Chapter Eleven:

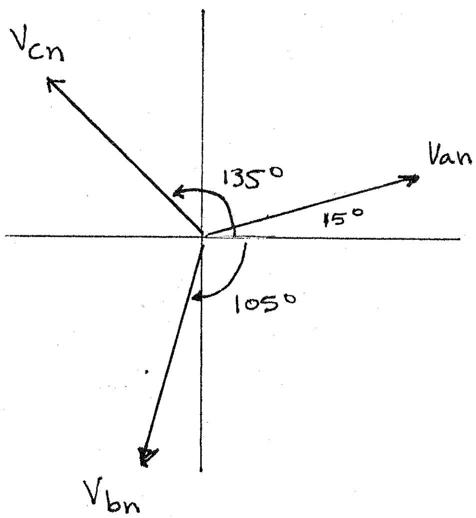
Polyphase Circuits

11.1 Sketch a phasor representation of an *abc*-sequence balanced three-phase Y-connected source, including $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$ if $\mathbf{V}_{an} = 120\,\underline{/15^\circ}$ V rms.

SOLUTION:

$V_{an} = 120\underline{/15^\circ}$ V rms $\quad V_{bn} = 120\,\underline{/-105^\circ}$ V rms

$V_{cn} = 120\,\underline{/135^\circ}$ V rms



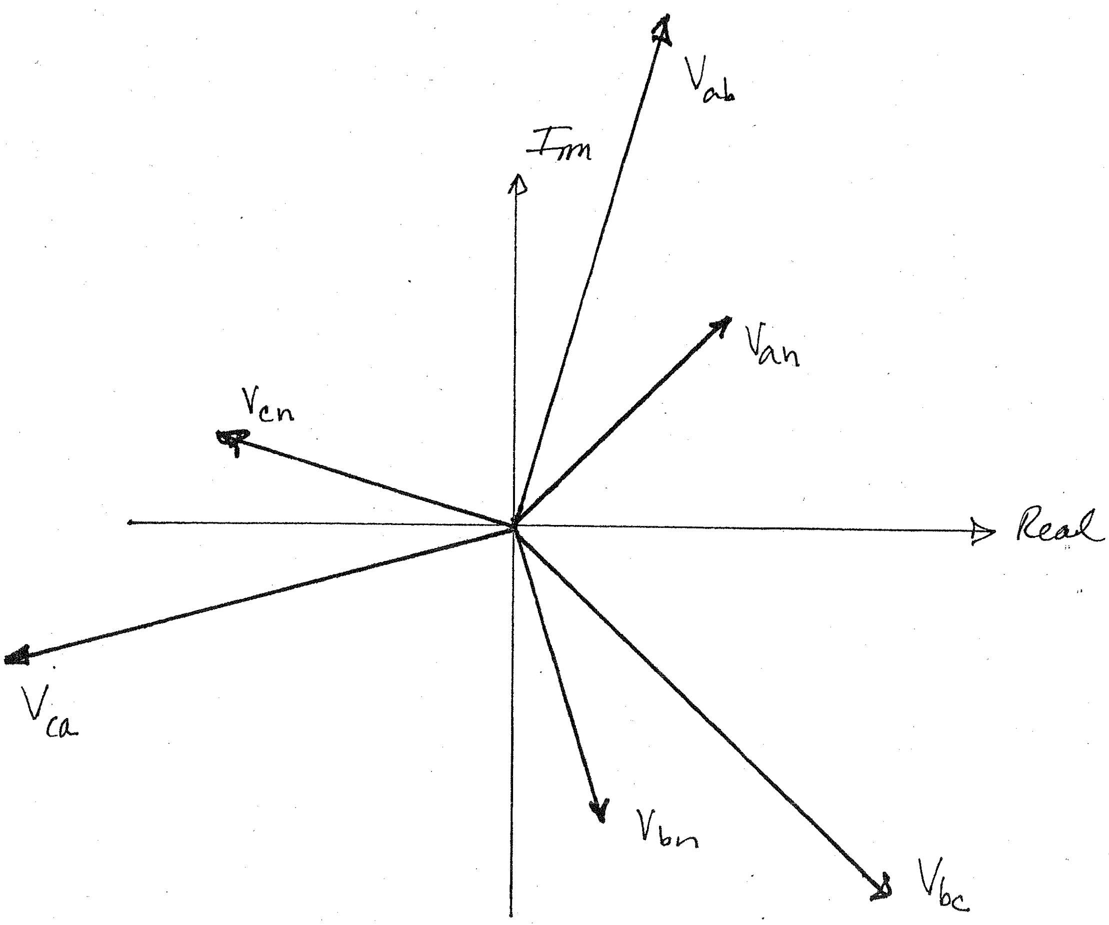
11.2 Sketch a phasor representation of a balanced three-phase system containing both phase voltages and line voltages if $\mathbf{V}_{an} = 100\angle 45°$ V rms. Label all magnitudes and assume an *abc*-phase sequence. CS

SOLUTION:

$V_{an} = 100\angle 45°$ Vrms $\quad V_{bn} = 100\angle -75°$ Vrms $\quad V_{cn} = 100\angle 165°$ Vrms

$V_{ab} = 100\sqrt{3}\angle 75°$ Vrms $= 173\angle 75°$ Vrms $\quad V_{bc} = 173\angle -45°$ Vrms

$V_{ca} = 173\angle -165°$ Vrms



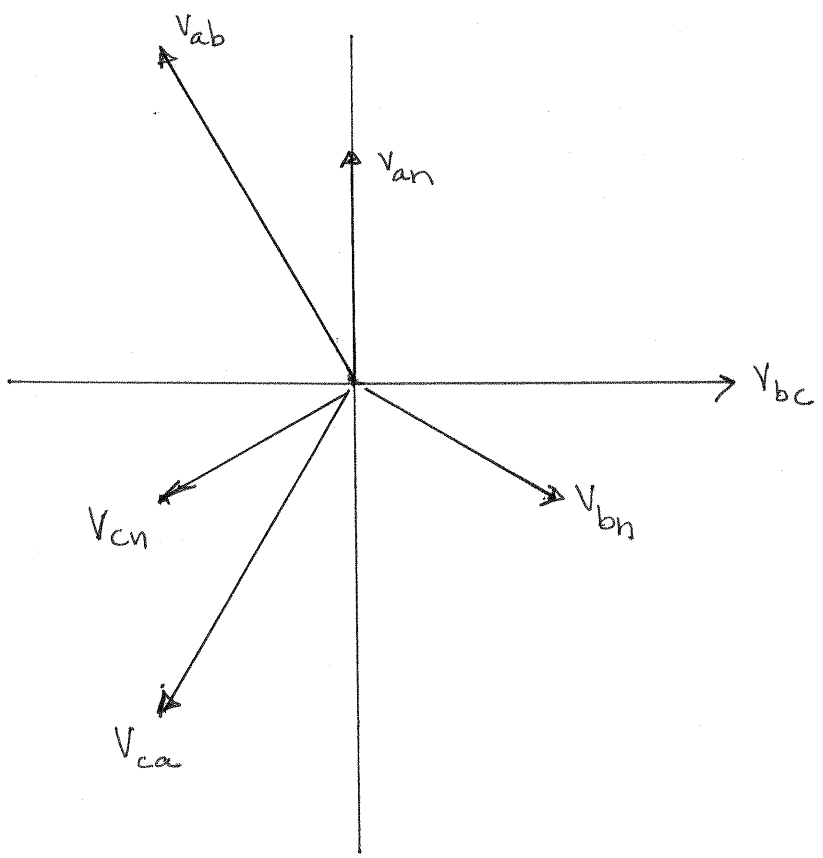
11.3 Sketch a phasor representation of a balanced three-phase system containing both phase voltages and line voltages if $\mathbf{V}_{an} = 120\angle 90^\circ$ V rms. Label all magnitudes and assume an *abc*-phase sequence.

SOLUTION:

$V_{an} = 120\angle 90^\circ\ V_{rms}$ $\quad V_{bn} = 120\angle -30^\circ\ V_{rms}$ $\quad V_{cn} = 120\angle -150^\circ\ V_{rms}$

$V_{ab} = V_{an} - V_{bn} = 120\sqrt{3}\angle 120^\circ\ V_{rms}$ $\quad V_{bc} = 120\sqrt{3}\angle 0^\circ\ V_{rms}$

$V_{ca} = 120\sqrt{3}\angle -120^\circ\ V_{rms}$



11.4 A positive-sequence three-phase balanced wye voltage source has a phase voltage of $\mathbf{V}_{an} = 240\angle 90^\circ$ V rms. Determine the line voltages of the source. CS

SOLUTION:

$V_{an} = 240\angle 90^\circ \, V_{rms} \Rightarrow V_{bn} = 240\angle -30^\circ \, V_{rms} \quad V_{cn} = 240\angle -150^\circ \, V_{rms}$

$V_{ab} = |V_{an}|\sqrt{3}\angle \theta_{V_{an}} + 30^\circ = 240\sqrt{3}\angle 120^\circ \, V_{rms}$

$V_{bc} = 240\sqrt{3}\angle 0^\circ \, V_{rms}$

$V_{ca} = 240\sqrt{3}\angle -120^\circ \, V_{rms}$

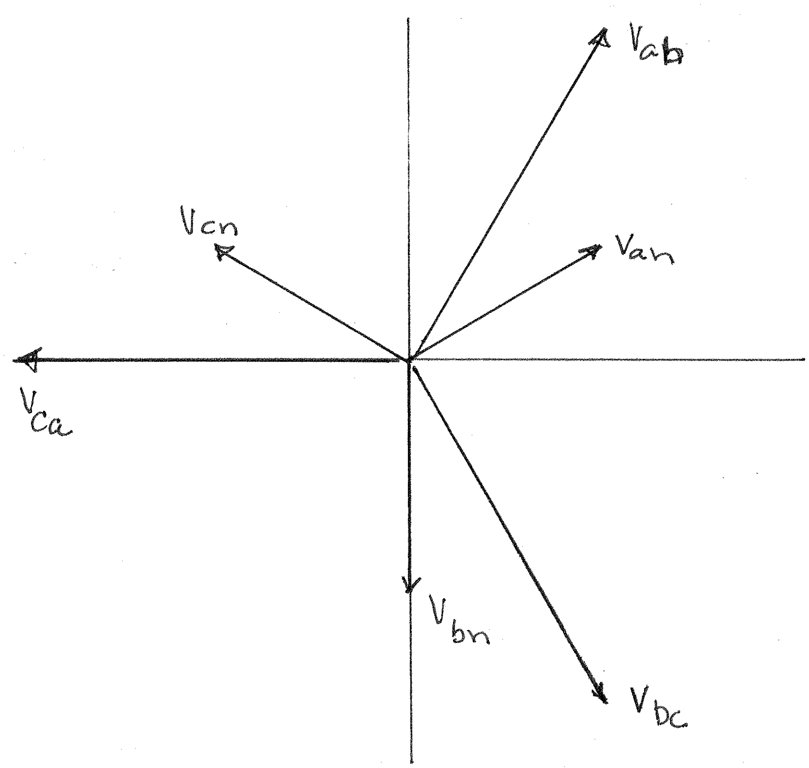
11.5 Sketch a phasor representation of a balanced three-phase system containing both phase voltages and line voltages if $\mathbf{V}_{ab} = 208\angle 60^\circ$ V rms. Label all phasors and assume an *abc*-phase sequence.

SOLUTION:

$V_{ab} = 208\angle 60^\circ\ V_{rms}$ $\quad V_{bc} = 208\angle -60^\circ\ V_{rms}$ $\quad V_{ca} = 208\angle 180^\circ\ V_{rms}$

$$V_{an} = \frac{|V_{ab}|}{\sqrt{3}} \angle \theta_{V_{ab}} - 30^\circ = 120\angle 30^\circ\ V_{rms}$$

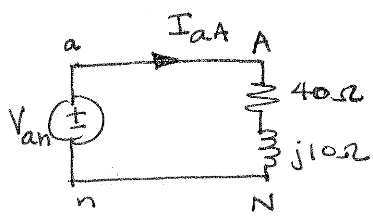
$V_{bn} = 120\angle -90^\circ\ V_{rms}$ $\quad V_{cn} = 120\angle 150^\circ\ V_{rms}$



11.6 A positive-sequence balanced three-phase wye-connected source with a phase voltage of 120 V rms supplies power to a balanced wye-connected load. The per phase load impedance is $40 + j10\ \Omega$. Determine the line currents in the circuit if $\angle \mathbf{V}_{an} = 0°$. **PSV**

SOLUTION:

Per phase Y circuit



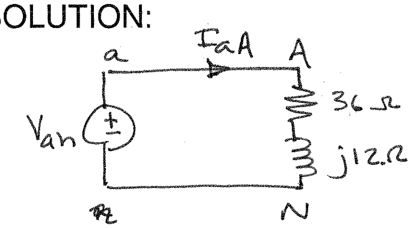
$V_{an} = 120 \angle 0°$ Vrms

$$I_{aA} = \frac{V_{an}}{40 + j10} = 2.91 \angle -14.0° \text{ Arms}$$

$$\boxed{\begin{aligned} I_{aA} &= 2.91 \angle -14.0° \text{ Arms} \\ I_{bB} &= 2.91 \angle -134° \text{ Arms} \\ I_{cC} &= 2.91 \angle 106° \text{ Arms} \end{aligned}}$$

11.7 A positive-sequence balanced three-phase wye-connected source supplies power to a balanced wye-connected load. The magnitude of the line voltages is 208 V rms. If the load impedance per phase is $36 + j12\ \Omega$, determine the line currents if $\underline{/\mathbf{V}_{an}} = 0°$.

SOLUTION:



Per phase Y circuit

$|V_{ab}| = 208\,V_{rms}$

$|V_{an}| = |V_{ab}|/\sqrt{3} = 120\,V_{rms}$

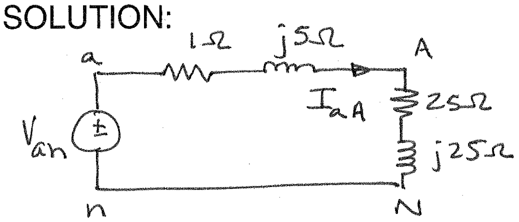
$V_{an} = 120\angle 0°\ V_{rms}$

$I_{aA} = \dfrac{V_{an}}{36+j12} = 3.16\angle -18.4°\ A_{rms}$

$$\boxed{\begin{aligned} I_{aA} &= 3.16\angle -18.4°\ A_{rms} \\ I_{bB} &= 3.16\angle -138°\ A_{rms} \\ I_{cC} &= 3.16\angle 102°\ A_{rms} \end{aligned}}$$

11.8 An *abc*-sequence balanced three-phase wye-connected source supplies power to a balanced wye-connected load. The line impedance per phase is $1 + j5\ \Omega$, and the load impedance per phase is $25 + j25\ \Omega$. If the source line voltage $\mathbf{V}_{ab}$ is $208\,\underline{/0^\circ}$ V rms, find the line currents.

