Chapter Thirteen:

The LaPlace Transform

13.1 Find the Laplace transform of the function

$$f(t) = te^{-at}\delta(t-1)$$

CS

SOLUTION:

$$\mathcal{L}[f(t)] = \int_0^\infty te^{-at}e^{-st}\delta(t-1)\,dt = te^{-(s+a)t}\Big|_{t=1}$$

$$\boxed{\mathcal{L}[f(t)] = e^{-(s+a)}}$$

13.2 Find the Laplace transform of the function

$$f(t) = te^{-at}\sin(\omega t)\delta(t-4).$$

SOLUTION:

$$\mathcal{L}[f(t)] = \int_0^{\infty} te^{-at}e^{-st}\sin(\omega t)\delta(t-4)\,dt$$

$$= te^{-(s+a)t}\sin(\omega t)\Big|_{t=4}$$

$$\boxed{\mathcal{L}[f(t)] = 4e^{-4(s+a)}\sin(4\omega)}$$

13.3 If $f(t) = e^{-at}$, show that $\mathbf{F}(s) = \dfrac{1}{s + a}$.

SOLUTION:

$$F(s) = \int_0^\infty e^{-at} e^{-st}\, dt = \int_0^\infty e^{-(s+a)t}\, dt = \left. \frac{e^{-(s+a)t}}{-(s+a)} \right|_0^\infty$$

$$F(s) = \frac{e^{-\infty(s+a)}}{-(s+a)} - \frac{e^{-0}}{-(s+a)} = \frac{1}{s+a}$$

$$\boxed{F(s) = \frac{1}{s+a}} \text{ for } \sigma > -a$$

13.4 If $f(t) = e^{-at} \sin \omega t$, show that $\mathbf{F}(s) = \dfrac{\omega}{(s+a)^2 + \omega^2}$.

SOLUTION:

$$F(s) = \int_0^\infty e^{-at} e^{-st} \sin \omega t \, dt = \int_0^\infty e^{-(s+a)t} \sin \omega t \, dt$$

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$F(s) = \int_0^\infty e^{-(s+a)t} \frac{e^{j\omega t} - e^{-j\omega t}}{2j} dt = \int_0^\infty \frac{e^{(j\omega - s - a)t} - e^{(-j\omega - s - a)t}}{2j} dt$$

$$F(s) = \left. \frac{e^{(j\omega - s - a)t}}{(j\omega - s - a)2j} + \frac{e^{(-j\omega - s - a)t}}{(j\omega + s + a)2j} \right|_0^\infty = \frac{-1}{(j\omega - s - a)2j} - \frac{1}{(j\omega + s + a)j2}$$

$$F(s) = \left[\frac{1}{\omega + j(s+a)} + \frac{1}{\omega - j(s+a)} \right] \frac{1}{2} = \frac{\omega}{(\omega + j(s+a))(\omega - j(s+a))}$$

$$\boxed{F(s) = \frac{\omega}{(s+a)^2 + \omega^2}}$$

13.5 If $f(t) = t\cos(\omega t)u(t-1)$, find $\mathbf{F}(s)$. CS

SOLUTION:

$$f(t) = e^{-at}\sin(\omega t)\,u(t-1)$$

$$F(s) = e^{-s}\mathcal{L}\left[e^{-a(t+1)}\sin[\omega(t+1)]\right] = e^{-(s+a)}\mathcal{L}\left[e^{-at}\sin[\omega(t+1)]\right]$$

$$F(s) = e^{-(s+a)}\mathcal{L}\left[e^{-at}(\sin\omega t\cos\omega + \cos\omega t\sin\omega)\right]$$

$$\boxed{F(s) = e^{-(s+a)}\left\{\frac{\omega\cos\omega}{(s+a)^2+\omega^2} + \frac{(s+a)\sin\omega}{(s+a)^2+\omega^2}\right\}}$$

13.6 Use the time-shifting theorem to determine $\mathcal{L}[f(t)]$, where $f(t) = [e^{-(t-2)} - e^{-2(t-2)}]u(t-2)$.

SOLUTION:

let $g(t) = (e^{-t} - e^{-2t})u(t) \quad G(s) = \frac{1}{s+1} - \frac{1}{s+2}$

$$F(s) = e^{-2s}G(s) = e^{-2s}\left(\frac{s+2-(s+1)}{(s+2)(s+1)}\right)$$

$$\boxed{F(s) = \frac{e^{-2s}}{(s+1)(s+2)}}$$

13.7 Use the time-shifting theorem to determine $\mathcal{L}[f(t)]$, where $f(t) = [t - 1 + e^{-(t-1)}]u(t-1)$. PSV

SOLUTION:

Let $g(t) = (t + e^{-t})u(t)$ $\quad G(s) = \frac{1}{s^2} + \frac{1}{s+1}$

$F(s) = e^{-s}G(s) \Rightarrow$ $\boxed{F(s) = e^{-s}\left[\frac{1}{s^2} + \frac{1}{s+1}\right]}$

13.8 Use property number 5 to find $\mathcal{L}[f(t)]$ if $f(t) = e^{-at}u(t-1)$. CS

SOLUTION:

$$\mathcal{L}[f(t)] = e^{-s}\mathcal{L}\left[e^{-a(t+1)}\right] = e^{-s}e^{-a}\mathcal{L}\left[e^{-at}\right]$$

$$F(s) = e^{-(s+a)}\mathcal{L}[e^{-at}]$$

$$\boxed{F(s) = \frac{e^{-(s+a)}}{s+1}}$$

13.9 Use property number 7 to find $\mathcal{L}[f(t)]$ if $f(t) = te^{-at}u(t-1)$.

SOLUTION:

let $g(t) = t\,u(t-1) \Rightarrow F(s) = G(s+a)$

$G(s) = \mathcal{L}[t\,u(t-1)] = e^{-s}\mathcal{L}[t+1] = e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right)$

$$\boxed{F(s) = e^{-(s+a)}\left[\frac{1}{(s+a)^2} + \frac{1}{s+a}\right]}$$

13.10 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $$\mathbf{F}(s) = \frac{4}{(s+3)(s+4)}$$

(b) $$\mathbf{F}(s) = \frac{10s}{(s+1)(s+6)}$$

SOLUTION:

a) $$F(s) = \frac{K_1}{s+3} + \frac{K_2}{s+4} \qquad \left\{ \begin{array}{l} K_1 = \dfrac{4}{-3+4} = 4 \\ K_2 = \dfrac{4}{-4+3} = -4 \end{array} \right.$$

$$F(s) = \frac{4}{s+3} - \frac{4}{s+4}$$

$$\boxed{f(t) = \left(4e^{-3t} - 4e^{-4t}\right) u(t)}$$

b) $$F(s) = \frac{K_1}{s+1} + \frac{K_2}{s+6} \qquad \left\{ \begin{array}{l} K_1 = \dfrac{-10}{-1+6} = -2 \\ K_2 = \dfrac{-60}{-5} = 12 \end{array} \right.$$

$$F(s) = \frac{-2}{s+1} + \frac{12}{s+6}$$

$$\boxed{f(t) = \left(12e^{-6t} - 2e^{-t}\right) u(t)}$$

13.11 Given the following functions $\mathbf{F}(s)$, find $f(t)$. PSV

(a) $\mathbf{F}(s) = \dfrac{s+1}{(s+2)(s+6)}$

(b) $\mathbf{F}(s) = \dfrac{24}{(s+2)(s+3)}$

SOLUTION:

a) $F(s) = \dfrac{K_1}{s+2} + \dfrac{K_2}{s+6}$ $\begin{cases} K_1 = \dfrac{-2+1}{-2+6} = -1/4 \\ K_2 = \dfrac{-6+1}{-6+2} = 5/4 \end{cases}$

$F(s) = \dfrac{5/4}{s+6} - \dfrac{1/4}{s+2}$ $\boxed{f(t) = \left(\frac{5}{4}e^{-6t} - \frac{1}{4}e^{-2t}\right)u(t)}$

b) $F(s) = \dfrac{K_1}{s+2} + \dfrac{K_2}{s+3}$ $\begin{cases} K_1 = \dfrac{24}{-2+3} = 24 \\ K_2 = \dfrac{24}{-3+2} = -24 \end{cases}$

