

Circuit Variables

Assessment Problems

AP 1.1 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

$$\text{\$100 billion} = \$100 \times 10^9$$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ mins}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} \cdot \frac{1 \text{ sec}}{1000 \text{ ms}} = \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100 \times 10^9}{1 \text{ year}} \cdot \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}} = \frac{100}{31.5576} = \$3.17/\text{ms}$$

AP 1.2 First, we recognize that $1 \text{ ns} = 10^{-9} \text{ s}$. The question then asks how far a signal will travel in 10^{-9} s if it is traveling at 80% of the speed of light. Remember that the speed of light $c = 3 \times 10^8 \text{ m/s}$. Therefore, 80% of c is $(0.8)(3 \times 10^8) = 2.4 \times 10^8 \text{ m/s}$. Now, we use a product of ratios to convert from meters/second to inches/nanosecond:

$$\frac{2.4 \times 10^8 \text{ m}}{1 \text{ s}} \cdot \frac{1 \text{ s}}{10^9 \text{ ns}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{(2.4 \times 10^8)(100)}{(10^9)(2.54)} = \frac{9.45 \text{ in}}{1 \text{ ns}}$$

Thus, a signal traveling at 80% of the speed of light will travel 9.45" in a nanosecond.

- AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. (1.2) to find an expression for charge in terms of current:

$$q(t) = \int_0^t i(x) dx$$

We are given the expression for current, i , which can be substituted into the above expression. To find the total charge, we let $t \rightarrow \infty$ in the integral. Thus we have

$$\begin{aligned} q_{\text{total}} &= \int_0^{\infty} 20e^{-5000x} dx = \frac{20}{-5000} e^{-5000x} \Big|_0^{\infty} = \frac{20}{-5000} (e^{\infty} - e^0) \\ &= \frac{20}{-5000} (0 - 1) = \frac{20}{5000} = 0.004 \text{ C} = 4000 \mu\text{C} \end{aligned}$$

- AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. (1.2):

$$\begin{aligned} i &= \frac{dq}{dt} = \frac{d}{dt} \left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2} \right) e^{-\alpha t} \right] \\ &= \frac{d}{dt} \left(\frac{1}{\alpha^2} \right) - \frac{d}{dt} \left(\frac{t}{\alpha} e^{-\alpha t} \right) - \frac{d}{dt} \left(\frac{1}{\alpha^2} e^{-\alpha t} \right) \\ &= 0 - \left(\frac{1}{\alpha} e^{-\alpha t} - \alpha \frac{t}{\alpha} e^{-\alpha t} \right) - \left(-\alpha \frac{1}{\alpha^2} e^{-\alpha t} \right) \\ &= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha} \right) e^{-\alpha t} \\ &= t e^{-\alpha t} \end{aligned}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t :

$$\frac{di}{dt} = \frac{d}{dt} (t e^{-\alpha t}) = e^{-\alpha t} + t(-\alpha) e^{-\alpha t} = (1 - \alpha t) e^{-\alpha t} = 0$$

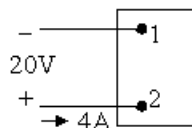
Since $e^{-\alpha t}$ never equals 0 for a finite value of t , the expression equals 0 only when $(1 - \alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t , the current is

$$i = \frac{1}{\alpha} e^{-\alpha/\alpha} = \frac{1}{\alpha} e^{-1}$$

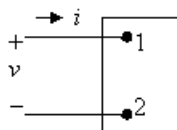
Remember in the problem statement, $\alpha = 0.03679$. Using this value for α ,

$$i = \frac{1}{0.03679} e^{-1} \cong 10 \text{ A}$$

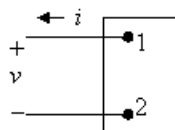
AP 1.5 Start by drawing a picture of the circuit described in the problem statement:



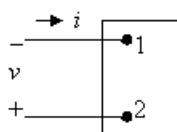
Also sketch the four figures from Fig. 1.6:



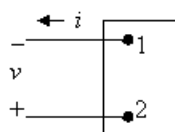
(a)



(b)



(c)



(d)

[a] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

$$(a) \ v = -20 \text{ V}, \quad i = -4 \text{ A}; \quad (b) \ v = -20 \text{ V}, \quad i = 4 \text{ A}$$

$$(c) \ v = 20 \text{ V}, \quad i = -4 \text{ A}; \quad (d) \ v = 20 \text{ V}, \quad i = 4 \text{ A}$$

[b] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80 \text{ W}$. Since the power is greater than 0, the box is absorbing power.

[c] From the calculation in part (b), the box is absorbing 80 W.

