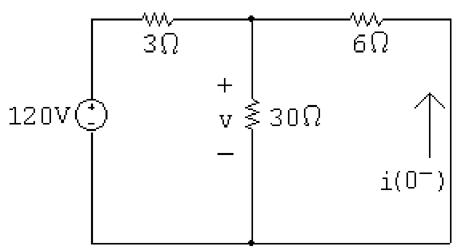
7

Response of First-Order RL and RC Circuits

Assessment Problems

AP 7.1 **[a]** The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.



First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:

$30\|6 = 5\,\Omega$

Use voltage division to find the voltage drop across the parallel resistors:

$$v = \frac{5}{5+3}(120) = 75\,\text{V}$$

Now find the current using Ohm's law:

$$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\,\text{A}$$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8 \times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,

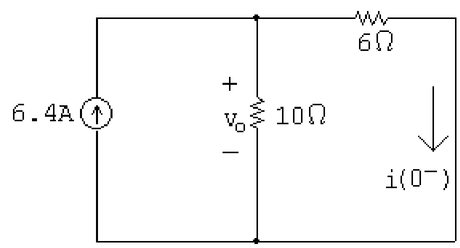
$$\tau = \frac{L}{R} = \frac{8 \times 10^{-3}}{2} = 4\,\text{ms}$$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

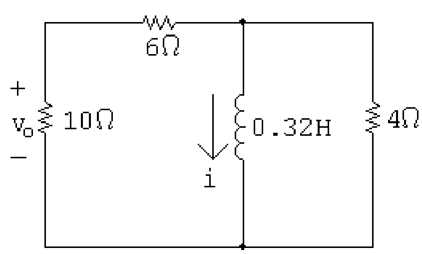
[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$

So $w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8)\times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$
$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$
$\%\text{ dissipated} = \left(\frac{573.7}{625}\right)100 = 91.8\%$

AP 7.2 **[a]** First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,
$$i(0^-) = \frac{10}{10+6}(6.4) = 4\,\text{A}$$
Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$$R_{\text{eq}} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \frac{L}{R_{\text{eq}}} = \frac{0.32}{3.2} = 0.1\,\text{s}$$
Use the initial inductor current and the time constant to find the current in the inductor:
$$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$$
Use current division to find the current in the $10\,\Omega$ resistor:
$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$
Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:
$$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$$

[b] The initial energy stored in the inductor is
$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$
Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:
$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

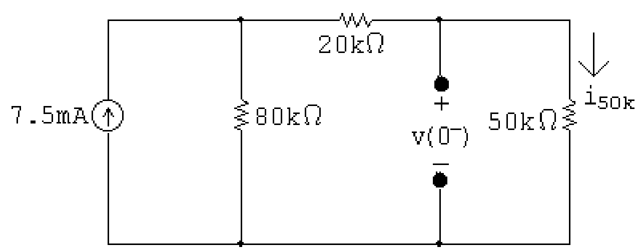
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\text{ dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 **[a]** The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\text{k}\Omega$ resistor. First use current division to find the current through the $50\,\text{k}\Omega$ resistor:

$$i_{50\text{k}} = \frac{80\times10^3}{80\times10^3 + 20\times10^3 + 50\times10^3}(7.5\times10^{-3}) = 4\,\text{mA}$$

Use Ohm's law to find the voltage drop:
$v(0^-) = (50\times10^3)i_{50\text{k}} = (50\times10^3)(0.004) = 200\,\text{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\text{k}\Omega$ resistor remains connected to the capacitor. Thus,
$\tau = RC = (50\times10^3)(0.4\times10^{-6}) = 20\,\text{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V}, \quad t \geq 0$

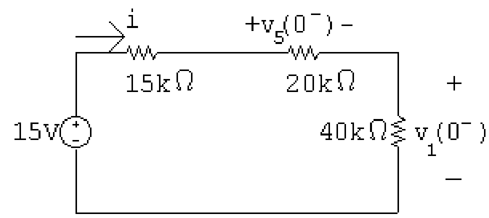
[d] $w(0) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4\times10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(0.4\times10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\text{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8\times10^{-3}e^{-100t} = 2\times10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

AP 7.4 **[a]** This circuit is actually two RC circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu$F – $20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu$F – $40\,\text{k}\Omega$ subcircuit:

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$

Therefore,

$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$

$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$

Finally,

$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$

$w_1(60\,\text{ms}) = \frac{1}{2}Cv_1^2(60\,\text{ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$

$w_5(60\,\text{ms}) = \frac{1}{2}Cv_5^2(60\,\text{ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$

$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$

Find the initial energy from the initial voltage:

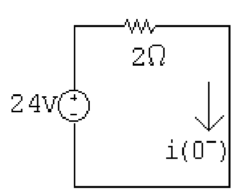
$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$

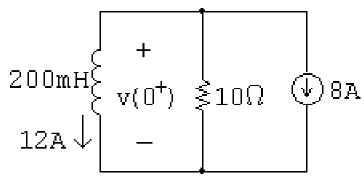
% dissipated $= (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$

AP 7.5 **[a]** Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$
Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

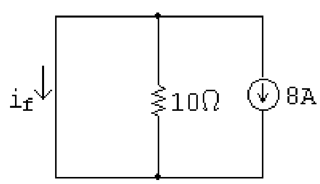
[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.



$v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,
$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:

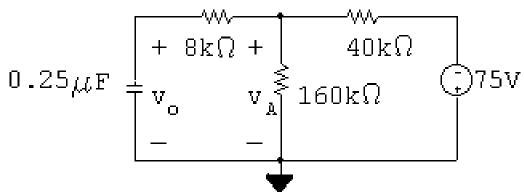


Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,
$i_f = -8\,\text{A}$
Now,
$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$
$= -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 **[a]**



From Example 7.6,

$v_o(t) = -60 + 90e^{-100t}$ V

Write a KVL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

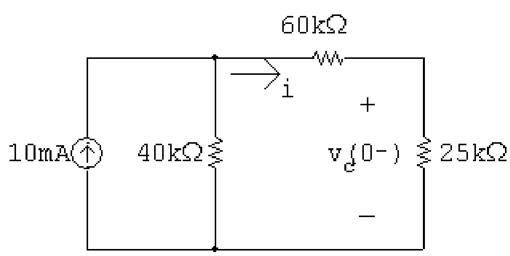
$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\text{ V}, \qquad t \geq 0^+$$

[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

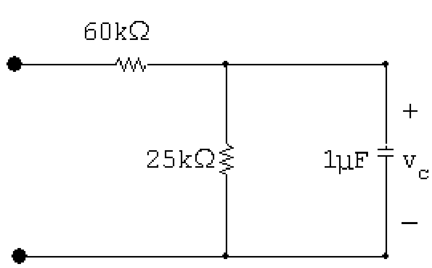
AP 7.7 **[a]** Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\text{ mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\text{ V} \quad \text{so} \quad v_c(0^+) = 80\text{ V}$$

Now use the next circuit, valid for $0 \leq t \leq 10$ ms, to calculate $v_c(t)$ for that interval:

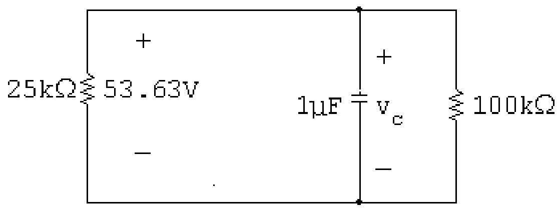


For $0 \le t \le 100\,\text{ms}$:

$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V}, \qquad 0 \le t \le 10\,\text{ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \ge 10\,\text{ms}$, using the capacitor voltage from the previous interval:
$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$
Now use the next circuit, valid for $t \ge 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $t \ge 10\,\text{ms}$:

$$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$$

Therefore $v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \ge 0.01\,\text{s}$

[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\,\text{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000}dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000}dt = 2.91\,\text{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

$$w_{100\,\text{k}\Omega} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000}dt = 0.29\,\text{mJ}$$

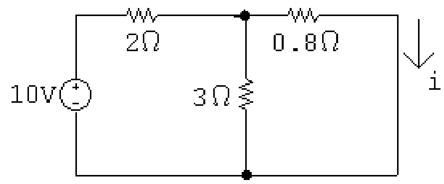
We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the $25\,\text{k}\Omega$ resistor and the $100\,\text{k}\Omega$ resistor.

Check: $w_{\text{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\text{mJ}$

$$w_{\text{diss}} = 2.91 + 0.29 = 3.2\,\text{mJ}$$

AP 7.8 **[a]** Note – the $30\,\Omega$ resistor should be a $3\,\Omega$ resistor; the resistor in parallel with the 8 A current source should be $9\,\Omega$.
Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.

For $0 \leq t \leq 1$ s,



The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\text{A}$$

The final current in the inductor, which is equal to the current in the $0.8\,\Omega$ resistor is

$$i(\infty) = \frac{3}{3 + 0.8}(3.8) = 3\,\text{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$0.8 + (2\|3) = 2\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2} = 1\,\text{s}$$

Therefore,

$$i = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau} = 3 - 3e^{-t}\,\text{A}, \quad 0 \leq t \leq 1\,\text{s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$$i(1) = 3 - 3e^{-1} = 1.896\,\text{A}$$

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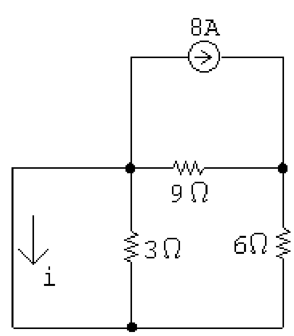
[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9 + 6}(-8) = -4.8\,\text{A}$$

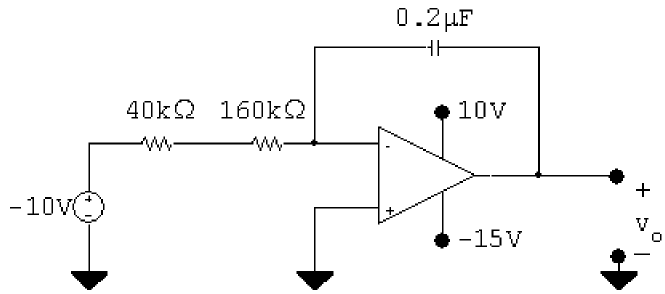
The equivalent resistance seen by the inductor is used to calculate the time constant:

$$3\|(9 + 6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\text{s}$$

Therefore,

$$i = i(\infty) + [i(1^+) - i(\infty)]e^{-(t-1)/\tau}$$
$$= -4.8 + 6.696e^{-1.25(t-1)}\,\text{A}, \quad t \geq 1\,\text{s}$$

AP 7.9 $0 \leq t \leq 32\,\text{ms}$:

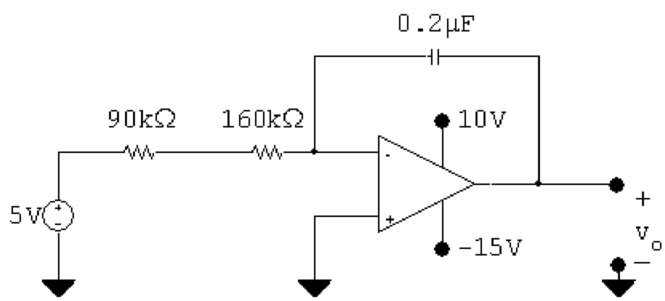


$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\text{V}$$

$t \geq 32$ ms:



$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} + 8 = -\frac{1}{RC_f}5(t - 32\times10^{-3}) + 8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t - 32\times10^{-3}) + 8 = -100t + 11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \quad \therefore \quad t = 262\,\text{ms}$$

AP 7.10 **[a]** Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0 + 2)e^{-t/\tau}$$

$$\tau = (160 \times 10^3)(10 \times 10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$$

[b] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1 + 2)e^{-625t} = -2 + 3e^{-625t}\,\text{V}$$

The analysis for v_o is the same as in part (a):

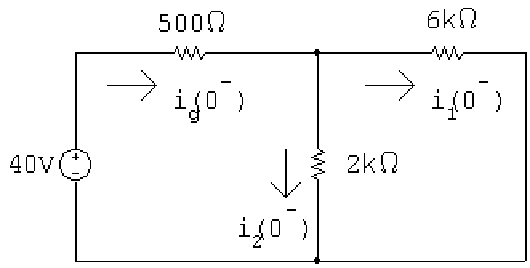
$$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$$

Problems

P 7.1 **[a]** $t < 0$



$2\,\text{k}\Omega \| 6\,\text{k}\Omega = 1.5\text{k}\Omega$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{40}{(1500 + 500)} = 20\,\text{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{2000}{8000}(0.02) = 5\,\text{mA}$$

$$i_2(0^-) = \frac{6000}{8000}(0.02) = 15\,\text{mA}$$

[b] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 5\,\text{mA}$$

$$i_2(0^+) = -i_1(0^+) = -5\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{0.4 \times 10^{-3}}{8 \times 10^3} = 5 \times 10^{-5}\,\text{s}; \qquad \dfrac{1}{\tau} = 20{,}000$

$$i_1(t) = i_1(0^+)e^{-t/\tau} = 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal $15\,\text{mA}$ and $i_2(0^+) = -5\,\text{mA}$.

P 7.2 **[a]** $i(0) = 60\,\text{V}/(10\,\Omega + 5\,\Omega) = 4\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{4}{45 + 5} = 80\,\text{ms}$

[c] $i = 4e^{-t/0.08} = 4e^{-12.5t}\,\text{A}, \qquad t \geq 0$

$$v_1 = -45i = -180e^{-12.5t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = L\frac{di}{dt} = (4)(-12.5)(4e^{-12.5t}) = -200e^{-12.5t}\,\text{V} \qquad t \geq 0^+$$

[d] $p_{\text{diss}} = i^2(45) = 720e^{-25t}\,\text{W}$

$$w_{\text{diss}} = \int_0^t 720e^{-25x}\,dx = 720\frac{e^{-25x}}{-25}\Bigg|_0^t = 28.8 - 28.8e^{-25t}\,\text{J}$$

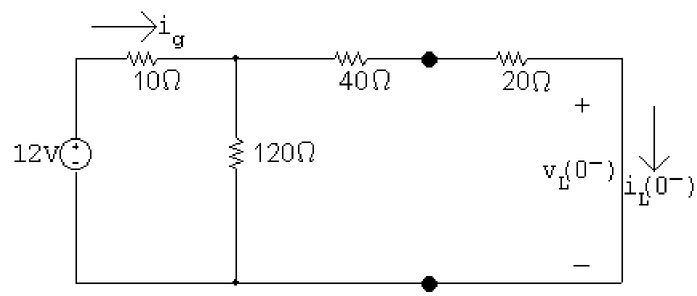
$$w_{\text{diss}}(40\,\text{ms}) = 28.8 - 28.8e^{-1} = 18.205\,\text{J}$$

$$w(0) = \frac{1}{2}(4)(4)^2 = 32\,\text{J}$$

$$\%\ \text{dissipated} = \frac{18.205}{32}(100) = 56.89\%$$

P 7.3 **[a]** $i_o(0^-) = 0$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

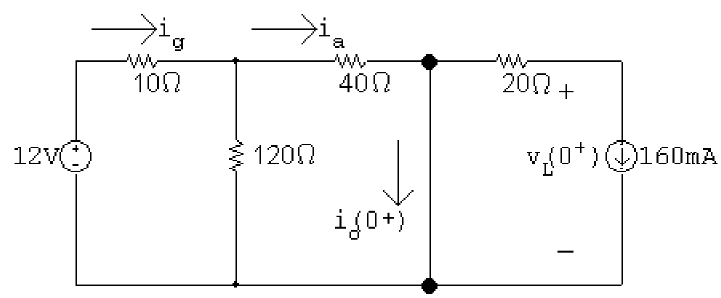


$$120\,\Omega \| 60\,\Omega = 40\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10+40} = 0.24\,\text{A} = 240\,\text{mA}$$

$$i_L(0^-) = \left(\frac{120}{180}\right) i_g = 160\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$120\,\Omega \| 40\,\Omega = 30\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10+30} = 0.30\,\text{A} = 300\,\text{mA}$$

$$i_\text{a} = \left(\frac{120}{160}\right)300 = 225\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 225 - 160 = 65\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 160\,\text{mA}$

[e] $i_o(\infty) = i_\text{a} = 225\,\text{mA}$

[f] $i_L(\infty) = 0,$ since the switch short circuits the branch containing the 20 Ω resistor and the 100 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{100\times 10^{-3}}{20} = 5\,\text{ms};\qquad \dfrac{1}{\tau} = 200$

$$\therefore \quad i_L = 0 + (160-0)e^{-200t} = 160e^{-200t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

$$20(0.16) + v_L(0^+) = 0; \qquad \therefore \quad v_L(0^+) = -3.2\,\text{V}$$

[j] $v_L(\infty) = 0,$ since the current in the inductor is a constant at $t = \infty$.

[k] $v_L(t) = 0 + (-3.2-0)e^{-200t} = -3.2e^{-200t}\,\text{V}, \qquad t \geq 0^+$

[l] $i_o = i_\text{a} - i_L = 225 - 160e^{-200t}\,\text{mA}, \qquad t \geq 0^+$

P 7.4 **[a]** $\dfrac{v}{i} = R = \dfrac{400e^{-5t}}{10e^{-5t}} = 40\,\Omega$

[b] $\tau = \dfrac{1}{5} = 200\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 200\times 10^{-3}$

$$L = (200\times 10^{-3})(40) = 8\,\text{H}$$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(8)(10)^2 = 400\,\text{J}$

[e] $w_\text{diss} = \displaystyle\int_0^t 4000e^{-10x}\,dx = 400 - 400e^{-10t}$

$$0.8w(0) = (0.8)(400) = 320\,\text{J}$$

$$400 - 400e^{-10t} = 320 \qquad \therefore \quad e^{10t} = 5$$

Solving, $t = 160.9$ ms.

P 7.5 **[a]** $i_L(0) = \frac{12}{6} = 2\,\text{A}$

$$i_o(0^+) = \frac{12}{2} - 2 = 6 - 2 = 4\,\text{A}$$

$$i_o(\infty) = \frac{12}{2} = 6\,\text{A}$$

[b] $i_L = 2e^{-t/\tau}; \qquad \tau = \frac{L}{R} = \frac{1}{4}\,\text{s}$

$$i_L = 2e^{-4t}\,\text{A}$$

$$i_o = 6 - i_L = 6 - 2e^{-4t}\,\text{A}, \qquad t \geq 0^+$$

[c] $6 - 2e^{-4t} = 5$

$$1 = 2e^{-4t}$$

$$e^{6t} = 2 \qquad \therefore \quad t = 173.3\,\text{ms}$$

P 7.6 $w(0) = \frac{1}{2}(30 \times 10^{-3})(3^2) = 135\,\text{mJ}$

$$\frac{1}{5}w(0) = 27\,\text{mJ}$$

$$i_R = 3e^{-t/\tau}$$

$$p_{\text{diss}} = i_R^2 R = 9Re^{-2t/\tau}$$

$$w_{\text{diss}} = \int_0^t R(9)e^{-2x/\tau}\,dx$$

$$w_{\text{diss}} = 9R\frac{e^{-2x/\tau}}{-2/\tau}\Bigg|_0^{t_o} = -4.5\tau R(e^{-2t_o/\tau} - 1) = 4.5L(1 - e^{-2t_o/\tau})$$

$$4.5L = (4.5)(30) \times 10^{-3} = 0.135; \qquad t_o = 15\,\mu\text{s}$$

$$1 - e^{-2t_o/\tau} = \frac{1}{5}$$

$$e^{2t_o/\tau} = 1.25; \qquad \frac{2t_o}{\tau} = \frac{2t_oR}{L} = \ln 1.25$$

$$R = \frac{L\ln 1.25}{2t_o} = \frac{30 \times 10^{-3}\ln 1.25}{30 \times 10^{-6}} = 223.14\,\Omega$$

P 7.7 **[a]** $w(0) = \frac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2 R e^{-2t/\tau}\, dt = I_g^2 R \frac{e^{-2t/\tau}}{(-2/\tau)} \Big|_0^{t_o}$$

$$= \frac{1}{2}I_g^2 R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2 L(1 - e^{-2t_o/\tau})$$

$$w_{\text{diss}} = \sigma w(0)$$

$$\therefore \quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \tau\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

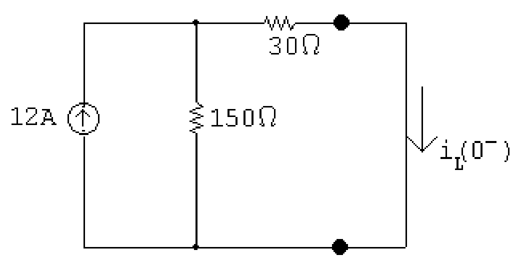
$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\,\ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(30 \times 10^{-3})\ln[1/0.8]}{30 \times 10^{-6}}$

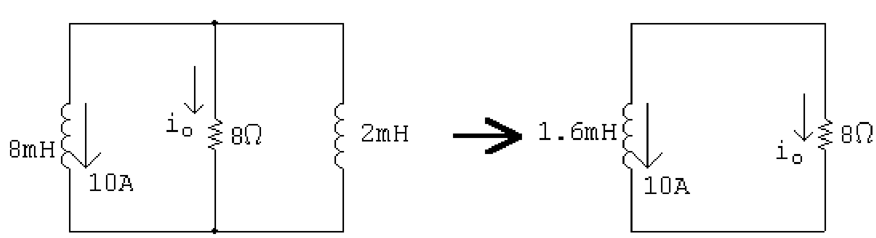
$$R = 223.14\,\Omega$$

P 7.8 **[a]** $t < 0$



$$i_L(0^-) = \frac{150}{180}(12) = 10\,\text{A}$$

$t \geq 0$



$$\tau = \frac{1.6 \times 10^{-3}}{8} = 200 \times 10^{-6}; \qquad 1/\tau = 5000$$

$$i_o = -10e^{-5000t}\,\text{A} \quad t \geq 0$$

[b] $w_{\text{del}} = \frac{1}{2}(1.6\times 10^{-3})(10)^2 = 80\,\text{mJ}$

[c] $0.95w_{\text{del}} = 76\,\text{mJ}$

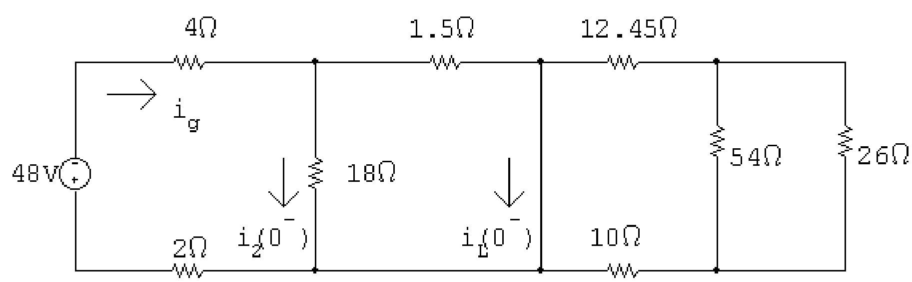
$$\therefore \quad 76\times 10^{-3} = \int_0^{t_o} 8(100e^{-10{,}000t})\,dt$$

$$\therefore \quad 76\times 10^{-3} = -80\times 10^{-3}e^{-10{,}000t}\Big|_0^{t_o} = 80\times 10^{-3}(1-e^{-10{,}000t_o})$$

$$\therefore \quad e^{-10{,}000t_o} = 4\times 10^{-3} \quad \text{so} \quad t_o = 552.1\,\mu\text{s}$$

$$\therefore \quad \frac{t_o}{\tau} = \frac{552.1\times 10^{-6}}{200\times 10^{-6}} = 2.76 \quad \text{so} \quad t_o \approx 2.76\tau$$

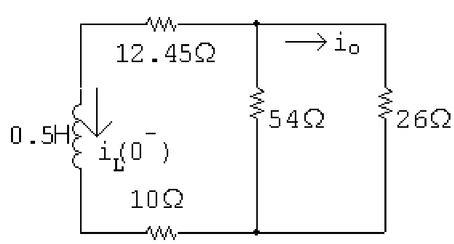
P 7.9 For $t < 0^+$



$$i_g = \frac{-48}{6+(18\|1.5)} = -6.5\,\text{A}$$

$$i_L(0^-) = \frac{18}{18+1.5}(-6.5) = -6\,\text{A} = i_L(0^+)$$

For $t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A}, \qquad t\geq 0$$

$$\tau = \frac{L}{R} = \frac{0.5}{10+12.45+(54\|26)} = 0.0125\,\text{s}; \qquad \frac{1}{\tau} = 80$$

$$i_L(t) = -6e^{-80t}\,\text{A}, \qquad t\geq 0$$

$$i_o(t) = \frac{54}{80}(-i_L(t)) = \frac{54}{80}(6e^{-80t}) = 4.05e^{-80t}\,\text{V}, \qquad t\geq 0^+$$

