6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 **[a]** $i_g = 8e^{-300t} - 8e^{-1200t}\,\mathrm{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\,\mathrm{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\mathrm{V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54\,\mathrm{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\mathrm{W}$

[d] $\dfrac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.45, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\mathrm{s}$$

$$x = 11.05, \qquad t = \frac{\ln 11.05}{900} = 2.67\,\mathrm{ms}$$

p is maximum at $t = 411.05\,\mu\mathrm{s}$

[e] $p_{\max} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\mathrm{W}$

[f] $i_{\max} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\mathrm{A}$

$$w_{\max} = (1/2)(4\times 10^{-3})(3.78)^2 = 28.6\,\mathrm{mJ}$$

[g] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\mathrm{ms}$.

AP 6.2 **[a]** $i = C\frac{dv}{dt} = 24 \times 10^{-6}\frac{d}{dt}[e^{-15{,}000t}\sin 30{,}000t]$

$$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15{,}000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$p = vi = -649.23\,\text{mW}$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 **[a]** $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$$= \frac{1}{0.6 \times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$$

[c] $w_{(\text{max})} = \left(\frac{1}{2}\right)Cv_{\text{max}}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 **[a]** $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$

$i_1 + i_2 = i$

AP 6.5 $v_1 = 0.5 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$v_2 = 0.125 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$

$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$

$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$

AP 6.6 **[a]** Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2 + i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i_g + 8\frac{di_g}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2 + i_g)}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di_g}{dt}$$

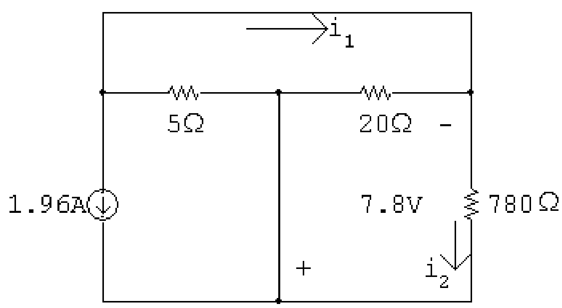
[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to



$$\therefore \quad i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

Also, $\quad \frac{di_g}{dt} = 7.84e^{-4t}$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

$$8\frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8\frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t}$$
$$+0.20 + 19.80e^{-4t} - 20e^{-5t} \stackrel{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5t}$$
$$+(185.60 - 290 + 31.68 + 19.80)e^{-4t} \stackrel{?}{=} -(9.8 + 52.92e^{-4t})$$

$$-9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \stackrel{?}{=} -9.8 - 52.92e^{-4t}$$

$$-9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16\frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16\frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t}$$
$$-8 - 792e^{-4t} + 800e^{-5t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5t}$$
$$+(371.20 + 232 + 63.36 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$-125.44e^{-4t} = -125.44e^{-4t} \quad \text{(OK)}$$

Problems

P 6.1 **[a]**

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 50t\,\text{A} && 0 \le t \le 5\,\text{ms} \\ i &= 0.5 - 50t\,\text{A} && 5 \le t \le 10\,\text{ms} \\ i &= 0 && 10\,\text{ms} < t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 20 \times 10^{-3}(50) = 1\,\text{V} \qquad 0 \le t \le 5\,\text{ms}$

$v = 20 \times 10^{-3}(-50) = -1\,\text{V} \qquad 5 \le t \le 10\,\text{ms}$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 1\,\text{V} && 0 < t < 5\,\text{ms} \\ v &= -1\,\text{V} && 5 < t < 10\,\text{ms} \\ v &= 0 && 10\,\text{ms} < t \end{aligned}$$

$p = vi$

$$\begin{aligned} p &= 0 && t < 0 \\ p &= (50t)(1) = 50t\,\text{W} && 0 < t < 5\,\text{ms} \\ p &= (0.5 - 50t)(-1) = 50t - 0.5\,\text{W} && 5 < t < 10\,\text{ms} \\ p &= 0 && 10\,\text{ms} < t \end{aligned}$$

$$\begin{aligned} w &= 0 && t < 0 \\ w &= \int_0^t (50x)\,dx = 50\frac{x^2}{2}\bigg|_0^t = 25t^2\,\text{J} && 0 < t < 5\,\text{ms} \\ w &= \int_{0.005}^t (50x - 0.5)\,dx + 0.625 \times 10^{-3} \\ &= 25x^2 - 0.5x\bigg|_{0.005}^t + 0.625 \times 10^{-3} \\ &= 25t^2 - 0.5t + 2.5 \times 10^{-3}\,\text{J} && 5 < t < 10\,\text{ms} \\ w &= 0 && 10\,\text{ms} < t \end{aligned}$$

P 6.2 **[a]** $0 \le t \le 2\,\text{ms}$:

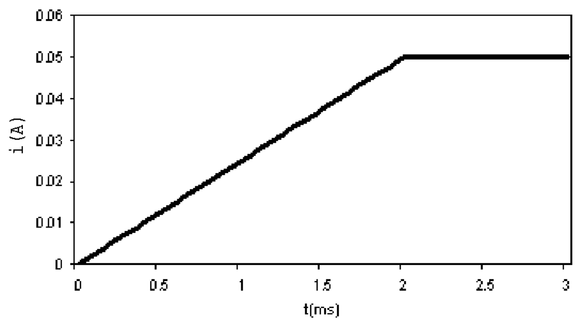
$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{1}{200 \times 10^{-6}}\int_0^t 5 \times 10^{-3}\,dx + 0$$

$$= 25x \Big|_0^t = 25t\,\text{A}$$

$2\,\text{ms} \leq t < \infty$:

$$i = \frac{1}{200 \times 10^{-6}} \int_{2\times 10^{-3}}^{t} (0)\,dx + 25(2 \times 10^{-3}) = 50\,\text{mA}$$

[b]



P 6.3 Note – the initial current should be 1 A.

$0 \leq t \leq 2$ s

$$i_L = \frac{1}{2.5 \times 10^{-4}} \int_0^t 3 \times 10^{-3} e^{-4x}\,dx + 0 = 1.2 \frac{e^{-4x}}{-4} \Big|_0^t + 0$$

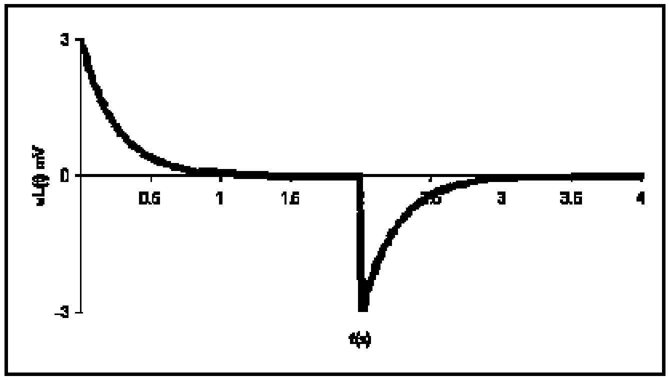
$$= 0.3 - 0.3e^{-4t}\,\text{A}, \qquad 0 \leq t \leq 2\text{ s}$$

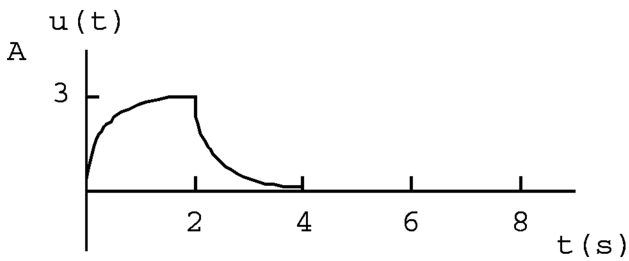
$$i_L(2) = 0.3\text{A}$$

2 s $< t < \infty$

$$i_L = -1.2 \left(\frac{e^{-4(x-2)}}{-4} \Big|_2^t + 0.3 \right)$$

$$= 0.3e^{-4(t-2)}\,\text{A}, \qquad 2\text{ s} \leq t < \infty$$





P 6.4 **[a]** $v = L\dfrac{di}{dt}$

$$\frac{di}{dt} = 18[t(-10e^{-10t}) + e^{-10t}] = 18e^{-10t}(1 - 10t)$$

$$v = (50 \times 10^{-6})(18)e^{-10t}(1 - 10t)$$

$$= 0.9e^{-10t}(1 - 10t)\,\text{mV}, \quad t > 0$$

[b] $p = vi$

$$v(200\,\text{ms}) = 0.9e^{-2}(1 - 2) = -121.8\,\mu\text{V}$$

$$i(200\,\text{ms}) = 18(0.2)e^{-2} = 487.2\,\text{mA}$$

$$p(200\,\text{ms}) = (-121.8 \times 10^{-6})(487.2 \times 10^{-3}) = -59.34\,\mu\text{W}$$

