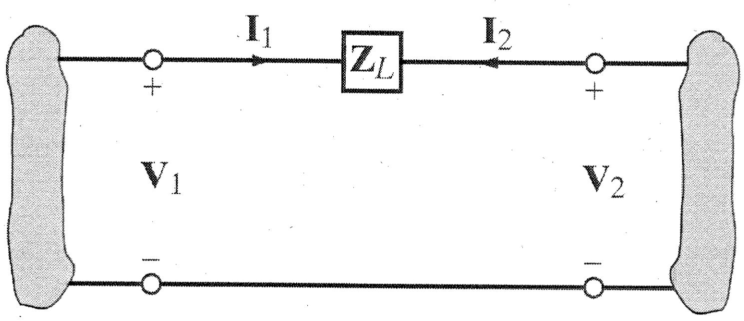
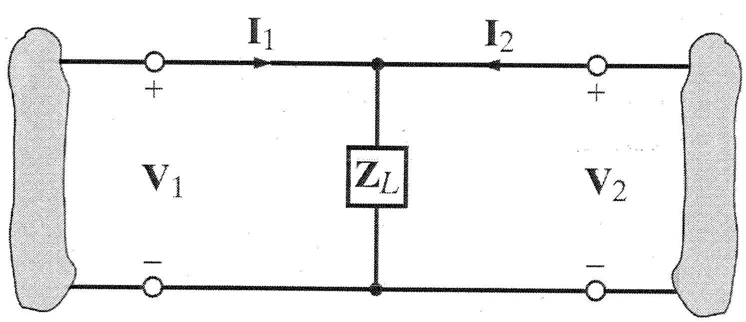
Chapter Sixteen

Two-Port Networks

16.1 Given the two networks in Fig. P16.1, find the Y parameters for the circuit in (a) and the Z parameters for the circuit in (b). CS



(a)



(b)

Figure P16.1

SOLUTION:

a) $Y_{11} = I_1/V_1 \big|_{V_2=0} = 1/Z_L$ $\quad Y_{21} = -\frac{1}{Z_L}$ $\quad Y_{12} = -\frac{1}{Z_L}$ $\quad Y_{22} = 1/Z_L$

b) $Z_{11} = V_1/I_1 \big|_{I_2=0} = Z_L$ $\quad Z_{21} = V_2/I_1 \big|_{I_2=0} = Z_L$

$Z_{12} = Z_L$ $\quad Z_{22} = Z_L$

16.2 Find the Y parameters for the two-port network shown in Fig. P16.2.

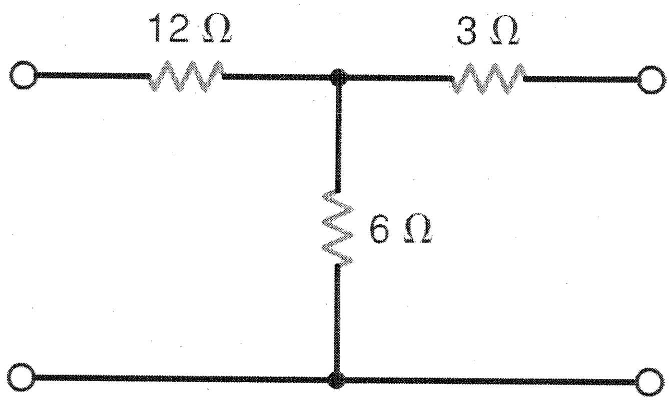
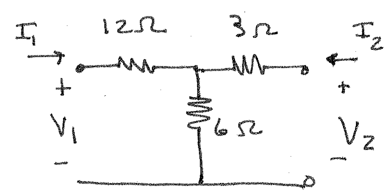


Figure P16.2

SOLUTION:



$$Y_{11} = I_1/V_1\Big|_{V_2=0} = \frac{1}{12 + (6/\!/3)} = \frac{1}{14}\ S$$

$$Y_{22} = I_2/V_2\Big|_{V_1=0} = \frac{1}{3 + (12/\!/6)} = \frac{1}{7}\ S$$

$$Y_{12} = I_1/V_2\Big|_{V_1=0} = \left[\frac{12/\!/6}{(12/\!/6)+3}\right]\left(\frac{-1}{12}\right) = -\frac{1}{21}\ S$$

$$Y_{21} = I_2/V_1\Big|_{V_2=0} = \left[\frac{3/\!/6}{(3/\!/6)+12}\right]\left(\frac{-1}{3}\right) = -\frac{1}{21}\ S$$

16.3 Find the Y parameters for the two-port network shown in Fig. P16.3. **PSV**

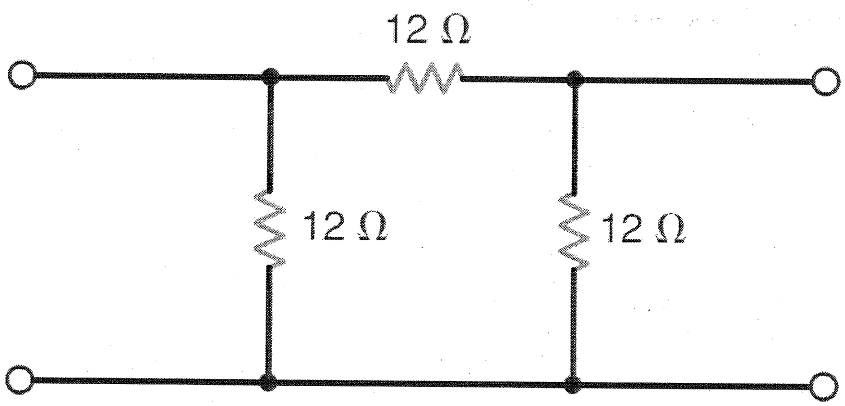
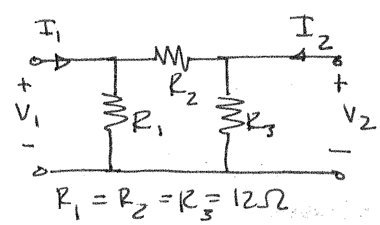


Figure P16.3

SOLUTION:



$$Y_{11} = I_1/V_1\big|_{V_2=0} = \frac{1}{R_1 // R_2} = \frac{1}{6}\ S$$

$$Y_{22} = \frac{1}{R_2 // R_3} = \frac{1}{6}\ S$$

$$Y_{21} = I_2/V_1\big|_{V_2=0} = \frac{-1}{R_2} = -\frac{1}{12}\ S$$

$$Y_{12} = I_1/V_2\big|_{V_1=0} = -\frac{1}{R_2} = -\frac{1}{12}\ S$$

16.4 Determine the Y parameters for the network shown in Fig. P16.4.

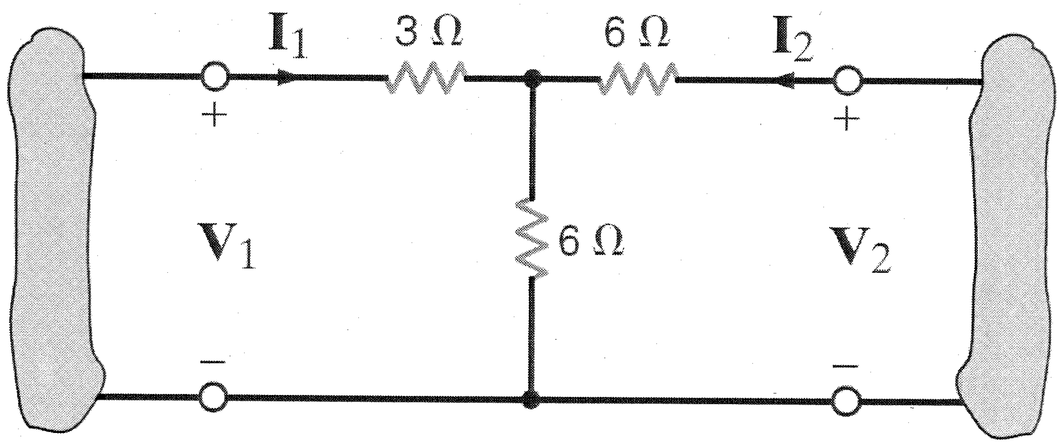


Figure P16.4

SOLUTION:

$$Y_{11} = I_1/V_1\big|_{V_2=0} = \frac{1}{3+(6\,//\,6)} = \frac{1}{6}\ S$$

$$Y_{22} = I_2/V_2\big|_{V_1=0} = \frac{1}{6+(6//3)} = \frac{1}{8}\ S$$

$$Y_{21} = I_2/V_1\big|_{V_2=0} = \frac{6//6}{(6//6)+3}\left(-\frac{1}{6}\right) = -\frac{1}{12}\ S$$

$$Y_{12} = I_1/V_2\big|_{V_1=0} = \frac{3//6}{(3//6)+6}\left(-\frac{1}{3}\right) = -\frac{1}{12}\ S$$

16.5 Find the Z parameters for the two-port network in Fig. P16.5.

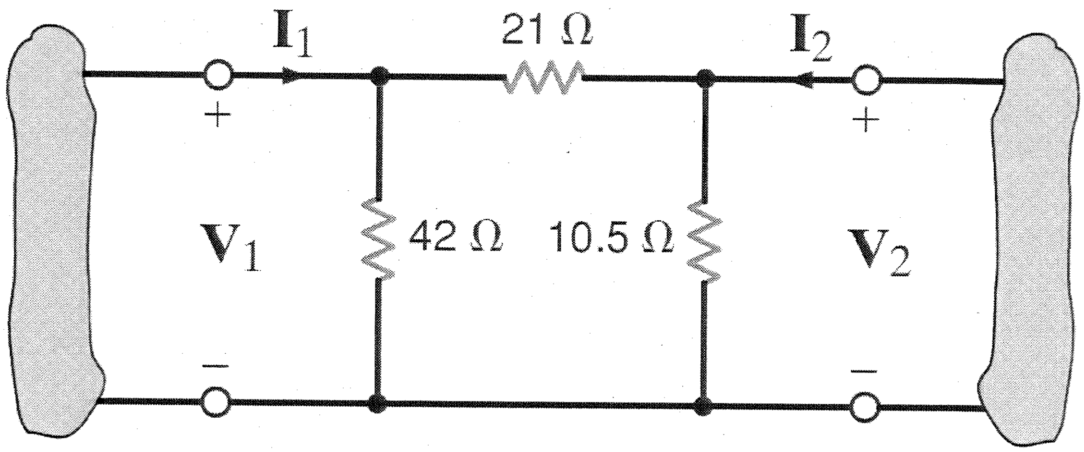


Figure P16.5

SOLUTION:

$$Z_{11} = V_1/I_1\big|_{I_2=0} = 42 \,//\, (21 + 10.5) = 18\,\Omega$$

$$Z_{22} = V_2/I_2\big|_{I_1=0} = 10.5 \,//\, (21 + 42) = 9\,\Omega$$

$$Z_{21} = V_2/I_1\big|_{I_2=0} = \frac{42}{42+21+10.5}(10.5) = 6\,\Omega$$

$$Z_{12} = V_1/I_2\big|_{I_1=0} = \frac{10.5}{10.5+21+42}(42) = 6\,\Omega$$

16.6 Determine the admittance parameters for the network shown in Fig. P16.6.

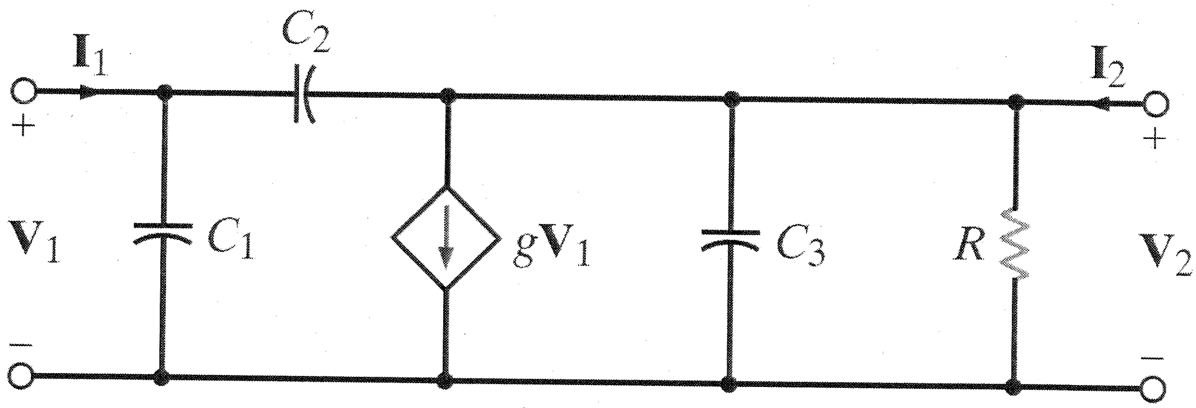


Figure P16.6

SOLUTION:

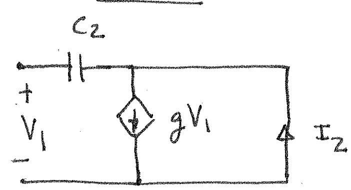
$Y_{11} = I_1/V_1\big|_{V_2=0} = j\omega(C_1 + C_2)$

$Y_{22} = I_2/V_2\big|_{V_1=0} = \frac{1}{R} + j\omega(C_2 + C_3)$

$Y_{21} = I_2/V_1\big|_{V_2=0} = g - j\omega C_2$

$Y_{12} = I_1/V_2\big|_{V_1=0} = -j\omega C_2$

For Y_{21}



$I_2 + V_1(j\omega C_2) = gV_1$

$I_2/V_1 = g - j\omega C_2$

16.7 Find the Y parameters for the two-port network in Fig. P16.7. CS

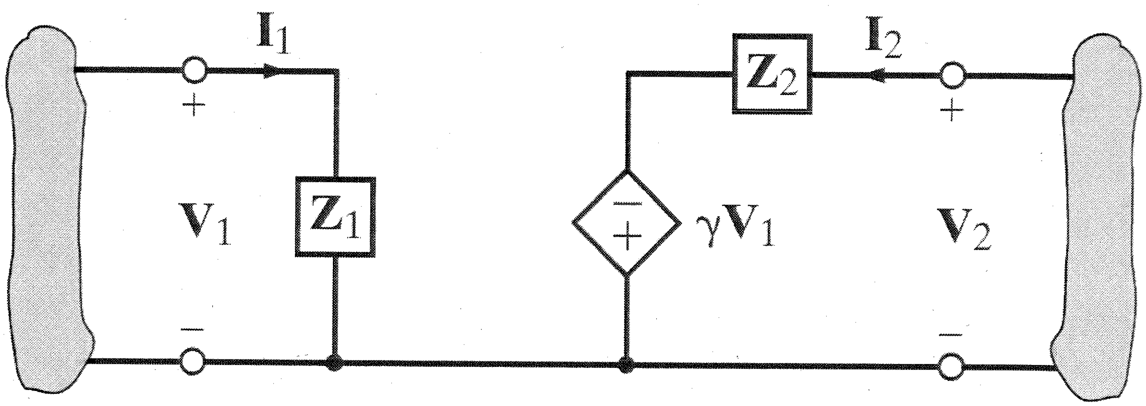


Figure P16.7

SOLUTION:

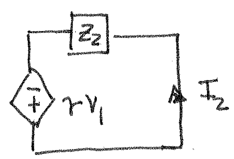
$Y_{11} = I_1/V_1 \big|_{V_2=0} = \frac{1}{Z_1}$

$Y_{22} = I_2/V_2 \big|_{V_1=0} = \frac{1}{Z_2}$

$Y_{21} = I_2/V_1 \big|_{V_2=0} = \gamma/Z_2$

$Y_{12} = I_1/V_2 \big|_{V_1=0} = 0$

For Y_{21}

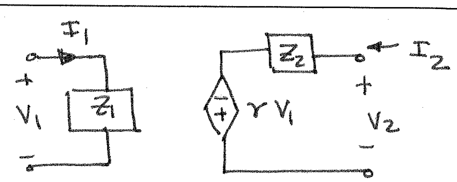


$I_2 = \frac{\gamma V_1}{Z_2}$

$\frac{I_2}{V_1} = \frac{\gamma}{Z_2}$

16.8 Find the Z parameters for the network in Fig. P16.7.
CS

SOLUTION:



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_1$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{-rV_1}{V_1/Z_1} = -rZ_1$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 0$$

16.9 Find the Z parameters for the two-port network shown in Fig. P16.9 and determine the voltage gain of the entire circuit with a 4-kΩ load attached to the output. **PSV**

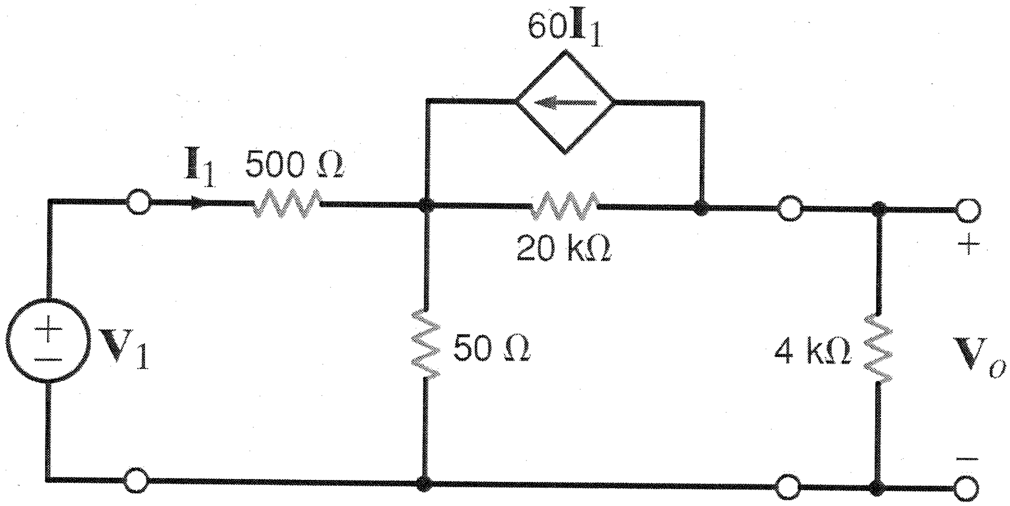
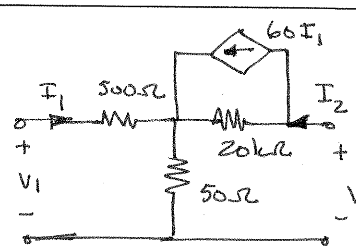


Figure P16.9

SOLUTION:



$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 550\Omega$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 20.05 k\Omega$

$Z_{12} = V_1/I_2\big|_{I_1=0} = 50\Omega$ $\quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{50I_1 - 60I_1(20\times10^3)}{I_1} = -1.2M\Omega$

w/ load

$\left.\begin{aligned} V_1 &= 550I_1 + 50I_2 \\ V_2 &= -1.2\times10^6 I_1 + 20.05\times10^3 I_2 \end{aligned}\right\} \quad V_2 = -4000 I_2$

Solving for I_2/V_1 yields $\quad I_2/V_1 = 16.39\times10^{-3}$

$\boxed{V_o/V_1 = 65.5}$

16.10 Find the Z parameters for the two-port network shown in Fig. P16.10.

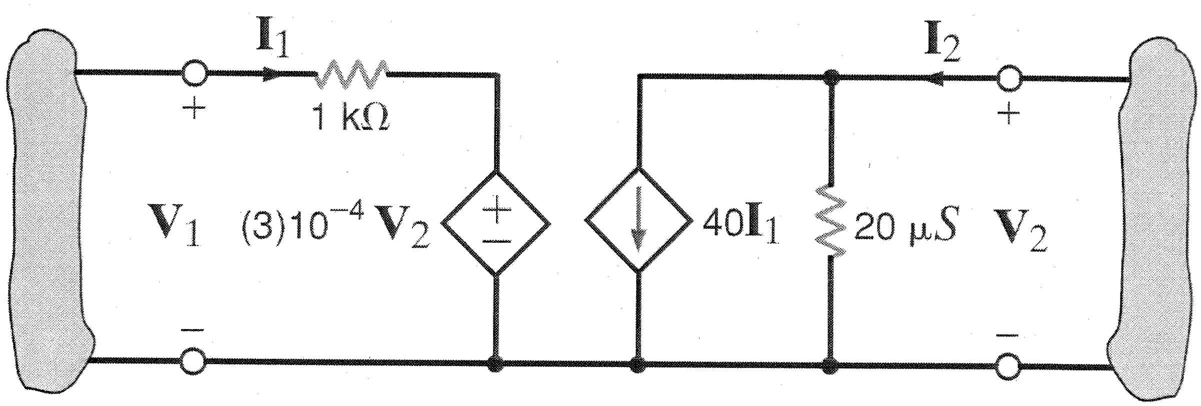


Figure P16.10

SOLUTION: Using h parameters,

$h_{11} = V_1/I_1 \big|_{V_2=0} = 1k\Omega$ $\quad h_{22} = I_2/V_2 \big|_{I_1=0} = 20\mu S$

$h_{12} = V_1/V_2 \big|_{I_1=0} = 3\times10^{-4}$ $\quad h_{21} = I_2/I_1 \big|_{V_2=0} = 40$