$F(s) = \dfrac{24}{s+2} - \dfrac{24}{s+3}$ $\boxed{f(t) = \left(24e^{-2t} - 24e^{-3t}\right)u(t)}$

13.12 Given the following functions $\mathbf{F}(s)$, find $f(t)$. CS

(a) $\mathbf{F}(s) = \dfrac{s+1}{s(s+2)(s+3)}$

(b) $\mathbf{F}(s) = \dfrac{s^2+s+1}{s(s+1)(s+2)}$

SOLUTION:

a) $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+2} + \dfrac{K_3}{s+3}$

$$\begin{cases} K_1 = \dfrac{1}{(2)(3)} = 1/6 \\ K_2 = \dfrac{-2+1}{-2(1)} = 1/2 = 3/6 \\ K_3 = \dfrac{-3+1}{-3(-3+2)} = -2/3 = -4/6 \end{cases}$$

$F(s) = \dfrac{1}{6}\left[\dfrac{1}{s} + \dfrac{3}{s+2} - \dfrac{4}{s+3}\right]$

$$\boxed{f(t) = \frac{1}{6}\left[1 + 3e^{-2t} - 4e^{-3t}\right]u(t)}$$

b) $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+1} + \dfrac{K_3}{s+2}$

$K_1 = \dfrac{1}{(1)(2)} = 1/2$

$K_2 = \dfrac{(-1)^2 - 1 + 1}{(-1)(-1+2)} = -1$

$K_3 = \dfrac{(-2)^2 - 2 + 1}{(-2)(-2+1)} = \dfrac{3}{2}$

$F(s) = \dfrac{1}{2}\left[\dfrac{1}{s} - \dfrac{2}{s+1} + \dfrac{3}{s+2}\right]$

$$\boxed{f(t) = \frac{1}{2}\left[1 - 2e^{-t} + 3e^{-2t}\right]u(t)}$$

13.13 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $$\mathbf{F}(s) = \frac{s^2 + 5s + 4}{(s+2)(s+4)(s+6)}$$

(b) $$\mathbf{F}(s) = \frac{(s+3)(s+6)}{s(s^2 + 8s + 12)}$$

SOLUTION:

a) $$F(s) = \frac{(s+4)(s+1)}{(s+2)(s+4)(s+6)} = \frac{(s+1)}{(s+2)(s+6)} = \frac{K_1}{s+2} + \frac{K_2}{s+6}$$

$$K_1 = \frac{-2+1}{-2+6} = -1/4 \qquad K_2 = \frac{-6+1}{-6+2} = 5/4$$

$$F(s) = \frac{1}{4}\left[\frac{5}{s+6} - \frac{1}{s+2}\right] \Rightarrow \boxed{f(t) = \frac{1}{4}\left[5e^{-6t} - e^{-2t}\right]u(t)}$$

b) $$F(s) = \frac{(s+3)(s+6)}{s(s+6)(s+2)} = \frac{s+3}{s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$$

$$K_1 = \frac{3}{2} \qquad K_2 = \frac{-2+3}{-2} = -1/2$$

$$F(s) = \frac{1}{2}\left[\frac{3}{s} - \frac{1}{s+2}\right] \Rightarrow \boxed{f(t) = \frac{1}{2}\left[3 - e^{-2t}\right]u(t)}$$

13.14 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $$\mathbf{F}(s) = \frac{s^2 + 7s + 12}{(s+2)(s+4)(s+6)}$$

(b) $$\mathbf{F}(s) = \frac{(s+3)(s+6)}{s(s^2 + 10s + 24)}$$

SOLUTION:

a) $$F(s) = \frac{(s+4)(s+3)}{(s+2)(s+4)(s+6)} = \frac{s+3}{(s+2)(s+6)} = \frac{K_1}{s+2} + \frac{K_2}{s+6}$$

$$K_1 = \frac{-2+3}{-2+6} = \frac{1}{4} \qquad K_2 = \frac{-6+3}{-6+2} = 3/4$$

$$F(s) = \frac{1}{4}\left[\frac{1}{s+2} + \frac{3}{s+6}\right] \Rightarrow \boxed{f(t) = \frac{1}{4}\left[e^{-2t} + 3e^{-6t}\right]u(t)}$$

b) $$F(s) = \frac{(s+3)(s+6)}{s(s+4)(s+6)} = \frac{s+3}{s(s+4)} = \frac{K_1}{s} + \frac{K_2}{s+4}$$

$$K_1 = \frac{3}{4} \qquad K_2 = \frac{-4+3}{-4} = +1/4$$

$$F(s) = \frac{1}{4}\left[\frac{3}{s} + \frac{1}{s+4}\right] \Rightarrow \boxed{f(t) = \frac{1}{4}\left[3 + e^{-4t}\right]u(t)}$$

13.15 Use MATLAB to solve Problem 13.14.

SOLUTION:

a)

```
EDU» syms s t
EDU» ilaplace((s^2+7*s+12)/((s+2)*(s+4)*(s+6)))

ans =

3/4*exp(-6*t)+1/4*exp(-2*t)
```

b)

```
EDU» syms s t
EDU» ilaplace((s+3)*(s+6)/(s*(s^2+10*s+24)))

ans =

1/4*exp(-4*t)+3/4
```

13.16 Given the following functions $\mathbf{F}(s)$, find $f(t)$

(a) $\mathbf{F}(s) = \dfrac{10}{s^2 + 2s + 2}$

(b) $\mathbf{F}(s) = \dfrac{10(s+2)}{s^2 + 4s + 5}$

SOLUTION:

a) $F(s) = \dfrac{10}{(s+1-j1)(s+1+j1)} = \dfrac{-5j}{s+1-j1} + \dfrac{5j}{s+1+j1}$

$K_1 = -j5 = 5\angle -90^\circ$

$$\boxed{f(t) = 10e^{-t}\cos(t - 90^\circ)\,u(t)}$$

b) $F(s) = \dfrac{10(s+2)}{(s+2-j1)(s+2+j1)} = \dfrac{K_1}{s+2-j1} + \dfrac{K_1^*}{s+2+j1}$

$K_1 = \dfrac{10(-2+j1+2)}{j2} = 5 \Rightarrow K_1^* = 5$

$F(s) = \dfrac{5}{s+2-j1} + \dfrac{5}{s+2+j1}$

$$\boxed{f(t) = 10e^{-2t}\cos(t)\,u(t)}$$

13.17 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $$\mathbf{F}(s) = \frac{s(s+6)}{(s+3)(s^2+6s+18)}$$

(b) $$\mathbf{F}(s) = \frac{(s+4)(s+8)}{s(s^2+4s+8)}$$

SOLUTION:

a) $$F(s) = \frac{s(s+6)}{(s+3)(s+3-j3)(s+3+j3)} = \frac{K_1}{s+3} + \frac{K_2}{s+3-j3} + \frac{K_2^*}{s+3+j3}$$

$$K_1 = \frac{-3(-3+6)}{(-j3)(j3)} = -1 \qquad K_2 = \frac{(-3+j3)(-3+j3+6)}{(-3+j3+3)(j6)} = 1 \Rightarrow K_2^* = 1$$

$$F(s) = \frac{-1}{s+3} + \frac{1}{s+3-j3} + \frac{1}{s+3+j3}$$

$$\boxed{f(t) = [-e^{-3t} + 2e^{-3t}\cos(3t)]u(t)}$$

b) $$F(s) = \frac{(s+4)(s+8)}{s(s+2-j2)(s+2+j2)} = \frac{K_1}{s} + \frac{K_2}{s+2-j2} + \frac{K_2^*}{s+2+j2}$$

$$K_1 = \frac{4(8)}{(2-j2)(2+j2)} = 4 \qquad K_2 = \frac{(2+j2)(6+j2)}{(-2+j2)(j4)} = 1.58\angle -162^\circ$$

$$\boxed{f(t) = [4 + 3.16e^{-2t}\cos(2t - 162^\circ)]u(t)}$$

13.18 Given the following functions $\mathbf{F}(s)$, find the inverse Laplace transform of each function.

(a) $\mathbf{F}(s) = \dfrac{10(s+1)}{s^2 + 2s + 2}$

(b) $\mathbf{F}(s) = \dfrac{s+1}{s(s^2 + 4s + 5)}$

SOLUTION:

a) $F(s) = \dfrac{K_1}{s+1-j1} + \dfrac{K_1^*}{s+1+j1}$ $\quad K_1 = \dfrac{10(-1+j1+1)}{j2} = 5 \Rightarrow K_1^* = 5$

$$\boxed{f(t) = [10\,e^{-t}\cos(t)]u(t)}$$

b) $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+2-j1} + \dfrac{K_2^*}{s+2+j1}$ $\quad K_1 = \dfrac{1}{5} = 0.2$

$$K_2 = \frac{-2+j1+1}{(-2+j1)(j2)} = \frac{-1+j1}{-2-j4} = 0.31\angle{-108^\circ}$$

$$\boxed{f(t) = [0.2 + 0.62e^{-2t}\cos(t - 108^\circ)]u(t)}$$

13.19 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $$\mathbf{F}(s) = \frac{s(s+6)}{(s+3)(s^2+6s+18)}$$

