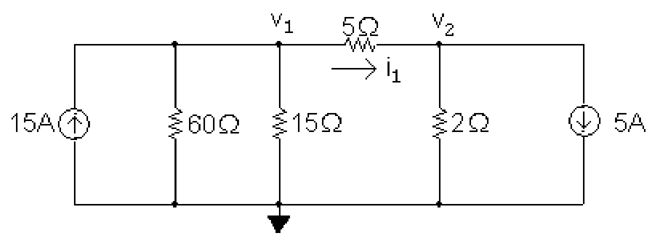
4

Techniques of Circuit Analysis

Assessment Problems

AP 4.1 **[a]** Redraw the circuit, labeling the reference node and the two node voltages:



The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) = 15$$

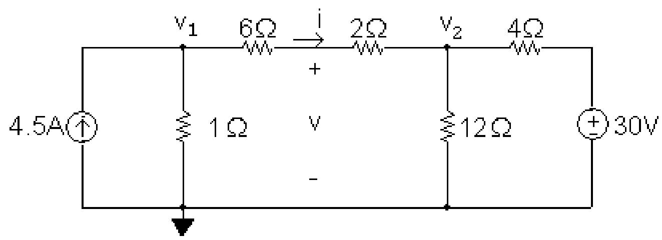
$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{2} + \frac{1}{5}\right) = -5$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[b] $p_{15\text{A}} = -(15\text{ A})v_1 = -(15\text{ A})(60\text{ V}) = -900\text{ W} = 900\text{ W}$(delivered)

[c] $p_{5\text{A}} = (5\text{ A})v_2 = (5\text{ A})(10\text{ V}) = 50\text{ W} = -50\text{ W}$(delivered)

AP 4.2 Redraw the circuit, choosing the node voltages and reference node as shown:



The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} = 0$$
$$\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1\left(1 + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 4.5$$
$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Solving, $v_1 = 6$ V $\qquad v_2 = 18$ V

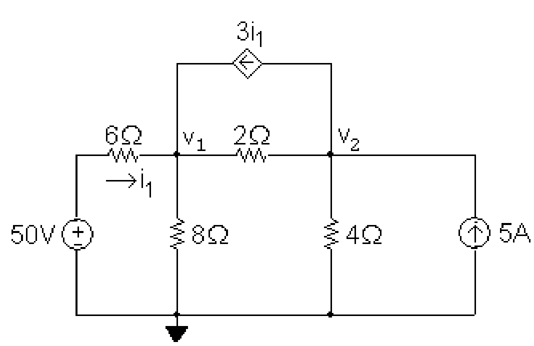
To find the voltage v, first find the current i through the series-connected $6\,\Omega$ and $2\,\Omega$ resistors:

$$i = \frac{v_1 - v_2}{6+2} = \frac{6 - 18}{8} = -1.5 \text{ A}$$

Using a KVL equation, calculate v:

$$v = 2i + v_2 = 2(-1.5) + 18 = 15 \text{ V}$$

AP 4.3 **[a]** Redraw the circuit, choosing the node voltages and reference node as shown:



The node voltage equations are:

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$
$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

The dependent source requires the following constraint equation:

$$i_1 = \frac{50 - v_1}{6}$$

Place these equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{6}+\frac{1}{8}+\frac{1}{2}\right) &+ v_2\left(-\frac{1}{2}\right) &+ i_1(-3) &= \frac{50}{6}\\
v_1\left(-\frac{1}{2}\right) &+ v_2\left(\frac{1}{4}+\frac{1}{2}\right) &+ i_1(3) &= 5\\
v_1\left(\frac{1}{6}\right) &+ v_2(0) &+ i_1(1) &= \frac{50}{6}
\end{aligned}$$

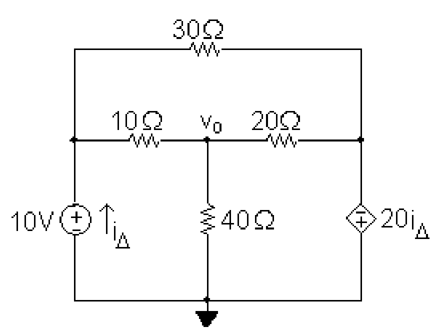
Solving, $v_1 = 32$ V; $v_2 = 16$ V; $i_1 = 3$ A

Using these values to calculate the power associated with each source:

$$\begin{aligned}
p_{50\text{V}} = -50i_1 &= -150 \text{ W}\\
p_{5\text{A}} = -5(v_2) &= -80 \text{ W}\\
p_{3i_1} = 3i_1(v_2 - v_1) &= -144 \text{ W}
\end{aligned}$$

[b] All three sources are delivering power to the circuit because the power computed in (a) for each of the sources is negative.

AP 4.4 Redraw the circuit and label the reference node and the node at which the node voltage equation will be written:



The node voltage equation is

$$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0$$

The constraint equation required by the dependent source is

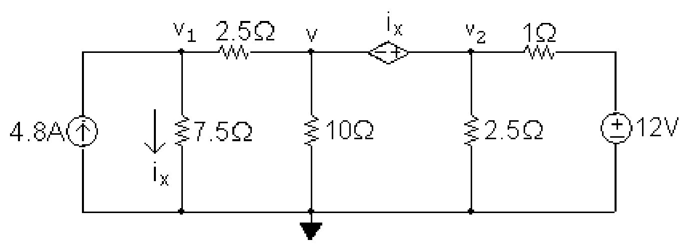
$$i_\Delta = i_{10\,\Omega} + i_{30\,\Omega} = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

Place these equations in standard form:

$$\begin{array}{llllll} v_o\left(\frac{1}{40}+\frac{1}{10}+\frac{1}{20}\right) & + & i_\Delta(1) & = & 1 \\ v_o\left(\frac{1}{10}\right) & + & i_\Delta\left(1-\frac{20}{30}\right) & = & 1+\frac{10}{30} \end{array}$$

Solving, $v_o = 24$ V $\quad i_\Delta = -3.2$ A

AP 4.5 Redraw the circuit identifying the three node voltages and the reference node:



Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5}+\frac{v_1-v}{2.5}-4.8=0$$

The supernode equation is

$$\frac{v-v_1}{2.5}+\frac{v}{10}+\frac{v_2}{2.5}+\frac{v_2-12}{1}=0$$

The constraint equation due to the dependent source is

$$i_x=\frac{v_1}{7.5}$$

The constraint equation due to the supernode is

$$v+i_x=v_2$$

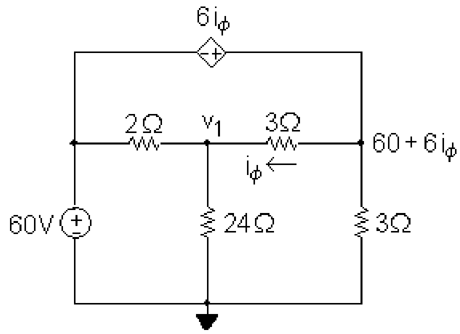
Place this set of equations in standard form:

$$\begin{array}{llllllllll} v_1\left(\frac{1}{7.5}+\frac{1}{2.5}\right) & + & v\left(-\frac{1}{2.5}\right) & + & v_2(0) & + & i_x(0) & = & 4.8 \\ v_1\left(-\frac{1}{2.5}\right) & + & v\left(\frac{1}{2.5}+\frac{1}{10}\right) & + & v_2\left(\frac{1}{2.5}+1\right) & + & i_x(0) & = & 12 \\ v_1\left(-\frac{1}{7.5}\right) & + & v(0) & + & v_2(0) & + & i_x(1) & = & 0 \\ v_1(0) & + & v(1) & + & v_2(-1) & + & i_x(1) & = & 0 \end{array}$$

Solving this set of equations for v gives $v = 8$ V

$v_1 = 15$ V, $\quad v_2 = 10$ V, $\quad i_x = 2$ A

AP 4.6 Redraw the circuit identifying the reference node and the two unknown node voltages. Note that the right-most node voltage is the sum of the 60 V source and the dependent source voltage.



The node voltage equation at v_1 is

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0$$

The constraint equation due to the dependent source is

$$i_\phi = \frac{60 + 6i_\phi - v_1}{3}$$

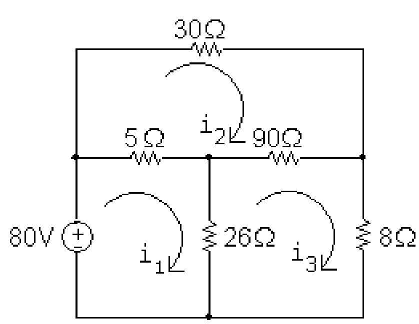
Place these two equations in standard form:

$$v_1\left(\frac{1}{2} + \frac{1}{24} + \frac{1}{3}\right) + i_\phi(-2) = 30 + 20$$

$$v_1\left(\frac{1}{3}\right) + i_\phi(1 - 2) = 20$$

Solving, $v_1 = 48$ V $\quad i_\phi = -4$ A

AP 4.7 **[a]** Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$-80 + 5(i_1 - i_2) + 26(i_1 - i_3) = 0$$

$$30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) = 0$$

$$8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) = 0$$

Place these equations in standard form:

$$31i_1 - 5i_2 - 26i_3 = 80$$
$$-5i_1 + 125i_2 - 90i_3 = 0$$
$$-26i_1 - 90i_2 + 124i_3 = 0$$

Solving,

$i_1 = 5$ A; $\quad i_2 = 2$ A; $\quad i_3 = 2.5$ A

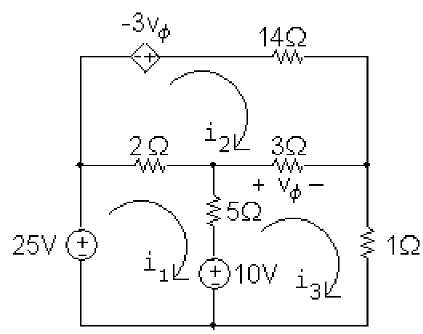
$p_{80V} = -(80)i_1 = -(80)(5) = -400$ W

Therefore the 80 V source is delivering 400 W to the circuit.

[b] $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the $8\,\Omega$ resistor dissipates 50 W.

AP 4.8 **[a]** $b = 8, \quad n = 6, \quad b - n + 1 = 3$

[b] Redraw the circuit identifying the three mesh currents:



The three mesh-current equations are

$$-25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 = 0$$
$$-(-3v_\phi) + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) = 0$$
$$1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) = 0$$

The dependent source constraint equation is

$v_\phi = 3(i_3 - i_2)$

Place these four equations in standard form:

$$7i_1 - 2i_2 - 5i_3 + 0v_\phi = 15$$
$$-2i_1 + 19i_2 - 3i_3 + 3v_\phi = 0$$
$$-5i_1 - 3i_2 + 9i_3 + 0v_\phi = 10$$
$$0i_1 + 3i_2 - 3i_3 + 1v_\phi = 0$$

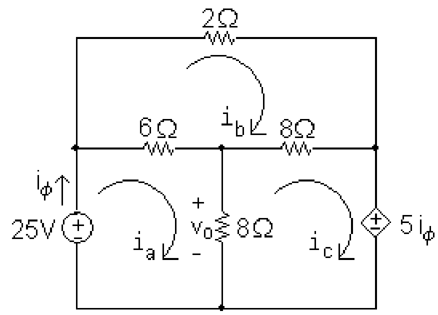
Solving

$i_1 = 4$ A; $\quad i_2 = -1$ A; $\quad i_3 = 3$ A; $\quad v_\phi = 12$ V

$$p_{\rm ds} = -(-3v_\phi)i_2 = 3(12)(-1) = -36\ \text{W}$$

Thus, the dependent source is delivering 36 W, or absorbing −36 W.

AP 4.9 Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -25 + 6(i_{\rm a} - i_{\rm b}) + 8(i_{\rm a} - i_{\rm c}) &= 0 \\ 2i_{\rm b} + 8(i_{\rm b} - i_{\rm c}) + 6(i_{\rm b} - i_{\rm a}) &= 0 \\ 5i_\phi + 8(i_{\rm c} - i_{\rm a}) + 8(i_{\rm c} - i_{\rm b}) &= 0 \end{aligned}$$

The dependent source constraint equation is $i_\phi = i_{\rm a}$. We can substitute this simple expression for i_ϕ into the third mesh equation and place the equations in standard form:

$$\begin{aligned} 14i_{\rm a} - 6i_{\rm b} - 8i_{\rm c} &= 25 \\ -6i_{\rm a} + 16i_{\rm b} - 8i_{\rm c} &= 0 \\ -3i_{\rm a} - 8i_{\rm b} + 16i_{\rm c} &= 0 \end{aligned}$$

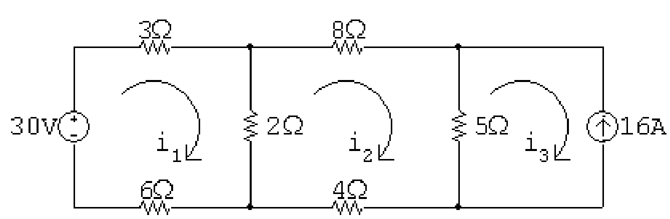
Solving,

$i_{\rm a} = 4$ A; $\quad i_{\rm b} = 2.5$ A; $\quad i_{\rm c} = 2$ A

Thus,

$$v_o = 8(i_{\rm a} - i_{\rm c}) = 8(4 - 2) = 16\ \text{V}$$

AP 4.10 Redraw the circuit identifying the mesh currents:



Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16$ A. The remaining two mesh equations are

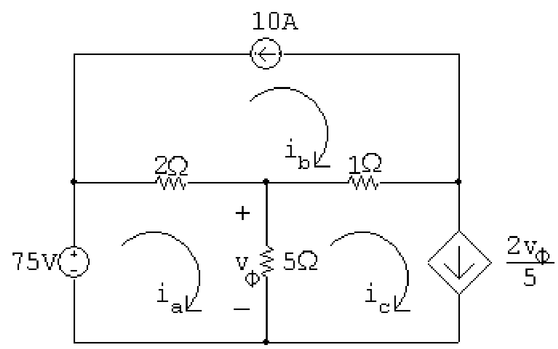
$$-30 + 3i_1 + 2(i_1 - i_2) + 6i_1 = 0$$
$$8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) = 0$$

Place these equations in standard form:

$$11i_1 - 2i_2 = 30$$
$$-2i_1 + 19i_2 = -80$$

Solving: $i_1 = 2$ A, $i_2 = -4$ A, $i_3 = -16$ A
The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6$ A $\therefore$ $p_{2\,\Omega} = (6)^2(2) = 72$ W
Thus, the $2\,\Omega$ resistors dissipates 72 W.

AP 4.11 Redraw the circuit and identify the mesh currents:



There are current sources on the perimeters of both the i_b mesh and the i_c mesh, so we know that

$$i_\mathrm{b} = -10 \text{ A}; \qquad i_\mathrm{c} = \frac{2v_\phi}{5}$$

The remaining mesh current equation is

$$-75 + 2(i_\mathrm{a} + 10) + 5(i_\mathrm{a} - 0.4v_\phi) = 0$$

The dependent source requires the following constraint equation:

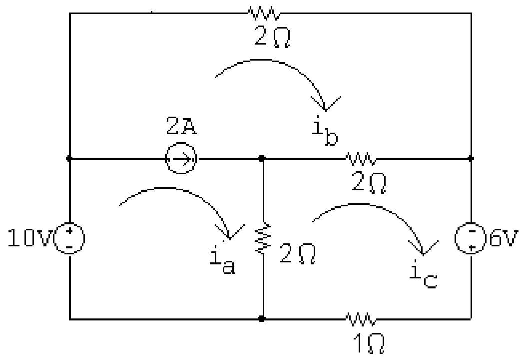
$$v_\phi = 5(i_\mathrm{a} - i_\mathrm{c}) = 5(i_\mathrm{a} - 0.4v_\phi)$$

Place the mesh current equation and the dependent source equation is standard form:

$$7i_\mathrm{a} - 2v_\phi = 55$$
$$5i_\mathrm{a} - 3v_\phi = 0$$

Solving: $i_\mathrm{a} = 15$ A; $i_\mathrm{b} = -10$ A; $i_\mathrm{c} = 10$ A; $v_\phi = 25$ V
Thus, $i_\mathrm{a} = 15$ A.

AP 4.12 Redraw the circuit and identify the mesh currents:



The 2 A current source is shared by the meshes i_a and i_b. Thus we combine these meshes to form a supermesh and write the following equation:

$$-10 + 2i_b + 2(i_b - i_c) + 2(i_a - i_c) = 0$$

The other mesh current equation is

$$-6 + 1i_c + 2(i_c - i_a) + 2(i_c - i_b) = 0$$

The supermesh constraint equation is

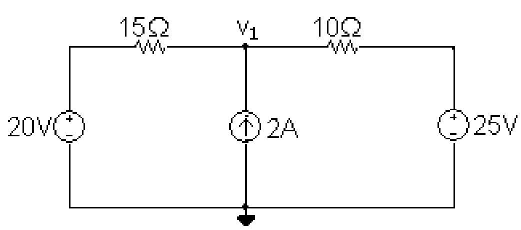
$$i_a - i_b = 2$$

Place these three equations in standard form:

$$2i_a + 4i_b - 4i_c = 10$$
$$-2i_a - 2i_b + 5i_c = 6$$
$$i_a - i_b + 0i_c = 2$$

Solving, $i_a = 7$ A; $i_b = 5$ A; $i_c = 6$ A
Thus, $p_{1\,\Omega} = i_c^2(1) = (6)^2(1) = 36$ W

AP 4.13 Redraw the circuit and identify the reference node and the node voltage v_1:



The node voltage equation is

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$$

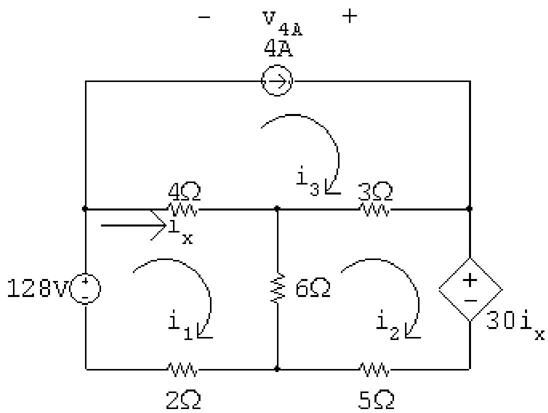
Rearranging and solving,

$$v_1\left(\frac{1}{15}+\frac{1}{10}\right)=2+\frac{20}{15}+\frac{25}{10} \qquad \therefore \quad v_1=35\text{ V}$$

$$p_{2A}=-35(2)=-70\text{ W}$$

Thus the 2 A current source delivers 70 W.

AP 4.14 Redraw the circuit and identify the mesh currents:



There is a current source on the perimeter of the i_3 mesh, so $i_3=4$ A. The other two mesh current equations are

$$\begin{aligned} -128+4(i_1-4)+6(i_1-i_2)+2i_1 &= 0 \\ 30i_x+5i_2+6(i_2-i_1)+3(i_2-4) &= 0 \end{aligned}$$

The constraint equation due to the dependent source is

$$i_x=i_1-i_3=i_1-4$$

Substitute the constraint equation into the second mesh equation and place the resulting two mesh equations in standard form:

$$\begin{aligned} 12i_1-6i_2 &= 144 \\ 24i_1+14i_2 &= 132 \end{aligned}$$

Solving,

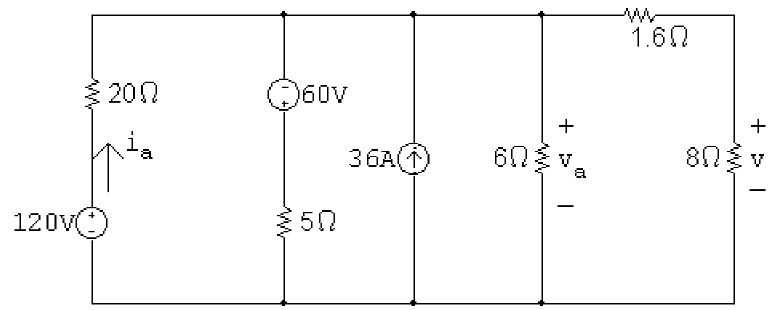
$$i_1=9\text{ A}; \qquad i_2=-6\text{ A}; \qquad i_3=4\text{ A}; \qquad i_x=9-4=5\text{ A}$$

$$\therefore \quad v_{4\text{A}}=3(i_3-i_2)-4i_x=10\text{ V}$$

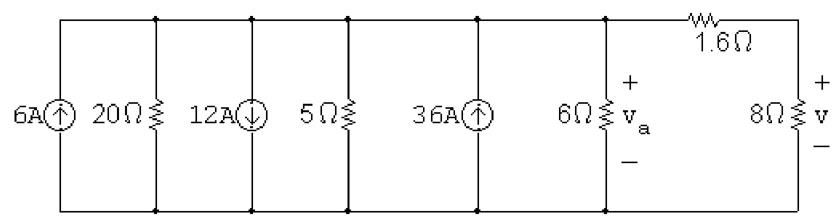
$$p_{4A}=-v_{4\text{A}}(4)=-(10)(4)=-40\text{ W}$$

Thus, the 2 A current source delivers 40 W.

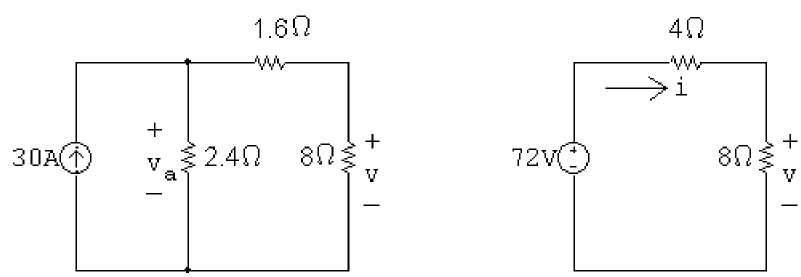
AP 4.15 **[a]** Redraw the circuit with a helpful voltage and current labeled:



Transform the 120 V source in series with the 20 Ω resistor into a 6 A source in parallel with the 20 Ω resistor. Also transform the −60 V source in series with the 5 Ω resistor into a −12 A source in parallel with the 5 Ω resistor. The result is the following circuit:



Combine the three current sources into a single current source, using KCL, and combine the 20 Ω, 5 Ω, and 6 Ω resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 Ω resistor into a 72 V source in series with the 2.4 Ω resistor. Combine the 2.4 Ω resistor in series with the 1.6 Ω resisor to get a very simple circuit that still maintains the voltage v. The resulting circuit is on the right.



Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48 \text{ V}$$

[b] Calculate i in the circuit on the right using Ohm's law:

$$i = \frac{v}{8} = \frac{48}{8} = 6 \text{ A}$$

Now use i to calculate v_{a} in the circuit on the left:

$v_{\mathrm{a}} = 6(1.6 + 8) = 57.6$ V

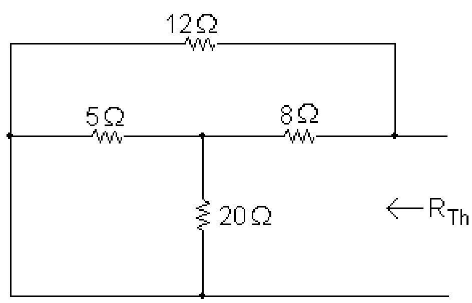
Returning back to the original circuit, note that the voltage v_{a} is also the voltage drop across the series combination of the 120 V source and $20\,\Omega$ resistor. Use this fact to calculate the current in the 120 V source, i_{a}:

$$i_{\mathrm{a}} = \frac{120 - v_{\mathrm{a}}}{20} = \frac{120 - 57.6}{20} = 3.12 \text{ A}$$

$$p_{120V} = -(120)i_{\mathrm{a}} = -(120)(3.12) = -374.40 \text{ W}$$

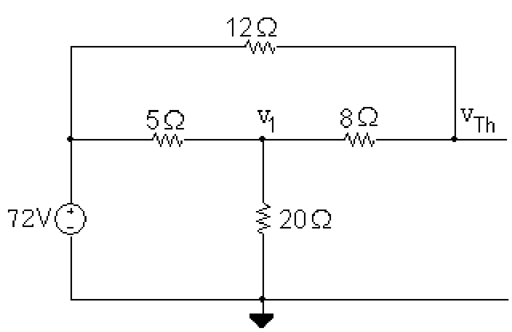
Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find R_{Th}, replace the 72 V source with a short circuit:



Note that the $5\,\Omega$ and $20\,\Omega$ resistors are in parallel, with an equivalent resistance of $5\|20 = 4\,\Omega$. The equivalent $4\,\Omega$ resistance is in series with the $8\,\Omega$ resistor for an equivalent resistance of $4 + 8 = 12\,\Omega$. Finally, the $12\,\Omega$ equivalent resistance is in parallel with the $12\,\Omega$ resistor, so $R_{\mathrm{Th}} = 12\|12 = 6\,\Omega$.

Use node voltage analysis to find v_{Th}. Begin by redrawing the circuit and labeling the node voltages:



The node voltage equations are

$$\frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{\mathrm{Th}}}{8} = 0$$

$$\frac{v_{\mathrm{Th}} - v_1}{8} + \frac{v_{\mathrm{Th}} - 72}{12} = 0$$

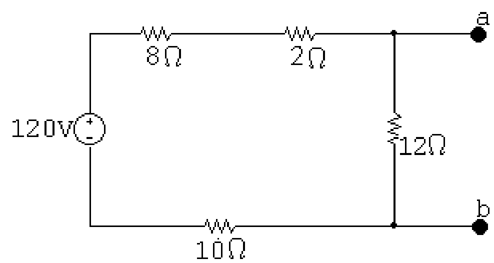
Place these equations in standard form:

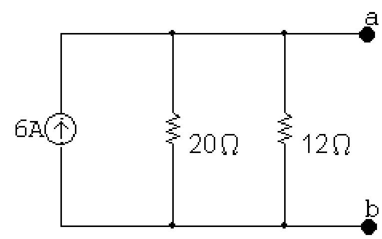
$$v_1\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{8}\right) + v_{\rm Th}\left(-\frac{1}{8}\right) = \frac{72}{5}$$

$$v_1\left(-\frac{1}{8}\right) + v_{\rm Th}\left(\frac{1}{8}+\frac{1}{12}\right) = 6$$

Solving, $v_1 = 60$ V and $v_{\rm Th} = 64.8$ V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a $6\,\Omega$ resistor.

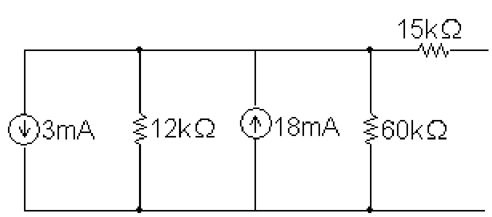
AP 4.17 We begin by performing a source transformation, turning the parallel combination of the 15 A source and $8\,\Omega$ resistor into a series combination of a 120 V source and an $8\,\Omega$ resistor, as shown in the figure on the left. Next, combine the $2\,\Omega$, $8\,\Omega$ and $10\,\Omega$ resistors in series to give an equivalent $20\,\Omega$ resistance. Then transform the series combination of the 120 V source and the $20\,\Omega$ equivalent resistance into a parallel combination of a 6 A source and a $20\,\Omega$ resistor, as shown in the figure on the right.





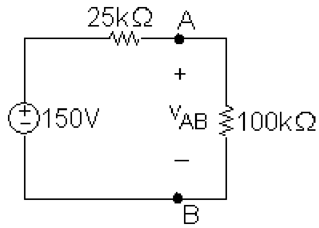
Finally, combine the $20\,\Omega$ and $12\,\Omega$ parallel resistors to give $R_{\rm N} = 20\|12 = 7.5\,\Omega$. Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a $7.5\,\Omega$ resistor.

AP 4.18 Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the -36 V source and 12 kΩ resistor into a parallel combination of a -3 mA source and 12 kΩ resistor. The resulting circuit is shown below:



Now combine the two parallel current sources and the two parallel resistors to give a $-3 + 18 = 15$ mA source in parallel with a 12 k$\|60$ k$= 10$ kΩ resistor. Then transform the 15 mA source in parallel with the 10 kΩ resistor into a 150 V source in series with a 10 kΩ resistor, and combine this 10 kΩ resistor in series with the 15 kΩ resistor. The Thévenin equivalent is thus a 150 V source in series with a 25 kΩ

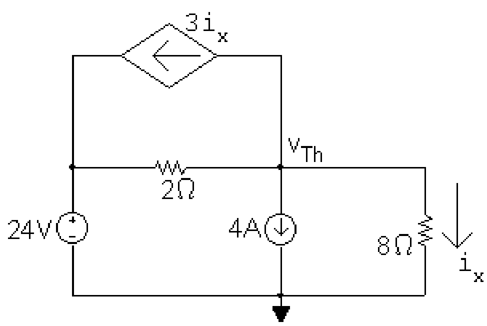
resistor, as seen to the left of the terminals A,B in the circuit below.



Now attach the voltmeter, modeled as a $100\ \text{k}\Omega$ resistor, to the Thévenin equivalent and use voltage division to calculate the meter reading v_{AB}:

$$v_{\text{AB}} = \frac{100{,}000}{125{,}000}(150) = 120\ \text{V}$$

AP 4.19 Begin by calculating the open circuit voltage, which is also v_{Th}, from the circuit below:



Summing the currents away from the node labeled v_{Th} We have

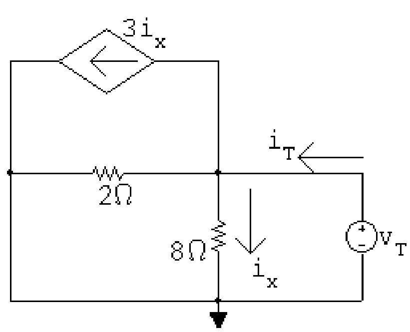
$$\frac{v_{\text{Th}}}{8} + 4 + 3i_x + \frac{v_{\text{Th}} - 24}{2} = 0$$

Also, using Ohm's law for the $8\,\Omega$ resistor,

$$i_x = \frac{v_{\text{Th}}}{8}$$

Substituting the second equation into the first and solving for v_{Th} yields $v_{\text{Th}} = 8$ V.

Now calculate R_{Th}. To do this, we use the test source method. Replace the voltage source with a short circuit, the current source with an open circuit, and apply the test voltage v_{T}, as shown in the circuit below:



Write a KCL equation at the middle node:

$$i_T = i_x + 3i_x + v_T/2 = 4i_x + v_T/2$$

Use Ohm's law to determine i_x as a function of v_T:

$$i_x = v_T/8$$

Substitute the second equation into the first equation:

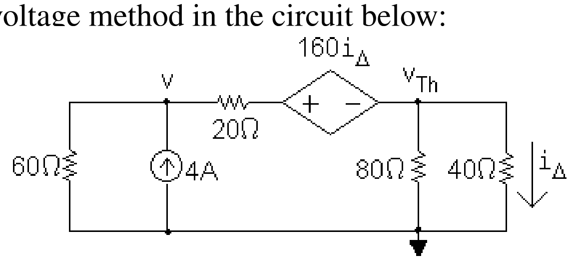
$$i_T = 4(v_T/8) + v_T/2 = v_T$$

Thus,

$$R_{Th} = v_T/i_T = 1\,\Omega$$

The Thévenin equivalent is an 8 V source in series with a $1\,\Omega$ resistor.

AP 4.20 Begin by calculating the open circuit voltage, which is also v_{Th}, using the node voltage method in the circuit below:



The node voltage equations are

$$\frac{v}{60} + \frac{v - (v_{Th} + 160i_\Delta)}{20} - 4 = 0,$$

$$\frac{v_{Th}}{40} + \frac{v_{Th}}{80} + \frac{v_{Th} + 160i_\Delta - v}{20} = 0$$

The dependent source constraint equation is

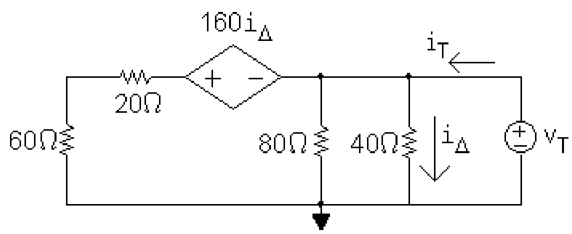
$$i_\Delta = \frac{v_{Th}}{40}$$

Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

$$v\left(\frac{1}{60}+\frac{1}{20}\right) + v_{\mathrm{Th}}\left(-\frac{5}{20}\right) = 4$$
$$v\left(-\frac{1}{20}\right) + v_{\mathrm{Th}}\left(\frac{1}{40}+\frac{1}{80}+\frac{5}{20}\right) = 0$$

Solving, $v = 172.5$ V and $v_{\mathrm{Th}} = 30$ V.

Now use the test source method to calculate the test current and thus R_{Th}. Replace the current source with a short circuit and apply the test source to get the following circuit:



Write a KCL equation at the rightmost node:

$$i_{\mathrm{T}} = \frac{v_{\mathrm{T}}}{80} + \frac{v_{\mathrm{T}}}{40} + \frac{v_{\mathrm{T}} + 160 i_\Delta}{80}$$

The dependent source constraint equation is

$$i_\Delta = \frac{v_{\mathrm{T}}}{40}$$

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

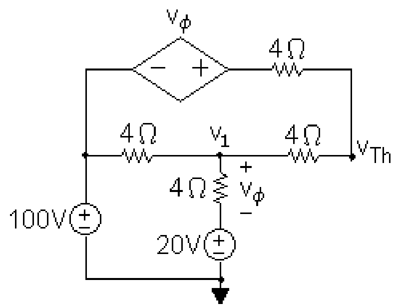
$$i_{\mathrm{T}} = \frac{v_{\mathrm{T}}}{10}$$

Therefore,

$$R_{\mathrm{Th}} = \frac{v_{\mathrm{T}}}{i_{\mathrm{T}}} = 10\,\Omega$$

Thus, the Thévenin equivalent is a 30 V source in series with a $10\,\Omega$ resistor.