[c] delivering

[d] $w = \dfrac{1}{2}Li^2 = \dfrac{1}{2}(50 \times 10^{-6})(487.2 \times 10^{-3})^2 = 5.93\,\mu\text{J}$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 18[t(-10)e^{-10t} + e^{-10t}) = 18e^{-10t}(1 - 10t)$$

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad t = 0.1\,\text{s}$$

$$i_{\max} = 18(0.1)e^{-1} = 662.2\,\text{mA}$$

$$w_{\max} = \frac{1}{2}(50 \times 10^{-6})(662.2 \times 10^{-3})^2 = 10.96\,\mu\text{J}$$

P 6.5 **[a]** $0 \le t \le 1\,\text{s}:$

$$v = -100t$$

$$i = \frac{1}{5}\int_0^t -100x\,dx + 0 = -20\frac{x^2}{2}\Big|_0^t$$

$$i = -10t^2\,\text{A}$$

$$1\,\text{s} \le t \le 3\,\text{s}:$$

$$v = -200 + 100t$$

$$i(1) = -10\,\text{A}$$

$$\begin{aligned}
\therefore\quad i &= \frac{1}{5}\int_1^t (100x - 200)\,dx - 10\\
&= 20\int_1^t x\,dx - 40\int_1^t dx - 10\\
&= 10(t^2 - 1) - 40(t - 1) - 10\\
&= 10t^2 - 40t + 20\,\text{A}
\end{aligned}$$

$$3\,\text{s} \le t \le 5\,\text{s}:$$

$$v = 100$$

$$i(3) = 10(9) - 120 + 20 = -10\,\text{A}$$

$$\begin{aligned}
i &= \frac{1}{5}\int_3^t 100\,dx - 10\\
&= 20t - 60 - 10 = 20t - 70\,\text{A}
\end{aligned}$$

$$5\,\text{s} \le t \le 6\,\text{s}:$$

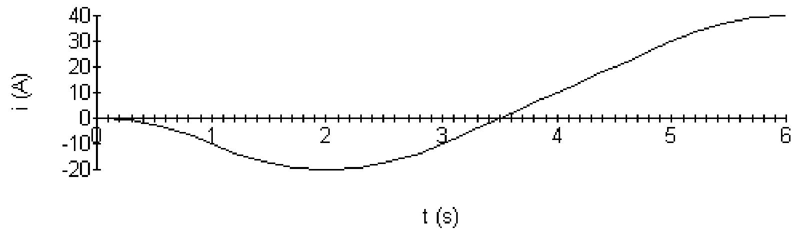
$$v = -100t + 600$$

$$i(5) = 100 - 70 = 30$$

$$\begin{aligned}
i &= \frac{1}{5}\int_5^t (-100x + 600)\,dx + 30\\
&= -20\int_5^t x\,dx + 120\int_5^t dx + 30\\
&= -10(t^2 - 25) + 120(t - 5) + 30\\
&= -10t^2 + 120t - 320\,\text{A}
\end{aligned}$$

[b] $i(6) = -10(36) + 120(6) - 320 = 720 - 680 = 40\,\text{A}, \qquad 6 \le t < \infty$

[c]



P 6.6 **[a]** $v_L = L\dfrac{di}{dt} = [125\sin 400t]e^{-200t}\,\text{V}$

$$\therefore \quad \frac{dv_L}{dt} = 25{,}000(2\cos 400t - \sin 400t)e^{-200t}\,\text{V/s}$$

$$\frac{dv_L}{dt} = 0 \quad \text{when} \quad \tan 400t = 2$$

$$\therefore \quad t = 2.77\,\text{ms}$$

Also $\quad 400t = 1.107 + \pi \qquad$ etc.

Because of the decaying exponential v_L will be maximum the first time the derivative is zero.

[b] $v_L(\text{max}) = [125\sin 1.107]e^{-0.554} = 64.27\,\text{V}$

v_L max $= 64.27\,\text{V}$

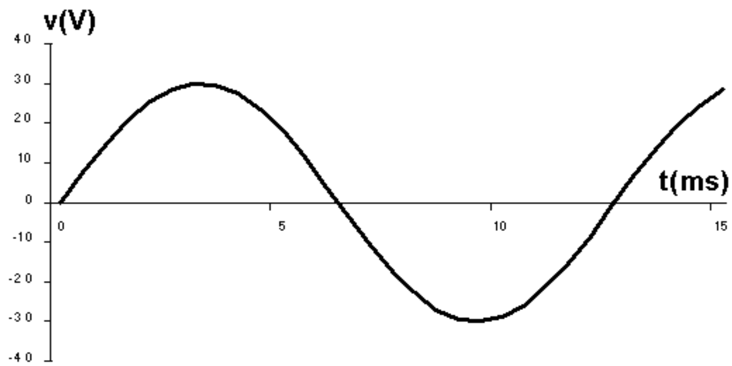
Note: When $\quad t = (1.107 + \pi)/400; \qquad v_L = -13.36\,\text{V}$

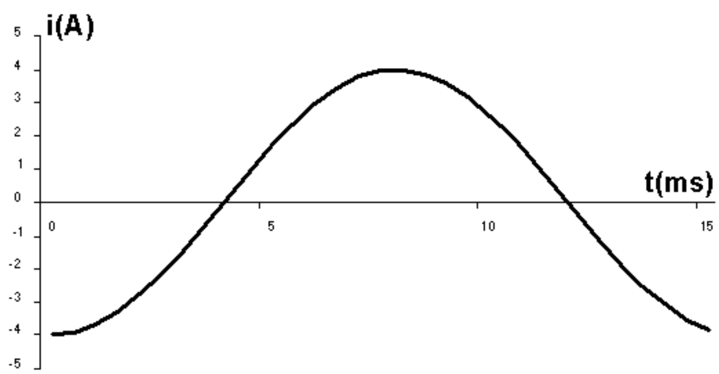
P 6.7 **[a]**

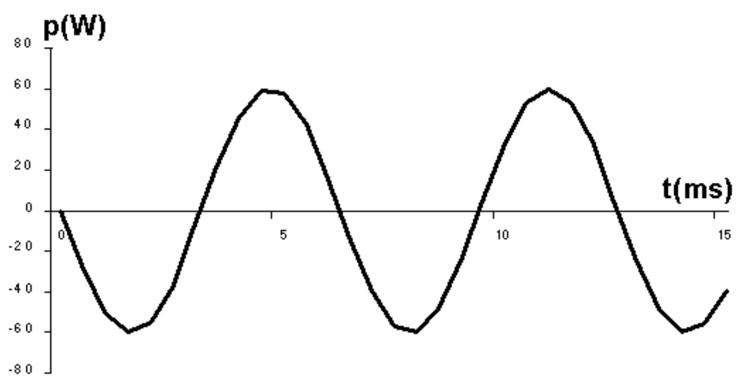
$$\begin{aligned} i &= \frac{1}{15 \times 10^{-3}}\int_0^t 30\sin 500x\,dx - 4 \\ &= 2000\int_0^t \sin 500x\,dx - 4 \\ &= 2000\left[\frac{-\cos 500x}{500}\right|_0^t - 4 \\ &= 4(1 - \cos 500t) - 4 \\ i &= -4\cos 500t\,\text{A} \end{aligned}$$

