Chapter Fourteen: Application of the LaPlace Transform to Circuit Analysis

14.1 Find the input impedance $\mathbf{Z}(s)$ of the network in Fig. P14.1. **CS**

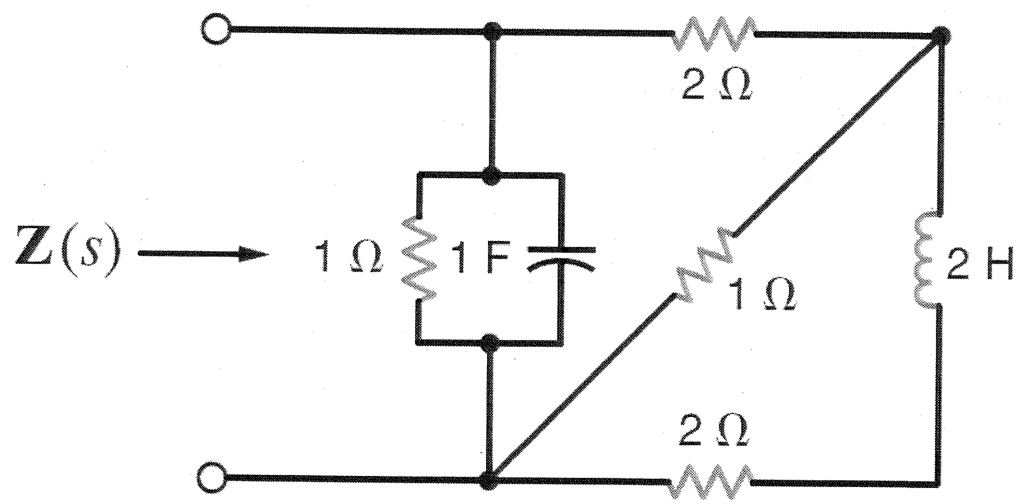
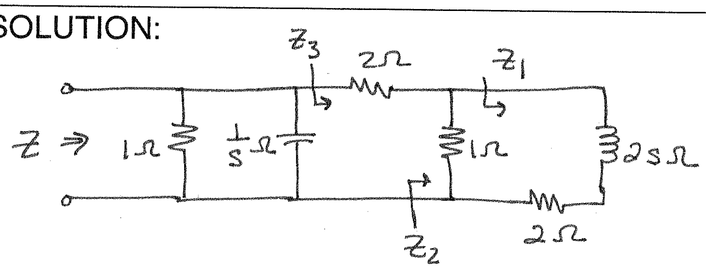


Figure P14.1

SOLUTION:



$Z_1 = 2 + 2s\ \Omega$

$Z_2 = \dfrac{Z_1}{1+Z_1} = \dfrac{2s+2}{2s+3}$

$Z_3 = 2 + Z_2 = 2 + \dfrac{2s+2}{2s+3} = \dfrac{4s+6+2s+2}{2s+3} = \dfrac{6s+8}{2s+3}$

$Z = \dfrac{1}{\frac{1}{1} + s + \frac{1}{Z_3}}$ $\qquad 1 + s + \dfrac{1}{Z_3} = s+1+\dfrac{2s+3}{6s+8} = \dfrac{6s^2+16s+11}{6s+8}$

$$\boxed{Z = \frac{6s+8}{6s^2+16s+11}}$$

14.2 Given the network in Fig. P14.2, determine the value of the output voltage as $t \to \infty$.

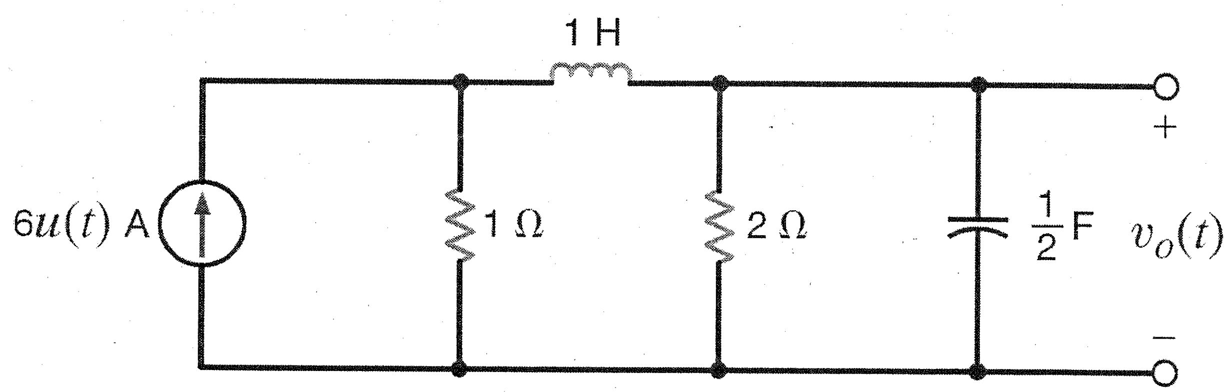


Figure P14.2

SOLUTION:

Since input is dc for $t>0$, all voltages & currents will eventually become dc as well. Thus $v_L \to 0$ & $i_c \to 0$ as $t \to \infty$.

$$v_o(\infty) = \frac{6\,((1)(2))}{1+2} = \frac{6\,(2)}{3} = 4 \qquad \boxed{v_o(\infty) = 4V}$$

14.3 For the network shown in Fig. P14.3, determine the value of the output voltage as $t \to \infty$.

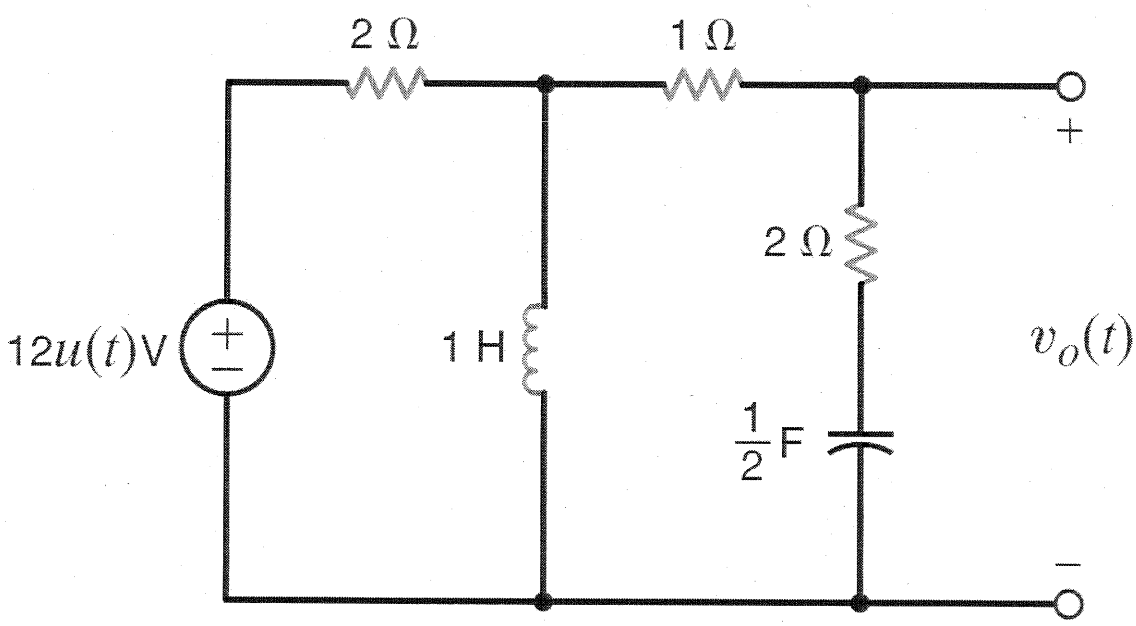
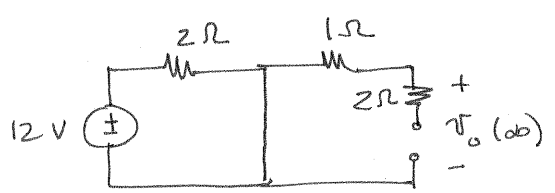


Figure P14.3

SOLUTION:

Since input is dc for $t>0$, all voltages & currents will eventually go to dc as well. Thus $v_L \to 0$ & $i_c \to 0$ as $t \to 0$.



$\boxed{v_o(\infty) = 0\text{ V}}$

14.4 Use Laplace transforms to find $v(t)$ for $t > 0$ in the network shown in Fig. P14.4. Assume zero initial conditions.

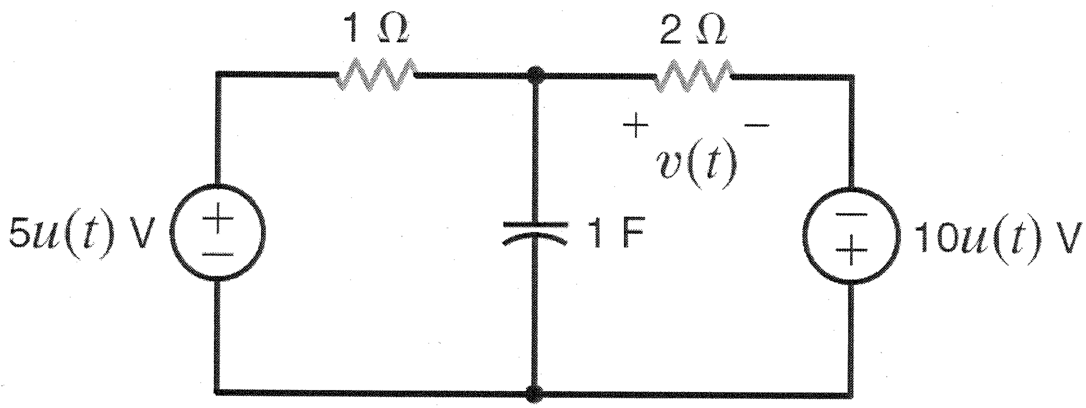
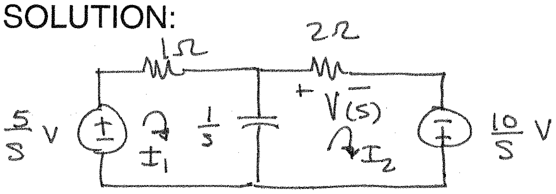


Figure P14.4

SOLUTION:



$$\frac{5}{s} = I_1(s)\left[1+\tfrac{1}{s}\right] - I_2(s)\left(\tfrac{1}{s}\right)$$

$$\frac{10}{s} = -I_1(s)\left(\tfrac{1}{s}\right) + I_2(s)\left[2+\tfrac{1}{s}\right]$$

or,

$$\left.\begin{aligned} 5 &= I_1(s+1) - I_2 \\ 10 &= -I_1 + I_2(2s+1) \end{aligned}\right\} \quad \begin{aligned} I_1 &= 5/s \\ I_2 &= 5/s \end{aligned}$$

$$V(s) = 2\,I_2(s) = \frac{10}{s}$$

$$\boxed{v(t) = 10\,u(t)}$$

14.5 Use Laplace transforms and nodal analysis to find $i_1(t)$ for $t > 0$ in the network shown in Fig. P14.5. Assume zero initial conditions.

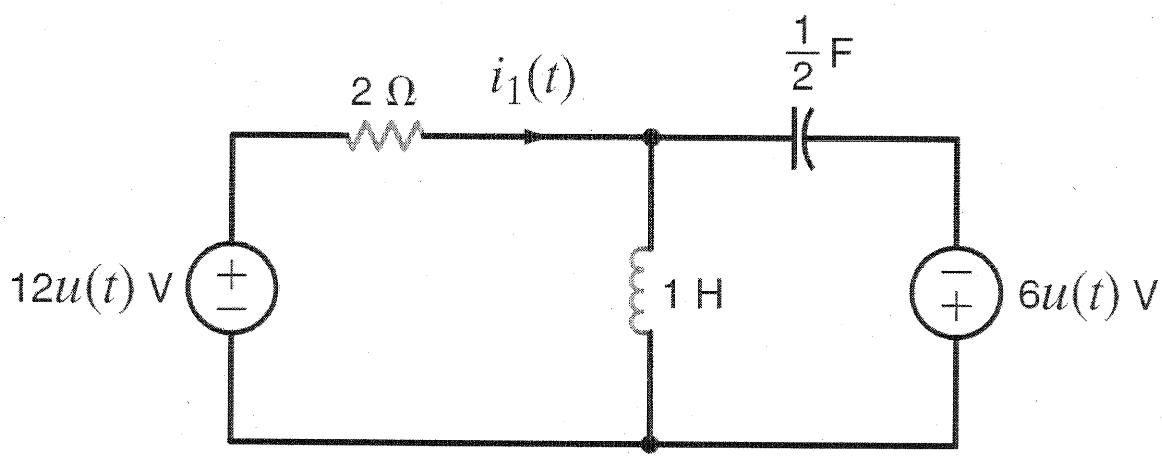
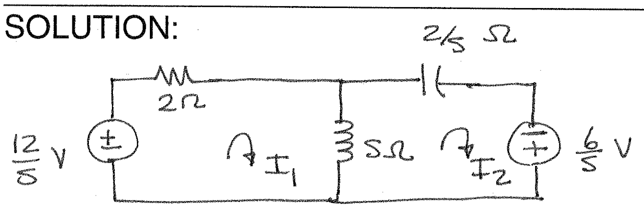


Figure P14.5

SOLUTION:



$$\frac{12}{s} = I_1\left[s+2\right] - sI_2$$

$$\frac{6}{s} = -sI_1 + I_2\left[s+\frac{2}{s}\right]$$

or,

$$\frac{12}{s} = I_1\left[s+2\right] - sI_2 \quad \& \quad \frac{6}{s} = -sI_1 + I_2\left[\frac{s^2+2}{s}\right]$$

Solve for I_1 yields $\quad I_1(s) = \dfrac{3(3s^2+4)}{s(s^2+s+2)}$

$$I_1(s) = \frac{K_1}{s} + \frac{K_2}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_2^*}{s+\frac{1}{2}+j\sqrt{7}/2} \qquad K_1 = 6$$

$$K_2 = \left.\frac{3(3s^2+4)}{s(s+\frac{1}{2}+j\sqrt{7}/2)}\right|_{s=-\frac{1}{2}+j\sqrt{7}/2} = 3.21\angle 62.1^\circ$$

$$i_1(t) = \left[6 + 6.42\,e^{-t/2}\cos\left(\sqrt{7}t/2 + 62.1^\circ\right)\right]u(t)\ \text{V}$$

14.6 For the network shown in Fig. P14.6, find $v_o(t)$, $t > 0$, using node equations. **PSV**

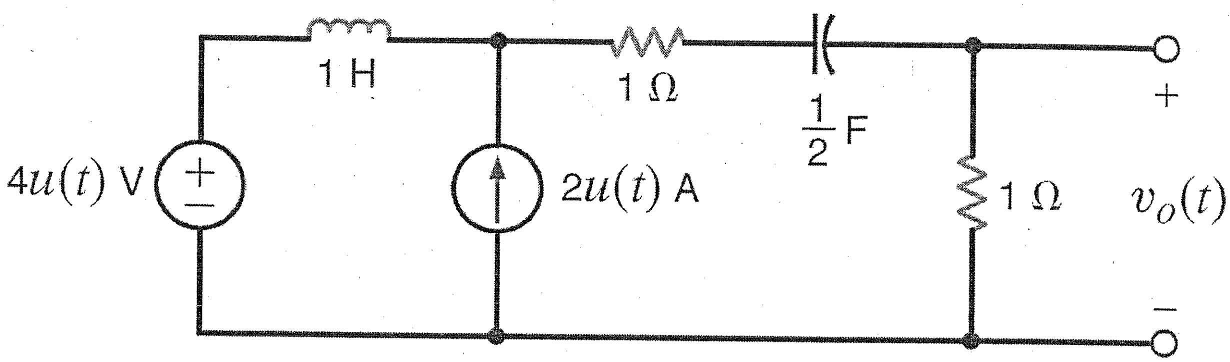
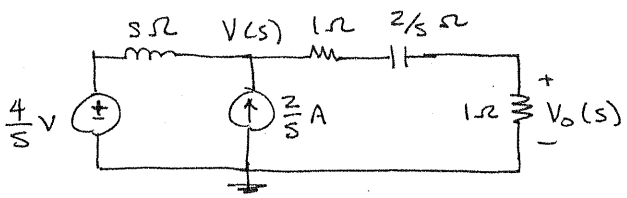


Figure P14.6

SOLUTION: at $t=0^-$, no excitation. So initial conditions $=0$.



$$\frac{V - 4/s}{s} + \frac{V}{2+\frac{2}{s}} = \frac{2}{s}$$

$$\frac{V}{s} + \frac{Vs}{2(s+1)} = \frac{2}{s} + \frac{4}{s^2}$$

$$V\left[s(s+1) + \frac{s^3}{2} \right] = 2s(s+1) + 4(s+1) = 2s^2 + 6s + 4$$

$$\frac{V}{2}\left[s^3 + 2s^2 + 2s \right] = 2(s^2 + 3s + 2) \Rightarrow V = \frac{4(s+1)(s+2)}{s(s^2+2s+2)}$$

$$V_o = V\left[\frac{1}{2 + 2/s} \right] = V\left[\frac{s}{2(s+1)} \right] = \frac{2(s+2)}{(s+1-j1)(s+1+j1)}$$

$$V_o = \frac{K_1}{s+1-j1} + \frac{K_1^*}{s+1+j1} \qquad K_1 = \frac{2(1+j1)}{j2} = \sqrt{2}\angle{-45^\circ}$$

$$v_o(t) = \left[\sqrt{2}\, e^{-t} \cos(t - 45^\circ) \right] u(t) \checkmark$$

14.7 For the network shown in Fig. P14.7, find $i_o(t)$, $t > 0$.
CS

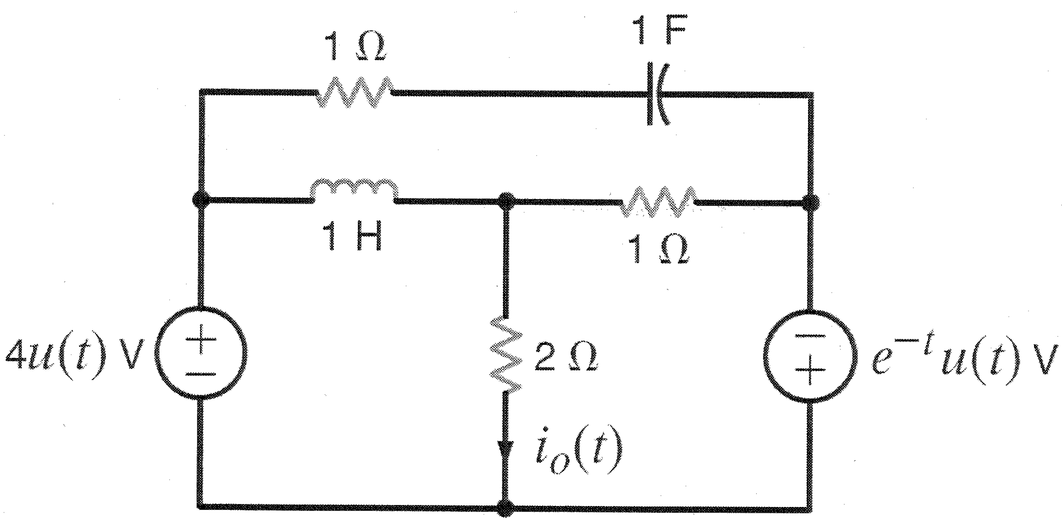
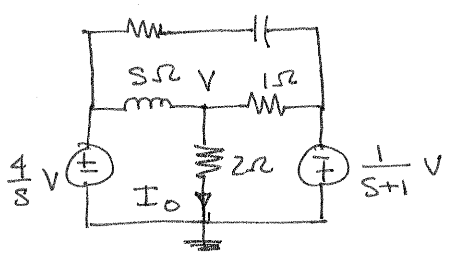


Figure P14.7

SOLUTION: $t = 0^-$: No excitation. So, all initial conditions $= 0$.



$$\frac{V - 4/s}{s} + \frac{V + \frac{1}{s+1}}{1} + \frac{V}{2} = 0$$

$$V\left(\frac{1}{s} + \frac{3}{2}\right) = \frac{4}{s^2} - \frac{1}{s+1}$$

$$V = \frac{2(-s^2 + 4s + 4)}{s(s+1)(3s+2)}$$

$$I_o = V/2 = \frac{1/3\,(-s^2+4s+4)}{s(s+1)(s+2/3)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2/3}$$

$K_1 = 2$ $\quad K_2 = -1$ $\quad K_3 = -4/3$

$$i_o(t) = \left[2 - e^{-t} - \frac{4}{3}e^{-(2/3)t}\right]u(t)$$

14.8 Find $v_o(t), t > 0$, in the network in Fig. P14.8 using node equations.

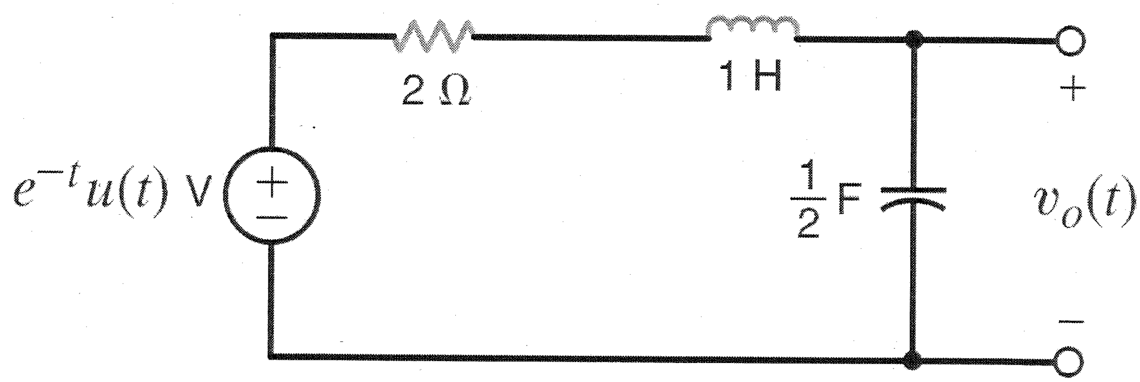
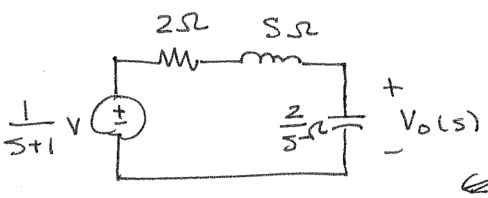


Figure P14.8

SOLUTION: $v_c(0^-) = 0\text{V}$



$$\frac{V_o(s)}{2/s} + \frac{V_o(s) - \frac{1}{s+1}}{s+2} = 0$$

$$\frac{s V_o}{2} + \frac{V_o}{s+2} - \frac{1}{(s+2)(s+1)} = 0$$

$$s(s+2)V_o + 2V_o = \frac{2}{s+1} = V_o\left[s^2+2s+2\right]$$

$$V_o = \frac{2}{(s+1)(s^2+2s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+1-j1} + \frac{K_2^*}{s+1+j1}$$

$$K_1 = 2 \qquad K_2 = \frac{2}{(j1)(j2)} = -1 \qquad K_2^* = -1$$

$$V_o = \frac{2}{s+1} - \frac{1}{s+1-j1} - \frac{1}{s+1+j1}$$

$$\boxed{v_o(t) = \left[2e^{-t} - 2e^{-t}\cos(t)\right]u(t)}$$

14.9 Find $v_o(t)$, $t > 0$, in the network shown in Fig. P14.9 using nodal analysis. CS

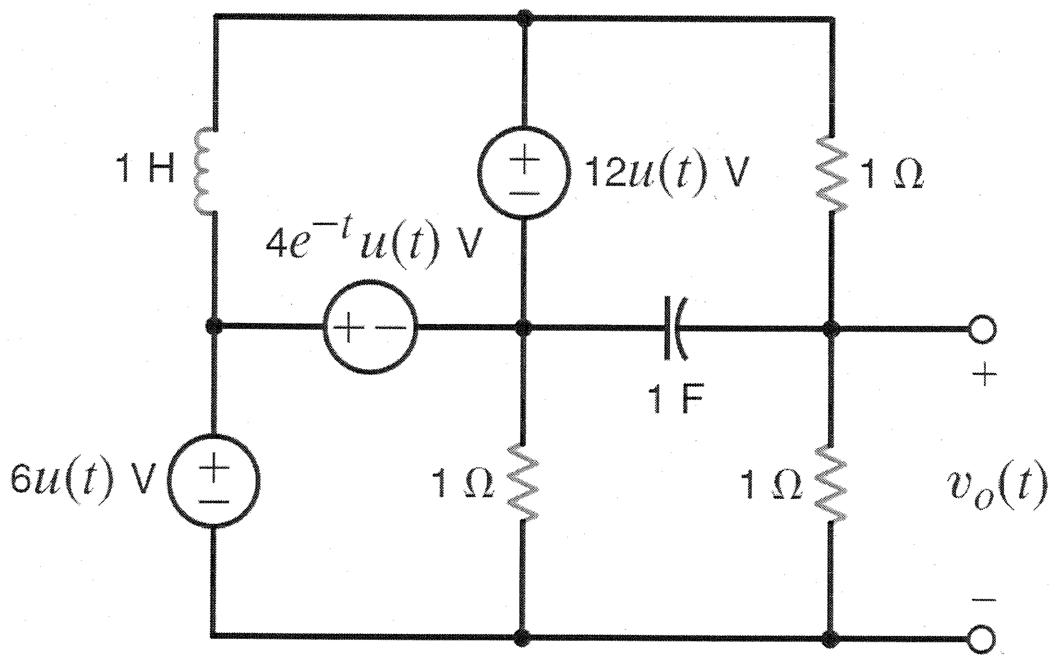
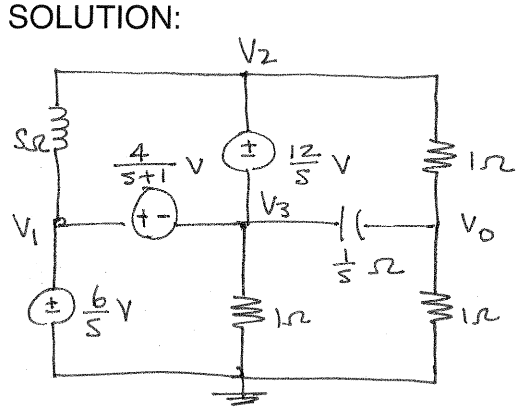