$\Delta h = h_{11}h_{22} - h_{12}h_{21} = 8\times10^{-3}$

$Z_{11} = \dfrac{\Delta h}{h_{22}} = 400\,\Omega$ $\quad Z_{12} = \dfrac{h_{12}}{h_{22}} = 15\,\Omega$

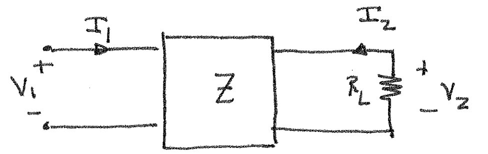
$Z_{21} = \dfrac{-h_{21}}{h_{22}} = -2\times10^6 = -2M\Omega$ $\quad Z_{22} = \dfrac{1}{h_{22}} = 50\,k\Omega$

16.11 Find the voltage gain of the two-port network in Fig. P16.10 if a 12-kΩ load is connected to the output port. CS

SOLUTION:

From P16.10, $Z_{11} = 400\,\Omega$ $Z_{12} = 15\,\Omega$ $Z_{21} = -2\,M\Omega$ $Z_{22} = 50\,k\Omega$

$V_1 = Z_{11} I_1 + Z_{12} I_2$



$R_L = 12\,k\Omega$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$V_2 = -I_2 R_L$

3 equations + unknowns. Eliminate I_1 + find V_2/V_1.

$$\boxed{\frac{V_2}{V_1} = -438}$$

16.12 Find the input impedance of the network in Fig. P16.10.

SOLUTION:

$$Z_{IN} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_{11}$$

from P16.10, $Z_{11} = 400\,\Omega$

$$\boxed{Z_{IN} = 400\,\Omega}$$

16.13 Find the Z parameters of the two-port network in Fig. P16.13.

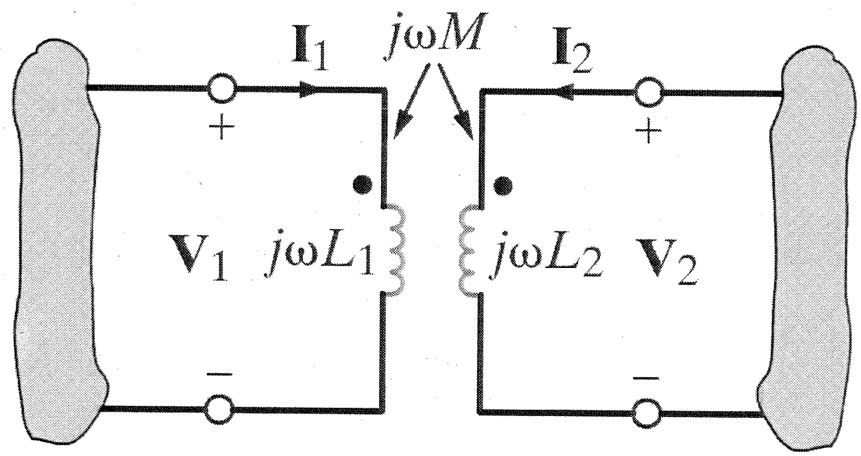


Figure P16.13

SOLUTION:

$V_1 = I_1(j\omega L_1) + I_2(j\omega M)$ $V_2 = I_1(j\omega M) + I_2(j\omega L_2)$

$Z_{11} = j\omega L_1$ $Z_{12} = j\omega M$ $Z_{21} = j\omega M$ $Z_{22} = j\omega L_2$

16.14 Determine the Z parameters for the two-port network in Fig. P16.14.

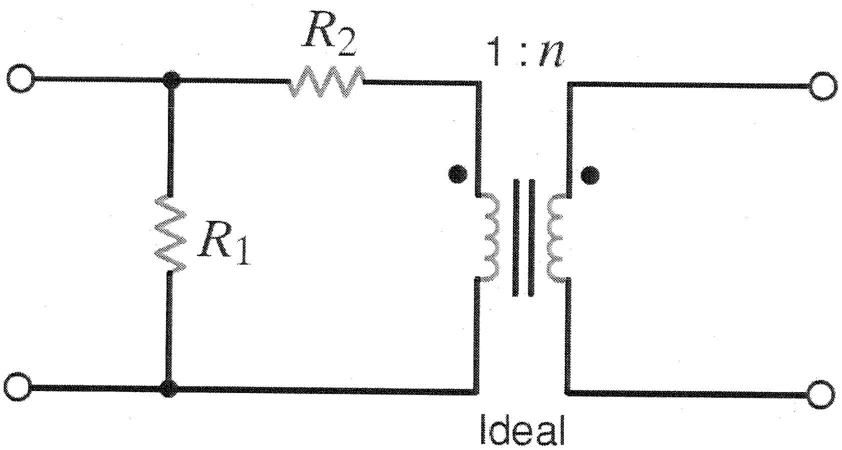
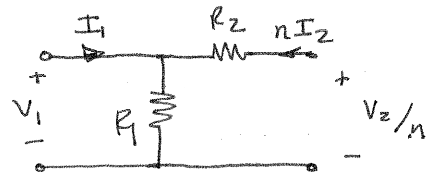


Figure P16.14

SOLUTION:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = R_1$$



$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$V_1 = nI_2R_1 \Rightarrow Z_{12} = nR_1$$

$$Z_{21} = V_2/I_1\Big|_{I_2=0} \Rightarrow \frac{V_2}{n} = I_1R_1 \Rightarrow Z_{21} = nR_1$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Rightarrow \frac{V_2}{n} = I_2n(R_1+R_2) \Rightarrow Z_{22} = n^2(R_1+R_2)$$

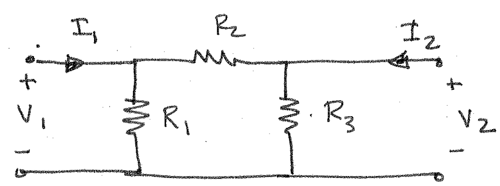
16.15 Draw the circuit diagram (with all passive elements in ohms) for a network that has the following Y parameters:

$$[Y] = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{6} \end{bmatrix}$$

SOLUTION:

$I_1 = V_1 Y_{11} + V_2 Y_{12}$

$I_2 = V_1 Y_{21} + V_2 Y_{22}$



$$I_1 = V_1 \left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2 \left(\frac{1}{R_2}\right)$$

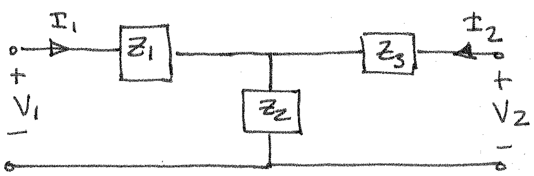
$$I_2 = -\frac{1}{R_2} V_1 + V_2 \left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$\boxed{R_2 = 2\Omega \qquad R_1 = 1\Omega \qquad R_3 = 3\Omega}$

16.16 Draw the circuit diagram for a network that has the following Z parameters:

$$[Z] = \begin{bmatrix} 6 - j2 & 4 - j6 \\ 4 - j6 & 7 + j2 \end{bmatrix}$$

SOLUTION:



$V_1 = I_1 (Z_1 + Z_2) + Z_2 I_2$

$V_2 = + Z_2 I_1 + I_2 (Z_2 + Z_3)$

$Z_2 = 4 - j6\,\Omega$ $\quad Z_1 = 2 + j4\,\Omega$

$Z_3 = 3 + j8\,\Omega$

16.17 Show that the network in Fig. P16.17 does not have a set of Y parameters unless the source has an internal impedance.

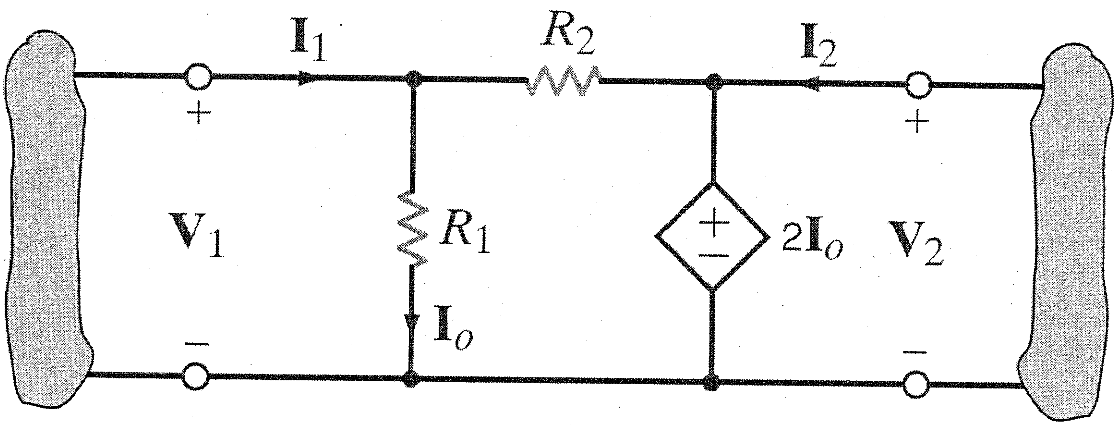
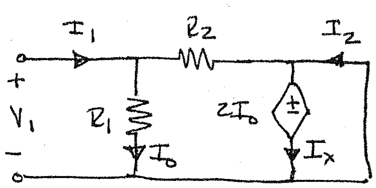


Figure P16.17

SOLUTION:

with $V_2 = 0$



$2I_o = V_2 = 0 \Rightarrow I_o = 0$

$V_1 = I_o R_1 = 0 \Rightarrow V_1 = 0$

$I_1 = \dfrac{V_1}{R_1} + \dfrac{V_1 - V_2}{R_2} = 0$

No power supplied by input port or consumed by R_1 & R_2

thus, $I_x = 0$ & $I_2 = 0$.

$Y_{11} = I_1/V_1 \big|_{V_2=0} = \frac{0}{0}$ undefined! $\quad Y_{21} = I_2/V_1 \big|_{V_2=0} = \frac{0}{0}$ undefined!

with $V_1 = 0$: $\quad I_o = V_1/R_1 = 0 \quad V_2 = 2I_o = 0$

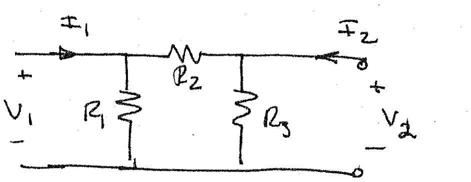
$I_1 = V_1/R_1 + (V_1 - V_2)/R_2 = 0$

no power at input port or consumed by R_1 & $R_2 \Rightarrow I_2 = 0$

$Y_{12} = I_1/V_2 \big|_{V_1=0} = \frac{0}{0}$ undefined $\quad Y_{22} = I_2/V_2 \big|_{V_1=0} = \frac{0}{0}$ undefined!