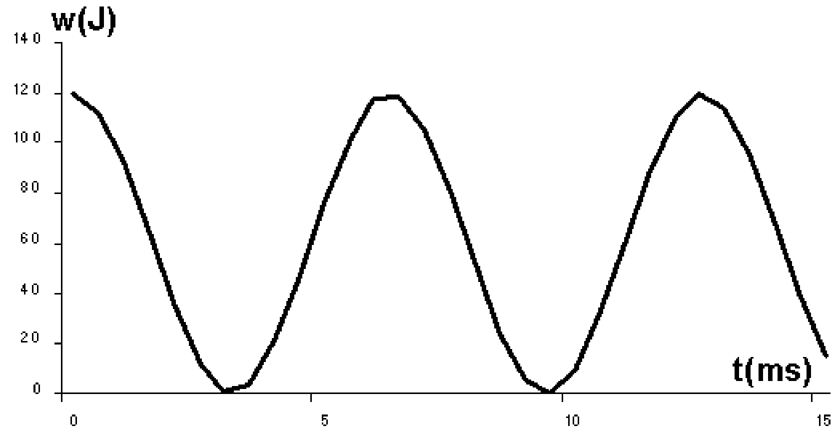
[b]

$$
\begin{aligned}
p &= vi = (30\sin 500t)(-4\cos 500t) \\
&= -120\sin 500t\cos 500t \\
p &= -60\sin 1000t\,\mathrm{W} \\
w &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(15\times 10^{-3})16\cos^2 500t \\
&= 120\cos^2 500t\,\mathrm{mJ} \\
w &= [60 + 60\cos 1000t]\,\mathrm{mJ}.
\end{aligned}
$$









[c] Absorbing power: Delivering power:

$\pi \leq t \leq 2\pi$ ms $\qquad 0 \leq t \leq \pi$ ms

$3\pi \leq t \leq 4\pi$ ms $\qquad 2\pi \leq t \leq 3\pi$ ms

P 6.8 **[a]** $i(0) = A_1 + A_2 = 0.04$

$$\frac{di}{dt} = -10{,}000 A_1 e^{-10{,}000t} - 40{,}000 A_2 e^{-40{,}000t}$$

$$v = -200 A_1 e^{-10{,}000t} - 800 A_2 e^{-40{,}000t}\,\text{V}$$

$$v(0) = -200A_1 - 800A_2 = 28$$

Solving, $A_1 = 0.1$ and $A_2 = -0.06$

Thus,

$$i = 0.1e^{-10{,}000t} - 0.06e^{-40{,}000t}\,\text{A}, \qquad t \geq 0$$

$$v = -20e^{-10{,}000t} + 48e^{-40{,}000t}\,\text{V}, \qquad t \geq 0$$

[b] If $p = 0$ then either $i = 0$ or $v = o$. Suppose $i = 0$:

$$i = 0.1e^{-10{,}000t} - 0.06e^{-40{,}000t} = 0$$

$$\therefore \quad 0.1e^{30{,}000t} = 0.06 \quad \text{so} \quad t = -17.03\,\mu\text{s}$$

This answer is impossible! So assume that $v = 0$:

$$v = -20e^{-10{,}000t} + 48e^{-40{,}000t} = 0$$

Then, $\quad -20e^{30{,}000t} = -48 \quad \therefore \quad t = 29.18\,\mu\text{s}$

This answer makes sense; therefore, the power is 0 at $t = 29.18\,\mu$s.

P 6.9 **[a]** From Problem 6.8 we have

$$A_1 + A_2 = 0.04$$

Now, we add the second equation for the coefficients:

$$-200A_1 - 800A_2 = -68$$

Solving, $\quad A_1 = -0.06; \qquad A_2 = 0.1$

Thus,

$$i = -0.06e^{-10{,}000t} + 0.1e^{-40{,}000t}\text{A} \quad t \geq 0$$

$$v = 12e^{-10{,}000t} - 80e^{-40{,}000t}\text{A} \quad t \geq 0$$

[b] $i = 0 \quad$ when $\quad 0.06e^{-10{,}000t} = 0.1e^{-40{,}000t}$

$$\therefore \quad e^{30{,}0000t} = 5/3 \quad \text{so} \quad t = 17.03\,\mu\text{s}$$

Thus,

$$i > 0 \quad \text{for} \quad 0 \leq t \leq 17.03\,\mu\text{s} \qquad \text{and} \qquad i < 0 \quad \text{for} \quad 17.03\,\mu\text{s} \leq t < \infty$$

$v = 0 \quad$ when $\quad 12e^{-10{,}000t} = 80e^{-40{,}000t}$

$$\therefore \quad e^{30{,}0000t} = 20/3 \quad \text{so} \quad t = 63.24\,\mu\text{s}$$

Thus,

$$v < 0 \quad \text{for} \quad 0 \leq t \leq 63.24\,\mu\text{s} \qquad \text{and} \qquad v > 0 \quad \text{for} \quad 63.24\,\mu\text{s} \leq t < \infty$$

Therefore,

$$p < 0 \quad \text{for} \quad 0 \leq t \leq 17.03\,\mu\text{s} \qquad \text{and} \qquad 63.24\,\mu\text{s} \leq t < \infty$$

(inductor delivers energy)

$$p > 0 \quad \text{for} \quad 17.03\,\mu\text{s} \leq t \leq 63.24\,\mu\text{s}$$ (inductor stores energy)

[c] The energy stored at $t = 0$ is

$$w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(20 \times 10^{-3})(40 \times 10^{-3})^2 = 16\,\mu\text{J}$$

The power for $t > 0$ is

$$p = vi = 6e^{-50{,}000t} - 8e^{-80{,}000t} - 0.72e^{-20{,}000t}$$

The energy for $t > 0$ is

$$w = \int_0^\infty p\,dt = \int_0^\infty 6e^{-50{,}000x}\,dx - \int_0^\infty 8e^{-80{,}000x}\,dx - \int_0^\infty 0.72e^{-20{,}000x}\,dx$$

$$= \frac{6}{50{,}000} - \frac{8}{80{,}000} - \frac{0.72}{20{,}000} = -16\,\mu\text{J}$$

Thus, the energy stored at $t = 0$ equals the energy extracted for $t > 0$.

P 6.10 $i = (B_1 \cos 1.6t + B_2 \sin 1.6t)e^{-0.4t}$

$i(0) = B_1 = 5\,\mathrm{A}$

$$\frac{di}{dt} = (B_1 \cos 1.6t + B_2 \sin 1.6t)(-0.4e^{-0.4t}) + e^{-0.4t}(-1.6B_1 \sin 1.6t + 1.6B_2 \cos 1.6t)$$

$$= [(1.6B_2 - 0.4B_1)\cos 1.6t - (1.6B_1 + 0.4B_2)\sin 1.6t]e^{-0.4t}$$

$$v = 2\frac{di}{dt} = [(3.2B_2 - 0.8B_1)\cos 1.6t - (3.2B_1 + 0.8B_2)\sin 1.6t]e^{-0.4t}$$

$$v(0) = 28 = 3.2B_2 - 0.8B_1 = 3.2B_2 - 4 \quad \therefore \quad B_2 = 32/3.2 = 10\,\mathrm{A}$$

Thus,

$$i = (5\cos 1.6t + 10\sin 1.6t)e^{-0.4t}\,\mathrm{A}, \qquad t \geq 0$$

$$v = (28\cos 1.6t - 24\sin 1.6t)e^{-0.4t}\,\mathrm{V}, \qquad t \geq 0$$

$$i(5) = 1.24\,\mathrm{A}; \qquad v(5) = -3.76\,\mathrm{V}$$

$$p(5) = (1.24)(-3.76) = -4.67\,\mathrm{W}$$

The power delivered is 4.67 W.

P 6.11 For $0 \leq t \leq 1.6\,\mathrm{s}$:

$$i_L = \frac{1}{5}\int_0^t 3 \times 10^{-3}\,dx + 0 = 0.6 \times 10^{-3}t$$

$$i_L(1.6\,\mathrm{s}) = (0.6 \times 10^{-3})(1.6) = 0.96\,\mathrm{mA}$$

$$R_m = (20)(1000) = 20\,\mathrm{k}\Omega$$

$$v_m(1.6\,\mathrm{s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2\,\mathrm{V}$$

P 6.12 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$\mathrm{W} = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\mathrm{J}$$

This is energy stored in the inductor at $t = \infty$.