AP 4.21 First find the Thévenin equivalent circuit. To find v_{Th}, create an open circuit between nodes a and b and use the node voltage method with the circuit below:



The node voltage equations are:

$$\frac{v_{\rm Th} - (100 + v_\phi)}{4} + \frac{v_{\rm Th} - v_1}{4} = 0$$

$$\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_{\rm Th}}{4} = 0$$

The dependent source constraint equation is

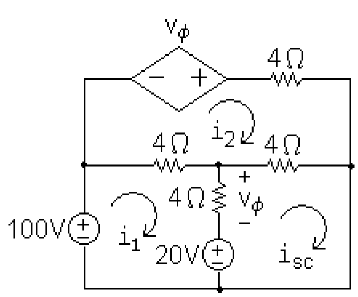
$$v_\phi = v_1 - 20$$

Place these three equations in standard form:

$$\begin{aligned} v_{\rm Th}\left(\frac{1}{4} + \frac{1}{4}\right) &+ v_1\left(-\frac{1}{4}\right) &+ v_\phi\left(-\frac{1}{4}\right) &= 25 \\ v_{\rm Th}\left(-\frac{1}{4}\right) &+ v_1\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) &+ v_\phi\,(0) &= 30 \\ v_{\rm Th}\,(0) &+ v_1\,(1) &+ v_\phi\,(-1) &= 20 \end{aligned}$$

Solving, $v_{\rm Th} = 120$ V, $v_1 = 80$ V, and $v_\phi = 60$ V.

Now create a short circuit between nodes a and b and use the mesh current method with the circuit below:



The mesh current equations are

$$-100 + 4(i_1 - i_2) + v_\phi + 20 = 0$$

$$-v_\phi + 4i_2 + 4(i_2 - i_{\rm sc}) + 4(i_2 - i_1) = 0$$

$$-20 - v_\phi + 4(i_{\rm sc} - i_2) = 0$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_{sc})$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_{sc} + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_{sc} - v_\phi &= 0 \\ 0i_1 - 4i_2 + 4i_{sc} - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_{sc} - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 45$ A, $i_2 = 30$ A, $i_{sc} = 40$ A, and $v_\phi = 20$ V. Thus,

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{120}{40} = 3\,\Omega$$

[a] For maximum power transfer, $R = R_{Th} = 3\,\Omega$

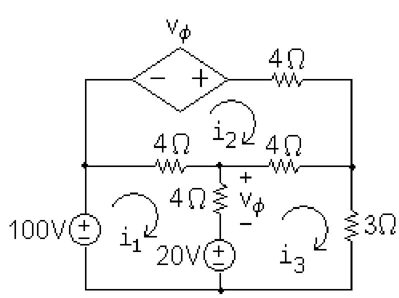
[b] The Thévenin voltage, $v_{Th} = 120$ V, splits equally between the Thévenin resistance and the load resistance, so

$$v_{load} = \frac{120}{2} = 60 \text{ V}$$

Therefore,

$$p_{max} = \frac{v_{load}^2}{R_{load}} = \frac{60^2}{3} = 1200 \text{ W}$$

AP 4.22 Sustituting the value $R = 3\,\Omega$ into the circuit and identifying three mesh currents we have the circuit below:



The mesh current equations are:

$$\begin{aligned} -100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\ -v_\phi + 4i_2 + 4(i_2 - i_3) + 4(i_2 - i_1) &= 0 \\ -20 - v_\phi + 4(i_3 - i_2) + 3i_3 &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_3)$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_3 + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_3 - v_\phi &= 0 \\ 0i_1 - 4i_2 + 7i_3 - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_3 - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 30$ A, $i_2 = 20$ A, $i_3 = 20$ A, and $v_\phi = 40$ V.

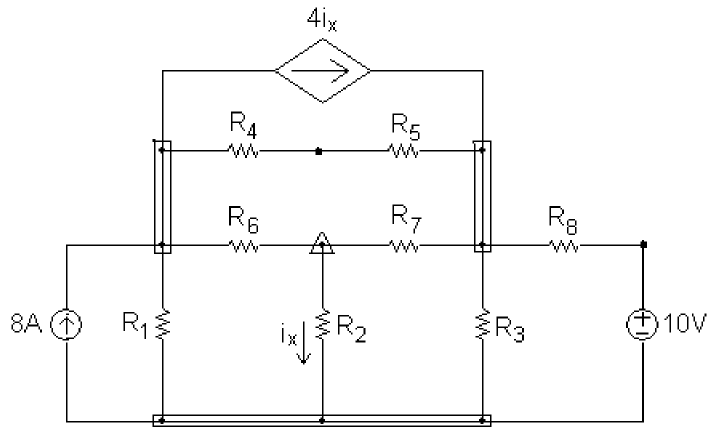
[a] $p_{100\text{V}} = -(100)i_1 = -(100)(30) = -3000$ W. Thus, the 100 V source is delivering 3000 W.

[b] $p_{\text{depsource}} = -v_\phi i_2 = -(40)(20) = -800$ W. Thus, the dependent source is delivering 800 W.

[c] From Assessment Problem 4.21(b), the power delivered to the load resistor is 1200 W, so the load power is $(1200/3800)100 = 31.58\%$ of the combined power generated by the 100 V source and the dependent source.

Problems

P 4.1



[a] 11 branches, 8 branches with resistors, 2 branches with independent sources, 1 branch with a dependent source

[b] The current is unknown in every branch except the one containing the 8 A current source, so the current is unknown in 10 branches.

[c] 9 essential branches – $R_4 - R_5$ forms an essential branch as does $R_8 - 10$ V. The remaining seven branches are essential branches that contain a single element.

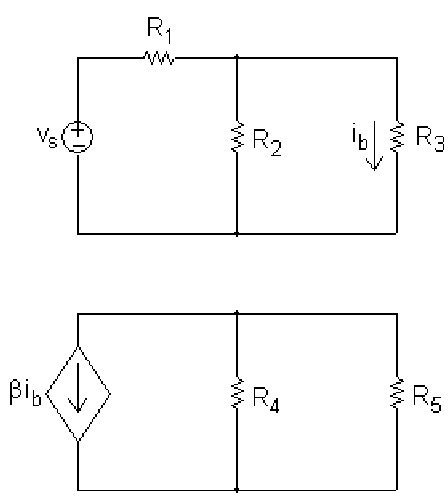
[d] The current is known only in the essential branch containing the current source, and is unknown in the remaining 8 essential branches

[e] From the figure there are 6 nodes – three identified by rectangular boxes, two identified with single black dots, and one identified by a triangle.

[f] There are 4 essential nodes, three identified with rectangular boxes and one identified with a triangle

[g] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

P 4.2



[a] As can be seen from the figure, the circuit has 2 separate parts.

[b] There are 5 nodes – the four black dots and the node betweem the voltage source and the resistor R_1.

[c] There are 7 branches, each containing one of the seven circuit components.

[d] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

P 4.3 **[a]** From Problem 4.1(d) there are 8 essential branches were the current is unknown, so we need 8 simultaneous equations to describe the circuit.

[b] From Problem 4.1(f), there are 4 essential nodes, so we can apply KCL at $(4 - 1) = 3$ of these essential nodes. These would also be a dependent source constraint equation.

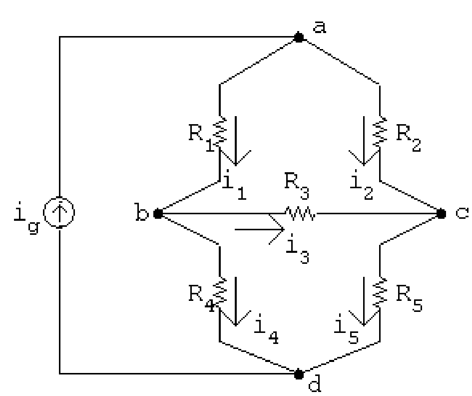
[c] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

[d] We must avoid using the topmost mesh and the leftmost mesh. Each of these meshes contains a current source, and we have no way of determining the voltage drop across a current source.

P 4.4 **[a]** There are six circuit components, five resistors and the current source. Since the current is known only in the current source, it is unknown in the five resistors. Therefore there are **five** unknown currents.

[b] There are four essential nodes in this circuit, identified by the dark black dots in Fig. P4.4. At three of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the fourth node would be dependent on the first three. Therefore there are **three** independent KCL equations.

[c]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$$-i_g + i_1 + i_2 = 0$$

$-i_1 + i_4 + i_3 = 0$

$i_5 - i_2 - i_3 = 0$

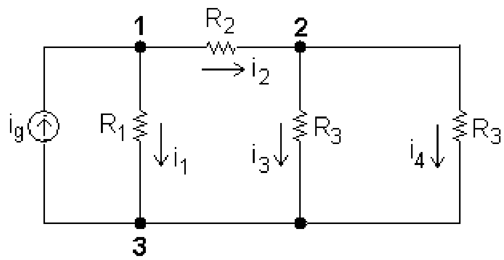
[d] There are three meshes in this circuit: one on the left with the components i_g, R_1, and R_4; one on the top right with components R_1, R_2, and R_3; and one on the bottom right with components R_3, R_4, and R_5. We cannot write a KVL equation for the left mesh because we don't know the voltage drop across the current source. Therefore, we can write KVL equations for the two meshes on the right, giving a total of **two** independent KVL equations.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$R_1 i_1 + R_3 i_3 - R_2 i_2 = 0$

$R_3 i_3 + R_5 i_5 - R_4 i_4 = 0$

P 4.5



[a] At node 1: $\quad -i_g + i_1 + i_2 = 0$

At node 2: $\quad -i_2 + i_3 + i_4 = 0$

At node 3: $\quad i_g - i_1 - i_3 - i_4 = 0$

[b] There are many possible solutions. For example, solve the equation at node 1 for i_g:

$i_g = i_1 + i_2$

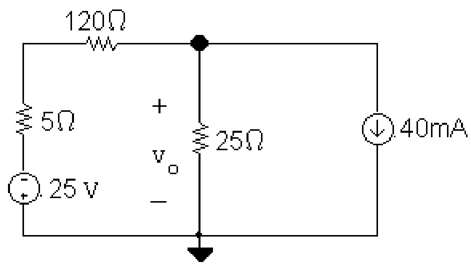
Substitute this expression for i_g into the equation at node 3:

$(i_1 + i_2) - i_1 - i_3 - i_4 = 0 \qquad \text{so} \qquad i_2 - i_3 - i_4 = 0$

Multiply this last equation by -1 to get the equation at node 2:

$-(i_2 - i_3 - i_4) = -0 \qquad \text{so} \qquad -i_2 + i_3 + i_4 = 0$

P 4.6



Note that we have chosen the lower node as the reference node, and that the voltage at the upper node with respect to the reference node is v_o. Write a KCL equation (node voltage equation)by summing the currents leaving the upper node:

$$\frac{v_o + 25}{120 + 5} + \frac{v_o}{25} + 0.04 = 0$$

Solve by multiplying both sides of the KCL equation by 125 and collecting the terms involving v_o on one side of the equation and the constants on the other side of the equation:

$v_o + 25 + 5v_o + 5 = 0$ $\quad\therefore\quad$ $6v_o = -30$ $\quad$ so $\quad$ $v_o = -30/6 = -5$ V

P 4.7 **[a]** From the solution to Problem 4.6 we know $v_o = -5$ V; therefore

$p_{40\text{mA}} = (-5)(0.04) = -0.2$ W

The power developed by the 40 mA source is 200 mW

[b] The current into the negative terminal of the 25 V source in the figure of Problem 4.6 is

$i_g = (-5 + 25)/125 = 160$ mA

The power in the 25 V source is

$p_{25\text{V}} = -(25)(0.16) = -4$ W

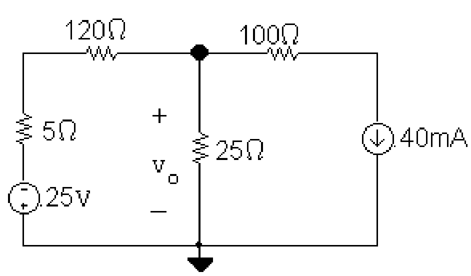
The power developed by the 25 V source is 4 W

[c]
$$p_{5\Omega} = (0.16)^2(5) = 128 \text{ mW}$$
$$p_{120\Omega} = (0.16)^2(120) = 3.072 \text{ W}$$
$$p_{25\Omega} = (-5)^2/25 = 1 \text{ W}$$

$\sum p_{\text{dis}} = 0.128 + 3.072 + 1 = 4.2$ W

$\sum p_{\text{dev}} = 0.2 + 4 = 4.2$ W (checks!)

P 4.8



[a] The node voltage equation is:

$$\frac{v_o + 25}{125} + \frac{v_o}{25} + 0.04 = 0$$

Solving,
$v_o + 25 + 5v_o + 5 = 0 \qquad \therefore \qquad 6v_o = -30 \qquad \text{so} \qquad v_o = -5 \text{ V}$

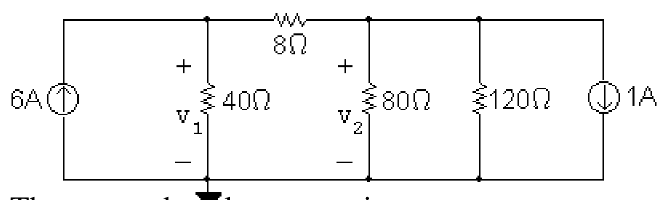
[b] Let v_x = voltage drop across 40 mA source:
$v_x = v_o - (100)(0.04) = -5 - 4 = -9$ V
$p_{40\text{mA}} = (-9)(0.04) = -360$ mW
The power developed by the 40 mA source is 360 mW

[c] Let i_g = current into negative terminal of 25 V source:
$i_g = (-5 + 25)/125 = 160$ mA
$p_{25\text{V}} = -(25)(0.16) = -4$ W
The power developed by the 25 V source is 4 W

[d]
$$p_{5\Omega} = (0.16)^2(5) = 128 \text{ mW}$$
$$p_{120\Omega} = (0.16)^2(120) = 3.072 \text{ W}$$
$$p_{25\Omega} = (-5)^2/25 = 1 \text{ W}$$
$$p_{100\Omega} = (0.04)^2(100) = 160 \text{ mW}$$
$\sum p_{\text{dis}} = 0.128 + 3.072 + 1 + 0.160 = 4.36$ W
$\sum p_{\text{dev}} = 0.360 + 4 = 4.36$ W (checks!)

[e] v_o is independent of any finite resistance connected in series with the 40 mA current source

P 4.9

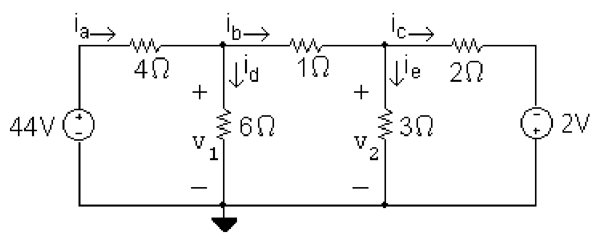


The two node voltage equations are:
$$-6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} = 0$$
$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$
Place these equations in standard form:
$$v_1\left(\frac{1}{40} + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 6$$
$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{8} + \frac{1}{80} + \frac{1}{120}\right) = -1$$
Solving, $v_1 = 120$ V and $v_2 = 96$ V.

Check this result by calculating the power associated with each component:

Component	Power Delivered (W)	Power Absorbed (W)
6A	$-(6\text{ A})(120\text{ V}) = -720$	
$40\,\Omega$		$\frac{120^2}{40} = 360$
$8\,\Omega$		$\frac{(120-96)^2}{8} = 72$
$80\,\Omega$		$\frac{96^2}{80} = 115.2$
$120\,\Omega$		$\frac{96^2}{120} = 76.8$
1 A		$(96\text{ V})(1\text{ A}) = 96$
Total	-720	720

P 4.10 **[a]**



The two node voltage equations are:

$$\frac{v_1}{6} + \frac{v_1 - 44}{4} + \frac{v_1 - v_2}{1} = 0$$

$$\frac{v_2}{3} + \frac{v_2 - v_1}{1} + \frac{v_2 + 2}{2} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{6} + \frac{1}{4} + 1\right) + v_2(-1) = \frac{44}{4}$$

$$v_1(-1) + v_2\left(\frac{1}{3} + 1 + \frac{1}{2}\right) = -\frac{2}{2}$$

Solving, $v_1 = 12$ V; $\quad v_2 = 6$ V

Now calculate the branch currents from the node voltage values:

$$i_\text{a} = \frac{44 - 12}{4} = 8\text{ A}$$

$$i_\text{b} = \frac{12}{6} = 2\text{ A}$$

$$i_\text{c} = \frac{12 - 6}{1} = 6\text{ A}$$

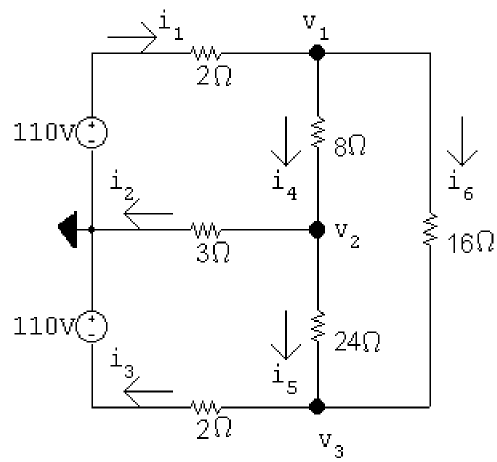
$$i_\text{d} = \frac{6}{3} = 2\text{ A}$$

$$i_\text{e} = \frac{6 + 2}{2} = 4\text{ A}$$

[b] $p_{\text{sources}} = p_{44\text{V}} + p_{2\text{V}} = -(44)i_{\text{a}} - (2)i_{\text{e}} = -(44)(8) - (2)(4) = -352 - 8 = -360$ W

Thus, the power developed in the circuit is 360 W. Note that the resistors cannot develop power!

P 4.11 **[a]**



$$\frac{v_1 - 110}{2} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{16} = 0 \qquad \text{so} \qquad 11v_1 - 2v_2 - v_3 = 880$$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{3} + \frac{v_2 - v_3}{24} = 0 \qquad \text{so} \qquad -3v_1 + 12v_2 - v_3 = 0$$

$$\frac{v_3 + 110}{2} + \frac{v_3 - v_2}{24} + \frac{v_3 - v_1}{16} = 0 \qquad \text{so} \qquad -3v_1 - 2v_2 + 29v_3 = -2640$$

Solving, $v_1 = 74.64$ V; $v_2 = 11.79$ V; $v_3 = -82.5$ V

Thus, $i_1 = \dfrac{110 - v_1}{2} = 17.68$ A $\quad i_4 = \dfrac{v_1 - v_2}{8} = 7.86$ A

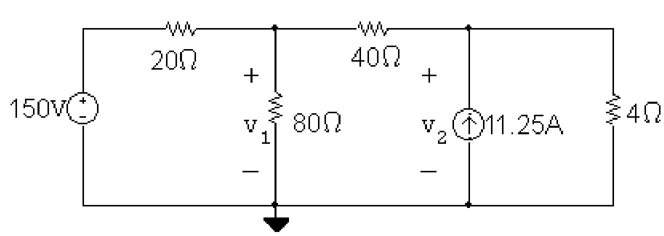
$i_2 = \dfrac{v_2}{3} = 3.93$ A $\quad i_5 = \dfrac{v_2 - v_3}{24} = 3.93$ A

$i_3 = \dfrac{v_3 + 110}{2} = 13.75$ A $\quad i_6 = \dfrac{v_1 - v_3}{16} = 9.82$ A

[b] $\sum P_{\text{dev}} = 110i_1 + 110i_3 = 3457.14$ W

$$\sum P_{\text{dis}} = i_1^2(2) + i_2^2(3) + i_3^2(2) + i_4^2(8) + i_5^2(24) + i_6^2(16) = 3457.14 \text{ W}$$

P 4.12



The two node voltage equations are:

$$\frac{v_1 - 150}{20} + \frac{v_1}{80} + \frac{v_1 - v_2}{40} = 0$$

$$\frac{v_2 - v_1}{40} - 11.25 + \frac{v_2}{4} = 0$$

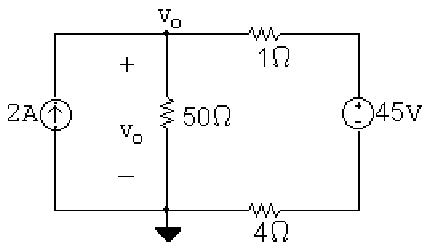
Place these equations in standard form:

$$v_1\left(\frac{1}{20}+\frac{1}{80}+\frac{1}{40}\right) + v_2\left(-\frac{1}{40}\right) = \frac{150}{20}$$

$$v_1\left(-\frac{1}{40}\right) + v_2\left(\frac{1}{40}+\frac{1}{4}\right) = 11.25$$

Solving, $v_1 = 100$ V; $v_2 = 50$ V

P 4.13



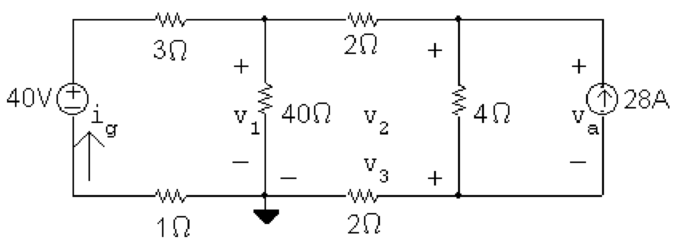
At v_o : $$-2 + \frac{v_o}{50} + \frac{v_o - 45}{4+1} = 0$$

Solving, $v_o = 50$ V

$p_{2\text{A}} = -(50)(2) = -100$ W

Thus, the 2 A current source delivers 100 W, or the current source extracts -100 W from the circuit.

P 4.14



The three node voltage equations are:

$$\frac{v_1 - 40}{4} + \frac{v_1}{40} + \frac{v_1 - v_2}{2} = 0$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{4} - 28 = 0$$

$$\frac{v_3}{2} + \frac{v_3 - v_2}{4} + 28 = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{4}+\frac{1}{40}+\frac{1}{2}\right) + v_2\left(-\frac{1}{2}\right) + v_3(0) = \frac{40}{4}$$

$$v_1\left(-\frac{1}{2}\right) + v_2\left(\frac{1}{2}+\frac{1}{4}\right) + v_3\left(-\frac{1}{4}\right) = 28$$

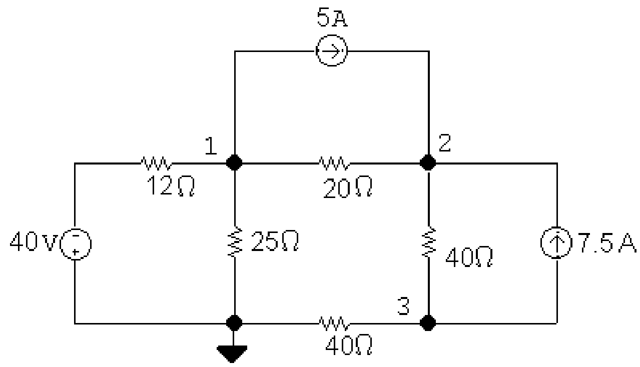
$$v_1(0) + v_2\left(-\frac{1}{4}\right) + v_3\left(\frac{1}{2}+\frac{1}{4}\right) = -28$$

Solving, $v_1 = 60$ V; $v_2 = 73$ V; $v_3 = -13$ V.

$$p_{28\text{A}} = -v_\text{a}(28\text{ A}) = -(v_2 - v_3)(28\text{ A}) = -(73 + 13)(28) = -2408\text{ W}$$

The 28 A source delivers 2408 W.

P 4.15



The node voltage equations are:

$$\frac{v_1 + 40}{12} + \frac{v_1}{25} + 5 + \frac{v_1 - v_2}{20} = 0$$
$$\frac{v_2 - v_1}{20} + \frac{v_2 - v_3}{40} - 7.5 - 5 = 0$$
$$\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{12} + \frac{1}{25} + \frac{1}{20}\right) + v_2\left(-\frac{1}{20}\right) + v_3(0) = -\frac{40}{12} - 5$$
$$v_1\left(-\frac{1}{20}\right) + v_2\left(\frac{1}{20} + \frac{1}{40}\right) + v_3\left(-\frac{1}{40}\right) = 12.5$$
$$v_1(0) + v_2\left(-\frac{1}{40}\right) + v_3\left(\frac{1}{40} + \frac{1}{40}\right) = -7.5$$

Solving, $v_1 = -10$ V; $v_2 = 132$ V; $v_3 = -84$ V.
Find the power:

$$i_{40\text{V}} = (-10 + 40)/12 = 2.5\text{ A}$$
$$p_{40\text{V}} = -(2.5)(40) = -100\text{ W} \quad \text{(del)}$$
$$p_{5\text{A}} = (5)(-10 - 132) = -710\text{ W} \quad \text{(del)}$$
$$p_{7.5\text{A}} = (7.5)(-84 - 132) = -1620\text{ W} \quad \text{(del)}$$
$$p_{12\Omega} = (-10 + 40)^2/12 = 75\text{ W} \quad \text{(abs)}$$
$$p_{25\Omega} = (-10)^2/25 = 4\text{ W} \quad \text{(abs)}$$

$$p_{20\Omega} = (132+10)^2/20 = 1008.2 \text{ W} \quad \text{(abs)}$$

$$p_{40\Omega} = (132+84)^2/40 = 1166.4 \text{ W} \quad \text{(abs)}$$

$$p_{40\Omega} = (-84)^2/40 = 176.4 \text{ W} \quad \text{(abs)}$$

$$\sum p_{\text{diss}} = 75+4+1008.2+1166.4+176.4 = 2430 \text{ W}$$

$$\sum p_{\text{dev}} = 100+710+1620 \text{ W} = 2430 \text{ W} \qquad \text{(CHECKS)}$$

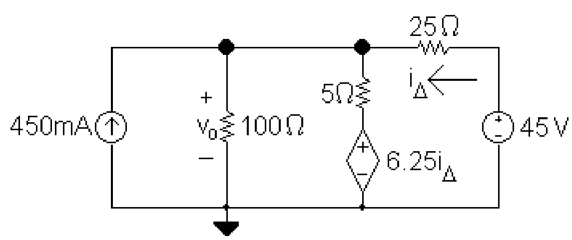
P 4.16 **[a]** $\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \frac{v_o - v_3}{R} + \cdots + \frac{v_o - v_n}{R} = 0$

$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$

$\therefore \quad v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n}\sum_{k=1}^{n} v_k$

[b] $v_o = \frac{1}{3}(120 + 60 - 30) = 50 \text{ V}$

P 4.17 **[a]**



The node voltage equation is:

$$-0.45 + \frac{v_o}{100} + \frac{v_o - 6.25i_\Delta}{5} + \frac{v_o - 45}{25} = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{45 - v_o}{25}$$

Place these equations in standard form:

$$v_o\left(\frac{1}{100} + \frac{1}{5} + \frac{1}{25}\right) + i_\Delta\left(-\frac{6.25}{5}\right) = \frac{45}{25} + 0.45$$

$$v_o\left(\frac{1}{25}\right) + i_\Delta(1) = \frac{45}{25}$$

Solving, $\quad v_o = 15 \text{ V}; \quad i_\Delta = 1.2 \text{ A}$

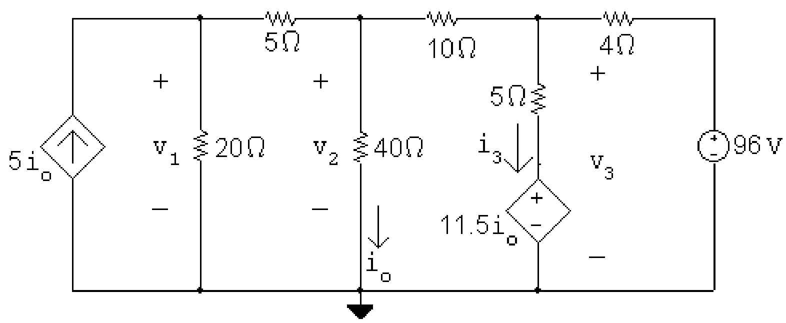
[b] $i_{\text{ds}} = \frac{v_o - 6.25i_\Delta}{5} = \frac{15 - 7.5}{5} = 1.5 \text{ A}$

$p_{\text{ds}} = [6.25(1.2)](1.5) = 11.25 \text{ W}$

Thus, the dependent source absorbs 11.25 W

[c] $p_{450\text{mA}} = -(0.45)(15) = -6.75$ W
$p_{45\text{V}} = -(1.2)(45) = -54$ W
$\sum p_{\text{dev}} = 6.75 + 54 = 60.75$ W
Thus the independent sources develop 60.75 W
Also,
$\sum p_{\text{dis}} = p_{\text{ds}} + p_{100\Omega} + p_{5\Omega} + p_{25\Omega}$
$= 11.25 + (15)^2/100 + (1.5)^2(5) + (1.2)^2(25)$
$= 11.25 + 2.25 + 11.25 + 36 = 60.75$ W (checks!)

P 4.18 **[a]**



The node voltage equations are:

$$-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$
$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} = 0$$
$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i_o}{5} + \frac{v_3 - 96}{4} = 0$$

The dependent source constraint equation is:
$i_o = v_2/40$
Place these equations in standard form:

$$v_1\left(\frac{1}{20} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) + v_3(0) + i_o(-5) = 0$$
$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5} + \frac{1}{40} + \frac{1}{10}\right) + v_3\left(-\frac{1}{10}\right) + i_o(0) = 0$$
$$v_1(0) + v_2\left(-\frac{1}{10}\right) + v_3\left(\frac{1}{10} + \frac{1}{5} + \frac{1}{4}\right) + i_o\left(-\frac{11.5}{5}\right) = \frac{96}{4}$$
$$v_1(0) + v_2\left(-\frac{1}{40}\right) + v_3(0) + i_o(1) = 0$$

Solving, $v_1 = 156$ V; $v_2 = 120$ V; $v_3 = 78$ V; $i_o = 3$ A

[b] Calculate the power:

$$p_{\rm cccs} = -[5(3)](156) = -2340 \text{ W}$$
$$p_{20\Omega} = (156)^2/20 = 1216.8 \text{ W}$$
$$p_{5\Omega} = (156-120)^2/5 = 259.2 \text{ W}$$
$$p_{40\Omega} = (120)^2/40 = 360 \text{ W}$$
$$p_{10\Omega} = (120-78)^2/10 = 176.4 \text{ W}$$
$$p_{5\Omega} = (78-11.5\cdot 3)^2/5 = 378.45 \text{ W}$$
$$p_{4\Omega} = (78-96)^2/4 = 81 \text{ W}$$
$$p_{96\rm V} = [(78-96)/4](96) = -432 \text{ W}$$
$$p_{\rm ccvs} = [(78-3\cdot 11.5)/5](11.5\cdot 3) = 300.15 \text{ W}$$

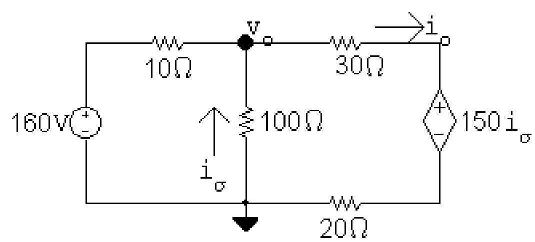
$\sum p_{\rm dev} = 2340 + 432 = 2772$ W

$\sum p_{\rm dis} = 1216.8 + 259.2 + 360 + 176.4 + 378.45 + 81 + 300.15 = 2772$ W

(checks)

Thus, the circuit dissipates 2772 W

P 4.19



The node voltage equation is

$$\frac{v_o - 160}{10} + \frac{v_o}{100} + \frac{v_o - 150i_\sigma}{30+20} = 0$$

The dependent source constraint equation is:

$$i_\sigma = -\frac{v_o}{100}$$

Place these equations in standard form:

$$v_o\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{50}\right) + i_\sigma\left(-\frac{150}{50}\right) = \frac{160}{10}$$
$$v_o\left(\frac{1}{100}\right) + i_\sigma(1) = 0$$

Solving, $v_o = 100$ V; $i_\sigma = -1$ A

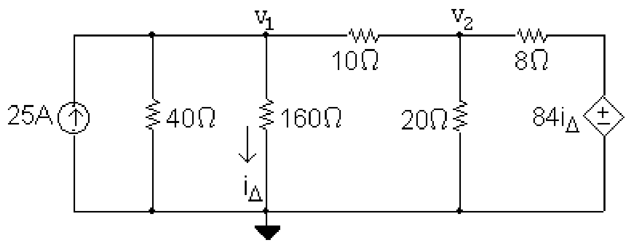
Now find the power:

$$i_o = \frac{160-100}{10} - 1 = 5 \text{ A}$$

$p_{\rm ds} = [150(-1)](5) = -750$ W.

