8

Natural and Step Responses of RLC Circuits

Assessment Problems

AP 8.1 **[a]** $\dfrac{1}{(2RC)^2} = \dfrac{1}{LC}$, therefore $C = 500\,\text{nF}$

[b] $\alpha = 5000 = \dfrac{1}{2RC}$, therefore $C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[c] $\dfrac{1}{\sqrt{LC}} = 20{,}000$, therefore $C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

AP 8.2
$$\begin{aligned}
i_\text{L} &= \frac{1}{50 \times 10^{-3}} \int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\
&= 20\left\{ \frac{-14e^{-5000x}}{-5000}\Bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\Bigg|_0^t \right\} + 30 \times 10^{-3} \\
&= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\
&= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\
&= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0
\end{aligned}$$

AP 8.3 From the given values of R, L, and C, $s_1 = -10$ krad/s and $s_2 = -40$ krad/s.

[a] $v(0^-) = v(0^+) = 0$, therefore $i_\text{R}(0^+) = 0$

[b] $i_{\mathrm{C}}(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4 + 0) = 4\,\mathrm{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = i_c(0^+) = 4,$ therefore $\dfrac{dv_c(0^+)}{dt} = \dfrac{4}{C} = 4 \times 10^8\,\mathrm{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\mathrm{V}, \qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

Therefore $A_1 + A_2 = 0, \qquad -A_1 - 4A_2 = 40{,}000; \qquad A_1 = 40{,}000/3\,\mathrm{V}$

[e] $A_2 = -40{,}000/3\,\mathrm{V}$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\mathrm{V}, \qquad t \geq 0$

AP 8.4 **[a]** $\dfrac{1}{2RC} = 8000,$ therefore $R = 62.5\,\Omega$

[b] $i_{\mathrm{R}}(0^+) = \dfrac{10\,\mathrm{V}}{62.5\,\Omega} = 160\,\mathrm{mA}$

$$i_{\mathrm{C}}(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240\,\mathrm{mA} = C\frac{dv(0^+)}{dt}$$

Therefore $\dfrac{dv(0^+)}{dt} = \dfrac{-240\,\mathrm{m}}{C} = -240\,\mathrm{kV/s}$

[c] $B_1 = v(0^+) = 10\,\mathrm{V}, \qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $6000B_2 - 8000B_1 = -240{,}000, \qquad B_2 = (-80/3)\,\mathrm{V}$

[d] $i_{\mathrm{L}} = -(i_{\mathrm{R}} + i_{\mathrm{C}}); \qquad i_{\mathrm{R}} = v/R; \qquad i_{\mathrm{C}} = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\mathrm{V}$$

Therefore $i_{\mathrm{R}} = e^{-8000t}[160\cos 6000t - \dfrac{1280}{3}\sin 6000t]\,\mathrm{mA}$

$$i_{\mathrm{C}} = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\mathrm{mA}$$

$$i_{\mathrm{L}} = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\mathrm{mA}, \qquad t \geq 0$$

AP 8.5 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4},$ therefore $\dfrac{1}{2RC} = 500, \quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5 \times 10^{-3},$ therefore $V_0 = 50\,\mathrm{V}$

[c] $0.5LI_0^2 = 12.5 \times 10^{-3}, \qquad I_0 = 250\,\mathrm{mA}$

[d] $D_2 = v(0^+) = 50, \qquad \dfrac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$$i_{\rm R}(0^+) = \frac{50}{100} = 500\,\text{mA}$$

Therefore $\quad i_{\rm C}(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\quad \dfrac{dv(0^+)}{dt} = -750 \times \dfrac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $\quad D_1 - \alpha D_2 = -75{,}000; \qquad \alpha = \dfrac{1}{2RC} = 500, \quad D_1 = -50{,}000$ V/s

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$$i_{\rm R} = \frac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}, \qquad t \geq 0^+$$

AP 8.6 **[a]** $i_{\rm R}(0^+) = \dfrac{V_0}{R} = \dfrac{40}{500} = 0.08\,\text{A}$

[b] $i_{\rm C}(0^+) = I - i_{\rm R}(0^+) - i_{\rm L}(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\dfrac{di_{\rm L}(0^+)}{dt} = \dfrac{V_o}{L} = \dfrac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \dfrac{1}{2RC} = 1000; \qquad \dfrac{1}{LC} = 1{,}562{,}500; \qquad s_{1,2} = -1000 \pm j750$ rad/s

[e] $i_{\rm L} = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t, \qquad i_f = I = -1\,\text{A}$

$i_{\rm L}(0^+) = 0.5 = i_f + B_1', \qquad$ therefore $\quad B_1' = 1.5\,\text{A}$

$\dfrac{di_{\rm L}(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad$ therefore $\quad B_2' = (25/12)\,\text{A}$

Therefore $\quad i_{\rm L}(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}, \qquad t \geq 0$

[f] $v(t) = \dfrac{Łdi_{\rm L}}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V \qquad t \geq 0$

AP 8.7 **[a]** $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_{\rm C}(0^-) = \left(\dfrac{15\,\text{k}}{15\,\text{k} + 9\,\text{k}}\right)(80) = 50\,\text{V}$

[c] $50 + 80i(0^+) + L\dfrac{di(0^+)}{dt} = 100, \qquad \dfrac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \dfrac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000$ rad/s

[e] $i = i_f + e^{-\alpha t}[B_1'\cos\omega_d t + B_2'\sin\omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $\quad B_1' = 0; \qquad \dfrac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $\quad B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t}\sin 6000t\,\text{A}, \qquad t \geq 0$

AP 8.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos \omega_d t + B_2' \sin \omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d} B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

$$\text{Therefore} \quad v_c(t) = 100 - e^{-8000t}[50 \cos 6000t + 66.67 \sin 6000t]\,\text{V}, \qquad t \geq 0$$

Problems

P 8.1 **[a]** $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(1000)(2 \times 10^{-6})} = 250$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(12.5)(2 \times 10^{-6})} = 40{,}000$$

$$s_{1,2} = -250 \pm \sqrt{250^2 - 40{,}000} = -250 \pm 150$$

$$s_1 = -100 \text{ rad/s}$$

$$s_2 = -400 \text{ rad/s}$$

[b] overdamped

[c] Note — we want $\omega_d = 120$ rad/s:

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 40{,}000 - (120)^2 = 25{,}600$$

$$\alpha = 160$$

$$\frac{1}{2RC} = 160; \qquad \therefore \quad R = \frac{1}{2(160)(2 \times 10^{-6})} = 1562.5\,\Omega$$

[d] $s_1,\ s_2 = -160 \pm \sqrt{160^2 - 40{,}000} = -160 \pm j120$ rad/s

[e] $\alpha = \sqrt{40{,}000} = \dfrac{1}{2RC}; \qquad \therefore \quad R = \dfrac{1}{2(200)(2 \times 10^{-6})} = 1250\,\Omega$

P 8.2 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -250$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -1000$$

Adding the above equations, $\qquad -2\alpha = -1250$

$$\alpha = 625\,\text{rad/s}$$

$$\frac{1}{2RC} = \frac{1}{2R(0.1\times 10^{-6})} = 625$$

$$R = 8\,\text{k}\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 750$$

$$4(\alpha^2 - \omega_o^2) = 562{,}500$$

$$\therefore \quad \omega_o = 500\,\text{rad/s}$$

$$\omega_o^2 = 25\times 10^4 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{(25\times 10^4)(0.1\times 10^{-6})} = 40\,\text{H}$$

[b] $i_\text{R} = \dfrac{v(t)}{R} = -1e^{-250t} + 4e^{-1000t}\,\text{mA}, \qquad t \ge 0^+$

$$i_\text{C} = C\frac{dv(t)}{dt} = 0.2e^{-250t} - 3.2e^{-1000t}\,\text{mA}, \qquad t \ge 0^+$$

$$i_\text{L} = -(i_\text{R} + i_\text{C}) = 0.8e^{-250t} - 0.8e^{-1000t}\,\text{mA}, \qquad t \ge 0$$

P 8.3 **[a]** $i_\text{R}(0) = \dfrac{15}{200} = 75\text{mA}$

$$i_\text{L}(0) = -45\text{mA}$$

$$i_\text{C}(0) = -i_\text{L}(0) - i_\text{R}(0) = 45 - 75 = -30\,\text{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(200)(0.2\times 10^{-6})} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50\times 10^{-3})(0.2\times 10^{-6})} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{1.5625\times 10^8 - 10^8} = -12{,}500 \pm 7500$$

$$s_1 = -5000\,\text{rad/s}; \qquad s_2 = -20{,}000\,\text{rad/s}$$

$$v = A_1e^{-5000t} + A_2e^{-20{,}000t}$$

$$v(0) = A_1 + A_2 = 15$$

$$\frac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \frac{-30\times10^{-3}}{0.2\times10^{-6}} = -15\times10^{4}\text{V/s}$$

Solving, $\quad A_1 = 10; \quad A_2 = 5$

$$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$$

[c]
$$\begin{aligned} i_{\text{C}} &= C\frac{dv}{dt} \\ &= 0.2\times10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}] \\ &= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA} \end{aligned}$$

$$i_{\text{R}} = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$$

$$i_{\text{L}} = -i_{\text{C}} - i_{\text{R}} = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.4
$$\frac{1}{2RC} = \frac{1}{2(312.5)(0.2\times10^{-6})} = 8000$$

$$\frac{1}{LC} = \frac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8$$

$$s_{1,2} = -8000 \pm \sqrt{8000^2 - 10^8} = -8000 \pm j6000\,\text{rad/s}$$

$\therefore$ response is underdamped

$$v(t) = B_1e^{-8000t}\cos 6000t + B_2e^{-8000t}\sin 6000t$$

$$v(0^+) = 15\,\text{V} = B_1; \qquad i_{\text{R}}(0^+) = \frac{15}{312.5} = 48\,\text{mA}$$

$$i_{\text{C}}(0^+) = [-i_{\text{L}}(0^+) + i_{\text{R}}(0^+)] = -[-45 + 48] = -3\,\text{mA}$$

$$\frac{dv(0^+)}{dt} = \frac{-3\times10^{-3}}{0.2\times10^{-6}} = -15{,}000\,\text{V/s}$$

$$\frac{dv(0)}{dt} = -8000B_1 + 6000B_2 = -15{,}000$$

$$6000B_2 = 8000(15) - 15{,}000; \qquad \therefore \quad B_2 = 17.5\,\text{V}$$

$$v(t) = 15e^{-8000t}\cos 6000t + 17.5e^{-8000t}\sin 6000t\,\text{V}, \qquad t \geq 0$$

P 8.5 $\alpha = \frac{1}{2RC} = \frac{1}{2(250)(0.2\times10^{-6})} = 10^4$

$\alpha^2 = 10^8; \qquad \therefore \quad \alpha^2 = \omega_o^2$

Critical damping:

$v = D_1te^{-\alpha t} + D_2e^{-\alpha t}$

$i_R(0^+) = \frac{15}{250} = 60\,\text{mA}$

$i_C(0^+) = -[i_L(0^+) + i_R(0^+)] = -[-45+60] = -15\,\text{mA}$

$v(0) = D_2 = 15$

$\frac{dv}{dt} = D_1[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}] - \alpha D_2e^{-\alpha t}$

$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{i_C(0)}{C} = \frac{-15\times10^{-3}}{0.2\times10^{-6}} = -75{,}000$

$D_1 = \alpha D_2 - 75{,}000 = (10^4)(15) - 75{,}000 = 75{,}000$

$v = (75{,}000t + 15)e^{-10{,}000t}\,\text{V}, \qquad t \geq 0$

P 8.6 $\alpha = 1000/2 = 500$

$R = \frac{1}{2\alpha C} = \frac{1}{2(500)(2.5\times10^{-6})} = 400\,\Omega$

$v(0^+) = 3(1+1) = 6\,\text{V}$

$i_\text{R}(0^+) = \frac{6}{400} = 15\,\text{mA}$

$\frac{dv}{dt} = -300e^{-100t} - 2700e^{-900t}$

$\frac{dv(0^+)}{dt} = -300 - 2700 = -3000\,\text{V/s}$

$i_\text{C}(0^+) = 2.5\times10^{-6}(-3000) = -7.5\,\text{mA}$

$i_\text{L}(0^+) = -[i_\text{R}(0^+) + i_\text{C}(0^+)] = -[15 - 7.5] = -7.5\,\text{mA}$

P 8.7 **[a]** $\alpha = 20{,}000; \qquad \omega_d = 15{,}000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 225 \times 10^6 + 400 \times 10^6 = 625 \times 10^6$$

$$\frac{1}{LC} = 625 \times 10^6$$

$$L = \frac{1}{(625 \times 10^6)(40 \times 10^{-9})} = 40\,\text{mH}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{1}{2(20{,}000)(40 \times 10^{-9})} = 625\,\Omega$$