SOLUTION:



$V_{ab} = 208\,\underline{/0^\circ}\ V_{rms}$

$V_{an} = 120\,\underline{/-30^\circ}\ V_{rms}$

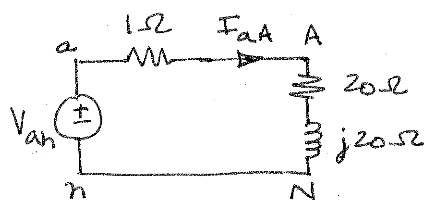
$$I_{aA} = \frac{V_{an}}{26 + j30} = 3.02\,\underline{/-79.1^\circ}\,A_{rms}$$

$$\boxed{\begin{aligned} I_{aA} &= 3.02\,\underline{/-79.1^\circ}\ A_{rms} \\ I_{bB} &= 3.02\,\underline{/161^\circ}\ A_{rms} \\ I_{cC} &= 3.02\,\underline{/40.9^\circ}\ A_{rms} \end{aligned}}$$

11.9 An *abc*-sequence balanced three-phase wye-connected source supplies power to a balanced wye-connected load. The line impedance per phase is $1 + j0\ \Omega$, and the load impedance per phase is $20 + j20\ \Omega$. If the source line voltage $\mathbf{V}_{ab}$ is $100\angle 0°$ V rms, find the line currents. CS

SOLUTION:

Per phase Y circuit



$$V_{ab} = 100 \angle 0° \text{ Vrms}$$

$$V_{an} = \frac{100}{\sqrt{3}} \angle -30° \text{ Vrms}$$

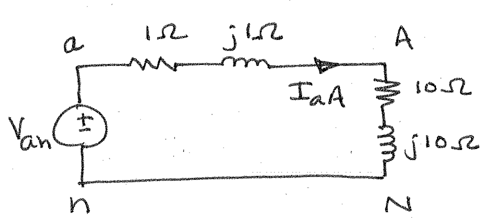
$$I_{aA} = \frac{V_{an}}{21 + j20} = 2.00 \angle -73.6° \text{ Arms}$$

$$\boxed{\begin{aligned} I_{aA} &= 2.00 \angle -73.6° \text{ Arms} \\ I_{bB} &= 2.00 \angle 166° \text{ Arms} \\ I_{cC} &= 2.00 \angle 46.4° \text{ Arms} \end{aligned}}$$

11.10 An *abc*-sequence set of voltages feeds a balanced three-phase wye–wye system. The line and load impedances are $1 + j1\ \Omega$ and $10 + j10\ \Omega$, respectively. If the load voltage on the *a* phase is $\mathbf{V}_{AN} = 110\underline{/30^\circ}$ V rms, determine the line voltages of the input. **PSV**

SOLUTION:

Per phase Y circuit



$V_{AN} = 110\ \underline{/30^\circ}$ Vrms

$$V_{AN} = V_{an}\left[\frac{10+j10}{11+j11}\right]$$

$V_{an} = 121\ \underline{/30^\circ}$ Vrms

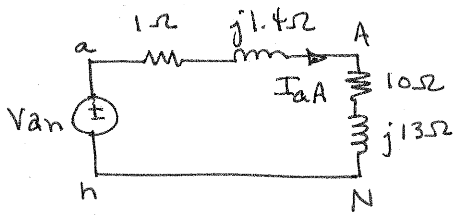
$V_{ab} = (V_{an})(\sqrt{3}\ \underline{/30^\circ}) = 210\ \underline{/60^\circ}$ Vrms

$$\boxed{\begin{aligned} V_{ab} &= 210\ \underline{/60^\circ}\ \text{Vrms} \\ V_{bc} &= 210\ \underline{/-60^\circ}\ \text{Vrms} \\ V_{ca} &= 210\ \underline{/180^\circ}\ \text{Vrms} \end{aligned}}$$

11.11 In a balanced three-phase wye–wye system, the source is an *abc*-sequence set of voltages. The load voltage on the *a* phase is $\mathbf{V}_{AN} = 110\angle 80^\circ$ V rms, $\mathbf{Z}_{\text{line}} = 1 + j1.4\ \Omega$, and $\mathbf{Z}_{\text{load}} = 10 + j13\ \Omega$. Determine the input sequence of the line-to-neutral voltages.

SOLUTION:

Per phase Y circuit



$V_{AN} = 110\angle 80^\circ$ Vrms

$$\frac{V_{AN}}{V_{an}} = \frac{10 + j13}{11 + j14.4} = 0.905\angle -0.19^\circ$$

$V_{an} = 122\angle 80.2^\circ$ Vrms

$$\boxed{\begin{array}{l} V_{an} = 122\angle 80.2^\circ \text{ Vrms} \\ V_{bn} = 122\angle -39.8^\circ \text{ Vrms} \\ V_{cn} = 122\angle -160^\circ \text{ Vrms} \\ abc \text{ sequence} \end{array}}$$

11.12 Find the equivalent impedances $\mathbf{Z}_{ab}$, $\mathbf{Z}_{bc}$, and $\mathbf{Z}_{ca}$ in the network in Fig. P11.12.

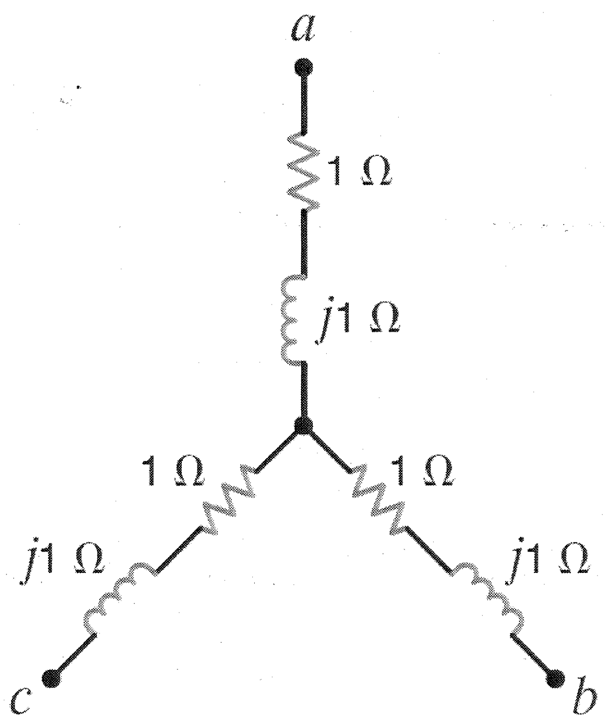


Figure P11.12

SOLUTION:

$Z_{an} = Z_{bn} = Z_{cn} = 1 + j1\,\Omega = Z_Y$

$Z_\Delta = 3 Z_Y = 3 + j3\,\Omega$

$\boxed{Z_{ab} = Z_{bc} = Z_{ca} = 3 + j3\,\Omega}$

11.13 Find the equivalent **Z** of the network in Fig. P11.13.

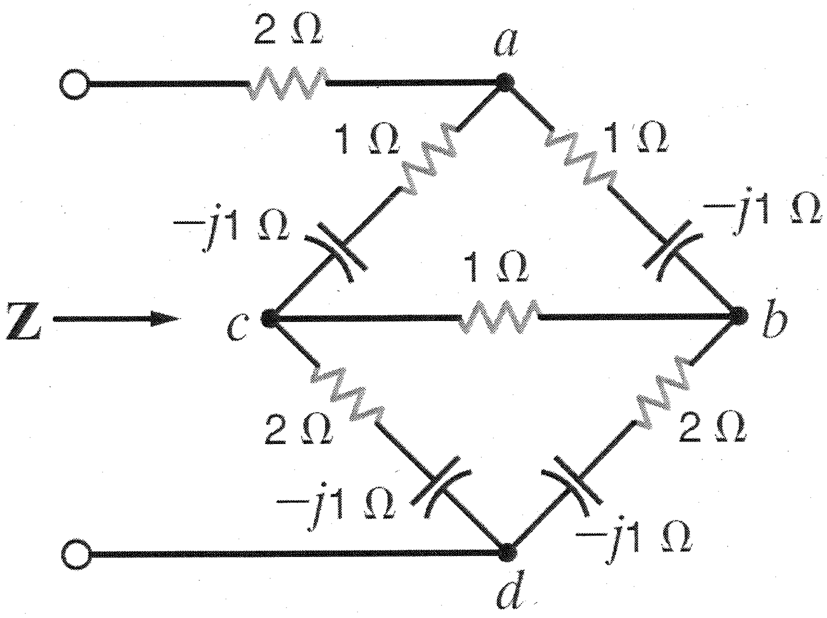
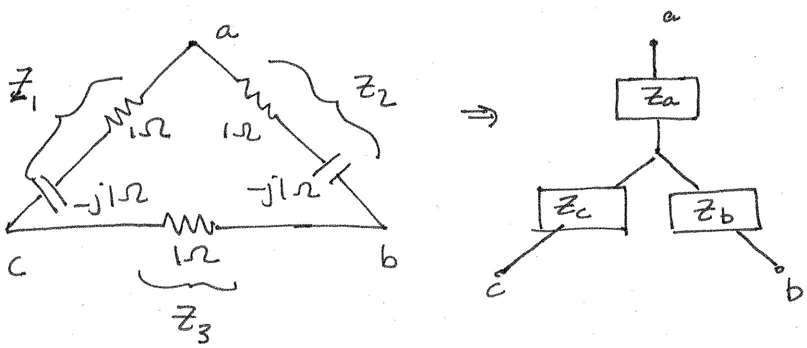


Figure P11.13

SOLUTION:

Use a $\Delta \Rightarrow Y$ transform



$Z_1 = Z_2 = 1 - j1\,\Omega$

$Z_3 = 1\,\Omega$

$$Z_a = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

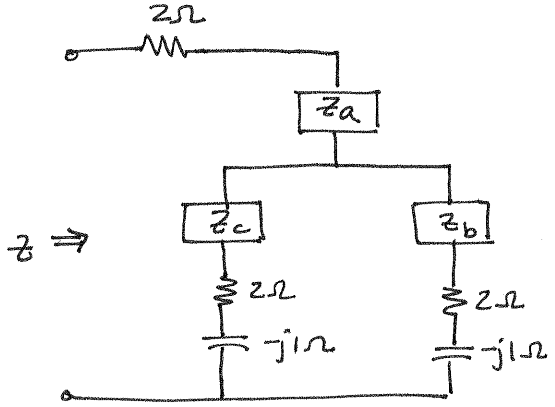
$$Z_a = \frac{4 - j6}{13}\,\Omega$$

$$Z_b = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_b = \frac{5 - j1}{13}\,\Omega$$

$$Z_c = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_c = \frac{5 - j1}{13}\,\Omega$$



$$Z_1 = Z_c + 2 - j1\,\Omega = \frac{31 - j14}{13}\,\Omega$$

$$Z_2 = Z_b + 2 - j1\,\Omega = \frac{31 - j14}{13}\,\Omega$$

$$Z_3 = \frac{Z_1 Z_2}{Z_1 + Z_2} = 1.31\angle -24.3^\circ\,\Omega$$

$$Z = 2 + Z_a + Z_3 = 3.5 - j1\,\Omega$$

$$\boxed{Z = 3.5 - j1\,\Omega}$$

11.14 Find the equivalent **Z** of the network in Fig. P11.14.

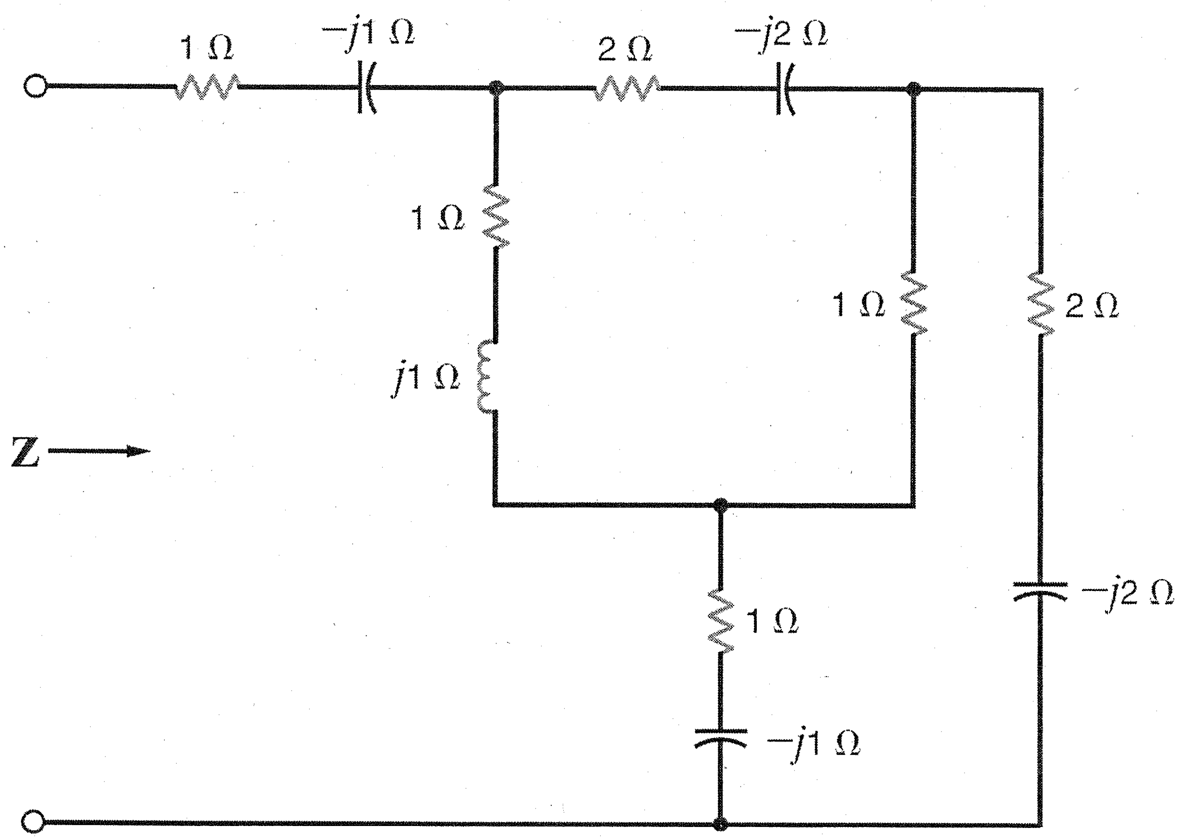
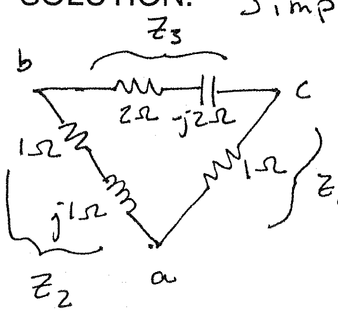
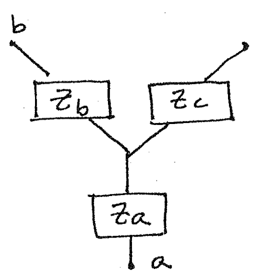


Figure P11.14

SOLUTION: Simplify using $\Delta \Rightarrow Y$ transformation.



