Chapter Twelve:

Variable-Frequency Network Performance

12.1 Determine the driving point impedance at the input terminals of the network shown in Fig. P12.1 as a function of s. **CS**

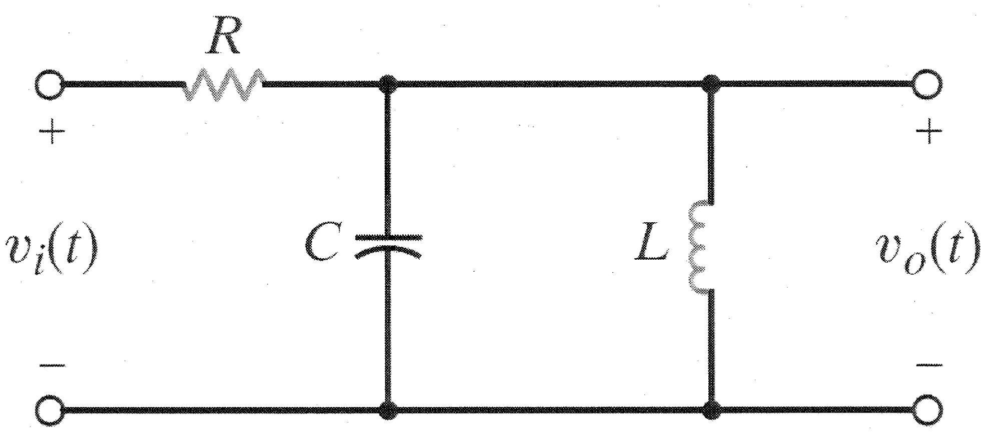
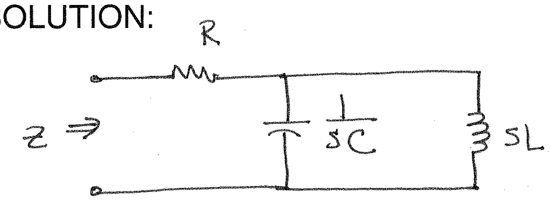


Figure P12.1

SOLUTION:



$$Z = R + \frac{(sL)(1/sC)}{sL + 1/sC}$$

$$= R + \frac{sL}{s^2LC+1} = \frac{s^2LCR + R + sL}{s^2LC+1}$$

$$\boxed{Z = R\left[\frac{s^2 + \frac{s}{RC} + \frac{1}{LC}}{s^2 + \frac{1}{LC}}\right]}$$

12.2 Determine the voltage transfer function $\mathbf{V}_o(s)/\mathbf{V}_i(s)$ as a function of s for the network shown in Fig. P12.2.
PSV

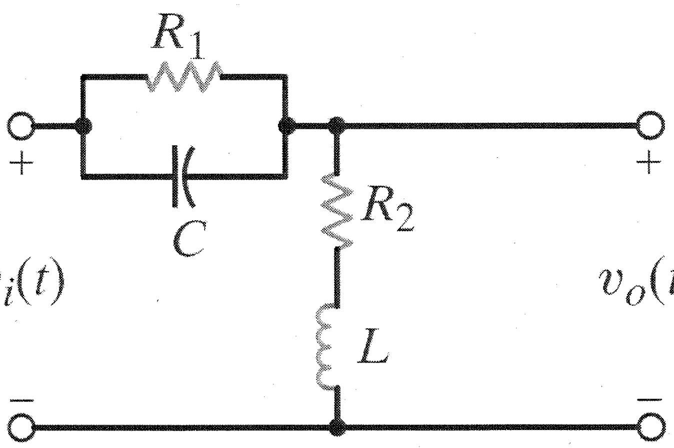
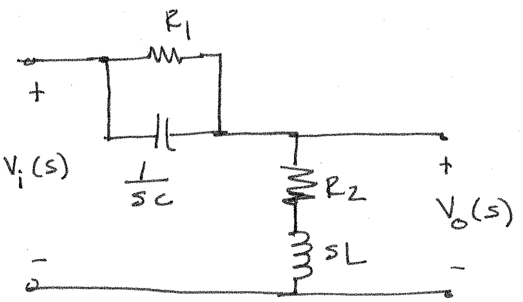


Figure P12.2

SOLUTION:



Let $Z_1 = \dfrac{R_1 /sC}{R_1 + 1/sC} = \dfrac{R_1}{sR_1C+1}$

$Z_2 = sL + R_2$

$$\frac{V_o}{V_i}(s) = \frac{Z_2}{Z_1+Z_2} = \frac{sL+R_2}{sL+R_2+\dfrac{R_1}{sR_1C+1}}$$

$$\boxed{\frac{V_o}{V_I}(s) = \frac{s^2+s\left[\frac{R_2}{L}+\frac{1}{R_1C}\right]+\frac{R_2/R_1}{LC}}{s^2+s\left[\frac{R_2}{L}+\frac{1}{R_1C}\right]+\frac{(R_1+R_2)/R_1}{LC}}}$$

12.3 Determine the driving point impedance at the input terminals of the network shown in Fig. P12.3 as a function of *s*.

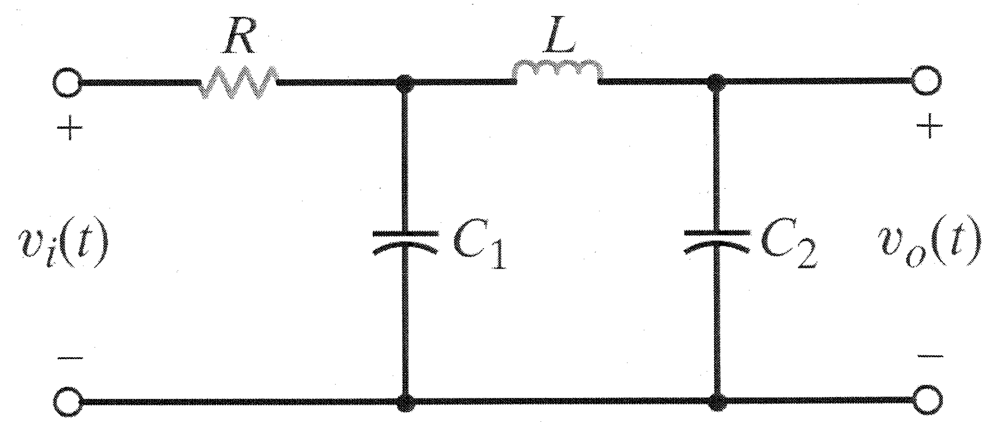
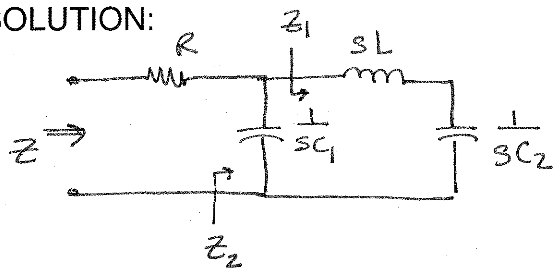


Figure P12.3

SOLUTION:



$$Z_1 = sL + 1/sC_2$$

$$Z_2 = \left(\frac{1}{sC_1}\right) Z_1 \Big/ \left(\frac{1}{sC_1} + Z_1\right)$$

$$Z_2 = \frac{s^2 + \frac{1}{LC_2}}{sC_1\left[s^2 + \frac{C_1 + C_2}{LC_1C_2}\right]}$$

$$Z = R + \frac{s^2 + \frac{1}{LC_2}}{sC_1\left[s^2 + \frac{C_1 + C_2}{LC_1C_2}\right]}$$

$$\boxed{Z = R\left\{\frac{s^3 + \frac{s^2}{C_1R} + \frac{(C_1 + C_2)}{LC_1C_2}s + \frac{1}{RLC_1C_2}}{s\left(s^2 + \frac{C_1 + C_2}{LC_1C_2}\right)}\right\}}$$

12.4 Find the transfer impedance $\mathbf{V}_o(s)/\mathbf{I}_S(s)$ for the network shown in Fig. P12.4.

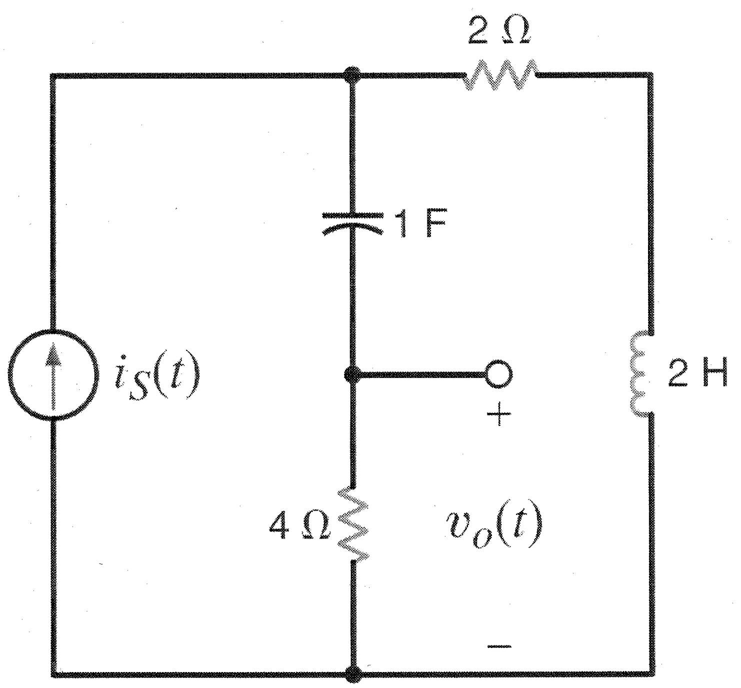
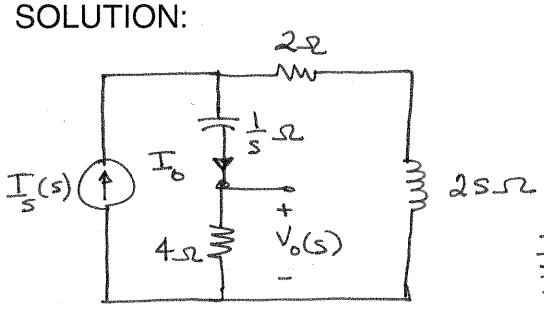


Figure P12.4

SOLUTION:



Let $Z_1 = 4 + 1/s = \dfrac{4s+1}{s}\ \Omega$

$Z_2 = 2s + 2\ \Omega$

$$\frac{I_o}{I_s} = \frac{Z_2}{Z_1 + Z_2} = \frac{2s(s+1)}{2s^2 + 6s + 1}$$

$$\frac{V_o}{I_s} = 4\frac{I_o}{I_s} = \boxed{\frac{8s(s+1)}{2s^2+6s+1} = \frac{V_o}{I_s}}$$

12.5 Find the driving point impedance at the input terminals of the circuit in Fig. P12.5 as a function of s. **CS**

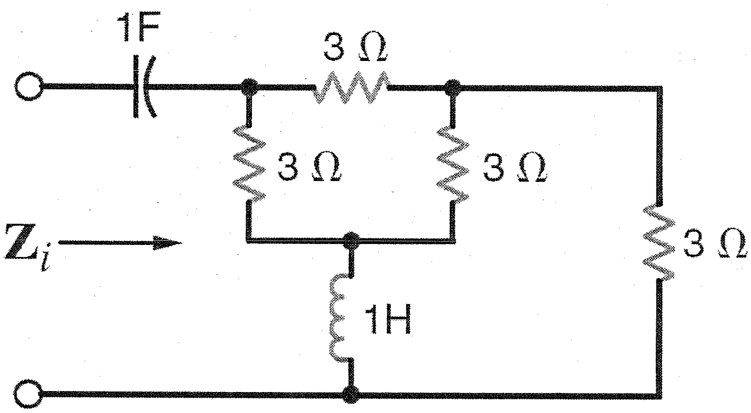
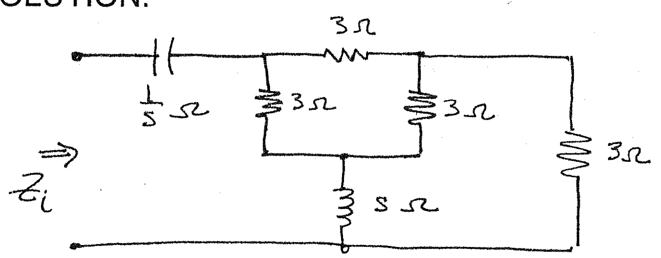
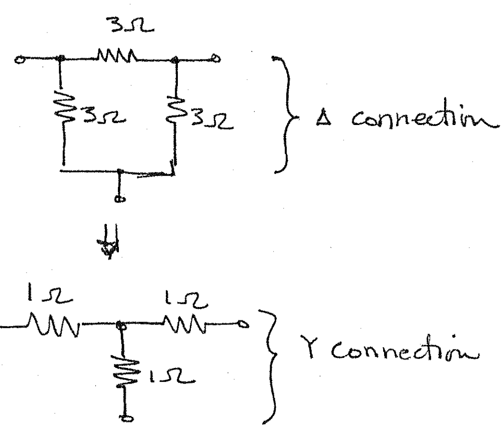
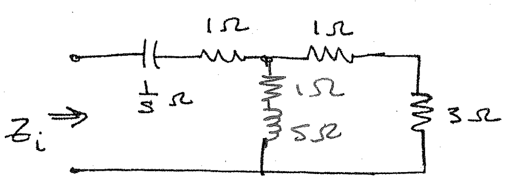


Figure P12.5

SOLUTION:







$Z_1 = 1 + 3 = 4\Omega$ $\quad Z_2 = (s+1)(Z_1)/(s+1+Z_1) = 4(s+1)/(s+5)$

$$Z_i = Z_2 + 1 + \frac{1}{s} = \frac{5(s^2+2s+1)}{s(s+5)}$$

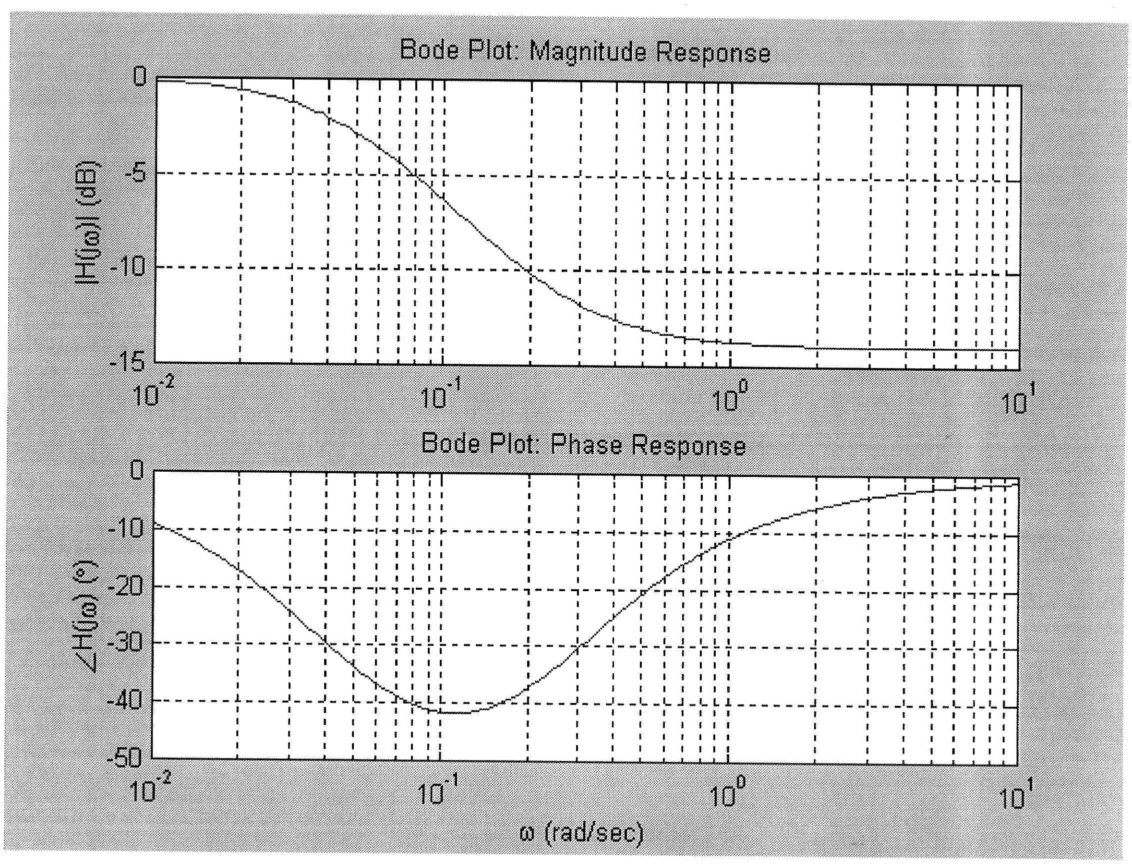
$$\boxed{Z_i = \frac{5(s^2+2s+1)}{s(s+5)}\ \Omega}$$

12.6 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{j\omega 4 + 1}{j\omega 20 + 1}$$

SOLUTION:

$$H(j\omega) = \frac{1}{5}\,\frac{j\omega + 1/4}{j\omega + 1/20}$$

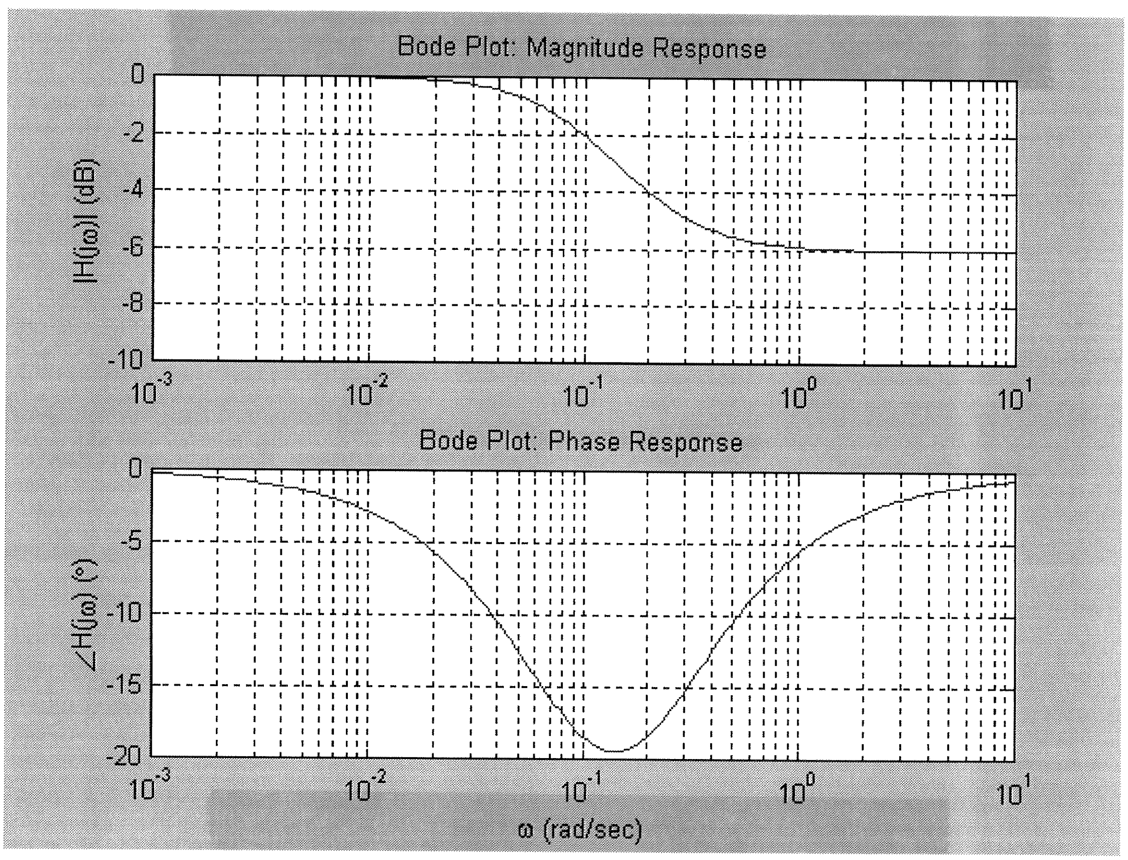


12.7 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{j\omega 5 + 1}{j\omega 10 + 1}$$

SOLUTION:

$$H(j\omega) = \frac{1}{2}\left[\frac{j\omega + 1/5}{j\omega + 1/10}\right]$$

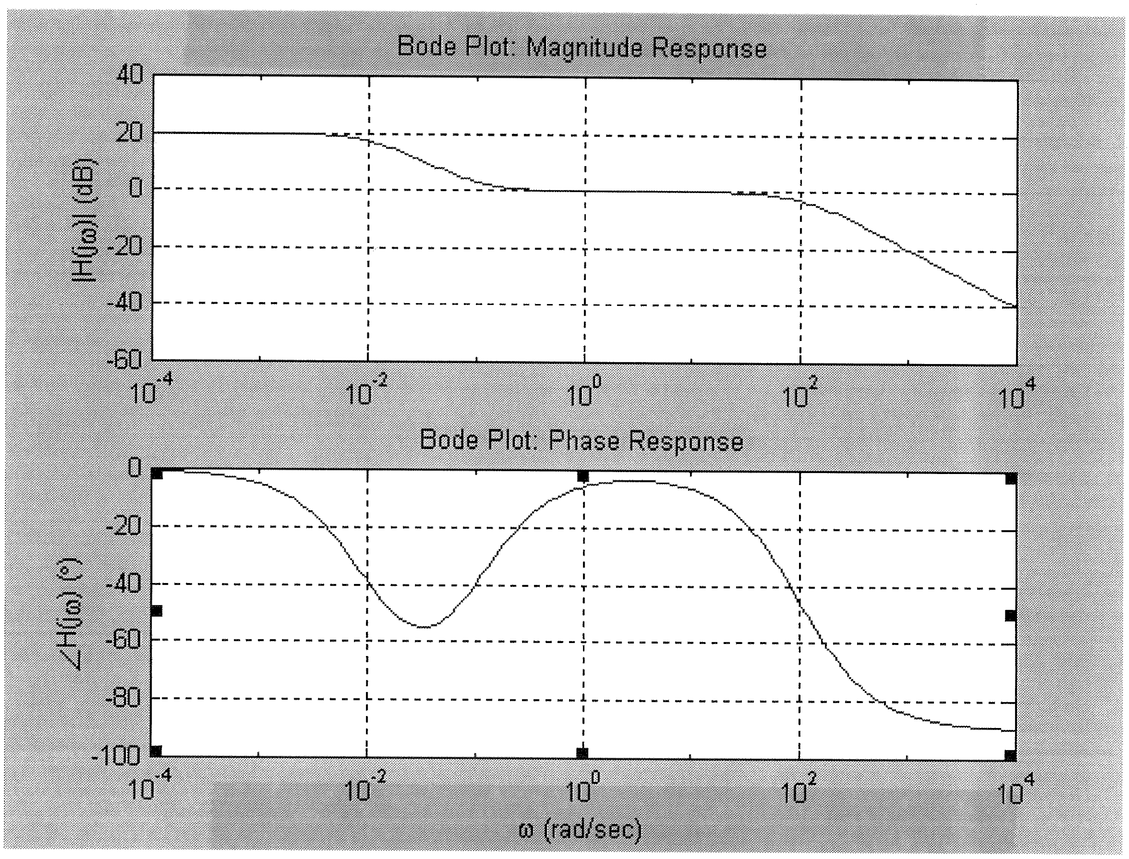


12.8 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{10(10j\omega + 1)}{(100j\omega + 1)(0.01j\omega + 1)}$$

SOLUTION:

$$H(j\omega) = \frac{100\left(j\omega + \frac{1}{10}\right)}{\left(j\omega + \frac{1}{100}\right)\left(j\omega + 100\right)}$$



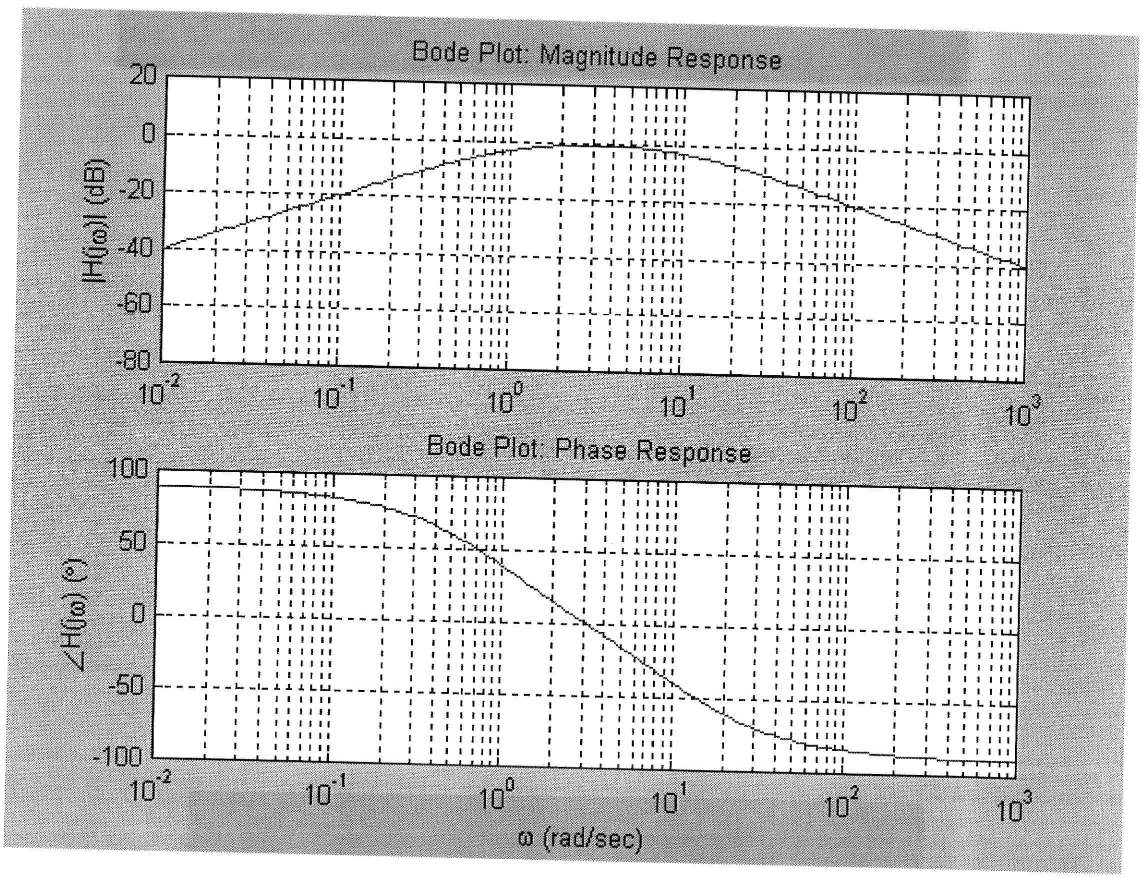
12.9 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{j\omega}{(j\omega + 1)(0.1j\omega + 1)}$$