Thus, the dependent source delivers 750 W

P 4.20 **[a]**



The node voltage equations are:

$$-25 + \frac{v_1}{40} + \frac{v_1}{160} + \frac{v_1 - v_2}{10} = 0$$

$$\frac{v_2 - v_1}{10} + \frac{v_2}{20} + \frac{v_2 - 84i_\Delta}{8} = 0$$

The dependent source constraint equation is:

$i_\Delta = v_1/160$

Place these three equations in standard form:

$$v_1\left(\frac{1}{40}+\frac{1}{160}+\frac{1}{10}\right) + v_2\left(-\frac{1}{10}\right) + i_\Delta(0) = 25$$

$$v_1\left(-\frac{1}{10}\right) + v_2\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{8}\right) + i_\Delta\left(-\frac{84}{8}\right) = 0$$

$$v_1\left(-\frac{1}{160}\right) + v_2(0) + i_\Delta(1) = 0$$

Solving, $v_1 = 352$ V; $v_2 = 212$ V; $i_\Delta = 2.2$ A

Now calculate the power. Only the two sources can develop power, so focus on the sources:

$$p_{25\text{A}} = -(352)(25) = -8800 \text{ W}$$

$$i_{\text{dep source}} = (v_2 - 84i_\Delta)/8 = (212 - 84 \cdot 2.2)/8 = 3.4 \text{ A}$$

$$p_{\text{dep source}} = (84 \cdot 2.2)(3.4) = 628.32 \text{ W}$$

Thus, only the current source develops power, so the total power developed in the circuit is 8800 W

[b] The dependent source and all of the resistors dissipate the power developed by the current source. Check that the power developed equals the power dissipated:

$$p_{40\Omega} = (352)^2/40 = 3097.6 \text{ W}$$

$$p_{160\Omega} = (352)^2/160 = 774.4 \text{ W}$$

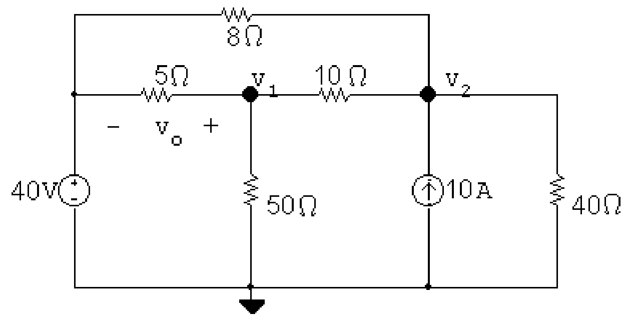
$$p_{10\Omega} = (352 - 212)^2/10 = 1960 \text{ W}$$

$$p_{20\Omega} = (212)^2/20 = 2247.2 \text{ W}$$

$$p_{8\Omega} = (212 - 84 \cdot 2.2)^2/8 = 92.48 \text{ W}$$

$\sum p_{\text{diss}} = 628.32 + 3097.6 + 774.4 + 1960 + 2247.2 + 92.48 = 8800$ W so the power balances.

P 4.21



The two node voltage equations are:

$$\frac{v_1 - 40}{5} + \frac{v_1}{50} + \frac{v_1 - v_2}{10} = 0$$

$$\frac{v_2 - v_1}{10} - 10 + \frac{v_2}{40} + \frac{v_2 - 40}{8} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{5} + \frac{1}{50} + \frac{1}{10}\right) + v_2\left(-\frac{1}{10}\right) = \frac{40}{5}$$

$$v_1\left(-\frac{1}{10}\right) + v_2\left(\frac{1}{10} + \frac{1}{40} + \frac{1}{8}\right) = 10 + \frac{40}{8}$$

Solving, $v_1 = 50$ V; $v_2 = 80$ V.

Thus, $v_o = v_1 - 40 = 50 - 40 = 10$ V.

POWER CHECK:

$$i_g = (50 - 40)/5 + (80 - 40)/8 = 7\text{ A}$$

$$p_{40\text{V}} = (40)(7) = 280\text{ W} \quad \text{(abs)}$$

$$p_{5\Omega} = (50 - 40)^2/5 = 20\text{ W} \quad \text{(abs)}$$

$$p_{8\Omega} = (80 - 40)^2/8 = 200\text{ W} \quad \text{(abs)}$$

$$p_{10\Omega} = (80 - 50)^2/10 = 90\text{ W} \quad \text{(abs)}$$

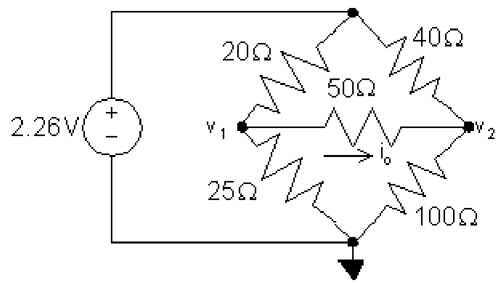
$$p_{50\Omega} = 50^2/50 = 50\text{ W} \quad \text{(abs)}$$

$$p_{40\Omega} = 80^2/40 = 160\text{ W} \quad \text{(abs)}$$

$$p_{10\text{A}} = -(80)(10) = -800\text{ W} \quad \text{(del)}$$

$$\sum p_{\text{abs}} = 280 + 20 + 200 + 90 + 50 + 160 = 800\text{ W} = \sum p_{\text{del}}$$

P 4.22



The node voltage equations are:

$$\frac{v_1 - 2.26}{20} + \frac{v_1 - v_2}{50} + \frac{v_1}{25} = 0$$

$$\frac{v_2 - 2.26}{40} + \frac{v_2 - v_1}{50} + \frac{v_2}{100} = 0$$

Place these equations in standard form:

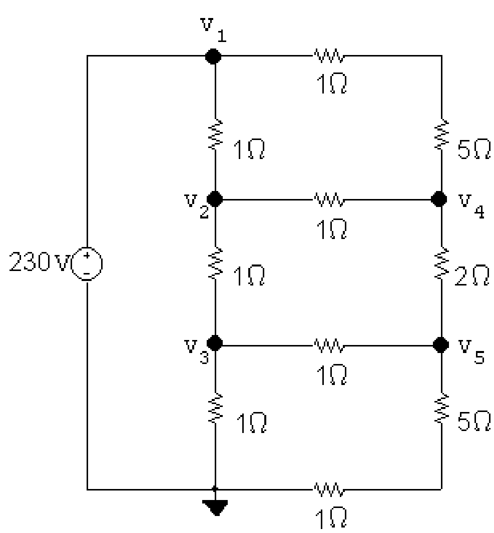
$$v_1\left(\frac{1}{20} + \frac{1}{50} + \frac{1}{25}\right) + v_2\left(-\frac{1}{50}\right) = \frac{2.26}{20}$$

$$v_1\left(-\frac{1}{50}\right) + v_2\left(\frac{1}{40} + \frac{1}{50} + \frac{1}{100}\right) = \frac{2.26}{40}$$

Solving, $v_1 = 1.3$ V; $v_2 = 1.5$ V.

Thus, $i_o = \dfrac{v_1 - v_2}{50} = \dfrac{1.3 - 1.5}{50} = -4$ mA

P 4.23 **[a]**



The node voltage equations are:

$$\frac{v_2-230}{1}+\frac{v_2-v_4}{1}+\frac{v_2-v_3}{1} = 0$$
$$\frac{v_3-v_2}{1}+\frac{v_3-v_5}{1}+\frac{v_3}{1} = 0$$
$$\frac{v_4-230}{5+1}+\frac{v_4-v_2}{1}+\frac{v_4-v_5}{2} = 0$$
$$\frac{v_5-v_4}{2}+\frac{v_5-v_3}{1}+\frac{v_5}{5+1} = 0$$

Place these equations in standard form:

$$v_2(1+1+1)+v_3(-1)+v_4(-1)+v_5(0) = 230$$
$$v_2(-1)+v_3(1+1+1)+v_4(0)+v_5(-1) = 0$$
$$v_2(-1)+v_3(0)+v_4\left(\frac{1}{6}+1+\frac{1}{2}\right)+v_5\left(-\frac{1}{2}\right) = \frac{230}{6}$$
$$v_2(0)+v_3(-1)+v_4\left(-\frac{1}{2}\right)+v_5\left(\frac{1}{2}+1+\frac{1}{6}\right) = 0$$

Solving, $v_2 = 150$ V; $v_3 = 80$ V; $v_4 = 140$ V; $v_5 = 90$ V

Find the power dissipated by the $2\,\Omega$ resistor:

$i_{2\Omega} = \frac{v_4 - v_5}{2} = \frac{140-90}{2} = 25$ A

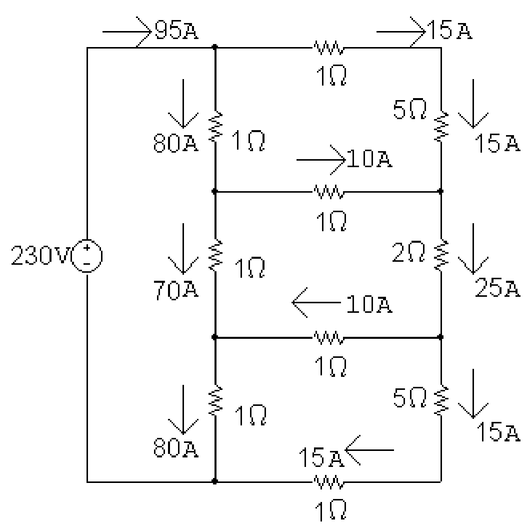
$p_{2\Omega} = (25)^2(2) = 1250$ W

[b] Find the power developed by the 230 V source:

$i_{230\text{V}} = \frac{v_2-230}{1}+\frac{v_4-230}{6} = -80-15 = -95$ A

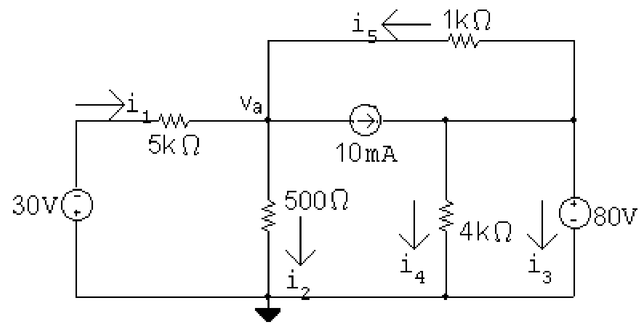
$p_{230\text{V}} = (230)(-95) = -21{,}850$ W, so the source supplies 21,850 W

Check:



$$\sum P_{\text{dis}} = (80)^2(1)+(15)^2(1)+(15)^2(5)+(70)^2(1)+(10)^2(1)$$
$$+(25)^2(2)+(10)^2(1)+(80)^2(1)+(15)^2(5)+(15)^2(1)$$
$$= 21{,}850 \text{ W(checks)}$$

P 4.24 **[a]**



There is only one node voltage equation:

$$\frac{v_a + 30}{5000} + \frac{v_a}{500} + \frac{v_a - 80}{1000} + 0.01 = 0$$

Solving,

$v_a + 30 + 10v_a + 5v_a - 400 + 50 = 0$ so $16v_a = 320$

$\therefore \quad v_a = 20$ V

Calculate the currents:

$$i_1 = (-30 - 20)/5000 = -10 \text{ mA}$$

$$i_2 = 20/500 = 40 \text{ mA}$$

$$i_4 = 80/4000 = 20 \text{ mA}$$

$$i_5 = (80 - 20)/1000 = 60 \text{ mA}$$

$i_3 + i_4 + i_5 - 10 \text{ mA} = 0$ so $i_3 = 0.01 - 0.02 - 0.06 = -0.07 = -70$ mA

[b]

$$p_{30V} = (30)(-0.01) = -0.3 \text{ W}$$

$$p_{10mA} = (20 - 80)(0.01) = -0.6 \text{ W}$$

$$p_{80V} = (80)(-0.07) = -5.6 \text{ W}$$

$$p_{5k} = (-0.01)^2(5000) = 0.5 \text{ W}$$

$$p_{500\Omega} = (0.04)^2(500) = 0.8 \text{ W}$$

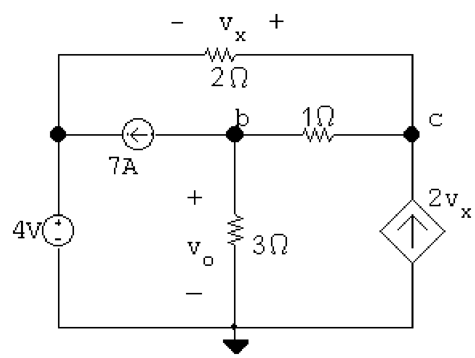
$$p_{1k} = (80 - 20)^2/(1000) = 3.6 \text{ W}$$

$$p_{4k} = (80)^2/(4000) = 1.6 \text{ W}$$

$\sum p_{abs} = 0.5 + 0.8 + 3.6 + 1.6 = 6.5$ W

$\sum p_{del} = 0.3 + 0.6 + 5.6 = 6.5$ W (checks!)

P 4.25



The two node voltage equations are:

$$7 + \frac{v_\text{b}}{3} + \frac{v_\text{b} - v_\text{c}}{1} = 0$$

$$-2v_x + \frac{v_\text{c} - v_\text{b}}{1} + \frac{v_\text{c} - 4}{2} = 0$$

The constraint equation for the dependent source is:

$v_x = v_\text{c} - 4$

Place these equations in standard form:

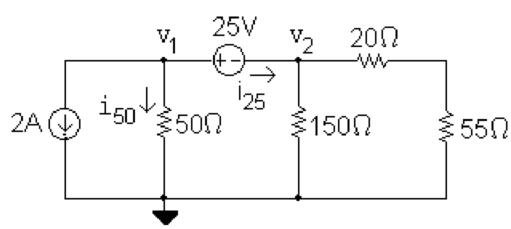
$$v_\text{b}\left(\frac{1}{3} + 1\right) + v_\text{c}(-1) + v_x(0) = -7$$

$$v_\text{b}(-1) + v_\text{c}\left(1 + \frac{1}{2}\right) + v_x(-2) = \frac{4}{2}$$

$$v_\text{b}(0) + v_\text{c}(1) + v_x(-1) = 4$$

Solving, $v_o = v_\text{b} = 1.5$ V Also, $v_c = 9$ V and $v_x = 5$ V.

P 4.26



This circuit has a supernode includes the nodes v_1, v_2 and the 25 V source. The supernode equation is

$$2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{20 + 55} = 0$$

The supernode constraint equation is

$$v_2 + 25 = v_1$$

Place these two equations in standard form:

$$v_1\left(\frac{1}{50}\right) + v_2\left(\frac{1}{150}+\frac{1}{75}\right) = -2$$
$$v_1(1) + v_2(-1) = 25$$

Solving, $v_1 = -37.5$ V and $v_2 = -62.5$ V.

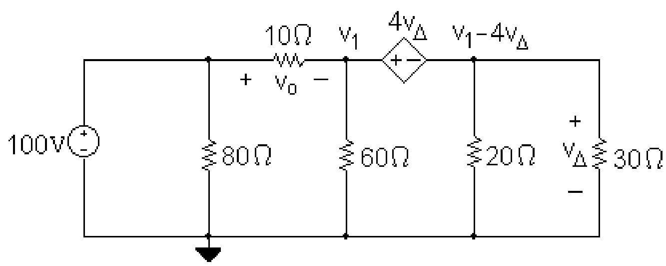
$p_{25\text{V}} = (25)i_{25}$

$$i_{25} = -2\text{ A} - i_{50} = -2\text{ A} - \frac{v_1}{50} = 2\text{ A} - \frac{-37.5}{50} = -2\text{ A} + 0.75\text{ A} = -1.25\text{ A}$$

Thus, $p_{25\text{V}} = (25)(-1.25) = -31.25$ W

The 25 V source delivers 31.25 W.

P 4.27



The supernode equation is:

$$\frac{v_1-100}{10}+\frac{v_1}{60}+\frac{v_1-4v_\Delta}{20}+\frac{v_1-4v_\Delta}{30}=0$$

The constraint equation for the dependent source is:

$4v_\Delta = v_1 - v_\Delta$

Place these equations in standard form:

$$v_1\left(\frac{1}{10}+\frac{1}{60}+\frac{1}{20}+\frac{1}{30}\right) + v_\Delta\left(-\frac{4}{20}-\frac{4}{30}\right) = \frac{100}{10}$$
$$v_1(1) + v_\Delta(-5) = 0$$

Solving, $v_1 = 75$ V; $v_\Delta = 15$ V

Thus, $v_o = 100 - v_1 = 25$ V

P 4.28 Calculate currents and voltages needed to calculate the power for the various components:

$$i_\phi = \frac{v_4 - v_3}{8} = \frac{81.6-108}{8} = -3.3\text{ A}$$
$$\frac{40}{3}i_\phi = \frac{40}{3}(-3.3) = -44\text{ V}$$
$$v_1 = v_4 + \frac{40}{3}i_\phi = 81.6 - 44 = 37.6\text{ V}$$

$$v_3 + v_\Delta = 120 \qquad \therefore \qquad v_\Delta = 120 - 108 = 12 \text{ V}$$

$$1.75 v_\Delta = (1.75)(12) = 21 \text{ A}$$

$$i_{120\text{V}} = \frac{v_1 - 120}{4} + \frac{v_3 - 120}{2} = \frac{37.6 - 120}{4} + \frac{108 - 120}{2} = -26.6 \text{ A}$$

$$i_{\text{ccvs}} = \frac{0 - v_1}{20} + \frac{v_2 - v_1}{4} = \frac{-37.6}{20} + \frac{120 - 37.6}{4} = 18.72 \text{ A}$$

Now calculate the power associated with each circuit element:

$$p_{20\Omega} = (37.6)^2/20 = 70.688 \text{ W}$$

$$p_{4\Omega} = (37.6 - 120)^2/4 = 1697.44 \text{ W}$$

$$p_{120\text{V}} = (120)(-26.6) = -3192 \text{ W}$$

$$p_{2\Omega} = (12)^2/2 = 72 \text{ W}$$

$$p_{40\Omega} = (108)^2/40 = 291.6 \text{ W}$$

$$p_{8\Omega} = (108 - 81.6)^2/8 = 87.12 \text{ W}$$

$$p_{80\Omega} = (81.6)^2/80 = 83.232 \text{ W}$$

$$p_{\text{vccs}} = (81.6)[1.75(12)] = 1713.6 \text{ W} \qquad \sum p_{\text{abs}} = \sum p_{\text{del}} = 4015.6 \text{ W}$$

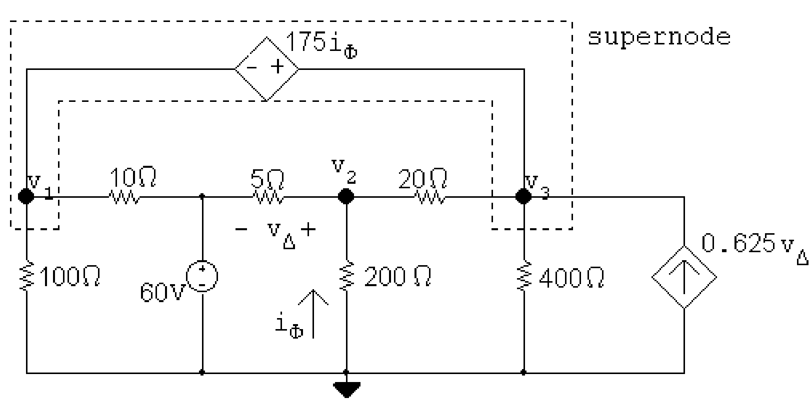
$$p_{\text{ccvs}} = (18.72)(-44) = -823.68 \text{ W}$$

Now sum the powers:

$$\sum p_{\text{total}} = 70.688 + 1697.44 - 3192 + 72 + 291.6 + 87.12 + 83.232 + 1712.6 - 823.68 = 0 \text{ W}$$

Thus, the power balances and the staff analyst has correctly calculated the voltage values

P 4.29



The supernode equation is:

$$\frac{v_1}{100} + \frac{v_1 - 60}{10} + \frac{v_3 - v_2}{20} + \frac{v_3}{400} - 0.625 v_\Delta = 0$$

The node voltage equation at v_2 is:

$$\frac{v_2-60}{5}+\frac{v_2}{200}+\frac{v_2-v_3}{20}=0$$

The supernode constraint equation is:

$v_3 - v_1 = 175 i_\phi$

The two dependent source constraint equations are:

$v_\Delta = v_2 - 60$

$i_\phi = -v_2/200$

Place the four equations above in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{100}+\frac{1}{10}\right)+v_2\left(-\frac{1}{20}\right)+v_3\left(\frac{1}{400}+\frac{1}{20}\right)+i_\phi(0)+v_\Delta(-0.625) &= \frac{60}{10}\\
v_1(0)+v_2\left(\frac{1}{5}+\frac{1}{200}+\frac{1}{20}\right)+v_3\left(-\frac{1}{20}\right)+i_\phi(0)+v_\Delta(0) &= \frac{60}{5}\\
v_1(1)+v_2(0)+v_3(-1)+i_\phi(175)+v_\Delta(0) &= 0\\
v_1(0)+v_2(1)+v_3(0)+i_\phi(0)+v_\Delta(-1) &= 60\\
v_1(0)+v_2\left(\frac{1}{200}\right)+v_3(0)+i_\phi(1)+v_\Delta(0) &= 0
\end{aligned}$$

Solving,

$v_1 = -60.75$ V $\quad v_2 = 30$ V; $\quad v_3 = -87$ V; $\quad i_\phi = -0.15$ A; $\quad v_\Delta = -30$ V

Calculate the power for the 60 V source:

$$\begin{aligned}
i_{60\text{V}} &= \frac{v_1-60}{10}+\frac{v_2-60}{5}\\
&= \frac{-60.75-60}{10}+\frac{30-60}{5}=-18.075 \text{ A}\\
p_{60\text{V}} &= (60)(-18.075)=-1084.5 \text{ W}
\end{aligned}$$

Thus, the 60 V source delivers 1084.5 W

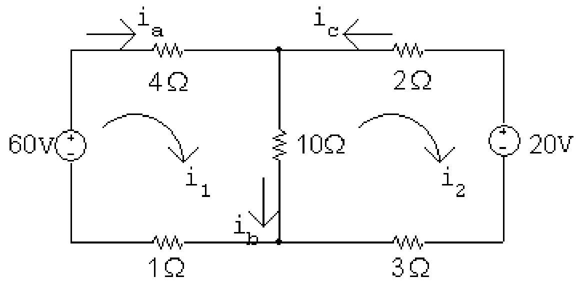
P 4.30 From Eq. 4.16, $\quad i_B = v_c/(1+\beta)R_E$

From Eq. 4.17, $\quad i_B = (v_b - V_o)/(1+\beta)R_E$

From Eq. 4.19,

$$\begin{aligned}
i_B &= \frac{1}{(1+\beta)R_E}\left[\frac{V_{CC}(1+\beta)R_ER_2+V_oR_1R_2}{R_1R_2+(1+\beta)R_E(R_1+R_2)}-V_o\right]\\
&= \frac{V_{CC}R_2-V_o(R_1+R_2)}{R_1R_2+(1+\beta)R_E(R_1+R_2)}=\frac{[V_{CC}R_2/(R_1+R_2)]-V_o}{[R_1R_2/(R_1+R_2)]+(1+\beta)R_E}
\end{aligned}$$

P 4.31 **[a]**



The mesh current equations are:

$$-60 + 4i_1 + 10(i_1 - i_2) + 1i_1 = 0$$

$$20 + 3i_2 + 10(i_2 - i_1) + 2i_2 = 0$$

Place the equations in standard form:

$$i_1(4 + 10 + 1) + i_2(-10) = 60$$

$$i_1(-10) + i_2(3 + 10 + 2) = -20$$

Solving, $i_1 = 5.6$ A; $i_2 = 2.4$ A

Now solve for the requested currents:

$i_a = i_1 = 5.6$ A; $i_b = i_1 - i_2 = 3.2$ A; $i_c = -i_2 = -2.4$ A

[b] If the polarity of the 60 V source is reversed, we have the following mesh current equations in standard form:

$$i_1(4 + 10 + 1) + i_2(-10) = -60$$

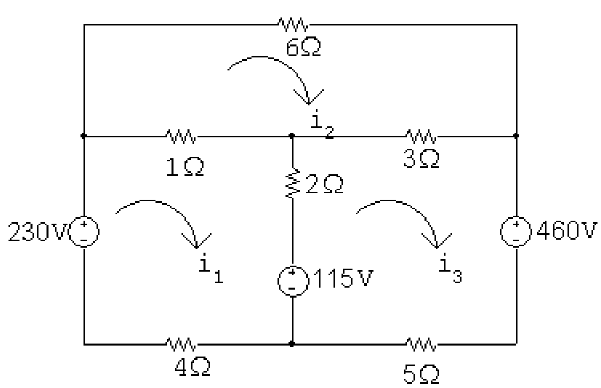
$$i_1(-10) + i_2(3 + 10 + 2) = -20$$

Solving, $i_1 = -8.8$ A; $i_2 = -7.2$ A

Now solve for the requested currents:

$i_a = i_1 = -8.8$ A; $i_b = i_1 - i_2 = -1.6$ A; $i_c = -i_2 = 7.2$ A

P 4.32 **[a]**



The mesh current equations are:

$$-230 + 1(i_1 - i_2) + 2(i_1 - i_3) + 115 + 4i_1 = 0$$
$$6i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) = 0$$
$$460 + 5i_3 - 115 + 2(i_3 - i_1) + 3(i_3 - i_2) = 0$$

Place these equations in standard form:

$$i_1(1 + 2 + 4) + i_2(-1) + i_3(-2) = 115$$
$$i_1(-1) + i_2(6 + 3 + 1) + i_3(-3) = 0$$
$$i_1(-2) + i_2(-3) + i_3(5 + 2 + 3) = -345$$

Solving, $i_1 = 4.4$ A; $i_2 = -10.6$ A; $i_3 = -36.8$ A

The only components that can develop power in the circuit are the sources:

$$p_{230\text{V}} = -(230)(4.4) = -1012 \text{ W}$$
$$p_{115\text{V}} = -(115)(-36.8 - 4.4) = 4738 \text{ W}$$
$$p_{460\text{V}} = (460)(-36.8) = -16{,}928 \text{ W}$$

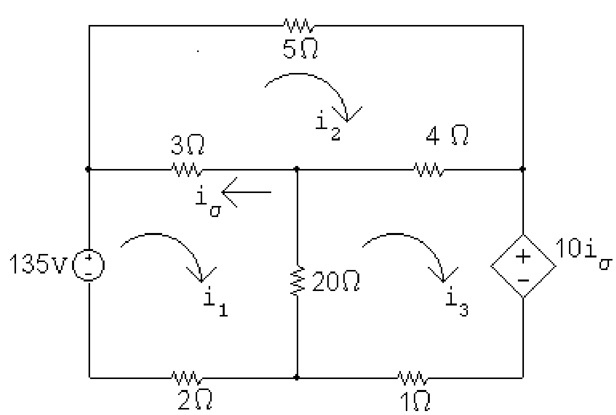
$\therefore \quad \sum p_{\text{dev}} = 1012 + 16{,}928 = 17940$ W

[b] From part (a) we know that the 115 V source is dissipating power; compute the power dissipated by the resistors:

$$p_{1\Omega} = (1)(4.4 + 10.6)^2 = 225 \text{ W}$$
$$p_{4\Omega} = (4)(4.4)^2 = 77.44 \text{ W}$$
$$p_{6\Omega} = (6)(-10.6)^2 = 674.16 \text{ W}$$
$$p_{2\Omega} = (2)(4.4 + 36.8)^2 = 3394.88 \text{ W}$$
$$p_{3\Omega} = (3)(-10.6 + 36.8)^2 = 2059.32 \text{ W}$$
$$p_{5\Omega} = (5)(-36.8)^2 = 6771.2 \text{ W}$$

$\therefore \quad \sum p_{\text{dis}} = 4738 + 225 + 77.44 + 674.16 + 3394.88 + 2059.32 + 6771.2$
$= 17940$ W (checks!)

P 4.33



The mesh current equations are:

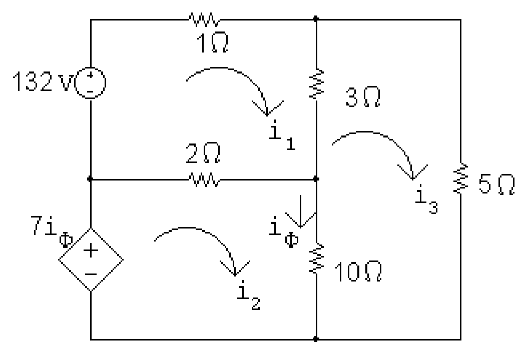
$$-135 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 = 0$$
$$5i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$
$$10i_\sigma + 1i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) = 0$$

The dependent source constraint equation is:
$i_\sigma = i_2 - i_1$
Place these equations in standard form:

$$i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i_\sigma(0) = 135$$
$$i_1(-3) + i_2(5 + 4 + 3) + i_3(-4) + i_\sigma(0) = 0$$
$$i_1(-20) + i_2(-4) + i_3(1 + 20 + 4) + i_\sigma(10) = 0$$
$$i_1(1) + i_2(-1) + i_3(0) + i_\sigma(1) = 0$$

Solving, $i_1 = 64.8$ A, $i_2 = 39$ A; $i_3 = 68.4$ A; $i_\sigma = -25.8$ A
Calculate the power:
$p_{20\Omega} = 20(68.4 - 64.8)^2 = 259.2$ W
Thus the $20\,\Omega$ resistor dissipates 259.2 W.

P 4.34



The mesh current equations:

$$-132 + 1i_1 + 3(i_1 - i_3) + 2(i_1 - i_2) = 0$$
$$-7i_\phi + 2(i_2 - i_1) + 10(i_2 - i_3) = 0$$
$$5i_3 + 10(i_3 - i_2) + 3(i_3 - i_1) = 0$$

The dependent source constraint equation:
$i_\phi = i_2 - i_3$

Place these equations in standard form:

$$i_1(1 + 3 + 2) + i_2(-2) + i_3(-3) + i_\phi(0) = 132$$
$$i_1(-2) + i_2(10 + 2) + i_3(-10) + i_\phi(-7) = 0$$
$$i_1(-3) + i_2(-10) + i_3(5 + 10 + 3) + i_\phi(0) = 0$$
$$i_1(0) + i_2(-1) + i_3(1) + i_\phi(1) = 0$$

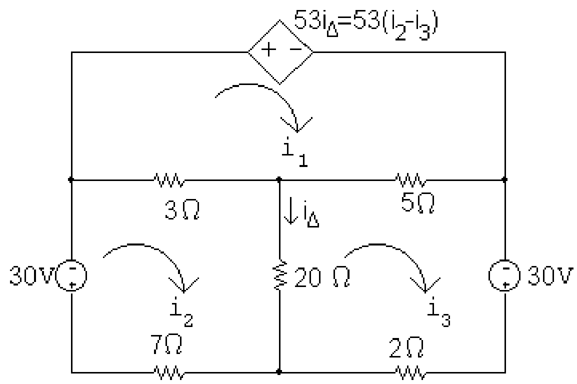
Solving, $i_1 = 48$ A; $i_2 = 36$ A; $i_3 = 28$ A; $i_\phi = 8$ A

Solve for the power:

$p_{\text{dep source}} = -7(i_\phi)i_2 = -7(8)(36) = -2016$ W

Thus, the dependent source is developing 2016 W.

P 4.35



The mesh current equations:

$$53(i_2 - i_3) + 5(i_1 - i_3) + 3(i_1 - i_2) = 0$$

$$30 + 3(i_2 - i_1) + 20(i_2 - i_3) + 7i_2 = 0$$

$$-30 + 2i_3 + 20(i_3 - i_2) + 5(i_3 - i_1) = 0$$

Place these equations in standard form:

$$i_1(5+3) + i_2(53-3) + i_3(-53-5) = 0$$

$$i_1(-3) + i_2(3+20+7) + i_3(-20) = -30$$

$$i_1(-5) + i_2(-20) + i_3(2+20+5) = 30$$

Solving, $i_1 = 186$ A; $i_2 = 81.6$ A; $i_3 = 96$ A

Calculate the power:

$$p_{30\text{V(left)}} = (30)(81.6) = 2448 \text{ W}$$

$$p_{30\text{V(right)}} = -(30)(96) = -2880 \text{ W}$$

$$p_{\text{dep source}} = 53(81.6 - 96)(186) = -141{,}955.2 \text{ W}$$

$$p_{3\Omega} = (3)(186 - 81.6)^2 = 32{,}698.08 \text{ W}$$

$$p_{5\Omega} = (5)(186 - 96)^2 = 40{,}500 \text{ W}$$

$$p_{20\Omega} = (20)(81.6 - 96)^2 = 4147.2 \text{ W}$$

$$p_{7\Omega} = (7)(81.6)^2 = 46{,}609.92 \text{ W}$$

$$p_{2\Omega} = (2)(96)^2 = 18{,}432 \text{ W}$$

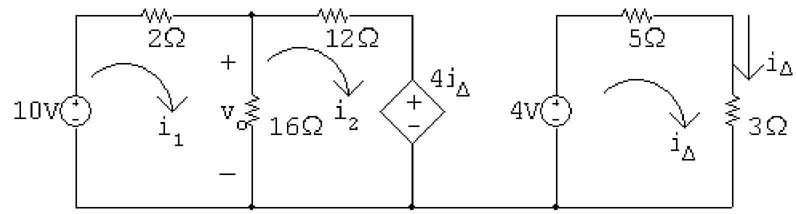
$$\sum p_{\text{dev}} = 2880 + 141{,}955.2 = 144{,}835.2 \text{ W}$$

$$\sum p_{\rm dis} = 2448 + 32{,}698.08 + 40{,}500 + 4147.2 + 46{,}609.92 + 18{,}432$$

$$= 144{,}835.2 \text{ W(checks)}$$

Thus the dependent source develops $141{,}955.2$ W.

