Chapter Fifteen:

Fourier Analysis Techniques

15.1 Find the exponential Fourier series for the periodic signal shown in Fig. P15.1.

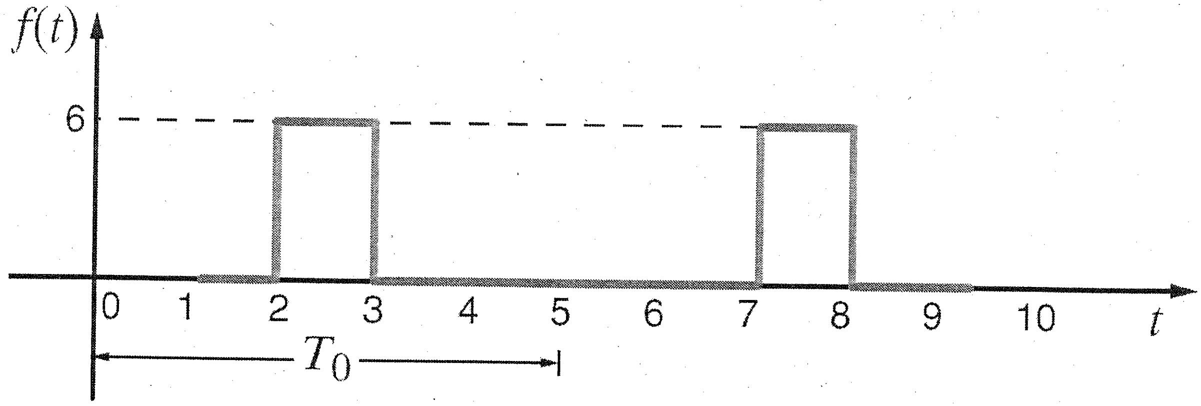


Figure P15.1

SOLUTION: $T_o = 5$ sec. $\quad \omega_o T_o = 2\pi$

$$C_n = \frac{1}{T_o}\int_0^{T_o/5} 6e^{-jn\omega_o t}\,dt = \frac{6}{jn2\pi}\left[e^{jn(\pi/5)} - e^{-jn(\pi/5)}\right]$$

$$C_n = \frac{6}{n\pi}e^{-jn(\pi/5)}\sin(n\pi/5) \quad \& \quad f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_o t}$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \frac{6}{n\pi} e^{-jn\pi} \sin(n\pi/5)\, e^{j0.4\pi t}}$$

15.2 Find the exponential Fourier series for the periodic pulse train shown in Fig. P15.2. **PSV**

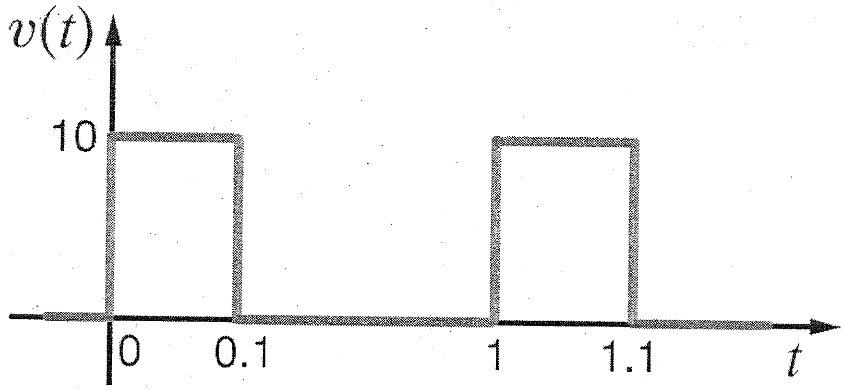


Figure P15.2

SOLUTION:

$$C_0 = \frac{10(0.1)}{1} = 1 \qquad T_0 = 1 \qquad \omega_0 = 2\pi$$

$$C_n = \frac{1}{T_0}\int_0^{T_0/10} 10\, e^{-jn\omega_0 t}\, dt = \frac{10 e^{-jn\pi/10}}{n\pi}\left[\frac{e^{jn\pi/10} - e^{-jn\pi/10}}{j2}\right]$$

$$C_n = \frac{10}{n\pi}\, e^{-jn\pi/10} \sin(n\pi/10)$$

$$\boxed{f(t) = \frac{10}{\pi}\sum_{n=-\infty}^{\infty} \frac{1}{n}\, e^{-jn\pi/10} \sin(n\pi/10)\, e^{jn\omega_0 t}}$$

15.3 Find the exponential Fourier series for the signal shown in Fig. P15.3. CS

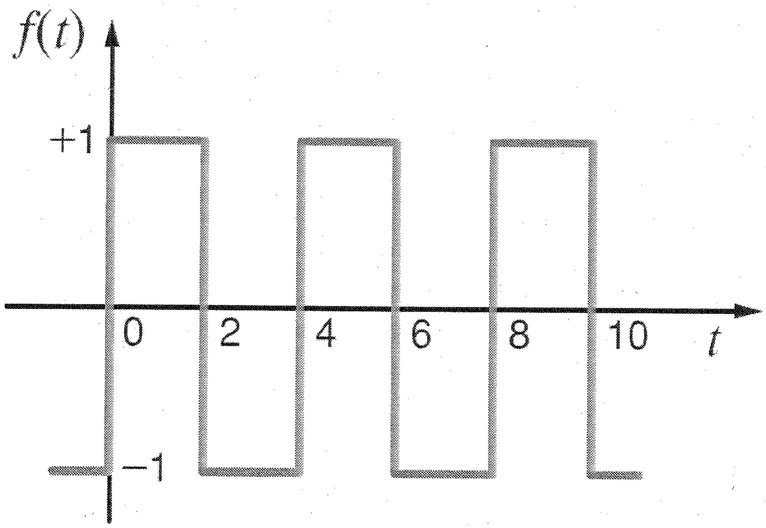


Figure P15.3

SOLUTION:

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{0} -e^{-jn\omega_0 t}\,dt + \frac{1}{T_0}\int_{0}^{T_0/2} e^{-jn\omega_0 t}\,dt \qquad \omega_0 = 2\pi/T_0$$

$$C_n = \frac{1}{T_0 jn\omega_0}\left[1 - e^{-jn\omega_0 T_0/2} + 1 - e^{jn\omega_0 T_0/2}\right] = \frac{1}{2jn\pi}\left[2 - e^{jn\pi} - e^{-jn\pi}\right]$$

$$C_n = \frac{1}{2jn\pi}\left[2 - 2\cos(n\pi)\right] = \frac{1 - \cos(n\pi)}{jn\pi} = \begin{cases} 0 & n \text{ even} \\ \frac{2}{jn\pi} & n \text{ odd} \end{cases}$$

$$\boxed{f(t) = \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{2}{jn\pi} e^{jn\omega_0 t}}$$

15.4 Find the exponential Fourier series for the signal shown in Fig. P15.4. CS

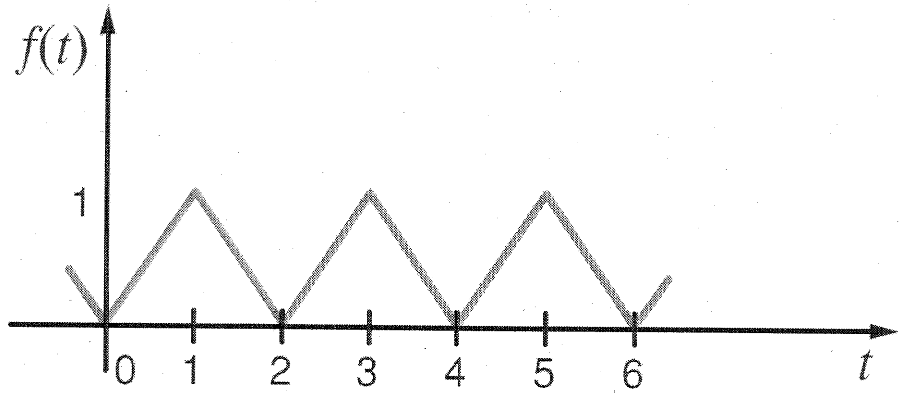


Figure P15.4

SOLUTION: $T_0 = 2\text{ sec}$ $\quad \omega_0 T_0 = 2\pi \Rightarrow \omega_0 = \pi$ $\quad$ let $a = jn\pi$

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\,dt = \frac{1}{2}\int_{-1}^{0} -te^{-jn\omega_0 t}dt + \frac{1}{2}\int_0^1 te^{-jn\omega_0 t}dt$$

$$C_n = \frac{1}{2}\left\{\int_{-1}^{0} -te^{-at}dt + \int_0^1 te^{-at}dt\right\} = \frac{1}{2}\left\{\left(\frac{1}{a^2}+\frac{t}{a}\right)e^{-at}\Big|_{-1}^{0} + \left(\frac{1}{a^2}+\frac{t}{a}\right)e^{-at}\Big|_{1}^{0}\right\}$$

$$C_n = \frac{1}{2}\left\{\frac{1}{a^2} - \left(\frac{1}{a^2}-\frac{1}{a}\right)e^{a} + \left(\frac{1}{a^2}\right) - \left(\frac{1}{a^2}+\frac{1}{a}\right)e^{-a}\right\}$$

$$C_n = \frac{1}{2a^2}\left\{2 + e^{a}(a-1) - e^{-a}(a+1)\right\} = -\frac{1}{2n^2\pi^2}\left\{2 + e^{jn\pi}(jn\pi-1) - e^{-jn\pi}(jn\pi+1)\right\}$$

$$C_n = \frac{-2}{n^2\pi^2} \text{ if } n \text{ is odd} \qquad C_n = 0 \text{ if } n \text{ is even}$$

$$C_0 = 1/2$$

$$\boxed{f(t) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n\neq 0 \\ n\text{ odd}}}^{\infty} \frac{-2}{n^2\pi^2}e^{jn\pi t}}$$

15.5 Compute the exponential Fourier series for the waveform that is the sum of the two waveforms in Fig. P15.5 by computing the exponential Fourier series of the two waveforms and adding them.

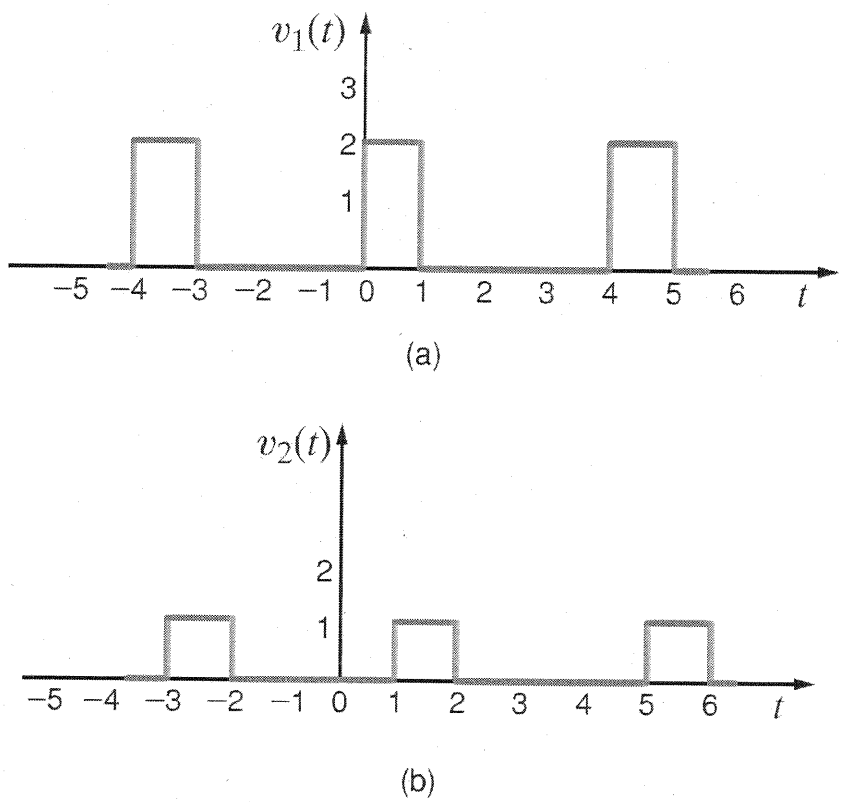


Figure P15.5

SOLUTION: $T_0 = 4$ $\quad \omega_0 T_0 = 2\pi \Rightarrow \omega_0 = \pi/2$

$$C_{n_1} = \frac{2}{T_0}\int_0^{T_0/4} e^{-jn\omega_0 t}\,dt = \frac{2}{jn2\pi}\left[1 - e^{-jn\pi/2}\right]$$

$C_{n2} = \frac{C_{n1}}{2} e^{-jn\omega_0}$ Since $v_2(t)$ is half $v_1(t)$ shifted by 1 sec.

$$C_n = C_{n1} + C_{n2} = \frac{1}{jn2\pi}\left\{2 - 2e^{-jn\pi/2} + e^{-jn\pi/2} - e^{-jn\pi}\right\}$$

$$C_n = \frac{1}{jn\pi}\left\{1 - e^{-jn3\pi/4}\left[\frac{e^{jn\pi/4} + e^{-jn\pi/4}}{2}\right]\right\} = \frac{1}{jn\pi}\left(1 - e^{-jn3\pi/4}\cos(n\pi/4)\right)$$

$$v(t) = \sum_{n=-\infty}^{\infty} \frac{1}{jn\pi}\left\{1 - e^{-jn3\pi/4}\cos(n\pi/4)\right\} e^{jn\pi t/2} \text{ V}$$

15.6 Given the waveform in Fig. P15.6, determine the type of symmetry that exists if the origin is selected at (a) l_1 and (b) l_2.