$\Rightarrow$

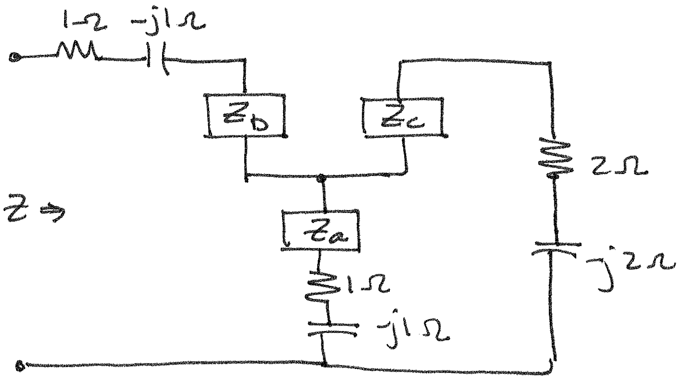


$$Z_a = \frac{Z_1 Z_2}{Z_1+Z_2+Z_3} = 0.343\angle 59^\circ\ \Omega$$

$$Z_b = \frac{Z_2 Z_3}{Z_1+Z_2+Z_3} = 0.97\angle 14^\circ\ \Omega$$

$$Z_c = \frac{Z_1 Z_3}{Z_1+Z_2+Z_3} = 0.686\angle -31^\circ\ \Omega$$

$Z_1 = 1\Omega$ $\quad Z_2 = 1+j1\ \Omega$ $\quad Z_3 = 2-j2\Omega$



Let $Z_x = Z_a + 1 - j1 = 1.37 \angle -31.0° \ \Omega$

$Z_y = Z_c + 2 - j2 = 3.50 \angle -42.3° \ \Omega$

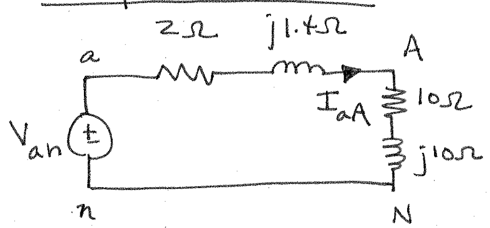
$Z_z = Z_x \,//\, Z_y = 0.989 \angle -34.1° \ \Omega$

$Z = 1 - j1 + Z_b + Z_z$ $\boxed{Z = 3.06 \angle -34.1° \ \Omega}$

11.15 In a balanced three-phase wye–wye system, the source is an *abc*-sequence set of voltages. The load voltage on the *a* phase is $\mathbf{V}_{AN} = 120\,\underline{/60^\circ}$ V rms, $\mathbf{Z}_{\text{line}} = 2 + j1.4\ \Omega$, and $\mathbf{Z}_{\text{load}} = 10 + j10\ \Omega$. Determine the input sequence of voltages.

SOLUTION:

Per phase Y circuit



$$V_{AN} = 120\,\underline{/60^\circ} = V_{an}\frac{(10+j10)}{12+j11.4}$$

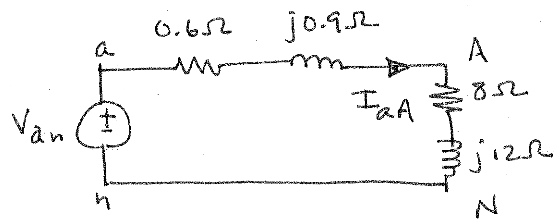
$$\boxed{\begin{aligned} V_{an} &= 140\,\underline{/58.5^\circ}\ V_{rms} \\ V_{bn} &= 140\,\underline{/-61.5^\circ}\ V_{rms} \\ V_{cn} &= 140\,\underline{/178.5^\circ}\ V_{rms} \end{aligned}}$$

11.16 A balance *abc*-sequence of voltages feeds a balanced three-phase wye–wye system. The line and load impedances are $0.6 + j0.9\ \Omega$ and $8 + j12\ \Omega$, respectively. The load voltage on the *a* phase is $\mathbf{V}_{AN} = 116.63\angle 10^\circ$ V rms.

Find the line voltage $\mathbf{V}_{ab}$. CS

SOLUTION:

Per phase Y circuit



$V_{AN} = 116.63\angle 10^\circ$ V_{rms}

$$\frac{V_{AN}}{V_{an}} = \frac{8 + j12}{8.6 + j12.9}$$

$V_{an} = 125.4\angle 10^\circ$ V_{rms}

$V_{ab} = V_{an}\left(\sqrt{3}\angle 30^\circ\right)$

$V_{ab} = 217\angle 40^\circ$ V_{rms}

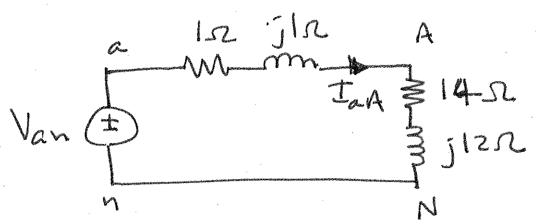
$V_{bc} = 217\angle -80^\circ$ V_{rms}

$V_{ca} = 217\angle 160^\circ$ V_{rms}

11.17 In a balanced three-phase wye–wye system, the source is an *abc*-sequence set of voltages. $\mathbf{Z}_{\text{line}} = 1 + j1\ \Omega$, $\mathbf{Z}_{\text{load}} = 14 + j12\ \Omega$, and the load voltage on the *a* phase is $\mathbf{V}_{AN} = 440\angle 30^\circ$ V rms. Find the line voltage $\mathbf{V}_{ab}$.

SOLUTION:

Per phase Y



$V_{AN} = 440\angle 30^\circ$ Vrms

$$\frac{V_{AN}}{V_{an}} = \frac{14+j12}{15+j13}$$

$V_{an} = 474\angle 30.3^\circ$ Vrms

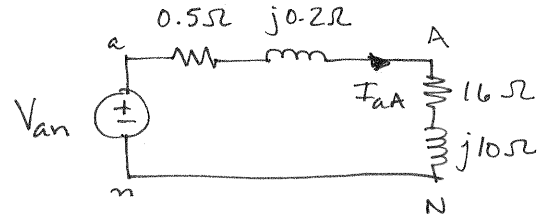
$V_{ab} = V_{an}\left(\sqrt{3}\angle 30^\circ\right)$

$$\boxed{V_{ab} = 821\angle 60.3^\circ \text{ Vrms}}$$

11.18 An *abc*-phase sequence balanced three-phase source feeds a balanced load. The system is connected wye–wye and $\angle \mathbf{V}_{an} = 0°$. The line impedance is $0.5 + j0.2\ \Omega$, the load impedance is $16 + j10\ \Omega$, and the total power absorbed by the load is 2000 W. Determine the magnitude of the source voltage $\mathbf{V}_{an}$.

SOLUTION:

Per phase Y



$P_L = 2000 = |I_{aA}|^2(16)$

$|I_{aA}| = 11.2$ Arms

$Z_{EQ} = 16 + j10 + 0.5 + j0.2$

$= 16.5 + j10.2\ \Omega$

$= 19.4\angle 31.7°\ \Omega$

$I_{aA} = \dfrac{V_{an}}{Z_{EQ}} \Rightarrow \angle I_{aA} = \angle V_{an} - \angle Z_{EQ}$

$\angle I_{aA} = 0° - 31.7° = -31.7°$

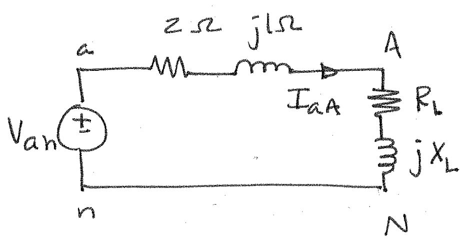
$I_{aA} = 11.2\angle -31.7°$ Arms

$V_{an} = I_{aA} Z_{EQ} \Rightarrow \boxed{V_{an} = 217.3\angle 0°\ \text{Vrms}}$

11.19 In a balanced three-phase wye–wye system, the total power loss in the lines is 400 W. $\mathbf{V}_{AN} = 105.28\,\angle 31.65^\circ$ V rms and the power factor of the load is 0.77 lagging. If the line impedance is $2 + j1\ \Omega$, determine the load impedance. **PSV**

SOLUTION:

Per phase Y



$V_{AN} = 105.28\,\angle 31.65^\circ$ Vrms

$P_\ell = 400 = |I_{aA}|^2 (R_\ell) = 2|I_{aA}|^2$

$|I_{aA}| = 14.14$ Arms

$\cos^{-1}(pf) = \theta_{V_{AN}} - \theta_{I_{aA}} = 39.6^\circ$

$\theta_{I_{aA}} = -8^\circ$

$V_{AN} = Z_L I_{aA}$

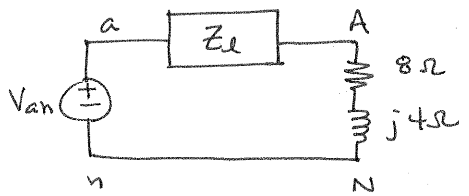
$Z_L = \dfrac{V_{AN}}{I_{aA}}$ $\Leftarrow$ $I_{aA} = 14.14\,\angle -8^\circ$ Arms

$$\boxed{Z_L = 7.45\,\angle 39.7^\circ\ \Omega}$$

11.20 In a balanced three-phase wye–wye system, the load impedance is $8 + j4\ \Omega$. The source has phase sequence *abc* and $\mathbf{V}_{an} = 120\angle 0^\circ$ V rms. If the load voltage is $\mathbf{V}_{AN} = 111.62\angle -1.33^\circ$ V rms, determine the line impedance. CS

SOLUTION:

Per phase Y



$V_{AN} = 111.62\angle -1.33^\circ\ V_{rms}$

$V_{an} = 120\angle 0^\circ\ V_{rms}$

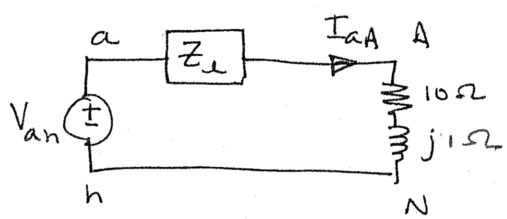
$$\frac{V_{AN}}{V_{an}} = \frac{8+j4}{8+j4+Z_\ell} = \frac{111.62\angle -1.33^\circ}{120\angle 0^\circ} = 0.930\angle -1.33^\circ$$

$$Z_L = \frac{8+j4}{0.93\angle -1.33} - 8 - j4 \qquad \boxed{Z_L = 0.5 + j0.5\,\Omega}$$

11.21 In a balanced three-phase wye–wye system, the load impedance is $10 + j1\Omega$. The source has phase sequence *abc* and the line voltage $\mathbf{V}_{ab} = 220\underline{/30^\circ}$ V rms. If the load voltage $\mathbf{V}_{AN} = 120\underline{/0^\circ}$ V rms, determine the line impedance.

SOLUTION:

Per phase Y



$V_{ab} = 220 \angle 30^\circ \ V_{rms}$

$V_{an} = \frac{220}{\sqrt{3}} \angle 0^\circ \ V_{rms}$

$V_{AN} = 120 \angle 0^\circ \ V_{rms}$

$$\frac{V_{AN}}{V_{an}} = \frac{120 \angle 0}{\frac{220}{\sqrt{3}} \angle 0} = \frac{10 + j1}{10 + j1 + Z_\ell} = 0.945$$

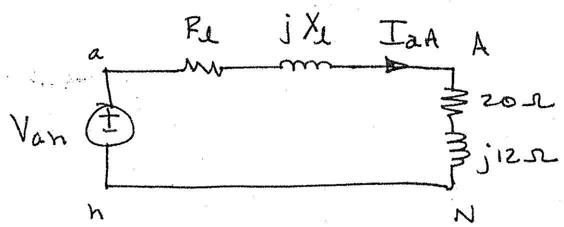
$$Z_\ell = \frac{10 + j1}{0.945} - 10 - j1$$

$$\boxed{Z_\ell = 585 + j\,58.5 \ m\Omega}$$

11.22 In a balanced three-phase wye–wye system, the load impedance is $20 + j12\ \Omega$. The source has an *abc*-phase sequence and $\mathbf{V}_{an} = 120\underline{/0^\circ}$ V rms. If the load voltage is $\mathbf{V}_{AN} = 111.49\underline{/-0.2^\circ}$ V rms, determine the magnitude of the line current if the load is suddenly short-circuited.

SOLUTION:

Per phase Y



$V_{an} = 120\underline{/0^\circ}$ Vrms

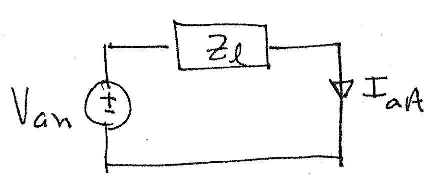
$V_{AN} = 111.49\underline{/-0.2^\circ}$ Vrms

$$\frac{V_{AN}}{V_{an}} = \frac{20+j12}{20+j12+Z_\ell} = 0.929\underline{/-0.2^\circ}$$

$$Z_\ell = \frac{20+j12}{0.929\underline{/-0.2}} - 20 - j12$$

$Z_\ell = 1.78\underline{/33.8^\circ}\ \Omega$

Shorted load



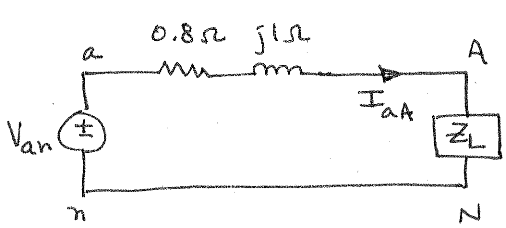
$I_{aA} = V_{an} / Z_\ell$

$$\boxed{I_{aA} = 67.4\underline{/-33.8^\circ}\ A_{rms}}$$

11.23 In a balanced three-phase wye–wye system, the source is an *abc*-sequence set of voltages and $\mathbf{V}_{an} = 120\,\underline{/40^\circ}$ V rms. If the *a*-phase line current and line impedance are known to be $7.10\,\underline{/-10.28^\circ}$ A rms and $0.8 + j1\ \Omega$, respectively, find the load impedance. CS

SOLUTION:

Per phase Y circuit



$V_{an} = 120\,\underline{/40^\circ}$ Vrms

$I_{aA} = 7.10\,\underline{/-10.28^\circ}$ Arms

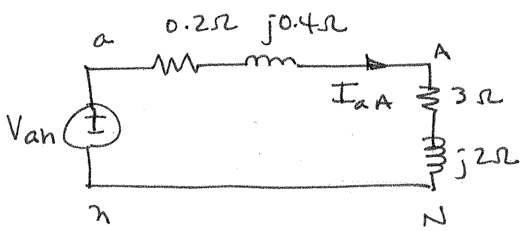
$V_{an} = I_{aA}\left[0.8 + j1 + Z_L\right]$

$$\boxed{Z_L = 10 + j12\,\Omega}$$

11.24 In a three-phase balanced system, a delta-connected source supplies power to a wye-connected load. If the line impedance is $0.2 + j0.4\ \Omega$, the load impedance $3 + j2\ \Omega$, and the source phase voltage $\mathbf{V}_{ab} = 208\angle 10^\circ$ V rms, find the magnitude of the line voltage at the load.

SOLUTION:

Per phase Y



$$V_{an} = V_{ab}\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right)$$

$$V_{an} = 120\angle -20^\circ \text{ V rms}$$

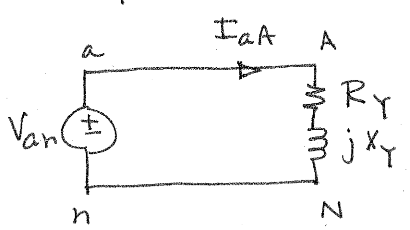
$$V_{AN} = V_{an}\frac{[3+j2]}{3.2+j2.4} \Rightarrow V_{AN} = 108\angle -23.2^\circ \text{ Vrms}$$

$$V_{AB} = V_{AN}\left(\sqrt{3}\angle 30^\circ\right) \Rightarrow \boxed{V_{AB} = 187\angle 6.8^\circ \text{ Vrms}}$$

11.25 An *abc*-phase-sequence three-phase balanced wye-connected 60-Hz source supplies a balanced delta-connected load. The phase impedance in the load consists of a 20-Ω resistor in series with a 20-mH inductor, and the phase voltage at the source is $\mathbf{V}_{an} = 120\,\underline{/30^\circ}$ V rms. If the line impedance is zero, find the line currents in the system.