[c] $V_o = v(0) = 100\,\text{V}$

[d] $I_o = i_{\text{L}}(0) = -i_{\text{R}}(0) - i_{\text{C}}(0)$

$$i_{\text{R}}(0) = \frac{V_o}{R} = \frac{100}{625} = 160\,\text{mA}$$

$$i_{\text{C}}(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 100\{e^{-20{,}000t}[-15{,}000\sin 15{,}000t - 30{,}000\cos 15{,}000t] -$$

$$20{,}000e^{-20{,}000t}[\cos 15{,}000t - 2\sin 15{,}000t]$$

$$\frac{dv}{dt}(0) = 100\{1(-30{,}000) - 20{,}000\} = -500 \times 10^4$$

$$C\frac{dv}{dt}(0) = -500 \times 10^4(40 \times 10^{-9}) = -200\,\text{mA}$$

$$\therefore \quad I_o = 200 - 160 = 40\,\text{mA}$$

[e]
$$\frac{dv}{dt} = 100e^{-20{,}000t}[25{,}000\sin 15{,}000t - 50{,}000\cos 15{,}000t]$$

$$= 25 \times 10^5 e^{-20{,}000t}[\sin 15{,}000t - 2\cos 15{,}000t]$$

$$C\frac{dv}{dt} = 0.1e^{-20{,}000t}(\sin 15{,}000t - 2\cos 15{,}000t)$$

$$i_{\text{C}}(t) = 0.1e^{-20{,}000t}(\sin 15{,}000t - 2\cos 15{,}000t)\,\text{A}$$

$$i_{\text{R}}(t) = 0.16e^{-20{,}000t}(\cos 15{,}000t - 2\sin 15{,}000t)\,\text{A}$$

$$i_{\text{L}}(t) = -i_{\text{R}}(t) - i_{\text{C}}(t)$$

$$= e^{-20{,}000t}(40\cos 15{,}000t + 220\sin 15{,}000t)\,\text{mA}, \quad t \geq 0$$

P 8.8 **[a]** $2\alpha = 1000; \qquad \alpha = 500\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 600; \qquad \omega_o = 400\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{2(500)(250)} = 4\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(400)^2(4 \times 10^{-6})} = 1.5625\,\text{H}$$

$$i_\text{C}(0^+) = A_1 + A_2 = 45\,\text{mA}$$

$$\frac{di_\text{C}}{dt} + \frac{di_\text{L}}{dt} + \frac{di_\text{R}}{dt} = 0$$

$$\frac{di_\text{C}(0)}{dt} = -\frac{di_\text{L}(0)}{dt} - \frac{di_\text{R}(0)}{dt}$$

$$\frac{di_\text{L}(0)}{dt} = \frac{15}{1.5625} = 9.6\,\text{A/s}$$

$$\frac{di_\text{R}(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_\text{C}(0)}{C} = \frac{45 \times 10^{-3}}{(250)(4 \times 10^{-6})} = 45\,\text{A/s}$$

$$\therefore \quad \frac{di_\text{C}(0)}{dt} = -9.6 - 45 = -54.6\,\text{A/s}$$

$$\therefore \quad 200A_1 + 800A_2 = 54.6 \quad A_1 + A_2 = 0.045$$

Solving, $A_1 = -31\,\text{mA}; \qquad A_2 = 76\,\text{mA}$

$$\therefore \quad i_\text{C} = -31e^{-200t} + 76e^{-800t}\,\text{mA}, \qquad t \geq 0^+$$

[b] By hypothesis

$$v = A_3e^{-200t} + A_4e^{-800t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 15$$

$$\frac{dv(0)}{dt} = \frac{45 \times 10^{-3}}{4 \times 10^{-6}} = 11{,}250\,\text{V/s}$$

$$-200A_3 - 800A_4 = 11{,}250; \qquad \therefore \quad A_3 = 38.75\,\text{V}; \qquad A_4 = -23.75\,\text{V}$$

$$v = 38.75e^{-200t} - 23.75e^{-800t}\,\text{V}, \qquad t \geq 0$$

[c] $i_\text{R}(t) = \dfrac{v}{250} = 155e^{-200t} - 95e^{-800t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_\text{L} = -i_\text{R} - i_\text{C}$

$$i_\text{L} = -124e^{-200t} + 19e^{-800t}\,\text{mA}, \qquad t \geq 0$$

P 8.9 **[a]** $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (500)^2$

$$\therefore \quad C = \frac{1}{(500)^2(4)} = 1\,\mu\text{F}$$

$$\frac{1}{2RC} = 500$$

$$\therefore \quad R = \frac{1}{2(500)(10^{-6})} = 1\,\text{k}\Omega$$

$$v(0) = D_2 = 8\,\text{V}$$

$$i_{\text{R}}(0) = \frac{8}{1000} = 8\text{mA}$$

$$i_{\text{C}}(0) = -8 + 10 = 2\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 500D_2 = \frac{2\times 10^{-3}}{10^{-6}} = 2000\,\text{V/s}$$

$$\therefore \quad D_1 = 2000 + 500(8) = 6000\ \text{V/s}$$

[b] $v = 6000te^{-500t} + 8e^{-500t}\,\text{V}, \quad t \geq 0$

$$\frac{dv}{dt} = [-3\times 10^6 t + 2000]e^{-500t}$$

$$i_{\text{C}} = C\frac{dv}{dt} = (-3000t + 2)e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

P 8.10 **[a]** $\alpha = \frac{1}{2RC} = 0.5$ rad/s

$$\omega_o^2 = \frac{1}{LC} = 25.25$$

$$\omega_d = \sqrt{25.25 - (0.5)^2} = 5\ \text{rad/s}$$

$$\therefore \quad v = B_1e^{-t/2}\cos 5t + B_2e^{-t/2}\sin 5t$$

$$v(0) = B_1 = 0; \qquad v = B_2e^{-t/2}\sin 5t$$

$$i_R(0^+) = 0\,\text{A}; \qquad i_C(0^+) = 4\,\text{A}; \qquad \frac{dv}{dt}(0^+) = \frac{4}{0.08} = 50\ \text{V/s}$$

$$50 = -\alpha B_1 + \omega_d B_2 = -0.5(0) + 5B_2$$

$$\therefore \quad B_2 = 10$$

$$\therefore \quad v = 10e^{-t/2}\sin 5t\,\text{V}, \qquad t \geq 0$$

[b] $\dfrac{dv}{dt} = -5e^{-t/2}\sin 5t + 10e^{-t/2}(5\cos 5t)$

$\dfrac{dv}{dt} = 0 \quad \text{when} \quad 10\cos 5t = \sin 5t \quad \text{or} \quad \tan 5t = 10$

$\therefore \quad 5t_1 = 1.47, \qquad t_1 = 294.23\,\text{ms}$

$5t_2 = 1.47 + \pi, \qquad t_2 = 922.54\,\text{ms}$

$5t_3 = 1.47 + 2\pi, \qquad t_3 = 1550.86\,\text{ms}$

[c] $t_3 - t_1 = 1256.6\,\text{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{5} = 1256.6\,\text{ms}$

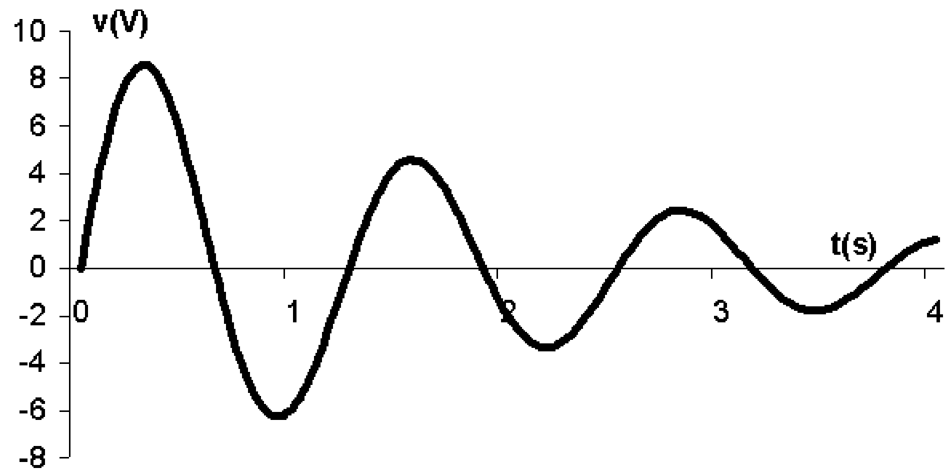
[d] $t_2 - t_1 = 628.3\,\text{ms}; \qquad \dfrac{T_d}{2} = \dfrac{1256.6}{2} = 628.3\,\text{ms}$

[e] $v(t_1) = 10e^{-(0.147115)}\sin 5(0.29423) = 8.59\,\text{V}$

$v(t_2) = 10e^{-(0.46127)}\sin 5(0.92254) = -6.27\,\text{V}$

$v(t_3) = 10e^{-(0.77543)}\sin 5(1.55086) = 4.58\,\text{V}$

[f]



P 8.11 **[a]** $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{25.25} = 5.02\,\text{rad/s}$

$v = B_1\cos\omega_o t + B_2\sin\omega_o t; \qquad v(0) = B_1 = 0; \qquad v = B_2\sin\omega_o t$

$C\dfrac{dv}{dt}(0) = -i_{\text{L}}(0) = 4$

$50 = -\alpha B_1 + \omega_d B_2 = -0 + \sqrt{25.25}B_2$

$\therefore \quad B_2 = 50/\sqrt{25.25} = 9.95\,\text{V}$

$v = 9.95\sin 5.02t\,\text{V}, \qquad t \geq 0$

[b] $2\pi f = 5.02; \qquad f = \dfrac{5.02}{2\pi} \cong 0.80\,\text{Hz}$

[c] $9.95\,\text{V}$

P 8.12 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(12.5)(3.2\times 10^{-9})} = 25\times 10^6$

$\omega_o = 5000$ rad/s

$\dfrac{1}{2RC} = 5000; \qquad R = \dfrac{1}{2(5000)(3.2\times 10^{-9})} = 31.25\,\text{k}\Omega$

[b] $v(t) = D_1te^{-5000t} + D_2e^{-5000t}$

$v(0) = 100\,\text{V} = D_2$

$\dfrac{dv}{dt} = (D_1t + 100)(-5000e^{-5000t}) + D_1e^{-5000t}$

$\dfrac{dv}{dt}(0) = -500\times 10^3 + D_1 = \dfrac{i_\text{C}(0)}{C}$

$i_\text{C}(0) = -i_\text{R}(0) - i_\text{L}(0)$

$i_\text{R}(0) = \dfrac{100}{31{,}500} = 3.2\,\text{mA}$

$\therefore \quad i_\text{C}(0) = -(3.2 + 6.4) = -9.6\,\text{mA}$

$\therefore \quad \dfrac{dv}{dt}(0) = -\dfrac{9.6\times 10^{-3}}{3.2\times 10^{-9}} = -3\times 10^6$

$\therefore \quad -500\times 10^3 + D_1 = -3\times 10^6$

$D_1 = -25\times 10^5\text{V/s}$

$\therefore \quad v(t) = (-25\times 10^5t + 100)e^{-5000t}\,\text{V}, \qquad t\geq 0$

[c] $i_\text{C}(t) = 0$ when $\dfrac{dv}{dt}(t) = 0$

$\dfrac{dv}{dt} = (-25\times 10^5t + 100)(-5000)e^{-5000t} + e^{-5000t}(-25\times 10^5)$

$= (125\times 10^8t - 30\times 10^5)e^{-5000t}$

$\dfrac{dv}{dt} = 0$ when $125\times 10^8t_1 = 3\times 10^6; \qquad \therefore \quad t_1 = 240\,\mu\text{s}$

$v(240\mu\text{s}) = e^{-1.2}[(-25\times 10^5)(240\times 10^{-6}) + 100] = -150.6\,\text{V}$

