9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 **[a]** $\mathbf{V} = 170\underline{/-40^\circ}\,\text{V}$

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\underline{/-70^\circ}\,\text{A}$$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ}\,\text{A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ}\,\text{mV}$$

AP 9.2 **[a]** $v = 18.6\cos(\omega t - 54^\circ)\,\text{V}$

[b] $\mathbf{I} = 20\underline{/45^\circ} - 50\underline{/-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\underline{/126.68^\circ}$$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)\,\text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15^\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\underline{/97.08^\circ}$$

$$v = 72.79\cos(\omega t + 97.08^\circ)\,\text{V}$$

AP 9.3 **[a]** $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 **[a]** $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5 \times 10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\underline{/25^\circ}}{50\underline{/-90^\circ}} = 0.6\underline{/115^\circ}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.72 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A},$ therefore $i_4 = 0\,\text{A}$

AP 9.6 **[a]** $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore\ \theta_Z = 45^\circ$

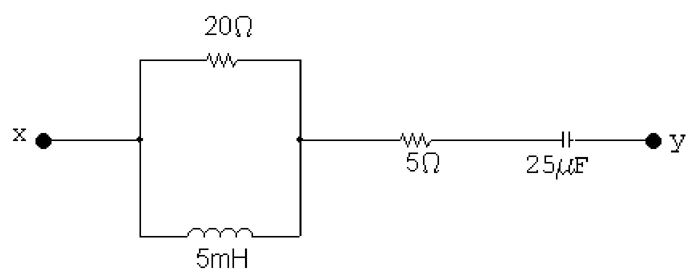
$Z = 90 + j160 + jX_C$

$\therefore\ X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore\ C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore\ |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 **[a]**



$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

$Z_{xy} = 20 \| j10 + 5 + j20 = \dfrac{20(j10)}{(20 + j10)} + 5 - j20$

$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$

[b] $\omega L = 40\,\Omega, \qquad \dfrac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20+j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\dfrac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \dfrac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000$ rad/s.

[d] $Z_{xy} = \dfrac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{\text{xy}} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

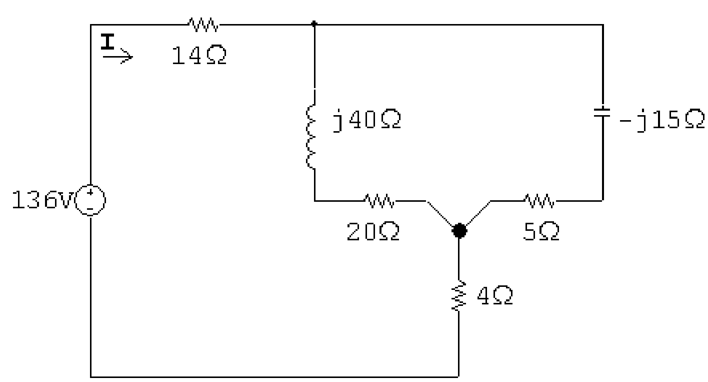
$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{\text{xy}}} = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$$(20 + j40)\|(5 - j15) = (12 - j16)\,\Omega$$

Therefore $\quad \mathbf{I} = \dfrac{136\underline{/0^\circ}}{14 + 12 - j16 + 4} = 4\underline{/28.07^\circ}\,\text{A}$

AP 9.10 $\mathbf{V}_1 = 240\underline{/53.13^\circ} = 144 + j192\,\text{V}$

$$\mathbf{V}_2 = 96\underline{/-90^\circ} = -j96\,\text{V}$$

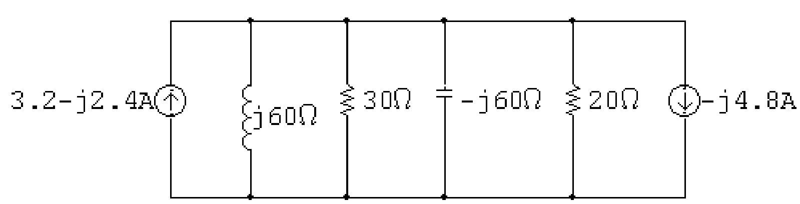
$$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$$

Perform source transformations:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4\,\text{A}$$

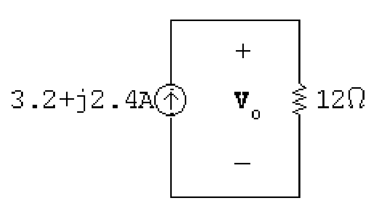
$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\text{A}$$



Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12\,\Omega$$



$$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\text{V} = 48\underline{/36.87^\circ}\,\text{V}$$

$$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$$

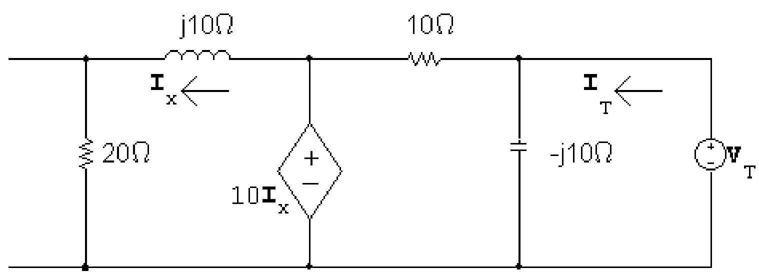
AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\underline{/45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\underline{/45^\circ} V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

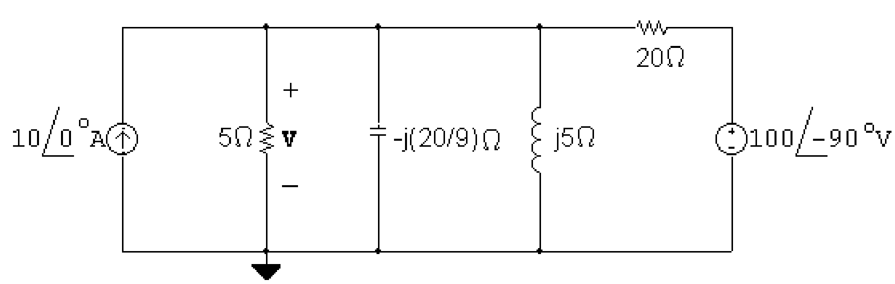
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

Therefore $\quad \mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $\quad v = 31.62\cos(50{,}000t - 71.57^\circ)\,\text{V}$

AP 9.13 Let $\mathbf{I}_\text{a}$, $\mathbf{I}_\text{b}$, and $\mathbf{I}_\text{c}$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_\text{a} + (3 - j5)(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})$$

and

$$0 = (3 - j5)(\mathbf{I}_\text{b} - \mathbf{I}_\text{a}) + 2(\mathbf{I}_\text{b} - \mathbf{I}_\text{c}).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b}),$$

therefore

$$\mathbf{I}_\text{c} = -0.75[-j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})].$$

Solving for $\mathbf{I} = \mathbf{I}_\text{a} = 29 + j2 = 29.07\underline{/3.95^\circ}$ A.

AP 9.14 **[a]** $M = 0.4\sqrt{0.0625} = 0.1\,\text{H}, \qquad \omega M = 80\,\Omega$

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300)\,\Omega$$

Therefore $\quad |Z_{22}| = 500\,\Omega, \qquad Z_{22}^* = (400 - j300)\,\Omega$

$$Z_r = \left(\frac{80}{500}\right)^2 (400 - j300) = (10.24 - j7.68)\,\Omega$$

[b] $\mathbf{I}_1 = \dfrac{245.20}{184 + 100 + j400 + Z_r} = 0.50\underline{/-53.13^\circ}\,\text{A}$

$$i_1 = 0.5\cos(800t - 53.13^\circ)\,\text{A}$$

[c] $\mathbf{I}_2 = \left(\dfrac{j\omega M}{Z_{22}}\right)\mathbf{I}_1 = \dfrac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ}) = 0.08\underline{/0^\circ}\,\text{A}$

$$i_2 = 80\cos 800t\,\text{mA}$$

AP 9.15 $$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + Z_2/a^2} = \frac{25 \times 10^3\underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5\underline{/36.87^\circ}\text{ A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}\text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/-143.13^\circ}\text{ A}$$

Also, $I_2 = -25I_1$

Problems

P 9.1 **[a]** $\omega = 2\pi f = 3769.91$ rad/s, $\quad f = \dfrac{\omega}{2\pi} = 600$ Hz

[b] $T = 1/f = 1.67$ ms

[c] $V_m = 10$ V

[d] $v(0) = 10\cos(-53.13^\circ) = 6$ V

[e] $\phi = -53.13^\circ; \quad \phi = \dfrac{-53.13^\circ(2\pi)}{360^\circ} = -0.9273$ rad

[f] $V = 0$ when $3769.91t - 53.13^\circ = 90^\circ$. Now resolve the units:

$$(3769.91 \text{ rad/s})t = \frac{143.13^\circ}{(180^\circ/\pi)} = 2.498 \text{ rad}, \qquad t = 662.64\,\mu\text{s}$$

[g] $(dv/dt) = (-10)3769.91\sin(3769.91t - 53.13^\circ)$

$$(dv/dt) = 0 \quad \text{when} \quad 3769.91t - 53.13^\circ = 0^\circ$$

$$\text{or} \qquad 3769.91t = \frac{53.13^\circ}{57.3^\circ/\text{rad}} = 0.9273 \text{ rad}$$

Therefore $t = 245.97\,\mu$s

P 9.2 $$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T}t\,dt}$$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}\right)t\,dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\,dt = \frac{V_m^2 T}{4}$$

Therefore $\quad V_{\text{rms}} = \sqrt{\dfrac{1}{T}\dfrac{V_m^2 T}{4}} = \dfrac{V_m}{2}$

P 9.3 **[a]** 40 V

[b] $2\pi f = 100\pi; \quad f = 50$Hz

[c] $\omega = 100\pi = 314.159$ rad/s

[d] $\theta(\text{rad}) = \dfrac{2\pi}{360^\circ}(60^\circ) = \dfrac{\pi}{3} = 1.05$ rad

[e] $\theta = 60^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{50} = 20$ ms

[g] $v = -40$ when

$$100\pi t + \frac{\pi}{3} = \pi; \qquad \therefore \quad t = 6.67 \text{ ms}$$

[h] $v = 40\cos\left[100\pi\left(t - \frac{0.01}{3}\right) + \frac{\pi}{3}\right]$

$= 40\cos[100\pi t - (\pi/3) + (\pi/3)]$

$= 40\cos 100\pi t$ V

[i] $100\pi(t - t_o) + (\pi/3) = 100\pi t - (\pi/2)$

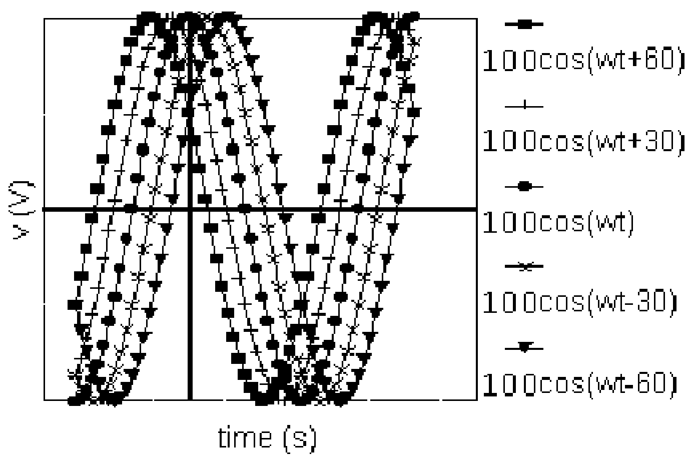
$\therefore \quad 100\pi t_o = \frac{5\pi}{6}; \qquad t_o = 8.33\,\text{ms}$

[j] $100\pi(t + t_o) + (\pi/3) = 100\pi t + 2\pi$

$\therefore \quad 100\pi t_o = \frac{5\pi}{3}; \qquad t_o = 16.67\,\text{ms}$

16.67 ms to the left

P 9.4



[a] Left as ϕ becomes more positive

[b] Left

P 9.5 **[a]** By hypothesis

$v = 80\cos(\omega t + \theta)$

$\frac{dv}{dt} = -80\omega\sin(\omega t + \theta)$

$\therefore \quad 80\omega = 80{,}000; \qquad \omega = 1000\,\text{rad/s}$

[b] $f = \frac{\omega}{2\pi} = 159.155$ Hz; $\qquad T = \frac{1}{f} = 6.28\,\text{ms}$

$\frac{-2\pi/3}{6.28} = -0.3333, \qquad \therefore \quad \theta = -90 - (-0.3333)(360) = 30^\circ$

$\therefore \quad v = 80\cos(1000t + 30^\circ)$ V

P 9.6 **[a]** $\dfrac{T}{2} = 8 + 2 = 10\,\text{ms}; \qquad T = 20\,\text{ms}$

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50\text{Hz}$$

[b] $v = V_m \sin(\omega t + \theta)$

$$\omega = 2\pi f = 100\pi \text{ rad/s}$$

$$100\pi(-2 \times 10^{-3}) + \theta = 0; \qquad \therefore \quad \theta = \frac{\pi}{5} \text{ rad} = 36^\circ$$

$$v = V_m \sin[100\pi t + 36^\circ]$$

$$80.9 = V_m \sin 36^\circ; \qquad V_m = 137.64\,\text{V}$$

$$v = 137.64 \sin[100\pi t + 36^\circ] = 137.64 \cos[100\pi t - 54^\circ]\,\text{V}$$

P 9.7

$$\begin{aligned}
u &= \int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\, dt \\
&= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\, dt \\
&= \frac{V_m^2}{2}\left\{ \textstyle\int_{t_o}^{t_o+T} dt + \displaystyle\int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\, dt \right\} \\
&= \frac{V_m^2}{2}\left\{ T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\ \Big|_{t_o}^{t_o+T} \right] \right\} \\
&= \frac{V_m^2}{2}\left\{ T + \frac{1}{2\omega}\left[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi) \right] \right\} \\
&= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right)
\end{aligned}$$

P 9.8 $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(120) = 169.71\,\text{V}$

P 9.9 **[a]** The numerical values of the terms in Eq. 9.8 are

$$V_m = 20, \qquad R/L = 1066.67, \qquad \omega L = 60$$

$$\sqrt{R^2 + \omega^2 L^2} = 100$$

$$\phi = 25^\circ, \qquad \theta = \tan^{-1} 60/80, \qquad \theta = 36.87^\circ$$

Substitute these values into Equation 9.9:

$$i = \left[-195.72e^{-1066.67t} + 200\cos(800t - 11.87^\circ)\right] \text{ mA}, \qquad t \geq 0$$

[b] Transient component $= -195.72e^{-1066.67t}$ mA
Steady-state component $= 200\cos(800t - 11.87^\circ)$ mA

[c] By direct substitution into Eq 9.9 in part (a), $\quad i(1.875\,\text{ms}) = 28.39\,\text{mA}$

[d] $200\,\text{mA}, \quad 800\,\text{rad/s}, \quad -11.87^\circ$

[e] The current lags the voltage by 36.87°.