CS

SOLUTION:

$$H(j\omega) = \frac{10(j\omega)}{(j\omega+1)(j\omega+10)}$$

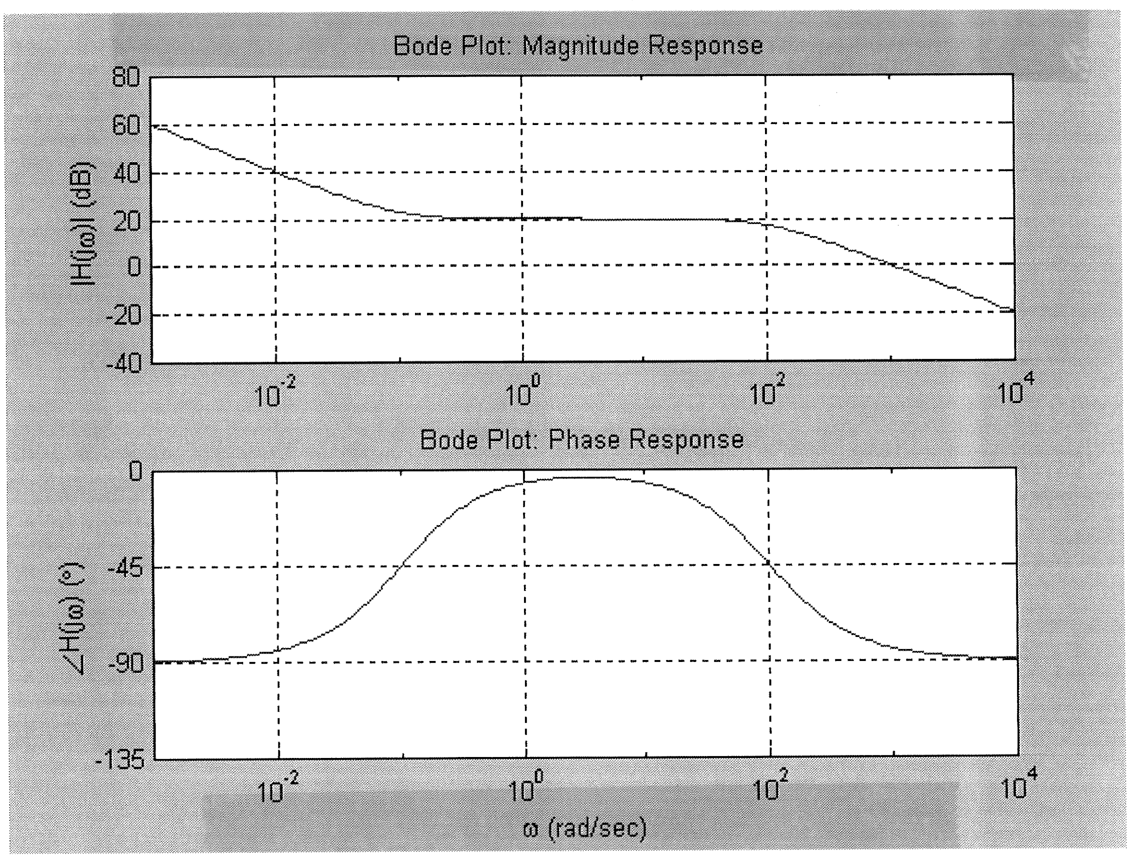


12.10 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{10j\omega + 1}{j\omega(0.01j\omega + 1)}$$

SOLUTION:

$$H(j\omega) = \frac{10(j\omega + 0.1)}{j\omega\frac{(j\omega+100)}{100}} = \frac{1000(j\omega+0.1)}{j\omega(j\omega+100)}$$



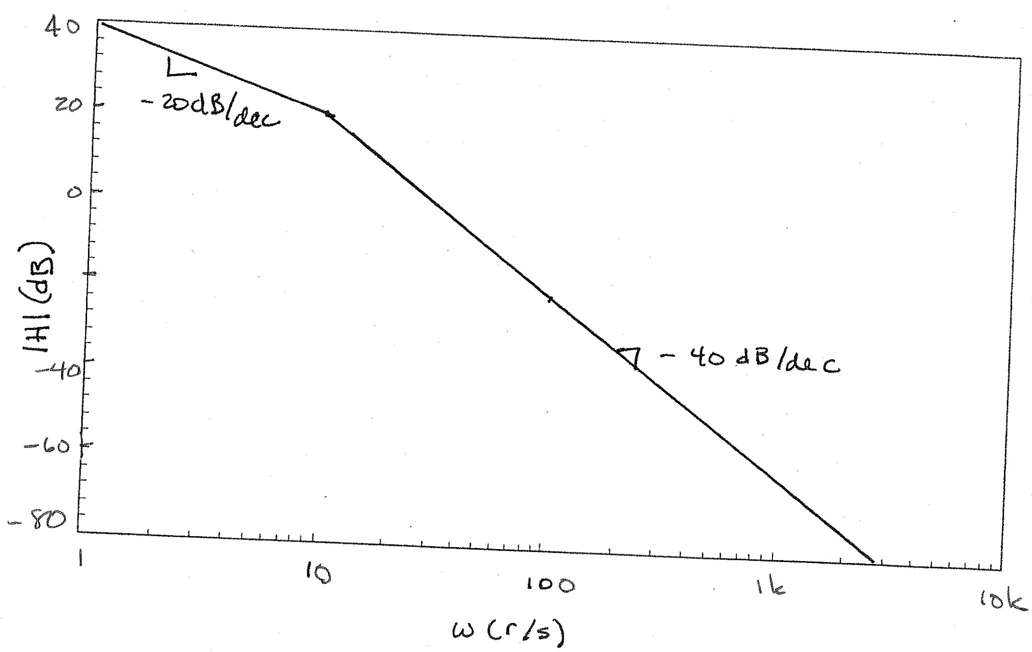
12.11 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{H}(j\omega) = \frac{100}{j\omega(0.1j\omega + 1)}$$ **PSV**

SOLUTION:

$$H(j\omega) = \frac{100}{j\omega\,\frac{(j\omega+10)}{10}} = \frac{1000}{j\omega(j\omega+10)}$$

$$|H|\Big|_{\omega=1} \approx \frac{1000}{(1)(10)} = 100 = 40\text{ dB}$$



12.12 Sketch the magnitude characteristic of the Bode plot for the transfer function

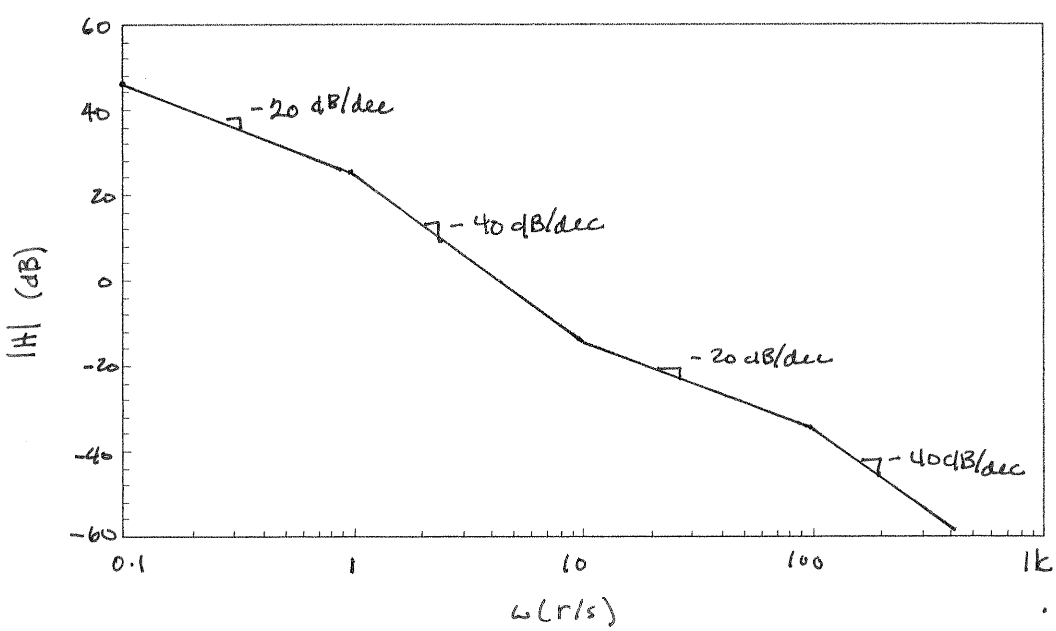
$$\mathbf{H}(j\omega) = \frac{20(0.1j\omega + 1)}{j\omega(j\omega + 1)(0.01j\omega + 1)}$$

SOLUTION:

$$H(j\omega) = \frac{20(0.1)(j\omega+10)}{j\omega(j\omega+1)(0.01)(j\omega+100)} = \frac{200(j\omega+10)}{j\omega(j\omega+1)(j\omega+100)}$$

As $\omega \to 0$, $|H| \to 20/\omega$

$$|H|\Big|_{\omega=0.1} \simeq \frac{200(10)}{(0.1)(1)(100)} = 200 = 46\,dB$$



12.13 Sketch the magnitude characteristic of the Bode plot for the transfer function

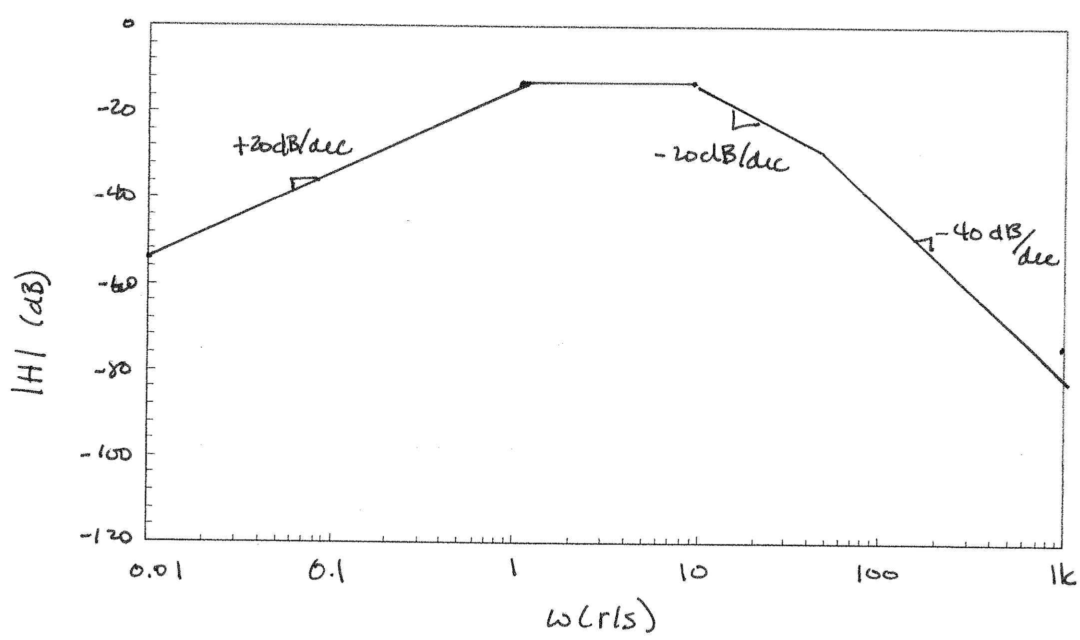
$$\mathbf{H}(j\omega) = \frac{100(j\omega)}{(j\omega + 1)(j\omega + 10)(j\omega + 50)}$$

CS

SOLUTION:

$$H(j\omega) = \frac{100\,(j\omega)}{(j\omega+1)(j\omega+10)(j\omega+50)}$$

$$|H|\Big|_{\omega=\frac{1}{100}} \simeq 2\times10^{-3} = -54\,dB$$

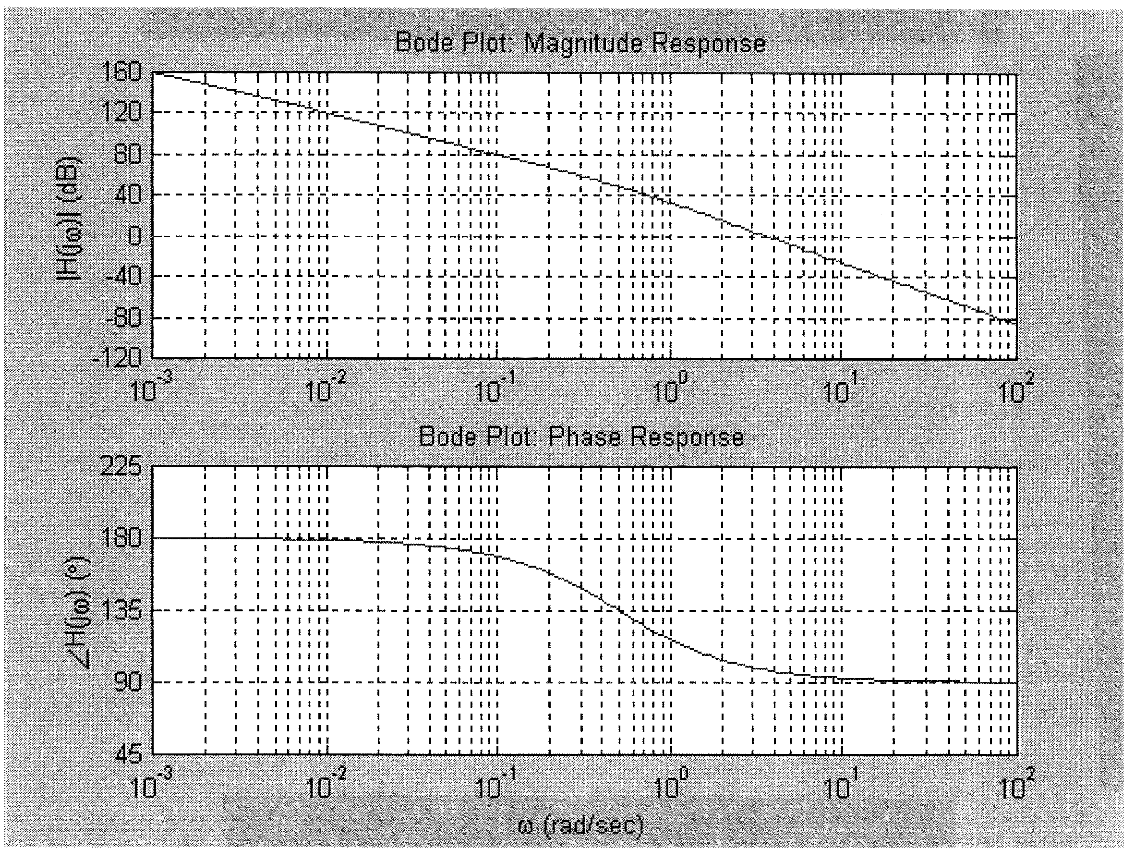


12.14 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{100}{(j\omega)^2(j\omega 2 + 1)}$$

SOLUTION:

$$H(j\omega) = \frac{100}{(j\omega)^2\,(j\omega + 1/2)2} = \frac{50}{(j\omega)^2(j\omega + 0.5)}$$



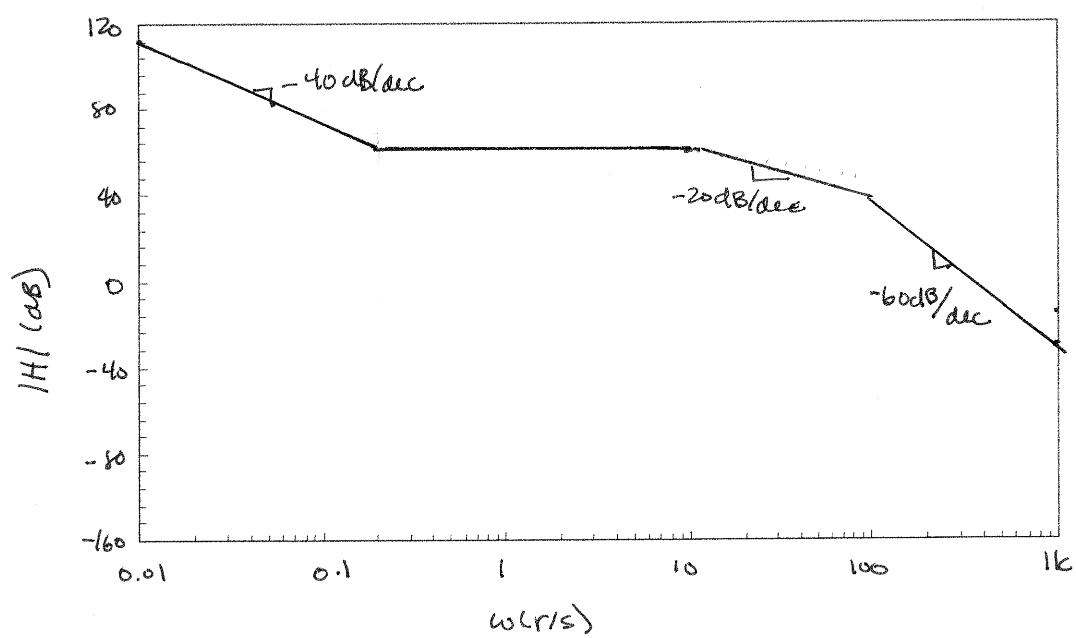
12.15 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{H}(j\omega) = \frac{36 \times 10^5(5j\omega + 1)^2}{(j\omega)^2(j\omega + 10)(j\omega + 100)^2}$$

SOLUTION:

$$H(j\omega) = \frac{36\times10^5 (25)(j\omega+0.2)^2}{(j\omega)^2(j\omega+10)(j\omega+100)^2} = \frac{9\times10^7(j\omega+0.2)^2}{(j\omega)^2(j\omega+10)(j\omega+100)^2}$$

$$|H|\Big|_{\omega=0.01} \simeq \frac{9\times10^7(0.2)^2}{(0.01)^2(10)(100)^2} = 3.60\times10^5 = 111\text{ dB}$$



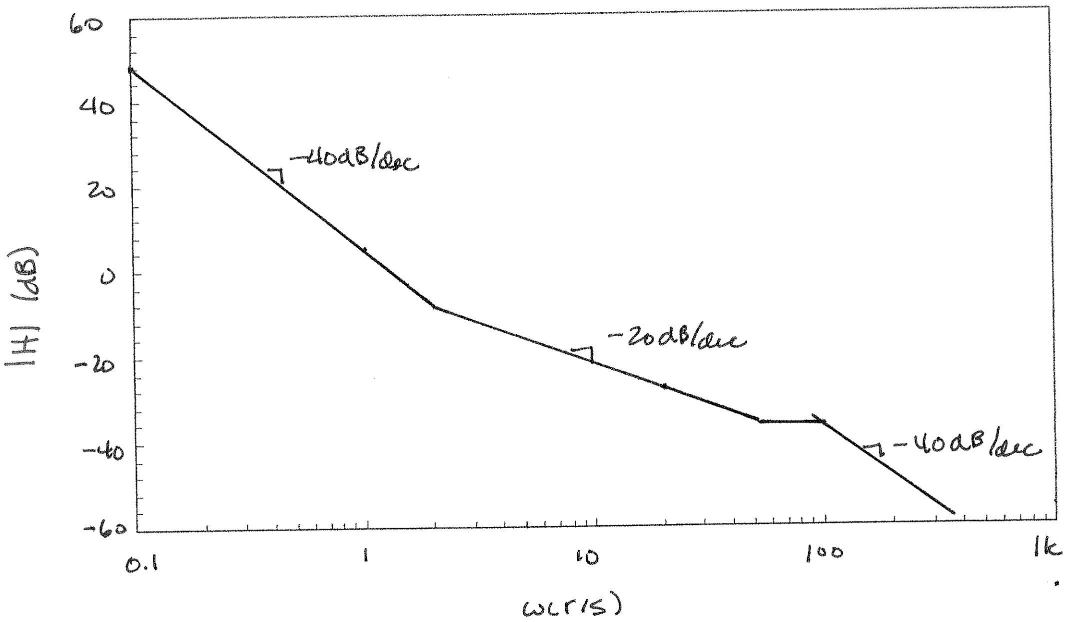
12.16 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{400(j\omega + 2)(j\omega + 50)}{-\omega^2(j\omega + 100)^2}$$

SOLUTION:

$$G(j\omega) = \frac{400\,(j\omega + 2)(j\omega + 50)}{-\omega^2\,(j\omega + 100)^2} \qquad |G|\Big|_{\omega = 0.1} \approx \frac{400\,(2)(50)}{(0.1)^2\,(100)^2} = 400$$

$$= 52.0\,dB$$



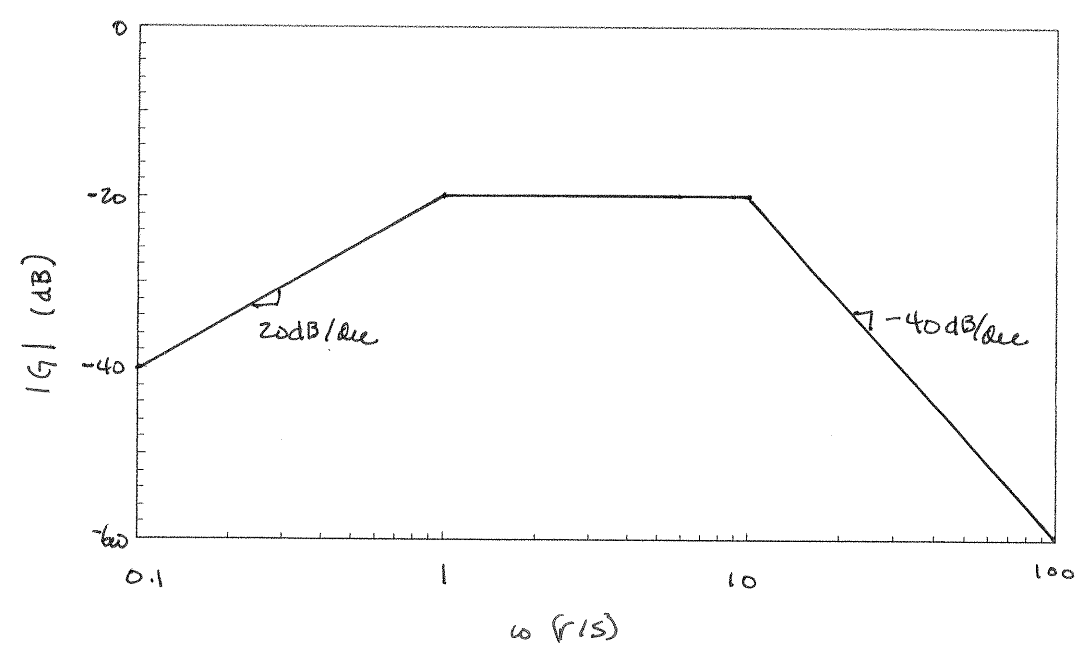
12.17 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{10j\omega}{(j\omega + 1)(j\omega + 10)^2}$$

CS

SOLUTION:

$$|G|\Big|_{\omega=0.1} \approx \frac{10(0.1)}{1(10)^2} = \frac{1}{100} = -40\text{ dB}$$

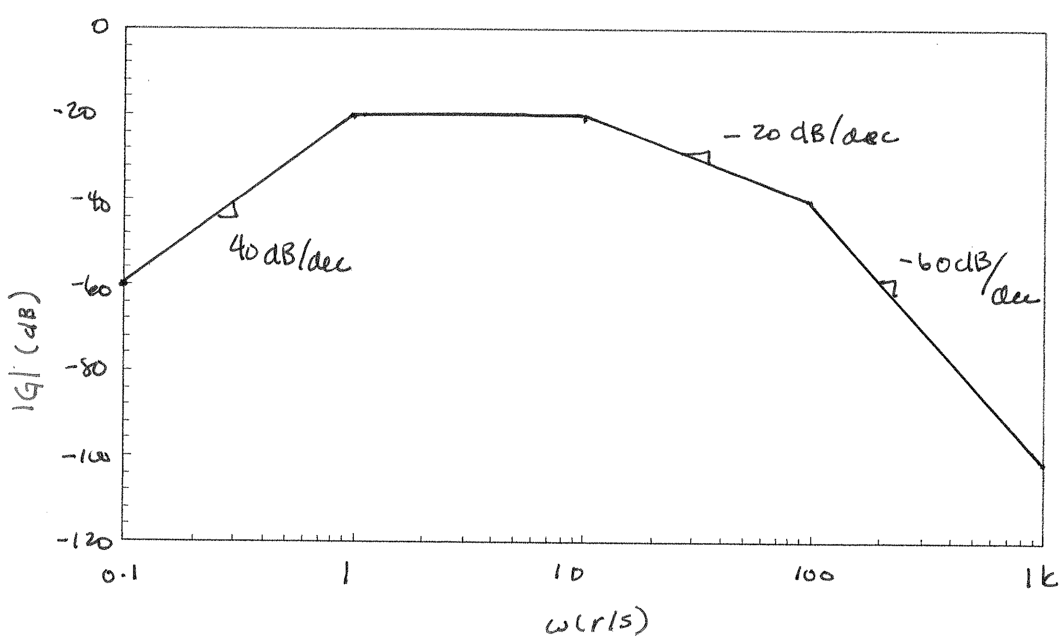


12.18 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{-\omega^2 10^4}{(j\omega + 1)^2(j\omega + 10)(j\omega + 100)^2}$$

