5

The Operational Amplifier

Assessment Problems

AP 5.1 **[a]** This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \qquad \text{so} \qquad v_o = -5v_s$$

v_s(V)	0.4	2.0	3.5	−0.6	−1.6	−2.4
v_o(V)	−2.0	−10.0	−15.0	3.0	8.0	10.0

Two of the v_s values, 3.5 V and -2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3 \text{ V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \qquad v_s = -2 \text{ V}$$

Therefore $\quad -2 \leq v_s \leq 3$ V

AP 5.2 From Assessment Problem 5.1

$$v_o = (-R_f/R_i)v_s = (-R_x/16{,}000)v_s$$
$$= (-R_x/16{,}000)(-0.640) = 0.64R_x/16{,}000 = 4\times10^{-5}R_x$$

Use the negative power supply value to determine one limit on the value of R_x:

$$4\times10^{-5}R_x = -15 \qquad \text{so} \qquad R_x = -15/4\times10^{-5} = -375\,\text{k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x:

$$4\times10^{-5}R_x = 10 \qquad \text{so} \qquad R_x = 10/4\times10^{-5} = 250\,\text{k}\Omega$$

Therefore,

$$0 \leq R_x \leq 250\,\text{k}\Omega$$

AP 5.3 **[a]** This is an inverting summing amplifier so

$$v_o = (-R_f/R_\mathrm{a})v_\mathrm{a} + (-R_f/R_\mathrm{b})v_\mathrm{b} = -(250/5)v_\mathrm{a} - (250/25)v_\mathrm{b} = -50v_\mathrm{a} - 10v_\mathrm{b}$$

Substituting the values for v_a and v_b:

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_\mathrm{a} - 10(0.25) = -50v_\mathrm{a} - 2.5 = -10 \ \text{V}$$

Therefore $\quad 50v_\mathrm{a} = 7.5, \quad$ so $\quad v_\mathrm{a} = 0.15 \ \text{V}$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_\mathrm{b} = -5 - 10v_\mathrm{b} = -10 \ \text{V};$$

Therefore $\quad 10v_\mathrm{b} = 5, \quad$ so $\quad v_\mathrm{b} = 0.5 \ \text{V}$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.
Repeat part (a):

$$v_o = -50v_\mathrm{a} + 10v_\mathrm{b} = -5 + 2.5 = -2.5 \ \text{V}$$

Repeat part (b):

$$v_o = -50v_\mathrm{a} + 2.5 = -10 \ \text{V}; \qquad 50v_\mathrm{a} = 12.5, \quad v_\mathrm{a} = 0.25 \ \text{V}$$

Repeat part (c):

$$v_o = -5 + 10v_\mathrm{b} = 15 \ \text{V}; \qquad 10v_\mathrm{b} = 20; \qquad v_\mathrm{b} = 2.0 \ \text{V}$$

AP 5.4 **[a]** Write a node voltage equation at v_n; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63{,}000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by $63{,}000$ and collecting terms:

$$14v_n + v_n - v_o = 0 \qquad \text{so} \qquad v_o = 15v_n$$

Now use voltage division to calculate v_p. We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R_x resistor:

$$v_p = \frac{R_x}{15{,}000 + R_x}(0.400)$$

Now substitute the value $R_x = 60\ \text{k}\Omega$:

$$v_p = \frac{60{,}000}{15{,}000 + 60{,}000}(0.400) = 0.32\ \text{V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_0

$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8\ \text{V}$$

[b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15\left(\frac{0.4R_x}{15{,}000 + R_x}\right) = 5$$

$$15(0.4R_x) = 5(15{,}000 + R_x) \quad \text{so} \quad R_x = 75\,\text{k}\Omega$$

AP 5.5 **[a]** Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_\text{b} - \frac{50}{10}v_\text{a}$$

Simplify this expression and substitute in the value for v_b:

$$v_o = 5(v_\text{b} - v_\text{a}) = 20 - 5v_\text{a}$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_\text{a} = 10\ \text{V} \quad \text{so} \quad v_\text{a} = 2\ \text{V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_\text{a} = -10\ \text{V} \quad \text{so} \quad v_\text{a} = 6\ \text{V}$$

Therefore $\quad 2 \leq v_\text{a} \leq 6\ \text{V}$

[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)}v_\text{b} - 5v_\text{a} = 4v_\text{b} - 5v_\text{a}$$

Now substitute the value for v_b:

$$v_o = 4(4) - 5v_\text{a} = 16 - 5v_\text{a}$$

Set this expression for v_o to the positive power supply value:

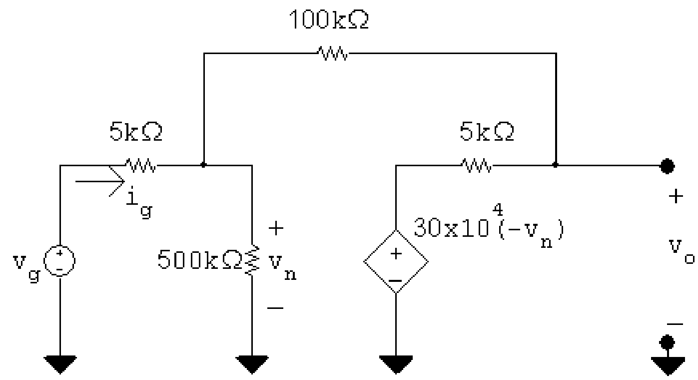
$$16 - 5v_\text{a} = 10\ \text{V} \quad \text{so} \quad v_\text{a} = 1.2\ \text{V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_\text{a} = -10\ \text{V} \quad \text{so} \quad v_\text{a} = 5.2\ \text{V}$$

Therefore $\quad 1.2 \leq v_\text{a} \leq 5.2\ \text{V}$

AP 5.6 **[a]** Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v_n}{500{,}000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} = 0$$

Multiply both sides by $500{,}000$ and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_0 = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300{,}000(-v_n)}{5000} + \frac{v_o - v_n}{100{,}000} = 0$$

Multiply through by $100{,}000$ and simplify:

$$20v_o + 6 \times 10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6 \times 10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o:

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6{,}000{,}445.2$$

$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985v_g; \qquad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n:

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v_g$$

$$v_g = 1 \text{ V}, \qquad v_n = 69.995\,\mu\text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35\,\Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

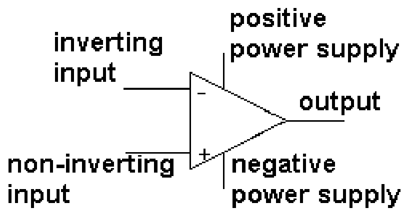
$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$
Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000\,\Omega$$

Problems

P 5.1 **[a]** The five terminals of the op amp are identified as follows:



[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.

[d] Write a node voltage equation at v_n:

$$\frac{v_n - 2.5}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-2.5}{10{,}000} - \frac{v_o}{40{,}000} = 0 \quad \text{so} \quad v_o = -10 \text{ V}$$

P 5.2 $$\frac{v_\mathrm{b} - v_\mathrm{a}}{20} + \frac{v_\mathrm{b} - v_o}{100} = 0, \qquad \text{therefore} \quad v_o = 6v_\mathrm{b} - 5v_\mathrm{a}$$

[a] $v_\mathrm{a} = 4$ V, $\quad v_\mathrm{b} = 0$ V, $\quad v_o = -15$ V (sat)

[b] $v_\mathrm{a} = 2$ V, $\quad v_\mathrm{b} = 0$ V, $\quad v_o = -10$ V

[c] $v_\mathrm{a} = 2$ V, $\quad v_\mathrm{b} = 1$ V, $\quad v_o = -4$ V

[d] $v_\mathrm{a} = 1$ V, $\quad v_\mathrm{b} = 2$ V, $\quad v_o = 7$ V

[e] If $v_\mathrm{b} = 1.6$ V, $\quad v_o = 9.6 - 5v_\mathrm{a} = \pm 15$

$$\therefore \quad -1.08 \leq v_\mathrm{a} \leq 4.92 \text{ V}$$

P 5.3 $$v_o = -(0.5 \times 10^{-3})(10 \times 10^3) = -5 \text{ V}$$

$$\therefore \quad i_o = \frac{-5}{5000} = -1 \text{ mA}$$

P 5.4 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $2.2\,\text{M}\Omega$ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V. Therefore the voltmeter reads 7.7 V.

P 5.5 **[a]** $i_{\rm a} = \dfrac{25 \times 10^{-3}}{5000} = 5\,\mu\text{A}$

$v_{\rm a} = -50 \times 10^3 i_{\rm a} = -250\,\text{mV}$

[b] $\dfrac{v_{\rm a}}{50{,}000} + \dfrac{v_{\rm a}}{10{,}000} + \dfrac{v_{\rm a} - v_o}{40{,}000} = 0$

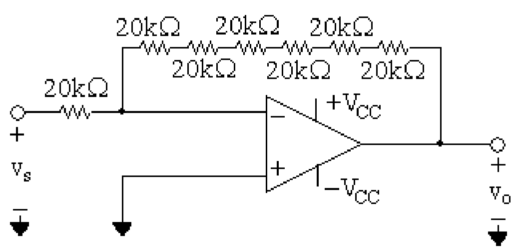
$\therefore \quad 4v_{\rm a} + 20v_{\rm a} + 5v_{\rm a} - 5v_o = 0$

$\therefore \quad v_o = 29v_{\rm a}/5 = -1.45\ \text{V}$

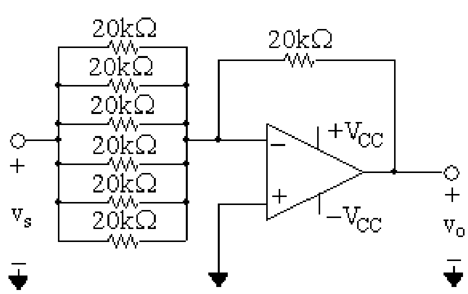
[c] $i_{\rm a} = 5\,\mu\text{A}$

[d] $i_o = \dfrac{-v_o}{30{,}000} + \dfrac{v_a - v_o}{40{,}000} = 78.33\,\mu\ \text{A}$

P 5.6 **[a]** The gain of an inverting amplifier is the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 6, the feedback resistor must be 6 times as large as the input resistor. There are many possible designs that use only 20 kΩ resistors. We present two here. Use a single 20 kΩ resistor as the input resistor, and use six 20 kΩ resistors in series as the feedback resistor to give a total of 120 kΩ.