P 9.10 **[a]** From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi-\theta)e^{-(R/L)t}}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m R\cos(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2+\omega^2L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$$V_m\cos(\omega t+\phi)$$

At $t=0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m\cos(\phi-\theta)}{\sqrt{R^2-\omega^2L^2}} + \frac{V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} = 0$$

[b] $i_{ss} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t+\phi-\theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

$$= V_m\cos(\omega t+\phi)$$

P 9.11 **[a]** $\mathbf{Y} = 50\angle 60^\circ + 100\angle -30^\circ = 111.8\angle -3.43^\circ$

$$y = 111.8\cos(500t - 3.43^\circ)$$

[b] $\mathbf{Y} = 200\angle 50^\circ - 100\angle 60^\circ = 102.99\angle 40.29^\circ$

$$y = 102.99\cos(377t + 40.29^\circ)$$

[c] $\mathbf{Y} = 80\angle 30^\circ - 100\angle -225^\circ + 50\angle -90^\circ = 161.59\angle -29.96^\circ$

$$y = 161.59\cos(100t - 29.96^\circ)$$

[d] $\mathbf{Y} = 250\underline{/0^\circ} + 250\underline{/120^\circ} + 250\underline{/-120^\circ} = 0$

$y = 0$

P 9.12 **[a]** 1000Hz

[b] $\theta_v = 0^\circ$

[c] $\mathbf{I} = \dfrac{200\underline{/0^\circ}}{j\omega L} = \dfrac{200}{\omega L}\underline{/-90^\circ} = 25\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$

[d] $\dfrac{200}{\omega L} = 25; \qquad \omega L = \dfrac{200}{25} = 8\,\Omega$

[e] $L = \dfrac{8}{2\pi(1000)} = 1.27\,\text{mH}$

[f] $Z_L = j\omega L = j8\,\Omega$

P 9.13 **[a]** $\omega = 2\pi f = 314{,}159.27\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{10\times10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(10\times10^{-3})\underline{/0^\circ} = 10\times10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $628.32\times10^{-6} = 10\times10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{10\times10^{-3}}{628.32\times10^{-6}} = 15.92\,\Omega, \quad \therefore \quad X_{\rm C} = -15.92\,\Omega$$

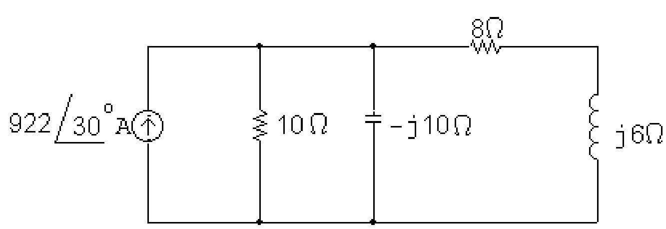
[d] $C = \dfrac{1}{15.92(\omega)} = \dfrac{1}{(15.92)(100\pi\times10^3)}$

$C = 0.2\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j15.92\,\Omega$

P 9.14 **[a]** $j\omega L = j(2\times10^4)(300\times10^{-6}) = j6\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(2\times10^4)(5\times10^{-6})} = -j10\,\Omega; \qquad \mathbf{I}_g = 922\underline{/30^\circ}\,\text{A}$$



[b] $\mathbf{V}_o = 922\underline{/30^\circ}Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{10} + j\frac{1}{10} + \frac{1}{8+j6}$$

$$Y_e = 0.18 + j0.04\,\text{S}$$

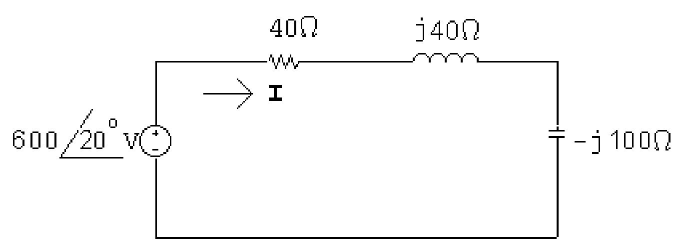
$$Z_e = \frac{1}{0.18 + j0.04} = 5.42\underline{/-12.53^\circ}\,\Omega$$

$$\mathbf{V}_o = (922\underline{/30^\circ})(5.42\underline{/-12.53^\circ}) = 5000.25\underline{/17.47^\circ}\,\text{V}$$

[c] $v_o = 5000.25\cos(2\times 10^4 t + 17.47^\circ)\,\text{V}$

P 9.15 **[a]** $Z_L = j(8000)(5\times 10^{-3}) = j40\,\Omega$

$$Z_C = \frac{-j}{(8000)(1.25\times 10^{-6})} = -j100\,\Omega$$



[b] $\mathbf{I} = \dfrac{600\underline{/20^\circ}}{40 + j40 - j100} = 8.32\underline{/76.31^\circ}\,\text{A}$

[c] $i = 8.32\cos(8000t + 76.31^\circ)\,\text{A}$

P 9.16 $Z = 4 + j(50)(0.24) - j\dfrac{1}{(50)(0.0025)} = 4 + j4 = 5.66\underline{/45^\circ}\,\Omega$

$$\mathbf{I}_o = \frac{\mathbf{V}}{Z} = \frac{0.1\underline{/-90^\circ}}{5.66\underline{/45^\circ}} = 17.68\underline{/-135^\circ}\,\text{mA}$$

$$i_o(t) = 17.68\cos(50t - 135^\circ)\,\text{mA}$$

P 9.17 **[a]** $Y = \dfrac{1}{3+j4} + \dfrac{1}{16-j12} + \dfrac{1}{-j4}$

$$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$$

$$= 0.16 + j0.12 = 200\underline{/36.87^\circ}\,\text{mS}$$

[b] $G = 160\,\text{mS}$

[c] $B = 120\,\text{mS}$

[d] $\mathbf{I} = 8\underline{/0^\circ}$ A, $\qquad \mathbf{V} = \dfrac{\mathbf{I}}{Y} = \dfrac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}$ V

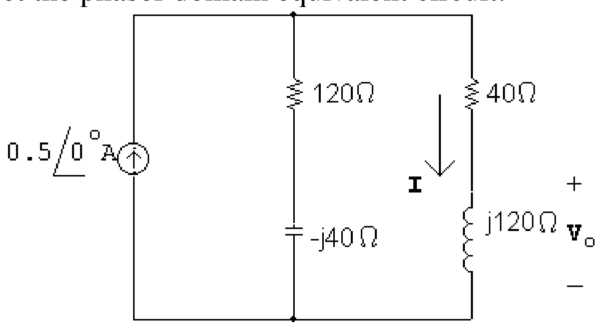
$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\text{ A}$$

$$i_C = 10\cos(\omega t + 53.13^\circ)\text{ A}, \qquad I_m = 10\text{ A}$$

P 9.18 $\quad Z_L = j(2000)(60\times 10^{-3}) = j120\,\Omega$

$$Z_C = \frac{-j}{(2000)(12.5\times 10^{-6})} = -j40\,\Omega$$

Construct the phasor domain equivalent circuit:



Using current division:

$$\mathbf{I} = \frac{(120 - j40)}{120 - j40 + 40 + j120}(0.5) = 0.25 - j0.25\text{ A}$$

$$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43\underline{/45^\circ}\text{ V}$$

$$v_o = 42.43\cos(2000t + 45^\circ)\text{ V}$$

P 9.19 **[a]** $\mathbf{V}_g = 300\underline{/78^\circ}; \qquad \mathbf{I}_g = 6\underline{/33^\circ}$

$$\therefore\quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\underline{/78^\circ}}{6\underline{/33^\circ}} = 50\underline{/45^\circ}\,\Omega$$

[b] i_g lags v_g by 45°:

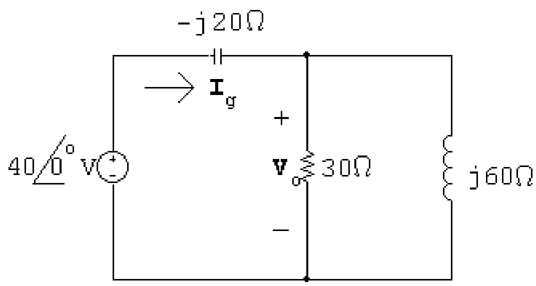
$$2\pi f = 5000\pi; \qquad f = 2500\text{ Hz}; \qquad T = 1/f = 400\,\mu\text{s}$$

$$\therefore\quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

P 9.20 $\dfrac{1}{j\omega C} = \dfrac{1}{(1\times 10^{-6})(50\times 10^{3})} = -j20\,\Omega$

$j\omega L = j50\times 10^{3}(1.2\times 10^{-3}) = j60\,\Omega$

$\mathbf{V}_g = 40\underline{/0^\circ}\,\text{V}$



$Z_e = -j20 + 30\|j60 = 24 - j8\,\Omega$

$\mathbf{I}_g = \dfrac{40\underline{/0^\circ}}{24 - j8} = 1.5 + j0.5\,\text{mA}$

$\mathbf{V}_o = (30\|j60)\mathbf{I}_g = \dfrac{30(j60)}{30 + j60}(1.5 + j0.5) = 30 + j30 = 42.43\underline{/45^\circ}\,\text{V}$

$v_o = 42.43\cos(50{,}000t + 45^\circ)\,\text{V}$

P 9.21 **[a]** $Z_1 = R_1 - j\dfrac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$Z_1 = Z_2$ when $R_1 = \dfrac{R_2}{1 + \omega^2 R_2^2 C_2^2}$ and

$\dfrac{1}{\omega C_1} = \dfrac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$ or $C_1 = \dfrac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$

[b] $R_1 = \dfrac{1000}{1 + (40\times 10^{3})^2(1000)^2(50\times 10^{-9})^2} = 200\,\Omega$

$$C_1 = \frac{1 + (40\times 10^{3})^2(1000)^2(50\times 10^{-9})^2}{(40\times 10^{3})^2(1000)^2(50\times 10^{-9})} = 62.5\,\text{nF}$$

P 9.22 **[a]** $Y_2 = \dfrac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $\quad Y_1 = Y_2 \quad$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $R_2 = \dfrac{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2}{(50 \times 10^3)^2(1000)(40 \times 10^{-9})^2} = 1250\,\Omega$

$$C_2 = \frac{40 \times 10^{-9}}{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2} = 8\,\text{nF}$$