16.18 Compute the hybrid parameters for the network in Fig. E16.1.

SOLUTION:



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_1 // R_2 = 14\Omega$$

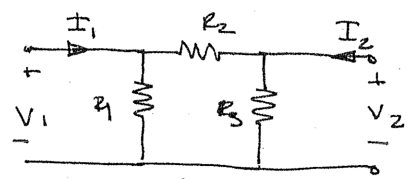
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-R_1}{R_1+R_2} = -\frac{2}{3}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{R_1}{R_1+R_2} = \frac{2}{3}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_3 // (R_1+R_2)} = \frac{1}{9}\ S$$

16.19 Find the hybrid parameters for the network in Fig. 16.3. **PSV**

SOLUTION:



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_1 // R_2 = 2/3\,\Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{R_1}{R_1+R_2} = -\frac{1}{3}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = +\frac{R_1}{R_1+R_2} = \frac{1}{3}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_3 // (R_1+R_2)} = \frac{2}{3}\ S$$

16.20 Consider the network in Fig. P16.20. The two-port network is a hybrid model for a basic transistor. Determine the voltage gain of the entire network, $\mathbf{V}_2/\mathbf{V}_S$, if a source $\mathbf{V}_S$ with internal resistance R_1 is applied at the input to the two-port network and a load R_L is connected at the output port.

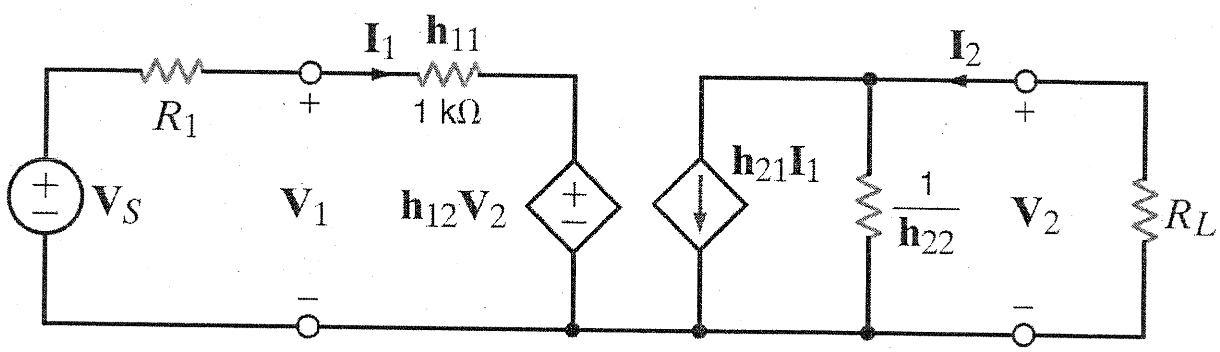


Figure P16.20

SOLUTION:

$$\left.\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right\} \quad \text{and,} \quad \left\{\begin{aligned} V_2 &= -R_L I_2 \Rightarrow I_2 = -V_2/R_L \\ V_S &= I_1 R_1 + V_1 \Rightarrow V_1 = V_S - I_1 R_1 \end{aligned}\right.$$

Now,

$$V_S = I_1 (h_{11} + R_1) + h_{12} V_2$$

$$0 = h_{21} I_1 + V_2 (h_{22} + 1/R_L)$$

$$V_2 = \frac{\begin{vmatrix} h_{11}+R_1 & V_S \\ h_{21} & 0 \end{vmatrix}}{\begin{vmatrix} h_{11}+R_1 & h_{12} \\ h_{21} & h_{22}+\frac{1}{R_L} \end{vmatrix}} \Rightarrow V_2 = \frac{-V_S h_{21}}{(h_{11}+R_1)\left(h_{22}+\frac{1}{R_L}\right) - h_{12} h_{21}}$$

$$\boxed{\frac{V_2}{V_S} = \frac{h_{21} R_L}{h_{12} h_{21} R_L - (1+h_{22} R_L)(h_{11}+R_1)}}$$

16.21 Determine the hybrid parameters for the network shown in Fig. P16.21.

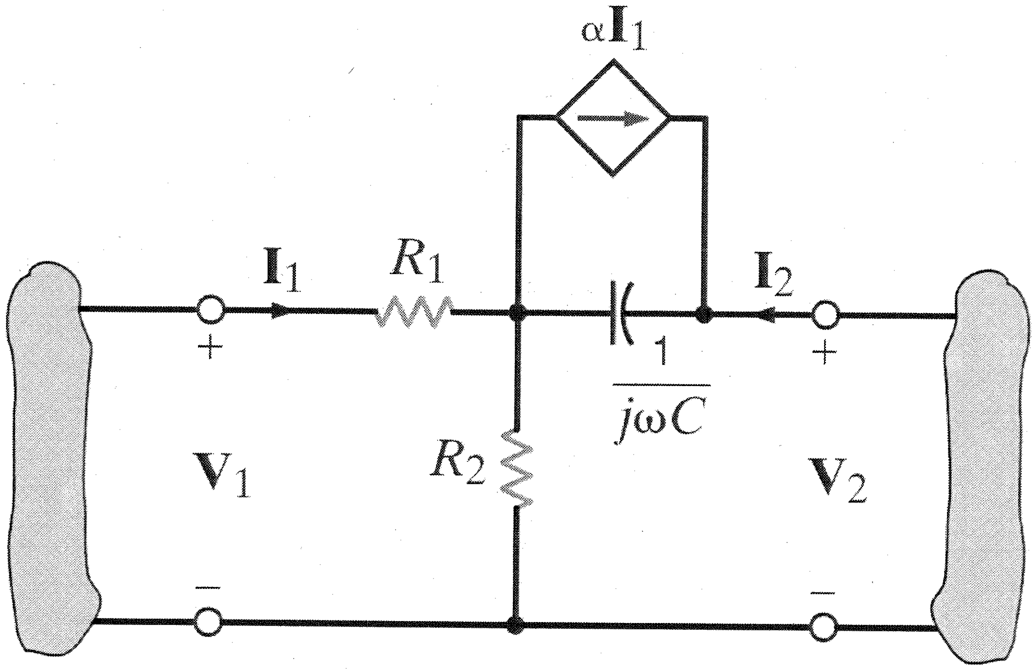
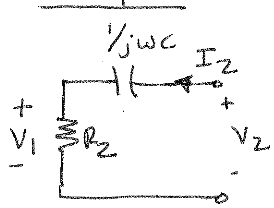


Figure P16.21

SOLUTION:

For $I_1 = 0$



$*$ $h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = \frac{j\omega C R_2}{1+j\omega C R_2}$

$*$ $h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = \frac{j\omega C}{1+j\omega C R_2}$

For $V_2=0$ Use loop analysis!

$(I_2+\alpha I_1)/j\omega C + (I_1+I_2)R_2 = 0$

yields $I_1(\alpha + j\omega C R_2) + I_2(1+j\omega C R_2) = 0$

$h_{21} = I_2/I_1 = -\frac{\alpha + j\omega C R_2}{1+j\omega C R_2}$ $*$

And,

$V_1 = I_1(R_1+R_2) + I_2R_2 = I_1\left\{R_1+R_2 - R_2\left(\frac{\alpha+j\omega C R_2}{1+j\omega C R_2}\right)\right\}$

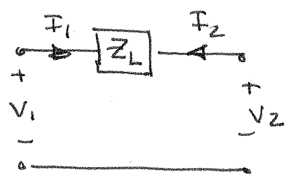
$V_1 = I_1\left\{\frac{R_1+R_2(1-\alpha)+j\omega R_1R_2C}{1+j\omega R_2 C}\right\}$

$h_{11} = \frac{V_1}{I_1} = \frac{R_1+R_2(1-\alpha)+j\omega R_1R_2C}{1+j\omega R_2C}$ $*$

16.22 Find the ABCD parameters for the networks in Fig. P16.1. CS

SOLUTION:

a) $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$

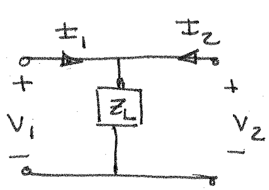


$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = Z_L$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$

b) $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$



$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 0$

$C = \left.I_1/V_2\right|_{I_2=0} = \frac{1}{Z_L}$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$

16.23 Find the transmission parameters for the network in Fig. P16.23.

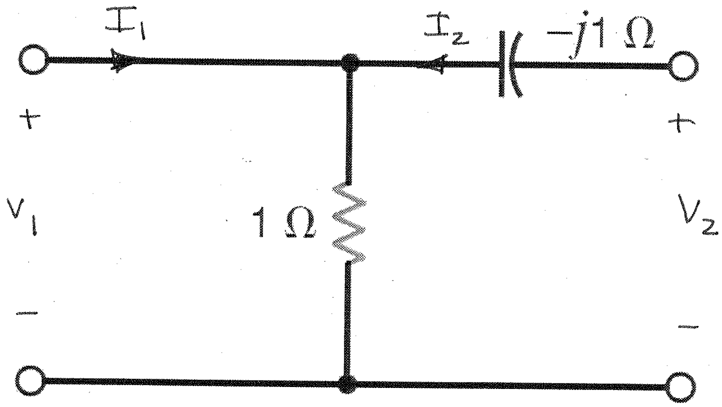


Figure P16.23

SOLUTION:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$$

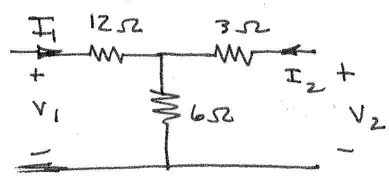
$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = -j1\Omega$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 1\ S$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \Rightarrow \quad -I_2 = I_1\left(\frac{1}{1-j1}\right) \Rightarrow D = 1-j1$$

16.24 Find the transmission parameters for the network shown in Fig. P16.2. **PSV**

SOLUTION:



$A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0} \Rightarrow V_2 = \dfrac{6}{18}V_1 \Rightarrow A = 3$

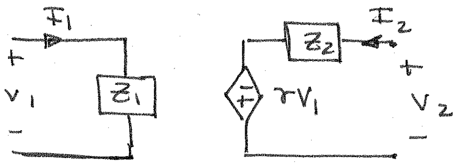
$B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0} \Rightarrow -I_2 = \dfrac{(3//6)V_1}{12+(3//6)}\left(\dfrac{1}{3}\right) \Rightarrow B = 21\Omega$

$C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0} = \dfrac{1}{6}\,S$

$D = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0} \Rightarrow -I_2 = I_1\left(\dfrac{6}{6+3}\right) \Rightarrow D = \dfrac{3}{2}$