P 6.13 **[a]**

$$\begin{aligned} v(20\,\mu\text{s}) &= 12.5 \times 10^9 (20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)} \\ v(20\,\mu\text{s}) &= 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10 \\ &= 5\,\text{V (start of second interval)} \\ v(40\,\mu\text{s}) &= 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10 \\ &= 10\,\text{V (end of second interval)} \end{aligned}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = 62.5\,\text{mW},$

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$

P 6.14 $i_C = C(dv/dt)$

$0 < t < 0.5:$

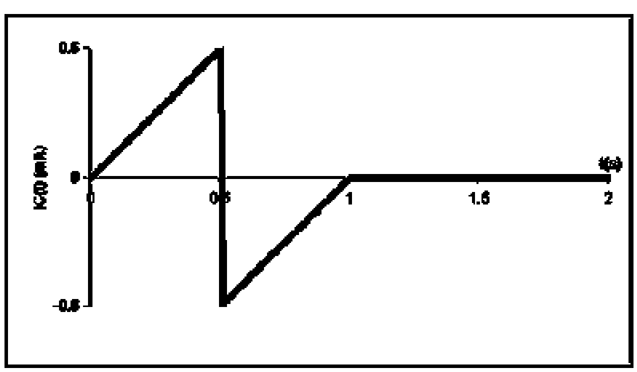
$v_c = 30t^2\,\text{V}$

$i_C = 20 \times 10^{-6}(60)t = 1.2t\,\text{mA}$

$0.5 < t < 1:$

$v_c = 30(t-1)^2\,\text{V}$

$i_C = 20 \times 10^{-6}(60)(t-1) = 1.2(t-1)\,\text{mA}$



P 6.15 **[a]** $0 \le t \le 5\,\mu\text{s}$

$$C = 5\,\mu\text{F} \qquad \frac{1}{C} = 2 \times 10^5$$

$$v = 2 \times 10^5 \int_0^t 4\,dx + 12$$

$$v = 8 \times 10^5 t + 12\,\text{V} \qquad 0 \le t \le 5\,\mu\text{s}$$

$$v(5\,\mu\text{s}) = 4 + 12 = 16\,\text{V}$$

[b] $5\,\mu\text{s} \le t \le 20\,\mu\text{s}$

$$v = 2 \times 10^5 \int_{5\times10^{-6}}^t -2\,dx + 16 = -4 \times 10^5 t + 2 + 16$$

$$v = -4 \times 10^5 t + 18\text{V} \qquad 5 \le t \le 20\,\mu\text{s}$$

$$v(20\,\mu\text{s}) = -4 \times 10^5 (20 \times 10^{-6}) + 18 = 10\,\text{V}$$

[c] $20\,\mu\text{s} \le t \le 25\,\mu\text{s}$

$$v = 2 \times 10^5 \int_{20\times10^{-6}}^t 6\,dx + 10 = 12 \times 10^5 t - 24 + 10$$

$$v = 12 \times 10^5 t - 14\,\text{V}, \qquad 20\,\mu\text{s} \le t \le 25\,\mu\text{s}$$

$$v(25\,\mu\text{s}) = 12 \times 10^5 (25 \times 10^{-6}) - 14 = 16\,\text{V}$$

[d] $25\,\mu\text{s} \le t \le 35\,\mu\text{s}$

$$v = 2 \times 10^5 \int_{25\times10^{-6}}^t 4\,dx + 16 = 8 \times 10^5 t - 20 + 16$$

$$v = 8 \times 10^5 t - 4\,\text{V}, \qquad 25\,\mu\text{s} \le t \le 35\,\mu\text{s}$$

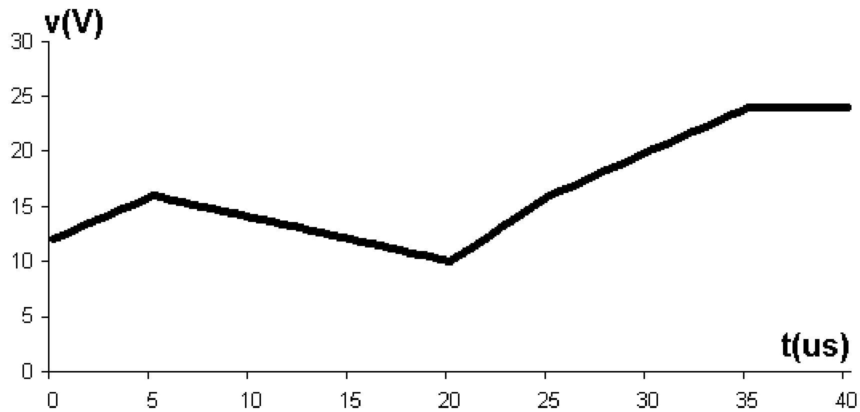
$$v(35\,\mu\text{s}) = 8 \times 10^5 (35 \times 10^{-6}) - 4 = 24\,\text{V}$$

[e] $35\,\mu\text{s} \le t < \infty$

$$v = 2 \times 10^5 \int_{35\times10^{-6}}^t 0\,dx + 24 = 24$$

$$v = 24\,\text{V}, \qquad 35\,\mu\text{s} \le t < \infty$$

[f]



P 6.16 $v = -10\,\text{V}, \quad t \le 0; \qquad C = 0.8\,\mu\text{F}$

$$v = 40 - e^{-1000t}(50\cos 500t + 20\sin 500t)\text{V}, \quad t \ge 0$$

[a] $i = 0, \quad t < 0$

[b]
$$\begin{aligned}
\frac{dv}{dt} &= 1000e^{-1000t}(50\cos 500t + 20\sin 500t) \\
&\quad -e^{-1000t}(-25{,}000\sin 500t + 10{,}000\cos 500t) \\
&= e^{-1000t}(50{,}000\cos 500t + 20{,}000\sin 500t \\
&\quad +25{,}000\sin 500t - 10{,}000\cos 500t) \\
&= (40{,}000\cos 500t + 45{,}000\sin 500t)e^{-1000t} \\
i &= C\frac{dv}{dt} = (32\cos 500t + 36\sin 500t)e^{-1000t}\text{ mA}
\end{aligned}$$

[c] no

[d] yes, from 0 to 32 mA

[e] $v(\infty) = 40\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.8\times 10^{-6})(40)^2 = 640\,\mu\text{J}$$

P 6.17 **[a]** $i = \dfrac{400\times 10^{-3}}{5\times 10^{-6}}t = 8\times 10^4 t \qquad 0 \le t \le 5\,\mu\text{s}$

$$i = 400\times 10^{-3} \qquad 5 \le t \le 20\,\mu\text{s}$$

$$\begin{aligned}
q &= \int_0^{5\times 10^{-6}} 8\times 10^4 t\,dt + \int_{5\times 10^{-6}}^{15\times 10^{-6}} 400\times 10^{-3}\,dt \\
&= 8\times 10^4 \frac{t^2}{2}\Bigg|_0^{5\times 10^{-6}} + 400\times 10^{-3}(10\times 10^{-6}) \\
&= 8\times 10^4(\tfrac{1}{2})(25\times 10^{-12}) + 4\times 10^{-6} \\
&= 5\,\mu\text{C}
\end{aligned}$$

[b]
$$\begin{aligned}
v &= \frac{1}{0.25\times 10^{-6}}\left[\int_0^{5\,\mu\text{s}} 8\times 10^4 x\,dx + \int_{5\,\mu\text{s}}^{20\,\mu\text{s}} 0.4x\,dx + \int_{20\,\mu\text{s}}^{30\,\mu\text{s}} (10^4 x - 0.5)\,dx\right] \\
&= \frac{1}{0.25\times 10^{-6}}\left[4\times 10^4 t^2\Bigg|_0^{5\,\mu\text{s}} + 0.4t\Bigg|_{5\,\mu\text{s}}^{20\,\mu\text{s}} + (5000t^2 - 0.5t)\Bigg|_{20\,\mu\text{s}}^{30\,\mu\text{s}}\right] \\
&= \frac{1}{0.25\times 10^{-6}}[1\times 10^{-6} + 6\times 10^{-6} - 10.5\times 10^{-6} + 8\times 10^{-6}] = 18\text{V}
\end{aligned}$$

[c] $v(50\,\mu\text{s}) = 18 + \dfrac{1}{0.25\times 10^{-6}}(5000t^2 - 0.5t)\Big|_{30\,\mu\text{s}}^{50\,\mu\text{s}}$

$$= 18 + \frac{1}{0.25\times 10^{-6}}(-12.5\times 10^{-6} + 10.5\times 10^{-6}) = 10\text{V}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.25\times 10^{-6})(10)^2 = 12.5\,\mu\text{J}$$