SOLUTION:

$$|G|\Big|_{\omega=0.1} \approx \frac{(0.1)^2(10^4)}{(1)^2(10)(100)^2} = 10^{-3} = -60\text{ dB}$$



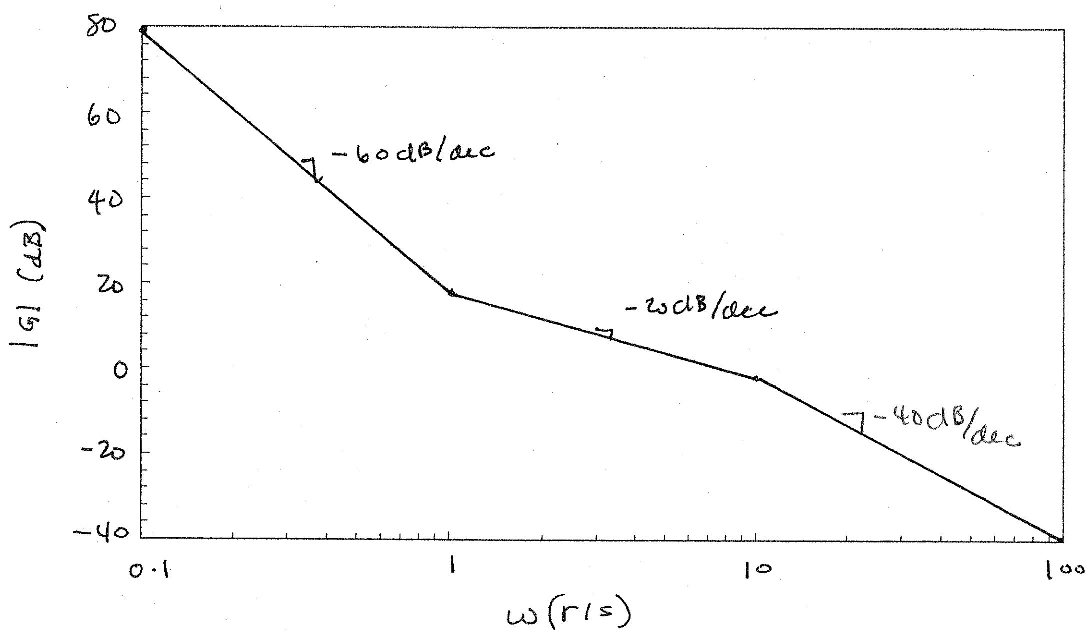
12.19 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{8(j\omega + 1)^2}{-j\omega^3(0.1j\omega + 1)}$$

SOLUTION:

$$|G(j\omega)|\Big|_{\omega=0.1} \approx \frac{8(1)^2}{(0.1)^3(1)} = 8000 = 78\text{dB}$$

$$G(j\omega) = \frac{80(j\omega+1)^2}{-j\omega^3(\omega+10)}$$

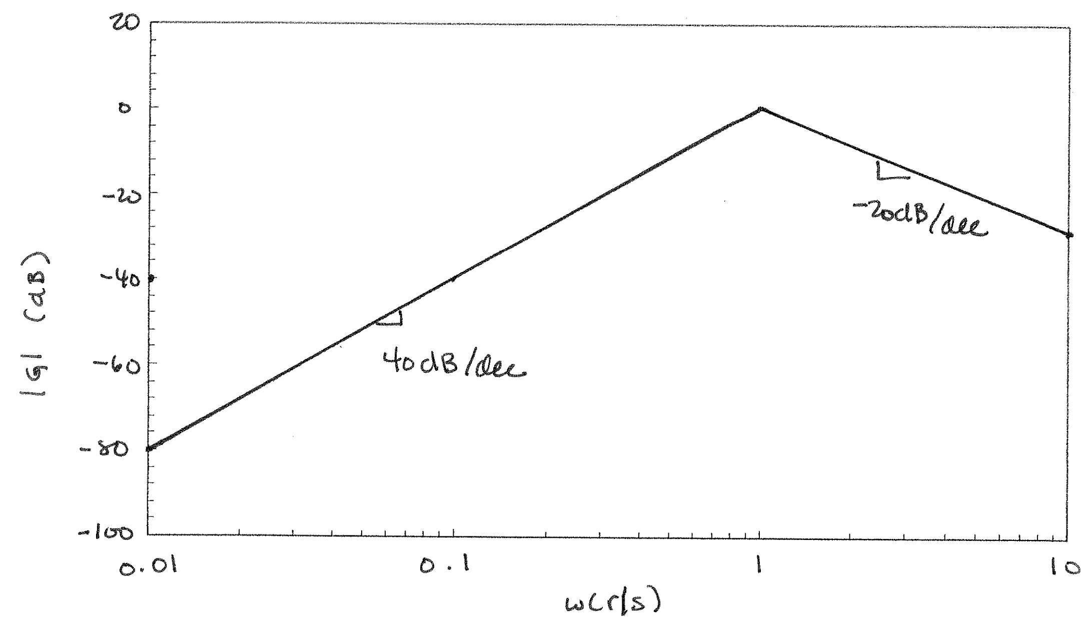


12.20 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$G(j\omega) = \frac{-\omega^2}{(j\omega + 1)^3} \quad \text{CS}$$

SOLUTION:

$$|G|\Big|_{\omega=0.1} \approx \frac{(0.1)^2}{(1)^3} = 10^{-2} = -40\,\text{dB}$$



12.21 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{10^4(j\omega + 1)(-\omega^2 + 6j\omega + 225)}{j\omega(j\omega + 50)^2(j\omega + 450)}$$

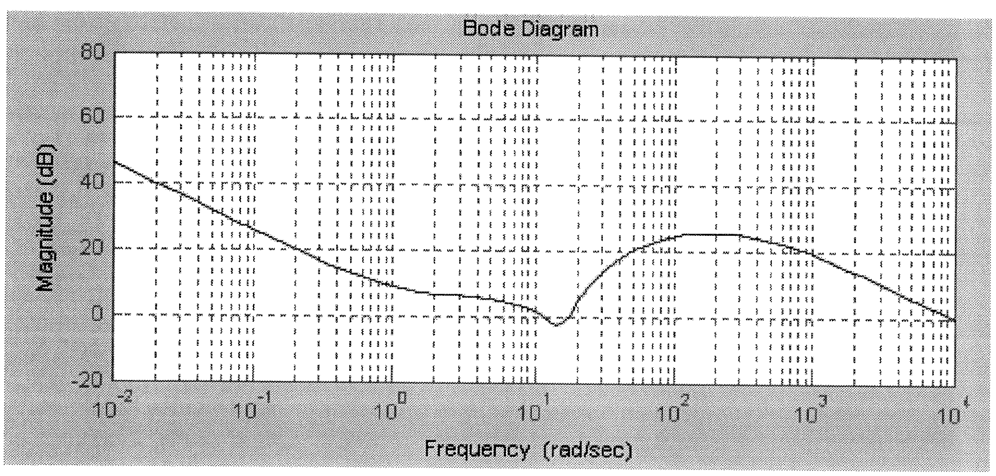
SOLUTION:

$$|G|\Big|_{\omega=0.1} \approx \frac{10^4(1)(225)}{(0.1)(50)^2(450)} = 20 = 26\text{dB}$$

$$-\omega^2 + 6j\omega + 225 = (j\omega)^2 + 6j\omega + 225 = (j\omega + 3 + j14.7)(j\omega + 3 - j14.7)$$

$$\text{or, } \frac{1}{225}\left[1 - \frac{\omega^2}{225} + \frac{6j\omega}{225}\right] = \frac{1}{225}\left[1 - (\omega\tau)^2 + 2j\zeta(\omega\tau)\right]$$

$$\text{So, } \tau = 1/\sqrt{225} = 1/15 \qquad \zeta = \frac{6}{225(2)\tau} = 0.2$$



12.22 Sketch the magnitude characteristic of the Bode plot for the transfer function

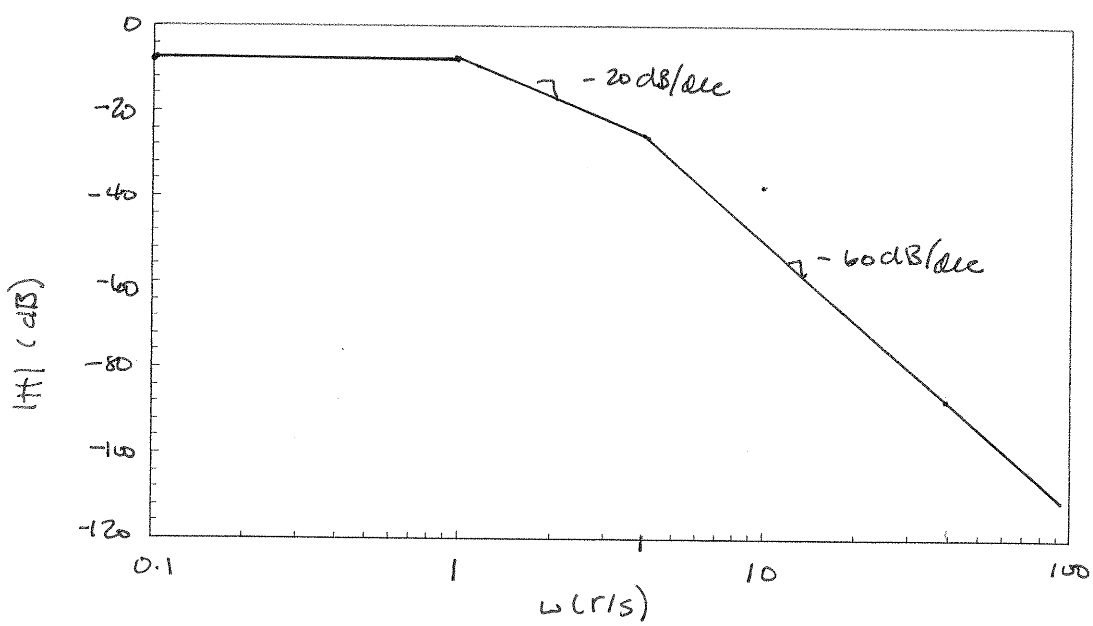
$$\mathbf{H}(j\omega) = \frac{+6.4}{(j\omega + 1)(-\omega^2 + 8j\omega + 16)}$$

SOLUTION:

$$-\omega^2 + 8j\omega + 16 = 16\left\{\left(1-\left(\frac{\omega}{4}\right)^2\right) + j\,\omega/2\right\} = 16\left\{1-(\omega\tau)^2 + 2j\,\zeta\omega\tau\right\}$$

$$\tau = 1/4 \text{ s} \qquad \zeta = \frac{1}{2(2)\tau} = 1$$

$$H(j\omega) = \frac{6.4}{(j\omega+1)(j\omega+4)^2} \qquad |H|\Big|_{\omega=0.1} \simeq \frac{6.4}{(1)(16)} = 0.4 = -8\text{dB}$$



12.23 Sketch the magnitude characteristic of the Bode plot for the transfer function

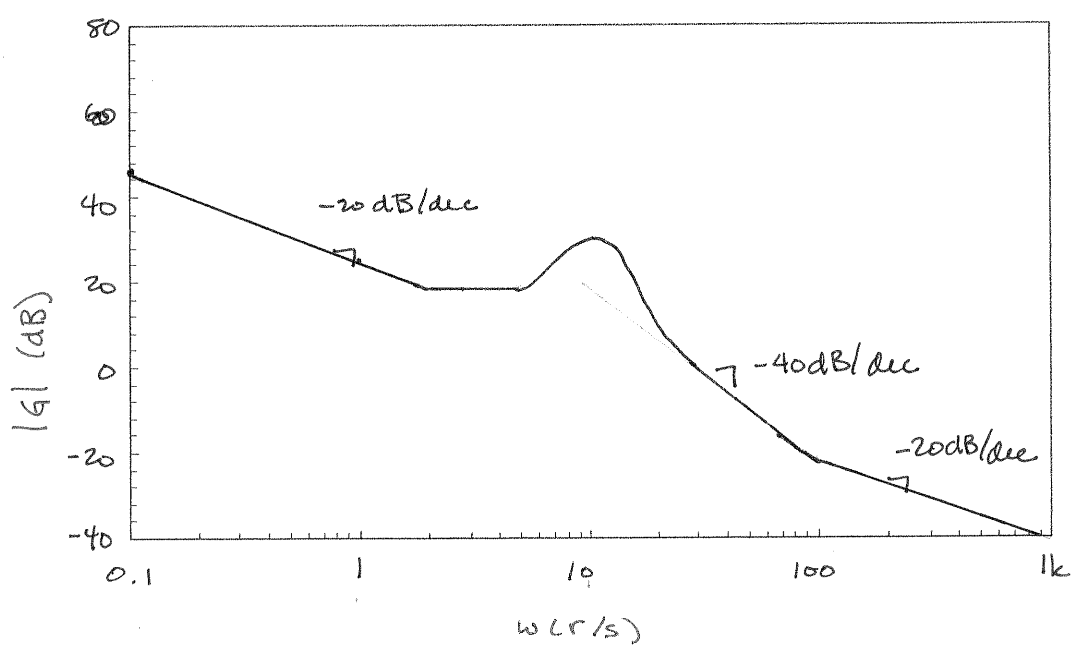
$$\mathbf{G}(j\omega) = \frac{10(j\omega + 2)(j\omega + 100)}{j\omega(-\omega^2 + 4j\omega + 100)}$$

SOLUTION:

$$100 - \omega^2 + 4j\omega = 100\left[1 - \left(\frac{\omega}{10}\right)^2 + \frac{4}{100}j\omega\right] = 100\left[1 - (\omega\tau)^2 + j2\xi(\omega\tau)\right]$$

$$\tau = \frac{1}{10}\text{ s} \qquad \xi = \frac{4}{100}\,\frac{1}{2}\,\frac{1}{\tau} = 0.2$$

$$|G|\Big|_{\omega=0.1} \simeq \frac{10(2)(100)}{(0.1)(100)} = 200 = 46\text{dB}$$



12.24 Sketch the magnitude characteristic of the Bode plot for the transfer function

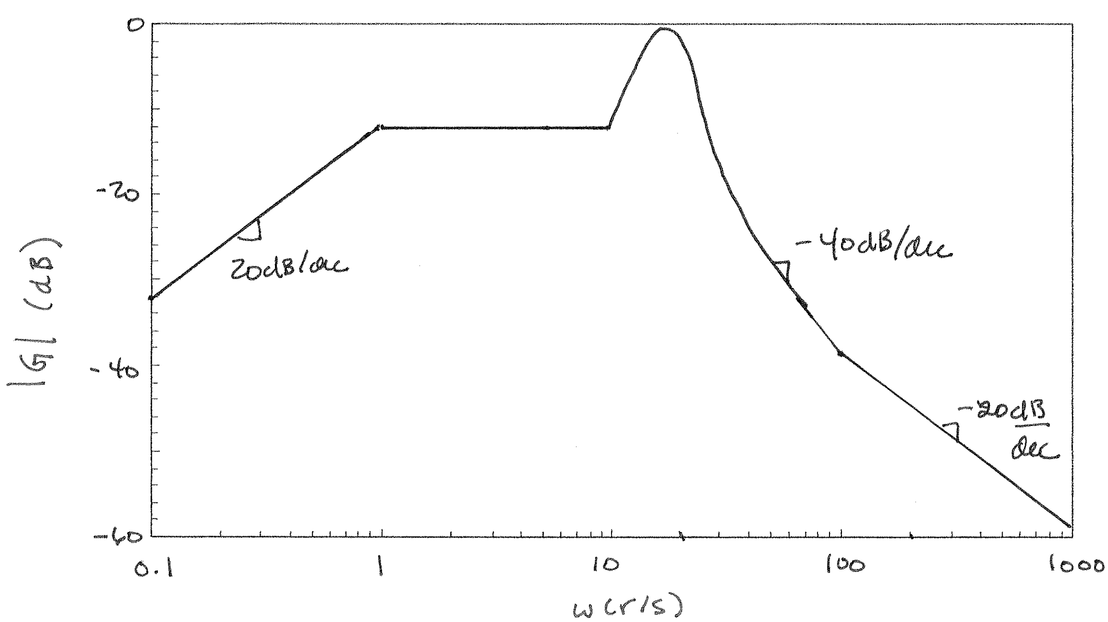
$$\mathbf{G}(j\omega) = \frac{j\omega(j\omega + 100)}{(j\omega + 1)(-\omega^2 + 6j\omega + 400)}$$

SOLUTION:

$$400 - \omega^2 + 6j\omega = 400\left[1 - \left(\frac{\omega}{20}\right)^2 + \frac{6}{400}(j\omega)\right] = 400\left[1 - (\omega\tau)^2 + 2j\xi\omega\tau\right]$$

$$\tau = \frac{1}{20} \qquad \xi = \frac{6}{400}\,\frac{1}{2}\,\frac{1}{\tau} = 0.15$$

$$|G|\Big|_{\omega=0.1} \approx \frac{(0.1)(100)}{(1)(400)} = \frac{1}{40} = -32\text{dB}$$



12.25 Sketch the magnitude characteristic of the Bode plot for the transfer function

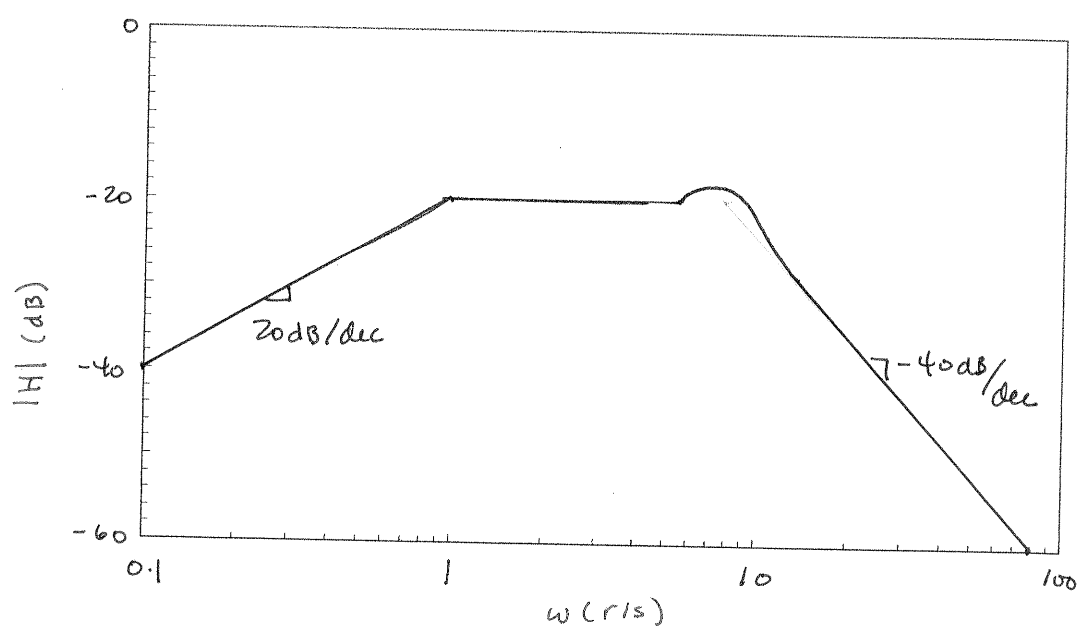
$$\mathbf{H}(j\omega) = \frac{+6.4(j\omega)}{(j\omega + 1)(-\omega^2 + 8j\omega + 64)}$$

SOLUTION:

$$64-\omega^2+8j\omega = 64\left[1-\left(\frac{\omega}{8}\right)^2+\left(\frac{1}{8}\right)j\omega\right] = 64\left[1-(\omega\tau)^2+2j\xi\omega\tau\right]$$

$$\tau = 1/8 \text{ s} \qquad \xi = \frac{\left(\frac{1}{8}\right)}{2\tau} = 0.5$$

$$|G|\Big|_{\omega=0.1} \approx \frac{6.4(0.1)}{(1)(64)} = 10^{-2} = -40dB$$



12.26 Use MATLAB to generate the Bode plot for the following transfer function over the frequency range from $\omega = 0.01$ to 1000 rad/s.

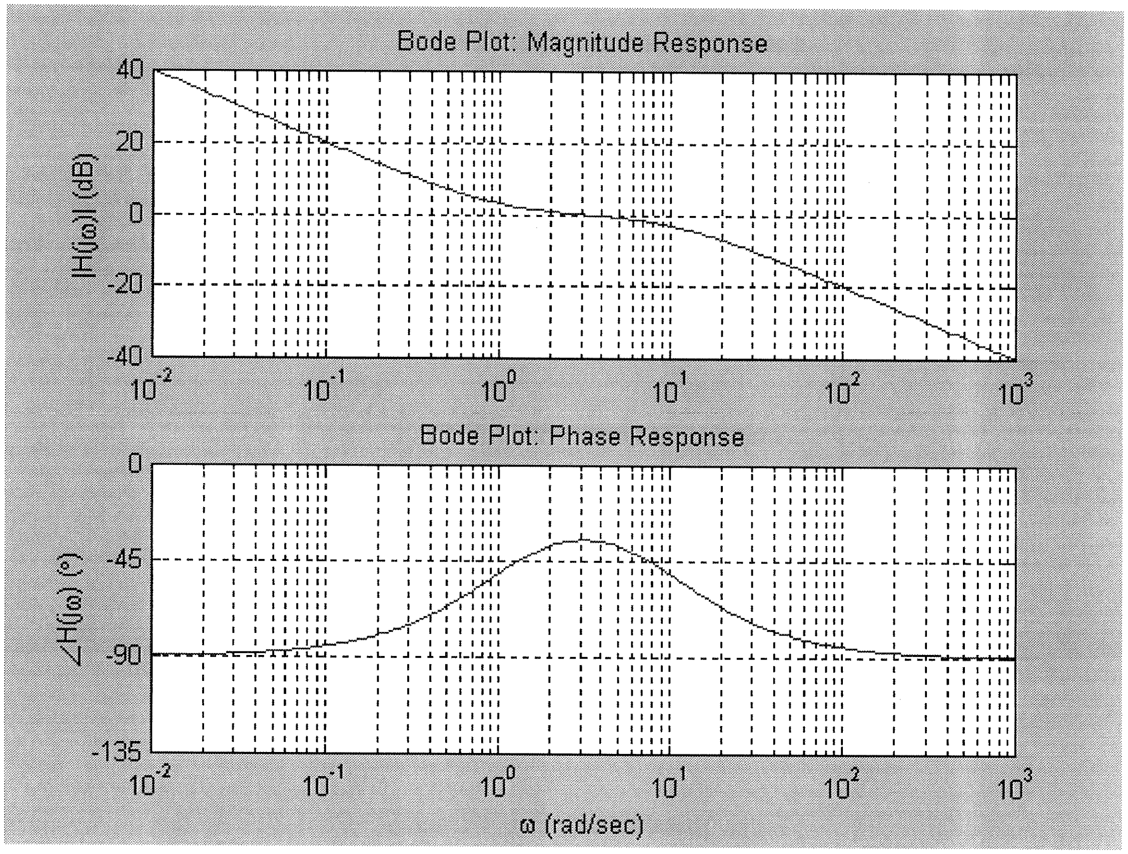
$$\mathbf{G}(j\omega) = \frac{10(j\omega + 1)}{(j\omega)(j\omega + 10)}$$

SOLUTION:

12.26

This sequence of commands works in both the commercial and student versions of MATLAB.

```
» figure(1);
» w = logspace(-2,3,200);
» H = 10*(j*w+1)./((10+j*w).*(j*w));
» subplot(2,1,1);
» semilogx(w,20*log10(abs(H)));
» grid; ylabel('|H(j\omega)| (dB)');
» title('Bode Plot: Magnitude Response');
» subplot(2,1,2);
» semilogx(w,unwrap(angle(H))*180/pi);
» grid; xlabel('\omega (rad/sec)'); ylabel('\angleH(j\omega) (\circ)');
» title('Bode Plot: Phase Response');
```



12.27 Use MATLAB to generate the Bode plot for the following transfer function over the frequency range from $\omega = 0.1$ to 10,000 rad/s.