[d] $i_{\rm L}(240\mu{\rm s}) = -i_{\rm R}(240\mu{\rm s}) = \dfrac{-150.6}{31{,}250} = -4.82\,{\rm mA}$

$$\omega_{\rm C}(240\mu{\rm s}) = \frac{1}{2}(3.2\times 10^{-9})(-150.6)^2 = 36.29\,\mu{\rm J}$$

$$\omega_{\rm L}(240\mu{\rm s}) = \frac{1}{2}(12.5)(-4.82\times 10^{-3})^2 = 145.2\,\mu{\rm J}$$

$$\omega(240\mu{\rm s}) = \omega_{\rm C} + \omega_{\rm L} = 181.49\,\mu{\rm J}$$

$$\omega(0) = \frac{1}{2}(12.5)(6.4\times 10^{-3})^2 + \frac{1}{2}(3.2\times 10^{-9})(100)^2 = 272\,\mu{\rm J}$$

$$\%\ \text{remaining} = \frac{181.49}{272}(100) = 66.72\%$$

P 8.13 **[a]** $\alpha = \dfrac{1}{2RC} = 1250, \qquad \omega_o = 10^3,$ therefore overdamped

$$s_1 = -500, \qquad s_2 = -2000$$

therefore $\quad v = A_1 e^{-500t} + A_2 e^{-2000t}$

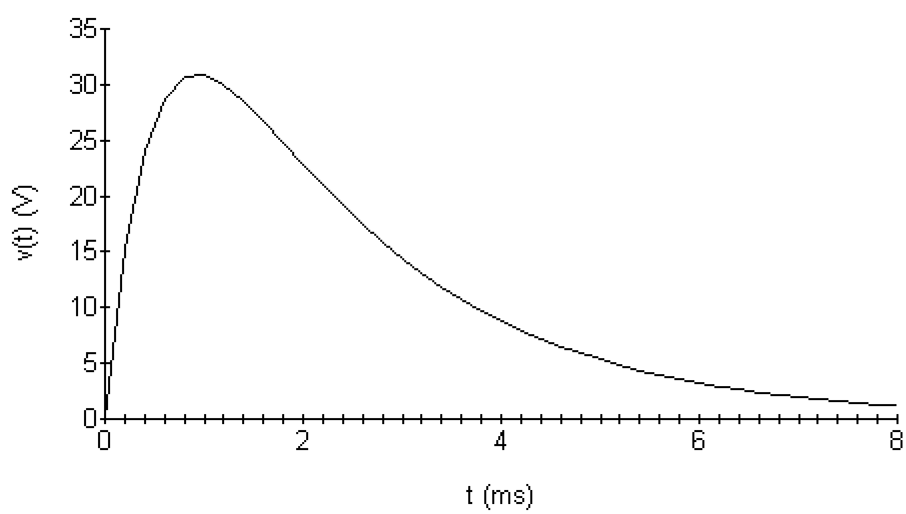
$$v(0^+) = 0 = A_1 + A_2; \qquad \left[\frac{dv(0^+)}{dt}\right] = \frac{i_{\rm C}(0^+)}{C} = 98{,}000\,{\rm V/s}$$

Therefore $\quad -500A_1 - 2000A_2 = 98{,}000$

$$A_1 = \frac{+980}{15}, \quad A_2 = \frac{-980}{15}$$

$$v(t) = \left[\frac{980}{15}\right][e^{-500t} - e^{-2000t}]\,{\rm V}, \qquad t \ge 0$$

[b]



Example 8.4: $\quad v_{\max} \cong 74.1\,{\rm V}$ at 1.4 ms

Example 8.5: $v_{\max} \cong 36.1\,\text{V}$ at 1.0 ms

Problem 8.13: $v_{\max} \cong 30.9$ at 0.92 ms

P 8.14 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinate is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinates are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $\quad N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$

It follows that $\quad A_1 = \dfrac{N_1}{\Delta} = \dfrac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$

and $\quad A_2 = \dfrac{N_2}{\Delta} = \dfrac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$

We see from these expressions that $\quad A_1 = A_2^*$

P 8.15 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.14 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.14 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

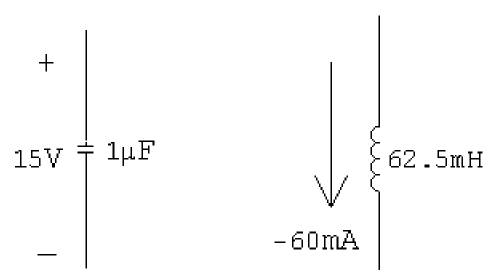
$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but} \quad K_2 = \frac{dv(0^+)}{dt} \quad \text{and} \quad K_1 = B_1$$

Thus we have

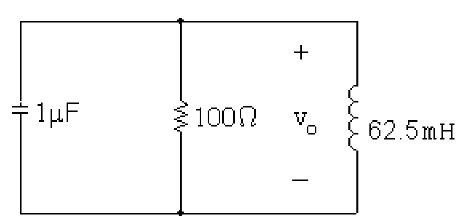
$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

P 8.16 $t < 0: \qquad V_o = 15\,\text{V}, \qquad I_o = -60\,\text{mA}$



$t > 0$:



$$i_R(0) = \frac{15}{100} = 150\,\text{mA}; \qquad i_\text{L}(0) = -60\,\text{mA}$$

$$i_\text{C}(0) = -150 - (-60) = -90\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(100)(10^{-6})} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 15$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-90 \times 10^{-3}}{10^{-6}} = -90{,}000$$

Solving, $\qquad A_1 = 5\,\text{V}, \qquad A_2 = 10\,\text{V}$

$$\therefore \quad v_o = 5e^{-2000t} + 10e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 8.17
$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(250)(10^{-6})} = 2500$$

$$s_{1,2} = -2500 \pm \sqrt{2500^2 - 16 \times 10^6} = -2500 \pm j3122.5\text{rad/s}$$

$$v_o(t) = B_1 e^{-2500t} \cos 3122.5t + B_2 e^{-2500t} \sin 3122.5t$$

$$v_o(0) = B_1 = 15\,\text{V}$$

$$i_R(0) = \frac{15}{200} = 75\,\text{mA}$$

$$i_\text{L}(0) = -60\,\text{mA}$$

$$i_\text{C}(0) = -i_R(0) - i_\text{L}(0) = -15\,\text{mA} \quad \therefore \quad \frac{i_\text{C}(0)}{C} = -15{,}000 \text{ V/s}$$

$$\frac{dv_o}{dt}(0) = -2500B_1 + 3122.5B_2 = -15{,}000 \text{ V/s}$$

$$\therefore \quad 3122.5B_2 = 2500(15) - 15{,}000 \quad \therefore \quad B_2 = 7.21 \text{ V}$$

$$v_o(t) = 15e^{-2500t} \cos 3122.5t + 7.21e^{-2500t} \sin 3122.5t\,\text{V}, \qquad t \geq 0$$

P 8.18 $\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$

$\alpha = \frac{1}{2RC} = \frac{1}{2(125)(10^{-6})} = 4000$

$\therefore \quad \alpha^2 = \omega_o^2$ (critical damping)

$v_o(t) = D_1 te^{-4000t} + D_2 e^{-4000t}$

$v_o(0) = D_2 = 15\,\text{V}$

$i_R(0) = \frac{15}{125} = 120\,\text{mA}$

$i_{\text{L}}(0) = -60\,\text{mA}$

$i_{\text{C}}(0) = -60\,\text{mA}$

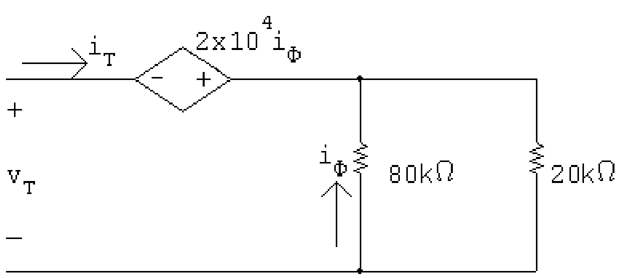
$\frac{dv_o}{dt}(0) = -4000D_2 + D_1$

$\frac{i_{\text{C}}(0)}{C} = \frac{-60 \times 10^{-3}}{10^{-6}} = -60{,}000$

$D_1 - 4000D_2 = -60{,}000; \qquad D_1 = 0$

$v_o(t) = 15e^{-4000t}\,\text{V}, \qquad t \geq 0$

P 8.19



$v_T = -2 \times 10^4 i_\phi + 16 \times 10^3 i_T; \qquad i_\phi = \frac{20}{100}(-i_T)$

$= 4000i_t + 16{,}000i_T = 20{,}000i_T$

$\frac{v_T}{i_T} = 20\,\text{k}\Omega$

$$V_o = \frac{3000}{5000}(50) = 30\,\text{V}; \qquad I_o = 0$$

$$i_\text{C}(0) = -i_R(0) - i_\text{L}(0) = -\frac{30}{20{,}000} = -1.5\,\text{mA}$$

$$\frac{i_\text{C}(0)}{C} = \frac{-1.5\times 10^{-3}}{0.25\times 10^{-6}} = -6000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(40)(0.25\times 10^{-6})} = 10^5$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(20\times 10^3)(0.25\times 10^{-6})} = 100\ \text{rad/s}$$

$$\omega_d = \sqrt{10^5 - 100^2} = 300\ \text{rad/s}$$

$$v_o = B_1 e^{-100t}\cos 300t + B_2 e^{-100t}\sin 300t$$

$$v_o(0) = B_1 = 30\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 300B_2 - 100B_1 = -6000$$

$$\therefore\quad 300B_2 = 100(30) - 6000; \qquad \therefore\quad B_2 = -10\,\text{V}$$

$$v_o = 30e^{-100t}\cos 300t - 10e^{-100t}\sin 300t\,\text{V}, \qquad t \geq 0$$

P 8.20 **[a]** $v = L\left(\frac{di_\text{L}}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_\text{R} = \frac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_\text{C} = I - i_\text{L} - i_\text{R} = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

P 8.21 **[a]** $v = L\left(\frac{di_\text{L}}{dt}\right) = 40e^{-32{,}000t}\sin 24{,}000t\,\text{V}, \qquad t \geq 0$

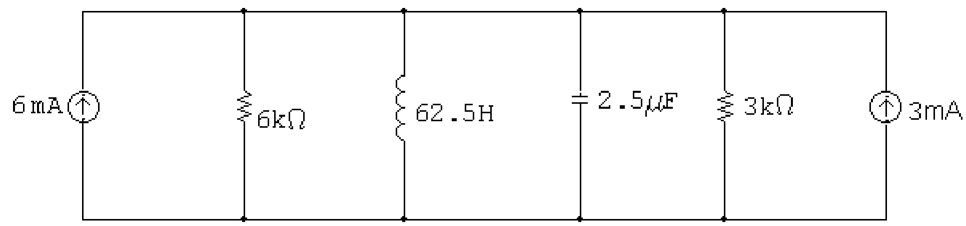
[b] $i_\text{C}(t) = I - i_\text{R} - i_\text{L} = 24\times 10^{-3} - \frac{v}{625} - i_\text{L}$

$$= [24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$$

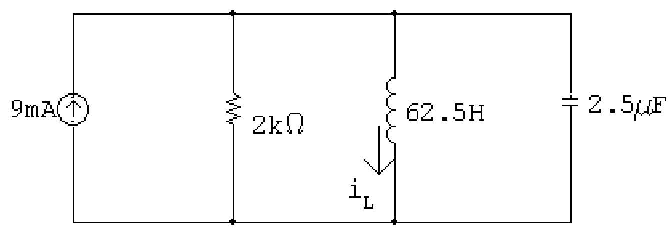
P 8.22 $v = L\left(\frac{di_\text{L}}{dt}\right) = 960{,}000te^{-40{,}000t}\,\text{V}, \qquad t \geq 0$

P 8.23 $t < 0: \qquad i_L = 9/3000 = 3\,\text{mA}$

$t > 0$:



$6\,\text{k}\|3\,\text{k} = 2\,\text{k}\Omega$



$i_L(0) = 3\,\text{mA}, \qquad i_L(\infty) = 9\,\text{mA}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5)(2.5 \times 10^{-6})} = 6400; \qquad \omega_o = 80 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(2000)(2.5 \times 10^{-6})} = 100; \qquad \alpha^2 = 10^4$$

$$\alpha^2 - \omega_o^2 = 10^4 - 6400 = 3600$$

$$s_{1,2} = -100 \pm 60 \text{ rad/s}$$

$$s_1 = -40 \text{ rad/s}; \qquad s_2 = -160 \text{ rad/s}$$

$$i_L = I_f + A_1' e^{-40t} + A_2' e^{-160t}$$

$$i_L(\infty) = I_f = 9\text{mA}$$

$$i_L(0) = A_1' + A_2' + I_f = 3\,\text{mA}$$

$$\therefore \quad A_1' + A_2' + 9\,\text{m} = 3\,\text{m} \quad \text{so} \quad A_1' + A_2' = -6\,\text{mA}$$

$$\frac{di_L}{dt}(0) = 0 = -40A_1' - 160A_2'$$

Solving, $\qquad A_1' = -8\,\text{mA}, \qquad A_2' = 2\,\text{mA}$

$$i_L = 9 - 8e^{-40t} + 2e^{-160t}\,\text{mA}, \qquad t \geq 0$$

P 8.24 $$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8; \qquad \omega_o = 10^4 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2 \times 10^{-6})} = 12{,}500 \text{ rad/s} \qquad \therefore \quad \text{overdamped}$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 \pm 7500 \text{ rad/s}$$

$$s_1 = -5000 \text{ rad/s}; \qquad s_2 = -20{,}000 \text{ rad/s}$$

$$I_f = 60\,\text{mA}$$

$$i_{\text{L}} = 60 \times 10^{-3} + A_1' e^{-5000t} + A_2' e^{-20{,}000t}$$

$$\therefore \quad -45 \times 10^{-3} = 60 \times 10^{-3} + A_1' + A_2'; \qquad A_1' + A_2' = -105 \times 10^{-3}$$

$$\frac{di_{\text{L}}}{dt} = -5000A_1' - 20{,}000A_2' = \frac{15}{0.05} = 300$$

Solving, $\quad A_1' = -120\,\text{mA}; \qquad A_2' = 15\,\text{mA}$

$$i_{\text{L}} = 60 - 120e^{-5000t} + 15e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.25 $$\alpha = \frac{1}{2RC} = \frac{1}{2(312.5)(0.2 \times 10^{-6})} = 8000; \qquad \alpha^2 = 64 \times 10^6$$