(b) $$\mathbf{F}(s) = \frac{(s+4)(s+8)}{s(s^2+8s+32)}$$

SOLUTION:

a) $$F(s) = \frac{s(s+6)}{(s+3)(s+3-j3)(s+3+j3)} = \frac{K_1}{s+3} + \frac{K_2}{s+3-j3} + \frac{K_2^*}{s+3+j3}$$

$$K_1 = \frac{(-3)(-3+6)}{(-3)^2-6(3)+18} = -1 \qquad K_2 = \frac{(-3+j3)(3+j3)}{(j3)(j6)} = 1 \Rightarrow K_2^* = 1$$

$$\boxed{f(t) = [-e^{-3t} + 2e^{-3t}\cos(3t)]u(t)}$$

b) $$F(s) = \frac{K_1}{s} + \frac{K_2}{s+4-j4} + \frac{K_2^*}{s+4+j4} \qquad K_1 = \frac{(4)(8)}{32} = 1$$

$$K_2 = \frac{j4(4+j4)}{(-4+j4)(j8)} = -j(1/2) = \tfrac{1}{2}\angle -90^\circ$$

$$\boxed{f(t) = [1 + e^{-4t}\cos(4t - 90^\circ)]u(t)}$$

13.20 Use MATLAB to solve Problem 13.19. CS

SOLUTION:

a)

```
EDU» syms s t
EDU» ilaplace(s*(s+6)/((s+3)*(s^2+6*s+18)))

ans =

-exp(-3*t)+2*exp(-3*t)*cos(3*t)
```

b)

```
EDU» syms s t
EDU» ilaplace((s+4)*(s+8)/(s*(s^2+8*s+32)))

ans =

1+exp(-4*t)*sin(4*t)
```

13.21 Given the following functions $\mathbf{F}(s)$, find $f(t)$. **PSV**

(a) $\mathbf{F}(s) = \dfrac{(s+1)(s+3)}{(s+2)(s^2+2s+2)}$

(b) $\mathbf{F}(s) = \dfrac{(s+2)^2}{s^2+4s+5}$

SOLUTION:

a) $F(s) = \dfrac{(s+1)(s+3)}{(s+2)(s+1-j1)(s+1+j1)} = \dfrac{K_1}{s+2} + \dfrac{K_2}{s+1-j1} + \dfrac{K_2^*}{s+1+j1}$

$K_1 = \dfrac{(-1)(1)}{-4+4+2} = -\frac{1}{2}$ $\quad K_2 = \dfrac{j1(2+j1)}{(1+j1)(j2)} = 0.79\angle -18.4^\circ$

$$\boxed{f(t) = \left[\tfrac{1}{2}e^{-2t} + 1.58\,e^{-t}\cos(t-18.4^\circ)\right]u(t)}$$

b)

$F(s) = \dfrac{s^2+4s+4}{s^2+4s+5} = 1 - \dfrac{1}{s^2+4s+5} = 1 - \left[\dfrac{K_1}{s+2-j1} + \dfrac{K_1^*}{s+2+j1}\right]$

$K_1 = \dfrac{1}{j2} = -j\frac{1}{2} = \frac{1}{2}\angle 90^\circ$

$F(s) = 1 - \dfrac{\frac{1}{2}\angle -90^\circ}{s+2-j1} - \dfrac{\frac{1}{2}\angle +90^\circ}{s+2+j1}$

$$\boxed{f(t) = \left[\delta(t) - e^{-2t}\cos(t-90^\circ)\right]u(t)}$$

13.22 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $\mathbf{F}(s) = \dfrac{s^2 + 4s + 8}{(s+1)(s+4)}$

(b) $\mathbf{F}(s) = \dfrac{s+4}{s^2}$

SOLUTION:

a) $F(s) = \dfrac{s^2+4s+8}{s^2+5s+4} = 1 + \dfrac{4-s}{(s+1)(s+4)} = 1 + \dfrac{K_1}{s+1} + \dfrac{K_2}{s+4}$

$K_1 = \dfrac{4-(-1)}{3} = \dfrac{5}{3}$ $\quad K_2 = \dfrac{4-(-4)}{-3} = -8/3$

$F(s) = 1 + \dfrac{5/3}{s+1} - \dfrac{8/3}{s+4}$ $\quad \boxed{f(t) = \left[\delta(t) + \frac{5}{3}e^{-t} - \frac{8}{3}e^{-4t}\right]u(t)}$

b) $F(s) = \dfrac{s+4}{s^2} = \dfrac{K_1}{s^2} + \dfrac{K_2}{s}$ $\quad K_1 = 4$

let $s = 1$: $\dfrac{1+4}{(1)^2} = \dfrac{4}{(1)^2} + \dfrac{K_2}{(1)} \Rightarrow K_2 = 1$

$F(s) = \dfrac{4}{s^2} + \dfrac{1}{s}$ $\quad \boxed{f(t) = (4t+1)\,u(t)}$

13.23 Given the following functions $\mathbf{F}(s)$, find the inverse Laplace transform of each function. **PSV**

(a) $\mathbf{F}(s) = \dfrac{s+6}{s^2(s+2)}$

(b) $\mathbf{F}(s) = \dfrac{s+3}{(s+1)^2(s+3)}$

SOLUTION:

a) $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+2}$ $\quad K_1 = \dfrac{6}{2} = 3$ $\quad K_3 = \dfrac{4}{(-2)^2} = 1$

Let $s=-1$: $F(-1) = \dfrac{5}{1^2(1)} = \dfrac{3}{1} + \dfrac{K_2}{(-1)} + \dfrac{1}{1} \Rightarrow K_2 = -1$

$F(s) = \dfrac{3}{s^2} - \dfrac{1}{s} + \dfrac{1}{s+2} \Rightarrow \boxed{f(t) = [3t - 1 + e^{-2t}]u(t)}$

b) $F(s) = \dfrac{1}{(s+1)^2}$ $\quad K_1 = 1$

Let $G(s) = 1/s^2$, $\quad f(t) = e^{-t} g(t)$

$g(t) = t\,u(t) \rightarrow \boxed{f(t) = te^{-t}u(t)}$

13.24 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $$\mathbf{F}(s) = \frac{s+4}{(s+2)^2}$$

(b) $$\mathbf{F}(s) = \frac{s+6}{s(s+1)^2}$$

SOLUTION:

a) $F(s) = \frac{K_1}{(s+2)^2} + \frac{K_2}{s+2}$ $\quad K_1 = 2$

let $s=-1$, $\quad F(-1) = \frac{3}{1} = \frac{2}{1} + K_2 \Rightarrow K_2 = 1$

$F(s) = \frac{2}{(s+2)^2} + \frac{1}{s+2} \Rightarrow \boxed{f(t) = \left[e^{-2t}(2t+1)\right]u(t)}$

b) $F(s) = \frac{K_1}{s} + \frac{K_2}{(s+1)^2} + \frac{K_3}{s+1}$ $\quad K_1 = 6 \quad K_2 = \frac{5}{-1} = -5$

let $s=-2$; $\quad F(-2) = \frac{4}{(-2)(-1)^2} = -\frac{6}{2} - \frac{5}{(-1)^2} - K_3 = -6$