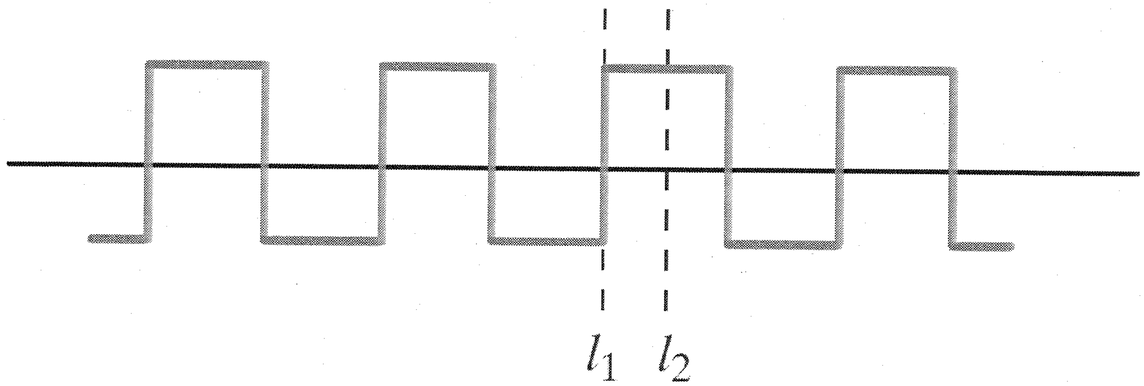


Figure P15.6

SOLUTION:

If origin is at l_1, $v(t) = -v(-t)$ odd symmetry

If origin is at l_2, $v(t) = v(-t)$ even symmetry

15.7 What type of symmetry is exhibited by the two waveforms in Fig. P15.7?

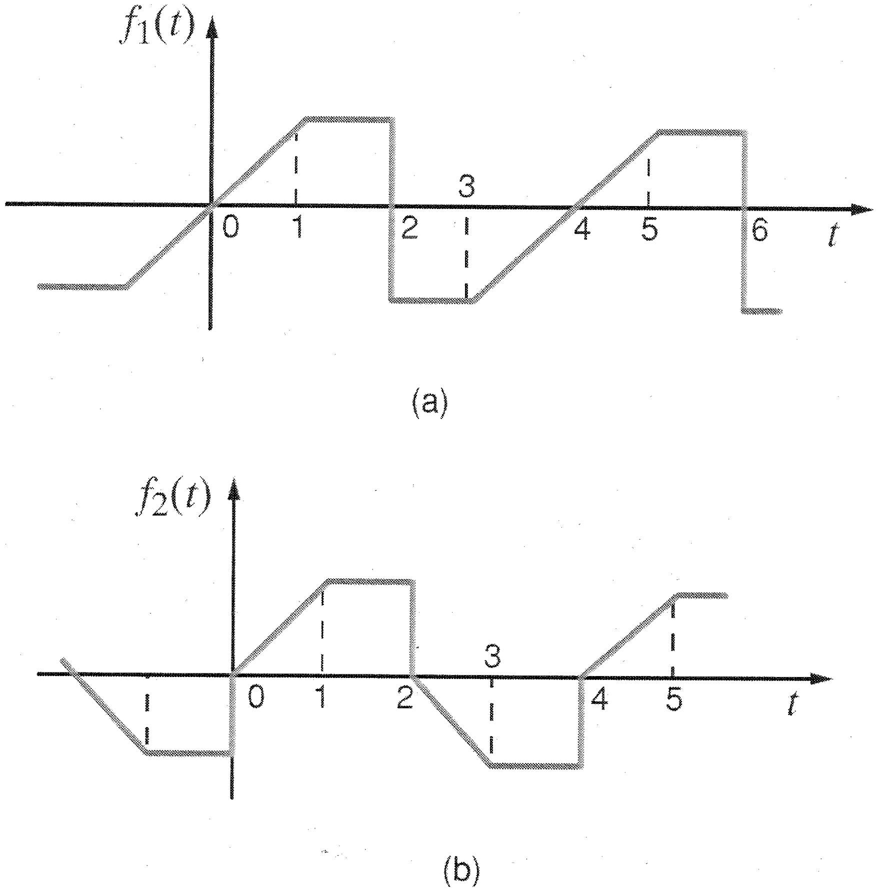


Figure P15.7

SOLUTION:

(a) $f_1(t) = -f_1(-t)$ odd symmetry

(b) $f_2(t) = -f_2(t - T_0/2)$ half wave symmetry

15.8 Find the trigonometric Fourier series for the waveform shown in Fig. P15.8.

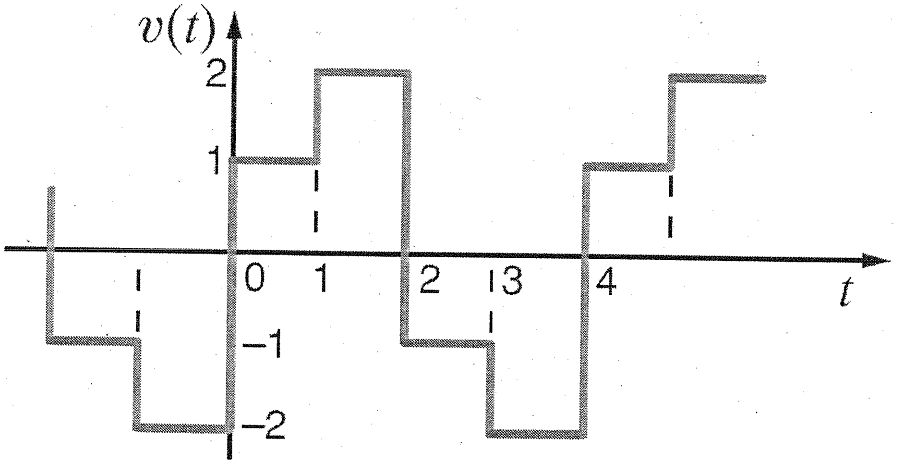


Figure P15.8

SOLUTION: $v(t)$ has half wave symmetry!

$a_0 = 0$ $\quad T_0 = 4$ sec $\quad \omega_0 = \pi/2 \quad a_n = b_n = 0$ for n even

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos(n\omega_0 t)\,dt = \int_0^1 \cos(n\omega_0 t\,dt) + 2\int_1^2 \cos(n\omega_0 t)\,dt$$

$$a_n = \frac{\sin(n\omega_0 t)}{n\omega_0}\Big|_0^1 + \frac{2\sin(n\omega_0 t)}{n\omega_0}\Big|_1^2 = \frac{\sin(n\pi/2) + 2\sin(n\pi) - 2\sin(n\pi/2)}{n\pi/2}$$

for n odd, $a_n = \frac{-2}{n\pi}\sin(n\pi/2)$

$$b_n = \int_0^1 \sin(n\omega_0 t)\,dt + 2\int_1^2 \sin(n\omega_0 t)\,dt = \frac{\cos(n\omega_0 t)}{n\omega_0}\Big|_1^0 + \frac{2\cos(n\omega_0 t)}{n\omega_0}\Big|_2^1$$

$$b_n = \frac{1 - \cos(n\pi/2) + 2\cos(n\pi/2) - 2\cos(n\pi)}{n\pi/2}$$

for n odd, $b_n = \frac{2}{n\pi}\left[1 \mp \cos(\pi/2) \pm 2\cos(\pi/2) + 2\cos(0)\right] = \frac{6}{n\pi}$

$$v(t) = \frac{2}{\pi}\sum_{\substack{n=1 \\ n\,\text{odd}}}^{\infty}\left[\frac{3}{n}\sin(n\pi t/2) - \frac{\sin(n\pi/2)}{n}\cos(n\pi/2)\right] \text{ V}$$

15.9 Find the trigonometric Fourier series for the periodic waveform shown in Fig. P15.9.

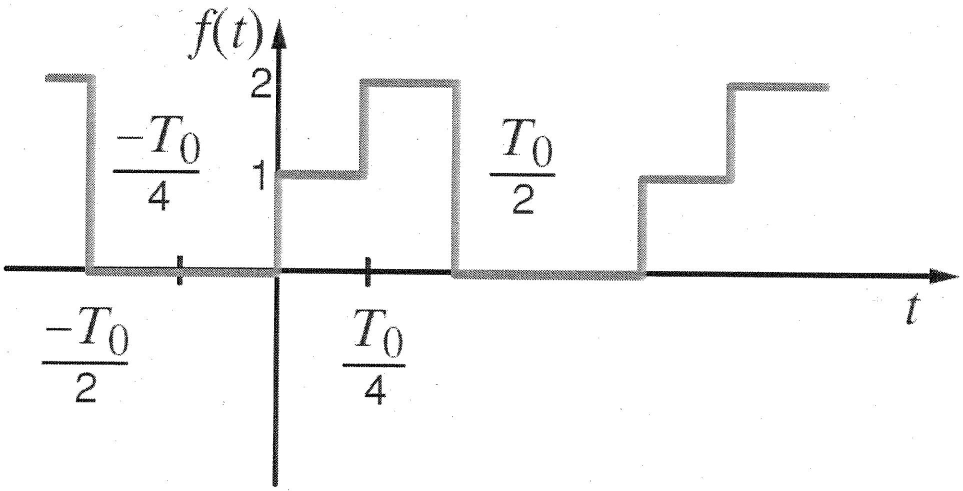


Figure P15.9

SOLUTION: $a_0 = \dfrac{1(T_0/4) + 2(T_0/4)}{T_0} = 3/4$

$$a_n = \frac{2}{T_0}\left\{\int_0^{T_0/4} \cos(n\omega_0 t)\,dt + 2\int_{T_0/4}^{T_0/2} \cos(n\omega_0 t)\,dt\right\}$$

$$a_n = \frac{2}{n\omega_0 T_0}\left\{ \sin(n\omega_0 t)\Big|_0^{T_0/4} + 2\sin(n\omega_0 t)\Big|_{T_0/4}^{T_0/2} \right\} = -\frac{\sin(n\pi/2)}{n\pi}$$

$$b_n = \frac{2}{T_0}\left\{\int_0^{T_0/4} \sin(n\omega_0 t)\,dt + 2\int_{T_0/4}^{T_0/2} \sin(n\omega_0 t)\,dt\right.$$

$$b_n = \frac{2}{n\omega_0 T_0}\left\{ \cos(n\omega_0 t)\Big|_{T_0/4}^{0} + 2\cos(n\omega_0 t)\Big|_{T_0/2}^{T_0/4} \right. = \frac{1}{n\pi}\Big[1 + \cos(n\pi/2) - 2\cos(n\pi)\Big]$$

$$f(t) = \frac{3}{4} + \sum_{n=1}^{\infty} \frac{1}{n\pi}\Big[\big(1 + \cos(n\pi/2) - 2\cos(n\pi)\big)\sin(n\omega_0 t) - \sin(n\pi/2)\cos(n\omega_0 t)\Big] \checkmark$$

15.10 Given the waveform in Fig. P15.10 show that

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-A}{n\pi} \sin \frac{2n\pi}{T_0} t$$

PSV

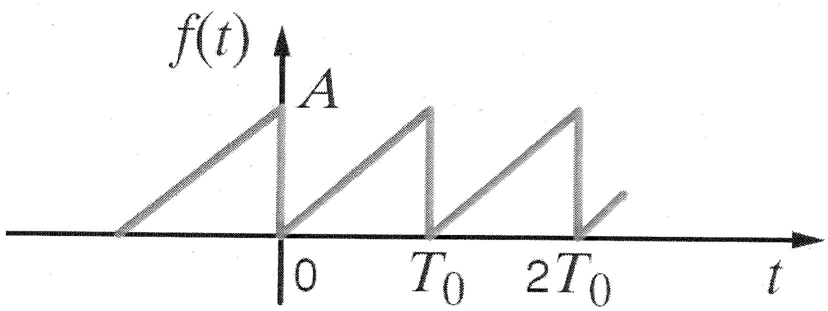


Figure P15.10

SOLUTION: $a_0 = A/2$ except for a_0, odd symmetry, so $a_n = 0$ $n \neq 0$

$$b_n = \frac{2A}{T_0} \int \frac{t}{T_0} \sin(n\omega_0 t)\,dt = \frac{2A}{T_0^2}\left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right]_0^{T_0}$$

$$b_n = \frac{2A}{T_0^2}\left[\frac{\sin(n2\pi)}{(n\omega_0)^2} - \frac{T_0\cos(n2\pi)}{n\omega_0}\right] = \frac{-2A}{T_0}\cdot\left(\frac{1}{n\omega_0}\right) = \frac{-2A}{n2\pi} = -\frac{A}{n\pi}$$

$$\boxed{f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-A}{n\pi} \sin\left(\frac{2n\pi}{T_0} t\right)}$$

15.11 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.11. **CS**

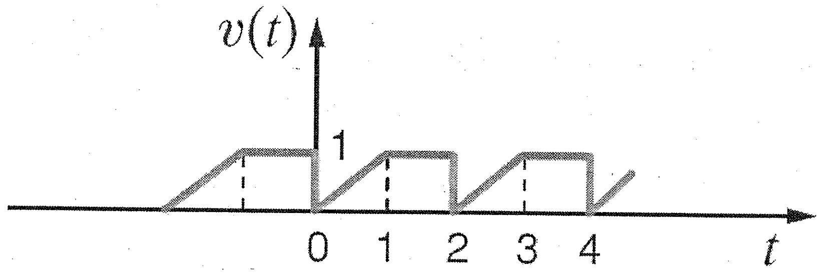


Figure P15.11

SOLUTION: $a_0 = \frac{(1/2)(1)(1) + (1)(1)}{2} = \frac{3}{4}$ $T_0 = 2\text{ sec}$ $\omega_0 = \pi$

$$a_n = \frac{2}{T_0}\int_0^1 t\cos(n\omega_0 t)\,dt + \frac{2}{T_0}\int_1^2 \cos(n\omega_0 t)\,dt = \int_0^1 t\cos(n\pi t)\,dt + \int_1^2 \cos(n\pi t)\,dt$$

$$a_n = \frac{\sin(n\pi)}{n\pi} + \frac{\cos(n\pi)}{(n\pi)^2} - \frac{1}{(n\pi)^2} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{\cos(n\pi) - 1}{(n\pi)^2}$$

$$b_n = \frac{2}{T_0}\left\{\int_0^1 t\sin(n\omega_0 t)\,dt + \int_1^2 \sin(n\omega_0 t)\,dt\right\} = \int_0^1 t\sin(n\pi t)\,dt + \int_1^2 \sin(n\pi)\,dt$$

$$b_n = \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} = -\frac{1}{n\pi}$$

$$\boxed{a_0 = 3/4 \qquad a_n = \frac{\cos(n\pi) - 1}{(n\pi)^2} \qquad b_n = -\frac{1}{n\pi}}$$

15.12 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.12.