SOLUTION:

Per phase Y



$R_\Delta = 20\,\Omega \qquad L_\Delta = 20\,\text{mH}$

$R_Y = R_\Delta/3 = 20/3\,\Omega \qquad L_Y = \frac{L_\Delta}{3} = \frac{20}{3}\,\text{mH}$

$Z_Y = \frac{20}{3} + j(377)\left(\frac{0.02}{3}\right) = \frac{20}{3} + j\,2.51\,\Omega$

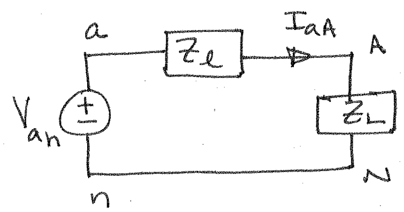
$I_{aA} = \frac{V_{an}}{Z_Y}$

$$\boxed{\begin{aligned} I_{aA} &= 16.8\,\underline{/9.37^\circ}\ \text{Arms} \\ I_{bB} &= 16.8\,\underline{/-110.6^\circ}\ \text{Arms} \\ I_{cC} &= 16.8\,\underline{/129.4^\circ}\ \text{Arms} \end{aligned}}$$

11.26 In a balanced three-phase wye–wye system, the source is an *abc*-sequence set of voltages and $\mathbf{V}_{an} = 120\,\underline{/50^\circ}$ V rms. The load voltage on the *a* phase is $110\,\underline{/50^\circ}$ V rms, and the load impedance is $16 + j20\ \Omega$. Find the line impedance.

SOLUTION:

Per phase Y



$V_{AN} = 110\,\underline{/50^\circ}\text{ V} \qquad Z_L = 16 + j20\ \Omega$

$$\frac{V_{AN}}{V_{an}} = \frac{Z_L}{Z_L + Z_\ell} = \frac{110}{120}$$

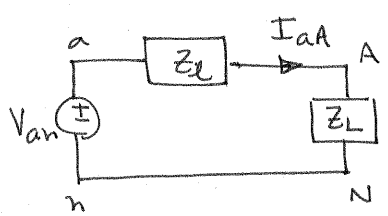
$$Z_\ell = \frac{12}{11} Z_L - Z_L = \frac{Z_L}{11}$$

$$\boxed{Z_\ell = 1.45 + j1.82\ \Omega}$$

11.27 In a balanced three-phase wye–wye system, the source is an *abc*-sequence set of voltages and $\mathbf{V}_{an} = 120\,\underline{/40^\circ}$ V rms. If the *a*-phase line current and line impedance are known to be $6\,\underline{/15^\circ}$ A rms and $1 + j1\ \Omega$, respectively, find the load impedance.

SOLUTION:

Per phase Y



$I_{aA} = 6\,\underline{/15^\circ}$ Arms $\quad Z_\ell = 1 + j1\,\Omega$

$V_{an} = I_{aA}\left[Z_\ell + Z_L\right]$

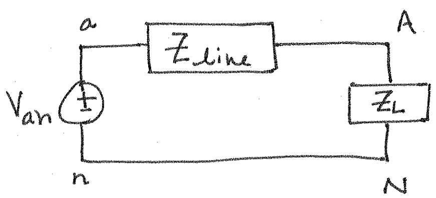
$Z_L = \dfrac{V_{an}}{I_{aA}} - Z_\ell$

$\boxed{Z_L = 18.7\,\underline{/23.5^\circ}\,\Omega}$

11.28 An *abc*-sequence set of voltages feeds a balanced three-phase wye-wye system.
If $\mathbf{V}_{an} = 440\angle 30°$ V rms,
$\mathbf{V}_{AN} = 413.28\angle 29.78°$ V rms, and $\mathbf{Z}_{\text{line}} = 2 + j1.5\ \Omega$,
find the load impedance.

SOLUTION:

Per phase Y



$$\frac{V_{AN}}{V_{an}} = \frac{Z_L}{Z_L + Z_{line}} = \frac{413.28\angle 29.78}{440\angle 30°}$$

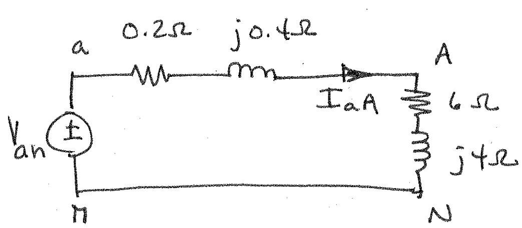
$$\frac{Z_L}{Z_L + Z_{line}} = 0.939\angle -0.22° = \alpha$$

$$Z_L(1-\alpha) = \alpha Z_{line} \qquad Z_L = \left(\frac{\alpha}{1-\alpha}\right) Z_{line} \qquad \boxed{Z_L = 38.4\angle 33.3°\ \Omega}$$

11.29 In a three-phase balanced system, a delta-connected source supplies power to a wye-connected load. If the line impedance is $0.2 + j0.4\ \Omega$, the load impedance $6 + j4\ \Omega$, and the source phase voltage $\mathbf{V}_{ab} = 210\,\underline{/40^\circ}$ V rms, find the magnitude of the line voltage at the load. **PSV**

SOLUTION:

Per phase Y



$$V_{an} = V_{ab}\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right) = 121\angle 10^\circ\ V_{rms}$$

$$V_{AN} = V_{an}\frac{(6+j4)}{6.2+j4.4} = 115\angle 8.33^\circ\ V_{rms}$$

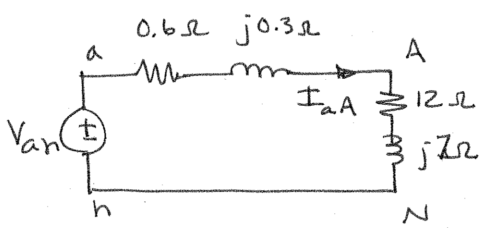
$$V_{AB} = V_{AN}\left(\sqrt{3}\angle 30^\circ\right)$$

$$\boxed{|V_{AB}| = 199\ V_{rms}}$$

11.30 In a balanced three-phase delta–wye system, the source has an *abc*-phase sequence. The line and load impedances are $0.6 + j0.3\ \Omega$ and $12 + j7\ \Omega$, respectively. If the line current $\mathbf{I}_{aA} = 9.6\,\underline{/-20^\circ}$ A rms, determine the phase voltages of the source. CS

SOLUTION:

Per phase Y



$V_{an} = I_{aA}\left[12.6 + j7.3\right]$

$V_{an} = 140\,\underline{/10^\circ}$ Vrms

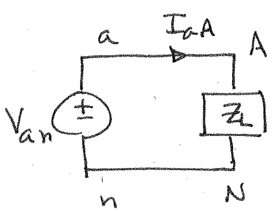
$V_{bn} = 140\,\underline{/-110^\circ}$ Vrms

$V_{cn} = 140\,\underline{/130^\circ}$ Vrms

11.31 An *abc*-phase-sequence three-phase balanced wye-connected source supplies a balanced delta-connected load. The impedance per phase in the delta load is $12 + j6\ \Omega$. The line voltage at the source is $\mathbf{V}_{ab} = 120\sqrt{3}\angle 40^\circ$ V rms. If the line impedance is zero, find the line currents in the balanced wye-delta system.

SOLUTION:

Per phase Y



$Z_\Delta = 12 + j6\ \Omega \qquad Z_Y = \frac{Z_\Delta}{3} = 4 + j2\ \Omega \qquad Z_L = Z_Y$

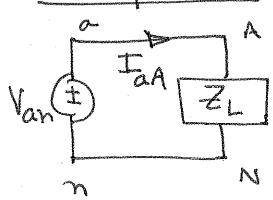
$V_{an} = V_{ab}\left(\frac{1}{\sqrt{3}}\angle -30\right) = 120\angle 10^\circ\ V_{rms}$

$I_{aA} = \frac{V_{an}}{Z_L} = \boxed{\begin{array}{l} 26.8\angle -16.6^\circ\ A_{rms} = I_{aA} \\ I_{bB} = 26.8\angle -136.6^\circ\ A_{rms} \\ I_{cC} = 26.8\angle 103.4^\circ\ A_{rms} \end{array}}$

11.32 An *abc*-phase-sequence three-phase balanced wye-connected source supplies power to a balanced delta-connected load. The impedance per phase in the load is $14 + j7\ \Omega$. If the source voltage for the *a* phase is $\mathbf{V}_{an} = 120\angle 80^\circ$ V rms and the line impedance is zero, find the phase currents in the wye-connected source.

SOLUTION:

Per phase Y



$Z_\Delta = 14 + j7\ \Omega$

$Z_Y = \frac{Z_\Delta}{3} = \frac{14}{3} + j\frac{7}{3} = Z_L$

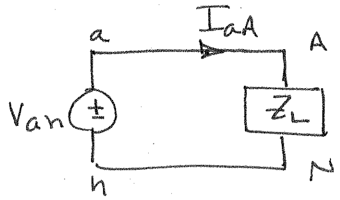
$I_{aA} = \frac{V_{an}}{Z_L}$

$$\boxed{\begin{aligned} I_{aA} &= 23\angle 53.4^\circ \text{ Arms} \\ I_{bB} &= 23\angle -66.6 \text{ Arms} \\ I_{cC} &= 23\angle 173.4^\circ \text{ Arms} \end{aligned}}$$

11.33 An *abc*-phase-sequence three-phase balanced wye-connected source supplies a balanced delta-connected load. The impedance per phase of the delta load is $20 + j4\ \Omega$. If $\mathbf{V}_{AB} = 115\underline{/35^\circ}$ V rms, find the line current.

SOLUTION:

Per phase Y



$Z_\Delta = 20 + j4\ \Omega \qquad Z_Y = \frac{Z_\Delta}{3} = Z_L = \frac{20}{3} + j\frac{4}{3}\ \Omega$

$V_{AB} = 115\angle 35^\circ$ Vrms

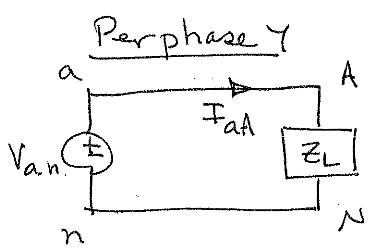
$V_{AN} = V_{AB}\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right) = 66.4\angle 5^\circ$ Vrms

$I_{aA} = \frac{V_{AN}}{Z_L}$

$$\boxed{\begin{aligned} I_{aA} &= 9.77\angle -6.31^\circ \text{ Arms} \\ I_{bB} &= 9.77\angle -126.3^\circ \text{ Arms} \\ I_{cC} &= 9.77\angle 113.7^\circ \text{ Arms} \end{aligned}}$$

11.34 An *abc*-phase-sequence three-phase balanced wye-connected source supplies a balanced delta-connected load. The impedance per phase of the delta load is $10 + j8\ \Omega$. If the line impedance is zero and the line current in the *a* phase is known to be $\mathbf{I}_{aA} = 28.10 \angle -28.66^\circ$ A rms, find the load voltage $\mathbf{V}_{AB}$. CS

SOLUTION:



$$Z_\Delta = 10 + j8\,\Omega \qquad Z_Y = \frac{Z_\Delta}{3} = Z_L = \frac{10}{3} + j\frac{8}{3}\,\Omega$$

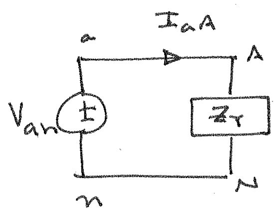
$$V_{AN} = I_{aA} Z_L = 120 \angle 10^\circ \text{ Vrms}$$

$$V_{AB} = V_{AN}\left(\sqrt{3} \angle 30^\circ\right) \qquad \boxed{V_{AB} = 208 \angle 40^\circ \text{ Vrms}}$$

11.35 In a balanced three-phase wye–delta system, the source has an *abc*-phase sequence and $\mathbf{V}_{an} = 120\,\underline{/0^\circ}$ V rms. If the line impedance is zero and the line current $\mathbf{I}_{aA} = 5\,\underline{/20^\circ}$ A rms, find the load impedance per phase in the delta.

SOLUTION:

Per phase Y



$Z_Y = V_{an} / I_{aA} = 24\,\underline{/-20^\circ}\ \Omega$

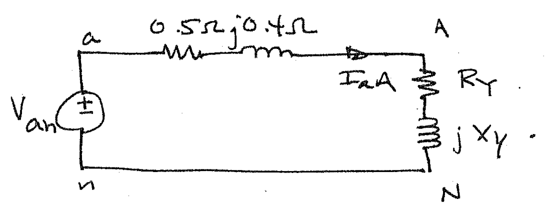
$Z_\Delta = 3\,Z_Y$

$\boxed{Z_\Delta = 72\,\underline{/-20^\circ}\ \Omega}$

11.36 In a balanced three-phase wye–delta system, the source has an *abc*-phase sequence and $\mathbf{V}_{an} = 120\angle 40^\circ$ V rms. The line and load impedance are $0.5 + j0.4\ \Omega$ and $36 + j18\ \Omega$, respectively. Find the delta currents in the load.

SOLUTION:

Per phase Y



$$Z_\Delta = 36 + j18\ \Omega$$

$$Z_Y = Z_\Delta / 3 = 12 + j6\ \Omega$$

$$I_{aA} = \frac{V_{an}}{Z_{line} + Z_Y} = \frac{120\angle 40^\circ}{12.5 + j6.4}$$

$$I_{aA} = 8.55\angle 12.9^\circ \text{ A}_{rms}$$

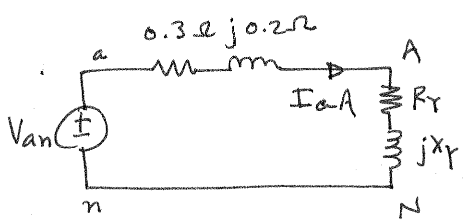
$$I_{AB} = \frac{I_{aA}}{\sqrt{3}} \angle 30^\circ$$

$$\boxed{\begin{aligned} I_{AB} &= 4.94\angle 42.9^\circ \text{ Arms} \\ I_{BC} &= 4.94\angle -77.1^\circ \text{ Arms} \\ I_{CA} &= 4.94\angle 162.9^\circ \text{ Arms} \end{aligned}}$$

11.37 In a three-phase balanced delta-delta system, the source has an *abc*-phase sequence. The line and load impedances are $0.3 + j0.2\ \Omega$ and $9 + j6\ \Omega$, respectively. If the load current in the delta is $\mathbf{I}_{AB} = 15\underline{/40^\circ}$ A rms, find the phase voltages of the source. CS

SOLUTION:

Per phase Y



$Z_\Delta = 9 + j6\ \Omega \qquad Z_Y = \frac{Z_\Delta}{3} = 3 + j2\ \Omega$

$I_{AB}\left(\sqrt{3}\ \underline{/-30^\circ}\right) = I_{aA}$

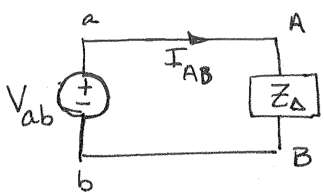
$I_{aA} = 26\ \underline{/10^\circ}$ Arms

$V_{an} = I_{aA}\left[Z_{line} + Z_L\right] = I_{aA}\left[3.3 + j\,2.2\right]$

$$\boxed{\begin{aligned} V_{an} &= 103\ \underline{/43.7^\circ}\ \text{Vrms} \\ V_{bn} &= 103\ \underline{/-76.3^\circ}\ \text{Vrms} \\ V_{cn} &= 103\ \underline{/163.7^\circ}\ \text{Vrms} \end{aligned}}$$

11.38 In a balanced three-phase delta–delta system, the source has an *abc*-phase sequence. The phase angle for the source voltage is $\angle\mathbf{V}_{ab} = 40°$ and $\mathbf{I}_{ab} = 4\angle 15°$ A rms. If the total power absorbed by the load is 1400 W, find the load impedance.