P 9.23 **[a]** $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \dfrac{(4000)^2(1.25)^2(5000)}{5000^2 + 4000^2(1.25)^2} = 2500\,\Omega$

$$L_1 = \frac{(5000)^2(1.25)}{5000^2 + 4000^2(1.25)^2} = 625\,\text{mH}$$

P 9.24 **[a]** $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \dfrac{8000^2 + 1000^2(4)^2}{8000} = 10\,\text{k}\Omega$

$$L_2 = \frac{8000^2 + 1000^2(4)^2}{1000^2(4)} = 20\,\text{H}$$

P 9.25 $\mathbf{V}_g = 500\underline{/30^\circ}\,\text{V}; \qquad \mathbf{I}_g = 0.1\underline{/83.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 5000\underline{/-53.13^\circ}\,\Omega = 3000 - j4000\,\Omega$$

$$z = 3000 + j\left(\omega - \frac{32\times10^3}{\omega}\right)$$

$$\omega - \frac{32\times10^3}{\omega} = -4000$$

$$\omega^2 + 4000\omega - 32\times10^3 = 0$$

$$\omega = 7.984\ \text{rad/s}$$

P 9.26 **[a]** $Z_{\text{eq}} = \dfrac{50{,}000}{3} + \dfrac{-j20\times10^6}{\omega}\|(1200 + j0.2\omega)$

$$= \frac{50{,}000}{3} + \frac{-j20\times10^6}{\omega}\,\frac{(1200 + j0.2\omega)}{1200 + j[0.2\omega - \frac{20\times10^6}{\omega}]}$$

$$= \frac{50{,}000}{3} + \frac{\frac{-j20\times10^6}{\omega}(1200 + j0.2\omega)\left[1200 - j\left(0.2\omega - \frac{20\times10^6}{\omega}\right)\right]}{1200^2 + \left(0.2\omega - \frac{20\times10^6}{\omega}\right)^2}$$

$$\mathbf{Im}(Z_{\text{eq}}) = -\frac{20\times10^6}{\omega}(1200)^2 - \frac{20\times10^6}{\omega}\left[0.2\omega\left(0.2\omega - \frac{20\times10^6}{\omega}\right)\right] = 0$$

$$-20\times10^6(1200)^2 - 20\times10^6\left[0.2\omega\left(0.2\omega - \frac{20\times10^6}{\omega}\right)\right] = 0$$

$$-(1200)^2 = 0.2\omega\left(0.2\omega - \frac{20\times10^6}{\omega}\right)$$

$$0.2^2\omega^2 - 0.2(20\times10^6) + 1200^2 = 0$$

$$\omega^2 = 64\times10^6 \qquad \therefore \qquad \omega = 8000\,\text{rad/s}$$

$$\therefore \qquad f = 1273.24\ \text{Hz}$$

[b] $Z_{\text{eq}} = \dfrac{50{,}000}{3} + -j2500\|(1200 + j1600)$

$$= \frac{50{,}000}{3} + \frac{(-j2500)(1200 + j1600)}{1200 - j900} = 20{,}000\,\Omega$$

$$\mathbf{I}_g = \frac{30\underline{/0^\circ}}{20{,}000} = 1.5\underline{/0^\circ}\,\text{mA}$$

$$i_g(t) = 1.5\cos 8000t\ \text{mA}$$

P 9.27 **[a]** Find the equivalent impedance seen by the source, as a function of L, and set the imaginary part of the equivalent impedance to 0, solving for L:

$$Z_C = \frac{-j}{(500)(2\times 10^{-6})} = -j1000\,\Omega$$

$$Z_{\rm eq} = -j1000 + j500L\|2000 = -j1000 + \frac{2000(j500L)}{2000 + j500L}$$

$$= -j1000 + \frac{2000(j500L)(2000 - j500L)}{2000^2 + (500L)^2}$$

$$\mathbf{Im}(Z_{\rm eq}) = -1000 + \frac{2000^2(500L)}{2000^2 + (500L)^2} = 0$$

$$\therefore \qquad \frac{2000^2(500L)}{2000^2 + (500L)^2} = 1000$$

$$\therefore \qquad 500^2L^2 - \frac{1}{2}2000^2L + 2000^2 = 0$$

Solving the quadratic equation, $\quad L = 4\,\mathrm{H}$

[b] $\mathbf{I}_g = \dfrac{100\underline{/0^\circ}}{-j1000 + j2000\|2000} = \dfrac{100\underline{/0^\circ}}{1000} = 0.1\underline{/0^\circ}\ \mathrm{A}$

$i_g(t) = 0.1\cos 500t\,\mathrm{A}$

P 9.28 **[a]** $j\omega L + R\|(-j/\omega C) = j\omega L + \dfrac{-jR/\omega C}{R - j/\omega C}$

$$= j\omega L + \frac{-jR}{\omega CR - j1}$$

$$= j\omega L + \frac{-jR(\omega CR + j1)}{\omega^2C^2R^2 + 1}$$

$$\mathbf{Im}(Z_{\rm ab}) = \omega L - \frac{\omega CR^2}{\omega^2C^2R^2 + 1} = 0$$

$$\therefore \qquad L = \frac{CR^2}{\omega^2C^2R^2 + 1}$$

$$\therefore \qquad \omega^2C^2R^2 + 1 = \frac{CR^2}{L}$$

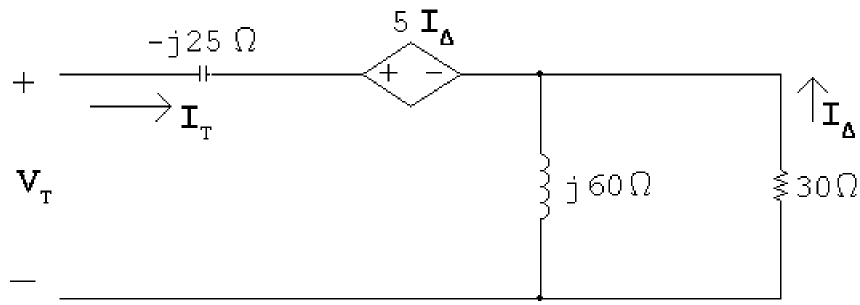
$$\therefore \qquad \omega^2 = \frac{(CR^2/L) - 1}{C^2R^2} = \frac{\frac{(25\times10^{-9})(100)^2}{160\times10^{-6}} - 1}{(25\times10^{-9})^2(100)^2} = 900\times 10^8$$

$\omega = 300\,\mathrm{krad/s}$

[b] $Z_{\rm ab}(300\times10^3) = j48 + \dfrac{(100)(-j133.33)}{100-j133.33} = 64\,\Omega$

P 9.29 $j\omega L = j100\times10^3(0.6\times10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(100\times10^3)(0.4\times10^{-6})} = -j25\,\Omega$$



$$\mathbf{V}_T = -j25\mathbf{I}_T + 5\mathbf{I}_\Delta - 30\mathbf{I}_\Delta$$

$$\mathbf{I}_\Delta = \frac{-j60}{30+j60}\mathbf{I}_T$$

$$\mathbf{V}_T = -j25\mathbf{I}_T + 25\frac{j60}{30+j60}\mathbf{I}_T$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\rm ab} = 20 - j15 = 25\underline{/-36.87^\circ}\,\Omega$$

P 9.30 **[a]** $Z_1 = 400 - j\dfrac{10^6}{500(2.5)} = 400 - j800\,\Omega$

$$Z_2 = 2000\|j500L = \frac{j10^6L}{2000+j500L}$$

$$Z_T = Z_1 + Z_2 = 400 - j800 + \frac{j10^6L}{2000+j500L}$$

$$= 400 + \frac{500\times10^6L^2}{2000^2+500^2L^2} - j800 + j\frac{2\times10^9L}{2000^2+500^2L^2}$$

Z_T is resistive when

$$\frac{2\times10^9L}{2000^2+500^2L^2} = 800 \quad \text{or} \quad 500^2L^2 - 25\times10^5L + 2000^2 = 0$$

Solving, $L_1 = 8$ H and $L_2 = 2$ H.

[b] When $L = 8$ H:

$$Z_T = 400 + \frac{500 \times 10^6(8)^2}{2000^2 + 500^2(8)^2} = 2000\,\Omega$$

$$\mathbf{I}_g = \frac{200\underline{/0^\circ}}{2000} = 100\underline{/0^\circ}\,\text{mA}$$

$$i_g = 100\cos 500t\,\text{mA}$$

When $L = 2$ H:

$$Z_T = 400 + \frac{500 \times 10^6(2)^2}{2000^2 + 500(2)^2} = 800\,\Omega$$

$$\mathbf{I}_g = \frac{200\underline{/0^\circ}}{800} = 250\underline{/0^\circ}\,\text{mA}$$

$$i_g = 250\cos 500t\,\text{mA}$$

P 9.31 **[a]** $Y_1 = \dfrac{11}{2500 \times 10^3} = 4.4 \times 10^{-6}\,\text{S}$

$$Y_2 = \frac{1}{14{,}000 + j5\omega}$$

$$= \frac{14{,}000}{196 \times 10^6 + 25\omega^2} - j\frac{5\omega}{196 \times 10^6 + 25\omega^2}$$

$$Y_3 = j\omega 2 \times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For i_g and v_o to be in phase the j component of Y_T must be zero; thus,

$$\omega 2 \times 10^{-9} = \frac{5\omega}{196 \times 10^6 + 25\omega^2}$$

or

$$25\omega^2 + 196 \times 10^6 = \frac{5}{2 \times 10^{-9}}$$

$$\therefore\quad 25\omega^2 = 2304 \times 10^6 \qquad \therefore\quad \omega = 9600\,\text{rad/s}$$

[b] $Y_T = 4.4 \times 10^{-6} + \dfrac{14{,}000}{196 \times 10^6 + 25(9600)^2} = 10 \times 10^{-6}\,\text{S}$

$$\therefore\quad Z_T = 100\,\text{k}\Omega$$

$$\mathbf{V}_o = (0.25 \times 10^{-3}\underline{/0^\circ})(100 \times 10^3) = 25\underline{/0^\circ}\,\text{V}$$

$$v_o = 25\cos 9600t\,\text{V}$$

P 9.32 **[a]** $Z_g = 500 - j\dfrac{10^6}{\omega} + \dfrac{10^3(j0.5\omega)}{10^3 + j0.5\omega}$

$$= 500 - j\frac{10^6}{\omega} + \frac{500j\omega(1000 - j0.5\omega)}{10^6 + 0.25\omega^2}$$

$$= 500 - j\frac{10^6}{\omega} + \frac{250\omega^2}{10^6 + 0.25\omega^2} + j\frac{5\times 10^5\omega}{10^6 + 0.25\omega^2}$$

$\therefore$ If Z_g is purely real, $\dfrac{10^6}{\omega} = \dfrac{5\times 10^5\omega}{10^6 + 0.25\omega^2}$

$2(10^6 + 0.25\omega^2) = \omega^2 \quad \therefore \quad 4\times 10^6 = \omega^2$

$\therefore \quad \omega = 2000$ rad/s

[b] When $\omega = 2000$ rad/s

$Z_g = 500 - j500 + (j1000\|1000) = 1000\,\Omega$

$\therefore \quad \mathbf{I}_g = \dfrac{20\underline{/0^\circ}}{1000} = 20\underline{/0^\circ}$ mA

$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$

$Z_1 = 500 - j500\,\Omega$

$\mathbf{V}_o = 20\underline{/0^\circ} - (0.02\underline{/0^\circ})(500 - j500) = 10 + j10 = 14.14\underline{/45^\circ}$ V

$v_o = 14.14\cos(2000t + 45^\circ)$ V

P 9.33 $Z_{\text{ab}} = 1 - j8 + (2 + j4)\|(10 - j20) + (40\|j20)$

$$= 1 - j8 + 3 + j4 + 8 + j16 = 12 + j12\,\Omega = 16.971\underline{/45^\circ}\ \Omega$$

P 9.34 First find the admittance of the parallel branches

$$Y_p = \frac{1}{2 - j6} + \frac{1}{12 + j4} + \frac{1}{2} + \frac{1}{j0.5} = 0.625 - j1.875\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.625 - j1.875} = 0.16 + j0.48\,\Omega$$

$$Z_{\text{ab}} = -j4.48 + 0.16 + j0.48 + 2.84 = 3 - j4\,\Omega$$

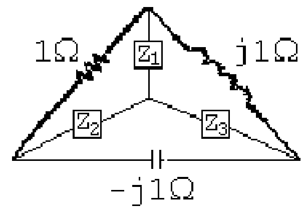
$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{3 - j4} = 120 + j160\,\text{mS}$$

$$= 200\underline{/53.13^\circ}\,\text{mS}$$

P 9.35 Simplify the top triangle using series and parallel combinations:

$(1 + j1)\|(1 - j1) = 1\,\Omega$

Convert the lower left delta to a wye:

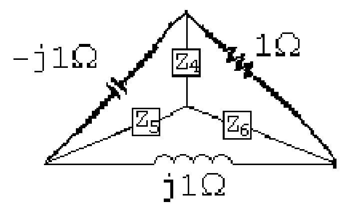


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

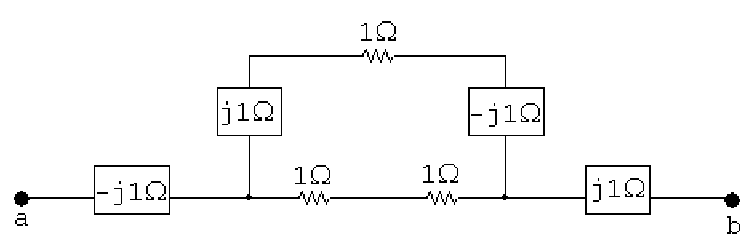


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:



Simplify the middle portion of the circuit by making series and parallel combinations:

$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$

$Z_{\text{ab}} = -j1 + 2/3 + j1 = 2/3\,\Omega$

P 9.36 $\mathbf{V}_o = \mathbf{V}_g \dfrac{Z_o}{Z_T} = \dfrac{500 - j1000}{300 + j1600 + 500 - j1000}(100\underline{/0^\circ}) = 111.8\underline{/-100.3^\circ}\,\text{V}$