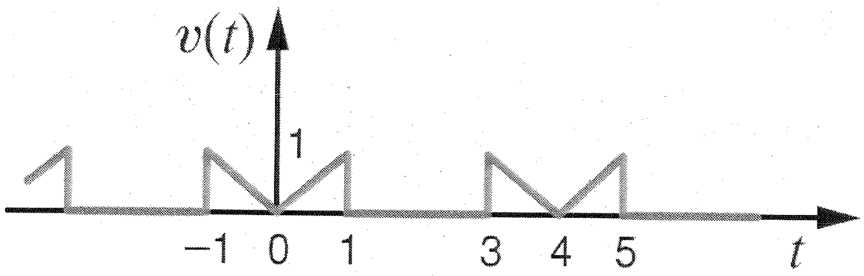


Figure P15.12

SOLUTION: $T_0 = 4$ $\quad \omega_0 = \pi/2$ $\quad a_0 = \frac{1}{4}\left[\frac{1}{2}(1)(1) + \frac{1}{2}(1)(1)\right] = 1/4$

Even symmetry, $b_n = 0$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} v(t)\cos(n\omega_0 t)\,dt = \int_0^1 t\cos(n\omega_0 t)\,dt = \left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t\sin(n\omega_0 t)}{n\omega_0}\right]_0^1$$

$$a_n = \frac{\cos(n\pi/2) - 1}{(n\pi/2)^2} + \frac{\sin(n\pi/2)}{n\pi/2} = \frac{4}{(n\pi)^2}\left(\cos(n\pi/2) - 1\right) + \frac{2}{n\pi}\sin(n\pi/2)$$

$$\boxed{a_0 = 1/4 \qquad b_n = 0 \qquad a_n = \frac{4}{(n\pi)^2}\left(\cos(n\pi/2) - 1\right) + \frac{2}{n\pi}\sin(n\pi/2)}$$

15.13 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.13.

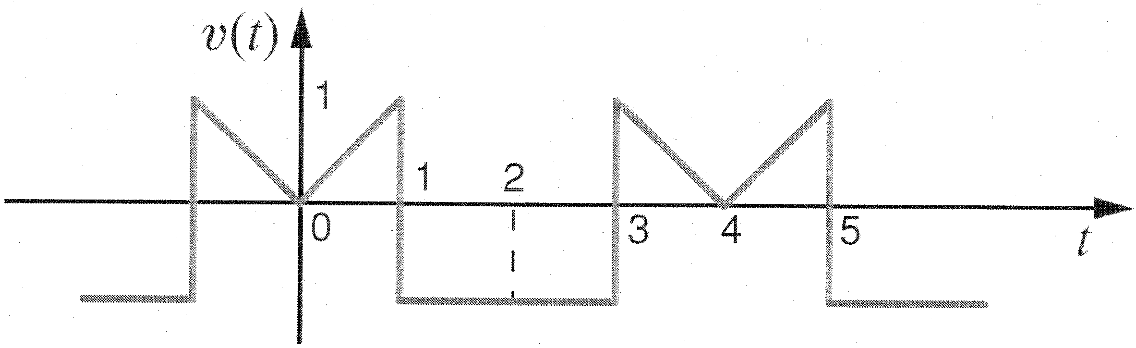


Figure P15.13

SOLUTION: $T_0 = 4 \quad \omega_0 = \pi/2 \quad a_0 = \dfrac{\frac{1}{2}(1)(1) - 2(1) + \frac{1}{2}(1)(1)}{4} = -\frac{1}{4}$

even symmetry, so $b_n = 0$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} v(t)\cos(n\omega_0 t)\,dt = \int_0^1 t\cos(n\omega_0 t)\,dt - \int_1^2 \cos(n\omega_0 t)\,dt$$

$$a_n = \left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t\sin(n\omega_0 t)}{n\omega_0}\right]_0^1 - \left.\frac{\sin(n\omega_0 t)}{n\omega_0}\right|_1^2$$

$$a_n = \frac{\cos(n\omega_0) - 1}{(n\omega_0)^2} + \frac{\sin(n\omega_0)}{n\omega_0} - \cancelto{0}{\frac{\sin(2n\omega_0)}{n\omega_0}} + \frac{\sin(n\omega_0)}{n\omega_0}$$

$$\boxed{a_n = \frac{4}{(n\pi)^2}\left(\cos(n\pi/2) - 1\right) + \frac{4}{n\pi}\sin(n\pi/2) \qquad a_0 = -\frac{1}{4} \qquad b_n = 0}$$

15.14 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.14.

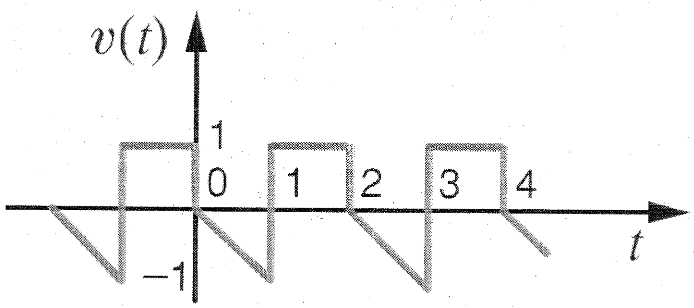


Figure P15.14

SOLUTION: $T_o = 2$ sec $\quad \omega_o = \pi$ r/s $\quad a_o = \dfrac{-(1/2)(1)(1) + (1)(1)}{2} = \dfrac{1}{4}$

$$a_n = \frac{2}{T_o}\left\{\int_0^1 -t\cos(n\omega_o t)\,dt + \int_1^2 \cos(n\omega_o t)\,dt\right\} = \frac{\cos(n\omega_o t)}{(n\pi)^2} + \frac{t\sin(n\omega_o t)}{n\omega_o}\Bigg|_1^0 + \frac{\sin(n\omega_o t)}{n\omega_o}\Bigg|_1^2$$

$$a_n = \frac{1-\cos(n\pi)}{(n\pi)^2} - \frac{\sin(n\pi)}{n\pi} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{1-\cos(n\pi)}{(n\pi)^2}$$

$$b_n = \frac{2}{T_o}\left\{\int_0^1 -t\sin(n\omega_o t)\,dt + \int_1^2 \sin(n\omega_o t)\,dt\right\} = \frac{t\cos(n\pi t)}{n\pi} - \frac{\sin(n\pi t)}{(n\pi)^2}\Bigg|_0^1 + \frac{\cos(n\pi t)}{n\pi}\Bigg|_2^1$$

$$b_n = \frac{\cos(n\pi)}{n\pi} - \frac{\sin(n\pi)}{(n\pi)^2} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} = \frac{2\cos(n\pi) - 1}{n\pi}$$

$$\boxed{a_o = 1/4 \qquad a_n = \frac{1-\cos(n\pi)}{(n\pi)^2} \qquad b_n = \frac{2\cos(n\pi)-1}{n\pi}}$$

15.15 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.15.

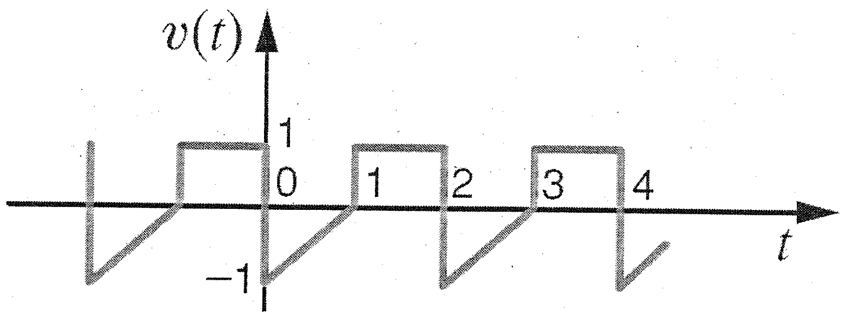


Figure P15.15

SOLUTION: $T_0 = 2\,sec$ $\quad \omega_0 = \pi\ r/s$ $\quad a_0 = \frac{1}{2}\left[-\frac{1}{2}(1)(1) + (1)(1)\right] = 1/4$

$$a_n = \frac{2}{T_0}\left\{\int_0^1 (t-1)\cos(n\pi t)\,dt + \int_1^2 \cos(n\pi t)\,dt\right\}$$

$$a_n = \left(\frac{\cos(n\pi t)}{(n\pi)^2} + \frac{t\sin(n\pi t)}{n\pi}\right)\Bigg|_0^1 - \frac{\sin(n\pi t)}{n\pi}\Bigg|_0^1 + \frac{\sin(n\pi t)}{n\pi}\Bigg|_1^2$$

$$a_n = \frac{\cos(n\pi)-1}{(n\pi)^2} + \frac{\sin(n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{\cos(n\pi)-1}{(n\pi)^2}$$

$$b_n = \int_0^1 (t-1)\sin(n\pi t)\,dt + \int_1^2 \sin(n\pi t)\,dt$$

$$b_n = \left(\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right)\Bigg|_0^1 + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_2^1 + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_0^1$$

$$b_n = \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{1}{n\pi} = \frac{\cos(n\pi)-2}{n\pi}$$

$$\boxed{a_0 = 1/4 \qquad a_n = \frac{\cos(n\pi)-1}{(n\pi)^2} \qquad b_n = \frac{\cos(n\pi)-2}{n\pi}}$$

15.16 Derive the trigonometric Fourier series for the waveform shown in Fig. P15.16.

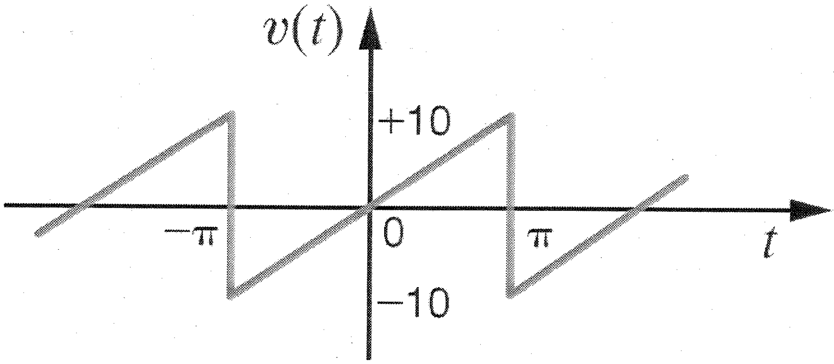


Figure P15.16

SOLUTION: $a_0 = 0 \qquad T_0 = 2\pi \qquad \omega_0 = 1$ r/s

odd symmetry $\Rightarrow a_n = 0$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} \frac{10}{\pi} t \sin(nt)\,dt = \frac{40}{\pi^2}\int_0^{\pi/2} t \sin(nt)\,dt$$

$$b_n = \frac{40}{\pi^2}\left[\frac{\sin(nt)}{n^2} - \frac{t\cos(nt)}{n}\right]_0^{\pi} = \frac{40}{\pi^2}\left[\frac{\sin(n\pi)}{n^2} - \frac{\pi\cos(n\pi)}{2n}\right]$$

$$b_n = \frac{40}{\pi^2}\left[\frac{-\pi\cos(n\pi)}{2n}\right] = -\frac{20}{n\pi}\cos(n\pi) = (-1)^{n+1}\left(\frac{20}{n\pi}\right)$$

$$\boxed{v(t) = \sum_{n=1}^{\infty} (-1)^{n+1}\frac{20}{n\pi}\sin(nt)}$$

15.17 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.17.

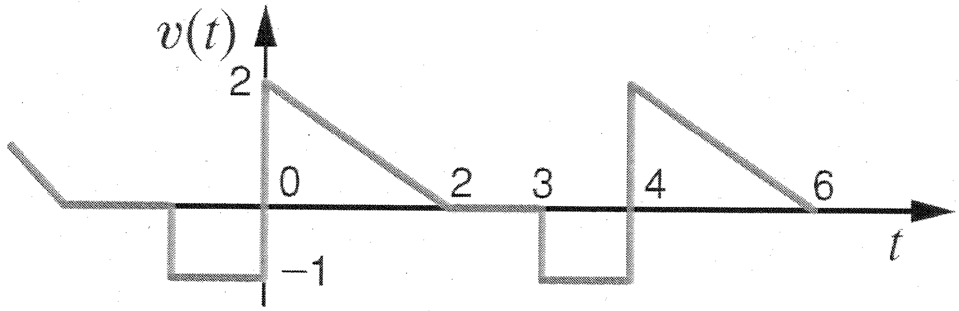


Figure P15.17

SOLUTION: $T_0 = 4 \qquad \omega_0 = \pi/2 \qquad a_0 = \frac{1}{4}\left[\frac{1}{2}(2)(2) - (1)(1)\right] = 1/4$

$$a_n = \frac{2}{T_0}\left\{\int_0^2 (2-t)\cos(n\omega_0 t)\,dt - \int_3^4 \cos(n\omega_0 t)\,dt\right\}$$

$$a_n = \left.\frac{\sin(n\omega_0 t)}{n\omega_0}\right|_0^2 + \left.\left(\frac{t\sin(n\omega_0 t)}{2n\omega_0} + \frac{\cos(n\omega_0 t)}{2(n\omega_0)^2}\right)\right|_2^0 + \left.\frac{\sin(n\omega_0 t)}{2n\omega_0}\right|_4^3$$

$$a_n = \frac{\sin(n\pi)}{n\omega_0} + \frac{1}{2(n\omega_0)^2} - \frac{\sin(n\pi)}{n\omega_0} - \frac{\cos(n\pi)}{2(n\omega_0)^2} + \frac{\sin(n3\pi/2)}{2n\omega_0} - \frac{\sin(n2\pi)}{2n\omega_0}$$

$$\boxed{a_n = \frac{2}{(n\pi)^2}(1-\cos(n\pi)) - \frac{\sin(n\pi/2)}{n\pi}} \qquad \boxed{a_0 = 1/4}$$