AP 1.6 Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. From Eq. (1.3), we know that power is the time rate of change of energy, or $p = \frac{dw}{dt}$. If we know the power, we can find the energy by integrating Eq. (1.3). To begin, find the expression for power:

$$p = vi = (10,000e^{-5000t})(20e^{-5000t}) = 200,000e^{-10,000t} = 2 \times 10^5 e^{-10,000t} \text{ W}$$

Now find the expression for energy by integrating Eq. (1.3):

$$w(t) = \int_0^t p(x) dx$$

Substitute the expression for power, p , above. Note that to find the total energy, we let $t \rightarrow \infty$ in the integral. Thus we have

$$\begin{aligned} w &= \int_0^{\infty} 2 \times 10^5 e^{-10,000x} dx = \frac{2 \times 10^5}{-10,000} e^{-10,000x} \bigg|_0^{\infty} \\ &= \frac{2 \times 10^5}{-10,000} (e^{-\infty} - e^0) = \frac{2 \times 10^5}{-10,000} (0 - 1) = \frac{2 \times 10^5}{10,000} = 20 \text{ J} \end{aligned}$$

AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440 \text{ MW}$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

Chapter Problems

P 1.1 To begin, we calculate the number of pixels that make up the display:

$$n_{\text{pixels}} = (1280)(1024) = 1,310,720 \text{ pixels}$$

Each pixel requires 24 bits of information. Since 8 bits comprise a byte, each pixel requires 3 bytes of information. We can calculate the number of bytes of information required for the display by multiplying the number of pixels in the display by 3 bytes per pixel:

$$n_{\text{bytes}} = \frac{1,310,720 \text{ pixels}}{1 \text{ display}} \cdot \frac{3 \text{ bytes}}{1 \text{ pixel}} = 3,932,160 \text{ bytes/display}$$

Finally, we use the fact that there are 10^6 bytes per MB:

$$\frac{3,932,160 \text{ bytes}}{1 \text{ display}} \cdot \frac{1 \text{ MB}}{10^6 \text{ bytes}} = 3.93 \text{ MB/display}$$

P 1.2 $c = 3 \times 10^8 \text{ m/s}$ so $\frac{1}{2}c = 1.5 \times 10^8 \text{ m/s}$

$$\frac{1.5 \times 10^8 \text{ m}}{1 \text{ s}} = \frac{5 \times 10^6 \text{ m}}{x \text{ s}} \quad \text{so} \quad x = \frac{5 \times 10^6}{1.5 \times 10^8} = 33.3 \text{ ms}$$

P 1.3 We can set up a ratio to determine how long it takes the bamboo to grow $10 \mu\text{m}$. First, recall that $1 \text{ mm} = 10^3 \mu\text{m}$. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250 \text{ mm}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456} \text{ mm/s}$$

Use a ratio to determine the time it takes for the bamboo to grow $10 \mu\text{m}$:

$$\frac{10/3456 \times 10^{-3} \text{ m}}{1 \text{ s}} = \frac{10 \times 10^{-6} \text{ m}}{x \text{ s}} \quad \text{so} \quad x = \frac{10 \times 10^{-6}}{10/3456 \times 10^{-3}} = 3.456 \text{ s}$$

P 1.4 Volume = area \times thickness

$$10^6 = (10 \times 10^6)(\text{thickness})$$

$$\Rightarrow \text{thickness} = \frac{10^6}{10 \times 10^6} = 0.10 \text{ mm}$$

P 1.5
$$\frac{300 \times 10^9 \text{ dollars}}{1 \text{ year}} \cdot \frac{100 \text{ pennies}}{1 \text{ dollar}} \cdot \frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \cdot \frac{1.5 \text{ mm}}{1 \text{ penny}} \cdot \frac{1 \text{ m}}{1000 \text{ mm}}$$

$$= 1426 \text{ m/s}$$

P 1.6 Our approach is as follows: To determine the area of a bit on a track, we need to know the height and width of the space needed to store the bit. The height of the space used to store the bit can be determined from the width of each track on the disk. The width of the space used to store the bit can be determined by calculating the number of bits per track, calculating the circumference of the inner track, and dividing the number of bits per track by the circumference of the track. The calculations are shown below.

$$\text{Width of track} = \frac{1 \text{ in}}{77 \text{ tracks}} \frac{25,400 \mu\text{m}}{\text{in}} = 329.87 \mu\text{m/track}$$

$$\text{Bits on a track} = \frac{1.4 \text{ MB } 8 \text{ bits}}{2 \text{ sides } \text{byte}} \frac{1 \text{ side}}{77 \text{ tracks}} = 72,727.273 \text{ bits/track}$$

$$\text{Circumference of inner track} = 2\pi(1/2'')(25,400 \mu\text{m/in}) = 79,796.453 \mu\text{m}$$

$$\text{Width of bit on inner track} = \frac{79,796.453 \mu\text{m}}{72,727.273 \text{ bits}} = 1.0972 \mu\text{m/bit}$$

$$\text{Area of bit on inner track} = (1.0972)(329.87) = 361.934 \mu\text{m}^2$$

P 1.7 $C/m^3 = 1.6022 \times 10^{-19} \times 10^{29} = 1.6022 \times 10^{10} C/m^3$

$$C/m = (1.6022 \times 10^{10})(5.4 \times 10^{-4}) = 8.652 \times 10^6 C/m$$

Therefore, $(8.652 \times 10^6) \frac{C}{m} \times \text{ave vel} \left(\frac{m}{s} \right) = i$

Thus, average velocity $= \frac{1400}{8.652} \times 10^{-6} = 161.81 \mu m/s$

P 1.8 [a] $n = \frac{20 \times 10^{-6} C/s}{1.6022 \times 10^{-19} C/elec} = 1.25 \times 10^{14} \text{ elec/s}$

[b] $m = 3303 \text{ mi} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{2.54 \text{ cm}}{1 \text{ in}} \cdot \frac{10^4 \mu m}{1 \text{ cm}} = 5.32 \times 10^{12} \mu m$

Therefore, $\frac{n}{m} = 23.5$

The number of electrons/second is approximately 23.5 times the number of micrometers between Sydney and San Francisco.

