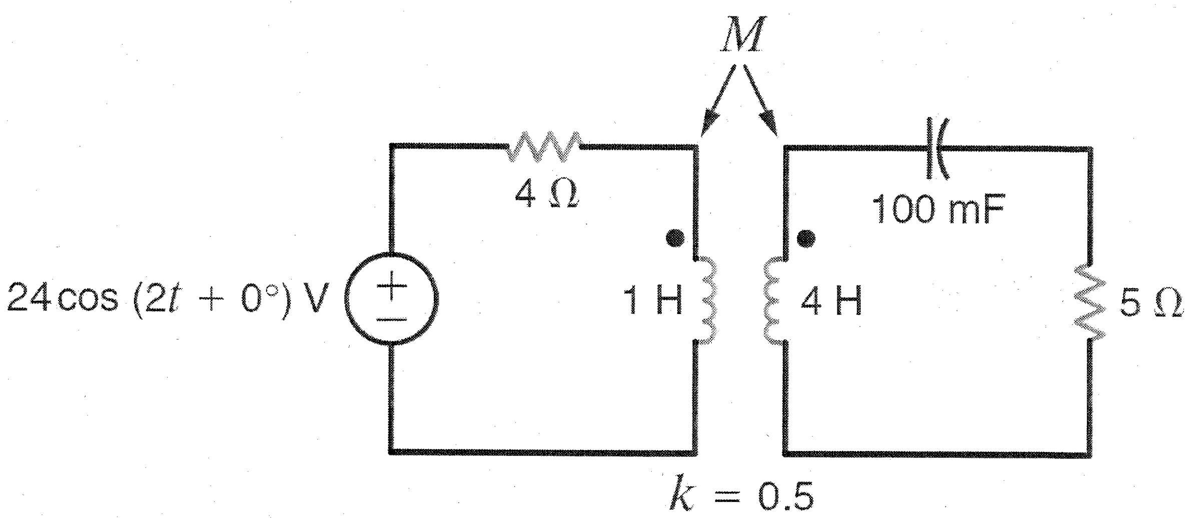
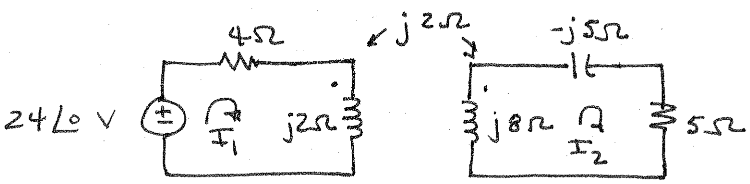


Figure 10PFE-1

SOLUTION: $\omega = 2\ r/s$ $M = k\sqrt{L_1 L_2} = 1\ H$



$$24\angle 0 = I_1(4+j2) - j2I_2$$

$$0 = -j2I_1 + I_2(5+j3)$$

$$\Rightarrow \begin{bmatrix} 4+j2 & -j2 \\ -j2 & 5+j3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \end{bmatrix}$$

$I_1 = 4.92\angle -19.7°\ A$

$$Z_{IN} = \frac{24\angle 0}{I_1}$$

$$\boxed{Z_{IN} = 4.87\angle 19.7°\ \Omega}$$

10FE-2 In the circuit in Fig. 10PFE-2, select the value of the transformer's turns ratio $n = N_2/N_1$ to achieve impedance matching for maximum power transfer. Using this value of n, calculate the power absorbed by the 3-Ω resistor.

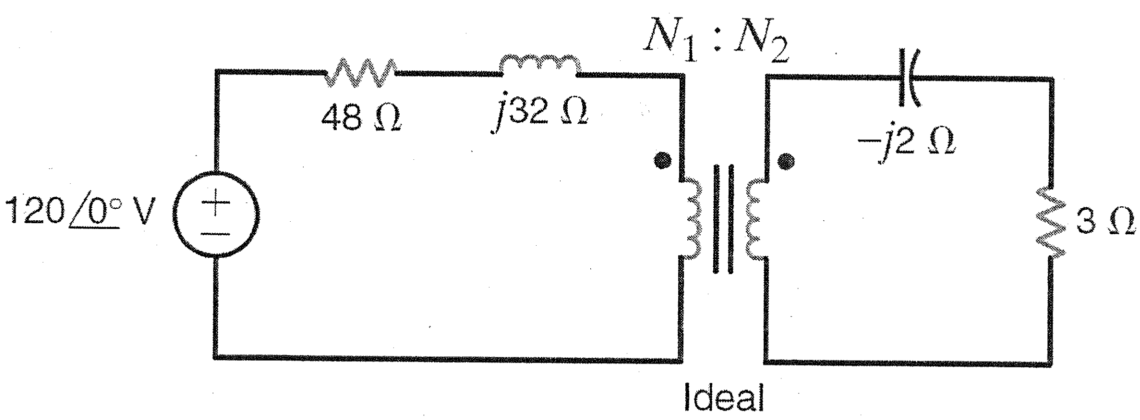
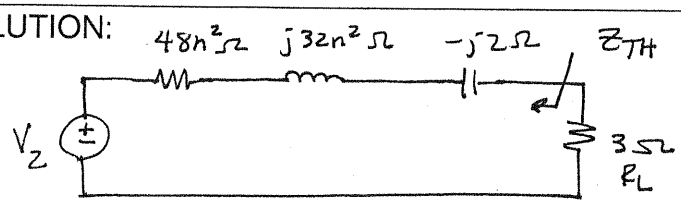