$$b_n = \frac{2}{T_0}\left\{\int_0^2 (2-t)\sin(n\omega_0 t)\,dt + \int_3^4 -\sin(n\omega_0 t)\,dt\right\}$$

$$b_n = \left.\frac{\cos(n\omega_0 t)}{n\omega_0}\right|_2^0 + \left.\left(\frac{t\cos(n\omega_0 t)}{2n\omega_0} - \frac{\sin(n\omega_0 t)}{2(n\omega_0)^2}\right)\right|_0^2 + \left.\frac{\cos(n\omega_0 t)}{2n\omega_0}\right|_3^4$$

$$b_n = \frac{1-\cos(n\pi)}{n\omega_0} + \frac{\cos(n\pi)}{n\omega_0} - \frac{\sin(n\pi)}{2(n\omega_0)^2} + \frac{\cos(n2\pi) - \cos(3n\pi/2)}{2n\omega_0}$$

$$\boxed{b_n = \frac{3 - \cos(n\pi/2)}{n\pi}}$$

15.18 Find the trigonometric Fourier series for the waveform shown in Fig. P15.18. CS

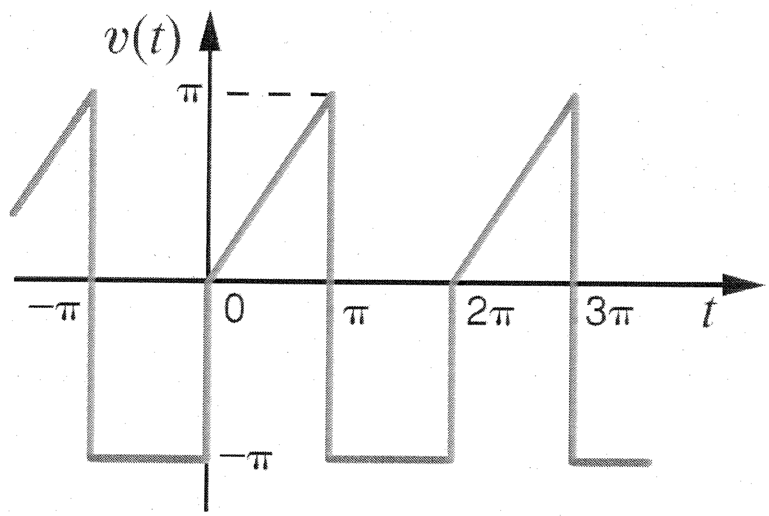


Figure P15.18

SOLUTION: $T_o = 2\pi$ $\quad \omega_o = 1$ r/s $\quad a_o = \frac{1}{2\pi}\left[\pi^2/2 - \pi^2\right] = -\pi/4$

$$a_n = \frac{1}{\pi}\int_0^{\pi} t\cos(n\omega_o t)\,dt - \frac{1}{\pi}\int_{\pi}^{2\pi} \pi\cos n\omega_o t\,dt$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(n\omega_o t)}{(n\omega_o)^2} + \frac{t\sin(n\omega_o t)}{n\omega_o}\right]\Bigg|_0^{\pi} - \left[\frac{\sin n\omega_o t}{n\omega_o}\right]\Bigg|_{\pi}^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(n\pi)}{n^2} - \frac{1}{n^2} + \frac{\pi\sin(n\pi)}{n}\right] - \frac{\sin(n2\pi)}{n} + \frac{\sin(n\pi)}{n} = \frac{1}{\pi n^2}\left[\cos(n\pi) - 1\right]$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} t\sin(n\omega_o t)\,dt - \int_{\pi}^{2\pi} \sin(n\omega_o t)\,dt$$

$$b_n = \frac{1}{\pi}\left[\frac{\sin(n\omega_o t)}{(n\omega_o)^2} - \frac{t\cos(n\omega_o t)}{n\omega_o}\right]_0^{\pi} + \frac{\cos(n\omega_o t)}{n\omega_o}\Bigg|_{\pi}^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{\sin(n\pi)}{n^2\pi^2} - \frac{\pi\cos(n\pi)}{n\omega_o}\right] + \frac{\cos(2n\pi) - \cos(n\pi)}{n\omega_o} = \frac{1}{n}\left(1 - 2\cos(n\pi)\right)$$

$$v(t) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2}\left(\cos(n\pi) - 1\right)\cos(nt) + \frac{1}{n}\left(1 - 2\cos(n\pi)\right)\sin(nt) \text{ V}$$ ✓

15.19 Derive the trigonometric Fourier series for the function shown in Fig. P15.19. CS

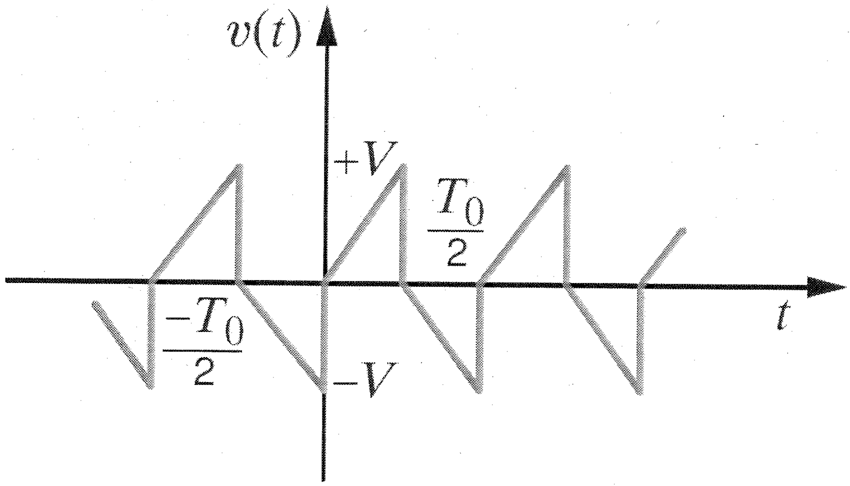


Figure P15.19

SOLUTION: Half wave symmetry $\Rightarrow a_n = b_n = 0$ for n even $\&$ $a_0 = 0$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} v(t)\cos(n\omega_0 t)\,dt = \frac{4}{T_0}\int_0^{T_0/2} \frac{2V}{T_0} t\cos(n\omega_0 t)\,dt = \frac{8V}{T_0^2}\int_0^{T_0/2} t\cos(n\omega_0 t)\,dt$$

$$a_n = \frac{8V}{T_0^2}\left[\frac{t\sin(n\omega_0 t)}{n\omega_0} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\right]\Bigg|_0^{T_0/2} = \frac{8V}{T_0^2}\left[\frac{T_0}{2}\left(\frac{\sin(n\pi)}{n\omega_0}\right) + \frac{\cos(n\pi)-1}{(n\omega_0)^2}\right]$$

$$a_n = 8V\left(\frac{\cos(n\pi)-1}{4(n\pi)^2}\right) \quad \text{for } n \text{ odd}, \quad a_n = \frac{-4V}{n^2\pi^2}$$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} v(t)\sin(n\omega_0 t)\,dt = \frac{8V}{T_0^2}\int_0^{T_0/2} t\sin(n\omega_0 t)\,dt$$

$$b_n = \frac{8V}{T_0^2}\left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right]\Bigg|_0^{T_0/2} = \frac{8V}{T_0^2}\left\{\frac{\sin(n\pi)}{(n\omega_0)^2} - \frac{T_0}{2}\frac{\cos(n\pi)}{n\omega_0}\right\}$$

$$b_n = \frac{-2V}{n\pi}\cos(n\pi) \quad \text{for } n \text{ odd}, \quad b_n = \frac{2V}{n\pi}$$

$$\boxed{v(t) = \sum_{\substack{n=1\\ n\ \text{odd}}}^{\infty} \frac{2V}{n\pi}\sin(n\omega_0 t) - \frac{4V}{n^2\pi^2}\cos(n\omega_0 t)\ \text{V}}$$

15.20 Derive the trigonometric Fourier series for the function $v(t) = A|\sin t|$ as shown in Fig. P15.20.

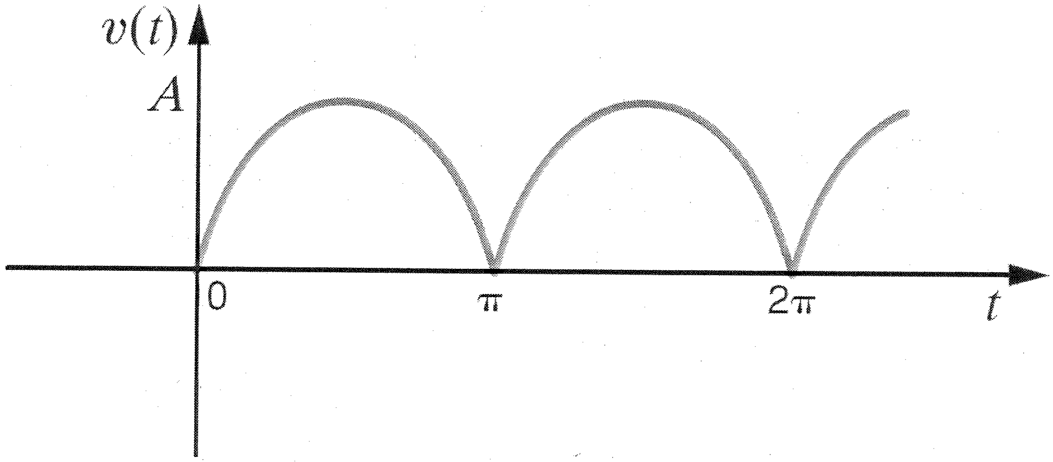


Figure P15.20

SOLUTION: $T_0 = \pi$ $\quad \omega_0 = 2$ $\quad$ even function $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} A\sin(t)\,dt = \frac{A}{T_0}\left[\cos(t)\right]\Big|_{T_0}^{0} = \frac{2A}{T_0} = \frac{2A}{\pi}$$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} A\sin(t)\cos(n\omega_0 t)\,dt = \frac{4A}{\pi}\int_0^{\pi/2} \sin(t)\cos(n2t)\,dt$$

$$\sin(t)\cos(2nt) = \left(\sin\left[(2n+1)t\right] + \sin\left((1-2n)t\right]\right)/2$$

$$a_n = \frac{2A}{\pi}\left[\frac{\cos(1+2n)t}{1+2n} + \frac{\cos(1-2n)t}{1-2n}\right]\Bigg|_{T_0/2}^{0}$$

$$a_n = \frac{2A}{\pi}\left[\frac{1-\cos\left[(1+2n)\pi/2\right]}{1+2n} + \frac{1-\cos\left[(1-2n)\pi/2\right]}{1-2n}\right] = \frac{2A}{\pi}\left[\frac{1}{1+2n} + \frac{1}{1-2n}\right]$$

$$a_n = \frac{4A}{\pi(1-4n^2)}$$

$$\boxed{v(t) = \frac{2A}{\pi}\left[1 + \sum_{n=1}^{\infty}\left(\frac{2}{1-4n^2}\right)\cos(n2t)\right] \text{ V}}$$

15.21 Derive the trigonometric Fourier series for the waveform shown in Fig. P15.21.

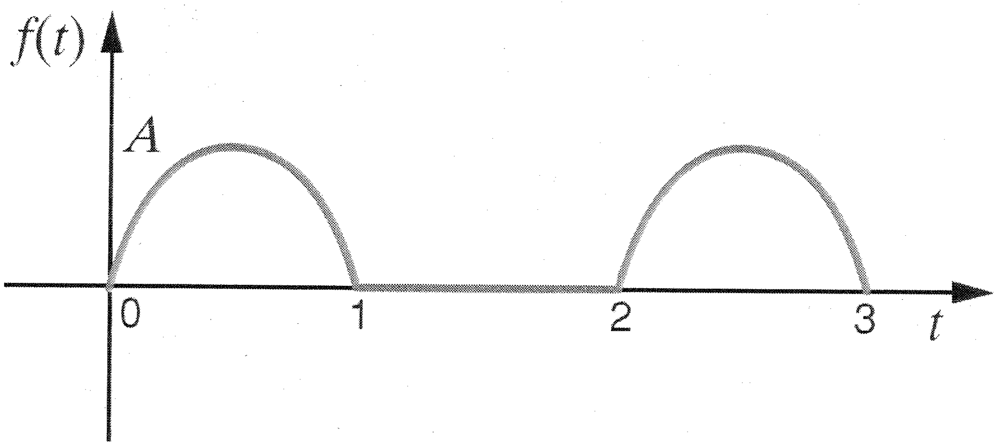


Figure P15.21

SOLUTION: $T_o = 2$ $\omega_o = \pi$

$$a_0 = \frac{1}{2}\int_0^1 A\sin(\pi t)\,dt = \frac{A}{2\pi}\cos(\pi t)\Big|_1^0 = A/\pi$$

$$a_n = A\int_0^1 \sin(\pi t)\cos(n\pi t)\,dt = \frac{A}{2}\int_0^1 \left[\sin(1-n)\pi t + \sin(1+n)\pi t\right]dt$$

$$a_n = \frac{A}{2}\left[\frac{\cos(1-n)\pi t}{(1-n)\pi} + \frac{\cos(1+n)\pi t}{(1+n)\pi}\right]\Bigg|_1^0 = \frac{A}{2}\left[\frac{1-\cos(1-n)\pi}{(1-n)\pi} + \frac{1-\cos(1+n)\pi}{(1+n)\pi}\right]$$

for n odd, $a_n = 0$ $\quad a_n = \frac{2A}{\pi(1-n^2)}$ for n even

$$b_n = A\int_0^1 \sin(\pi t)\sin(n\pi t)\,dt = \begin{cases} 0 & n \neq 1 \\ A\int_0^1 \sin^2(\pi t)\,dt & \text{for } n = 1 \end{cases}$$

$$b_1 = \frac{A}{2}\int_0^1 (1-\cos(2\pi t))\,dt = \frac{A}{2}\left[t - \frac{\sin(2\pi t)}{2\pi}\right]\Bigg|_0^1 = \frac{A}{2}[1] = A/2$$

$$\boxed{f(t) = \frac{A}{\pi} + \frac{A}{2}\sin(\pi t) + \sum_{\substack{n=2 \\ n \text{ even}}}^{\infty} \frac{2A}{\pi(1-n^2)}\cos(n\pi t)}$$