$\omega_o = 10^4 \qquad$ underdamped

$$s_{1,2} = -8000 \pm j\sqrt{8000^2 - 10^8} = -8000 \pm j6000 \text{ rad/s}$$

$$i_{\text{L}} = 60 \times 10^{-3} + B_1' e^{-8000t} \cos 6000t + B_2' e^{-8000t} \sin 6000t$$

$$-45 \times 10^{-3} = 60 \times 10^{-3} + B_1' \qquad \therefore \quad B_1' = -105\,\text{mA}$$

$$\frac{di_{\text{L}}}{dt}(0) = -8000B_1' + 6000B_2' = 300$$

$$\therefore \quad B_2' = -90\,\text{mA}$$

$$i_{\text{L}} = 60 - 105e^{-8000t} \cos 6000t - 90e^{-8000t} \sin 6000t\,\text{mA}, \quad t \geq 0$$

P 8.26 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(250)(0.2 \times 10^{-6})} = 10^4$

$\alpha^2 = 10^8 = \omega_o^2$ critical damping

$i_{\rm L} = I_f + D_1' te^{-10^4 t} + D_2' e^{-10^4 t} = 60 \times 10^{-3} + D_1' te^{-10^4 t} + D_2' e^{-10^4 t}$

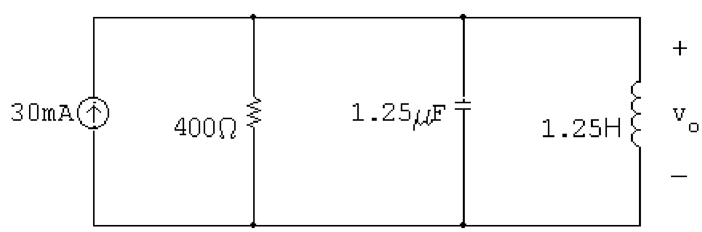
$i_{\rm L}(0) = -45 \times 10^{-3} = 60 \times 10^{-3} + D_2'; \qquad \therefore \quad D_2' = -105\,\text{mA}$

$\dfrac{di_{\rm L}}{dt}(0) = -10^4 D_2' + D_1' = 300\,\text{A/s}$

$\therefore \quad D_1' = 300 + 10^4(-105 \times 10^{-3}) = -750\,\text{A/s}$

$i_{\rm L} = 60 - 750{,}000te^{-10^4 t} - 105e^{-10^4 t}\,\text{mA}, \quad t \geq 0$

P 8.27 For $t > 0$



$\alpha = \dfrac{1}{2RC} = 1000; \qquad \dfrac{1}{LC} = 64 \times 10^4$

$s_{1,2} = -1000 \pm 600$ rad/s

$s_1 = -400$ rad/s; $\qquad s_2 = -1600$ rad/s

$v_o = V_f + A_1' e^{-400t} + A_2' e^{-1600t}$

$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_{\rm C}(0^+) = 30\,\text{mA}$

$\therefore \quad A_1' + A_2' = 0$

$\dfrac{dv_o(0^+)}{dt} = \dfrac{i_{\rm C}(0^+)}{1.25 \times 10^{-6}} = 24{,}000\,\text{V/s}$

$\dfrac{dv_o(0^+)}{dt} = -400A_1' - 1600A_2' = 24{,}000$

Solving,

$A_1' = 20\,\text{V}; \qquad A_2' = -20\,\text{V}$

$v_o = 20e^{-400t} - 20e^{-1600t}\,\text{V}, \qquad t \geq 0$

P 8.28 **[a]** From the solution to Prob. 8.27 $s_1 = -400\,\text{rad/s}$ and $s_2 = -1600\,\text{rad/s}$, therefore

$$i_o = I_f + A_1'e^{-400t} + A_2'e^{-1600t}$$

$$I_f = 30\,\text{mA}; \qquad i_o(0^+) = 0; \qquad \frac{di_o(0^+)}{dt} = 0$$

$$\therefore \quad 0 = 30 \times 10^{-3} + A_1' + A_2'; \qquad -400A_1' - 1600A_2' = 0$$

Solving

$$A_1' = -40\,\text{mA}; \qquad A_2' = 10\,\text{mA}$$

$$\therefore \quad i_o = 30 - 40e^{-400t} + 10e^{-1600t}\,\text{mA}, \qquad t \geq 0$$

[b] $\dfrac{di_o}{dt} = 16e^{-400t} - 16e^{-1600t}$

$$v_o = L\frac{di_o}{dt} = 20e^{-400t} - 20e^{-1600t}\,\text{V}, \qquad t \geq 0$$

This agrees with the solution to Problem 8.27

P 8.29 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(400)(1.25 \times 10^{-6})} = 1000$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1.25 \times 10^{-6})(1.25)} = 64 \times 10^4$$

$$s_{1,2} = -1000 \pm \sqrt{1000^2 - 64 \times 10^4} = -1000 \pm 600\ \text{rad/s}$$

$$s_1 = -400\ \text{rad/s}; \qquad s_2 = -1600\ \text{rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$\therefore \quad v_o = A_1'e^{-400t} + A_2'e^{-1600t}$$

$$v_o(0) = 12 = A_1' + A_2'$$

Note: $\qquad i_C(0^+) = 0$

$$\therefore \quad \frac{dv_o}{dt}(0) = 0 = -400A_1' - 1600A_2'$$

Solving, $\qquad A_1' = 16\,\text{V}, \qquad A_2' = -4\,\text{V}$

$$v_o(t) = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \qquad t > 0$$

P 8.30 **[a]** $i_o = I_f + A_1' e^{-400t} + A_2' e^{-1600t}$

$$I_f = \frac{12}{400} = 30\text{mA}; \qquad i_o(0) = 0$$

$$0 = 30 \times 10^{-3} + A_1' + A_2', \qquad \therefore \quad A_1' + A_2' = -30 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = \frac{12}{1.25} = -400A_1' - 1600A_2'$$

Solving, $\quad A_1' = -32\,\text{mA}; \qquad A_2' = 2\,\text{mA}$

$$i_o = 30 - 32e^{-400t} + 2e^{-1600t}\,\text{mA}, \quad t \geq 0$$

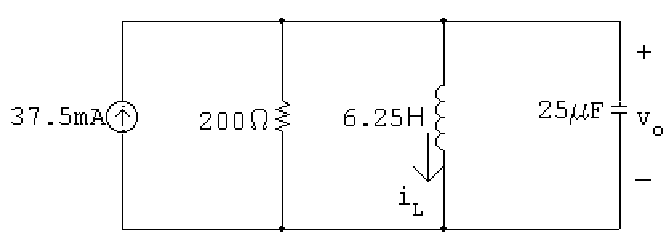
[b] $\dfrac{di_o}{dt} = [12.8e^{-400t} - 3.2e^{-1600t}]$

$$v_o = L\frac{di_o}{dt} = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \quad t \geq 0$$

This agrees with the solution to Problem 8.29.

P 8.31 $i_{\text{L}}(0^-) = i_{\text{L}}(0^+) = 37.5\,\text{mA}$

For $t > 0$



$$i_{\text{L}}(0^-) = i_{\text{L}}(0^+) = 37.5\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = 100\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = 6400$$

$$s_1 = -40\,\text{rad/s} \qquad s_2 = -160\,\text{rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$v_o = A_1' e^{-40t} + A_2' e^{-160t}$$

$$i_{\text{C}}(0^+) = -37.5 + 37.5 + 0 = 0$$

$$\therefore \quad \frac{dv_o}{dt} = 0$$

$$\frac{dv_o}{dt}(0) = -40A_1' - 160A_2'$$

$$\therefore \quad A_1' + 4A_2' = 0; \qquad A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 0; \qquad A_2' = 0$$

$$\therefore \quad v_o = 0 \text{ for } t \geq 0$$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \dfrac{dv_o(0)}{dt} = 0$

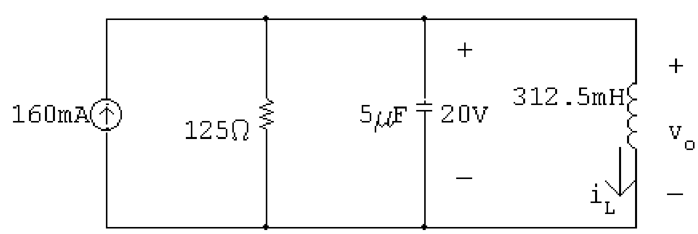
Hence the 37.5 mA current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit the 7.5 V source sustains a current of 37.5 mA in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 8.32 $\quad t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{625}{781.25}(25) = 20\,\text{V}$$

$$i_L(0^-) = i_L(0^+) = 0$$

$t > 0$



$$-160 \times 10^{-3} + \frac{20}{125} + i_C(0^+) + 0 = 0; \qquad \therefore \quad i_C(0^+) = 0$$

$$\frac{1}{2RC} = \frac{1}{2(125)(5 \times 10^{-6})} = 800\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(312.5 \times 10^{-3})(5 \times 10^{-6})} = 64 \times 10^4$$

$$\therefore \quad \alpha^2 = \omega_o^2 \quad \text{critically damped}$$

[a] $v_o = V_f + D_1' te^{-800t} + D_2' e^{-800t}$

$$V_f = 0$$

$$\frac{dv_o(0)}{dt} = -800D_2' + D_1' = 0$$

$$v_o(0^+) = 20 = D_2'$$

$$D_1' = 800D_2' = 16{,}000\,\text{V/s}$$

$$\therefore \quad v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}, \quad t \geq 0^+$$

[b] $i_\text{L} = I_f + D_3' te^{-800t} + D_4' e^{-800t}$

$$i_\text{L}(0^+) = 0; \qquad I_f = 160\,\text{mA}; \qquad \frac{di_\text{L}(0^+)}{dt} = \frac{20}{312.5 \times 10^{-3}} = 64\ \text{A/s}$$

$$\therefore \quad 0 = 160 + D_4'; \qquad D_4' = -160\,\text{mA};$$

$$-800D_4' + D_3' = 64; \qquad D_3' = -64\,\text{A/s}$$

$$\therefore \quad i_\text{L} = 160 - 64{,}000te^{-800t} - 160e^{-800t}\,\text{mA} \qquad t \geq 0$$

P 8.33 **[a]** $w_\text{L} = \int_0^\infty p dt = \int_0^\infty v_o i_\text{L}\, dt$

$$v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}$$

$$i_\text{L} = 0.16 - 64te^{-800t} - 0.16e^{-800t}\,\text{A}$$

$$p = 3.2e^{-800t} + 2560te^{-800t} - 3840te^{-1600t}$$
$$-1{,}024{,}000t^2e^{-1600t} - 3.2e^{-1600t}\,\text{W}$$

$$w_\text{L} = 3.2\int_0^\infty e^{-800t}\,dt + 2560\int_0^\infty te^{-800t}\,dt - 3480\int_0^\infty te^{-1600t}\,dt$$
$$-1{,}024{,}000\int_0^\infty t^2e^{-1600t}\,dt - 3.2\int_0^\infty e^{-1600t}\,dt$$
$$= 3.2\frac{e^{-800t}}{-800}\Bigg|_0^\infty + \frac{2560}{(800)^2}e^{-800t}(-800t-1)\Bigg|_0^\infty$$
$$- \frac{3840}{(1600)^2}e^{-1600t}(-1600t-1)\Bigg|_0^\infty$$
$$- \frac{1{,}024{,}000}{(-1600)^3}e^{-1600t}(1600^2t^2 + 3200t + 2)\Bigg|_0^\infty$$
$$- 3.2\frac{e^{-1600t}}{(-1600)}\Bigg|_0^\infty$$

All the upper limits evaluate to zero hence

$$w_L = \frac{3.2}{800} + \frac{2560}{800^2} - \frac{3840}{1600^2} - \frac{(1{,}024{,}000)(2)}{1600^3} - \frac{3.2}{1600} = 4\,\text{mJ}$$

Note this value corresponds to the final energy stored in the inductor, i.e.

$$w_L(\infty) = \frac{1}{2}(312.5 \times 10^{-3})(0.16)^2 = 4\,\text{mJ}.$$

[b] $v = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}$

$$i_R = \frac{v}{125} = 128te^{-800t} + 0.16e^{-800t}\,\text{A}$$

$$p_R = vi_R = 2{,}048{,}000t^2e^{-1600t} + 5120te^{-1600t} + 3.2e^{-1600t}$$

$$w_R = \int_0^\infty p_R\,dt$$

$$= 2{,}048{,}000\int_0^\infty t^2e^{-1600t}\,dt + 5120\int_0^\infty te^{-1600t}\,dt + 3.2\int_0^\infty e^{-1600t}\,dt$$

$$= \frac{2{,}048{,}000e^{-1600t}}{-1600^3}[1600^2t^2 + 3200t + 2]\Big|_0^\infty + \frac{5120e^{-1600t}}{1600^2}(-1600t - 1)\Big|_0^\infty + \frac{3.2e^{-1600t}}{(-1600)}\Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$w_R = \frac{2{,}048{,}000(2)}{1600^3} + \frac{5120}{1600^2} + \frac{3.2}{1600} = 5\,\text{mJ}$$