Alternately, Use a single 20 kΩ resistor as the feedback resistor and use six 20 kΩ resistors in parallel as the input resistor to give a total of 3.33 kΩ.



[b] To amplify a 3 V signal without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain. Thus, the power supplies should have a magnitude of $(3)(6) = 18$ V.

P 5.7 **[a]** The circuit shown is a non-inverting amplifier.

[b] We assume the op amp to be ideal, so $v_n = v_p = 3$V. Write a KCL equation at v_n:

$$\frac{3}{40{,}000} + \frac{3 - v_o}{80{,}000} = 0$$

Solving,

$v_o = 9$ V.

P 5.8 $\quad v_p = \frac{18}{24}(12) = 9\ \text{V} = v_n$

$$\frac{v_n - 24}{30} + \frac{v_n - v_o}{20} = 0$$

$$v_o = (45 - 48)/3 = -1.0\ \text{V}$$

$$i_{\text{L}} = \frac{v_o}{5} \times 10^{-3} = -\frac{1}{5} \times 10^{-3} = -200 \times 10^{-6}$$

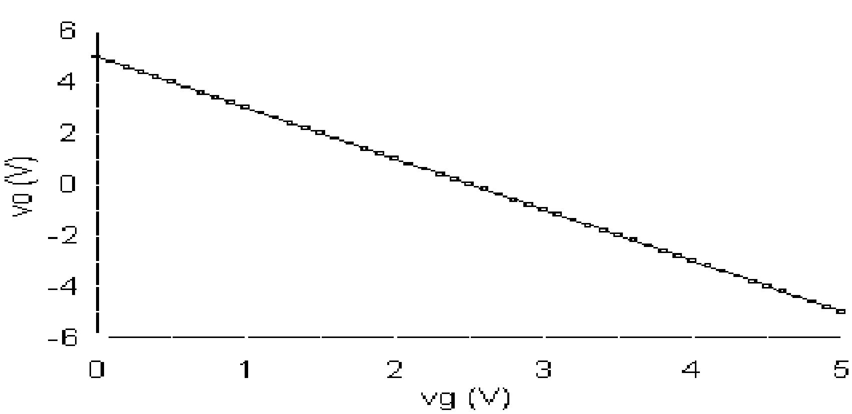
$$i_{\text{L}} = -200\,\mu\text{A}$$

P 5.9 **[a]** First, note that $\quad v_n = v_p = 2.5\ \text{V}$

Let v_{o1} equal the voltage output of the op-amp. Then

$$\frac{2.5 - v_g}{5000} + \frac{2.5 - v_{o1}}{10{,}000} = 0, \qquad \therefore\ \ v_{o1} = 7.5 - 2v_g$$

Also note that $\quad v_{o1} - 2.5 = v_o, \qquad \therefore\ \ v_o = 5 - 2v_g$



[b] Yes, the circuit designer is correct!

P 5.10 **[a]** Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{2000} + \frac{0 - v_\Delta}{50{,}000} = 0$$

$$-25v_g - v_\Delta = 0, \qquad \therefore \quad v_\Delta = -25(40 \times 10^{-3}) = -1 \text{ V}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{50{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 2v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 2 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore \quad v_o = -1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.2, \qquad v_o = -1(1 + 1.6 + 4) = -6.6 \text{ V}$

When $\alpha = 1, \qquad v_o = -1(1 + 0 + 0) = -1 \text{ V}$

$\therefore \quad -6.6 \text{ V} \le v_o \le -1 \text{ V}$

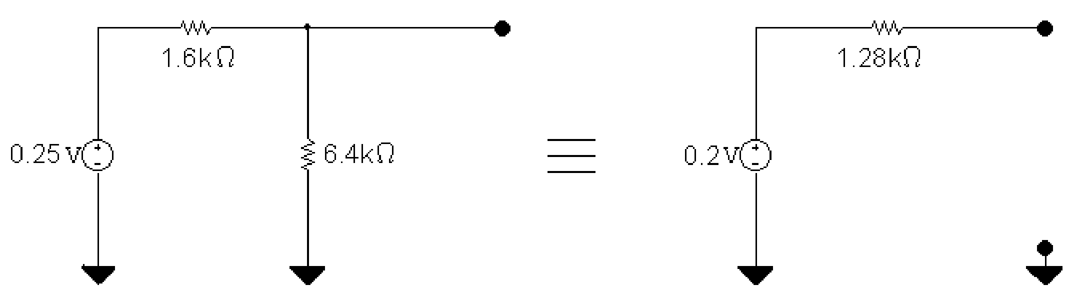
[b] $$-1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right] = -7$$

$$\alpha + 2\alpha(1-\alpha) + (1-\alpha) = 7\alpha$$

$$\alpha + 2\alpha - 2\alpha^2 + 1 - \alpha = 7\alpha$$

$\therefore \quad 2\alpha^2 + 5\alpha - 1 = 0 \quad$ so $\quad \alpha \cong 0.186$

P 5.11 **[a]** Replace the combination of v_g, $1.6\,\text{k}\Omega$, and the $6.4\,\text{k}\Omega$ resistors with its Thévenin equivalent.



Then $\quad v_o = \dfrac{-[12 + \sigma 50]}{1.28}(0.2)$

At saturation $\quad v_o = -5 \text{ V}; \qquad$ therefore

$$-\left(\frac{12 + \sigma 50}{1.28}\right)(0.2) = -5, \quad \text{or} \quad \sigma = 0.4$$

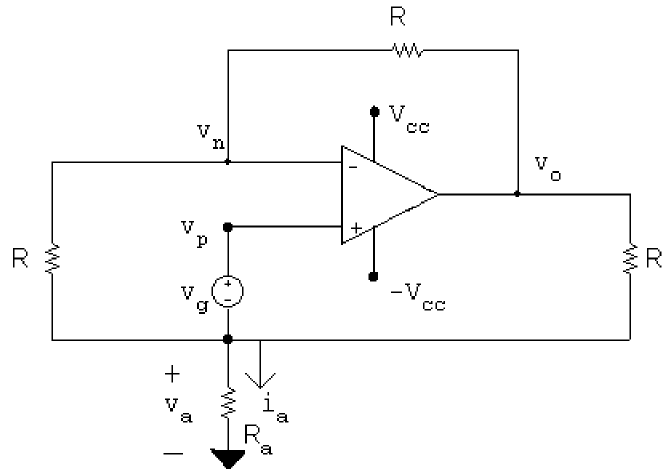
Thus for $0 \le \sigma < 0.40$ the operational amplifier will not saturate.

[b] When $\sigma = 0.272,\qquad v_o = \dfrac{-(12+13.6)}{1.28}(0.2) = -4\ \text{V}$

Also $\dfrac{v_o}{10} + \dfrac{v_o}{25.6} + i_o = 0$

$\therefore\quad i_o = -\dfrac{v_o}{10} - \dfrac{v_o}{25.6} = \dfrac{4}{10} + \dfrac{4}{25.6}\,\text{mA} = 556.25\,\mu\text{A}$

P 5.12 **[a]**



$$\frac{v_n - v_\text{a}}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_\text{a} = v_o$$

$$\frac{v_\text{a}}{R_\text{a}} + \frac{v_\text{a} - v_n}{R} + \frac{v_\text{a} - v_o}{R} = 0$$

$$v_\text{a}\left[\frac{1}{R_\text{a}} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_\text{a}\left(2 + \frac{R}{R_\text{a}}\right) - v_n = v_o$$

$$v_n = v_p = v_\text{a} + v_g$$

$$\therefore\quad 2v_n - v_\text{a} = 2v_\text{a} + 2v_g - v_\text{a} = v_\text{a} + 2v_g$$

$$\therefore\quad v_\text{a} - v_o = -2v_g \qquad (1)$$

$$2v_\text{a} + v_\text{a}\left(\frac{R}{R_\text{a}}\right) - v_\text{a} - v_g = v_o$$

$$\therefore\quad v_\text{a}\left(1 + \frac{R}{R_\text{a}}\right) - v_o = v_g \qquad (2)$$

Now combining equations (1) and (2) yields

$$-v_\text{a}\frac{R}{R_\text{a}} = -3v_g$$

or $v_a = 3v_g \dfrac{R_a}{R}$

Hence $i_a = \dfrac{v_a}{R_a} = \dfrac{3v_g}{R}$ Q.E.D.