15.22 Use PSPICE to determine the Fourier series of the waveform in Fig. P15.22 in the form

$$v_S(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega_t + \theta_n)$$

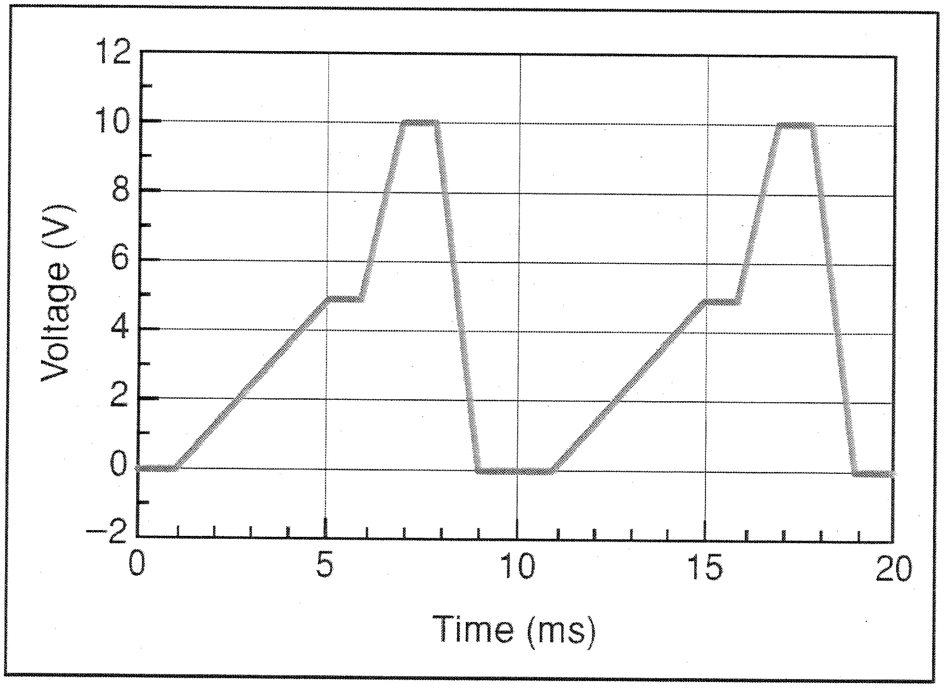


Figure P15.22

SOLUTION: Final time = 10 ms Step ceiling = 10 μs Center freq = 100 Hz
Number of harmonics = 10.

Use VPWL source to produce $v_s(t)$

15.22

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(Vs)

DC COMPONENT = 3.750000E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000E+02	3.954E+00	1.000E+00	-1.424E+02	0.000E+00
2	2.000E+02	2.016E+00	5.099E-01	-8.774E+01	1.971E+02
3	3.000E+02	1.247E+00	3.154E-01	-8.735E+00	4.185E+02
4	4.000E+02	6.417E-01	1.623E-01	6.991E+01	6.395E+02
5	5.000E+02	2.027E-01	5.126E-02	9.000E+01	8.020E+02
6	6.000E+02	2.852E-01	7.213E-02	1.101E+02	9.645E+02
7	7.000E+02	2.291E-01	5.793E-02	-1.713E+02	8.256E+02
8	8.000E+02	1.260E-01	3.188E-02	-9.226E+01	1.047E+03
9	9.000E+02	4.883E-02	1.235E-02	-3.759E+01	1.244E+03
10	1.000E+03	1.993E-08	5.040E-09	-1.206E+02	1.303E+03

TOTAL HARMONIC DISTORTION = 6.310255E+01 PERCENT

15.23 Use PSPICE to determine the Fourier series of the waveform in Fig. P15.23 in the form

$$i_S(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t + \theta_n)$$ CS

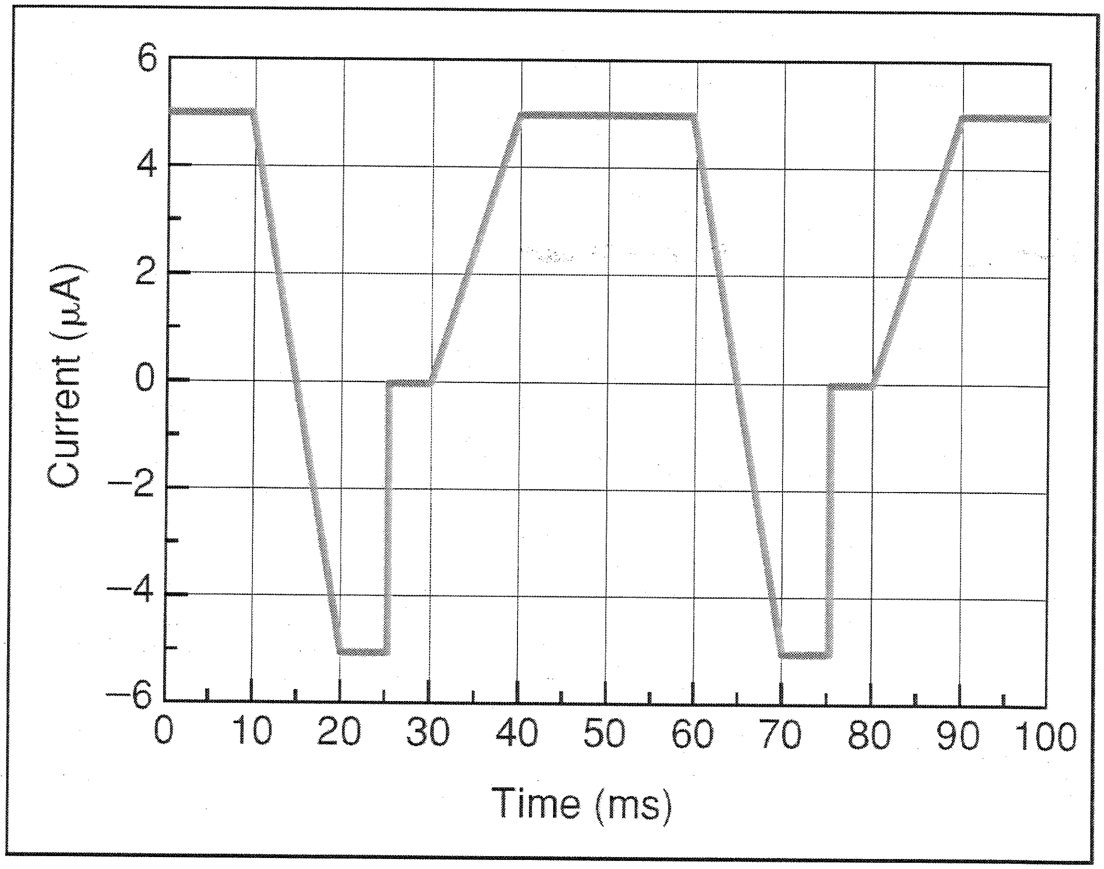


Figure P15.23

SOLUTION:

Final time = 50 ms   Step Ceiling = 50 μs   # harmonics = 10

Use IPWL to create $i_s(t)$

15.23

FOURIER COMPONENTS OF TRANSIENT RESPONSE I(I_Is)

DC COMPONENT = 1.997500E-06

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	2.000E+01	4.401E-06	1.000E+00	1.049E+02	0.000E+00
2	4.000E+01	1.669E-06	3.792E-01	-3.975E+01	-2.495E+02
3	6.000E+01	8.810E-07	2.002E-01	-1.480E+02	-4.627E+02
4	8.000E+01	4.518E-07	1.027E-01	3.521E+01	-3.844E+02
5	1.000E+02	3.183E-07	7.233E-02	1.791E+02	-3.454E+02
6	1.200E+02	3.001E-07	6.819E-02	2.212E+01	-6.073E+02
7	1.400E+02	2.046E-07	4.648E-02	-1.565E+02	-8.907E+02
8	1.600E+02	1.829E-07	4.157E-02	-2.297E+01	-8.621E+02
9	1.800E+02	1.913E-07	4.347E-02	1.625E+02	-7.816E+02
10	2.000E+02	1.592E-07	3.617E-02	-1.800E+00	-1.051E+03

TOTAL HARMONIC DISTORTION = 4.597326E+01 PERCENT

15.24 The discrete line spectrum for a periodic function $f(t)$ is shown in Fig. P15.24. Determine the expression for $f(t)$.
PSV

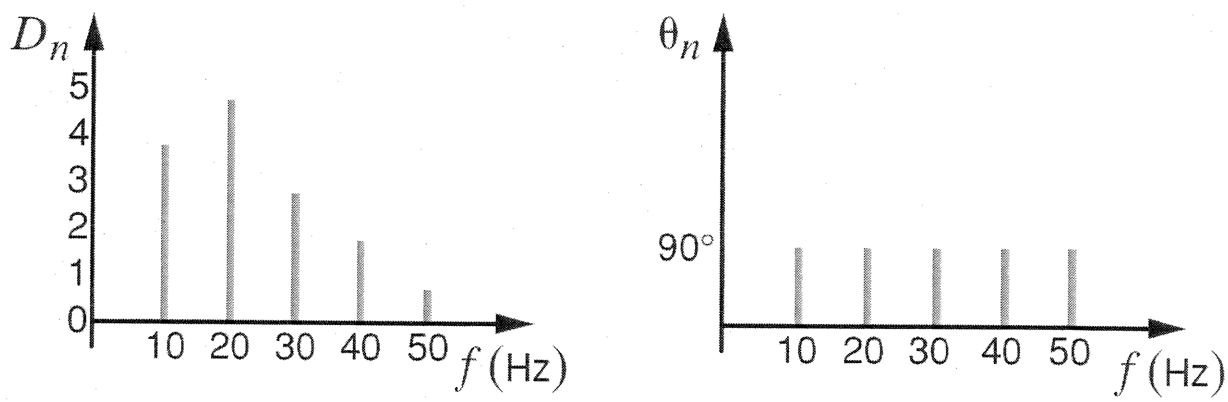


Figure P15.24

SOLUTION: $f_o = 10\,\text{Hz}$ $\omega_o = 20\pi$ r/s $D_n = a_n - jb_n$

all $\theta_n = 90°$, so $a_n = 0$ & $b_n = -|D_n|$

$f(t) = -4\sin(20\pi t) - 5\sin(40\pi t) - 3\sin(60\pi t) - 2\sin(80\pi t) - \sin(100\pi t)$ ✓

15.25 The amplitude and phase spectra for a periodic function $v(t)$ that has only a small number of terms is shown in Fig. P15.25. Determine the expression for $v(t)$ if $T_0 = 0.1$ s.

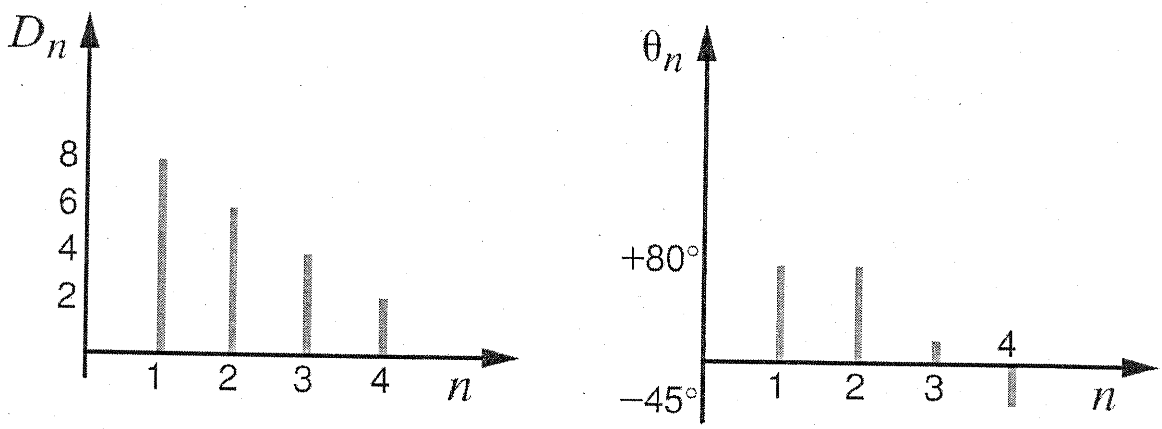


Figure P15.25

SOLUTION: $T_0 = 0.1\,s$ $\quad f_0 = 10\,Hz$ $\quad \omega_0 = 20\pi\,r/s$

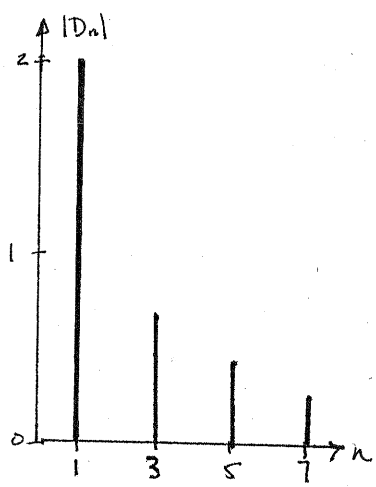
$v(t) = 8\cos(20\pi t + 80°) + 6\cos(40\pi t + 80°) + 4\cos(60\pi t + 15°) + 2\cos(80\pi t - 45°)$

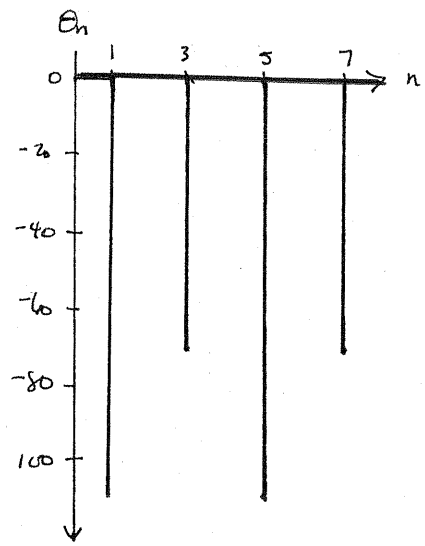
15.26 Plot the first four terms of the amplitude and phase spectra for the signal

$$f(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{-2}{n\pi} \sin \frac{n\pi}{2} \cos n\omega_0 t + \frac{6}{n\pi} \sin n\omega_0 t$$

SOLUTION: $D_n = a_n - jb_n$

n	a_n	b_n	$\lvert D_n \rvert$	θ_n (°)
1	-0.637	1.91	2.01	-108
3	0.212	0.637	0.671	-71.6
5	-0.127	0.382	0.403	-108
7	0.091	0.273	0.288	-71.6





15.27 Determine the steady-state response of the current $i_o(t)$ in the circuit shown in Fig. P15.27 if the input voltage is described by the waveform shown in Problem 15.16.