$v_o = 111.8\cos(8000t - 100.3^\circ)\,\text{V}$

P 9.37 $\dfrac{1}{j\omega C} = -j400\,\Omega$

$j\omega L = j1200\,\Omega$

Let $Z_1 = 200 - j400\,\Omega$; $\qquad Z_2 = 600 + j1200\,\Omega$

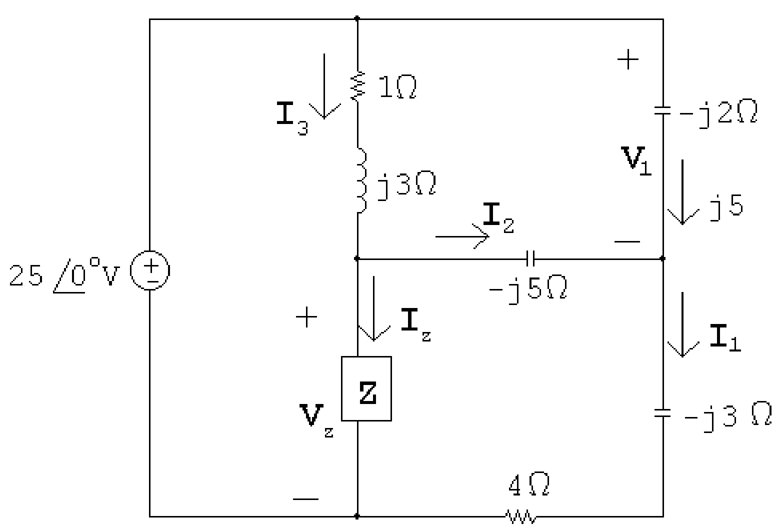
$\mathbf{I}_g = 400\underline{/0^\circ}\,\text{mA}$

$\mathbf{I}_o = \dfrac{Z_2}{Z_1 + Z_2}\mathbf{I}_g = \dfrac{600 + j1200}{800 + j800}(0.4\underline{/0^\circ})$

$= 450 + j150\,\text{mA} = 474.34\underline{/18.43^\circ}\,\text{mA}$

$i_o = 474.34\cos(20{,}000t + 18.43^\circ)\,\text{mA}$

P 9.38



$\mathbf{V}_1 = j5(-j2) = 10\,\text{V}$

$-25 + 10 + (4 - j3)\mathbf{I}_1 = 0 \quad \therefore \quad \mathbf{I}_1 = \dfrac{15}{4 - j3} = 2.4 + j1.8\,\text{A}$

$\mathbf{I}_2 = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\text{A}$

$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\text{V}$$

$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \qquad \therefore \qquad \mathbf{I}_3 = 6.2 - j6.6\,\text{A}$$

$$\mathbf{I}_Z = \mathbf{I}_3 - \mathbf{I}_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4\,\text{A}$$

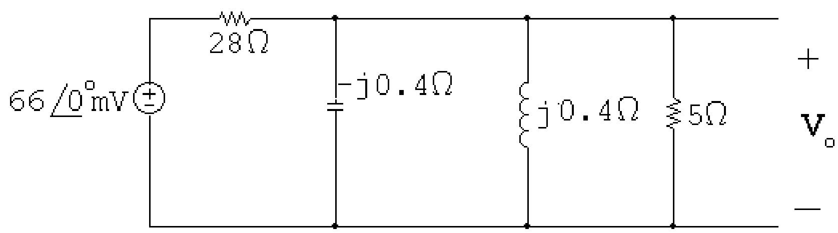
$$Z = \frac{\mathbf{V}_Z}{\mathbf{I}_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88\,\Omega$$

P 9.39 $\mathbf{I}_s = 3\underline{/0^\circ}\,\text{mA}$

$$\frac{1}{j\omega C} = -j0.4\,\Omega$$

$$j\omega L = j0.4\,\Omega$$

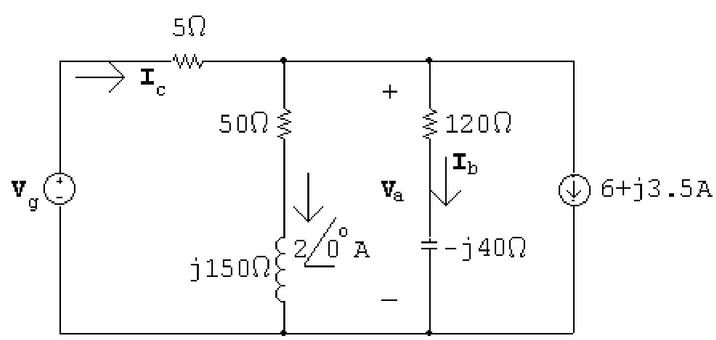
After source transformation we have



$$\mathbf{V}_o = \frac{-j0.4\|j0.4\|5}{28 + -j0.4\|j0.4\|5}(66 \times 10^{-3}) = 10\,\text{mV}$$

$$v_o = 10\cos 200t\,\text{mV}$$

P 9.40 **[a]**



$$\mathbf{V}_\text{a} = (50 + j150)(2\underline{/0^\circ}) = 100 + j300\,\text{V}$$

$$\mathbf{I}_\text{b} = \frac{100 + j300}{120 - j40} = j2.5\,\text{A} = 2.5\underline{/90^\circ}\,\text{A}$$

$$\mathbf{I}_\text{c} = 2\underline{/0^\circ} + j2.5 + 6 + j3.5 = 8 + j6\,\text{A} = 10\underline{/36.87^\circ}\,\text{A}$$

$$\mathbf{V}_g = 5\mathbf{I}_\text{c} + \mathbf{V}_\text{a} = 5(8 + j6) + 100 + j300 = 140 + j330\,\text{V} = 358.47\underline{/67.01^\circ}\,\text{V}$$

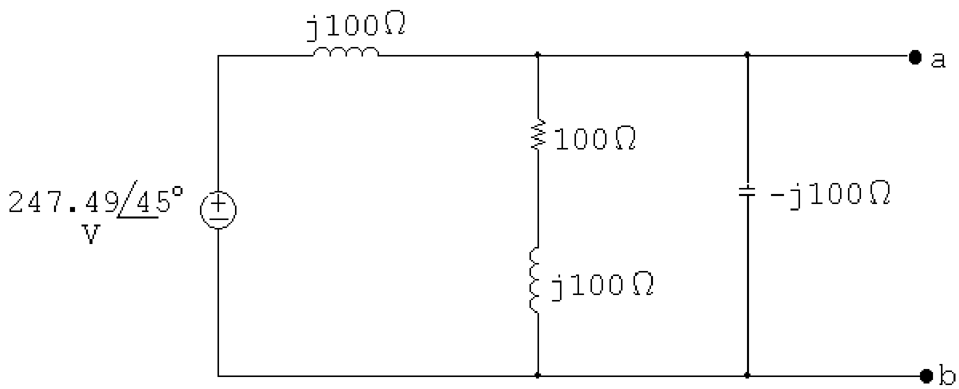
[b] $i_{\rm b} = 2.5\cos(800t + 90^\circ)$ A

$i_{\rm c} = 10\cos(800t + 36.87^\circ)$ A

$v_g = 358.47\cos(800t + 67.01^\circ)$ V

P 9.41 **[a]** $j\omega L = j(1000)(100) \times 10^{-3} = j100\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(1000)(10)} = -j100\,\Omega$$



Using voltage division,

$$\mathbf{V}_{\rm ab} = \frac{(100 + j100)\|(-j100)}{j100 + (100 + j100)\|(-j100)}(247.49\underline{/45^\circ}) = 350\underline{/0^\circ}$$

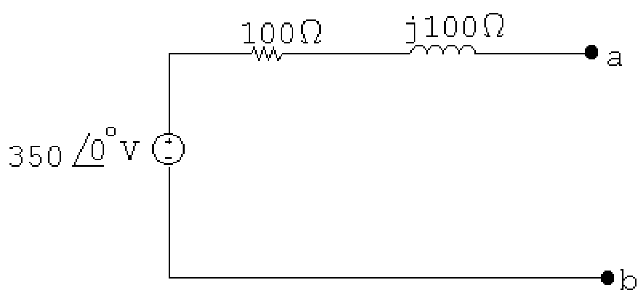
$$\mathbf{V}_{\rm Th} = \mathbf{V}_{\rm ab} = 350\underline{/0^\circ}\text{ V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{\rm Th} = Z_{\rm ab}$:

$$Y_{\rm ab} = \frac{1}{j100} + \frac{1}{100 + j100} + \frac{1}{-j100} = 5 - j5\text{ mS}$$

$$Z_{\rm Th} = Z_{\rm ab} = \frac{1}{Y_{\rm ab}} = 100 + j100\,\Omega$$

[c]



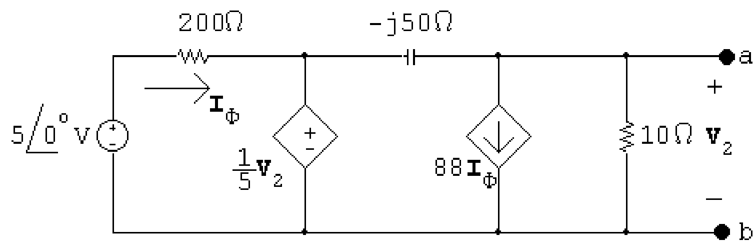
P 9.42 Using voltage division:

$$\mathbf{V}_{\text{Th}} = \frac{36}{36 + j60 - j48}(240) = 216 - j72 = 227.68\underline{/-18.43^\circ}\,\text{V}$$

Remove the source and combine impedances in series and in parallel:

$$Z_{\text{Th}} = 36\|(j60 - j48) = 3.6 + j10.8\,\Omega$$

P 9.43 Open circuit voltage:



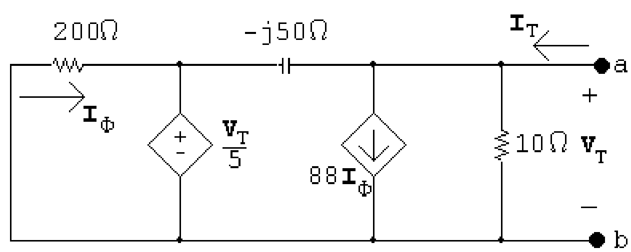
$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\,\text{V} = \mathbf{V}_{\text{Th}}$$

Find the Thévenin equivalent impedance using a test source:



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

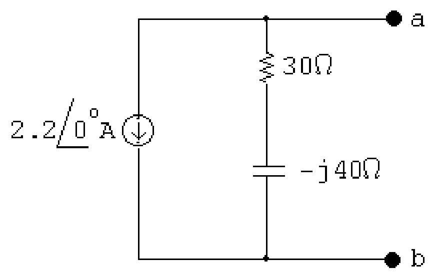
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{1/5}{200} + \frac{0.8}{-j50}\right)$$

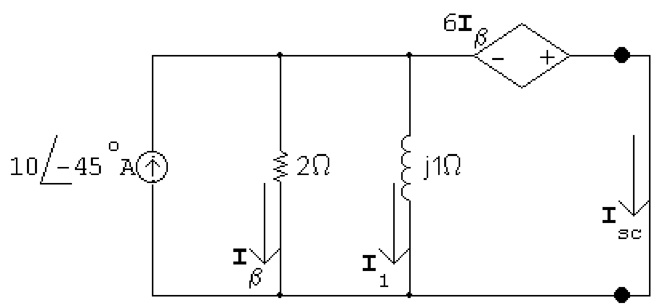
$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\text{Th}}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\text{A} = 2.2\underline{/180^\circ}\ \text{A}$$

The Norton equivalent circuit:



P 9.44 Short circuit current

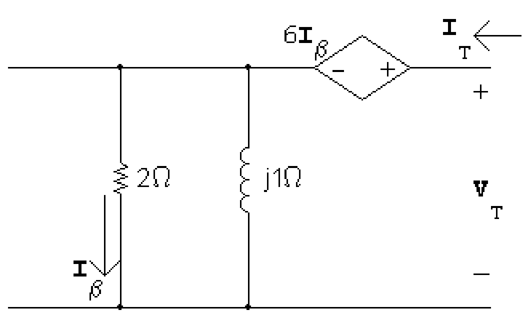


$$\mathbf{I}_\beta = \frac{-6\mathbf{I}_\beta}{2}$$

$$2\mathbf{I}_\beta = -6\mathbf{I}_\beta; \qquad \therefore \quad \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \qquad \therefore \quad \mathbf{I}_{sc} = 10\underline{/-45^\circ}\,\text{A} = \mathbf{I}_N$$

The Norton impedance is the same as the Thévenin impedance. Find it using a test source



$$\mathbf{V}_T = 6\mathbf{I}_\beta + 2\mathbf{I}_\beta = 8\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{j1}{2 + j1}\mathbf{I}_T$$

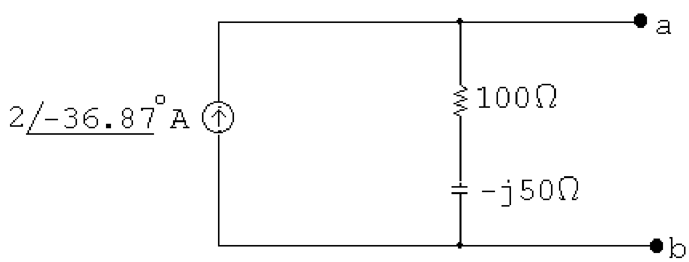
$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{8\mathbf{I}_\beta}{[(2+j1)/j1]\mathbf{I}_\beta} = \frac{j8}{2+j1} = 1.6 + j3.2\,\Omega$$

P 9.45 Using current division:

$$\mathbf{I}_{\mathrm{N}} = \mathbf{I}_{\mathrm{sc}} = \frac{50}{80+j60}(4) = 1.6 - j1.2 = 2\underline{/-36.87^\circ}\,\mathrm{A}$$