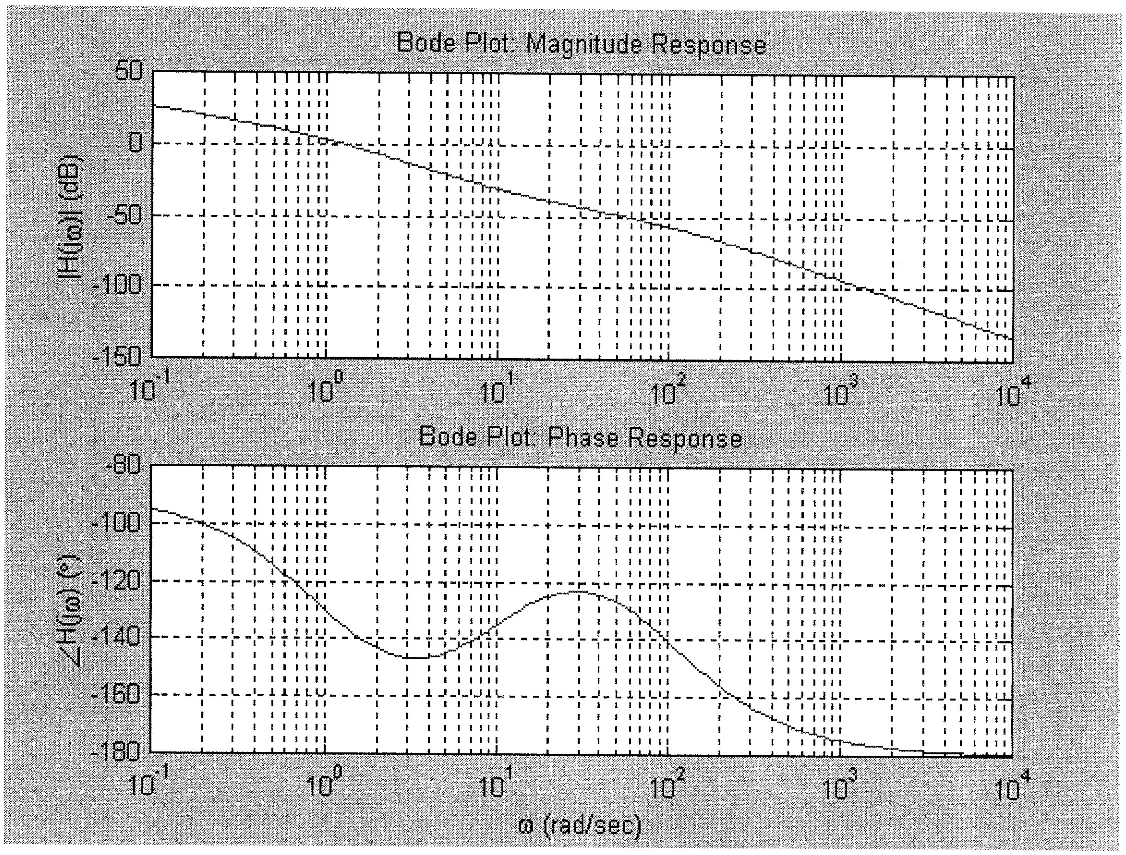
$$\mathbf{G}(j\omega) = \frac{20(j\omega + 10)}{(j\omega)(j\omega + 1)(j\omega + 100)}$$

SOLUTION:

12.27

This sequence of commands works in both the commercial and student versions of MATLAB.

```
EDU» figure(1);
EDU» w = logspace(-2,3,200);
EDU» H = 20*(j*w+10)./((1+j*w).*(j*w).*(100+j*w));
EDU» figure(1);
EDU» w = logspace(-1,4,200);
EDU» H = 20*(j*w+10)./((1+j*w).*(j*w).*(100+j*w));
EDU» subplot(2,1,1);
EDU» semilogx(w,20*log10(abs(H)));
EDU» grid; ylabel('|H(j\omega)| (dB)');
EDU» title('Bode Plot: Magnitude Response');
EDU» subplot(2,1,2);
EDU» semilogx(w,unwrap(angle(H))*180/pi);
EDU» grid; xlabel('\omega (rad/sec)'); ylabel('\angleH(j\omega) (\circ)');
EDU» title('Bode Plot: Phase Response');
```



12.28 The magnitude characteristic of a band-elimination filter is shown in Fig. P12.28. Determine $\mathbf{H}(j\omega)$.

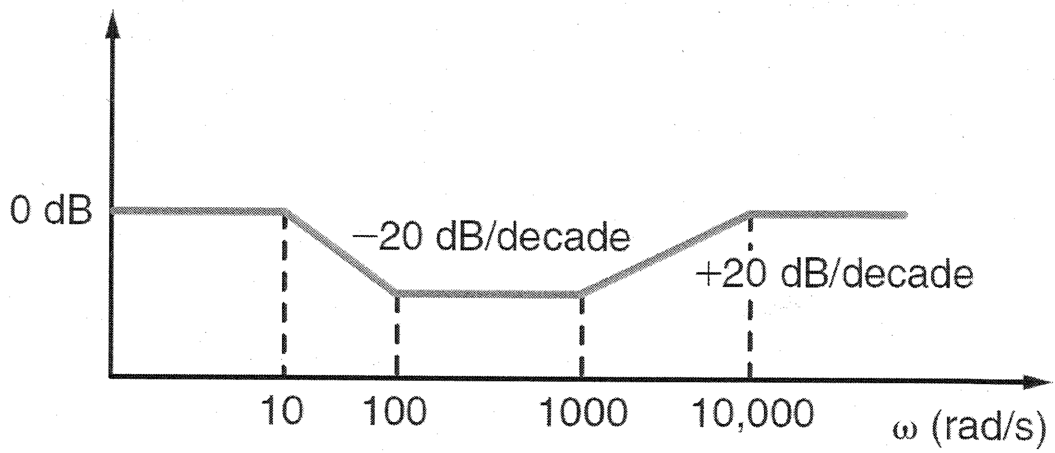


Figure P12.28

SOLUTION:

poles at 10 and 10,000 r/s zeros at 100 and 1000 r/s

$$H(j\omega) = \frac{K(j\omega + 100)(j\omega + 1000)}{(j\omega + 10)(j\omega + 10000)} \qquad 0\,dB = 1$$

$$|H|\Big|_{\omega \ll 10} = \frac{K(100)(1000)}{10(10^4)} = 1 \Rightarrow K = 1$$

$$\boxed{H(j\omega) = \frac{(j\omega + 100)(j\omega + 10^3)}{(j\omega + 10)(j\omega + 10^4)}}$$

12.29 Find $\mathbf{H}(j\omega)$ if its magnitude characteristic is shown in Fig. P12.29. **PSV**

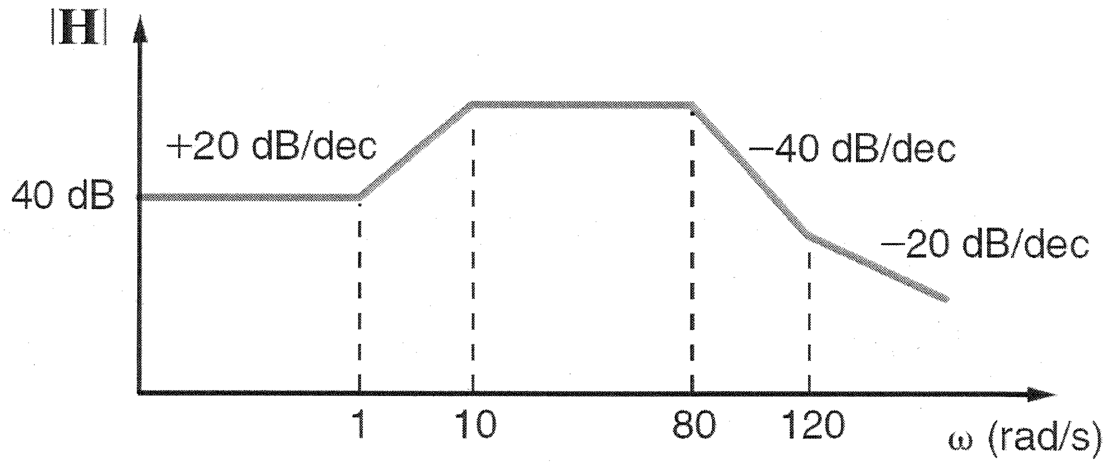


Figure P12.29

SOLUTION:

poles at $\left\{\begin{array}{l} 10 \text{ r/s} \\ 80 \text{ r/s (double)} \end{array}\right\}$ zeros at $\left\{\begin{array}{l} 1 \text{ r/s} \\ 120 \text{ r/s} \end{array}\right.$

$$H(j\omega) = \frac{K(j\omega+1)(j\omega+120)}{(j\omega+10)(j\omega+80)^2}$$

for $\omega \ll 1$, $|H| = 40\text{dB} = 100 = \dfrac{K(1)(120)}{10(80)^2} \Rightarrow K = 5.33\times10^4$

$$\boxed{H(j\omega) = \frac{5.33\times10^4(j\omega+1)(j\omega+120)}{(j\omega+10)(j\omega+80)^2}}$$

12.30 Find $\mathbf{H}(j\omega)$ if its magnitude characteristic is shown in Fig. P12.30. CS

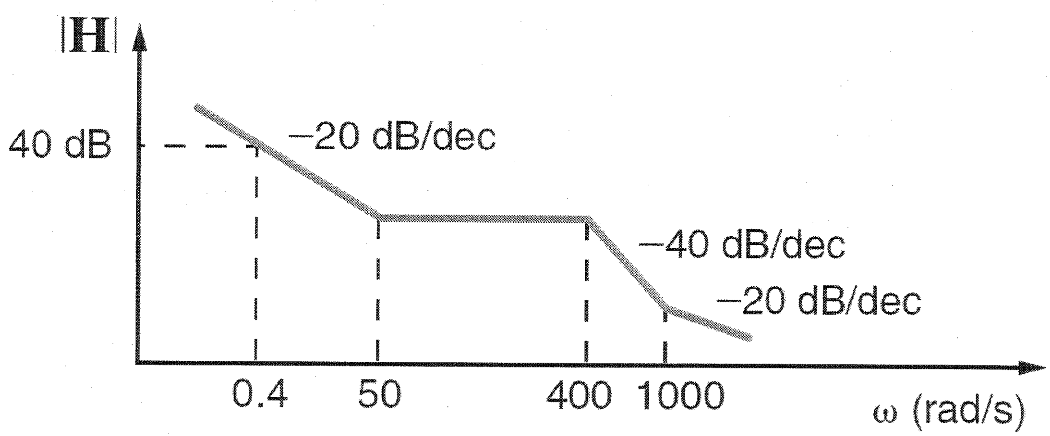


Figure P12.30

SOLUTION:

poles at: $\left\{ \begin{array}{l} \text{origin } (\omega = 0) \\ 400 \text{ r/s (double} \end{array} \right\}$ zeros at: $\left\{ \begin{array}{l} 50 \text{ r/s} \\ 1000 \text{ r/s} \end{array} \right\}$

$$H(j\omega) = \frac{K(j\omega + 50)(j\omega + 1000)}{j\omega(j\omega + 400)^2}$$

at $\omega = 0.4$ r/s, $|H| = 40\text{dB} = 100 \cong \dfrac{K(50)(1000)}{0.4(400)^2} \Rightarrow K = 128$

$$\boxed{H(j\omega) = \frac{128(j\omega + 50)(j\omega + 1000)}{j\omega(j\omega + 400)^2}}$$

12.31 Find $\mathbf{H}(j\omega)$ if its amplitude characteristic is shown in Fig. P12.31.

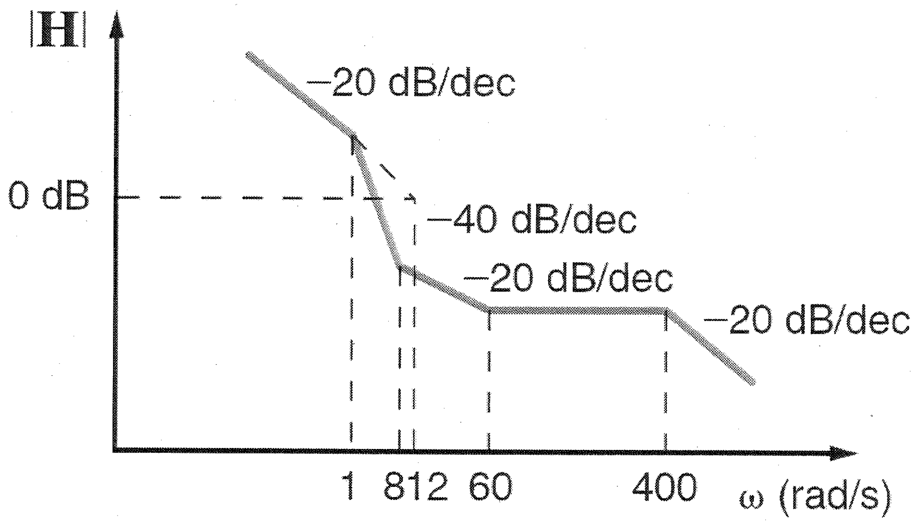


Figure P12.31

SOLUTION:

Poles at $\left\{ \begin{array}{l} \omega = 0 \\ \omega = 1 \text{ r/s} \\ \omega = 400 \text{ r/s} \end{array} \right\}$ Zeros at : $\left\{ \begin{array}{l} \omega = 8 \text{ r/s} \\ \omega = 60 \text{ r/s} \end{array} \right\}$

$$H(j\omega) = \frac{K(j\omega + 8)(j\omega + 60)}{j\omega(j\omega + 1)(j\omega + 400)}$$

At $\omega \simeq 2 \text{ r/s}$, $|H| = 0\text{dB} = 1 \cong \dfrac{K(8)(60)}{2\sqrt{2^2+1^2}\,(400)} \Rightarrow K = 3.73$

$$\boxed{H(j\omega) = \frac{3.73(j\omega + 8)(j\omega + 60)}{j\omega(j\omega + 1)(j\omega + 400)}}$$

12.32 Given the magnitude characteristic in Fig. P12.32, find $\mathbf{H}(j\omega)$.

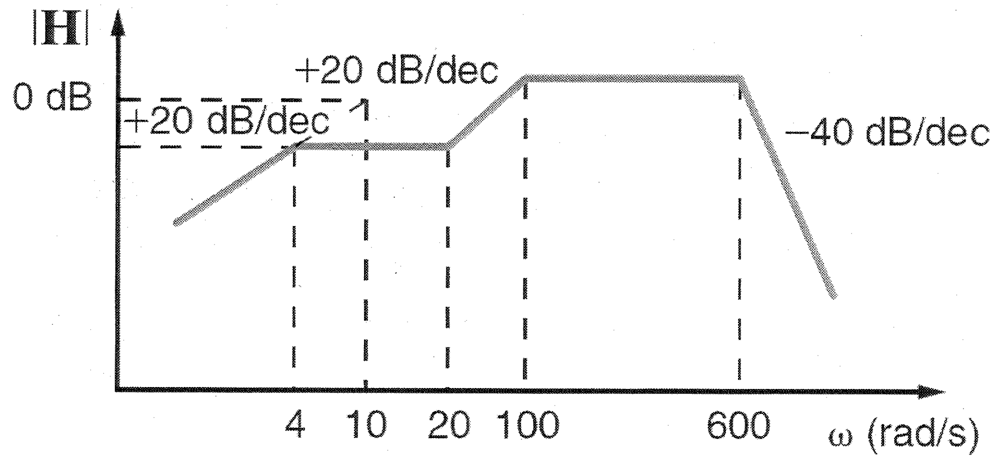


Figure P12.32

SOLUTION:

poles at: $\left\{ \begin{array}{l} \omega = 4 \text{ r/s} \\ 100 \text{ r/s} \\ 600 \text{ r/s (double)} \end{array} \right\}$ zeros at: $\left\{ \begin{array}{l} \omega = 0 \text{ r/s} \\ 20 \text{ r/s} \end{array} \right\}$

$$H(j\omega) = \frac{K(j\omega)(j\omega+20)}{(j\omega+4)(j\omega+100)(j\omega+600)^2}$$

at $\omega = 1$ r/s, $|H| = -20\text{dB} = 0.1 = \dfrac{K(1)(20)}{4(100)(600)^2} \Rightarrow K = 7.2\times10^5$

$$\boxed{H(j\omega) = \frac{7.2\times10^5 (j\omega)(j\omega+20)}{(j\omega+4)(j\omega+100)(j\omega+600)^2}}$$

12.33 Determine $\mathbf{H}(j\omega)$ if its magnitude characteristic is shown in Fig. P12.33.

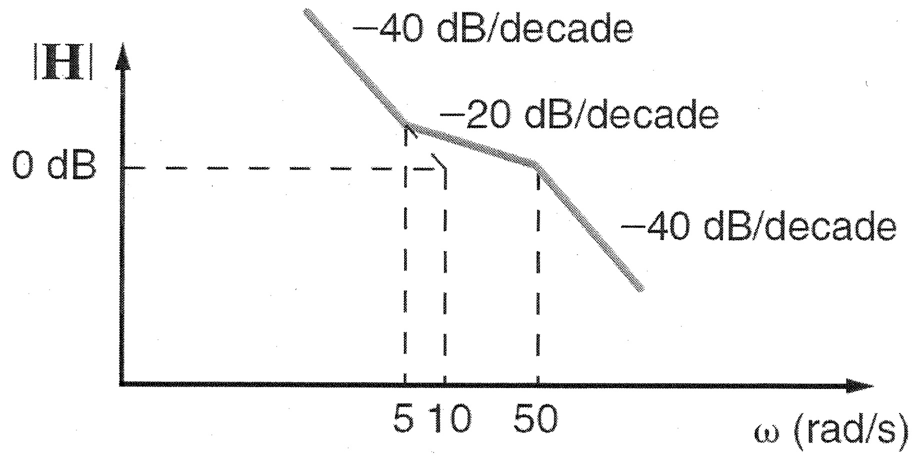


Figure P12.33

SOLUTION:

poles at: $\left\{ \begin{array}{l} \omega = 0 \text{ r/s (double)} \\ \omega = 50 \text{ r/s} \end{array} \right\}$ zero at: $\{5 \text{ r/s}\}$

$$H(j\omega) = \frac{K(j\omega + 5)}{(j\omega)^2(j\omega + 50)}$$

at $\omega = 1$ r/s, $|H| = 40\text{dB} = 100 = \dfrac{K(5)}{(1)^2(50)} \Rightarrow K = 1000$

$$\boxed{H(j\omega) = \frac{1000(j\omega + 5)}{(j\omega)^2(j\omega + 50)}}$$

12.34 Find $\mathbf{G}(j\omega)$ for the magnitude characteristic shown in Fig. P12.34. CS

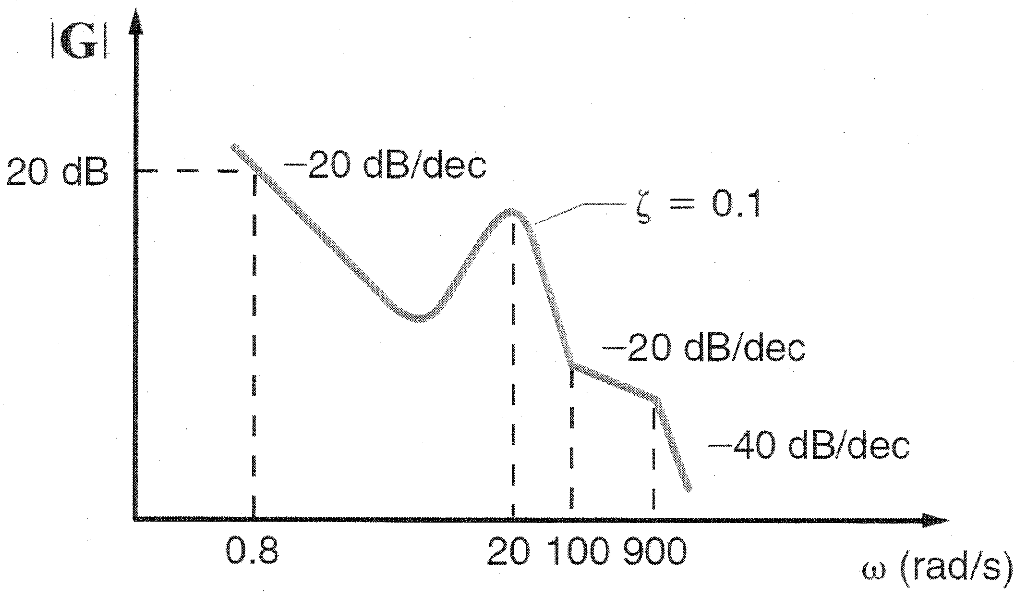


Figure P12.34

SOLUTION:

Simple poles: $\left\{ \begin{array}{l} \omega = 0 \text{ r/s} \\ \omega = 900 \text{ r/s} \end{array} \right\}$ zeros at 100 r/s (double)

Complex conjugate poles at $\omega = 20$ r/s with $\zeta = 0.1$

or $1 - \left(\frac{\omega}{20}\right)^2 + 2j(0.1)/20 \rightarrow ((j\omega)^2 + j4\omega + 400)/400$

$$G(j\omega) = \frac{K(j\omega + 100)^2}{j\omega(j\omega + 900)\left[(j\omega)^2 + j4\omega + 400\right]}$$

at $\omega = 0.8$ r/s, $|G| = 20\text{dB} = 10 = K(100)^2 / \{(0.8)(900)(400)\}$

$K = 288$

$$\boxed{G(j\omega) = \frac{288(j\omega + 100)^2}{j\omega(j\omega + 900)\left[(j\omega)^2 + j4\omega + 400\right]}}$$

12.35 A series *RLC* circuit resonates at 1000 rad/s. If $C = 20\ \mu F$, and it is known that the impedance at resonance is $2.4\ \Omega$, compute the value of L, the Q of the circuit, and the bandwidth.

SOLUTION:

$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

at resonance, $\omega_o = \dfrac{1}{\sqrt{LC}} = 1000 \Rightarrow \boxed{L = 50\,mH}$

also $Z(j\omega_o) = R = 2.4\Omega$

$$Q = \frac{\omega_o L}{R} = \frac{1000(50 \times 10^{-3})}{2.4} \qquad \boxed{Q = 20.8}$$

$$BW = \frac{\omega_o}{Q} \qquad \boxed{BW = 48\ r/s}$$

12.36 A series resonant circuit has a Q of 120 and a resonant frequency of 10,000 rad/s. Determine the half-power frequencies and the bandwidth of the circuit.

SOLUTION: $\omega_o = 10k\ r/s$

$BW = \omega_o / Q$ $\boxed{BW = 83.3\ r/s}$

$\omega_{LO} = \omega_o \left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$ $\boxed{\omega_{LO} = 9958\ r/s}$

$\omega_{HI} = \omega_o^2 / \omega_{LO} \Rightarrow$ $\boxed{\omega_{HI} = 10.04\ kr/s}$

12.37 The series *RLC* circuit in Fig. P12.37 is driven by a variable-frequency source. If the resonant frequency of the network is selected as $\omega_0 = 1600$ rad/s, find the value of C. In addition, compute the current at resonance and at $\omega_0/4$ and $4\,\omega_0$. **PSV**

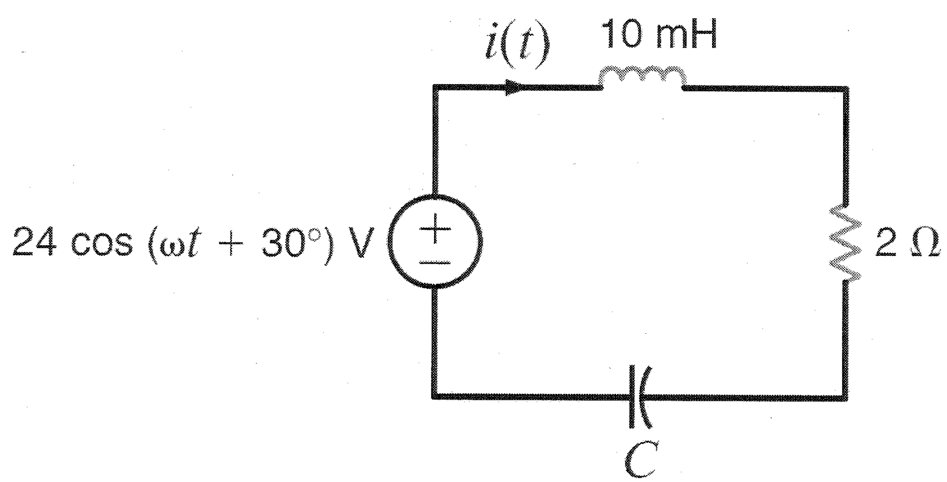
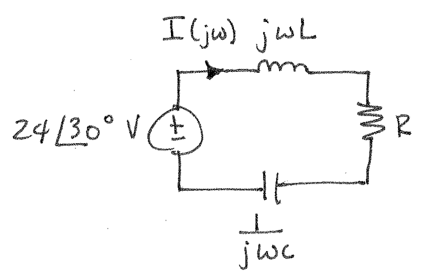


Figure P12.37

SOLUTION:

$$\omega_0 = 1600 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{C = 39\,\mu F}$$



$$I(j\omega) = \frac{24\angle 30^\circ}{j\left(\omega L - \frac{1}{\omega C}\right) + R}$$

ω/ω_0	$I(j\omega)$
1	$12\angle 30^\circ$ A
1/4	$0.400\angle 118^\circ$ A
4	$0.400\angle -58^\circ$ A

12.38 Given the series *RLC* circuit in Fig. P12.38, (a) derive the expression for the half-power frequencies, the resonant frequency, the bandwidth, and the quality factor for the transfer characteristic $\mathbf{I}/\mathbf{V}_{\text{in}}$ in *R*, *L*, *C*. (b) Compute the quantities in part (a) if $R = 10\ \Omega$, $L = 50$ mH, and $C = 10\ \mu$F.