[b] At saturation $V_o = \pm V_{cc}$

$\therefore \quad v_a = \pm V_{cc} - 2v_g \qquad (3)$

and

$\therefore \quad v_a\left(1 + \dfrac{R}{R_a}\right) = \pm V_{cc} + v_g \qquad (4)$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g}$$

$$\therefore \quad \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g} - 1 = \frac{3v_g}{\pm V_{cc} - 2v_g}$$

or $R_a = \dfrac{(\pm V_{cc} - 2v_g)}{3v_g} R$ Q.E.D.

P 5.13 **[a]** Assume the op-amp is operating within its linear range, then

$$i_L = \frac{8}{4000} = 2\,\text{mA}$$

For $R_L = 4\,\text{k}\Omega \qquad v_o = (4+4)(2) = 16$ V

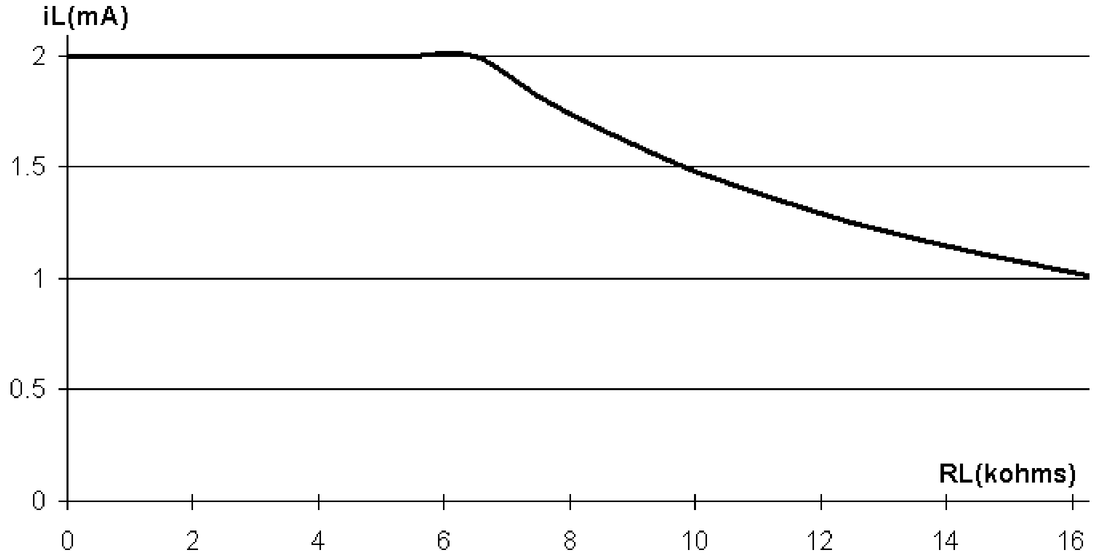
Now since $v_o < 20$ V our assumption of linear operation is correct, therefore

$i_L = 2\,\text{mA}$

[b] $20 = 2(4 + R_L); \qquad R_L = 6\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 6\,\text{k}\Omega$. Therefore when $R_L = 16\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 20/(4{,}0000 + 16{,}000) = 1\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 50 kΩ resistor is negligible, and the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i_p = i_n = (8-4)/(500 \times 10^3) = 8\,\mu\text{A}$. But $8\,\mu\text{A} \ll 1\,\text{mA}$, hence our assumption is reasonable.

[d]



P 5.14 **[a]** Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \qquad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V}$$

$$i_{\rm a} = \frac{v_{o2} - v_{o1}}{1000} = 5.7\,\text{mA}$$

[b] $i_{\rm a} = 0$ when $v_{o1} = v_{02}$ so from (a) $v_{o2} = 1$ V

Thus

$$\frac{-47}{10}(v_{\rm L}) = 1$$

$$v_{\rm L} = -\frac{10}{47} = -212.77 \text{ mV}$$

P 5.15 **[a]** $p_{600\Omega} = \dfrac{(60 \times 10^{-3})^2}{(600)} = 6\,\mu\text{W}$

[b] $v_{600\Omega} = \dfrac{600}{30{,}000}(60 \times 10^{-3}) = 1.2\,\text{mV}$

$$p_{600\Omega} = \frac{(1.2 \times 10^{-3})^2}{(600)} = 2.4\,\text{nW}$$

[c] $\dfrac{p_{\rm a}}{p_{\rm b}} = \dfrac{6 \times 10^{-6}}{2.4 \times 10^{-9}} = 2500$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control $2,500$ times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 5.16 **[a]** This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\frac{220}{33}v_\text{a} - \frac{220}{22}v_\text{b} - \frac{220}{80}v_\text{c} = -8 + 15 - 11 = -4\ \text{V}$

[c] $v_o = -19 - 10v_\text{b} = \pm 6$

$$\therefore\quad v_\text{b} = -1.3\ \text{V}\quad\text{when}\quad v_o = -6\ \text{V};$$

$$v_\text{b} = -2.5\ \text{V}\quad\text{when}\quad v_o = 6\ \text{V}$$

$$\therefore\quad -2.5\ \text{V} \leq v_\text{b} \leq -1.3\ \text{V}$$

P 5.17 **[a]** Write a KCL equation at the inverting input to the op amp:

$$\frac{v_\text{d} - v_\text{a}}{40{,}000} + \frac{v_\text{d} - v_\text{b}}{22{,}000} + \frac{v_\text{d} - v_\text{c}}{100{,}000} + \frac{v_\text{d}}{352{,}000} + \frac{v_\text{d} - v_o}{220{,}000} = 0$$

Multiply through by 220,000, plug in the values of input voltages, and rearrange to solve for v_o:

$$v_o = 220{,}000\left(\frac{4}{40{,}000} + \frac{-1}{22{,}000} + \frac{-5}{100{,}000}\right.$$
$$\left. + \frac{8}{352{,}000} + \frac{8}{220{,}000}\right) = 14\ \text{V}$$

[b] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{8 - v_\text{a}}{40{,}000} + \frac{8 - 9}{22{,}000} + \frac{8 - 13}{100{,}000} + \frac{8}{352{,}000} + \frac{8 - v_o}{220{,}000} = 0$$

Simplify and solve for v_o:

$$44 - 5.5v_\text{a} - 10 - 11 + 5 + 8 - v_o = 0\quad\text{so}\quad v_o = 36 - 5.5v_\text{a}$$

Set v_o to the positive power supply voltage and solve for v_a:

$$36 - 5.5v_\text{a} = 15\quad\therefore\quad v_\text{a} = 3.818\ \text{V}$$

Set v_o to the negative power supply voltage and solve for v_a:

$$36 - 5.5v_\text{a} = -15\quad\therefore\quad v_\text{a} = 9.273\ \text{V}$$

Therefore,

$3.818 \text{ V} \leq v_\text{a} \leq 9.273 \text{ V}$

P 5.18 **[a]** $\dfrac{8-4}{40{,}000} + \dfrac{8-9}{22{,}000} + \dfrac{8-13}{100{,}000} + \dfrac{8}{352{,}000} + \dfrac{8-v_0}{R_f} = 0$

$$\frac{8-v_o}{R_f} = -2.7272 \times 10^{-5} \quad \text{so} \quad R_f = \frac{8-v_o}{-2.727 \times 10^{-5}}$$

For $v_o = 15$ V, $R_f = 256.7\,\text{k}\Omega$

For $v_o = -15$ V, $R_f < 0$ so this solution is not possible.

[b] $i_o = -(i_f + i_{10\text{k}}) = -\left[\dfrac{15-8}{256.7 \times 10^3} + \dfrac{15}{10{,}000}\right] = -1.527$ mA

P 5.19 We want the following expression for the output voltage:

$$v_o = -(2v_a + 4v_b + 6v_c + 8v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage:

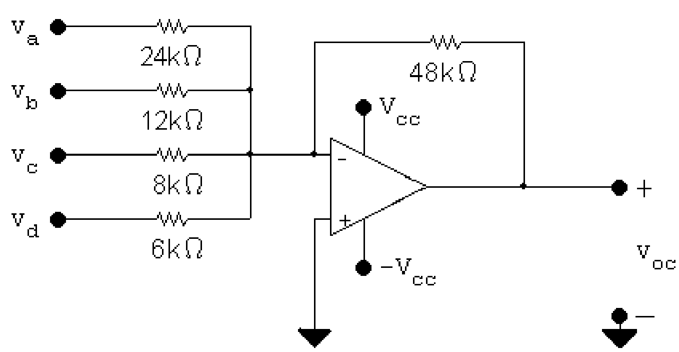
$$v_o = -\left[\frac{48}{R_\text{a}}v_\text{a} + \frac{48}{R_\text{b}}v_\text{b} + \frac{48}{R_\text{c}}v_\text{c} + \frac{48}{R_\text{d}}v_\text{d}\right]$$

Solve for each input resistance value to yield the desired gain:

$\therefore \quad R_\text{a} = 48{,}000/2 = 24\,\text{k}\Omega \qquad R_\text{c} = 48{,}000/6 = 8\,\text{k}\Omega$

$R_\text{b} = 48{,}000/4 = 12\,\text{k}\Omega \qquad R_\text{d} = 48{,}000/8 = 6\,\text{k}\Omega$

The final circuit is shown here:



P 5.20 **[a]** $v_p = v_s, \qquad v_n = \dfrac{R_1 v_o}{R_1 + R_2}, \qquad v_n = v_p$

Therefore $\quad v_o = \left(\dfrac{R_1 + R_2}{R_1}\right) v_s = \left(1 + \dfrac{R_2}{R_1}\right) v_s$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 5.21 $$v_o = -\left[\frac{R_\mathrm{f}}{3000}(0.15) + \frac{R_\mathrm{f}}{5000}(0.1) + \frac{R_\mathrm{f}}{25{,}000}(0.25)\right]$$

$$-6 = -8\times 10^{-5}R_\mathrm{f}; \qquad R_\mathrm{f} = 75\,\mathrm{k}\Omega; \qquad \therefore \quad 0 \le R_\mathrm{f} \le 75\,\mathrm{k}\Omega$$

P 5.22 **[a]** This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v_p:

$$v_p = \frac{10{,}000}{10{,}000 + 30{,}000}v_s = \frac{v_s}{4}$$

Write a KCL equation at $v_n = v_p = v_s/4$:

$$\frac{v_s/4}{4000} + \frac{v_s/4 - v_o}{28{,}000} = 0$$

Solving,

$$v_o = 7v_s/4 + v_s/4 = 2v_s$$

[c] $2v_s = 8$ so $v_s = 4$ V

$2v_s = -12$ so $v_s = -6$ V

Thus, -6 V $\le v_s \le 4$ V.