P 7.10 From the solution to Problem 7.9,

$$i_{54\Omega} = \frac{26}{80}(-i_L) = -1.95e^{-80t}\,\text{A}$$

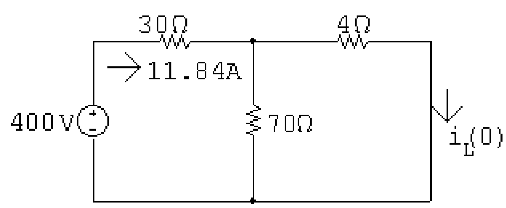
$$P_{54\Omega} = 54(i_{54\Omega})^2 = 205.335e^{-160t}\text{W}$$

$$\begin{aligned} w_{\text{diss}} &= \int_0^{0.0125} 205.335e^{-160t}\,dt \\ &= \frac{205.335}{-160}e^{-160t}\Big|_0^{0.0125} \\ &= 1.28(1-e^{-2}) = 1.11\,\text{J} \end{aligned}$$

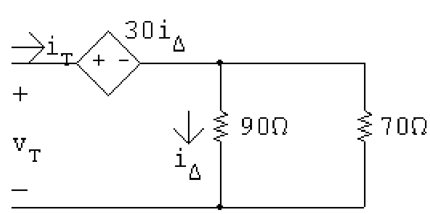
$$w_{\text{stored}} = \frac{1}{2}(0.5)(-6)^2 = 9\,\text{mJ}.$$

$$\%\text{ diss } = \frac{1.11}{9} \times 100 = 12.3\%$$

P 7.11 **[a]** $t < 0$:



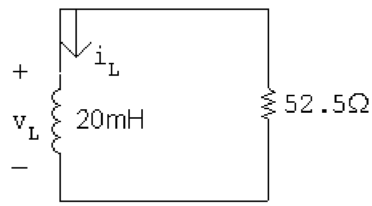
$$i_L(0^-) = i_L(0^+) = \frac{70}{70+4}(11.84) = 11.2\,\text{A}$$



$$i_\Delta = \frac{70}{160}i_T = 0.4375i_T$$

$$v_T = 30i_\Delta + i_T\frac{(90)(70)}{160} = 30(0.4375)i_T + \frac{(90)(70)}{160}i_T = 52.5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = 52.5\,\Omega$$

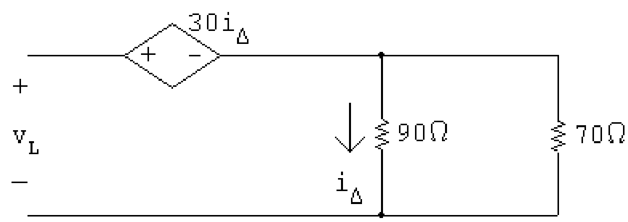


$$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{52.5} = \quad \therefore \qquad \frac{1}{\tau} = 2625$$

$$i_L = 11.2e^{-2625t}\,\mathrm{A}, \qquad t \geq 0$$

[b] $v_L = L\dfrac{di_L}{dt} = 20 \times 10^{-3}(-2625)(11.2e^{-2625t}) = -588e^{-2625t}\,\mathrm{V}, \quad t \geq 0^+$

[c]



$$v_L = 30i_\Delta + 90i_\Delta = 120i_\Delta$$

$$i_\Delta = \frac{v_L}{120} = -4.9e^{-2625t}\,\mathrm{A} \qquad t \geq 0^+$$

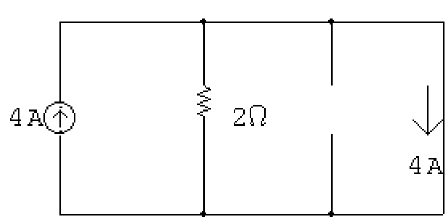
P 7.12 $\quad w(0) = \dfrac{1}{2}(20 \times 10^{-3})(11.2)^2 = 1254.4\,\mathrm{mJ}$

$$p_{30i_\Delta} = -30i_\Delta i_L = -30(-4.9e^{-2625t})(11.2e^{-2625t}) = 1646.4e^{-5250t}\,\mathrm{W}$$

$$w_{30i_\Delta} = \int_0^\infty 1646.4e^{-5250t}\,dt = 1646.4\frac{e^{-5250t}}{-5250}\Big|_0^\infty = 313.6\mathrm{mJ}$$

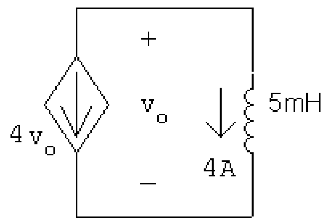
$$\%\text{ dissipated } = \frac{313.6}{1254.4}(100) = 25\%$$

P 7.13 $\quad t < 0$

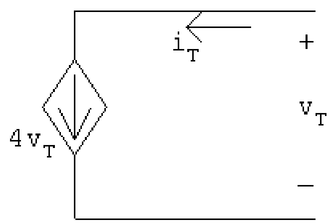


$$i_L(0^-) = i_L(0^+) = 4\,\mathrm{A}$$

$t > 0$

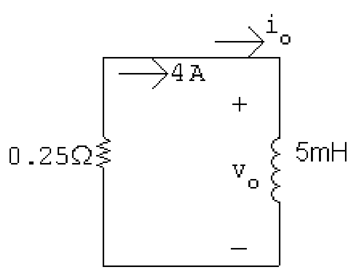


Find Thévenin resistance seen by inductor



$$i_T = 4v_T; \qquad \frac{v_T}{i_T} = R_{\rm Th} = \frac{1}{4} = 0.25\,\Omega$$

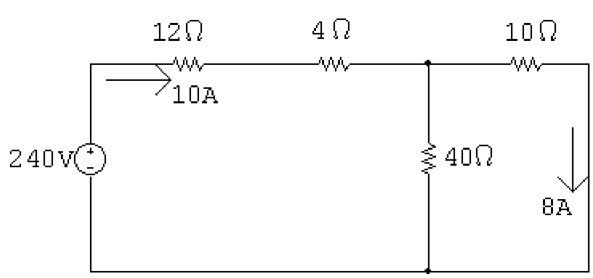
$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{0.25} = 20\,\text{ms}; \qquad 1/\tau = 50$$



$$i_o = 4e^{-50t}\,\text{A}, \qquad t \geq 0$$

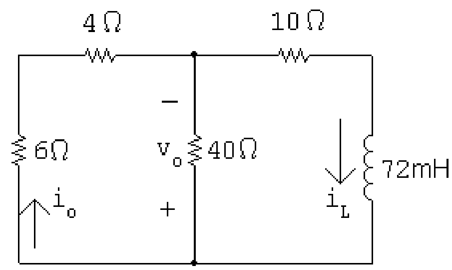
$$v_o = L\frac{di_o}{dt} = (5 \times 10^{-3})(-200e^{-50t}) = -e^{-50t}\,\text{V}, \quad t \geq 0^+$$

P 7.14 $t < 0$:



$$i_L(0^+) = 8\,\text{A}$$

$t > 0$:



$$R_e = \frac{(10)(40)}{50} + 10 = 18\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{0.072}{18} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

$$\therefore \quad i_L = 8e^{-250t}\,\text{A}$$

$$\therefore \quad v_o = -10i_L - 0.072\frac{di_L}{dt} = -80e^{-250t} + 144e^{-250t}$$

$$= 64e^{-250t}\,\text{A} \quad t \geq 0^+$$

P 7.15 $w(0) = \frac{1}{2}(72 \times 10^{-3})(8)^2 = 2304\,\text{mJ}$

$$p_{40\Omega} = \frac{v_o^2}{40} = \frac{64^2}{40}e^{-500t} = 102.4e^{-500t}\,\text{W}$$

$$w_{40\Omega} = \int_0^\infty 102.4e^{-500t}\,dt = 204.8\,\text{mJ}$$

$$\%\text{diss} = \frac{204.8}{2304}(100) = 8.89\%$$

P 7.16 **[a]** $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-1\times10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore \quad e^{1\times10^{-3}/\tau} = 2$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{1 \times 10^{-3}}{\ln 2}$$

$$\therefore \quad L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43\,\text{mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10(1/10)30 \times 10^{-3} = -30\,\text{mV}$

$$\therefore \qquad v_o = -0.03e^{-t/\tau}\,\text{V}, \quad t \geq 0^+$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5}e^{-2t/\tau}$$

$$w_{10\Omega}(1\,\text{ms}) = \int_{0^+}^{10^{-3}} 9 \times 10^{-5}e^{-2t/\tau}\,dt$$

$$= 4.5\tau \times 10^{-5}(1 - e^{-2(0.001)/\tau})$$

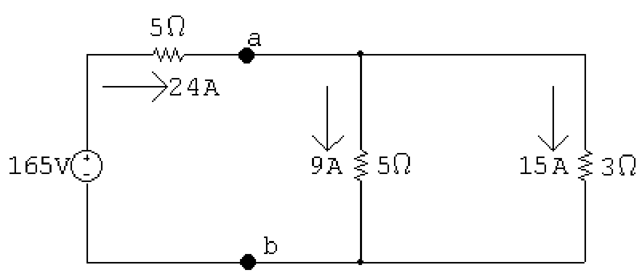
$$\tau = \frac{1}{1000\ln 2}$$

$$\therefore \qquad w_{10\Omega}(1\,\text{ms}) = 48.69\,\text{nJ}$$

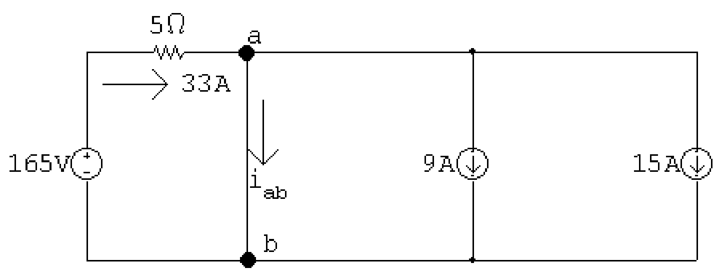
$$w_L(0) = \frac{1}{2}Li_L^2(0) = \frac{1}{2}(14.43 \times 10^{-3})(3 \times 10^{-3})^2 = 64.92\,\text{nJ}$$

$$\%\text{dissipated in 1 ms} = \frac{48.69}{64.92}(100) = 75\%$$

P 7.17 **[a]** $t < 0$:

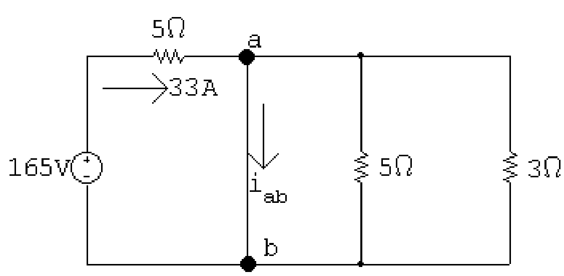


$t = 0^+$:

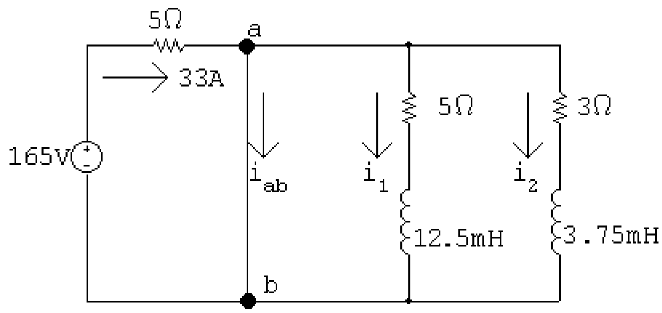


$$33 = i_{\text{ab}} + 9 + 15, \qquad i_{\text{ab}} = 9\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:



$$i_{\text{ab}} = 165/5 = 33\,\text{A}, \quad t = \infty$$



[c] $i_1(0) = 9, \qquad \tau_1 = \dfrac{12.5 \times 10^{-3}}{5} = 2.5\,\text{ms}$

$$i_2(0) = 15, \qquad \tau_2 = \frac{3.75 \times 10^{-3}}{3} 1.25\,\text{ms}$$

$$i_1(t) = 9e^{-400t}\,\text{A}, \quad t \geq 0$$

$$i_2(t) = 15e^{-800t}\,\text{A}, \quad t \geq 0$$

$$i_{\text{ab}} = 33 - 9e^{-400t} - 15e^{-800t}\,\text{A}, \quad t \geq 0^+$$

$$33 - 9e^{-400t} - 15e^{-800t} = 19$$

$$14 = 9e^{-400t} + 15e^{-800t}$$

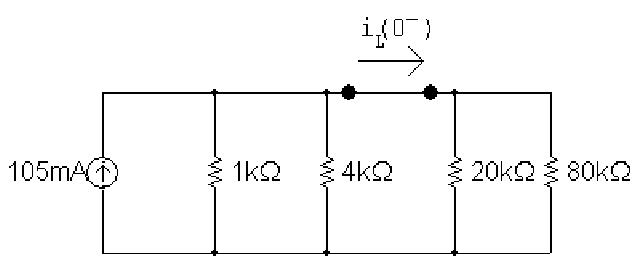
Let $\quad x = e^{-400t} \quad \therefore \quad x^2 = e^{-800t}$

Substituting,

$15x^2 + 9x - 14 = 0 \quad$ so $\quad x = 0.7116 = e^{-400t}$

$$\therefore \qquad t = \frac{[\ln(1/0.7116)]}{400} = 850.6\,\mu\text{s}$$

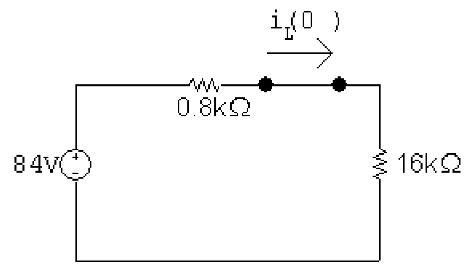
P 7.18 **[a]** $t < 0$



$$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$$

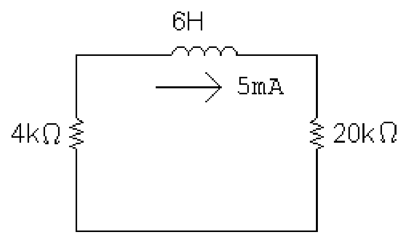
$$20\,\text{k}\Omega \| 80\,\text{k}\Omega = 16\,\text{k}\Omega$$

$$(105 \times 10^{-3})(0.8 \times 10^{3}) = 84\,\text{V}$$



$$i_L(0^-) = \frac{84}{16{,}800} = 5\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{4k} = 25 \times 10^{-6}e^{-8000t}(4000) = 0.10e^{-8000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\text{J}$$

$$0.10w(0) = 7.5\,\mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

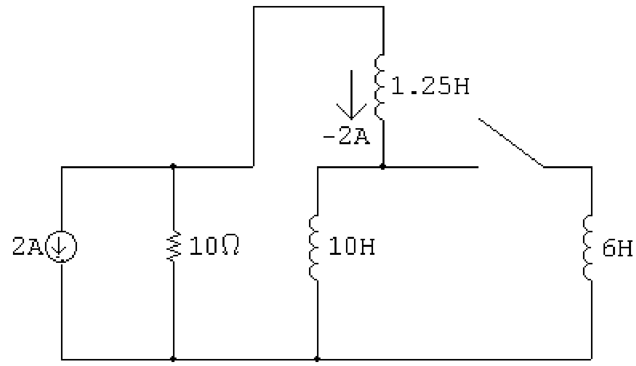
$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t})\,\mu\text{J}$

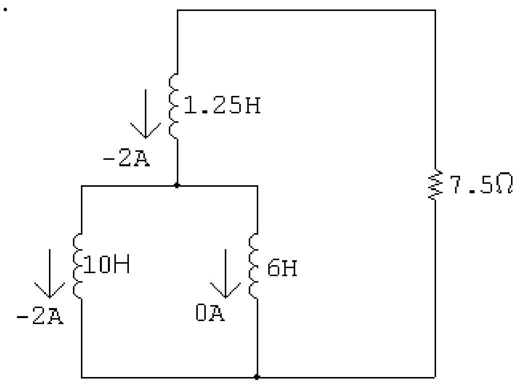
$$w_{\text{diss}}(114.54\,\mu\text{s}) = 45\,\mu\text{J}$$

$$\% = (45/75)(100) = 60\%$$

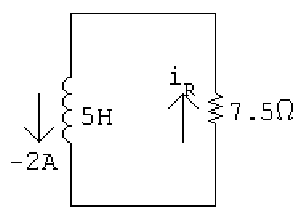
P 7.19 **[a]** $t < 0$:



$t = 0^+$:

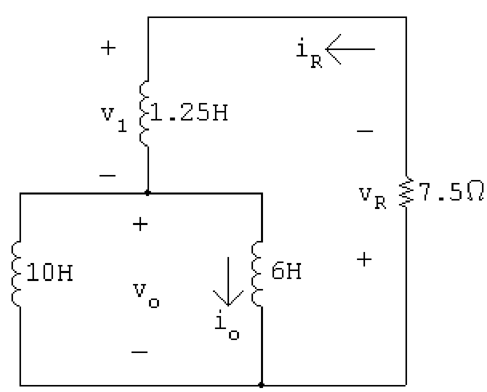


$t > 0$:



$i_R = -2e^{-t/\tau}$ A; $\qquad \tau = \dfrac{L}{R} = \dfrac{5}{7.5} = 666.67\,\text{ms} \quad \therefore \quad \dfrac{1}{\tau} = 1.5$

$i_R = -2e^{-1.5t}$ A



$v_R = (7.5)(-2e^{-1.5t}) = -15e^{-1.5t}$ V

$$v_1 = 1.25[(-1.5)(-2e^{-1.5t})] = 3.75e^{-1.5t}\,\text{V},$$

$$v_o = -v_1 - v_R = 11.25e^{-1.5t}\,\text{V} \quad t \geq 0^+$$

[b] $i_o = \dfrac{1}{6}\displaystyle\int_0^t 11.25e^{-1.5x}\,dx + 0 = 1.25 - 1.25e^{-1.5t}\,\text{A} \quad t \geq 0$

P 7.20 **[a]** From the solution to Problem 7.19,

$$i_R = -2e^{-1.5t}\,\text{A}$$

$$p_R = (-2e^{-1.5t})^2(7.5) = 30e^{-3t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^\infty 30e^{-3t}\,dt$$

$$= 30\frac{e^{-3t}}{-3}\Big|_0^\infty = 10\,\text{J}$$

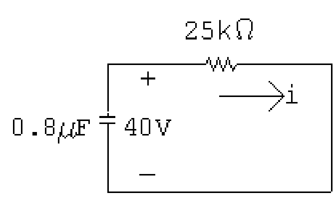
[b] $w_{\text{trapped}} = \frac{1}{2}(10)(-1.25)^2 + \frac{1}{2}(6)(1.25)^2 = 12.5\,\text{J}$

CHECK: $w(0) = \frac{1}{2}(1.25)(2)^2 + \frac{1}{2}(10)(2)^2 = 22.5\,\text{J}$

$\therefore \quad w(0) = w_{\text{diss}} + w_{\text{trapped}}$

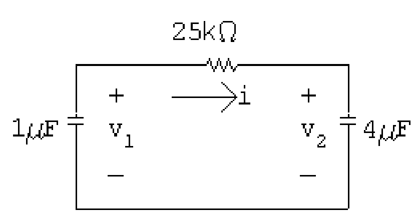
P 7.21 **[a]** $v_1(0^-) = v_1(0^+) = 40\,\text{V} \qquad v_2(0^+) = 0$

$$C_{\text{eq}} = (1)(4)/5 = 0.8\,\mu\text{F}$$



$$\tau = (25\times 10^3)(0.8\times 10^{-6}) = 20\text{ms}; \qquad \frac{1}{\tau} = 50$$

$$i = \frac{40}{25{,}000}e^{-50t} = 1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-1}{10^{-6}}\int_0^t 1.6\times 10^{-3}e^{-50x}\,dx + 40 = 32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{4\times 10^{-6}}\int_0^t 1.6\times 10^{-3}e^{-50x}\,dx + 0 = -8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(10^{-6})(40)^2 = 800\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(10^{-6})(8)^2 + \frac{1}{2}(4 \times 10^{-6})(8)^2 = 160\,\mu\text{J}.$

The energy dissipated by the 25 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors (final voltage on the equivalent capacitor is zero):

$$w_{\text{diss}} = \frac{1}{2}(0.8 \times 10^{-6})(40)^2 = 640\,\mu\text{J}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 160 + 640 = 800\,\mu\text{J}; \qquad w(0) = 800\,\mu\text{J}.$

P 7.22 **[a]** Calculate the initial voltage drop across the capacitor:

$$v(0) = (2.7\,\text{k}\|3.3\,\text{k})(40\,\text{mA}) = (1485)(40 \times 10^{-3}) = 59.4\,\text{V}$$

The equivalent resistance seen by the capacitor is

$$R_e = 3\,\text{k}\|(2.4\,\text{k} + 3.6\,\text{k}) = 3\,\text{k}\|6\,\text{k} = 2\,\text{k}\Omega$$

$$\tau = R_eC = (2000)(0.5) \times 10^{-6} = 1000\,\mu\text{s}; \qquad \frac{1}{\tau} = 1000$$

$$v = v(0)e^{-t/\tau} = 59.4e^{-1000t}\,\text{V} \qquad t \geq 0$$

$$i_o = \frac{v}{2.4\,\text{k} + 3.6\,\text{k}} = 9.9e^{-1000t}\,\text{mA}, \quad t \geq 0^+$$

[b] $w(0) = \frac{1}{2}(0.5 \times 10^{-6})(59.4)^2 = 882.09\,\mu\text{J}$

$$i_{3k} = \frac{59.4e^{-1000t}}{3000} = 19.8e^{-1000t}\,\text{mA}$$

$$p_{3k} = [(19.8 \times 10^{-3})e^{-1000t}]^2(3000) = 1.176e^{-2000t}$$

$$w_{3k}(500\,\mu\text{s}) = 1.176\frac{e^{-2000x}}{-2000}\Big|_0^{500\times10^{-6}} = \frac{1.176}{-2000}(e^{-1} - 1) = 371.72\,\mu\text{J}$$

$$\% = \frac{371.72}{882.09} \times 100 = 42.14\%$$

P 7.23 **[a]** $R = \frac{v}{i} = 4\,\text{k}\Omega$

[b] $\frac{1}{\tau} = \frac{1}{RC} = 25; \qquad C = \frac{1}{(25)(4 \times 10^3)} = 10\,\mu\text{F}$

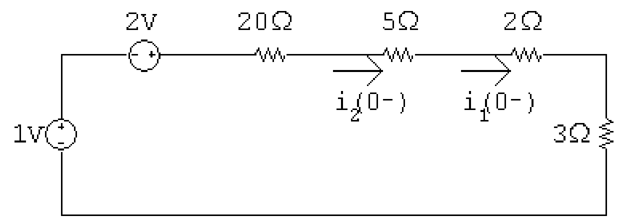
[c] $\tau = \frac{1}{25} = 40\,\text{ms}$

[d] $w(0) = \frac{1}{2}(10 \times 10^{-6})(48)^2 = 11.52\,\text{mJ}$

[e] $w_{\text{diss}}(60\,\text{ms}) = \int_0^{0.06} \frac{v^2}{R}dt = \int_0^{0.06} \frac{(48e^{-25t})^2}{(4\times 10^3)}dt$

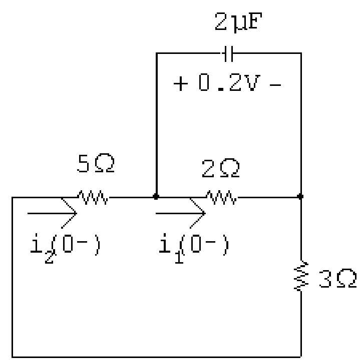
$$= 0.576\frac{e^{-50t}}{-50}\Big|_0^{0.06} = -5.74\times 10^{-4} + 0.01152 = 10.95\,\text{mJ}$$

P 7.24 **[a]** $t < 0$:



$$i_1(0^-) = i_2(0^-) = \frac{3\,\text{V}}{30\,\Omega} = 100\,\text{mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{0.2}{2} = 100\,\text{mA}$$

$$i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 100\,\text{mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 100\,\text{mA} \quad \text{and} \quad i_2(0^+) = -25\,\text{mA}$$

[e] $v_c = 0.2e^{-t/\tau}\,\text{V}, \quad t \geq 0 \qquad R_e = 2||(5+3) = 1.6\,\Omega$