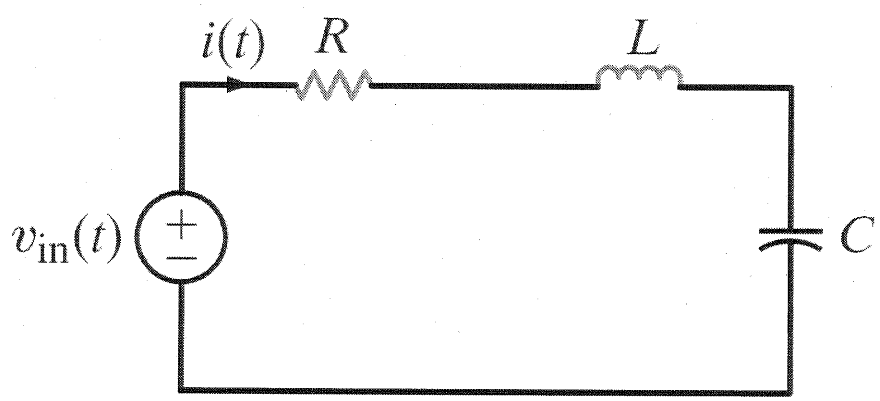


Figure P12.38

SOLUTION: a. Let $Z = R + j\omega L + \frac{1}{j\omega C}$

at resonance, $Z = R$. So $\omega_o L + \frac{1}{\omega_o C} = 0 \Rightarrow \boxed{\omega_o = \frac{1}{\sqrt{LC}}}$

Q is defined as $\frac{\omega_o L}{R} \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$

$\frac{I}{V_{IN}} = \frac{1}{Z(j\omega)}$ At half power, $\left|\frac{I}{V_{IN}}\right| = \frac{1}{\sqrt{2}R} = \frac{1}{|Z(j\omega)|}$

so, $|R + j(\omega L - 1/\omega C)| = \sqrt{2}R = R\left|1 + j\left(\omega L/R - \frac{1}{\omega RC}\right)\right|$

$\sqrt{2} = \left|1 + jQ(\omega/\omega_o - \omega_o/\omega)\right|$ or $Q(\omega/\omega_o - \omega_o/\omega) = \pm 1$

Quadratic equation yields $\omega_{LO} = \omega_o\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$

$\omega_{HI} = \omega_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$

In terms of R, L, C $\Rightarrow$ $\frac{\omega_0}{Q} = R/L$

$$\boxed{\begin{aligned} \omega_{LO} &= \left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ \omega_{HI} &= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \end{aligned}}$$

$BW = \omega_{HI} - \omega_{LO} = \omega_0/Q$ $\quad \boxed{BW = R/L}$

b. $R = 10\,\Omega$ $\quad L = 50\,mH$ $\quad C = 10\,\mu F$

$\boxed{\omega_0 = 1414 \text{ r/s}}$ $\quad \boxed{Q = 7.07}$ $\quad \boxed{BW = 200 \text{ r/s}}$

$\boxed{\omega_{HI} = 1518 \text{ r/s}}$ $\quad \boxed{\omega_{LO} = 1318 \text{ r/s}}$

12.39 Given the network in Fig. P12.39, find ω_0, Q, ω_{max}, and $|\mathbf{V}_o|_{max}$. CS

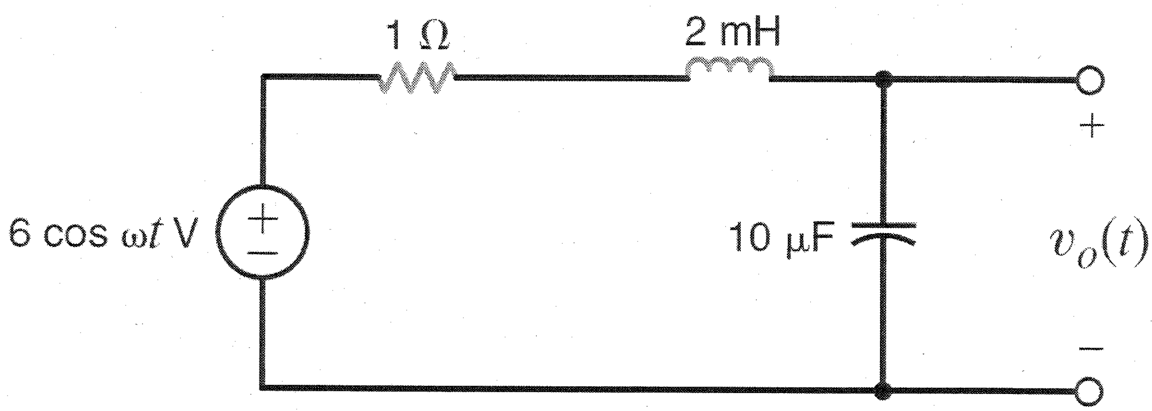


Figure P12.39

SOLUTION:

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_0 = 7.07 \text{ kr/s}}$

$Q = \omega_0 L / R \rightarrow \boxed{Q = 14.14}$

$\omega_{max} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} \Rightarrow \boxed{\omega_{max} = 7.06 \text{ kr/s}}$

$|V_{omax}| = \frac{Q|V_s|}{\sqrt{1 - \frac{1}{4Q^2}}}$ $\quad |V_s| = 6\text{V}$

$\boxed{|V_{omax}| = 84.9 \text{ V}}$

12.40 A series *RLC* circuit is driven by a signal generator. The resonant frequency of the network is known to be 1600 rad/s, and at that frequency the impedance seen by the signal generator is 5 Ω. If $C = 20\ \mu F$, find L, Q, and the bandwidth.

SOLUTION:

$Z = R + j(\omega L - 1/\omega C)$ at $\omega = 1600$ r/s, $Z = 5 = R$

So, $1600 \text{ r/s} = \omega_o = \dfrac{1}{\sqrt{LC}} \Rightarrow L = \dfrac{1}{\omega_o^2 C}$ $\boxed{L = 19.5\text{mH}}$

$Q = \dfrac{\omega_o L}{R} \Rightarrow \boxed{Q = 6.24}$

$BW = \dfrac{\omega_o}{Q} \Rightarrow \boxed{BW = 256 \text{ r/s}}$

12.41 A variable-frequency voltage source drives the network in Fig. P12.41. Determine the resonant frequency, Q, BW, and the average power dissipated by the network at resonance.

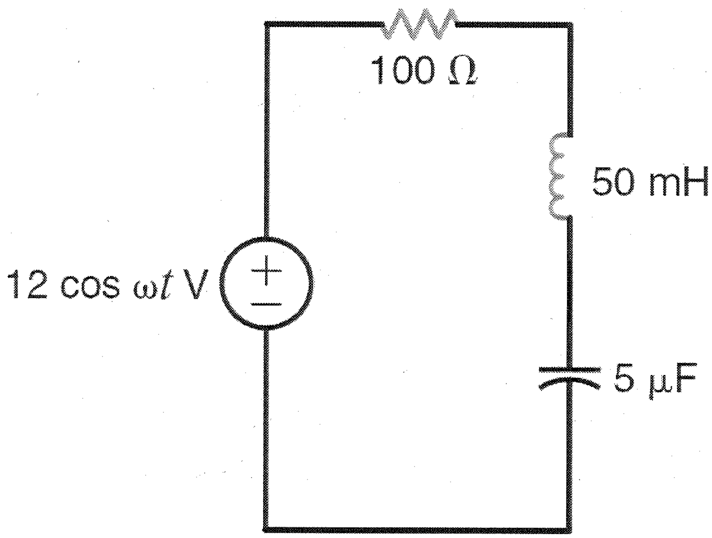
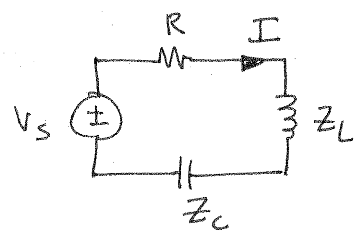


Figure P12.41

SOLUTION:

$\omega_o = \dfrac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_o = 2\ kr/s}$



$Q = \omega_o L / R \Rightarrow \boxed{Q = 1}$

$BW = \omega_o / Q \Rightarrow \boxed{BW = 2kr/s}$

at resonance, $Z = R = 100\Omega$

$I = \dfrac{V_s}{R} = \dfrac{12\angle 0}{100} = 0.12\angle 0°A$

$P = \dfrac{|I|^2 R}{2}$ $\boxed{P = 720\,mW}$

12.42 A parallel *RLC* resonant circuit with a resonant frequency of 20,000 rad/s has an admittance at resonance of 1 mS. If the capacitance of the network is 2 μF, find the values of *R* and *L*. **PSV**

SOLUTION: $\omega_o = 20\text{ kr/s}$ $\quad Y(j\omega_o) = 10^{-3}\text{ S} = 1/R \Rightarrow \boxed{R = 1\text{k}\Omega}$

$$\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_o^2 C} \Rightarrow \boxed{L = 1.25\text{ mH}}$$

12.43 A parallel *RLC* resonant circuit has a resistance of 200 Ω. If it is known that the bandwidth is 80 rad/s and the lower half-power frequency is 800 rad/s, find the values of the parameters L and C.

SOLUTION: $R = 200\,\Omega$ $\quad BW = 80$ r/s $\quad \omega_{LO} = 800$ r/s

$$BW = \frac{1}{RC} \Rightarrow \boxed{C = 62.5\,\mu F}$$

$$\omega_{HI} = \omega_{LO} + BW = 880 \text{ r/s} \qquad \omega_o = \sqrt{\omega_{HI}\,\omega_{LO}} = 839 \text{ r/s}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_o^2 C} \Rightarrow \boxed{L = 22.7\,mH}$$

12.44 In the network in Fig. P12.44, the inductor value is 10 mH, and the circuit is driven by a variable-frequency source. If the magnitude of the current at resonance is 12 A, $\omega_0 = 1000$ rad/s, and $L = 10$ mH, find C, Q, and the bandwidth of the circuit. CS

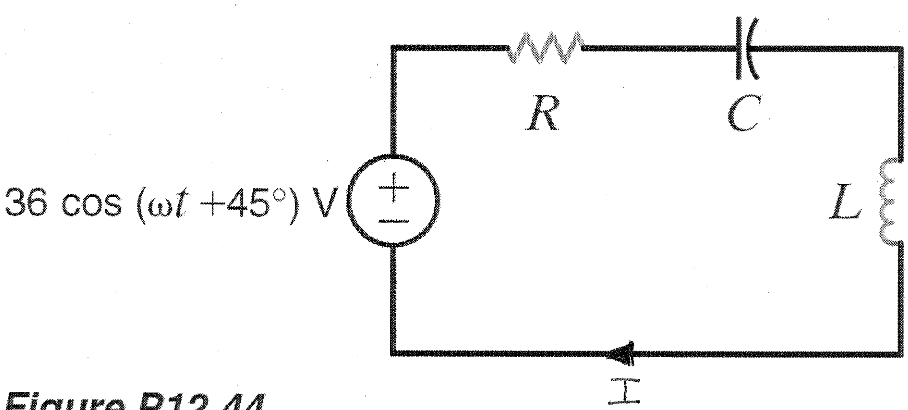


Figure P12.44

SOLUTION: $Z = R + j(\omega L - 1/\omega C)$

at $\omega = \omega_0$, $Z = R$ and $|I| = 36/R = 12 \Rightarrow R = 3\,\Omega$

$Q = \omega_0 L / R \Rightarrow \boxed{Q = 3.33}$

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L} \Rightarrow \boxed{C = 100\,\mu F}$

$B\omega = \omega_0 / Q \Rightarrow \boxed{BW = 300 \text{ r/s}}$

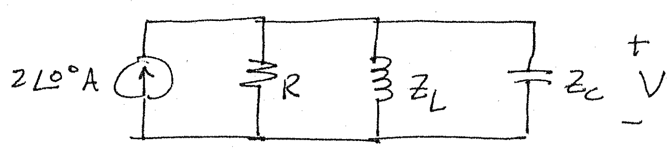
12.45 A parallel *RLC* circuit, which is driven by a variable-frequency 2-A current source, has the following values: $R = 1\ \text{k}\Omega$, $L = 400$ mH, and $C = 10\ \mu$F. Find the bandwidth of the network, the half-power frequencies, and the voltage across the network at the half-power frequencies. **CS**

SOLUTION:

$BW = \frac{1}{RC} \Rightarrow \boxed{BW = 100\ r/s}$ $\omega_o = \frac{1}{\sqrt{LC}} = 500\ r/s$

$Q = \omega_o / BW = 5$ $\omega_{HI} = \omega_o \left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right] = 552\ r/s$

$\boxed{\omega_{HI} = 552\ r/s}$ $\omega_{LO} = \omega_o^2 / \omega_{HI} \Rightarrow \boxed{\omega_{LO} = 452\ r/s}$



$Y = \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}$ $V = I/Y = 2\angle 0 / Y$

$Y(j\omega_{HI}) = 1 + j1$ mS $\boxed{V(j\omega_{HI}) = 1414 \angle{-45°}\ V}$

$Y(j\omega_{LO}) = 1 - j1$ mS $\boxed{V(j\omega_{LO}) = 1414 \angle{+45°}\ V}$

12.46 A parallel *RLC* circuit, which is driven by a variable-frequency 2-A current source, has the following values: $R = 1\ \text{k}\Omega$, $L = 100$ mH, and $C = 10\ \mu$F. Find the bandwidth of the network, the half-power frequencies, and the voltage across the network at the half-power frequencies. CS

SOLUTION:

$BW = 1/RC \Rightarrow \boxed{BW = 100\ r/s}$ $\qquad \omega_0 = \dfrac{1}{\sqrt{LC}} = 1000\ r/s$

$Q = \omega_0 / BW = 10$

$\omega_{HI} = \omega_0 \left[\dfrac{1}{2Q} + \sqrt{\left(\dfrac{1}{2Q}\right)^2 + 1} \right] \Rightarrow \boxed{\omega_{HI} = 1051\ r/s}$

$\omega_{LO} = \omega_0^2 / \omega_{HI} \Rightarrow \boxed{\omega_{LO} = 951\ r/s}$

$Y = \dfrac{1}{R} + \dfrac{1}{Z_L} + Z_C \qquad V = I/Y \qquad I = 2\angle 0^\circ\ A$

$Y(j\omega_{HI}) = 1 + j1\ mS$

$Y(j\omega_{LO}) = 1 - j1\ mS$

$\boxed{V = \sqrt{2}\angle -45^\circ\ kV\ @\ \omega_{HI}}$

$\boxed{V = \sqrt{2}\angle +45^\circ\ kV\ @\ \omega_{LO}}$

12.47 Consider the network in Fig. P12.47. If $R = 1\ \text{k}\Omega$, $L = 20\ \text{mH}$, $C = 50\ \mu\text{F}$, and $R_S = \infty$, determine the resonant frequency ω_0, the Q of the network, and the bandwidth of the network. What impact does an R_S of $10\ \text{k}\Omega$ have on the quantities determined?

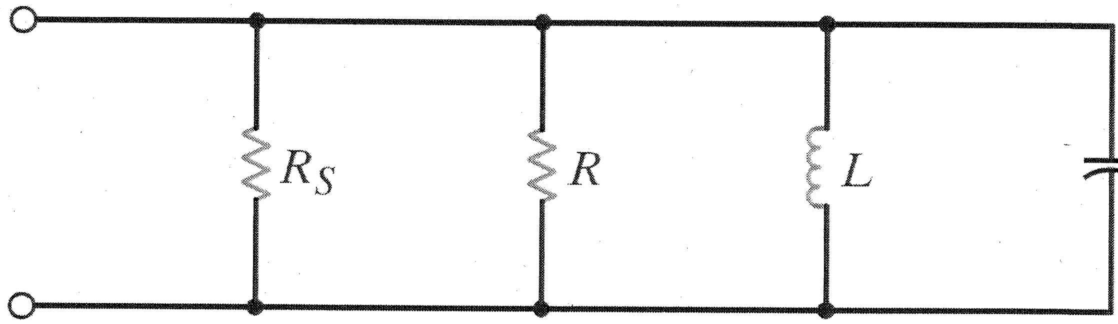


Figure P12.47

SOLUTION:

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_0 = 1000\ r/s}$ $\quad R_{eq} = R_S // R = 1\text{k}\Omega$

$Q = \omega_0 R_{eq} C = 50 \Rightarrow \boxed{Q = 50}$ $\quad BW = \omega_0 / Q \Rightarrow \boxed{BW = 20\ r/s}$

If $R_S = 10\text{k}\Omega$, $R_{eq} = 909\Omega$.

ω_0 is unchanged. $\boxed{\omega_0 = 1000\ r/s}$

Q changes $\boxed{Q = 45.5}$

BW changes $\boxed{BW = 22.0\ r/s}$

12.48 The source in the network in Fig. P12.48 is $i_S(t) = \cos 1000t + \cos 1500t$ A. $R = 200\ \Omega$ and $C = 500\ \mu F$. If $\omega_0 = 1000$ rad/s, find L, Q, and the BW. Compute the output voltage $v_o(t)$ and discuss the magnitude of the output voltage at the two input frequencies.

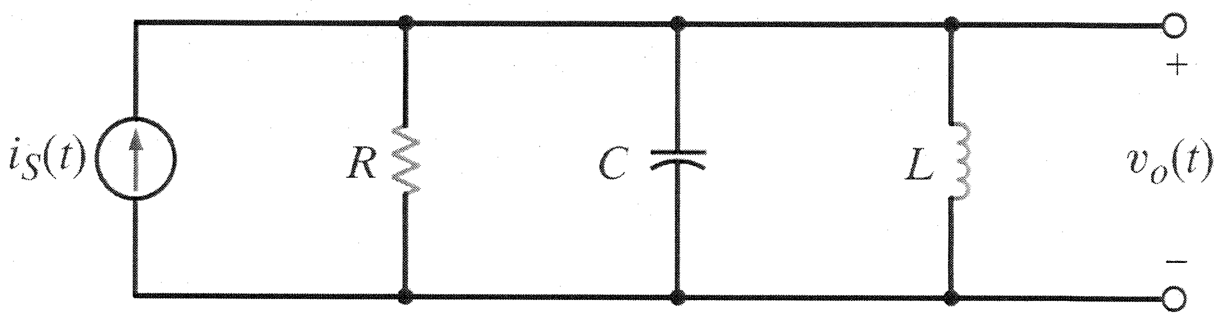


Figure P12.48

SOLUTION:

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} \Rightarrow \boxed{L = 2mH}$

$BW = \frac{1}{RC} \Rightarrow \boxed{BW = 10\ r/s}$ $\quad Q = \omega_0 / BW \Rightarrow \boxed{Q = 100}$

Use superposition. Let $I_{S1} = 1\angle 0°$ at 1000 r/s and $I_{S2} = 1\angle 0°$ A @ 1500 r/s.

$Y = \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}$ $\quad Y(j1000) = 1/R = 5\ mS = Y_1$

$V_{o1} = I_{S1}/Y_1 = 1\angle 0° / 5\times10^{-3} = 200\angle 0°\ V$

$Y(j1500) = Y_2 = 5 + j417\ mS$ $\quad V_{o2} = 1\angle 0°/Y_2 = 2.4\angle -89.3°\ V$

$v_{o1}(t) = 200 \cos 1000t\ V$ $\quad v_{o2}(t) = 2.4 \cos(1500t - 89.3°)\ V$

$v_o(t) = v_{o1} + v_{o2} \Rightarrow \boxed{v_o(t) = 200 \cos 1000t + 2.4 \cos(1500t - 89.3°)\ V}$

Magnitude difference (200V versus 2.4V) is due to $|Y|$ at the two frequencies!

12.49 Determine the parameters of a parallel resonant circuit that has the following properties: $\omega_0 = 2$ Mrad/s, BW = 20 rad/s, and an impedance at resonance of 2000 Ω. CS

SOLUTION:

At Resonance, $Z = R = 2000\Omega$ $\boxed{R = 2000\,\Omega}$

$BW = \frac{1}{RC} \Rightarrow C = \frac{1}{(BW)R} \Rightarrow \boxed{C = 25\,\mu F}$

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} \Rightarrow \boxed{L = 10\,nH}$

12.50 Determine the value of C in the network shown in Fig. P12.50 for the circuit to be in resonance.

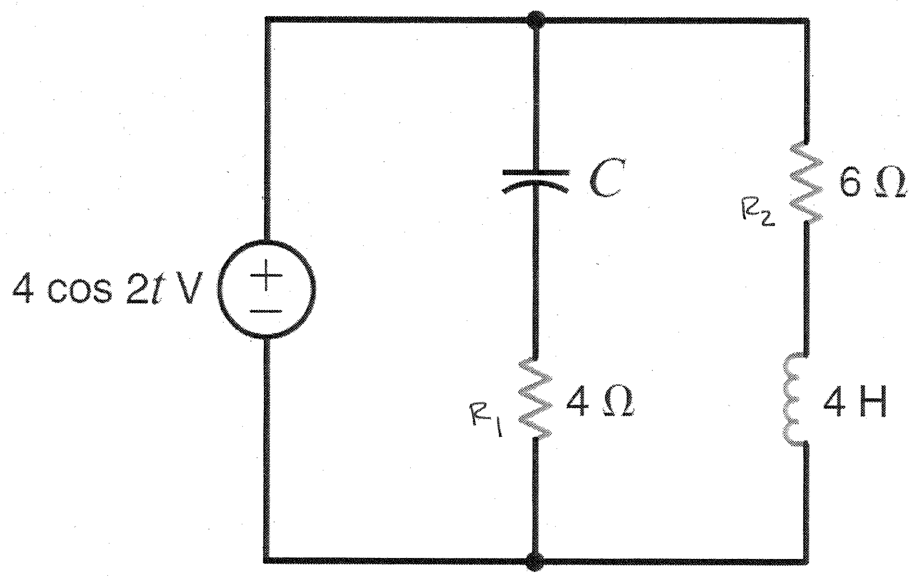
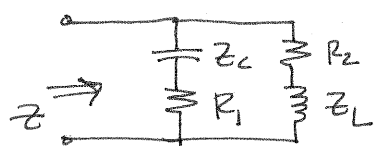


Figure P12.50

SOLUTION: $\omega = 2\ r/s$ $\qquad Z_L = j8\ \Omega \qquad Z_C = -j/2C\ \Omega$



Let $Z_1 = Z_C + R_1$ & $Z_2 = R_2 + Z_L = 6 + j8\ \Omega$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{R_1R_2 + L/C + j(\omega L R_1 - R_2/\omega C)}{R_1 + R_2 + j(\omega L - 1/\omega C)}$$

at resonance, Z is real. So, phase angle of numerator and denominator are equal.

$$\frac{\omega L R_1 - R_2/\omega C}{R_1 R_2 + L/C} = \frac{\omega L - 1/\omega C}{R_1 + R_2} \Rightarrow \frac{32 - 3/C}{24 + 4/C} = \frac{8 - 1/2C}{10}$$

$$\frac{32C - 3}{24C + 4} = \frac{16C - 1}{20C} \Rightarrow \frac{32C - 3}{6C + 1} = \frac{16C - 1}{5C} \Rightarrow 64C^2 - 25C + 1 = 0$$

$$\boxed{C = \begin{cases} 345\ mF \\ 46\ mF \end{cases}}$$

12.51 Determine the equation for the nonzero resonant frequency of the impedance shown in Fig. P12.51.

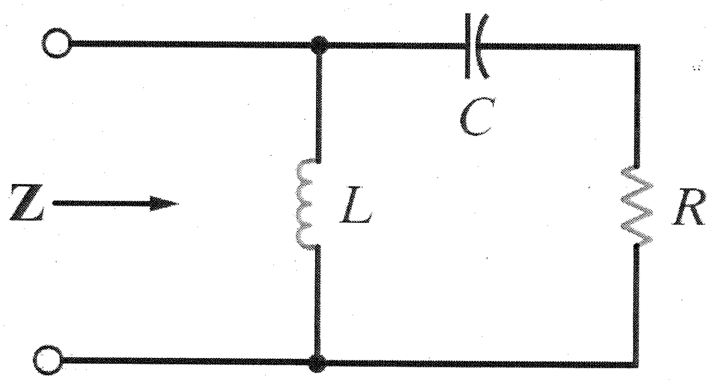


Figure P12.51

SOLUTION:

$$Z = \frac{j\omega L\,(R - j/\omega C)}{R + j(\omega L - 1/\omega C)} = \frac{L/C + j\omega LR}{R + j(\omega L - 1/\omega C)} = \frac{L\left[1/C + j\omega R\right]}{R + j(\omega L - 1/\omega C)}$$

at resonance, $\angle Z = 0$. So,

$$\frac{\omega_0 R}{1/C} = \frac{\omega_0 L - 1/\omega_0 C}{R} \Rightarrow \omega_0 R^2 C = \omega_0 L - \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 (R^2C^2 - LC) = -1$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC - (RC)^2}}}$$

12.52 Determine the new parameters of the network shown in Fig. P12.52 if $\mathbf{Z}_{new} = 10^4\mathbf{Z}_{old}$.

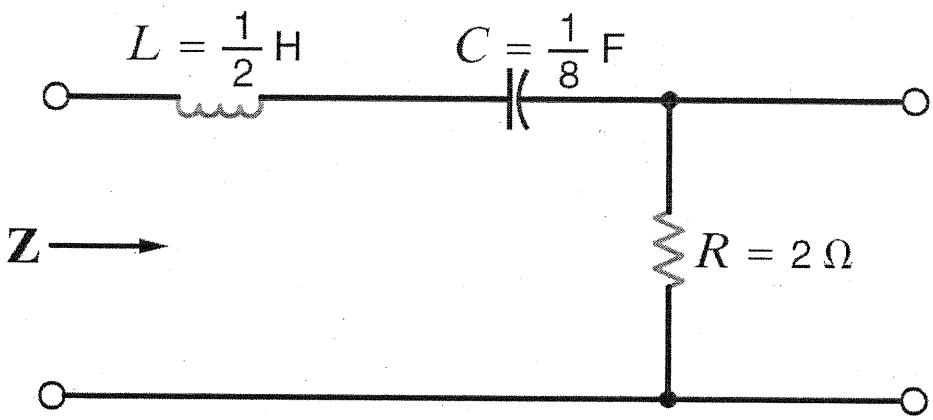


Figure P12.52

SOLUTION:

scale factor $= K_M = Z_{new} / Z_{old} = 10^4$

$R_{new} = K_M R_{old}$

$L_{new} = K_m L_{old}$

$C_{new} = C_{old} / K_m$

$$\boxed{\begin{aligned} R_{new} &= 20\,k\Omega \\ L_{new} &= 5\,kH \\ C_{new} &= 12.5\,\mu F \end{aligned}}$$

12.53 Determine the new parameters of the network in Problem 12.52 if $\omega_{new} = 10^4 \omega_{old}$. CS

SOLUTION:

$L_{old} = 0.5\text{H}$ $C_{old} = 0.125\text{F}$ $R_{old} = 2\Omega$

Let $K_F = \omega_{new} / \omega_{old} = 10^4$

$L_{new} = L_{old} / K_F$

$C_{new} = C_{old} / K_F$

$R_{new} = R_{old}$

$$\boxed{\begin{aligned} L_{new} &= 50\mu\text{H} \\ C_{new} &= 12.5\mu\text{F} \\ R &= 2\Omega \end{aligned}}$$

12.54 Given the network in Fig. P12.54, sketch the magnitude characteristic of the transfer function

$$\mathbf{G}_v(j\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_1}(j\omega)$$

Identify the type of filter.