P 1.9 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 20 \cos 5000t$$

Therefore, $dq = 20 \cos 5000t dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 20 \int_0^t \cos 5000y dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 20 \frac{\sin 5000y}{5000} \Big|_0^t = \frac{20}{5000} \sin 5000t - \frac{20}{5000} \sin 5000(0) = \frac{20}{5000} \sin 5000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 4 \times 10^{-3} \sin 5000t C = 4 \sin 5000t \text{ mC}$

P 1.10 $w = qV = (1.6022 \times 10^{-19})(9) = 14.42 \times 10^{-19} = 1.442 \text{ aJ}$

P 1.11 $p = (6)(100 \times 10^{-3}) = 0.6 \text{ W}; \quad 3 \text{ hr} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} = 10,800 \text{ s}$

$$w(t) = \int_0^t p \, dt \quad w(10,800) = \int_0^{10,800} 0.6 \, dt = 0.6(10,800) = 6480 \text{ J}$$

P 1.12 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = vi$, since the current i is flowing into the + terminal of the voltage v . Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

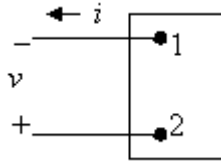
[a] $p = (20)(15) = 300 \text{ W}$ 300 W from A to B

[b] $p = (100)(-5) = -500 \text{ W}$ 500 W from B to A

[c] $p = (-50)(4) = -200 \text{ W}$ 200 W from B to A

[d] $p = (-25)(-16) = 400 \text{ W}$ 400 W from A to B

P 1.13 [a]



$$p = vi = (-20)(5) = -100 \text{ W}$$

Power is being delivered by the box.

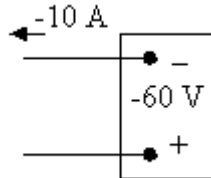
[b] Leaving

[c] Gaining

P 1.14 [a] $p = vi = (-20)(-5) = 100 \text{ W}$, so power is being absorbed by the box.

[b] Entering

[c] Losing



P 1.15 [a] In Car A, the current i is in the direction of the voltage drop across the 12 V battery (the current i flows into the + terminal of the battery of Car A).

Therefore using the passive sign convention, $p = vi = (-40)(12) = -480 \text{ W}$.

Since the power is negative, the battery in Car A is generating power, so Car B must have the "dead" battery.

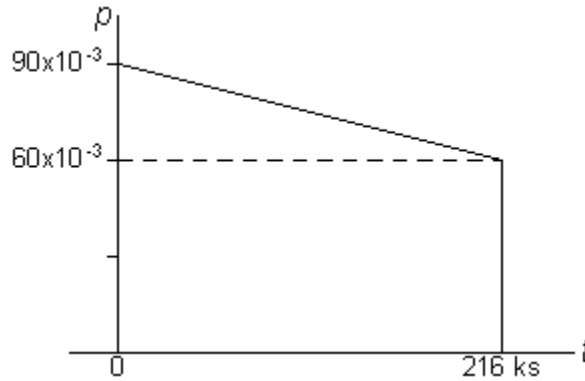
$$\text{[b]} \quad w(t) = \int_0^t p \, dx; \quad 1.5 \text{ min} = 1.5 \cdot \frac{60 \text{ s}}{1 \text{ min}} = 90 \text{ s}$$

$$w(90) = \int_0^{90} 480 \, dx$$

$$w = 480(90 - 0) = 480(90) = 43,200 \text{ J} = 43.2 \text{ kJ}$$

$$\text{P 1.16} \quad p = vi; \quad w = \int_0^t p \, dx$$

Since the energy is the area under the power vs. time plot, let us plot p vs. t .



Note that in constructing the plot above, we used the fact that $60 \text{ hr} = 216,000 \text{ s} = 216 \text{ ks}$

$$p(0) = (6)(15 \times 10^{-3}) = 90 \times 10^{-3} \text{ W}$$

$$p(216 \text{ ks}) = (4)(15 \times 10^{-3}) = 60 \times 10^{-3} \text{ W}$$

$$w = (60 \times 10^{-3})(216 \times 10^3) + \frac{1}{2}(90 \times 10^{-3} - 60 \times 10^{-3})(216 \times 10^3) = 16.2 \text{ kJ}$$