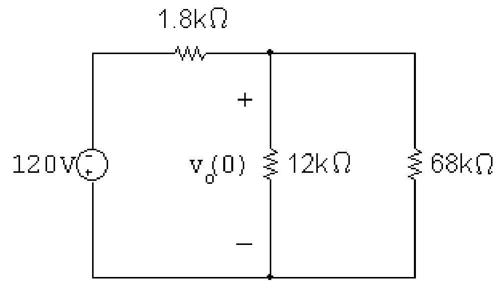
$$\tau = 1.6(2\times 10^{-6}) = 3.2\times 10^{-6}\,\text{s}$$

$$v_c = 0.2e^{-312{,}500t}\,\text{V}, \qquad t \geq 0$$

$$i_1 = \frac{v_c}{2} = 0.1e^{-312{,}500t}\,\text{A}, \qquad t \geq 0$$

[f] $i_2 = \frac{-v_c}{8} = -25e^{-312{,}500t}\,\text{mA}, \qquad t \geq 0^+$

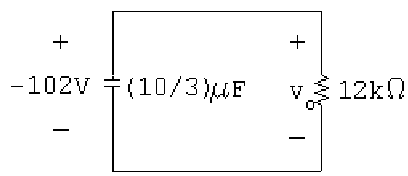
P 7.25 **[a]** $t < 0$:



$$R_e = 12\text{ k}||68\text{ k} = 10.2\text{ k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:



$$\tau = [(10/3)\times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867\times 10^{-3}e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12\times 10^{-3}} 867\times 10^{-3}e^{-50t}\,dt$$
$$= 17.34\times 10^{-3}(1 - e^{-50(12\times 10^{-3})}) = 7.82\,\text{mJ}$$

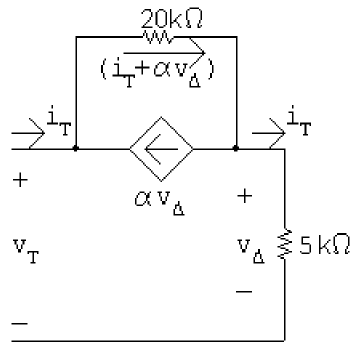
[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$$0.75w(0) = 13\,\text{mJ}$$

$$\int_0^{t_o} 867\times 10^{-3}e^{-50x}\,dx = 13\times 10^{-3}$$

$$\therefore \quad 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,ms$$

P 7.26 **[a]**



$$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$$

$$v_\Delta = 5 \times 10^3 i_T$$

$$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha (5 \times 10^3 i_T)$$

$$R_{\text{Th}} = 25{,}000 + 100 \times 10^6 \alpha$$

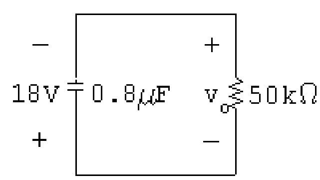
$$\tau = R_{\text{Th}} C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$$

$$R_{\text{Th}} = 50\,\text{k}\Omega = 25{,}000 + 100 \times 10^6 \alpha$$

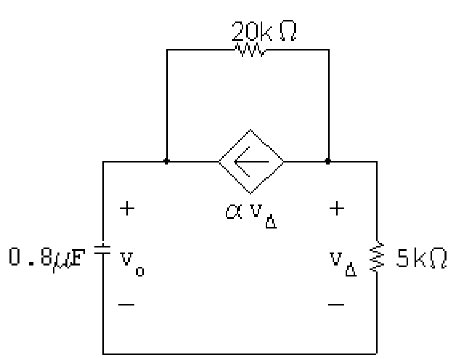
$$\alpha = \frac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4}\,\text{A/V}$$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = -18e^{-25t}\,\text{V}, \quad t \geq 0$$

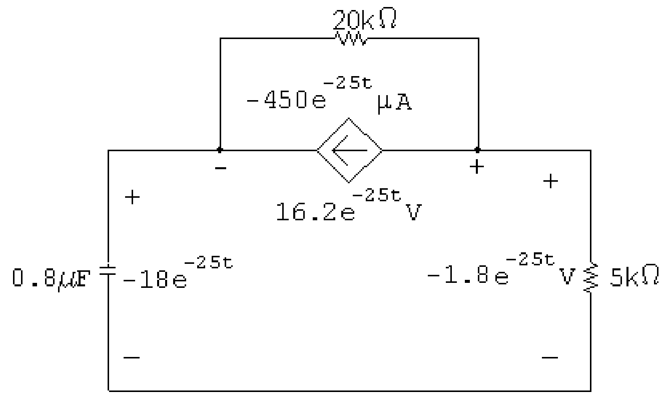


$$\frac{v_\Delta}{5000} + \frac{v_\Delta - v_o}{20{,}000} + 2.5 \times 10^{-4} v_\Delta = 0$$

$$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$$

$$\therefore \quad v_\Delta = \frac{v_o}{10} = -1.8e^{-25t}\,\text{V}, \quad t \geq 0^+$$

P 7.27 **[a]**



$$p_{ds} = (16.2e^{-25t})(-450 \times 10^{-6}e^{-25t}) = -7290 \times 10^{-6}e^{-50t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -145.8\,\mu\text{J}.$$

$\therefore$ dependent source is delivering $145.8\,\mu$J

[b] $$w_{5\text{k}} = \int_0^\infty (5000)(0.36 \times 10^{-3}e^{-25t})^2\,dt = 648 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 12.96\,\mu\text{J}$$

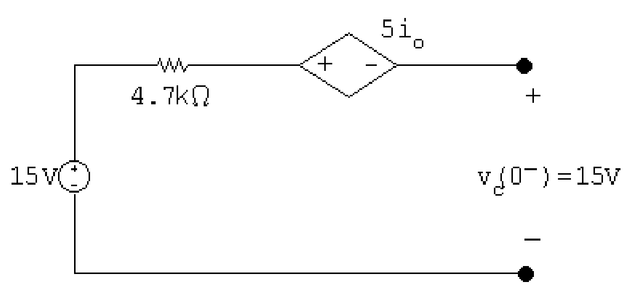
$$w_{20\text{k}} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20{,}000}\,dt = 13{,}122 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 262.44\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.6\,\mu\text{J}$$

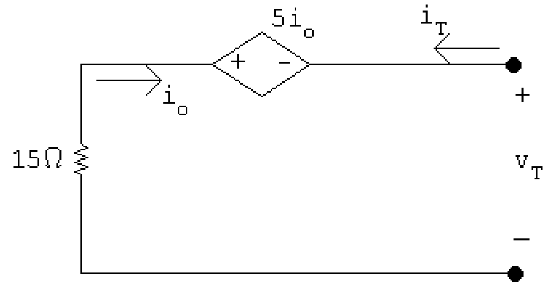
$$\sum w_{\text{diss}} = 12.96 + 262.44 = 275.4\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 145.8 + 129.6 = 275.4\,\mu\text{J}.$$

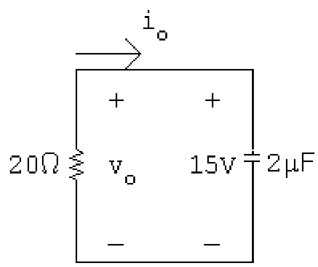
P 7.28 $t < 0$



$t > 0$



$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \qquad \therefore \qquad R_{\rm Th} = \frac{v_T}{i_T} = 20\,\Omega$$

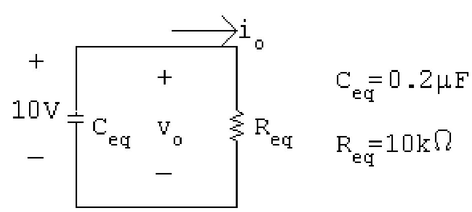


$$\tau = RC = 40\,\mu\text{s}; \qquad \frac{1}{\tau} = 25{,}000$$

$$v_o = 15e^{-25{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.29 **[a]** The equivalent circuit for $t > 0$:



$$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{24k\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$p_{24k\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$$

$$w_{24k\Omega} = \int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0-1) = 3.84\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25 \times 10^{-6})(40)^2 + \frac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45\,\text{mJ}$$

$$\%\ \text{diss}\,(24\,\text{k}\Omega) = \frac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$$

[b] $p_{400\Omega} = 400(1 \times 10^{-3}e^{-500t})^2 = 0.4 \times 10^{-3}e^{-1000t}$

$$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$$

$$\%\ \text{diss}\,(400\Omega) = \frac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$$

$$i_{16k\Omega} = e^{-500t}\left(\frac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

$$p_{16k\Omega} = (0.6 \times 10^{-3}e^{-500t})^2(16{,}000) = 5.76 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{16k\Omega} = \int_0^\infty 5.76 \times 10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$$

$$\%\ \text{diss}\,(16\text{k}\Omega) = 0.4\%$$

[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44\,\text{mJ}$$

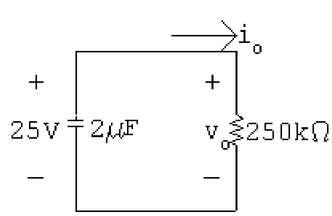
$$\%\ \text{trapped} = \frac{1.44}{1.45} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

P 7.30 **[a]** $C_e = \dfrac{(2+1)6}{2+1+6} = 2\,\mu\text{F}$

$$v_o(0) = -5 + 30 = 25\,\text{V}$$

$$\tau = (2 \times 10^{-6})(250 \times 10^3) = 0.5\,\text{s}; \qquad \frac{1}{\tau} = 2$$



$$v_o = 25e^{-2t}\,\text{V}, \qquad t > 0^+$$

[b] $w_o = \frac{1}{2}(3\times10^{-6})(30)^2 + \frac{1}{2}(6\times10^{-6})(5)^2 = 1425\,\mu\text{J}$

$$w_{\text{diss}} = \frac{1}{2}(2\times10^{-6})(25)^2 = 625\,\mu\text{J}$$

$$\%\text{ diss } = \frac{1425-625}{1425}\times100 = 56.14\%$$

[c] $i_o = \frac{v_o}{250\times10^{-3}} = 100e^{-2t}\,\mu\text{A}$

$$v_1 = -\frac{1}{6\times10^{-6}}\int_0^t 100\times10^{-6}e^{-2x}\,dx - 5 = -16.67\int_0^t e^{-2x}\,dx - 5$$

$$= -16.67\frac{e^{-2x}}{-2}\Big|_0^t -5 = 8.33e^{-2t} - 13.33\,\text{V} \qquad t\ge0$$

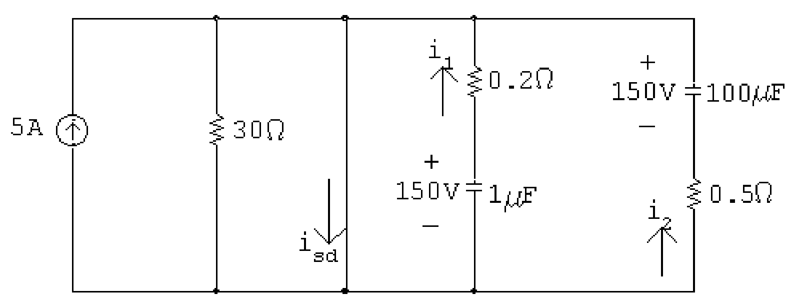
[d] $v_1 + v_2 = v_o$

$$v_2 = v_o - v_1 = 25e^{-2t} - 8.33e^{-2t} + 13.33 = 16.67e^{-2t} + 13.33\,\text{V} \quad t\ge0$$

[e] $w_{\text{trapped}} = \frac{1}{2}(6\times10^{-6})(13.33)^2 + \frac{1}{2}(3\times10^{-6})(13.33)^2 = 800\,\mu\text{J}$

$$w_{\text{diss}} + w_{\text{trapped}} = 625 + 800 = 1425\,\mu\text{J} \qquad \text{(check)}$$

P 7.31 **[a]** At $t = 0^-$ the voltage on each capacitor will be $150\,\text{V}(5\times30)$, positive at the upper terminal. Hence at $t\ge0^+$ we have



$$\therefore\quad i_{sd}(0^+) = 5 + \frac{150}{0.2} + \frac{150}{0.5} = 1055\,\text{A}$$

At $t=\infty$, both capacitors will have completely discharged.

$$\therefore\quad i_{sd}(\infty) = 5\,\text{A}$$

[b] $i_{sd}(t) = 5 + i_1(t) + i_2(t)$

$$\tau_1 = 0.2(10^{-6}) = 0.2\,\mu\text{s}$$

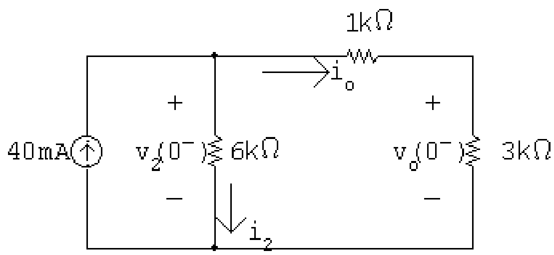
$$\tau_2 = 0.5(100\times10^{-6}) = 50\,\mu\text{s}$$

$$\therefore \quad i_1(t) = 750e^{-5\times10^6 t}\,\text{A}, \qquad t \geq 0^+$$

$$i_2(t) = 300e^{-20{,}000t}\,\text{A}, \qquad t \geq 0$$

$$\therefore \quad i_{sd} = 5 + 750e^{-5\times10^6 t} + 300e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

P 7.32 **[a]** $t < 0$:



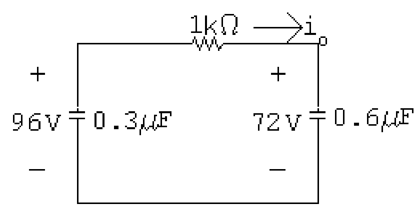
$$i_o(0^-) = \frac{6000}{6000 + 4000}(40\,\text{m}) = 24\,\text{mA}$$

$$v_o(0^-) = (3000)(24\,\text{m}) = 72\,\text{V}$$

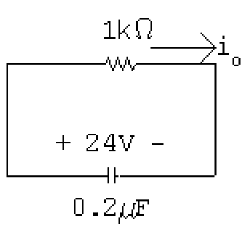
$$i_2(0^-) = 40 - 24 = 16\,\text{mA}$$

$$v_2(0^-) = (6000)(16\,\text{m}) = 96\,\text{V}$$

$t > 0$

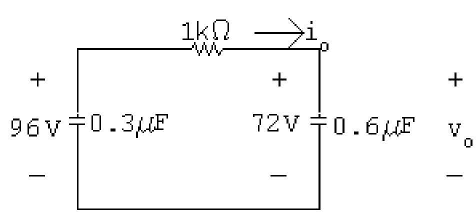


$$\tau = RC = (1000)(0.2 \times 10^{-6}) = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{24}{1 \times 10^3}e^{-t/\tau} = 24e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$v_o = \frac{1}{0.6 \times 10^{-6}} \int_0^t 24 \times 10^{-3} e^{-5000x}\, dx + 72$$
$$= (40{,}000) \frac{e^{-5000x}}{-5000} \Big|_0^t + 72$$
$$= -8e^{-5000t} + 8 + 72$$
$$v_o = [-8e^{-5000t} + 80]\,\text{V}, \qquad t \geq 0$$

[c] $w_{\text{trapped}} = (1/2)(0.3 \times 10^{-6})(80)^2 + (1/2)(0.6 \times 10^{-6})(80)^2$

$w_{\text{trapped}} = 2880\,\mu\text{J}.$

Check:

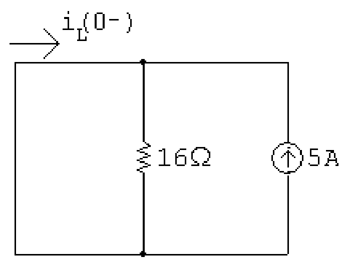
$$w_{\text{diss}} = \frac{1}{2}(0.2 \times 10^{-6})(24)^2 = 57.6\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.3 \times 10^{-6})(96)^2 + \frac{1}{2}(0.6 \times 10^{-6})(72)^2 = 2937.6\,\mu\text{J}.$$

$$w_{\text{trapped}} + w_{\text{diss}} = w(0)$$

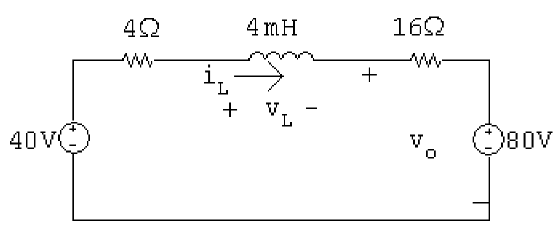
$2880 + 57.6 = 2937.6$ OK.

P 7.33 **[a]** $t < 0$



$i_L(0^-) = -5\,\text{A}$

$t > 0$



$$i_L(\infty) = \frac{40 - 80}{4 + 16} = -2\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{4 \times 10^{-3}}{4 + 16} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -2 + (-5+2)e^{-5000t} = -2 - 3e^{-5000t}\,\text{A}, \qquad t \geq 0$$

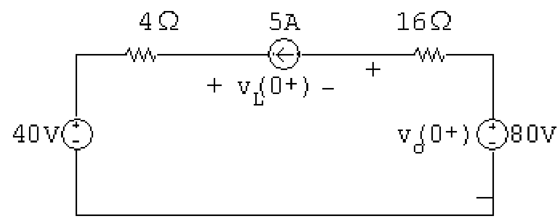
$$v_o = 16i_L + 80 = 16(-2 - 3e^{-5000t}) + 80 = 48 - 48e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_L = L\dfrac{di_L}{dt} = 4\times 10^{-3}(-5000)[-3e^{-5000t}] = 60e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

$v_L(0^+) = 60\,\text{V}$

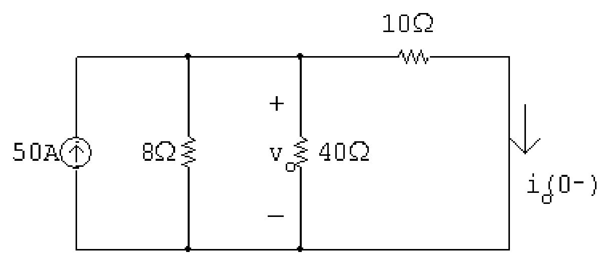
From part (a) $\qquad v_o(0^+) = 0\,\text{V}$

Check: at $t = 0^+$ the circuit is:



$v_L(0^+) = 40 + (5\,\text{A})(4\,\Omega) = 60\,\text{V}, \qquad v_o(0^+) = 80 - (16\,\Omega)(5\,\text{A}) = 0\,\text{V}$

P 7.34 **[a]** $t < 0$



KVL equation at the top node:

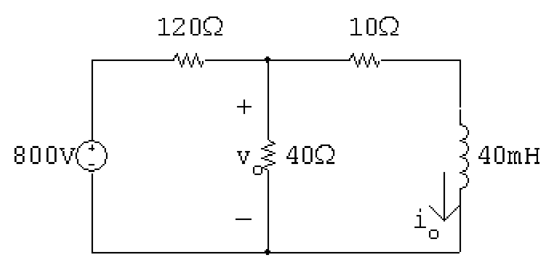
$$50 = \frac{v_o}{8} + \frac{v_o}{40} + \frac{v_o}{10}$$

Multiply by 40 and solve:

$2000 = (5+1+4)v_o; \qquad v_o = 200\,\text{V}$

$\therefore \quad i_o(0^-) = \dfrac{v_o}{10} = 200/10 = 20\,\text{A}$

$t > 0$



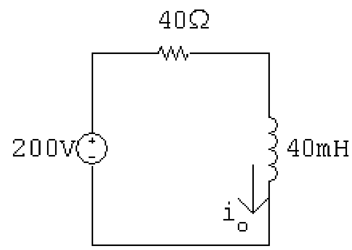
Use voltage division to find the Thévenin voltage:

$$V_{\rm Th} = v_o = \frac{40}{40+120}(800) = 200\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\rm Th} = 10 + 120\|40 = 10 + 30 = 40\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{40\times 10^{-3}}{40} = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

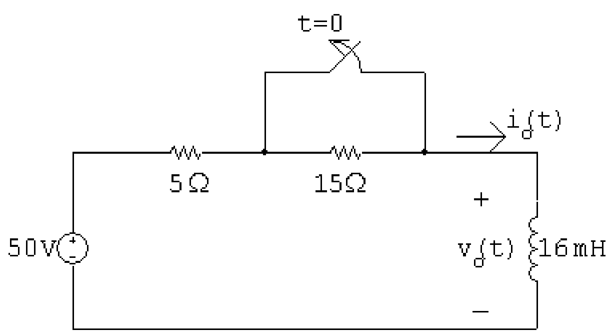
$$i_o(\infty) = \frac{200}{40} = 5\,\text{A}$$

$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 5 + (20-5)e^{-1000t} = 5 + 15e^{-1000t}\,\text{A}, \qquad t \geq 0$$

[b]
$$\begin{aligned} v_o &= 10i_o + L\frac{di_o}{dt} \\ &= 10(5+15e^{-1000t}) + 0.04(-1000)(15e^{-1000t}) \\ &= 50 + 150e^{-1000t} - 600e^{-1000t} \\ v_o &= 50 - 450e^{-1000t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

P 7.35 After making a Thévenin equivalent we have



For $t < 0$, the $15\,\Omega$ resistor is bypassed:

$$i_o(0^-) = i_o(0^+) = 50/5 = 10\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{16\times 10^{-3}}{5+15} = 8\times 10^{-4}; \qquad \frac{1}{\tau} = 1250$$

$$i(\infty) = \frac{V}{R_{\text{eq}}} = \frac{50}{5+15} = 2.5\,\text{A}$$

$$i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau} = 2.5 + (10-2.5)e^{-1250t} = 2.5 + 7.5e^{-1250t}\,\text{A}, t \geq 0$$

$$v_o = L\frac{di_o}{dt} = 16\times 10^{-3}(-1250)(7.5e^{-1250t}) = -150e^{-1250t}\,\text{V}, \quad t \geq 0^+$$

P 7.36 **[a]** $v_o(0^+) = -I_gR_2; \qquad \tau = \dfrac{L}{R_1+R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_gR_2e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_o = -(10)(15)e^{-\frac{(5+15)}{0.016}t} = -150e^{-1250t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o(0^+) \to \infty,$ and the duration of $v_o(t) \to$ zero

[d] $v_{sw} = R_2i_o; \qquad \tau = \dfrac{L}{R_1+R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\frac{R_1}{R_1+R_2}$$

Therefore $\quad i_o(t) = \frac{I_gR_1}{R_1+R_2} + \left[I_g - \frac{I_gR_1}{R_1+R_2}\right]e^{-[(R_1+R_2)/L]t}$

$$i_o(t) = \frac{R_1I_g}{(R_1+R_2)} + \frac{R_2I_g}{(R_1+R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{sw} = \frac{R_1I_g}{(1+R_1/R_2)} + \frac{R_2I_g}{(1+R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[e] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.37 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [e] of Problem 7.36) causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.38 **[a]** From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}$$

$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 4; \qquad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_oR = -80; \qquad \frac{R}{L} = 40$$

$$\therefore \quad I_o = 4 + \frac{V_s}{R} = 8\,\text{A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \qquad R = 20\,\Omega$$

$$V_s = 80\,\text{V}; \qquad L = \frac{R}{40} = 0.5\,\text{H}$$

[b] $i = 4 + 4e^{-40t}; \qquad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore \quad 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

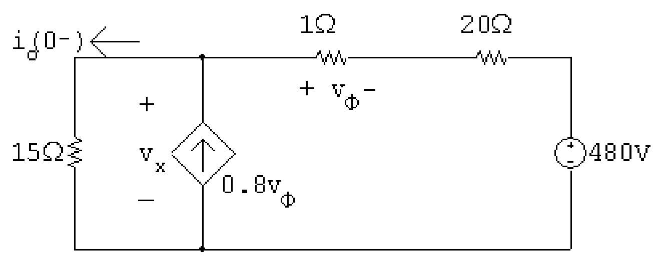
Let $x = e^{-40t}$:

$x^2 + 2x - 1.25 = 0;$ Solving, $x = 0.5; \quad x = -2.5$

But $x \geq 0$ for all t. Thus,

$$e^{-40t} = 0.5; \qquad e^{40t} = 2; \qquad t = 25\ln 2 = 17.33\,\text{ms}$$

P 7.39 For $t < 0$



$$\frac{v_x}{15} - 0.8v_\phi + \frac{v_x - 480}{21} = 0$$

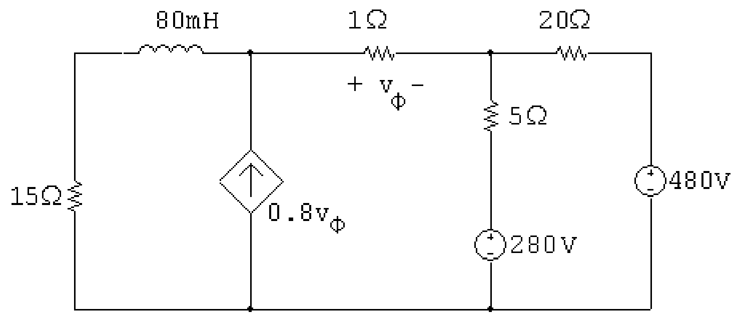
$$v_\phi = \frac{v_x - 480}{21}$$

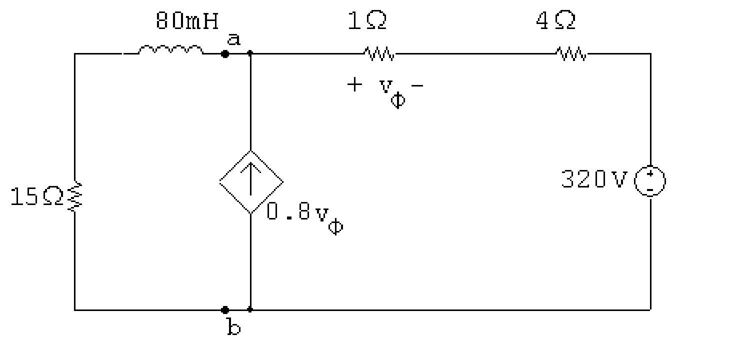
$$\frac{v_x}{15} - 0.8\left(\frac{v_x - 480}{21}\right) + \left(\frac{v_x - 480}{21}\right)$$

$$= \frac{v_x}{15} + 0.2\left(\frac{v_s - 480}{21}\right) = 21v_x + 3(v_x - 480) = 0$$