CS

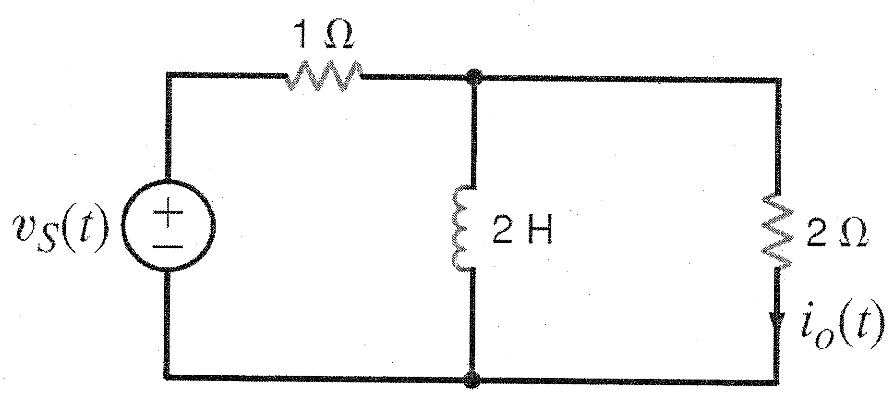
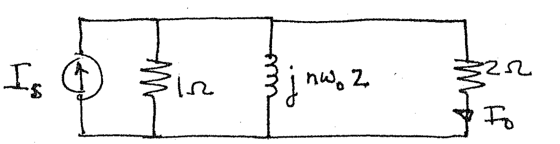


Figure P15.27

SOLUTION:



From problem 15.16 $\quad \omega_0 = 1$ r/s

$$V_S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{20}{n\pi} \cos(nt - 90^\circ)\ V$$

$$I_S = V_S / 1\Omega = V_S$$

$$\frac{I_o}{I_S} = \frac{I_o}{V_S} = \frac{1/2}{1/2 + 1 + \frac{1}{j2n}} = \frac{jn}{1 + j3n}$$

Let $G(n) = \dfrac{jn}{1 + j3n}$ $\quad \Rightarrow I_o(n) = G(n)\, V_S(n)$

$$i_o(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{20}{n\pi}\right) |G(n)| \cos(nt - 90^\circ + \theta_{G(n)})\ A$$

$$G(n) = \frac{n}{\sqrt{1+9n^2}} \angle 90^\circ - \tan^{-1}(3n)$$

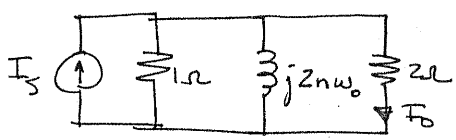
$$\boxed{i_o(t) = \sum_{n=1}^{\infty} \left[(-1)^{n+1} (20/n\pi) \frac{n}{\sqrt{1+9n^2}} \cos(nt - \tan^{-1}(3n)) \right] A}$$

15.28 If the input voltage in Problem 15.27 is

$$v_S(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 0.2\pi nt \text{ V}$$

find the expression for the steady-state current $i_o(t)$.

SOLUTION:



$$I_S = V_S/1 = V_S$$

$$\frac{I_o}{I_S} = \frac{I_o}{V_S} = G(n) = |G(n)| \angle \theta(n)$$

$$G(n) = \frac{1/2}{1/2 + 1 + \frac{1}{j2n\omega_o}} = \frac{jn\omega_o}{1 + j3n\omega_o} \qquad I_o(n) = G(n)\, V_S(n)$$

$$v_S(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\cos(0.2\pi nt - 90^\circ) \qquad \omega_o = 0.2\pi = \pi/5$$

$$G(n) = \frac{jn\pi}{5 + j3n\pi} = |G(n)| \angle \theta(n)$$

for $n=0$, inductor is a short & $i_o \to 0$.

$$\boxed{i_o = \sum_{n=1}^{\infty} -\frac{2}{\pi n}|G(n)| \cos\left(\frac{\pi n t}{5} - 90^\circ + \theta(n)\right) \text{ A}}$$

15.29 Determine the first three terms of the steady-state voltage $v_o(t)$ in Fig. P15.29 if the input voltage is a periodic signal of the form

$$v(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi}(\cos n\pi - 1)\sin nt \text{ V}$$ **PSV**

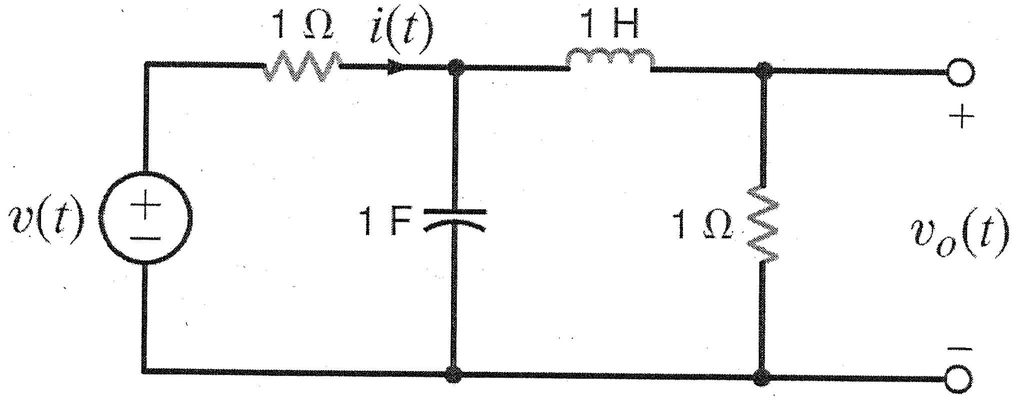
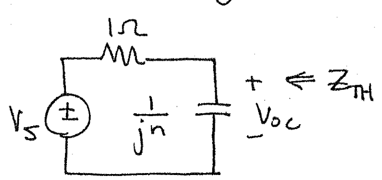


Figure P15.29

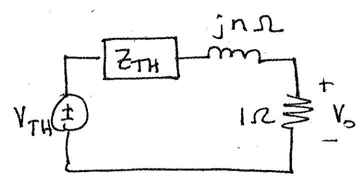
SOLUTION: $\omega_o = 1$ r/s

Thevenin eq.:



$V_{OC} = \dfrac{V_S}{1+jn}$

$Z_{TH} = \dfrac{1}{1+jn}$



$\dfrac{V_o}{V_{OC}} = \dfrac{1}{1+jn+\dfrac{1}{1+jn}} = \dfrac{1+jn}{(1+jn)^2+1}$

$\dfrac{V_o}{V_S} = \dfrac{V_o}{V_{OC}}\dfrac{V_{OC}}{V_S} = \dfrac{1}{(1+jn)^2+1} = \dfrac{1}{2-n^2+j2n} = H(n) = |H(n)|\angle\theta(n)$

for $n=0$, cap → open & inductor → short

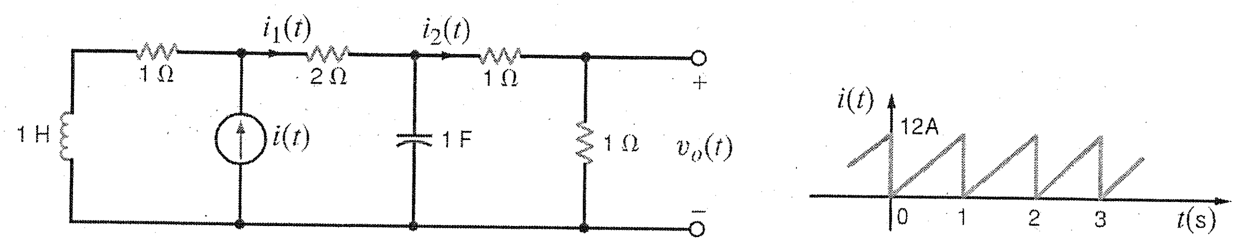
$V_o(0) = V(0)\left[\left(\tfrac{1}{2}\right)\right] = \tfrac{1}{4}$ V

for $n=1$, $V_o(1) = V(1)\,H(1) = \dfrac{\cos\pi - 1}{\pi}\,\dfrac{1}{1+j2} = 0.285\angle 116°$

for $n=2$, $V_o(2) = V(2)\,H(2) = 0$ for $n=3$, $V_o(3) = 0.023\angle 41.0°$

$$\boxed{v_o(t) = \frac{1}{4} + 0.285\cos(t+26.6) + 0.023\cos(3t-49°) \text{ V}}$$

15.30 Determine the steady-state voltage $v_o(t)$ in the network in Fig. P15.30a if the input current is given in Fig. P15.30b.



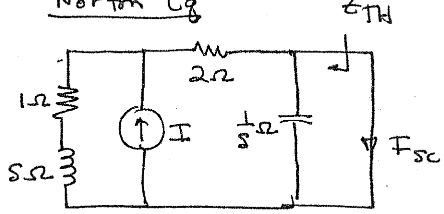
(a) (b)

Figure P15.30

SOLUTION: $T_o = 1\,s \qquad \omega_o = 2\pi \ r/s$

From Table 15.2, $\quad i(t) = 6 + \sum_{n=1}^{\infty} \frac{-12}{n\pi} \sin n2\pi t = 6 + \sum_{n=1}^{\infty} \frac{-12}{n\pi} \cos(n2\pi t - 90^\circ)\, A$

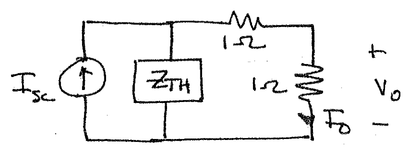
Norton Eq



$$I_{sc} = I\left\{\frac{S+1}{S+3}\right\}$$

$$Z_{TH} = \frac{(1/S)(S+3)}{S+3+1/S}$$

$$Z_{TH} = \frac{S+3}{S^2+3S+1}$$



$$\frac{I_o}{I_{sc}} = \frac{Z_{TH}}{2+Z_{TH}}$$

$$V_o = (1)\, I_o$$

$$\frac{V_o}{I_{sc}} = \frac{S+3}{S+3+2(S^2+3S+1)} = \frac{3+S}{(2S+5)(S+1)}$$

$$\frac{V_o}{I} = \frac{S+3}{(2S+5)(S+1)} \cdot \frac{S+1}{S+3} = \frac{1}{2S+5}$$

let $S = j2\pi n$, $\quad \frac{V_o}{I} = \frac{1}{5+j4\pi n} = |G(n)| \angle \theta(n)$

for $n=0$, $\quad i(t) = 6 \quad \& \quad G(0) = 1/5$

$$v_o(t) = \frac{6}{5} + \sum_{n=1}^{\infty} \frac{-12}{n\pi} |G(n)| \cos(2\pi n t - 90^\circ + \theta(n))\ V$$

15.31 Find the average power absorbed by the network in Fig. P15.31 if

$$v(t) = 12 + 6\cos(377t - 10^\circ) + 4\cos(754t - 60^\circ)\ \text{V}$$

$$i(t) = 0.2 + 0.4\cos(377t - 150^\circ)$$
$$-0.2\cos(754t - 80^\circ) + 0.1\cos(1131t - 60^\circ)\ \text{A}$$

PSV

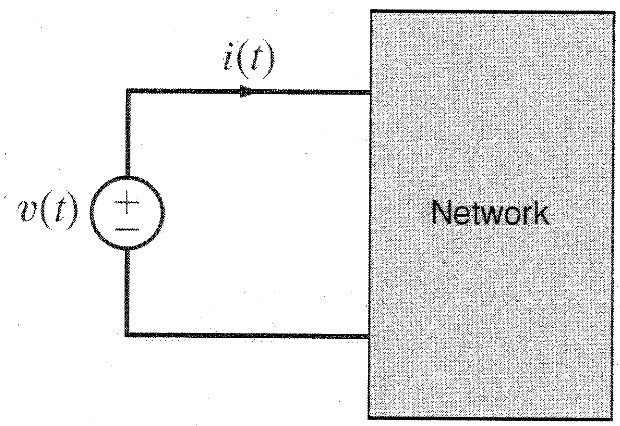


Figure P15.31

SOLUTION:

$$P = V_{DC} I_{DC} + \frac{1}{2}\sum_{n=1}^{\infty} |V(n)||I(n)| \cos(\theta_V - \theta_I)$$

$$P = 12(0.2) + \frac{1}{2}\{(6)(0.4)\cos(140^\circ) + 4(0.2)\cos(-160^\circ)\}$$

$$\boxed{P = 1.1\ \text{W}}$$

15.32 Find the average power absorbed by the network in Fig. P15.32 if $v(t) = 60 + 36\cos(377t + 45°) + 24\cos(754t - 60°)$ V. **CS**