$F(s) = \frac{6}{s} - \frac{5}{(s+1)^2} - \frac{6}{s+1} \Rightarrow \boxed{f(t) = \left[6 - 5te^{-t} - 6e^{-t}\right]u(t)}$

13.25 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $\mathbf{F}(s) = \dfrac{s+8}{s^2(s+4)}$

(b) $\mathbf{F}(s) = \dfrac{1}{s^2(s+1)^2}$

SOLUTION:

a) $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+4}$ $\quad K_1 = 2 \quad K_3 = \dfrac{4}{(-4)^2} = 1/4$

let $s=-2$, $F(-2) = \dfrac{6}{4(2)} = \dfrac{3}{4} = \dfrac{2}{4} - \dfrac{K_2}{2} + \dfrac{1/4}{2} \Rightarrow K_2 = -1/4$

$F(s) = \dfrac{2}{s^2} - \dfrac{1/4}{s} + \dfrac{1/4}{s+4} \Rightarrow \boxed{f(t) = \left[2t - 1/4 + 1/4\, e^{-4t}\right]u(t)}$

b) $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{(s+1)^2} + \dfrac{K_4}{s+1}$ $\quad K_1 = 1 \quad K_3 = 1$

let $s=1$, $F(1) = \dfrac{1}{4} = 1 + K_2 + \dfrac{1}{4} + \dfrac{K_4}{2} \Rightarrow K_2 + \dfrac{K_4}{2} = -1$

let $s=-2$, $F(-2) = \dfrac{1}{4} = \dfrac{1}{4} - \dfrac{K_2}{2} + \dfrac{1}{1} - K_4 \Rightarrow K_2 + 2K_4 = 2$

$\left.\begin{matrix} \\ \end{matrix}\right\} K_2 = -2, \quad K_4 = 2$

$F(s) = \dfrac{1}{s^2} - \dfrac{2}{s} + \dfrac{1}{(s+1)^2} + \dfrac{2}{s+1} \Rightarrow \boxed{f(t) = \left[t - 2 + te^{-t} + 2e^{-t}\right]u(t)}$

13.26 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $\mathbf{F}(s) = \dfrac{s+3}{(s+2)^2}$

(b) $\mathbf{F}(s) = \dfrac{s+6}{s(s+2)^2}$

SOLUTION:

a) $F(s) = \dfrac{K_1}{(s+2)^2} + \dfrac{K_2}{s+2}$ $\quad K_1 = 1$

let $s = -1$, $F(-1) = 2 = 1 + K_2 \Rightarrow K_2 = 1$

$F(s) = \dfrac{1}{(s+2)^2} + \dfrac{1}{s+2} \Rightarrow \boxed{f(t) = \left[t e^{-2t} + e^{-2t}\right] u(t)}$

b) $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+2)^2} + \dfrac{K_3}{s+2}$ $\quad K_1 = \dfrac{3}{2}$ $\quad K_2 = \dfrac{4}{-2} = -2$

let $s = -1$, $F(-1) = \dfrac{5}{-1} = -5 = -\dfrac{3}{2} - 2 + K_3 \Rightarrow K_3 = -3/2$

$F(s) = \dfrac{3}{2}/s - \dfrac{2}{(s+2)^2} - \dfrac{3/2}{s+2} \Rightarrow \boxed{f(t) = \left[3/2 - 2t e^{-2t} - \frac{3}{2} e^{-2t}\right] u(t)}$

13.27 Given the following functions $\mathbf{F}(s)$, find $f(t)$. CS

(a) $\mathbf{F}(s) = \dfrac{s^2}{(s+1)^2(s+2)}$

(b) $\mathbf{F}(s) = \dfrac{s^2+9s+20}{s(s+4)^3(s+5)}$

SOLUTION:

a) $F(s) = \dfrac{K_1}{(s+1)^2} + \dfrac{K_2}{s+1} + \dfrac{K_3}{s+2}$ $\quad K_1 = 1 \quad K_3 = \dfrac{4}{1} = 4$

let $s = 0$, $F(0) = 0 = 1 + K_2 + 2 \Rightarrow K_2 = -3$

$F(s) = \dfrac{1}{(s+1)^2} - \dfrac{3}{s+1} + \dfrac{4}{s+2} \Rightarrow \boxed{f(t) = \left[te^{-t} - 3e^{-t} + 4e^{-2t}\right]u(t)}$

b) $F(s) = \dfrac{(s+4)(s+5)}{s(s+4)^3(s+5)} = \dfrac{1}{s(s+4)^2} = \dfrac{K_1}{s} + \dfrac{K_2}{(s+4)^2} + \dfrac{K_3}{s+4}$

$K_1 = 1/16 \quad K_2 = -1/4$

let $s=-2$; $F(-2) = \dfrac{1}{-2(2)^2} = \dfrac{-1}{8} = -\dfrac{1}{32} - \dfrac{1}{16} + \dfrac{K_3}{2} \Rightarrow K_3 = -1/16$

$F(s) = \dfrac{1}{16}\left[\dfrac{1}{s} - \dfrac{4}{(s+4)^2} - \dfrac{1}{s+4}\right] \Rightarrow \boxed{f(t) = \dfrac{1}{16}\left[1 - 4te^{-4t} - e^{-4t}\right]u(t)}$

13.28 Find $f(t)$ if $\mathbf{F}(s)$ is given by the expression

$$\mathbf{F}(s) = \frac{s(s+1)}{(s+2)^3(s+3)}$$

SOLUTION:

$$F(s) = \frac{K_1}{(s+2)^3} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2} + \frac{K_4}{s+3}$$

$$K_1 = \frac{(-2)(-1)}{1} = 2 \qquad K_4 = \frac{(-3)(-2)}{(-1)^3} = -6$$

let $s=-1$, $F(-1) = 0 = 2 + K_2 + K_3 - 3 \Rightarrow \quad K_2 + K_3 = 1$

let $s = 0$, $F(0) = 0 = \frac{2}{8} + \frac{K_2}{4} + \frac{K_3}{2} - 2 \Rightarrow \quad K_2 + 2K_3 = 7$

yields $K_2 = -5$ & $K_3 = 6$

$$F(s) = \frac{2}{(s+2)^3} - \frac{5}{(s+2)^2} + \frac{6}{s+2} - \frac{6}{s+3}$$

$$\boxed{f(t) = \left[t^2 e^{-2t} - 5te^{-2t} + 6e^{-2t} - 6e^{-3t}\right]u(t)}$$

13.29 Use MATLAB to solve Problem 13.27.

SOLUTION:

a)

```
EDU» syms s t
EDU» ilaplace(s^2/((s+2)*(s^2+2*s+1)))

ans =

4*exp(-2*t)+t*exp(-t)-3*exp(-t)
```

b)

```
EDU» syms s t
EDU» ilaplace(1/(s*(s+4)^2))

ans =

1/16-1/16*(1+4*t)*exp(-4*t)
```

13.30 Find the inverse Laplace transform of the following functions. CS

(a) $\mathbf{F}(s) = \dfrac{e^{-s}}{s+1}$

(b) $\mathbf{F}(s) = \dfrac{1 - e^{-2s}}{s}$

(c) $\mathbf{F}(s) = \dfrac{1 - e^{-s}}{s+2}$

SOLUTION:

a) Let $G(s) = \frac{1}{s+1} \Rightarrow g(t) = e^{-t} u(t)$

$F(s) = e^{-s} G(s) \Rightarrow f(t) = g(t-1)\, u(t-1)$

$\boxed{f(t) = e^{-(t-1)} u(t-1)}$

b) $G(s) = 1/s \Rightarrow g(t) = 1\, u(t) \quad F(s) = G(s) - e^{-2s} G(s)$

$\boxed{f(t) = 1\, u(t) - 1\, u(t-2)}$

c) $G(s) = \frac{1}{s+2} \Rightarrow g(t) = e^{-2t} u(t) \quad F(s) = G(s) - e^{-s} G(s)$