P 4.36 **[a]**



$$10 = 18i_1 - 16i_2$$

$$0 = -16i_1 + 28i_2 + 4i_\Delta$$

$$4 = 8i_\Delta$$

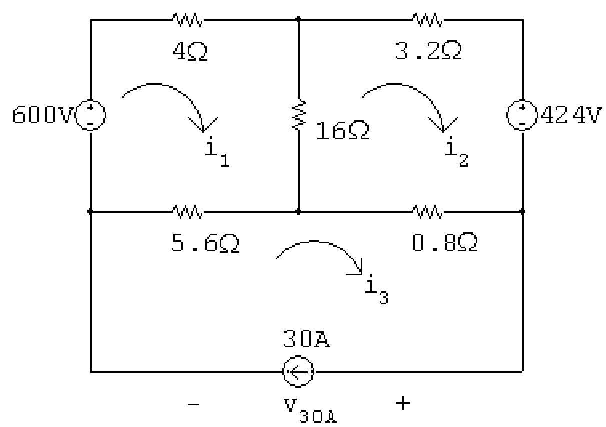
Solving, $i_1 = 1$ A; $i_2 = 0.5$ A; $i_\Delta = 0.5$ A

$v_0 = 16(i_1 - i_2) = 16(0.5) = 8$ V

[b] $p_{4i_\Delta} = 4i_\Delta i_2 = (4)(0.5)(0.5) = 1$ W (abs)

$\therefore$ p_{4i_Δ} (deliver) $= -1$ W

P 4.37



$$600 = 25.6i_1 - 16i_2 - 5.6i_3$$

$$-424 = -16i_1 + 20i_2 - 0.8i_3$$

$$30 = i_3$$

Solving, $i_1 = 35$ A; $i_2 = 8$ A; $i_3 = 30$ A

[a] $v_{30\text{A}} = 0.8(i_2 - i_3) + 5.6(i_1 - i_3)$

$= 0.8(8 - 30) + 5.6(35 - 30) = 10.4$ V

$p_{30\text{A}} = 30v_{30\text{A}} = 30(10.4) = 312$ W (abs)

Therefore, the 30 A source delivers -312 W.

[b] $p_{600\text{V}} = -600(35) = -21{,}000$ W(del)

$p_{424\text{V}} = 424(8) = 3392$ W(abs)

Therefore, the total power delivered is $21{,}000$ W

[c] $p_{4\Omega} = (35)^2(4) = 4900$ W

$p_{3.2\Omega} = (8)^2(3.2) = 204.8$ W

$p_{16\Omega} = (35 - 8)^2(16) = 11{,}664$ W

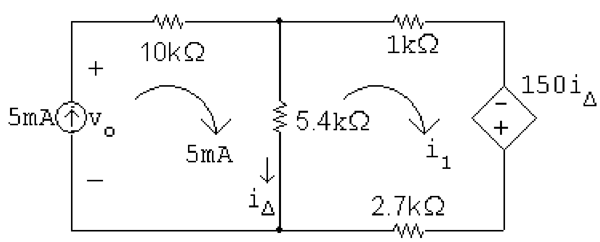
$p_{5.6\Omega} = (35 - 30)^2(5.6) = 140$ W

$p_{0.8\Omega} = (-30 + 8)^2(0.8) = 387.2$ W

$\sum p_{\text{resistors}} = 17{,}296$ W

$\sum p_{\text{abs}} = 17{,}296 + 312 + 3392 = 21{,}000$ W (CHECKS)

P 4.38 **[a]**



The mesh current equation for the right mesh is:
$5400(i_1 - 0.005) + 3700i_1 - 150(0.005 - i_1) = 0$
Solving, $9250i_1 = 27.75$ $\therefore$ $i_1 = 3$ mA
Then, $i_\Delta = 0.005 - i_1 = 0.005 - 0.003 = 0.002 = 2$ mA

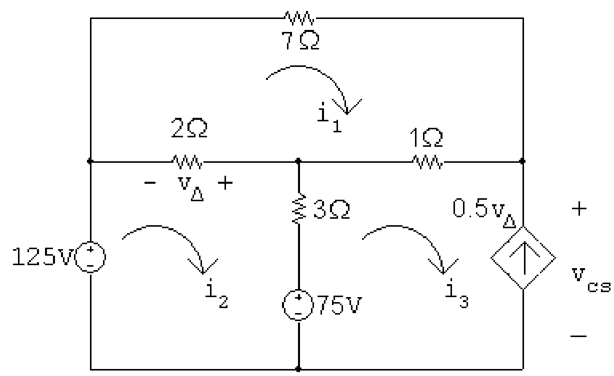
[b] $v_o = (0.005)(10{,}000) + (0.002)(5400) = 60.8$ V
$p_{5\text{mA}} = -(60.8)(0.005) = -304$ mW
Thus, the 5 mA source delivers 304 mW

[c] $150i_\Delta = 150(0.002) = 0.3$ V

$p_{\text{dep source}} = 150i_\Delta i_1 = -(0.3)(0.003) = -0.9$ mW

The dependent source delivers 0.9 mW.

P 4.39



Mesh equations:

$$7i_1 + 1(i_1 - i_3) + 2(i_1 - i_2) = 0$$

$$-125 + 2(i_2 - i_1) + 3(i_2 - i_3) + 75 = 0$$

Constraint equations:

$i_3 = -0.5v_\Delta;$ $\quad v_\Delta = 2(i_1 - i_2)$

Place these equations in standard form:

$$i_1(7+1+2) + i_2(-2) + i_3(-1) + v_\Delta(0) = 0$$

$$i_1(-2) + i_2(2+3) + i_3(-3) + v_\Delta(0) = 50$$

$$i_1(0) + i_2(0) + i_3(1) + v_\Delta(0.5) = 0$$

$$i_1(2) + i_2(-2) + i_3(0) + v_\Delta(-1) = 0$$

Solving, $i_1 = 6$ A; $\quad i_2 = 22$ A; $\quad i_3 = 16$ A; $\quad v_\Delta = -32$ V

Solve the outer loop KVL equation to find v_{cs}:

$-125 + 7i_1 + v_{\text{cs}} = 0;$ $\quad\therefore\quad v_{\text{cs}} = 125 - 7(6) = 83$ V

Calculate the power:

$$p_{125\text{V}} = -(125)(22) = -2750 \text{ W}$$

$$p_{75\text{V}} = (75)(22 - 16) = 450 \text{ W}$$

$$p_{\text{dep source}} = -(83)[0.5(-32)] = 1328 \text{ W}$$

Thus, the total power developed is 2750 W.

CHECK:

$$p_{7\Omega} = (6)^2(7) = 252 \text{ W}$$

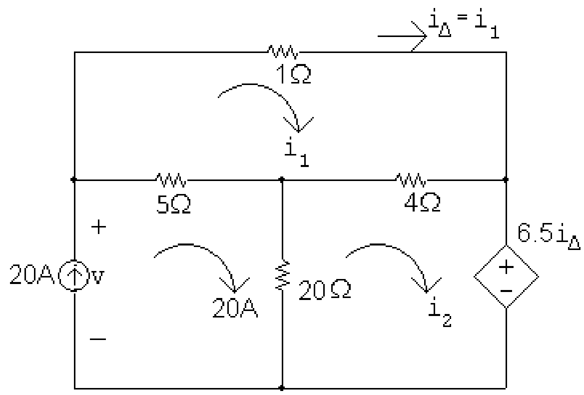
$$p_{2\Omega} = (22 - 6)^2(2) = 512 \text{ W}$$

$$p_{3\Omega} = (22 - 16)^2(3) = 108 \text{ W}$$

$$p_{1\Omega} = (16 - 6)^2(1) = 100 \text{ W}$$

$\therefore \quad \sum p_{\text{abs}} = 450 + 1328 + 252 + 512 + 108 + 100 = 2750$ W (checks!)

P 4.40



Since the bottom left mesh current value is known, we need only two mesh current equations:

$$1i_1 + 4(i_1 - i_2) + 5(i_1 - 20) = 0$$

$$6.5i_1 + 20(i_2 - 20) + 4(i_2 - i_1) = 0$$

Place these equations in standard form:

$$i_1(1 + 4 + 5) + i_2(-4) = 100$$

$$i_1(6.5 - 4) + i_2(20 + 4) = 400$$

Solving, $\quad i_1 = 16$ A; $\quad i_2 = 15$ A

Find v:

$-v + 5(20 - i_1) + 20(20 - i_2) = 0 \quad \therefore \quad v = 5(4) + 20(5) = 120$ V

Calculate the power:

$$p_{20\text{A}} = -(120)(20) = -2400 \text{ W}$$

$$p_{\text{dep source}} = [6.5(16)](15) = 1560 \text{ W}$$

$$p_{1\Omega} = 1(16)^2 = 256 \text{ W}$$

$$p_{5\Omega} = 5(20 - 16)^2 = 80 \text{ W}$$

$$p_{4\Omega} = 4(16 - 15)^2 = 4 \text{ W}$$

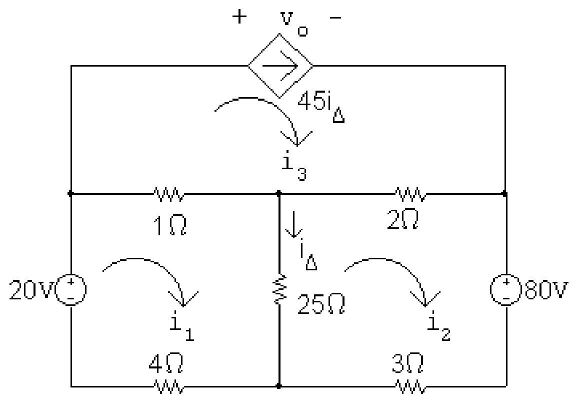
$$p_{20\Omega} = 20(20 - 15)^2 = 500 \text{ W}$$

$\sum p_{\text{dev}} = 2400$ W

$\sum p_{\text{dis}} = 1560 + 256 + 80 + 4 + 500 = 2400$ W (checks)

The power developed by the 20 A source is 2400 W

P 4.41 **[a]**



The mesh current equations are:

$$-20 + 1(i_1 - i_3) + 25(i_1 - i_2) + 4i_1 = 0$$

$$80 + 3i_2 + 25(i_2 - i_1) + 2(i_2 - i_3) = 0$$

The constraint equation is:

$i_3 = 45i_\Delta = 45(i_1 - i_2)$

Place these equations in standard form:

$$i_1(1 + 25 + 4) + i_2(-25) + i_3(-1) = 20$$

$$i_1(-25) + i_2(3 + 25 + 2) + i_3(-2) = -80$$

$$i_1(-45) + i_2(45) + i_3(1) = 0$$

Solving, $i_1 = 8$ A; $i_2 = 7$ A; $i_3 = 45$ A

Find the power in the $2\,\Omega$ resistor:

$p_{2\Omega} = 2(i_2 - i_3)^2 = 2(-38)^2 = 2888$ W

The $2\,\Omega$ resistor dissipates 2888 W.

[b] Find the power developed by the sources:

$v_o + 80 + 3(7) + 4(8) - 20 = 0 \qquad \therefore \qquad v_o = 20 - 80 - 21 - 32 = -113$ V

$$p_{\text{dep source}} = (-113)[45(8 - 7)] = -5085 \text{ W}$$

$$p_{80\text{V}} = (80)(7) = 560 \text{ W}$$

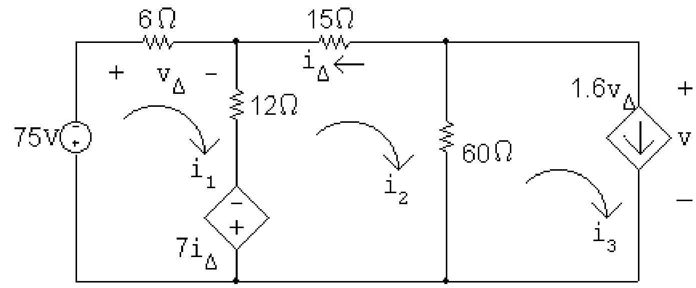
$$p_{20\text{V}} = -(20)(8) = -160 \text{ W}$$

$$\sum p_{\text{dev}} = 5085 + 160 = 5245 \text{ W}$$

The percent of the power developed that is deliverd to the $2\,\Omega$ resistor is:

$$\frac{2888}{5245} \times 100 = 55.06\%$$

P 4.42 **[a]**



The mesh current equations are:

$$75 + 6i_1 + 12(i_1 - i_2) - 7i_\Delta = 0$$

$$15i_2 + 60(i_2 - i_3) + 7i_\Delta + 12(i_2 - i_1) = 0$$

The two constraint equations are:

$$i_\Delta = -i_2$$

$$i_3 = 1.6v_\Delta = 1.6(6i_1) = 9.6i_1$$

Place these equations in standard form:

$$i_1(6 + 12) + i_2(-12) + i_3(0) + i_\Delta(-7) = -75$$

$$i_1(-12) + i_2(15 + 60 + 12) + i_3(-60) + i_\Delta(7) = 0$$

$$i_1(0) + i_2(1) + i_3(0) + i_\Delta(1) = 0$$

$$i_1(9.6) + i_2(0) + i_3(-1) + i_\Delta(0) = 0$$

Solving, $i_1 = 4$ A; $i_2 = 29.4$ A; $i_3 = 38.4$ A; $i_\Delta = -29.4$ A

Calculate the power associated with the three sources:

$$v = 60(i_2 - i_3) = -540 \text{ V}$$

$$v_\Delta = 6i_1 = 6(4) = 24 \text{ V}$$

$$p_{75\text{V}} = (75)(4) = 300 \text{ W}$$

$$p_{\text{CCVS}} = -7(-29.4)(4 - 29.4) = -5227.32 \text{ W}$$

$$p_{\text{VCCS}} = (-540)[1.6(24)] = -20{,}736 \text{ W}$$

The two dependent sources are generating a total of $5227.32 + 20{,}736 = 25{,}963.32$ W.

[b] Find the power dissipated. Remember that the 75 V source is generating 300 W, as calculated in part (a):

$$p_{6\Omega} = (6)(4)^2 = 96 \text{ W}$$

$$p_{12\Omega} = (12)(4 - 29.4)^2 = 7741.92 \text{ W}$$

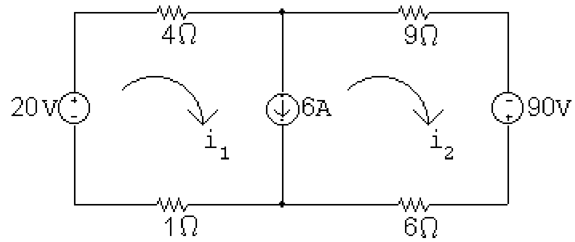
$$p_{15\Omega} = (15)(29.4)^2 = 12{,}965.4 \text{ W}$$

$$p_{60\Omega} = (60)(29.4 - 38.4)^2 = 4860 \text{ W}$$

$$\sum p_{\text{dis}} = 300 + 96 + 7741.92 + 12{,}965.4 + 4860 = 25{,}963.32 \text{ W(checks)}$$

Thus the power dissipated in the circuit is $25{,}963.32$ W.

P 4.43



The supermesh equation is:
$-20 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0$
The supermesh constraint equation is :
$i_1 - i_2 = 6$
Place these equations in standard form:

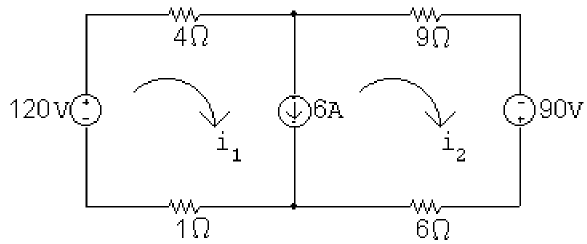
$$i_1(4+1) + i_2(9+6) = 20 + 90$$
$$i_1(1) + i_2(-1) = 6$$

Solving, $i_1 = 10$ A; $i_2 = 4$ A
Now find the power:

$$p_{4\Omega} = 10^2(4) = 400 \text{ W}$$
$$p_{1\Omega} = 10^2(1) = 100 \text{ W}$$
$$p_{9\Omega} = 4^2(9) = 144 \text{ W}$$
$$p_{6\Omega} = 4^2(6) = 96 \text{ W}$$
$$p_{20\text{V}} = -(20)(10) = -200 \text{ W}$$
$$v_{6\text{A}} = 9i_2 - 90 + 6i_2 = (9)(4) - 90 + (6)(4) = -30 \text{ V}$$
$$p_{6\text{A}} = (-30)(6) = -180 \text{ W}$$
$$p_{90\text{V}} = -(90)(4) = -360 \text{ W}$$

In summary:
$\sum p_{\text{dev}} = 200 + 180 + 360 = 740$ W
$\sum p_{\text{diss}} = 400 + 100 + 144 + 96 = 740$ W
Thus the power dissipated in the circuit is 740 W

P 4.44



The supermesh equation is:
$-120 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0$
The supermesh constraint equation is :
$i_1 - i_2 = 6$
Place these equations in standard form:

$$i_1(4+1) + i_2(9+6) = 120 + 90$$

$$i_1(1) + i_2(-1) = 6$$

Solving, $\quad i_1 = 15$ A; $\quad i_2 = 9$ A
Now find the power:

$$p_{4\Omega} = 15^2(4) = 900 \text{ W}$$

$$p_{1\Omega} = 15^2(1) = 225 \text{ W}$$

$$p_{9\Omega} = 9^2(9) = 729 \text{ W}$$

$$p_{6\Omega} = 9^2(6) = 486 \text{ W}$$

$$p_{120\text{V}} = -(120)(15) = -1800 \text{ W}$$

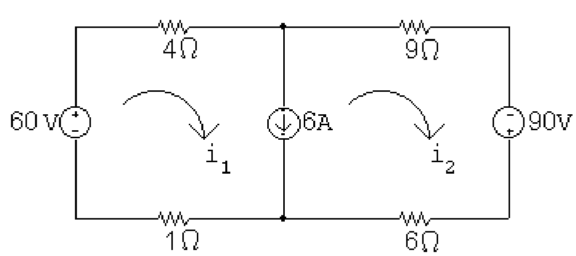
$$v_o = 9i_2 - 90 + 6i_2 = 9(9) - 90 + 6(9) = 45 \text{ V}$$

$$p_{6\text{A}} = (45)(6) = 270 \text{ W}$$

$$p_{90\text{V}} = -(90)(9) = -810 \text{ W}$$

In summary:
$\sum p_{\text{dev}} = 900 + 225 + 729 + 486 + 270 = 2610$ W (note that the 6 A source is now dissipating power!)
$\sum p_{\text{diss}} = 1800 + 810 = 2610$ W
Thus the power dissipated in the circuit is 2610 W

P 4.45 **[a]**



The supermesh equation is:
$-60 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0$
The supermesh constraint equation is :
$i_1 - i_2 = 6$
Place these equations in standard form:

$$i_1(4+1) + i_2(9+6) = 60 + 90$$
$$i_1(1) + i_2(-1) = 6$$

Solving, $i_1 = 12$ A; $i_2 = 6$ A
Now find the power:

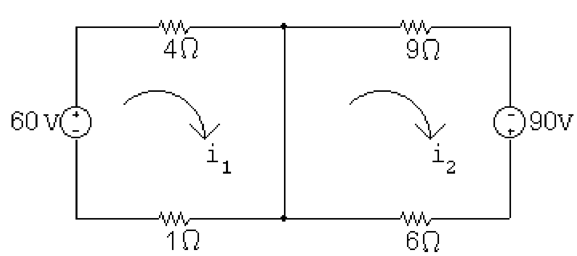
$$p_{4\Omega} = 12^2(4) = 576 \text{ W}$$
$$p_{1\Omega} = 12^2(1) = 144 \text{ W}$$
$$p_{9\Omega} = 6^2(9) = 324 \text{ W}$$
$$p_{6\Omega} = 6^2(6) = 216 \text{ W}$$
$$p_{60\text{V}} = -(60)(20) = -720 \text{ W}$$
$$v_o = 9i_2 - 90 + 6i_2 = 9(6) - 90 + 6(6) = 0 \text{ V}$$

(the 6 A source acts like a short circuit carrying 6 A of current)

$$p_{6\text{A}} = (0)(6) = 0 \text{ W}$$
$$p_{90\text{V}} = -(90)(6) = -540 \text{ W}$$

In summary:
$\sum p_{\text{dev}} = 576 + 144 + 324 + 216 = 1260$ W (note that the power of the 6 A source is zero)
$\sum p_{\text{diss}} = 720 + 540 = 1260$ W
Thus the power dissipated in the circuit is 1260 W

[b]



Now there is no longer a supermesh. The two simple mesh current equations are:

$$-60 + 4i_1 + 1i_1 = 0$$
$$-90 + 6i_2 + 9i_2 = 0$$

Since these equations are uncoupled, each can be solved separately:

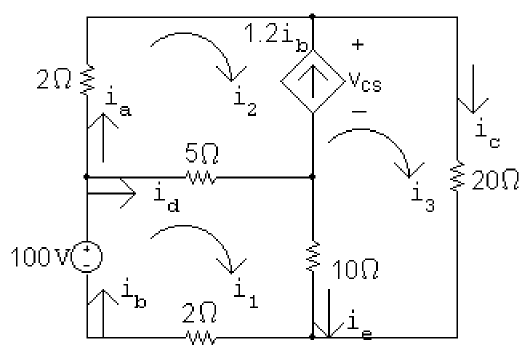
$$5i_1 = 60 \quad \therefore \quad i_1 = 60/5 = 12 \text{ A}$$

$$15i_2 = 90 \quad \therefore \quad i_2 = 90/15 = 6 \text{ A}$$

Since the currents are the same as in part (a), the power will be the same as calculated in part (a). Thus, the power dissipated in the circuit is again 1260 W.

[c] As noted in part (a), the 6 A source has zero voltage drop, so is equivalent to a short circuit (which has no voltage drop by definition) carrying 6 A of current, as in the circuit of part (b).

P 4.46 **[a]**



The i_1 mesh current equation:
$-100 + 5(i_1 - i_2) + 10(i_1 - i_3) + 2i_1 = 0$
The i_2 — i_3 supermesh equationa:
$2i_2 + 20i_3 + 10(i_3 - i_1) + 5(i_2 - i_1) = 0$
The supermesh constraint:
$i_3 - i_2 = 1.2i_\text{b} = 1.2i_1$
Place these equations in standard form:

$$\begin{aligned} i_1(5 + 10 + 2) + i_2(-5) + i_3(-10) &= 100 \\ i_1(-10 - 5) + i_2(2 + 5) + i_3(20 + 10) &= 0 \\ i_1(1.2) + i_2(1) + i_3(-1) &= 0 \end{aligned}$$

Solving, $\quad i_1 = 7.4$ A; $\quad i_2 = -4.2$ A; $\quad i_3 = 4.68$ A
Solve for the requested currents:

$$\begin{aligned} i_a &= i_2 = -4.2 \text{ A} \\ i_b &= i_1 = 7.4 \text{ A} \\ i_c &= i_3 = 4.68 \text{ A} \\ i_d &= i_1 - i_2 = 11.6 \text{ A} \\ i_e &= i_1 - i_3 = 2.72 \text{ A} \end{aligned}$$

[b] Find v_cs:
$2i_2 + v_\text{cs} + 5(i_2 - i_1) = 0 \quad \therefore \quad v_\text{cs} = -2(-4.2) - 5(-4.2 - 7.4) = 66.4$ V

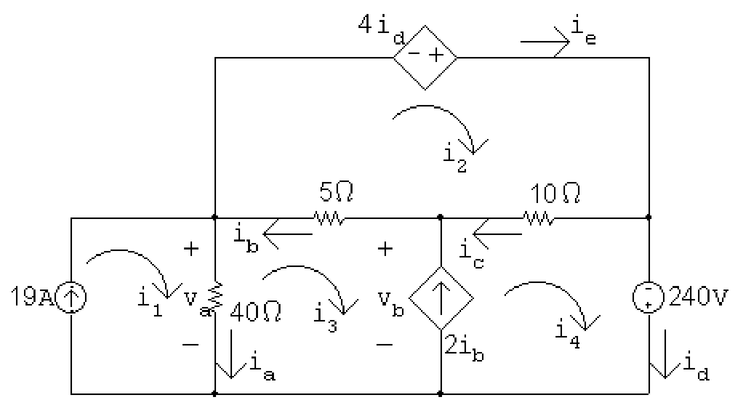
Calculate the power:

$$p_{100\text{V}} = -(100)(7.4) = -740 \text{ W}$$
$$p_{\text{dep source}} = -(66.4)[1.2(7.4)] = -589.632 \text{ W}$$
$$p_{2\Omega} = 2(-4.2)^2 = 35.28 \text{ W}$$
$$p_{5\Omega} = 5(7.4 + 4.2)^2 = 672.8 \text{ W}$$
$$p_{2\Omega} = 2(7.4)^2 = 109.52 \text{ W}$$
$$p_{10\Omega} = 10(7.4 - 4.68)^2 = 73.984 \text{ W}$$
$$p_{20\Omega} = 20(4.68)^2 = 438.048 \text{ W}$$

$$\sum p_{\text{dev}} = 740 + 589.632 = 1329.632 \text{ W}$$

$$\sum p_{\text{dis}} = 35.28 + 672.8 + 109.52 + 73.984 + 438.048 = 1329.632 \text{ W}$$

P 4.47 **[a]**



The i_2 mesh current equation:
$-4i_{\text{d}} + 10(i_2 - i_4) + 5(i_2 - i_3) = 0$
The i_3 — i_4 supermesh equation:
$40(i_3 - 19) + 5(i_3 - i_2) + 10(i_4 - i_2) - 240 = 0$
The supermesh constraint equation:
$i_4 - i_3 = 2i_{\text{b}} = 2(i_2 - i_3)$
Place the equations in standard form:

$$i_2(10 + 5) + i_3(-5) + i_4(-10 - 4) = 0$$
$$i_2(-5 - 10) + i_3(40 + 5) + i_4(10) = 240 + (40)(19)$$
$$i_2(2) + i_3(-1) + i_4(-1) = 0$$

Solving, $i_2 = 18$ A; $i_3 = 26$ A; $i_4 = 10$ A
Solve for the requested currents:

$i_a = 19 - i_3 = 19 - 26 = -7$ A

$i_b = i_2 - i_3 = 18 - 26 = -8$ A

$i_c = i_2 - i_4 = 18 - 10 = 8$ A

$i_d = i_4 = 10$ A

$i_e = i_2 = 18$ A

[b] Find the power in the circuit:

$v_a = 40i_a = 40(-7) = -280$ V

$v_b = -10i_c - 240 = -10(8) - 240 = -320$ V

$p_{19A} = -(-280)(19) = 5320$ W

$p_{CCCS} = -(-320)(2)(-8) = -5120$ W

$p_{CCVS} = -(4)(10)(18) = -720$ W

$p_{240V} = -(240)(10) = -2400$ W

$p_{40\Omega} = (40)(-7)^2 = 1960$ W

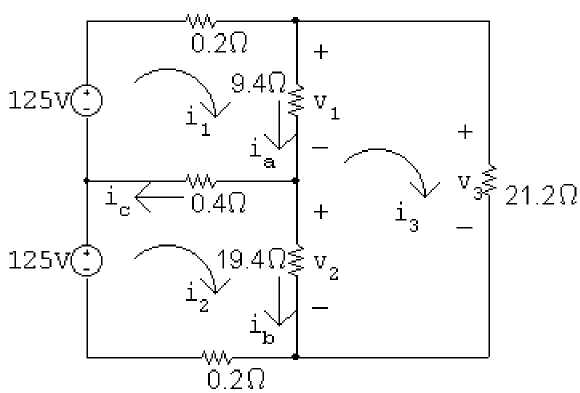
$p_{5\Omega} = (5)(-8)^2 = 320$ W

$p_{10\Omega} = (10)(8)^2 = 640$ W

$\sum p_{dev} = 5120 + 720 + 2400 = 8240$ W

$\sum p_{dis} = 5320 + 1960 + 320 + 640 = 8240$ W(checks)

P 4.48 **[a]**



$125 = 10i_1 - 0.4i_2 - 9.4i_3$

$125 = -0.4i_1 + 20i_2 - 19.4i_3$

$0 = -9.4i_1 - 19.4i_2 + 50i_3$

Solving, $i_1 = 23.93$ A; $i_2 = 17.79$ A; $i_3 = 11.40$ A

$$v_1 = 9.4(i_1 - i_3) = 117.76 \text{ V}$$
$$v_2 = 19.4(i_2 - i_3) = 123.90 \text{ V}$$
$$v_3 = 21.2i_3 = 241.66 \text{ V}$$

[b] $p_{R1} = (i_1 - i_3)^2(9.4) = 1475.22$ W

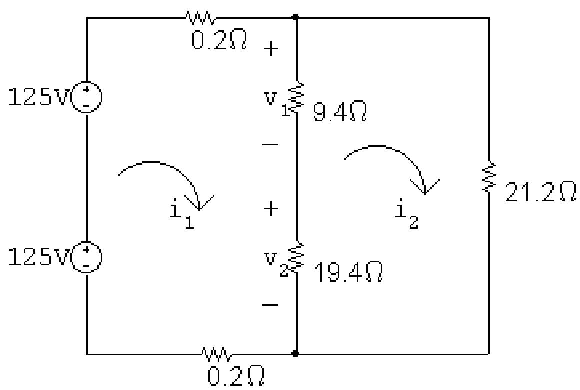
$$p_{R2} = (i_2 - i_3)^2(19.4) = 791.29 \text{ W}$$
$$p_{R3} = i_3^2(21.2) = 2754.64 \text{ W}$$

[c] $\sum p_{\text{dev}} = 125(i_1 + i_2) = 5213.99$ W

$\sum p_{\text{load}} = 5021.15$ W

$$\% \text{ delivered } = \frac{5021.15}{5213.99} \times 100 = 96.3\%$$

[d]



$$250 = 29.2i_1 - 28.8i_2$$

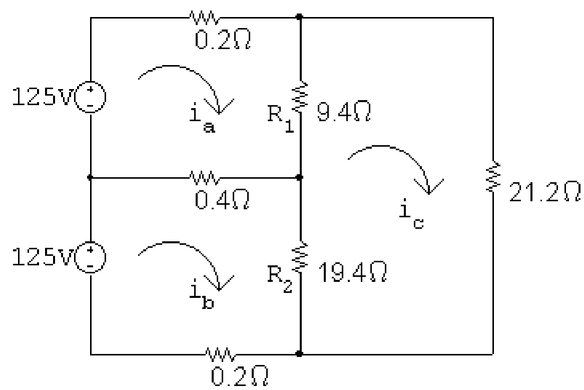
$$0 = -28.8i_1 + 50i_2$$

Solving, $i_1 = 19.82$ A; $i_2 = 11.42$ A

$$i_1 - i_2 = 8.41 \text{ A}$$
$$v_1 = (8.41)(9.4) = 79.01 \text{ V}$$
$$v_2 = 8.41(19.4) = 163.06 \text{ V}$$

Note v_1 is low and v_2 is high. Therefore, loads designed for 125 V would not function properly, and could be damaged.