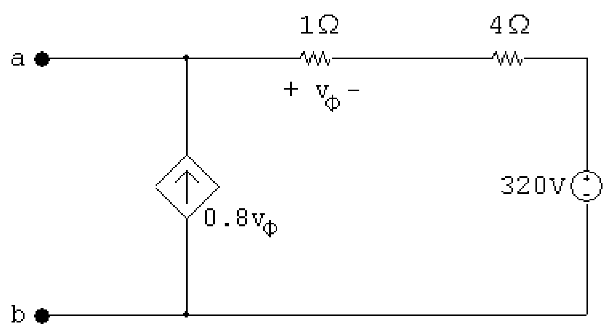
$$\therefore \qquad 24v_x = 1440 \quad \text{so} \quad v_x = 60\,\text{V} \quad i_o(0^-) = \frac{v_x}{15} = 4\text{ A}$$

$t > 0$

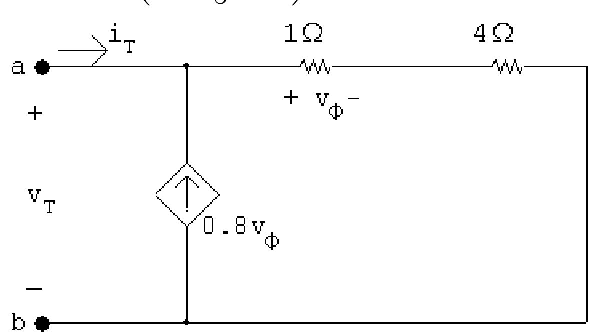




Find Thévenin equivalent with respect to a, b



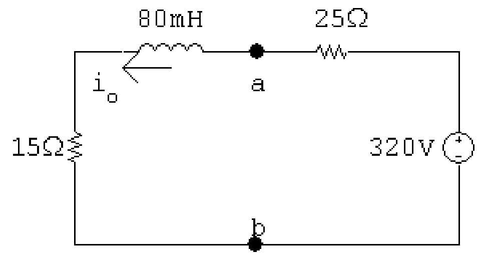
$$\frac{V_{\text{Th}} - 320}{5} - 0.8\left(\frac{V_{\text{Th}} - 320}{5}\right) = 0 \qquad V_{\text{Th}} = 320\,\text{V}$$



$$v_T = (i_T + 0.8v_\phi)(5) = \left(i_T + 0.8\frac{v_T}{5}\right)(5)$$

$$v_T = 5i_T + 0.8v_T \qquad \therefore \quad 0.2v_T = 5i_T$$

$$\frac{v_T}{i_T} = R_{\rm Th} = 25\,\Omega$$

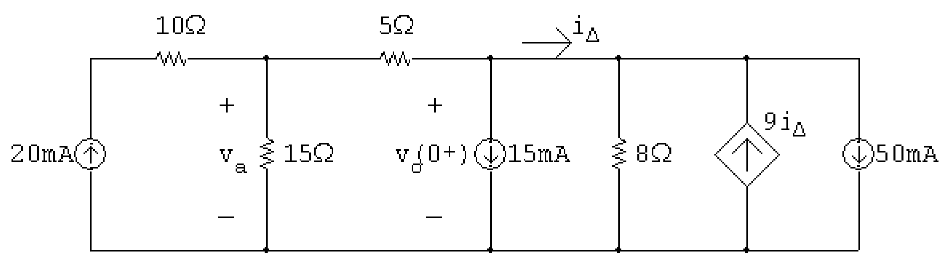


$$i_o(\infty) = 320/40 = 8\,\text{A}$$

$$\tau = \frac{80\times 10^{-3}}{40} = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$i_o = 8 + (4-8)e^{-500t} = 8 - 4e^{-500t}\,\text{A}, \qquad t \geq 0$$

P 7.40 $t > 0$;



$$\frac{v_{\rm a}}{15} + \frac{v_{\rm a} - v_o(0^+)}{5} = 20\times 10^{-3}$$

$$\therefore \quad v_{\rm a} = 0.75v_o(0^+) + 75\times 10^{-3}$$

$$15\times 10^{-3} + \frac{v_o(0^+) - v_{\rm a}}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50\times 10^{-3} = 0$$

$$13v_o(0^+) - 8v_{\rm a} - 360i_\Delta = -2600\times 10^{-3}$$

$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50\times 10^{-3}$$

$$\therefore \quad i_\Delta = \frac{v_o(0^+)}{80} + 5\times 10^{-3}$$

$$\therefore \quad 360i_\Delta = 4.5v_o(0^+) + 1800\times 10^{-3}$$

$$8v_a = 6v_o(0^+) + 600 \times 10^{-3}$$

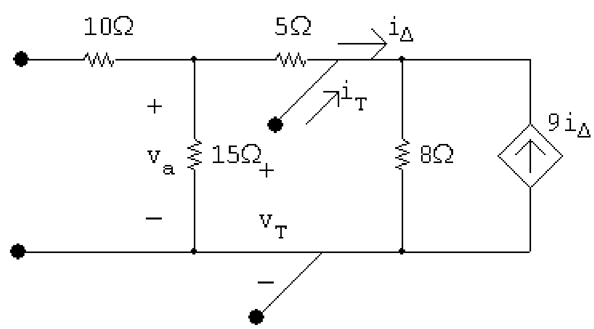
$$\therefore \quad 13v_o(0^+) - 6v_o(0^+) - 600 \times 10^{-3} - 4.5v_o(0^+) -$$

$$1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5v_o(0^+) = -200 \times 10^{-3}; \qquad v_o(0^+) = -80\,\text{mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \qquad \therefore \quad 10i_\Delta = \frac{v_T}{8}; \qquad i_\Delta = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

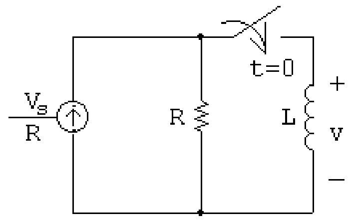
$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\,\text{S}$$

$$\therefore \quad R_{\text{Th}} = 16\Omega$$

$$\tau = \frac{4 \times 10^{-3}}{16} = 0.25\,\text{ms}; \qquad 1/\tau = 4000$$

$$\therefore \quad v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t}\,\text{mV}, \qquad t \geq 0^+$$

P 7.41 **[a]**



$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,dx = \frac{V_s}{R}$$

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

$$\frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $\frac{dv}{dt} = -\frac{R}{L}v$

$$\frac{dv}{dt}\,dt = -\frac{R}{L}v\,dt$$

$$\therefore \quad \frac{dv}{v} = -\frac{R}{L}\,dt$$

$$\int_{v(0^+)}^{v(t)} \frac{dy}{y} = -\frac{R}{L}\int_{0^+}^{t} dx$$

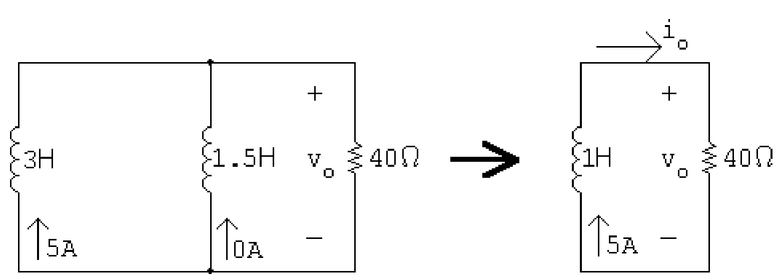
$$\ln y \Big|_{v(0^+)}^{v(t)} = -\left(\frac{R}{L}\right)t$$

$$\ln\left[\frac{v(t)}{v(0^+)}\right] = -\left(\frac{R}{L}\right)t$$

$$v(t) = v(0^+)e^{-(R/L)t}; \qquad v(0^+) = \left(\frac{V_s}{R} - I_o\right)R = V_s - I_oR$$

$$\therefore \quad v(t) = (V_s - I_oR)e^{-(R/L)t}$$

P 7.42 $t > 0$



$$\tau = \frac{1}{40}$$

$$i_o = 5e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 40i_o = 200e^{-40t}\,\text{V}, \qquad t > 0^+$$

$$200e^{-40t} = 100; \qquad e^{40t} = 2$$

$$\therefore \quad t = \frac{1}{40}\ln 2 = 17.33\,\text{ms}$$

P 7.43 **[a]** $w_{\text{diss}} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(1)(5)^2 = 12.5\,\text{J}$

[b] $i_{3H} = \frac{1}{3}\int_0^t (200)e^{-40x}\,dx - 5$

$$= 1.67(1 - e^{-40t}) - 5 = -1.67e^{-40t} - 3.33\,\text{A}$$

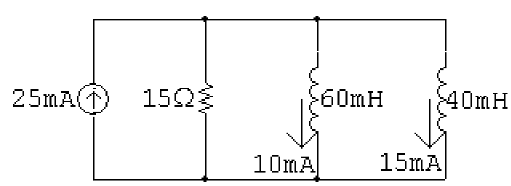
$$i_{1.5H} = \frac{1}{1.5}\int_0^t (200)e^{-40x}\,dx + 0$$

$$= -3.33e^{-40t} + 3.33\,\text{A}$$

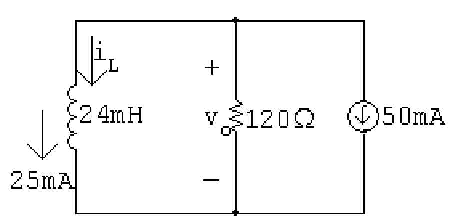
$$w_{\text{trapped}} = \frac{1}{2}(4.5)(3.33)^2 = 25\,\text{J}$$

[c] $w(0) = \frac{1}{2}(3)(5)^2 = 37.5\,\text{J}$

P 7.44 **[a]** $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 25\,\text{mA}; \qquad \tau = \frac{24 \times 10^{-3}}{120} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$i_L(\infty) = -50\,\text{mA}$$

$$i_L = -50 + (25 + 50)e^{-5000t} = -50 + 75e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = -120[75 \times 10^{-3}e^{-5000t}] = -9e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_1 = \dfrac{1}{60\times 10^{-3}}\displaystyle\int_0^t -9e^{-5000x}\,dx + 10\times 10^{-3} = (30e^{-5000t} - 20)\,\text{mA}, \qquad t \geq 0$

[c] $i_2 = \dfrac{1}{40\times 10^{-3}}\displaystyle\int_0^t -9e^{-5000x}\,dx + 15\times 10^{-3} = (45e^{-5000t} - 30)\,\text{mA}, \qquad t \geq 0$

P 7.45 **[a]** Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $v = I_gR_ge^{-t/\tau}; \qquad \tau = L_e/R_g$

Thus

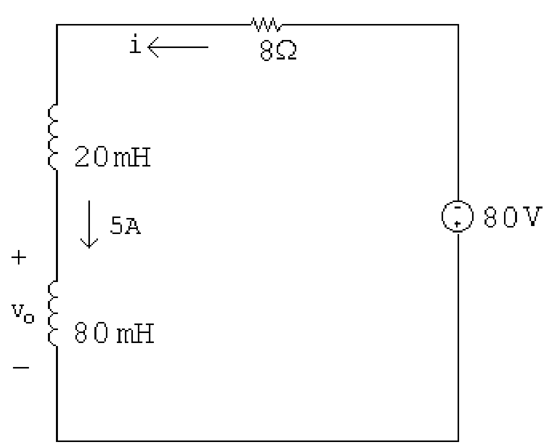
$$i_1 = \frac{1}{L_1}\int_0^t I_gR_ge^{-x/\tau}\,dx = \frac{I_gR_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\Big|_0^t = \frac{I_gL_e}{L_1}(1 - e^{-t/\tau})$$

$$i_1 = \frac{I_gL_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_gL_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \dfrac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \dfrac{L_1}{L_1 + L_2}I_g$

P 7.46 For $t < 0$, $\quad i_{80\text{mH}}(0) = 50\,\text{V}/10\,\Omega = 5\,\text{A}$

For $t > 0$, after making a Thévenin equivalent we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-t/\tau}$$

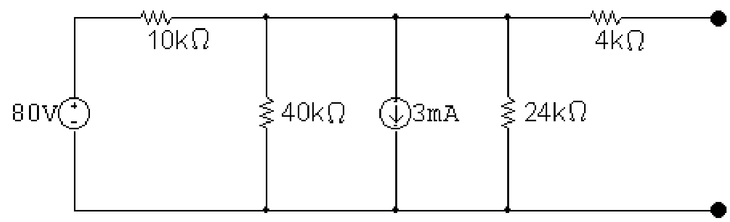
$$\frac{1}{\tau} = \frac{R}{L} = \frac{8}{100 \times 10^{-3}} = 80$$

$$I_o = 5\,\text{A}; \qquad I_f = \frac{V_s}{R} = \frac{-80}{8} = -10\,\text{A}$$

$$i = -10 + (5 + 10)e^{-80t} = -10 + 15e^{-80t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 0.08\frac{di}{dt} = 0.08(-1200e^{-80t}) = -96e^{-80t}\,\text{V}, \qquad t > 0^+$$

P 7.47 For $t < 0$



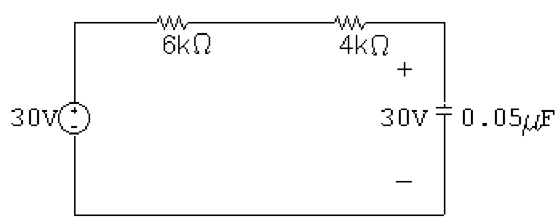
Simplify the circuit:

$$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$$

$$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$$

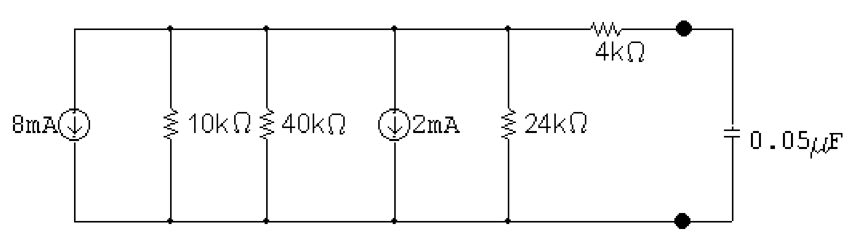
$$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$$

Thus, for $t < 0$



$$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$$

$t > 0$



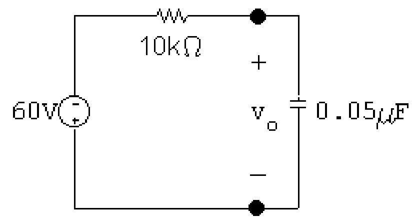
Simplify the circuit:

$8\,\text{mA} + 2\,\text{mA} = 10\,\text{mA}$

$10\,\text{k}\|40\,\text{k}\|24\,\text{k} = 6\,\text{k}\Omega$

$(10\,\text{mA})(6\,\text{k}\Omega) = 60\,\text{V}$

Thus, for $t > 0$



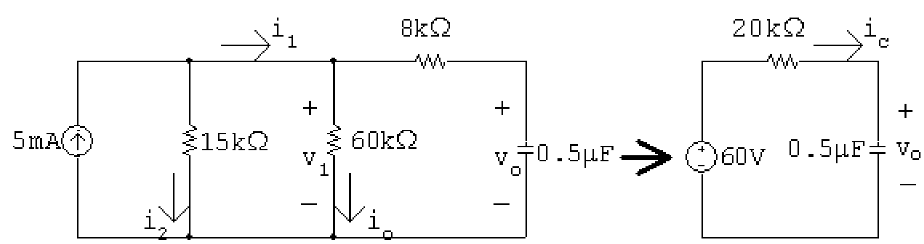
$v_o(\infty) = -60\,\text{V}$

$\tau = RC = (10\,\text{k})(0.05\,\mu) = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t}$

$\quad = -60 + 90e^{-2000t}\,\text{V} \qquad t \geq 0$

P 7.48 **[a]** Simplify the circuit for $t > 0$ using source transformation:



Since there is no source connected to the capacitor for $t < 0$

$v_o(0^-) = v_o(0^+) = 0\,\text{V}$

From the simplified circuit,

$v_o(\infty) = 60\,\text{V}$

$\tau = RC = (20 \times 10^3)(0.5 \times 10^{-6}) = 10\,\text{ms} \qquad 1/\tau = 100$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = (60 - 60e^{-100t})\,\text{V}, \qquad t \geq 0$

[b] $i_c = C\dfrac{dv_o}{dt}$

$$i_c = 0.5 \times 10^{-6}(-100)(-60e^{-100t}) = 3e^{-100t}\,\text{mA}$$

$$v_1 = 8000i_c + v_o = (8000)(3 \times 10^{-3})e^{-100t} + (60 - 60e^{-100t}) = 60 - 36e^{-100t}\,\text{V}$$

$$i_o = \frac{v_1}{60 \times 10^3} = 1 - 0.6e^{-100t}\,\text{mA}, \qquad t \geq 0^+$$

[c] $i_1(t) = i_o + i_c = 1 + 2.4e^{-100t}\,\text{mA} \qquad t \geq 0^+$

[d] $i_2(t) = \dfrac{v_1}{15 \times 10^3} = 4 - 2.4e^{-100t}\,\text{mA} \qquad t \geq 0^+$

[e] $i_1(0^+) = 1 + 2.4 = 3.4\,\text{mA}$

At $t = 0^+$:

$$R_e = 15\,\text{k}\|60\,\text{k}\|8\,\text{k} = 4800\,\Omega$$

$$v_1(0^+) = (5 \times 10^{-3})(4800) = 24\,\text{V}$$

$$i_1(0^+) = \frac{v_1(0^+)}{60{,}000} + \frac{v_1(0^+)}{8000} = 0.4\,\text{m} + 3\,\text{m} = 3.4\text{mA} \quad \text{(checks)}$$

P 7.49 **[a]** $v = I_sR + (V_o - I_sR)e^{-t/RC} \qquad i = \left(I_s - \dfrac{V_o}{R}\right)e^{-t/RC}$

$$\therefore \quad I_sR = 40, \qquad V_o - I_sR = -24$$

$$\therefore \quad V_o = 16\,\text{V}$$

$$I_s - \frac{V_o}{R} = 3 \times 10^{-3}; \qquad I_s - \frac{16}{R} = 3 \times 10^{-3}; \qquad R = \frac{40}{I_s}$$

$$\therefore \quad I_s - 0.4I_s = 3 \times 10^{-3}; \qquad I_s = 5\,\text{mA}$$

$$R = \frac{40}{5} \times 10^3 = 8\,\text{k}\Omega$$

$$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20 \times 10^3} = 50\,\text{nF}; \quad \tau = RC = \frac{1}{2500} = 400\,\mu\text{s}$$

[b] $v(\infty) = 40\,\text{V}$

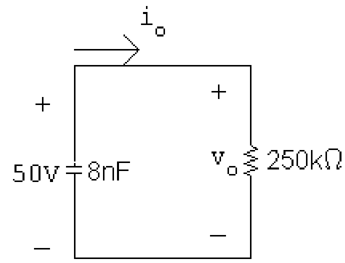
$$w(\infty) = \frac{1}{2}(50 \times 10^{-9})(1600) = 40\,\mu\text{J}$$

$$0.81w(\infty) = 32.4\,\mu\text{J}$$

$$v^2(t_o) = \frac{32.4 \times 10^{-6}}{25 \times 10^{-9}} = 1296; \qquad v(t_o) = 36\,\text{V}$$

$$40 - 24e^{-2500t_o} = 36; \qquad e^{2500t_o} = 6; \qquad \therefore \quad t_o = 716.70\,\mu\text{s}$$

P 7.50 **[a]** For $t > 0$:



$$\tau = RC = 250 \times 10^3 \times 8 \times 10^{-9} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 50e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $$i_o = \frac{v_o}{250{,}000} = \frac{50e^{-500t}}{250{,}000} = 200e^{-500t}\,\mu\text{A}$$

$$v_1 = \frac{-1}{40 \times 10^{-9}} \times 200 \times 10^{-6} \int_0^t e^{-500x}\,dx + 50 = 10e^{-500t} + 40\,\text{V}, \quad t \geq 0$$

P 7.51 **[a]** $$w = \frac{1}{2}C_{\text{eq}}v_o^2 = \frac{1}{2}(8 \times 10^{-9})(50^2) = 10\,\mu\text{J}$$

[b] $$w_{\text{trapped}} = \frac{1}{2}(40)^2(50 \times 10^{-9}) = 40\,\mu\text{J}$$

[c] $$w(0) = \frac{1}{2}(40 \times 10^{-9})(50^2) = 50\,\mu\text{J}$$

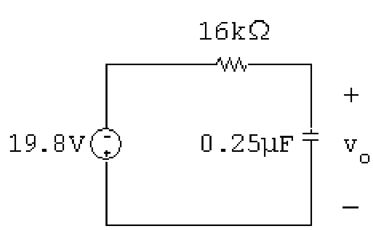
P 7.52 For $t > 0$

$$V_{\text{Th}} = (-25)(16{,}000)i_{\text{b}} = -400 \times 10^3 i_{\text{b}}$$

$$i_{\text{b}} = \frac{33{,}000}{80{,}000}(120 \times 10^{-6}) = 49.5\,\mu\text{A}$$

$$V_{\text{Th}} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8\,\text{V}$$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$



$$v_o(\infty) = -19.8\,\text{V}; \qquad v_o(0^+) = 0$$

$$\tau = (16,000)(0.25 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_o = -19.8 + 19.8e^{-250t}\,\text{V}, \qquad t \geq 0$$

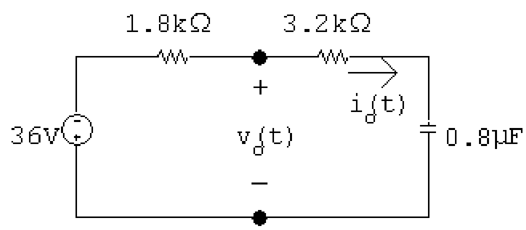
$$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2\,\text{J}$$

$$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$$

$$1 - e^{-250t} = 0.6$$

$$e^{-250t} = 0.4 \quad \therefore \qquad t = 3.67\,\text{ms}$$

P 7.53 **[a]**



$$i_o(0^+) = \frac{-36}{5000} = -7.2\,\text{mA}$$

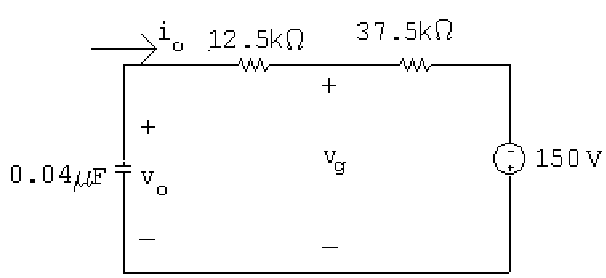
[b] $i_o(\infty) = 0$

[c] $\tau = RC = (5000)(0.8 \times 10^{-6}) = 4\,\text{ms}$

[d] $i_o = 0 + (-7.2)e^{-250t} = -7.2e^{-250t}\,\text{mA}, \qquad t \geq 0^+$

[e] $v_o = -[36 + 1800(-7.2 \times 10^{-3}e^{-250t})] = -36 + 12.96e^{-250t}\,\text{V}, \qquad t \geq 0^+$

P 7.54 **[a]** $v_o(0^-) = v_o(0^+) = 120\,\text{V}$



$$v_o(\infty) = -150\,\text{V}; \qquad \tau = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = -150 + (120 - (-150))e^{-500t}$$

$$v_o = -150 + 270e^{-500t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -0.04 \times 10^{-6}(-500)[270e^{-500t}] = 5.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

[c] $v_g = v_o - 12.5 \times 10^3 i_o = -150 + 202.5e^{-500t}\,\text{V}$

[d] $v_g(0^+) = -150 + 202.5 = 52.5\,\text{V}$

Checks:

$v_g(0^+) = i_o(0^+)[37.5 \times 10^3] - 150 = 202.5 - 150 = 52.5\,\text{V}$

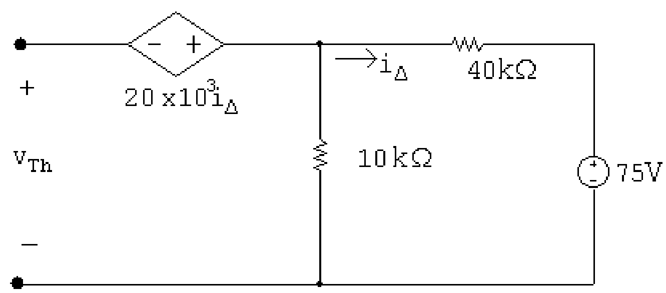
$i_{50k} = \dfrac{v_g}{50\text{k}} = -3 + 4.05e^{-500t}\,\text{mA}$

$i_{150k} = \dfrac{v_g}{150\text{k}} = -1 + 1.35e^{-500t}\,\text{mA}$

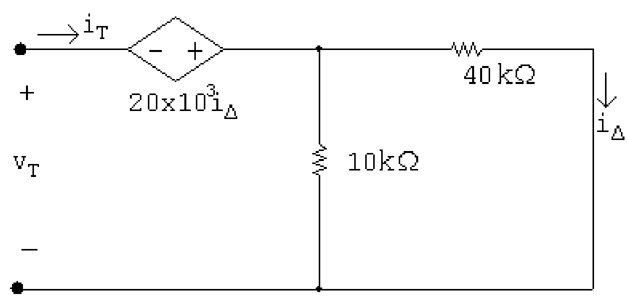
-$i_o + i_{50k} + i_{150k} + 4 = 0$ (ok)

P 7.55 For $t < 0$, $v_o(0) = (-3\,\text{m})(15\,\text{k}) = -45\,\text{V}$

$t > 0$:



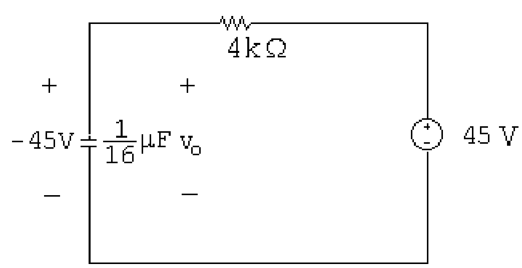
$$V_{\text{Th}} = -20 \times 10^3 i_\Delta + \frac{10}{50}(75) = -20 \times 10^3 \left(\frac{-75}{50 \times 10^3}\right) + 15 = 45\,\text{V}$$



$$v_T = -20 \times 10^3 i_\Delta + 8 \times 10^3 i_T = -20 \times 10^3 (0.2) i_T + 8 \times 10^3 i_T = 4 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 4\,\text{k}\Omega$$

$t > 0$



$v_o = 45 + (-45 - 45)e^{-t/\tau}$

$$\tau = RC = (4000)\left(\frac{1}{16} \times 10^{-6}\right) = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$v_o = 45 - 90e^{-4000t}\,\text{V}, \quad t \geq 0$$

P 7.56 $v_o(0) = 45\,\text{V}; \qquad v_o(\infty) = -45\,\text{V}$

$$R_{\text{Th}} = 20\,\text{k}\Omega$$

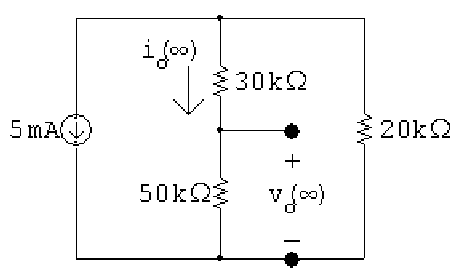
$$\tau = (20 \times 10^3)\left(\frac{1}{16} \times 10^{-6}\right) = 1.25 \times 10^{-3}; \qquad \frac{1}{\tau} = 800$$

$$v = -45 + (45 + 45)e^{-800t} = -45 + 90e^{-800t}\,\text{V}, \quad t \geq 0$$

P 7.57 $t < 0$;

$$i_o(0^-) = \frac{20}{100}(10 \times 10^{-3}) = 2\,\text{mA}; \qquad v_o(0^-) = (2 \times 10^{-3})(50{,}000) = 100\,\text{V}$$

$t = \infty$:

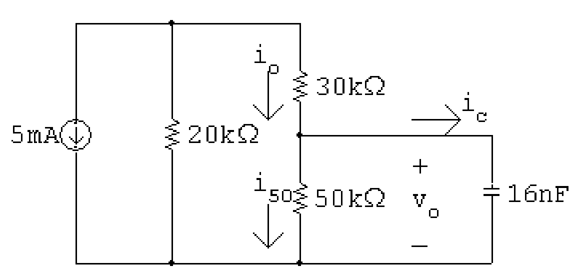


$$i_o(\infty) = -5 \times 10^{-3}\left(\frac{20}{100}\right) = -1\,\text{mA}; \qquad v_o(\infty) = i_o(\infty)(50{,}000) = -50\,\text{V}$$

$$R_{\text{Th}} = 50\,\text{k}\Omega \| 50\,\text{k}\Omega = 25\,\text{k}\Omega; \qquad C = 16\,\text{nF}$$

$$\tau = (25{,}000)(16 \times 10^{-9}) = 0.4\,\text{ms}; \qquad \frac{1}{\tau} = 2500$$

$$\therefore \quad v_o(t) = -50 + 150e^{-2500t}\,\text{V}, \qquad t \geq 0$$



$$i_c = C\frac{dv_o}{dt} = -6e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{50k} = \frac{v_o}{50{,}000} = -1 + 3e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_o = i_c + i_{50k} = -(1 + 3e^{-2500t})\,\text{mA}, \qquad t \geq 0^+$$

P 7.58 **[a]** Let i be the current in the clockwise direction around the circuit. Then

$$V_g = iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx$$

$$= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $\quad i = \dfrac{V_g}{R_g}e^{-t/R_gC_e} = \dfrac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

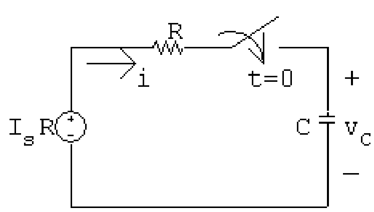
$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\,\frac{e^{-x/\tau}}{-1/\tau}\bigg|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

$$v_2(t) = \frac{V_gC_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

[b] $v_1(\infty) = \dfrac{C_2}{C_1 + C_2}V_g; \qquad v_2(\infty) = \dfrac{C_1}{C_1 + C_2}V_g$

P 7.59 **[a]**



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^t i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\dfrac{di}{dt} = -\dfrac{i}{RC}; \qquad \dfrac{di}{i} = -\dfrac{dt}{RC}$

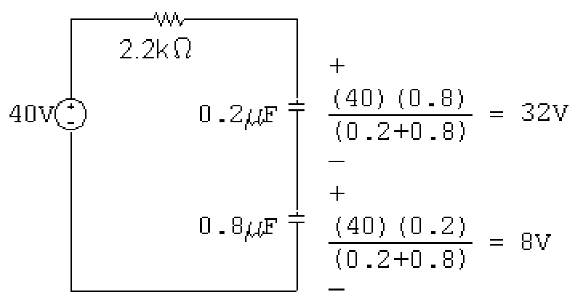
$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

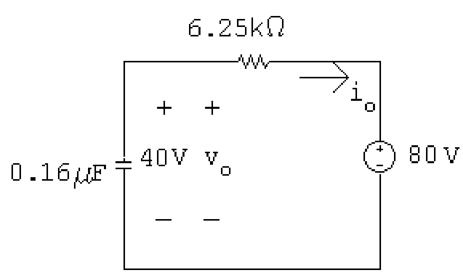
$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.60 **[a]** $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16\times 10^{-6})(6.25\times 10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -C\dfrac{dv_o}{dt} = -0.16\times 10^{-6}[40{,}000e^{-1000t}]$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $v_1 = \dfrac{-1}{0.2\times10^{-6}}\displaystyle\int_0^t -6.4\times10^{-3}e^{-1000x}\,dx + 32$

$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$

[d] $v_2 = \dfrac{-1}{0.8\times10^{-6}}\displaystyle\int_0^t -6.4\times10^{-3}e^{-1000x}\,dx + 8$

$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$

[e] $w_{\text{trapped}} = \frac{1}{2}(0.2\times10^{-6})(64)^2 + \frac{1}{2}(0.8\times10^{-6})(16)^2 = 512\,\mu\text{J}.$

P 7.61 **[a]** $v_c(0^+) = 50\,\text{V}$

[b] Use voltage division to find the final value of voltage:

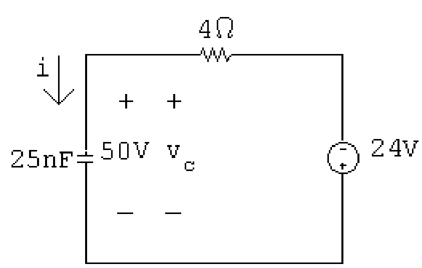
$$v_c(\infty) = \frac{20}{20+5}(-30) = -24\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$V_{\text{Th}} = -24\,\text{V}, \qquad R_{\text{Th}} = 20\|5 = 4\,\Omega,$

Therefore $\tau = R_{\text{eq}}C = 4(25\times10^{-9}) = 0.1\,\mu\text{s}$

The simplified circuit for $t > 0$ is:



[d] $i(0^+) = \dfrac{-24-50}{4} = -18.5\,\text{A}$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$= -24 + [50 - (-24)]e^{-t/\tau} = -24 + 74e^{-10^7t}\,\text{V}, \qquad t \geq 0$

[f] $i = C\dfrac{dv_c}{dt} = (25\times10^{-9})(-10^7)(74e^{-10^7t}) = -18.5e^{-10^7t}\,\text{A}, \qquad t \geq 0^+$

P 7.62 **[a]** Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\text{k}}{9\,\text{k}+3\,\text{k}}(120) = 90\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5\times10^{-3})(40\times10^3) = -60\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -60\,\text{V}, \qquad R_{\text{Th}} = 10\,\text{k} + 40\,\text{k} = 50\,\text{k}\Omega$$

$$\tau = R_{\text{Th}}C = 1\,\text{ms} = 1000\,\mu\text{s}$$

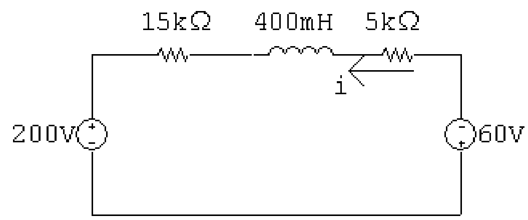
[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\text{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

Therefore $\quad t = \dfrac{\ln(150/60)}{1000} = 916.3\,\mu\text{s}$

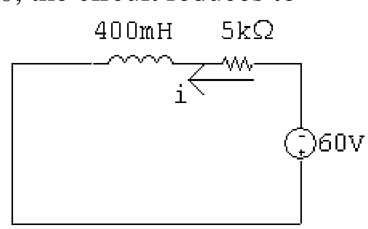
P 7.63 **[a]** For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 400-mH inductor. We get



$$i(0^-) = \frac{-60 - 200}{15\,\text{k} + 5\,\text{k}} = -13\,\text{mA}$$

$$i(0^-) = i(0^+) = -13\,\text{mA}$$

[b] For $t > 0$, the circuit reduces to



Therefore $\quad i(\infty) = -60/5{,}000 = -12\,\text{mA}$

[c] $\tau = \dfrac{L}{R} = \dfrac{400 \times 10^{-3}}{5000} = 80\,\mu\text{s}$

[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= -12 + [-13 + 12]e^{-12{,}500t} = -12 - e^{-12{,}500t}\,\text{mA}, \qquad t \geq 0$$

P 7.64 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{36 - 16}{20 - 8} = \frac{5}{3}\,\text{H}$$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{(5/3)}{(50/3)} = \frac{1}{10}$$

$$i_o = \frac{100}{(50/3)} - \frac{100}{(50/3)}e^{-10t} = 6 - 6e^{-10t}\,\text{A} \quad t \geq 0$$

[b] $v_o = 100 - \frac{50}{3}i_o = 100 - \frac{50}{3}(6 - 6e^{-10t}) = 100e^{-10t}\,\text{V}, \quad t \geq 0^+$

[c] $v_o = 2\frac{di_1}{dt} + 4\frac{di_2}{dt}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{di_2}{dt} = \frac{di_o}{dt} - \frac{di_1}{dt} = 60e^{-10t} - \frac{di_1}{dt}$$

$$\therefore \quad 100e^{-10t} = 2\frac{di_1}{dt} + 4\left(60e^{-10t} - \frac{di_1}{dt}\right)$$

$$\therefore \quad \frac{di_1}{dt} = 70e^{-10t}$$

$$di_1 = 70e^{-10t}\,dt$$

$$\int_0^{i_1} dx = 70\int_0^t e^{-10y}\,dy$$

$$\therefore \quad i_1 = 70\frac{e^{-10y}}{-10}\Big|_0^t = 7 - 7e^{-10t}\,\text{A}, \quad t \geq 0$$

[d]
$$\begin{aligned} i_2 &= i_o - i_1 \\ &= 6 - 6e^{-10t} - 7 + 7e^{-10t} \\ &= -1 + e^{-10t}\,\text{A}, \quad t \geq 0 \end{aligned}$$

[e]
$$\begin{aligned} v_o &= L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \\ &= 18(-10e^{-10t}) + 4(70e^{-10t}) \\ &= 100e^{-10t}\,\text{V}, \quad t \geq 0^+ \quad \text{(checks)} \end{aligned}$$

Also,

$$\begin{aligned} v_o &= L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ &= 2(70e^{-10t}) + 4(-10e^{-10t}) \\ &= 100e^{-10t}\,\text{V}, \quad t \geq 0^+ \quad \text{CHECKS} \end{aligned}$$

$i_1(0) = 7 - 7 = 0$; agrees with initial conditions;
$i_2(0) = -1 + 1 = 0$; agrees with initial conditions;
The final values of i_o, i_1, and i_2 can be checked via the conservation of Wb-turns:

$$i_o(\infty)L_{\text{eq}} = 6 \times (5/3) = 10 \text{ Wb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = 7(2) - 1(4) = 10 \text{ Wb-turns}$$

$$i_2(\infty)L_2 + i_1(\infty)M = -1(18) + 7(4) = 10 \text{ Wb-turns}$$

Thus our solutions make sense in terms of known circuit behavior.

P 7.65 **[a]** $L_{\text{eq}} = \dfrac{(3)(15)}{3+15} = 2.5\,\text{H}$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore \quad i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$\therefore \quad \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40 \text{ Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40 \text{ Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40 \text{ Wb-turns}$$

$\therefore\quad i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.66 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{50-25}{15+10} = 1\,\text{H}$$

$$\tau = \frac{L}{R} = \frac{1}{20}; \qquad \frac{1}{\tau} = 20$$

$$\therefore\quad i_o(t) = 4 - 4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o = 80 - 20i_o = 80 - 80 + 80e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 5\dfrac{di_1}{dt} - 5\dfrac{di_2}{dt} = 80e^{-20t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 80e^{-20t}\ \text{A/s}$$

$$\therefore\quad \frac{di_2}{dt} = 80e^{-20t} - \frac{di_1}{dt}$$

$$\therefore\quad 80e^{-20t} = 5\frac{di_1}{dt} - 400e^{-20t} + 5\frac{di_1}{dt}$$

$$\therefore\quad 10\frac{di_1}{dt} = 480e^{-20t}; \qquad di_1 = 48e^{-20t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 48e^{-20y}\,dy$$

$$i_1 = \frac{48}{-20}e^{-20y}\Big|_0^t = 2.4 - 2.4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 4 - 4e^{-20t} - 2.4 + 2.4e^{-20t}$

$$= 1.6 - 1.6e^{-20t}\,\text{A}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = 1(80)e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 10\frac{di_2}{dt} - 5\frac{di_1}{dt} = 80e^{-20t}\,\text{V}, \qquad t \ge 0^+ \text{ (checks)}$$

$$v_o(0^+) = 80\,\text{V}, \text{ which agrees with } i_o(0^+) = 0\,\text{A}$$

$$i_o(\infty) = 4\,\text{A}; \qquad i_o(\infty)L_{\text{eq}} = (4)(1) = 4 \text{ Wb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = (2.4)(5) + (1.6)(-5) = 4 \text{ Wb-turns (ok)}$$

$$i_2(\infty)L_2 + i_1(\infty)M = (1.6)(10) + (2.4)(-5) = 4 \text{ Wb-turns (ok)}$$

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.67 **[a]** $L_{\text{eq}} = 5 + 10 - 2.5(2) = 10\,\text{H}$

$$\tau = \frac{L}{R} = \frac{10}{40} = \frac{1}{4}; \qquad \frac{1}{\tau} = 4$$

$$i = 2 - 2e^{-4t}\,\text{A}, \quad t \ge 0$$

[b] $v_1(t) = 5\frac{di_1}{dt} - 2.5\frac{di}{dt} = 2.5\frac{di}{dt} = 2.5(8e^{-4t}) = 20e^{-4t}\,\text{V}, \quad t \ge 0^+$

[c] $v_2(t) = 10\frac{di_1}{dt} - 2.5\frac{di}{dt} = 7.5\frac{di}{dt} = 7.5(8e^{-4t}) = 60e^{-4t}\,\text{V}, \quad t \ge 0^+$

[d] $i(0) = 2 - 2 = 0$, which agrees with initial conditions.

$$80 = 40i_1 + v_1 + v_2 = 40(2 - 2e^{-4t}) + 20e^{-4t} + 60e^{-4t} = 80\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \ge 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.68 **[a]** $L_{\text{eq}} = 5 + 10 + 2.5(2) = 20\,\text{H}$

$$\tau = \frac{L}{R} = \frac{20}{40} = \frac{1}{2}; \qquad \frac{1}{\tau} = 2$$

$$i = 2 - 2e^{-2t}\,\text{A}, \quad t \ge 0$$

[b] $v_1(t) = 5\frac{di_1}{dt} + 2.5\frac{di}{dt} = 7.5\frac{di}{dt} = 7.5(4e^{-2t}) = 30e^{-2t}\,\text{V}, \quad t \ge 0^+$

[c] $v_2(t) = 10\frac{di_1}{dt} + 2.5\frac{di}{dt} = 12.5\frac{di}{dt} = 12.5(4e^{-2t}) = 50e^{-2t}\,\text{V}, \quad t \ge 0^+$

[d] $i(0) = 0$, which agrees with initial conditions.

$$80 = 40i_1 + v_1 + v_2 = 40(2 - 2e^{-2t}) + 30e^{-2t} + 50e^{-2t} = 80\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \ge 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.69 Use voltage division to find the initial voltage:

$$v_o(0) = \frac{60}{40+60}(50) = 30\,\text{V}$$

Use Ohm's law to find the final value of voltage:

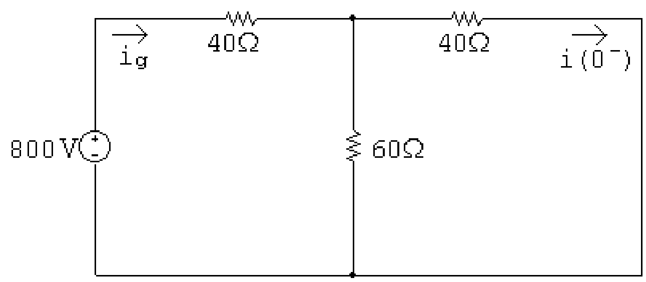
$$v_o(\infty) = (-5\,\text{mA})(20\,\text{k}\Omega) = -100\,\text{V}$$

$$\tau = RC = (20\times 10^3)(250\times 10^{-9}) = 5\,\text{ms}; \qquad \frac{1}{\tau} = 200$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau}$$

$$= -100 + (30+100)e^{-200t} = -100 + 130e^{-200t}\,\text{V}, \qquad t \geq 0$$

P 7.70 **[a]** $t < 0$:



Using Ohm's law,

$$i_g = \frac{800}{40 + 60\|40} = 12.5\,\text{A}$$

Using current division,

$$i(0^-) = \frac{60}{60+40}(12.5) = 7.5\,\text{A} = i(0^+)$$

[b] $0 \leq t \leq 1\,\text{ms}$:

$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

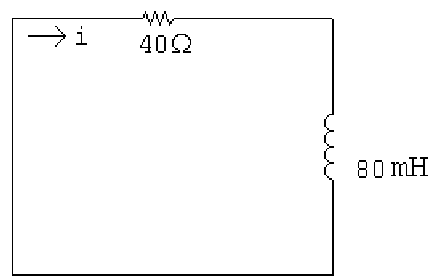
$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120\|60}{80\times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200\times 10^{-6})} = 7.5e^{-0.2} = 6.14\,\text{A}$$

[c] $i(1\text{ms}) = 7.5e^{-1} = 2.7591\,\text{A}$

$1\,\text{ms} \le t < \infty$



$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1\,\text{ms})e^{-(t-1\,ms)/\tau} = 2.7591e^{-500(t-0.001)}\,\text{A}$$

$$i(6\text{ms}) = 2.7591e^{-500(0.005)} = 2.7591e^{-2.5} = 226.48\,\text{mA}$$

[d] $0 \le t \le 1\,\text{ms}$:

$$i = 7.5e^{-1000t}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5e^{-1000t}) = -600e^{-1000t}\,\text{V}$$

$$v(1^{-}\text{ms}) = -600e^{-1} = -220.73\,\text{V}$$

[e] $1\,\text{ms} \le t \le \infty$:

$$i = 2.7591e^{-500(t-0.001)}$$

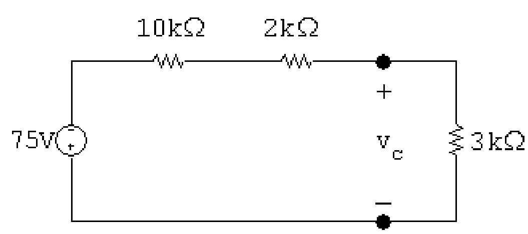
$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-500)(2.591e^{-500(t-0.001)})$$

$$= -110.4e^{-500(t-0.001)}\,\text{V}$$

$$v(1^{+}\text{ms}) = -110.4\,\text{V}$$

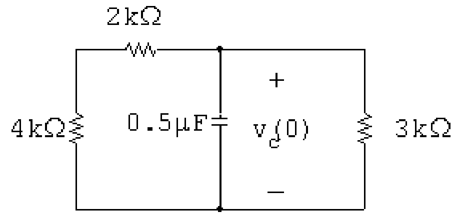
P 7.71 Note that for $t > 0$, $v_o = (4/6)v_c$, where v_c is the voltage across the $0.5\,\mu$F capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{3}{15}(-75) = -15\,\text{V}$$

$0 \le t \le 800\,\mu$s:



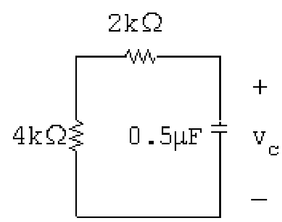
$$\tau = R_eC, \qquad R_e = \frac{(6000)(3000)}{9000} = 2\,\text{k}\Omega$$

$$\tau = (2 \times 10^3)(0.5 \times 10^{-6}) = 1\,\text{ms}, \qquad \frac{1}{\tau} = 1000$$

$$v_{\text{c}} = -15e^{-1000t}\,\text{V}, \qquad t \ge 0$$

$$v_{\text{c}}(800\,\mu\text{s}) = -15e^{-0.8} = -6.74\,\text{V}$$

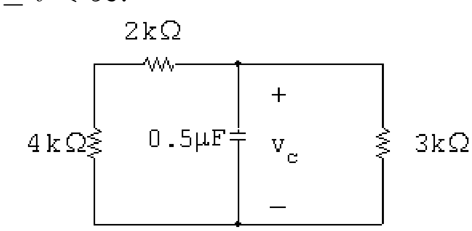
$800\,\mu\text{s} \le t \le 1.1\,$ms:



$$\tau = (6 \times 10^3)(0.5 \times 10^{-6}) = 3\,\text{ms}, \qquad \frac{1}{\tau} = 333.33$$

$$v_{\text{c}} = -6.74e^{-333.33(t-800\times10^{-6})}\,\text{V}$$

$1.1\,\text{ms} \le t < \infty$:



$$\tau = 1\,\text{ms}, \qquad \frac{1}{\tau} = 1000$$

$$v_{\text{c}}(1.1\text{ms}) = -6.74e^{-333.33(1100-800)10^{-6}} = -6.74e^{-0.1} = -6.1\,\text{V}$$

$$v_c = -6.1e^{-1000(t-1.1\times10^{-3})}\,\text{V}$$

$$v_c(1.5\text{ms}) = -6.1e^{-1000(1.5-1.1)10^{-3}} = -6.1e^{-0.4} = -4.09\,\text{V}$$

$$v_o = (4/6)(-4.09) = -2.73\,\text{V}$$

P 7.72 $w(0) = \frac{1}{2}(0.5\times10^{-6})(-15)^2 = 56.25\,\mu\text{J}$

$0 \le t \le 800\,\mu$s:

$$v_c = -15e^{-1000t}; \qquad v_c^2 = 225e^{-2000t}$$

$$p_{3k} = 75e^{-2000t}\,\text{mW}$$

$$w_{3k} = \int_0^{800\times10^{-6}} 75\times10^{-3}e^{-2000t}\,dt$$

$$= 75\times10^{-3}\frac{e^{-2000t}}{-2000}\Big|_0^{800\times10^{-6}}$$

$$= -37.5\times10^{-6}(e^{-1.6}-1) = 29.93\,\mu\text{J}$$

$1.1\,\text{ms} \le t \le \infty$:

$$v_c = -6.1e^{-1000(t-1.1\times10^{-3})}\,\text{V}; \qquad v_c^2 = 37.19e^{-2000(t-1.1\times10^{-3})}$$

$$p_{3k} = 12.4e^{-2000(t-1.1\times10^{-3})}\,\text{mW}$$

$$w_{3k} = \int_{1.1\times10^{-3}}^{\infty} 12.4\times10^{-3}e^{-2000(t-1.1\times10^{-3})}\,dt$$