P 1.17 [a] To find the power at $625 \mu\text{s}$, we substitute this value of time into both the equations for $v(t)$ and $i(t)$ and multiply the resulting numbers to get $p(625 \mu\text{s})$:

$$v(625 \mu\text{s}) = 50e^{-1600(625 \times 10^{-6})} - 50e^{-400(625 \times 10^{-6})} = 18.394 - 38.94 = -20.546 \text{ V}$$

$$i(625 \mu\text{s}) = 5 \times 10^{-3}e^{-1600(625 \times 10^{-6})} - 5 \times 10^{-3}e^{-400(625 \times 10^{-6})}$$

$$= 0.0018394 - 0.003894 = -0.0020546 \text{ A}$$

$$p(625 \mu\text{s}) = (-20.546)(-0.0020546) = 42.2 \text{ mW}$$

[b] To find the energy at $625 \mu\text{s}$, we need to integrate the equation for $p(t)$ from 0 to $625 \mu\text{s}$. To start, we need an expression for $p(t)$:

$$p(t) = v(t)i(t) = (50)(5 \times 10^{-3})(e^{-1600t} - e^{-400t})(e^{-1600t} - e^{-400t})$$

$$= \frac{1}{4}(e^{-3200t} - 2e^{-2000t} + e^{-800t})$$

Now we integrate this expression for $p(t)$ to get an expression for $w(t)$. Note we substitute x for t on the right side of the integral.

$$\begin{aligned} w(t) &= \frac{1}{4} \int_0^t (e^{-3200x} - 2e^{-2000x} + e^{-800x}) dx \\ &= \frac{1}{4} \left[\frac{e^{-3200x}}{-3200} + \frac{e^{-2000x}}{1000} - \frac{e^{-800x}}{800} \right] \Big|_0^t \\ &= \frac{1}{4} \left[\frac{e^{-3200t}}{-3200} + \frac{e^{-2000t}}{1000} - \frac{e^{-800t}}{800} - \left(\frac{1}{-3200} + \frac{1}{1000} - \frac{1}{800} \right) \right] \\ &= \frac{1}{4} \left[\frac{e^{-3200t}}{-3200} + \frac{e^{-2000t}}{1000} - \frac{e^{-800t}}{800} + 5.625 \times 10^{-4} \right] \end{aligned}$$

Finally, substitute $t = 625 \mu\text{s}$ into the equation for $w(t)$:

$$\begin{aligned} w(625 \mu\text{s}) &= \frac{1}{4} [-4.2292 \times 10^{-5} + 2.865 \times 10^{-4} - 7.5816 \times 10^{-4} + 5.625 \times 10^{-4}] \\ &= 12.137 \mu\text{J} \end{aligned}$$

[c] To find the total energy, we let $t \rightarrow \infty$ in the above equation for $w(t)$. Note that this will cause all expressions of the form e^{-nt} to go to zero, leaving only the constant term 5.625×10^{-4} . Thus,

$$w_{\text{total}} = \frac{1}{4} [5.625 \times 10^{-4}] = 140.625 \mu\text{J}$$

P 1.18 [a] $v(20 \text{ ms}) = 100e^{-1} \sin 3 = 5.19 \text{ V}$
 $i(20 \text{ ms}) = 20e^{-1} \sin 3 = 1.04 \text{ A}$ $p(20 \text{ ms}) = vi = 5.39 \text{ W}$

[b] $p = vi = 2000e^{-100t} \sin^2 150t$
 $= 2000e^{-100t} \left[\frac{1}{2} - \frac{1}{2} \cos 300t \right]$
 $= 1000e^{-100t} - 1000e^{-100t} \cos 300t$
 $w = \int_0^\infty 1000e^{-100t} dt - \int_0^\infty 1000e^{-100t} \cos 300t dt$
 $= 1000 \frac{e^{-100t}}{-100} \Big|_0^\infty$
 $\quad - 1000 \left\{ \frac{e^{-100t}}{(100)^2 + (300)^2} [-100 \cos 300t + 300 \sin 300t] \right\} \Big|_0^\infty$
 $= 10 - 1000 \left[\frac{100}{1 \times 10^4 + 9 \times 10^4} \right] = 10 - 1$
 $w = 9 \text{ J}$

P 1.19 [a] $0 \text{ s} \leq t < 1 \text{ s}$:

$$v = 5 \text{ V}; \quad i = 20t \text{ A}; \quad p = 100t \text{ W}$$

 $1 \text{ s} < t \leq 2 \text{ s}$:

$$v = 0 \text{ V}; \quad i = 20 \text{ A}; \quad p = 0 \text{ W}$$

 $2 \text{ s} \leq t < 3 \text{ s}$:

$$v = 0 \text{ V}; \quad i = 20 \text{ A}; \quad p = 0 \text{ W}$$

 $3 \text{ s} < t \leq 4 \text{ s}$:

$$v = -5 \text{ V}; \quad i = 80 - 20t \text{ A}; \quad p = -400 + 100t \text{ W}$$

 $4 \text{ s} \leq t < 5 \text{ s}$:

$$v = -5 \text{ V}; \quad i = 80 - 20t \text{ A}; \quad p = -400 + 100t \text{ W}$$

 $5 \text{ s} < t \leq 6 \text{ s}$:

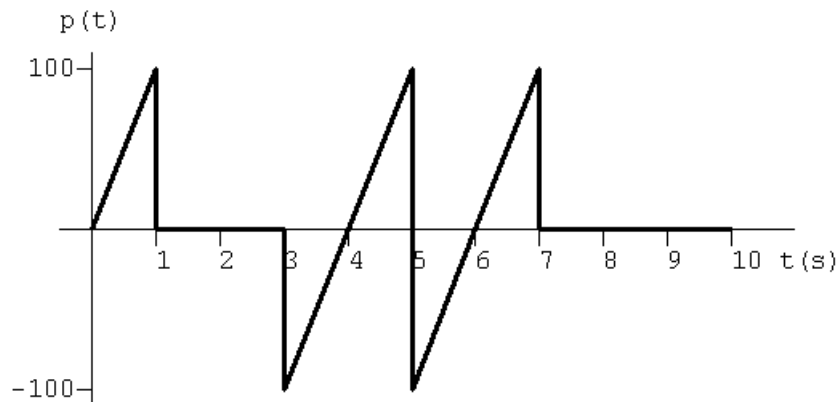
$$v = 5 \text{ V}; \quad i = -120 + 20t \text{ A}; \quad p = -600 + 100t \text{ W}$$

 $6 \text{ s} \leq t < 7 \text{ s}$:

$$v = 5 \text{ V}; \quad i = -120 + 20t \text{ A}; \quad p = -600 + 100t \text{ W}$$

 $t > 7 \text{ s}$:

$$v = 0 \text{ V}; \quad i = 20 \text{ A}; \quad p = 0 \text{ W}$$



[b] Calculate the area under the curve from zero up to the desired time:

$$w(1) = \frac{1}{2}(1)(100) = 50 \text{ J}$$

$$w(6) = \frac{1}{2}(1)(100) - \frac{1}{2}(1)(100) + \frac{1}{2}(1)(100) - \frac{1}{2}(1)(100) = 0 \text{ J}$$

$$w(10) = \frac{1}{2}(1)(100) - \frac{1}{2}(1)(100) + \frac{1}{2}(1)(100) - \frac{1}{2}(1)(100) + \frac{1}{2}(1)(100) = 50 \text{ J}$$

P 1.20 [a] $p = vi = (100e^{-500t})(0.02 - 0.02e^{-500t}) = (2e^{-500t} - 2e^{-1000t}) \text{ W}$

$$\frac{dp}{dt} = -1000e^{-500t} + 2000e^{-1000t} = 0 \quad \text{so} \quad 2e^{-1000t} = e^{-500t}$$

$$2 = e^{500t} \quad \text{so} \quad \ln 2 = 500t \quad \text{thus} \quad p \text{ is maximum at } t = 1.4 \text{ ms}$$

$$p_{\max} = p(1.4 \text{ ms}) = 0.5 \text{ W}$$

$$\begin{aligned} \text{[b]} \quad w &= \int_0^\infty [2e^{-500t} - 2e^{-1000t}] dt = \left[\frac{2}{-500} e^{-500t} - \frac{2}{-1000} e^{-1000t} \right]_0^\infty \\ &= \frac{4}{1000} - \frac{2}{1000} = 2 \text{ mJ} \end{aligned}$$

P 1.21 [a] $p = vi = 200 \cos(500\pi t) 4.5 \sin(500\pi t) = 450 \sin(1000\pi t) \text{ W}$

Therefore, $p_{\max} = 450 \text{ W}$

[b] $p_{\max}(\text{extracting}) = 450 \text{ W}$

[c]

$$\begin{aligned} p_{\text{avg}} &= \frac{1}{4 \times 10^{-3}} \int_0^{4 \times 10^{-3}} 450 \sin(1000\pi x) dx = \frac{450}{4 \times 10^{-3}} \left[\frac{-\cos 1000\pi t}{1000\pi} \right]_0^{4 \times 10^{-3}} \\ &= \frac{-450}{4\pi} [\cos 4\pi - \cos 0] = \frac{-450}{4\pi} [1 - 1] = 0 \text{ W} \end{aligned}$$

[d] $p_{\text{avg}} = \frac{-450}{4\pi} [\cos 15\pi - \cos 0] = \frac{-450}{4\pi} [-1 - 1] = \frac{900}{4\pi} = 71.62 \text{ W}$

P 1.22 [a] $q =$ area under i vs. t plot

$$\begin{aligned} &= \frac{6(5000)}{2} + 6(5000) + \frac{6(10,000)}{2} + 8(15,000) + \frac{8(5000)}{2} \\ &= 15,000 + 30,000 + 30,000 + 120,000 + 20,000 = 215,000 \text{ C} \end{aligned}$$

[b] $w = \int p dt = \int vi dt$

$$v = 0.2 \times 10^{-3}t + 8 \quad 0 \leq t \leq 20 \text{ ks}$$

$$0 \leq t \leq 5000 \text{ s}$$

$$i = 20 - 1.2 \times 10^{-3}t$$

$$p = (8 + 0.2 \times 10^{-3}t)(20 - 1.2 \times 10^{-3}t)$$

$$= 160 - 5.6 \times 10^{-3}t - 2.4 \times 10^{-7}t^2$$

$$\begin{aligned} w_1 &= \int_0^{5000} (160 - 5.6 \times 10^{-3}t - 2.4 \times 10^{-7}t^2) dt \\ &= \left(160t - \frac{5.6 \times 10^{-3}}{2}t^2 - \frac{2.4 \times 10^{-7}}{3}t^3 \right) \bigg|_0^{5000} = 720 \text{ kJ} \end{aligned}$$