$\boxed{f(t) = e^{-2t} u(t) - e^{-2(t-1)} u(t-1)}$

13.31 Find the inverse Laplace transform of the following functions.

(a) $\mathbf{F}(s) = \dfrac{(s+2)e^{-s}}{s(s+2)}$

(b) $\mathbf{F}(s) = \dfrac{e^{-10s}}{(s+2)(s+3)}$

SOLUTION:

a) $F(s) = \dfrac{e^{-s}}{s} \Rightarrow f(t) = u(t-1)$

b) $F(s) = e^{-10s}\left[\dfrac{K_1}{s+2} + \dfrac{K_2}{s+3}\right]$ $\quad K_1 = 1$ $\quad K_2 = -1$

$F(s) = e^{-10s}\left[\dfrac{1}{s+2} - \dfrac{1}{s+3}\right] \Rightarrow \boxed{f(t) = \left\{e^{-2[t-10]} - e^{-3[t-10]}\right\} u(t-10)}$

13.32 Find the inverse Laplace transform $f(t)$ if $\mathbf{F}(s)$ is

$$\mathbf{F}(s) = \frac{se^{-s}}{(s+4)(s+8)}$$

SOLUTION:

$$F(s) = e^{-s}\left[\frac{K_1}{s+4} + \frac{K_2}{s+8}\right] \quad \begin{cases} K_1 = \dfrac{-4}{4} = -1 \\ K_2 = \dfrac{-8}{-4} = 2 \end{cases}$$

$$F(s) = e^{-s}\left[\frac{-1}{s+4} + \frac{2}{s+8}\right] \qquad \boxed{f(t) = 2e^{-8(t-1)} - e^{-4(t-1)}\,u(t-1)}$$

13.33 Find $f(t)$ if $\mathbf{F}(s)$ is given by the following functions:

(a) $$\mathbf{F}(s) = \frac{(s^2 + 2s + 1)e^{-2s}}{s(s+1)(s+2)}$$

(b) $$\mathbf{F}(s) = \frac{(s+1)e^{-4s}}{s^2(s+2)}$$

SOLUTION:

a) Let $G(s) = \dfrac{s^2+2s+1}{s(s+1)(s+2)} = \dfrac{s+1}{s(s+2)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+2}$

$K_1 = 1/2 \qquad K_2 = {-1}/{-2} = 1/2 \qquad G(s) = \frac{1}{2}\left[\frac{1}{s} + \frac{1}{s+2}\right]$

$F(s) = e^{-2s} G(s) \qquad g(t) = \frac{1}{2}\left[\, 1 + e^{-2t}\right] u(t)$

$\boxed{f(t) = \left[\frac{1}{2}\left(1 + e^{-2(t-2)}\right) u(t-2)\right.}$

b) $G(s) = \dfrac{s+1}{s^2(s+2)} = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+2} \qquad K_1 = \frac{1}{2} \qquad K_3 = \dfrac{-1}{(-2)^2} = -1/4$

let $s = -1$, $G(-1) = 0 = K_1 - K_2 + K_3 \Rightarrow K_2 = 1/4$

$G(s) = \frac{1}{4}\left[\frac{2}{s^2} + \frac{1}{s} - \frac{1}{s+2}\right] \Rightarrow g(t) = \frac{1}{4}\left[2t + 1 - e^{-2t}\right] u(t)$

$F(s) = e^{-4s} G(s) \rightarrow \boxed{f(t) = \frac{1}{4}\left[2(t-4) + 1 - e^{-2(t-4)} u(t-4)\right]}$

13.34 Find $f(t)$ if $\mathbf{F}(s)$ is given by the following functions:

(a) $\mathbf{F}(s) = \dfrac{2(s+1)e^{-s}}{(s+2)(s+4)}$

(b) $\mathbf{F}(s) = \dfrac{10(s+2)e^{-2s}}{(s+1)(s+4)}$ CS

SOLUTION:

a) $G(s) = \dfrac{2(s+1)}{(s+2)(s+4)} = \dfrac{K_1}{s+2} + \dfrac{K_2}{s+4}$ $\begin{cases} K_1 = \dfrac{2(-1)}{2} = -1 \\ K_2 = \dfrac{2(-3)}{-2} = 3 \end{cases}$

$G(s) = \dfrac{3}{s+4} - \dfrac{1}{s+2} \Rightarrow g(t) = (3e^{-4t} - e^{-2t})\,u(t)$

$F(s) = G(s)\,e^{-s} \Rightarrow \boxed{f(t) = \left[3e^{-4(t-1)} - e^{-2(t-1)}\right]u(t-1)}$

b) $G(s) = \dfrac{10(s+2)}{(s+1)(s+4)} = \dfrac{K_1}{s+1} + \dfrac{K_2}{s+4}$ $\begin{cases} K_1 = 10/3 \\ K_2 = -20/-3 = 20/3 \end{cases}$

$G(s) = \dfrac{10}{3}\left[\dfrac{1}{s+1} + \dfrac{2}{s+4}\right] \Rightarrow g(t) = \left[\dfrac{10}{3}e^{-t} + \dfrac{20}{3}e^{-4t}\right]u(t)$

$F(s) = e^{-2s}G(s) \Rightarrow \boxed{f(t) = \left[\dfrac{10}{3}e^{-(t-2)} + \dfrac{20}{3}e^{-4(t-2)}\right]u(t-2)}$

13.35 Solve the following differential equations using Laplace transforms.

(a) $$\frac{dx(t)}{dt} + 4x(t) = e^{-2t}, \qquad x(0) = 1$$

(b) $$\frac{dx(t)}{dt} + 6x(t) = 4u(t), \qquad x(0) = 2$$

SOLUTION:

a) $$\frac{dx(t)}{dt} \Rightarrow s\,X(s) - x(0)$$

so, $$s\,X(s) - 1 + 4X(s) = \frac{1}{s+2} \Rightarrow X(s)[s+4] = \frac{1}{s+2} + 1 = \frac{s+3}{s+2}$$

$$X(s) = \frac{s+3}{(s+2)(s+4)} = \frac{K_1}{s+2} + \frac{K_2}{s+4} = \frac{1/2}{s+2} + \frac{1/2}{s+4}$$

$$\boxed{x(t) = \frac{1}{2}\left[e^{-2t} + e^{-4t}\right]u(t)}$$

b) $$s\,X(s) - x(0) + 6X(s) = \frac{4}{s} \Rightarrow X(s)[s+6] = \frac{4}{s} + 2 = \frac{(s+2)2}{s}$$

$$X(s) = \frac{2(s+2)}{s(s+6)} = \frac{K_1}{s} + \frac{K_2}{s+6} = \frac{2/3}{s} + \frac{4/3}{s+6}$$

$$\boxed{x(t) = \frac{2}{3}\left[1 + 2e^{-6t}\right]u(t)}$$

13.36 Solve the following differential equations using Laplace transforms.

(a) $\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = e^{-2t},$

$y(0) = y'(0) = 0$

(b) $\frac{d^2y(t)}{dt^2} + \frac{4dy(t)}{dt} + 4y(t) = u(t), \quad y(0) = 0,$

$y'(0) = 1$

SOLUTION:

a) $s^2 Y(s) + 2sY(s) + Y(s) = \frac{1}{s+2} \Rightarrow Y(s)\left[s^2+2s+1\right] = \frac{1}{s+2}$

$Y(s) = \frac{1}{(s+2)(s+1)^2} = \frac{K_1}{s+2} + \frac{K_2}{(s+1)^2} + \frac{K_3}{s+1} \qquad K_1 = 1, \; K_2 = 1$

let $s = 0$, $Y(0) = \frac{1}{2} = \frac{K_1}{2} + K_2 + K_3 \Rightarrow K_3 = -1$

$Y(s) = \frac{1}{s+2} + \frac{1}{(s+1)^2} - \frac{1}{s+1} \Rightarrow \boxed{y(t) = \left[e^{-2t} + te^{-t} - e^{-t}\right]u(t)}$

b) $s^2 Y(s) - sy'(0) + 4sY(s) + 4Y(s) = \frac{1}{s} \Rightarrow Y(s)\left[s^2+4s+4\right] = \frac{1}{s} + s = \frac{s^2+1}{s}$