16.25 Find the ABCD parameters for the circuit in Fig. P16.7.

SOLUTION:



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \Rightarrow V_2 = -rV_1 \Rightarrow A = -\frac{1}{r}$

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \Rightarrow I_2 = \frac{rV_1}{Z_2} \Rightarrow B = -Z_2/r$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \Rightarrow V_2 = -rV_1$ & $V_1 = I_1 Z_1 \Rightarrow C = -\frac{1}{rZ_1}$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \Rightarrow I_2 = \frac{rV_1}{Z_2}$ & $I_1 = V_1/Z_1 \Rightarrow D = -\frac{Z_2}{rZ_1}$

16.26 Determine the transmission parameters for the network in Fig. P16.26. CS

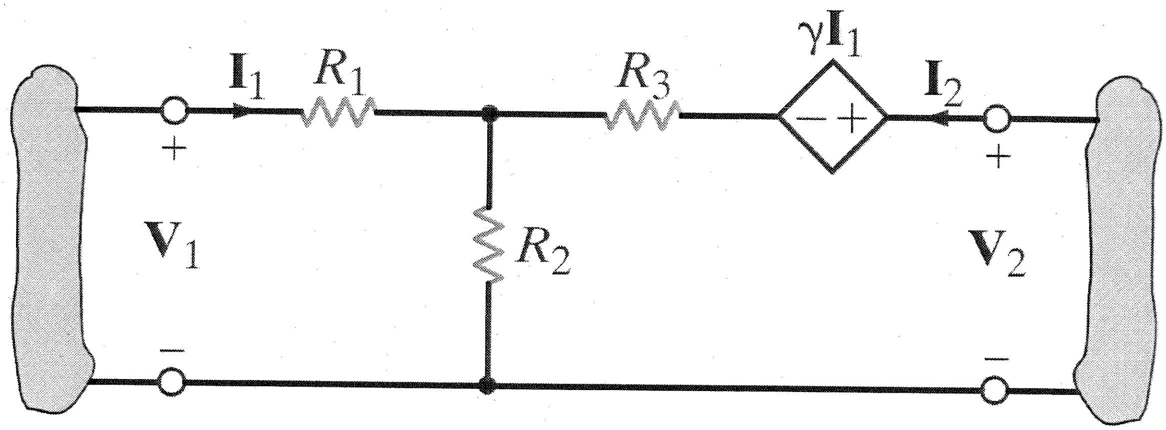


Figure P16.26

SOLUTION: Loop analysis!

$V_1 = I_1(R_1+R_2) + R_2 I_2$

$V_2 = (\gamma + R_2) I_1 + (R_2 + R_3) I_2$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{R_1+R_2}{\gamma + R_1}$ *

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{\gamma + R_2}$ *

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{R_2+R_3}{\gamma+R_2}$ *

So, $I_1 = -I_2(R_2+R_3)/(\gamma+R_2)$

and, $V_1 = I_2\left[R_2 - (R_1+R_2)\frac{R_2+R_3}{\gamma+R_2}\right] = -I_2\left[\frac{R_1R_2+R_1R_3+R_2R_3-\gamma R_2}{\gamma+R_2}\right]$

* $B = \frac{R_1R_2 + R_1R_3 + R_2R_3 - \gamma R_2}{\gamma + R_2}$

16.27 Find the transmission parameters for the circuit in Fig. P16.27.

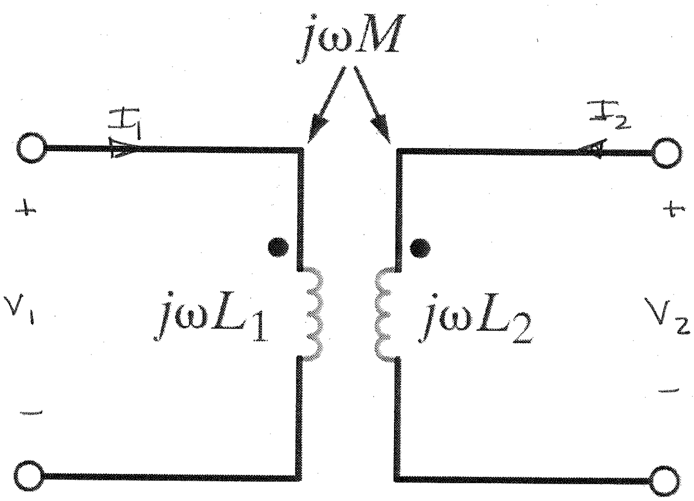


Figure P16.27

SOLUTION:

$V_1 = j\omega L_1 I_1 + j\omega M I_2$

$V_2 = j\omega M I_1 + j\omega L_2 I_2$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = L_1/M$ *

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{j\omega M}$ *

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = L_2/M$ *

or, $I_1 = -I_2\,(L_2/M)$

& $V_1 = -I_2\left(\frac{j\omega L_1 L_2}{M} - j\omega M\right) = -I_2\left(\frac{j\omega L_1 L_2 - j\omega M^2}{M}\right)$

* $B = j\omega\left(\frac{L_1 L_2 - M^2}{M}\right)$

16.28 Given the network in Fig. P16.28, find the transmission parameters for the two-port network and then find $\mathbf{I}_o$ using the terminal conditions.

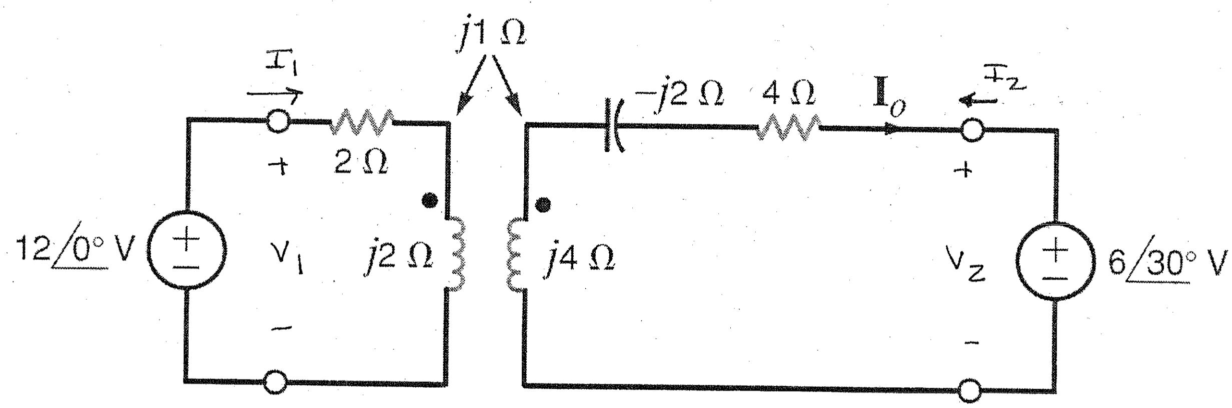


Figure P16.28

SOLUTION:

$V_1 = I_1(2+j2) + j1\,I_2$

$V_2 = j1\,I_1 + I_2(4+j2)$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{2+j2}{j1} = 2-j2$ *

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{4+j2}{j1} = 2-j4$ *

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

$C = \left. I_1/V_2\right|_{I_2=0} = -j1\ S$ *

now, $I_1 = -I_2(2-j4)$

and $V_1 = -I_2[(2-j4)(2+j2) - j1] = -I_2(12-j5)$

* $B = 12-j5\ \Omega$

Terminal conditions, $V_1 = 12\angle 0°\,V$ $V_2 = 6\angle 30°\,V$ $I_o = -I_2$

$$\begin{bmatrix} 2+j2 & -j1 \\ j1 & -4-j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_o \end{bmatrix} = \begin{bmatrix} 12\angle 0° \\ 6\angle 30° \end{bmatrix} \Rightarrow I_o = 0.48\angle 157.6°\ A \ *$$

16.29 Find the input admittance of the two-port in Fig. P16.29 in terms of the Y parameters and the load $\mathbf{Y}_L$.

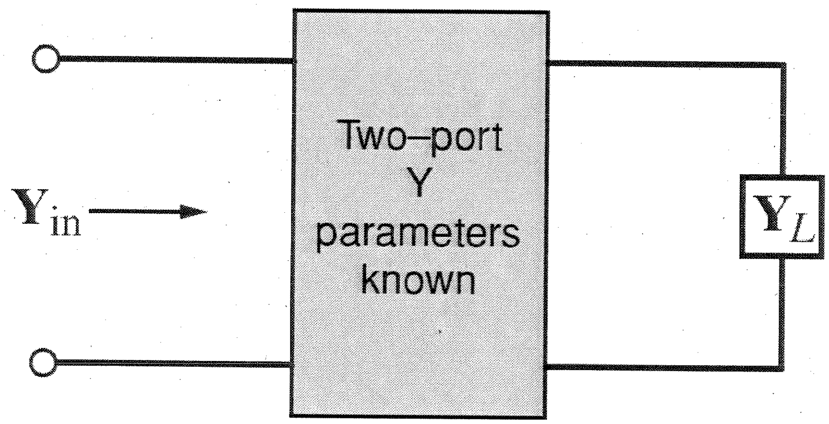


Figure P16.29

SOLUTION:

$$\left.\begin{aligned} I_1 &= V_1 Y_{11} + V_2 Y_{12} \\ I_2 &= Y_{21} V_1 + Y_{22} V_2 \end{aligned}\right\} \qquad I_2 = -V_2 Y_L$$

$$Y_{in} = \frac{I_1}{V_1} = Y_{11} + Y_{12}\left(V_2/V_1\right)$$

$$-V_2 Y_L = Y_{21} V_1 + Y_{22} V_2$$

$$0 = Y_{21} V_1 + V_2(Y_{22} + Y_L) \Rightarrow V_2/V_1 = -\frac{Y_{21}}{Y_{22} + Y_L}$$

$$\boxed{Y_{in} = Y_{11} - \frac{Y_{12} Y_{21}}{Y_{22} + Y_L}}$$

16.30 Find the voltage gain $\mathbf{V}_2/\mathbf{V}_1$ for the network in Fig. P16.30 using the Z parameters.

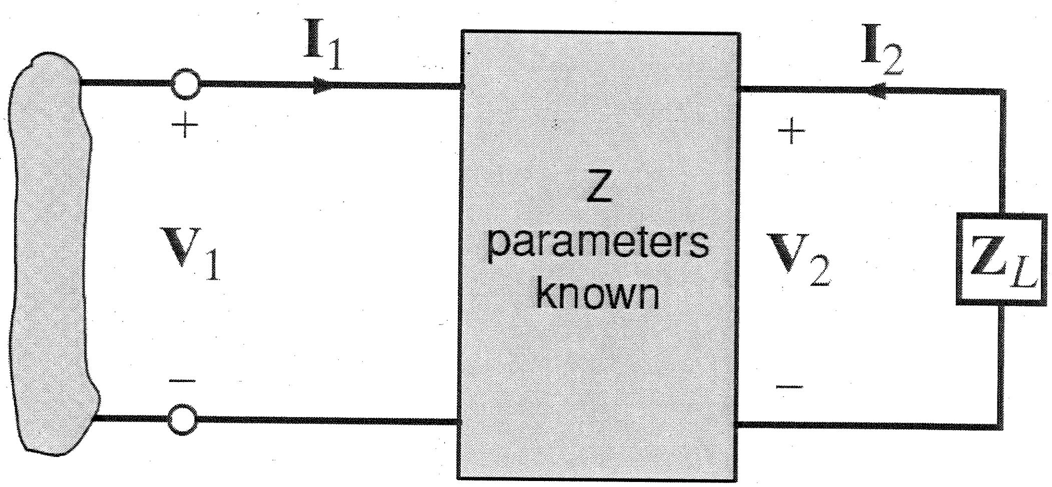


Figure P16.30

SOLUTION:

$V_1 = I_1 Z_{11} + I_2 Z_{12}$ $\quad V_2 = I_1 Z_{21} + I_2 Z_{22}$ $\quad V_2 = -I_2 Z_L \Rightarrow I_2 = -\frac{V_2}{Z_L}$

now, $V_1 = I_1 Z_{11} - V_2 \left(Z_{12}/Z_L \right)$

$V_2 = I_1 Z_{21} - V_2 \left(Z_{22}/Z_L \right) \Rightarrow I_1 = V_2 \frac{(Z_L + Z_{22})}{Z_L Z_{21}}$

and, $V_1 = V_2 \left[\frac{Z_{11}(Z_L + Z_{22})}{Z_L \cdot Z_{21}} - \frac{Z_{12}}{Z_L} \right] = V_2 \left[\frac{Z_{11} Z_L + Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_L Z_{21}} \right]$

$$\boxed{\frac{V_2}{V_1} = \frac{Z_{11} Z_L + Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_L Z_{21}}}$$

16.31 Following are the hybrid parameters for a network.

$$\begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Determine the Y parameters for the network.

SOLUTION:

* $y_{11} = \frac{1}{h_{11}} = \frac{5}{11}\ S$ $\qquad y_{21} = \frac{h_{21}}{h_{11}} = -\frac{2}{11}\ S$ *

* $y_{12} = \frac{-h_{12}}{h_{11}} = -\frac{2}{11}\ S$

$y_{22} = \frac{\Delta_H}{h_{11}}$ $\qquad \Delta_H = \frac{11}{5}\left(\frac{1}{5}\right) - \left(-\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{11}{25} + \frac{4}{25} = \frac{15}{25} = \frac{3}{5}$

* $y_{22} = \frac{3}{11}\ S$

16.32 If the Y parameters for a network are known to be

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

find the Z parameters. CS

SOLUTION:

$$\Delta_Y = \frac{5}{11}\left(\frac{3}{11}\right) - \left(-\frac{2}{11}\right)^2 = \frac{15}{121} - \frac{4}{121} = \frac{1}{11}$$

$z_{11} = y_{22}/\Delta_Y = 3\,\Omega$ $\qquad z_{21} = -y_{21}/\Delta_Y = 2\,\Omega$

$z_{12} = -y_{12}/\Delta_Y = 2\,\Omega$ $\qquad z_{22} = y_{11}/\Delta_Y = 5\,\Omega$

16.33 Find the Z parameters in terms of the ABCD parameters.

SOLUTION:

ABCD Parameters

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

Z Parameters

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$V_2 = I_1/C + (D/C) I_2$

$V_1 = \frac{AI_1}{C} + \frac{AD}{C} I_2 - BI_2 = \frac{A}{C} I_1 + \left(\frac{AD - BC}{C}\right) I_2$

By comparisim,

$Z_{21} = \frac{1}{C}$ $\quad Z_{22} = D/C$ $\quad Z_{11} = A/C$

$Z_{12} = \frac{AD - BC}{C} = \frac{\Delta_T}{C}$

16.34 Find the hybrid parameters in terms of the impedance parameters.

SOLUTION:

$V_1 = Z_{11} I_1 + Z_{12} I_2 \qquad (1)$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \qquad (2) \quad \Rightarrow \quad I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \qquad (3)$

Put (3) into (1)

$V_1 = I_1 \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}} \right) + \frac{Z_{12}}{Z_{22}} V_2 \qquad (4)$

from (3) & (4):

$$\boxed{h_{11} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}} \qquad h_{12} = Z_{12}/Z_{22} \qquad h_{21} = -\frac{Z_{21}}{Z_{22}} \qquad h_{22} = \frac{1}{Z_{22}}}$$

16.35 Find the Y parameters for the network in Fig. P16.35.

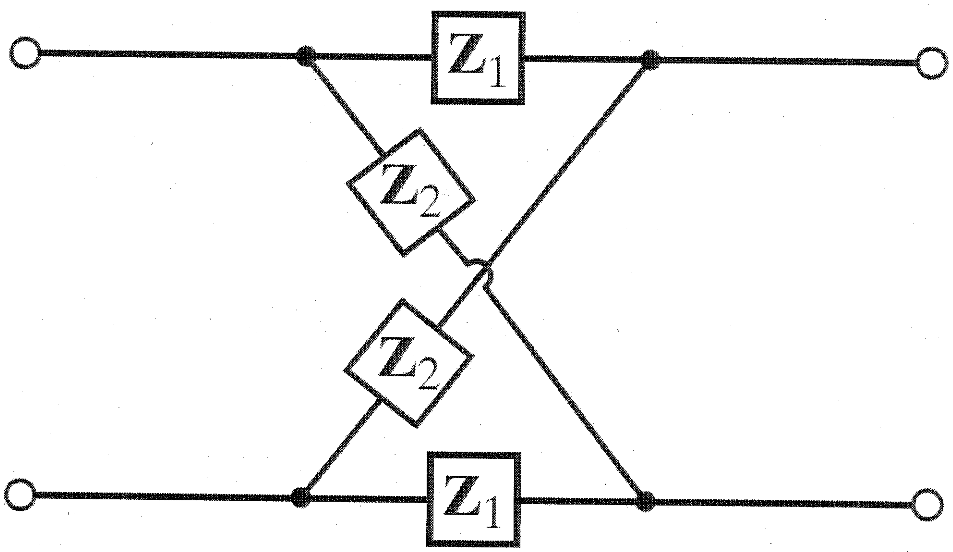
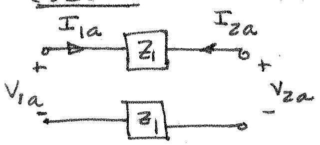


Figure P16.35

SOLUTION: Use parallel connections.

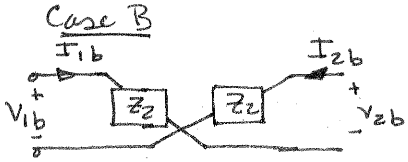
Case A



$$y_{11a} = \frac{1}{2Z_1} \qquad y_{12a} = -\frac{1}{2Z_1}$$

$$y_{21a} = -\frac{1}{2Z_1} \qquad y_{22a} = \frac{1}{2Z_1}$$

Case B



$$y_{11b} = \frac{1}{2Z_2} \qquad y_{12b} = \frac{1}{2Z_2}$$

$$y_{21b} = \frac{1}{2Z_2} \qquad y_{22b} = \frac{1}{2Z_2}$$

Total network: $y_{ij} = y_{ija} + y_{ijb}$

$$y_{11} = \frac{1}{2Z_1} + \frac{1}{2Z_2} \qquad y_{12} = \frac{1}{2Z_2} - \frac{1}{2Z_1}$$

$$y_{21} = \frac{1}{2Z_2} - \frac{1}{2Z_1} \qquad y_{22} = \frac{1}{2Z_1} + \frac{1}{2Z_2}$$

16.36 Determine the Y parameters for the network shown in Fig. P16.36.

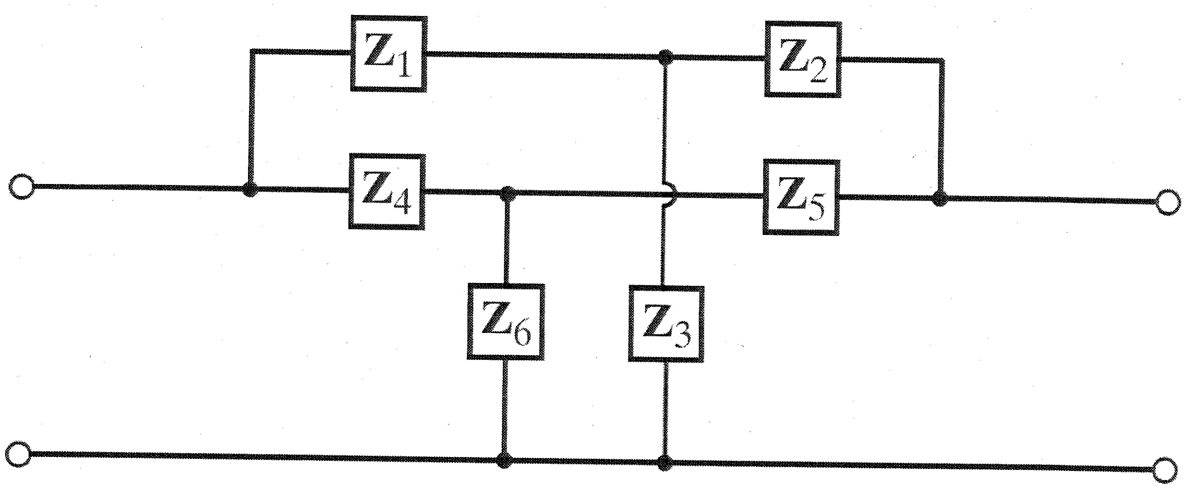
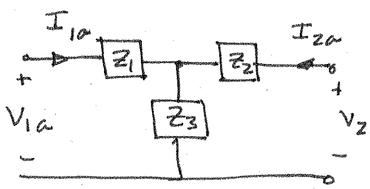


Figure P16.36

SOLUTION: 2 parallel T networks.