Figure P14.9

SOLUTION:



$V_1 = \frac{6}{s}$ $\quad V_1 - V_3 = \frac{4}{s+1} \Rightarrow V_3 = \frac{6}{s} - \frac{4}{s+1}$

$V_2 - V_3 = \frac{12}{s} \Rightarrow V_2 = \frac{12}{s} + V_3 = \frac{18}{s} - \frac{4}{s+1}$

$$\frac{V_o - V_2}{1} + s(V_o - V_3) + \frac{V_o}{1} = 0$$

$$V_o(s+2) = V_2 + sV_3$$

$$V_o(s+2) = \frac{2(s+9)}{s} \Rightarrow V_o(s) = \frac{2(s+9)}{s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$$

$K_1 = 9 \qquad K_2 = -7$

$$\boxed{v_o(t) = [9 - 7e^{-2t}]u(t)}$$

14.10 Use nodal analysis to find $v_o(t)$, $t > 0$, in the network in Fig. P14.10. **PSV**

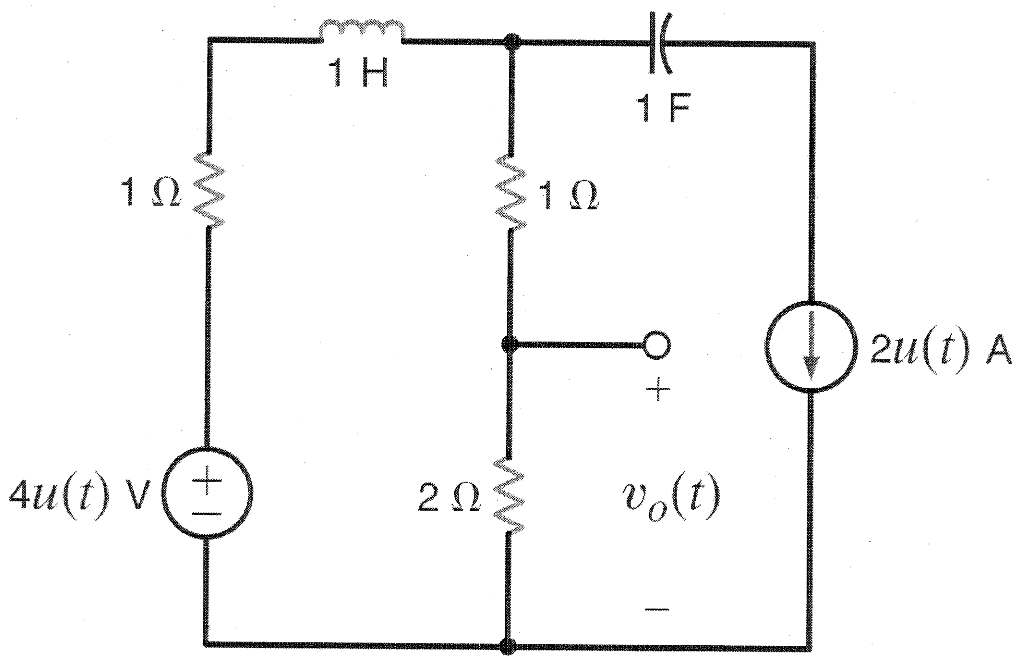
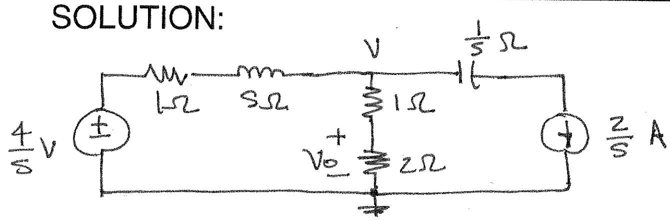


Figure P14.10

SOLUTION:



$$\frac{V - 4/s}{s+1} + \frac{V}{3} + \frac{2}{s} = 0$$

$$V\left[\frac{1}{s+1} + \frac{1}{3}\right] = \frac{4}{s(s+1)} - \frac{2}{s}$$

$$V\left[\frac{3+s+1}{3(s+1)}\right] = \frac{-2s+2}{s(s+1)} \Rightarrow \quad V(s) = \frac{6(-s+1)}{s(s+4)} \qquad V_o = \frac{2}{3}V$$

$$V_o(s) = \frac{4(-s+1)}{s(s+4)} = \frac{1}{s} - \frac{5}{s+4}$$

$$\boxed{v_o(t) = [1 - 5e^{-4t}]u(t)}$$

14.11 For the network shown in Fig. P14.11, find $v_o(t)$, $t > 0$, using loop equations.

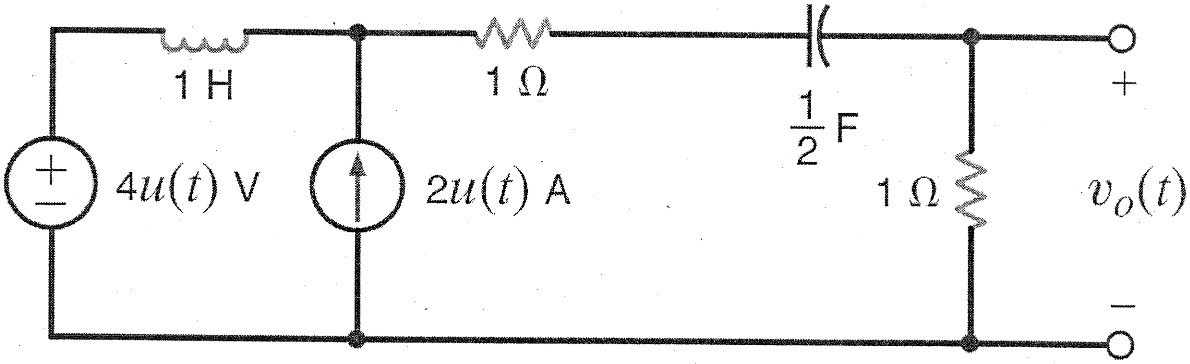
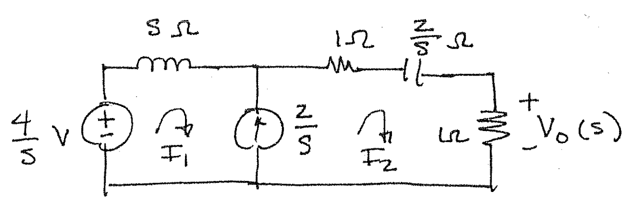


Figure P14.11

SOLUTION:



$$\frac{4}{s} = sI_1 + \left(2 + \frac{2}{s}\right) I_2$$

or, $4 = s^2 I_1 + (2s+2) I_2$

and, $I_2 - I_1 = 2/s \Rightarrow I_1 = I_2 - \frac{2}{s}$

$$4 = s^2 I_2 - 2s + (2s+2)I_2 = I_2(s^2+2s+2) - 2s$$

$$I_2 = \frac{2s+4}{s^2+2s+2} \qquad V_o = (1)I_2 = \frac{2(s+2)}{(s+1-j1)(s+1+j1)} = \frac{K_1}{s+1-j1} + \frac{K_1^*}{s+1+j1}$$

$$K_1 = \frac{2(-1+j1+2)}{j2} = \sqrt{2}\angle{-45^\circ}$$

$$\boxed{v_o(t) = [2\sqrt{2}\, e^{-t} \cos(t - 45^\circ)]\, u(t) \text{ V}}$$

14.12 For the network shown in Fig. P14.12, find $v_o(t)$, $t > 0$, using mesh equations.

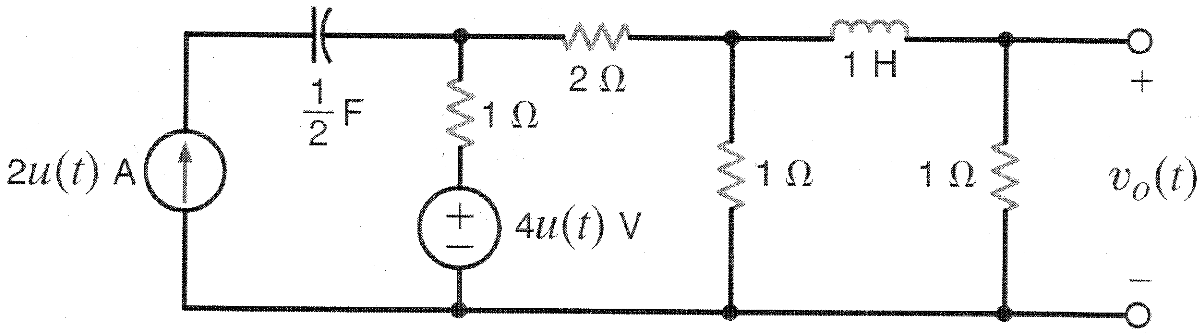
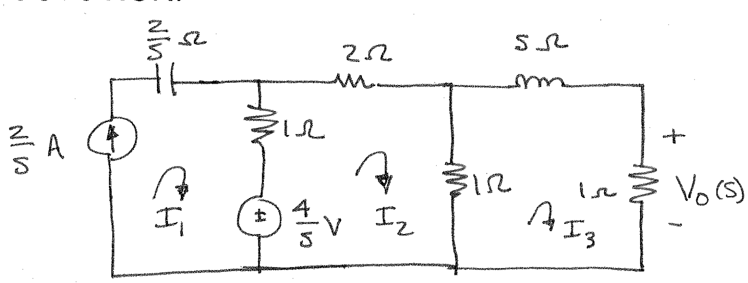


Figure P14.12

SOLUTION:



$I_1 = \frac{2}{s}$ A

$\frac{4}{s} = -I_1 + I_2(4) - I_3$

$0 = -I_2 + I_3(s+2)$

$\frac{4}{s} = -\frac{2}{s} + 4I_2 - I_3 \Rightarrow \frac{6}{s} = 4I_2 - I_3$

and $0 = -I_2 + I_3(s+2)$

$$\left.\begin{array}{l} \frac{6}{s} = 4I_2 - I_3 \\ 0 = -I_2 + I_3(s+2) \end{array}\right\} \quad I_3(s)\left[4(s+2)-1\right] = 6/s$$

$I_3(s) = \dfrac{6}{s(4s+7)}$ $\qquad V_o = (1)\,I_3 = \dfrac{3/2}{s(s+7/4)}$

$V_o(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+7/4}$ $\qquad K_1 = \left(\frac{3}{2}\right)\left(\frac{4}{7}\right) = \frac{6}{7}$ $\qquad K_2 = \frac{3}{2}\left(-\frac{4}{7}\right) = -\frac{6}{7}$

$V_o(s) = \frac{6}{7}\left[\frac{1}{s} - \frac{1}{s+7/4}\right]$ $\qquad \boxed{v_o(t) = \left[\frac{6}{7}\left(1 - e^{-1.75t}\right)\right]u(t)}$

14.13 Use mesh equations to find $v_o(t)$, $t > 0$, in the network in Fig. P14.13. CS

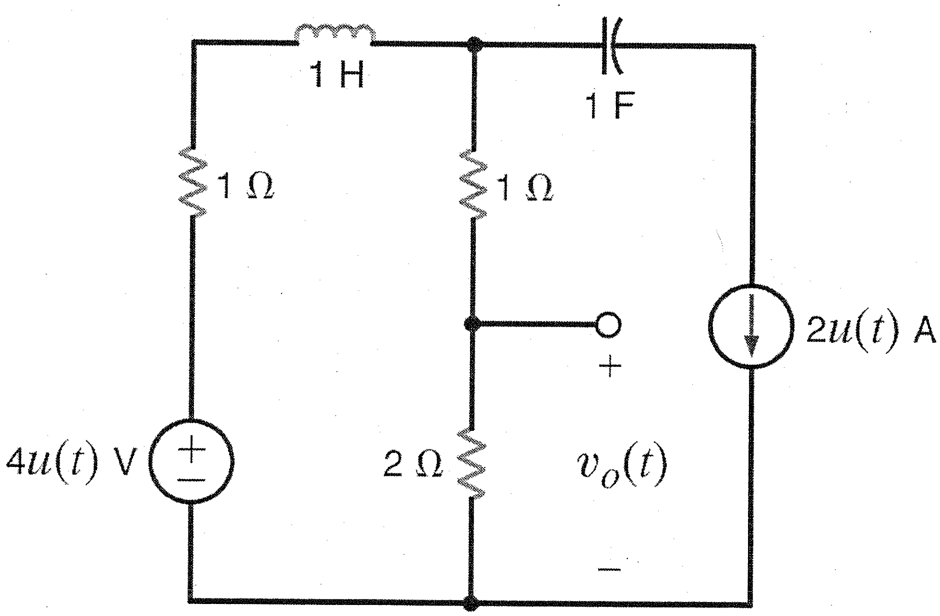
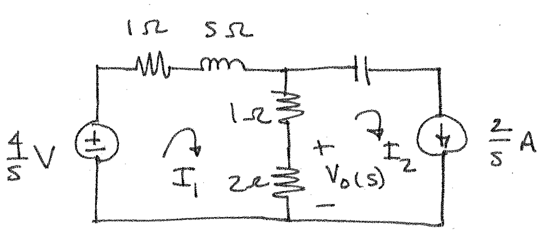


Figure P14.13

SOLUTION:



$\frac{4}{s} = I_1(s+4) - 3I_2 \qquad I_2 = 2/s$

$\frac{4}{s} = I_1(s+4) - \frac{6}{s}$

$\frac{10}{s} = I_1(s+4)$

$I_1 = \frac{10}{s(s+4)}$

$V_o = 2(I_1 - I_2) = 2\left[\frac{10}{s(s+4)} - \frac{2}{s}\right] = \frac{4(-s+1)}{s(s+4)} = \frac{1}{s} - \frac{5}{s+4}$

$$\boxed{v_o(t) = \left[1 - 5e^{-4t}\right]u(t)\ \text{V}}$$

14.14 Use loop equations to find $i_o(t)$, $t > 0$, in the network shown in Fig. P14.14.

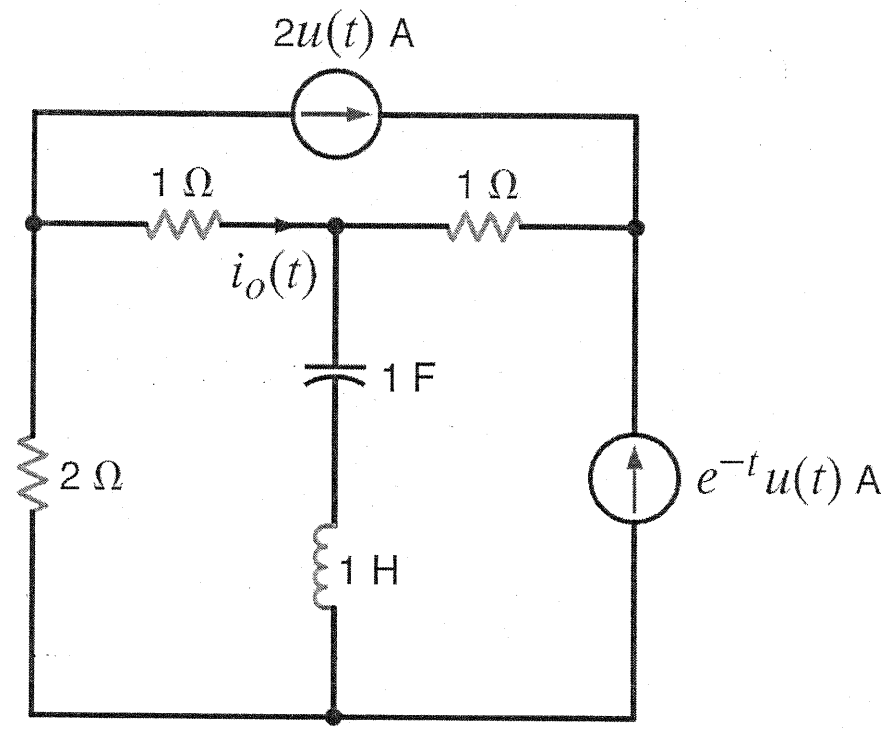
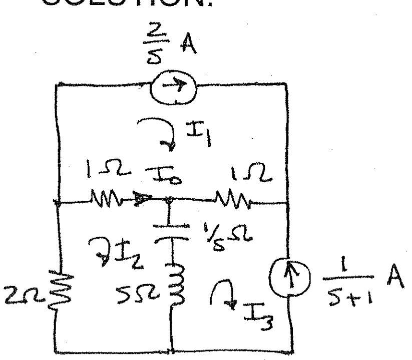


Figure P14.14

SOLUTION:



$$I_1 = \frac{2}{s}\text{ A} \quad \& \quad I_3 = \frac{-1}{s+1}\text{ A}$$

$$I_2\left(s + 3 + 1/s\right) - I_1(1) - I_3\left(s + \frac{1}{s}\right) = 0$$

$$\text{or,}\quad I_2\left(s^2 + 3s + 1\right) = sI_1 + \left(s^2+1\right)I_3$$

$$I_2\left(s^2+3s+1\right) = 2 - \frac{s^2+1}{s+1} = \frac{-s^2+2s+1}{s+1}$$

$$I_2 = \frac{-s^2+2s+1}{\left(s^2+3s+1\right)(s+1)}$$

$$I_o = I_2 - I_1 = \frac{-s^2+2s+1}{s^2+3s+1} - \frac{2}{s} = \frac{-\left(3s^3+6s^2+7s+2\right)}{s(s+1)\left(s^2+3s+1\right)}$$

$$I_o(s) = \frac{-\left(3s^3+6s^2+7s+2\right)}{s(s+1)(s+0.382)(s+2.62)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+0.382} + \frac{K_4}{s+2.62}$$

$$K_1 = \frac{-2}{(1)(1)} = -2$$

$$K_2 = \frac{-(-3+6-7+2)}{(-1)(1-3+1)} = 2$$

$$K_3 = \left.\frac{-(3s^3+6s^2+7s+2)}{s(s+1)(s+2.62)}\right|_{s=-0.382} = 0.065$$

$$K_4 = \left.\frac{-(3s^2+6s^2+7s+2)}{s(s+1)(s+0.382)}\right|_{s=2.62} = -3.065$$

$$i_o(t) = \left[2 + 2e^{-t} + 0.065e^{-0.382t} - 3.065e^{-2.62t}\right]u(t)$$ ✓

14.15 Use loop analysis to find $v_o(t)$ for $t > 0$ in the network in Fig. P14.15.

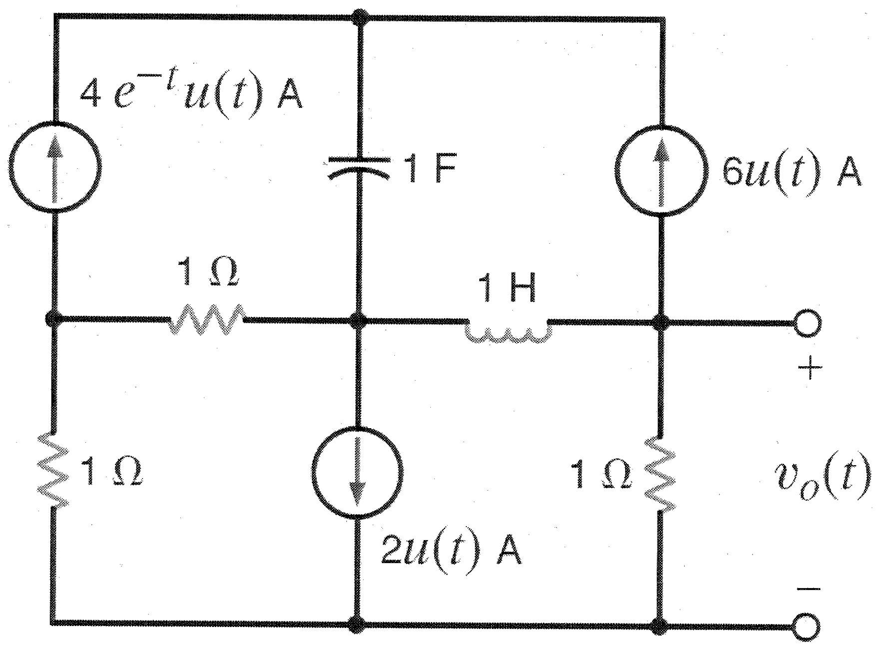
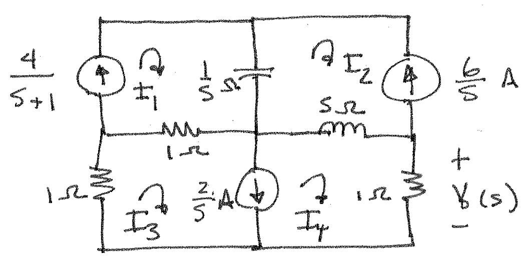


Figure P14.15

SOLUTION:



$I_1 = \frac{4}{s+1} \qquad I_2 = -\frac{6}{s} \qquad I_3 - I_4 = \frac{2}{s}$

$I_3(2) - I_1 + I_4(s+1) - s I_2 = 0$

$V_o = (1) I_4 \qquad I_3 = \frac{2}{s} + I_4$

$$2\left[\frac{2}{s} + I_4\right] - \frac{4}{s+1} + I_4(s+1) + 6 = 0 \Rightarrow I_4(s+3) = \frac{4}{s+1} - \frac{4}{s} - 6$$

$$I_4(s+3) = \frac{4s - 4s - 4 - 6s^2 - 6s}{s(s+1)} = -\frac{(6s^2+6s+4)}{s(s+1)}$$

$$V_o = \frac{-(6s^2+6s+4)}{s(s+1)(s+3)} = \frac{-4/3}{s} + \frac{2}{s+1} - \frac{20/3}{s+3}$$

$$v_o(t) = \left[2e^{-t} - 4/3 - \tfrac{20}{3} e^{-3t}\right] u(t)$$

14.16 Use mesh analysis to find $v_o(t)$, $t > 0$, in the network in Fig. P14.16. CS

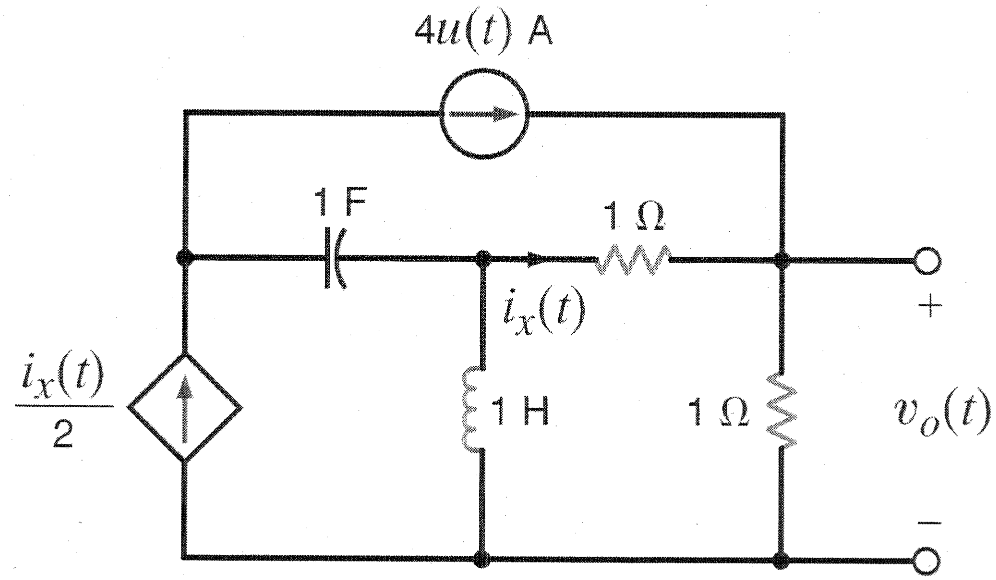
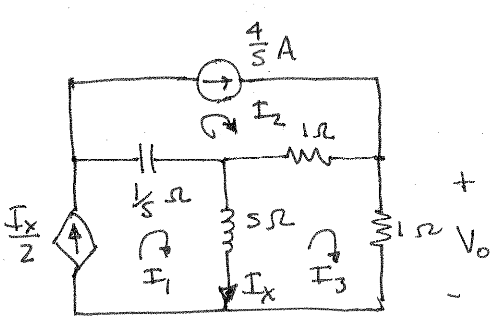