P 5.23 **[a]** $v_p = v_n = \dfrac{68{,}000}{80{,}000}v_g = 0.85v_g$

$$\therefore \quad \frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{63{,}000} = 0;$$

$$\therefore \quad v_o = 2.635v_g = 2.635(4), \quad v_o = 10.54 \text{ V}$$

[b] $v_o = 2.635v_g = \pm 12$

$$v_g = \pm 4.554 \text{ V}, \qquad -4.554 \le v_g \le 4.554 \text{ V}$$

[c] $$\frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{R_\mathrm{f}} = 0$$

$$\left(\frac{0.85R_\mathrm{f}}{30{,}000} + 0.85\right)v_g = v_o = \pm 12$$

$$\therefore \quad 1.7\times 10^{-3}R_\mathrm{f} + 51 = \pm 360; \quad 1.7\times 10^{-3}R_\mathrm{f} = 360 - 51; \quad R_\mathrm{f} = 181.76\,\mathrm{k}\Omega$$

P 5.24 **[a]** This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v_p and solve for v_p in terms of v_s:

$$\frac{v_p - v_s}{15{,}000} + \frac{v_p - 6}{30{,}000} = 0$$

$$2v_p - 2v_s + v_p - 6 = 0 \qquad \text{so} \qquad v_p = 2v_s/3 + 2$$

Now write a KCL equation at v_n and solve for v_o:

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{60{,}000} = 0 \qquad \text{so} \qquad v_o = 4v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$$v_o = 4(2v_s/3 + 2) = 8v_s/3 + 8$$

[c] $8v_s/3 + 8 = 16 \qquad \text{so} \qquad v_s = 3\ \text{V}$

$$8v_s/3 + 8 = -12 \qquad \text{so} \qquad v_s = -7.5\ \text{V}$$

Thus, $-7.5\ \text{V} \leq v_s \leq 3\ \text{V}$.

P 5.25 **[a]** The circuit is a non-inverting summing amplifier.

[b] $\dfrac{v_p - v_\text{a}}{3.3 \times 10^3} + \dfrac{v_p - v_\text{b}}{4.7 \times 10^3} = 0$

$$\therefore \quad v_p = 0.5875v_\text{a} + 0.4125v_\text{b}$$

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{100{,}000} = 0$$

$$\therefore \quad v_o = 11v_n = 11v_p = 6.4625v_\text{a} + 4.5375v_\text{b} = 8.03\ \text{V}$$

[c] $v_p = v_n = \dfrac{v_o}{11} = 730\ \text{mV}$

$$i_\text{a} = \frac{v_\text{a} - v_p}{3.3 \times 10^3} = -100\,\mu\text{A}$$

$$i_\text{b} = \frac{v_\text{b} - v_p}{4.7 \times 10^3} = 100\,\mu\text{A}$$

[d] 6.4625 for v_a

4.5375 for v_b

P 5.26 **[a]** $\dfrac{v_p - v_\text{a}}{R_\text{a}} + \dfrac{v_p - v_\text{b}}{R_\text{b}} + \dfrac{v_p - v_\text{c}}{R_\text{c}} + \dfrac{v_p}{R_g} = 0$

$$\therefore \quad v_p = \frac{R_\text{b}R_\text{c}R_g}{D}v_\text{a} + \frac{R_\text{a}R_\text{c}R_g}{D}v_\text{b} + \frac{R_\text{a}R_\text{b}R_g}{D}v_\text{c}$$

where $\quad D = R_\text{b}R_\text{c}R_g + R_\text{a}R_\text{c}R_g + R_\text{a}R_\text{b}R_g + R_\text{a}R_\text{b}R_\text{c}$

$$\frac{v_n}{R_s} + \frac{v_n - v_o}{R_\mathrm{f}} = 0$$

$$v_n\left(\frac{1}{R_s} + \frac{1}{R_\mathrm{f}}\right) = \frac{v_o}{R_\mathrm{f}}$$

$$\therefore \quad v_o = \left(1 + \frac{R_\mathrm{f}}{R_s}\right) v_n = kv_n$$

where $\quad k = \left(1 + \frac{R_\mathrm{f}}{R_s}\right)$

$$v_p = v_n$$

$$\therefore \quad v_o = kv_p$$

or

$$v_o = \frac{kR_gR_\mathrm{b}R_\mathrm{c}}{D}v_\mathrm{a} + \frac{kR_gR_\mathrm{a}R_\mathrm{c}}{D}v_\mathrm{b} + \frac{kR_gR_\mathrm{a}R_\mathrm{b}}{D}v_\mathrm{c}$$

$$\frac{kR_gR_\mathrm{b}R_\mathrm{c}}{D} = 3 \qquad \therefore \quad \frac{R_\mathrm{b}}{R_\mathrm{a}} = 1.5$$

$$\frac{kR_gR_\mathrm{a}R_\mathrm{c}}{D} = 2 \qquad \therefore \quad \frac{R_\mathrm{c}}{R_\mathrm{b}} = 2$$

$$\frac{kR_gR_\mathrm{a}R_\mathrm{b}}{D} = 1 \qquad \therefore \quad \frac{R_\mathrm{c}}{R_\mathrm{a}} = 3$$

Since $\quad R_\mathrm{a} = 2\,\mathrm{k}\Omega \qquad R_\mathrm{b} = 3\,\mathrm{k}\Omega \qquad R_\mathrm{c} = 6\,\mathrm{k}\Omega$

$$\therefore \quad D = [(3)(6)(4) + (2)(6)(4) + (2)(3)(4) + (2)(3)(6)] \times 10^9 = 180 \times 10^9$$

$$\frac{k(4)(3)(6) \times 10^9}{180 \times 10^9} = 3$$

$$k = \frac{540 \times 10^9}{72 \times 10^9} = 7.5$$

$$\therefore \quad 7.5 = 1 + \frac{R_\mathrm{f}}{R_s}$$

$$\frac{R_\mathrm{f}}{R_s} = 6.5$$

$$R_\mathrm{f} = (6.5)(12{,}000) = 78\,\mathrm{k}\Omega$$

[b] $v_o = 3(0.8) + 2(1.5) + 2.10 = 7.5\ \mathrm{V}$

$$v_n = v_p = \frac{7.5}{7.5} = 1.0\ \mathrm{V}$$

$$i_\mathrm{a} = \frac{0.8 - 1}{2000} = \frac{-0.2}{2000} = -0.1\,\mathrm{mA} = -100\,\mu\mathrm{A}$$

$$i_{\rm b} = \frac{1.5 - 1.0}{3000} = \frac{0.5}{3000} = 166.67\,\mu\text{A}$$

$$i_{\rm c} = \frac{2.10 - 1.0}{6000} = \frac{1.1}{6000} = 183.33\,\mu\text{A}$$

$$i_g = \frac{1}{4000} = 250\,\mu\text{A}$$

$$i_s = \frac{v_n}{12{,}000} = \frac{1}{12{,}000} = 83.33\,\mu\text{A}$$

P 5.27 **[a]** $\dfrac{v_p - v_{\rm a}}{R_{\rm a}} + \dfrac{v_p - v_{\rm b}}{R_{\rm b}} + \dfrac{v_p - v_{\rm c}}{R_{\rm c}} = 0$

$$\therefore \quad v_p = \frac{R_{\rm b}R_{\rm c}}{D}v_{\rm a} + \frac{R_{\rm a}R_{\rm c}}{D}v_{\rm b} + \frac{R_{\rm a}R_{\rm b}}{D}v_{\rm c}$$

where $\quad D = R_{\rm b}R_{\rm c} + R_{\rm a}R_{\rm c} + R_{\rm a}R_{\rm b}$

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{R_{\rm f}} = 0$$

$$\left(\frac{R_{\rm f}}{10{,}000} + 1\right)v_n = v_o$$

Let $\quad \dfrac{R_{\rm f}}{10{,}000} + 1 = k$

$$v_o = kv_n = kv_p$$

$$\therefore \quad v_o = \frac{kR_{\rm b}R_{\rm c}}{D}v_{\rm a} + \frac{kR_{\rm a}R_{\rm c}}{D}v_{\rm b} + \frac{kR_{\rm a}R_{\rm b}}{D}v_{\rm c}$$

$$\therefore \quad \frac{kR_{\rm b}R_{\rm c}}{D} = 5 \qquad\qquad \therefore \quad \frac{R_{\rm c}}{R_{\rm a}} = 5$$

$$\frac{kR_{\rm a}R_{\rm c}}{D} = 4$$

$$\frac{kR_{\rm a}R_{\rm b}}{D} = 1 \qquad\qquad \therefore \quad \frac{R_{\rm c}}{R_{\rm b}} = 4$$

$$\therefore \quad R_{\rm c} = 5R_{\rm a} = 5\,\text{k}\Omega$$

$$R_{\rm b} = R_{\rm c}/4 = 1.25\,\text{k}\Omega$$

$$\therefore \quad D = (1.25)(5) + (1)(5) + (1.25)(1) = 12.5 \times 10^6$$

$$\therefore \quad k = \frac{5D}{R_{\rm b}R_{\rm c}} = \frac{(5)(12.5) \times 10^6}{(1.25)(5) \times 10^6} = 10$$

$$\therefore \quad \frac{R_{\rm f}}{10{,}000} + 1 = 10, \qquad R_{\rm f} = 90\,\text{k}\Omega$$