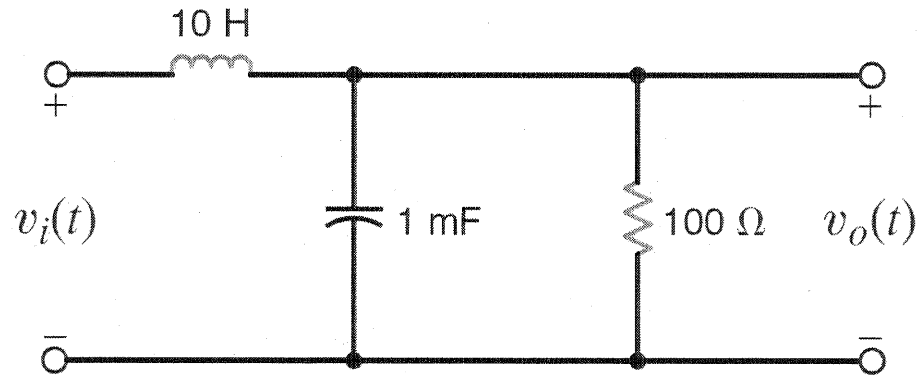
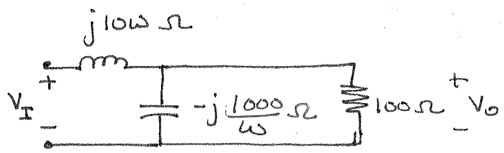


Figure P12.54

SOLUTION:



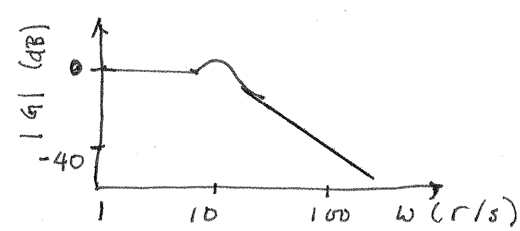
Let $Z = \dfrac{100\,(-j1000/\omega)}{100 - j\frac{1000}{\omega}} = \dfrac{10^3}{10 + j\omega}$

$$\frac{V_o}{V_s} = \frac{Z}{Z + j10\omega} = \frac{10^3}{10^3 + j10\omega(10 + j\omega)} = \frac{10^3}{10^3 - 10\omega^2 + j1000\omega} = \frac{100}{100 - \omega^2 + j10\omega}$$

$$\frac{V_o}{V_s} = \frac{100}{(j\omega)^2 + j\omega(10) + 100}$$

complex conjugate poles : $\tau = 1/10$ $\quad \frac{10}{100} = 2\zeta\tau \Rightarrow \zeta = 0.5$

$\boxed{\text{filter is low-pass}}$



12.55 Given the network in Fig. P12.55, sketch the magnitude characteristic of the transfer function

$$\mathbf{G}_v(j\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_1}(j\omega)$$

Identify the type of filter.

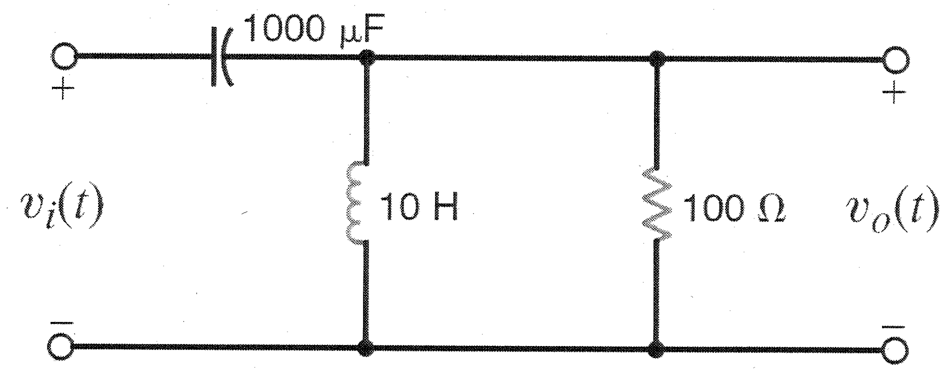
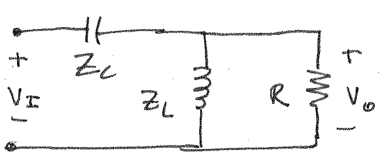


Figure P12.55

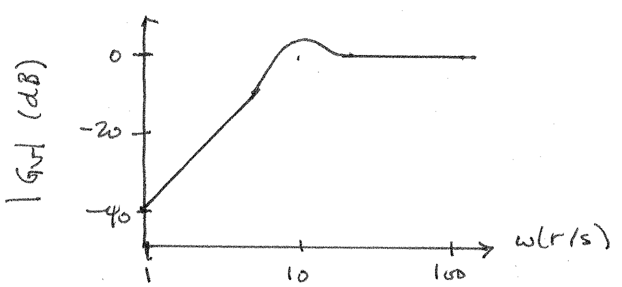
SOLUTION: $Z_L = j\omega(10)\ \Omega$ $\quad Z_C = 1000/(j\omega)\ \Omega$ $\quad R = 100\Omega$



Let $Z = RZ_L / (R + Z_L) = 100(j\omega)/(10 + j\omega)$

$$V_o/I_I = G_v(j\omega) = Z/(Z+Z_C) = \frac{(j\omega)^2}{(j\omega)^2 + 10(j\omega) + 100}$$

Complex conjugate poles at: $\tau = 1/10$ $\quad 2\xi\tau = 0.1 \Rightarrow \xi = 0.5$



$\boxed{\text{filter is high pass}}$

12.56 Determine what type of filter the network shown in Fig. P12.56 represents by determining the voltage transfer function. CS

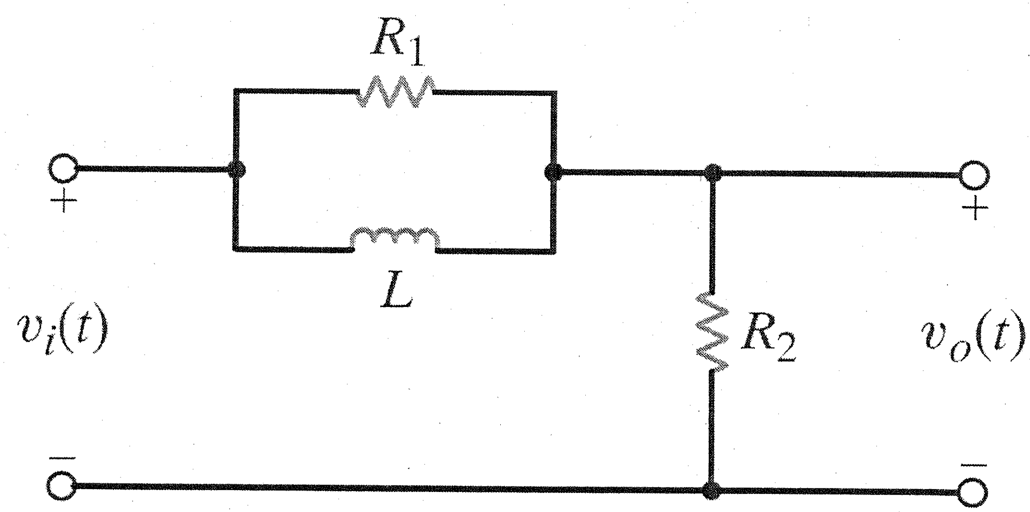
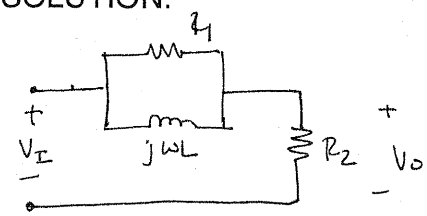


Figure P12.56

SOLUTION:

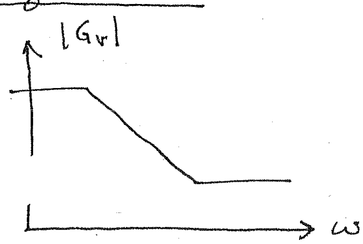


Let $Z = R_1(j\omega L)/(R_1 + j\omega L)$

$$\frac{V_o}{V_I} = G_v(j\omega) = \frac{R_2}{R_2 + Z}$$

$$G_v(j\omega) = \frac{j\omega L R_1 + R_1 R_2}{j\omega L R_1 + R_1 R_2 + j\omega L R_2} = \frac{(j\omega L + R_1)R_2}{R_1 R_2 + j\omega L(R_1 + R_2)} = \frac{R_2}{R_1 + R_2}\left[\frac{j\omega + R_1/L}{j\omega + \left(\frac{R_1 R_2}{R_1 + R_2}\right)/L}\right]$$

Rough sketch



$\boxed{\text{filter is lowpass}}$

12.57 Determine what type of filter the network shown in Fig. P12.57 represents by determining the voltage transfer function. **PSV**

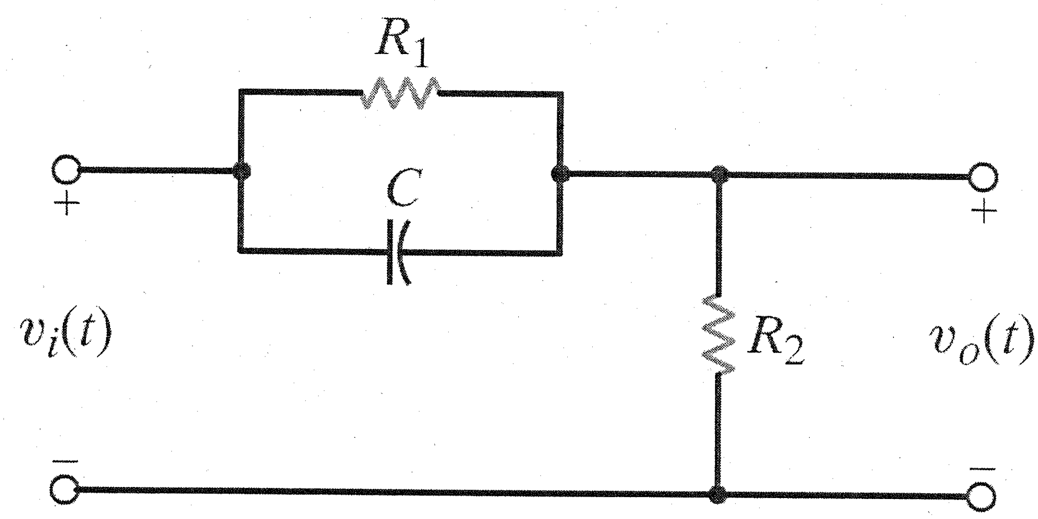
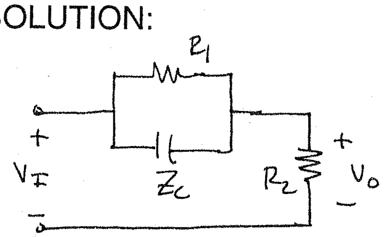


Figure P12.57

SOLUTION:

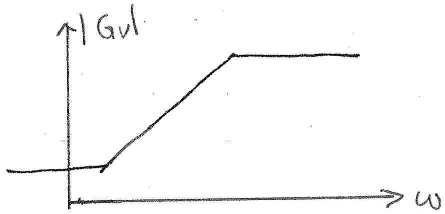


$Z_C = \frac{1}{j\omega C}$ $\qquad Z = R_1 Z_C / (R_1 + Z_C) = \frac{R_1}{j\omega R_1 C + 1}$

$G_V(j\omega) = \frac{V_o}{V_I} = \frac{R_2}{R_2 + Z}$

$$G_V(j\omega) = \frac{R_2}{R_2 + \frac{R_1}{j\omega R_1 C + 1}} = \frac{R_2(j\omega R_1 C + 1)}{j\omega R_1 R_2 C + R_1 + R_2} = \frac{j\omega + 1/R_1 C}{j\omega + 1/R_p C} \qquad R_p = R_1 // R_2$$

Rough sketch



$\boxed{\text{filter is highpass}}$

12.58 Given the lattice network shown in Fig. P12.58, determine what type of filter this network represents by determining the voltage transfer function.

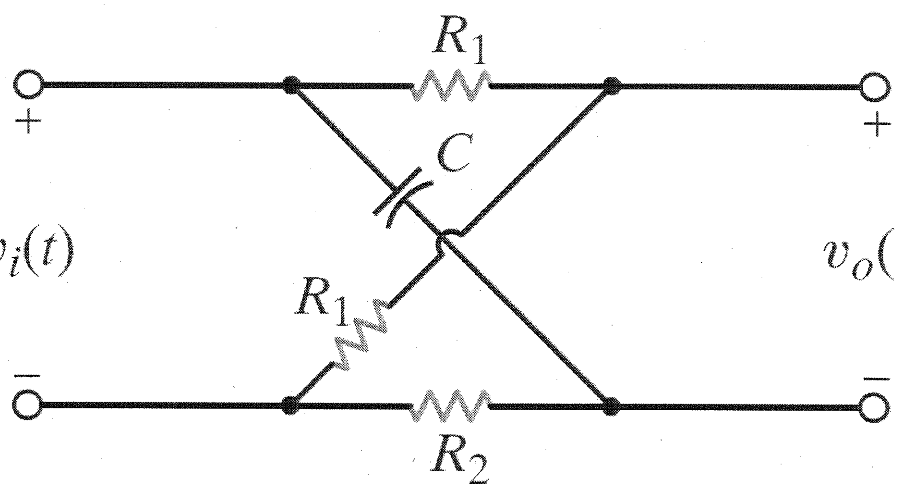
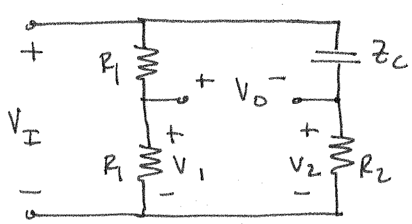


Figure P12.58

SOLUTION: $Z_C = 1/j\omega C$



$$V_1 = \frac{V_I R_1}{R_1 + R_1} = V_I/2$$

$$V_2 = \frac{V_I R_2}{R_2 + Z_C} = \frac{V_I\,(j\omega R_2 C)}{j\omega R_2 C + 1}$$

$$V_o = V_1 - V_2 = V_I\left[\frac{1}{2} - \frac{j\omega R_2 C}{j\omega R_2 C + 1}\right] = V_I\left\{\frac{1}{2}\left[\frac{1 - j\omega C R_2}{1 + j\omega C R_2}\right]\right\}$$

$$G_v = V_o/V_I = \frac{1}{2}\left[\frac{1 - j\omega C R_2}{1 + j\omega C R_2}\right]$$

$|G_v|$ is independent of ω!

$\boxed{\text{All pass filter}}$

12.59 Given the network in Fig. P12.59, and employing the voltage follower analyzed in Chapter 4, determine the voltage transfer function and its magnitude characteristic. What type of filter does the network represent?

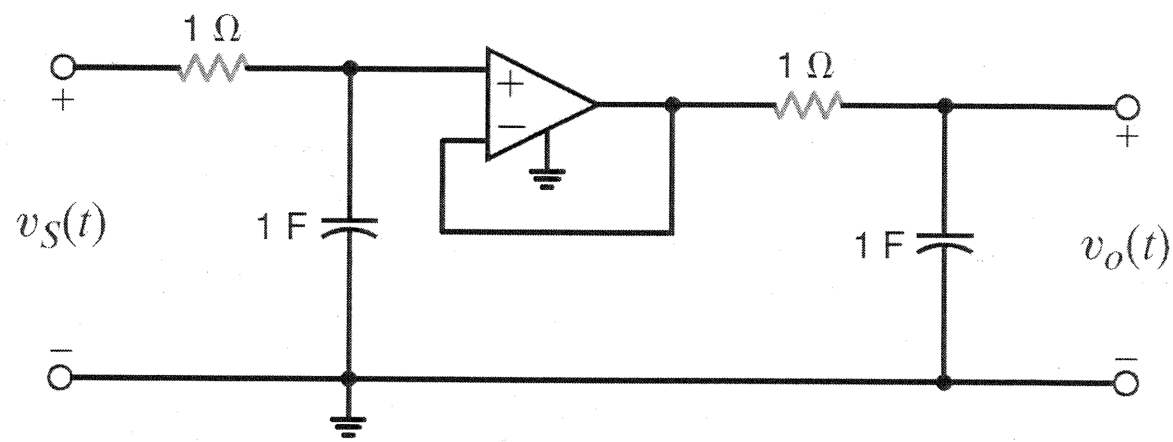
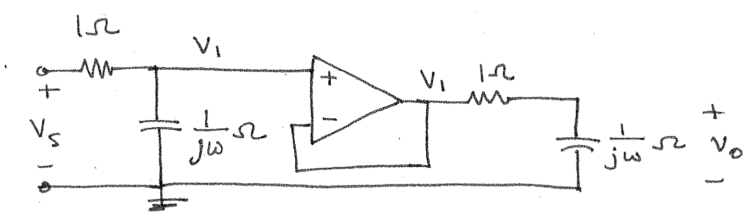


Figure P12.59

SOLUTION:

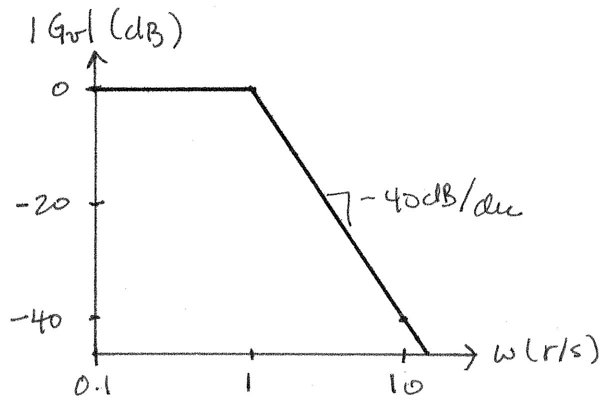


$$\frac{V_1}{V_S} = \frac{1/j\omega}{1 + 1/j\omega} = \frac{1}{j\omega + 1}$$

$$\frac{V_o}{V_1} = \frac{1/j\omega}{1 + 1/j\omega} = \frac{1}{j\omega + 1}$$

$$V_o/V_S = \frac{1}{(j\omega + 1)^2} = G_v$$

$\boxed{\text{low pass filter}}$



12.60 Given the network in Fig. P12.60, find the transfer function

$$\frac{\mathbf{V}_o}{\mathbf{V}_1}(j\omega)$$

and determine what type of filter the network represents.

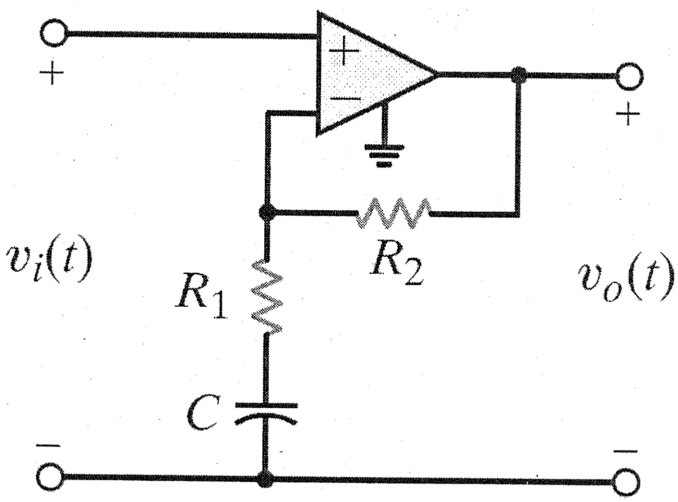


Figure P12.60

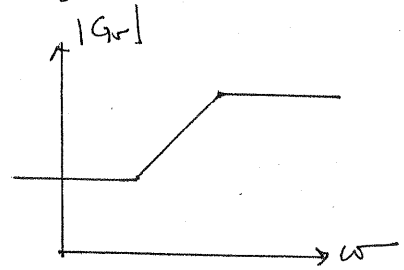
SOLUTION: Let $Z_1 = R_1 + \frac{1}{j\omega C}$

$$\frac{V_o}{V_I} = G_v(j\omega) = 1 + \frac{R_2}{Z_1} = 1 + \frac{j\omega C R_2}{j\omega C R_1 + 1} = \frac{j\omega C(R_1+R_2)+1}{j\omega C R_1 + 1} = G_v(j\omega)$$

$$\boxed{G_v = \left(1+\frac{R_2}{R_1}\right)\frac{j\omega + \frac{1}{C(R_1+R_2)}}{j\omega + \frac{1}{CR_1}}}$$

$\boxed{\text{High pass filter}}$

Rough sketch



12.61 Repeat Problem 12.55 for the network shown in Fig. P12.61.

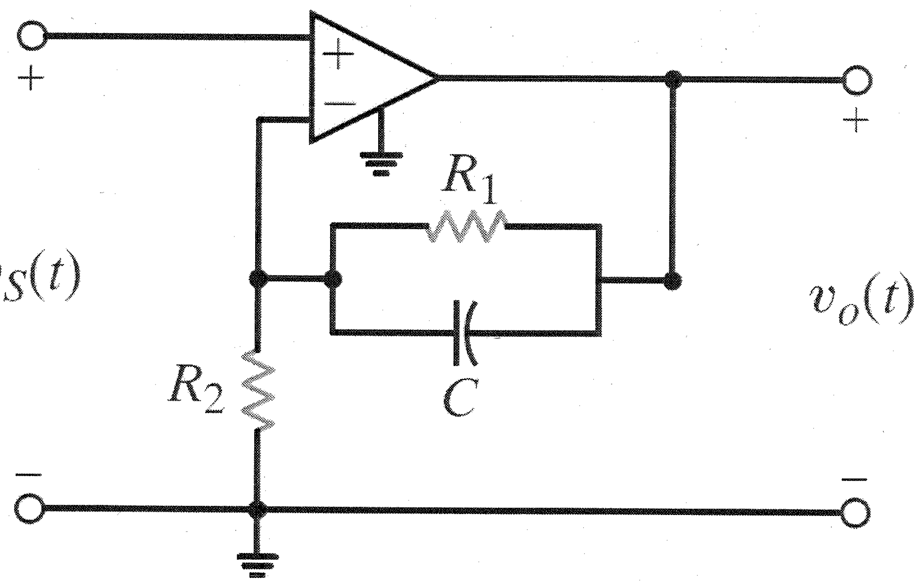


Figure P12.61

SOLUTION:

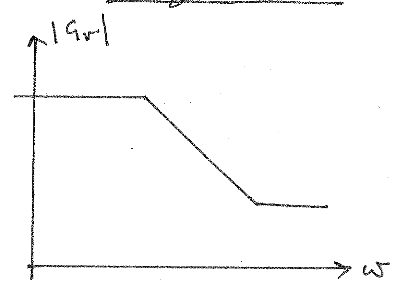
Let $Z_2 = R_1\,(1/j\omega C)\,/\,(R_1 + 1/j\omega C) = R_1\,/\,(j\omega C R_1 + 1)$

$$\frac{V_o}{V_I} = G_v(j\omega) = 1 + \frac{Z_2}{R_2} = 1 + \frac{R_1/R_2}{j\omega C R_1 + 1} = \frac{j\omega C R_1 + 1 + R_1/R_2}{j\omega C R_1 + 1}$$

$$\boxed{G_v(j\omega) = \frac{j\omega + \frac{1}{R_P C}}{j\omega + \frac{1}{R_1 C}} \qquad R_P = R_1 \,//\, R_2}$$

$\boxed{\text{Lowpass Filter}}$

Rough sketch



12.62 Determine the voltage transfer function and its magnitude characteristic for the network shown in Fig. P12.62 and identify the filter properties.