[c] $160 = i_R + i_C + i_L \quad \text{(mA)}$

$$i_R + i_L = 160 + 64{,}000te^{-800t}\,\text{mA}$$

$$\therefore\quad i_C = 160 - (i_R + i_L) = -64{,}000te^{-800t}\,\text{mA} = -64te^{-800t}\,\text{A}$$

$$p_C = vi_C = [16{,}000te^{-800t} + 20e^{-800t}][-64te^{-800t}]$$

$$= -1{,}024{,}000t^2e^{-1600t} - 1280e^{-1600t}$$

$$w_C = -1{,}024{,}000\int_0^\infty t^2e^{-1600t}\,dt - 1280\int_0^\infty te^{-1600t}\,dt$$

$$w_C = \frac{-1{,}024{,}000e^{-1600t}}{-1600^3}[1600^2t^2 + 3200t + 2]\Big|_0^\infty - \frac{1280e^{-1600t}}{1600^2}(-1600t - 1)\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_C = \frac{-1{,}024{,}000(2)}{1600^3} - \frac{1280(1)}{1600^2} = -1\,\text{mJ}$$

Note this 1 mJ corresponds to the initial energy stored in the capacitor, i.e.,

$$w_C(0) = \frac{1}{2}(5\times10^{-6})(20)^2 = 1\,\text{mJ}.$$

Thus $w_C(\infty) = 0$ mJ which agrees with the final value of $v = 0$.

[d] $i_s = 160\,\text{mA}$

$$\begin{aligned} p_s(\text{del}) &= 160v\,\text{mW} \\ &= 0.16[16{,}000te^{-800t} + 20e^{-800t}] \\ &= 3.2e^{-800t} + 2560te^{-800t}\,\text{W} \\ w_s &= 3.2\int_0^\infty e^{-800t}\,dt + \int_0^\infty 2560te^{-800t}\,dt \\ &= \frac{3.2e^{-800t}}{-800}\Big|_0^\infty + \frac{2560e^{-800t}}{800^2}(-800t-1)\Big|_0^\infty \\ &= \frac{3.2}{800} + \frac{2560}{800} = 8\,\text{mJ} \end{aligned}$$

[e] $w_L = 4\,\text{mJ}$ (absorbed)

$w_R = 5\,\text{mJ}$ (absorbed)

$w_C = 1\,\text{mJ}$ (delivered)

$w_S = 8\,\text{mJ}$ (delivered)

$\sum w_{\text{del}} = w_{\text{abs}} = 9\,\text{mJ}.$

P 8.34 $$v_C(0^+) = \frac{3.75\times10^3}{11.25\times10^3}(150) = 50\,\text{V}$$

$$i_L(0^+) = 100\,\text{mA}; \qquad i_L(\infty) = \frac{150}{7500} = 20\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(2500)(0.25\times10^{-6})} = 800$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(4)(0.25\times10^{-6})} = 10^6$$

$$\alpha^2 = 64\times10^4; \qquad \alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$$

$$s_{1,2} = -800 \pm j\sqrt{800^2 - 10^6} = -800 \pm j600\ \text{rad/s}$$

$$\begin{aligned} i_{\rm L} &= I_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t \\ &= 20 + B_1' e^{-800t}\cos 600t + B_2' e^{-800t}\sin 600t \end{aligned}$$

$$i_{\rm L}(0) = 20\times 10^{-3} + B_1'; \qquad B_1' = 100\,\text{m} - 20\,\text{m} = 80\,\text{mA}$$

$$\frac{di_{\rm L}}{dt}(0) = 600B_2' - 800B_1' = \frac{50}{4} = 12.5$$

$$\therefore\quad 600B_2 = 800(80\times 10^{-3}) + 12.5; \qquad B_2' = 127.5\,\text{mA}$$

$$\therefore\quad i_{\rm L} = 20 + 80e^{-800t}\cos 600t + 127.5e^{-800t}\sin 600t\,\text{mA}, \qquad t\geq 0$$

P 8.35 **[a]** $2\alpha = 5000; \quad \alpha = 2500\,\text{rad/s}$

$$\sqrt{\alpha^2-\omega_o^2} = 1500; \qquad \omega_o^2 = 4\times 10^6; \qquad \omega_o = 2000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 2500; \qquad R = 5000L$$

$$\omega_o^2 = \frac{1}{LC} = 4\times 10^6; \qquad L = \frac{10^9}{4\times 10^6(50)} = 5\text{H}$$

$$R = 25{,}000\,\Omega$$

[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(50)\times 10^{-9}v_c^2(0) = 90\times 10^{-6}$$

$$\therefore\quad v_c^2(0) = 3600; \qquad v_c(0) = 60\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{60}{5} = 12\,\text{A/s}$$

[c] $i(t) = A_1e^{-1000t} + A_2e^{-4000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -1000A_1 - 4000A_2 = 12$$

Solving,

$$\therefore\quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-1000t} - 4e^{-4000t}\,\text{mA} \qquad t\geq 0$$

[d] $\dfrac{di(t)}{dt} = -4e^{-1000t} + 16e^{-4000t}$

$\dfrac{di}{dt} = 0$ when $16e^{-4000t} = 4e^{-1000t}$

or $e^{3000t} = 4$

$\therefore \quad t = \dfrac{\ln 4}{3000}\mu\text{s} = 462.10\,\mu\text{s}$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 5\dfrac{di}{dt} = [-20e^{-1000t} + 80e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 8.36 $\alpha = 2000\,\text{rad/s}; \qquad \omega_d = 1500\,\text{rad/s}$

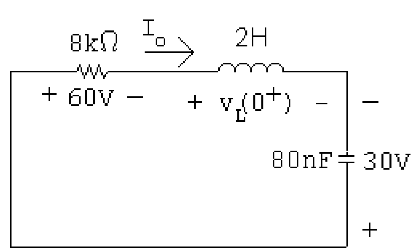
$\omega_o^2 - \alpha^2 = 225 \times 10^4; \qquad \omega_o^2 = 625 \times 10^4; \quad w_o = 2500\,\text{rad/s}$

$\alpha = \dfrac{R}{2L} = 2000; \qquad R = 4000L$

$\dfrac{1}{LC} = 625 \times 10^4; \qquad L = \dfrac{1}{(625 \times 10^4)(80 \times 10^{-9})} = 2\,\text{H}$

$\therefore \quad R = 8\,\text{k}\Omega$

$i(0^+) = B_1 = 7.5\,\text{mA}; \qquad$ at $t = 0^+$



$60 + v_{\text{L}}(0^+) - 30 = 0; \qquad \therefore \qquad v_{\text{L}}(0^+) = -30\,\text{V}$

$\dfrac{di(0^+)}{dt} = \dfrac{-30}{2} = -15\,\text{A/s}$

$\therefore \quad \dfrac{di(0^+)}{dt} = 1500B_2 - 2000B_1 = -15$

$\therefore \quad 1500B_2 = 2000(7.5 \times 10^{-3}) - 15; \qquad \therefore \quad B_2 = 0\,\text{A}$

$\therefore \quad i = 7.5e^{-2000t}\sin 1500t\,\text{mA}, \quad t \geq 0$

P 8.37 From Prob. 8.36 we know v_c will be of the form

$$v_c = B_3 e^{-2000t}\cos 1500t + B_4 e^{-2000t}\sin 1500t$$

From Prob. 8.36 we have

$$v_c(0) = -30\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_C(0)}{C} = \frac{7.5\times 10^{-3}}{80\times 10^{-9}} = 93.75\times 10^3$$

$$\frac{dv_c(0)}{dt} = 1500B_4 - 2000B_3 = 93{,}750$$

$$\therefore\quad 1500B_4 = 2000(-30) + 93{,}750; \qquad B_4 = 22.5\,\text{V}$$

$$v_c(t) = -30e^{-2000t}\cos 1500t + 22.5e^{-2000t}\sin 1500t\,\text{V} \qquad t\geq 0$$

P 8.38 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(80\times 10^{-3})(0.5\times 10^{-6})} = 25\times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\,\text{rad/s}$$

$$\therefore\quad R = (5000)(2)L = 800\,\Omega$$

[b] $i(0) = i_L(0) = 30\,\text{mA}$

$$v_c(0) = 800i(0) + 80\times 10^{-3}\frac{di(0)}{dt}$$

$$\frac{20 - 800(30\times 10^{-3})}{80\times 10^{-3}} = \frac{di(0)}{dt}$$

$$\therefore\qquad \frac{di(0)}{dt} = -50\,\text{A/s}$$

[c] $v_C = D_1 t e^{-5000t} + D_2 e^{-5000t}$

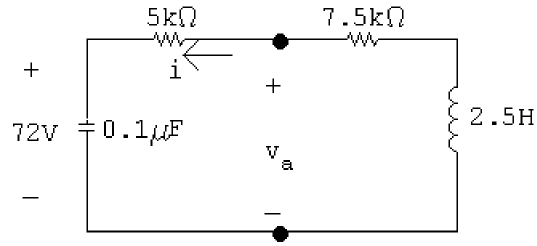
$$v_C(0) = D_2 = 20\,\text{V}$$

$$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C}$$

$$D_1 - 100{,}000 = -\frac{30\times 10^{-3}}{0.5\times 10^{-6}} = -60{,}000 \qquad \therefore \qquad D_1 = 40{,}000\,\text{V/s}$$

$$v_C = 40{,}000te^{-5000t} + 20e^{-5000t}\,\text{V}, \qquad t\geq 0$$

P 8.39 **[a]** For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$v_a(0^+) = 72\,\text{V}$

[b] $v_a = 5000i + \dfrac{1}{0.1 \times 10^{-6}} \displaystyle\int_0^t i\,dx + 72$

$$\frac{dv_a}{dt} = 5000\frac{di}{dt} + 10 \times 10^6 i$$

$$\frac{dv_a(0^+)}{dt} = 5000\frac{di(0^+)}{dt} + 10 \times 10^6 i(0^+) = 5000\frac{di(0^+)}{dt}$$

$$-L\frac{di(0^+)}{dt} = 72$$

$$\frac{di(0^+)}{dt} = -\frac{72}{2.5} = -28.8\,\text{A/s}$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = -144{,}000\,\text{V/s}$$

[c] $\alpha = \dfrac{R}{2L} = \dfrac{12{,}500}{2(2.5)} = 2500\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5)(0.1 \times 10^{-6})} = 4 \times 10^6$$

$$s_{1,2} = -2500 \pm \sqrt{2500^2 - 4 \times 10^6} = -2500 \pm 1500\,\text{rad/s}$$

Overdamped:

$$v_a = A_1 e^{-1000t} + A_2 e^{-4000t}$$

$$v_a(0) = 72 = A_1 + A_2$$

$$\frac{dv_a(0)}{dt} = -144{,}000 = -1000A_1 - 4000A_2$$

Solving, $\quad A_1 = 48; \quad A_2 = 24$

$$v_a = 48e^{-1000t} + 24e^{-4000t}\,\text{V}, \quad t \geq 0^+$$

P 8.40 $\omega_o^2 = \frac{1}{LC} = \frac{1}{(10)(4\times 10^{-3})} = 25$

$$\alpha = \frac{R}{2L} = \frac{80}{2(10)} = 4; \qquad \alpha^2 = 16$$

$$\alpha^2 < \omega_o^2 \quad \therefore \qquad \text{underdamped}$$

$$s_{1,2} = -4 \pm j\sqrt{9} = -4 \pm j3 \text{ rad/s}$$

$$i = B_1 e^{-4t}\cos 3t + B_2 e^{-4t}\sin 3t$$

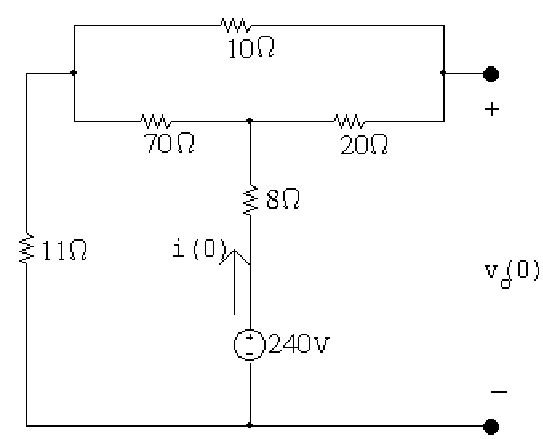
$$i(0) = B_1 = -240/100 = -2.4\,\text{A}$$

$$\frac{di}{dt}(0) = 3B_2 - 4B_1 = 0$$

$$\therefore \quad B_2 = -3.2\,\text{A}$$

$$i = -2.4e^{-4t}\cos 3t - 3.2\sin 3t\,\text{A}, \qquad t \geq 0$$

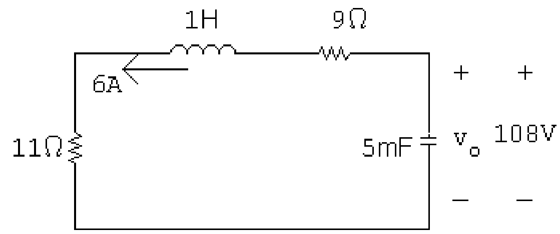
P 8.41 $t < 0$:



$$i(0) = \frac{240}{8 + 30\|70 + 11} = \frac{240}{40} = 6\,\text{A}$$

$$v_o(0) = 240 - 8(6) - \frac{70}{100}(6)(20) = 108\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{2(1)} = 10, \qquad \alpha^2 = 100$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1)(5 \times 10^{-3})} = 200$$