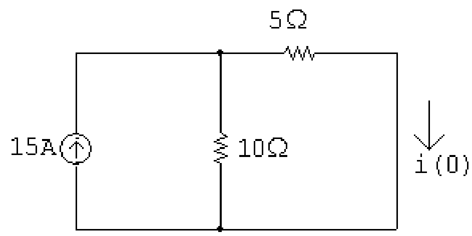
$$= 12.4\times10^{-3}\frac{e^{-2000(t-1.1\times10^{-3})}}{-2000}\Big|_{1.1\times10^{-3}}^{\infty}$$

$$= -6.2\times10^{-6}(0-1) = 6.2\,\mu\text{J}$$

$$w_{3k} = 29.93 + 6.2 = 36.13\,\mu\text{J}$$

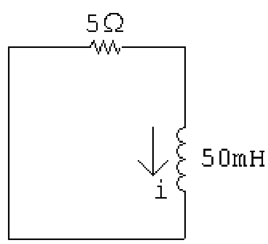
$$\% = \frac{36.13}{56.25}(100) = 64.23\%$$

P 7.73 For $t < 0$:



$$i(0) = \frac{10}{15}(15) = 10\,\text{A}$$

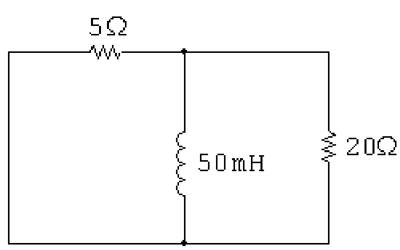
$0 \leq t \leq 10\,\text{ms}$:



$$i = 10e^{-100t}\,\text{A}$$

$$i(10\text{ms}) = 10e^{-1} = 3.68\,\text{A}$$

$10\,\text{ms} \leq t \leq 20\,\text{ms}$:



$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$$i = 3.68e^{-80(t-0.01)}\,\text{A}$$

$20\,\text{ms} \leq t \leq \infty$:

$$i(20\text{ms}) = 3.68e^{-80(0.02-0.01)} = 1.65\,\text{A}$$

$$i = 1.65e^{-100(t-0.02)}\,\text{A}$$

$$v_o = L\frac{di}{dt}; \qquad L = 50\,\text{mH}$$

$$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$$

$$v_o = (50\times 10^{-3})(-165)e^{-100(t-0.02)}$$

$$= -8.26e^{-100(t-0.02)}\,\text{V}, \qquad t > 20^{+}\,\text{ms}$$

$$v_o(25\text{ms}) = -8.26e^{-100(0.025-0.02)} = -5\,\text{V}$$

P 7.74 From the solution to Problem 7.73, the initial energy is

$$w(0) = \frac{1}{2}(50\,\text{mH})(10\,\text{A})^2 = 2.5\,\text{J}$$

$$0.04w(0) = 0.1\,\text{J}$$

$$\therefore \quad \frac{1}{2}(50\times 10^{-3})i_L^2 = 0.1 \quad \text{so} \quad i_L = 2\,\text{A}$$

Again, from the solution to Problem 7.73, t must be between 10 ms and 20 ms since

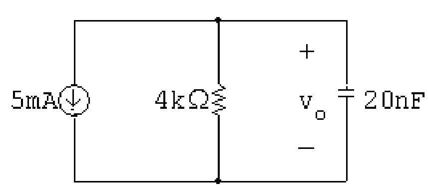
$$i(10\,\text{ms}) = 3.68\,\text{A} \quad \text{and} \quad i(20\,\text{ms}) = 1.65\,\text{A}$$

For $10\,\text{ms} \le t \le 20\,\text{ms}$:

$$i = 3.68e^{-80(t-0.01)} = 2$$

$$e^{80(t-0.01)} = \frac{3.68}{2} \quad \text{so} \quad t - 0.01 = 0.0076 \quad \therefore \qquad t = 17.6\,\text{ms}$$

P 7.75 $0 \le t \le 10\,\mu$s:

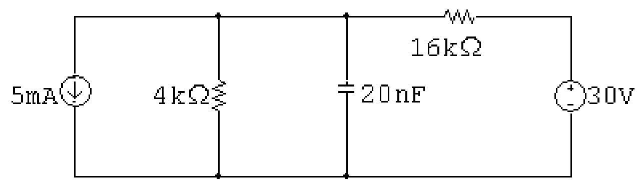


$$\tau = RC = (4\times 10^{3})(20\times 10^{-9}) = 80\,\mu\text{s}; \qquad 1/\tau = 12{,}500$$

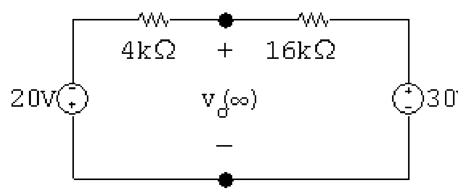
$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = -20\,\text{V}$

$v_o = -20 + 20e^{-12{,}500t}\,\text{V} \qquad 0 \le t \le 10\,\mu\text{s}$

$10\,\mu\text{s} \le t \le \infty$:



$t = \infty$:



$$i = \frac{-50\,\text{V}}{20\,\text{k}\Omega} = -2.5\,\text{mA}$$

$v_o(\infty) = (-2.5 \times 10^{-3})(16{,}000) + 30 = -10\,\text{V}$

$v_o(10\,\mu\text{s}) = -20 + 20^{-0.125} = -2.35\,\text{V}$

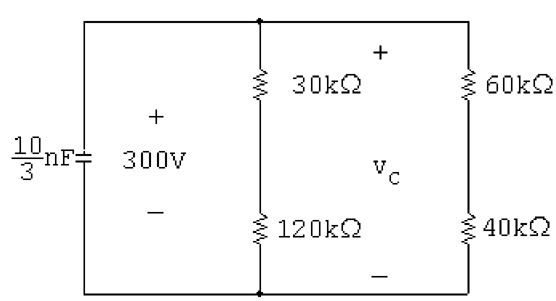
$v_o = -10 + (-2.35 + 10)e^{-(t-10\times10^{-6})/\tau}$

$R_{\text{Th}} = 4\,\text{k}\Omega \| 16\,\text{k}\Omega = 3.2\,\text{k}\Omega$

$\tau = (3200)(20 \times 10^{-9}) = 64\,\mu\text{s}; \qquad 1/\tau = 15{,}625$

$v_o = -10 + 7.65e^{-15{,}625(t-10\times10^{-6})} \qquad 10\,\mu\text{s} \le t \le \infty$

P 7.76 $\quad 0 \le t \le 200\,\mu\text{s}$;

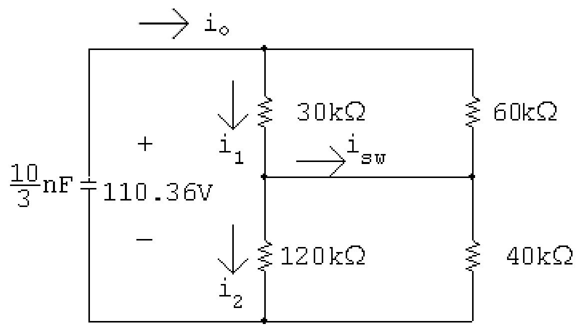


$R_e = 150\|100 = 60\,\text{k}\Omega; \qquad \tau = \left(\frac{10}{3} \times 10^{-9}\right)(60{,}000) = 200\,\mu\text{s}$

$v_c = 300e^{-5000t}\,\text{V}$

$v_c(200\,\mu\text{s}) = 300e^{-1} = 110.36\,\text{V}$

$200\,\mu\text{s} \le t \le \infty$:



$R_e = 30\|60 + 120\|40 = 20 + 30 = 50\,\text{k}\Omega$

$\tau = \left(\frac{10}{3} \times 10^{-9}\right)(50{,}000) = 166.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 6000$

$v_c = 110.36e^{-6000(t\,-\,200\,\mu s)}\,\text{V}$

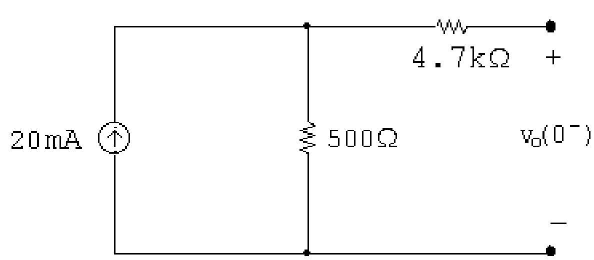
$v_c(300\,\mu\text{s}) = 110.36e^{-6000(100\,\mu\text{S})} = 60.57\,\text{V}$

$i_o(300\,\mu\text{s}) = \frac{60.57}{50{,}000} = 1.21\,\text{mA}$

$i_1 = \frac{60}{90}i_o = \frac{2}{3}i_o; \qquad i_2 = \frac{40}{160}i_o = \frac{1}{4}i_o$

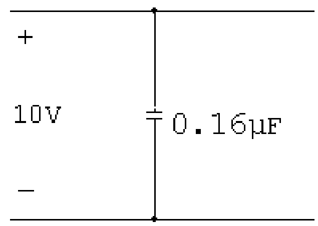
$i_{\text{sw}} = i_1 - i_2 = \frac{2}{3}i_o - \frac{1}{4}i_o = \frac{5}{12}i_o = \frac{5}{12}(1.21 \times 10^{-3}) = 0.50\,\text{mA}$

P 7.77 $t < 0$:



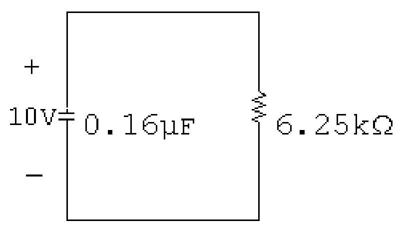
$v_c(0^-) = (20 \times 10^{-3})(500) = 10\,\text{V} = v_c(0^+)$

$0 \leq t \leq 50\,\text{ms}$:



$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = 10e^{-0} = 10\,\text{V}$

$50\,\text{ms} \leq t \leq \infty$:



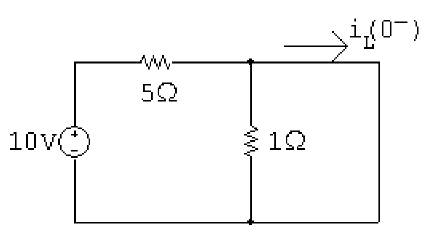
$\tau = (6.25\,\text{k})(0.16\,\mu) = 1\,\text{ms}; \qquad 1/\tau = 1000; \qquad v_o = 10e^{-1000(t\,-\,0.05)}\,\text{V}$

Summary:

$v_o = 10\,\text{V}, \qquad 0 \leq t \leq 50\,\text{ms}$

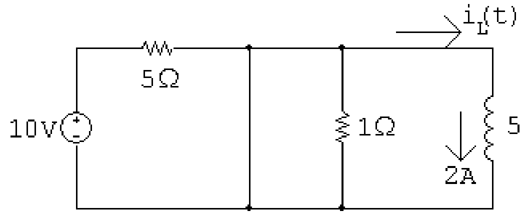
$v_o = 10e^{-1000(t\,-\,0.05)}\,\text{V}, \qquad 50\,\text{ms} \leq t \leq \infty$

P 7.78 $t < 0$:



$i_L(0^-) = 10\,\text{V}/5\,\Omega = 2\,\text{A} = i_L(0^+)$

$0 \leq t \leq 5$:

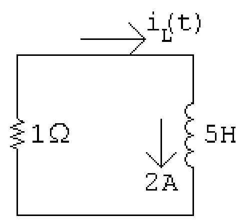


$$\tau = 5/0 = \infty$$

$$i_L(t) = 2e^{-t/\infty} = 2e^{-0} = 2$$

$$i_L(t) = 2\,\text{A}, \quad 0 \leq t \leq 5\,\text{s}$$

$5 \leq t \leq \infty$:



$$\tau = \frac{5}{1} = 5\,\text{s}; \qquad 1/\tau = 0.2$$

$$i_L(t) = 2e^{-0.2(t-5)}\,\text{A}, \quad t \geq 5\,\text{s}$$

P 7.79 **[a]** $0 \leq t \leq 2.5\,\text{ms}$

$$v_o(0^+) = 80\,\text{V}; \qquad v_o(\infty) = 0$$

$$\tau = \frac{L}{R} = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o(t) = 80e^{-500t}\,\text{V}, \qquad 0^+ \leq t \leq 2.5\,\text{ms}$$

$$v_o(2.5^-\,\text{ms}) = 80e^{-1.25} = 22.92\,\text{V}$$

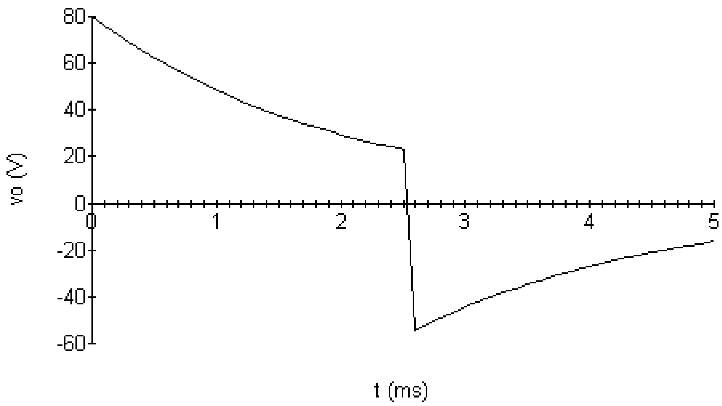
$$i_o(2.5^-\,\text{ms}) = \frac{(80 - 22.92)}{20} = 2.85\,\text{A}$$

$$v_o(2.5^+\,\text{ms}) = -20(2.85) = -57.08\,\text{V}$$

$$v_o(\infty) = 0; \qquad \tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = -57.08e^{-500(t-0.0025)}\,\text{V} \qquad 2.5^+\,\text{ms} \leq t \leq \infty$$

[b]



[c] $v_o(5\,\text{ms}) = -16.35\,\text{V}$

$$i_o = \frac{+16.35}{20} = 817.68\,\text{mA}$$

P 7.80 **[a]** $i_o(0) = 0; \qquad i_o(\infty) = 25\,\text{mA}$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{2000}{250} \times 10^3 = 8000$$

$$i_o = (25 - 25e^{-8000t})\,\text{mA}, \qquad 0 \le t \le 75\,\mu\text{s}$$

$$v_o = 0.25\frac{di_o}{dt} = 50e^{-8000t}\,\text{V}, \qquad 0^+ \le t \le 75^-\,\mu s$$

$75^+\,\mu\text{s} \le t \le \infty$:

$$i_o(75\mu\text{s}) = 25 - 25e^{-0.6} = 11.28\,\text{mA}; \qquad i_o(\infty) = 0$$

$$i_o = 11.28e^{-8000(t-75\times10^{-6})}\,\text{mA}$$

$$v_o = (0.25)\frac{di_o}{dt} = -22.56e^{-8000(t-75\mu s)}$$

$$\therefore \quad t < 0: \quad v_o = 0$$

$$0^+ \le t \le 75^-\,\mu\text{s}: \quad v_o = 50e^{-8000t}\,\text{V}$$

$$75^+\,\mu\text{s} \le t \le \infty: \quad v_o = -22.56e^{-8000(t-75\mu s)}$$

[b] $v_o(75^-\mu\text{s}) = 50e^{-0.6} = 27.44\,\text{V}$

$$v_o(75^+\mu\text{s}) = -22.56\,\text{V}$$

[c] $i_o(75^-\mu\text{s}) = i_o(75^+\mu\text{s}) = 11.28\,\text{mA}$

P 7.81 **[a]** $0 \leq t < 1\,\text{ms}$:

$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$

$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$

$v_c = 50 - 50e^{-250t}$

$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \leq t \leq 1\,\text{ms}$

$1\,\text{ms} < t \leq \infty$:

$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$

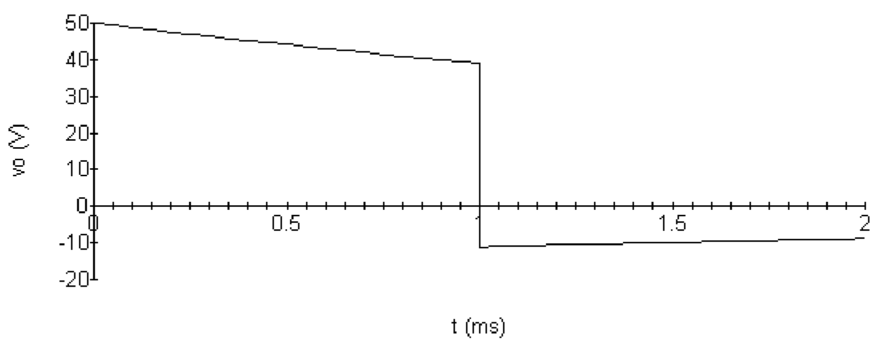
$v_c(\infty) = 0\,\text{V}$

$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$

$v_c = 11.06e^{-250(t-0.001)}\,\text{V}$

$v_o = -v_c = -11.06e^{-250(t-0.001)}\,\text{V}, \qquad 1\,\text{ms} < t \leq \infty$

[b]



P 7.82 **[a]** $t < 0; \qquad v_o = 0$

$0 \leq t \leq 4\,\text{ms}$:

$\tau = (200 \times 10^3)(0.025 \times 10^{-6}) = 5\,\text{ms}; \qquad 1/\tau = 200$

$v_o = 100 - 100e^{-200t}\,\text{V}, \qquad 0 \leq t \leq 4\,\text{ms}$

$v_o(4\,\text{ms}) = 100(1 - e^{-0.8}) = 55.07\,\text{V}$

$4\,\text{ms} \leq t \leq 8\,\text{ms}$:

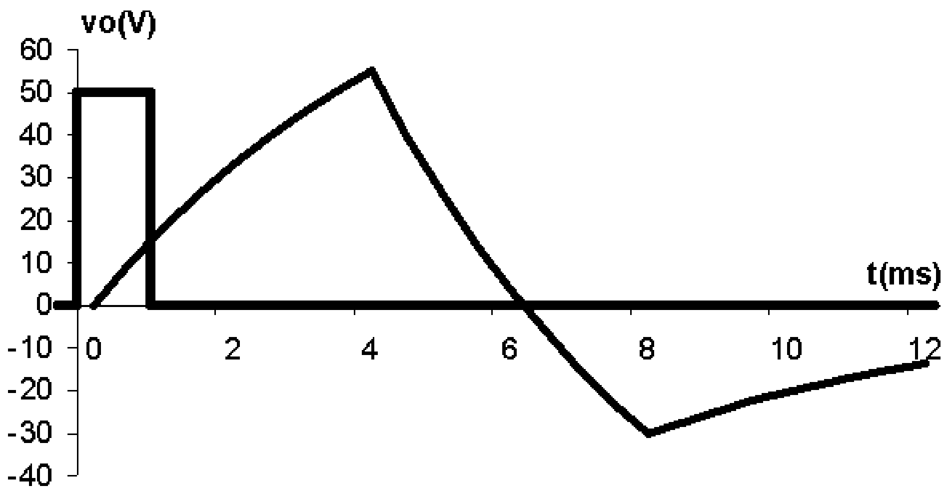
$v_o = -100 + 155.07e^{-200(t-0.004)}\,\text{V}$

$v_o(8\,\text{ms}) = -100 + 155.07e^{-0.8} = -30.32\,\text{V}$

$8\,\text{ms} \leq t \leq \infty$:

$v_o = -30.32e^{-200(t-0.008)}\,\text{V}$

[b]



[c] $t \leq 0$: $v_o = 0$

$0 \leq t \leq 4\,\text{ms}$:

$$\tau = (50 \times 10^3)(0.025 \times 10^{-6}) = 1.25\,\text{ms} \quad 1/\tau = 800$$

$$v_o = 100 - 100e^{-800t}\,\text{V}, \qquad 0 \leq t \leq 4\,\text{ms}$$

$$v_o(4\,\text{ms}) = 100 - 100e^{-3.2} = 95.92\,\text{V}$$

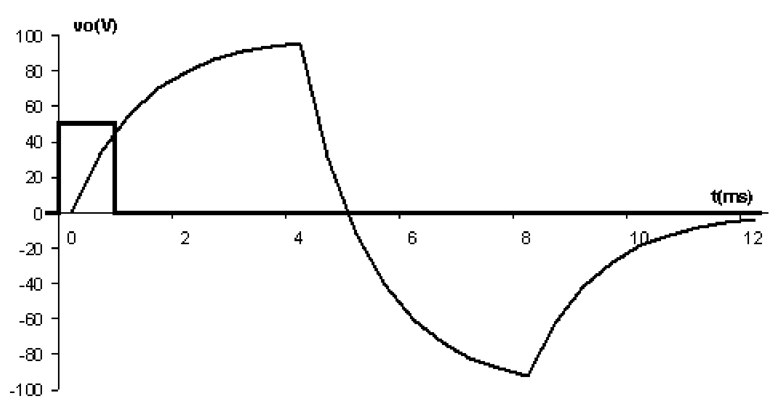
$4\,\text{ms} \leq t \leq 8\,\text{ms}$:

$$v_o = -100 + 195.92e^{-800(t-0.004)}\,\text{V}, \qquad 4\,\text{ms} \leq t \leq 8\,\text{ms}$$

$$v_o(8\,\text{ms}) = -100 + 195.92e^{-3.2} = -92.01\,\text{V}$$

$8\,\text{ms} \leq t \leq \infty$:

$$v_o = -92.01e^{-800(t-0.008)}\,\text{V}, \qquad 8\,\text{ms} \leq t \leq \infty$$



P 7.83 **[a]** $\tau = RC = (20{,}000)(0.2 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$

$i_o = v_o = 0 \qquad t < 0$

$i_o(0^+) = 20\left(\frac{16}{20}\right) = 16\,\text{mA}, \qquad i_o(\infty) = 0$

$\therefore \quad i_o = 16e^{-250t}\,\text{mA} \qquad 0^+ \leq t \leq 2^-\,\text{ms}$

$i_{16k\Omega} = 20 - 16e^{-250t}\,\text{mA}$

$\therefore \quad v_o = 320 - 256e^{-250t}\,\text{V} \qquad 0^+ \leq t \leq 2^-\,\text{ms}$

$v_c = v_o - 4 \times 10^3 i_o = 320 - 320e^{-250t}\,\text{V} \qquad 0 \leq t \leq 2\,\text{ms}$

$v_c(2\,\text{ms}) = 320 - 320e^{-0.5} = 125.91\,\text{V}$

$\therefore \quad i_o(2^+\,\text{ms}) = 16e^{-0.5} = 9.7\,\text{mA}$

$i_o(\infty) = 0$

$v_c = 125.91e^{-250(t-0.002)}, \quad 2^+\,\text{ms} \leq t \leq \infty$

$$i_o = C\frac{dv_c}{dt} = (0.2 \times 10^{-6})(-250)(125.91)e^{-250(t-0.002)}$$

$$= -6.3e^{-250(t-0.002)}\,\text{mA}, \quad 2^+\,\text{ms} \leq t \leq \infty$$

$v_o = 4000 i_o + v_c = 100.73e^{-250(t-0.002)}\,\text{V} \qquad 2^+\,\text{ms} \leq t \leq \infty$

Summary part (a)

$i_o = 0 \qquad t < 0$

$i_o = 16e^{-250t}\,\text{mA} \qquad (0^+ \leq t \leq 2^-\,\text{ms})$

$i_o = -6.3e^{-250(t-0.002)}\,\text{mA} \qquad 2^+\,\text{ms} \leq t \leq \infty$