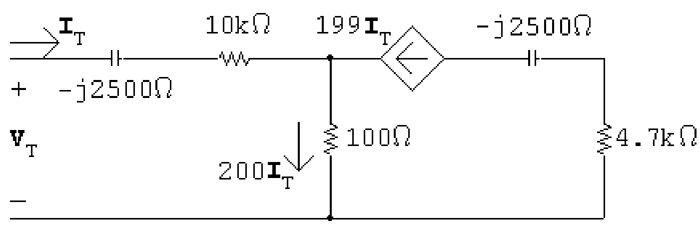
$$Z_{\mathrm{N}} = -j100 \| (80+j60) = 100 - j50\,\Omega$$

The Norton equivalent circuit:



P 9.46 $\omega = 2\pi(200/\pi) = 400$ rad/s

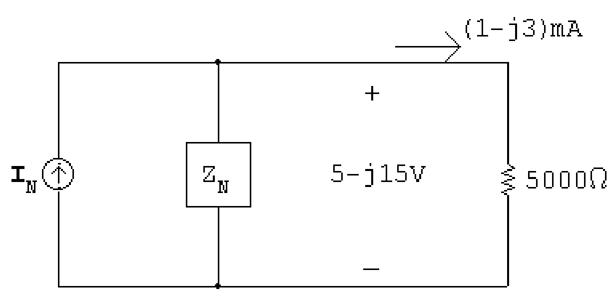
$$Z_c = \frac{-j}{400(10^{-6})} = -j2500\,\Omega$$



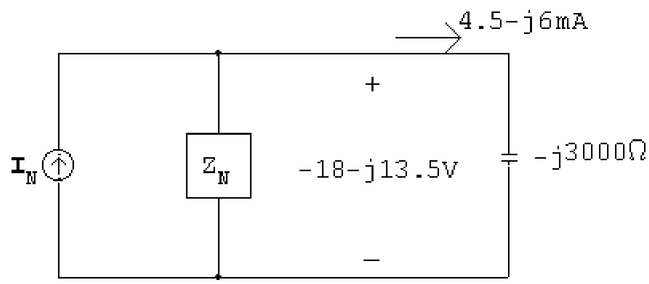
$$\mathbf{V}_T = (10{,}000 - j2500)\mathbf{I}_T + 100(200)\mathbf{I}_T$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j2.5\,\mathrm{k}\Omega$$

P 9.47



$$\mathbf{I}_N = \frac{5-j15}{Z_{\mathrm{N}}} + (1-j3)\,\mathrm{mA}, \quad Z_N \text{ in k}\Omega$$

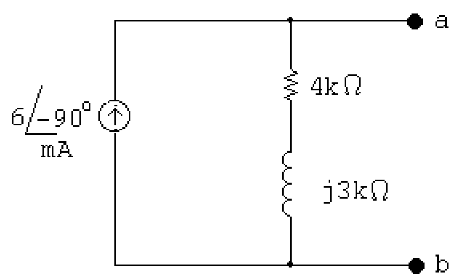


$$\mathbf{I}_N = \frac{-18 - j13.5}{Z_N} + 4.5 - j6\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

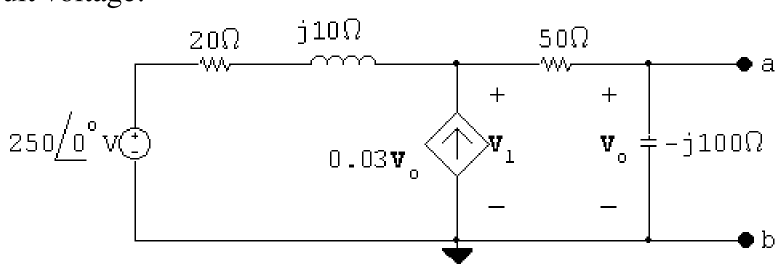
$$\frac{5 - j15}{Z_N} + 1 - j3 = \frac{-18 - j13.5}{Z_N} + (4.5 - j6)$$

$$\frac{23 - j1.5}{Z_N} = 3.5 - j3 \quad \therefore \qquad Z_N = 4 + j3\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{5 - j15}{4 + j3} + 1 - j3 = -j6\,\text{mA} = 6\underline{/-90^\circ}\ \text{mA}$$



P 9.48 Open circuit voltage:



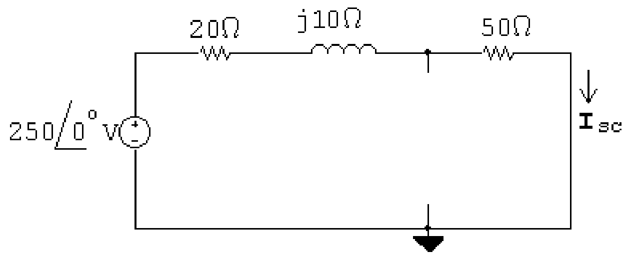
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20+j10}+\frac{j3\mathbf{V}_1}{50-j100}+\frac{\mathbf{V}_1}{50-j100}=\frac{250}{20+j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}} = 500\underline{/-53.13^\circ}\,\text{V}$$

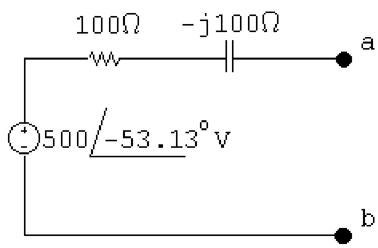
Short circuit current:



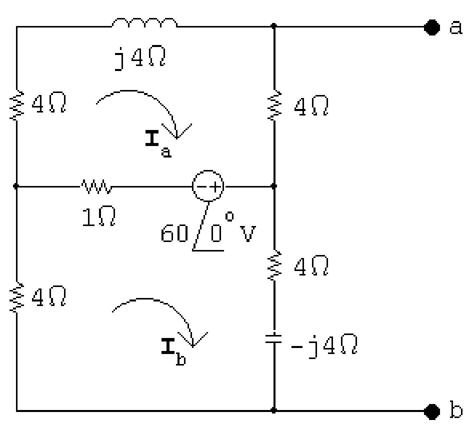
$$\mathbf{I}_{\text{sc}} = \frac{250\underline{/0^\circ}}{70+j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300-j400}{3.5-j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:



P 9.49 Open circuit voltage:



$$(9+j4)\mathbf{I}_{\text{a}} - \mathbf{I}_{\text{b}} = -60\underline{/0^\circ}$$

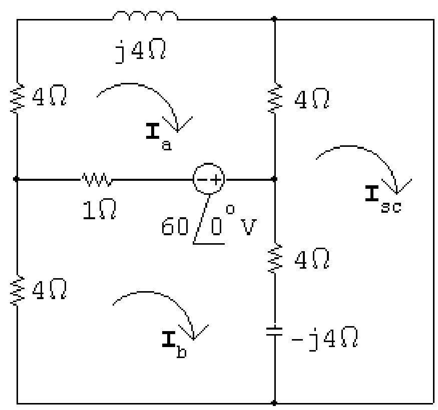
$-\mathbf{I}_\mathrm{a} + (9 - j4)\mathbf{I}_\mathrm{b} = 60\underline{/0^\circ}$

Solving,

$\mathbf{I}_\mathrm{a} = -5 + j2.5\,\mathrm{A}; \qquad \mathbf{I}_\mathrm{b} = 5 + j2.5\,\mathrm{A}$

$\mathbf{V}_\mathrm{Th} = 4\mathbf{I}_\mathrm{a} + (4 - j4)\mathbf{I}_\mathrm{b} = 10\underline{/0^\circ}\,\mathrm{V}$

Short circuit current:



$(9 + j4)\mathbf{I}_\mathrm{a} - 1\mathbf{I}_\mathrm{b} - 4\mathbf{I}_\mathrm{sc} = -60$

$-1\mathbf{I}_\mathrm{a} + (9 - j4)\mathbf{I}_\mathrm{b} - (4 - j4)\mathbf{I}_\mathrm{sc} = 60$

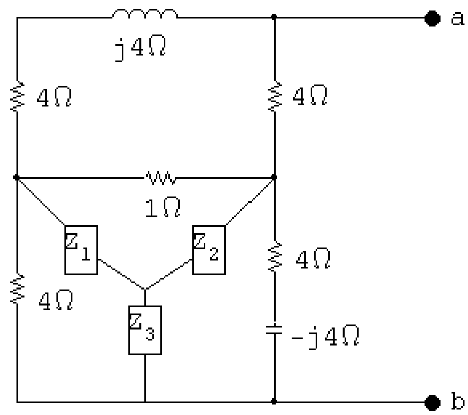
$-4\mathbf{I}_\mathrm{a} - (4 - j4)\mathbf{I}_\mathrm{b} + (8 - j4)\mathbf{I}_\mathrm{sc} = 0$

Solving,

$\mathbf{I}_\mathrm{sc} = 2.07\underline{/0^\circ}$

$$Z_\mathrm{Th} = \frac{\mathbf{V}_\mathrm{Th}}{\mathbf{I}_\mathrm{sc}} = \frac{10\underline{/0^\circ}}{2.07\underline{/0^\circ}} = 4.83\,\Omega$$

Alternate calculation for Z_{Th}:

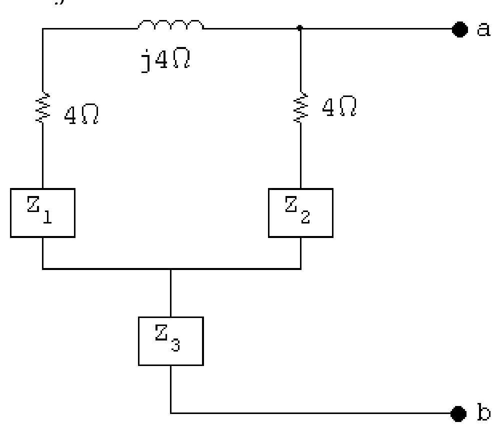


$$\sum Z = 4 + 1 + 4 - j4 = 9 - j4$$

$$Z_1 = \frac{4}{9 - j4}$$

$$Z_2 = \frac{4 - j4}{9 - j4}$$

$$Z_3 = \frac{16 - j16}{9 - j4}$$



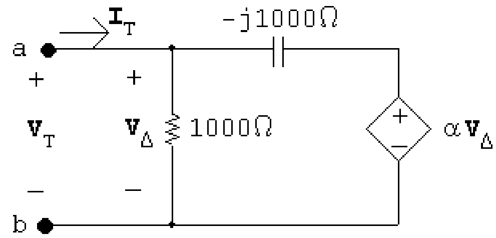
$$Z_{\mathrm{a}} = 4 + j4 + \frac{4}{9 - j4} = \frac{56 + j20}{9 - j4}$$

$$Z_{\mathrm{b}} = 4 + \frac{4 - j4}{9 - j4} = \frac{40 - j20}{9 - j4}$$

$$Z_{\mathrm{a}} \| Z_{\mathrm{b}} = \frac{2640 - j320}{864 - j384}$$

$$Z_3 + Z_{\mathrm{a}} \| Z_{\mathrm{b}} = \frac{16 - j16}{9 - j4} + \frac{2640 - j320}{864 - j384} = \frac{4176 - j1856}{864 - j384} = 4.83\,\Omega$$

P 9.50 **[a]**



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T}{-j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} - \frac{(1-\alpha)}{j1000} = \frac{j-1+\alpha}{j1000}$$

$$\therefore \quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{j1000}{\alpha - 1 + j}$$

Z_{Th} is real when $\alpha = 1$.

[b] $Z_{\text{Th}} = 1000\,\Omega$

[c] $Z_{\text{Th}} = 500 - j500 = \dfrac{j1000}{\alpha - 1 + j}$

$$= \frac{1000}{(\alpha-1)^2+1} + j\frac{1000(\alpha-1)}{(\alpha-1)^2+1}$$

Equate the real parts:

$$\frac{1000}{(\alpha-1)^2+1} = 500 \quad \therefore \quad (\alpha-1)^2 + 1 = 2$$

$$\therefore \quad (\alpha-1)^2 = 1 \quad \text{so} \quad \alpha = 0$$

Check the imaginary parts:

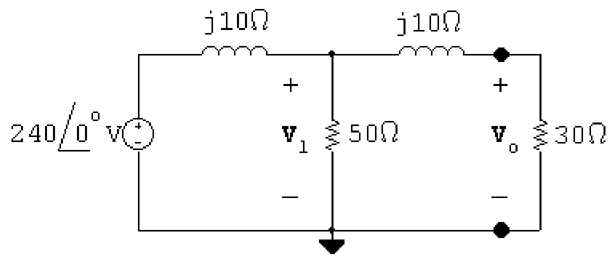
$$\left.\frac{(\alpha-1)1000}{(\alpha-1)^2+1}\right|_{\alpha=1} = -500$$

Thus, $\alpha = 0$.

[d] $Z_{\text{Th}} = \dfrac{1000}{(\alpha-1)^2+1} + j\dfrac{1000(\alpha-1)}{(\alpha-1)^2+1}$

For $\mathbf{Im}(Z_{\text{Th}}) > 0$, α must be greater than 1. So Z_{Th} is inductive for $1 < \alpha \leq 10$.