[b] $v_o = 5(0.5) + 4(1) + 1.5 = 8\ \text{V}$

$$v_n = v_o/10 = 0.8\ \text{V} = v_p$$

$$i_\text{a} = \frac{v_\text{a} - v_p}{1000} = \frac{0.5 - 0.8}{1000} = -300\,\mu\text{A}$$

$$i_\text{b} = \frac{v_\text{b} - v_p}{1250} = \frac{1 - 0.8}{1250} = 160\,\mu\text{A}$$

$$i_\text{c} = \frac{v_\text{c} - v_p}{5000} = \frac{1.5 - 0.8}{5000} = 140\,\mu\text{A}$$

P 5.28 **[a]** Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_\text{a}}{R_\text{a}} + \frac{0 - v_o'}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

Therefore

$$\frac{v_o'}{R_\text{b}} = -\frac{v_\text{a}}{R_\text{a}}, \qquad v_o' = -\frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_\text{b} R_\text{d}}{R_\text{c} + R_\text{d}}$$

$$\frac{v_n}{R_\text{a}} + \frac{v_n - v_o''}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

$$\left(\frac{1}{R_\text{a}} + \frac{1}{R_\text{b}}\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{v_o''}{R_\text{b}} = 0$$

$$v_o'' = \left(\frac{R_\text{b}}{R_\text{a}} + 1\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b}$$

$$v_o = v_o' + v_o'' = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

[b] $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) = \dfrac{R_\text{b}}{R_\text{a}}$, therefore $R_\text{d}(R_\text{a} + R_\text{b}) = R_\text{b}(R_\text{c} + R_\text{d})$

$R_\text{d} R_\text{a} = R_\text{b} R_\text{c}$, therefore $\dfrac{R_\text{a}}{R_\text{b}} = \dfrac{R_\text{c}}{R_\text{d}}$

When $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) = \dfrac{R_\text{b}}{R_\text{a}}$

Eq. (5.22) reduces to $v_o = \dfrac{R_\text{b}}{R_\text{a}} v_\text{b} - \dfrac{R_\text{b}}{R_\text{a}} v_\text{a} = \dfrac{R_\text{b}}{R_\text{a}}(v_\text{b} - v_\text{a})$.

P 5.29 Use voltage division to find v_p:

$$v_p = \frac{2000}{2000 + 8000}(5) = 1 \text{ V}$$

Write a KCL equation at v_n and solve it for v_o:

$$\frac{v_n - v_a}{5000} + \frac{v_n - v_o}{R_f} = 0 \qquad \text{so} \qquad \left(\frac{R_f}{5000} + 1\right) v_n - \frac{R_f}{5000} v_a = v_o$$

Since the op amp is ideal, $v_n = v_p = 1$V, so

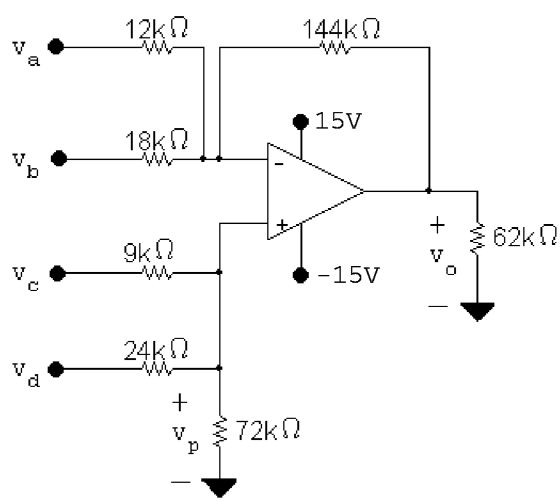
$$v_o = \left(\frac{R_f}{5000} + 1\right) - \frac{R_f}{5000} v_a$$

To satisfy the equation,

$$\left(\frac{R_f}{5000} + 1\right) = 5 \qquad \text{and} \qquad \frac{R_f}{5000} = 4$$

Thus, $R_f = 20$ kΩ.

P 5.30 **[a]**



$$\frac{v_p}{72{,}000} + \frac{v_p - v_c}{9{,}000} + \frac{v_p - v_d}{24{,}000} = 0$$

$$\therefore \quad v_p = (2/3)v_c + 0.25v_d = v_n$$

$$\frac{v_n - v_a}{12{,}000} + \frac{v_n - v_b}{18{,}000} + \frac{v_n - v_o}{144{,}000} = 0$$

$$\begin{aligned} \therefore \quad v_o &= 21v_n - 12v_a - 8v_b \\ &= 21[(2/3)v_c + 0.25v_d] - 12v_a - 8v_b \\ &= 21(0.4 + 0.2) - 12(0.5) - 8(0.3) = 4.2 \text{ V} \end{aligned}$$

[b] $v_o = 14v_c + 4.2 - 6 - 2.4$

$\pm 15 = 14v_c - 4.2$

$\therefore \quad 14v_c = \pm 15 + 4.2$

$\therefore \quad v_c = 1.371\ \text{V}$ and $v_c = -0.771\ \text{V}$

$\therefore \quad -771 \leq v_c \leq 1371\,\text{mV}$

P 5.31 **[a]** $v_o = \dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \dfrac{R_b}{R_a}v_a = \dfrac{47(110)}{10(80)}(0.80) - 10(0.67)$

$v_o = 5.17 - 6.70 = -1.53\ \text{V}$

[b] $v_n = v_p = \dfrac{(800)(47)}{80} = 470\,\text{mV}$

$i_a = \dfrac{(670 - 470)10^{-3}}{10 \times 10^3} = 20\,\mu\text{A}$

$R_a = \dfrac{v_a}{i_a} = \dfrac{670 \times 10^{-3}}{20 \times 10^{-6}} = 33.5\,\text{k}\Omega$

[c] $R_{\text{in b}} = R_c + R_d = 80\,\text{k}\Omega$

P 5.32 $v_p = \dfrac{v_b R_b}{R_a + R_b} = v_n$

$$\frac{v_n - v_a}{4700} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left(\frac{R_f}{4700} + 1\right) - \frac{v_a R_f}{4700} = v_o$$

$$\therefore \quad \left(\frac{R_f}{4700} + 1\right)\frac{R_b}{R_a + R_b}v_b - \frac{R_f}{4700}v_a = v_o$$

$$\therefore \quad \frac{R_f}{4700} = 10; \qquad R_f = 47\,\text{k}\Omega$$

$$\therefore \quad \frac{R_f}{4700} + 1 = 11$$

$$\therefore \quad 11\left(\frac{R_b}{R_a + R_b}\right) = 10$$

$$11R_b = 10R_b + 10R_a \qquad R_b = 10R_a$$

$$R_\mathrm{a} + R_\mathrm{b} = 220\,\mathrm{k}\Omega$$

$$11R_\mathrm{a} = 220\,\mathrm{k}\Omega$$

$$R_\mathrm{a} = 20\,\mathrm{k}\Omega$$

$$R_\mathrm{b} = 220 - 20 = 200\,\mathrm{k}\Omega$$

P 5.33 $v_p = v_n = R_\mathrm{b}i_\mathrm{b}$

$$\frac{R_\mathrm{b}i_\mathrm{b} - 3000i_\mathrm{a}}{3000} + \frac{R_\mathrm{b}i_\mathrm{b} - v_o}{R_\mathrm{f}} = 0$$

$$\left(\frac{R_\mathrm{b}}{3000} + \frac{R_\mathrm{b}}{R_\mathrm{f}}\right) i_\mathrm{b} - i_\mathrm{a} = \frac{v_o}{R_\mathrm{f}}$$

$$v_o = \left[\frac{R_\mathrm{b}R_\mathrm{f}}{3000} + R_\mathrm{b}\right] i_\mathrm{b} - R_\mathrm{f}i_\mathrm{a}$$

$$\therefore \quad R_\mathrm{f} = 2000\,\Omega$$

$$(2/3)R_\mathrm{b} + R_\mathrm{b} = 2000$$

$$\therefore \quad R_\mathrm{b} = 1200\,\Omega$$

P 5.34 $v_o = \dfrac{R_\mathrm{d}(R_\mathrm{a} + R_\mathrm{b})}{R_\mathrm{a}(R_\mathrm{c} + R_\mathrm{d})}v_\mathrm{b} - \dfrac{R_\mathrm{b}}{R_\mathrm{a}}v_\mathrm{a}$