Figure P14.16

SOLUTION:



$$I_1 = \frac{I_x}{2} = \frac{I_1 - I_3}{2} \Rightarrow I_1 = -I_3$$

$$I_2 = 4/s$$

$$I_3(s+2) - sI_1 - I_2 = 0$$

$$V_o = (1)I_3$$

$$I_3(s+2) + sI_3 = 4/s \Rightarrow I_3 = \frac{4}{s(2s+2)} = \frac{2}{s(s+1)}$$

$$V_o = \frac{2}{s(s+1)} = \frac{2}{s} - \frac{2}{s+1}$$

$$\boxed{v_o(t) = [2(1-e^{-t})]u(t)}$$

14.17 Use superposition to find $v_o(t)$, $t > 0$, in the network shown in Fig. P14.17.

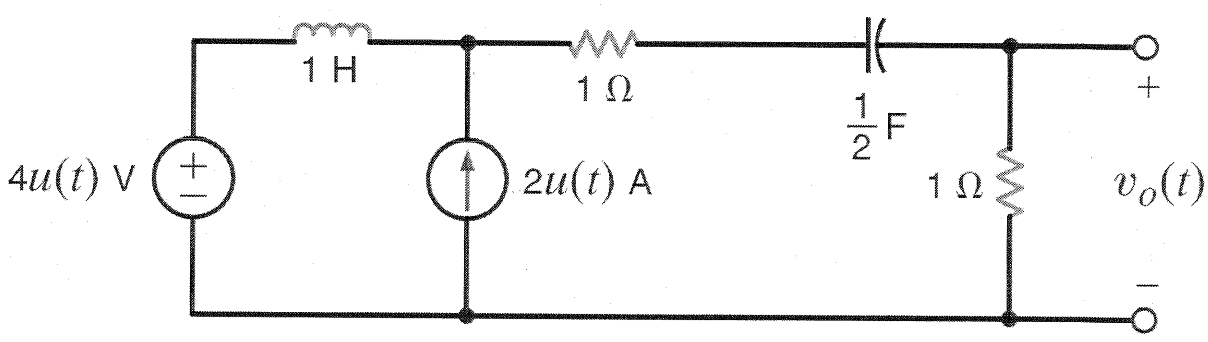
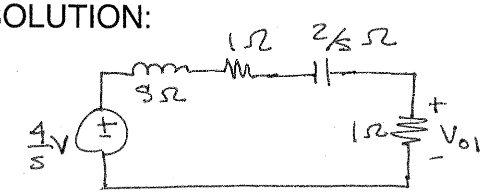
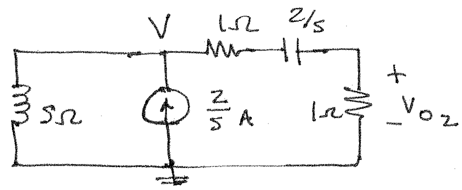


Figure P14.17

SOLUTION:





$$V_{01} = \frac{4}{s}\left[\frac{1}{s+1+\frac{2}{s}+1}\right]$$

$$V_{01} = \frac{4}{s^2+2s+2}$$

$$\frac{V}{s} + \frac{V}{2+2/s} = \frac{2}{s} \Rightarrow V\left[1+\frac{s^2}{2(s+1)}\right] = 2$$

$$V = \frac{4(s+1)}{s^2+2s+2} \qquad \frac{V_{02}}{V} = \frac{1}{2+\frac{2}{s}} = \frac{s}{2(s+1)}$$

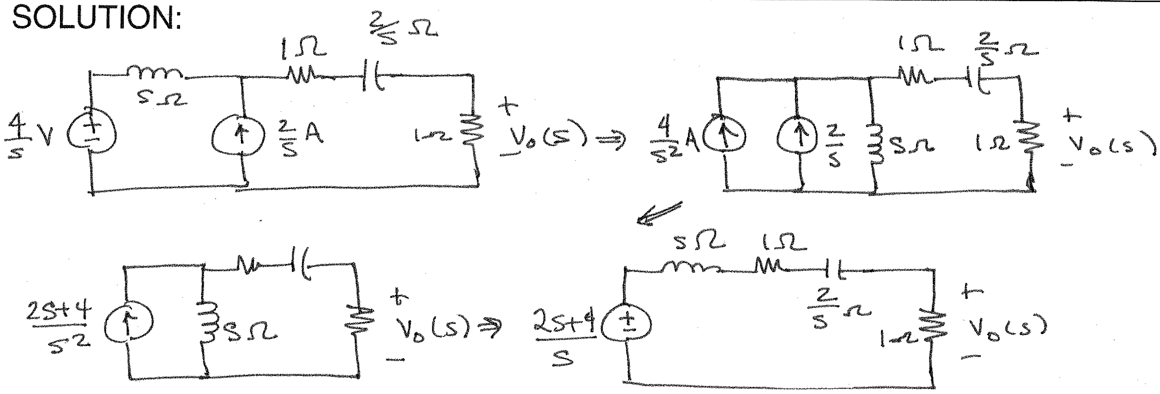
$$V_{02} = \frac{2s}{s^2+2s+2}$$

$$V_0 = V_{01}+V_{02} = \frac{2(s+2)}{s^2+2s+2} = \frac{K_1^*}{s+1+j1} + \frac{K_1}{s+1-j1} \qquad K_1 = \sqrt{2}\angle -45^\circ$$

$$\boxed{v_o(t) = \left[2\sqrt{2}\, e^{-t}\cos(t-45^\circ)\, V\right]u(t)}$$

14.18 Use source transformation to solve Problem 14.17.

SOLUTION:



$$V_o(s) = \frac{2s+4}{s}\left[\frac{1}{s+1+\frac{2}{s}+1}\right] = \frac{2s+4}{s}\left(\frac{s}{s^2+2s+2}\right) = \frac{2(s+2)}{s^2+2s+2}$$

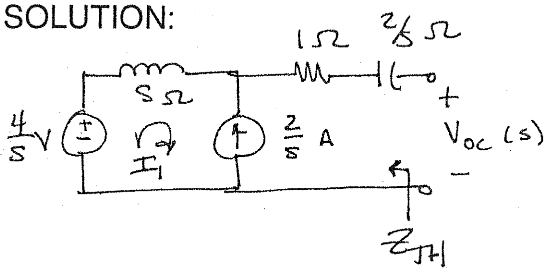
$$V_o = \frac{K_1}{s+1-j1} + \frac{K_1^*}{s+1+j1} \qquad K_1 = \sqrt{2}\,\angle -45^\circ$$

$$\boxed{v_o(t) = \left[2\sqrt{2}\, e^{-t}\cos(t-45^\circ)\right]u(t) \text{ V}}$$

14.19 Use Thévenin's theorem to solve Problem 14.17.

CS

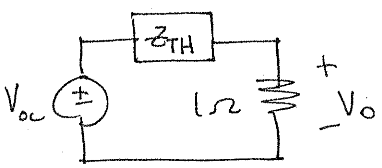
SOLUTION:



$$\frac{4}{s} = sI_1 + V_{oc} \qquad I_1 = -2/s$$

$$V_{oc} = \frac{4}{s} + 2 = \frac{2s+4}{s}$$

$$Z_{TH} = s+1+\frac{2}{s} = \frac{s^2+s+2}{s}$$



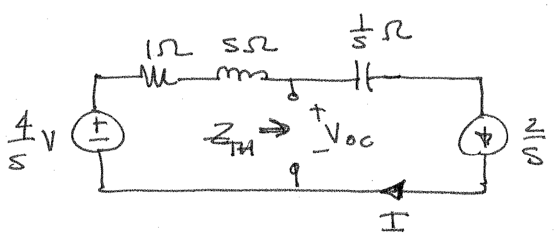
$$V_o = V_{oc}\left[\frac{1}{1+Z_{TH}}\right] = \frac{2(s+2)}{s^2+2s+2}$$

$$V_o = \frac{K_1}{s+1-j1} + \frac{K_1^*}{s+1+j1} \qquad K_1 = \sqrt{2}\angle -45°$$

$$\boxed{v_o(t) = \left[2\sqrt{2}\, e^{-t}\cos(t-45°)\right]u(t) \quad V}$$

14.20 Use Thévenin's theorem to solve Problem 14.13.

SOLUTION:

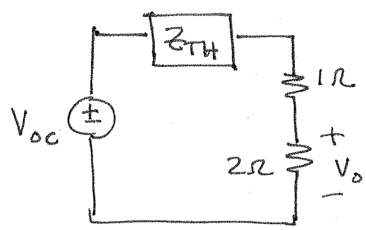


$$I = \frac{2}{s}$$

$$\frac{4}{s} = (1)I + sI + V_{oc}$$

$$V_{oc} = \frac{4}{s} - \frac{2}{s} - 2 = \frac{2}{s} - 2 = \frac{(-s+1)2}{s}$$

$$Z_{TH} = s + 1\ \Omega$$



$$V_o = \frac{V_{oc}(2)}{2 + 1 + Z_{TH}} = \frac{4(-s+1)}{s\left[3+s+1\right]} = \frac{4(-s+1)}{s(s+4)}$$

$$V_o = \frac{1}{s} - \frac{5}{s+4}$$

$$\boxed{v_o(t) = \left[1 - 5e^{-4t}\right]u(t)\ \mathrm{V}}$$

14.21 Use Thévenin's theorem to find $v_o(t)$, $t > 0$, in the network in Fig. P14.21. CS

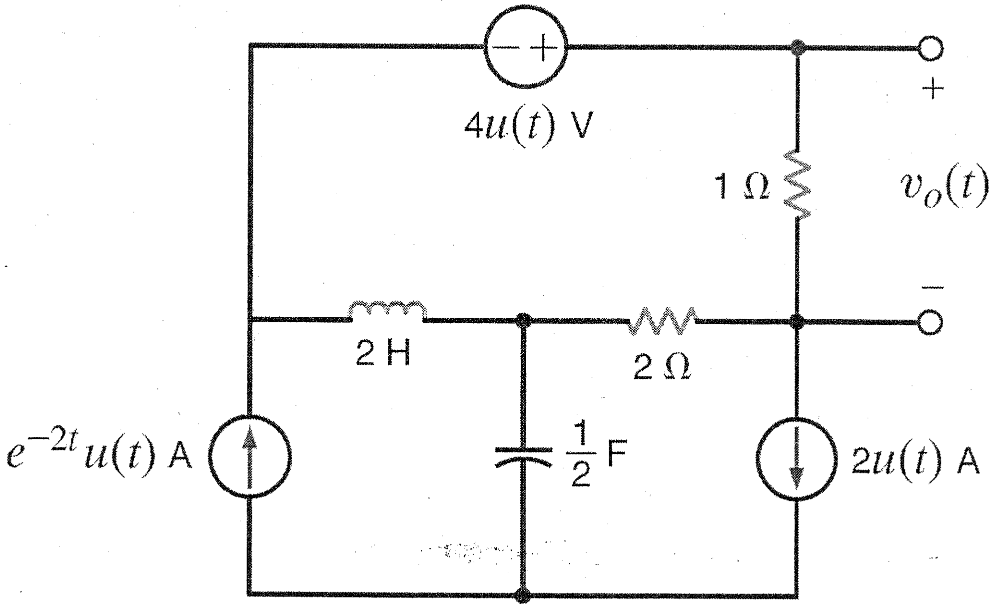
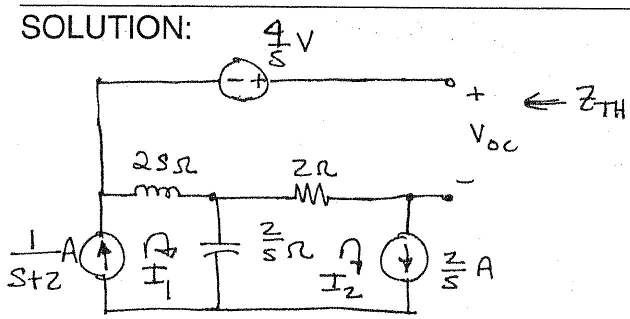


Figure P14.21

SOLUTION:

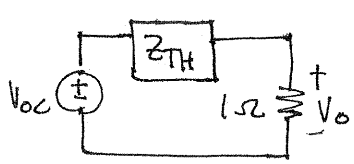


$$I_1 = \frac{1}{s+2} \qquad I_2 = \frac{2}{s}$$

$$\frac{4}{s} = V_{oc} - 2I_2 - 2sI_1$$

$$V_{oc} = \frac{4}{s} + \frac{4}{s} + \frac{2s}{s+2} = \frac{8}{s} + \frac{2s}{s+2}$$

$$V_{oc} = \frac{2s^2+8s+16}{s(s+2)}$$



$$Z_{TH} = 2s+2$$

$$V_o = \frac{V_{oc}(1)}{1+Z_{TH}} = \frac{2(s^2+4s+8)}{s(s+2)(s+1.5)2} = \frac{s^2+4s+8}{s(s+1.5)(s+2)} = \frac{8/3}{s} - \frac{17/3}{s+1.5} + \frac{4}{s+2}$$

$$\boxed{v_o(t) = \left[\frac{8}{3} - \frac{17}{3}e^{-1.5t} + 4e^{-2t}\right]u(t)\ \text{V}}$$

14.22 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.22 using Thévenin's theorem.

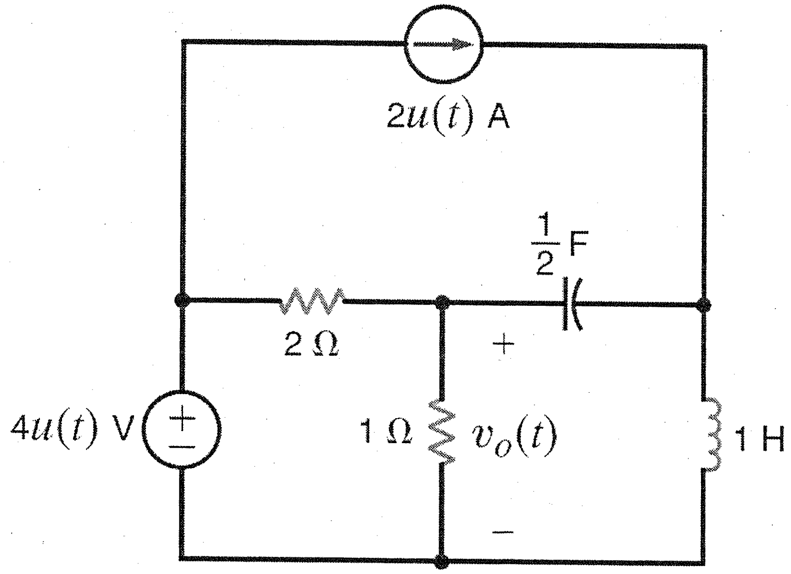
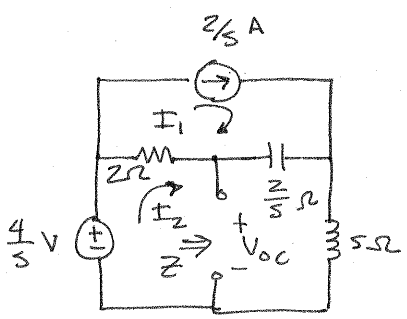


Figure P14.22

SOLUTION:



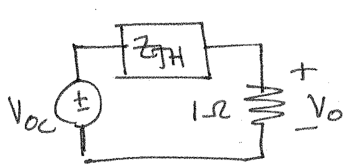
$$I_1 = \frac{2}{s} \quad ; \quad \frac{4}{s} = I_2\left[2 + \frac{2}{s} + s\right] - I_1\left[2 + \frac{2}{s}\right]$$

or, $$\frac{4}{s} = I_2\left(\frac{s^2+2s+2}{s}\right) - \frac{2}{s}\left(\frac{2s+2}{s}\right)$$

$$I_2 = \frac{8s+4}{s(s^2+2s+2)}$$

$$V_{oc} = \frac{4}{s} - 2(I_2 - I_1) = \frac{8s^2+8}{s(s^2+2s+2)}$$

$$Z = \frac{2\left(s+\frac{2}{s}\right)}{2+s+2/s} = \frac{2s^2+4}{s^2+2s+2}$$



$$V_o = \frac{V_{oc}\,(1)}{1 + Z_{TH}} = \frac{(8/3)(s^2+1)}{s(s^2+\frac{2}{3}s+2)}$$

$$V_o = \frac{4/3}{s} + \frac{K_1}{s+\frac{1}{3}-j\frac{\sqrt{17}}{3}} + \frac{K_1^*}{s+\frac{1}{3}+j\sqrt{17}/3}$$

$$K_1 = 0.825 \angle 36.0°$$

$$v_o(t) = \left[\frac{4}{3} + 1.65\, e^{-t/3} \cos\left(\frac{\sqrt{17}}{3}t + 36°\right)\right]u(t) \text{ V}$$

14.23 Use Thévenin's theorem to determine $i_o(t)$, $t > 0$, in the circuit shown in Fig. P14.23. **PSV**

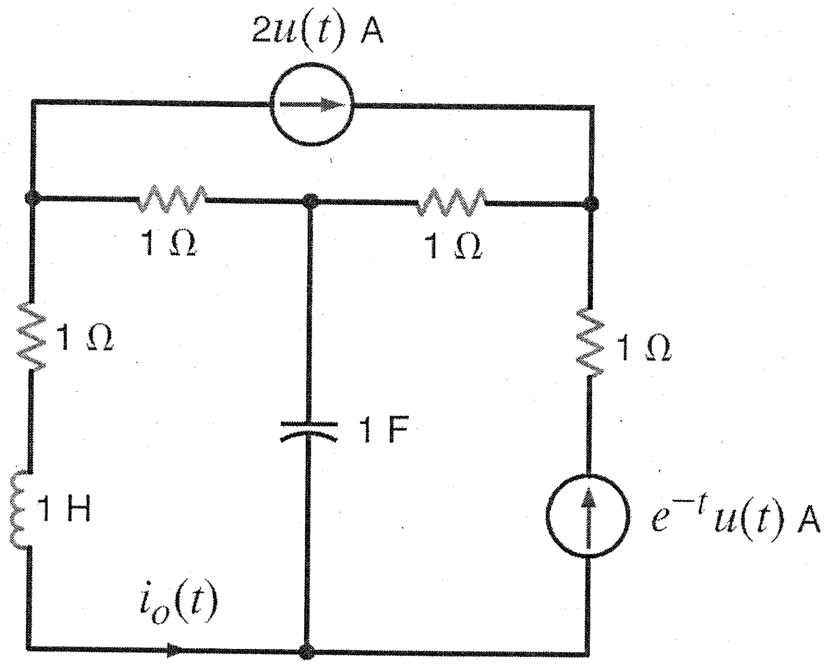
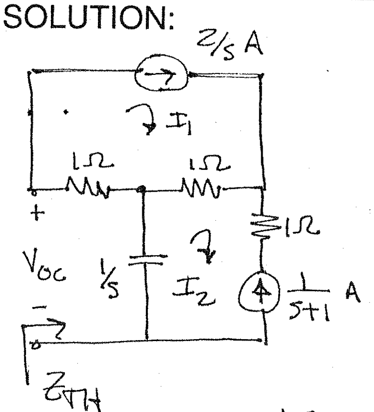


Figure P14.23

SOLUTION:

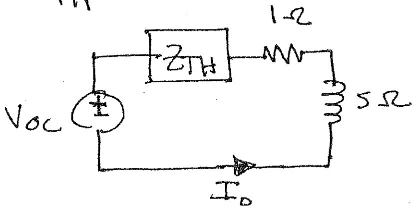


$I_1 = \frac{2}{s}$ $\quad I_2 = -\frac{1}{s+1}$

$V_{oc} = (1)(-I_1) - \frac{1}{s} I_2 = -\frac{2}{s} + \frac{1}{s(s+1)} = \frac{-(2s+1)}{s(s+1)}$

$Z = 1 + 1/s = (s+1)/s$

$I_o = \frac{-V_{oc}}{Z + 1 + s} = \frac{(2s+1)}{(s+1)^3}$



$I_o = \frac{K_1}{(s+1)^3} + \frac{K_2}{(s+1)^2} + \frac{K_3}{s+1} = \frac{-1}{(s+1)^3} + \frac{2}{(s+1)^2}$

$\boxed{i_o(t) = \left[2te^{-t} - \frac{1}{2}t^2e^{-t}\right]u(t) \text{ A}}$

14.24 Use Thévenin's theorem to find $v_o(t)$, $t > 0$, in the network in Fig. P14.24.

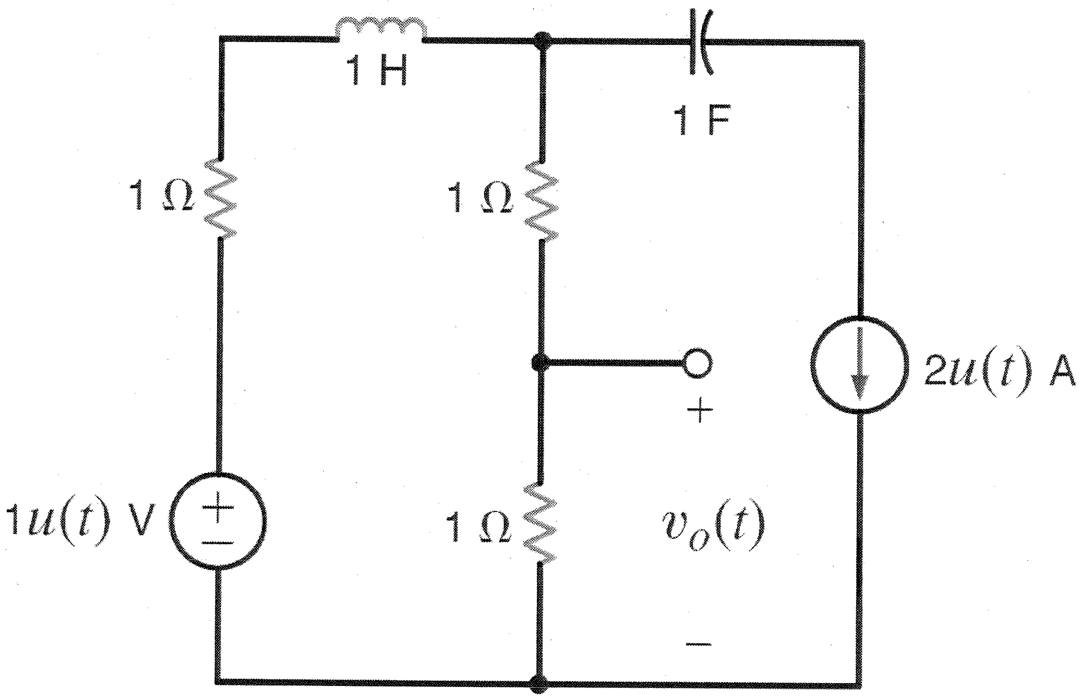
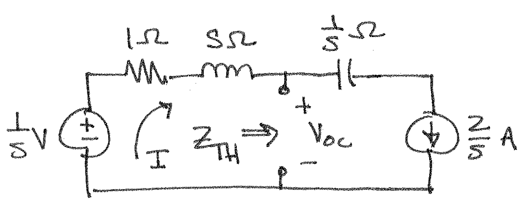