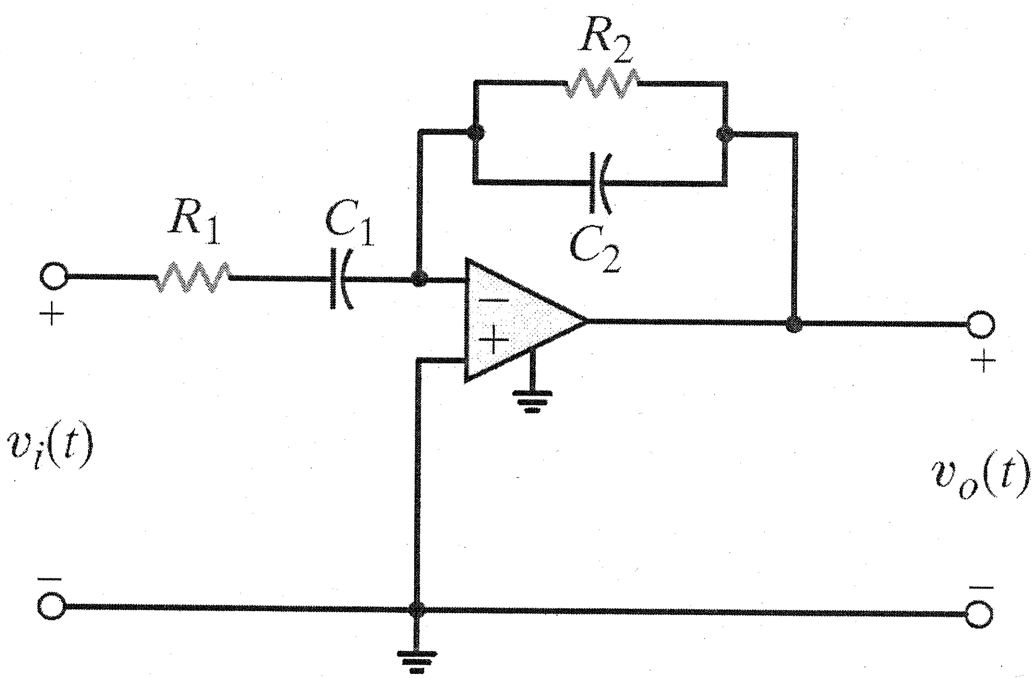


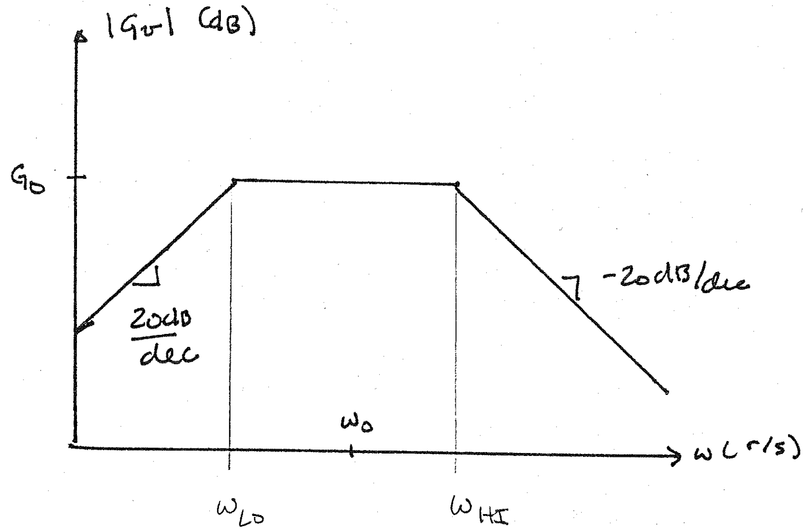
Figure P12.62

SOLUTION: Let $Z_1 = R_1 + 1/j\omega C_1 = \dfrac{j\omega C_1 R_1 + 1}{j\omega C_1}$

Let $Z_2 = \dfrac{R_2 (1/j\omega C_2)}{R_2 + 1/j\omega C_2} = \dfrac{R_2}{j\omega C_2 R_2 + 1}$

$$G_v(j\omega) = \frac{V_o}{V_I} = -\frac{Z_2}{Z_1} = \boxed{-\frac{j\omega C_1 R_2}{(j\omega C_2 R_2 + 1)(j\omega C_1 R_1 + 1)} = G_v(j\omega)}$$

Bandpass filter



$$G_v(j\omega) = \frac{j\omega C_1 R_2 / C_1 C_2 R_1 R_2}{(j\omega)^2 + j\omega\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}}$$

center freq $= \boxed{\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}}$

$$\frac{\omega_0}{Q} = \frac{1}{R_1C_1} + \frac{1}{R_2C_2} \Rightarrow \boxed{Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_2}}$$

$$BW = \frac{\omega_0}{Q} \Rightarrow \boxed{BW = \frac{1}{R_1C_1} + \frac{1}{R_2C_2}}$$

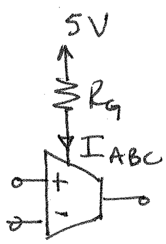
12.63 An OTA with a transconductance of 1 mS is required. A 5-V supply is available, and the sensitivity of g_m to I_{ABC} is 20.

(a) What values of I_{ABC} and R_G do you recommend?

(b) If R_G has a tolerance of +5%, what is the possible range of g_m in the final circuit?

SOLUTION:

a)



$I_{ABC} = 5/R_G$ $\quad$ $g_m = 20\, I_{ABC} = 1\text{mS}$

$I_{ABC} = \dfrac{10^{-3}}{20} \Rightarrow \boxed{I_{ABC} = 50\mu A}$

$R_G = 5/I_{ABC} \rightarrow \boxed{R_G = 100\,k\Omega}$

b) $g_m = 20\left(\dfrac{5}{R_G}\right) = \dfrac{100}{R_G}$

$95k\Omega \leq R_G \leq 105k\Omega$ $\quad$ $\boxed{0.952\text{mS} \leq g_m \leq 1.053\text{ mS}}$

12.64 A particular OTA has a maximum transconductance of 5 mS with a range of 6 decades.

(a) What is the minimum possible transconductance?

(b) What is the range of I_{ABC}?

(c) Using a 5-V power supply and resistor to set I_{ABC}, what is the range of values for the resistor and the power it consumes?

SOLUTION:

a) $\frac{G_{max}}{G_{min}} = 10^6 \Rightarrow G_{min} = \frac{G_{max}}{10^6} = 5\,nS$ $\quad \boxed{G_{min} = 5\,nS}$

b) $G_m = 20\, I_{ABC} \Rightarrow$ $\boxed{250\,pA \le I_{ABC} < 250\,\mu A}$

c) $5 = R\, I_{ABC} \Rightarrow R = \frac{5}{I_{ABC}}$ $\quad \boxed{20\,k\Omega \le R \le 20\,G\Omega}$

$P_R = I_{ABC}^2 R$ $\quad \boxed{1.25\,nW \le P \le 1.25\,mW}$

12.65 The OTA and 5-V source described in Problem 12.64 are used to create a transconductance of 2.5 mS.

(a) What resistor value is required?

(b) If the input voltage to the amplifier is $v_{in}(t) = 1.5\cos(\omega t)\text{V}$, what is the output current function?

SOLUTION:

$G_{max} = 5\,mS \qquad G_m = 20 I_{ABC} \qquad I_{ABC} = 5/R_G$

a) $G_m = \dfrac{100}{R_G} = 2.5\,mS \Rightarrow \boxed{R_G = 40\,k\Omega}$

b) $\dfrac{i_o(t)}{v_{in}(t)} = G_m = 2.5\,mS \qquad \boxed{i_o(t) = 3.75\cos(\omega t)\ mA}$

12.66 A fluid level sensor, used to measure water level in a reservoir, outputs a voltage directly proportional to fluid level. Unfortunately, the sensitivity of the sensor drifts about 10% over time. Some means for tuning the sensitivity is required. Your engineering team produces the simple OTA circuit in Fig. P12.66.

(a) Show that either V_G or R_G can be used to vary the sensitivity.

(b) List all pros and cons you can think of for these two options.

(c) What's your recommendation?

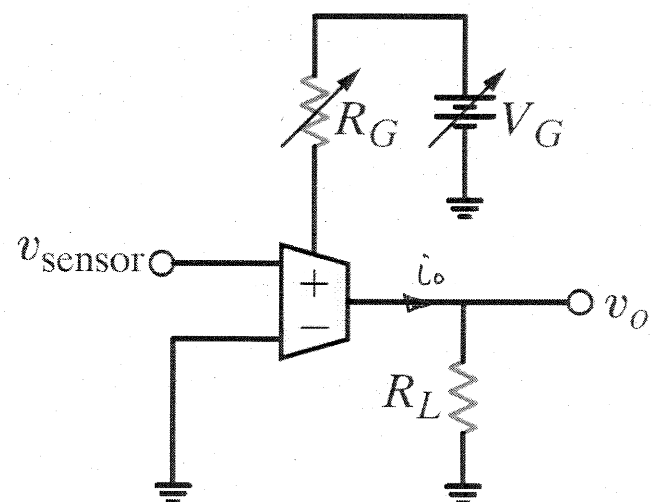


Figure P12.66

SOLUTION:

a) $v_o = i_o R_L \qquad i_o = G_m v_{sensor} \qquad G_m = 20\, I_{ABC} \qquad I_{ABC} = V_G / R_G$

So, $v_o = \dfrac{20\, V_G R_L}{R_G} v_{sensor}$ changing V_G or R_G will change $v_o(t)$!

b) Tuning with V_G requires a variable voltage source. These are much more costly than a fixed source. On the plus side, V_G and v_o are directly related - double V_G and double v_o. Tuning with R_G requires only an inexpensive potentiometer, but the $R_G - v_o$ relation is indirect.

c) Based primarily on cost considerations, recommend tuning with R_G!

12.67 A circuit is required that can double the frequency of a sinusoidal voltage.

(a) If $v_{in}(t) = 1\sin(\omega t)$, show that the multiplier circuit in Fig. P12.67 can produce an output that contains a sinusoid at frequency 2ω.

(b) We want the magnitude of the double-frequency sinusoid to be 1 V. Determine values for R_G and R_L if the transconductance range is limited between 10 μS and 10 mS.

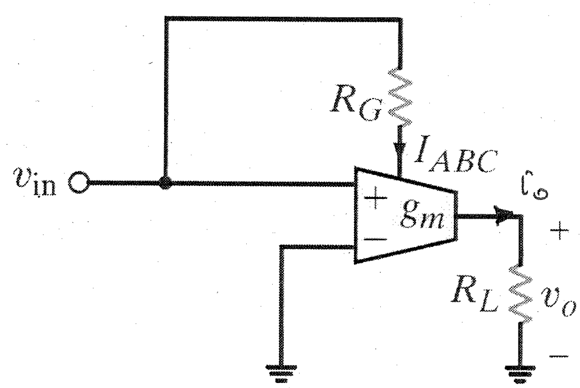


Figure P12.67

SOLUTION:

a) $v_o = i_o R_L$ $\quad i_o = g_m v_{in}$ $\quad g_m = 20 I_{ABC}$ $\quad I_{ABC} = v_{in}/R_G$

So, $v_o = R_L \frac{20}{R_G} v_{in}^2 = R_L \frac{20}{R_G} \sin^2(\omega t) = \boxed{\frac{10R_L}{R_G}\left[1 - \cos(2\omega t)\right] = v_o}$

b) $|v_{in}| = 1\text{V}$ $\quad g_{max} = 10\text{ mS} = 20|v_{in}|/R_G = 20(1)/R_G$

$R_G = 20/10^{-2} \Rightarrow \boxed{R_G = 2\text{k}\Omega}$

$|v_o| = 1 = \frac{10R_L}{R_G} \Rightarrow \boxed{R_L = 200\Omega}$ (applies to $\sin(2\omega t)$ signal only)

12.68 The frequency doubler in Problem 12.67 uses a two-quadrant multiplier.

(a) What effect does this have on the output signal?

(b) The circuit in Fig. P12.68 is one solution. Show that v_o has a double-frequency term.

(c) How would you propose to eliminate the other terms?

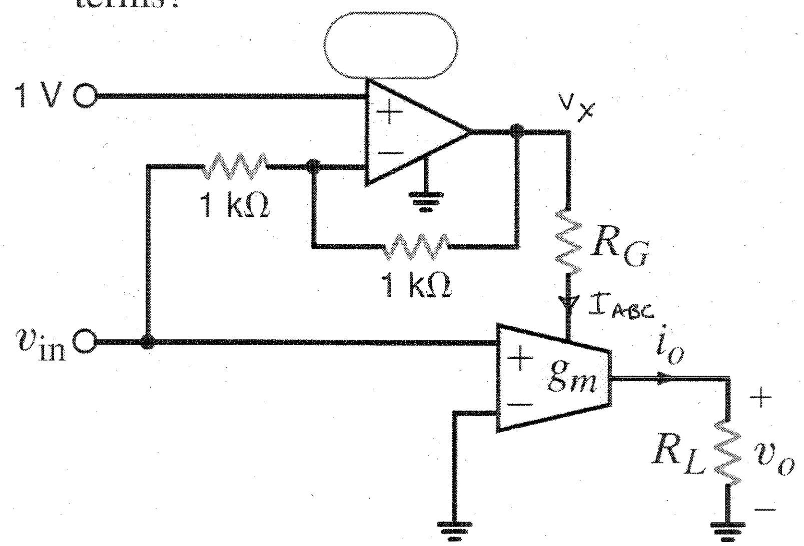


Figure P12.68

SOLUTION:

a) In problem 12.67, the negative half-cycles of $v_{in}(t)$ "try" to create a $I_{ABC} < 0$. This is not possible. The multiplier works only when $v_{in}(t) > 0$.

b) $v_X = 1\left(1 + \frac{1000}{1000}\right) - v_{in}\left(\frac{1000}{1000}\right) = 2 - v_{in}$

If as in 12.67, $v_{in}(t) = 1 \sin(\omega t)$ V, v_X is always > 0 and so is I_{ABC}.

$$v_o = i_o R_L = g_m v_{in} R_L = 20 I_{ABC} v_{in} R_L = 20 v_X R_L v_{in} / R_G$$

$$\boxed{v_o = 20(R_L/R_G)\left[2\sin(\omega t) - 1/2 + \tfrac{1}{2}\cos(2\omega t)\right]}$$

c) use a highpass filter to reject the dc & sin(ωt) terms!

12.69 In Fig. P12.69, V_x is a dc voltage. The circuit is intended to be a dc wattmeter where the output voltage value equals the power consumed by R_L in watts.

(a) The $g_m - I_{ABC}$ sensitivity is 20 S/A. Find R_G such that $I_x/I_1 = 10^4$.

(b) Choose R such that 1 V at V_o corresponds to 1 W dissipated in R_L.

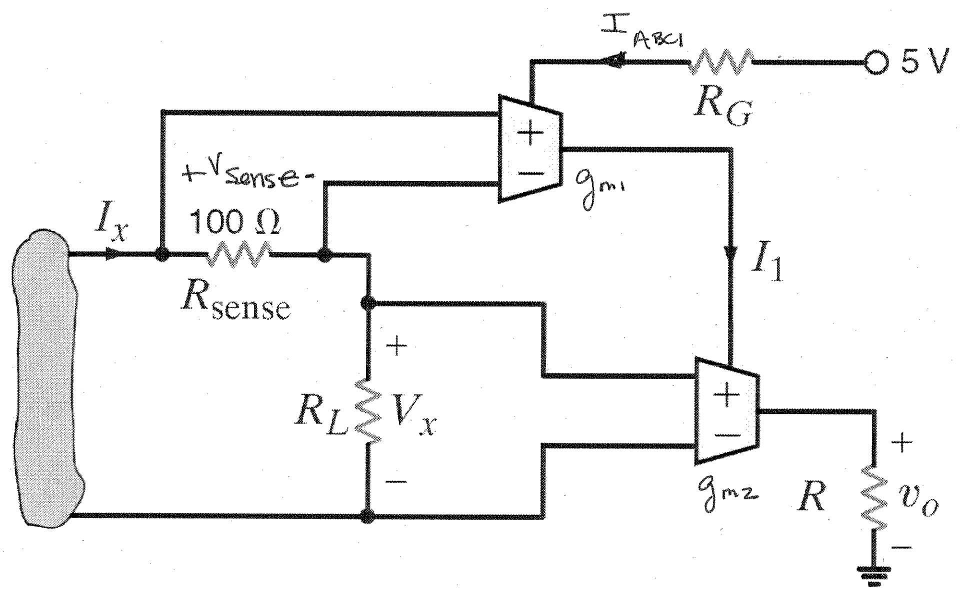


Figure P12.69

SOLUTION:

a) $V_{sense} = 100\, I_x \qquad I_1 = g_{m1} v_{sense} = \frac{20(5)}{R_G} v_{sense} = \frac{10^4 I_x}{R_G}$

$\frac{I_x}{I_1} = 10^4 = \frac{R_G}{10^4} \Rightarrow \boxed{R_G = 100\,M\Omega}$

b) $v_o = V_x\, g_{m2} R = V_x (20 I_1) R = V_x \frac{I_x}{10^4} (20R) = P_L \left(\frac{20R}{10^4}\right)$

$1V = (1W)\left(20R/10^4\right) \Rightarrow \boxed{R = 500\,\Omega}$

12.70 The automatic gain control circuit in Fig. P12.70 is used to limit the transconductance, i_o/v_{in}.

(a) Find an expression for v_o in terms of v_{in}, R_G, and R_L.

(b) Express the asymptotic transconductance, i_o/v_{in}, in terms of R_G and R_L at $v_{in} = 0$ and as v_{in} approaches infinity. Given R_L and R_G values in the circuit diagram, what are the values of the asymptotic transconductance?

(c) What are the consequences of your results in (b)?

(d) If v_{in} must be no more than V_{CC} for proper operation, what is the minimum transconductance for the functional circuit?

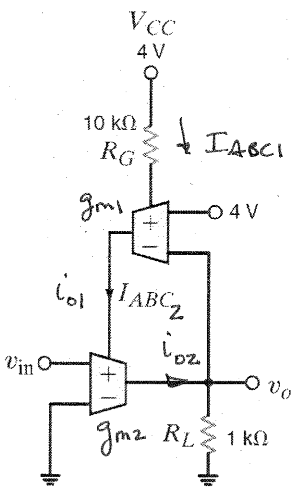


Figure P12.70

SOLUTION: a) $v_o = i_{o2} R_L = v_{in} g_{m2} R_L = 20 I_{ABC2} v_{in} R_L = 20 i_{o1} v_{in} R_L$

$i_{o1} = g_{m1}(4 - v_o) = 20 I_{ABC1}(4 - v_o) = 20(4/R_G)(4 - v_o)$

So, $v_o = 20 v_{in} R_L \left[\frac{80}{R_G}(4 - v_o)\right] \Rightarrow 6400 v_{in} \frac{R_L}{R_G} - 1600 v_{in} v_o \frac{R_L}{R_G}$

yields $v_o(1 + 160 v_{in}) = 640 v_{in} \Rightarrow \boxed{v_o = \frac{640 v_{in}}{1 + 160 v_{in}}}$

b) $G_m = i_o/v_{in} = \frac{v_o}{v_{in}}(1/R_L) = \frac{0.64}{1 + 160 v_{in}}$

as $v_{in} \to 0$, $G_m \to 0.64$ S	$G_m \to 6400/R_G$
as $v_{in} \to \infty$, $G_m \to \frac{1}{250 v_{in}}$	$G_m \to \frac{4}{R_L v_{in}}$

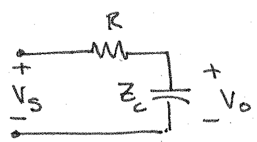
c) When v_{in} is small, G_m is high at 0.64 S.

But as v_{in} increases, G_m decreases. Eventually, i_o is limited to $4/R_L$.

d) $G_m \rightarrow \dfrac{0.64}{1+160(4)} = \boxed{\text{minimum } G_m = 998\,\mu S}$

12.71 Design a low-pass filter using one resistor and one capacitor that will produce a 4.24-volt output at 159 Hz when 6 volts at 159 Hz is applied at the input.

SOLUTION:



$V_S = 6\angle 0°$ V $f = 159$ Hz $\omega = 2\pi f = 1$ kr/s

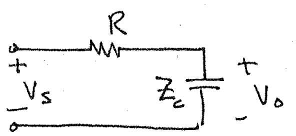
$$V_o = \frac{V_S Z_C}{Z_C + R} = \frac{6\angle 0}{j\omega CR + 1}$$

$$|V_o| = \frac{6}{\sqrt{(\omega RC)^2 + 1}} = 4.24 \Rightarrow RC = 10^{-3}\text{ s}$$

$\boxed{\text{Arbitrarily select } C = 1\,\mu F \Rightarrow R = 1\,k\Omega}$

12.72 Design a low-pass filter with a cutoff frequency between 15 and 16 kHz.

SOLUTION:



$$\frac{V_o}{V_s} = \frac{Z_C}{R+Z_C} = \frac{1}{j\omega RC+1} = \frac{1/RC}{j\omega + 1/RC}$$

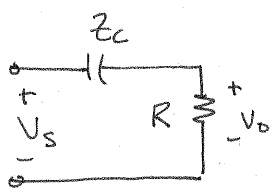
$$\omega_{cutoff} = \frac{1}{RC}.$$

Arbitrarily select $\omega_{cutoff} = 2\pi(15.5)$ kr/s & $C = 10$ nF

yields, $R = 1.03\text{k}\Omega$

12.73 Design a high-pass filter with a half-power frequency between 159 and 161 Hz.

SOLUTION:



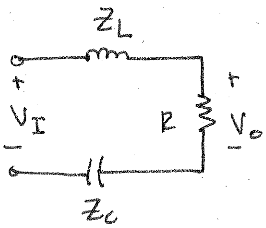
$$\frac{V_o}{V_S} = \frac{R}{R+Z_C} = \frac{j\omega}{j\omega + \frac{1}{RC}} \qquad \omega_{cutoff} = \frac{1}{RC}$$

Arbitrarily select $\omega_{cutoff} = 160\,Hz$ and $C = 1\mu F$

yields, $R = 6.25\,k\Omega$

12.74 Design a band-pass filter with a low cutoff frequency of approximately 4535 Hz and a high cutoff frequency of approximately 5535 Hz.

SOLUTION:



$$\frac{V_o}{V_I} = \frac{R}{j\omega L + R + \frac{1}{j\omega C}} = \frac{j\omega (R/L)}{(j\omega)^2 + j\omega (R/L) + \frac{1}{LC}}$$

Series RLC circut, $BW = \frac{R}{L} = 2\pi(1000)$

$$\omega_0 = \sqrt{\omega_{HI}\,\omega_{LO}} = 2\pi\sqrt{f_{HI} f_{LO}} = 31.5 \text{ kr/s} = \frac{1}{\sqrt{LC}}$$

Arbitrarily select $C = 100$ nF

yields $L = 10$ mH

and $R = 62.8\,\Omega$

12.75 An engineer has proposed the circuit shown in Fig. P12.75 to filter out high-frequency noise. Determine the values of the capacitor and resistor to achieve a 3-dB voltage drop at 23.16 kHz.

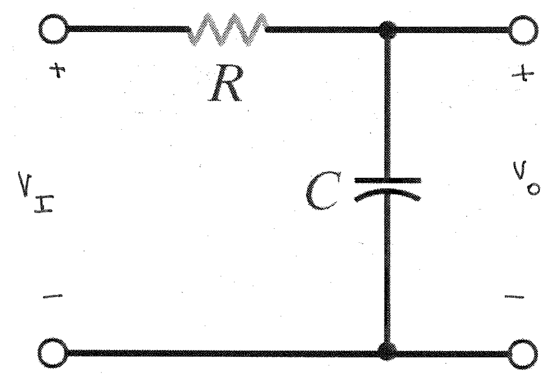


Figure P12.75

SOLUTION: $Z_C = 1/j\omega C$

$$\frac{V_o}{V_I} = \frac{Z_C}{Z_C + R} = \frac{1/RC}{j\omega + 1/RC}$$

3dB down at $\omega = \frac{1}{RC} = 2\pi(23.16 \times 10^3)$

$\boxed{\text{Arbitrarily select } C = 1\,\text{nF, yields } R = 6.87\,\text{k}\Omega}$

12.76 For the low-pass active filter in Fig. P12.76, choose R_2 and C such that $H_o = -7$ and $f_c = 10$ kHz.

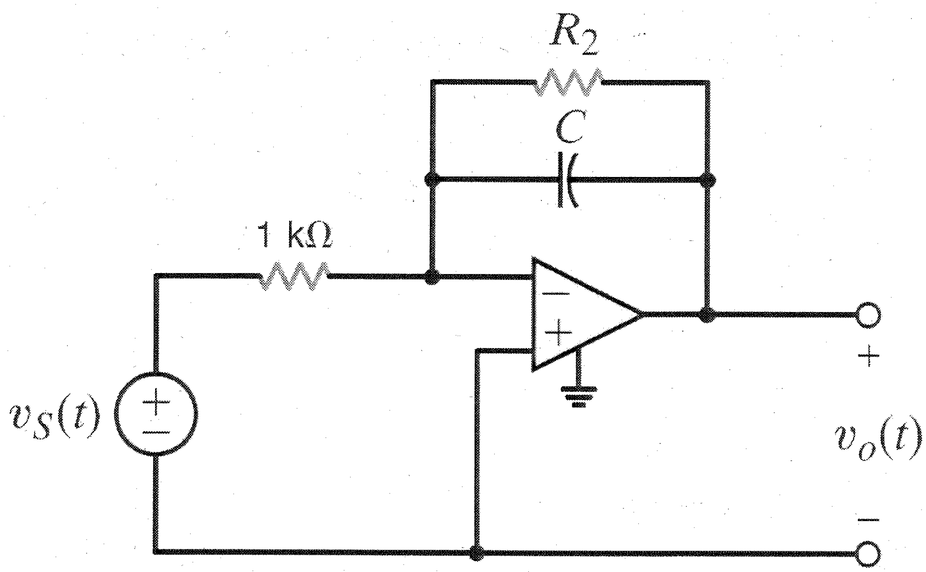


Figure P12.76

SOLUTION: Let $Z_2 = \dfrac{R_2/j\omega C}{R_2 + \frac{1}{j\omega C}} = \dfrac{R_2}{j\omega C R_2 + 1}$ & $Z_1 = 1k\Omega$

$$H(j\omega) = \frac{V_o}{V_s} = -\frac{Z_2}{Z_1} = \frac{-R_2(10^{-3})}{j\omega C R_2 + 1} = -\left(\frac{1}{10^3 C}\right)\left(\frac{1}{j\omega + \frac{1}{CR_2}}\right)$$

$$H_o = -7 = -\frac{1(CR_2)}{CR_1} = \frac{-R_2}{R_1} \Rightarrow \boxed{R_2 = 7k\Omega}$$

$$\omega_c = \frac{1}{CR_2} = 2\pi f_c \Rightarrow \boxed{C = 2.27\,nF}$$

12.77 For the high-pass active filter in Fig. P12.77, choose C, R_1, and R_2 such that $H_o = 5$ and $f_c = 3$ kHz.

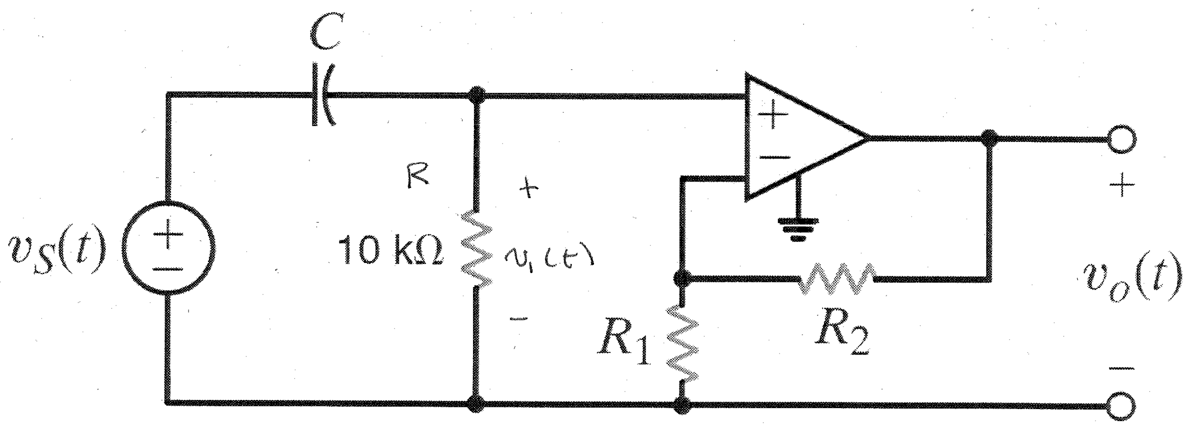


Figure P12.77

SOLUTION:

$$\frac{V_1}{V_S} = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega}{j\omega + \frac{1}{RC}} \qquad \frac{V_o}{V_1} = 1 + R_2/R_1$$

$$\frac{V_o}{V_S} = \left(1 + \frac{R_2}{R_1}\right)\frac{j\omega}{j\omega + \frac{1}{RC}} \qquad H_o = 1 + \frac{R_2}{R_1} = 5 \qquad \omega_c = 2\pi f_c = \frac{1}{RC}$$

$$C = \frac{1}{2\pi f_c R} \Rightarrow \boxed{C = 5.31\text{ nF}}$$

$$\boxed{\text{Arbitrarily select } R_1 = 10\text{k}\Omega\text{, yields } R_2 = 40\text{k}\Omega}$$

12.78 Given the second-order low-pass filter in Fig. P12.78, design a filter that has $H_o = 100$ and $f_c = 5$ kHz. Set $R_1 = R_3 = 1\ \text{k}\Omega$, and let $R_2 = R_4$ and $C_1 = C_2$. Use an op-amp model with $R_i = \infty$, $R_o = 0$, and $A = (2)10^5$.