By hypothesis: $R_\mathrm{b}/R_\mathrm{a} = 4; \quad R_\mathrm{c} + R_\mathrm{d} = 470\,\mathrm{k}\Omega; \quad \dfrac{R_\mathrm{d}(R_\mathrm{a} + R_\mathrm{b})}{R_\mathrm{a}(R_\mathrm{c} + R_\mathrm{d})} = 3$

$$\therefore \quad \frac{R_\mathrm{d}}{R_\mathrm{a}}\frac{(R_\mathrm{a} + 4R_\mathrm{a})}{470{,}000} = 3 \quad \text{so} \quad R_\mathrm{d} = 282\,\mathrm{k}\Omega; \quad R_\mathrm{c} = 188\,\mathrm{k}\Omega$$

Also, when $v_o = 0$ we have

$$\frac{v_n - v_\mathrm{a}}{R_\mathrm{a}} + \frac{v_n}{R_\mathrm{b}} = 0$$

$$\therefore \quad v_n\left(1 + \frac{R_\mathrm{a}}{R_\mathrm{b}}\right) = v_\mathrm{a}; \qquad v_n = 0.8v_\mathrm{a}$$

$$i_\mathrm{a} = \frac{v_\mathrm{a} - 0.8v_\mathrm{a}}{R_\mathrm{a}} = 0.2\frac{v_\mathrm{a}}{R_\mathrm{a}}; \qquad R_\mathrm{in} = \frac{v_\mathrm{a}}{i_\mathrm{a}} = 5R_\mathrm{a} = 22\,\mathrm{k}\Omega$$

$$\therefore \quad R_\mathrm{a} = 4.4\,\mathrm{k}\Omega; \qquad R_\mathrm{b} = 17.6\,\mathrm{k}\Omega$$

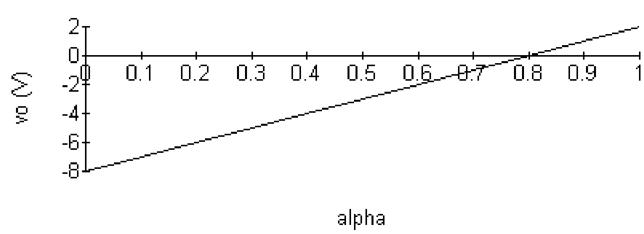
P 5.35 **[a]** $v_n = v_p = \alpha v_g$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_\mathrm{f}} = 0$$

$$(v_n - v_g)\frac{R_\mathrm{f}}{R_1} + v_n - v_o = 0$$

$$\begin{aligned} v_o &= (\alpha v_g - v_g)4 + \alpha v_g \\ &= [(\alpha - 1)4 + \alpha]v_g \\ &= (5\alpha - 4)v_g \\ &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	−8 V	0.4	−4 V	0.8	0 V
0.1	−7 V	0.5	−3 V	0.9	1 V
0.2	−6 V	0.6	−2 V	1.0	2 V
0.3	−5 V	0.7	−1 V		



[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g \alpha + - \left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope } = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad \text{intercept } = -\left(\frac{R_f}{R_1}\right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \qquad \frac{R_f}{R_1} = 2$$

P 5.36 $v_p = \dfrac{1500}{9000}(-18) = -3 \text{ V} = v_n$

$$\frac{18 - 3}{1600} + \frac{-3 - v_o}{R_\mathrm{f}} = 0$$

$$\therefore \quad v_o = \frac{15}{1600}R_\text{f} - 3$$

$v_o = 9$ V; $\qquad R_\text{f} = 1280\,\Omega$

$v_o = -9$ V; $\qquad R_\text{f} = -640\,\Omega$

But $\quad R_\text{f} \geq 0, \qquad \therefore \quad R_\text{f} = 1280\,\Omega$

P 5.37 **[a]** $A_\text{dm} = \dfrac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[b] $A_\text{cm} = \dfrac{(1)(24) - 25(1)}{1(25)} = -0.04$

[c] CMRR $= \left|\dfrac{24.98}{0.04}\right| = 624.50$

P 5.38 $A_\text{cm} = \dfrac{(20)(50) - (50)R_x}{20(50 + R_x)}$

$$A_\text{dm} = \frac{50(20 + 50) + 50(50 + R_x)}{2(20)(50 + R_x)}$$

$$\frac{A_\text{dm}}{A_\text{cm}} = \frac{R_x + 120}{2(20 - R_x)}$$

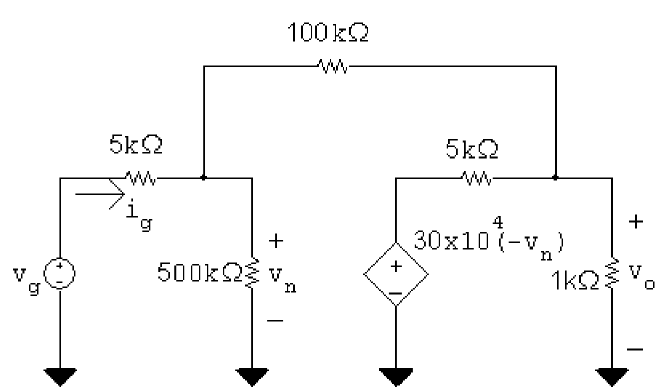
$\therefore \quad \dfrac{R_x + 120}{2(20 - R_x)} = \pm 1000$ for the limits on the value of R_x

If we use $+1000 \quad R_x = 19.93\,\text{k}\Omega$

If we use $-1000 \quad R_x = 20.07\,\text{k}\Omega$

$19.93\,\text{k}\Omega \leq R_x \leq 20.07\,\text{k}\Omega$

P 5.39 **[a]** Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} + \frac{v_n}{500{,}000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100{,}000} + \frac{v_o}{1000} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6{\times}10^6 - 1)v_n + 121v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$$v_o = -19.9915 \text{ V}; \qquad v_n = 403.2\,\mu\text{V}$$

[b] From the solution in part (a), $v_n = 403.2\,\mu$V.

[c] $i_g = \dfrac{v_g - v_n}{5000} = \dfrac{v_g - 403.2 \times 10^{-6} v_g}{5000}$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 403.2 \times 10^{-6}} = 5002.02\,\Omega$$

[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite. Therefore,

$$v_n = v_p = 0 \text{ V}; \qquad R_g = 5000\,\Omega$$

P 5.40 Note – the load resistor should have the value $4\,\text{k}\Omega$.

[a] Replace the op amp with the model shown in Fig. 5.15. The node voltage equation at the inverting input:

$$\frac{v_n}{40{,}000} + \frac{v_n - v_g}{500{,}000} + \frac{v_n - v_o}{80{,}000} = 0$$

Simplify:

$$12.5v_n + v_n - v_g + 6.25v_n - 6.25v_o = 0$$

The node voltage equation at the op amp output:

$$\frac{v_o}{4000} + \frac{v_o - 20{,}000(v_p - v_n)}{5000} + \frac{v_o - v_n}{80{,}000} = 0$$

Simplify:

$$20v_o + 16v_o - 320{,}000(v_p - v_n) + v_o - v_n = 0$$

From the input,

$$v_p - v_n = 0.8(v_g - v_n)$$

Substituting into the equation written at the output,

$$20v_o + 16v_o - 256{,}000(v_g - v_n) + v_o - v_n = 0$$

Now let $v_g = 1$ V; plug this value into both the input and output equations and simplify into two simultaneous equations:

$$19.75v_n - 6.25v_o = 1$$

$$255{,}999v_n + 37v_o = 256{,}000$$

These equations are in standard form, so solve them to yield
$v_o = 2.9986$ V; $\quad v_n = 999.571$ mV
Thus,

$$\frac{v_o}{v_g} = \frac{2.9986}{1} = 2.9986$$

[b] From part (a), $v_n = 999.571$ mV. Use this value to solve for v_p:

$$v_p = 0.8(1 - v_n) + v_n = 999.914 \text{ mV}$$

[c] $v_p - v_n = 343.6\,\mu$ V

[d] $i_g = \dfrac{v_g - v_p}{100{,}000} = \dfrac{1 - 999.914 \times 10^{-3}}{100{,}000} = 859$ pA

[e] For an ideal op amp, $v_n = v_p = v_g$, so the KVL equation at the inverting node is

$$\frac{v_o}{40{,}000} + \frac{v_g - v_o}{80{,}000} = 0$$

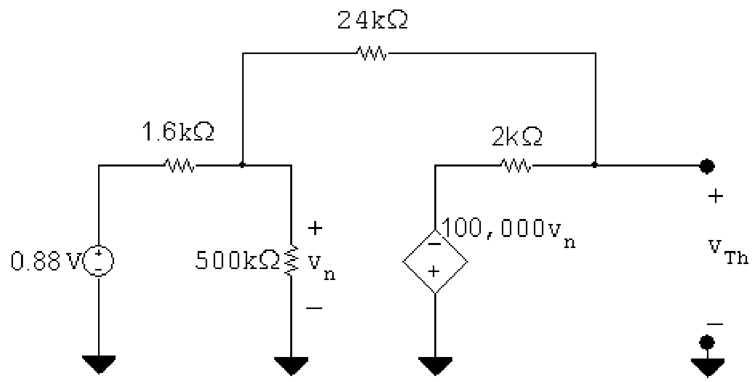
Then,

$$v_o = 3v_g \quad \text{so} \quad \frac{v_o}{v_g} = 3$$

Also,

$$v_n = v_p = 1 \text{ V}; \quad v_p - v_n = 0 \text{ V}; \quad i_g = 0 \text{ A}$$