SOLUTION:



$$P_L = |V_{ab}|\,|I_{AB}| \cos\left(\theta_{V_{ab}} - \theta_{I_{AB}}\right) = 1400$$

$$|V_{ab}| = \frac{1400}{4\cos(25°)} = 386\ V_{rms}$$

$$V_{ab} = 386 \angle 40°\ V_{rms}$$

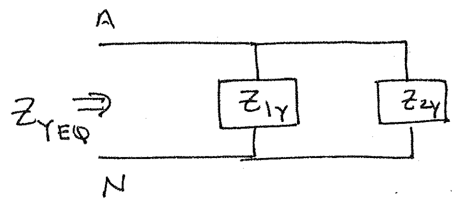
$$Z_\Delta = \frac{V_{ab}}{I_{AB}}$$

$$\boxed{Z_\Delta = 96.5\angle 25°\ \Omega}$$

11.39 A three-phase load impedance consists of a balanced wye in parallel with a balanced delta. What is the equivalent wye load and what is the equivalent delta load if the phase impedances of the wye and delta are $6 + j3\ \Omega$ and $15 + j10\ \Omega$, respectively?

SOLUTION:

Per phase Y circuit



$$Z_{YEQ} = \frac{Z_{1Y} Z_{2Y}}{Z_{1Y} + Z_{2Y}}$$

$$Z_{\Delta EQ} = 3 Z_{YEQ}$$

$Z_{1Y} = 6 + j3\,\Omega$ in wye

$Z_{2\Delta} = 15 + j10$ in delta

convert Z_2 to wye $Z_{2Y} = \frac{Z_{2\Delta}}{3}$

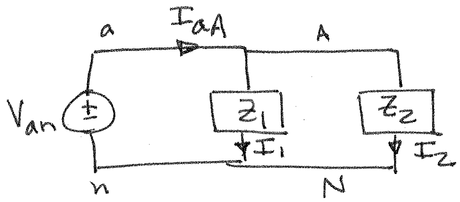
$Z_{2Y} = 5 + j\,3.33\,\Omega$

$$\boxed{Z_{YEQ} = 3.18\angle 30.3^\circ\,\Omega}$$

$$\boxed{Z_{\Delta EQ} = 9.53\angle 30.3^\circ\,\Omega}$$

11.40 In a balanced three-phase system, the *abc*-phase-sequence source is wye connected and $\mathbf{V}_{an} = 120\angle 20^\circ$ V rms. The load consists of two balanced wyes with phase impedances of $8 + j2\ \Omega$ and $12 + j3\ \Omega$. If the line impedance is zero, find the line currents and the phase current in each load.

SOLUTION: Per phase Y



$$I_{AN1} = \frac{V_{an}}{Z_1} = \frac{120\angle 20}{8+j2}$$

$$I_{AN1} = 14.6\angle 5.96^\circ \text{ Arms}$$

$$I_{AN2} = \frac{V_{an}}{Z_2} = 9.70\angle 5.96^\circ \text{ Arms}$$

$I_{aA} = I_{AN1} + I_{AN2}$

$I_{aA} = 24.3\angle 5.96^\circ$ Arms

$I_{bB} = 24.3\angle -114^\circ$ Arms

$I_{cC} = 24.3\angle 126^\circ$ Arms

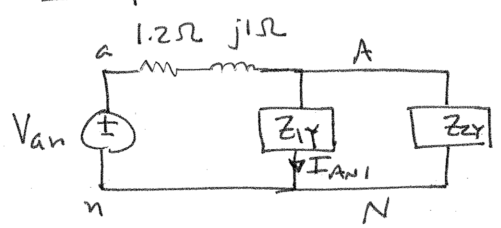
$I_{AN1} = 14.6\angle 5.96^\circ$ Arms

$I_{AN2} = 14.6\angle 5.96^\circ$ Arms

11.41 In a balanced three-phase system, the source is a balanced wye with an *abc*-phase sequence and $\mathbf{V}_{ab} = 208\angle 60^\circ$ V rms. The load consists of a balanced wye with a phase impedance of $8 + j5\ \Omega$ in parallel with a balanced delta with a phase impedance of $21 + j12\ \Omega$. If the line impedance is $1.2 + j1\ \Omega$, find the phase currents in the balanced wye load. CS

SOLUTION:

Per phase Y



$$V_{an} = V_{ab}\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right)$$

$$V_{an} = 120\angle 30^\circ\ V_{rms}$$

$$Z_{1Y} = 8 + j5\ \Omega \qquad Z_{2\Delta} = 21 + j12\ \Omega$$

$$Z_{2Y} = Z_{2\Delta}/3 = 7 + j4\ \Omega$$

$$\text{Let } Z = \frac{Z_{1Y}Z_{2Y}}{Z_{1Y} + Z_{2Y}} = 4.35\angle 30.8^\circ\ \Omega = 3.74 + j\,2.23\ \Omega$$

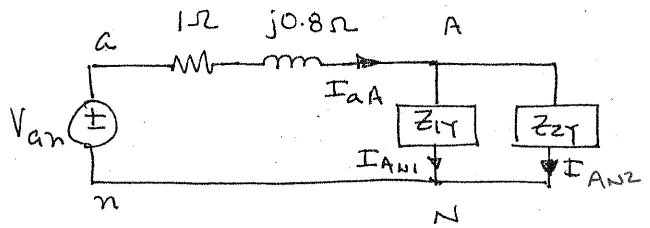
$$V_{AN} = \frac{V_{an}\,Z}{Z + Z_{line}} = \frac{V_{an}\,(4.35\angle 30.8)}{3.74 + 1.2 + j(2.23 + 1)} = 88.5\angle 27.7^\circ\ V_{rms}$$

$$I_{AN1} = \frac{V_{AN}}{Z_{1Y}} = \frac{V_{AN}}{8 + j5}$$

$$\boxed{\begin{aligned} I_{AN1} &= 9.38\angle -4.39^\circ\ A_{rms} \\ I_{BN1} &= 9.38\angle -124.4^\circ\ A_{rms} \\ I_{CN1} &= 9.38\angle 115.6^\circ\ A_{rms} \end{aligned}}$$

11.42 In a balanced three-phase system, the source is a balanced wye with an *abc*-phase sequence and $\mathbf{V}_{ab} = 215\angle 50°$ V rms. The load is a balanced wye in parallel with a balanced delta. The phase impedance of the wye is $5 + j3\ \Omega$, and the phase impedance of the delta is $18 + j12\ \Omega$. If the line impedance is $1 + j0.8\ \Omega$, find the line currents and the phase currents in the loads.

SOLUTION: Per phase Y



$V_{an} = V_{ab}\left[\frac{1}{\sqrt{3}}\angle -30°\right] = 124\angle 20°\ V_{rms}$

$Z_{1Y} = 5+j3\ \Omega$

$Z_{2\Delta} = 18+j12\ \Omega \qquad Z_{2Y} = Z_{2\Delta}/3$

$Z_{2Y} = 6+j4\ \Omega$

Let $Z = \dfrac{Z_{1Y}Z_{2Y}}{Z_{1Y}+Z_{2Y}} = 3.22\angle 32.3°\ \Omega$

$I_{aA} = V_{an}/(Z+1+j0.8) = 27.6\angle -14.0°$ Arms

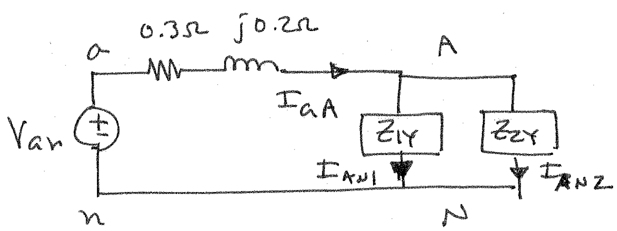
$I_{AN1} = I_{aA}Z_{2Y}/(Z_{1Y}+Z_{2Y}) = 15.2\angle -12.8°$ Arms

$I_{AN2} = I_{aA}Z_{1Y}/(Z_{1Y}+Z_{2Y}) = 12.3\angle -15.5°$ Arms

$I_{aA} = 27.6\angle -14°$ Arms	$I_{AN1} = 15.2\angle -12.8°$ Arms	$I_{AN2} = 12.3\angle -15.5$ Arms
$I_{bB} = 27.6\angle -134$ Arms	$I_{BN1} = 15.2\angle -133°$ Arms	$I_{BN2} = 12.3\angle -136°$ Arms
$I_{cC} = 27.6\angle 106$ Arms	$I_{CN1} = 15.2\angle 107°$ Arms	$I_{CN2} = 12.3\angle 105°$ Arms

11.43 In a balanced three-phase system, the source has an *abc*-phase sequence and is connected in delta. There are two parallel wye-connected loads. The phase impedance of load 1 and load 2 is $4 + j4\ \Omega$ and $10 + j4\ \Omega$, respectively. The line impedance connecting the source to the loads is $0.3 + j0.2\ \Omega$. If the current in the *a* phase of load 1 is $\mathbf{I}_{AN_1} = 10\angle 20°$ A rms, find the delta currents in the source.

SOLUTION: Per phase Y



$Z_{1Y} = 4 + j4\ \Omega \qquad Z_{2Y} = 10 + j4\ \Omega$

$V_{AN} = I_{AN1} Z_{1Y} = 10\angle 20 \left[4\sqrt{2}\angle 45\right]$

$V_{AN} = 56.6\angle 65°$ Vrms

$Z = Z_{1Y} Z_{2Y} / (Z_{1Y} + Z_{2Y})$

$Z = 3.78\angle 37.1°\ \Omega$

$$\frac{V_{AN}}{V_{an}} = \frac{Z}{Z + 0.3 + j0.2} = 0.913\angle 0.293°$$

$$V_{an} = \frac{V_{AN}}{0.913\angle 0.293°} = 62.0\angle 64.7° \text{ Vrms}$$

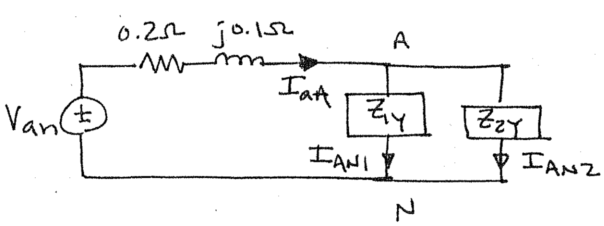
$$I_{aA} = \frac{V_{an}}{Z + 0.3 + j0.2} = 15.0\angle 27.9° \text{ Arms}$$

$$I_{ab} = I_{aA}\left(\frac{1}{\sqrt{3}}\angle 30°\right) = 8.66\angle 57.9° \text{ Arms}$$

$$\boxed{\begin{aligned} I_{ab} &= 8.66\angle 57.9° \text{ Arms} \\ I_{bc} &= 8.66\angle -62.1° \text{ Arms} \\ I_{ca} &= 8.66\angle 178° \text{ Arms} \end{aligned}}$$

11.44 In a balanced three-phase system, the source has an *abc*-phase sequence and is connected in delta. There are two loads connected in parallel. The line connecting the source to the loads has an impedance of $0.2 + j0.1\ \Omega$. Load 1 is connected in wye, and the phase impedance is $4 + j2\ \Omega$. Load 2 is connected in delta, and the phase impedance is $12 + j9\ \Omega$. The current $\mathbf{I}_{AB}$ in the delta load is $16\underline{/45^\circ}$ A rms. Find the phase voltages of the source. CS

SOLUTION: Per phase Y



$Z_{1Y} = 4 + j2\,\Omega \quad Z_{2\Delta} = 12 + j9\,\Omega$

$Z_{2Y} = Z_{2\Delta}/3 = 4 + j3\,\Omega$

$I_{AN2} = I_{AB2}\left(\sqrt{3}\,\underline{/-30^\circ}\right)$

$I_{AN2} = 27.7\,\underline{/15^\circ}$ Arms

$V_{AN} = I_{AN2} Z_{2Y}$

$V_{AN} = 139\,\underline{/51.9^\circ}$ Vrms $\qquad I_{AN1} = V_{AN}/Z_{1Y} = 31.0\,\underline{/25.3^\circ}$ Arms

$I_{aA} = I_{AN1} + I_{AN2} = 58.5\,\underline{/20.4^\circ}$ Arms

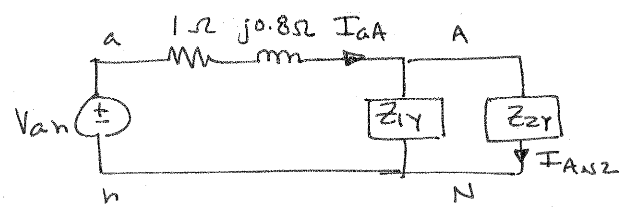
$V_{an} = I_{aA}(0.2 + j0.1) + V_{AN} = 152\,\underline{/51.5^\circ}$ Vrms

$V_{ab} = V_{an}\left(\sqrt{3}\,\underline{/30^\circ}\right) = 263\,\underline{/81.5^\circ}$ Vrms

$$\boxed{\begin{aligned} V_{ab} &= 263\,\underline{/81.5^\circ}\ \text{Vrms} \\ V_{bc} &= 263\,\underline{/-38.5^\circ}\ \text{Vrms} \\ V_{ca} &= 263\,\underline{/-158.5^\circ}\ \text{Vrms} \end{aligned}}$$

11.45 A balanced three-phase delta-connected source supplies power to a load consisting of a balanced delta in parallel with a balanced wye. The phase impedance of the delta is $24 + j12\ \Omega$, and the phase impedance of the wye is $12 + j8\ \Omega$. The *abc*-phase-sequence source voltages are $\mathbf{V}_{ab} = 440\underline{/60^\circ}$ V rms, $\mathbf{V}_{bc} = 440\underline{/-60^\circ}$ V rms, and $\mathbf{V}_{ca} = 440\underline{/-180^\circ}$ V rms, and the line impedance per phase is $1 + j0.8\ \Omega$. Find the line currents and the power absorbed by the wye-connected load. **PSV**

SOLUTION: Per phase Y



$$V_{an} = V_{ab}\left[\frac{1}{\sqrt{3}}\underline{/-30^\circ}\right]$$

$$V_{an} = 254\underline{/-30^\circ}\ V_{rms}$$

$Z_{1\Delta} = 24 + j12\Omega \qquad Z_{1Y} = Z_{1\Delta}/3 = 8 + j4\Omega \qquad Z_{2Y} = 12 + j8\Omega$

Let $Z = Z_{1Y}Z_{2Y}/(Z_{1Y} + Z_{2Y}) = 5.53\underline{/29.3^\circ}\ \Omega$

$I_{aA} = V_{an}/(Z + 1 + j0.8) = 37.4\underline{/-1.05^\circ}$ Arms

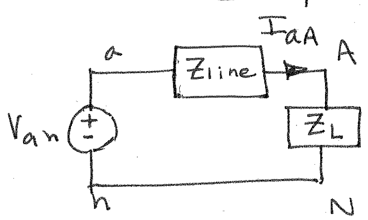
$I_{AN2} = I_{aA}Z_{1Y}/(Z_{1Y} + Z_{2Y}) = 14.3\underline{/-5.45^\circ}$ Arms

$P_{L_2} = |I_{AN2}|^2 R_{2Y} \qquad R_{2Y} = 12\Omega \qquad P_{L2} = 2.45$ kW

$I_{aA} = 37.4\underline{/-1.05^\circ}$ Arms
$I_{bB} = 37.4\underline{/-121^\circ}$ Arms
$I_{cC} = 37.4\underline{/119^\circ}$ Arms
$P_{L2} = 2.45$ kW

11.46 An *abc*-sequence wye-connected source having a phase-*a* voltage of $120\,\underline{/0^\circ}$ V rms is attached to a wye-connected load having a per-phase impedance of $100\,\underline{/70^\circ}\ \Omega$. If the line impedance is $1\,\underline{/20^\circ}\ \Omega$, determine the total complex power produced by the voltage sources and the real and reactive power dissipated by the load.