Figure 10PFE-2

SOLUTION:

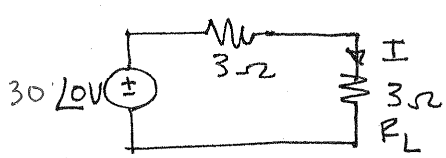


$V_2 = 120n\,\angle 0°\ V$

For max. power transfer, $Z_{TH} = 3\,\Omega$

$Z_{TH} = 48n^2 + j\,(32n^2 - 2) = 3$

if $n = 1/4$, $Z_{TH} = 48/16 + j\,(32/16 - 2) = 3\,\Omega$ ✓



$I = \dfrac{30\angle 0}{6} = 5\,\angle 0\ A$

$P_L = \frac{1}{2} I_M^2 R_L$

$\boxed{P_L = 37.5\ W}$

$\boxed{n = 1/4}$

10FE-3 In the circuit in Fig. 10FE-3, select the turns ratio of the ideal transformer that will match the output of the transistor amplifier to the speaker represented by the 16-Ω load. CS

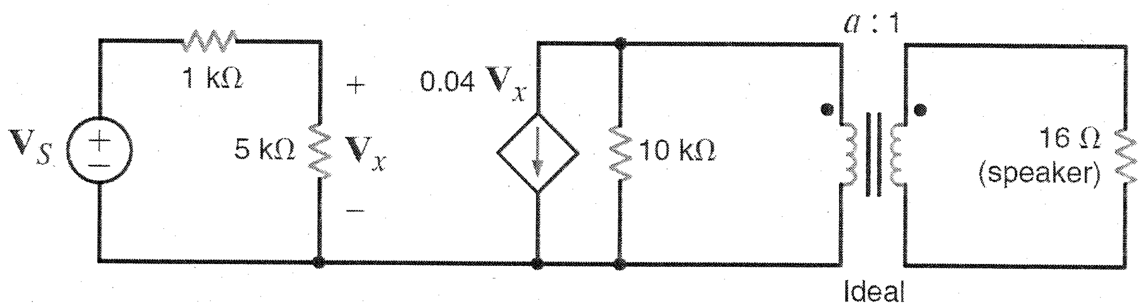
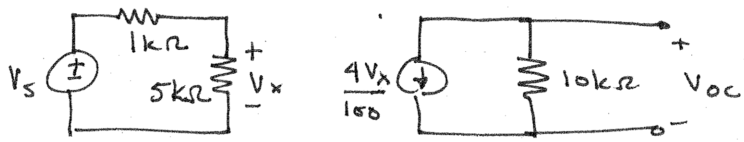


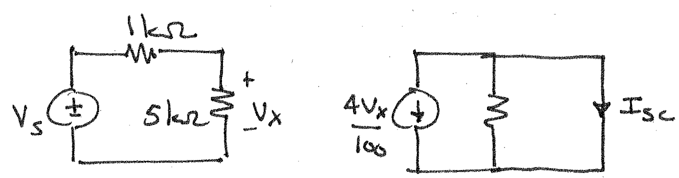
Figure 10PFE-3

SOLUTION:

Find R_{out} of amplifier



$$V_{oc} = \left(\frac{5}{6}V_s\right)\left(-\frac{4}{100}\right)\left(10^4\right)$$



$$I_{sc} = \left(\frac{5}{6}V_s\right)\left(-\frac{4}{100}\right)$$

$$R_{OUT} = V_{oc}/I_{sc} = 10\,k\Omega$$

$$16a^2 = 10^4 \quad \Rightarrow \quad \boxed{a = 25}$$