$\omega_o^2 > \alpha^2$ underdamped

$$s_{1,2} = -100 \pm \sqrt{100 - 200} = -10 \pm j10 \text{ rad/s}$$

$$v_o = B_1 e^{-10t} \cos 10t + B_2 e^{-10t} \sin 10t$$

$$v_o(0) = B_1 = 108\,\text{V}$$

$$C\frac{dv_o}{dt}(0) = -6, \qquad \frac{dv_o}{dt} = \frac{-6}{5 \times 10^{-3}} = -1200\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -10B_1 + 10B_2 = -1200$$

$$10B_2 = -1200 + 10B_1 = -1200 + 1080; \qquad B_2 = -120/10 = -12\,\text{V}$$

$$\therefore \quad v_o = 108e^{-10t} \cos 10t - 12e^{-10t} \sin 10t\,\text{V}, \qquad t \geq 0$$

P 8.42 **[a]** $t < 0$:

$$i_o = \frac{80}{800} = 100\,\text{mA}; \qquad v_o = 500i_o = (500)(0.01) = 50\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(2.5 \times 10^{-3})} = 10^5 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5 \times 10^{-3})(40 \times 10^{-9})} = 100 \times 10^8$$

$\alpha^2 = \omega_o^2 \quad \therefore \qquad$ critically damped

$$\therefore \quad i_o(t) = D_1te^{-10^5t} + D_2e^{-10^5t}$$

$$i_o(0) = D_2 = 100\,\text{mA}$$

$$\frac{di_o}{dt}(0) = -\alpha D_2 + D_1 = 0$$

$$\therefore \qquad D_1 = 10^5(100 \times 10^{-3}) = 10{,}000$$

$$i_o(t) = 10{,}000te^{-10^5t} + 0.1e^{-10^5t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o(t) = D_3te^{-10^5t} + D_4e^{-10^5t}$

$$v_o(0) = D_4 = 50$$

$$C\frac{dv_o}{dt}(0) = -0.1$$

$$\frac{dv_o}{dt}(0) = \frac{-0.1}{40 \times 10^{-9}} = -25 \times 10^5\,\text{V/s} = -\alpha D_4 + D_3$$

$$\therefore \qquad D_3 = 10^5(50) - 25 \times 10^5 = 25 \times 10^5$$

$$v_o(t) = 25 \times 10^5te^{-10^5t} + 50e^{-10^5t}\,\text{V}, \quad t \geq 0$$

P 8.43 $\alpha = \dfrac{R}{2L} = \dfrac{8000}{2(1)} = 4000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1)(50 \times 10^{-9})} = 20 \times 10^6$$

$$s_{1,2} = -4000 \pm \sqrt{4000^2 - 20 \times 10^6} = -4000 \pm j2000\,\text{rad/s}$$

$$v_o = V_f + B_1'e^{-4000t}\cos 2000t + B_2'e^{-4000t}\sin 2000t$$

$$v_o(0) = 0 = V_f + B_1'$$

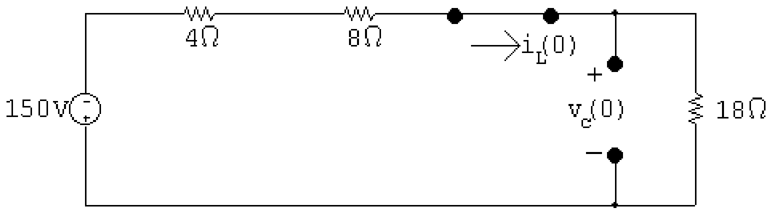
$$v_o(\infty) = 80\,\text{V}; \qquad \therefore \quad B_1' = -80\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = 2000B_2' - 4000B_1'$$

$$\therefore \quad 2000B_2' = 4000(-80) \quad \therefore \qquad B_2' = -160\,\text{V}$$

$$v_o = 80 - 80e^{-4000t}\cos 2000t - 160e^{-4000t}\sin 2000t\,\text{V}, \quad t \geq 0$$

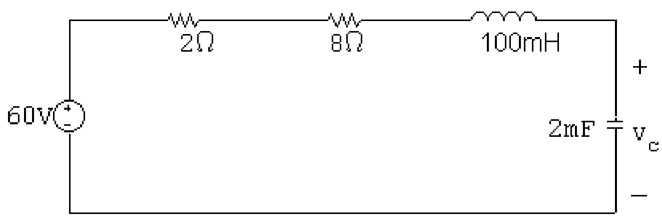
P 8.44 $t < 0$:



$$i_{\text{L}}(0) = \frac{-150}{30} = -5\,\text{A}$$

$$v_{\text{C}}(0) = 18 i_{\text{L}}(0) = -90\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{10}{2(0.1)} = 50 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.1)(2\times 10^{-3})} = 5000$$

$$\omega_o > \alpha^2 \quad \therefore \quad \text{underdamped}$$

$$s_{1,2} = -50 \pm \sqrt{50^2 - 5000} = -50 \pm j50$$

$$v_c = 60 + B_1' e^{-50t}\cos 50t + B_2' e^{-50t}\sin 50t$$

$$v_c(0) = -90 = 60 + B_1' \quad \therefore \quad B_1' = -150$$

$$C\frac{dv_c}{dt}(0) = -5; \qquad \frac{dv_c}{dt}(0) = \frac{-5}{2\times 10^{-3}} = -2500$$

$$\frac{dv_c}{dt}(0) = -50B_1' + 50B_2 = -2500 \quad \therefore \quad B_2' = -200$$

$$v_c = 60 - 150e^{-50t}\cos 50t - 200e^{-50t}\sin 50t\,\text{V}, \quad t \geq 0$$

P 8.45 $i_C(0) = 0; \qquad v_o(0) = 50\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{8000}{2(160 \times 10^{-3})} = 25{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(160 \times 10^{-3})(10 \times 10^{-9})} = 625 \times 10^6$$

$\therefore \quad \alpha^2 = \omega_o^2; \qquad$ critical damping

$$v_o(t) = V_f + D_1' t e^{-25{,}000t} + D_2' e^{-25{,}000t}$$

$$V_f = 250\,\text{V}$$

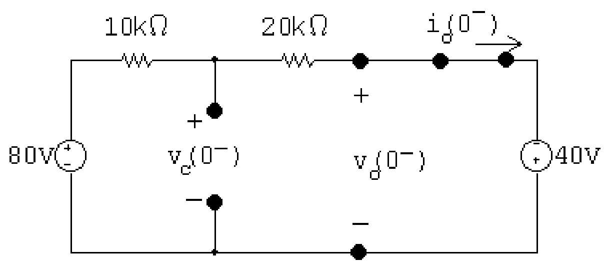
$$v_o(0) = 250 + D_2' = 50; \qquad D_2' = -200\,V$$

$$\frac{dv_o}{dt}(0) = -25{,}000 D_2' + D_1' = 0$$

$$D_1' = 25{,}000 D_2' - 5 \times 10^6 \text{ V/s}$$

$$v_o = 250 - 5 \times 10^6 t e^{-25{,}000t} - 200 e^{-25{,}000t}\,\text{V}, \quad t \geq 0$$

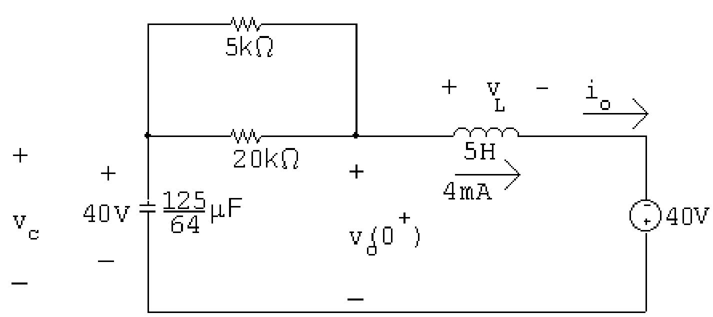
P 8.46 **[a]** $t < 0$:



$$i_o(0^-) = \frac{120}{30{,}000} = 4\,\text{mA}$$

$$v_C(0^-) = 80 - (10{,}000)(0.004) = 40\,\text{V}$$

$t = 0^+$:



$$5\,\text{k}\Omega \| 20\,\text{k}\Omega = 4\,\text{k}\Omega$$

$$\therefore \quad v_o(0^+) = -(0.004)(4000) + 40 = 40 - 16 = 24\,\text{V}$$

[b] $v_o(t) = v_{\rm c} - 4000 i_o$

$$\frac{dv_o}{dt}(0^+) = \frac{dv_c}{dt}(0^+) - 4000\frac{di_o}{dt}(0^+)$$

$$\frac{dv_c}{dt}(0^+) = \frac{-4\times 10^{-3}}{(125/64)\times 10^{-6}} = -2048\,\text{V/s}$$

$$-v_{\rm L}(0^+) + v_o(0^+) + 40 = 0 \quad v_{\rm L} = 64\ \text{V}$$

$$\frac{di_o}{dt}(0^+) = \frac{64}{5} = 12.8\,\text{A/s}$$

$$\frac{dv_o}{dt}(0^+) = -2048 - 4000(12.8) = -53{,}248\,\text{V/s}$$

[c] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(5)[(125/64)\times 10^{-6}]} = 10.24\times 10^4$

$$\alpha = \frac{R}{2L} = \frac{4000}{2(5)} = 400\ \text{rad/s}; \qquad \alpha^2 = 16\times 10^4$$

$\alpha^2 > \omega_o^2$ overdamped

$$s_{1,2} = -400 \pm 240\ \text{rad/s}$$

$$v_o(t) = V_f + A_1' e^{-160t} + A_2' e^{-640t}$$

$$V_f = v_o(\infty) = -40\,\text{V}$$

$$-40 + A_1' + A_2' = 24$$

$$-160A_1' - 640A_2' = -53{,}248$$

Solving, $A_1' = -25.6$; $A_2' = 89.6$

$$\therefore\ v_o(t) = -40 - 25.6e^{-160t} + 89.6e^{-640t}\,\text{V}, \qquad t \ge 0^+$$

P 8.47 **[a]** $v_c = V_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

It follows that $B_1' = -V_f$ and $B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t} \sin \omega_d t$$

But $\quad V_f = V \quad$ and $\quad \dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\quad \dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) V e^{-\alpha t} \sin \omega_d t$

[b] $\dfrac{dv_c}{dt} = 0 \quad$ when $\quad \sin \omega_d t = 0, \quad$ or $\quad \omega_d t = n\pi$

where $\quad n = 0, 1, 2, 3, \dots$

Therefore $\quad t = \dfrac{n\pi}{\omega_d}$

[c] When $\quad t_n = \dfrac{n\pi}{\omega_d}, \quad \cos \omega_d t_n = \cos n\pi = (-1)^n$

and $\quad \sin \omega_d t = \sin n\pi = 0$

Therefore $\quad v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$v(t_1) = V + V e^{-(\alpha\pi/\omega_d)}$ $\quad$ and $\quad v_c(t_3) = V + V e^{-(3\alpha\pi/\omega_d)}$

Therefore $\quad \dfrac{v_c(t_1) - V}{v_c(t_3) - V} = \dfrac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$

But $\quad \dfrac{2\pi}{\omega_d} = t_3 - t_1 = T_d, \quad$ thus $\quad \alpha = \dfrac{1}{T_d} \ln \dfrac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$

P 8.48 $\quad \alpha = \dfrac{1}{T_d} \ln \left\{\dfrac{v_c(t_1) - V}{v_c(t_3) - V}\right\}; \qquad T_d = t_3 - t_1 = \dfrac{3\pi}{7} - \dfrac{\pi}{7} = \dfrac{2\pi}{7}$ ms

$$\alpha = \frac{7000}{2\pi} \ln \left[\frac{63.84}{26.02}\right] \approx 1000; \qquad \omega_d = \frac{2\pi}{T_d} = 7000 \text{ rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 49 \times 10^6 + 10^6 = 50 \times 10^6$$

$$L = \frac{1}{(50 \times 10^6)(0.1 \times 10^{-6})} = 200 \text{ mH}; \qquad R = 2\alpha L = 400\,\Omega$$