SOLUTION: Per phase Y



$Z_{line} = 1\,\underline{/20^\circ}\ \Omega \qquad Z_L = 100\,\underline{/70^\circ} = 34.2 + j\,94.0\ \Omega$

$V_{an} = 120\,\underline{/0^\circ}$ Vrms

$I_{aA} = \dfrac{V_{AN}}{Z_{line} + Z_L} = 1.19\,\underline{/-69.6^\circ}$ Arms

$S_{1\phi} = V_{an} I_{aA}^{*} = 143\,\underline{/69.6^\circ}$ VA

$S_{3\phi} = 3 S_{1\phi} = 429\,\underline{/69.6^\circ}$ VA

produced by sources

$P_L = 3\,|I_{aA}|^2 R_L = 3(1.19)^2(34.2) = 145$ W

$Q_L = 3\,|I_{aA}|^2 X_L = 3(1.19)^2(94) = 399$ VAR

dissipated by load

11.47 The magnitude of the complex power (apparent power) supplied by a three-phase balanced wye–wye system is 3600 VA. The line voltage is 208 V rms. If the line impedance is negligible and the power factor angle of the load is 25°, determine the load impedance. CS

SOLUTION:

$S_{3\phi} = 3600\ VA$ supplied

$|V_{ab}| = 208\ V_{rms}$ $\quad |V_{an}| = \dfrac{|V_{ab}|}{\sqrt{3}} = 120\ V_{rms}$

$\theta_{Z_L} = 25°$

$S_{3\phi} = 3600 \angle 25° = 3|V_{an}||I_{aA}| \angle \theta_{V_{an}} - \theta_{I_{aA}} = 3|V_{an}||I_{aA}| \angle \theta_{Z_L}$

$|I_{aA}| = \dfrac{3600}{3(120)} = 10\ A_{rms}$

$Z_L = \dfrac{|V_{an}|}{|I_{aA}|} \angle \theta_{Z_L} = \dfrac{120}{10} \angle 25°$ $\quad \boxed{Z_L = 12 \angle 25°\ \Omega}$

11.48 A three-phase *abc*-sequence wye-connected source supplies 14 kVA with a power factor of 0.75 lagging to a delta load. If the delta load consumes 12 kVA at a power factor of 0.7 lagging and has a phase current of $10\,\underline{/-30^\circ}$ A rms, determine the per-phase impedance of the load and the line.

SOLUTION:

$S_{3\phi} = 14\text{kVA}$ supplied $\quad$ pf at source $= 0.75$ lagging $= pf_S$

$S_{3\phi_L} = 12\text{kVA}$ at load $\quad$ pf at load $= 0.7$ lagging $= pf_L$

$I_{AB} = 10\,\underline{/-30^\circ}$ Arms

At load $\quad |S_{1\phi_L}| = \frac{|S_{3\phi L}|}{3} = 4\text{kVA} \qquad \theta_S = \cos^{-1}(0.7) = 45.6^\circ$

$\theta_{V_{AB}} - \theta_{I_{AB}} = \theta_S \Rightarrow \theta_{V_{AB}} = 15.6^\circ \qquad |V_{AB}| = \frac{|S_{1\phi L}|}{|I_{AB}|} = 400\text{Vrms}$

$V_{AB} = 400\,\underline{/15.6}$ Vrms $\qquad Z_\Delta = \frac{V_{AB}}{I_{AB}} \qquad \boxed{Z_\Delta = 40\,\underline{/45.6^\circ}\ \Omega}$

$I_{aA} = I_{AB}\left[\sqrt{3}\,\underline{/-30^\circ}\right] = 17.3\,\underline{/-60^\circ}$

$S_{1\phi S}$ = per phase power supplied $= |I_{aA}|^2\left(Z_{line} + \frac{Z_\Delta}{3}\right)$

$S_{1\phi S} = \frac{14{,}000}{3}\,\underline{/\cos^{-1}(pf_S)} = 4.67\,\underline{/41.4^\circ}$ kVA

$$\boxed{Z_{line} = 2.37 + j0.786\,\Omega}$$

11.49 A three-phase balanced wye–wye system has a line voltage of 208 V rms. The line current is 6 A rms and the total real power absorbed by the load is 1800 W. Determine the load impedance per-phase, if the line impedance is negligible.

SOLUTION:

$|V_{ab}| = 208\ V_{rms}$ $\quad |I_{aA}| = 6\ A_{rms}$ $\quad P_{3\phi} = 3\,|V_{an}|\,|I_{aA}|\,pf$

$|V_{an}| = |V_{ab}|/\sqrt{3} = 120\ V_{rms}$ $\quad pf = \dfrac{1800}{3(120)(6)} = 0.833$

$Z_L = \dfrac{|V_{an}|}{|I_{aA}|}\angle \cos^{-1}(pf)$ assuming pf is lagging

$$\boxed{Z_L = 20\angle 33.6^\circ\ \Omega}$$

11.50 A balanced three-phase source serves two loads:

Load 1: 36 kVA at 0.8 pf lagging

Load 2: 18 kVA at 0.6 pf lagging

The line voltage at the load is 208 V rms at 60 Hz. Find the line current and the combined power factor at the load. CS

SOLUTION:

$|V_{AB}| = 208\ \text{Vrms}$ $\quad |V_{AN}| = \dfrac{|V_{ab}|}{\sqrt{3}} = 120\ \text{Vrms}$

assume $\theta_{V_{AN}} = 0°$ $\quad V_{AN} = 120\angle 0°\ \text{Vrms}$

LOAD 1: $S_1 = 3V_{AN} I_{AN1}^*$ $\quad I_{AN1} = \left[\dfrac{36000\angle \cos^{-1}(0.8)}{3(120\angle 0)}\right]^*$

$I_{AN1} = 100\angle -36.9°\ \text{Arms}$

LOAD 2: $S_2 = 3V_{AN} I_{AN2}^*$ $\quad I_{AN2} = 50\angle -53.1°\ \text{Arms}$

TOTAL LOAD: $I_{aA} = I_{AN1} + I_{AN2} = 148\angle -42.3°\ \text{Arms}$

$\theta_{Z_L} = \theta_{V_{AN}} - \theta_{I_{aA}} = 42.3°$

$pf_L = \cos(\theta_{Z_L}) = 0.740$ lagging

$$\boxed{\begin{array}{l} I_{aA} = 148\angle -42.3°\ \text{Arms} \\ pf_L = 0.740\ \text{lagging} \\ \text{assumed } \theta_{V_{AN}} = 0° \end{array}}$$

11.51 A balanced three-phase source serves the following loads:

Load 1: 60 kVA at 0.8 pf lagging

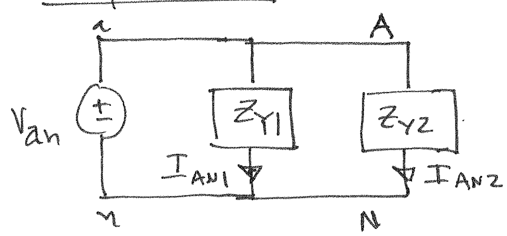
Load 2: 30 kVA at 0.75 pf lagging

The line voltage at the load is 208 V rms at 60 Hz. Determine the line current and the combined power factor at the load.

SOLUTION:

$|V_{ab}| = 208\ V_{rms}$ $\quad |V_{an}| = |V_{ab}|/\sqrt{3} = 120\ V_{rms}$

Per phase Y



LOAD 1: $S_{3\phi_1} = 60\,\angle\cos^{-1}(pf_1)$ kVA

$= 60\,\angle 36.9^\circ$ kVA

$S_{1\phi_1} = S_{3\phi}/3 = 20\,\angle 36.9^\circ$ kVA

assume $\theta_{V_{an}} = 0^\circ$

$I_{AN1} = (S_{1\phi_1}/V_{an})^* = 167\,\angle -36.9^\circ$ Arms

Load 2: $S_{3\phi_2} = 30\,\angle\cos^{-1}(pf_2)$ kVA

$S_{3\phi_2} = 30\,\angle 41.4^\circ$ kVA

$S_{1\phi_2} = S_{3\phi_2}/3 = 10\,\angle 41.4^\circ$ kVA

$I_{AN2} = (S_{1\phi_2}/V_{an})^* = 83.3\,\angle -41.4^\circ$ Arms

$I_{aA} = I_{AN1} + I_{AN2} = 250\,\angle -38.4^\circ$ Arms $= I_{aA}$

$pf = \cos(38.4^\circ) = 0.784$ $\quad$ pf = 0.784 lagging

11.52 A small shopping center contains three stores that represent three balanced three-phase loads. The power lines to the shopping center represent a three-phase source with a line voltage of 13.8 kV rms. The three loads are

Load 1: 400 kVA at 0.9 pf lagging

Load 2: 200 kVA at 0.85 pf lagging

Load 3: 100 kVA at 0.90 pf lagging

Find the power line current.

SOLUTION:

$|V_{AB}| = 13.8\,\text{kVrms} \qquad |V_{AN}| = |V_{AB}|/\sqrt{3} = 7.97\,\text{kV rms}$

LOAD 1: $S_1 = 400{,}000 \angle \cos^{-1}(0.9) = 3 V_{AN} I_{AN1}^*$ assume $\theta_{V_{AN}} = 0$

$$I_{AN1} = \left[\frac{400{,}000 \angle 25.8}{3(7970 \angle 0°)}\right]^* = 16.7 \angle -25.8° \text{ Arms}$$

LOAD 2: $S_2 = 200{,}000 \angle \cos^{-1}(0.85) = 3 V_{AN} I_{AN2}^*$

$I_{AN2} = 8.38 \angle -31.8°$ Arms

LOAD 3: $S_3 = 3 V_{AN} I_{AN3}^* \Rightarrow I_{AN3} = 4.18 \angle -25.8°$ Arms

$I_{aA} = I_{AN1} + I_{AN2} + I_{AN3}$

$$\boxed{\begin{array}{l} I_{aA} = 29.2 \angle -27.5° \text{ Arms} \\ \text{assumed } \theta_{V_{AN}} = 0° \end{array}}$$

11.53 The following loads are served by a balanced three-phase source:

Load 1: 20 kVA at 0.8 pf lagging

Load 2: 4 kVA at 0.8 pf leading

Load 3: 10 kVA at 0.75 pf lagging

The load voltage is 208 V rms at 60 Hz. If the line impedance is negligible, find the power factor at the source.

SOLUTION:

$|V_{AB}| = 208 \text{ V}_{rms}$ $\quad |V_{AN}| = |V_{AB}| / \sqrt{3} = 120 \text{ V}_{rms} = |V_{an}|$

assume $\theta_{V_{AN}} = 0°$ since $Z_{line} \approx 0$, $\theta_{V_{an}} = 0$ also.

LOAD 1: $S_1 = 20{,}000 \angle \cos^{-1}(0.8) = 3 V_{AN} I_{AN1}^* = 3(120\angle 0) I_{AN1}^*$

$I_{AN1}^* = 55.6 \angle -36.9° \text{ Arms}$

LOAD 2: $S_2 = 4000 \angle -\cos^{-1}(0.8) = 3 V_{AN} I_{AN2}^*$

$I_{AN2}^* = 11.1 \angle +36.9° \text{ Arms}$

LOAD 3: $S_3 = 10{,}000 \angle \cos^{-1}(0.75) = 3 V_{AN} I_{AN3}^* \Rightarrow I_{AN3} = 27.8 \angle -41.4° \text{ Arms}$

$I_{aA} = I_{AN1} + I_{AN2} + I_{AN3} = 86.8 \angle -31.3° \text{ Arms}$

$pf_L = \cos(\theta_{V_{AN}} - \theta_{I_{aA}}) = 0.854$ lagging

Since $V_{an} = V_{AN}$ (no line impedance)

$$\boxed{pf_s = pf_L = 0.854 \text{ lagging}}$$

11.54 A balanced three-phase source supplies power to three loads. The loads are

Load 1: 30 kVA at 0.8 pf lagging

Load 2: 24 kW at 0.6 pf leading

Load 3: unknown

If the line voltage and total complex power at the load are 208 V rms and $60\underline{/0^\circ}$ kVA, respectively, find the unknown load. CS

SOLUTION:

$|V_{AB}| = 208\ V_{rms}$ $\quad |V_{AN}| = |V_{AB}|/\sqrt{3} = 120\ V_{rms}$

$S_{3\phi} = 60\underline{/0^\circ}\ kVA = S_1 + S_2 + S_3$

$S_1 = 30{,}000\ \underline{/\cos^{-1}(0.8)} = 30\underline{/36.9^\circ}\ kVA$

$P_2 = 24{,}000 = |S_2|\,(pf_2) \Rightarrow |S_2| = 40\,kVA \quad \theta_{S_2} = -\cos^{-1}(0.6) = -53.1^\circ$

$S_2 = 40\underline{/-53.1^\circ}\ kVA$

$S_3 = 60\underline{/0^\circ} - 30\underline{/36.9} - 40\underline{/-53.1^\circ}$ in kVA

$$\boxed{\begin{array}{l} S_3 = 18.4\underline{/49.4^\circ}\ kVA \\ S_3 = 18.4 @ 0.65\ \text{pf lagging} \end{array}}$$

11.55 A balanced three-phase source serves the following loads:

Load 1: 20 kVA at 0.8 pf lagging

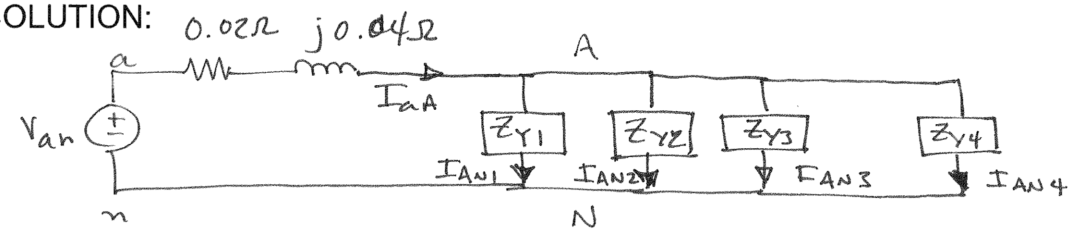
Load 2: 10 kVA at 0.7 pf leading

Load 3: 10 kW at unity pf

Load 4: 16 kVA at 0.6 pf lagging

The line voltage at the load is 208 V rms at 60 Hz, and the line impedance is $0.02 + j0.04\ \Omega$. Find the line voltage and power factor at the source. **PSV**

SOLUTION:



$|V_{AB}| = 208\ V_{rms}$ $\quad |V_{AN}| = |V_{ab}|/\sqrt{3} = 120 V_{rms}$ $\quad \theta_{V_{AN}} = 0°$

$S_{3\phi_1} = 20 \angle \cos^{-1}(0.8)$ kVA $\quad S_{1\phi_1} = 6.67 \angle 36.9°$ kVA

$I_{AN1} = \left(\frac{S_{1\phi 1}}{V_{AN}}\right)^* = 55.6 \angle -36.9°$ Arms

$S_{1\phi 2} = \frac{|S_{3\phi 2}|}{3} \angle -\cos^{-1}(pf_2) = \frac{10^4}{3} \angle -\cos^{-1}(0.7) = 3.33 \angle -45.6°$ kVA

$I_{AN_2} = (S_{1\phi 2}/V_{AN})^* = 27.8 \angle 45.6°$ Arms

$I_{AN3} = (S_{1\phi 3}/V_{AN})^* = \left[\frac{10^4}{3} \angle 0° / 120 \angle 0\right]^* = 27.8 \angle 0°$ Arms

$I_{AN4} = \left(\frac{S_{1\phi 4}}{V_{AN}}\right)^* = \left(\frac{16 \times 10^3}{3} \angle \cos^{-1}(0.6) \Big/ 120 \angle 0°\right)^* = 44.4 \angle -53.1°$ Arms

$I_{aA} = I_{AN1} + I_{AN2} + I_{AN3} + I_{AN4} = 128 \angle -22.5°$ Arms

$$V_{an} = I_{aA}\,(0.02 + j0.04) + V_{AN}$$

$$= 128\angle{-22.5^\circ}\,(0.02 + j0.04) + 120\angle{0^\circ}$$

$$V_{an} = 124\angle{1.72^\circ} \text{ Vrms}$$

line voltage $= |V_{ab}| = |V_{an}|\sqrt{3}$ $\boxed{|V_{ab}| = 215 \text{ Vrms}}$

pf at source $= \cos(\theta_{V_{an}} - \theta_{I_{aA}}) = \cos(1.72 - (-22.5))$

$$\boxed{\text{pf @ source} = 0.912 \text{ lagging}}$$

11.56 A balanced three-phase source supplies power to three loads. The loads are

Load 1: 24 kW at 0.8 pf lagging

Load 2: 10 kVA at 0.7 pf leading

Load 3: unknown

If the line voltage at the load is 208 V rms, the magnitude of the total complex power is 41.93 kVA, and the combined power factor at the load is 0.86 lagging, find the unknown load.