P 5.41 **[a]**

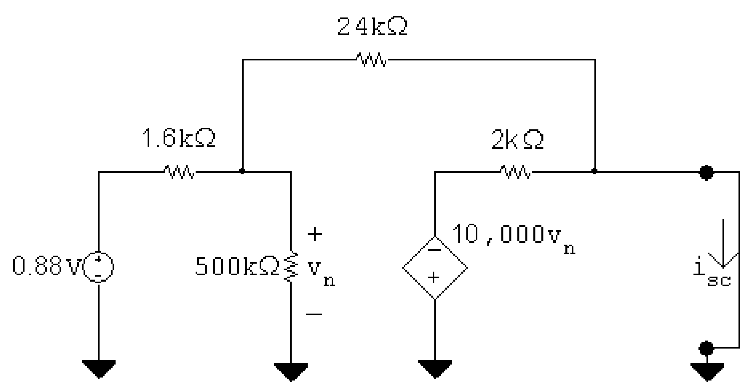


$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n - v_{\mathrm{Th}}}{24{,}000} = 0$$

$$\frac{v_{\mathrm{Th}} + 10^5 v_n}{2000} + \frac{v_{\mathrm{Th}} - v_n}{24{,}000} = 0$$

Solving, $v_{\mathrm{Th}} = -13.198 \text{ V}$

Short-circuit current calculation:

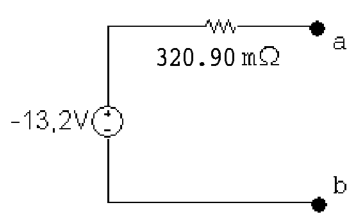


$$\frac{v_n}{500{,}000} + \frac{v_n - 0.88}{1600} + \frac{v_n - 0}{24{,}000} = 0$$

$$\therefore \quad v_n = 0.823 \text{ V}$$

$$i_{\mathrm{sc}} = \frac{v_n}{24{,}000} - \frac{10^5}{2000} v_n = -41.13 \text{ A}$$

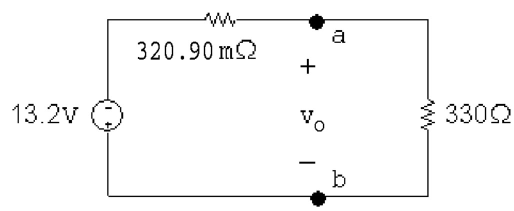
$$R_{\mathrm{Th}} = \frac{v_{\mathrm{Th}}}{i_{\mathrm{sc}}} = 320.90\,\mathrm{m}\Omega$$



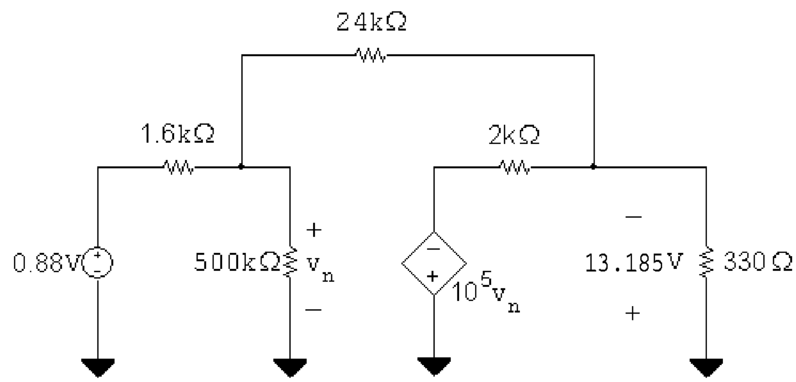
[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{\text{Th}} = 320.90\,\text{m}\Omega$$

[c]



$$v_o = \left(\frac{330}{330.32}\right)(-13.198) = -13.185 \text{ V}$$



$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n + 13.185}{24{,}000} = 0$$

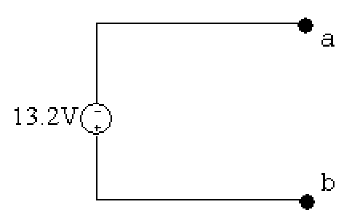
$$\therefore \quad v_n = 941.92\,\mu\text{V}$$

$$i_g = \frac{0.88 - 941.92 \times 10^{-6}}{1600} = 549.41\,\mu\text{A}$$

$$R_g = \frac{0.88}{0.88 - 941.92 \times 10^{-6}}(1600) = 1601.7\,\Omega$$

P 5.42 **[a]** $v_{\text{Th}} = \dfrac{-24}{1.6}(0.88) = -13.2 \text{ V}$

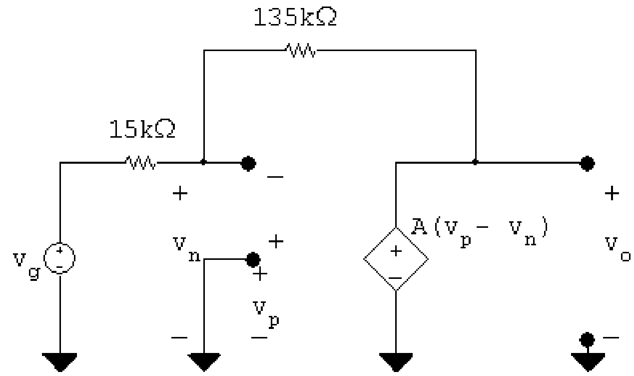
$$R_{\text{Th}} = 0, \text{ since op-amp is ideal}$$



[b] $R_o = R_{\text{Th}} = 0\,\Omega$

[c] $R_g = 1.6\,\text{k}\Omega \quad \text{since} \quad v_n = 0$

P 5.43 **[a]**



$$\frac{v_n - v_g}{15{,}000} + \frac{v_n - v_o}{135{,}000} = 0$$

$$\therefore \quad v_o = 10v_n - 9v_g$$

Also $\quad v_o = A(v_p - v_n) = -Av_n$

$$\therefore \quad v_n = \frac{-v_o}{A}$$

$$\therefore \quad v_o\left(1 + \frac{10}{A}\right) = -9v_g$$

$$v_o = \frac{-9A}{(10 + A)}v_g$$

[b] $v_o = \dfrac{-9(90)(0.4)}{(10 + 90)} = -3.24\ \text{V}$

[c] $v_o = -9(0.4) = -3.60\ \text{V}$

[d] $-3.42 = \dfrac{-9(0.4)A}{10 + A}$

$$\therefore \quad A = 190$$

P 5.44 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R + \Delta R} = v_n\left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f}\right) - \frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R + \Delta R} = \frac{v_{\text{ref}}\left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f}\right)}{(R - \Delta R)\left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f}\right)} - \frac{v_o}{R_f}$$

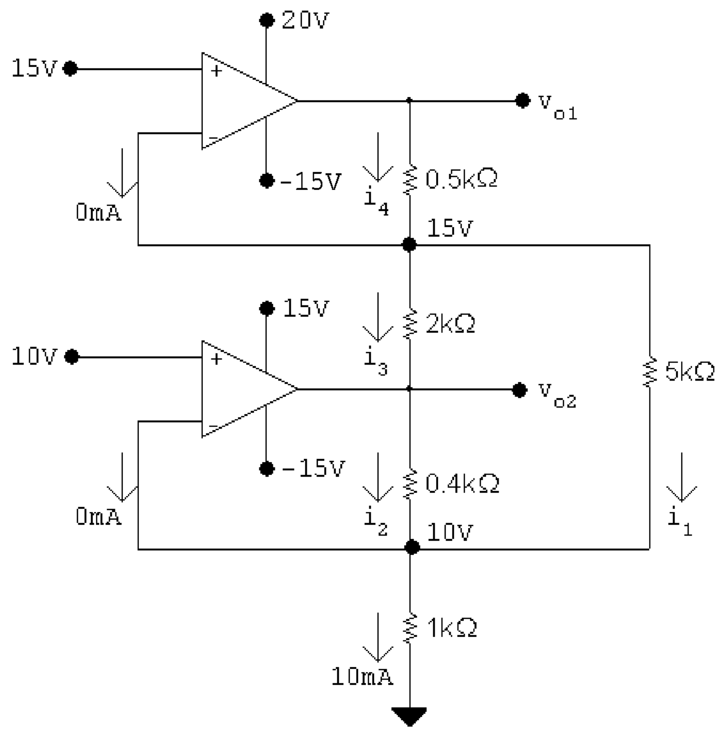
Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}}\left(\frac{1}{R-\Delta R} - \frac{1}{R+\Delta R}\right)$$

Thus,

$$v_o = v_{\text{ref}}\left(\frac{2\Delta R}{R^2 - \Delta R^2}\right)R_f$$

P 5.45



$$i_1 = \frac{15-10}{5000} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 10\,\text{mA}; \qquad i_2 = 9\,\text{mA}$$

$$v_{o2} = 10 + (400)(9)\times 10^{-3} = 13.6\ \text{V}$$

$$i_3 = \frac{15-13.6}{2000} = 0.7\,\text{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\text{mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85\ \text{V}$$