Figure P14.24

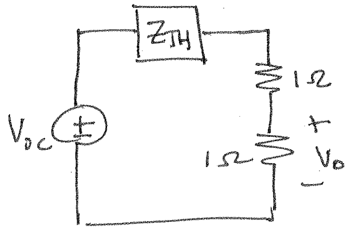
SOLUTION:



$$I = \frac{2}{s}\,A \qquad \frac{1}{s} = (s+1)I + V_{OC}$$

So, $$V_{OC} = \frac{-(2s+1)}{s}$$

$$Z_{TH} = s + 1\,\Omega$$



$$V_o = \frac{V_{OC}\,(1)}{2 + Z_{TH}} = \frac{-(2s+1)}{s}\,\frac{1}{s+3} = \frac{-(2s+1)}{s(s+3)}$$

$$V_o = \frac{-1/3}{s} - \frac{5/3}{s+3}$$

$$\boxed{v_o(t) = \left[-\frac{1}{3} - \frac{5}{3}e^{-3t}\right]u(t)\ V}$$

14.25 Use Thévenin's theorem to find $v_o(t)$, $t > 0$, in the network shown in Fig. P14.25. **PSV**

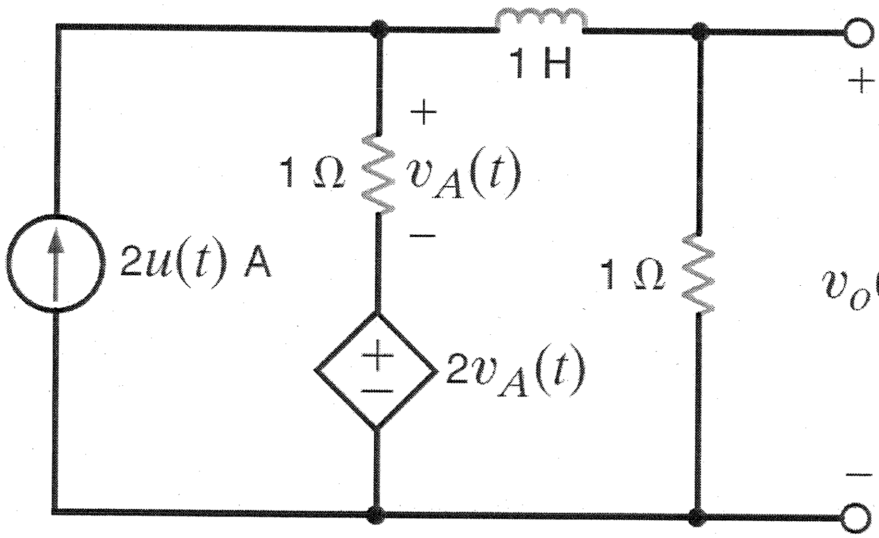
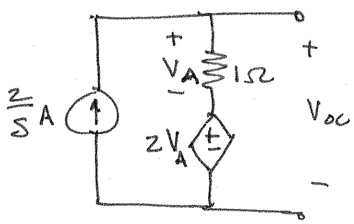
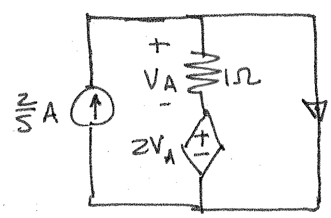


Figure P14.25

SOLUTION:





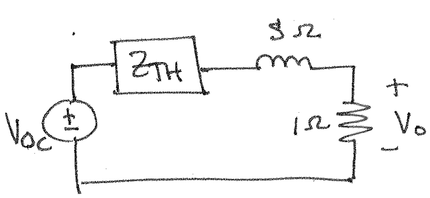
$Z_{TH} = V_{oc}/I_{sc}$

$Z_{TH} = 3\,\Omega$

$V_{oc} = 3V_A$; $V_A = (1)(2/s)$

$V_{oc} = 6/s$

$I_{sc} = \frac{2}{s}$



$$V_o = \frac{V_{oc}\,(1)}{s+1+Z_{TH}} = \frac{6}{s(s+4)} = \frac{3/2}{s} - \frac{3/2}{s+4}$$

$$\boxed{v_o(t) = [1.5(1-e^{-4t})]u(t)\ \text{V}}$$

14.26 Find $v_o(t)$, $t > 0$, in the network shown in Fig. P14.26 using Laplace transforms. Assume that the circuit has reached steady state at $t = 0-$.

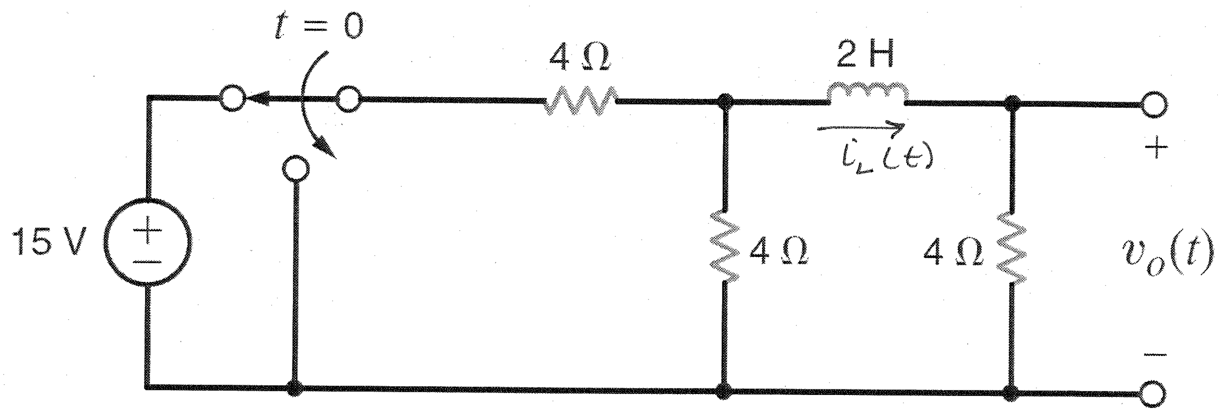
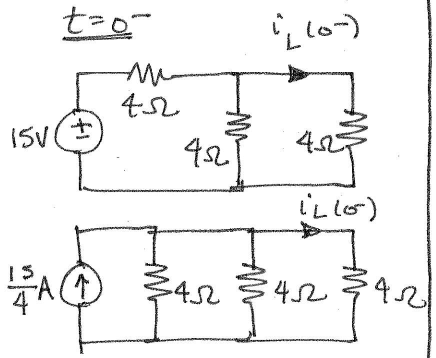


Figure P14.26

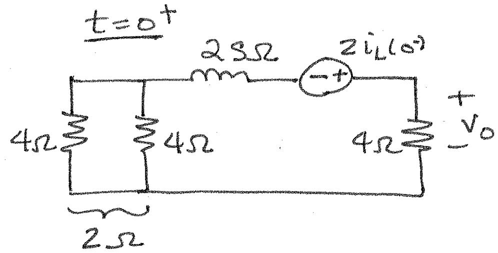
SOLUTION:

$t = 0^-$



$$i_L(0^-) = \left(\frac{15}{4}\right)\frac{1}{3} = 1.25\text{A}$$

$t = 0^+$



$$V_o = 2\,i_L(0^-)\left[\frac{4}{4+2+2s}\right] = \frac{5}{s+3}$$

$$\boxed{v_o(t) = 5e^{-3t}\ \text{V}}$$

14.27 Find $i_o(t)$, $t > 0$, in the network shown in Fig. P14.27.

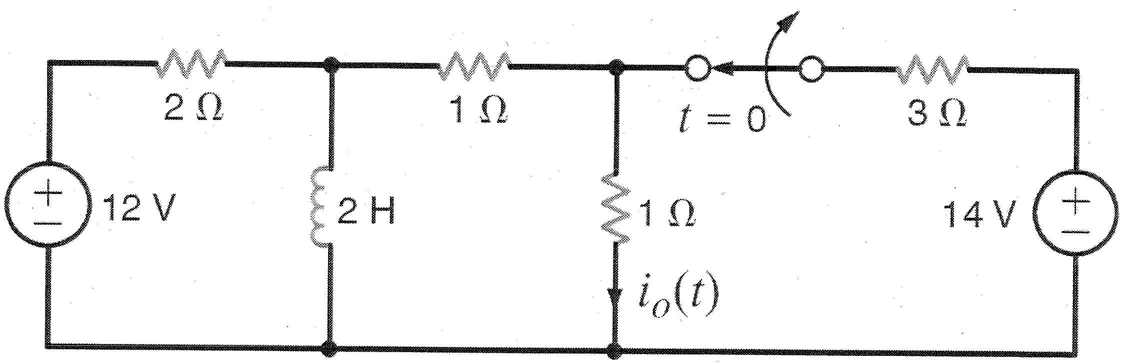
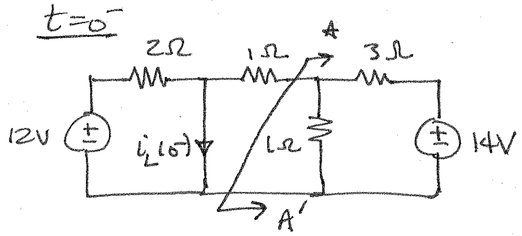


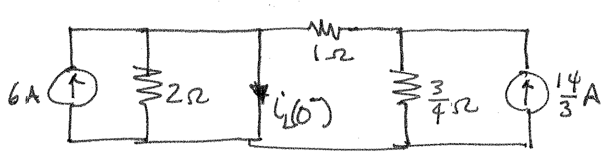
Figure P14.27

SOLUTION:

$t = 0^-$



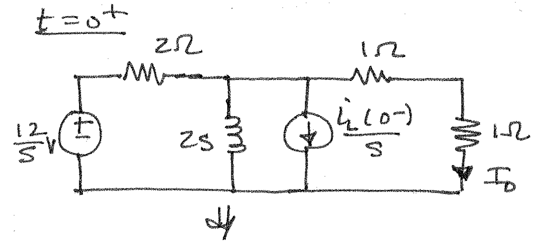
Use source transform & Norton's

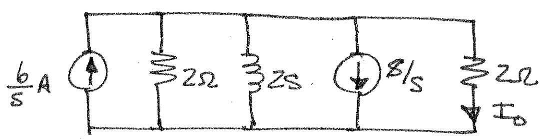


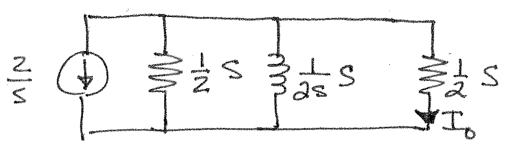
By superposition:

$$i_L(0^-) = 6 + \frac{14}{3}\left[\frac{3/4}{1 + 3/4}\right] = 8A$$

$t = 0^+$







$$I_o = -\frac{2}{s}\left[\frac{1/2}{1/2 + 1/2 + 1/2s}\right] = \frac{-1}{s + 1/2}$$

$$\boxed{i_o(t) = -e^{-t/2}u(t) \text{ A}}$$

14.28 Find $i_o(t)$, $t > 0$, in the network shown in Fig. P14.28.
CS

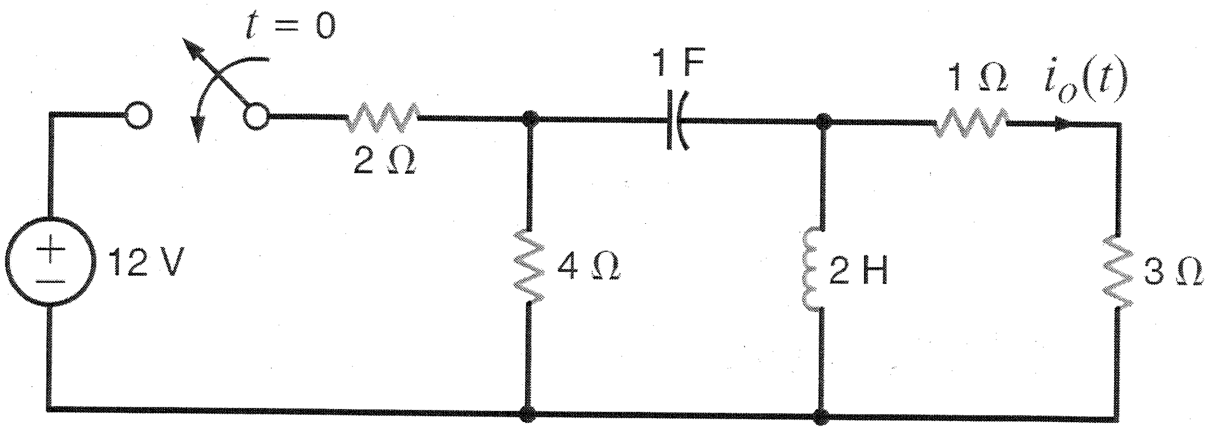


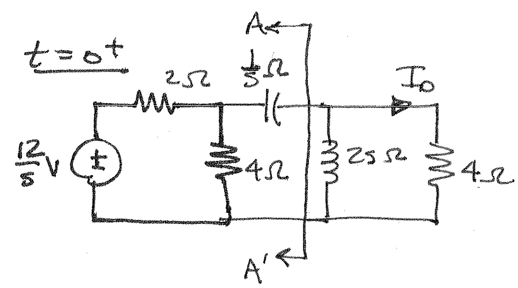
Figure P14.28

SOLUTION:

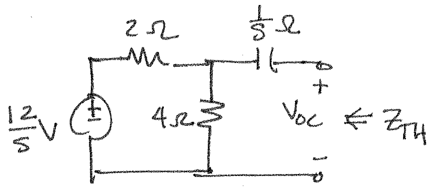
$t = 0^-$, $i_L(0^-) = 0$

$v_c(0^-) = 0$

$t = 0^+$

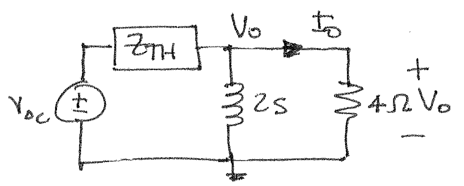


Thevenin at A-A'



$$V_{oc} = \frac{12}{s}\left(\frac{4}{6}\right) = 8/s$$

$$Z_{TH} = \frac{1}{s} + \frac{2(4)}{6} = \frac{1}{s} + \frac{4}{3} = \frac{4s+3}{3s}$$



$$\frac{V_o - V_{oc}}{Z_{TH}} + \frac{V_o}{4} + \frac{V_o}{2s} = 0 \qquad I_o = \frac{V_o}{4}$$

Let $Z_1 = 2s(4)/(2s+4) = 4s/(s+2)$

$$V_o = V_{oc} Z_1 / (Z_1 + Z_{TH}) \Rightarrow I_o = V_o/4 = \frac{1.5s}{s^2 + \left(\frac{11}{16}\right)s + \frac{6}{16}}$$

$$I_o = \frac{K}{s + \frac{11}{32} - j\frac{\sqrt{263}}{32}} + \frac{K^*}{s + \frac{11}{32} + j\frac{\sqrt{263}}{32}} \qquad K = 0.906\angle 34.2^\circ$$

$$i_o(t) = 1.81e^{-0.344t}\cos(0.507t + 34.2^\circ)\,u(t) \text{ A}$$

14.29 Find $v_o(t)$, $t > 0$, in the circuit shown in Fig. P14.29.

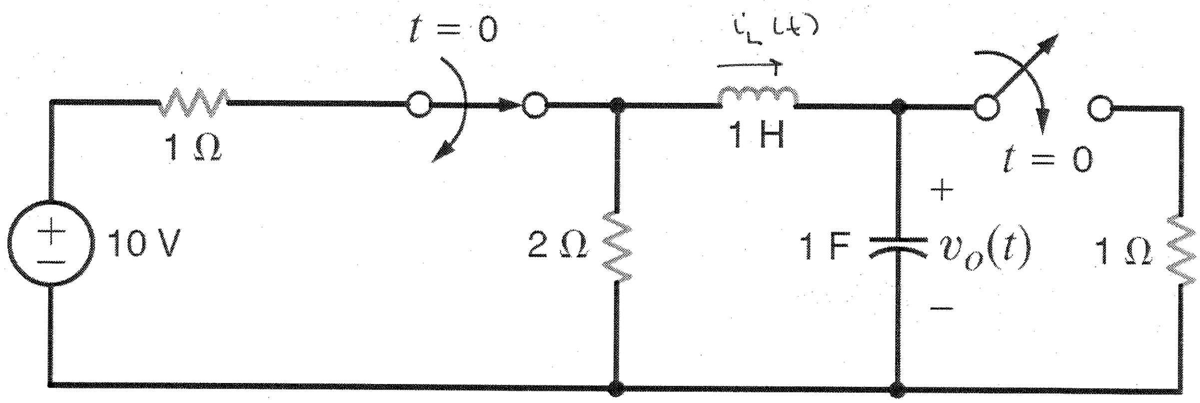
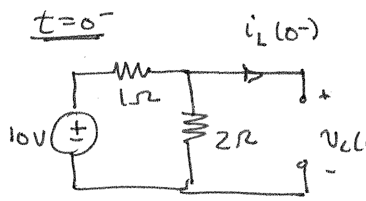


Figure P14.29

SOLUTION:

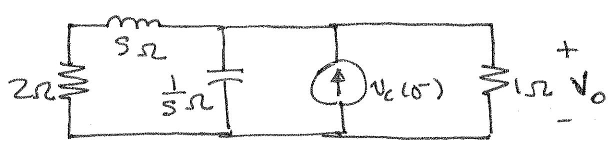
$t=0^-$



$i_L(0^-) = 0$

$v_c(0^-) = \frac{10(2)}{3} = \frac{20}{3}$ V

$t=0^+$



$$\frac{V_o}{s+2} + V_o(s) + \frac{V_o}{1} = \frac{20}{3} = V_o\left[s+1+\frac{1}{s+2}\right]$$

$$V_o\left[\frac{s^2+3s+3}{s+2}\right] = 20/3$$

$$V_o = \frac{20/3\,(s+2)}{s^2+3s+3} = \frac{K}{s+\frac{3}{2}-j\frac{\sqrt{3}}{2}} + \frac{K^*}{s+\frac{3}{2}+j\frac{\sqrt{3}}{2}}\ ; \quad K = 3.85\angle{-30^\circ}$$

$$\boxed{v_o(t) = 7.7e^{-(3/2)t}\cos\left[(\sqrt{3}/2)t - 30^\circ\right]u(t)\ \text{V}}$$

14.30 Find $v_o(t)$, $t > 0$, in the circuit in Fig. P14.30. CS

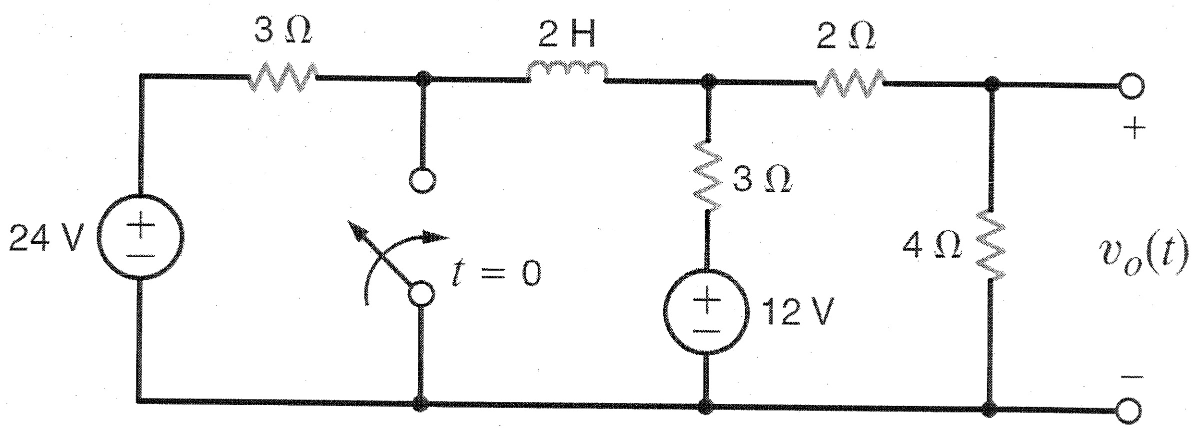
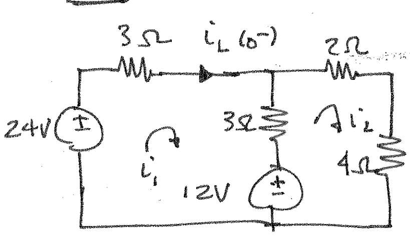


Figure P14.30

SOLUTION:

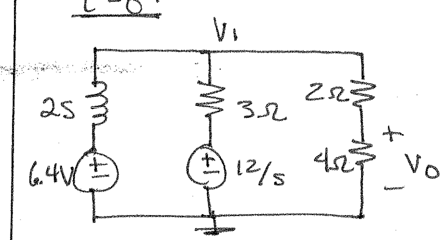
$\underline{t = 0^-}$



$$12 = 6i_1 - 3i_2$$
$$12 = -3i_1 + 9i_2$$

yields $i_L(0^-) = i_1 = 3.2A$

$\underline{t = 0^+}$



$$\frac{V_1 - 6.4}{2s} + \frac{V_1 - 12/s}{3} + \frac{V_1}{6} = 0$$

$$V_1(3 + 3s) = 6(3.2) + 24 = 43.2$$

$$V_1 = 14.4/(s+1)$$

$$V_o = V_1(4/6) = 9.6/(s+1)$$

$$\boxed{v_o(t) = 9.6e^{-t}u(t)\ V}$$

14.31 Find $i_o(t)$, $t > 0$, in the network in Fig. P14.31.
PSV

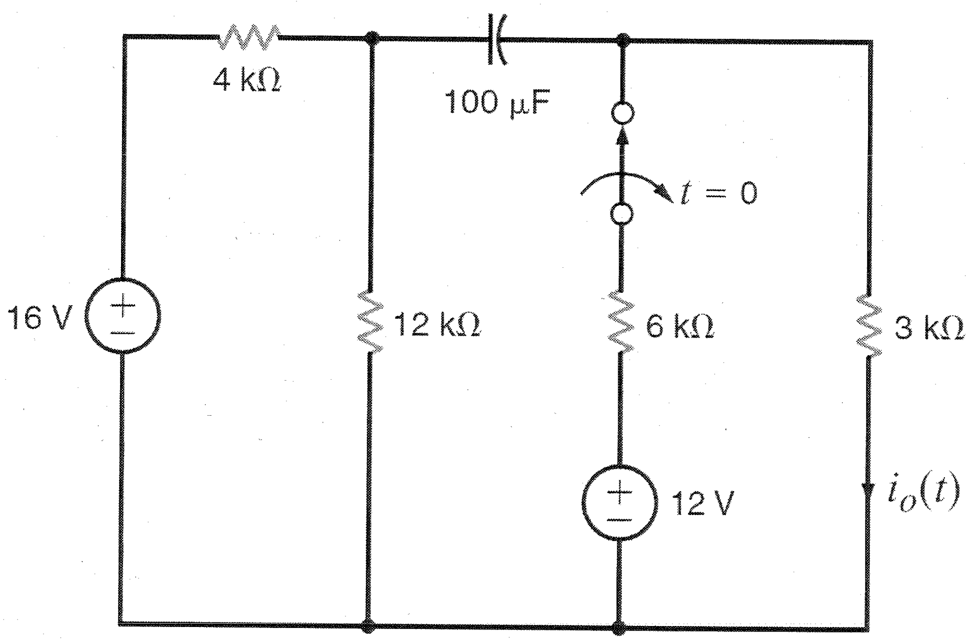
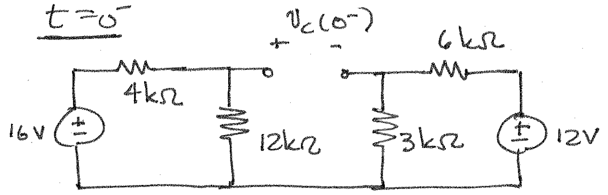


Figure P14.31

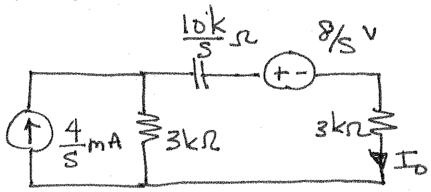
SOLUTION:

$t = 0^-$

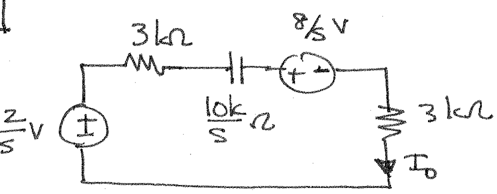


$$v_c(0^-) = 16\left(\frac{12}{16}\right) - 12\left(\frac{3}{9}\right) = 8V$$

$t = 0^+$ (w/ a source transformation)