SOLUTION:

$|V_{AB}| = 208\ V_{rms}$ $\quad |V_{AN}| = |V_{AB}|/\sqrt{3} = 120\ V_{rms}$ $\quad$ assumed $\theta_{V_{AN}} = 0°$

$S_L = 41.93 \angle \cos^{-1}(0.86)\ \text{kVA} = 41.93 \angle 30.68°\ \text{kVA}$

$S_1 = P_1 + jQ_1 \quad P_1 = 24\text{kW} \quad |S_1| = \dfrac{P_1}{pf_1} = 30\text{kVA} \quad Q_1 = 18\text{kVAR}$

$S_1 = 24 + j18\ \text{kVA}$

$S_2 = 10 \angle -\cos^{-1}(0.7)\ \text{kVA} = 10 \angle -45.6°\ \text{kVA}$

$S_3 = S_L - S_1 - S_2 = 11.7 \angle 64.3°\ \text{kVA} \qquad pf = \cos(64.3) = 0.434$

$\boxed{S_3 = 11.7\ \text{kVA} \text{ @ } pf = 0.434 \text{ lagging}}$

11.57 A balanced three-phase source supplies power to three loads. The loads are

Load 1: 24 kVA at 0.6 pf lagging

Load 2: 10 kW at 0.75 pf lagging

Load 3: unknown

If the line voltage at the load is 208 V rms, the magnitude of the total complex power is 35.52 kVA, and the combined power factor at the load is 0.88 lagging, find the unknown load.

SOLUTION:

$$S_{TOTAL} = 35.52 \angle \cos^{-1}(0.88)\ \text{kVA} = 35.52\angle 28.4^\circ\ \text{kVA}$$

$$S_{TOTAL} = S_1 + S_2 + S_3$$

$$S_1 = 24\angle \cos^{-1}(0.6)\ \text{kVA} = 24\angle 53.1^\circ\ \text{kVA}$$

$$S_2 = \frac{10}{0.75}\angle \cos^{-1}(0.75)\ \text{kVA} = 13.3\angle 41.4^\circ\ \text{kVA}$$

$$S_3 = S_{TOTAL} - S_1 - S_2 \Rightarrow \boxed{S_3 = 13.0\angle -58.4^\circ\ \text{kVA}}$$

11.58 A standard practice for utility companies is to divide its customers into single-phase users and three-phase users. The utility must provide three-phase users, typically industries, with all three phases. However, single-phase users, residential, and light commercial are connected to only one phase. To reduce cable costs, all single-phase users in a neighborhood are connected together. This means that even if the three-phase users present perfectly balanced loads to the power grid, the single-phase loads will never be in balance, resulting in current flow in the neutral connection. Consider the 60-Hz, *abc*-sequence network in Fig. P11.58. With a line voltage of $416\angle 30^\circ$ V rms, phase *a* supplies the single-phase users on A Street, phase *b* supplies B Street, and phase *c* supplies C Street. Furthermore, the three-phase industrial load, which is connected in delta, is balanced. Find the neutral current. CS

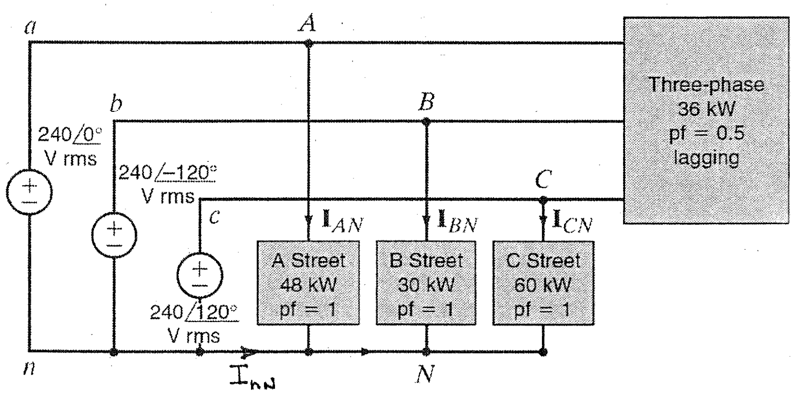


Figure P11.58

SOLUTION:

S_A = Complex power consumed on A street. $= 48\text{kW} = 240\angle 0 \; I_{AN} = 48\times 10^3$

$$I_{AN} = \left[\frac{48\times 10^3\angle 0}{240\angle 0}\right]^* = 200\angle 0^\circ \text{ Arms} \qquad V_{AN} = 240\angle 0^\circ \text{ Vrms}$$

$$I_{BN} = \left[30\times 10^3\angle 0^\circ / 240\angle -120^\circ\right]^* = 125\angle -120^\circ \text{ Arms}$$

$$I_{CN} = \left[60\times 10^3\angle 0^\circ / 240\angle 120^\circ\right]^* = 250\angle +120^\circ \text{ Arms}$$

for the 3ϕ load, $P_{3\phi} = 36\text{kw}$ $\quad S_{3\phi} = \dfrac{P_{3\phi}}{pf}\angle \cos^{-1}(pf)$

$$S_{3\phi} = 72\angle 60^\circ \text{ kVA}$$

let I_A be the A-phase line current for 3ϕ load.

$$S_{3\phi} = 3 V_{AN} I_A^* \qquad I_A = 100\angle -60^\circ \text{ Arms}$$

$$I_B = 100\angle 180^\circ \text{ Arms} \qquad I_C = 100\angle 60^\circ \text{ Arms}$$

$$I_{aA} = I_A + I_{AN} = 264\angle -19.1^\circ \text{ Arms}$$

$$I_{bB} = I_B + I_{BN} = 195\angle -146^\circ \text{ Arms}$$

$$I_{cC} = I_C + I_{CN} = 312\angle 104^\circ \text{ Arms}$$

$$I_{nN} = -I_{aA} - I_{bB} - I_{cC}$$

$$\boxed{I_{nN} = 109\angle -96.6^\circ \text{ Arms}}$$

11.59 A three-phase *abc*-sequence wye-connected source with $\mathbf{V}_{an} = 220\,\underline{/0^\circ}$ V rms supplies power to a wye-connected load that consumes 50 kW of power in each phase at a pf of 0.8 lagging. Three capacitors are found that each have an impedance of $-j2.0\ \Omega$, and they are connected in parallel with the previous load in a wye configuration. Determine the power factor of the combined load as seen by the source.

SOLUTION:

$P_{1\phi} = 50\text{kw} \qquad pf = 0.8 \text{ lagging}$

$P_{1\phi} = |V_{an}||I_{aA}|pf \Rightarrow |I_{aA}| = \dfrac{50{,}000}{0.8(220)} = 284 \text{ Arms}$

$\theta_{V_{an}} - \theta_{I_{aA}} = \cos^{-1}(pf) = 36.9^\circ \qquad \theta_{I_{aA}} = -36.9^\circ$

$I_{aA} = 284\,\underline{/-36.9^\circ} \text{ Arms} \qquad \theta_Z = \theta_S = 36.9^\circ$

$Q_{1\phi} = |S_{1\phi}|\sin(\theta_S) \qquad |S_{1\phi}| = P_{1\phi}/pf = 62.5 \text{ kVA}$

$Q_{1\phi} = 37.5 \text{ kVAR}$

$Q_C = |V_{an}|^2/X_C = 220^2/(-2) = -24.2 \text{ kVAR}$

$Q_{new} = Q_{1\phi} + Q_C = 13.3 \text{ kVAR}$

$S_{new} = P_{1\phi} + jQ_{new} = 50 + j13.3 \text{ kVA}$

$\theta_{S_{new}} = \tan^{-1}\left(\dfrac{Q_{new}}{P_{1\phi}}\right) = 14.9^\circ \qquad pf_{new} = \cos(\theta_{S_{new}})$

$\boxed{pf_{new} = 0.966 \text{ lagging}}$

11.60 If the three capacitors in the network in Problem 11.59 are connected in a delta configuration, determine the power factor of the combined load as seen by the source.

SOLUTION:

In delta: $Z_{cap} = -j2\Omega$

In wye: $Z_{cap} = -j2/3\ \Omega$

From 11.59: $V_{an} = 220\angle 0°\ V_{rms}$ $\quad P_{1\phi} = 50kW \quad pf = 0.8$ lagging

$$S_{old} = \frac{P_{1\phi}}{pf}\angle\cos^{-1}(pf) = \frac{50}{0.8}\angle\cos^{-1}(0.8)\ kVA = 62.5\angle 36.9°\ kVA$$

$Q_{old} = 37.5$ kVAR

$$Q_{cap} = -|V_{an}|^2/|Z_{cap}| = -\frac{220^2}{\frac{2}{3}} = -72.6\ kVAR$$

$$Q_{new} = Q_{old} + Q_{cap} = -35.1\ kVAR$$

$$S_{new} = P_{1\phi} + jQ_{new} = 50 - j35.1\ kVA$$

$$pf = \cos\left[\tan^{-1}\left(\frac{-35.1}{50}\right)\right]$$ $\boxed{pf = 0.818 \text{ leading}}$

11.61 Find C in the network in Fig. P11.61 such that the total load has a power factor of 0.9 lagging.

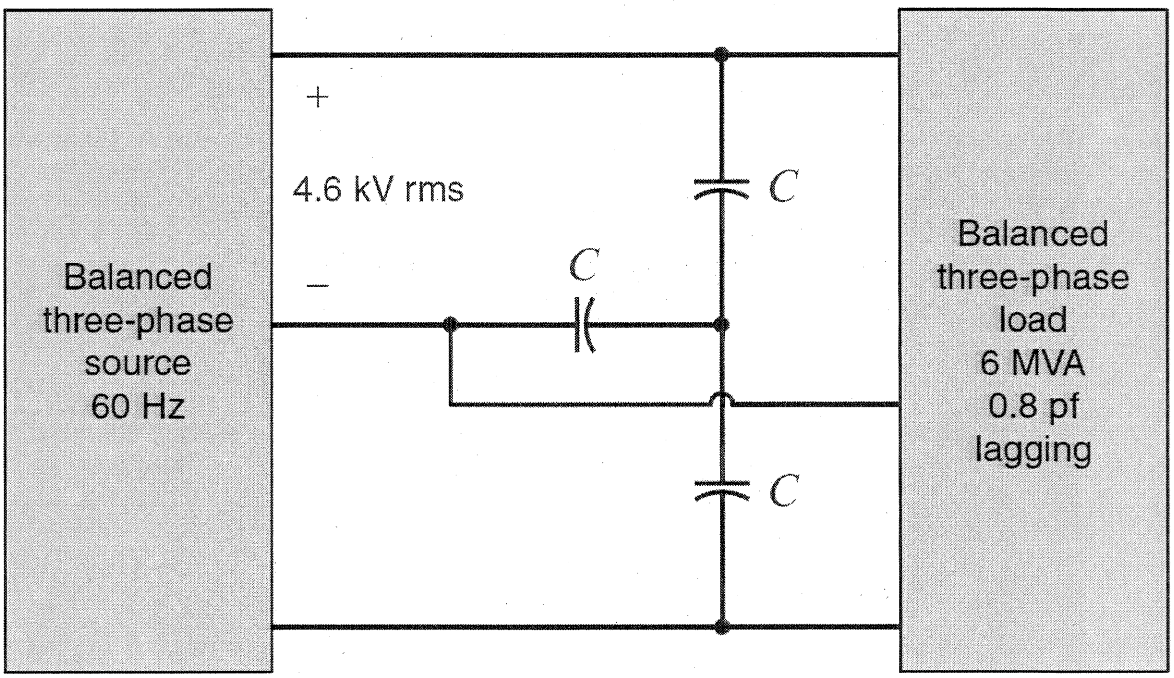


Figure P11.61

SOLUTION:

old case: $P_{1\phi} = \frac{S_{3\phi}}{3}(pf_{old}) = \frac{6\times10^6}{3}(0.8) = 1.6\,MW$

$\theta_S = \cos^{-1}(pf) = \cos^{-1}(0.8) = 36.9°$

$Q_{1\phi} = \frac{S_{3\phi}}{3}\sin(\theta_S) = 1.2\,MVAR$

new case: $P_{1\phi} = 1.6\,MW$

$Q_{new} = \frac{P_{1\phi}}{pf_{new}}\sin\left[\cos^{-1}(pf_{new})\right] = \frac{1.6\times10^6}{0.9}\sin\left[\cos^{-1}(0.9)\right]$

$Q_{new} = 775\,kVAR$

capacitor: $Q_C = Q_{new} - Q_{old} = -425\,kVAR = -|V_{ab}|^2\omega C$

$|V_{ab}| = 4.6\,kV \qquad \omega = 377\,r/s$

$\boxed{C = 53.3\,\mu F}$

11.62 Find C in the network in Fig. P11.61 so that the total load has a power factor of 0.9 leading. CS

SOLUTION:

$|V_{ab}| = 4.6\text{ kV}_{rms}$ $\quad \omega = 377\ r/s$

At load: $|S_{3\phi}| = 6\text{ MVA}$ $\quad$ pf $= 0.8$ lagging

$|S_{1\phi}| = \dfrac{|S_{3\phi}|}{3} = 2\text{ MVA}$ $\quad P_{1\phi} = (S_{1\phi})\text{pf} = 1.6\text{ MW}$ $\quad \theta_S = \theta_Z = \cos^{-1}(0.8) = 36.9°$

$S_{1\phi} = 2\,\angle 36.9°\text{ MVA}$ $\quad Q_{1\phi} = 1.2\text{ MVAR} = Q_{old}$

after pf correction: $P_{1\phi} = 1.6\text{ mW}$ $\quad \theta_S = \theta_Z = -\cos^{-1}(0.9) = -25.8°$

$|S_{new}| = \dfrac{P_{1\phi}}{\text{pf}_{new}} = \dfrac{1.6}{0.9} = 1.78\text{ MVA}$