P 6.18 **[a]**

$$\begin{aligned} v &= \frac{1}{0.5\times 10^{-6}}\int_0^{500\times 10^{-6}} 50\times 10^{-3}e^{-2000t}\,dt - 20 \\ &= 100\times 10^3\frac{e^{-2000t}}{-2000}\Big|_0^{500\times 10^{-6}} -20 \\ &= 50(1-e^{-1}) - 20 = 11.61\,\text{V} \\ w &= \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7\,\mu\text{J} \end{aligned}$$

[b] $v(\infty) = 50 - 20 = 30\text{V}$

$$w(\infty) = \frac{1}{2}(0.5\times 10^{-6})(30)^2 = 225\,\mu\text{J}$$

P 6.19 **[a]** $w(0) = \dfrac{1}{2}C[v(0)]^2 = \dfrac{1}{2}(0.25)\times 10^{-6}(50)^2 = 312.5\,\mu\text{J}$

[b] $v = (A_1t + A_2)e^{-4000t}$

$$v(0) = A_2 = 50\,\text{V}$$

$$\begin{aligned} \frac{dv}{dt} &= -4000e^{-4000t}(A_1t + A_2) + e^{-4000t}(A_1) \\ &= (-4000A_1t - 4000A_2 + A_1)e^{-4000t} \end{aligned}$$

$$\frac{dv}{dt}(0) = A_1 - 4000A_2$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore\quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{400\times 10^{-3}}{0.25\times 10^{-6}} = 16\times 10^5$$

$$\therefore\quad 16\times 10^5 = A_1 - 4000(50)$$

Thus, $A_1 = 16\times 10^5 + 2\times 10^5 = 18\times 10^5\,\dfrac{\text{V}}{\text{s}}$

[c] $v = (18 \times 10^5 t + 50)e^{-4000t}$

$$i = C\frac{dv}{dt} = 0.25 \times 10^{-6}\frac{d}{dt}(18 \times 10^5 t + 50)e^{-4000t}$$

$$\begin{aligned} i &= \frac{d}{dt}[(0.45t + 12.5 \times 10^{-6})e^{-4000t}] \\ &= (0.45t + 12.5 \times 10^{-6})(-4000)e^{-4000t} + e^{-4000t}(0.45) \\ &= (-1800t - 0.05 + 0.45)e^{-4000t} \\ &= (0.40 - 1800t)e^{-4000t}\,\text{A}, \qquad t \ge 0 \end{aligned}$$

P 6.20 $5\|(12 + 8) = 4\,\text{H}$

$4\|4 = 2\,\text{H}$

$15\|(8 + 2) = 6\,\text{H}$

$3\|6 = 2\,\text{H}$

$6 + 2 = 8\,\text{H}$

P 6.21 $30\|20 = 12\,\text{H}$

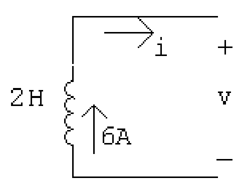
$80\|(8 + 12) = 16\,\text{H}$

$60\|(14 + 16) = 20\,\text{H}$

$15\|(20 + 10) = 20\,\text{H}$

$L_{\text{ab}} = 5 + 10 = 15\,\text{H}$

P 6.22 **[a]**



$$\begin{aligned} i(t) &= -\frac{1}{2}\int_0^t 12e^{-x}\,dx + 6 \\ &= 6e^{-x}\Big|_0^t + 6 \\ &= 6e^{-t} - 6 + 6 \\ i(t) &= 6e^{-t}\,\text{A}, \qquad t \ge 0 \end{aligned}$$

[b]
$$\begin{aligned} i_1(t) &= -\frac{1}{3}\int_0^t 12e^{-x}\,dx + 2 \\ &= 4e^{-x}\Big|_0^t + 2 \\ &= 4(e^{-t}-1)+2 \\ i_1(t) &= 4e^{-t} - 2\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[c]
$$\begin{aligned} i_2(t) &= -\frac{1}{6}\int_0^t 12e^{-x}\,dx + 4 \\ &= 2e^{-x}\Big|_0^t + 4 \\ &= 2(e^{-t}-1)+4 \\ i_2(t) &= 2e^{-t} + 2\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[d] $p = vi = (12e^{-t})(6e^{-t}) = 72e^{-2t}\,\text{W}$

$$\begin{aligned} w &= \int_0^\infty p\,dt = \int_0^\infty 72e^{-2t}\,dt \\ &= 72\frac{e^{-2t}}{-2}\Big|_0^\infty \\ &= 36\,\text{J} \end{aligned}$$

[e] $w = \frac{1}{2}(3)(2)^2 + \frac{1}{2}(6)(4)^2 = 54\,\text{J}$

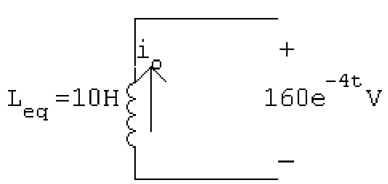
[f] $w_{\text{trapped}} = \frac{1}{2}(3)(-2)^2 + \frac{1}{2}(6)(2)^2 = 18\,\text{J}$

$w_{\text{trapped}} = 54 - 36 = 18\,\text{J}$ checks

[g] Yes, they agree.

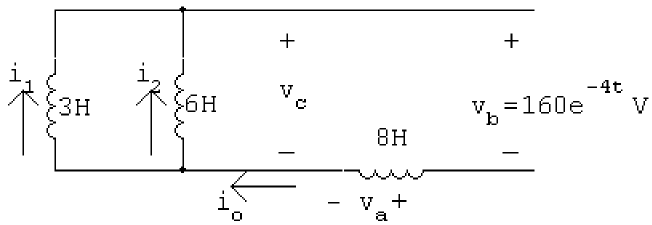
P 6.23 **[a]** $i_o(0) = i_1(0) + i_2(0) = 4\,\text{A}$

[b]



$$\begin{aligned} i_o &= -\frac{1}{10}\int_0^t 160e^{-4x}\,dx + 4 = -16\left[\frac{e^{-4x}}{-4}\right|_0^t + 4 \\ &= 4(e^{-4t}-1)+4 = 4e^{-4t}\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[c]



$$v_a = 8\frac{d}{dt}(4e^{-4t}) = -128e^{-4t}\,\text{V}$$

$$\begin{aligned} v_c &= v_a + v_b = -128e^{-4t} + 160e^{-4t} \\ &= 32e^{-4t}\,\text{V} \\ i_1 &= -\frac{1}{3}\int_0^t 32e^{-4x}\,dx + 1 \\ &= 2.67e^{-4t} - 2.67 + 1 \\ i_1 &= 2.67e^{-4t} - 1.67\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[d] $$\begin{aligned} i_2 &= -\frac{1}{6}\int_0^t 32e^{-4x}\,dx + 3 \\ &= 1.33e^{-4t} - 1.33 + 3 \\ i_2 &= 1.33e^{-4t} + 1.67\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[e] $w(0) = \frac{1}{2}(3)(1)^2 + \frac{1}{2}(6)(3)^2 + \frac{1}{2}(8)(4)^2 = 92.5\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(10)(4)^2 = 80\,\text{J}$

[g] $w_{\text{trapped}} = 92.5 - 80 = 12.5\,\text{J}$

P 6.24 $v_{\text{b}} = 160e^{-4t}\,\text{V}$

$i_o = 4e^{-4t}\,\text{A}$

$p = 640e^{-8t}\,\text{W}$

$$w = \int_0^t 640e^{-8x}\,dx = 640\frac{e^{-8x}}{-8}\Big|_0^t = 80(1 - e^{-8t})\,\text{W}$$