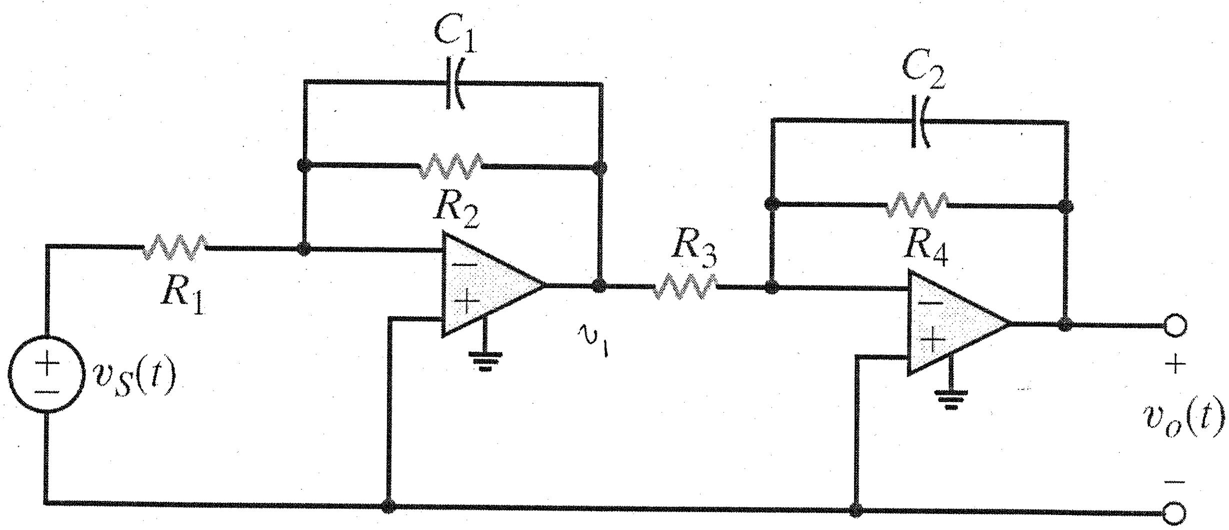


Figure P12.78

SOLUTION: Constraints yield

$$\frac{V_1}{V_S} = G_1(j\omega) = \frac{V_o}{V_1} = G_2(j\omega)$$

Let $$Z = \frac{R_2/j\omega C}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1} = \frac{1/C}{j\omega + 1/R_2C}$$

$$*\ G_1(j\omega) = -Z/R_1 = \frac{-1/R_1C}{j\omega + 1/R_2C}$$

$$G(j\omega) = G_1 G_2 = \frac{(1/R_1C)^2}{\left(j\omega + \frac{1}{R_2C}\right)^2}$$

$$\begin{cases} H_o = (R_2/R_1)^2 = 100 \Rightarrow R_2/R_1 = 10 \\ \omega_c = 2\pi f_c = \dfrac{1}{R_2C} = \dfrac{1}{10^4 C} \end{cases}$$

$C = 3.18$nF, yields $R_2 = 10\ \text{k}\Omega$ and $R_4 = 10\ \text{k}\Omega$

* For each op amp, $R_2/R_1 = 10$ which is << A for op amp (2×10^5). Including A in the analysis would only affect beyond 4 digits!

12.79 The second-order low-pass filter shown in Fig. P12.79 has the transfer function

$$\frac{\mathbf{V}_o}{\mathbf{V}_1}(s) = \frac{\dfrac{-R_3}{R_1}\left(\dfrac{1}{R_2R_3C_1C_2}\right)}{s^2 + \dfrac{s}{C_1}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right) + \dfrac{1}{R_2R_3C_1C_2}}$$

Design a filter with $H_o = -10$ and $f_c = 5$ kHz, assuming that $C_1 = C_2 = 10$ nF and $R_1 = 1$ kΩ.

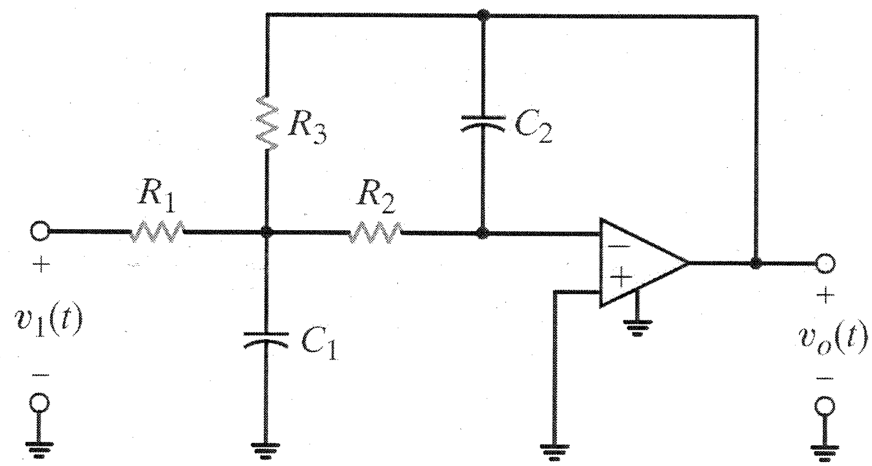


Figure P12.79

SOLUTION:

$$H_o = -\frac{R_3}{R_1}\left(\frac{1}{R_2R_3C_1C_2}\right) \Big/ \frac{1}{R_2R_3C_1C_2} = -R_3/R_1 = -10 \Rightarrow \boxed{R_3 = 10\,k\Omega}$$

R_2 affects f_c. Look at characteristic equation

$$s^2 + Bs + C = 0 \qquad B = \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad C = \frac{1}{R_2R_3C_1C_2}$$

Roots at

$$\left(s + \frac{B}{2} - \sqrt{\frac{B^2}{4} - C}\right)\left(s + B/2 + \sqrt{\frac{B^2}{4} - C}\right) = 0 = (s + \omega_{P1})(s + \omega_{P2})$$

where ω_{P1} & ω_{P2} are the poles. Since $\omega_{P1} < \omega_{P2}$,

$\omega_{P1} = 2\pi f_c = \omega_c$.

$$B/2 - \sqrt{B^2/4 - C} = \omega_c \Rightarrow \left(\frac{B}{2} - \omega_c\right)^2 = B^2/4 - C$$

$$\frac{B^2}{4} - B\omega_c + \omega_c^2 = B^2/4 - C \Rightarrow B\omega_c - C = \omega_c^2$$

Using R & C values and $f_c = 500\text{Hz}$, $B = 1.1\times10^5 + 10^8/R_2$

and $C = 10^{12}/R_2$ & $\omega_c = 1000\pi$

$$\left(1.1\times10^5 + \frac{10^8}{R_2}\right)1000\pi - \frac{10^{12}}{R_2} = 10^6\pi^2$$

$$1.1\times10^8\pi + \frac{10^{11}\pi}{R_2} - \frac{10^{12}}{R_2} = 10^6\pi^2$$

$$110\pi - \pi^2 = \frac{10^6 - 10^5\pi}{R_2} \Rightarrow R_2 = \frac{10^5(10-\pi)}{\pi(110-\pi)}$$

$$\boxed{R_2 = 2043\,\Omega}$$

As an aside: $\omega_{p1} = 500(2\pi)$ r/s

$\omega_{p2} = 2\pi\,(24.8)$ kr/s

12.80 Given the circuit in Figure 12.57, design a second-order bandpass filter with a center frequency gain of -5, $\omega_0 = 50$ krad/s, and a BW $= 10$ krad/s. Let $C_1 = C_2 = C$ and $R_1 = 1\ \text{k}\Omega$. What is the Q of this filter? Sketch the Bode plot for the filter. Use the ideal op-amp model.

SOLUTION:

From the text,

$$\omega_o = \left(\frac{1 + R_1/R_3}{R_1 R_2 C^2}\right)^{1/2} = 50\ \text{kr/s}$$

$$\frac{\omega_o}{Q} = BW = \frac{2C}{R_2 C^2} = \frac{2}{R_2 C} = BW = 10\ \text{kr/s}$$

$$\text{center freq gain} = \frac{-\frac{1}{R_1 C}}{\frac{1}{R_2 C} + \frac{1}{R_2 C}} = -\frac{R_2}{2R_1} = -5 = H_o$$

$R_2 = 10 R_1$ $\qquad \boxed{R_2 = 10\text{k}\Omega}$

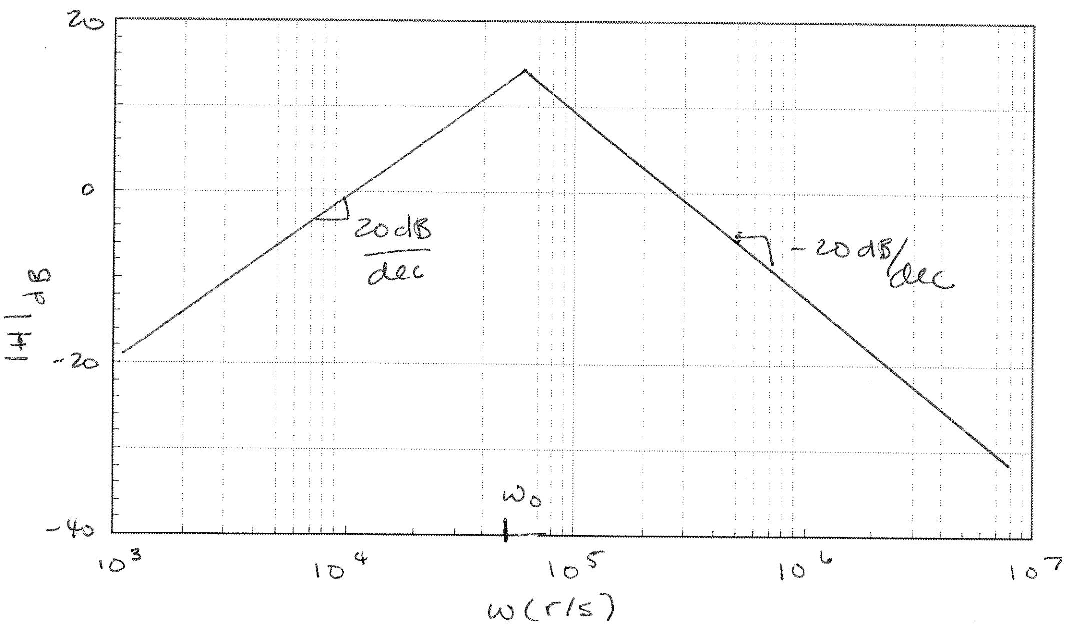
$C = \dfrac{2}{R_2 (BW)}$ $\qquad \boxed{C = 20\text{nF}}$

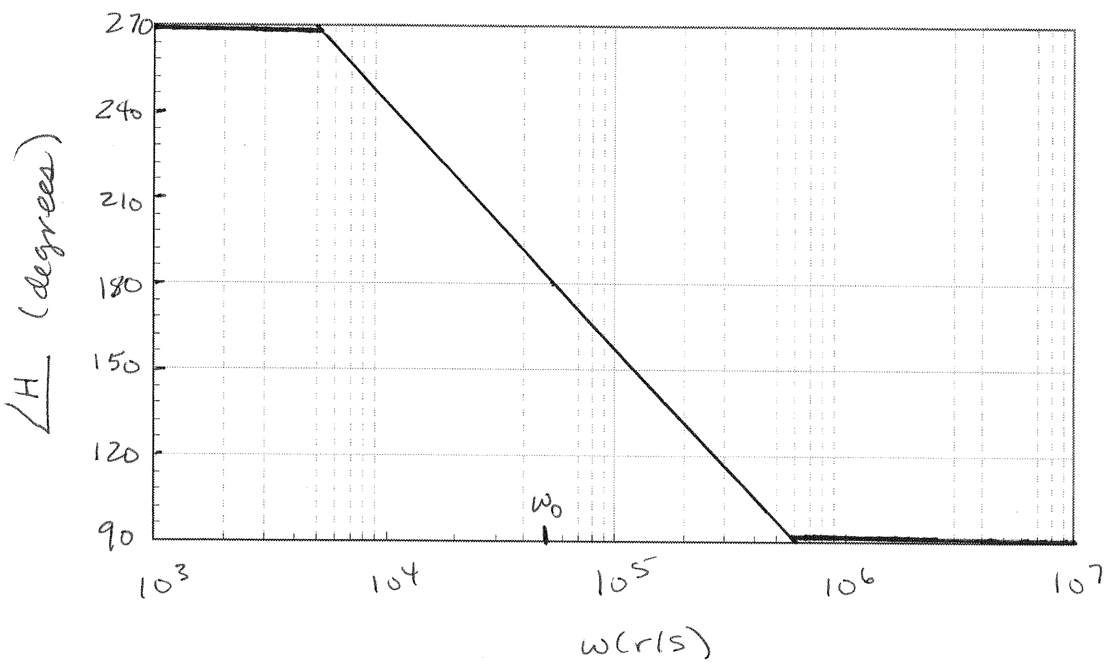
$R_3 = \dfrac{R_1}{\omega_o^2 R_1 R_2 C^2 - 1}$ $\qquad \boxed{R_3 = 111\Omega}$

$Q = \omega_o / BW$ $\qquad \boxed{Q = 5}$

$$H(s) = \frac{H_o (2BW) s}{s^2 + 2BW s + \omega_o^2} = \frac{-10^5 s}{s^2 + 2\times10^4 s + 2.5\times10^9}$$

poles at $s = -10^4 \pm j4.90\times10^4$ r/s


20
0
-20
-40
|H| dB
20dB/dec
-20dB/dec
ω0
10^3
10^4
10^5
10^6
10^7
ω(r/s)


270
240
210
180
150
120
90
∠H (degrees)
ω0
10^3
10^4
10^5
10^6
10^7
ω(r/s)

12.81 Referring to Example 12.38, design a notch filter for the tape deck for use in Europe, where power utilities generate at 50 Hz.

SOLUTION:

From Ex. 12.38, $\omega_z = \frac{1}{\sqrt{LC}}$

We need $\omega_z = 2\pi(50) = 100\pi$ r/s

Arbitrarily, we select $C = 1000\,\mu F$
yielding $L = 10\,mH$

12FE-1 Determine the resonant frequency of the circuit in Fig. 12PFE-1, and find the voltage $\mathbf{V}_o$ at resonance.
CS

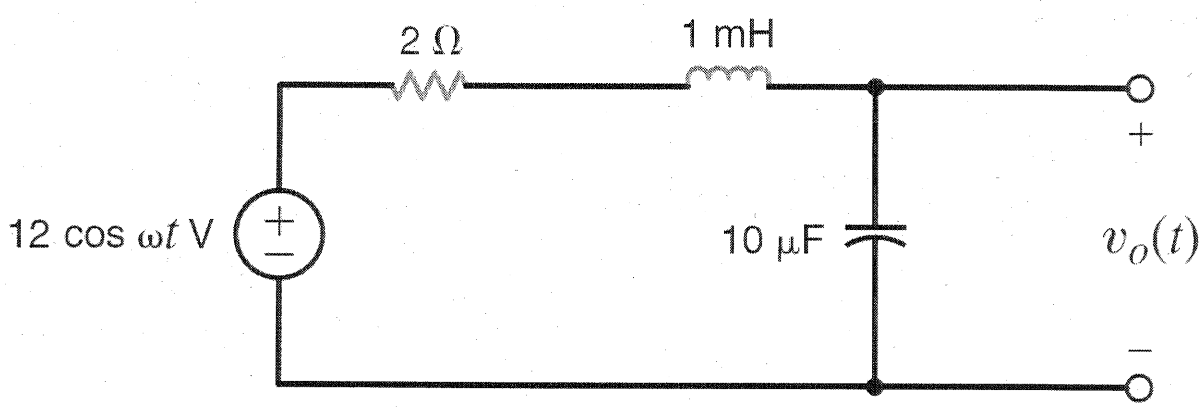


Figure 12PFE-1

SOLUTION:

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_0 = 10k \text{ r/s}}$$

$$V_o(j\omega_0) = 12\angle 0 \left[\frac{Z_C(j\omega_0)}{2}\right] = 6\angle 0 \left(\frac{1}{\omega_0 C \angle 90}\right)$$

$$\boxed{V_o(j\omega_0) = 60\angle -90^\circ \text{ V}}$$

12FE-2 Given the series circuit in Fig. 12PFE-2, determine the resonant frequency, and find the value of R so that the BW of the network about the resonant frequency is 200 rad/s.

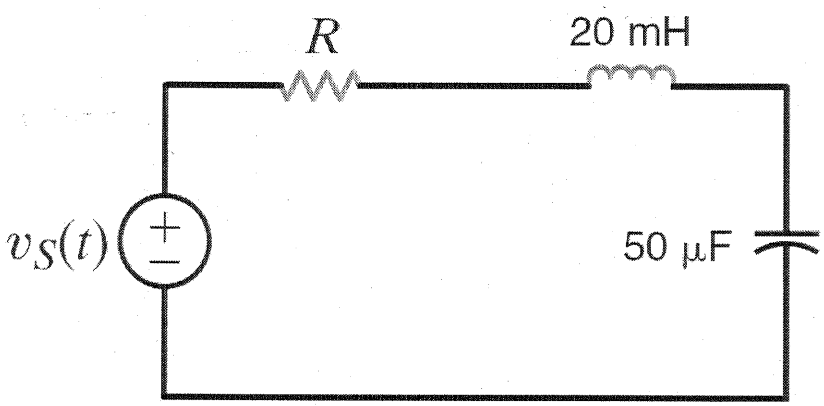


Figure 12PFE-2

SOLUTION:

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_0 = 1000 \text{ r/s}}$$

$$BW = R/L = 200 \text{ r/s} \Rightarrow \boxed{R = 4\Omega}$$

12FE-3 Given the low-pass filter circuit shown in Fig. 12PFE-3, find the frequency in Hz at which the output is down 3 dB from the dc, or very low frequency, output. CS

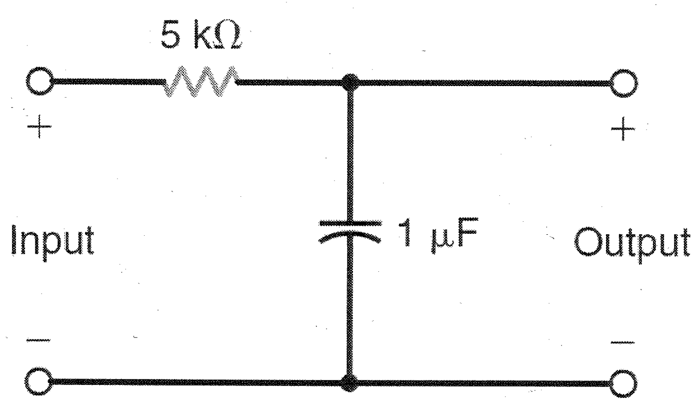


Figure 12PFE-3

SOLUTION:

$$\frac{V_o}{V_I} = \frac{Z_C}{Z_C + R} = \frac{1/j\omega C}{1/j\omega C + R} = \frac{1/RC}{j\omega + 1/RC} = G_v(j\omega)$$

at dc, $G_v = 1 = 0\,\text{dB}$

at 3dB down, $\omega = \frac{1}{RC} \Rightarrow \boxed{\omega = 200\ \text{r/s}}$

12FE-4 Given the band-pass filter shown in Fig. 12PFE-4, find the components L and R necessary to provide a resonant frequency of 1000 rad/s and a BW of 100 rad/s.

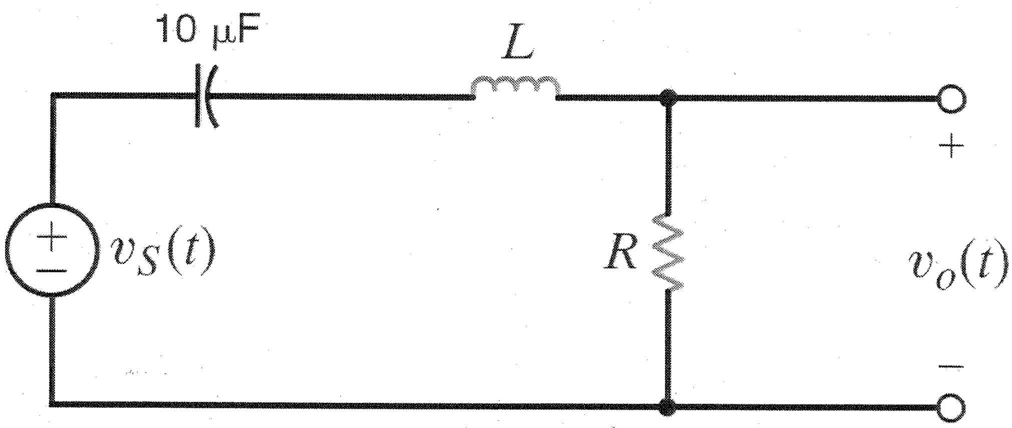


Figure 12PFE-4

SOLUTION:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1000 \rightarrow L = \frac{1}{\omega_0^2 C} \Rightarrow \boxed{L = 100\,mH}$$

$$BW = R/L = 100\ r/s \Rightarrow \boxed{R = 10\,\Omega}$$

12FE-5 Given the low-pass filter shown in Fig. 12PFE-5, find the half-power frequency and the gain of this circuit, if the source frequency is 8 Hz. CS

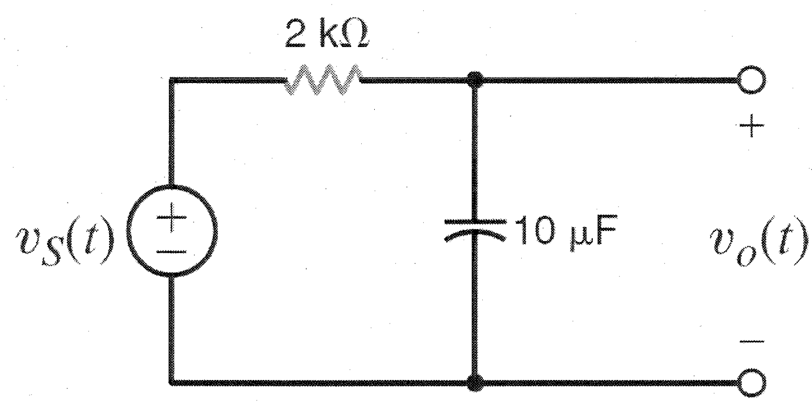


Figure 12PFE-5

SOLUTION:

$f = 8\text{ Hz} \qquad \omega_1 = 16\pi \text{ r/s} \qquad Z_C = \dfrac{1}{j\omega C} = -j2.0\text{ k}\Omega$

$$\frac{V_o}{V_s} = G_v(j\omega_1) = \frac{Z_C}{Z_C + R} = \frac{-j\,2000}{2000 - j\,2000} \Rightarrow \boxed{G_v(j\omega_1) = 0.707\,\angle -45^\circ}$$

In general, $G_v(j\omega) = \dfrac{1/j\omega C}{R + 1/j\omega C} = \dfrac{1/RC}{j\omega + 1/RC}$

Half power freq $= f_c = \dfrac{1}{2\pi RC} \Rightarrow \boxed{f_c = 8.0\text{ Hz}}$