$S_{new} = 1.78\,\angle -25.8°\text{ MVA}$ $\quad Q_{new} = -775\text{ kVAR}$

Capacitor: $Q_C = Q_{new} - Q_{old} = -1.975\text{ MVAR} = -|V_{ab}|^2 \omega C$

$C = \dfrac{1.975 \times 10^6}{377\,(4600)^2}$ $\quad \boxed{C = 248\,\mu F}$

11.63 Find C in the network in Fig. P11.63 such that the total load has a power factor of 0.87 leading. **PSV**

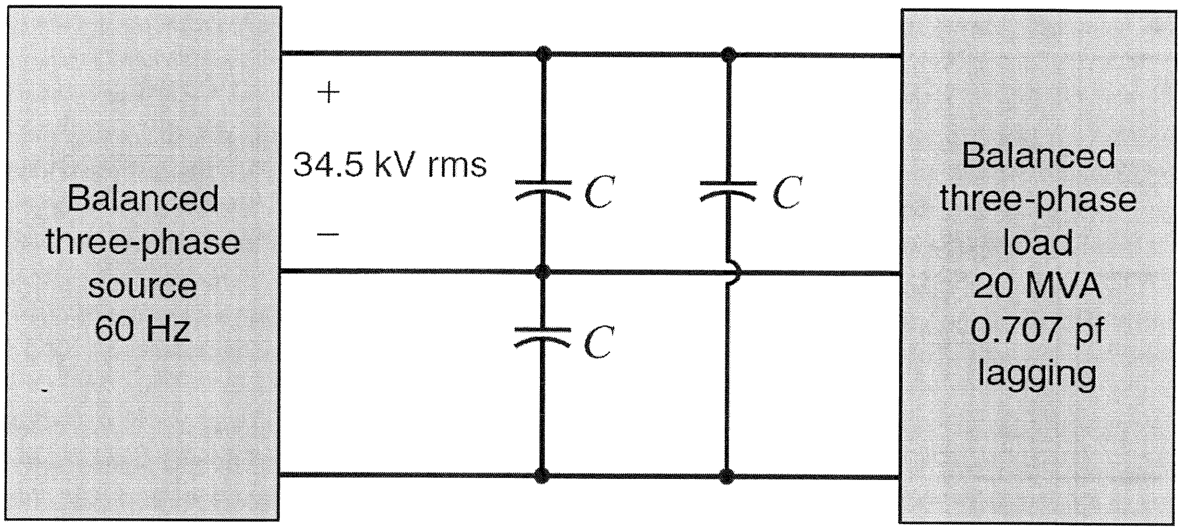


Figure P11.63

SOLUTION:

$|V_{ab}| = 34.5\,kV_{rms}$ At load: $|S_{3\phi}| = 20\,MVA$ pf$=0.707$ lagging

$\theta_{S\,old} = \cos^{-1}(pf) = 45°$ $S_{1\phi} = \frac{|S_{3\phi}|}{3} \angle \theta_{S\,old} = 6.67 \angle 45°$ MVA

$S_{1\phi} = 4.71 + j\,4.71$ MVA $\omega = 377\,r/s$

$Q_{old} = 4.71$ MVAR

After pf corrections: $|S_{1\phi}| = P_{1\phi}/pf_{new} = 4.71 \times 10^6 / 0.87 = 5.42$ MW

$\theta_{S\,new} = -\cos^{-1}(0.87) = -29.5°$ $S_{new} = 5.42 \angle -29.5°$ MVA

$Q_{new} = -2.67$ MVAR

Capacitor: $Q_C = Q_{new} - Q_{old} = -7.38\,MVAR = -|V_{ab}|^2 \omega C$

$\boxed{C = 16.4\,\mu F}$

11.64 Find the value of C in Problem 11.63 such that the total load has a power factor of 0.87 lagging.

SOLUTION:

$|V_{ab}| = 34.5\text{ kV}_{rms}$ At load: $|S_{3\phi}| = 20\text{MVA}$ pf $= 0.707$ lagging

$S_{3\phi} = 20\angle\theta_s$ MVA $\theta_s = \cos^{-1}(0.707) = 45°$ $\omega = 377\text{ r/s}$

$Q_{3\phi} = 20\sin(45°) = 14.14$ MVAR $Q_{old} = \frac{Q_{3\phi}}{3} = 4.71$ MVAR

After pf correction

$$P_{1\phi} = \frac{|S_{3\phi old}|}{3}(pf_{old}) = \frac{20}{3}(0.707) = 4.71\text{ MW}$$

$$S_{1\phi new} = \frac{P_{1\phi}}{pf_{new}}\angle\cos^{-1}(pf_{new}) = \frac{4.71\times10^6}{0.87}\angle\cos^{-1}(0.87) = 5.42\angle 29.5\text{ MVA}$$

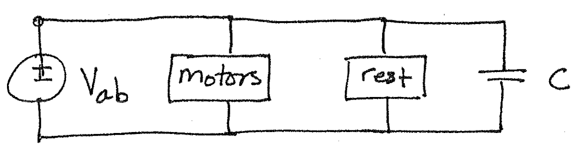
$$Q_{new} = S_{1\phi new}\sin(\theta_{new}) = 5.42\sin(29.5) = 2.67\text{ MVAR}$$

Capacitor: $Q_c = Q_{new} - Q_{old} = -2.04\text{ MVAR} = -|V_{ab}|^2\omega C$

$$\boxed{C = 4.54\,\mu F}$$

11.65 The U.S. Department of Energy estimates that in 2001 the electrical energy consumption in the United States was roughly 3.6 TW. (Check the URL http://www.eia.doe.gov/emeu/aer/txt/ptb0805.html for a complete breakdown.) Of that total, 1 TW was consumed in the industrial sector where large electric motor loads represent 60% of the usage. Suppose that the average uncorrected power factor for these motor loads is 0.7 lagging and that the power factor for the remaining loads is 0.9 lagging. Determine the total capacitance required nationwide to correct everything to unity power factor. Assume that all capacitors are connected to 480 V rms line voltages.

SOLUTION:



$P_{rest} = 3.6\times10^{12} - 0.6\times10^{12} = 3\text{ TW}$

$pf_{rest} = 0.9$ lagging

$|S_{rest}| = P_{rest}/pf_{rest} = 3.33\text{ TVA}$

$\theta_{rest} = \cos^{-1}(pf_{rest}) = 25.8°$

$|V_{ab}| = 480\text{ V rms}$

$P_{motor} = (0.6)(1000\times10^6) = 600\text{ MW}$

$pf_{motor} = 0.7$ lagging

$|S_{motor}| = \dfrac{P_{motor}}{pf_{motor}} = 857\text{ MVA}$

$\theta_{motor} = \cos^{-1}(pf_{motor}) = 45.6°$

$S_{motor} = 857\angle 45.6\text{ MVA}$

$S_{3\phi} = S_{rest} + S_{motor} = 3.33\times10^9\angle 25.8° + 857\times10^6\angle 45.6°$

$S_{3\phi} = 4.15\angle 29.8°\text{ TVA}$ $S_{1\phi} = S_{3\phi}/3 = 1.38\angle 29.8°\text{ TVA}$

$Q_{1\phi} = 688\text{ MVAR}$ $Q_C = -Q_{1\phi} = -688\times10^6 = -|V_{ab}|^2\omega C$

$C = 7925\text{ F}$ $C_{TOTAL} = 3C$ $\boxed{C_{TOTAL} = 23775\text{ F}}$

11.66 The power utilities typically transmit and distribute power as shown in Fig. P11.66 with high-voltage transmission for long distances and lower voltage distribution within a city or town. Also, the largest loads are, if possible, located "upstream" of lesser loads List at least two advantages of this arrangement.

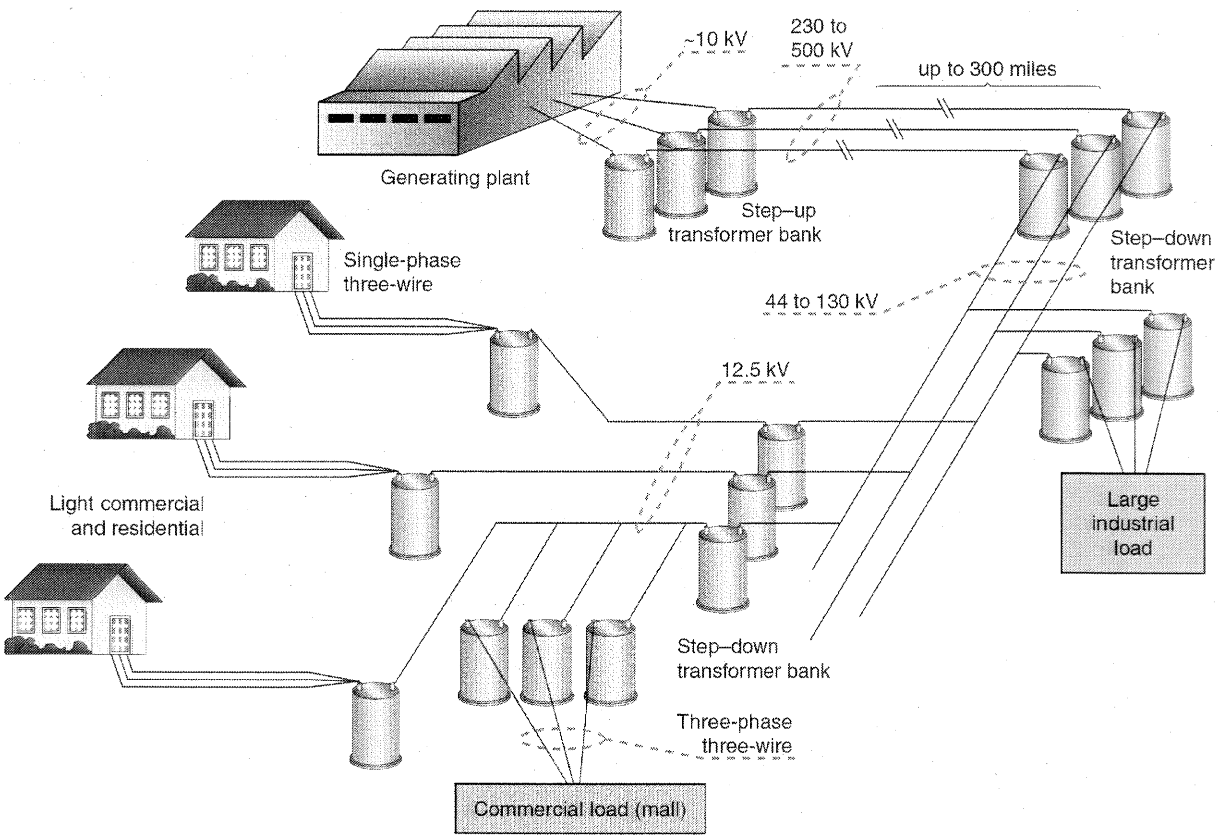


Figure P11.66

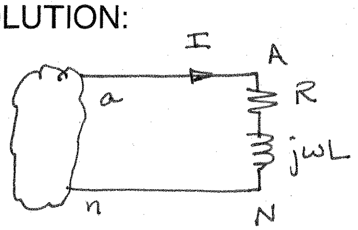
SOLUTION:

① Transmission at high voltages & low currents means transmission cables can be smaller. This decreases weight of cabling which lower cable costs and tower costs.

② Having larger loads "upstream" decreases the impact of large load changes (changes in current supplied to the larger loads) on "downstream" lesser loads.

11FE-1 A wye-connected load consists of a series *RL* impedance. Measurements indicate that the rms voltage across each element is 84.85 V. If the rms line current is 6 A, find the total complex power for the three-phase load configuration. CS

SOLUTION:



$|V_R| = 84.85 \text{ Vrms}$

$|V_L| = 85.85 \text{ V}_{rms}$

$|I| = 6 \text{ A}_{rms}$

$R = |V_R| / |I| = 14.14\,\Omega$ $\quad \omega L = |V_L| / |I| = 14.14\,\Omega$

$V_{AN} = V_R + V_L$ $\quad S_{1\phi} = V_{AN} I = |I|^2 Z_{LOAD}$

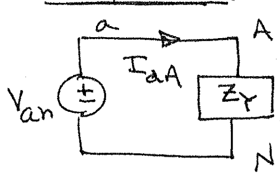
$Z_{LOAD} = 14.14 + j\,14.14\,\Omega = 20 \angle 45^\circ\,\Omega$

$S_{1\phi} = 720 \angle 45^\circ \text{ VA}$ $\quad S_{3\phi} = 3 S_{1\phi}$ $\quad \boxed{S_{3\phi} = 2.16 \angle 45^\circ \text{ kVA}}$

11FE-2 A balanced three-phase delta-connected load consists of an impedance of 12 + j12 Ω. If the line voltage at the load is measured to be 230 V rms, find the magnitude of the line current and the total real power absorbed by the three-phase configuration.

SOLUTION:

per phase Y



$|V_{AB}| = 230\ V_{rms}$ $\quad |V_{AN}| = |V_{AB}| / \sqrt{3} = 133\ V_{rms}$

$Z_\Delta = 12 + j12\ \Omega$ $\quad Z_Y = \frac{Z_\Delta}{3} = 4 + j4\ \Omega$

$|V_{AN}| = |I_{aA}||Z_Y| \Rightarrow |I_{aA}| = \frac{133}{4\sqrt{2}} = 23.5\ A_{rms}$

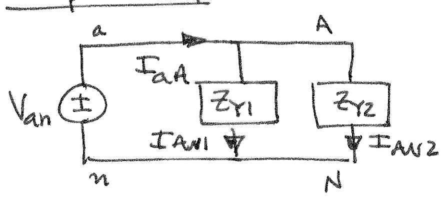
$P_{1\phi} = |I_{aA}|^2 R_Y = (23.5)^2(4) = 2.21\ kW$

$\boxed{|I_{aA}| = 23.5\ A_{rms}}$ $\quad P_{3\phi} = 3P_{1\phi} \Rightarrow \boxed{P_{3\phi} = 6.63\ kW}$

11FE-3 Two balanced three-phase loads are connected in parallel. One load with a phase impedance of $24 + j18\ \Omega$ is connected in delta, and the other load has a phase impedance of $6 + j4\ \Omega$ and is connected in wye. If the line-to-line voltage is 208 V rms, determine the line current.

SOLUTION:

Per phase Y



$Z_{\Delta 1} = 24 + j18\ \Omega \qquad Z_{Y1} = \frac{Z_{\Delta 1}}{3} = 8 + j6\ \Omega$

$Z_{Y2} = 6 + j4\ \Omega \qquad |V_{AB}| = 208\ V_{rms}$

$|V_{AN}| = \frac{|V_{AB}|}{\sqrt{3}} = 120\ V_{rms}$

$I_{AN2} = \frac{V_{AN}}{Z_{Y2}} = \frac{120\angle 0^\circ}{6+j4} = 16.6\angle -33.7^\circ\ A_{rms}$ assume $\theta_{V_{AN}} = 0^\circ$!

$I_{AN1} = \frac{V_{AN}}{Z_{Y1}} = \frac{120\angle 0^\circ}{8+j6} = 12\angle -36.9^\circ\ A_{rms}$

$I_{aA} = I_{AN1} + I_{AN2}$ $\boxed{I_{aA} = 28.6\angle -35.0^\circ\ A_{rms}}$

11FE-4 The total complex power at the load of a three-phase balanced system is $24 \angle 30°$ kVA. Find the real power per phase.

SOLUTION:

$S_{3\phi} = 24 \angle 30°$ kVA $S_{1\phi} = \frac{S_{3\phi}}{3} = 8 \angle 30°$ kVA

$P_{1\phi} = |S_{1\phi}| \cos(30°) = 8(0.866) = 6.93$ kW

$\boxed{P_{1\phi} = 6.93 \text{ kW}}$