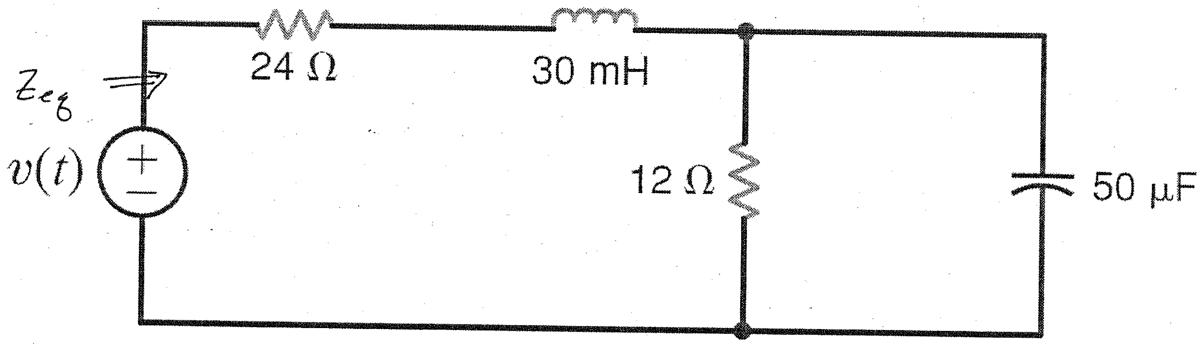


Figure P15.32

SOLUTION: $\omega_o = 377$ r/s

$$Z_{eq} = 24 + sL + \frac{12/sC}{12 + 1/sC} = 24 + sL + \frac{12}{1 + 12sC} = \frac{12LCs^2 + s(288C + L) + 36}{12sC + 1}$$

Let $s \to jn\omega_o = jn377$ $\quad Z_{eq} = \dfrac{36 - 2.56n^2 + jn(16.7)}{1 + jn(0.226)}$

$I(n) = V(n)/Z_{eq}(n)$

$$P = \frac{V(0)^2}{Z_{eq}(0)} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{|V(n)|^2}{|Z(n)|}\cos\theta_{Z(n)}$$

$$P = \frac{60^2}{36} + \frac{1}{2}\left\{\frac{36^2}{36.4}\cos(13.8°) + \frac{24^2}{38.4}\cos(28.1)\right\}$$

$\boxed{P = 123.8 \text{ W}}$

15.33 Determine the Fourier transform of the waveform shown in Fig. P15.33.

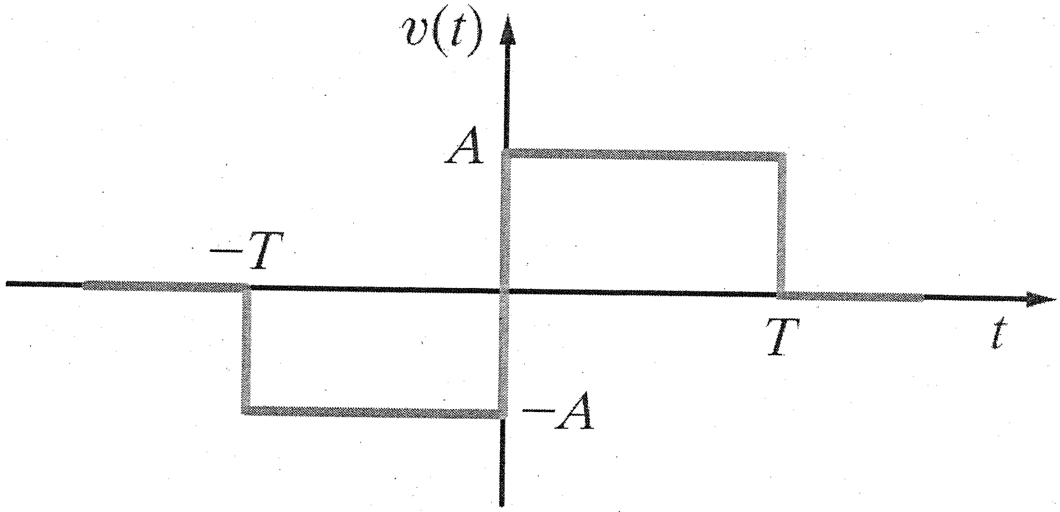


Figure P15.33

SOLUTION:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-T}^{0} -Ae^{-j\omega t}dt + \int_{0}^{T} Ae^{-j\omega t}dt$$

$$F(\omega) = \frac{A}{j\omega}\left[e^{-j\omega t}\Big|_{-T}^{0} - e^{-j\omega t}\Big|_{0}^{T}\right] = \frac{A}{j\omega}\left[1 - e^{j\omega T} + 1 - e^{-j\omega T}\right]$$

$$F(\omega) = \frac{A}{j\omega}\left[2 - \left(e^{j\omega T} + e^{-j\omega T}\right)\right] = \frac{2A}{j\omega}\left[1 - \frac{e^{j\omega T} + e^{-j\omega T}}{2}\right]$$

$$\boxed{F(\omega) = \frac{2A}{j\omega}\left[1 - \cos(\omega T)\right]}$$

15.34 Derive the Fourier transform for the following functions:

(a) $f(t) = e^{-2t} \cos 4t u(t)$

(b) $f(t) = e^{-2t} \sin 4t u(t)$

SOLUTION:

a) $$F(\omega) = \int_0^\infty e^{-2t} \cos(4t)\, e^{-j\omega t}\, dt = \frac{1}{2}\int_0^\infty \left(e^{-(2-j4+j\omega)t} + e^{-(2+j4+j\omega)t}\right) dt$$

$$F(\omega) = \frac{1}{2}\left[\frac{e^{-(2-j4+j\omega)t}}{2-j4+j\omega}\Bigg|_\infty^0 + \frac{e^{-(2+j4+j\omega)t}}{2+j4+j\omega}\Bigg|_\infty^0 \right] = \frac{1}{2}\left[\frac{1}{2-j4+j\omega} + \frac{1}{2+j4+j\omega}\right]$$

$$\boxed{F(\omega) = \frac{2+j\omega}{(2+j\omega)^2+16}}$$

b) $$F(\omega) = \int_0^\infty e^{-2t} \sin(4t)\, e^{-j\omega t}\, dt = \frac{1}{2j}\int_0^\infty \left[e^{-(2-j4+j\omega)t} - e^{-(2+j4+j\omega)t}\right] dt$$

$$F(\omega) = \frac{1}{2j}\left[\frac{e^{-(2-j4+j\omega)t}}{2-j4+j\omega}\Bigg|_\infty^0 - \frac{e^{-(2+j4+j\omega)t}}{2+j4+j\omega}\Bigg|_\infty^0 \right] = \frac{1}{2j}\left[\frac{1}{2-j4+j\omega} - \frac{1}{2+j4+j\omega}\right]$$

$$F(\omega) = \frac{1}{j2}\left[\frac{j8}{4+(j\omega)^2+4j\omega+16}\right] \qquad \boxed{F(\omega) = \frac{4}{(2+j\omega)^2+16}}$$

15.35 Show that

$$\mathcal{F}[f_1(t)f_2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{F}_1(x)\mathbf{F}_2(\omega - x)\,dx$$ CS

SOLUTION:

Let $G = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(x)\,F_2(\omega - x)\,dx$

$$F^{-1}[G] = \frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty} F_1(x)\int_{\omega=-\infty}^{\infty} F_2(\omega - x)\,e^{j\omega t}\,d\omega\,dx$$

let $u = \omega - x \Rightarrow du = d\omega$

$$F^{-1}[G] = \frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty} F_1(x)\int_{u=-\infty}^{\infty} F_2(u)\,e^{jut}\,e^{jxt}\,du\,dx$$

$$F^{-1}[G] = \frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty} F_1(x)\,e^{jxt}\,dt\int_{-\infty}^{\infty} F_2(u)\,e^{jut}\,du = f_1(t)\,f_2(t)$$

Thus,

$$F[f_1(t)\,f_2(t)] = G = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(x)\,F_2(\omega - x)\,dx \quad \checkmark$$

15.36 Find the Fourier transform of the function $f(t) = 12e^{-2|t|}\cos 4t$.

SOLUTION: Let $g(t) = 12e^{-2|t|}$

From Table 15.3, $G(\omega) = \dfrac{48}{4+\omega^2}$

$\cos 4t = \dfrac{e^{j4t} + e^{-j4t}}{2} \Rightarrow F\left[g(t)\cos 4t\right] = F\left[\dfrac{48}{4+\omega^2}\left(\dfrac{e^{j4t}+e^{-j4t}}{2}\right)\right]$

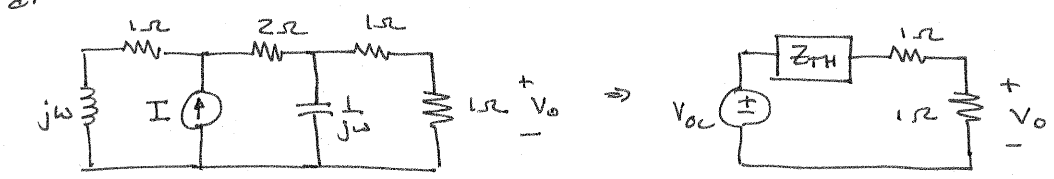
From Table 15.4, $F(\omega) = \dfrac{1}{2}\left[\dfrac{48}{4+(\omega-4)^2} + \dfrac{48}{4+(\omega+4)^2}\right]$

$$\boxed{F(\omega) = \dfrac{24}{4+(\omega-4)^2} + \dfrac{24}{4+(\omega+4)^2}}$$

15.37 Use the transform technique to find $v_o(t)$ in the network in Fig. P15.30a if (a) $i(t) = 4(e^{-t} - e^{-2t})u(t)$ A and (b) $i(t) = 12\cos 4t$ A. **PSV**

SOLUTION:

a)



$$I(\omega) = 4\left[\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right]$$

$$I(\omega) = \frac{4}{(1+j\omega)(2+j\omega)}$$

$$V_{oc} = I\left[\frac{1+j\omega}{3+j\omega+1/j\omega}\right]1/j\omega$$

$$V_{oc} = I\left[\frac{1+j\omega}{1-\omega^2+j3\omega}\right]$$

$$Z_{TH} = \frac{1}{j\omega} // (3+j\omega) = \frac{3+j\omega}{1-\omega^2+j3\omega}$$

$$V_o = V_{oc}\left[\frac{1}{2+Z_{TH}}\right] = \frac{I(1+j\omega)}{5-2\omega^2+j7\omega} = \frac{I}{5+j2\omega}$$

$$H(\omega) = \frac{V_o(\omega)}{I(\omega)} = \frac{1/2}{2.5+j\omega} \qquad V_o(\omega) = \frac{2}{(1+j\omega)(2+j\omega)(2.5+j\omega)}$$

$$V_o(\omega) = \frac{4/3}{1+j\omega} - \frac{4}{2+j\omega} + \frac{8/3}{2.5+j\omega}$$

$$\boxed{v_o(t) = \frac{4}{3}e^{-t} - 4e^{-2t} + \frac{8}{3}e^{-2.5t} \quad u(t)}$$

b) $I(\omega) = 12\pi\left[\delta(\omega-4) + \delta(\omega+4)\right] \qquad H(\omega) = \frac{1}{5+2\omega}$

$$V_o(\omega) = 6(2\pi)\left[\frac{\delta(\omega-4)}{5+j8} + \frac{\delta(\omega+4)}{5-j8}\right] = \frac{6(2\pi)}{K}\left[e^{-j\theta}\delta(\omega-4) + e^{j\theta}\delta(\omega+4)\right]$$

$K = 9.43 \qquad \theta = 58° \qquad v_o(t) = \frac{6}{K}\left[e^{j(4t-\theta)} + e^{-j(4t-\theta)}\right]$

$v_o(t) = \frac{12}{K}\cos(4t-\theta)$ V $\qquad \boxed{v_o(t) = 1.27\cos(4t - 58°)\text{ V}}$

15.38 The input signal to a network is $v_i(t) = e^{-3t}u(t)$ V. The transfer function of the network is $\mathbf{H}(j\omega) = 1/(j\omega + 4)$. Find the output of the network $v_o(t)$ if the initial conditions are zero. CS

SOLUTION:

$V_i(\omega) = \frac{1}{3+j\omega}$ $\quad V_o(\omega) = V_i(\omega)\,H(\omega) = \frac{1}{(3+j\omega)(4+j\omega)} = \frac{1}{3+j\omega} - \frac{1}{4+j\omega}$

$$\boxed{v_o(t) = \left(e^{-3t} - e^{-4t}\right)u(t)\ \text{V}}$$

15.39 Determine $v_o(t)$ in the circuit shown in Fig. P15.39 using the Fourier transform if the input signal is $i_S(t) = (e^{-2t} + \cos t)u(t)$ A.

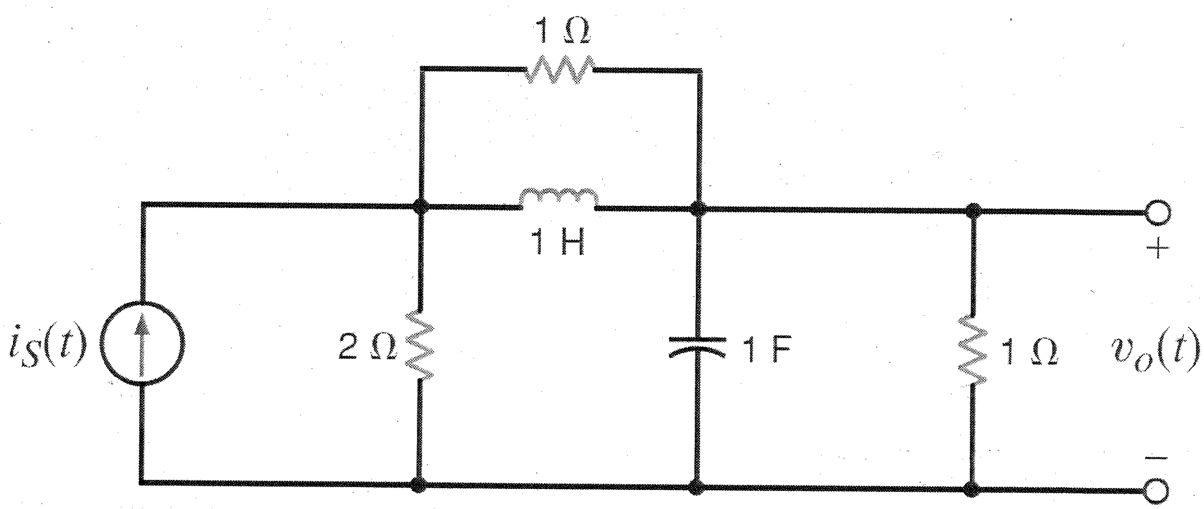
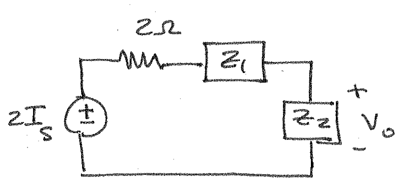


Figure P15.39

SOLUTION: Use source transformation!



$$Z_1 = 1 // j\omega = \frac{j\omega}{1+j\omega} \qquad Z_2 = 1 // \frac{1}{j\omega} = \frac{1}{1+j\omega}$$

$$\frac{V_o}{2I_S} = \frac{Z_2}{2+Z_1+Z_2} = \frac{1}{3(1+j\omega)} \qquad H(\omega) = \frac{V_o}{I_S} = \frac{2/3}{j\omega+1}$$

$$I_S = \frac{1}{2+j\omega} + \pi\delta(\omega-1) + \pi\delta(\omega+1)$$

$$V_o(\omega) = \frac{2/3}{(2+j\omega)(1+j\omega)} + \frac{\pi(2/3)\delta(\omega-1)}{1+j1} + \frac{\pi(2/3)\delta(\omega+1)}{1-j1}$$

$$V_o(\omega) = \frac{-2/3}{2+j\omega} + \frac{2/3}{1+j\omega} + \frac{2}{3}\pi\left\{\frac{\delta(\omega-1)}{1+j1} + \frac{\delta(\omega+1)}{1-j1}\right\}$$

$$\boxed{v_o(t) = \frac{2}{3}\left[e^{-t} - e^{-2t} + \frac{1}{\sqrt{2}}\cos(t-45°)\right]u(t)\ \text{V}}$$

15.40 The input signal for the network in Fig. P15.40 is $v_i(t) = 10e^{-5t}u(t)$ V. Determine the total 1-Ω energy content of the output $v_o(t)$.