⇕



$$\frac{12}{s} = \left(3000 + 3000 + \frac{10^4}{s}\right) I_o + \frac{8}{s} \Leftarrow$$

$$I_o = \frac{4}{6s+10} \text{ mA} = \frac{2/3}{s + 5/3}$$

$$\boxed{i_o(t) = \frac{2}{3} e^{-(5/3)t} u(t) \text{ mA}}$$

14.32 Find $v_o(t)$, $t > 0$, in the network in Fig. P14.32.

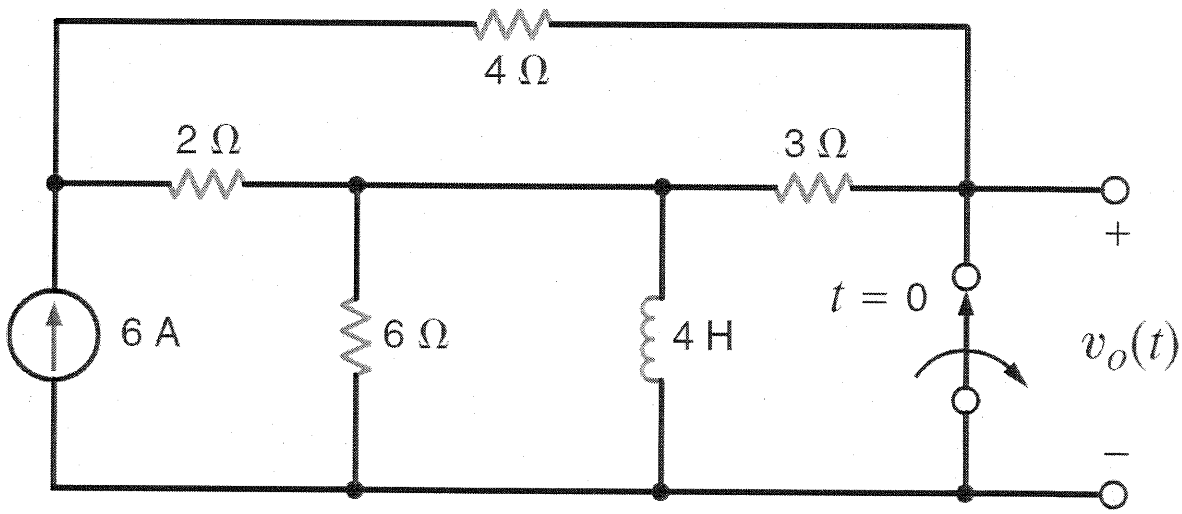
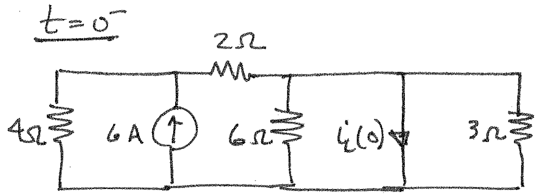


Figure P14.32

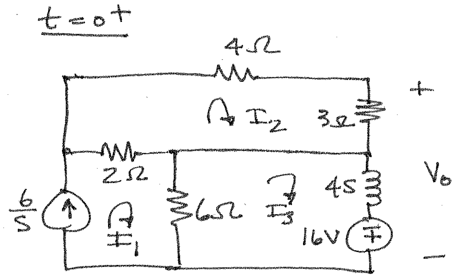
SOLUTION:

$t=0^-$



$$i_L(0) = 6\left[\frac{4}{4+2}\right] = 4A$$

$t=0^+$



$$I_1 = 6/s \qquad I_2(9) - 2I_1 = 0$$

$$16 = I_3(4s+6) - 6I_1$$

$$\Leftarrow \text{yields} \quad I_2 = \frac{4/3}{s} \qquad I_3 = \frac{8s+18}{s(2s+3)}$$

$$V_o = 3I_2 + 4sI_3 - 16$$

$$V_o = \frac{4}{s} + \frac{32s+72}{2s+3} - 16$$

$$V_o = \frac{16s+6}{s(s+1.5)} = \frac{4}{s} + \frac{12}{s+1.5} \Rightarrow \boxed{v_o(t) = \left[4 + 12e^{-1.5t}\right]u(t)\ V}$$

14.33 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.33.

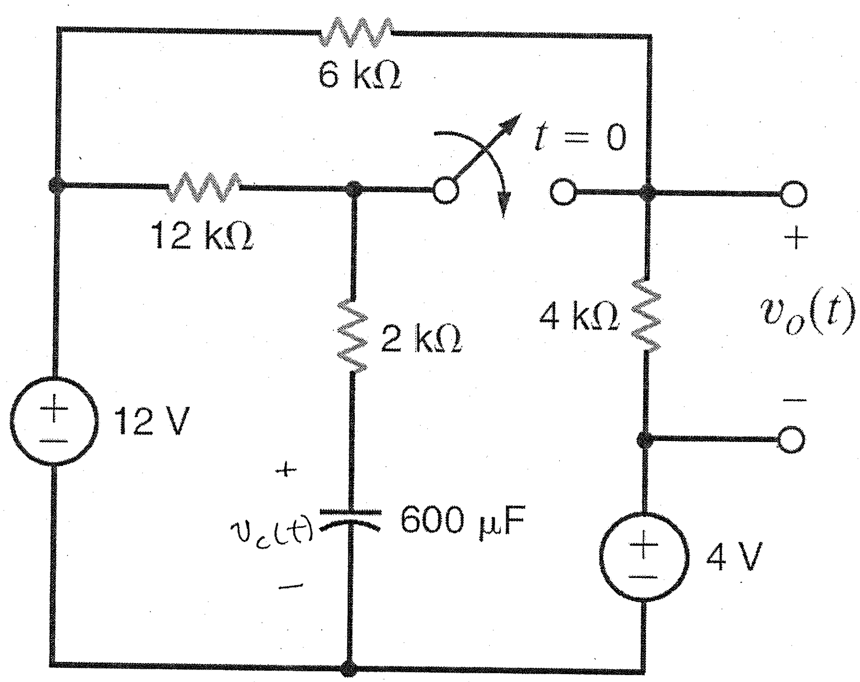
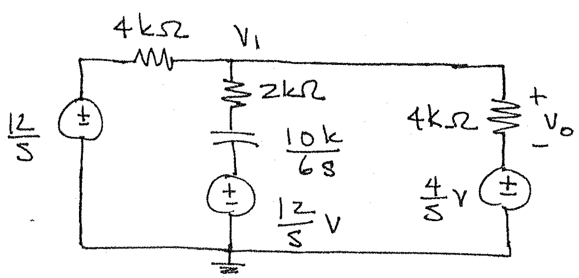


Figure P14.33

SOLUTION: $v_C(0^-) = 12V$

$t=0^+$ (12k ∥ 6k in parallel!)



$$\frac{V_1 - 12/s}{4\times10^3} + \frac{V_1 - 12/s}{\left(2+\frac{10}{6s}\right)\times10^3} + \frac{V_1 - 4/s}{4\times10^3} = 0$$

or, $$\frac{V_1}{4} + \frac{V_1}{2+\frac{10}{6s}} + \frac{V_1}{4} = \frac{3}{s} + \frac{1}{s} + \frac{12}{s\left(2+\frac{10}{6s}\right)}$$

$$V_1\left[\frac{12s+5}{12s+10}\right] = \frac{120s+40}{s(12s+10)}$$

$$V_1 = \frac{120s+40}{s(12s+5)}$$

$$V_o = V_1 - \frac{4}{s} = \frac{72s+20}{s(12s+5)}$$

$$V_o = \frac{(72s+20)/12}{s(s+5/12)} = \frac{4}{s} + \frac{2}{s+5/12}$$

$$\boxed{v_o(t) = \left[4 + 2e^{-(5/12)t}\right]u(t)\ \text{V}}$$

14.34 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.34.

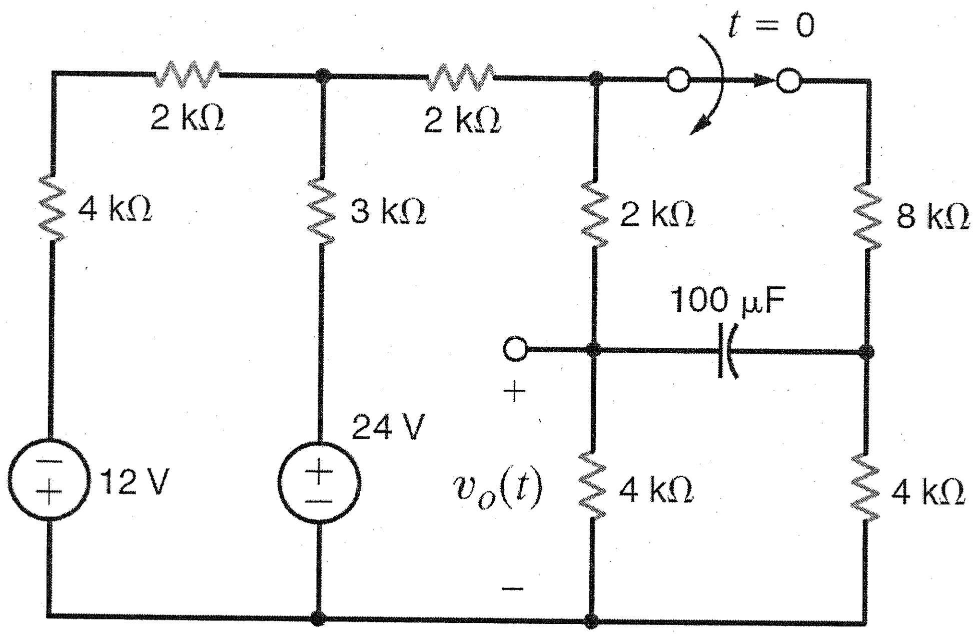
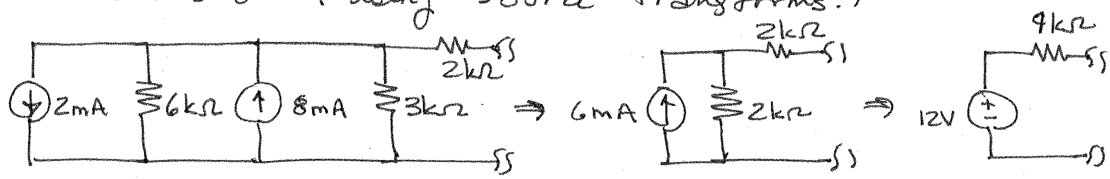
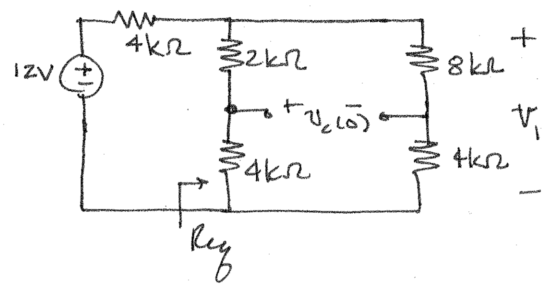


Figure P14.34

SOLUTION: $t = 0^-$ (using source transforms!)



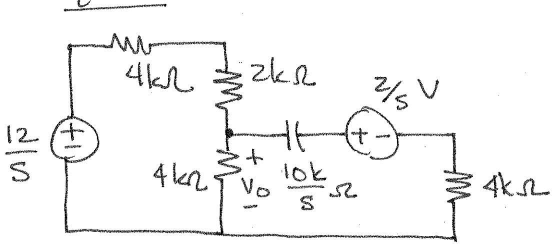


$R_{eq} = (6000)(12{,}000)/18{,}000 = 4k\Omega$

$v_1 = 12 R_{eq}/(4000 + R_{eq}) = 6V$

$v_c(0) = v_1 \left[\frac{4000}{6000} - \frac{4000}{12{,}000}\right] = 2V$

$t = 0^+$



$$\frac{V_o - 12/s}{6000} + \frac{V_o}{4000} + \frac{V_o - 2/s}{4000 + \frac{10{,}000}{s}} = 0$$

$$\frac{V_o}{6} + \frac{V_o}{4} + \frac{V_o s}{4s+10} = \frac{2}{s} + \frac{2}{4s+10}$$

$$V_o\left[\frac{s}{12}+\frac{s}{4s+10}\right]=\frac{10s+20}{s(4s+10)}=V_o\left[\frac{32s+50}{12(4s+10)}\right]$$

$$V_o=\frac{\frac{15}{4}(s+2)}{s\left(s+\frac{25}{16}\right)}=\frac{24/5}{s}-\frac{21/20}{s+25/16}$$

$$\boxed{v_o(t)=\left[\frac{24}{5}-\frac{21}{20}e^{-(25/16)t}\right]u(t)\ \text{V}}$$

14.35 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.35.
CS

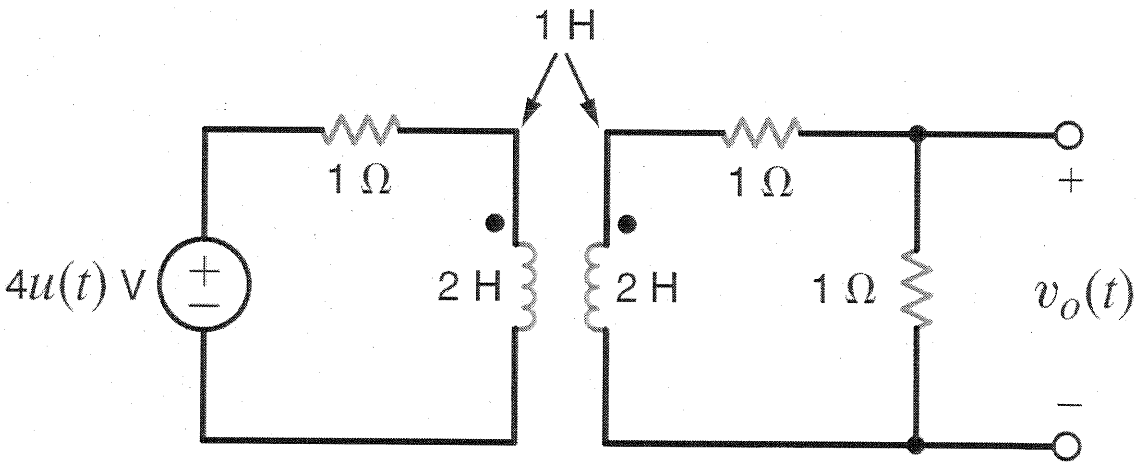
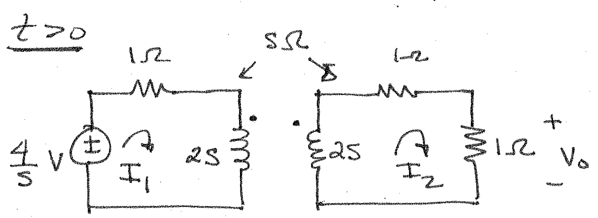


Figure P14.35

SOLUTION: $t=0^-$: no excition → ∅ intial conditions

$t>0$



$\frac{4}{s} = I_1(2s+1) - sI_2$

$0 = -sI_1 + I_2(2s+2)$

or, $I_1 = I_2(2s+2)/s$

yields, $I_2 = \dfrac{4/3}{s^2+2s+2/3}$

$V_o = (1)I_2 = \dfrac{4/3}{(s+0.42)(s+1.58)}$

$V_o = \dfrac{1.15}{s+0.42} - \dfrac{1.15}{s+1.58}$

$$\boxed{v_o(t) = 1.15\left[e^{-0.42t} - e^{-1.58t}\right]u(t) \text{ V}}$$

14.36 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.36.
PSV

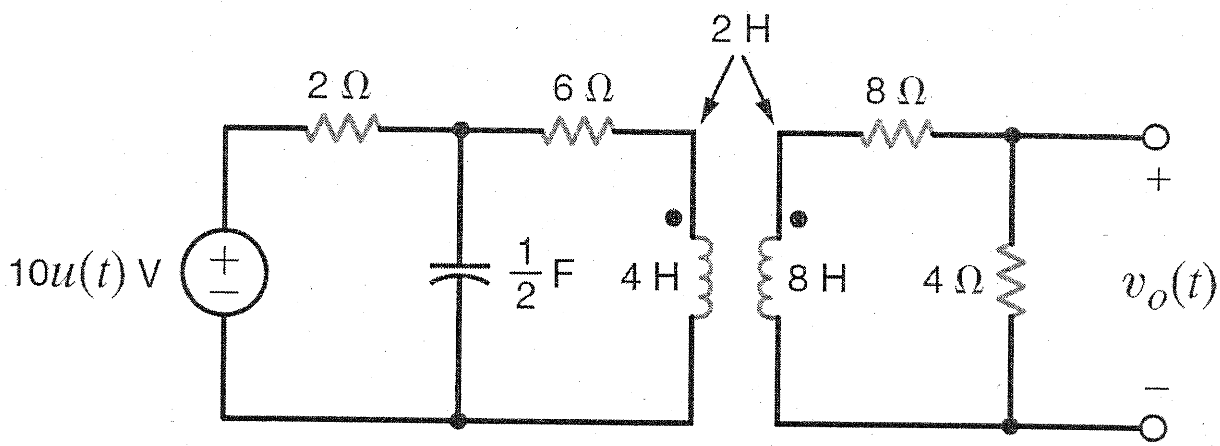
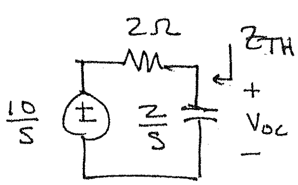


Figure P14.36

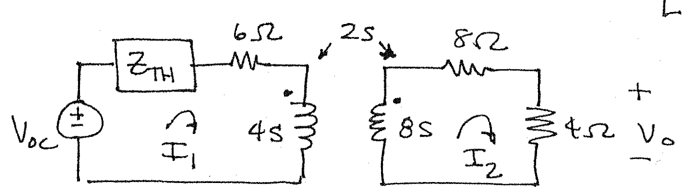
SOLUTION: $t=0^-$: no excitation $\Rightarrow$ $\emptyset$ initial conditions

$t=0^+$ (use Thevenin 1st)



$$V_{oc} = \frac{10}{s}\left[\frac{2/s}{2/s + 2}\right] = \frac{10}{s(s+1)} \text{ V}$$

$$Z_{TH} = 2(2/s) \Big/ \left[2 + 2/s\right] = \frac{2}{s+1} \ \Omega$$



$$V_{oc} = I_1(4s+6+Z_{TH}) - 2sI_2$$

$$0 = -2sI_1 + I_2(8s+12)$$

yields, $I_1 = I_2(4s+6)/s \Rightarrow V_{oc} = I_2\left[(4s+6+Z_{TH})(4s+6) - 2s\right]$

solve for I_2 and use $V_o = 4I_2$

$$V_o = \frac{20/7}{s^3 + \left(\frac{31}{7}\right)s^2 + \left(\frac{46}{7}\right)s + \frac{24}{7}}$$

Using the ROOTS function in MATLAB yields

$$V_0 = \frac{20/7}{(s+2)(s+1.21-j0.5)(s+1.21+j0.5)} = \frac{A}{s+2} + \frac{K}{s+1.21-j/2} + \frac{K^*}{s+1.21+j/2}$$

$A = 3.33 \qquad K = 3.15 \angle -122$

$$v_0(t) = \left[3.33e^{-2t} + 6.30\, e^{-1.21t} \cos\left(t/2 - 122^\circ\right]u(t)\ \text{V}\right.$$

14.37 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.37.

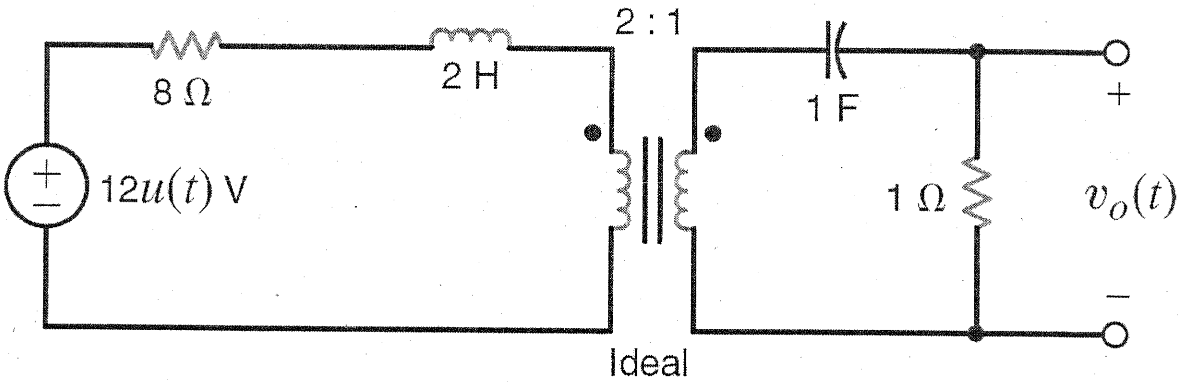
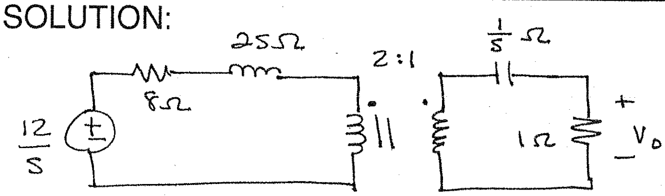
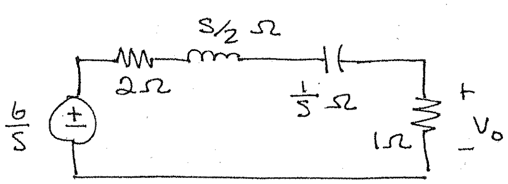


Figure P14.37

SOLUTION:



$n = 1/2$



$$V_o = \frac{6}{s}\left[\frac{1}{2 + s/2 + \frac{1}{s} + 1}\right] = \frac{12}{s^2+6s+2}$$

$$V_o = \frac{A}{s+0.35} + \frac{B}{s+5.65}$$

$A = 2.28$ & $B = -2.28$

$$\boxed{v_o(t) = 2.28\left[e^{-0.35t} - e^{-5.65t}\right]u(t)\ \text{V}}$$

14.38 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.38.
CS

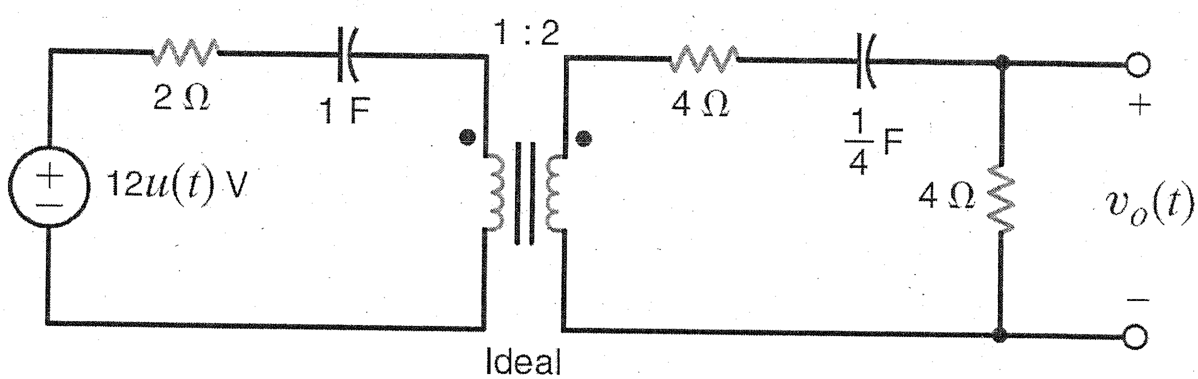
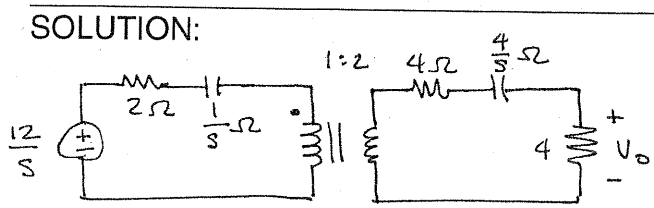
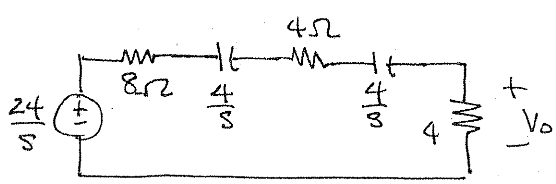


Figure P14.38

SOLUTION:



$n = 2/1$



$$V_o = \frac{24}{s}\left[\frac{4}{(8/s)+16}\right] \Rightarrow V_o = \frac{6}{s+1/2}$$

$$\boxed{v_o(t) = 6e^{-t/2}u(t)\ \text{V}}$$

14.39 Determine the initial and final values of the voltage $v_o(t)$ in the network in Fig. P14.39.

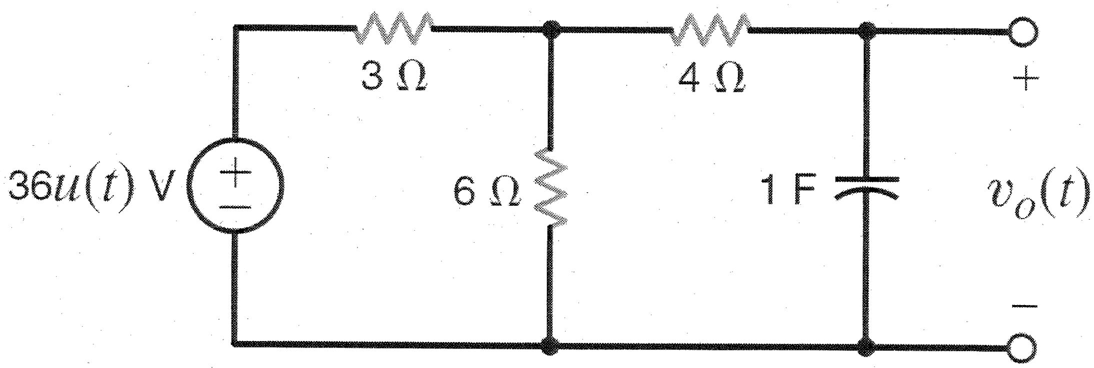
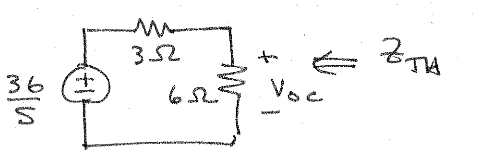


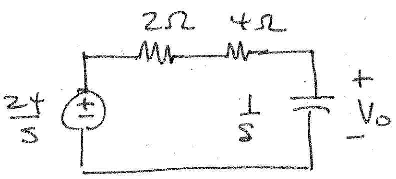
Figure P14.39

SOLUTION: Use Thevenin's



$V_{oc} = \frac{6}{9}\left(36/s\right) = 24/s\ \text{V}$

$Z_{TH} = 3(6)/9 = 2\Omega$



$V_o = \frac{24}{s}\left[\frac{1/s}{6 + 1/s}\right] = \frac{24}{s(6s+1)}$

$\lim_{t\to 0} v_o(t) = \lim_{s\to\infty} s\,V_o(s) = \frac{24}{6(\infty)} = 0$ $\boxed{v_o(0) \to 0\,\text{V}}$

$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} s\,V_o(s) = \frac{24}{1} = 24\text{V}$ $\boxed{v_o(\infty) \to 24\text{V}}$

14.40 Determine the initial and final values of the voltage $v_o(t)$ in the network in Fig. P14.40.

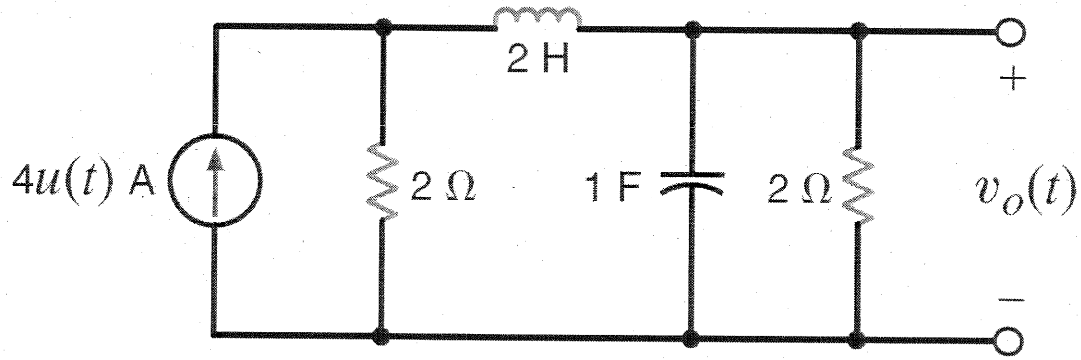
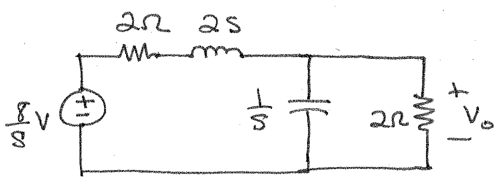


Figure P14.40

SOLUTION: Use source transformation,



Let $Z_1 = 2s + 2\,\Omega$ and

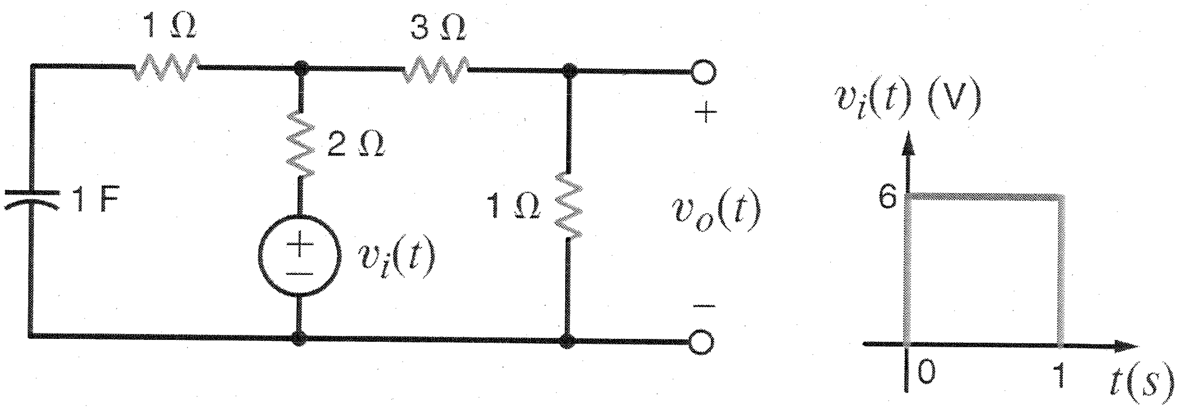
$Z_2 = 2(1/s) \,/\, (2 + 1/s) = \dfrac{2}{2s+1}\ \Omega$

$$V_o = \frac{8}{s}\left[\frac{Z_2}{Z_1+Z_2}\right] = \frac{8}{s}\left[\frac{2}{2+(2s+2)(2s+1)}\right] = \frac{16}{s(4s^2+6s+4)}$$

$$\lim_{t\to 0} v_o(t) = \lim_{s\to\infty} sV_o(s) = \frac{16}{4\infty^2} = 0 \qquad \boxed{v_o(0) \to 0}$$

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} sV_o(s) = \frac{16}{4} = 4 \qquad \boxed{v_o(\infty) \to 4V}$$

14.41 Determine the output voltage $v_o(t)$ in the network in Fig. P14.41a if the input is given by the source in Fig. P14.41b. **PSV**

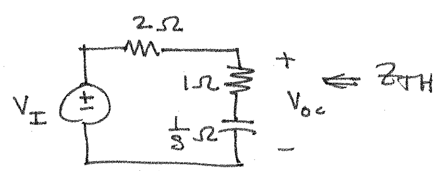


(a) (b)

Figure P14.41

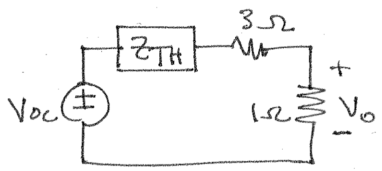
SOLUTION: $v_i(t) = 6u(t) - 6u(t-1)$ $\quad V_I(s) = \frac{6}{s}(1 - e^{-s})$

Use Thevenin eq.



$$V_{oc} = V_I\left[\frac{1 + 1/s}{3 + 1/s}\right] = V_I\left(\frac{s+1}{3s+1}\right)$$

$$Z_{TH} = \frac{(1 + 1/s)(2)}{3 + 1/s} = \frac{2(s+1)}{3s+1}$$



$$V_o = V_{oc}\left[\frac{1}{4 + Z_{TH}}\right] = V_I\left(\frac{s+1}{3s+1}\right)\left(\frac{3s+1}{4(3s+1) + 2s + 2}\right)$$

$$V_o = \frac{(6/14)(s+1)}{s(s + 6/14)}(1 - e^{-s}) = \left[\frac{1}{s} - \frac{4/7}{s + 6/14}\right](1 - e^{-s})$$

$$v_o(t) = \left[1 - \frac{4}{7}e^{-(6/14)t}\right]u(t) - \left[1 - \frac{4}{7}e^{-(6/14)(t-1)}\right]u(t-1) \text{ V}$$

14.42 Find the output voltage, $v_o(t)$, $t > 0$, in the network in Fig. P14.42a if the input is represented by the waveform shown in Fig. P14.42b.

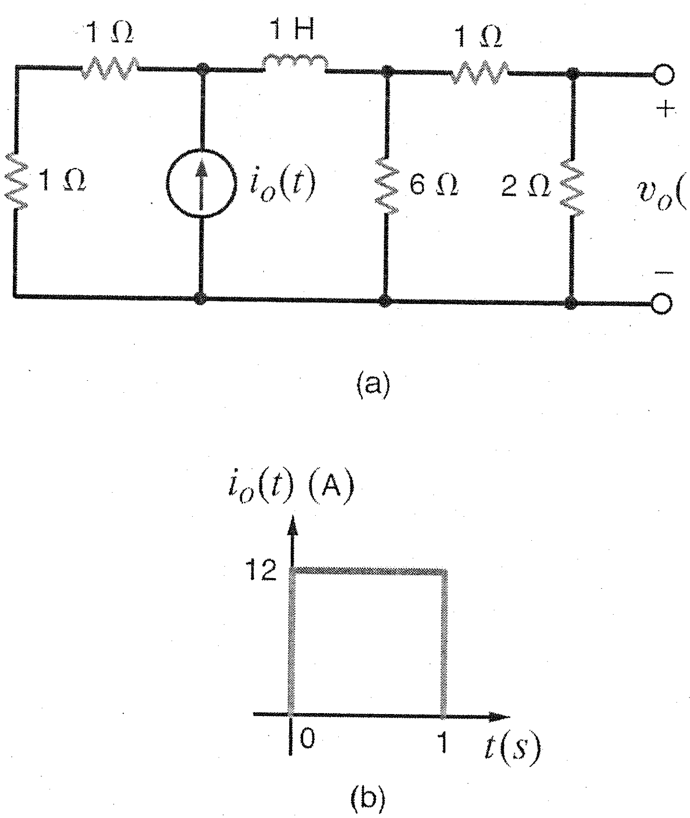
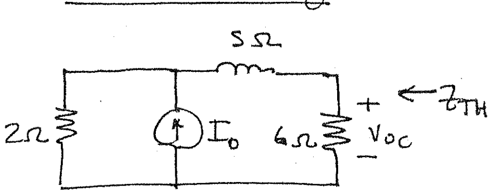


Figure P14.42

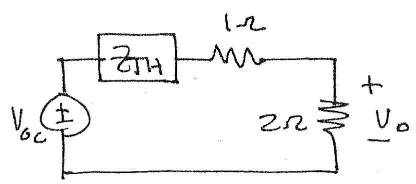
SOLUTION: $i_o(t) = 12u(t) - 12u(t-1)$ A $\Rightarrow$ $I_o(s) = \frac{12}{s}(1 - e^{-s})$ A

Use Thevenin Eq:



$$V_{oc} = I_o\left[\frac{2(6)}{s+8}\right] = I_o\left(\frac{12}{s+8}\right)$$

$$Z_{TH} = \frac{6(s+2)}{s+8}$$



$$V_o = V_{oc}\left(\frac{2}{3+Z_{TH}}\right) = I_o\left[\frac{24}{3(s+8)+6(s+2)}\right]$$

$$V_o = I_o\left[\frac{24}{9s+36}\right] = \frac{(8/3)(12)}{s(s+4)}(1-e^{-s})$$

$$V_o = \left(\frac{8}{s} - \frac{8}{s+4}\right)(1-e^{-s})$$

$$v_o(t) = \left[8 - 8e^{-4t}\right]u(t) - \left[8 - 8e^{-4(t-1)}\right]u(t-1) \text{ V}$$

14.43 Determine the output voltage, $v_o(t)$, in the circuit in Fig. P14.43a if the input is given by the source described in Fig. P14.43b.

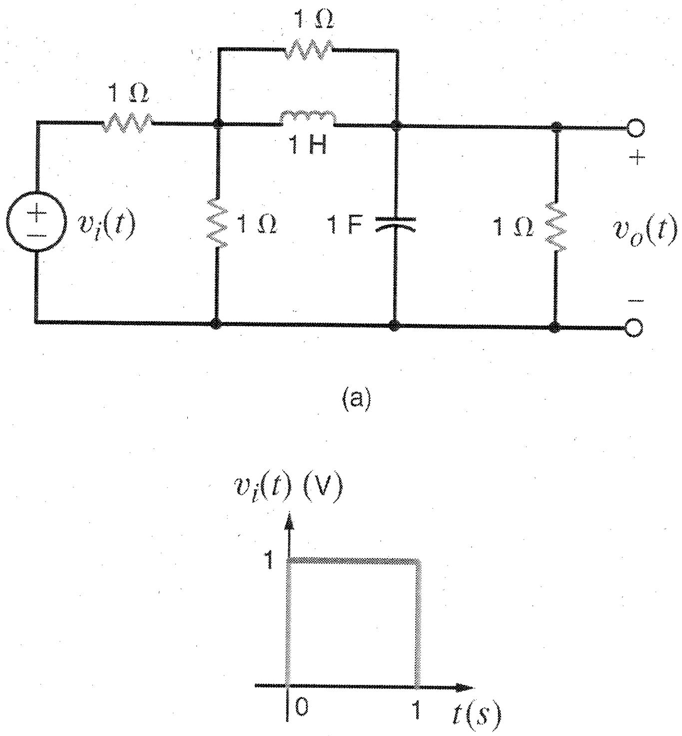
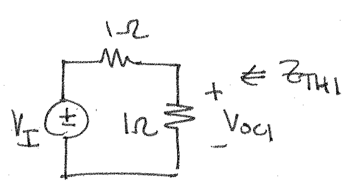


Figure P14.43

SOLUTION: $v_i(t) = u(t) - u(t-1) \Rightarrow V_I(s) = \frac{1}{s}(1 - e^{-s})$ V

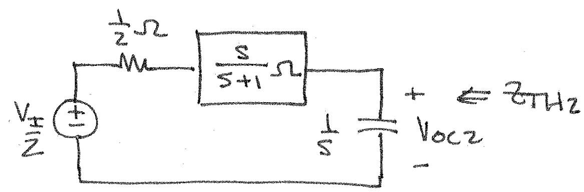
Use Thevenin's twice!



$V_{oc1} = \frac{V_I}{2}$

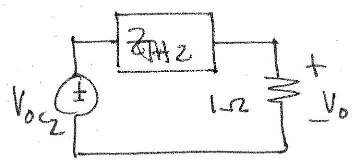
$Z_{TH1} = \frac{1}{2}\Omega$

⇒



$$V_{oc2} = \frac{V_I}{2}\left[\frac{1/s}{\frac{1}{2} + \frac{s}{s+1} + \frac{1}{s}}\right] = \frac{V_I(s+1)}{3s^2 + 3s + 2}$$

$$Z_{TH2} = \frac{\frac{1}{s}\left(\frac{1}{2} + \frac{s}{s+1}\right)}{\frac{1}{2} + \frac{1}{s} + \frac{s}{s+1}}$$



$$V_o = V_{oc2}\left(\frac{1}{1 + Z_{TH2}}\right)$$

$$Z_{TH2} = \frac{3s+1}{3s^2+3s+2}$$

$$V_o = \frac{(1/3)}{s(s+1)}\,(1-e^{-s}) = \left(\frac{1/3}{s} - \frac{1/3}{s+1}\right)(1-e^{-s})$$

$$v_o(t) = \frac{1}{3}\left[1-e^{-t}\right]u(t) - \frac{1}{3}\left[1-e^{-(t-1)}\right]u(t-1)\ \text{V}$$

14.44 Determine the transfer function $\mathbf{I}_o(s)/\mathbf{I}_i(s)$ for the network shown in Fig. P14.44.

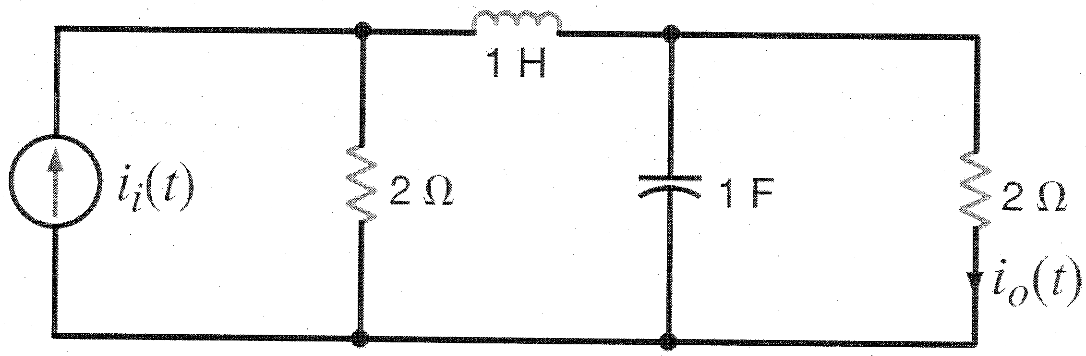
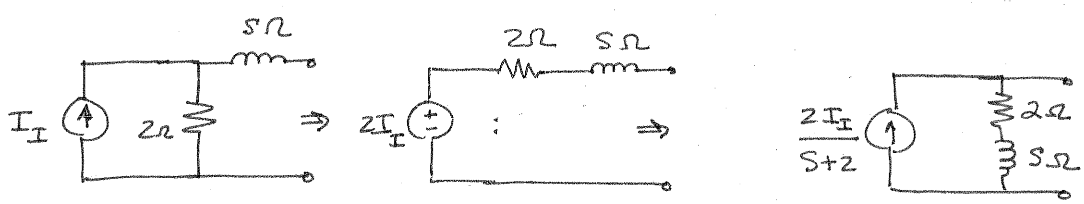
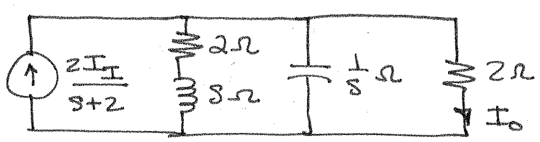


Figure P14.44

SOLUTION: Use source transformations





Current Division: $I_o = \frac{2I_I}{S+2}\left[\frac{1/2}{1/2 + S + \frac{1}{S+2}}\right] = \frac{I_I}{(S+2)(S+1/2)+1}$

$$\boxed{\frac{I_o}{I_I} = \frac{1}{s^2+2.5s+2}}$$

14.45 Find the transfer function $\mathbf{V}_o(s)/\mathbf{V}_i(s)$ for the network shown in Fig. P14.45. CS

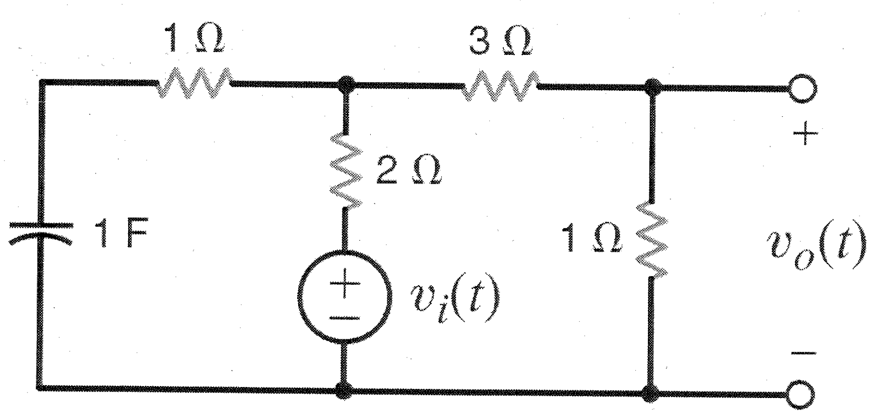
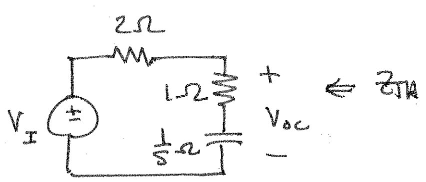


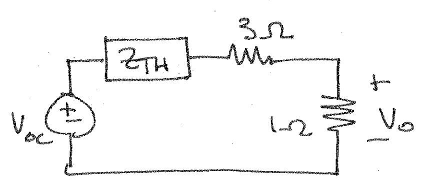
Figure P14.45

SOLUTION: Use Thevenin eq



$$V_{oc} = \frac{V_I\left(1+1/s\right)}{3+1/s} = \frac{V_I\,(s+1)}{3s+1}$$

$$Z_{TH} = \frac{2\left(1+1/s\right)}{3+1/s} = \frac{2(s+1)}{3s+1}$$



$$V_o = V_{oc}\left[\frac{1}{4+Z_{TH}}\right] = \frac{V_I\,(s+1)}{3s+1}\left(\frac{3s+1}{4(3s+1)+2(s+1)}\right)$$

$$V_o = V_I\left[\frac{s+1}{14s+6}\right]$$

$$\boxed{\frac{V_o}{V_I} = \frac{s+1}{14s+6}}$$

14.46 Find the transfer function for the network shown in Fig. P14.46.

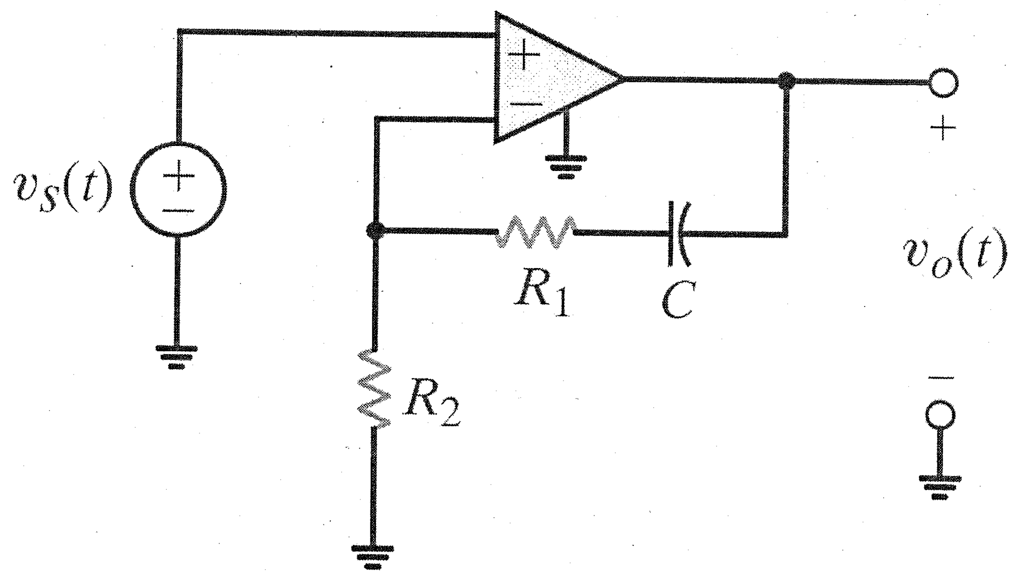


Figure P14.46

SOLUTION:

Let $Z_2 = R_1 + \frac{1}{sC}$ & $Z_1 = R_2$ $\qquad \frac{V_o}{V_s} = 1 + \frac{Z_2}{Z_1}$

$$\frac{V_o}{V_s} = 1 + \frac{R_1 + 1/sC}{R_2} = 1 + \frac{R_1 Cs + 1}{R_2 Cs} = \frac{(R_1+R_2)Cs + 1}{R_2 Cs}$$

$$\boxed{\frac{V_o}{V_s} = \left(1 + \frac{R_1}{R_2}\right)\left[\frac{s + \frac{1}{C(R_1+R_2)}}{s}\right]}$$