P 8.49 **[a]** Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $\quad i = B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]\, e^{-\alpha t}$$

Therefore $\quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \frac{\alpha}{\omega_d}B_1' = \frac{\alpha}{\omega_d}\frac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right) e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d} e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0^+$$

[b] $\displaystyle \frac{dv_o}{dt} = -\frac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\quad \omega_d t = \tan^{-1}(\omega_d/\alpha) \quad$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 8.50 **[a]** From Problem 8.49 we have

$$v_o = \frac{-V_g}{RC\omega_d} e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{4800}{2(64\times 10^{-3})} = 37{,}500\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(64\times 10^{-3})(4\times 10^{-9})} = 3906.25\times 10^6$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 50\,\text{krad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-72)}{(4800)(4\times10^{-9})(50\times10^3)} = 75$$

$$\therefore \quad v_o = 75e^{-37{,}500t}\sin 50{,}000t\,\text{V}, \quad t \geq 0$$

[b] From Problem 8.49

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{50{,}000}\tan^{-1}\left(\frac{50{,}000}{37{,}500}\right)$$

$$t_d = 18.55\,\mu\text{s}$$

[c] $v_{\text{max}} = 75e^{-0.0375(18.55)}\sin[(0.05)(18.55)] = 29.93\,\text{V}$

[d] $R = 480\,\Omega; \qquad \alpha = 3750\,\text{rad/s}$

$$\omega_d = 62{,}387.4\,\text{rad/s}$$

$$v_o = 601.08e^{-3750t}\sin 62{,}387.4t\,\text{V}, \quad t \geq 0$$

$$t_d = 24.22\,\mu\text{s}$$

$$v_{\text{max}} = 547.92\,\text{V}$$

P 8.51 **[a]** $\dfrac{d^2v_o}{dt^2} = \dfrac{1}{R_1C_1R_2C_2}v_g$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2)\times10^{-6}\times10^{-6}} = 250$$

$$\therefore \quad \frac{d^2v_o}{dt^2} = 250v_g$$

$0 \leq t \leq 0.5^-$:

$$v_g = 80\,\text{mV}$$

$$\frac{d^2v_o}{dt^2} = 20$$

Let $g(t) = \dfrac{dv_o}{dt}$, then $\dfrac{dg}{dt} = 20$ or $dg = 20\,dt$

$$\int_{g(0)}^{g(t)} dx = 20\int_0^t dy$$

$$g(t) - g(0) = 20t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 20t$$

$$dv_o = 20t\,dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = 20\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$v_o(t) = 10t^2\,\mathrm{V}, \quad 0 \le t \le 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g = -1.6$$

$$dv_{o1} = -1.6\,dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6\int_0^t dy$$

$$v_{o1}(t) - v_{o1}(0) = -1.6t, \qquad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t\,\mathrm{V}, \qquad 0 \le t \le 0.5^-$$

$0.5^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10; \qquad dg(t) = -10\,dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10\int_{0.5}^t dy$$

$$g(t) - g(0.5^+) = -10(t-0.5) = -10t + 5$$

$$g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400\times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80\,\mathrm{V}$$

$$\therefore \quad C\frac{dv_o(0.5^+)}{dt} = \frac{0.80}{0.4\times 10^6} = 2\,\mu\mathrm{A}$$

$$\frac{dv_o}{dt}(0.5^+) = \frac{2\times 10^{-6}}{0.2\times 10^{-6}} = 10\,\mathrm{V/s}$$

$$\therefore \quad g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore \quad dv_o = -10t\,dt + 15\,dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^t -10y\,dy + \int_{0.5^+}^t 15\,dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2 \Big|_{0.5}^{t} + 15y \Big|_{0.5}^{t}$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5\,\text{V}$$

$$\therefore \quad v_o(t) = -5t^2 + 15t - 3.75\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-0.04) = 0.8, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$dv_{o1} = 0.8\,dt; \qquad \int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8\int_{0.5^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \qquad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8\,\text{V}$$

$$\therefore \quad v_{o1}(t) = 0.8t - 1.2\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

Summary:

$0 \le t \le 0.5^-\text{s}:$ $\qquad v_{o1} = -1.6t\,\text{V}, \quad v_o = 10t^2\,\text{V}$

$0.5^+\text{s} \le t \le t_{\text{sat}}:$ $\qquad v_{o1} = 0.8t - 1.2\,\text{V}, \quad v_o = -5t^2 + 15t - 3.75\,\text{V}$

[b] $-12.5 = -5t_{\text{sat}}^2 + 15t_{\text{sat}} - 3.75$

$$\therefore \quad 5t_{\text{sat}}^2 - 15t_{\text{sat}} - 8.75 = 0$$

Solving, $\qquad t_{\text{sat}} = 3.5\,\text{sec}$

$$v_{o1}(t_{\text{sat}}) = 0.8(3.5) - 1.2 = 1.6\,\text{V}$$

P 8.52 $\quad \tau_1 = (10^6)(0.5 \times 10^{-6}) = 0.50\,\text{s}$

$$\frac{1}{\tau_1} = 2; \qquad \tau_2 = (5 \times 10^6)(0.2 \times 10^{-6}) = 1\,\text{s}; \qquad \therefore \quad \frac{1}{\tau_2} = 1$$

$$\therefore \quad \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 20$$

$$s^2 + 3s + 2 = 0$$

$$(s+1)(s+2) = 0; \qquad s_1 = -1, \quad s_2 = -2$$

$$v_o = V_f + A_1'e^{-t} + A_2'e^{-2t}; \qquad V_f = \frac{20}{2} = 10\,\text{V}$$

$$v_o = 10 + A_1'e^{-t} + A_2'e^{-2t}$$

$$v_o(0) = 0 = 10 + A_1' + A_2'; \qquad \frac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' = -20, \qquad A_2' = 10\,\text{V}$$

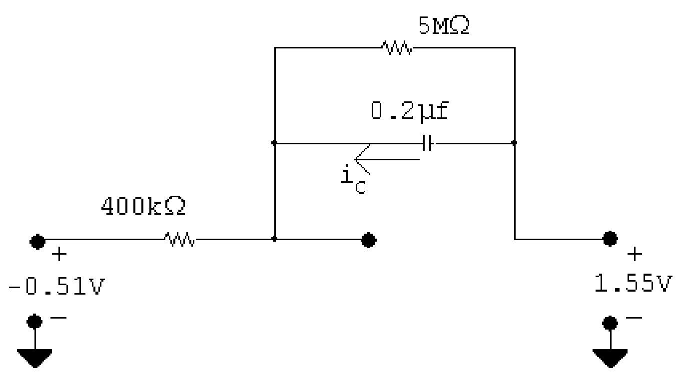
$$v_o(t) = 10 - 20e^{-t} + 10e^{-2t}\,\text{V}, \qquad 0 \le t \le 0.5\,\text{s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = -1.6; \qquad \therefore \quad v_{o1} = -0.8 + 0.8e^{-2t}\,\text{V}, \quad 0 \le t \le 0.5\,\text{s}$$

$$v_o(0.5) = 10 - 20e^{-0.5} + 10e^{-1} = 1.55\,\text{V}$$

$$v_{o1}(0.5) = -0.8 + 0.8e^{-1} = -0.51\,\text{V}$$

At $t = 0.5\,\text{s}$



$$i_C = \frac{0 + 0.51}{400 \times 10^3} - \frac{1.55 - 0}{5 \times 10^6} = 0.954\,\mu\text{A}$$

$$C\frac{dv_o}{dt} = 0.954\,\mu\text{A}; \qquad \frac{dv_o}{dt} = \frac{0.954}{0.2} = 4.773\ \text{V/s}$$

$t \ge 0.5\,\text{s}$

$$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = -10$$

$$v_o(\infty) = -5$$

$$\therefore \quad v_o = -5 + A_1'e^{-(t-0.5)} + A_2'e^{-2(t-0.5)}$$

$$1.55 = -5 + A_1' + A_2'$$

$$\frac{dv_o}{dt}(0.5) = 4.773 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' + A_2' = 6.55; \qquad -A_1' - 2A_2' = 4.773$$

Solving,

$$A_1' = 17.87\text{ V}; \qquad A_2' = -11.32\text{ V}$$

$$\therefore \quad v_o = -5 + 17.87e^{-(t-0.5)} - 11.32e^{-2(t-0.5)}\text{ V}, \qquad t \geq 0.5\text{ s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 0.8$$

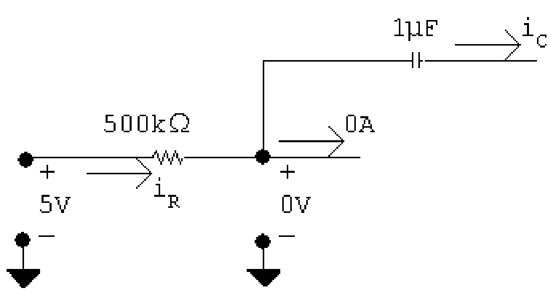
$$\therefore \quad v_{o1} = 0.4 + (-0.51 - 0.4)e^{-2(t-0.5)} = 0.4 - 0.91e^{-2(t-0.5)}\text{ V}, \qquad t \geq 0.5\text{ s}$$

P 8.53 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the $500\,\text{k}\Omega$ is zero. Therefore there cannot be an instantaneous change in the current in the $1\,\mu$F capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.54 **[a]** From Example 8.13 $\displaystyle\frac{d^2v_o}{dt^2} = 2$

therefore $\displaystyle\frac{dg(t)}{dt} = 2, \quad g(t) = \frac{dv_o}{dt}$

$$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$$



$$i_R = \frac{5}{500} \times 10^{-3} = 1\,\mu\text{A} = i_C = -C\frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-1 \times 10^{-6}}{1 \times 10^{-6}} = -1 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 1$$

$$dv_o = 2t\,dt - \,dt$$

$$v_o - v_o(0) = t^2 - t; \qquad v_o(0) = 8\text{ V}$$

$$v_o = t^2 - t + 8, \qquad 0 \leq t \leq t_{\text{sat}}$$

[b] $t^2 - t + 8 = 9$

$t^2 - t - 1 = 0$

$t = (1/2) \pm (\sqrt{5}/2) \cong 1.62\,\text{s}, \quad t_{\text{sat}} \cong 1.62\,\text{s}$

(Negative value has no physical significance.)

P 8.55 Part (1) — Example 8.14, with R_1 and R_2 removed:

[a] $R_{\text{a}} = 100\,\text{k}\Omega; \qquad C_1 = 0.1\,\mu\text{F}; \qquad R_{\text{b}} = 25\,\text{k}\Omega; \qquad C_2 = 1\,\mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_{\text{a}}C_1}\right)\left(\frac{1}{R_{\text{b}}C_2}\right)v_g; \qquad \frac{1}{R_{\text{a}}C_1} = 100 \quad \frac{1}{R_{\text{b}}C_2} = 40$$

$$v_g = 250 \times 10^{-3}; \qquad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[b] Since $v_o(0) = 0 = \dfrac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$ V

The second op-amp will saturate when

$v_o = 6\,\text{V}, \quad \text{or} \quad t_{\text{sat}} = \sqrt{6/500} \cong 0.1095\,\text{s}$

[c] $\dfrac{dv_{o1}}{dt} = -\dfrac{1}{R_{\text{a}}C_1}v_g = -25$

[d] Since $v_{o1}(0) = 0, \quad v_{o1} = -25t\,\text{V}$

At $t = 0.1095\,\text{s}, \quad v_{o1} \cong -2.74\,\text{V}$

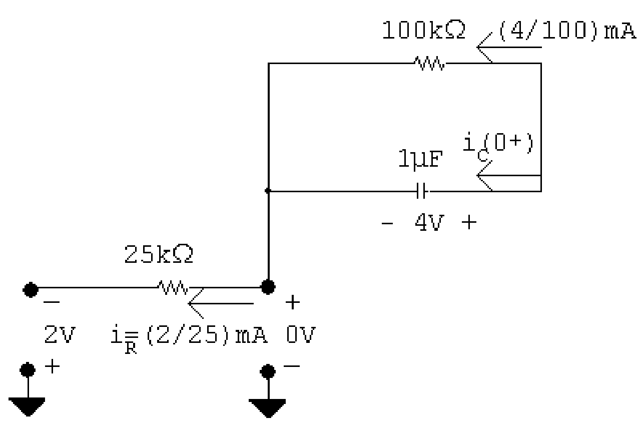
Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \leq t \leq 0.1095$ s. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2$ V and $v_o(0) = 4$ V:

[a] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

[b] $v_o = 5 + A_1'e^{-10t} + A_2'e^{-20t}$ (from Example 8.14)