Case A



$$y_{11a} = \frac{1}{Z_1 + (Z_2 // Z_3)} = \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3} = \frac{Z_2 + Z_3}{Z_A}$$

$$y_{22a} = \frac{1}{Z_2 + (Z_1 // Z_3)} = \frac{Z_1 + Z_3}{Z_A}$$

$$y_{12a} = \frac{Z_1 // Z_3}{Z_2 + (Z_1 // Z_3)} \left(\frac{-1}{Z_1}\right) = -\frac{Z_3}{Z_A} \qquad y_{21a} = -\frac{Z_3}{Z_A}$$

Similarly for the $Z_4 - Z_5 - Z_6$ T network,

$$y_{11b} = \frac{Z_5 + Z_6}{Z_B} \qquad y_{22b} = \frac{Z_4 + Z_6}{Z_B} \qquad y_{21b} = y_{12b} = -\frac{Z_6}{Z_B} \qquad Z_B = Z_4 Z_5 + Z_5 Z_6 + Z_4 Z_6$$

Total y parameters : $y_{ij} = y_{ija} + y_{ijb}$

$$y_{11} = \frac{Z_2 + Z_3}{Z_A} + \frac{Z_5 + Z_6}{Z_B} \qquad y_{12} = -\frac{Z_3}{Z_A} - \frac{Z_6}{Z_B}$$

$$y_{21} = -\frac{Z_3}{Z_A} - \frac{Z_6}{Z_B} \qquad y_{22} = \frac{Z_1 + Z_3}{Z_A} + \frac{Z_4 + Z_6}{Z_B}$$

16.37 Find the Y parameters of the two-port network in Fig. P16.37. Find the input admittance of the network when the capacitor is connected to the output port. CS

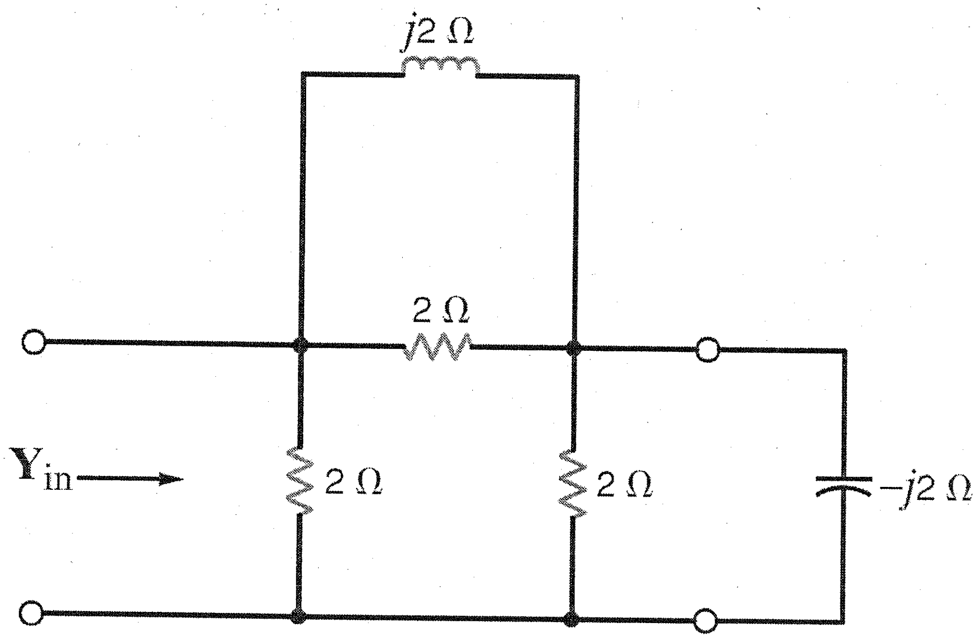
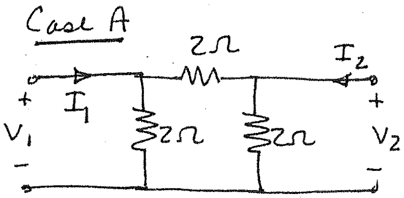


Figure P16.37

SOLUTION: Use 2 parallel networks

Case A

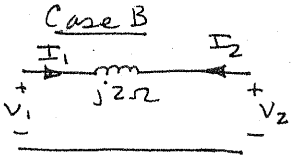


$y_{11a} = \frac{1}{2 // 2} = 1\ S$

$y_{12a} = y_{21a} = -\frac{1}{2}\ S$

$y_{22a} = \frac{1}{2//2} = 1\ S$

Case B



$y_{11b} = \frac{1}{j2}\ S$

$y_{12b} = y_{21b} = -\frac{1}{j2}\ S$

$y_{22} = \frac{1}{j2}\ S$

$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22}+Y_L}$

$Y_{in} = 1 - j\frac{1}{2} - \frac{\frac{1}{4}(1-j1)^2}{(1-j0.5)+j0.5}$

$y_{in} = 1 - \frac{j}{2} - \frac{(1-j1)^2}{4}$

$\boxed{y_{in} = 1\ S}$

$y_{11} = 1 + \frac{1}{j2} = y_{22} \qquad y_{12} = y_{21} = -\frac{1}{2} - \frac{1}{j2} = -\frac{1}{2}(1-j1)$

16.38 Find the Z parameters of the network in Fig. E16.3 by considering the circuit to be a series interconnection of two two-port networks as shown in Fig. P16.38.

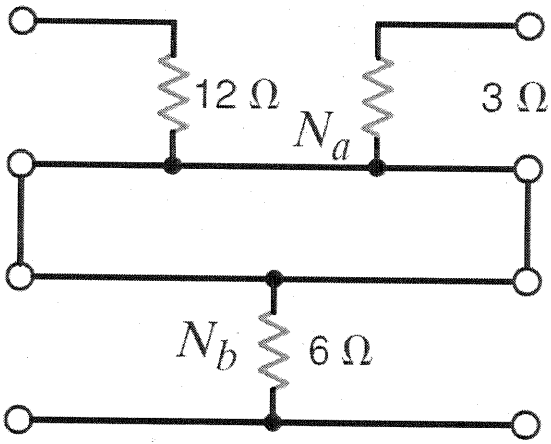
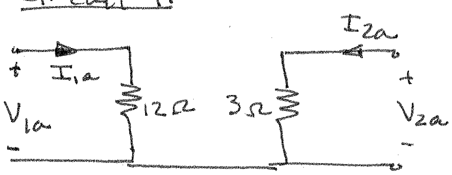


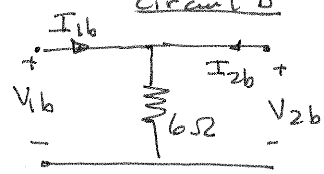
Figure P16.38

SOLUTION: Network consists of 2 series connected subcircuits

Circuit A



Circuit B



$Z_{11a} = 12\Omega$ $\quad Z_{22a} = 3\Omega$ $\quad Z_{11b} = 6\Omega$ $\quad Z_{22b} = 6\Omega$

$Z_{12a} = 0\Omega$ $\quad Z_{21a} = 0\Omega$ $\quad Z_{12b} = 6\Omega$ $\quad Z_{21b} = 6\Omega$

$Z_{ij} = Z_{ija} + Z_{ijb}$

$$\boxed{Z_{11} = 18\Omega \quad Z_{12} = 6\Omega \quad Z_{21} = 6\Omega \quad Z_{22} = 9\Omega}$$

16.39 Find the transmission parameters of the network in Fig. E16.3 by considering the circuit to be a cascade interconnection of three two-port networks as shown in Fig. P16.39. CS

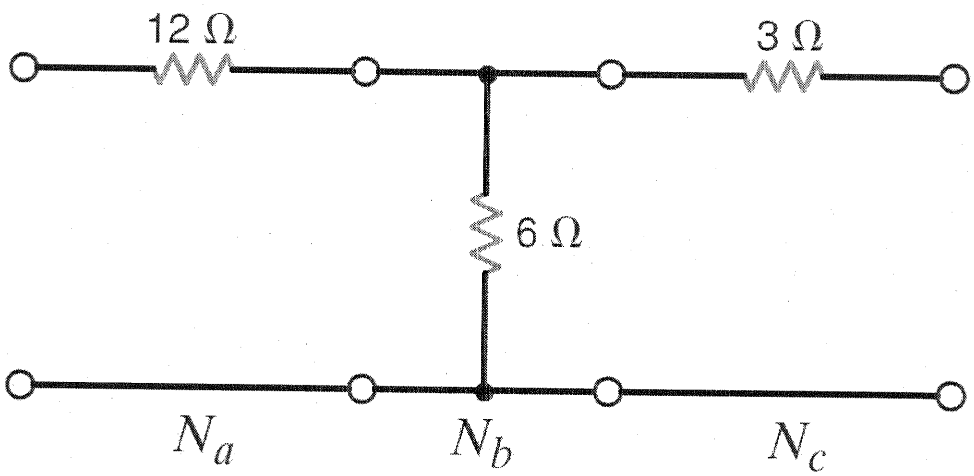
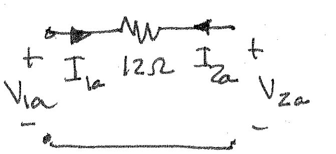


Figure P16.39

SOLUTION: Network consists of 3 two-ports!

Circuit A



$A_a = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$

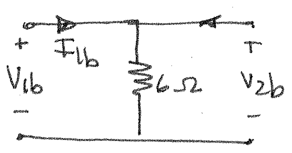
$B_a = \left.V_1/-I_2\right|_{V_2=0} = 12\Omega$

$C_a = \left.I_1/V_2\right|_{I_2=0} = 0$

$D_a = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$

Circuit B

I_{2b}



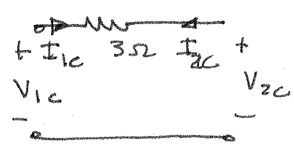
$A_b = 1$

$B_b = 0 \quad (I_{2b} = \infty)$

$C_b = 1/6\ S$

$D_b = 1$

Circuit C



$A_c = 1$

$B_c = 3\Omega$

$C_c = 0$

$D_c = 1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 21 \\ 1/6 & 3/2 \end{bmatrix}$$

16.40 Find the ABCD parameters for the circuit in Fig. P16.40.

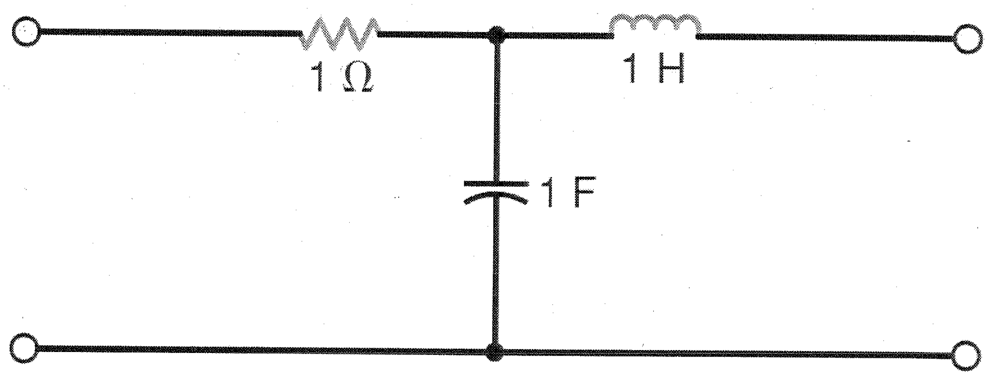
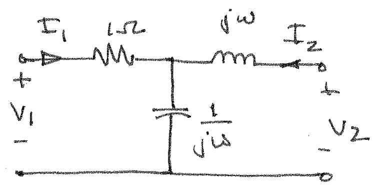