14.47 Find the transfer function for the network shown in Fig. P14.47.

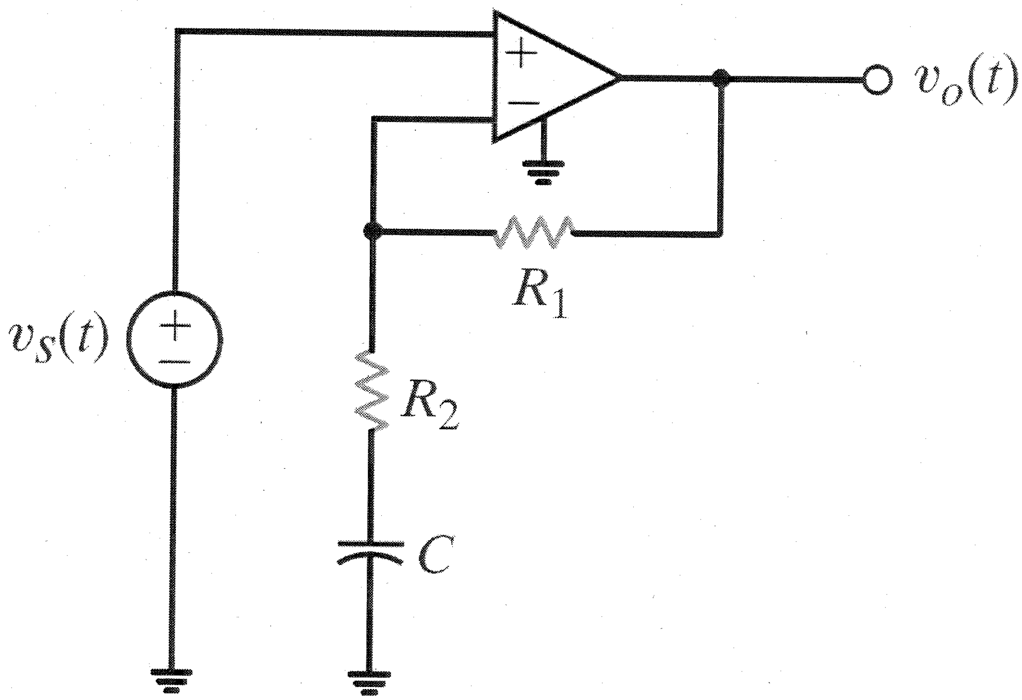


Figure P14.47

SOLUTION: Let $Z_2 = R_1$ & $Z_1 = R_2 + \frac{1}{sC} = \frac{R_2Cs+1}{sC}$

$$\frac{V_o}{V_s} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{R_1Cs}{R_2Cs+1} = \frac{(R_1+R_2)Cs+1}{R_2Cs+1}$$

$$\boxed{\frac{V_o}{V_s} = \left(1+\frac{R_1}{R_2}\right)\left(\frac{s+\frac{1}{CR}}{s+\frac{1}{CR_2}}\right) \quad R = R_1 + R_2}$$

14.48 Find the transfer function for the network in Fig. P14.48. **PSV**

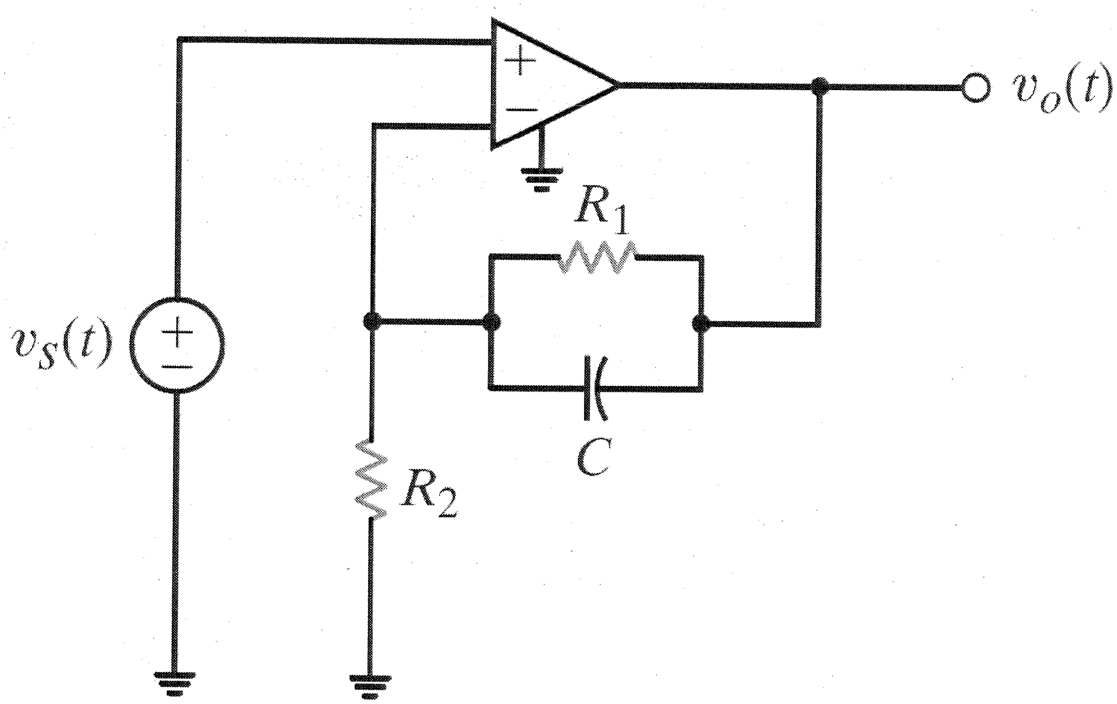


Figure P14.48

SOLUTION: Let $Z_1 = R_2$ & $Z_2 = \frac{R_1/sC}{R_1 + 1/sC} = \frac{R_1}{sCR_1+1}$

$$\frac{V_o}{V_s} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{R_1/R_2}{sCR_1+1} = \frac{sCR_1 + 1 + R_1/R_2}{sCR_1+1} = \frac{1}{R_2}\left[\frac{sCR_1R_2 + R_1 + R_2}{sCR_1+1}\right]$$

$$\boxed{\frac{V_o}{V_s} = \left(1 + \frac{R_1}{R_2}\right)\left(\frac{sCR_p+1}{sCR_1+1}\right) \quad R_p = \frac{R_1R_2}{R_1+R_2}}$$

14.49 Find the transfer function for the network in Fig. P14.49. If a step function is applied to the network, will the response be overdamped, underdamped, or critically damped?

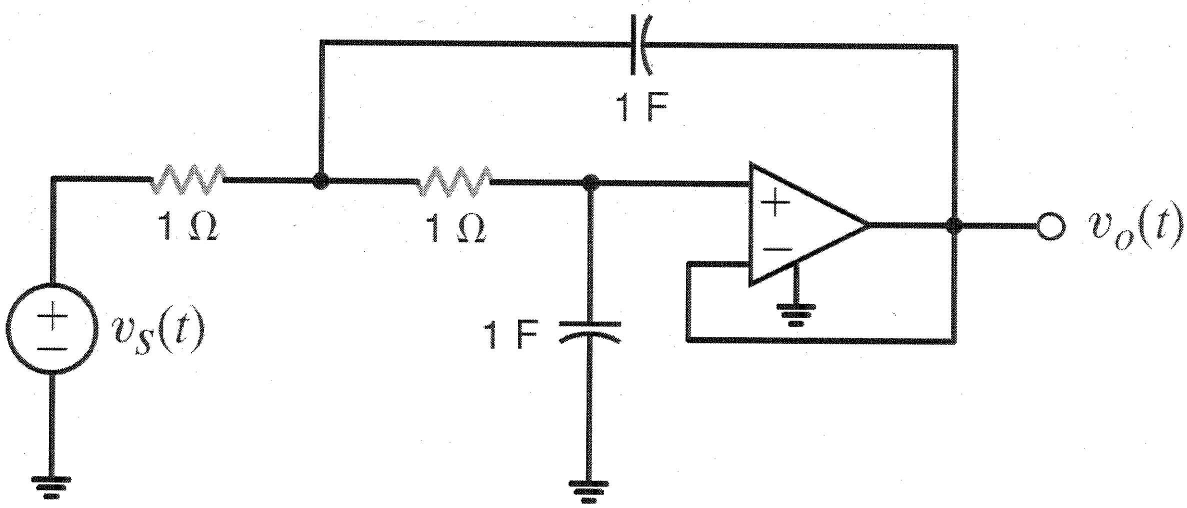
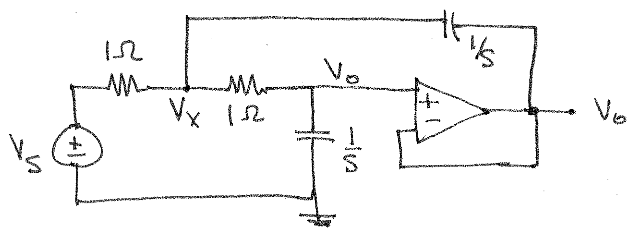


Figure P14.49

SOLUTION: Op amp is in unity gain configuration.



$$\frac{V_X - V_O}{1} = \frac{V_O}{1/s} \Rightarrow V_X = V_O(s+1)$$

$$\frac{V_S - V_X}{1} = \frac{V_X - V_O}{1} + \frac{V_X - V_O}{1/s}$$

yields $V_S = V_O(s+1)^2$

$$\frac{V_O}{V_S} = \frac{1}{(s+1)^2}$$

poles are real & identical, so,

CRITICALLY DAMPED!

14.50 Find the transfer function for the network in Fig. P14.50.

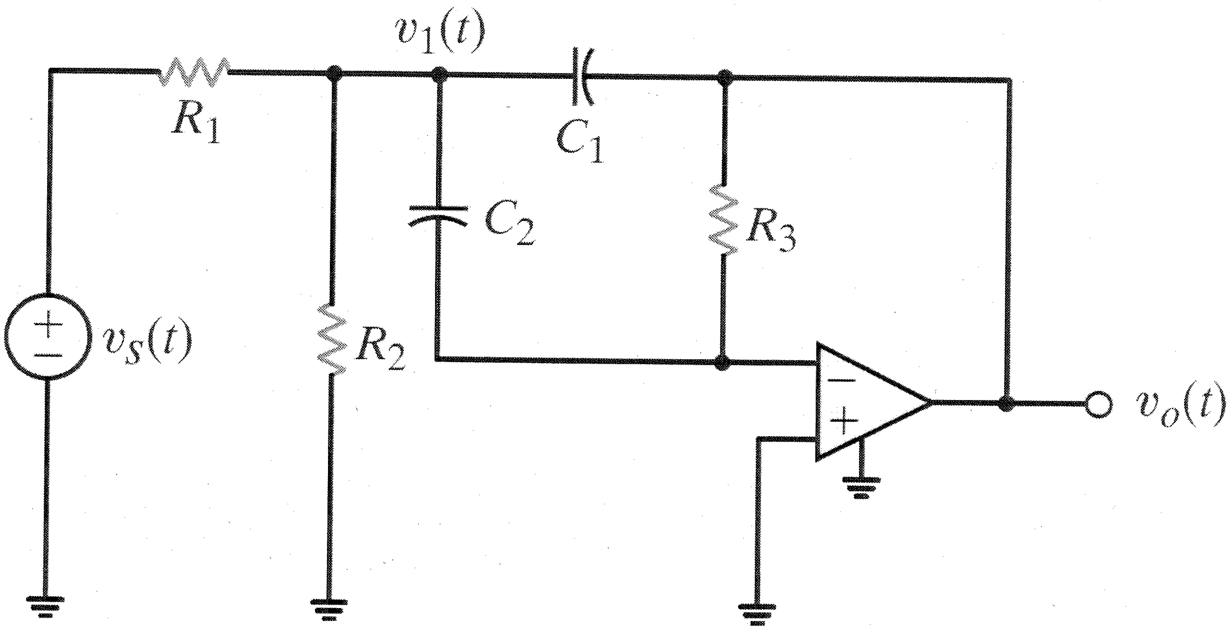
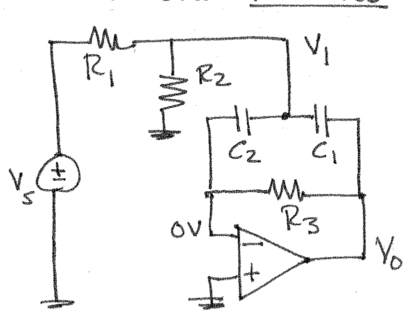


Figure P14.50

SOLUTION: Redraw



Nodal analysis

$$V_1 sC_2 + V_o/R_3 = 0 \Rightarrow V_1 = -V_o/sC_2R_3$$

$$\frac{V_s - V_1}{R_1} = \frac{V_1}{R_2} + V_1 sC_2 + (V_1 - V_o)sC_1$$

$$\frac{V_s}{R_1} = -V_o\left[sC_1 + \frac{1}{sC_2R_3}\left(\frac{1}{R_1} + \frac{1}{R_2} + s(C_2+C_1)\right)\right]$$

$$\frac{V_o}{V_s} = \frac{-1/R_1}{sC_1 + \frac{C_1+C_2}{C_2R_3} + \frac{R_1+R_2}{sR_1R_2R_3C_2}}$$

$$\boxed{\frac{V_o}{V_s} = \frac{-(1/C_1R_1)s}{s^2 + s\left(\frac{C_1+C_2}{C_1C_2R_3}\right) + \frac{R_1+R_2}{C_1C_2R_1R_2R_3}}}$$

14.51 Determine the transfer function for the network shown in Fig. P14.51. If a step function is applied to the network, what type of damping will the network exhibit?

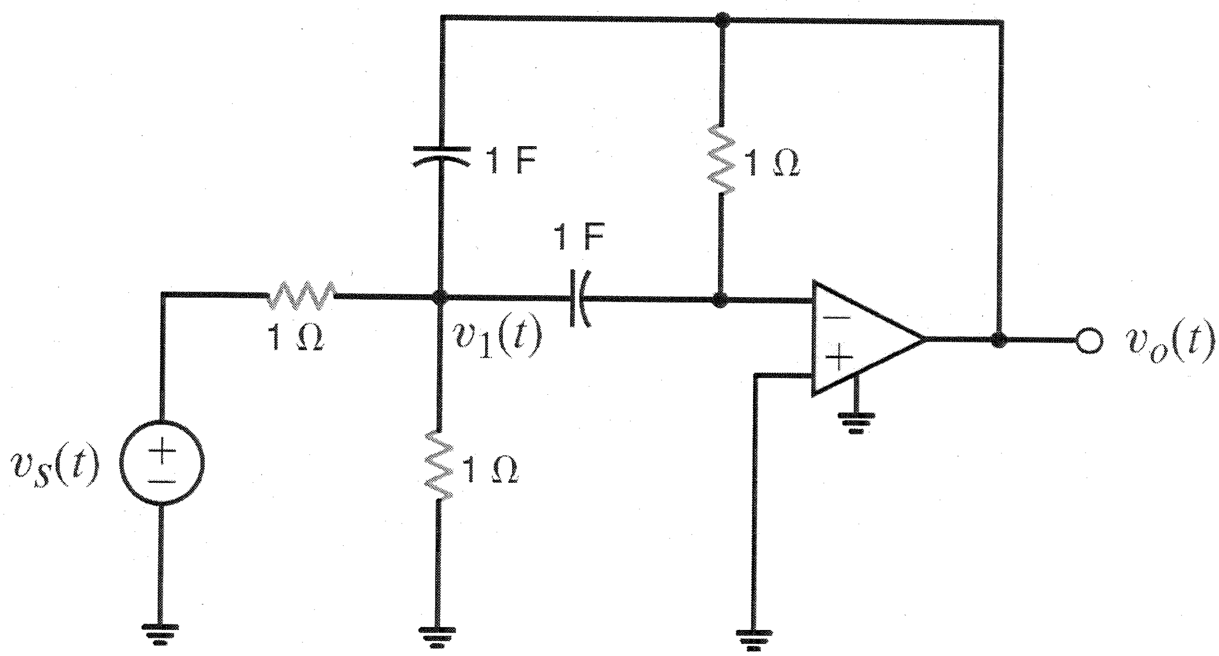
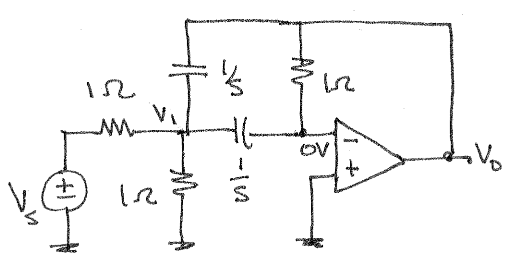


Figure P14.51

SOLUTION:



$$\frac{V_s - V_1}{1} = \frac{V_1}{1} + V_1 s + (V_1 - V_o) s$$

$$\text{\&} \quad \frac{V_o}{1} + V_1 s = 0 \Rightarrow \quad V_1 = -V_o/s$$

$$V_s = V_1(2s+2) - sV_o = -V_o\left(\frac{s^2+2s+2}{s}\right)$$

$$\boxed{\frac{V_o}{V_s} = \frac{-s}{s^2+2s+2}}$$

Roots at $s = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j1$ { complex conjugate poles. Network is UNDERDAMPED! }

14.52 The voltage response of the network to a unit step input is

$$\mathbf{V}_o(s) = \frac{2(s+1)}{s(s^2+10s+25)}$$

Is the response overdamped?

SOLUTION:

3 poles at $s = \begin{cases} 0 \\ -\frac{10}{2} \pm \frac{\sqrt{100-100}}{2} = -5 \end{cases}$ ⇐ [These poles are real & equal]

$\boxed{\text{System is critically damped, not overdamped}}$

14.53 The transfer function of the network is given by the expression

$$\mathbf{G}(s) = \frac{100s}{s^2 + 13s + 40}$$

Determine the damping ratio, the undamped natural frequency, and the type of response that will be exhibited by the network.

SOLUTION:

Char. eq. is $s^2 + 13s + 40 = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$

$$\boxed{\omega_0 = \sqrt{40} \text{ r/s}}$$

$2\zeta\omega_0 = 13 \Rightarrow \zeta = \frac{13}{2\sqrt{40}} \Rightarrow \boxed{\zeta = 1.03}$

$\boxed{\zeta > 1 \text{ (barely), so system is overdamped}}$

14.54 The transfer function of the network is given by the expression

$$\mathbf{G}(s) = \frac{100s}{s^2 + 22s + 40}$$

Determine the damping ratio, the undamped natural frequency, and the type of response that will be exhibited by the network. **CS**

SOLUTION:

Char. eq. is: $s^2 + 22s + 40 = s^2 + 2\zeta\omega_0 s + \omega_0^2$

$\boxed{\omega_0 = \sqrt{40}\ r/s}$

$2\zeta\omega_0 = 22 \Rightarrow \boxed{\zeta = 1.74}$ $\boxed{\text{overdamped}}$

14.55 The voltage response of a network to a unit step input is

$$\mathbf{V}_o(s) = \frac{10}{s(s^2 + 8s + 18)}$$

Is the response critically damped?

SOLUTION:

$V_I(s) = \frac{1}{s}$ $\qquad H(s) = \frac{V_o(s)}{V_I(s)} = \frac{10}{s^2+8s+18}$

Char. eq. is: $s^2 + 8s + 18 = s^2 + 2\zeta\omega_o s + \omega_o^2 = 0$

$\omega_o = \sqrt{18}$ r/s

$\zeta = \frac{8}{2\sqrt{18}} = 0.94$ $\qquad$ Under damped! Not critically damped!

14.56 For the network in Fig. P14.56, choose the value of C for critical damping.

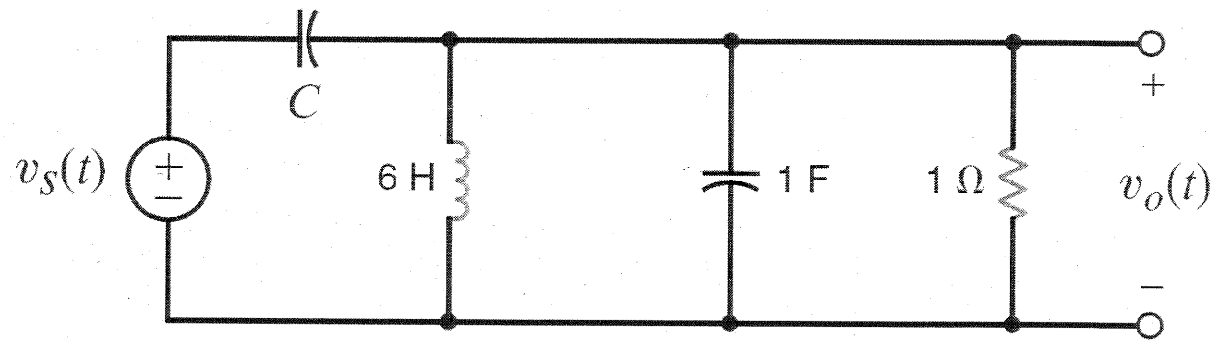
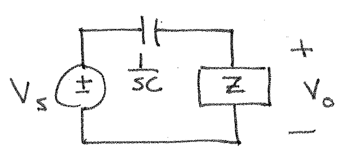


Figure P14.56

SOLUTION:



$$H(s) = \frac{V_o}{V_s} = \frac{Z}{Z + \frac{1}{sC}} \qquad \frac{1}{Z} = \frac{1}{6s} + s + 1 = \frac{6s^2 + 6s + 1}{6s}$$

$$H(s) = \frac{C6s^2}{6Cs^2 + 6s^2 + 6s + 1} = \frac{6Cs^2}{6(C+1)s^2 + 6s + 1}$$

$$H(s) = \frac{\left(\frac{C}{C+1}\right)s}{s^2 + \frac{s}{C+1} + \frac{1}{6(C+1)}}$$

$$\omega_o = \frac{1}{\sqrt{6(C+1)}} \qquad \zeta = 1$$

$$2\zeta\omega_o = \frac{1}{C+1} = \frac{2}{\sqrt{6(C+1)}}$$

$$\sqrt{C+1} = \sqrt{6}/2 \Rightarrow \boxed{C = 0.5F}$$

14.57 For the filter in Fig. P14.57, choose the values of C_1 and C_2 to place poles at $s = -2$ and $s = -5$ rad/s.

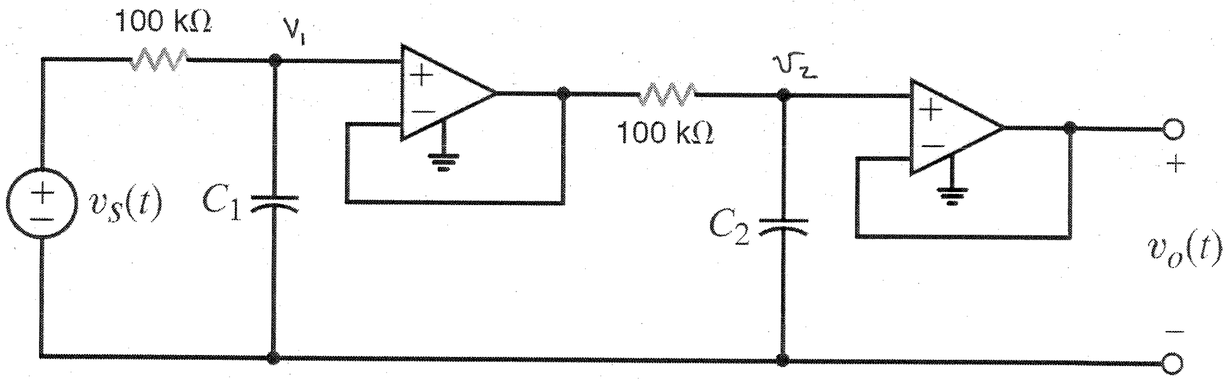
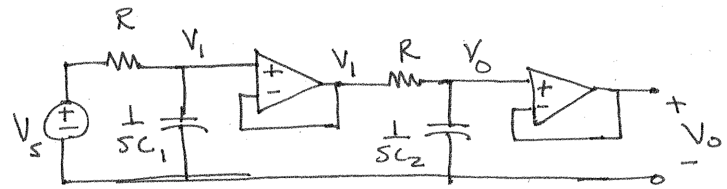


Figure P14.57

SOLUTION:



$R = 100\text{k}\Omega$

Both op-amps in unity gain configuration!

$$\frac{V_1}{V_S} = \frac{1/sC_1}{R + 1/sC_1} = \frac{1}{sC_1R + 1} = \frac{1/RC_1}{s + 1/RC_1}$$

$$\frac{V_o}{V_1} = \frac{1/RC_2}{s + 1/RC_2} \qquad \frac{V_o}{V_S} = \frac{\frac{1}{R^2C_1C_2}}{\left(s + \frac{1}{RC_1}\right)\left(s + \frac{1}{RC_2}\right)}$$

$$\frac{1}{RC_1} = 2 \quad \& \quad \frac{1}{RC_2} = 5 \quad \Rightarrow \quad \boxed{\begin{array}{l} C_1 = 5\mu F \\ C_2 = 2\mu F \end{array}}$$

14.58 Find the steady-state response $v_o(t)$ for the network in Fig. P14.58.

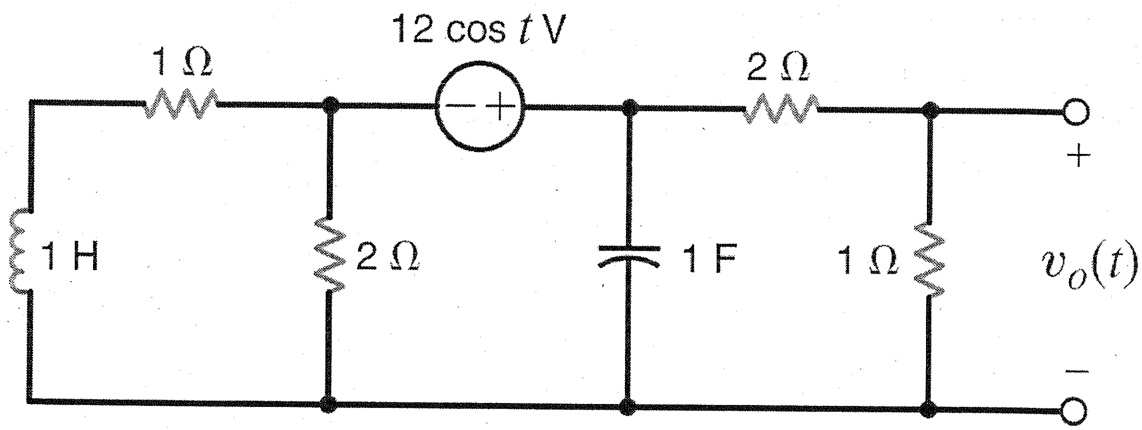
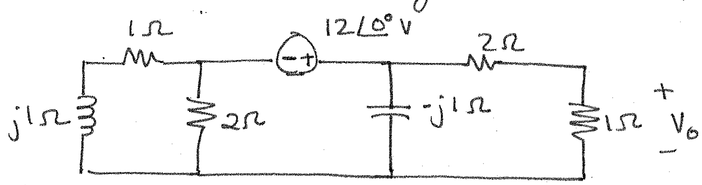


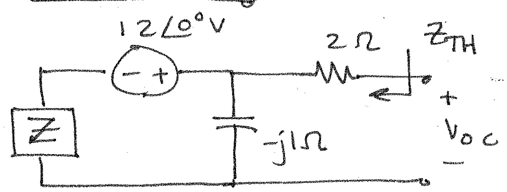
Figure P14.58

SOLUTION: In steady state $s \to j\omega$ and $\cos\omega t \Rightarrow$ phasor.