P 4.49



$$125 = (R_1 + 0.6)i_a - 0.4i_b - R_1 i_c$$

$$125 = -0.4i_a + (R_2 + 0.6)i_b - R_2 i_c$$

$$0 = -R_1 i_a - R_2 i_b + (R_1 + R_2 + 21.2)i_c$$

$$\Delta = \begin{vmatrix} (R_1 + 0.6) & -0.4 & -R_1 \\ -0.4 & (R_2 + 0.6) & -R_2 \\ -R_1 & -R_2 & (R_1 + R_2 + 21.2) \end{vmatrix}$$

When $R_1 = R_2$, Δ reduces to

$$\Delta = 21.6R_1^2 + 25.84R_1 + 4.24.$$

$$\begin{aligned} N_a &= \begin{vmatrix} 125 & -0.4 & -R_1 \\ 125 & (R_2 + 0.6) & -R_2 \\ 0 & -R_2 & (R_1 + R_2 + 21.2) \end{vmatrix} \\ &= 125\,[2R_1R_2 + R_1 + 22.2R_2 + 21.2] \end{aligned}$$

$$\begin{aligned} N_b &= \begin{vmatrix} (R_1 + 0.6) & 125 & -R_1 \\ -0.4 & 125 & -R_2 \\ -R_1 & 0 & (R_1 + R_2 + 21.2) \end{vmatrix} \\ &= 125\,[2R_1R_2 + 22.2R_1 + R_2 + 21.2] \end{aligned}$$

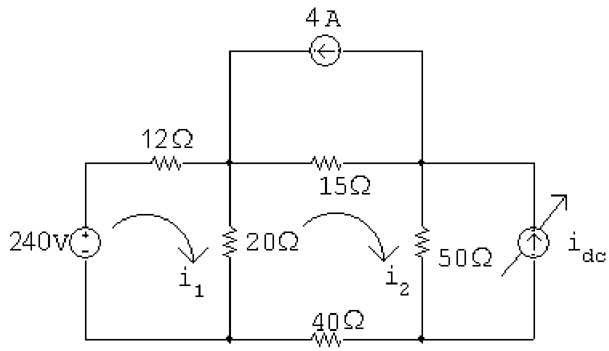
$$i_a = \frac{N_a}{\Delta}, \qquad i_b = \frac{N_b}{\Delta}$$

$$i_{\text{neutral}} = i_a - i_b = \frac{N_a - N_b}{\Delta} = \frac{125[(R_1 - R_2) + 22.2(R_2 - R_1)]}{\Delta}$$

Now note that when $R_1 = R_2$, i_{neutral} reduces to

$$i_{\text{neutral}} = \frac{0}{\Delta} = 0$$

P 4.50



The mesh current equations:

$$-240 + 12i_1 + 20(i_1 - i_2) = 0$$

$$20(i_2 - i_1) + 15(i_2 + 4) + 50(i_2 + i_{\text{dc}}) + 40i_2 = 0$$

Place these equations in standard form:

$$i_1(12 + 20) + i_2(-20) + i_{\text{dc}}(0) = 240$$

$$i_1(-20) + i_2(20 + 15 + 50 + 40) + i_{\text{dc}}(50) = -60$$

But if the power associated with the 4 A source is zero, the voltage drop across the source must be zero. This means that the voltage drop across the $15\,\Omega$ resistor is also zero, so the $15\,\Omega$ resistor is effectively removed from the circuit. Once this happens, $i_2 = -4$ A. Substitute this value into the first equation and solve for i_1:

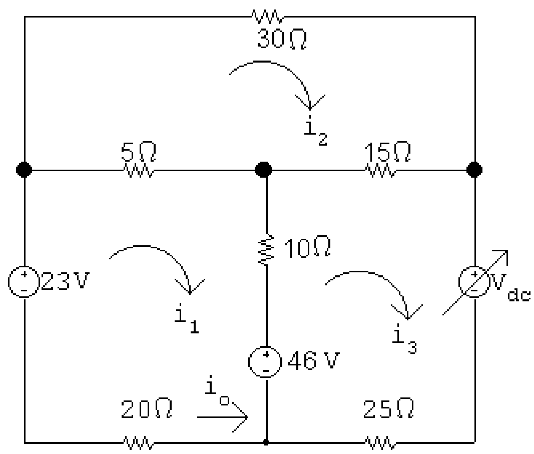
$32i_1 - 20(-4) = 240 \qquad \therefore \qquad 32i_1 = 160 \quad \text{so} \quad i_1 = 5$ A

Now substitute this value for i_1 into the second equation and solve for i_{dc}:

$-20(5) + 125(-4) + 50i_{\text{dc}} = -60 \quad \text{so} \quad 50i_{\text{dc}} = -60 + 100 + 500 = 540$

$\therefore \qquad i_{\text{dc}} = 540/50 = 10.8$ A

P 4.51 **[a]**



Write the mesh current equations. Note that if $i_o = 0$, then $i_1 = 0$:

$$-23 + 5(-i_2) + 10(-i_3) + 46 = 0$$

$$30i_2 + 15(i_2 - i_3) + 5i_2 = 0$$

$$V_{\rm dc} + 25i_3 - 46 + 10i_3 + 15(i_3 - i_2) = 0$$

Place the equations in standard form:

$$i_2(-5) + i_3(-10) + V_{\rm dc}(0) = -23$$

$$i_2(30 + 15 + 5) + i_3(-15) + V_{\rm dc}(0) = 0$$

$$i_2(-15) + i_3(25 + 10 + 15) + V_{\rm dc}(1) = 46$$

Solving, $i_2 = 0.6$ A; $i_3 = 2$ A; $V_{\rm dc} = -45$ V

Thus, the value of $V_{\rm dc}$ required to make $i_o = 0$ is -45 V.

[b] Calculate the power:

$$p_{23\rm V} = -(23)(0) = 0 \text{ W}$$

$$p_{46\rm V} = -(46)(2) = -92 \text{ W}$$

$$p_{\rm Vdc} = (-45)(2) = -90 \text{ W}$$

$$p_{30\Omega} = (30)(0.6)^2 = 10.8 \text{ W}$$

$$p_{5\Omega} = (5)(0.6)^2 = 1.8 \text{ W}$$

$$p_{15\Omega} = (15)(2 - 0.6)^2 = 29.4 \text{ W}$$

$$p_{10\Omega} = (10)(2)^2 = 40 \text{ W}$$

$$p_{20\Omega} = (20)(0)^2 = 0 \text{ W}$$

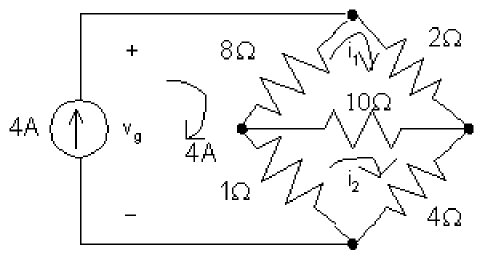
$$p_{25\Omega} = (25)(2)^2 = 100 \text{ W}$$

$$\sum p_{\rm dev} = 92 + 90 = 182 \text{ W}$$

$$\sum p_{\rm dis} = 10.8 + 1.8 + 29.4 + 40 + 0 + 100 = 182 \text{ W(checks)}$$

P 4.52 **[a]** There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2i_1 + 10(i_1 - i_2) + 8(i_1 - 4) = 0$$

$$4i_2 + 1(i_2 - 4) + 10(i_2 - i_1) = 0$$

Place the equations in standard form:

$$i_1(2 + 10 + 8) + i_2(-10) = 32$$

$$i_1(-10) + i_2(4 + 1 + 10) = 4$$

Solving, $i_1 = 2.6$ A; $\quad i_2 = 2$ A

Find the power in the $10\,\Omega$ resistor:

$i_{10\Omega} = i_1 - i_2 = 0.6$ A

$p_{10\Omega} = (0.6)^2(10) = 3.6$ W

[c] No, the voltage across the 4 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

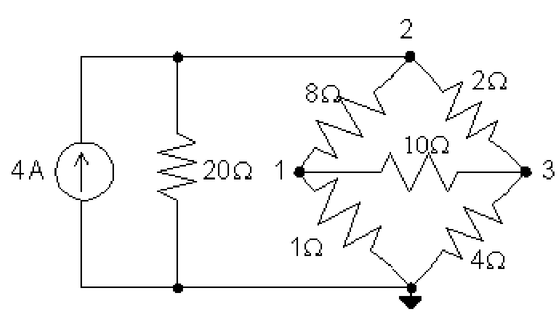
[d] $v_g = 2i_1 + 4i_2 = 2(2.6) + 4(2) = 13.2$ V

$p_{4\text{A}} = -(13.2)(4) = -52.8$ W

Thus the 4 A source develops 52.8 W.

P 4.53 **[a]** There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[b]



The node voltage equations are:

$$\frac{v_1}{1} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{10} = 0$$

$$-4 + \frac{v_2}{20} + \frac{v_2 - v_1}{8} + \frac{v_2 - v_3}{2} = 0$$

$$\frac{v_3 - v_1}{10} + \frac{v_3 - v_2}{2} + \frac{v_3}{4} = 0$$

Put the equations in standard form:

$$v_1\left(1 + \frac{1}{8} + \frac{1}{10}\right) + v_2\left(-\frac{1}{8}\right) + v_3\left(-\frac{1}{10}\right) = 0$$

$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{20} + \frac{1}{8} + \frac{1}{2}\right) + v_3\left(-\frac{1}{2}\right) = 4$$

$$v_1\left(-\frac{1}{10}\right) + v_2\left(-\frac{1}{2}\right) + v_3\left(\frac{1}{2} + \frac{1}{10} + \frac{1}{4}\right) = 0$$

Solving, $v_1 = 1.72$ V; $v_2 = 11.33$ V; $v_3 = 6.87$ V

$p_{4\text{A}} = -(11.33)(4) = -45.32$ W

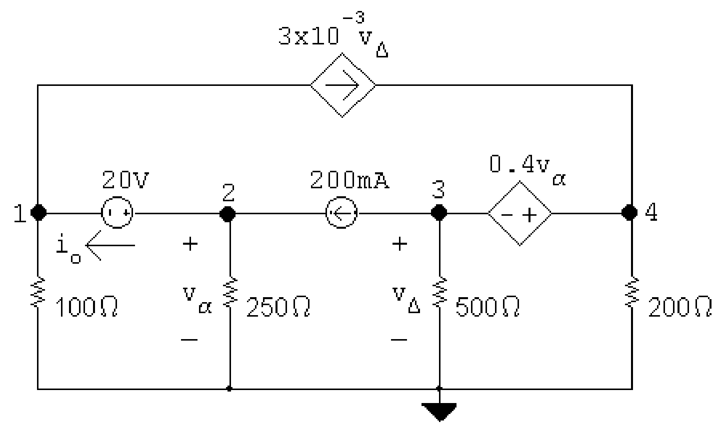
Therefore, the 4 A source is developing 45.32 W

P 4.54 **[a]** The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + \frac{v_2}{250} - 0.2 + 3 \times 10^{-3} v_3 = 0$$

$$\frac{v_3}{500} + \frac{v_4}{200} - 3 \times 10^{-3} v_3 + 0.2 = 0$$

Constraints:

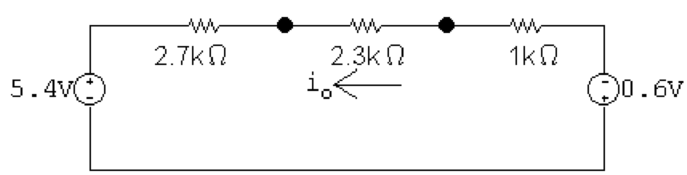
$v_2 - v_1 = 20; \qquad v_4 - v_3 = 0.4 v_\alpha; v_\alpha = v_2$

Solving, $v_2 = 44$ V

$i_o = 0.2 - 44/250 = 24$ mA

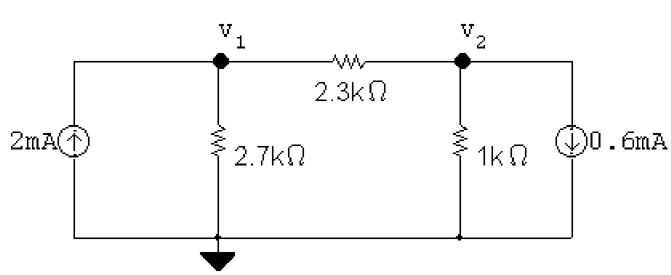
$p_{20\text{V}} = 20 i_o = 480$ mW (abs)

P 4.55 **[a]** Apply source transformations to both current sources to get



$$i_o = \frac{-(5.4 + 0.6)}{2700 + 2300 + 1000} = -1 \text{ mA}$$

[b]



The node voltage equations:

$$-2\times 10^{-3} + \frac{v_1}{2700} + \frac{v_1 - v_2}{2300} = 0$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{2300} + 0.6\times 10^{-3} = 0$$

Place these equations in standard form:

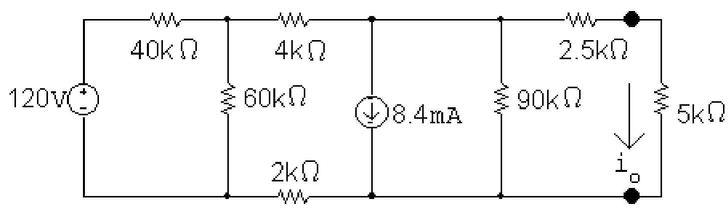
$$v_1\left(\frac{1}{2700} + \frac{1}{2300}\right) + v_2\left(-\frac{1}{2300}\right) = 2\times 10^{-3}$$

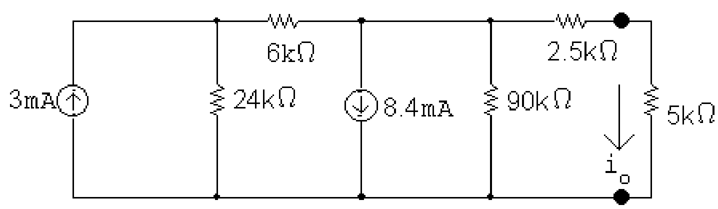
$$v_1\left(-\frac{1}{2300}\right) + v_2\left(\frac{1}{1000} + \frac{1}{2300}\right) = -0.6\times 10^{-3}$$

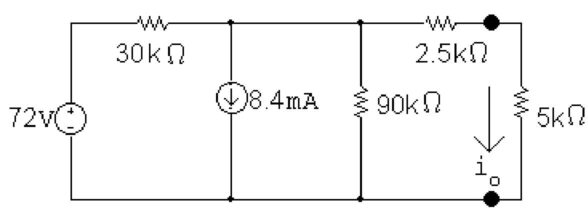
Solving, $v_1 = 2.7$ V; $v_2 = 0.4$ V

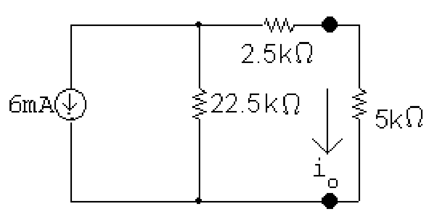
$$\therefore\ i_o = \frac{v_2 - v_1}{2300} = -1\text{ mA}$$

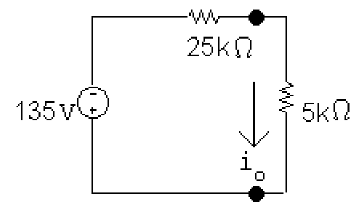
P 4.56 **[a]**





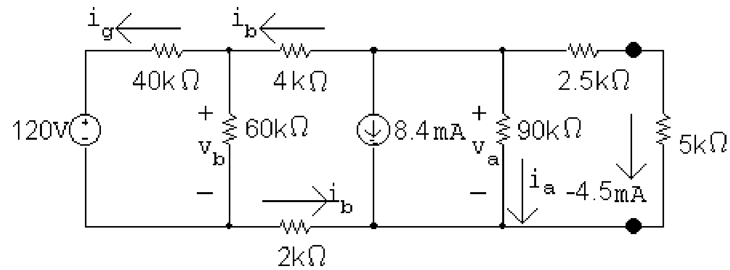






$i_o = -135/30{,}000 = -4.5$ mA

[b]

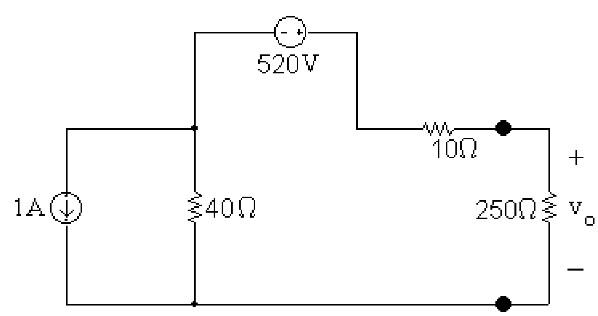


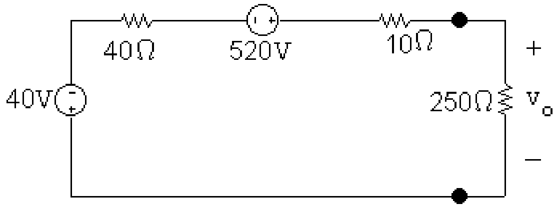
$$v_a = (7500)(-0.0045) = -33.75 \text{ V}$$
$$i_a = \frac{v_a}{90{,}000} = \frac{-33.75}{90{,}000} = -0.375 \text{ mA}$$
$$i_b = -8.4 \times 10^{-3} + 0.375 \times 10^{-3} + 4.5 \times 10^{-3} = -3.525 \text{ mA}$$
$$v_b = (6000)(3.525 \times 10^{-3}) - 33.75 = -12.6 \text{ V}$$
$$i_g = \frac{-12.6 - 120}{40{,}000} = -3.315 \text{ mA}$$
$$p_{120V} = (120)(-3.315 \times 10^{-3}) = -397.8 \text{ mW}$$

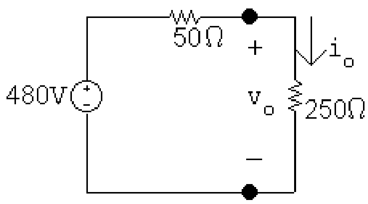
Check:

$$p_{8.4mA} = (-33.75)(8.4 \times 10^{-3}) = -283.5 \text{ mW}$$
$$\sum P_{dev} = 397.8 + 283.5 = 681.3 \text{ mW}$$
$$\sum P_{dis} = (40{,}000)(-3.315 \times 10^{-3})^2 + \frac{(-12.6)^2}{60{,}000} + \frac{(-33.75)^2}{90{,}000}$$
$$+(6000)(-3.525 \times 10^{-3})^2 + (7500)(-4.5 \times 10^{-3})^2$$
$$= 681.3 \text{ mW}$$

P 4.57 **[a]**

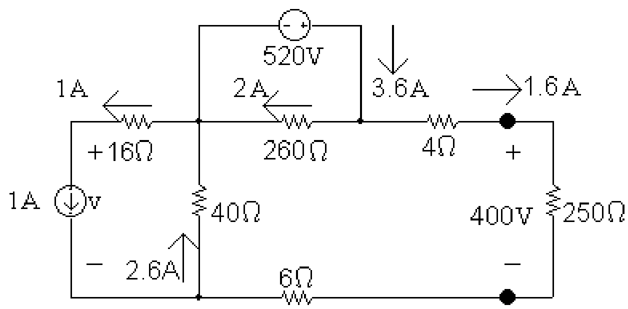






$$\therefore \quad v_o = \frac{250}{300}(480) = 400 \text{ V}; \qquad i_o = \frac{400}{250} = 1.6 \text{ A}$$

[b]



$p_{520\text{V}} = -(520)(3.6) = -1872$ W
Therefore, the 520 V source is developing 1872 kW.

[c] $v = -(16)(1) - 40(2.6) = -120$ V
$p_{1\text{A}} = (-120)(1) = -120$ W
Therefore the 1 A source is developing 120 W.

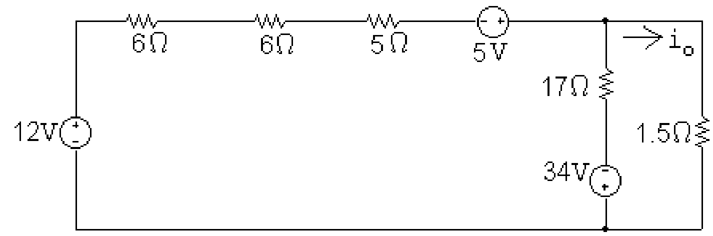
[d] Calculate the power dissipated by the resistors:

$$p_{16\Omega} = (16)(1)^2 = 16 \text{ W}$$
$$p_{260\Omega} = (260)(2)^2 = 1040 \text{ W}$$
$$p_{40\Omega} = (40)(2.6)^2 = 270.4 \text{ W}$$
$$p_{4\Omega} = (4)(1.6)^2 = 10.24 \text{ W}$$
$$p_{250\Omega} = (250)(1.6)^2 = 640 \text{ W}$$
$$p_{6\Omega} = (6)(1.6)^2 = 15.36 \text{ W}$$

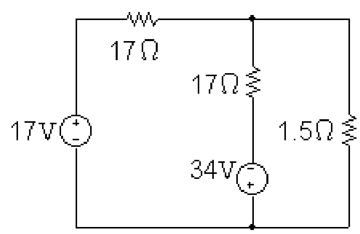
$$\sum p_{\text{dev}} = 120 + 1872 = 1992 \text{ W}$$

$$\sum p_{\text{dev}} = 16 + 1040 + 270.4 + 10.24 + 640 + 15.36 = 1992 \text{ W (CHECKS)}$$

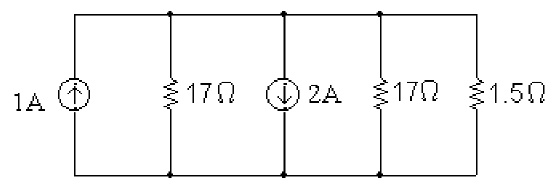
P 4.58 **[a]** Applying a source transformation to each current source yields



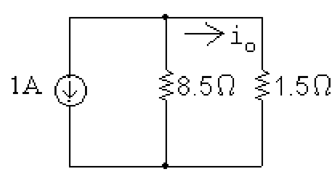
Now combine the 12 V and 5 V sources into a single voltage source and the 6 Ω, 6 Ω and 5 Ω resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

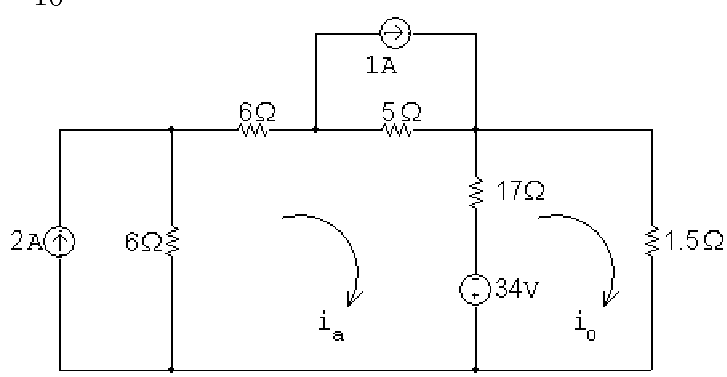


which can be reduced to



$$\therefore \quad i_o = \frac{8.5}{10}(-1) = -0.85 \text{ A}$$

[b]



The mesh current equations are:

$$6(i_a - 2) + 6i_a + 5(i_a - 1) + 17(i_a - i_o) - 34 = 0$$
$$1.5i_o + 34 + 17(i_o - i_a) = 0$$

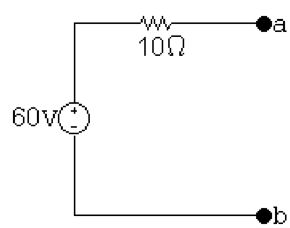
Put these equations in standard form:

$$i_a(6 + 6 + 5 + 17) + i_o(-17) = 12 + 5 + 34$$
$$i_a(-17) + i_o(1.5 + 17) = -34$$

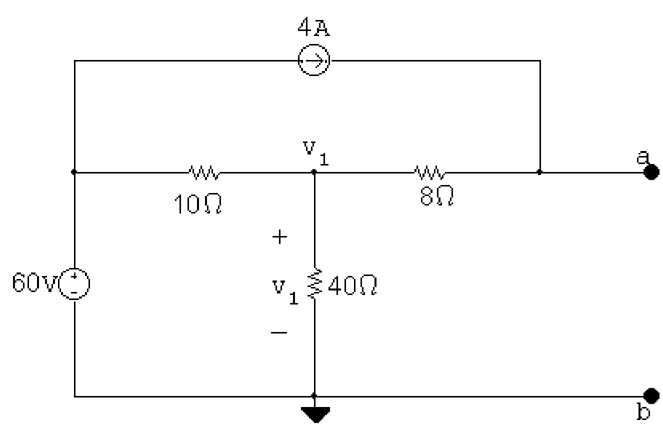
Solving, $i_a = 1.075$ A; $i_o = -0.85$ A

P 4.59 $V_{Th} = \frac{30}{30 + 10}(80) = 60$ V

$R_{Th} = 10\|30 + 2.5 = 10\,\Omega$



P 4.60



Write and solve the node voltage equation at v_1:

$$\frac{v_1 - 60}{10} + \frac{v_1}{40} - 4 = 0$$

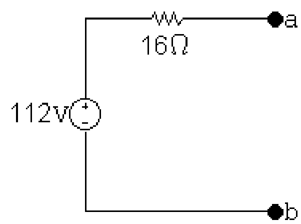
$4v_1 - 240 + v_1 - 160 = 0 \quad \therefore \quad v_1 = 400/5 = 80$ V

Calculate V_{Th}:

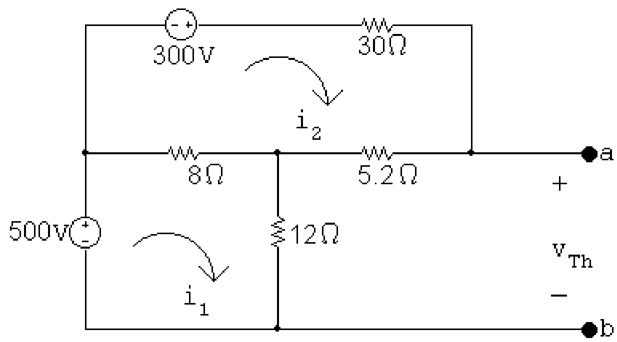
$V_{Th} = v_1 + (8)(4) = 80 + 32 = 112$ V

Calculate R_{Th} by removing the independent sources and making series and parallel combinations of the resistors:

$R_{Th} = 8 + 40\|10 = 8 + 8 = 16\,\Omega$



P 4.61 After making a source transformation the circuit becomes



The mesh current equations are:

$$-500 + 8(i_1 - i_2) + 12i_1 = 0$$

$$-300 + 30i_2 + 5.2i_2 + 8(i_2 - i_1) = 0$$

Put the equations in standard form:

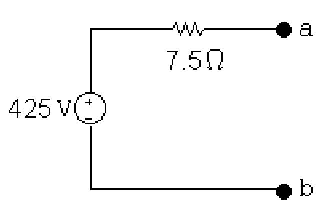
$$i_1(8 + 12) + i_2(-8) = 500$$

$$i_1(-8) + i_2(30 + 5.2 + 8) = 300$$

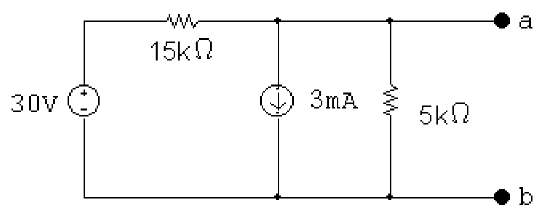
Solving, $i_1 = 30$ A; $i_2 = 12.5$ A

$V_{\rm Th} = 5.2i_2 + 12i_1 = 425$ V

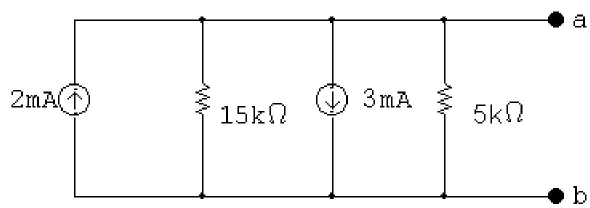
$R_{\rm Th} = (8\|12 + 5.2)\|30 = 7.5\,\Omega$



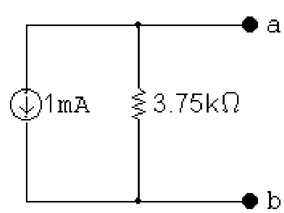
P 4.62 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



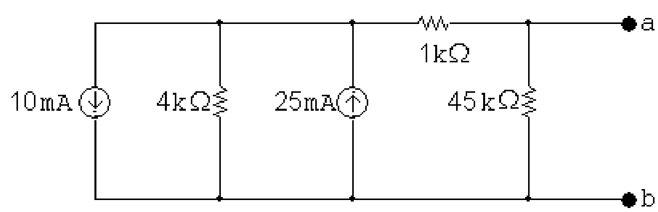
or

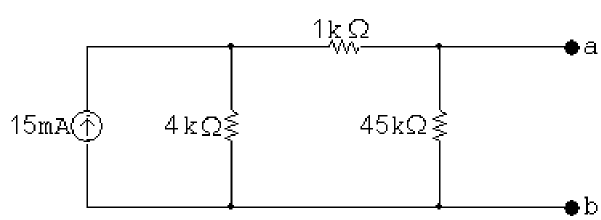


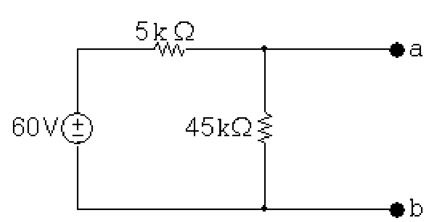
Therefore the Norton equivalent is determined by adding the current sources and combining the resistors in parallel:

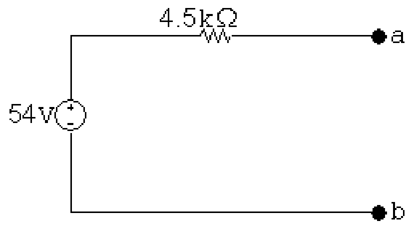


P 4.63 **[a]** First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.

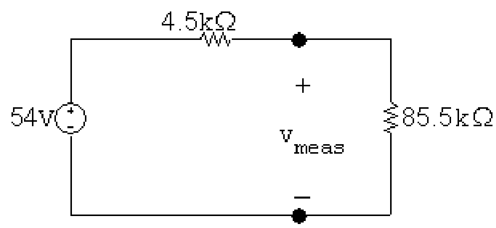








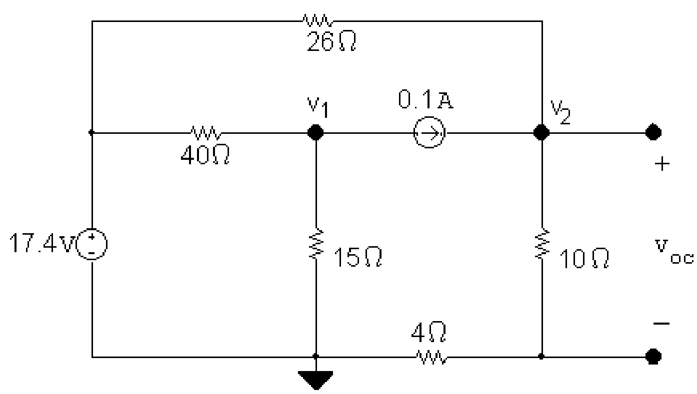
$\therefore \quad V_{\text{Th}} = 54 \text{ V} \qquad R_{\text{Th}} = 4.5 \text{ k}\Omega$



$$v_{\text{meas}} = \frac{85.5}{90}(54) = 51.3 \text{ V}$$

[b] $\%\text{error} = \left(\frac{51.3 - 54}{54}\right) \times 100 = -5\%$

P 4.64 **[a]** Open circuit:



The node voltage equations are:

$$\frac{v_1 - 17.4}{40} + \frac{v_1}{15} + 0.1 = 0$$

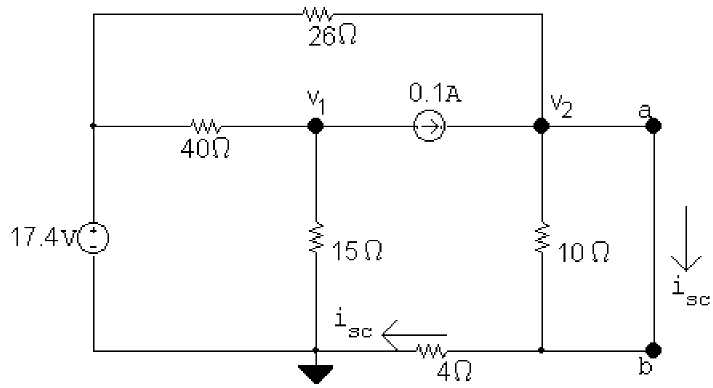
$$-0.1 + \frac{v_2}{14} + \frac{v_2 - 17.4}{26} = 0$$

The above equations are decoupled, so just solve the second equation for v_2 and use v_2 to solve for v_{oc}:

$-36.4 + 26v_2 + 14v_2 - 243.6 = 0 \quad \therefore \quad v_2 = 280/40 = 7 \text{ V}$

$$v_{\text{oc}} = \frac{10}{10 + 4}(7) = 5 \text{ V}$$

Short circuit:



Write a node voltage equation at v_2:

$$-0.1 + \frac{v_2 - 17.4}{26} + \frac{v_2}{4} = 0$$

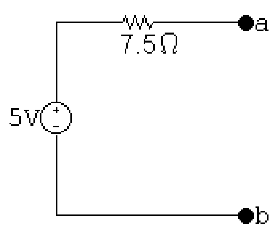
Solving,

$$-5.2 + 2v_2 - 34.8 + 13v_2 = 0 \quad \therefore \quad v_2 = 40/15 \text{ V}$$

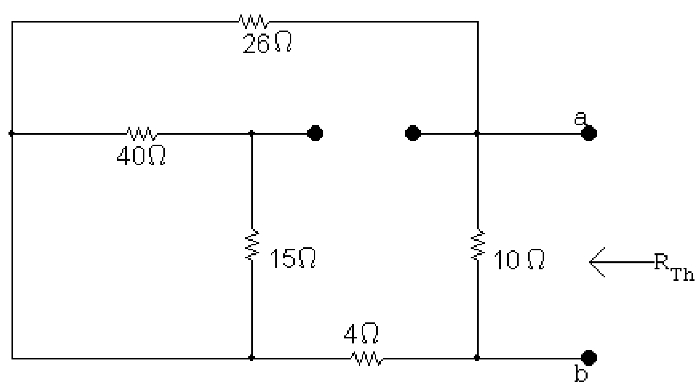
Calculate the short circuit current:

$i_{sc} = (40/15)/4 = 2/3$ A

Therefore, $R_{Th} = 5/(2/3) = 7.5\,\Omega$

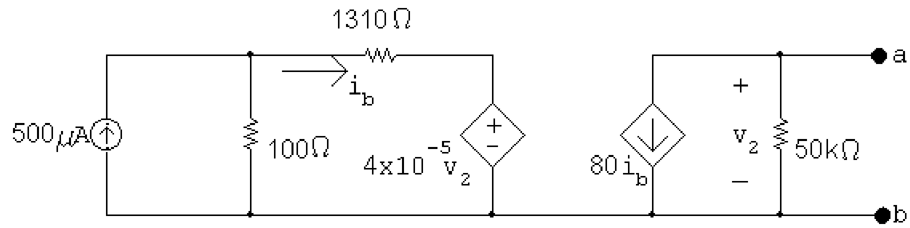


[b]



$R_{Th} = 10\|(26 + 4) = 7.5\,\Omega$ (CHECKS)

P 4.65



OPEN CIRCUIT

Use Ohm's law to solve for v_2 on the right hand side of the circuit:

$v_2 = -80i_{\text{b}}(50{,}000) = -40 \times 10^5 i_{\text{b}}$

Use this value of v_2 to express the value of the dependent voltage source in terms of i_{b}:

$4 \times 10^{-5} v_2 = 4 \times 10^{-5}(-40 \times 10^5 i_{\text{b}}) = -160 i_{\text{b}}$

Write the mesh current equation for the i_{b} mesh:

$1310 i_{\text{b}} - 160 i_{\text{b}} + 100(i_{\text{b}} - 500 \times 10^{-6}) = 0$

Solving,

$1250 i_{\text{b}} = 0.05 \quad \therefore \quad i_{\text{b}} = 0.05/1250 = 40\,\mu\text{ A}$

Thus,

$V_{\text{Th}} = v_2 = -40 \times 10^5 i_{\text{b}} = -40 \times 10^5(40 \times 10^{-6}) = -160\text{ V}$

SHORT CIRCUIT

$$v_2 = 0; \qquad i_{\text{sc}} = -80 i_b$$

Calculate i_{b} using current division on the left hand side of the circuit:

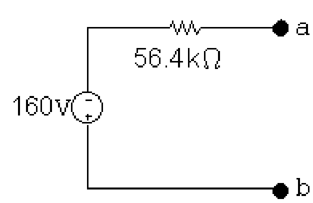
$$i_b = \frac{100}{100 + 1310} 500 \times 10^{-6} = 35.461\,\mu\text{ A}$$

Calculate the short circuit current from the right hand side of the circuit:

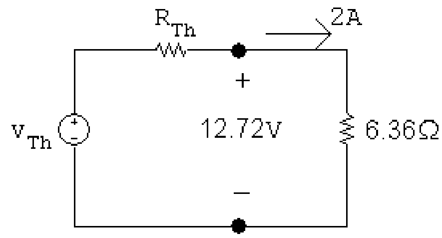
$$i_{\text{sc}} = -80(35.461 \times 10^{-6}) = -2.8369 \times 10^{-3}\,\text{mA}$$

Calculate R_{Th} from the short circuit current and open circuit voltage:

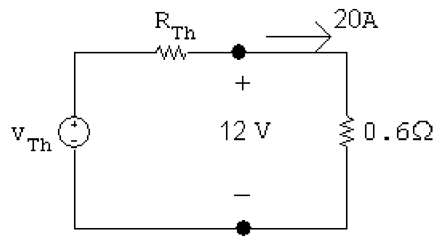
$$R_{\text{Th}} = \frac{-160}{-2.8369 \times 10^{-3}} = 56.4\text{ k}\Omega$$



P 4.66



$12.72 = V_{\text{Th}} - 2R_{\text{Th}}$



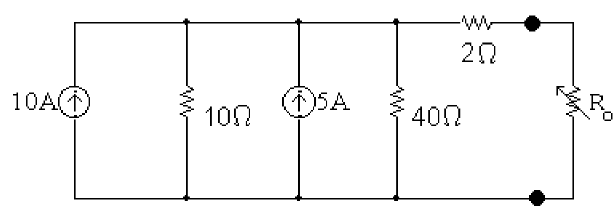
$12 = V_{\text{Th}} - 20R_{\text{Th}}$

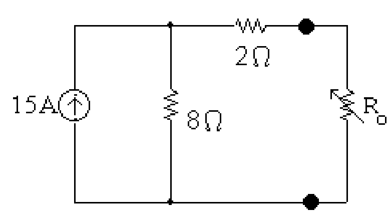
Solving the above equations for V_{Th} and R_{Th} yields

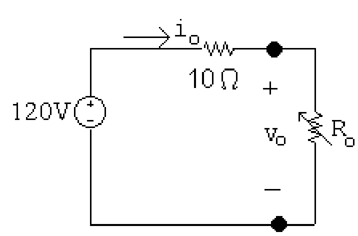
$V_{\text{Th}} = 12.8$ V, $\qquad R_{\text{Th}} = 40$ mΩ

$\therefore \quad I_N = 320$ A, $\qquad R_N = 40$ mΩ

P 4.67 First, find the Thévenin equivalent with respect to R_o.