P 5.46 $v_p = \dfrac{5.6}{8.0}v_g = 0.7v_g = 7\sin(\pi/3)t\ \text{V}$

$$\frac{v_n}{15{,}000} + \frac{v_n - v_o}{75{,}000} = 0$$

$$6v_n = v_o; \qquad v_n = v_p$$

$$\therefore \quad v_o = 42\sin(\pi/3)t\ \text{V} \qquad 0 \le t \le \infty$$

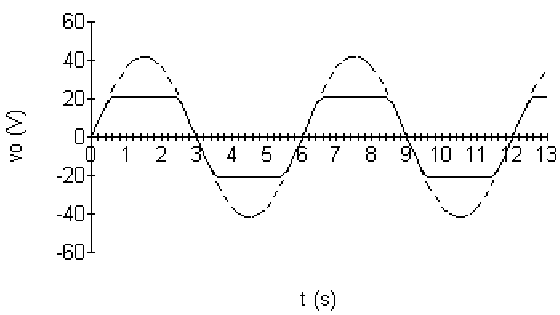
$$v_o = 0 \qquad t \le 0$$

At saturation

$$42\sin\left(\frac{\pi}{3}\right)t = \pm 21; \qquad \sin\frac{\pi}{3}t = \pm 0.5$$

$$\therefore \quad \frac{\pi}{3}t = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}, \quad \text{etc.}$$

$$t = 0.50\,\text{s}, \quad 2.50\,\text{s}, \quad 3.50\,\text{s}, \quad 5.50\,\text{s}, \quad \text{etc.}$$



P 5.47 It follows directly from the circuit that $v_o = -16v_g$
From the plot of v_g we have $\quad v_g = 0, \quad t < 0$

$$v_g = t \qquad 0 \le t \le 0.5$$

$$v_g = -t + 1 \qquad 0.5 \le t \le 1.5$$

$$v_g = t - 2 \qquad 1.5 \le t \le 2.5$$

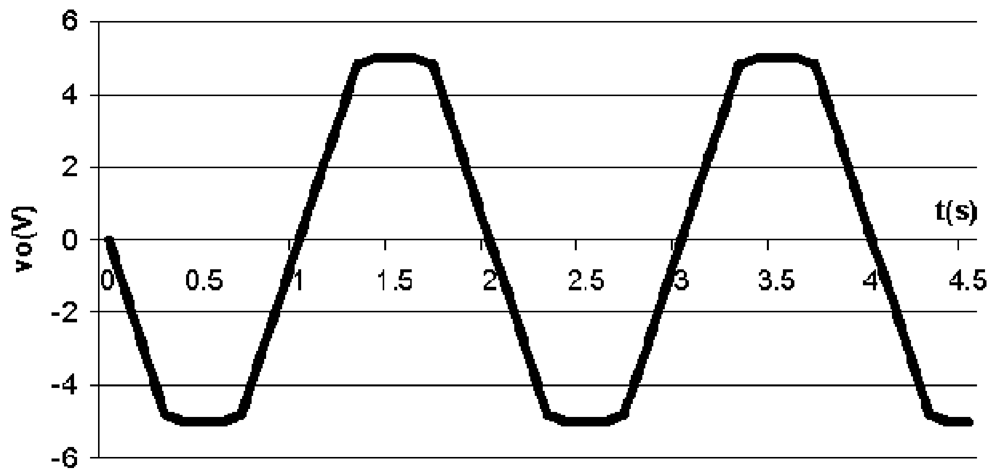
$$v_g = -t + 3 \qquad 2.5 \le t \le 3.5$$

$$v_g = t - 4 \qquad 3.5 \le t \le 4.5, \quad \text{etc.}$$

Therefore

$$
\begin{aligned}
v_o &= -16t & 0 \le t \le 0.5 \\
v_o &= 16t - 16 & 0.5 \le t \le 1.5 \\
v_o &= -16t + 32 & 1.5 \le t \le 2.5 \\
v_o &= 16t - 48 & 2.5 \le t \le 3.5 \\
v_o &= -16t + 64 & 3.5 \le t \le 4.5, \quad \text{etc.}
\end{aligned}
$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 5, the output is clipped at ± 5. The plot is shown below.



P 5.48 **[a]** Use Eq. 5.61 to solve for R_f; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2\,\text{k}\Omega$$

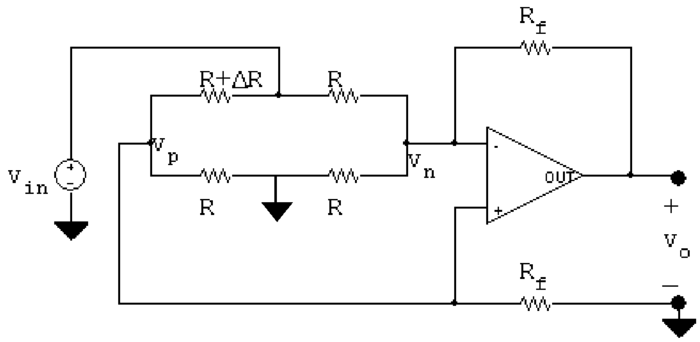
[b] Now solve for Δ given $v_o = 50$ mV:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12\ \text{m}\Omega$$

P 5.49 **[a]**



Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\rm in}}{R_1} = 0$$

$$\therefore \quad v_p\left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1}\right] = \frac{v_{\rm in}}{R_1}$$

$$\therefore \quad v_p = \frac{RR_f v_{\rm in}}{RR_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\rm in}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_f}\right] - \frac{v_o}{R_f} = \frac{v_{\rm in}}{R}$$

$$\therefore \quad v_n\left[\frac{R + 2R_f}{RR_f}\right] - \frac{v_{\rm in}}{R} = \frac{v_o}{R_f}$$

$$\therefore \quad \frac{v_o}{R_f} = \left[\frac{R + 2R_f}{RR_f}\right]\frac{RR_f v_{\rm in}}{[RR_1 + R_f R_1 + R_f R]} - \frac{v_{\rm in}}{R}$$

$$\therefore \quad \frac{v_o}{R_f} = \left[\frac{R + 2R_f}{RR_1 + R_f R_1 + R_f R} - \frac{1}{R}\right] v_{\rm in}$$

$$\therefore \quad v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f}{R[R_1(R + R_f) + RR_f]} v_{\rm in}$$

Now substitute $R_1 = R + \Delta R$ and get

$$v_o = \frac{-\Delta R(R + R_f)R_f v_{\rm in}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

If $\Delta R \ll R$

$$v_o \approx \frac{(R + R_f)R_f(-\Delta R)v_{\rm in}}{R^2(R + 2R_f)}$$

[b] $v_o \approx \dfrac{47 \times 10^4(48 \times 10^4)(-95)15}{10^8(95 \times 10^4)} \approx -3.384 \text{ V}$

[c] $v_o = \dfrac{-95(48 \times 10^4)(47 \times 10^4)15}{10^4[(1.0095)10^4(48 \times 10^4) + 47 \times 10^8]} = -3.368 \text{ V}$

P 5.50 **[a]** $v_o \approx \dfrac{(R + R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R + 2R_f)}$

$$v_o = \frac{(R + R_f)(-\Delta R)R_f v_{\text{in}}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R + \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \quad \text{Error} = \frac{R[(R + \Delta R)(R + R_f) + RR_f] - R^2(R + 2R_f)}{R^2(R + 2R_f)}$$

$$= \frac{\Delta R}{R}\frac{(R + R_f)}{(R + 2R_f)}$$

$$\therefore \quad \%\text{ error} = \frac{\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

[b] $\%\text{ error} = \dfrac{95(48 \times 10^4) \times 100}{10^4(95 \times 10^4)} = 0.48\%$

P 5.51 $1 = \dfrac{\Delta R(48 \times 10^4)}{10^4(95 \times 10^4)} \times 100$

$$\therefore \quad \Delta R = \frac{9500}{48} = 197.91667\,\Omega$$

$$\therefore \quad \%\text{ change in } R = \frac{197.19667}{10^4} \times 100 \approx 1.98\%$$

P 5.52 **[a]** It follows directly from the solution to Problem 5.49 that

$$v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f v_{\text{in}}}{R[R_1(R + R_f) + RR_f]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v_o = \frac{(R + R_f)R_f(\Delta R)v_{\text{in}}}{R[(R - \Delta R)(R + R_f) + RR_f]}$$

Now let $\Delta R \ll R$ and get

$$v_o \approx \frac{(R + R_f)R_f\Delta R v_{\text{in}}}{R^2(R + 2R_f)}$$

[b] It follows directly from the solution to Problem 5.49 that

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R-\Delta R)(R+R_f)+RR_f]}{R^2(R+2R_f)}$$

$$\therefore \quad \text{Error} = \frac{(R-\Delta R)(R+R_f)+RR_f-R(R+2R_f)}{R(R+2R_f)}$$

$$= \frac{-\Delta R(R+R_f)}{R(R+2R_f)}$$

$$\therefore \quad \%\ \text{error} = \frac{-\Delta R(R+R_f)}{R(R+2R_f)} \times 100$$

[c] $R-\Delta R = 9810\,\Omega \qquad \therefore \quad \Delta R = 10{,}000 - 9810 = 190\,\Omega$

$$\therefore \quad v_o \approx \frac{(48\times 10^4)(47\times 10^4)(190)(15)}{10^8(95\times 10^4)} \approx 6.768\ \text{V}$$

[d] $\%\ \text{error} = \dfrac{-190(48\times 10^4)(100)}{10^4(95\times 10^4)} = -0.96\%$