$v_o(0) = 4 = 5 + A_1' + A_2'$



$$\frac{4}{100} + i_C(0^+) - \frac{2}{25} = 0$$

$$i_C(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04\times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$

$$\frac{dv_o}{dt} = -10A_1'e^{-10t} - 20A_2'e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A_1' - 20A_2' = 40$$

Therefore $-A_1' - 2A_2' = 4$ and $A_1' + A_2' = -1$
Thus, $A_1' = 2$ and $A_2' = -3$

$$v_o = 5 + 2e^{-10t} - 3e^{-20t}\,\text{V}, \quad t \geq 0$$

[c] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

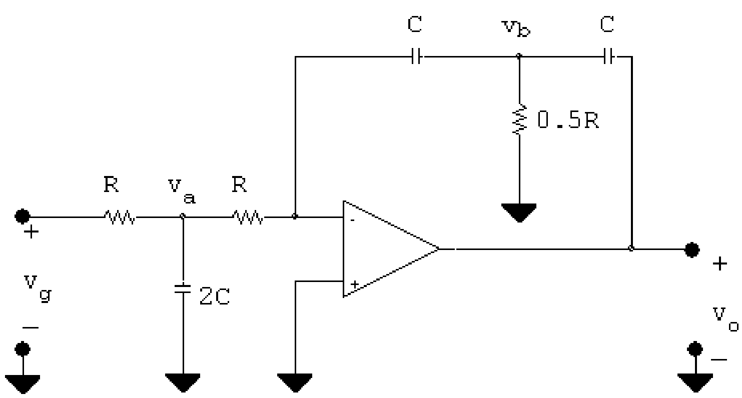
[d] From Example 8.14:

$v_{o1}(\infty) = -1.25\,\text{V}; \qquad v_1(0) = -2\,\text{V} \quad$ (given)

Therefore

$$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t}\,\text{V}, \quad t \geq 0$$

P 8.56 **[a]**



$$2C\frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

(1) Therefore $\dfrac{dv_a}{dt} + \dfrac{v_a}{RC} = \dfrac{v_g}{2RC}; \qquad \dfrac{0 - v_a}{R} + C\dfrac{d(0 - v_b)}{dt} = 0$

(2) Therefore $\dfrac{dv_b}{dt} + \dfrac{v_a}{RC} = 0, \quad v_a = -RC\dfrac{dv_b}{dt}$

$$\frac{2v_\mathrm{b}}{R} + C\frac{dv_\mathrm{b}}{dt} + C\frac{d(v_\mathrm{b} - v_o)}{dt} = 0$$

(3) Therefore $\quad \dfrac{dv_\mathrm{b}}{dt} + \dfrac{v_\mathrm{b}}{RC} = \dfrac{1}{2}\dfrac{dv_o}{dt}$

From (2) we have $\quad \dfrac{dv_\mathrm{a}}{dt} = -RC\dfrac{d^2v_\mathrm{b}}{dt^2} \quad$ and $\quad v_\mathrm{a} = -RC\dfrac{dv_\mathrm{b}}{dt}$

When these are substituted into (1) we get

(4) $-RC\dfrac{d^2v_\mathrm{b}}{dt^2} - \dfrac{dv_\mathrm{b}}{dt} = \dfrac{v_g}{2RC}$

Now differentiate (3) to get

(5) $\dfrac{d^2v_\mathrm{b}}{dt^2} + \dfrac{1}{RC}\dfrac{dv_\mathrm{b}}{dt} = \dfrac{1}{2}\dfrac{d^2v_o}{dt^2}$

But from (4) we have

(6) $\dfrac{d^2v_\mathrm{b}}{dt^2} + \dfrac{1}{RC}\dfrac{dv_\mathrm{b}}{dt} = -\dfrac{v_g}{2R^2C^2}$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

[b] When $\quad R_1C_1 = R_2C_2 = RC: \quad \dfrac{d^2v_o}{dt^2} = \dfrac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[c] Two integrations of the input signal with one operational amplifier.

P 8.57 **[a]** $f(t)$ = inertial force + frictional force + spring force

$\qquad = M[d^2x/dt^2] + D[dx/dt] + Kx$

[b] $\dfrac{d^2x}{dt^2} = \dfrac{f}{M} - \left(\dfrac{D}{M}\right)\left(\dfrac{dx}{dt}\right) - \left(\dfrac{K}{M}\right)x$

Given $\quad v_A = \dfrac{d^2x}{dt^2}, \quad$ then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right) dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_\mathrm{C} = -\frac{1}{R_2C_2}\int_0^t v_B\, dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5 + R_6}{R_6}\right] v_{\rm C} = \left[\frac{R_5 + R_6}{R_6}\right] \cdot \frac{1}{R_1 R_2 C_1 C_2} \cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right] f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $$\frac{d^2x}{dt^2} = \left[\frac{R_8}{R_7}\right] f(t) - \left[\frac{R_3}{R_4 R_1 C_1}\right] \frac{dx}{dt} - \left[\frac{R_5 + R_6}{R_6 R_1 R_2 C_1 C_2}\right] x$$

Therefore $M = \dfrac{R_7}{R_8}$, $\qquad D = \dfrac{R_3 R_7}{R_8 R_4 R_1 C_1}$ $\quad$ and $\quad K = \dfrac{R_7(R_5 + R_6)}{R_8 R_6 R_1 R_2 C_1 C_2}$

Box Number	Function
1	inverting and scaling
2	inverting and scaling
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.58 **[a]** Given that the current response is underdamped we know i will be of the form

$$i = I_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]e^{-\alpha t}$$

where $$\alpha = \frac{R}{2L}$$

and $$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \alpha^2}$$

The capacitor will force the final value of i to be zero, therefore $I_f = 0$.
By hypothesis $i(0^+) = V_{dc}/R$ therefore $B_1' = V_{dc}/R$.
At $t = 0^+$ the voltage across the primary winding is zero hence $di(0^+)/dt = 0$.
From our equation for i we have

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1') \cos\omega_d t - (\omega_d B_1' + \alpha B_2') \sin\omega_d t]e^{-\alpha t}$$

Hence

$$\frac{di(0^+)}{dt} = \omega_d B_2' - \alpha B_1' = 0$$

Thus

$$B_2' = \frac{\alpha}{\omega_d} B_1' = \frac{\alpha V_{dc}}{\omega_d R}$$

It follows directly that

$$i = \frac{V_{dc}}{R}\left[\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right] e^{-\alpha t}$$

[b] Since $\omega_d B_1' - \alpha B_1' = 0$ it follows that

$$\frac{di}{dt} = -(\omega_d B_1' + \alpha B_2')e^{-\alpha t}\, \sin\omega_d t$$

But $\quad \alpha B_2' = \dfrac{\alpha^2 V_{dc}}{\omega_d R} \quad$ and $\quad \omega_d B_1' = \dfrac{\omega_d V_{dc}}{R}$

Therefore

$$\omega_d B_1' + \alpha B_2' = \frac{\omega_d V_{dc}}{R} + \frac{\alpha^2 V_{dc}}{\omega_d R} = \frac{V_{dc}}{R}\left[\frac{\omega_d^2 + \alpha^2}{\omega_d}\right]$$

But $\quad \omega_d^2 + \alpha^2 = \omega_o^2 = \dfrac{1}{LC}$

Hence

$$\omega_d B_1' + \alpha B_2' = \frac{V_{dc}}{\omega_d RLC}$$

Now since $\quad v_1 = L\dfrac{di}{dt} \quad$ we get

$$v_1 = -L\frac{V_{dc}}{\omega_d RLC}e^{-\alpha t}\, \sin\omega_d t = -\frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\, \sin\omega_d t$$

[c] $v_c = V_{dc} - iR - L\dfrac{di}{dt}$

$$iR = V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\, \sin\omega_d t\right)e^{-\alpha t}$$

$$v_c = V_{dc} - V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\, \sin\omega_d t\right)e^{-\alpha t} + \frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc} - V_{dc}e^{-\alpha t}\cos\omega_d t + \left(\frac{V_{dc}}{\omega_d RC} - \frac{\alpha V_{dc}}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right)e^{-\alpha t}\sin\omega_d t\right]$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t\right]$$

P 8.59 $\quad v_{sp} = V_{dc}\left[1 - \dfrac{a}{\omega_d RC}e^{-\alpha t}\, \sin\omega_d t\right]$

$$\frac{dv_{sp}}{dt} = \frac{-aV_{dc}}{\omega_d RC}\,\frac{d}{dt}[e^{-\alpha t}\sin\omega_d t]$$

$$= \frac{-aV_{dc}}{\omega_d RC}[-\alpha e^{-\alpha t}\sin\omega_d t + \omega_d\, \cos\omega_d t e^{-\alpha t}]$$

$$= \frac{aV_{dc}e^{-\alpha t}}{\omega_d RC}[\alpha\, \sin\omega_d t - \omega_d\, \cos\omega_d t]$$

$$\frac{dv_{sp}}{dt} = 0 \quad \text{when} \quad \alpha \sin\omega_d t = \omega_d \cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}; \qquad \omega_d t = \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

$$\therefore \quad t_{\max} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

Note that because $\tan\theta$ is periodic, i.e., $\tan\theta = \tan(\theta \pm n\pi)$, where n is an integer, there are an infinite number of solutions for t where $dv_{sp}/dt = 0$, that is

$$t = \frac{\tan^{-1}(\omega_d/\alpha) \pm n\pi}{\omega_d}$$

Because of $e^{-\alpha t}$ in the expression for v_{sp} and knowing $t \geq 0$ we know v_{sp} will be maximum when t has its smallest positive value. Hence

$$t_{\max} = \frac{\tan^{-1}(\omega_d/\alpha)}{\omega_d}.$$

P 8.60 **[a]** $v_c = V_{dc}[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t]$

$$\frac{dv_c}{dt} = V_{dc}\frac{d}{dt}[1 + e^{-\alpha t}(K\sin\omega_d t - \cos\omega_d t)]$$

$$= V_{dc}\{(-\alpha e^{-\alpha t})(K\sin\omega_d t - \cos\omega_d t) +$$
$$e^{-\alpha t}[\omega_d K\cos\omega_d t + \omega_d\sin\omega_d t]\}$$
$$= V_{dc}e^{-\alpha t}[(\omega_d - \alpha K)\sin\omega_d t + (\alpha + \omega_d K)\cos\omega_d t]$$

$$\frac{dv_c}{dt} = 0 \quad \text{when} \quad (\omega_d - \alpha K)\sin\omega_d t = -(\alpha + \omega_d K)\cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$\therefore \quad \omega_d t \pm n\pi = \tan^{-1}\left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$t_c = \frac{1}{\omega_d}\left\{tan^{-1}\left(\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right) \pm n\pi\right\}$$

$$\alpha = \frac{R}{2L} = \frac{4\times 10^3}{6} = 666.67\,\text{rad/s}$$

$$\omega_d = \sqrt{\frac{10^9}{1.2} - (666.67)^2} = 28{,}859.81\,\text{rad/s}$$

$$K = \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right) = 21.63$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}(-43.29) + n\pi\right\} = \frac{1}{\omega_d}\{-1.55 + n\pi\}$$

The smallest positive value of t occurs when $n = 1$, therefore

$t_{c\,\max} = 55.23\,\mu$s

[b] $v_c(t_{c\,\max}) = 12[1 - e^{-\alpha t_{c\,\max}}\cos\omega_d t_{c\,\max} + Ke^{-\alpha t_{c\,\max}}\sin\omega_d t_{c\,\max}]$

$= 262.42\,$V

[c] From the text example the voltage across the spark plug reaches its maximum value in $53.63\,\mu$s. If the spark plug does not fire the capacitor voltage peaks in $55.23\,\mu$s. When v_{sp} is maximum the voltage across the capacitor is $262.15\,$V. If the spark plug does not fire the capacitor voltage reaches $262.42\,$V.

P 8.61 **[a]** $w = \frac{1}{2}L[i(0^+)]^2 = \frac{1}{2}(5)(16)\times 10^{-3} = 40\,$mJ

[b] $\alpha = \frac{R}{2L} = \frac{3\times 10^3}{10} = 300\,$rad/s

$$\omega_d = \sqrt{\frac{10^9}{1.25} - (300)^2} = 28{,}282.68\,\text{rad/s}$$

$$\frac{1}{RC} = \frac{10^6}{0.75} = \frac{4\times 10^6}{3}$$

$$t_{\max} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = 55.16\,\mu\text{s}$$

$$v_{sp}\,(t_{\max}) = 12 - \frac{12(50)(4\times 10^6)}{3(28{,}282.68)}e^{-\alpha t_{\max}}\sin\omega_d t_{\max} = -27{,}808.04\,\text{V}$$

[c] $v_c\,(t_{\max}) = 12[1 - e^{-\alpha t_{\max}}\cos\omega_d t_{\max} + Ke^{-\alpha t_{\max}}\sin\omega_d t_{\max}]$

$$K = \frac{1}{\omega_d}\left[\frac{1}{RC} - \alpha\right] = 47.13$$

$v_c\,(t_{\max}) = 568.15\,$V