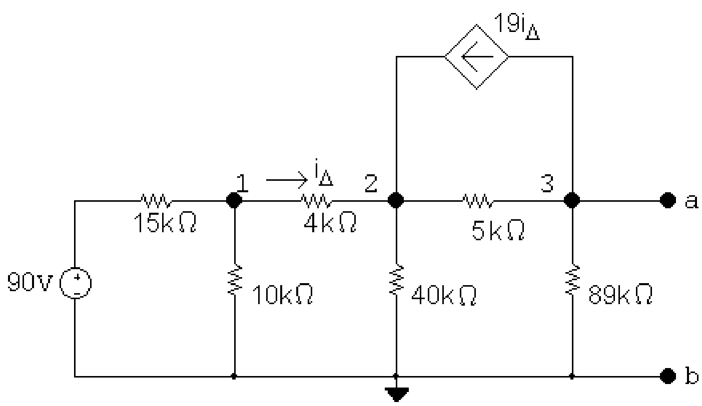






R_o	i_o	v_o	R_o	i_o	v_o
0	12	0	20	4	80
2	10	20	30	3	90
6	7.5	45	40	2.4	96
10	6	60	50	2	100
15	4.8	72	70	1.5	105

P 4.68



The node voltage equations are:

$$\frac{v_1 - 90}{15{,}000} + \frac{v_1}{10{,}000} + \frac{v_1 - v_2}{4000} = 0$$

$$\frac{v_2 - v_1}{4000} + \frac{v_2}{40{,}000} + \frac{v_2 - v_3}{5000} - 19i_\Delta = 0$$

$$\frac{v_3 - v_2}{5000} + \frac{v_3}{89{,}000} + 19i_\Delta = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1 - v_2}{4000}$$

Substitute the constraint equation into the node voltage equations and put the three remaining equations in standard form:

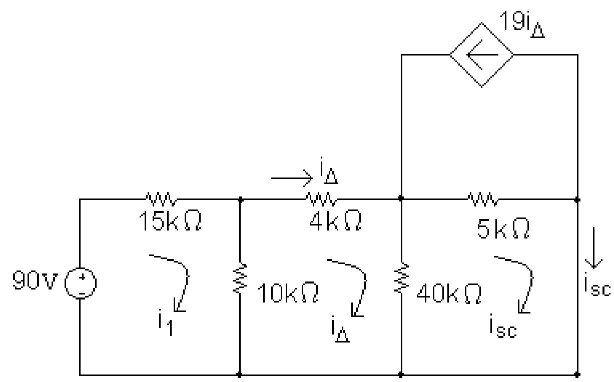
$$v_1\left(\frac{1}{15{,}000} + \frac{1}{10{,}000} + \frac{1}{4000}\right) + v_2\left(-\frac{1}{4000}\right) + v_3(0) = \frac{90}{15{,}000}$$

$$v_1\left(-\frac{1}{4000} - \frac{19}{4000}\right) + v_2\left(\frac{1}{4000} + \frac{1}{40{,}000} + \frac{1}{5000} + \frac{19}{4000}\right) + v_3\left(-\frac{1}{5000}\right) = 0$$

$$v_1\left(\frac{19}{4000}\right) + v_2\left(-\frac{1}{5000} - \frac{19}{4000}\right) + v_3\left(\frac{1}{5000} + \frac{1}{89{,}000}\right) = 0$$

Solving, $v_1 = 32.75$ V; $v_2 = 30.58$ V; $v_3 = -19.8$ V

$V_{\text{Th}} = v_3 = -19.8$ V



The mesh current equations are:

$$-90 + 15{,}000 i_1 + 10{,}000(i_1 - i_\Delta) = 0$$

$$4000 i_\Delta + 40{,}000(i_\Delta - i_{\text{sc}}) + 10{,}000(i_\Delta - i_1) = 0$$

$$40{,}000(i_{\text{sc}} - i_\Delta) + 5000(i_{\text{sc}} + 19 i_\Delta) = 0$$

Put these equations in standard form:

$$i_1(25{,}000) + i_\Delta(-10{,}000) + i_{\text{sc}}(0) = +90$$

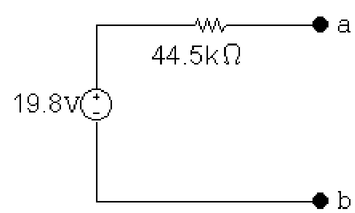
$$i_1(-10{,}000) + i_\Delta(54{,}000) + i_{\text{sc}}(-40{,}000) = 0$$

$$i_1(0) + i_\Delta(55{,}000) + i_{\text{sc}}(45{,}000) = 0$$

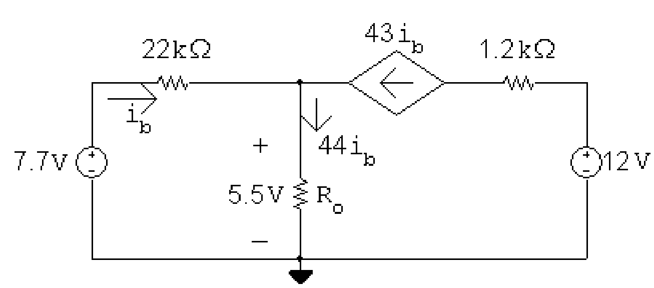
Solving, $i_1 = 3745.62\,\mu\text{A}$; $i_\Delta = 364.04\,\mu$ A; $i_{\text{sc}} = -444.94\,\mu$ A

$i_{\text{sc}} = -444.94\,\mu$ A

$R_{\text{Th}} = -19.8/-444.94 \times 10^{-6} = 44.5\ \text{k}\Omega$



P 4.69 **[a]** Use source transformations to simplify the left side of the circuit.



$$i_b = \frac{7.7 - 5.5}{22{,}000} = 0.1\ \text{mA}$$

Let $R_o = R_{\text{meter}}\|1.3\text{ k}\Omega = 5.5/4.4 \times 10^{-3} = 1250\,\Omega$

$$\therefore \quad \frac{(R_{\text{meter}})(1300)}{R_{\text{meter}} + 1300} = 1250; \qquad R_{\text{meter}} = \frac{(1250)(1300)}{50} = 32.5\text{ k}\Omega$$

[b] Actual value of v_e:

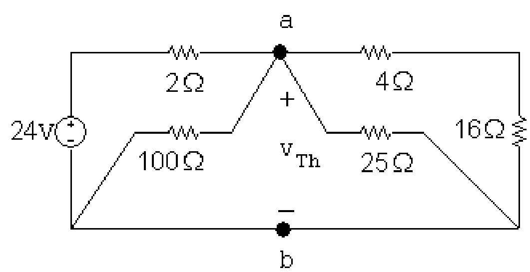
$$i_b = \frac{7.7}{22{,}000 + 44(1300)} = 97.22\,\mu\text{ A}$$

$$v_e = 44i_b(1300) = 5.56\text{ V}$$

$$\%\text{ error } = \left(\frac{5.5 - 5.56}{5.56}\right) \times 100 = -1.10\%$$

P 4.70 **[a]** Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the $4.8\,\Omega$ resistor.

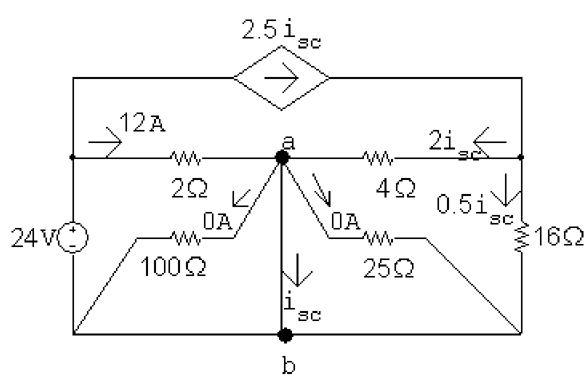
Thévenin voltage: note i_ϕ is zero.



$$\frac{V_{\text{Th}} - 24}{2} + \frac{V_{\text{Th}}}{100} + \frac{V_{\text{Th}}}{25} + \frac{V_{\text{Th}}}{20} = 0$$

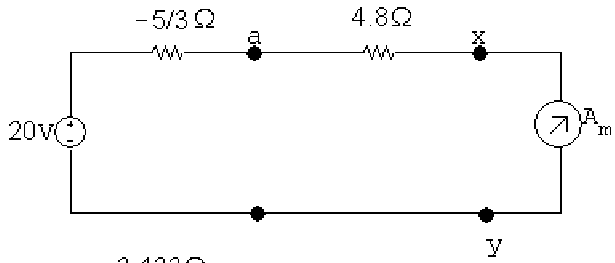
$$50V_{\text{Th}} + V_{\text{Th}} + 4V_{\text{Th}} + 5V_{\text{Th}} = 50(24) \qquad \therefore \qquad V_{\text{Th}} = 50(24)/60 = 20\text{ V}$$

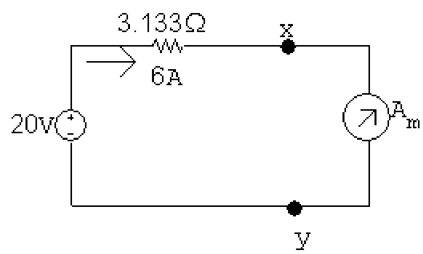
Short-circuit current:



$$i_{\text{sc}} = 12 + 2i_{\text{sc}}, \qquad \therefore \quad i_{\text{sc}} = -12\text{ A}$$

$$R_{\text{Th}} = \frac{20}{-12} = -1.67\,\Omega$$

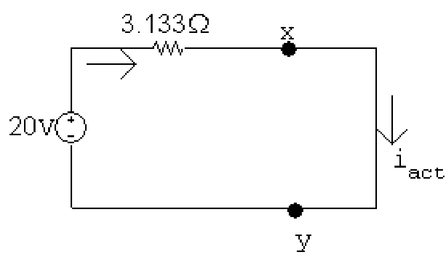




$$R_{\text{total}} = \frac{20}{6} = 3.333\,\Omega$$

$$R_{\text{meter}} = 3.333 - 3.133 = 0.20\,\Omega$$

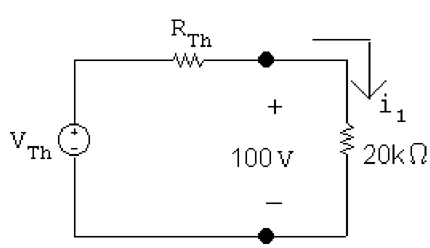
[b] Actual current:



$$i_{\text{actual}} = \frac{20}{3.133} = 6.383 \text{ A}$$

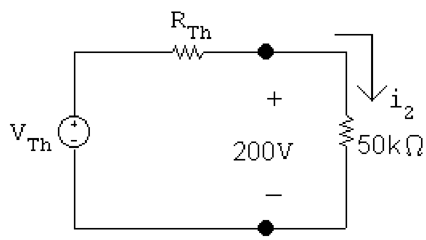
$$\% \text{ error } = \frac{6 - 6.383}{6.383} \times 100 = -6\%$$

P 4.71



$i_1 = 100/20{,}000 = 5$ mA

$100 = V_{\text{Th}} - 0.005R_{\text{Th}}, \qquad V_{\text{Th}} = 100 + 0.005R_{\text{Th}}$

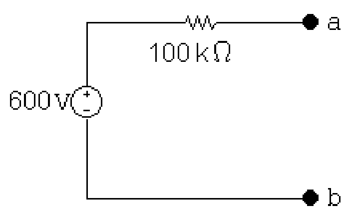


$i_2 = 200/50{,}000 = 4$ mA

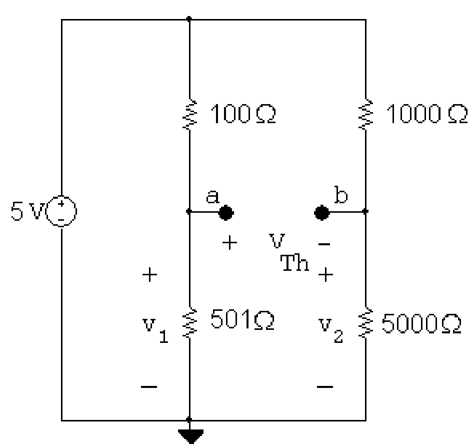
$200 = V_{\rm Th} - 0.004R_{\rm Th}, \qquad V_{\rm Th} = 200 + 0.004R_{\rm Th}$

$\therefore \quad 100 + 0.005R_{\rm Th} = 200 + 0.004R_{\rm Th}$ so $R_{\rm Th} = 100$ kΩ

$V_{\rm Th} = 100 + 500 = 600$ V



P 4.72



Use voltage division to calculate v_1 and v_2:

$$v_1 = \frac{501}{501+100}(5) = 4.168053 \text{ V}$$

$$v_2 = \frac{5000}{5000+1000}(5) = 4.1666667 \text{ V}$$

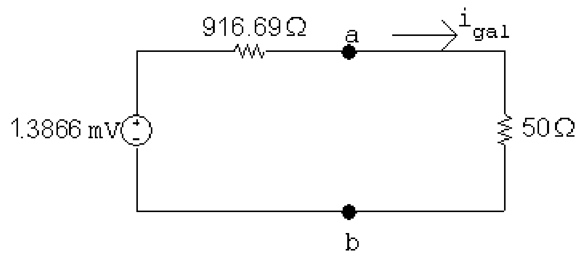
Now calculate $V_{\rm Th}$:

$V_{\rm Th} = v_1 - v_2 = 4.168053 - 4.1666667 = 1.3866$ mV

Calculate $R_{\rm Th}$ by removing the voltage source and creating series and parallel combinations of the resisitors:

$R_{\text{Th}} = 100\|501 + 1000\|5000 = \dfrac{(100)(501)}{601} + \dfrac{(1000)(5000)}{6000} = 916.69\,\Omega$

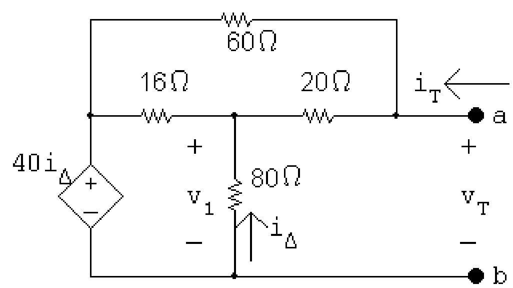
The resulting Thévenin equivalent circuit is shown below:



Use KVL to calculate i_{gal}:

$$i_{\text{gal}} = \frac{1.3866 \times 10^{-3}}{916.69 + 50} = 1.43\,\mu\text{A}$$

P 4.73 $V_{\text{Th}} = 0$, since circuit contains no independent sources.



$$i_{\text{T}} = \frac{v_{\text{T}} - v_1}{20} + \frac{v_{\text{T}} - 40i_\Delta}{60}$$

$$\frac{v_1 - 40i_\Delta}{16} + \frac{v_1}{80} + \frac{v_1 - v_{\text{T}}}{20} = 0$$

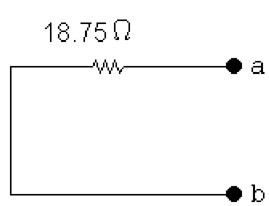
$$\therefore \quad 10v_1 - 200i_\Delta = 4v_{\text{T}} \qquad i_\Delta = \frac{-v_1}{80}, \qquad 200i_\Delta = -2.5v_1$$

$$\therefore \quad 12.5v_1 = 4v_{\text{T}}; \qquad v_1 = 0.32v_{\text{T}}$$

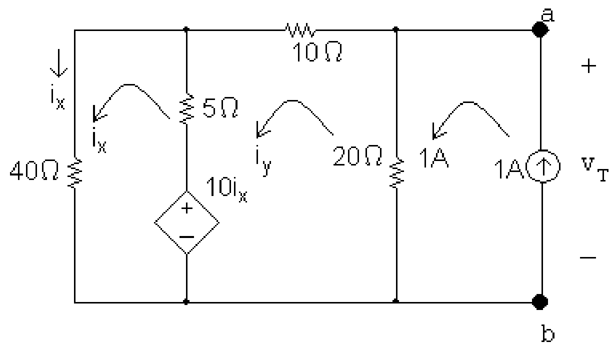
$$60i_{\text{T}} = 4v_{\text{T}} - 2.5v_1 = 3.2v_{\text{T}}$$

$$\therefore \quad \frac{v_{\text{T}}}{i_{\text{T}}} = \frac{60}{3.2} = 18.75\,\Omega$$

$$R_{\text{Th}} = 18.75\,\Omega$$



P 4.74 $V_{\text{Th}} = 0$ since there are no independent sources in the circuit. To find R_{Th}, apply a 1 A test source and calculate the voltage drop across the test source. Use the mesh current method.



The mesh current equations for the two meshes on the left:

$$-10i_x + 5(i_x - i_y) + 40i_x = 0$$

$$10i_x + 20(i_y - 1) + 10i_y + 5(i_y - i_x) = 0$$

Place these equations in standard form:

$$i_x(-10 + 5 + 40) + i_y(-5) = 0$$

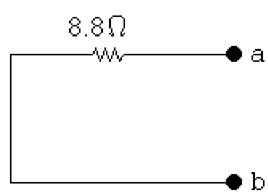
$$i_x(10 - 5) + i_y(20 + 10 + 5) = 20$$

Solving, $i_x = 80$ mA; $i_y = 560$ mA

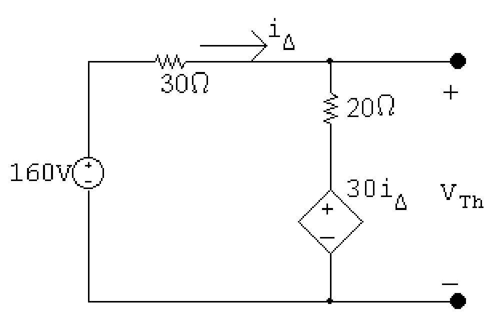
Find the voltage drop across the 1 A source:

$v_T = 20(1 - 0.56) = 8.8$ V

$\therefore \quad R_{\text{Th}} = v_T/1\text{ A} = 8.8/1 = 8.8\,\Omega$



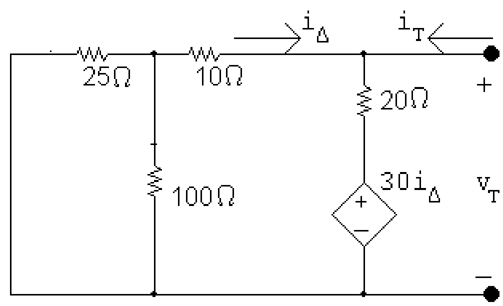
P 4.75 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2\text{ A}$$

$V_{\text{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100$ V

Using the test-source method to find the Thévenin resistance gives

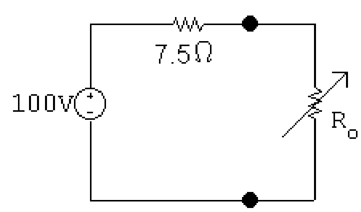


$$i_{\text{T}} = \frac{v_{\text{T}}}{30} + \frac{v_{\text{T}} - 30(-v_{\text{T}}/30)}{20}$$

$$\frac{i_{\text{T}}}{v_{\text{T}}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\text{Th}} = \frac{v_{\text{T}}}{i_{\text{T}}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25} R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

$$R_o^2 - 25R_o + 56.25 = 0$$

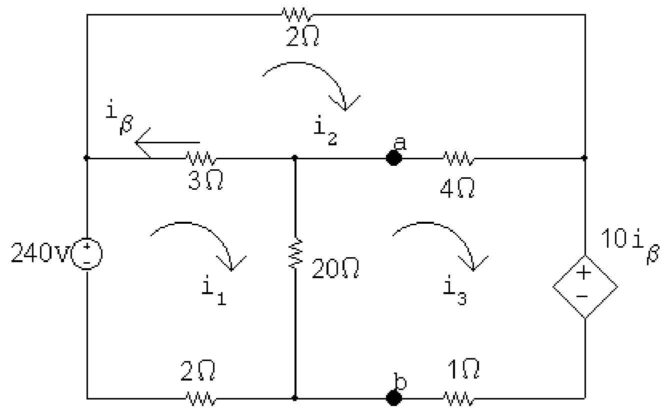
$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

$$R_o = 2.5\,\Omega$$

P 4.76 **[a]** Find the Thévenin equivalent with respect to the terminals of R_L.

Open circuit voltage:



The mesh current equations are:

$$\begin{aligned} -240 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 &= 0 \\ 2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) &= 0 \\ 10i_\beta + 1i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is:

$i_\beta = i_2 - i_1$

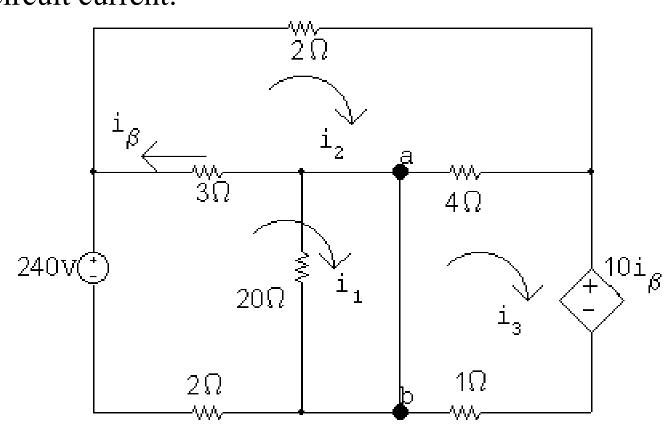
Place these equations in standard form:

$$\begin{aligned} i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i_\beta(0) &= 240 \\ i_1(-3) + i_2(2 + 4 + 3) + i_3(-4) + i_\beta(0) &= 0 \\ i_1(-20) + i_2(-4) + i_3(4 + 1 + 20) + i_\beta(10) &= 0 \\ i_1(1) + i_2(-1) + i_3(0) + i_\beta(1) &= 0 \end{aligned}$$

Solving, $i_1 = 99.6$ A; $i_2 = 78$ A; $i_3 = 100.8$ A; $i_\beta = -21.6$ A

$V_{\text{Th}} = 20(i_1 - i_3) = -24$ V

Short-circuit current:



The mesh current equations are:

$$-240 + 3(i_1 - i_2) + 2i_1 = 0$$
$$2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$
$$10i_\beta + 1i_3 + 4(i_3 - i_2) = 0$$

The dependent source constraint equation is:

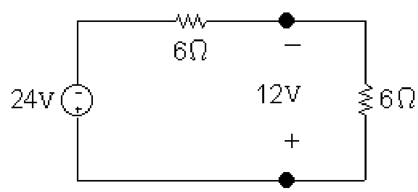
$i_\beta = i_2 - i_1$

Place these equations in standard form:

$$i_1(3+2) + i_2(-3) + i_3(0) + i_\beta(0) = 240$$
$$i_1(-3) + i_2(2+4+3) + i_3(-4) + i_\beta(0) = 0$$
$$i_1(0) + i_2(-4) + i_3(4+1) + i_\beta(10) = 0$$
$$i_1(1) + i_2(-1) + i_3(0) + i_\beta(1) = 0$$

Solving, $i_1 = 92$ A; $i_2 = 73.33$ A; $i_3 = 96$ A; $i_\beta = -18.67$ A

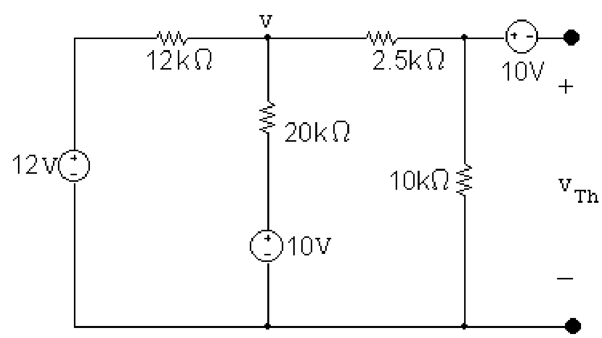
$i_{\rm sc} = i_1 - i_3 = -4$ A; $R_{\rm Th} = \dfrac{V_{\rm Th}}{i_{\rm sc}} = \dfrac{-24}{-4} = 6\,\Omega$



$R_{\rm L} = R_{\rm Th} = 6\,\Omega$

[b] $p_{\max} = \dfrac{12^2}{6} = 24$ W

P 4.77 **[a]**

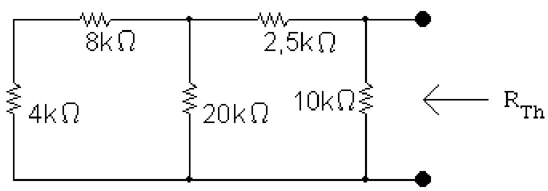


$$\frac{v-12}{12{,}000} + \frac{v-10}{20{,}000} + \frac{v}{12{,}500} = 0$$

Solving, $v = 7.03125$ V

$$v_{10\rm k} = \frac{10{,}000}{12{,}500}(7.03125) = 5.625 \text{ V}$$

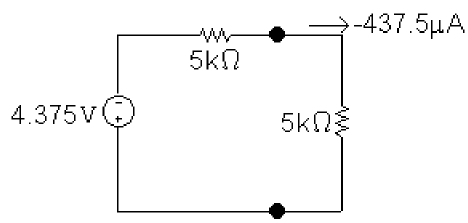
$$\therefore\quad V_{\rm Th} = v - 10 = -4.375 \text{ V}$$



$$R_{\text{Th}} = [(12{,}000\|20{,}000) + 2500] = 5\ \text{k}\Omega$$

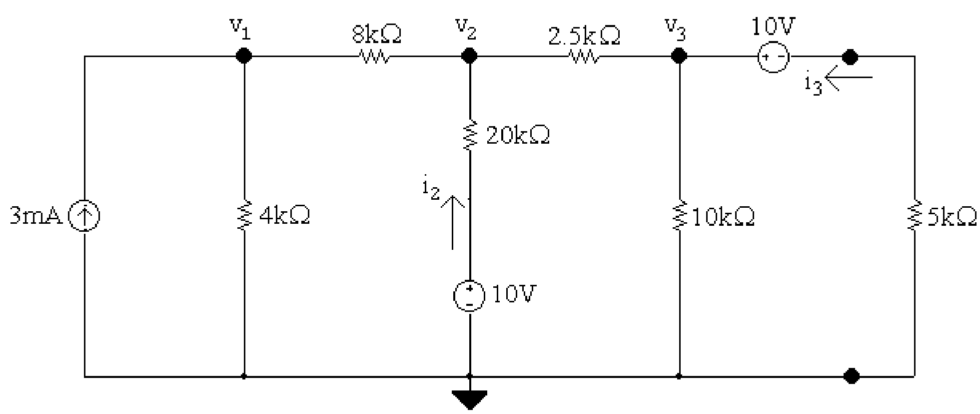
$$R_o = R_{\text{Th}} = 5\ \text{k}\Omega$$

[b]



$$p_{\max} = (-437.5 \times 10^{-6})^2(5000) = 957.03\,\mu\ \text{W}$$

P 4.78 Write KCL equations at each of the labeled nodes, place them in standard form, and solve:



At v_1: $$-3 \times 10^{-3} + \frac{v_1}{4000} + \frac{v_1 - v_2}{8000} = 0$$

At v_2: $$\frac{v_2 - v_1}{8000} + \frac{v_2 - 10}{20{,}000} + \frac{v_2 - v_3}{2500} = 0$$

At v_3: $$\frac{v_3 - v_2}{2500} + \frac{v_3}{10{,}000} + \frac{v_3 - 10}{5000} = 0$$

Standard form:

$$v_1\left(\frac{1}{4000}+\frac{1}{8000}\right)+v_2\left(-\frac{1}{8000}\right)+v_3(0)=0.003$$

$$v_1\left(-\frac{1}{8000}\right)+v_2\left(\frac{1}{8000}+\frac{1}{20{,}000}+\frac{1}{2500}\right)+v_3\left(-\frac{1}{2500}\right)=\frac{10}{20{,}000}$$

$$v_1(0)+v_2\left(-\frac{1}{2500}\right)+v_3\left(\frac{1}{2500}+\frac{1}{10{,}000}+\frac{1}{5000}\right)=\frac{10}{5000}$$

Calculator solution:

$v_1 = 10.890625\ \text{V} \qquad v_2 = 8.671875\ \text{V} \qquad v_3 = 7.8125\ \text{V}$

Calculate currents:

$$i_2=\frac{10-v_2}{20{,}000}=66.40625\,\mu\text{ A} \qquad i_3=\frac{10-v_3}{5000}=437.5\,\mu\text{ A}$$