$$5000 \leq t \leq 15,000 \text{ s}$$

$$i = 17 - 0.6 \times 10^{-3}t$$

$$p = (8 + 0.2 \times 10^{-3}t)(17 - 0.6 \times 10^{-3}t)$$

$$= 136 - 1.4 \times 10^{-3}t - 1.2 \times 10^{-7}t^2$$

$$\begin{aligned} w_2 &= \int_{5000}^{15,000} (136 - 1.4 \times 10^{-3}t - 1.2 \times 10^{-7}t^2) dt \\ &= \left(136t - \frac{1.4 \times 10^{-3}}{2}t^2 - \frac{1.2 \times 10^{-7}}{3}t^3 \right) \bigg|_{5000}^{15,000} = 1090 \text{ kJ} \end{aligned}$$

$$15,000 \leq t \leq 20,000 \text{ s}$$

$$i = 32 - 1.6 \times 10^{-3}t$$

$$p = (8 + 0.2 \times 10^{-3}t)(32 - 1.6 \times 10^{-3}t)$$

$$= 256 - 6.4 \times 10^{-3}t - 3.2 \times 10^{-7}t^2$$

$$\begin{aligned} w_3 &= \int_{15,000}^{20,000} (256 - 6.4 \times 10^{-3}t - 3.2 \times 10^{-7}t^2) dt \\ &= \left(256t - \frac{6.4 \times 10^{-3}}{2}t^2 - \frac{3.2 \times 10^{-7}}{3}t^3 \right) \bigg|_{15,000}^{20,000} = 226,666.67 \text{ J} \end{aligned}$$

$$w_T = w_1 + w_2 + w_3 = 720,000 + 1,090,000 + 226,666.67 = 2036.67 \text{ kJ}$$

P 1.23 [a]
$$\begin{aligned} p &= vi = [10^4t + 5]e^{-400t}[(40t + 0.05)e^{-400t}] \\ &= 400 \times 10^3 t^2 e^{-800t} + 700t e^{-800t} + 0.25e^{-800t} \\ &= e^{-800t}[400,000t^2 + 700t + 0.25] \\ \frac{dp}{dt} &= \{e^{-800t}[800 \times 10^3 t + 700] - 800e^{-800t}[400,000t^2 + 700t + 0.25]\} \\ &= [-3,200,000t^2 + 2400t + 5]100e^{-800t} \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $3,200,000t^2 - 2400t - 5 = 0$
so p_{\max} occurs at $t = 1.68 \text{ ms}$.

[b]
$$\begin{aligned} p_{\max} &= [400,000(.00168)^2 + 700(.00168) + 0.25]e^{-800(.00168)} \\ &= 666.34 \text{ mW} \end{aligned}$$

[c]
$$\begin{aligned} w &= \int_0^t p dx \\ w &= \int_0^t 400,000x^2 e^{-800x} dx + \int_0^t 700x e^{-800x} dx + \int_0^t 0.25e^{-800x} dx \\ &= \frac{400,000e^{-800x}}{-512 \times 10^6} [64 \times 10^4 x^2 + 1600x + 2] \bigg|_0^t + \\ &\quad \frac{700e^{-800x}}{64 \times 10^4} (-800x - 1) \bigg|_0^t + 0.25 \frac{e^{-800x}}{-800} \bigg|_0^t \end{aligned}$$

When $t \rightarrow \infty$ all the upper limits evaluate to zero, hence

$$w = \frac{(400,000)(2)}{512 \times 10^6} + \frac{700}{64 \times 10^4} + \frac{0.25}{800} = 2.97 \text{ mJ}.$$

P 1.24 [a] We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t . The calculations are shown below:

$$p = 0 \quad t < 0, \quad p = 0 \quad t > 40 \text{ s}$$

$$p = vi = (t - 0.025t^2)(4 - 0.2t) = 4t - 0.3t^2 + 0.005t^3 \text{ W} \quad 0 < t < 40 \text{ s}$$

$$\frac{dp}{dt} = 4 - 0.6t + 0.015t^2 = 0$$

Use a calculator to find the two solutions to this quadratic equation:

$$t_1 = 8.453 \text{ s}; \quad t_2 = 31.547 \text{ s}$$

Now we must find which of these two times gives the minimum power by substituting each of these values for t into the equation for $p(t)$:

$$p(t_1) = (8.453 - 0.025(8.453)^2)(4 - 0.2 \cdot 8.453) = 15.396 \text{ W}$$

$$p(t_2) = (31.547 - 0.025(31.547)^2)(4 - 0.2 \cdot 31.547) = -15.396 \text{ W}$$

Therefore, maximum power is being delivered at $t = 8.453 \text{ s}$.

[b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\max} = 15.396 \text{ W}$ (delivered)

[c] As we saw in part (a), the other “maximum” power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 31.547 \text{ s}$.