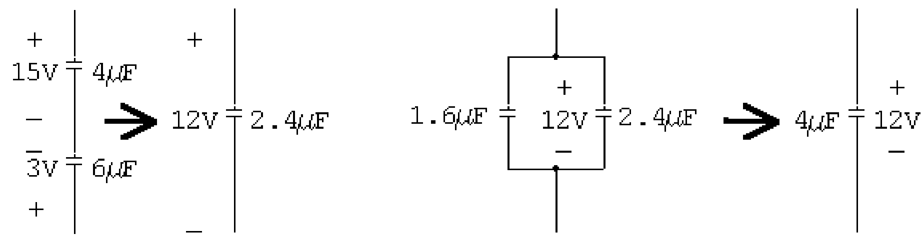
$w_{\text{total}} = 80\,\text{J}$

$w(0.2) = 80(1 - e^{-1.6}) = 63.85\,\text{J}$

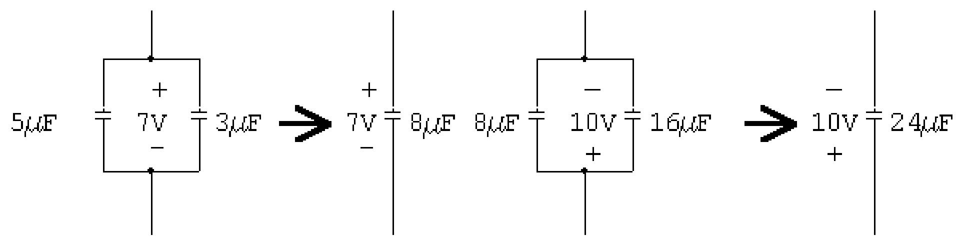
Thus,

$$\%\text{ delivered} = \frac{63.85}{80}(100) = 79.8\%$$

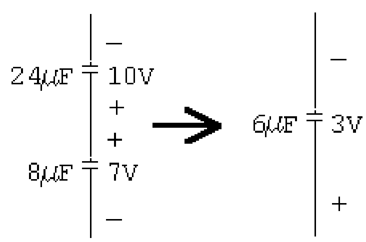
P 6.25 $\quad \dfrac{1}{4}+\dfrac{1}{6}=\dfrac{5}{12} \qquad \therefore \quad C_{\text{eq}}=2.4\,\mu\text{F}$



$$\frac{1}{4}+\frac{1}{12}=\frac{4}{12} \qquad \therefore \quad C_{\text{eq}}=3\,\mu\text{F}$$



$$\frac{1}{24}+\frac{1}{8}=\frac{4}{24} \qquad \therefore \quad C_{\text{eq}}=6\,\mu\text{F}$$



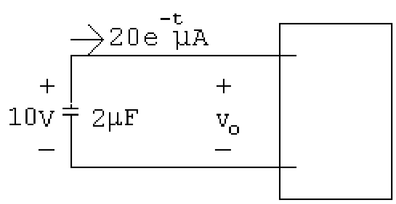
P 6.26 Work from the right hand side of the circuit, simplifying step by step:

1. $48\,\mu$F in series with $16\,\mu$F : $\quad 1/C=1/16\,\mu+1/48\,\mu \quad \therefore \quad C=12\,\mu$F
   The voltages add in series, so the $12\,\mu$F capacitor has a voltage of 20 V, negative at the top.
2. Previous $12\,\mu$F in parallel with $3\,\mu$F : $\quad C=12\,\mu+3\,\mu=15\,\mu$F
   The voltage is 20 V, negative at the top.
3. Previous $15\,\mu$F in series with $30\,\mu$F :
   $1/C=1/15\,\mu+1/30\,\mu \quad \therefore \quad C=10\,\mu$F
   The voltages add in series, so the $10\,\mu$F capacitor has a voltage of 10 V, positive at the right.

4. Previous $10\,\mu$F in parallel with $10\,\mu$F : $C = 10\,\mu + 10\,\mu = 20\,\mu$F
   The voltage is 10 V, negative at the top.
5. Previous $20\,\mu$F in series with $5\,\mu$F and $4\,\mu$F :
   $1/C = 1/20\,\mu + 1/5\,\mu + 1/4\,\mu \quad \therefore \quad C = 2\,\mu$F
   The voltages in series add: $5\text{V} - 10V + 30\text{V} = 25\text{V}$ positive at the top.

The equivalent capacitance is $2\,\mu$F with a voltage of 25 V, positive at the top.

P 6.27 **[a]**



$$v_o = -\frac{1}{2\times 10^{-6}}\int_0^t 20\times 10^{-6}e^{-x}\,dx + 10$$

$$= 10e^{-x}\Big|_0^t + 10$$

$$= 10e^{-t}\,\text{V}, \qquad t \geq 0$$

[b] $$v_1 = -\frac{1}{3\times 10^{-6}}(20\times 10^{-6})e^{-x}\Big|_0^t + 4$$

$$= 6.67e^{-t} - 2.67\,\text{V}, \qquad t \geq 0$$

[c] $$v_2 = -\frac{1}{6\times 10^{-6}}(20\times 10^{-6})e^{-x}\Big|_0^t + 6$$

$$= 3.33e^{-t} + 2.67\,\text{V}, \qquad t \geq 0$$

[d] $$p = vi = (10e^{-t})(20\times 10^{-6})e^{-t}$$

$$= 200\times 10^{-6}e^{-2t}$$

$$w = \int_0^\infty 200\times 10^{-6}e^{-2t}\,dt$$

$$= 200\times 10^{-6}\frac{e^{-2t}}{-2}\Big|_0^\infty$$

$$= -100\times 10^{-6}(0-1) = 100\,\mu\text{J}$$

[e] $$w = \tfrac{1}{2}(3\times 10^{-6})(4)^2 + \tfrac{1}{2}(6\times 10^{-9})(6)^2$$

$$= 132\,\mu\text{J}$$

[f] $$w_{\text{trapped}} = \tfrac{1}{2}(3\times 10^{-6})(8/3)^2 + \tfrac{1}{2}(6\times 10^{-6})(8/3)^2$$

$$= 32\,\mu\text{J}$$

CHECK: $100 + 32 = 132\,\mu$J

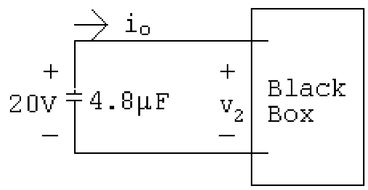
[g] Yes, they agree.

P 6.28 $C_1 = 10 + 2 = 12\,\mu$F

$$\frac{1}{C_2} = \frac{1}{12\,\mu} + \frac{1}{8\,\mu} \qquad \therefore \qquad C_2 = 4.8\,\mu\text{F}$$

$$v_o(0) + v_1(0) = -5 + 25 = 20\,\text{V}$$

[a]



$$\begin{aligned} v_2 &= -\frac{1}{4.8\times10^{-6}}\int_0^t 1.92\times10^{-3}e^{-20x}\,dx + 20 \\ &= -400\frac{e^{-20x}}{-20}\Big|_0^t + 20 \\ &= 20(e^{-20t} - 1) + 20 \\ &= 20e^{-20t}\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[b]
$$\begin{aligned} v_o &= -\frac{1}{8\times10^{-6}}\int_0^t 1.92\times10^{-3}e^{-20x}\,dx - 5 \\ &= -240\frac{e^{-20x}}{-20}\Big|_0^t - 5 \\ &= 12(e^{-20t} - 1) - 5 \\ &= 12e^{-20t} - 17\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[c]
$$\begin{aligned} v_1 &= -\frac{1}{12\times10^{-6}}\int_0^t 1.92\times10^{-3}e^{-20x}\,dx + 25 \\ &= -160\frac{e^{-20x}}{-20}\Big|_0^t + 25 \\ &= 8(e^{-20t} - 1) + 25 \\ &= 8e^{-20t} + 17\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[d]
$$i_1 = -10 \times 10^{-6} \frac{d}{dt}\,[8e^{-20t} + 17]$$
$$= -10 \times 10^{-6}(-20)8e^{-20t}$$
$$= 1.6e^{-20t}\,\text{mA}, \qquad t > 0$$

[e]
$$i_2 = -2 \times 10^{-6} \frac{d}{dt}\,[8e^{-20t} + 17]$$
$$= -2 \times 10^{-6}(-20)8e^{-20t}$$
$$= 0.32e^{-20t}\,\text{mA}, \qquad t > 0$$

CHECK: $i_1 + i_2 = 1.92e^{-20t}\,\text{mA} = i_o$

P 6.29 **[a]**
$$w(0) = [\tfrac{1}{2}(8 \times 10^{-6})(-5)^2 + \tfrac{1}{2}(10 \times 10^{-6})(25)^2 + \tfrac{1}{2}(2 \times 10^{-6})(25)^2]$$
$$= 3850\,\mu\text{J}$$

[b]
$$v_o(\infty) = -17\,\text{V}$$
$$v_1(\infty) = 17\,\text{V}$$
$$w(\infty) = [\tfrac{1}{2}(8 \times 10^{-6})(-17)^2 + \tfrac{1}{2}(12 \times 10^{-6})(17)^2]$$
$$= 2890\,\mu\text{J}$$

[c] $w = \int_0^\infty (20e^{-20t})(1.92 \times 10^{-3}e^{-20t})\,dt = 960\,\mu\text{J}$

CHECK: $3850 - 2890 = 960\,\mu\text{J}$

[d] % delivered $= \dfrac{960}{3850} \times 100 = 24.9\%$

[e]
$$w(40\,\text{ms}) = \int_0^{0.04} (20e^{-20t})(1.92 \times 10^{-3}e^{-20t})\,dt$$
$$= 0.0384\frac{e^{-40t}}{-40}\Big|_0^{0.04}$$
$$= 960 \times 10^{-6}(1 - e^{-1.6}) = 766.2\,\mu\text{J}$$

% delivered $= \dfrac{766.2}{960}(100) = 79.8\%$

P 6.30 From Figure 6.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

Therefore $\frac{1}{C_{\rm eq}} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right], \qquad v_{\rm eq}(0) = v_1(0) + v_2(0) + \cdots$

P 6.31 From Fig. 6.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{\rm eq} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on $C_{\rm eq}$.