Calculate power delivered by the sources:

$$p_{\text{3mA}}=(3\times10^{-3})v_1=(3\times10^{-3})(10.890625)=32.671875\text{ mW}$$

$$p_{\text{10Vmiddle}}=i_2(10)=(66.40625\times10^{-6})(10)=0.6640625\text{ mW}$$

$$p_{\text{10Vtop}}=i_3(10)=(437.5\times10^{-6})(10)=4.375\text{ mW}$$

$$p_{\text{deliveredtotal}}=32.671875+0.6640625+4.375=37.7109375\text{ mW}$$

Calculate power absorbed by the $5\text{ k}\Omega$ resistor and the percentage power:

$$p_{\text{5k}}=i_3^2(5000)=(437.5\times10^{-6})^2(5000)=0.95703125\text{ mW}$$

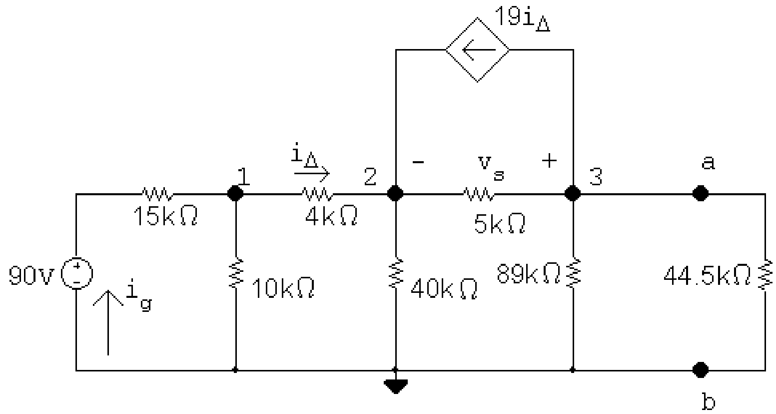
% delivered to R_o: $\quad \dfrac{0.95793125}{37.7109375}(100)=2.54\%$

P 4.79 **[a]** From the solution of Problem 4.68 we have $R_{\text{Th}}=44.5\text{ k}\Omega$ and $V_{\text{Th}}=-19.8$ V. Therefore

$$R_o=R_{\text{Th}}=44.5\text{ k}\Omega$$

[b] $p=\dfrac{(-9.9)^2}{44{,}500}=2.2\text{ mW}$

[c]



The node voltage equations are:

$$\frac{v_1 - 90}{15{,}000} + \frac{v_1}{10{,}000} + \frac{v_1 - v_2}{4000} = 0$$

$$\frac{v_2 - v_1}{4000} + \frac{v_2}{40{,}000} + \frac{v_2 - v_3}{5000} - 19i_\Delta = 0$$

$$\frac{v_3 - v_2}{5000} + \frac{v_3}{89{,}000} + 19i_\Delta + \frac{v_3}{44{,}500} = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1 - v_2}{4000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{15{,}000} + \frac{1}{10{,}000} + \frac{1}{4000}\right) + v_2\left(-\frac{1}{4000}\right) + v_3(0) + i_\Delta(0) = \frac{90}{15{,}000}$$

$$v_1\left(-\frac{1}{4000}\right) + v_2\left(\frac{1}{4000} + \frac{1}{40{,}000} + \frac{1}{5000}\right) + v_3\left(-\frac{1}{5000}\right) + i_\Delta(-19) = 0$$

$$v_1(0) + v_2\left(-\frac{1}{5000}\right) + v_3\left(\frac{1}{5000} + \frac{1}{89{,}000} + \frac{1}{44{,}500}\right) + i_\Delta(19) = 0$$

$$v_1\left(\frac{1}{4000}\right) + v_2\left(-\frac{1}{4000}\right) + v_3(0) + i_\Delta(-1) = 0$$

Solving,

$v_1 = 33.2818$ V; $\quad v_2 = 31.4697$ V; $\quad v_3 = -9.9$ V; $\quad i_\Delta = 453\,\mu$A

Calculate the power:

$$i_g = \frac{90 + 33.2818}{15{,}000} = 3.78 \text{ mA}$$

$$p_{90\text{V}} = -(90)(3.78 \times 10^{-3}) = -340.31 \text{ mW}$$

$$p_{\text{dep source}} = (v_3 - v_2)(19i_\Delta) = -356.07 \text{ mW}$$

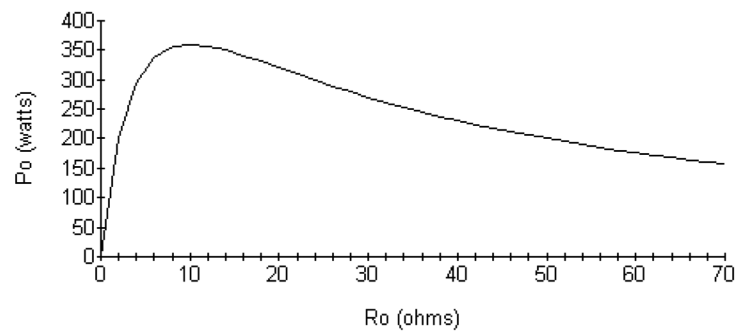
$$\sum p_{\text{dev}} = 340.31 + 356.07 = 696.38 \text{ mW}$$

$$\% \text{ delivered} = \frac{2.2 \times 10^{-3}}{696.38 \times 10^{-3}} \times 100 = 0.316\%$$

P 4.80 **[a]** From the solution to Problem 4.67 we have

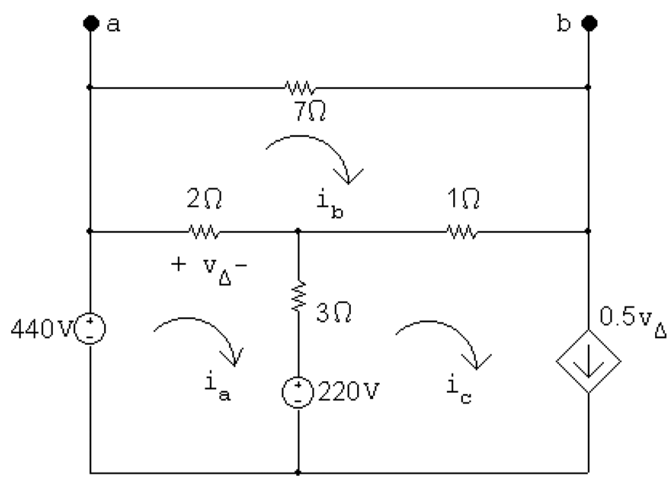
$R_o(\Omega)$	P_o(W)	$R_o(\Omega)$	P_o(W)
0	0	20	320.00
2	200.00	30	270.00
6	337.50	40	230.40
10	360.00	50	200.00
15	345.60	70	157.50

[b]



[c] $R_o = 10\,\Omega$, $\qquad P_o\,(\text{max}) = 360$ W

P 4.81 Find the Thévenin equivalent with respect to the terminals of R_o.
Open circuit voltage:



$$(440 - 220) = 5i_{\rm a} - 2i_{\rm b} - 3i_{\rm c}$$

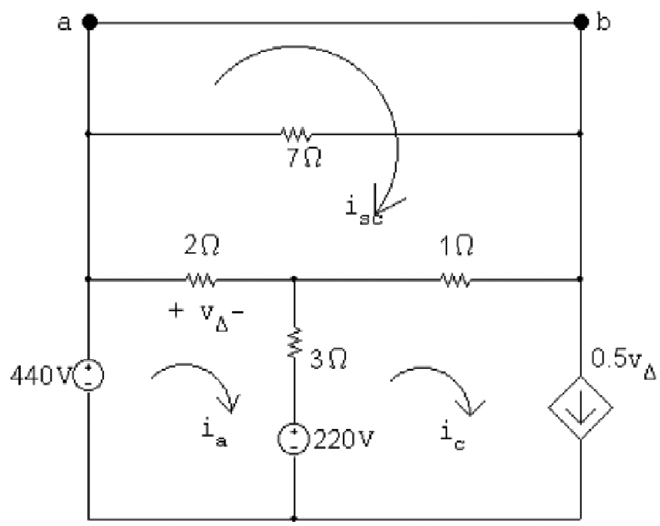
$$0 = -2i_{\rm a} + 10i_{\rm b} - i_{\rm c}$$

$$i_{\rm c} = 0.5v_\Delta; \qquad v_\Delta = 2(i_{\rm a} - i_{\rm b}); \qquad i_{\rm c} = i_{\rm a} - i_{\rm b}$$

Solving, $i_a = 96.8$ A; $i_b = 26.4$ A; $i_c = 70.4$ A; $v_\Delta = 140.8$ V

$\therefore \quad V_{Th} = 7i_b = 184.8$ V

Short circuit current:



$440 - 220 = 5i_a - 2i_{sc} - 3i_c$

$0 = -2i_a + 3i_{sc} - 1i_c$

$i_c = 0.5v_\Delta; \qquad v_\Delta = 2(i_a - i_{sc}) \qquad \therefore \qquad i_c = i_a - i_{sc}$

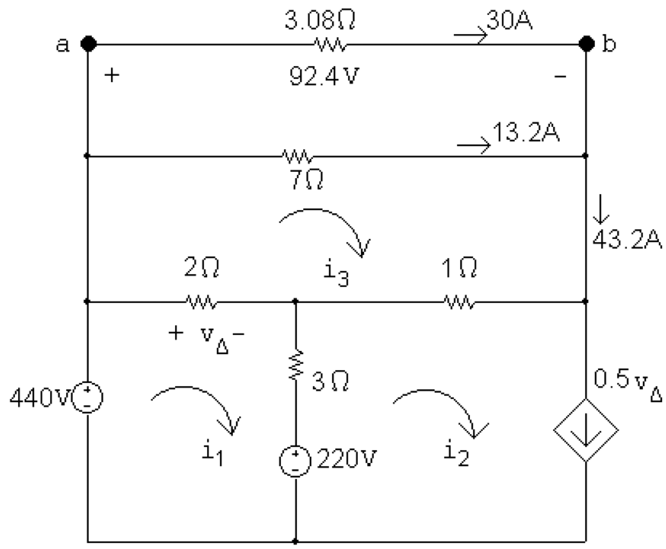
Solving, $i_{sc} = 60$ A; $\quad i_a = 80$ A; $\quad i_c = 20$ A; $\quad v_\Delta = 40$ V

$R_{Th} = V_{Th}/i_{sc} = 184.8/60 = 3.08\,\Omega$

$R_o = 3.08\,\Omega$

$$p_{R_o} = \frac{(92.4)^2}{3.08} = 2772 \text{ W}$$

With R_o equal to $3.08\,\Omega$ the circuit becomes



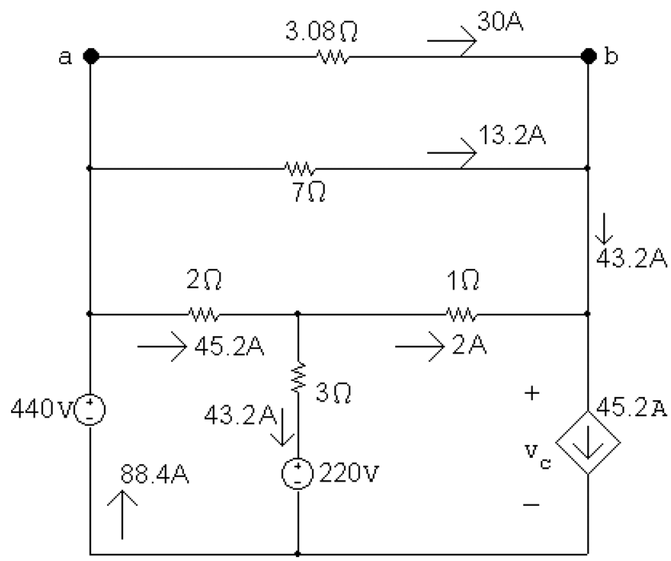
$$220 = 5i_1 - 3(0.5)(2)(i_1 - i_3) - 2i_3 = 2i_1 + i_3$$

$$\therefore \quad 2i_1 = 220 - i_3 = 220 - 43.2 = 176.8 \quad \therefore \; i_1 = 88.4 \text{ A}$$

$$v_\Delta = 2(i_1 - i_3) = 90.4 \text{ V}$$

$$i_2 = 0.5v_\Delta = 45.2 \text{ A}$$

Thus we have



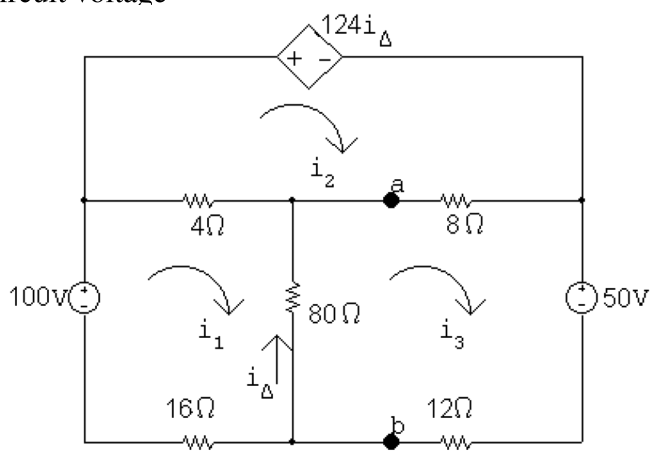
$$v_c = 220 + 3(43.2) - 2 = 347.6 \text{ V}$$

Therefore, the only source developing power is the 440 V source.

$p_{440\text{V}} = -(440)(88.4) = -38{,}896$ W Power delivered is 38,896 W

$$\%\text{ delivered } = \frac{2772}{38{,}896}(100) = 7.13\%$$

P 4.82 **[a]** We begin by finding the Thévenin equivalent with respect to the terminals of R_o.
Open circuit voltage



The mesh current equations are:

$$-100 + 4(i_1 - i_2) + 80(i_1 - i_3) + 16i_1 = 0$$

$$124i_\Delta + 8(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$50 + 12i_3 + 80(i_3 - i_1) + 8(i_3 - i_2) = 0$$

The constraint equation is:
$i_\Delta = i_3 - i_1$
Place these equations in standard form:

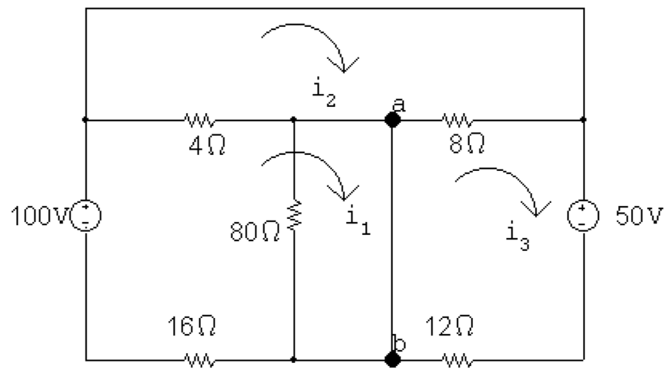
$$i_1(4 + 80 + 16) + i_2(-4) + i_3(-80) + i_\Delta(0) = 100$$

$$i_1(-4) + i_2(8 + 4) + i_3(-8) + i_\Delta(124) = 0$$

$$i_1(-80) + i_2(-8) + i_3(12 + 80 + 8) + i_\Delta(0) = -50$$

$$i_1(1) + i_2(0) + i_3(-1) + i_\Delta(1) = 0$$

Solving, $i_1 = 4.7$ A; $i_2 = 10.5$ A; $i_3 = 4.1$ A; $i_\Delta = -0.6$ A
Also, $V_{\text{Th}} = v_{\text{ab}} = -80i_\Delta = 48$ V
Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $124i_\Delta$ is also zero.

The mesh currents are:

$$-100 + 4(i_1 - i_2) + 16i_1 = 0$$

$$8(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$50 + 12i_3 + 8(i_3 - i_2) = 0$$

Place these equations in standard form:

$$i_1(4 + 16) + i_2(-4) + i_3(0) = 100$$

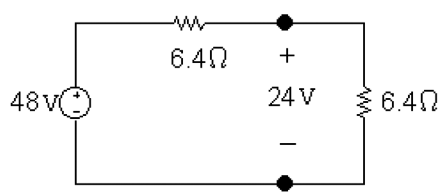
$$i_1(-4) + i_2(8 + 4) + i_3(-8) = 0$$

$$i_1(0) + i_2(-8) + i_3(12 + 8) = -50$$

Solving, $i_1 = 5$ A; $i_2 = 0$ A; $i_3 = -2.5$ A

Then, $i_{\text{sc}} = i_1 - i_3 = 7.5$ A

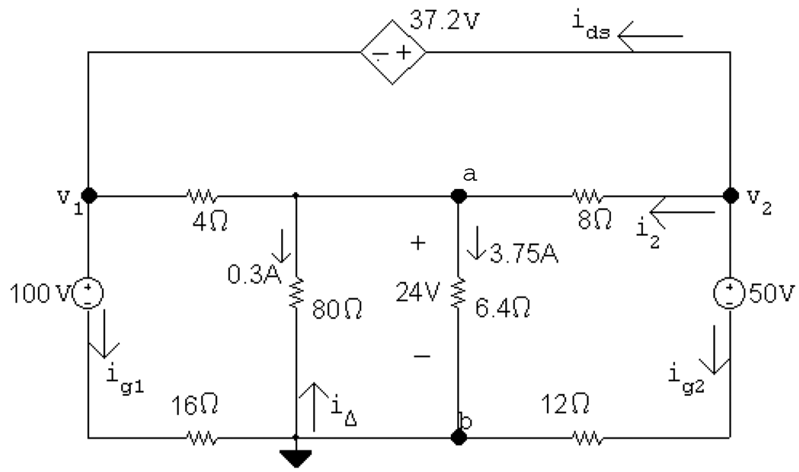
$R_{\text{Th}} = 48/7.5 = 6.4\,\Omega$



For maximum power transfer $R_o = R_{\text{Th}} = 6.4\,\Omega$

[b] $p_{\max} = \dfrac{24^2}{6.4} = 90$ W

P 4.83 From the solution of Problem 4.82 we know that when R_o is $6.4\,\Omega$, the voltage across R_o is 24 V, positive at the upper terminal. Therefore our problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is -0.3 A, and hence $124i_\Delta$ is -37.2 V.



Using the node voltage method to find v_1 and v_2 yields

$$4.05 + \frac{24 - v_1}{4} + \frac{24 - v_2}{8} = 0$$

$$2v_1 + v_2 = 104.4; \qquad v_1 + 37.2 = v_2$$

Solving, $v_1 = 22.4$ V; $\qquad v_2 = 59.6$ V.
It follows that

$$i_{g_1} = \frac{22.4 - 100}{16} = -4.85 \text{ A}$$

$$i_{g_2} = \frac{59.6 - 50}{12} = 0.8 \text{ A}$$

$$i_2 = \frac{59.6 - 24}{8} = 4.45 \text{ A}$$

$$i_{\text{ds}} = -4.45 - 0.8 = -5.25 \text{ A}$$

$$p_{100\text{V}} = 100 i_{g_1} = -485 \text{ W}$$

$$p_{50\text{V}} = 50 i_{g_2} = 40 \text{ W}$$

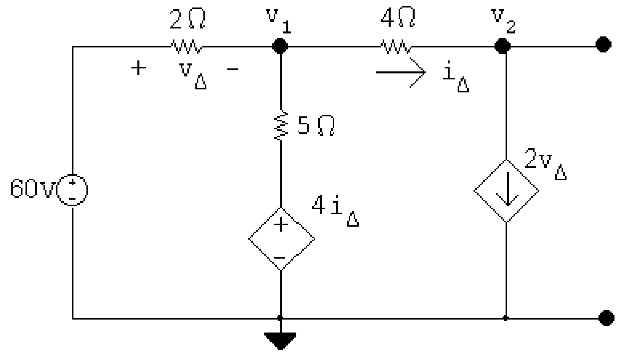
$$p_{\text{ds}} = 37.2 i_{\text{ds}} = -195.3 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}} = 485 + 195.3 = 680.3 \text{ W}$$

$$\therefore \quad \% \text{ delivered } = \frac{90}{680.3}(100) = 13.23\%$$

$\therefore \quad 13.23\%$ of developed power is delivered to load

P 4.84 **[a]** Open circuit voltage



Node voltage equations:

$$\frac{v_1 - 60}{2} + \frac{v_1 - 4i_\Delta}{5} + \frac{v_1 - v_2}{4} = 0$$

$$\frac{v_2 - v_1}{4} + 2v_\Delta = 0$$

Constraint equations:

$$v_\Delta = 60 - v_1$$

$$i_\Delta = \frac{v_1 - v_2}{4}$$

Place the equations in standard form:

$$v_1\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{4}\right) + v_2\left(-\frac{1}{4}\right) + i_\Delta\left(-\frac{4}{5}\right) + v_\Delta(0) = 30$$

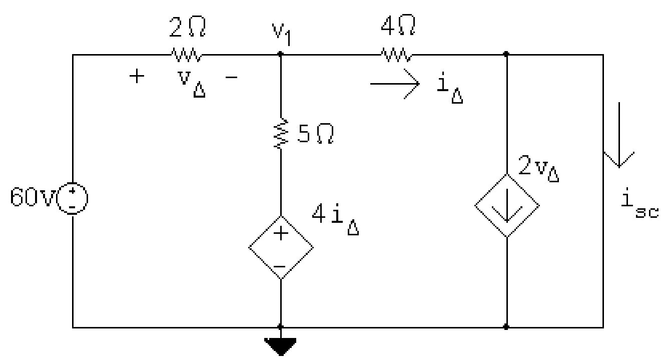
$$v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{1}{4}\right) + i_\Delta(0) + v_\Delta(2) = 0$$

$$v_1(1) + v_2(0) + i_\Delta(0) + v_\Delta(1) = 60$$

$$v_1(1) + v_2(-1) + i_\Delta(-4) + v_\Delta(0) = 0$$

Solving, $v_1 = 20$ V; $v_2 = -300$ V; $i_\Delta = 80$ A; $v_\Delta = 40$ V

Short circuit current:



The node voltage equation:

$$\frac{v_1 - 60}{2} + \frac{v_1 - 4i_\Delta}{5} + \frac{v_1}{4} = 0$$

The constraint equation:

$i_\Delta = v_1/4$

Place these equations in standard form:

$$v_1\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{4}\right)+i_\Delta\left(-\frac{4}{5}\right) = 30$$

$$v_1\left(\frac{1}{4}\right)+i_\Delta(-1) = 0$$

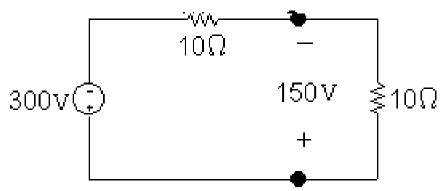
Solving, $v_1 = 40$ V; $i_\Delta = 10$ A

Then, $v_\Delta = 60 - 40 = 20$ V

and $i_{sc} = i_\Delta - 2v_\Delta = 10 - 40 = -30$ A

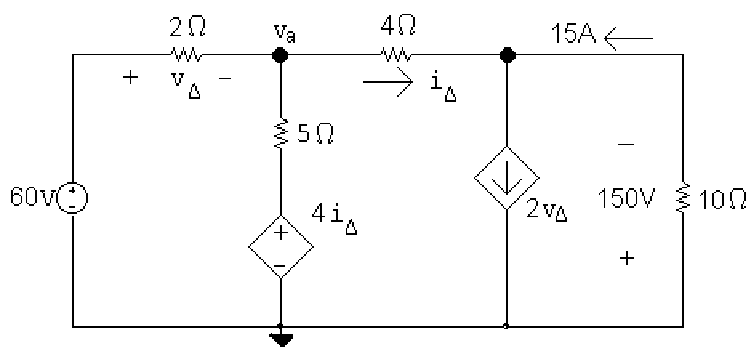
Thus, $R_{Th} = -300/-30 = 10\,\Omega$

[b]



$$p_{max} = \frac{(150)^2}{10} = 2250 \text{ W}$$

[c]



The node voltage equation:

$$\frac{v_a - 60}{2} + \frac{v_a - 4i_\Delta}{5} + \frac{v_a + 150}{4} = 0$$

The constraint equation is:

$$i_\Delta = \frac{v_a + 150}{4}$$

Place the equations in standard form:

$$v_a\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{4}\right)+i_\Delta\left(-\frac{4}{5}\right) = 30 - \frac{150}{4}$$

$$v_a\left(-\frac{1}{4}\right)+i_\Delta(1) = \frac{150}{4}$$

Solving, $v_a = 30$ V; $i_\Delta = 45$ A

Calculate the power:

$$i_{60\text{V}} = \frac{v_\text{a} - 60}{2} = -15 \text{ A}$$

$$p_{60\text{V}} = (60)(-15) = -900 \text{ W}$$

$$i_\text{ccvs} = \frac{v_\text{a} - 4i_\Delta}{5} = -30 \text{ A}$$

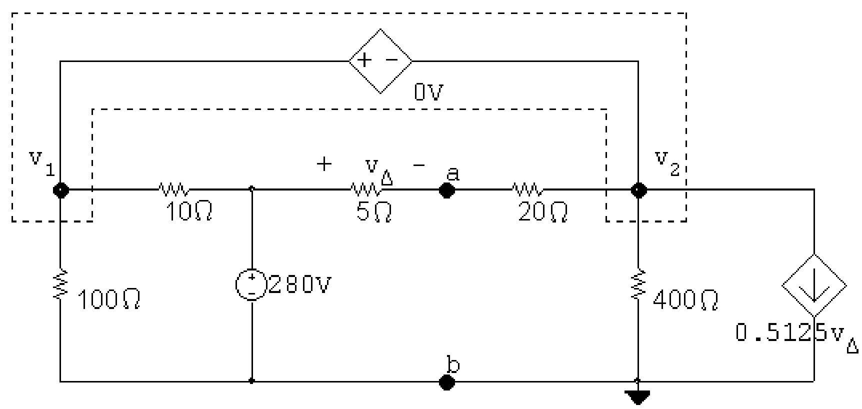
$$p_\text{ccvs} = 4(45)(-30) = -5400 \text{ W}$$

$$p_\text{vccs} = (-150)[2(30)] = -9000 \text{ W}$$

$$\sum p_\text{dev} = 900 + 5400 + 9000 = 15{,}300 \text{ W}$$

$$\% \text{ delivered } = \frac{2250}{15{,}3000} \times 100 = 14.7\%$$

P 4.85 **[a]** First find the Thévenin equivalent with respect to R_o.
Open circuit voltage: $i_\phi = 0$; $50i_\phi = 0$



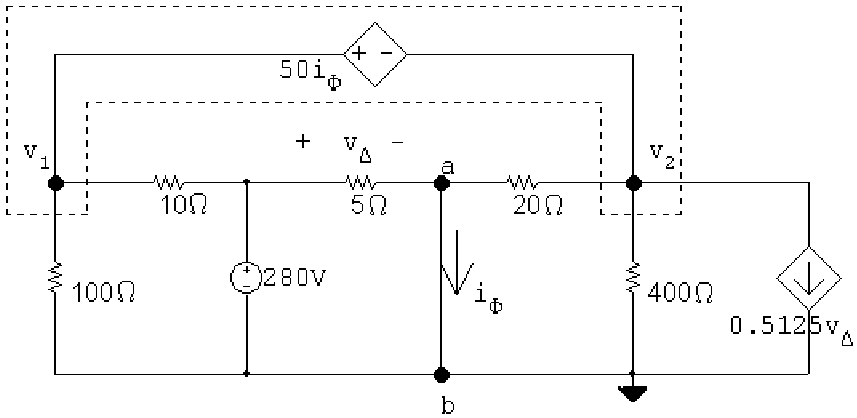
$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$$v_1 = 210 \text{ V}; \qquad v_\Delta = 14 \text{ V}$$

$$V_\text{Th} = 280 - v_\Delta = 280 - 14 = 266 \text{ V}$$

Short circuit current



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$v_\Delta = 280$ V

$v_2 + 50i_\phi = v_1$

$$i_\phi = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

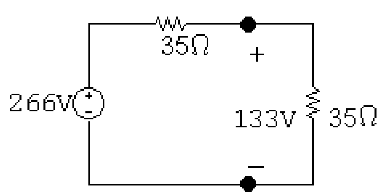
$v_2 = -968$ V; $\quad v_1 = -588$ V

$i_\phi = i_{\rm sc} = 56 + 0.05(-968) = 7.6$ A

$R_{\rm Th} = V_{\rm Th}/i_{\rm sc} = 266/7.6 = 35\,\Omega$

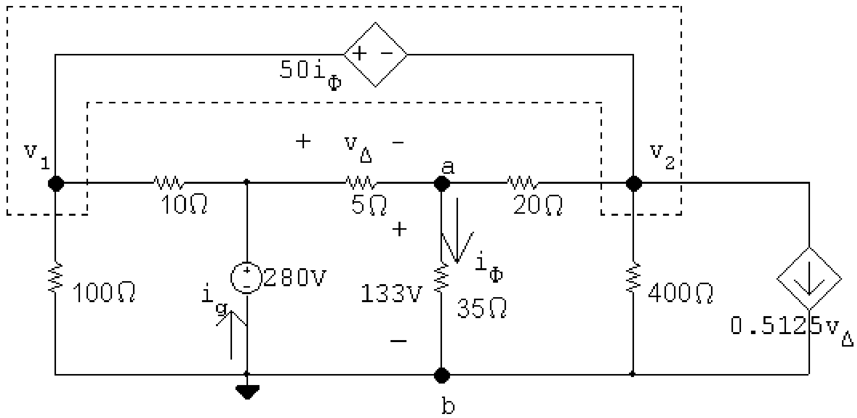
$\therefore \quad R_o = 35\,\Omega$

[b]



$p_{\rm max} = (133)^2/35 = 505.4$ W

[c]



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

$v_2 + 50i_\phi = v_1; \qquad i_\phi = 133/35 = 3.8$ A

Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

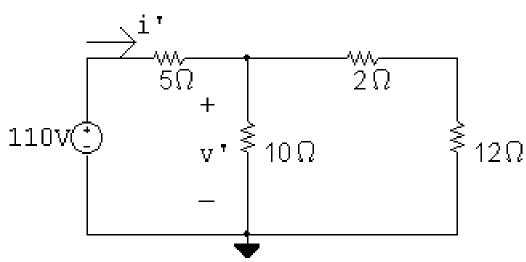
$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

$p_{280\text{V}}$ (dev) $= (280)(76.3) = 21{,}364$ W

P 4.86 **[a]** Since $0 \leq R_o < \infty$ maximum power will be delivered to the 8 Ω resistor when $R_o = 0$.