[d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\max\text{ext}} = 15.396 \text{ W}$ (extracted)

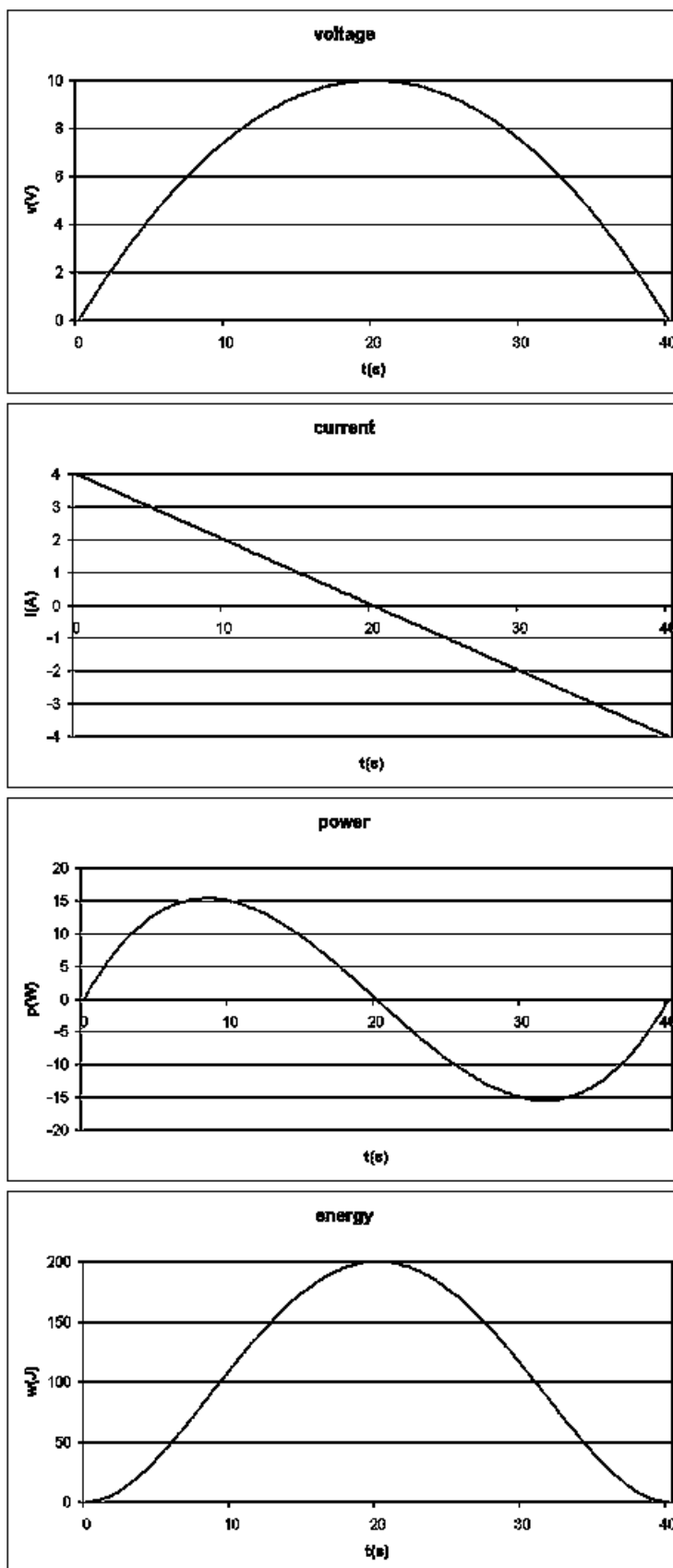
[e] $w = \int_0^t p dx = \int_0^t (4x - 0.3x^2 + 0.005x^3) dx = 2t^2 - 0.1t^3 + 0.00125t^4$

$$w(0) = 0 \text{ J} \qquad w(30) = 112.50 \text{ J}$$

$$w(10) = 112.50 \text{ J} \qquad w(40) = 0 \text{ J}$$

$$w(20) = 200 \text{ J}$$

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:



P 1.25 [a] $p = vi = (8 \times 10^4 t e^{-500t})(15 t e^{-500t}) = 12 \times 10^5 t^2 e^{-1000t} \text{ W}$

$$\frac{dp}{dt} = 12 \times 10^5 [t^2(-1000)e^{-1000t} + e^{-1000t}(2t)]$$

$$= 12 \times 10^5 e^{-1000t} [t(2 - 1000t)]$$

$$\frac{dp}{dt} = 0 \text{ at } t = 0, \quad t = 2 \text{ ms}$$

We know p is a minimum at $t = 0$ since v and i are zero at $t = 0$.

[b] $p_{\max} = 12 \times 10^5 (2 \times 10^{-3})^2 e^{-2} = 649.61 \text{ mW}$

[c] $w = 12 \times 10^5 \int_0^\infty t^2 e^{-1000t} dt$
 $= 12 \times 10^5 \left\{ \frac{e^{-1000t}}{(-1000)^3} [10^6 t^2 + 2,000t + 2] \right\} \Big|_0^\infty = 2400 \mu\text{J}$

P 1.26 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i , as shown below:

$$p_a = -v_a i_a = -(-18)(-51) = -918 \text{ W}$$

$$p_b = v_b i_b = (-18)(45) = -810 \text{ W}$$

$$p_c = v_c i_c = (2)(-6) = -12 \text{ W}$$

$$p_d = -v_d i_d = -(20)(-20) = 400 \text{ W}$$

$$p_e = -v_e i_e = -(16)(-14) = 224 \text{ W}$$

$$p_f = v_f i_f = (36)(31) = 1116 \text{ W}$$

Remember that if the power is positive, the circuit element is absorbing power, whereas if the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:

$$\sum P_{\text{dev}} = 918 + 810 + 12 = 1740 \text{ W};$$

$$\sum P_{\text{abs}} = 400 + 224 + 1116 = 1740 \text{ W}$$

Thus, the power balances and the total power developed in the circuit is 1740 W.