$Y(s) = \frac{s^2+1}{s(s+2)^2} = \frac{K_1}{s} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2} \qquad K_1 = \frac{1}{4} \qquad K_2 = -\frac{5}{2}$

let $s = -1$, $Y(-1) = \frac{2}{-1} = -2 = -K_1 + K_2 + K_3 \Rightarrow K_3 = 3/4$

$Y(s) = \frac{1}{4}\left[\frac{1}{s} - \frac{10}{(s+2)^2} + \frac{3}{s+2}\right] \Rightarrow \boxed{y(t) = \frac{1}{4}\left[1 - 10te^{-2t} + 3e^{-2t}\right]u(t)}$

13.37 Use Laplace transforms to find $y(t)$ if

$$\frac{dy(t)}{dt} + 3y(t) + 2\int_0^t y(x)\,dx = u(t), \quad y(0) = 0, \quad t > 0$$

PSV

SOLUTION:

$$sY(s) + 3Y(s) + \frac{2Y(s)}{s} = \frac{1}{s} \qquad Y(s)\left[s^2+3s+2\right] = 1$$

$$Y(s) = \frac{1}{(s+2)(s+1)} = \frac{K_1}{s+2} + \frac{K_2}{s+1} \qquad K_1 = -1 \qquad K_2 = 1$$

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2} \quad \Rightarrow \quad \boxed{y(t) = \left[e^{-t} - e^{-2t}\right]u(t)}$$

13.38 Solve the following integrodifferential equation using Laplace transforms. CS

$$\frac{dy(t)}{dt} + 2y(t) + \int_0^t y(\lambda)\, d\lambda = 1 - e^{-2t}, \quad y(0) = 0, \quad t > 0$$

SOLUTION:

$$sY(s) + 2Y(s) + \frac{Y(s)}{s} = \frac{1}{s} - \frac{1}{s+2} = \frac{2}{s(s+2)}$$

$$Y(s)\left[s^2+2s+1\right] = \frac{2}{s+2} \Rightarrow Y(s) = \frac{2}{(s+2)(s+1)^2}$$

$$Y(s) = \frac{K_1}{s+2} + \frac{K_2}{(s+1)^2} + \frac{K_3}{s+1} \qquad K_1 = 2 \qquad K_2 = 2$$

let $s = 0$, $Y(0) = 1 = \frac{2}{2} + 2 + K_3 \Rightarrow K_3 = -2$

$$Y(s) = \frac{2}{s+2} + \frac{2}{(s+1)^2} - \frac{2}{s+1}$$

$$\boxed{y(t) = \left[2e^{-2t} + 2te^{-2t} - 2e^{-t}\right]u(t)}$$

13.39 Find $f(t)$ using convolution if $\mathbf{F}(s)$ is

$$\mathbf{F}(s) = \frac{1}{(s+1)(s+4)}$$

SOLUTION:

Let $F_1(s) = \frac{1}{s+1} \Rightarrow f_1(t) = e^{-t}$

$F_2(s) = \frac{1}{s+4} \Rightarrow f_2(t) = e^{-4t}$

$$f(t) = \int_0^t e^{-(t-\lambda)} e^{-4\lambda} d\lambda = e^{-t} \int_0^t e^{-3\lambda} d\lambda = \frac{e^{-t}}{3} e^{-3\lambda} \Big|_t^0$$

$$f(t) = \frac{e^{-t}}{3}\left[1 - e^{-3t}\right] u(t) \quad \boxed{f(t) = \frac{1}{3}\left[e^{-t} - e^{-4t}\right] u(t)}$$

13.40 Use convolution to find $f(t)$ if

$$\mathbf{F}(s) = \frac{10}{(s+1)(s+3)^2}$$

SOLUTION:

$$F_1(s) = \frac{10}{s+1} \Rightarrow f_1(t) = 10e^{-t}$$

$$F_2(s) = \frac{1}{(s+3)^2} \Rightarrow te^{-3t} = f_2(t)$$

$$f(t) = \int_0^t 10e^{-(t-\lambda)} \lambda e^{-3\lambda} d\lambda = 10e^{-t} \int_0^t \lambda e^{-2\lambda} d\lambda$$

$$= 10e^{-t} \left[\frac{\lambda}{2} e^{-2\lambda} + \frac{e^{-2\lambda}}{4} \right] \Bigg|_t^0 = \left(\frac{10}{4} e^{-t} - 5te^{-3t} - \frac{10}{4} e^{-3t} \right) u(t)$$

$$\boxed{f(t) = \left[2.5e^{-t} - 5te^{-3t} - 2.5e^{-3t} \right] u(t)}$$

13.41 Determine the initial and final values of $f(t)$ if $\mathbf{F}(s)$ is given by the expressions

(a) $\mathbf{F}(s) = \dfrac{2(s+2)}{s(s+1)}$

(b) $\mathbf{F}(s) = \dfrac{2(s^2+2s+6)}{(s+1)(s+2)(s+3)}$

(c) $\mathbf{F}(s) = \dfrac{2s^2}{(s+1)(s^2+2s+2)}$ CS

SOLUTION:

Initial values

a) $\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{2(s+2)}{s+1} = \frac{2(\infty)}{(\infty)} = 2$ $\boxed{\lim_{t\to 0} f(t) = 2}$

b) $\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{2(\infty^3)}{(\infty)^3} = 2$ $\boxed{\lim_{t\to 0} f(t) = 2}$

c) $\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{2(\infty)^3}{\infty^3} = 2$ $\boxed{\lim_{t\to 0} f(t) = 2}$

final values

a) $\lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{2(s+2)}{(s+1)} = \frac{2(2)}{1} = 4$ $\boxed{\lim_{t\to\infty} f(t) = 4}$

b) $\lim_{s\to 0} sF(s) = \frac{2(6)(0)}{1(2)(3)} = 0$ $\boxed{\lim_{t\to\infty} f(t) = 0}$

c) $\lim_{s\to 0} sF(s) = 0$ $\boxed{\lim_{t\to\infty} f(t) = 0}$

13.42 Find the initial and final values of the time function $f(t)$ if $\mathbf{F}(s)$ is given as

(a) $\mathbf{F}(s) = \dfrac{10(s+2)}{(s+1)(s+4)}$

(b) $\mathbf{F}(s) = \dfrac{s^2+2s+2}{(s+6)(s^3+4s^2+8s+4)}$

(c) $\mathbf{F}(s) = \dfrac{2s}{s^2+2s+3}$ **PSV**

SOLUTION:

Initial values

a) $\lim_{s\to\infty} sF(s) = \dfrac{10(\infty)^2}{\infty^2} = 10$ $\boxed{\lim_{t\to 0} f(t) = 10}$

b) $\lim_{s\to\infty} sF(s) = \dfrac{\infty^3}{\infty^4} = 0$ $\boxed{\lim_{t\to 0} f(t) = 0}$

c) $\lim_{s\to\infty} sF(s) = \dfrac{2(\infty)^2}{\infty^2} = 2$ $\boxed{\lim_{t\to 0} f(t) = 2}$

Final value

a) $\lim_{s\to 0} sF(s) = 0$ $\boxed{\lim_{t\to\infty} f(t) = 0}$

b) $\lim_{s\to 0} sF(s) = 0$ $\boxed{\lim_{t\to\infty} f(t) = 0}$

c) $\lim_{s\to 0} sF(s) = 0$ $\boxed{\lim_{t\to\infty} f(t) = 0}$

13.43 Find the final values of the time function $f(t)$ given that

(a) $\mathbf{F}(s) = \dfrac{10(s+6)}{(s+2)(s+3)}$

(b) $\mathbf{F}(s) = \dfrac{2}{s^2+4s+8}$

SOLUTION:

Initial value

a) $\lim_{s\to\infty} sF(s) = \dfrac{10(\infty)^2}{\infty^2} = 10$ $\boxed{\lim_{t\to 0} f(t) = 10}$

b) $\lim_{s\to\infty} sF(s) = \dfrac{2(\infty)}{\infty^2} = 0$ $\boxed{\lim_{t\to 0} f(t) = 0}$

final values

a) $\lim_{s\to 0} sF(s) = 0$ $\boxed{\lim_{t\to\infty} f(t) = 0}$

b) $\lim_{s\to 0} sF(s) = 0$ $\boxed{\lim_{t\to\infty} f(t) = 0}$

13.44 In the network in Fig. P13.44, the switch opens at $t = 0$. Use Laplace transforms to find $i(t)$ for $t > 0$.

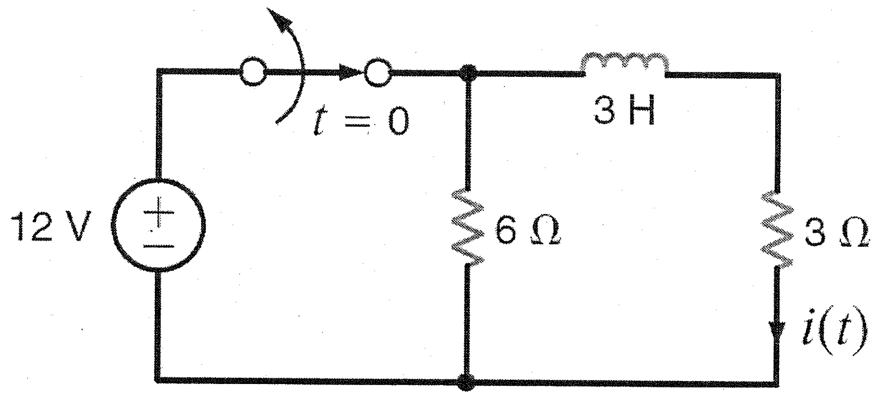
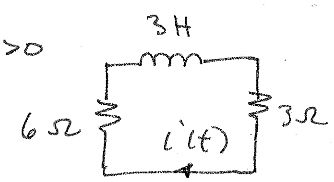


Figure P13.44

SOLUTION:

for $t < 0$ $\quad i(0^-) = 12/3 = 4A$

for $t > 0$



$$3\frac{di}{dt} + 3i(t) + 6i(t) = 0$$

$$3sI(s) - 3i(0^-) + 3I(s) + 6I(s) = 0$$

$$3I(s)[s+3] = 12$$

$$I(s) = \frac{4}{s+3} \qquad \boxed{i(t) = 4e^{-3t}u(t)}$$

13.45 The switch in the circuit in Fig. P13.45 opens at $t = 0$. Find $i(t)$ for $t > 0$ using Laplace transforms. **CS**

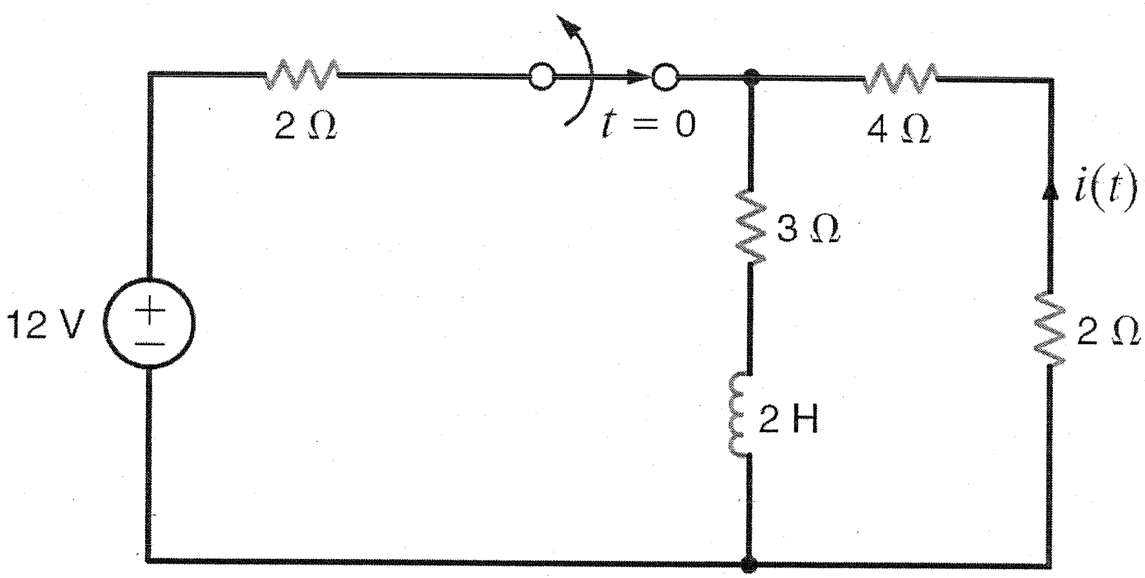
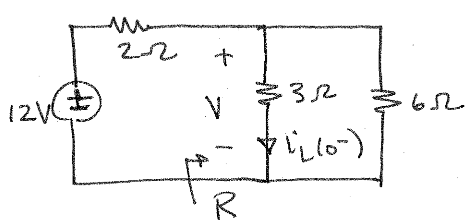


Figure P13.45

SOLUTION: for $t < 0$,

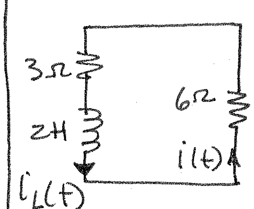


$R = 3 // 6 = 2\Omega$

$V = 12\left(\frac{2}{2+2}\right) = 6V$

$i_L(0^-) = \frac{V}{3} = 2A$

for $t > 0$



$$3i(t) + 6i(t) + 2\frac{di}{dt} = 0$$

$$3I(s) + 6I(s) + 2sI(s) - 2i(0^-) = 0$$

$$(9 + 2s)I(s) = 4$$

$$I(s) = \frac{4}{2s+9} = \frac{2}{s+4.5}$$

$$\boxed{i(t) = 2e^{-4.5t}u(t)}$$

13.46 In the circuit in Fig. P13.46, the switch moves from position 1 to position 2 at $t = 0$. Use Laplace transforms to find $v(t)$ for $t > 0$. **PSV**

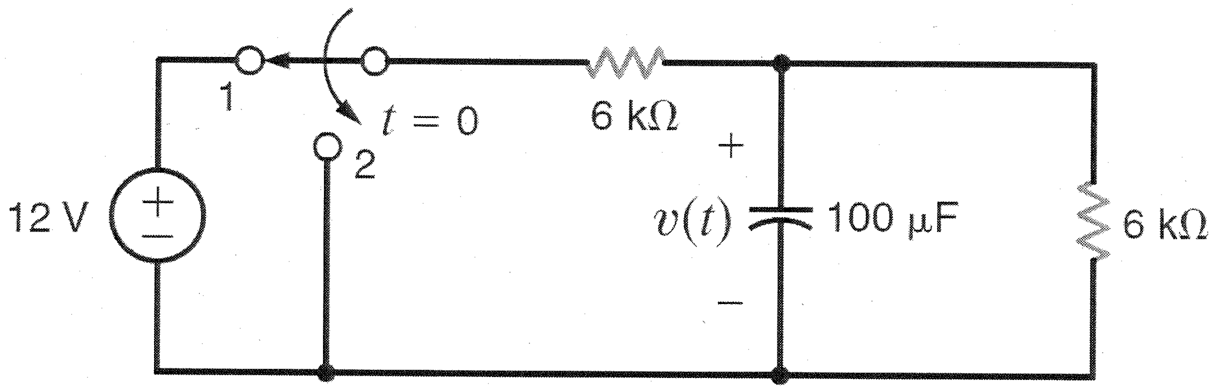


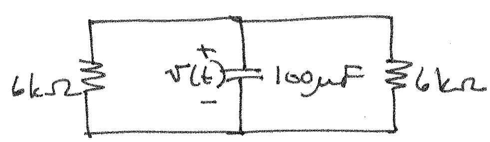
Figure P13.46

SOLUTION:

for $t<0$,

$$v(0^-) = \frac{12\,(6k)}{6k+6k} = 6V$$

for $t>0$



$$\frac{v(t)}{6000} + \frac{v(t)}{6000} + 10^{-4}\frac{dv}{dt} - 10^{-4}v(0^-) = 0$$

$$v(t) + v(t) + 0.6\frac{dv}{dt} = 0.6\,(6) = 3.6$$

$$V(s)\left[0.6s + 2\right] = 3.6 \Rightarrow V(s) = \frac{6}{s + 10/3}$$

$$\boxed{v(t) = 6e^{-3.33t}u(t)}$$

13.47 In the network in Fig. P13.47, the switch opens at $t = 0$. Use Laplace transforms to find $i_L(t)$ for $t > 0$.
PSV