P 6.32

$$\begin{aligned}
\frac{di_o}{dt} &= 5\{e^{-2000t}[-8000\sin 4000t + 4000\cos 4000t] \\
&\quad -2000e^{-2000t}[2\cos 4000t + \sin 4000t]\} \\
\frac{di_o}{dt}(0^+) &= 5[1(4000) + (-2000)(2)] = 0 \\
v_2(0^+) &= 10\times 10^{-3}\frac{di_o}{dt}(0^+) = 0 \\
v_1(0^+) &= 40i_o(0^+) + v_2(0^+) = 40(10) + 0 = 400\text{V}
\end{aligned}$$

P 6.33

$$\begin{aligned}
v_{\rm c} &= -\frac{1}{0.625\times 10^{-6}}\left(\int_0^t 1.5e^{-16{,}000x}\,dx - \int_0^t 0.5e^{-4000x}\,dx\right) - 50 \\
&= 150(e^{-16{,}000t} - 1) - 200(e^{-4000t} - 1) - 50 \\
&= 150e^{-16{,}000t} - 200e^{-4000t}\ \text{V} \\
v_L &= 25\times 10^{-3}\frac{di_o}{dt} \\
&= 25\times 10^{-3}(-24{,}000e^{-16{,}000t} + 2000e^{-4000t}) \\
&= -600e^{-16{,}000t} + 50e^{-4000t}\ \text{V} \\
v_o &= v_{\rm c} - v_L \\
&= (150e^{-16{,}000t} - 200e^{-4000t}) - (-600e^{-16{,}000t} + 50e^{-4000t}) \\
&= 750e^{-16{,}000t} - 250e^{-4000t}\ \text{V}, \qquad t > 0
\end{aligned}$$

P 6.34 **[a]** $-2\frac{di_g}{dt} + 16\frac{di_2}{dt} + 32i_2 = 0$

$$16\frac{di_2}{dt} + 32i_2 = 2\frac{di_g}{dt}$$

[b] $i_2 = e^{-t} - e^{-2t}\,\text{A}$

$$\frac{di_2}{dt} = -e^{-t} + 2e^{-2t}\,\text{A/s}$$

$$i_g = 8 - 8e^{-t}\,\text{A}$$

$$\frac{di_g}{dt} = 8e^{-t}\,\text{A/s}$$

$$\therefore\quad -16e^{-t} + 32e^{-2t} + 32e^{-t} - 32e^{-2t} = 16e^{-t}$$

[c]

$$\begin{aligned} v_1 &= 4\frac{di_g}{dt} - 2\frac{di_2}{dt} \\ &= 4(8e^{-t}) - 2(-e^{-t} + 2e^{-2t}) \\ &= 34e^{-t} - 4e^{-2t}\,\text{V}, \qquad t > 0 \end{aligned}$$

[d] $v_1(0) = 34 - 4 = 30\,\text{V}$; Also

$$\begin{aligned} v_1(0) &= 4\frac{di_g}{dt}(0) - 2\frac{di_2}{dt}(0) \\ &= 4(8) - 2(-1+2) = 32 - 2 = 30\,\text{V} \end{aligned}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.

P 6.35 **[a]** Yes, $v_o = 20(i_2 - i_1) + 60i_2$

[b]

$$\begin{aligned} v_o &= 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 20(-3 - 116e^{-5t} + 119e^{-4t}) + 60 - 3120e^{-5t} + 3060e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t}\,\text{V} \end{aligned}$$

[c]

$$\begin{aligned} v_o &= L_2\frac{d}{dt}(i_g - i_2) + M\frac{di_1}{dt} \\ &= 16\frac{d}{dt}(15 + 36e^{-5t} - 51e^{-4t}) + 8\frac{d}{dt}(4 + 64e^{-5t} - 68e^{-4t}) \\ &= -2880e^{-5t} + 3264e^{-4t} - 2560e^{-5t} + 2176e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t}\,\text{V} \end{aligned}$$

P 6.36 **[a]**

$$\begin{aligned} v_g &= 5(i_g - i_1) + 20(i_2 - i_1) + 60i_2 \\ &= 5(16 - 16e^{-5t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 60 + 5780e^{-4t} - 5840e^{-5t}\,\text{V} \end{aligned}$$

[b] $v_g(0) = 60 + 5780 - 5840 = 0\,\text{V}$

[c] $$p_{\text{dev}} = v_g i_g$$
$$= 960 + 92{,}480e^{-4t} - 94{,}400e^{-5t} - 92{,}480e^{-9t} + 93{,}440e^{-10t}\,\text{W}$$

[d] $p_{\text{dev}}(\infty) = 960\,\text{W}$

[e] $i_1(\infty) = 4\,\text{A}; \quad i_2(\infty) = 1\,\text{A}; \quad i_g(\infty) = 16\,\text{A};$

$$p_{5\Omega} = (16-4)^2(5) = 720\,\text{W}$$

$$p_{20\Omega} = 3^2(20) = 180\,\text{W}$$

$$p_{60\Omega} = 1^2(60) = 60\,\text{W}$$

$$\sum p_{\text{abs}} = 720 + 180 + 60 = 960\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 960\,\text{W}$$

P 6.37 **[a]** Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8\frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8\frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$
$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \stackrel{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$
$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \stackrel{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad \text{(OK)}$$

$$8\frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16\frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16\frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$
$$+80 - 4160e^{-5t} + 4080e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$
$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad \text{(OK)}$$

P 6.38 **[a]** $L_2 = \left(\dfrac{M^2}{k^2 L_1}\right) = \dfrac{(0.09)^2}{(0.75)^2(0.288)} = 50\,\text{mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{288}{50}} = 2.4$$

[b] $\mathcal{P}_1 = \dfrac{L_1}{N_1^2} = \dfrac{0.288}{(1200)^2} = 0.2 \times 10^{-6}$ Wb/A

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.05}{(500)^2} = 0.2 \times 10^{-6} \text{ Wb/A}$$

P 6.39 $\mathcal{P}_1 = \dfrac{L_1}{N_1^2} = 2$ nWb/A; $\quad \mathcal{P}_2 = \dfrac{L_2}{N_2^2} = 2$ nWb/A; $\quad M = k\sqrt{L_1 L_2} = 180\,\mu\text{H}$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1 N_2} = 1.2 \text{ nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 0.8 \text{ nWb/A}$$

P 6.40 **[a]** $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{7.2}{\sqrt{81}} = 0.8$

[b] $M = \sqrt{81} = 9\,\text{mH}$

[c] $\dfrac{L_1}{L_2} = \dfrac{N_1^2\mathcal{P}_1}{N_2^2\mathcal{P}_2} = \left(\dfrac{N_1}{N_2}\right)^2$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{27}{3} = 9$$

$$\frac{N_1}{N_2} = 3$$

P 6.41 **[a]** $M = k\sqrt{L_1L_2} = 0.8\sqrt{324} = 14.4\,\text{mH}$

$$\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{36\times10^{-3}}{(200)^2} = 900\ \text{nWb/A}$$

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.1; \qquad \mathcal{P}_{21} = 10\mathcal{P}_{11}$$

$$\mathcal{P}_1 = \mathcal{P}_{11} + \mathcal{P}_{21} = 11\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = \frac{1}{11}\mathcal{P}_1 = 81.82\ \text{nWb/A}$$

$$\mathcal{P}_{21} = 10\mathcal{P}_{11} = 818.18\ \text{nWb/A}$$

$$N_2 = \frac{M}{N_1\mathcal{P}_{21}} = \frac{14.4\times10^{-3}}{(200)(818.18\times10^{-9})} = 88\ \text{turns}$$