$v_o = 0 \qquad t < 0$

$v_o = 320 - 256e^{-250t}\,\text{V}, \qquad 0 \leq t \leq 2^-\,\text{ms}$

$v_o = 100.73e^{-250(t-0.002)}\,\text{V}, \qquad 2^+\,\text{ms} \leq t \leq \infty$

[b] $i_o(0^-) = 0$

$i_o(0^+) = 16\,\text{mA}$

$i_o(2^-\,\text{ms}) = 16e^{-0.5} = 9.7\,\text{mA}$

$i_o(2^+\,\text{ms}) = -6.3\,\text{mA}$

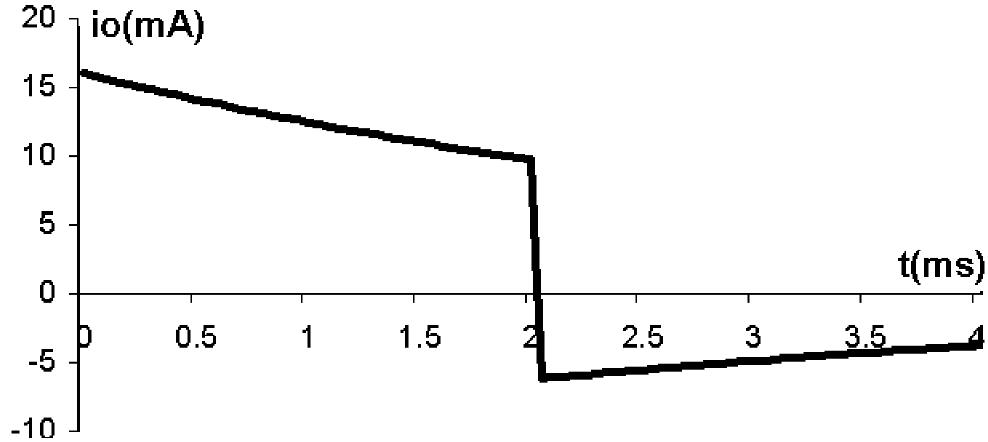
[c] $v_o(0^-) = 0$

$v_o(0^+) = 64\,\text{V}$

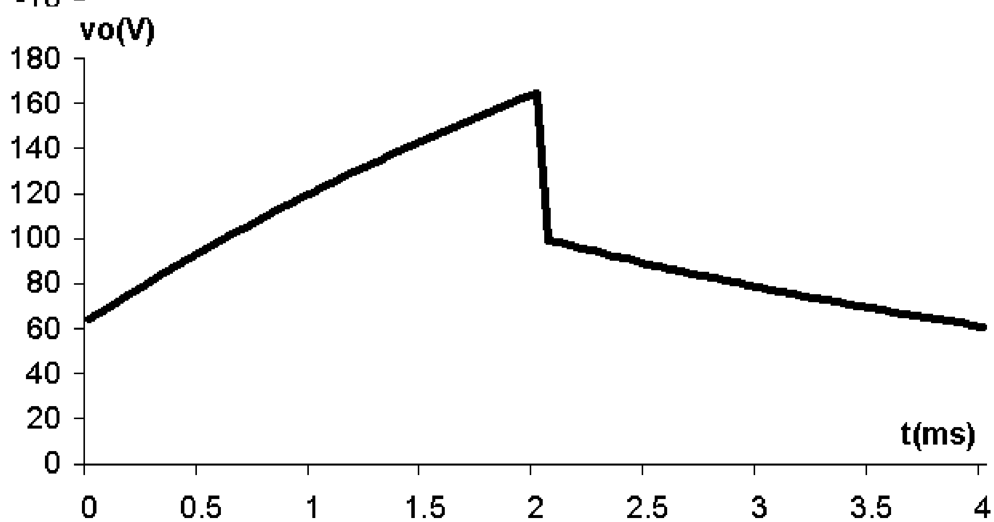
$v_o(2^-\,\text{ms}) = 320 - 256e^{-0.5} = 164.73\,\text{V}$

$v_o(2^+\,\text{ms}) = 100.73$

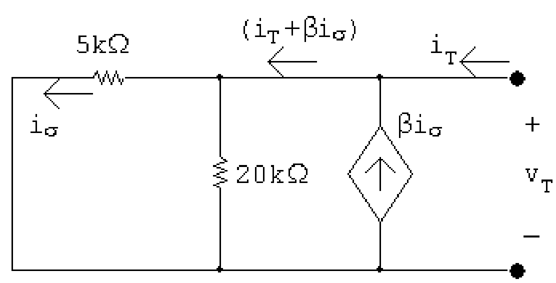
[d]



[e]



P 7.84 **[a]**



Using Ohm's law,

$v_T = 5000 i_\sigma$

Using current division,

$$i_\sigma = \frac{20{,}000}{20{,}000 + 5000}(i_T + \beta i_\sigma) = 0.8 i_T + 0.8\beta i_\sigma$$

Solve for i_σ:

$$i_\sigma(1 - 0.8\beta) = 0.8 i_T$$

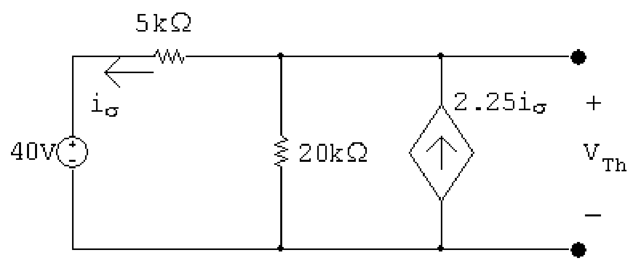
$$i_\sigma = \frac{0.8 i_T}{1 - 0.8\beta}; \qquad v_T = 5000 i_\sigma = \frac{4000 i_T}{(1 - 0.8\beta)}$$

Find β such that $R_{\text{Th}} = -5\,\text{k}\Omega$:

$$R_{\text{Th}} = \frac{v_T}{i_T} = \frac{4000}{1 - 0.8\beta} = -5000$$

$$1 - 0.8\beta = -0.8 \qquad \therefore \quad \beta = 2.25$$

[b] Find V_{Th};



Write a KCL equation at the top node:

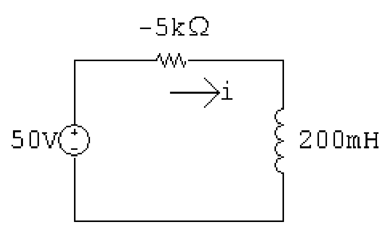
$$\frac{V_{\text{Th}} - 40}{5000} + \frac{V_{\text{Th}}}{20{,}000} - 2.25 i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{\text{Th}} - 40)}{5000} = 0$$

Solving,

$$V_{\text{Th}} = 50\,\text{V}$$



Write a KVL equation around the loop:

$$50 = -5000 i + 0.2\frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 250 + 25{,}000 i = 25{,}000(i + 0.01)$$

Separate the variables and integrate to find i;

$$\frac{di}{i+0.01} = 25{,}000\,dt$$

$$\int_0^i \frac{dx}{x+0.01} = \int_0^t 25{,}000\,dx$$

$$\therefore \quad i = -10 + 10e^{25{,}000t}\ \text{mA}$$

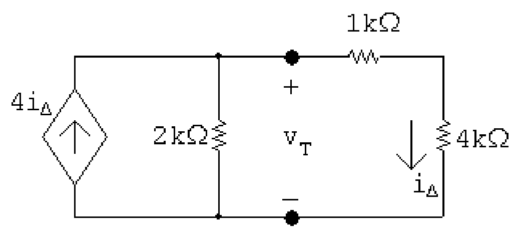
$$\frac{di}{dt} = (10\times 10^{-3})(25{,}000)e^{25{,}000t} = 250e^{25{,}000t}$$

Solve for the arc time:

$$v = 0.2\frac{di}{dt} = 50e^{25{,}000t} = 45{,}000; \qquad e^{25{,}000t} = 900$$

$$\therefore \quad t = \frac{\ln 900}{25{,}000} = 272.1\,\mu\text{s}$$

P 7.85 Find the Thévenin equivalent with respect to the terminals of the capacitor.
R_{Th} calculation:

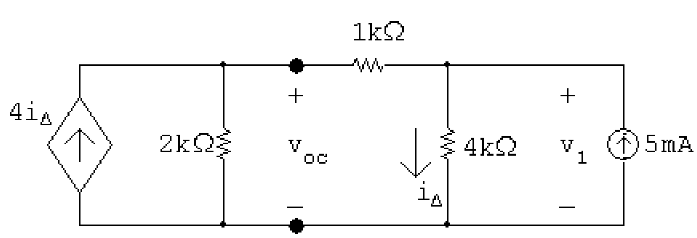


$$i_T = \frac{v_T}{2000} + \frac{v_T}{5000} - 4\frac{v_T}{5000}$$

$$\therefore \quad \frac{i_T}{v_T} = \frac{5+2-8}{10{,}000} = -\frac{1}{10{,}000}$$

$$\frac{v_T}{i_T} = -\frac{10{,}000}{1} = -10\,\text{k}\Omega$$

Open circuit voltage calculation:



The node voltage equations:

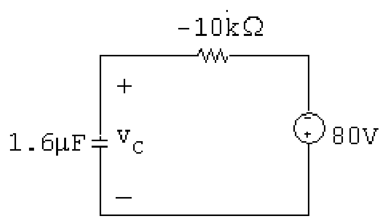
$$\frac{v_{\rm oc}}{2000} + \frac{v_{\rm oc} - v_1}{1000} - 4i_\Delta = 0$$

$$\frac{v_1 - v_{\rm oc}}{1000} + \frac{v_1}{4000} - 5 \times 10^{-3} = 0$$

The constraint equation:

$$i_\Delta = \frac{v_1}{4000}$$

Solving, $v_{\rm oc} = -80\,\text{V}, \quad v_1 = -60\,\text{V}$



$$v_{\rm c}(0) = 0; \qquad v_{\rm c}(\infty) = -80\,\text{V}$$

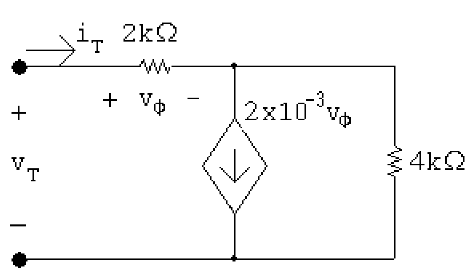
$$\tau = RC = (-10{,}000)(1.6 \times 10^{-6}) = -16\,\text{ms}; \qquad \frac{1}{\tau} = -62.5$$

$$v_{\rm c} = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = -80 + 80e^{62.5t} = 14{,}400$$

Solve for the time of the maximum voltage rating:

$$e^{62.5t} = 181; \qquad 62.5t = \ln 181; \qquad t = 83.18\,\text{ms}$$

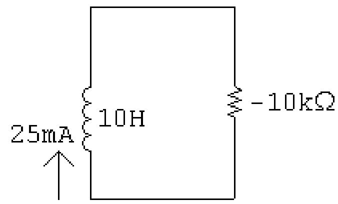
P 7.86



$$v_T = 2000i_T + 4000(i_T - 2 \times 10^{-3}v_\phi) = 6000i_T - 8v_\phi$$

$$= 6000i_T - 8(2000i_T)$$

$$\frac{v_T}{i_T} = -10{,}000$$

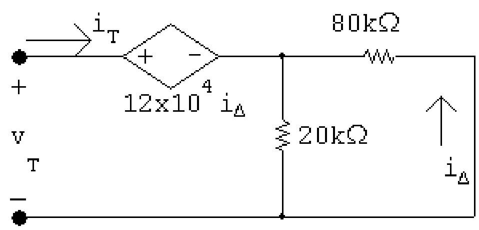


$$\tau = \frac{10}{-10{,}000} = -1\,\text{ms}; \qquad 1/\tau = -1000$$

$$i = 25e^{1000t}\,\text{mA}$$

$$\therefore \quad 25e^{1000t} \times 10^{-3} = 5; \qquad t = \frac{\ln 200}{1000} = 5.3\,\text{ms}$$

P 7.87 $t > 0$:



$$v_T = 12 \times 10^4 i_\Delta + 16 \times 10^3 i_T$$

$$i_\Delta = -\frac{20}{100} i_T = -0.2 i_T$$

$$\therefore \quad v_T = -24 \times 10^3 i_T + 16 \times 10^3 i_T$$

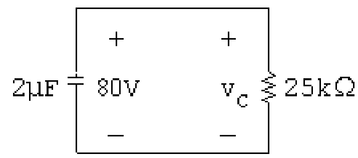
$$R_{\text{Th}} = \frac{v_T}{i_T} = -8\,\text{k}\Omega$$

$$\tau = RC = (-8 \times 10^3)(2.5 \times 10^{-6}) = -0.02 \quad 1/\tau = -50$$

$$v_c = 20e^{50t}\,\text{V}; \qquad 20e^{50t} = 20{,}000$$

$$50t = \ln 1000 \quad \therefore \quad t = 138.16\,\text{ms}$$

P 7.88 **[a]**



$$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$$

$$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$$

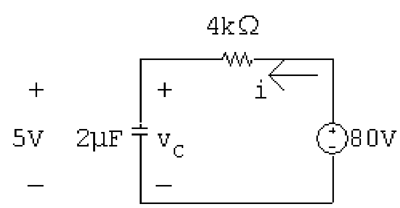
$$v_c = 80e^{-20t}\,\text{V}$$

$$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \frac{\ln 16}{20} = 138.63\,\text{ms}$$

[b] $0^+ < t < 138.63\,\text{ms}$:

$$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$$

$138.63^+\ \text{ms} < t \leq \infty$:



$$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$$

$$v_c(138.63^+\ \text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad 138.63^+\ \text{ms} \leq t \leq \infty$$

$$i = 2 \times 10^{-6}(9375)e^{-125(t-0.13863)}$$
$$= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad 138.63^+\ \text{ms} \leq t \leq \infty$$

[c] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \qquad \Delta t = \frac{\ln 6.25}{12.5} \cong 14.66\,\text{ms}$$

P 7.89 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36\,\text{V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36-14}{80{,}000} + 2.5\times10^{-6}\frac{d}{dt}(-36-v_o) = 0$$

$$\therefore \qquad 2.5\times10^{-6}\frac{dv_o}{dt} = \frac{-50}{80{,}000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \quad \therefore \quad dv_o = -250dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250\int_0^t dy \quad \therefore \quad v_o(t) - v_o(0) = -250t$$

$$v_o(0) = -36 + 56 = 20\,\text{V}$$

$$v_o(t) = -250t + 20$$

Find the time when the voltage reaches 0:

$$0 = -250t + 20 \quad \therefore \quad t = \frac{20}{250} = 80\,\text{ms}$$

P 7.90 The equation for an integrating amplifier:

$$v_o = \frac{1}{RC}\int_0^t (v_\text{b} - v_\text{a})\,dy + v_o(0)$$

Find the values and substitute them into the equation:

$$RC = (100\times10^3)(0.05\times10^{-6}) = 5\,\text{ms}$$

$$\frac{1}{RC} = 200; \qquad v_\text{b} - v_\text{a} = -15 - (-7) = -8\,\text{V}$$

$$v_o(0) = -4 + 12 = 8\,\text{V}$$

$$v_o = 200\int_0^t -8\,dx + 8 = (-1600t + 8)\,\text{V}, \quad 0 \le t \le t_\text{sat}$$

RC circuit analysis for v_2:

$$v_2(0^+) = -4\,\text{V}; \qquad v_2(\infty) = -15\,\text{V}; \qquad \tau = RC = (100\,\text{k})(0.05\,\mu) = 5\,\text{ms}$$

$$v_2 = v_2(\infty) + [v_2(0^+) - v_2(\infty)]e^{-t/\tau}$$

$$= -15 + (-4 + 15)e^{-200t} = -15 + 11e^{-200t}\,\text{V}, \quad 0 \le t \le t_{\text{sat}}$$

$$v_f + v_2 = v_o \quad \therefore \quad v_f = v_o - v_2 = 23 - 1600t - 11e^{-200t}\,\text{V}, \quad 0 \le t \le t_{\text{sat}}$$

Note that

$$-1600t_{\text{sat}} + 8 = -20 \quad \therefore \quad t_{\text{sat}} = \frac{-28}{-1600} = 17.5\,\text{ms}$$

so the op amp operates in its linear region until it saturates at 17.5 ms.

P 7.91 $$v_o = -\frac{1}{R(0.5 \times 10^{-6})}\int_0^t 4\,dx + 0 = \frac{-4t}{R(0.5 \times 10^{-6})}$$

$$\frac{-4(15 \times 10^{-3})}{R(0.5 \times 10^{-6})} = -10$$

$$\therefore \quad R = \frac{-4(15 \times 10^{-3})}{-10(0.5 \times 10^{-6})} = 12\,\text{k}\Omega$$

P 7.92 $$v_o = \frac{-4t}{R(0.5 \times 10^{-6})} + 6 = \frac{-4(40 \times 10^{-3})}{R(0.5 \times 10^{-6})} + 6 = -10$$

$$\therefore \quad R = \frac{-4(40 \times 10^{-3})}{-16(0.5 \times 10^{-6})} = 20\,\text{k}\Omega$$

P 7.93 **[a]** $$\frac{Cdv_p}{dt} + \frac{v_p - v_{\text{b}}}{R} = 0; \qquad \text{therefore} \quad \frac{dv_p}{dt} + \frac{1}{RC}v_p = \frac{v_{\text{b}}}{RC}$$

$$\frac{v_n - v_{\text{a}}}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

$$\text{therefore} \quad \frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_{\text{a}}}{RC}$$

But $v_n = v_p$

$$\text{Therefore} \quad \frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_{\text{b}}}{RC}$$

$$\text{Therefore} \quad \frac{dv_o}{dt} = \frac{1}{RC}(v_{\text{b}} - v_{\text{a}}); \qquad v_o = \frac{1}{RC}\int_0^t (v_{\text{b}} - v_{\text{a}})\,dy$$

[b] The output is the integral of the difference between v_{b} and v_{a} and then scaled by a factor of $1/RC$.

[c] $v_o = \frac{1}{RC}\int_0^t (v_\text{b} - v_\text{a})\,dx$

$$RC = (50 \times 10^3)(10 \times 10^{-9}) = 0.5\,\text{ms}$$

$$v_\text{b} - v_\text{a} = -25\,\text{mV}$$

$$v_o = \frac{1}{0.0005}\int_0^t -25 \times 10^{-3} dx = -50t$$

$$-50t_\text{sat} = -6; \qquad t_\text{sat} = 120\,\text{ms}$$

P 7.94 **[a]** $RC = (25 \times 10^3)(0.4 \times 10^{-6}) = 10\,\text{ms}; \qquad \frac{1}{RC} = 100$

$$v_o = 0, \quad t < 0$$

[b] $0 \leq t \leq 250\,\text{ms}:$

$$v_o = -100\int_0^t -0.20\,dx = 20t\,\text{V}$$

[c] $250\,\text{ms} \leq t \leq 500\,\text{ms};$

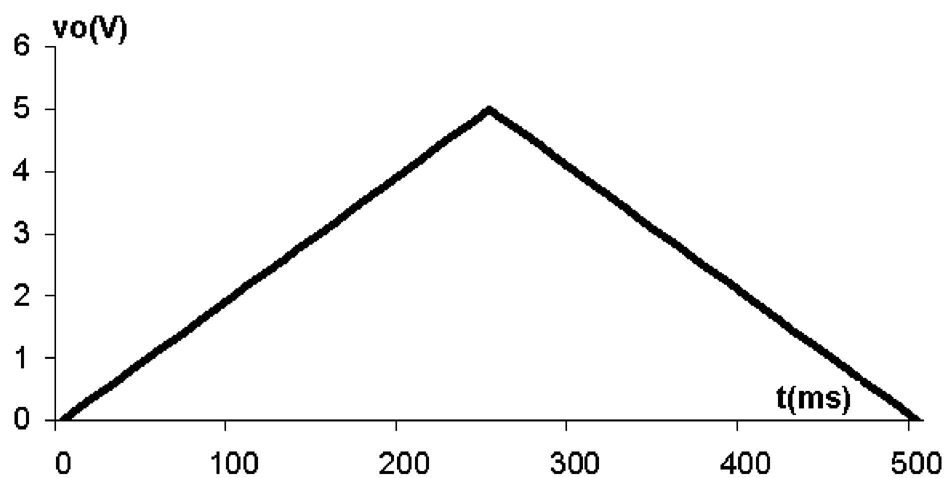
$$v_o(0.25) = 20(0.25) = 5\,\text{V}$$

$$v_o(t) = -100\int_{0.25}^t 0.20\,dx + 5 = -20(t - 0.25) + 5 = -20t + 10\,\text{V}$$

[d] $500\,\text{ms} \leq t \leq \infty:$

$$v_o(0.5) = -10 + 10 = 0\,\text{V}$$

$$v_o(t) = 0\,\text{V}$$



P 7.95 **[a]** $v_o = 0, \qquad t < 0$

$$RC = (25 \times 10^3)(0.4 \times 10^{-6}) = 10\,\text{ms} \qquad \frac{1}{RC} = 100$$

[b] $R_fC_f = (5 \times 10^6)(0.4 \times 10^{-6}) = 2; \qquad \frac{1}{R_fC_f} = 0.5$

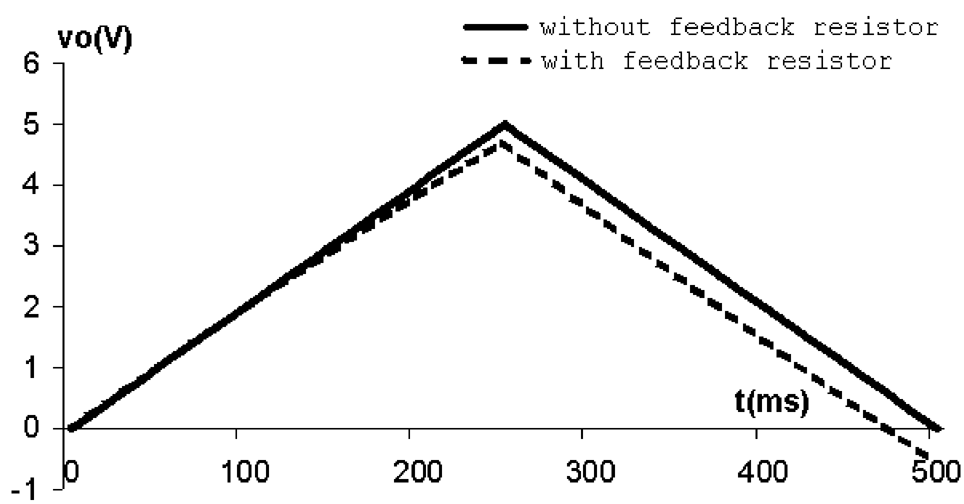
$$v_o = \frac{-5 \times 10^6}{25 \times 10^3}(-0.2)[1 - e^{-0.5t}] = 40(1 - e^{-0.5t})\,\text{V}, \qquad 0 \le t \le 250\,\text{ms}$$

[c] $v_o(0.25) = 40(1 - e^{-0.125}) \cong 4.70\,\text{V}$

$$\begin{aligned} v_o &= \frac{-V_mR_f}{R_s} + \frac{V_mR_f}{R_s}(2 - e^{-0.125})e^{-0.5(t-0.25)} \\ &= -40 + 40(2 - e^{-0.125})e^{-0.5(t-0.25)} \\ &= -40 + 44.70e^{-0.5(t-0.25)}\,\text{V}, \qquad 250\,\text{ms} \le t \le 500\,\text{ms} \end{aligned}$$

[d] $v_o(0.5) = -40 + 44.70e^{-0.125} \cong -0.55\,\text{V}$

$$v_o = -0.55e^{-0.5(t-0.5)}\,\text{V}, \qquad 500\,\text{ms} \le t \le \infty$$



P 7.96 **[a]** $RC = (1000)(800 \times 10^{-12}) = 800 \times 10^{-9}; \qquad \frac{1}{RC} = 1{,}250{,}000$