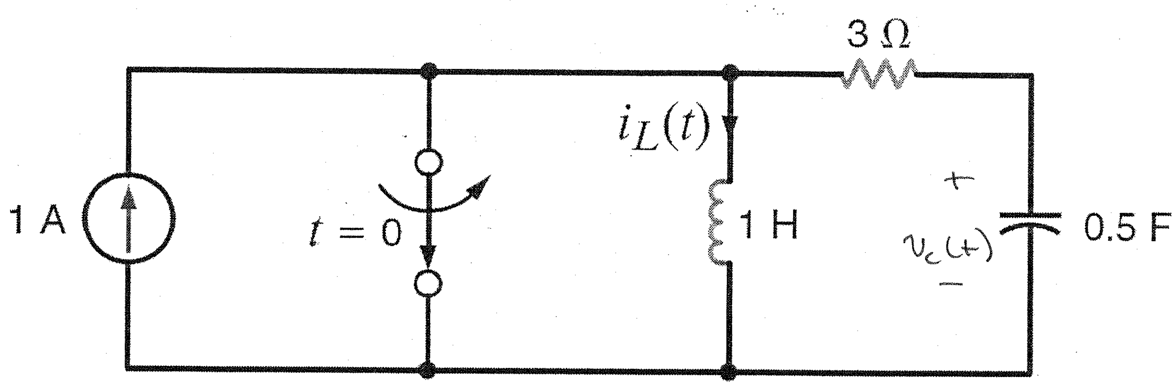


Figure P13.47

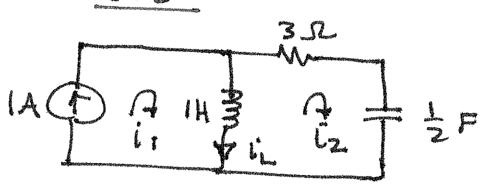
SOLUTION:

$\underline{t=0^-}$

$i_L(0^-)=0$

$v_c(0^-)=0$

$\underline{t=0^+}$



$i_L = i_1 - i_2$

$i_2 = 1 - i_L$

$$3i_2 + 2\int i_2\,dt - \frac{di_L}{dt} = 0 \Rightarrow 3(1-i_L) + 2\int dt - 2\int i_L\,dt - \frac{di_L}{dt} = 0$$

$$3 - 3I_L(s) + \frac{2}{s} - \frac{2I_L(s)}{s} - sI_L(s) = 0$$

$$I_L(s)\left[s^2+3s+2\right] = 3s+2 \Rightarrow I_L(s) = \frac{3s+2}{(s+1)(s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

$K_1 = -1$ $\quad K_2 = 4$

$$\boxed{i_L(t) = \left[4e^{-2t} - e^{-t}\right]u(t)}$$

13.48 In the network in Fig. P13.48, the switch opens at $t = 0$. Use Laplace transforms to find $v_o(t)$ for $t > 0$.

CS

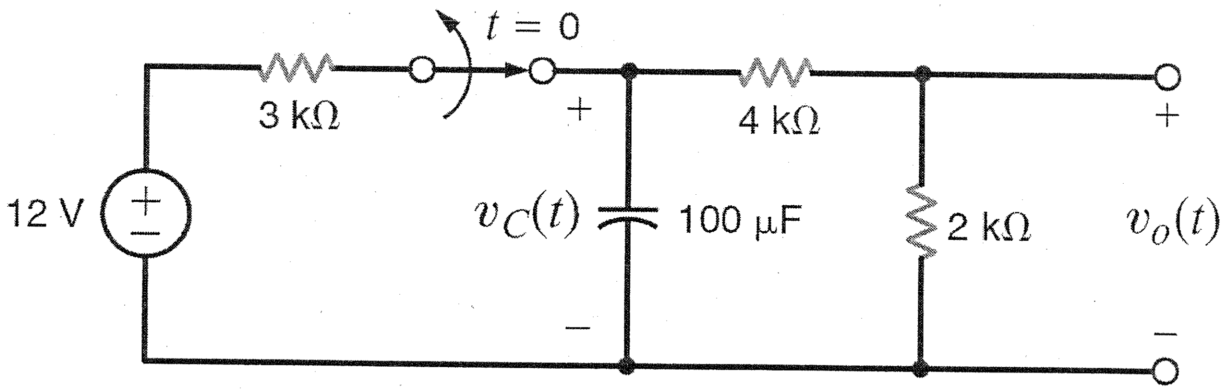


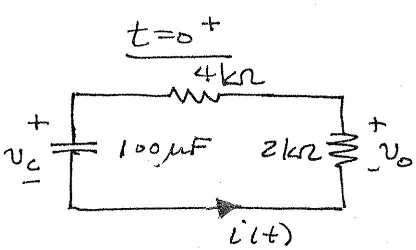
Figure P13.48

SOLUTION:

$t = 0^-$

$$V_c(0^-) = \frac{12(6000)}{6000 + 3000} = 8\,V$$

$t = 0^+$



$$6000\, i(t) + 10^4 \int i(t) = 0 \quad \text{or}, \quad 10^{-4}\frac{dv_c}{dt} + \frac{v_c(t)}{6000} = 0$$

$$0.6\left[sV_c(s) - v_c(0^-)\right] + V_c(s) = 0 \Rightarrow V_c(s)\left[0.6s + 1\right] = 4.8$$

But $\dfrac{V_o(s)}{V_c(s)} = \dfrac{2000}{2000 + 4000} = \dfrac{1}{3} \Rightarrow V_o(s) = \dfrac{V_c(s)}{3}$

$$3V_o(s)\left[0.6s + 1\right] = 4.8 \Rightarrow V_o(s)\left[s + 1.67\right] = 2.67$$

$$V_o(s) = \frac{2.67}{s + 1.67}$$

$$\boxed{v_o(t) = 2.67\, e^{-1.67t}\, u(t)}$$

13FE-1 The output function of a network is expressed using Laplace transforms, in the following form. CS

$$\mathbf{V}_o(s) = \frac{12}{s(s^2 + 3s + 2)}$$

Find the output $v_o(t)$ as a function of time.

SOLUTION:

$$V_o(s) = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$$K_1 = \frac{12}{2} = 6 \qquad K_2 = \frac{12}{-1} = -12 \qquad K_3 = \frac{12}{(-2)(-1)} = 6$$

$$V_o(s) = \frac{6}{s} - \frac{12}{s+1} + \frac{6}{s+2}$$

$$\boxed{v_o(t) = [6 - 12e^{-t} + 6e^{-2t}]u(t)}$$

13FE-2 The Laplace transform function representing the output voltage of a network is expressed as

$$\mathbf{V}_o(s) = \frac{120}{s(s+10)(s+20)}$$

Determine the time-domain function and the value of $v_o(t)$ at $t = 100$ ms.

SOLUTION:

$$V_o(s) = \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+20} \qquad K_1 = \frac{120}{10(20)} = 0.6$$

$$K_2 = \frac{120}{(-10)(10)} = -1.2 \qquad K_3 = \frac{120}{(-20)(-10)} = 0.6$$

$$V_o(s) = \frac{0.6}{s} - \frac{1.2}{s+10} + \frac{0.6}{s+20}$$

$$\boxed{v_o(t) = \left[0.6 - 1.2e^{-10t} + 0.6e^{-20t}\right]u(t)}$$

$$v_o(0.1) = 0.6 - 1.2e^{-1} + 0.6e^{-2} \qquad \boxed{v_o(0.1) = 0.24\,V}$$

13FE-3 The Laplace transform function for the output voltage of a network is expressed in the following form.

$$\mathbf{V}_o(s) = \frac{12(s+2)}{s(s+1)(s+3)(s+4)}$$

Determine the final value of this voltage, that is, $v_o(t)$ as $t \to \infty$. CS

SOLUTION:

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} sV_o(s) = \lim_{s\to 0} \frac{12(s+2)}{(s+1)(s+3)(s+4)} = \frac{12(2)}{1(3)(4)} = 2$$

$$\boxed{\lim_{t\to\infty} v_o(t) = 2V}$$