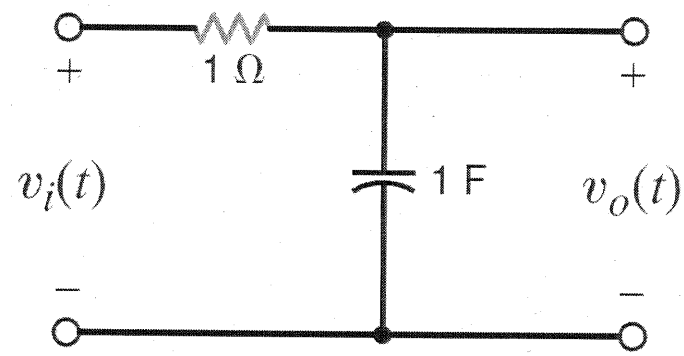


Figure P15.40

SOLUTION:

$$V_i(\omega) = \frac{10}{5+j\omega} \qquad H(\omega) = \frac{1/j\omega}{1+1/j\omega} = \frac{1}{1+j\omega} \qquad V_o = \frac{10}{(1+j\omega)(5+j\omega)}$$

$$|V_o(\omega)|^2 = \frac{100}{(1+\omega^2)(25+\omega^2)} = \frac{100}{24}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right] = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$W = \frac{1}{2\pi}\int_{-\infty}^{\infty} |V_o(\omega)|^2 d\omega = \frac{25}{12\pi}\left\{\int_{-\infty}^{\infty}\frac{d\omega}{1+\omega^2} - \int_{-\infty}^{\infty}\frac{d\omega}{25+\omega^2}\right\}$$

$$W = \frac{25}{12\pi}\left\{\tan^{-1}(\omega)\Big|_{-\infty}^{\infty} - \frac{1}{5}\tan^{-1}\left(\frac{\omega}{5}\right)\Big|_{-\infty}^{\infty}\right\} = \frac{25}{12\pi}\left[\pi - \frac{\pi}{5}\right] = \frac{25}{12\pi}\left(\frac{4\pi}{5}\right)$$

$$\boxed{W = 5/3 \text{ J}}$$

15.41 Compute the 1-Ω energy content of the signal $v_o(t)$ in Fig. P15.40 in the frequency range from $\omega = 2$ to $\omega = 4$ rad/s. **CS**

SOLUTION:

From problem 15.40,

$$|V_o|^2 = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$W = \frac{25}{6\pi}\left[\int_2^4 \frac{d\omega}{1+\omega^2} - \int_2^4 \frac{d\omega}{25+\omega^2}\right] = \frac{25}{6\pi}\left\{\tan^{-1}(\omega)\Big|_2^4 - \frac{\tan^{-1}}{5}(\omega/5)\Big|_2^4\right\}$$

$$W = \frac{25}{6\pi}\left\{1.326 - 1.107 - \left(\frac{0.675 - 0.381}{5}\right)\right\}$$

$$\boxed{W = 0.212\,\text{J}}$$

15.42 Determine the 1-Ω energy content of the signal $v_o(t)$ in Fig. P15.40 in the frequency range from 0 to 1 rad/s.

SOLUTION:

From problem 15.40

$$|V_o|^2 = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$W = \frac{2}{2\pi}\int_0^1 \frac{25}{6}\left(\frac{d\omega}{1+\omega^2}\right) - \frac{2}{2\pi}\int_0^1 \frac{25}{6}\left(\frac{d\omega}{25+\omega^2}\right)$$

$$W = \frac{25}{6\pi}\left[\tan^{-1}(\omega)\Big|_0^1 - \frac{\tan^{-1}(\omega/5)}{5}\Big|_0^1\right] = \frac{25}{6\pi}\left[0.785 - 0.039\right]$$

$$\boxed{W = 0.990\ \text{J}}$$

15.43 Compare the 1-Ω energy at both the input and output of the network in Fig. P15.43 for the given input forcing function $i_i(t) = 2e^{-4t}u(t)$ A.

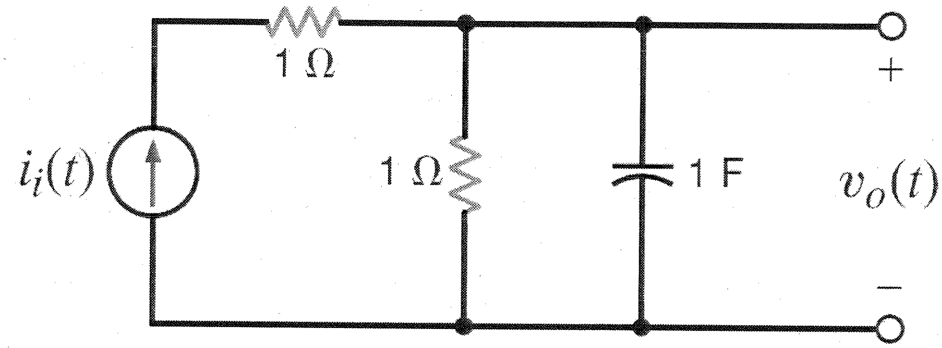
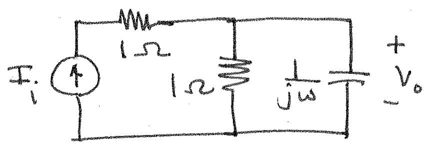


Figure P15.43

SOLUTION:



$$H(\omega) = \frac{V_o(\omega)}{I_i(\omega)} = 1 \,//\, \left(\frac{1}{j\omega}\right) = \frac{1}{1+j\omega}$$

$$I_i = \frac{2}{4+j\omega} \qquad V_o = \frac{2}{(1+j\omega)(4+j\omega)}$$

$$V_o = \frac{2/3}{1+j\omega} - \frac{2/3}{4+j\omega} \qquad |I_i|^2 = \frac{4}{16+\omega^2}$$

$$W_{in} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{4}{16+\omega^2}\,d\omega = \frac{1}{2\pi}\left[\tan^{-1}(\omega/4)\right]\Big|_{-\infty}^{\infty} = \frac{1}{2}\ \text{J}$$

$$|V_o|^2 = \frac{4}{(1+\omega^2)(16+\omega^2)} = \frac{4/15}{1+\omega^2} - \frac{4/15}{16+\omega^2}$$

$$W_{OUT} = \frac{1}{2\pi}\left(\frac{4}{15}\right)\int_{-\infty}^{\infty}\left(\frac{1}{1+\omega^2} - \frac{1}{16+\omega^2}\right)d\omega = \frac{2}{15\pi}\left[\tan^{-1}(\omega) - \frac{\tan^{-1}(\omega/4)}{4}\right]\Big|_{-\infty}^{\infty}$$

$$W_{OUT} = \frac{2}{15\pi}\left[\pi - \pi/4\right]$$

$$\boxed{W_{OUT} = 0.1\ \text{J}}$$

$$\boxed{W_{IN} = 0.5\ \text{J}}$$

15.44 The waveform shown in Fig. P15.44 demonstrates what is called the duty cycle; that is, D illustrates the fraction of the total period that is occupied by the pulse. Determine the average value of this waveform.

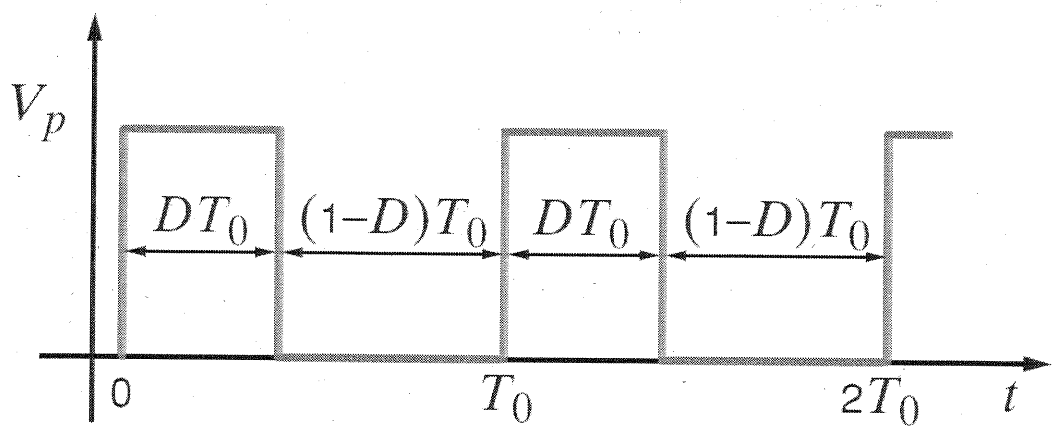


Figure P15.44

SOLUTION:

$$\text{average value} = a_0 = \frac{1}{T_0}\int_0^{T_0} v(t)\,dt = \frac{1}{T_0}\int_0^{DT_0} V_p\,dt$$

$$a_0 = \frac{V_P}{T_0}\, t\Big|_0^{DT_0} \qquad \boxed{a_0 = V_P D}$$

15FE-1 Given the waveform in Fig. 15PFE-1, determine which of the trigonometric Fourier coefficients have zero value, which have nonzero value, and why.
CS

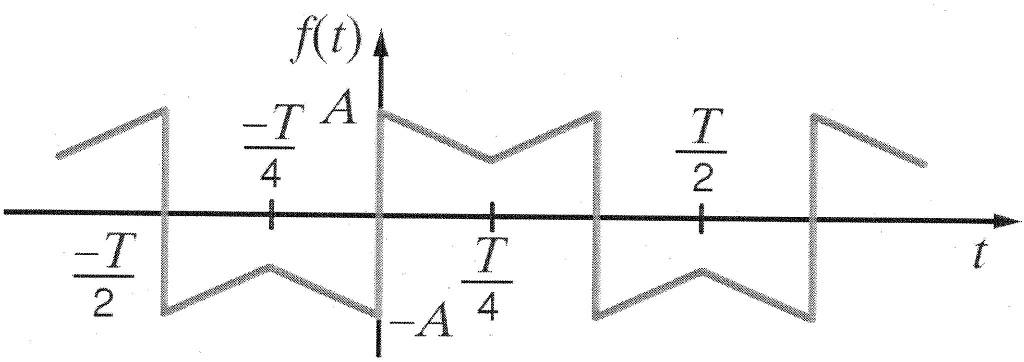


Figure 15PFE-1

SOLUTION:

Average value $= 0 \Rightarrow a_0 = 0$

$a_n = 0$ for all n since waveform has odd symmetry

$b_n = 0$ for n even since waveform has halfwave symmetry

b_n is finite & non zero for n odd.

15FE-2 Given the waveform in Fig. 15PFE-2, describe the type of symmetry and its impact on the trigonometric coefficients in the Fourier series—that is, a_0, a_n, and b_n.

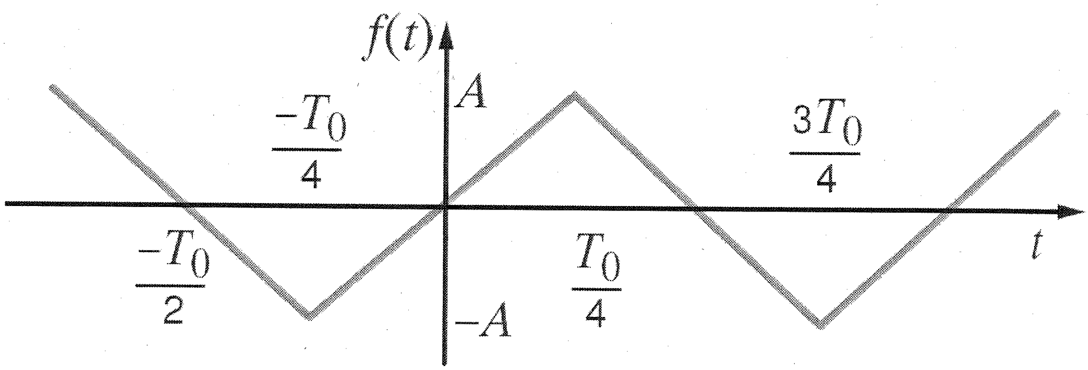


Figure 15PFE-2

SOLUTION:

Average value $= 0 \Rightarrow a_0 = 0$

Odd symmetry $\Rightarrow$ all $a_n = 0$

Halfwave symmetry $\Rightarrow$ $b_n = 0$ for n even

b_n is non zero only for n-odd