$0 \le t \le 1\,\mu\text{s}$:

$$v_g = 2 \times 10^6 t$$

$$\begin{aligned} v_o &= -1.25 \times 10^6 \int_0^t 2 \times 10^6 x\,dx + 0 \\ &= -2.5 \times 10^{12}\frac{x^2}{2}\Big|_0^t = -125 \times 10^{10}t^2\,\text{V}, \quad 0 \le t \le 1\,\mu\text{s} \end{aligned}$$

$$v_o(1\,\mu\text{s}) = -125\times10^{10}(1\times10^{-6})^2 = -1.25\,\text{V}$$

$1\,\mu\text{s} \le t \le 3\,\mu\text{s}$:

$$v_g = 4 - 2\times10^6 t$$

$$v_o = -125\times10^4\int_{1\times10^{-6}}^{t}(4-2\times10^6x)\,dx - 1.25$$
$$= -125\times10^4\left[4x\Big|_{1\times10^{-6}}^{t} - 2\times10^6\frac{x^2}{2}\Big|_{1\times10^{-6}}^{t}\right] - 1.25$$
$$= -5\times10^6t + 5 + 125\times10^{10}t^2 - 1.25 - 1.25$$
$$= 125\times10^{10}t^2 - 5\times10^6t + 2.5\,\text{V}, \quad 1\,\mu\text{s} \le t \le 3\,\mu\text{s}$$

$$v_o(3\,\mu\text{s}) = 125\times10^{10}(3\times10^{-6})^2 - 5\times10^6(3\times10^{-6}) + 2.5$$
$$= -1.25$$

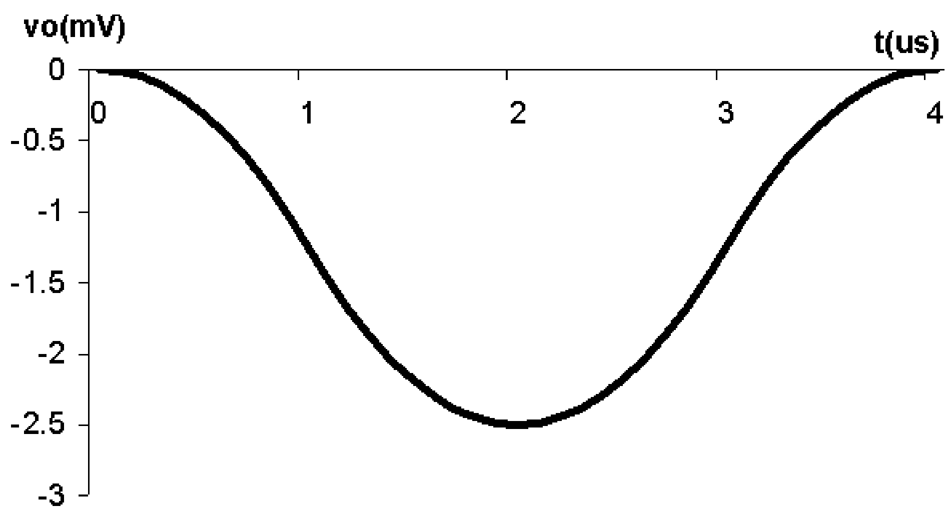
$3\,\mu\text{s} \le t \le 4\,\mu\text{s}$:

$$v_g = -8 + 2\times10^6 t$$

$$v_o = -125\times10^4\int_{3\times10^{-6}}^{t}(-8+2\times10^6x)\,dx - 1.25$$
$$= -125\times10^4\left[-8x\Big|_{3\times10^{-6}}^{t} + 2\times10^6\frac{x^2}{2}\Big|_{3\times10^{-6}}^{t}\right] - 1.25$$
$$= 10^7t - 30 - 125\times10^{10}t^2 + 11.25 - 1.25$$
$$= -125\times10^{10}t^2 + 10^7t - 20\,\text{V}, \quad 3\,\mu\text{s} \le t \le 4\,\mu\text{s}$$

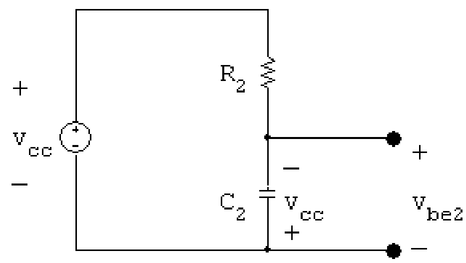
$$v_o(4\,\mu\text{s}) = -125\times10^{10}(4\times10^{-6})^2 + 10^7(4\times10^{-6}) - 20 = 0$$

[b]



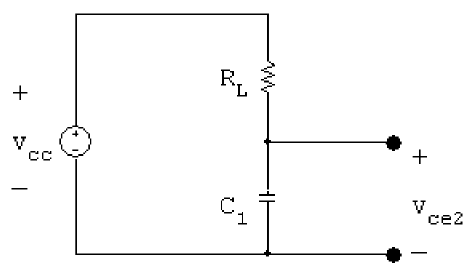
[c] The output voltage will also repeat. This follows from the observation that at $t = 4\,\mu$s the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\,\mu$s as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.97 **[a]** While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{\text{be2}} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, v_{be2} is zero, T_2 turns ON. This makes $v_{\text{be1}} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{\text{be2}} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage v_{ce2} is the same as the voltage across C_1, thus



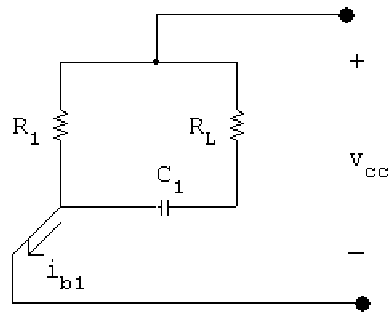
It follows that $v_{\text{ce2}} = V_{CC} - V_{CC}e^{-t/R_\text{L}C_1}$.

[c] T_2 will be OFF until v_{be2} reaches zero. As soon as v_{be2} is zero, i_{b2} will become positive and turn T_2 ON. $v_{\text{be2}} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

[d] When $t = R_2C_2 \ln 2$, we have

$$v_{\text{ce2}} = V_{CC} - V_{CC}e^{-[(R_2C_2 \ln 2)/(R_\text{L}C_1)]} = V_{CC} - V_{CC}e^{-10 \ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, i_{b1} is zero. At the instant T_1 turns ON, we have



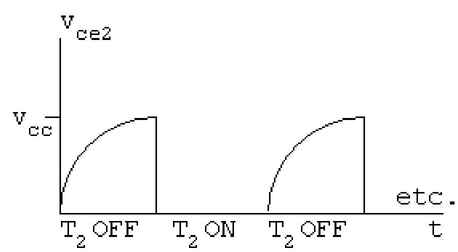
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\rm L}} e^{-t/R_{\rm L}C_1}$$

[f] At the instant T_2 turns back ON, $t = R_2C_2 \ln 2$; therefore

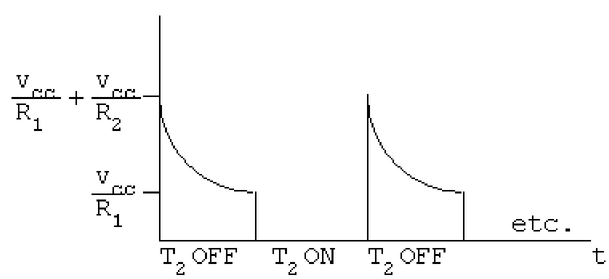
$$i_{\rm b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\rm L}} e^{-10 \ln 2} \cong \frac{V_{CC}}{R_1}$$

When T_2 turns OFF, $i_{\rm b1}$ drops to zero instantaneously.

[g]



[h]



P 7.98 **[a]** $t_{\rm OFF2} = R_2C_2 \ln 2 = 14.43 \times 10^3(1 \times 10^{-9}) \ln 2 \cong 10\,\mu$s

[b] $t_{\rm ON2} = R_1C_1 \ln 2 \cong 10\,\mu$s

[c] $t_{\rm OFF1} = R_1C_1 \ln 2 \cong 10\,\mu$s

[d] $t_{\rm ON1} = R_2C_2 \ln 2 \cong 10\,\mu$s

[e] $i_{\rm b1} = \frac{10}{1000} + \frac{10}{14{,}430} \cong 10.69\,$mA

[f] $i_{\rm b1} = \frac{10}{14{,}430} + \frac{10}{1000} e^{-10} \cong 0.693\,$mA

[g] $v_{\text{ce2}} = 10 - 10e^{-10} \cong 10\,\text{V}$

P 7.99 **[a]** $t_{\text{OFF2}} = R_2C_2 \ln 2 = (14.43 \times 10^3)(0.8 \times 10^{-9}) \ln 2 \cong 8\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 10\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 10\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 = 8\,\mu\text{s}$

[e] $i_{\text{b1}} = 10.69\,\text{mA}$

[f] $i_{\text{b1}} = \dfrac{10}{14{,}430} + \dfrac{10}{1000}e^{-8} \cong 0.693\,\text{mA}$

[g] $v_{\text{ce2}} = 10 - 10e^{-8} \cong 10\,\text{V}$

Note in this circuit T_2 is OFF $8\,\mu$s and ON $10\,\mu$s of every cycle, whereas T_1 is ON $8\,\mu$s and OFF $10\,\mu$s every cycle.

P 7.100 If $R_1 = R_2 = 50R_{\text{L}} = 100\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\text{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\text{pF}$$

If $R_1 = R_2 = 6R_{\text{L}} = 12\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \le C_1 \le 5.77\,\text{nF}$ and $519.37\,\text{pF} \le C_2 \le 4.33\,\text{nF}$

P 7.101 **[a]** T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{\text{b2}} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{\text{ce2}} = 0$, therefore $i_{\text{b1}} = 0$. When $i_{\text{b1}} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

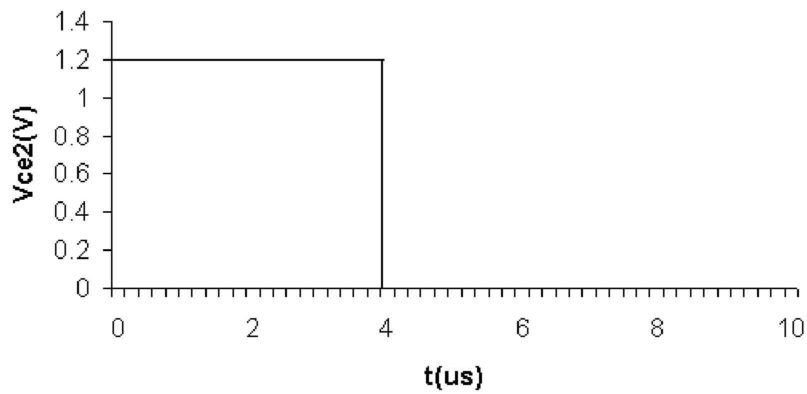
[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_{\text{L}})$ and i_{b1} jumps to $V_{CC}/(R_1 + R_{\text{L}})$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{\text{be2}} = 0$. The equation for v_{be2} is $v_{\text{be2}} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{\text{be2}} = 0$ when $t = RC \ln 2$, therefore T_2 stays OFF for $RC \ln 2$ seconds.

P 7.102 **[a]** For $t < 0$, $v_{\text{ce2}} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{\text{ce2}} = \left(\frac{V_{CC}}{R_1 + R_{\text{L}}}\right) R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

T_2 remains open for $(23{,}083)(250) \times 10^{-12} \ln 2 \cong 4\,\mu$s.

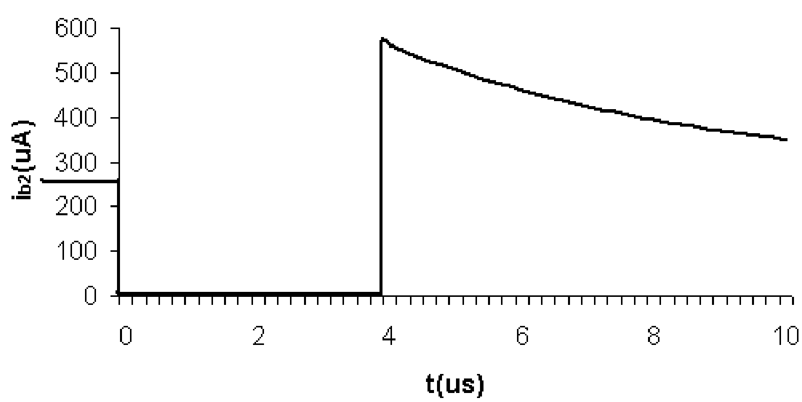


[b] $i_{b2} = \dfrac{V_{CC}}{R} = 259.93\,\mu\text{A}, \qquad -5 \le t \le 0\,\mu\text{s}$

$i_{b2} = 0, \qquad 0 < t < RC \ln 2$

$$i_{b2} = \frac{V_{CC}}{R} + \frac{V_{CC}}{R_L}e^{-(t-RC\ln 2)/R_L C}$$

$$= 259.93 + 300e^{-0.2\times 10^6(t-4\times 10^{-6})}\,\mu\text{A}, \qquad RC \ln 2 < t$$



P 7.103 **[a]** We want the lamp to be in its nonconducting state for no more than 10 s, the value of t_o:

$$10 = R(10 \times 10^{-6}) \ln \frac{1-6}{4-6} \quad \text{and} \quad R = 1.091\ \text{M}\Omega$$

[b] When the lamp is conducting

$$V_{Th} = \frac{20 \times 10^3}{20 \times 10^3 + 1.091 \times 10^6}(6) = 0.108\,\text{V}$$

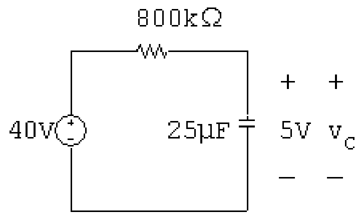
$$R_{Th} = 20\ \text{k}||1.091\ \text{M} = 19{,}640\ \Omega$$

So,

$$(t_c - t_o) = (19{,}640)(10 \times 10^{-6}) \ln \frac{4 - 0.108}{1 - 0.108} = 0.289\,\text{s}$$

The flash lasts for 0.289 s.

P 7.104 **[a]** At $t = 0$ we have



$\tau = (800)(25) \times 10^{-3} = 20$ sec; $\quad 1/\tau = 0.05$

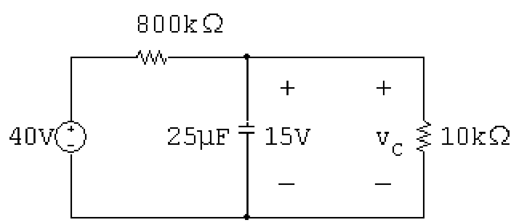
$v_c(\infty) = 40\,\text{V}; \quad v_c(0) = 5\,\text{V}$

$v_c = 40 - 35e^{-0.05t}\,\text{V}, \quad 0 \le t \le t_o$

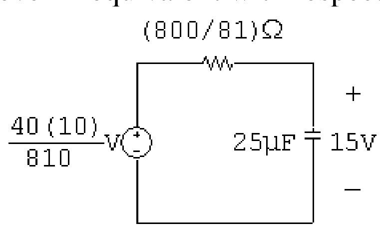
$40 - 35e^{-0.05t_o} = 15; \quad \therefore \quad e^{0.05t_o} = 1.4$

$t_o = 20\ \ln 1.4\,\text{s} = 6.73\,\text{s}$

At $t = t_o$ we have



The Thévenin equivalent with respect to the capacitor is



$$\tau = \left(\frac{800}{81}\right)(25) \times 10^{-3} = \frac{20}{81}\,\text{s}; \qquad \frac{1}{\tau} = \frac{81}{20} = 4.05$$

$$v_c(t_o) = 15\,\text{V}; \qquad v_c(\infty) = \frac{40}{81}\,\text{V}$$

$$v_c(t) = \frac{40}{81} + \left(15 - \frac{40}{81}\right) e^{-4.05(t-t_o)}\,\text{V} = \frac{40}{81} + \frac{1175}{81} e^{-4.05(t-t_o)}$$

$$\therefore \quad \frac{40}{81} + \frac{1175}{81} e^{-4.05(t-t_o)} = 5$$

$$\frac{1175}{81}e^{-4.05(t-t_o)} = \frac{365}{81}$$

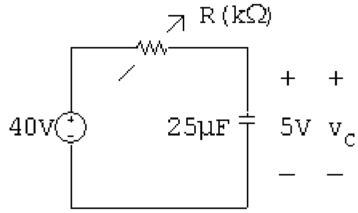
$$e^{4.05(t-t_o)} = \frac{1175}{365} = 3.22$$

$$t - t_o = \frac{1}{4.05}\ \ln 3.22 \cong 0.29\,\text{s}$$

One cycle = 7.02 seconds.

$N = 60/7.02 = 8.55$ flashes per minute

[b] At $t = 0$ we have



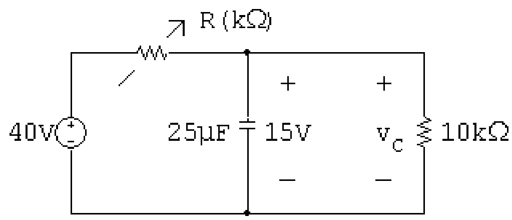
$$\tau = 25R \times 10^{-3}; \qquad 1/\tau = 40/R$$

$$v_c = 40 - 35e^{-(40/R)t}$$

$$40 - 35e^{-(40/R)t_o} = 15$$

$$\therefore\quad t_o = \frac{R}{40}\ \ln 1.4, \qquad R \quad \text{in} \quad \text{k}\Omega$$

At $t = t_o$:



$$v_{\text{Th}} = \frac{10}{R+10}(40) = \frac{400}{R+10}; \qquad R_{\text{Th}} = \frac{10R}{R+10}\,\text{k}\Omega$$

$$\tau = \frac{(25)(10R)\times 10^{-3}}{R+10} = \frac{0.25R}{R+10}; \qquad \frac{1}{\tau} = \frac{4(R+10)}{R}$$

$$v_c = \frac{400}{R+10} + \left(15 - \frac{400}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)}$$

$$\therefore\quad \frac{400}{R+10} + \left[\frac{15R-250}{R+10}\right]e^{-\frac{4(R+10)}{R}(t-t_o)} = 5$$

$$\text{or}\quad \left(\frac{15R-250}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)} = \frac{5R-350}{(R+10)}$$

$$\therefore \quad e^{\frac{4(R+10)}{R}(t-t_o)} = \frac{3R-50}{R-70}$$

$$\therefore \quad t - t_o = \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right)$$

At 12 flashes per minute $\quad t_o + (t - t_o) = 5\,\text{s}$

$$\therefore \quad \underbrace{\frac{R}{40} \ln 1.4}_{\text{dominant term}} + \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right) = 5$$

Start the trial-and-error procedure by setting $(R/40)\ \ln 1.4 = 5$, then $R = 200/(\ln 1.4)$ or $594.40\,\text{k}\Omega$. If $R = 594.40\,\text{k}\Omega$ then $t - t_o \cong 0.29\,\text{s}$. Second trial set $(R/40)\ \ln 1.4 = 4.7\,\text{s}$ or $R = 558.74\,\text{k}\Omega$.

With $\quad R = 558.74\,\text{k}\Omega, \quad t - t_o \cong 0.30\,\text{s}$

The procedure converges to $R = 559.3\ \text{k}\Omega$

P 7.105 **[a]** $t_o = RC \ln\left(\frac{V_{\text{min}} - V_s}{V_{\text{max}} - V_s}\right) = (3700)(250 \times 10^{-6}) \ln\left(\frac{-700}{-100}\right)$

$$= 1.80\,\text{s}$$

$$t_c - t_o = \frac{RCR_{\text{L}}}{R + R_{\text{L}}} \ln\left(\frac{V_{\text{max}} - V_{\text{Th}}}{V_{\text{min}} - V_{\text{Th}}}\right)$$

$$\frac{R_{\text{L}}}{R + R_{\text{L}}} = \frac{1.3}{1.3 + 3.7} = 0.26 \quad RC = (3700)(250 \times 10^{-6}) = 0.925\ \text{s}$$

$$V_{\text{Th}} = \frac{1000(1.3)}{1.3 + 3.7} = 260\,\text{V} \quad R_{\text{Th}} = 3.7\ \text{k}||1.3\ \text{k} = 962\ \Omega$$

$$\therefore \quad t_c - t_o = (0.925)(0.26) \ln(640/40) = 0.67\,\text{s}$$

$$\therefore \quad t_c = 1.8 + 0.67 = 2.47\,\text{s}$$

$$\text{flashes/min} = \frac{60}{2.47} = 24.32$$

[b] $0 \le t \le t_o$:

$$v_L = 1000 - 700e^{-t/\tau_1}$$

$$\tau_1 = RC = 0.925\,\text{s}$$

$t_o \le t \le t_c$:

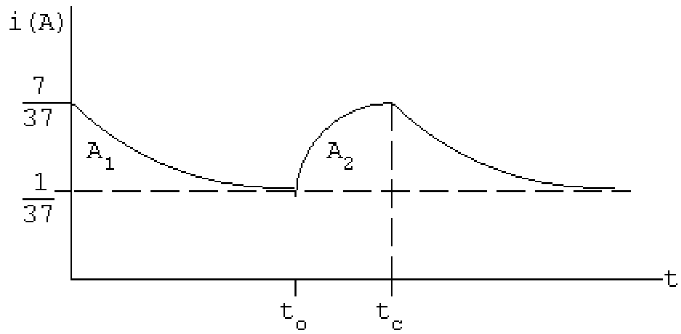
$$v_L = 260 + 640e^{-(t-t_o)/\tau_2}$$

$$\tau_2 = R_{\text{Th}}C = 962(250) \times 10^{-6} = 0.2405\,\text{s}$$

$$0 \le t \le t_o: \qquad i = \frac{1000 - v_L}{3700} = \frac{7}{37}e^{-t/0.925}\text{ A}$$

$$t_o \le t \le t_c: \qquad i = \frac{1000 - v_L}{3700} = \frac{74}{370} - \frac{64}{370}e^{-(t-t_o)/0.2405}$$

Graphically, i versus t is



The average value of i will equal the areas $(A_1 + A_2)$ divided by t_c.

$$\therefore \quad i_{\text{avg}} = \frac{A_1 + A_2}{t_c}$$

$$\begin{aligned} A_1 &= \frac{7}{37}\int_0^{t_o} e^{-t/0.925}\,dt \\ &= \frac{6.475}{37}(1 - e^{-\ln 7}) = 0.15\text{ A–s} \\ A_2 &= \int_{t_o}^{t_c} \frac{74 - 64e^{-(t-t_o)/0.2405}}{370}\,dt \\ &= \frac{74}{370}(t_c - t_o) + \frac{15.392}{370}(e^{-\ln 16} - 1) \\ &= \frac{17.797}{370}\ln 16 - \frac{15.392}{370}(1 - e^{-\ln 16}) \\ &= 0.09436\text{ A–s} \end{aligned}$$

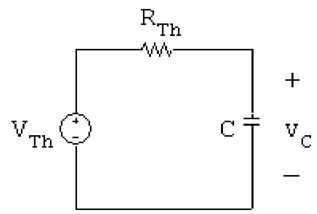
$$i_{\text{avg}} = \frac{(0.15 + 0.09436)}{0.925\ln 7 + 0.2405\ln 16}(1000) = 99.06\,\text{mA}$$

[c] $P_{\text{avg}} = (1000)(99.06 \times 10^{-3}) = 99.06\,\text{W}$

$$\text{No. of kw hrs/yr} = \frac{(99.06)(24)(365)}{1000} = 867.77$$

$$\text{Cost/year} = (867.77)(0.05) = 43.39\text{ dollars/year}$$

P 7.106 **[a]** Replace the circuit attached to the capacitor with its Thévenin equivalent, where the equivalent resistance is the parallel combination of the two resistors, and the open-circuit voltage is obtained by voltage division across the lamp resistance. The resulting circuit is



$$R_{\text{Th}} = R \| R_{\text{L}} = \frac{RR_{\text{L}}}{R + R_{\text{L}}}; \qquad V_{\text{Th}} = \frac{R_{\text{L}}}{R + R_{\text{L}}} V_s$$

From this circuit,

$$v_{\text{C}}(\infty) = V_{\text{Th}}; \qquad v_{\text{C}}(0) = V_{\text{max}}; \qquad \tau = R_{\text{Th}} C$$

Thus,

$$v_{\text{C}}(t) = V_{\text{Th}} + (V_{\text{max}} - V_{\text{Th}}) e^{-(t-t_o)/\tau}$$

where

$$\tau = \frac{RR_{\text{L}}C}{R + R_{\text{L}}}$$

[b] Now, set $v_{\text{C}}(t_c) = V_{\text{min}}$ and solve for $(t_c - t_o)$:

$$V_{\text{Th}} + (V_{\text{max}} - V_{\text{Th}}) e^{-(t_c - t_o)/\tau} = V_{\text{min}}$$

$$e^{-(t_c - t_o)/\tau} = \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$\frac{-(t_c - t_o)}{\tau} = \ln \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$(t_c - t_o) = -\frac{RR_{\text{L}}C}{R + R_{\text{L}}} \ln \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$(t_c - t_o) = \frac{RR_{\text{L}}C}{R + R_{\text{L}}} \ln \frac{V_{\text{max}} - V_{\text{Th}}}{V_{\text{min}} - V_{\text{Th}}}$$

P 7.107 **[a]** $0 \leq t \leq 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right) e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15 e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15 e^{-30/L} = 0.40$$

$$\therefore \quad e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\text{H}$$

[b] $0 \le t \le t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right)e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore \quad 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31\ln 1.25}{60} \cong 0.10\,\text{s}$$