P 1.27 [a] From the diagram and the table we have

$$p_a = -v_a i_a = -(900)(-22.5) = 20,250 \text{ W}$$

$$p_b = -v_b i_b = -(105)(-52.5) = 5512.5 \text{ W}$$

$$p_c = -v_c i_c = -(-600)(-30) = -18,000 \text{ W}$$

$$p_d = v_d i_d = (585)(-52.5) = -30,712.5 \text{ W}$$

$$p_e = -v_e i_e = -(-120)(30) = 3600 \text{ W}$$

$$p_f = v_f i_f = (300)(60) = 18,000 \text{ W}$$

$$p_g = -v_g i_g = -(585)(82.5) = -48,262.5 \text{ W}$$

$$p_h = -v_h i_h = -(-165)(82.5) = 13,612.5 \text{ W}$$

$$\sum P_{\text{del}} = 18,000 + 30,712.5 + 48,262.5 = 96,975 \text{ W}$$

$$\sum P_{\text{abs}} = 20,250 + 5512.5 + 3600 + 18,000 + 13,612.5 = 60,975 \text{ W}$$

Therefore, $\sum P_{\text{del}} \neq \sum P_{\text{abs}}$ and the subordinate engineer is correct.

[b] The difference between the power delivered to the circuit and the power absorbed by the circuit is

$$96,975 - 60,975 = 36,000$$

One-half of this difference is 18,000W, so it is likely that p_c or p_f is in error. Either the voltage or the current probably has the wrong sign. (In Chapter 2, we will discover that using KCL at the top node, the current i_c should be 30 A, not -30 A!) If the sign of p_c is changed from negative to positive, we can recalculate the power delivered and the power absorbed as follows:

$$\sum P_{\text{del}} = 30,712.5 + 48,262.5 = 78,975 \text{ W}$$

$$\sum P_{\text{abs}} = 20,250 + 5512.5 + 18,000 + 3600 + 18,000 + 13,612.5 = 78,975 \text{ W}$$

Now the power delivered equals the power absorbed and the power balances for the circuit.

P 1.28 $p_a = v_a i_a = (9)(1.8) = 16.2 \text{ W}$

$$p_b = -v_b i_b = -(-15)(1.5) = 22.5 \text{ W}$$

$$p_c = -v_c i_c = -(45)(-0.3) = 13.5 \text{ W}$$

$$p_d = -v_d i_d = -(54)(-2.7) = 145.8 \text{ W}$$

$$p_e = v_e i_e = (-30)(-1) = 30 \text{ W}$$

$$p_f = -v_f i_f = -(-240)(4) = 960 \text{ W}$$

$$p_g = -v_g i_g = -(294)(4.5) = -1323 \text{ W}$$

$$p_h = v_h i_h = (-270)(-0.5) = 135 \text{ W}$$

Therefore,

$$\sum P_{\text{abs}} = 16.2 + 22.5 + 13.5 + 145.8 + 3 - +960 + 135 = 1323 \text{ W}$$

$$\sum P_{\text{del}} = 1323 \text{ W}$$

$$\sum P_{\text{abs}} = \sum P_{\text{del}}$$

Thus, the interconnection satisfies the power check

P 1.29 $p_a = v_a i_a = (-160)(-10) = 1600 \text{ W}$

$$p_b = v_b i_b = (-100)(-20) = 2000 \text{ W}$$

$$p_c = -v_c i_c = -(-60)(6) = 360 \text{ W}$$

$$p_d = v_d i_d = (800)(-50) = -40,000 \text{ W}$$

$$p_e = -v_e i_e = -(800)(-20) = 16,000 \text{ W}$$

$$p_f = -v_f i_f = -(-700)(14) = 9800 \text{ W}$$

$$p_g = -v_g i_g = -(640)(-16) = 10,240 \text{ W}$$

$$\sum P_{\text{del}} = 40,000 \text{ W}$$

$$\sum P_{\text{abs}} = 1600 + 2000 + 360 + 16,000 + 9800 + 10,000 = 40,000 \text{ W}$$

Therefore, $\sum P_{\text{del}} = \sum P_{\text{abs}} = 40,000 \text{ W}$

P 1.30 [a] From an examination of reference polarities, the following elements employ the passive convention: a , c , e , and f .

[b] $p_a = -56 \text{ W}$

$$p_b = -14 \text{ W}$$

$$p_c = 150 \text{ W}$$

$$p_d = -50 \text{ W}$$

$$p_e = -18 \text{ W}$$

$$p_f = -12 \text{ W}$$

$$\sum P_{\text{abs}} = 150 \text{ W}; \quad \sum P_{\text{del}} = 56 + 14 + 50 + 18 + 12 = 150 \text{ W}.$$