Figure P16.40

SOLUTION:



$$A = V_1/V_2\Big|_{I_2=0} \Rightarrow \frac{V_2}{V_1} = \frac{1/j\omega}{1+1/j\omega} = \frac{1}{j\omega+1}$$

$$* \quad A = j\omega + 1$$

$$B = -V_1/I_2\Big|_{V_2=0} \Rightarrow \frac{-I_2}{V_1} = \frac{j\omega /\!/ 1/j\omega}{(j\omega /\!/ 1/j\omega)+1}\left(\frac{1}{j\omega}\right) = \frac{1}{1+j\omega-\omega^2}$$

$$* \quad B = 1 + j\omega - \omega^2 \ \Omega$$

$$C = I_1/V_2\Big|_{I_2=0} \Rightarrow V_2/I_1 = \frac{1}{j\omega} \Rightarrow C = j\omega \quad *$$

$$D = -I_1/I_2\Big|_{V_2=0} \Rightarrow -I_2/I_1 = \frac{1/j\omega}{j\omega + 1/j\omega} = \frac{1}{1-\omega^2}$$

$$* \quad D = 1 - \omega^2$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} j\omega+1 & 1+j\omega-\omega^2 \\ j\omega & 1-\omega^2 \end{bmatrix}}$$

16.41 Find the Z parameters for the two-port network in Fig. P16.41 and then determine $\mathbf{I}_o$ for the specified terminal conditions.

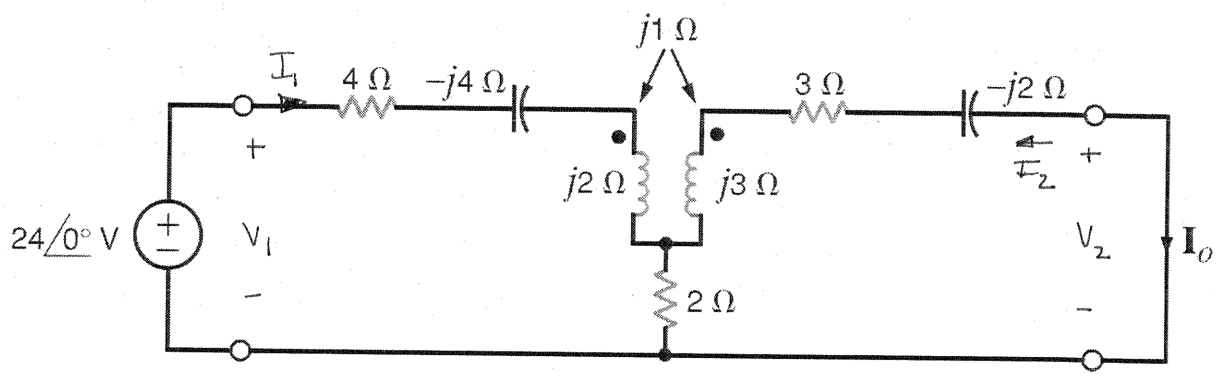


Figure P16.41

SOLUTION:

$V_1 = I_1(6-j2) + I_2(2+j1)$ $\qquad V_2 = I_1(2+j1) + I_2(5+j1)$

$$\boxed{\begin{array}{ll} Z_{11} = 6-j2\,\Omega & Z_{12} = 2+j1\,\Omega \\ Z_{21} = 2+j1\,\Omega & Z_{22} = 5+j1\,\Omega \end{array}}$$

$V_1 = 24\angle 0°\ V$ $\qquad I_2 = -I_o$

$V_2 = 0$

$$\begin{bmatrix} 6-j2 & 2+j1 \\ 2+j1 & 5+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24\angle 0 \\ 0 \end{bmatrix} \Rightarrow \quad I_2 = 1.78\angle -138°\ A$$

$$\boxed{I_o = 1.78\angle 42°\ A}$$

16.42 Determine the output voltage $\mathbf{V}_o$ in the network in Fig. P16.42 if the Z parameters for the two-port are

$$\mathbf{Z} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$ PSV

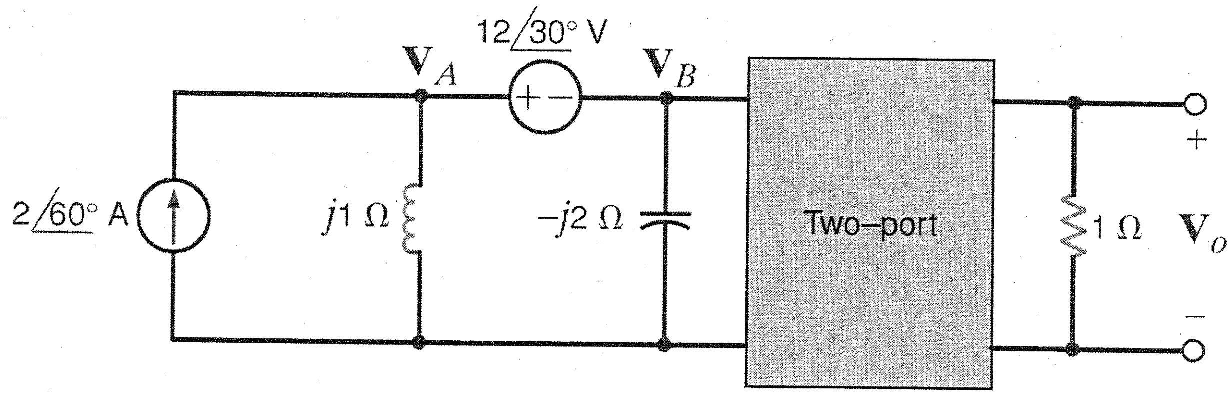
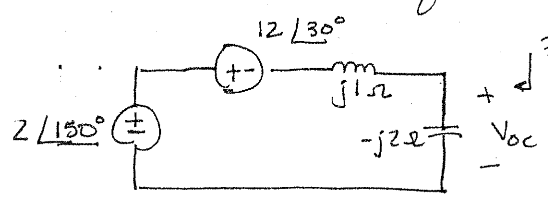


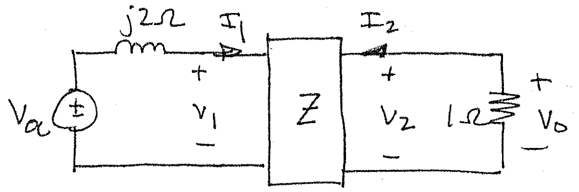
Figure P16.42

SOLUTION: Thevenin eq. at V_B. Use source exchange first.



$Z_{TH} = -j2 // j1 = j2\Omega$

$V_{oc} = (2\angle 150 - 12\angle 30)\left(\frac{-j2}{-j2+j1}\right) = 26.2\angle -158^\circ$



$V_1 = V_{oc} - j2I_1 \qquad V_o = V_2$

$V_o = (-1)I_2 = -I_2$

$V_1 = 3I_1 + 2I_2 = V_{oc} - j2I_1 \Rightarrow I_1(3+j2) + 2I_2 = V_{oc}$

$V_2 = 2I_1 + 3I_2 = -I_2 \Rightarrow I_1 + 2I_2 = 0 \Rightarrow I_1 = -2I_2$

now, $[(3-j2)(-2)+2]I_2 = V_{oc} \Rightarrow I_2 = -V_{oc}/(4+j4)$

$V_o = \frac{V_{oc}}{4+j4}$

$\boxed{V_o = 4.64\angle 157^\circ \text{ V}}$

16FE-1 A two-port network is known to have the following parameters:

$$y_{11} = \frac{1}{14}\text{S} \qquad y_{12} = y_{21} = -\frac{1}{21}\text{S} \qquad y_{22} = \frac{1}{7}\text{S}$$

If a 2-A current source is connected to the input terminals as shown in Fig. 16PFE-1, find the voltage across this current source. CS

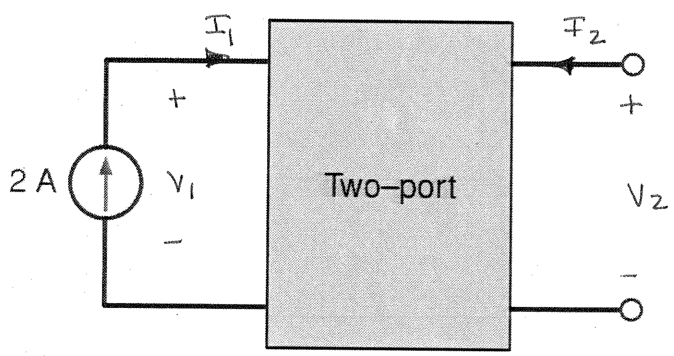


Figure 16PFE-1

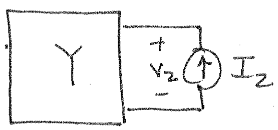
SOLUTION: $I_1 = 2A$ $I_2 = 0$

$$\left.\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 = 2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 = 0 \end{aligned}\right\} \quad \begin{aligned} (1/14)V_1 - (1/21)V_2 = 2 &\Rightarrow 3V_1 - 2V_2 = 84 \\ -(1/21)V_1 + (1/7)V_2 = 0 &\Rightarrow -V_1 + 3V_2 = 0 \end{aligned}$$

find V_1. $\boxed{V_1 = 36\text{ V}}$

16FE-2 Find the Thévenin equivalent circuit at the output terminals of the network in Fig. 16PFE-1.

SOLUTION:



$$Y = \begin{bmatrix} 1/14 & -1/21 \\ -1/21 & 1/7 \end{bmatrix}$$

$$I_1 = 0 = y_{11} V_1 + y_{12} V_2 \Rightarrow V_1 = -V_2 \left(y_{12}/y_{11} \right)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \Rightarrow I_2 = V_2 \left(\frac{y_{11} y_{22} - y_{12} y_{21}}{y_{11}} \right)$$

$$Z_{TH} = \frac{V_2}{I_2} = \frac{y_{11}}{y_{11} y_{22} - y_{12} y_{21}} = \frac{y_{11}}{\Delta_Y}$$

$$\boxed{Z_{TH} = 1.575\,\Omega}$$