[b] $\mathcal{P}_2 = \dfrac{L_2}{N_2^2} = \dfrac{9\times10^{-3}}{(88)^2} = 1162.19\ \text{nWb/A}$

[c] $\mathcal{P}_{11} = 81.82$ nWb/A [see part (a)]

[d] $\dfrac{\phi_{22}}{\phi_{12}} = \dfrac{P_{22}}{P_{12}}$

$$P_{12} = P_{21} = 818.18\ \text{nWb/A}$$

$$P_{22} = P_2 - P_{12} = 1162.19\times10^{-9} - 818.18\times10^{-9} = 344.01\ \text{nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{344.01}{818.18} = 0.4205$$

P 6.42 **[a]** Dot terminal 1; the flux is up in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

[b] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[c] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[d] Dot terminal 1; the flux is down in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

P 6.43 **[a]** $$\frac{1}{k^2} = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11}N_1i_1 + \mathcal{P}_{21}N_1i_1 = N_1i_1(\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2i_2(\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2i_2}\right)$$

Therefore

$$k^2 = \frac{(\phi_{12}/N_2i_2)(\phi_{21}/N_1i_1)}{(\phi_1/N_1i_1)(\phi_2/N_2i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.44 **[a]** $$v_{\text{ab}} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$$

It follows that $L_{\text{ab}} = (L_1 + L_2 + 2M)$

[b] $$v_{\text{ab}} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$$

Therefore $L_{\text{ab}} = (L_1 + L_2 - 2M)$

P 6.45 When the switch is opened the induced voltage is negative at the dotted terminal. Since the voltmeter kicks upscale, the induced voltage across the voltmeter must be positive at its positive terminal. Therefore, the voltage is negative at the negative terminal of the voltmeter.

Thus, the lower terminal of the unmarked coil has the same instantaneous polarity as the dotted terminal. Therefore, place a dot on the lower terminal of the unmarked coil.

P 6.46 **[a]** $v_{\rm ab} = L_1\frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$

$$0 = L_1\frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{\rm ab} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{\rm ab}$$

from which we have

$$v_{\rm ab} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore \quad L_{\rm ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{\rm ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

P 6.47 **[a]** $W = (0.5)L_1i_1^2 + (0.5)L_2i_2^2 + Mi_1i_2$

$$M = 0.85\sqrt{(18)(32)} = 20.4\,\text{mH}$$

$$W = [9(36) + 16(81) + 20.4(54)] = 2721.6\,\text{mJ}$$

[b] $W = [324 + 1296 + 1101.6] = 2721.6\,\text{mJ}$

[c] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

[d] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

P 6.48 **[a]** $M = 1.0\sqrt{(18)(32)} = 24\,\text{mH}, \qquad i_1 = 6\,\text{A}$

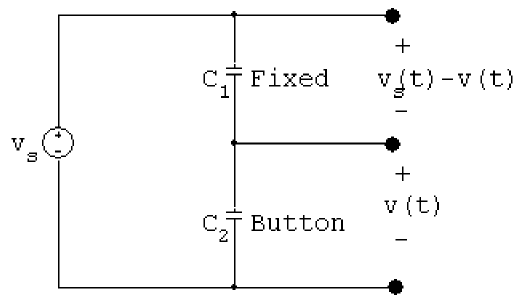
Therefore $\quad 16i_2^2 + 144i_2 + 324 = 0, \qquad i_2^2 + 9i_2 + 20.25 = 0$

Therefore $\quad i_2 = -\left(\frac{9}{2}\right) \pm \sqrt{\left(\frac{9}{2}\right)^2 - 20.25} = -4.5 \pm \sqrt{0}$

Therefore $\quad i_2 = -4.5\,\text{A}$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

P 6.49 When the button is not pressed we have



$$C_2\frac{dv}{dt} = C_1\frac{d}{dt}(v_s - v)$$

or

$$(C_1 + C_2)\frac{dv}{dt} = C_1\frac{dv_s}{dt}$$

$$\frac{dv}{dt} = \frac{C_1}{(C_1 + C_2)}\frac{dv_s}{dt}$$

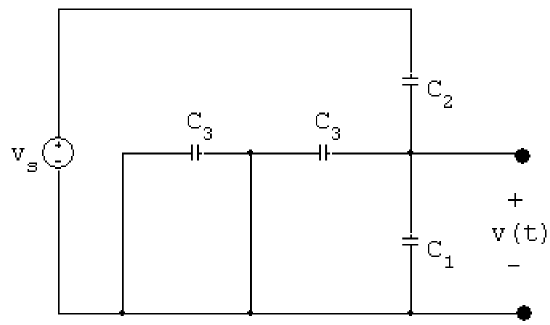
Assuming $\quad C_1 = C_2 = C$

$$\frac{dv}{dt} = 0.5\frac{dv_s}{dt}$$

or

$$v = 0.5v_s(t) + v(0)$$

When the button is pressed we have



$$C_1\frac{dv}{dt} + C_3\frac{dv}{dt} + C_2\frac{d(v - v_s)}{dt} = 0$$

$$\therefore \quad \frac{dv}{dt} = \frac{C_2}{C_1 + C_2 + C_3}\,\frac{dv_s}{dt}$$

Assuming $\quad C_1 = C_2 = C_3 = C$

$$\frac{dv}{dt} = \frac{1}{3}\,\frac{dv_s}{dt}$$

$$v = \frac{1}{3}v_s(t) + v(0)$$

Therefore interchanging the fixed capacitor and the button has no effect on the change in $v(t)$.

P 6.50 With no finger touching and equal 10 pF capacitors

$$v(t) = \frac{10}{20}(v_s(t)) + 0 = 0.5v_s(t)$$

With a finger touching

Let $\quad C_e =$ equivalent capacitance of person touching lamp

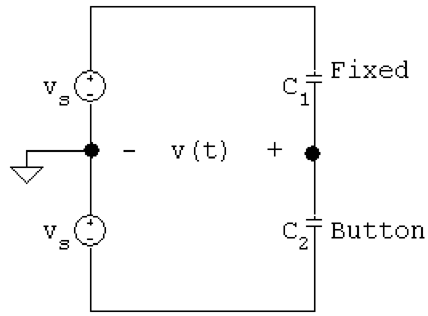
$$C_e = \frac{(10)(100)}{110} = 9.091 \text{ pF}$$

Then $\qquad C + C_e = 10 + 9.091 = 19.091$ pF

$$\therefore \quad v(t) = \frac{10}{29.091}v_s = 0.344v_s$$

$$\therefore \quad \Delta v(t) = (0.5 - 0.344)v_s = 0.156v_s$$

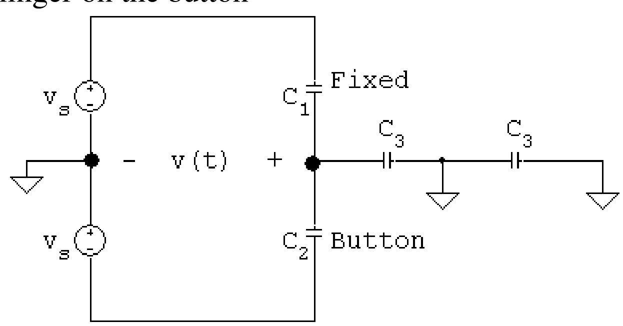
P 6.51 With no finger on the button the circuit is



$$C_1\frac{dv}{dt}(v - v_s) + C_2\frac{d}{dt}(v + v_s) = 0$$

when $\qquad C_1 = C_2 = C \qquad (2C)\dfrac{dv}{dt} = 0$

With a finger on the button



$$C_1\frac{d(v - v_s)}{dt} + C_2\frac{d(v + v_s)}{dt} + C_3\frac{dv}{dt} = 0$$

$$(C_1 + C_2 + C_3)\frac{dv}{dt} + C_2\frac{dv_s}{dt} - C_1\frac{dv_s}{dt} = 0$$

when $\qquad C_1 = C_2 = C_3 = C \qquad (3C)\dfrac{dv}{dt} = 0$

$\therefore$ there is no change in the output voltage of this circuit.