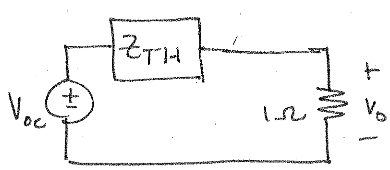
$Z = 2(1+j1)/(3+j1)$

Thevenin eq



$$Z_{TH} = 2 + \frac{Z(-j1)}{Z-j1} = \frac{8-j4}{-j1+3}\ \Omega$$

$$V_{oc} = 12\angle 0\left(\frac{-j1}{-j1+Z}\right) = \frac{12(1-j3)}{3-j1}$$



$$V_o = \frac{V_{oc}}{1+Z_{TH}} = \frac{12(1-j3)}{17-j7} = \frac{12(1-j3)}{11-j5}$$

$$V_o = 3.13\angle -47.2^\circ \text{V}$$

$$\boxed{v_o(t) = 3.13\cos(t-47.2^\circ)\text{V}}$$

14.59 Find the steady-state response $v_o(t)$ for the circuit shown in Fig. P14.59. **PSV**

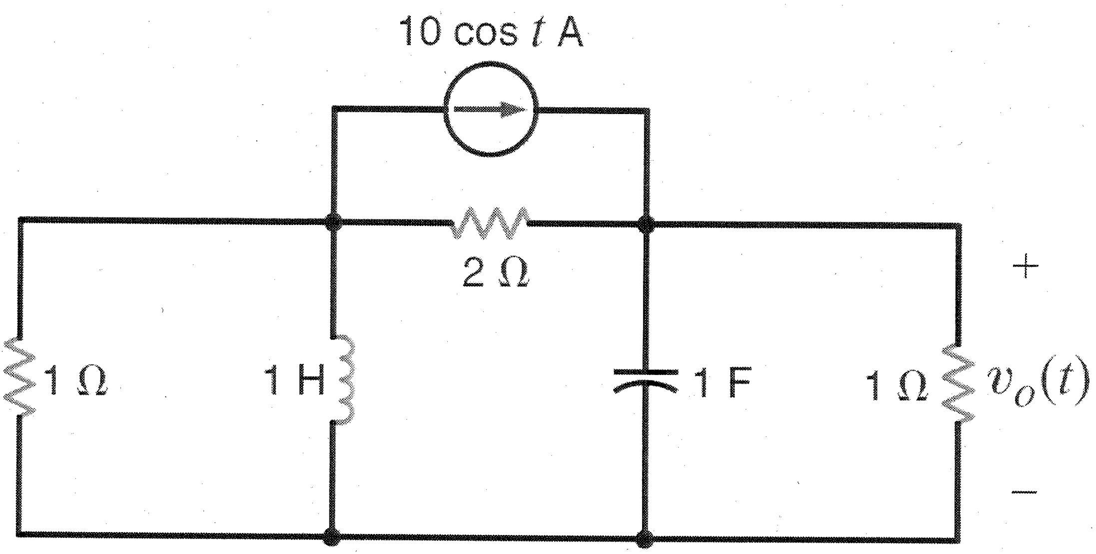
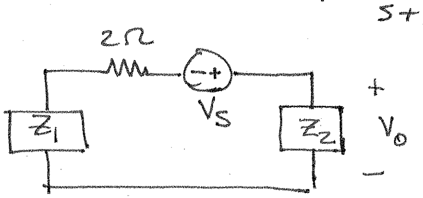


Figure P14.59

SOLUTION: Let $Z_1 = \frac{s}{s+1}$ & $Z_2 = \frac{1}{s+1}$



Eventually V_s is a phasor $\Rightarrow 20\angle 0°$ V

$$V_o = V_s\left(\frac{Z_2}{Z_1+Z_2+2}\right) = V_s\left(\frac{1}{s+1+2s+2}\right) = V_s\left(\frac{1/3}{s+1}\right)$$

In steady state, $s \to j\omega$

$$V_o = 20\angle 0°\left[\frac{1/3}{1+j\omega}\right] \qquad V_o = 4.17\angle -45° \text{ V}$$

$$\boxed{v_o(t) = 4.71\cos(t-45°)\text{ V}}$$

14.60 Determine the steady-state response $i_o(t)$ for the network in Fig. P14.60.

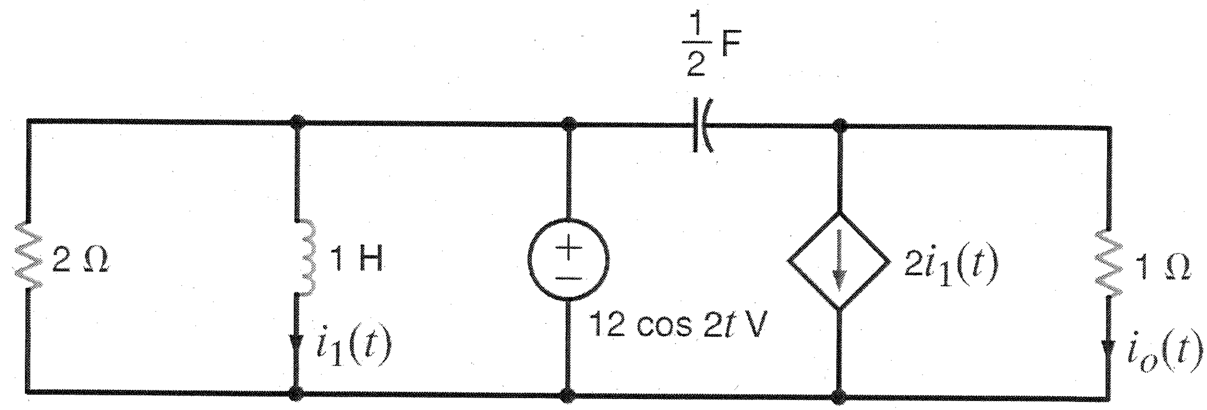
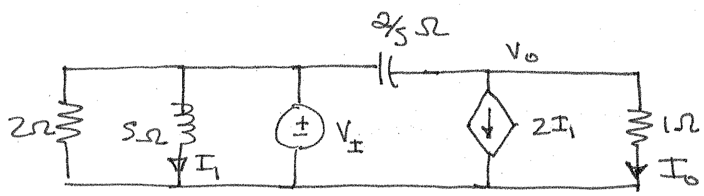


Figure P14.60

SOLUTION: Use KCL,



$$I_1 = \frac{V_I}{s} \qquad I_o = \frac{V_o}{1}$$

$$\frac{V_o - V_I}{2/s} + 2I_1 + \frac{V_o}{1} = 0 \Rightarrow V_o\left(\frac{s}{2}+1\right) = V_I\left(\frac{s}{2} - \frac{2}{s}\right)$$

$$\frac{V_o}{V_I} = \frac{s-2}{s} \Rightarrow I_o = \frac{V_I\,(s-2)}{s}$$

At steady-state, $V_I = 12\angle 0°\ V$ & $s = j2$

$$I_o = 12\sqrt{2}\ \angle 45°\ A$$

$$\boxed{i_o(t) = 12\sqrt{2}\cos(2t + 45°)\ A}$$

14.61 Find the steady-state response $i_o(t)$ for the network shown in Fig. P14.61. CS

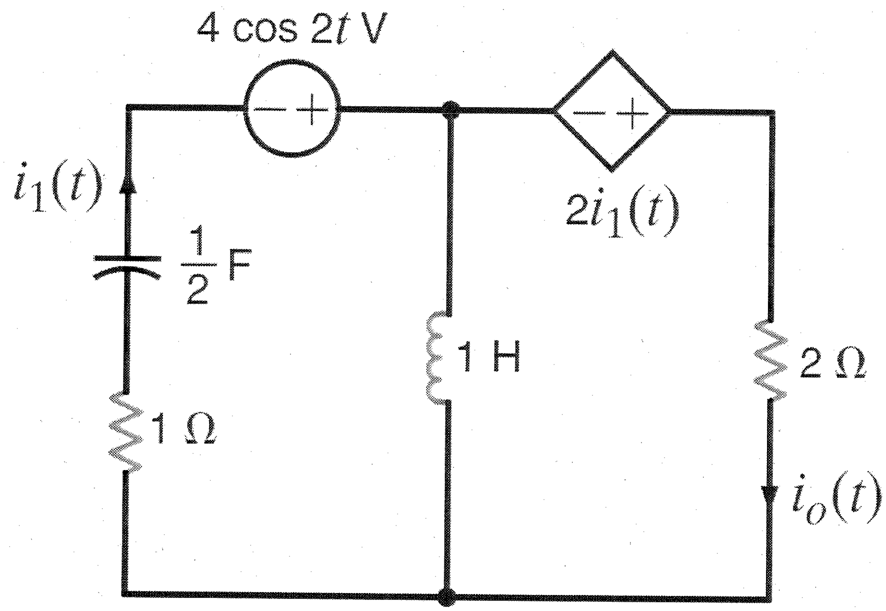
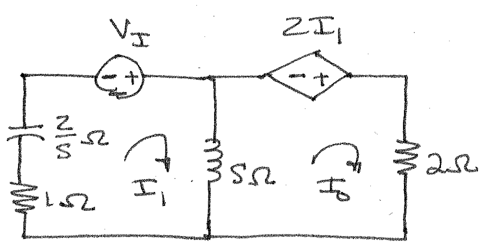


Figure P14.61

SOLUTION:



$$V_I = I_1\left(s+1+\tfrac{2}{s}\right) - sI_o$$

$$= I_1\left(\frac{s^2+s+2}{s}\right) - sI_o$$

and, $2I_1 = -sI_1 + (s+2)I_o$

or, $0 = -I_1(s+2) + I_o(s+2)$

⇐ yields $I_1 = I_o$

$$V_I = I_o\left[\frac{s^2+s+2-s^2}{s}\right]$$

$$I_o = \frac{V_I(s)}{s+2}$$

In steady state, $V_I = 4\angle 0°\,V$ $\quad s \to j2$

$$I_o = \frac{4\angle 0°\,(j2)}{2+j2} = 2\sqrt{2}\angle 45°\,A$$

$$\boxed{i_o(t) = 2\sqrt{2}\cos(2t+45°)\,A}$$

14.62 Find the steady-state response $v_o(t)$, for $t > 0$, in the network in Fig. P14.62.

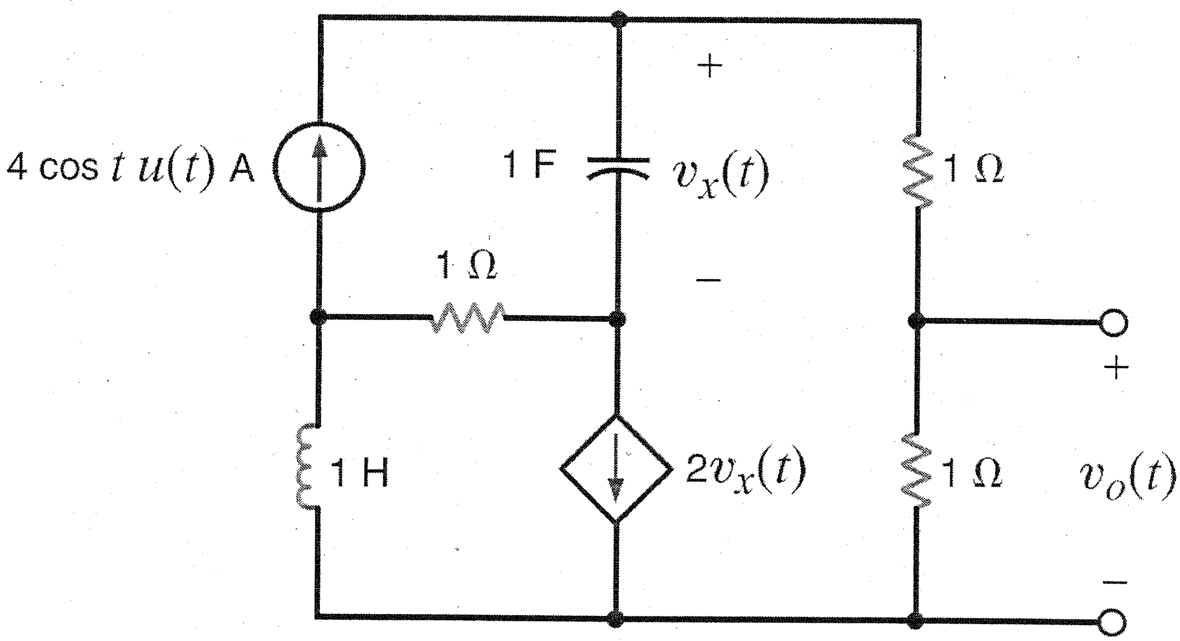
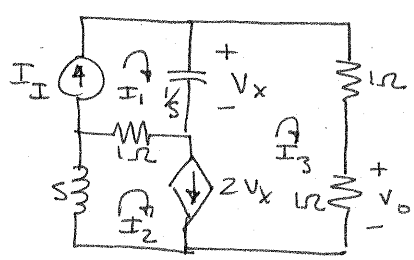


Figure P14.62

SOLUTION:



$I_1 = I_I \qquad I_2 - I_3 = 2V_x = 2\,(I_1 - I_3)/s$

yields $\quad I_2 = \dfrac{2I_1}{s} + I_3\left(1 - \dfrac{2}{s}\right)$

and

$I_2(s+1) + I_3(2 + 1/s) - I_1\,(1 + 1/s) = 0$

$$I_1\left[\frac{2}{s}(s+1) - \frac{s+1}{s}\right] + I_3\left[\frac{s-2}{s}(s+1) + \frac{2s+1}{s}\right] = 0 \qquad V_o = (1)\,I_3$$

$$I_I\left[2s+2-s-1\right] + V_o\left[s^2 - s - 2 + 2s + 1\right] = 0$$

$$V_o = -I_I\,(s+1)/(s^2+s-1)$$

In steady state, $I_I = 4\angle 0°\,A$ & $s = j1 \Rightarrow V_o = 2.53\angle 71.6°$ V

$$\boxed{v_o(t) = 2.53\cos(t + 71.6°)\ V}$$

14.63 Find the steady-state response $v_o(t)$, for $t > 0$, in the network in Fig. P14.63.

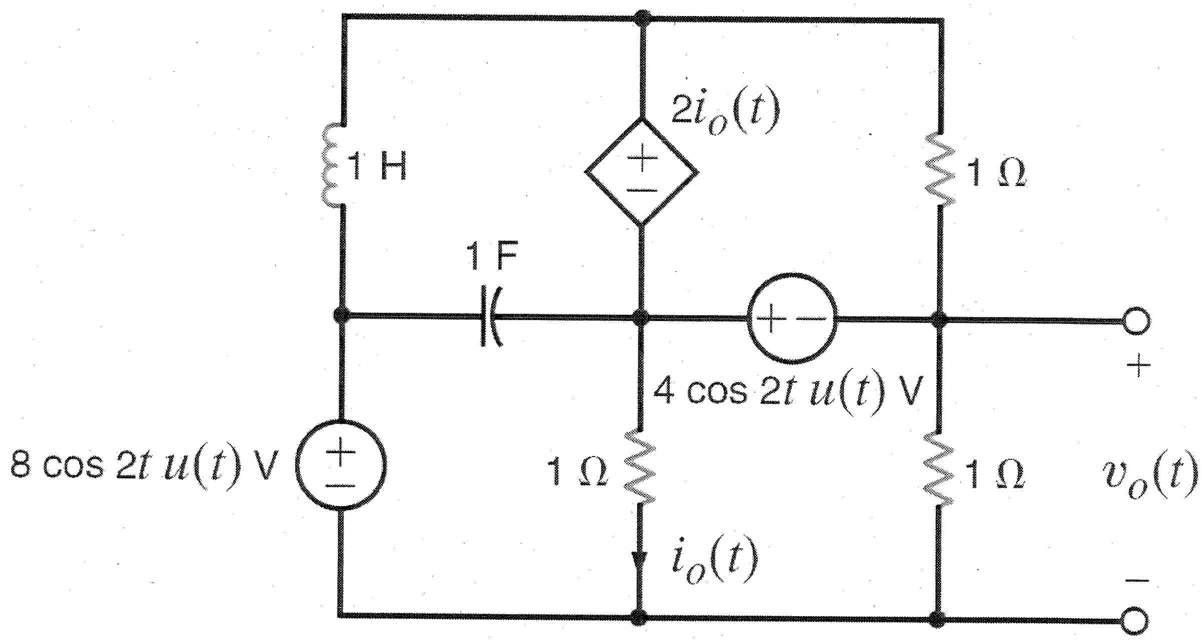
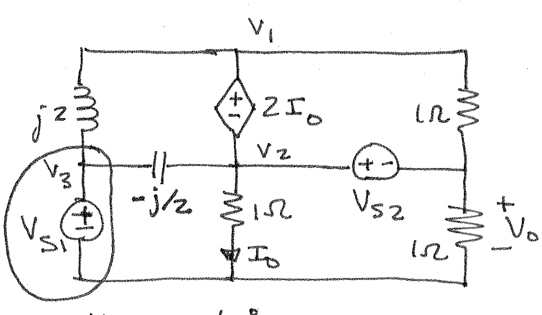


Figure P14.63

SOLUTION: Go straight to freq domain. $s \to j2$ & sources → phasors.



$V_{S1} = 8\angle 0° V$ $V_{S2} = 4\angle 0° V$

$V_3 = 8\angle 0° V$ $V_2 - V_o = 4\angle 0° V$

$V_1 - V_2 = 2I_o = 2(V_2/1) \Rightarrow V_1 = 3V_2$

At super node:

$$\frac{V_1 - V_3}{j2} + \frac{V_2 - V_3}{-j/2} + \frac{V_2}{1} + \frac{V_o}{1} = 0$$

yields $V_o = 5.22 \angle 97.8° V$

$$\boxed{v_o(t) = 5.22 \cos(2t + 97.8°) V}$$

14FE-1 A single loop, second-order circuit is described by the following differential equation.

$$2\frac{dv^2(t)}{dt^2} + 4\frac{dv(t)}{dt} + 4v(t) = 12u(t) \qquad t > 0$$

Which is the correct form of the total (natural plus forced) response? **CS**

(a) $v(t) = K_1 + K_2e^{-t}$

(b) $v(t) = K_1 \cos t + K_2 \sin t$

(c) $v(t) = K_1 + K_2te^{-t}$

(d) $v(t) = K_1 + K_2e^{-t}\cos t + K_3e^{-t}\sin t$

SOLUTION:

Natural response has char eq: $s^2+2s+2=0$

roots are at $s=-1\pm j1 \Rightarrow$ natural response is sinusodial!

Forced response is constant $= K_1$

$\boxed{\text{Answer is (d)}}$

14FE-2 If all initial conditions are zero in the network in Fig. 14PFE-2, find the transfer function $\mathbf{V}_o(s)/\mathbf{V}_S(s)$, and determine the type of damping exhibited by the network.

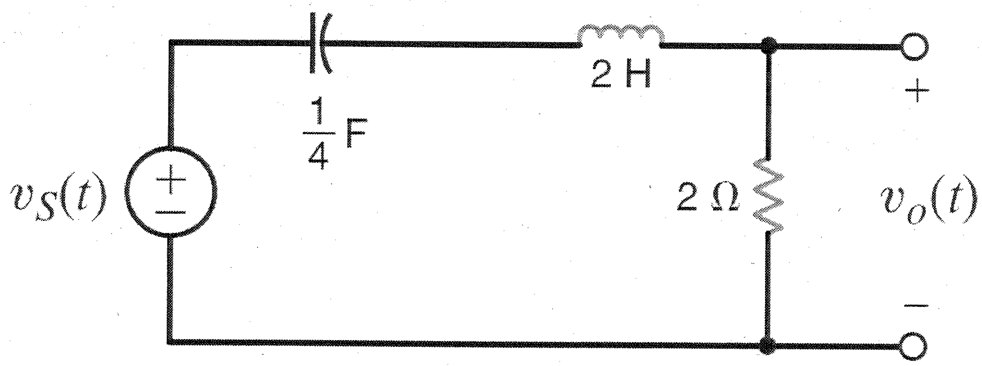


Figure 14PFE-2

SOLUTION:

$$\frac{V_o(s)}{V_s(s)} = \frac{2}{2+2s+4/s} = \frac{2s}{2s^2+2s+4} = \frac{s}{s^2+s+2}$$

$$\boxed{\frac{V_o}{V_s} = \frac{s}{s^2+s+2}}$$

Poles at $s = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$

since poles are complex, the circuit is underdamped

14FE-3 The initial conditions in the circuit in Fig. 14PFE-3 are zero. Find the transfer function $\mathbf{I}_o(s)/\mathbf{I}_s(s)$, and determine the type of damping exhibited by the circuit.

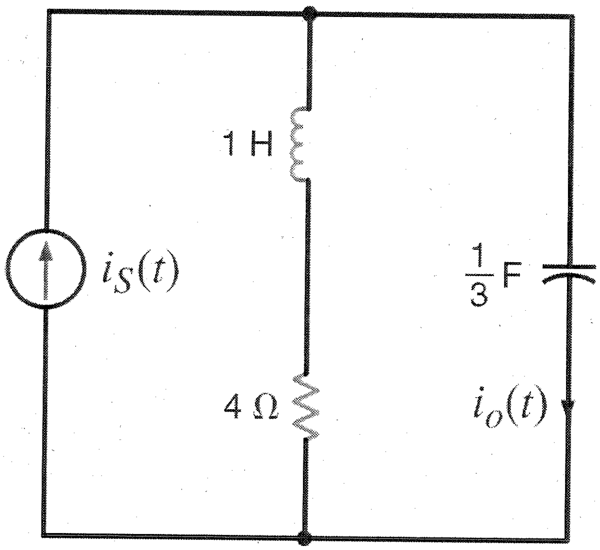
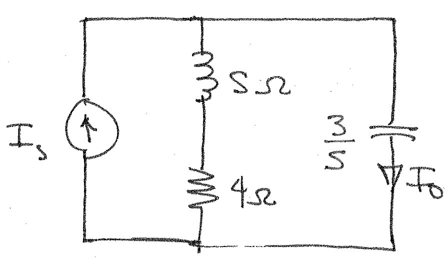


Figure 14PFE-3

SOLUTION:



$$\frac{I_o}{I_s} = \frac{3/s}{3/s + 4 + s} = \frac{3}{s^2+4s+3}$$

$$\boxed{\frac{I_o}{I_s} = \frac{3}{s^2+4s+3}}$$

Char equation: s^2+4s+3

Poles at $s = -2 \pm 1 = \begin{cases} -1 \\ -3 \end{cases}$

$\boxed{\text{Poles are real and unequal, network is over damped}}$