[b] $P = \dfrac{24^2}{8} = 72$ W

P 4.87 **[a]** 110 V source acting alone:

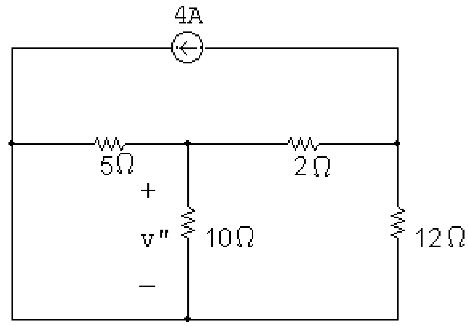


$$R_e = \frac{10(14)}{24} = \frac{35}{6}\ \Omega$$

$$i' = \frac{110}{5 + 35/6} = \frac{132}{13} \text{ A}$$

$$v' = \left(\frac{35}{6}\right)\left(\frac{132}{13}\right) = \frac{770}{13} \text{ V}$$

4 A source acting alone:

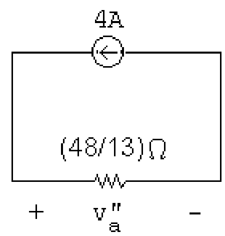


$5\,\Omega \| 10\,\Omega = 50/15 = 10/3\,\Omega$

$10/3 + 2 = 16/3\,\Omega$

$16/3 \| 12 = 48/13\,\Omega$

Hence our circuit reduces to:



It follows that

$v_a'' = 4(48/13) = (192/13)$ V

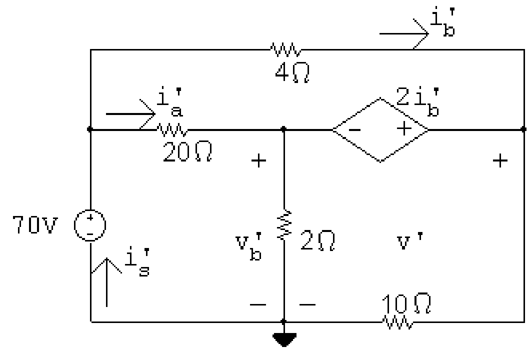
and

$$v'' = \frac{-v_a''}{(16/3)}(10/3) = -\frac{5}{8}v_a'' = -(120/13) \text{ V}$$

$$\therefore \qquad v = v' + v'' = \frac{770}{13} - \frac{120}{13} = 50 \text{ V}$$

[b] $p = \dfrac{v_o^2}{10} = 250$ W

P 4.88 70-V source acting alone:



$$v' = 70 - 4i'_b$$

$$i'_s = \frac{v'_b}{2} + \frac{v'}{10} = i'_a + i'_b$$

$$70 = 20i'_a + v'_b$$

$$i'_a = \frac{70 - v'_b}{20}$$

$$\therefore \quad i'_b = \frac{v'_b}{2} + \frac{v'}{10} - \frac{70 - v'_b}{20} = \frac{11}{20}v'_b + \frac{v'}{10} - 3.5$$

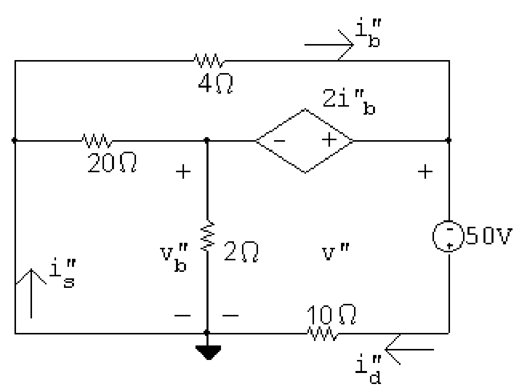
$$v' = v'_b + 2i'_b$$

$$\therefore \quad v'_b = v' - 2i'_b$$

$$\therefore \quad i'_b = \frac{11}{20}(v' - 2i'_b) + \frac{v'}{10} - 3.5 \quad \text{or} \quad i'_b = \frac{13}{42}v' - \frac{70}{42}$$

$$\therefore \quad v' = 70 - 4\left(\frac{13}{42}v' - \frac{70}{42}\right) \quad \text{or} \quad v' = \frac{3220}{94} = \frac{1610}{47}\text{ V}$$

50-V source acting alone:



$$v'' = -4i''_b$$

$$v'' = v_b'' + 2i_b''$$

$$v'' = -50 + 10i_d''$$

$$\therefore \quad i_d'' = \frac{v'' + 50}{10}$$

$$i_s'' = \frac{v_b''}{2} + \frac{v'' + 50}{10}$$

$$i_b'' = \frac{v_b''}{20} + i_s'' = \frac{v_b''}{20} + \frac{v_b''}{2} + \frac{v'' + 50}{10} = \frac{11}{20}v_b'' + \frac{v'' + 50}{10}$$

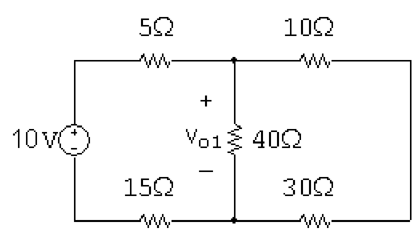
$$v_b'' = v'' - 2i_b''$$

$$\therefore \quad i_b'' = \frac{11}{20}(v'' - 2i_b'') + \frac{v'' + 50}{10} \quad \text{or} \quad i_b'' = \frac{13}{42}v'' + \frac{100}{42}$$

Thus, $v'' = -4\left(\frac{13}{42}v'' + \frac{100}{42}\right)$ or $v'' = -\frac{200}{47}$ V

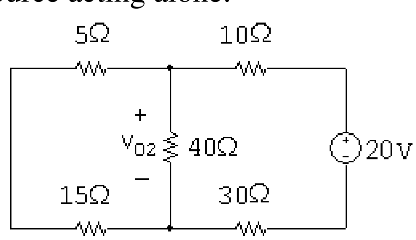
Hence, $v = v' + v'' = \frac{1610}{47} - \frac{200}{47} = \frac{1410}{47} = 30$ V

P 4.89 10 V source acting alone:



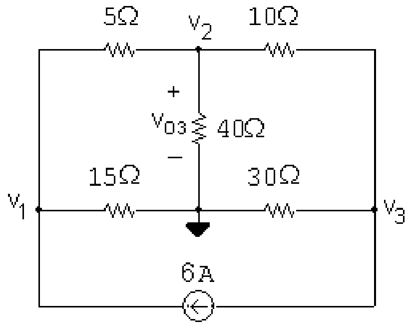
$$v_{o1} = \frac{20}{20 + 5 + 15}(10) = 5 \text{ V}$$

20 V source acting alone:



$$v_{o2} = \frac{13.333}{13.333 + 10 + 30}(20) = 5 \text{ V}$$

6 A current source acting alone:



Node voltage equations:

$$\frac{v_1}{15} + \frac{v_1 - v_2}{5} - 6 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3}{30} + 6 = 0$$

In standard form:

$$v_1\left(\frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) + v_3(0) = 6$$

$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5} + \frac{1}{40} + \frac{1}{10}\right) + v_3\left(-\frac{1}{10}\right) = 0$$

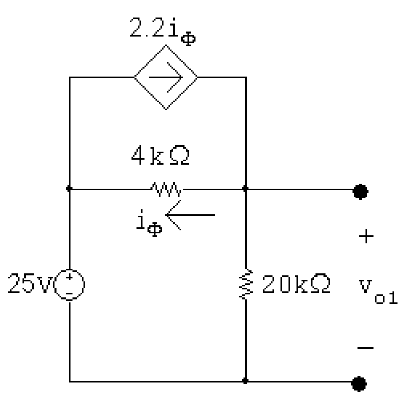
$$v_1(0) + v_2\left(-\frac{1}{10}\right) + v_3\left(\frac{1}{10} + \frac{1}{30}\right) = -6$$

Solving, $v_1 = 22.5$ V; $v_2 = 0$ V; $v_3 = -45$ V

Note that $v_{o3} = v_2 = 0$ V

Finally, $v_o = v_{o1} + v_{o2} + v_{o3} = 5 + 5 + 0 = 10$ V

P 4.90 Voltage source acting alone:

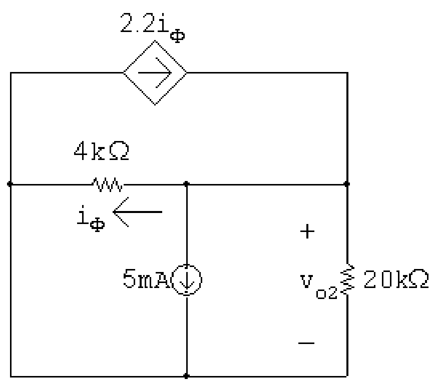


$$\frac{v_{o1} - 25}{4000} + \frac{v_{o1}}{20{,}000} - 2.2\left(\frac{v_{o1} - 25}{4000}\right) = 0$$

Simplifying $\quad 5v_{o1} - 125 + v_{o1} - 11v_{o1} + 275 = 0$

$\therefore \quad v_{o1} = 30$ V

Current source acting alone:



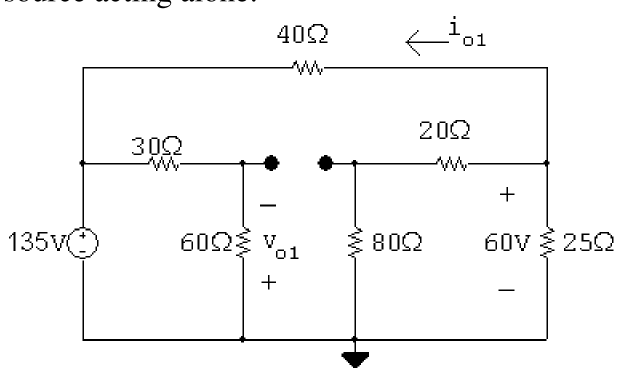
$$\frac{v_{o2}}{4000} + \frac{v_{o2}}{20{,}000} + 0.005 - 2.2\left(\frac{v_{o2}}{4000}\right) = 0$$

Simplifying $\quad 5v_{o2} + v_{o2} + 100 - 11v_{o2} = 0$

$\therefore \quad v_{o2} = 20$ V

$v_o = v_{o1} + v_{o2} = 30 + 20 = 50$ V

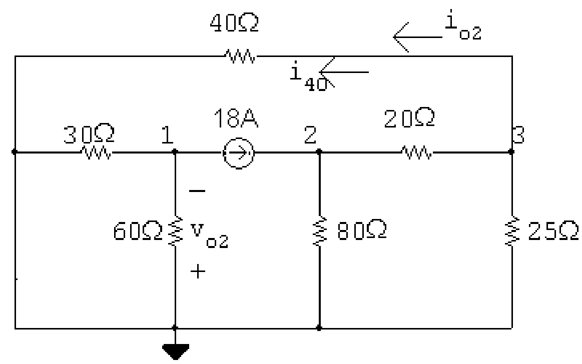
P 4.91 Voltage source acting alone:



$$i_{o1} = \frac{-135}{40 + 100\|25} = -2.25 \text{ A}$$

$$v_{o1} = \frac{60}{90}(-135) = -90 \text{ V}$$

Current source acting alone:



$$\frac{v_1}{30} + \frac{v_1}{60} + 18 = 0 \quad \therefore \quad v_1 = -360 \text{ V}; \quad v_{o2} = 360 \text{ V}$$

$$-18 + \frac{v_2}{80} + \frac{v_2 - v_3}{20} = 0$$

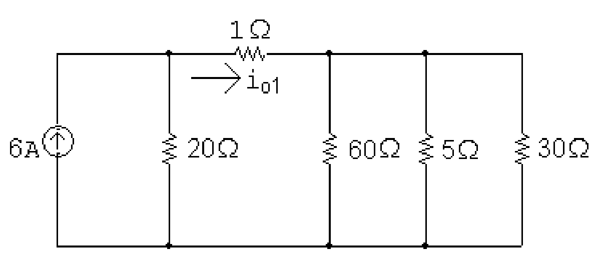
$$\frac{v_3 - v_2}{20} + \frac{v_3}{25} + \frac{v_3}{40} = 0$$

$$\therefore \quad v_2 = 441.6 \text{ V}; \quad v_3 = 192 \text{ V}; \quad i_{o2} = 192/40 = 4.8 \text{ A}$$

$$\therefore \quad v_o = v_{o1} + v_{o2} = -90 + 360 = 270 \text{ V}$$

$$i_o = i_{o1} + i_{o2} = -2.25 + 4.8 = 2.55 \text{ A}$$

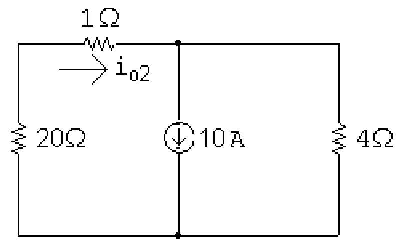
P 4.92 6 A source:



$$30\,\Omega \| 5\,\Omega \| 60\,\Omega = 4\,\Omega$$

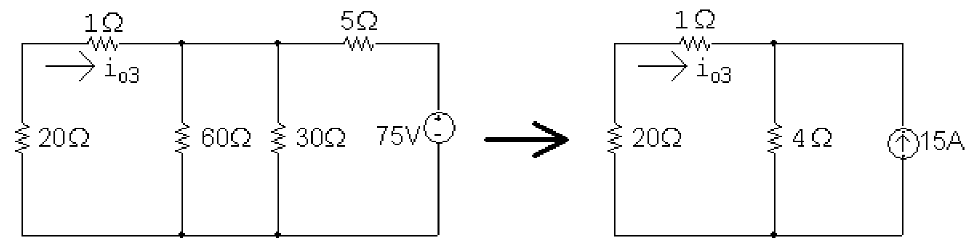
$$\therefore \quad i_{o1} = \frac{20}{20+5}(6) = 4.8 \text{ A}$$

10 A source:



$$i_{o2} = \frac{4}{25}(10) = 1.6 \text{ A}$$

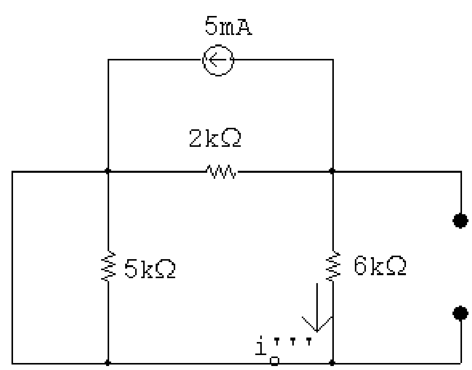
75 V source:



$$i_{o3} = -\frac{4}{25}(15) = -2.4 \text{ A}$$

$$i_o = i_{o1} + i_{o2} + i_{o3} = 4.8 + 1.6 - 2.4 = 4 \text{ A}$$

P 4.93 **[a]** By hypothesis $i_o' + i_o'' = 3.5$ mA.



$$i_o''' = \frac{2000}{8000}(-0.005) = -1.25 \text{ mA}; \qquad \therefore \quad i_o = 3.5 - 1.25 = 2.25 \text{ mA}$$

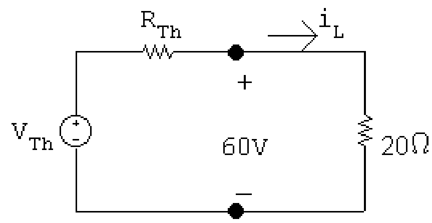
[b] With all three sources in the circuit write a single node voltage equation.

$$\frac{v_b - 8}{2000} + \frac{v_b}{6000} + 0.005 - 0.010 = 0$$

$$\therefore \quad v_b = 13.5 \text{ V}$$

$$i_o = \frac{v_b}{6000} = \frac{13.5}{6000} = 2.25 \text{ mA}$$

P 4.94 **[a]**



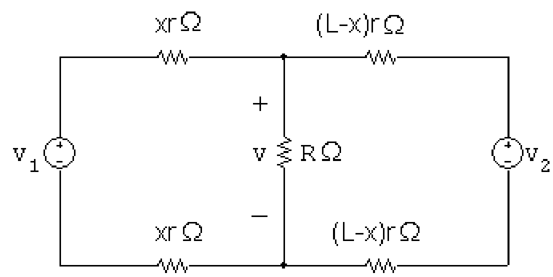
$$v_{\text{oc}} = V_{\text{Th}} = 75 \text{ V}; \qquad i_L = \frac{60}{20} = 3 \text{ A}; \qquad i_L = \frac{75-60}{R_{\text{Th}}} = \frac{15}{R_{\text{Th}}}$$

Therefore $\quad R_{\text{Th}} = \dfrac{15}{3} = 5\,\Omega$

[b] $i_L = \dfrac{v_o}{R_L} = \dfrac{V_{\text{Th}} - v_o}{R_{\text{Th}}}$

Therefore $\quad R_{\text{Th}} = \dfrac{V_{\text{Th}} - v_o}{v_o/R_L} = \left(\dfrac{V_{\text{Th}}}{v_o} - 1\right) R_L$

P 4.95 **[a]**



$$\frac{v - v_1}{2xr} + \frac{v}{R} + \frac{v - v_2}{2r(L-x)} = 0$$

$$v\left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(L-x)}\right] = \frac{v_1}{2xr} + \frac{v_2}{2r(L-x)}$$

$$v = \frac{v_1 RL + xR(v_2 - v_1)}{RL + 2rLx - 2rx^2}$$

[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx} = \frac{(RL + 2rLx - 2rx^2)R(v_2 - v_1) - [v_1 RL + xR(v_2 - v_1)]2r(L - 2x)}{D^2}$$

$\dfrac{dv}{dx} = 0 \quad$ when numerator is zero.

The numerator simplifies to

$$x^2 + \frac{2L - v_1}{(v_2 - v_1)}x + \frac{RL(v_2 - v_1) - 2rv_1L^2}{2r(v_2 - v_1)} = 0$$

Solving for the roots of the quadratic yields

$$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1 v_2 - \frac{R}{2rL}(v_2 - v_1)^2}\right\}$$

[c] $$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1 v_2 - \frac{R}{2rL}(v_1 - v_2)^2}\right\}$$

$v_2 = 1200$ V, $\qquad v_1 = 1000$ V, $\qquad L = 16$ km

$r = 5 \times 10^{-5}\,\Omega/m; \qquad R = 3.9\,\Omega$

$$\frac{L}{v_2 - v_1} = \frac{16{,}000}{1200 - 1000} = 80; \qquad v_1 v_2 = 1.2 \times 10^6$$

$$\frac{R}{2rL}(v_1 - v_2)^2 = \frac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$$

$$x = 80\{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\}$$

$$= 80\{-1000 \pm 1050\} = 80(50) = 4000 \text{ m}$$

[d]

$$v_{\min} = \frac{v_1 RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2}$$

$$= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)}$$

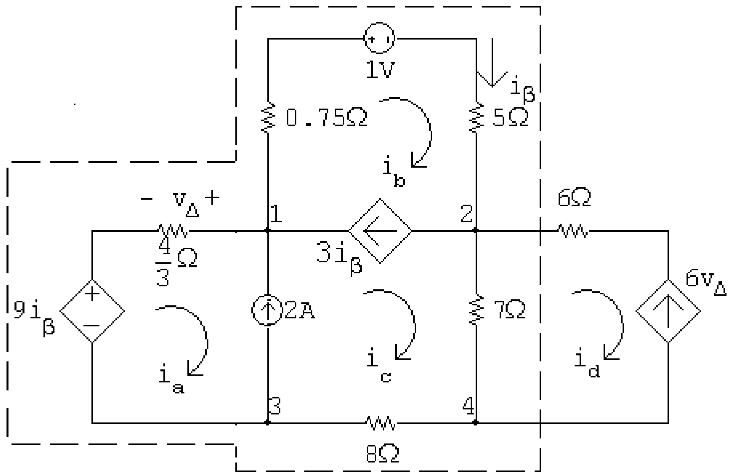
$$= 975 \text{ V}$$

P 4.96 **[a]** In studying the circuit in Fig. P4.96 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node Voltage approach will require solving three node Voltage equations along with equations involving v_Δ and i_β.

The mesh-current approach will require writing one supermesh equation plus three constraint equations involving the three current sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 1 V source, we will retain the mesh current i_b and eliminate the mesh currents i_a, i_c And i_d.

The supermesh is denoted by the dashed line in the following figure.



[b] Summing the voltages around the supermesh yields

$$-9i_\beta + \frac{4}{3}i_\mathrm{a} + 0.75i_\mathrm{b} + 1 + 5i_\mathrm{b} + 7(i_\mathrm{c} - i_\mathrm{d}) + 8i_\mathrm{c} = 0$$

Note that $i_\beta = i_\mathrm{b}$ And multiply the equation by 12:

$$-108i_\mathrm{b} + 16i_\mathrm{a} + 9i_\mathrm{b} + 12 + 60i_\mathrm{b} + 84(i_\mathrm{c} - i_\mathrm{d}) + 96i_\mathrm{c} = 0$$

or

$$16i_\mathrm{a} - 39i_\mathrm{b} + 180i_\mathrm{c} - 84i_\mathrm{d} = -12$$

Now note:

$$i_\mathrm{b} - i_\mathrm{c} = 3i_\beta = 3i_\mathrm{b}; \qquad \therefore \quad i_\mathrm{c} = -2i_\mathrm{b}$$

whence

$$16i_\mathrm{a} - 39i_\mathrm{b} - 360i_\mathrm{b} - 84i_\mathrm{d} = -12$$

Now use the constraint that

$$i_\mathrm{a} - i_\mathrm{c} = -2$$

$$i_\mathrm{a} = -2 + i_\mathrm{c} = -2 - 2i_\mathrm{b}$$

Therefore

$$-32 - 32i_\mathrm{b} - 399i_\mathrm{b} - 84i_\mathrm{d} = -12$$

$$-431i_\mathrm{b} - 84i_\mathrm{d} = 20$$

Now use the constraint

$$i_\mathrm{d} = -6v_\Delta = -6\left(\frac{-4}{3}i_\mathrm{a}\right) = 8i_\mathrm{a} = -16 - 16i_\mathrm{b}$$

Therefore

$-431i_{\rm b} - 84(-16 - 16i_{\rm b}) = 20$

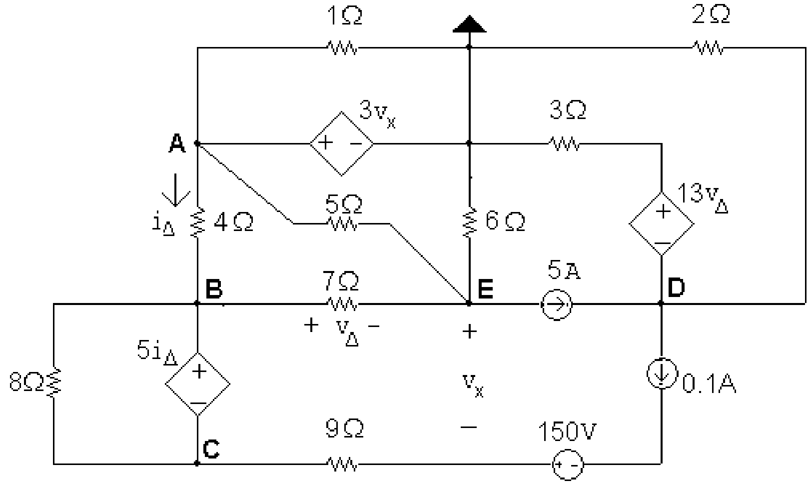
or

$913i_{\rm b} = -1324$

$\therefore \quad i_{\rm b} \approx -1.45$ A

$p_{\rm 1V} = 1i_{\rm b} \cong -1.45$ W; $\qquad \therefore \quad p_{\rm 1V}$ (developed) $\cong 1.45$ W

P 4.97



B–C supernode: $\dfrac{v_{\rm B} - 3v_x}{4} + \dfrac{v_{\rm B} - v_{\rm E}}{7} - 0.1 = 0$

At node E: $\dfrac{v_{\rm E}}{6} + \dfrac{v_{\rm E} - 3v_x}{5} + \dfrac{v_{\rm E} - v_{\rm B}}{7} + 5 = 0$

At node D: $\dfrac{v_{\rm D} + 13v_\Delta}{3} - 5 + 0.1 + \dfrac{v_{\rm D}}{2} = 0$

Constraint: $v_\Delta = v_{\rm B} - v_{\rm E}$

Constraint: $v_x = -v_\Delta + 5i_\Delta - 0.9$

Constraint: $i_\Delta = (3v_x - v_{\rm B})/4$

In standard form:

$$v_{\rm B}\left(\frac{1}{4} + \frac{1}{7}\right) + v_{\rm D}(0) + v_{\rm E}\left(-\frac{1}{7}\right) + v_\Delta(0) + v_x\left(-\frac{3}{4}\right) + i_\Delta(0) = 0.1$$

$$v_{\rm B}(0) + v_{\rm D}\left(\frac{1}{2} + \frac{1}{3}\right) + v_{\rm E}(0) + v_\Delta\left(\frac{13}{3}\right) + v_x(0) + i_\Delta(0) = 4.9$$

$$v_{\rm B}\left(-\frac{1}{7}\right) + v_{\rm D}(0) + v_{\rm E}\left(\frac{1}{6} + \frac{1}{5} + \frac{1}{7}\right) + v_\Delta(0) + v_x\left(-\frac{3}{5}\right) + i_\Delta(0) = -5$$

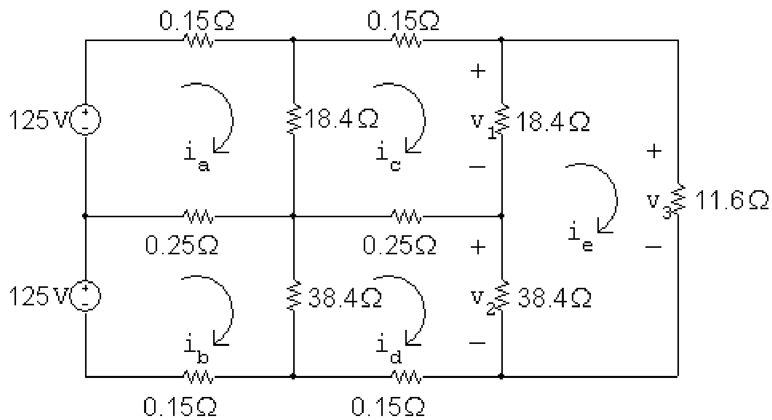
$$v_{\rm B}(-1) + v_{\rm D}(0) + v_{\rm E}(1) + v_\Delta(1) + v_x(0) + i_\Delta(0) = 0$$

$$v_{\rm B}(0) + v_{\rm D}(0) + v_{\rm E}(0) + v_\Delta(1) + v_x(1) + i_\Delta(-5) = -0.9$$

$$v_{\rm B}(1) + v_{\rm D}(0) + v_{\rm E}(0) + v_\Delta(0) + v_x(-3) + i_\Delta(4) = 0$$

Solving, $v_{\rm B} = -11.17$ V; $v_{\rm D} = -20.95$ V; $v_{\rm E} = -16.33$ V;
$v_\Delta = 5.16$ V; $v_x = -2.87$ V; $i_\Delta = 0.64$ A
$p_{\rm 5A} = (v_{\rm E} - v_{\rm D})(5) = 23.1$ W
The 5 A source absorbs 23.1 W

P 4.98



The mesh equations are:

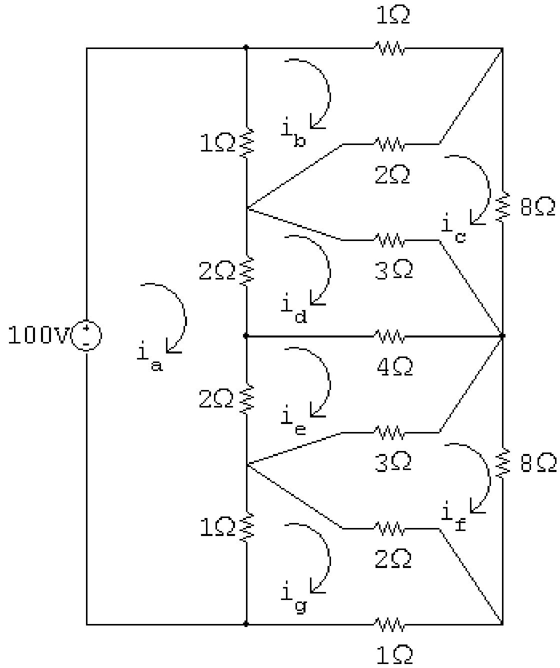
$$\begin{aligned}
-125 + 0.15i_{\rm a} + 18.4(i_{\rm a} - i_{\rm c}) + 0.25(i_{\rm a} - i_{\rm b}) &= 0\\
-125 + 0.25(i_{\rm b} - i_{\rm a}) + 38.4(i_{\rm b} - i_{\rm d}) + 0.15i_{\rm b} &= 0\\
0.15i_{\rm c} + 18.4(i_{\rm c} - i_{\rm e}) + 0.25(i_{\rm c} - i_{\rm d}) + 18.4(i_{\rm c} - i_{\rm a}) &= 0\\
0.15i_{\rm d} + 38.4(i_{\rm d} - i_{\rm b}) + 0.25(i_{\rm d} - i_{\rm c}) + 38.4(i_{\rm d} - i_{\rm e}) &= 0\\
11.6i_{\rm e} + 38.4(i_{\rm e} - i_{\rm d}) + 18.4(i_{\rm e} - i_{\rm c}) &= 0
\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}
i_{\rm a}(18.8) + i_{\rm b}(-0.25) + i_{\rm c}(-18.4) + i_{\rm d}(0) + i_{\rm e}(0) &= 125\\
i_{\rm a}(-0.25) + i_{\rm b}(38.8) + i_{\rm c}(0) + i_{\rm d}(-38.4) + i_{\rm e}(0) &= 125\\
i_{\rm a}(-18.4) + i_{\rm b}(0) + i_{\rm c}(37.2) + i_{\rm d}(-0.25) + i_{\rm e}(-18.4) &= 0\\
i_{\rm a}(0) + i_{\rm b}(-38.4) + i_{\rm c}(-0.25) + i_{\rm d}(77.2) + i_{\rm e}(-38.4) &= 0\\
i_{\rm a}(0) + i_{\rm b}(0) + i_{\rm c}(-18.4) + i_{\rm d}(-38.4) + i_{\rm e}(68.4) &= 0
\end{aligned}$$

Solving,
$i_{\rm a} = 32.77$ A; $i_{\rm b} = 26.46$ A; $i_{\rm c} = 26.33$ A; $i_{\rm d} = 23.27$ A; $i_{\rm e} = 20.14$ A
Find the requested voltages:
$v_1 = 18.4(i_{\rm c} - i_{\rm e}) = 113.90$ V
$v_2 = 38.4(i_{\rm d} - i_{\rm e}) = 120.19$ V
$v_3 = 11.6i_{\rm e} = 233.62$ V

P 4.99



$$100 = 6i_a - 1i_b + 0i_c - 2i_d - 2i_e + 0i_f - 1i_g$$
$$0 = -1i_a + 4i_b - 2i_c + 0i_d + 0i_e + 0i_f + 0i_g$$
$$0 = 0i_a - 2i_b + 13i_c - 3i_d + 0i_e + 0i_f + 0i_g$$
$$0 = -2i_a + 0i_b - 3i_c + 9i_d - 4i_e + 0i_f + 0i_g$$
$$0 = -2i_a + 0i_b + 0i_c - 4i_d + 9i_e - 3i_f + 0i_g$$
$$0 = 0i_a + 0i_b + 0i_c + 0i_d - 3i_e + 13i_f - 2i_g$$
$$0 = -1i_a + 0i_b + 0i_c + 0i_d + 0i_e - 2i_f + 4i_g$$

A computer solution yields

$i_a = 30$ A; $\quad i_e = 15$ A;

$i_b = 10$ A; $\quad i_f = 5$ A;

$i_c = 5$ A; $\quad i_g = 10$ A;

$i_d = 15$ A

$\therefore \quad i = i_d - i_e = 0$ A

CHECK: $p_{1\mathrm{T}} = p_{1\mathrm{B}} = (i_b)^2 = (i_g)^2 = 100$ W

$$p_{1\mathrm{L}} = (i_a - i_b)^2 = (i_a - i_g)^2 = 400 \text{ W}$$
$$p_{2\mathrm{C}} = 2(i_b - i_c)^2 = (i_g - i_f)^2 = 50 \text{ W}$$
$$p_3 = 3(i_c - i_d)^2 = 3(i_e - i_f)^2 = 300 \text{ W}$$
$$p_4 = 4(i_d - i_e)^2 = 0 \text{ W}$$
$$p_8 = 8(i_c)^2 = 8(i_f)^2 = 200 \text{ W}$$
$$p_{2\mathrm{L}} = 2(i_a - i_d)^2 = 2(i_a - i_e)^2 = 450 \text{ W}$$

$$\sum p_{\text{abs}} = 100 + 400 + 50 + 200 + 300 + 450 + 0 + 450 + 300 + 200 + 50 + 400 + 100 = 3000 \text{ W}$$

$$\sum p_{\text{gen}} = 100 i_a = 100(30) = 3000 \text{ W (CHECKS)}$$

P 4.100 $$\frac{dv_1}{dI_{g1}} = \frac{-R_1[R_2(R_3+R_4)+R_3R_4]}{(R_1+R_2)(R_3+R_4)+R_3R_4}$$

$$\frac{dv_1}{dI_{g2}} = \frac{R_1R_3R_4}{(R_1+R_2)(R_3+R_4)+R_3R_4}$$

$$\frac{dv_2}{dI_{g1}} + \frac{-R_1R_3R_4}{(R_1+R_2)(R_3+R_4)+R_3R_4}$$

$$\frac{dv_2}{dI_{g2}} = \frac{R_3R_4(R_1+R_2)}{(R_1+R_2)(R_3+R_4)+R_3R_4}$$

P 4.101 From the solution to Problem 4.100 we have

$$\frac{dv_1}{dI_{g1}} = \frac{-25[5(125)+3750]}{30(125)+3750} = -\frac{175}{12} \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g1}} = \frac{-(25)(50)(75)}{30(125)+3750} = -12.5 \text{ V/A}$$

By hypothesis, $\Delta I_{g1} = 11 - 12 = -1$ A

$$\therefore \quad \Delta v_1 = (-\frac{175}{12})(-1) = \frac{175}{12} = 14.5833 \text{ V}$$

Thus, $v_1 = 25 + 14.5833 = 39.5833$ V
Also,

$$\Delta v_2 = (-12.5)(-1) = 12.5 \text{ V}$$

Thus, $v_2 = 90 + 12.5 = 102.5$ V
The PSpice solution is

$$v_1 = 39.5830 \text{ V}$$

and

$$v_2 = 102.5000 \text{ V}$$

These values are in agreement with our predicted values.

P 4.102 From the solution to Problem 4.100 we have

$$\frac{dv_1}{dI_{g2}} = \frac{(25)(50)(75)}{30(125) + 3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g2}} = \frac{(50)(75)(30)}{30(125) + 3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_{g2} = 17 - 16 = 1$ A

$$\therefore \quad \Delta v_1 = (12.5)(1) = 12.5 \text{ V}$$

Thus, $v_1 = 25 + 12.5 = 37.5$ V
Also,

$$\Delta v_2 = (15)(1) = 15 \text{ V}$$

Thus, $v_2 = 90 + 15 = 105$ V
The PSpice solution is

$$v_1 = 37.5 \text{ V}$$

and

$$v_2 = 105 \text{ V}$$

These values are in agreement with our predicted values.

P 4.103 From the solutions to Problems 4.100 — 4.102 we have

$$\frac{dv_1}{dI_{g1}} = -\frac{175}{12} \text{ V/A}; \qquad \frac{dv_1}{dI_{g2}} = 12.5 \text{ V/A}$$

$$\frac{dv_2}{dI_{g1}} = -12.5 \text{ V/A}; \qquad \frac{dv_2}{dI_{g2}} = 15 \text{ V/A}$$

By hypothesis,

$$\Delta I_{g1} = 11 - 12 = -1 \text{ A}$$

$$\Delta I_{g2} = 17 - 16 = 1 \text{ A}$$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833 \text{ V}$$

$\Delta v_2 = 12.5 + 15 = 27.5$ V

Hence

$v_1 = 25 + 27.0833 = 52.0833$ V

$v_2 = 90 + 27.5 = 117.5$ V

The PSpice solution is

$v_1 = 52.0830$ V

and

$v_2 = 117.5$ V

These values are in agreement with our predicted values.

P 4.104 By hypothesis,

$\Delta R_1 = 27.5 - 25 = 2.5\,\Omega$

$\Delta R_2 = 4.5 - 5 = -0.5\,\Omega$

$\Delta R_3 = 55 - 50 = 5\,\Omega$

$\Delta R_4 = 67.5 - 75 = -7.5\,\Omega$

So

$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168$ V

$\therefore \quad v_1 = 25 + 4.9168 = 29.9168$ V

$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1$ V

$\therefore \quad v_2 = 90 - 1.1 = 88.9$ V

The PSpice solution is

$v_1 = 29.6710$ V

and

$v_2 = 88.5260$ V

Note our predicted values are within a fraction of a volt of the actual values.