P 9.51



$$\frac{\mathbf{V}_1 - 240}{j10} + \frac{\mathbf{V}_1}{50} + \frac{\mathbf{V}_1}{30 + j10} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 198.63\underline{/-24.44^\circ}\,\text{V}$$

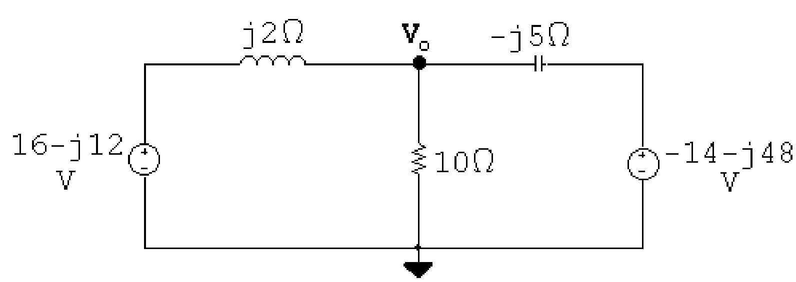
$$\mathbf{V}_o = \frac{30}{30 + j10}(\mathbf{V}_1) = 188.43\underline{/-42.88^\circ}\,\text{V}$$

P 9.52 $j\omega L = j(2000)(1 \times 10^{-3}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(2000)(100)} = -j5\,\Omega$$

$$\mathbf{V}_{g1} = 20\underline{/-36.87^\circ} = 16 - j12\,\text{V}$$

$$\mathbf{V}_{g2} = 50\underline{/-106.26^\circ} = -14 - j48\,\text{V}$$



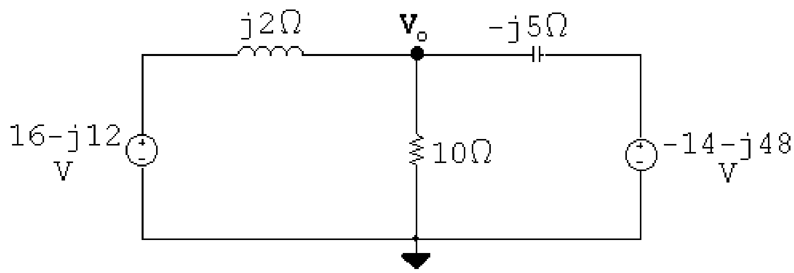
$$\frac{\mathbf{V}_o - (16 - j12)}{j2} + \frac{\mathbf{V}_o}{10} + \frac{\mathbf{V}_o - (-14 - j48)}{-j5} = 0$$

Solving,

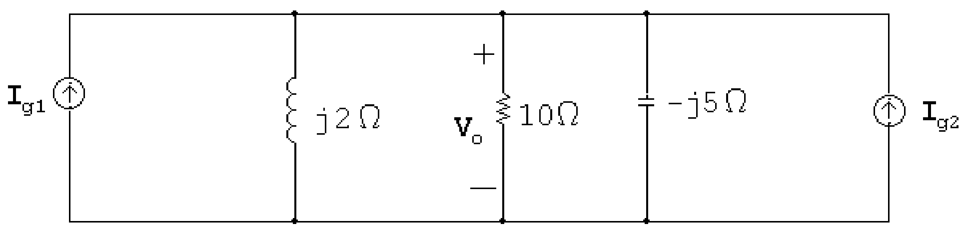
$$\mathbf{V}_o = 36\underline{/0^\circ}\,\text{V}$$

$$v_o(t) = 36\cos 2000t\,\text{V}$$

P 9.53 From the solution to Problem 9.52 the phasor-domain circuit is



Making two source transformations yields



$$\mathbf{I}_{g1} = \frac{16 - j12}{j2} = -6 - j8\,\mathrm{A}$$

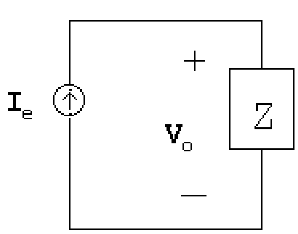
$$\mathbf{I}_{g2} = \frac{-14 - j48}{-j5} = 9.6 - j2.8\,\mathrm{A}$$

$$Y = \frac{1}{j2} + \frac{1}{10} + \frac{1}{-j5} = (0.1 - j0.3)\,\mathrm{S}$$

$$Z = \frac{1}{Y} = 1 + j3\,\Omega$$

$$\mathbf{I}_e = \mathbf{I}_{g1} + \mathbf{I}_{g2} = 3.6 - j10.8\,\mathrm{A}$$

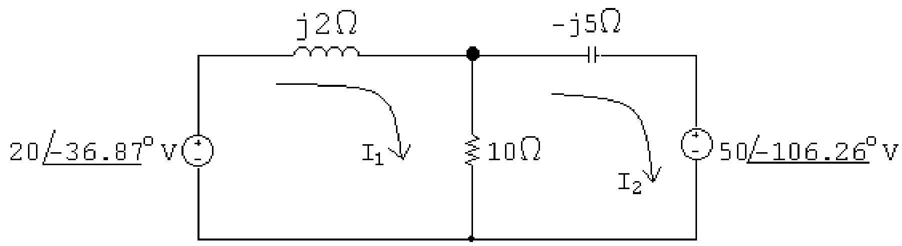
Hence the circuit reduces to



$$\mathbf{V}_o = Z\mathbf{I}_e = (1 + j3)(3.6 - j10.8) = 36\underline{/0^\circ}\ \mathrm{V}$$

$$\therefore \quad v_o(t) = 36\cos 2000t\,\mathrm{V}$$

P 9.54 The circuit with the mesh currents identified is shown below:



The mesh current equations are:

$$-20\underline{/-36.87^\circ} + j2\mathbf{I}_1 + 10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$50\underline{/-106.26^\circ} + 10(\mathbf{I}_2 - \mathbf{I}_1) - j5\mathbf{I}_2 = 0$$

In standard form:

$$\mathbf{I}_1(10 + j2) + \mathbf{I}_2(-10) = 20\underline{/-36.87^\circ}$$

$$\mathbf{I}_1(-10) + \mathbf{I}_2(10 - j5) = -50\underline{/-106.26^\circ} = 50\underline{/73.74^\circ}$$

Solving on a calculator yields:

$\mathbf{I}_1 = -6 + j10\text{A};\qquad \mathbf{I}_2 = -9.6 + j10\text{A}$

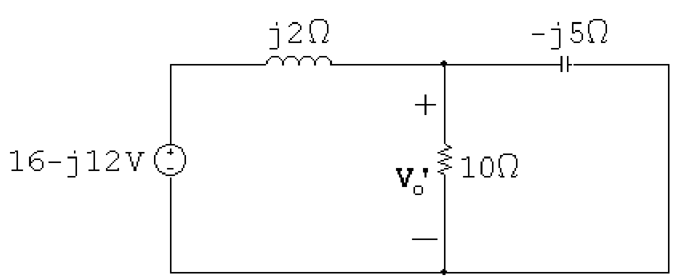
Thus,

$$\mathbf{V}_o = 10(\mathbf{I}_1 - \mathbf{I}_2) = 36\text{V}$$

and

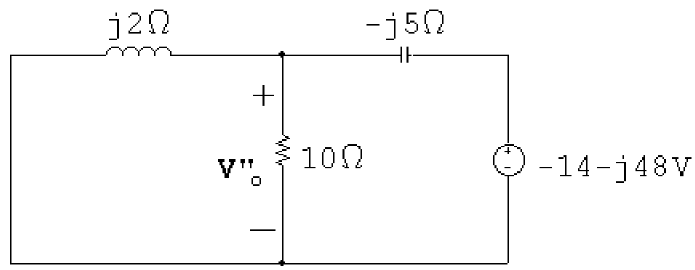
$$v_o(t) = 36\cos 2000t\text{V}$$

P 9.55 From the solution to Problem 9.52 the phasor-domain circuit with the right-hand source removed is



$$\mathbf{V}_o' = \frac{10\| - j5}{j2 + 10\| - j5}(16 - j12) = 18 - j26\,\text{V}$$

With the left hand source removed



$$\mathbf{V}_o'' = \frac{10\|j2}{-j5+10\|j2}(-14-j48) = 18+j26\,\text{V}$$

$$\mathbf{V}_o = \mathbf{V}_o' + \mathbf{V}_o'' = 18 - j26 + 18 + j26 = 36\,\text{V}$$

$$v_o(t) = 36\cos 2000t\,\text{V}$$

P 9.56 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10+j10) = 0$$

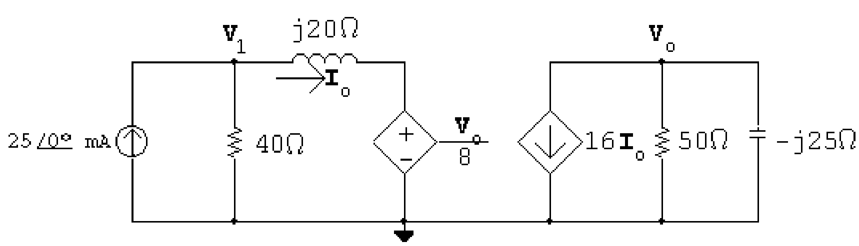
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\,\text{V}$$

P 9.57



Write node voltage equations:

Left Node:

$$\frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1 - \mathbf{V}_o/8}{j20} = 0.025\underline{/0^\circ}$$

Right Node:

$$\frac{\mathbf{V}_o}{50} + \frac{\mathbf{V}_o}{j25} + 16\mathbf{I}_o = 0$$

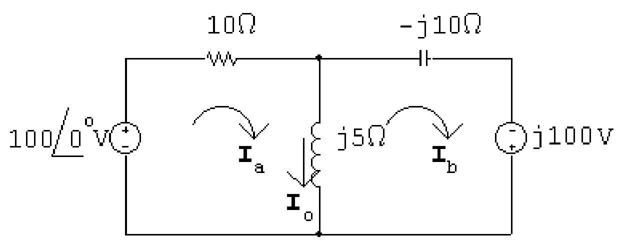
The constraint equation is

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_o/8}{j20}$$

Solution:

$$\mathbf{V}_o = (4 + j4) = 5.66\underline{/45^\circ}\text{ V}$$
$$\mathbf{V}_1 = (0.8 + j0.6) = 1.0\underline{/36.87^\circ}\text{ V}$$
$$\mathbf{I}_o = (5 - j15) = 15.81\underline{/-71.57^\circ}\text{ mA}$$

P 9.58



$$(10 + j5)\mathbf{I}_\text{a} - j5\mathbf{I}_\text{b} = 100\underline{/0^\circ}$$

$$-j5\mathbf{I}_\text{a} - j5\mathbf{I}_\text{b} = j100$$

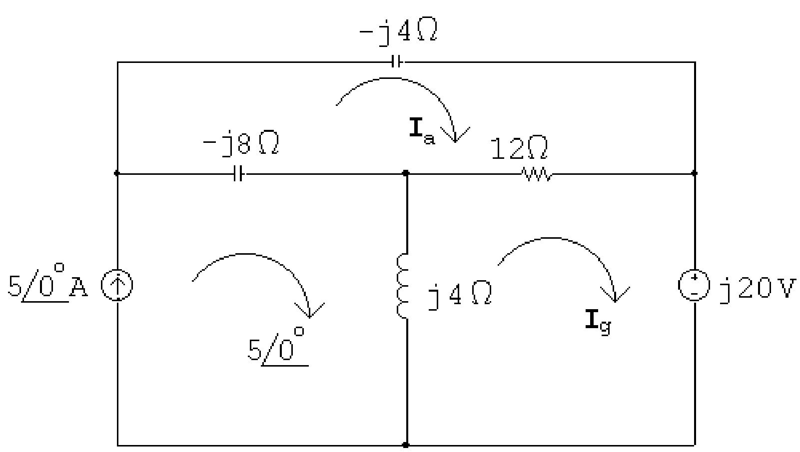
Solving,

$$\mathbf{I}_\text{a} = -j10\text{ A};\qquad \mathbf{I}_\text{b} = -20 + j10\text{ A}$$

$$\mathbf{I}_o = \mathbf{I}_\text{a} - \mathbf{I}_\text{b} = 20 - j20 = 28.28\underline{/-45^\circ}\text{ A}$$

$$i_o(t) = 28.28\cos(50{,}000t - 45^\circ)\text{ A}$$

P 9.59



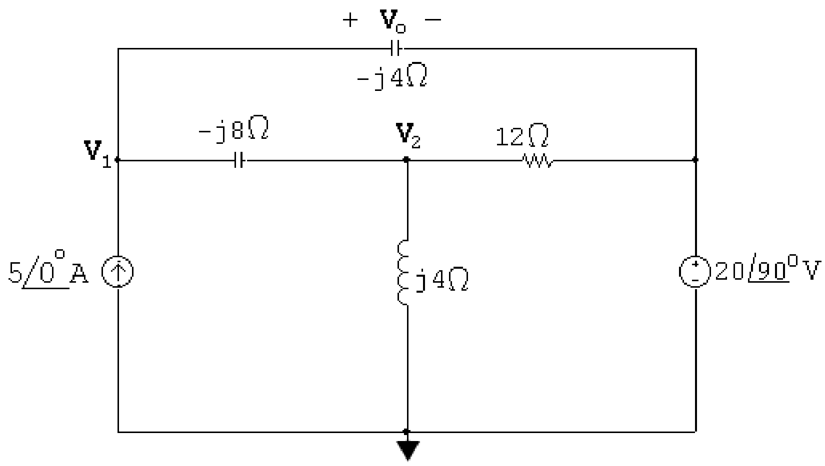
$$(12 - j12)\mathbf{I}_\text{a} - 12\mathbf{I}_g - 5(-j8) = 0$$

$$-12\mathbf{I}_a + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$$

Solving,

$$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}\,\text{A}$$

P 9.60 Set up the frequency domain circuit to use the node voltage method:



At $\mathbf{V}_1$: $$-5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$$

At $\mathbf{V}_2$: $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$$

In standard form:

$$\mathbf{V}_1\left(\frac{1}{-j8} + \frac{1}{-j4}\right) + \mathbf{V}_2\left(-\frac{1}{-j8}\right) = 5\underline{/0^\circ} + \frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right) + \mathbf{V}_2\left(\frac{1}{-j8} + \frac{1}{j4} + \frac{1}{12}\right) = \frac{20\underline{/90^\circ}}{12}$$

Solving on a calculator:

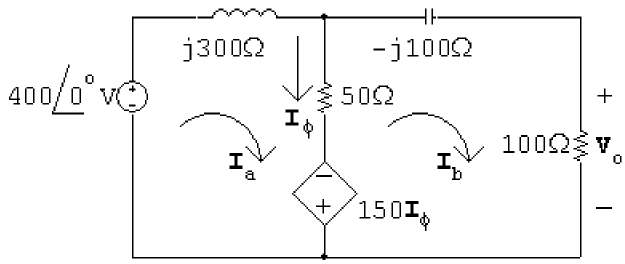
$$\mathbf{V}_1 = -\frac{8}{3} + j\frac{4}{3}\,\text{V} \qquad \mathbf{V}_2 = -8 + j4\,\text{V}$$

Thus

$$\mathbf{V}_0 = \mathbf{V}_1 - 20\underline{/90^\circ} = -\frac{8}{3} - j\frac{56}{3} = 18.86\underline{/-98.13^\circ}\,\text{V}$$

P 9.61 $j\omega L = j5000(60 \times 10^{-3}) = j300\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(2 \times 10^{-6})} = -j100\,\Omega$$



$$-400\underline{/0^\circ} + (50 + j300)\mathbf{I}_\mathrm{a} - 50\mathbf{I}_\mathrm{b} - 150(\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b}) = 0$$

$$(150 - j100)\mathbf{I}_\mathrm{b} - 50\mathbf{I}_\mathrm{a} + 150(\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b}) = 0$$

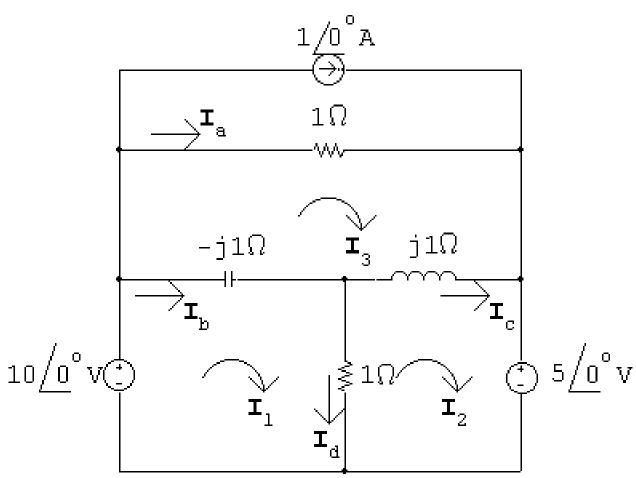
Solving,

$\mathbf{I}_\mathrm{a} = -0.8 - j1.6\,\mathrm{A}$; $\quad \mathbf{I}_\mathrm{b} = -1.6 + j0.8\,\mathrm{A}$

$$\mathbf{V}_o = 100\mathbf{I}_\mathrm{b} = -160 + j80 = 178.89\underline{/153.43^\circ}\ \mathrm{V}$$

$$v_o = 178.89\cos(5000t + 153.43^\circ)\,\mathrm{V}$$

P 9.62



$$10\underline{/0^\circ} = (1 - j1)\mathbf{I}_1 - 1\mathbf{I}_2 + j1\mathbf{I}_3$$

$$-5\underline{/0^\circ} = -1\mathbf{I}_1 + (1 + j1)\mathbf{I}_2 - j1\mathbf{I}_3$$

$$1 = j1\mathbf{I}_1 - j1\mathbf{I}_2 + \mathbf{I}_3$$

Solving,

$\mathbf{I}_1 = 11 + j10\,\text{A};\qquad \mathbf{I}_2 = 11 + j5\,\text{A};\qquad \mathbf{I}_3 = 6\,\text{A}$

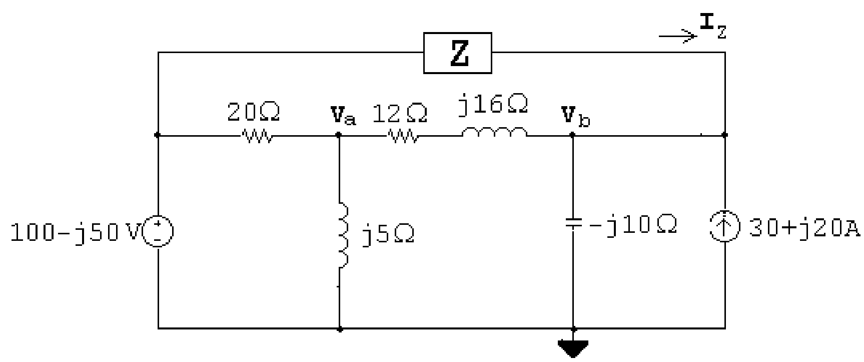
$$\mathbf{I}_\text{a} = \mathbf{I}_3 - 1 = 5\,\text{A} = 5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_3 = 5 + j10\,\text{A} = 11.18\underline{/63.43^\circ}\,\text{A}$$

$$\mathbf{I}_\text{c} = \mathbf{I}_2 - \mathbf{I}_3 = 5 + j5\,\text{A} = 7.07\underline{/45^\circ}\,\text{A}$$

$$\mathbf{I}_\text{d} = \mathbf{I}_1 - \mathbf{I}_2 = j5\,\text{A} = 5\underline{/90^\circ}\,\text{A}$$

P 9.63



$$\frac{\mathbf{V}_\text{a} - (100 - j50)}{20} + \frac{\mathbf{V}_\text{a}}{j5} + \frac{\mathbf{V}_\text{a} - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{V}_\text{a} = 40 + j30\,\text{V}$$

$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{I}_Z = -30 - j10\,\text{A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$

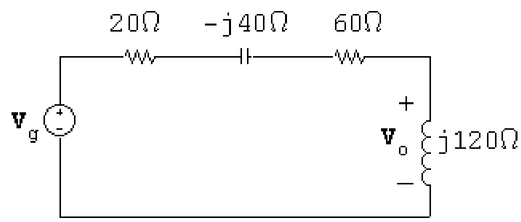
P 9.64 **[a]** $\frac{1}{j\omega C} = -j50\,\Omega$

$j\omega L = j120\,\Omega$

$Z_e = 100\| - j50 = 20 - j40\,\Omega$

$\mathbf{I}_g = 2\underline{/0^\circ}$

$\mathbf{V}_g = \mathbf{I}_g Z_e = 2(20 - j40) = 40 - j80\,\mathrm{V}$



$\mathbf{V}_o = \frac{j120}{80 + j80}(40 - j80) = 90 - j30 = 94.87\underline{/-18.43^\circ}\,\mathrm{V}$

$v_o = 94.87\cos(16\times 10^5 t - 18.43^\circ)\,\mathrm{V}$

[b] $\omega = 2\pi f = 16\times 10^5; \qquad f = \frac{8\times 10^5}{\pi}$

$T = \frac{1}{f} = \frac{\pi}{8\times 10^5} = 1.25\pi\,\mu\mathrm{s}$

$\therefore \quad \frac{18.43}{360}(1.25\pi\,\mu\mathrm{s}) = 201.09\,\mathrm{ns}$

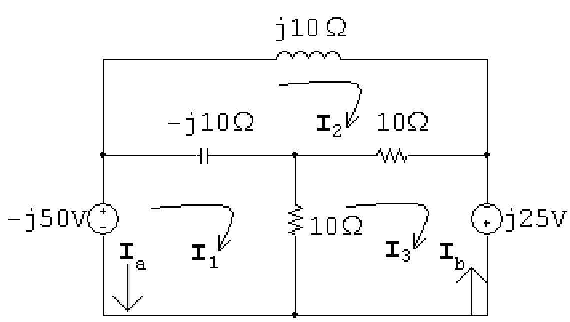
$\therefore \quad v_o$ lags i_g by 201.09 ns

P 9.65 $j\omega L = j10^6(10\times 10^{-6}) = j10\,\Omega$

$\frac{1}{j\omega C} = \frac{-j}{(10^6)(0.1\times 10^{-6})} = -j10\,\Omega$

$\mathbf{V}_\mathrm{a} = 50\underline{/-90^\circ} = -j50\,\mathrm{V}$

$\mathbf{V}_\mathrm{b} = 25\underline{/90^\circ} = j25\,\mathrm{V}$



$(10 - j10)\mathbf{I}_1 + j10\mathbf{I}_2 - 10\mathbf{I}_3 = -j50$

$j10\mathbf{I}_1 + 10\mathbf{I}_2 - 10\mathbf{I}_3 = 0$

$-10\mathbf{I}_1 - 10\mathbf{I}_2 + 20\mathbf{I}_3 = j25$

Solving,

$\mathbf{I}_1 = 0.5 - j1.5\,\text{A}; \qquad \mathbf{I}_3 = -1 + j0.5\,\text{A} \qquad \mathbf{I}_2 = -2.5\ \text{A}$

$\mathbf{I}_\text{a} = -\mathbf{I}_1 = -0.5 + j1.5 = 1.58\underline{/108.43^\circ}\,\text{A}$

$\mathbf{I}_\text{b} = -\mathbf{I}_3 = 1 - j0.5 = 1.12\underline{/-26.57^\circ}\,\text{A}$

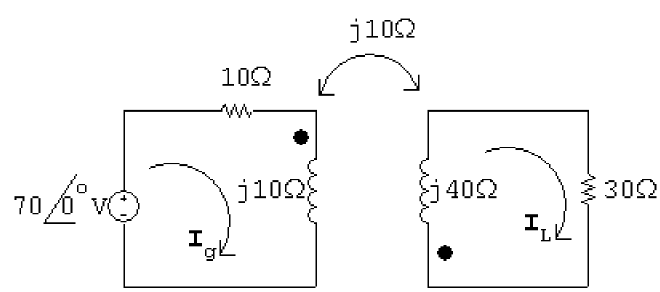
$i_\text{a} = 1.58\cos(10^6 t + 108.43^\circ)\,\text{A}$

$i_\text{b} = 1.12\cos(10^6 t - 26.57^\circ)\,\text{A}$

P 9.66 **[a]** $j\omega L_1 = j(5000)(2\times 10^{-3}) = j10\,\Omega$

$j\omega L_2 = j(5000)(8\times 10^{-3}) = j40\,\Omega$

$j\omega M = j10\,\Omega$



$70 = (10 + j10)\mathbf{I}_g + j10\mathbf{I}_L$

$0 = j10\mathbf{I}_g + (30 + j40)\mathbf{I}_L$

Solving,

$\mathbf{I}_g = 4 - j3\,\text{A}; \qquad \mathbf{I}_L = -1\,\text{A}$

$i_g = 5\cos(5000t - 36.87^\circ)\,\text{A}$

$i_\text{L} = 1\cos(5000t - 180^\circ)\,\text{A}$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{2}{\sqrt{16}} = 0.5$

[c] When $t = 100\pi\ \mu$s,

$$5000t = (5000)(100\pi) \times 10^{-6} = 0.5\pi = \pi/2 \text{ rad} = 90^\circ$$

$$i_g(100\pi\mu s) = 5\cos(53.13^\circ) = 3\text{ A}$$

$$i_L(100\pi\mu s) = 1\cos(-90^\circ) = 0\text{ A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(2\times 10^{-3})(9) + 0 + 0 = 9\text{ mJ}$$

When $t = 200\pi\ \mu$s,

$$5000t = \pi \text{ rad} = 180^\circ$$

$$i_g(200\pi\mu s) = 5\cos(180^\circ - 36.87^\circ) = -4\text{ A}$$

$$i_L(200\pi\mu s) = 1\cos(180^\circ - 180^\circ) = 1\text{ A}$$

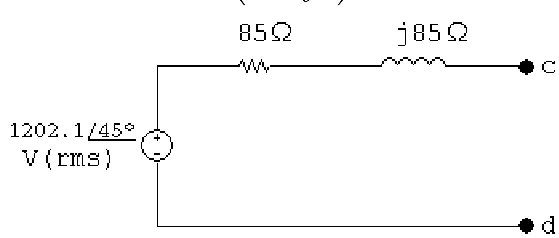
$$w = \frac{1}{2}(2\times 10^{-3})(16) + \frac{1}{2}(8\times 10^{-3})(1) + 2\times 10^{-3}(-4)(1) = 12\text{ mJ}$$

P 9.67 Remove the voltage source to find the equivalent impedance:

$$Z_{\text{Th}} = 45 + j125 + \left(\frac{20}{|5+j5|}\right)^2 (5 - j5) = 85 + j85\,\Omega$$

Using voltage division:

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{cd}} = j20\mathbf{I}_1 = j20\left(\frac{425}{5+j5}\right) = 850 + j850\text{ V} = 1202.1\underline{/45^\circ}\text{ V}$$



P 9.68 **[a]** $j\omega L_1 = j(200\times 10^3)(10^{-3}) = j200\,\Omega$

$$j\omega L_2 = j(200\times 10^3)(4\times 10^{-3}) = j800\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(200\times 10^3)(12.5\times 10^{-9})} = -j400\,\Omega$$

$$\therefore\quad Z_{22} = 100 + 200 + j800 - j400 = 300 + j400\,\Omega$$

$$\therefore\quad Z_{22}^* = 300 - j400\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 2k \times 10^{-3}$$

$$\omega M = (200 \times 10^3)(2k \times 10^{-3}) = 400k$$

$$Z_r = \left[\frac{400k}{500}\right]^2 (300 - j400) = k^2(192 - j256)\,\Omega$$

$$Z_{\text{in}} = 200 + j200 + 192k^2 - j256k^2$$

$$|Z_{\text{in}}| = [(200 + 192k^2)^2 + (200 - 256k^2)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\text{in}}|}{dk} = \frac{1}{2}[(200 + 192k^2)^2 + (200 - 256k^2)^2]^{-\frac{1}{2}} \times$$

$$[2(200 + 192k^2)384k + 2(200 - 256k^2)(-512k)]$$

$$\frac{d|Z_{\text{in}}|}{dk} = 0 \text{ when}$$

$$768k(200 + 192k^2) - 1024k(200 - 256k^2) = 0$$

$$\therefore \quad k^2 = 0.125; \qquad \therefore \quad k = \sqrt{0.125} = 0.3536$$

[b] Z_{in} (min) $= 200 + 192(0.125) + j[200 - 0.125(256)]$

$$= 224 + j168 = 280\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1 \text{ (max)} = \frac{560\underline{/0^\circ}}{224 + j168} = 2\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore \quad i_1 \text{ (peak)} = 2\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{\text{in}}|/dk = 0$ leads to a minimum by noting $0 \leq k \leq 1$. If $k = 1$,

$$Z_{\text{in}} = 392 - j56 = 395.98\underline{/-8.13^\circ}\,\Omega$$

Thus,

$$|Z_{\text{in}}|_{k=1} > |Z_{\text{in}}|_{k=\sqrt{0.125}}$$

If $k = 0$,

$$Z_{\text{in}} = 200 + j200 = 282.84\underline{/45^\circ}\,\Omega$$

Thus,

$$|Z_{\text{in}}|_{k=0} > |Z_{\text{in}}|_{k=\sqrt{0.125}}$$

P 9.69 $j\omega L_1 = j50\,\Omega$

$$j\omega L_2 = j32\,\Omega$$

$$\frac{1}{j\omega C} = -j20\,\Omega$$

$$j\omega M = j(4 \times 10^3)k\sqrt{(12.5)(8)} \times 10^{-3} = j40k\,\Omega$$

$$Z_{22} = 5 + j32 - j20 = 5 + j12\,\Omega$$

$$Z_{22}^* = 5 - j12\,\Omega$$

$$Z_r = \left[\frac{40k}{|5 + j12|}\right]^2 (5 - j12) = 47.337k^2 - j113.609k^2$$

$$Z_{\text{ab}} = 20 + j50 + 47.337k^2 - j113.609k^2 = (20 + 47.337k^2) + j(50 - 113.609k^2)$$

Z_{ab} is resistive when

$50 - 113.609k^2 = 0$ or $k^2 = 0.44$ so $k = 0.66$

$\therefore \quad Z_{\text{ab}} = 20 + (47.337)(0.44) = 40.83\,\Omega$

P 9.70 **[a]** $j\omega L_L = j100\,\Omega$

$$j\omega L_2 = j500\,\Omega$$

$$Z_{22} = 300 + 500 + j100 + j500 = 800 + j600\,\Omega$$

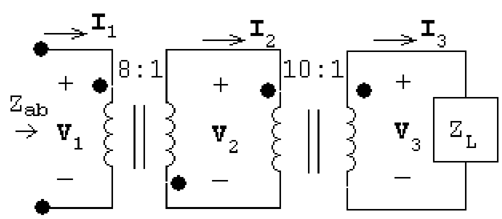
$$Z_{22}^* = 800 - j600\,\Omega$$

$$\omega M = 270\,\Omega$$

$$Z_r = \left(\frac{270}{1000}\right)^2 [800 - j600] = 58.32 - j43.74\,\Omega$$

[b] $Z_{\text{ab}} = R_1 + j\omega L_1 + Z_r = 41.68 + j180 + 58.32 - j43.74 = 100 + j136.26\,\Omega$

P 9.71



$$Z_L = \frac{\mathbf{V}_3}{\mathbf{I}_3} = 80\underline{/60^\circ}\,\Omega$$

$$\frac{\mathbf{V}_2}{10} = \frac{\mathbf{V}_3}{1}; \qquad 10\mathbf{I}_2 = 1\mathbf{I}_3$$

$$\frac{\mathbf{V}_1}{8} = -\frac{\mathbf{V}_2}{1}; \qquad 8\mathbf{I}_1 = -1\mathbf{I}_2$$

$$Z_{\rm ab} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

Substituting,

$$Z_{\rm ab} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{-8\mathbf{V}_2}{-\mathbf{I}_2/8} = \frac{8^2\mathbf{V}_2}{\mathbf{I}_2}$$

$$= \frac{8^2(10\mathbf{V}_3)}{\mathbf{I}_3/10} = \frac{(8)^2(10)^2\mathbf{V}_3}{\mathbf{I}_3} = (8)^2(10)^2 Z_L = 512,000\underline{/60^\circ}\ \Omega$$

P 9.72 In Eq. 9.69 replace $\omega^2 M^2$ with $k^2\omega^2 L_1 L_2$ and then write $X_{\rm ab}$ as

$$X_{\rm ab} = \omega L_1 - \frac{k^2\omega^2 L_1 L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}$$

$$= \omega L_1 \left\{1 - \frac{k^2\omega L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}\right\}$$

For $X_{\rm ab}$ to be negative requires

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2\omega L_2(\omega L_2 + \omega L_L)$$

or

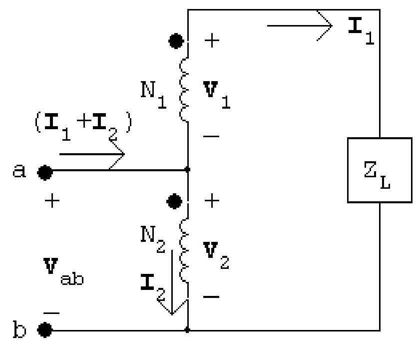
$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2\omega L_2(\omega L_2 + \omega L_L) < 0$$

which reduces to

$$R_{22}^2 + \omega^2 L_2^2(1 - k^2) + \omega L_2 \omega L_L(2 - k^2) + \omega^2 L_L^2 < 0$$

But $k \leq 1$ hence it is impossible to satisfy the inequality. Therefore $X_{\rm ab}$ can never be negative if X_L is an inductive reactance.

P 9.73 **[a]**



$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \qquad \mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2$$

$$Z_{ab} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore \quad Z_{ab} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[b] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2}\mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2}\mathbf{I}_1$$

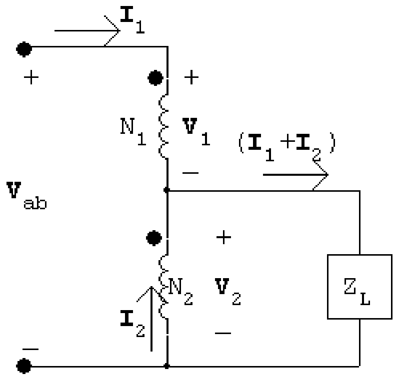
As before

$$Z_{ab} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{ab} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L\mathbf{I}_1}{[1 - (N_1/N_2)]^2\mathbf{I}_1}$$

$$Z_{ab} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.74 **[a]**



$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)Z_L = \mathbf{I}_1\left(1 + \frac{N_1}{N_2}\right)Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2 = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{(1 + N_1/N_2)^2 Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L \quad \text{Q.E.D.}$$

[b] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_1 = \frac{-N_1}{N_2}\mathbf{V}_2$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{-N_1}{N_2}\mathbf{I}_1$$

As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1)Z_L \quad \text{and} \quad Z_{\text{ab}} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{(1 - N_1/N_2)\mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2)Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = [1 - (N_1/N_2)]^2 Z_L \quad \text{Q.E.D.}$$

P 9.75 **[a]** $\mathbf{I} = \dfrac{240}{24} + \dfrac{240}{j32} = (10 - j7.5)\,\text{A}$

$\mathbf{V}_s = 240\underline{/0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11\underline{/1.68^\circ}\,\text{V}$

[b] Use the capacitor to eliminate the j component of $\mathbf{I}$, therefore

$\mathbf{I}_c = j7.5\,\text{A}, \qquad Z_c = \dfrac{240}{j7.5} = -j32\,\Omega$

$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13\underline{/1.90^\circ}\,\text{V}$

[c] Let $\mathbf{I}_c$ denote the magnitude of the current in the capacitor branch. Then

$\mathbf{I} = (10 - j7.5 + j\mathbf{I}_c) = 10 + j(\mathbf{I}_c - 7.5)\,\text{A}$

$$\begin{aligned}\mathbf{V}_s &= 240\underline{/\alpha} = 240 + (0.1 + j0.8)[10 + j(\mathbf{I}_c - 7.5)]\\ &= (247 - 0.8\mathbf{I}_c) + j(7.25 + 0.1\mathbf{I}_c)\end{aligned}$$

It follows that

$240\cos\alpha = (247 - 0.8\mathbf{I}_c) \quad \text{and} \quad 240\sin\alpha = (7.25 + 0.1\mathbf{I}_c)$

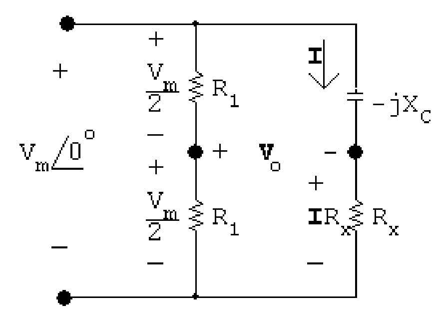
Now square each term and then add to generate the quadratic equation

$\mathbf{I}_c^2 - 605.77\mathbf{I}_c + 5325.48 = 0; \qquad \mathbf{I}_c = 302.88 \pm 293.96$

Therefore

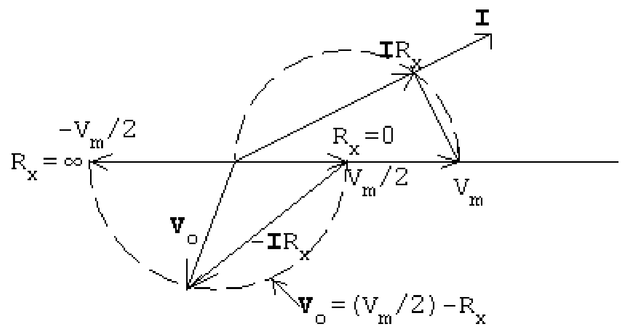
$\mathbf{I}_c = 8.92\,\text{A}$ (smallest value) and $Z_c = 240/j8.92 = -j26.90\,\Omega.$

P 9.76 The phasor domain equivalent circuit is

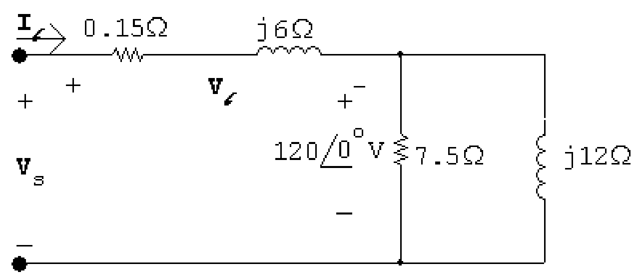


$$V_o = \frac{V_m\underline{/0^\circ}}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m\underline{/0^\circ}}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle decreases from 0° to -180°, as shown in the following phasor diagram:



P 9.77 **[a]**

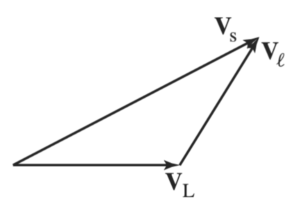


$$\mathbf{I}_\ell = \frac{120}{7.5} + \frac{120}{j12} = 16 - j10\,\text{A}$$

$$\mathbf{V}_\ell = (0.15 + j6)(16 - j10) = 62.4 + j94.5 = 113.24\underline{/56.56^\circ}\,\text{V}$$

$$\text{V}_\text{s} = 120\underline{/0^\circ} + \mathbf{V}_\ell = 205.43\underline{/27.39^\circ}\,\text{V}$$

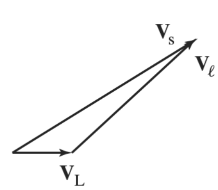
[b]



[c] $\mathbf{I}_\ell = \dfrac{120}{2.5} + \dfrac{120}{j4} = 48 - j30\,\text{A}$

$$\mathbf{V}_\ell = (0.15 + j6)(48 - j30) = 339.73\underline{/56.56^\circ}\,\text{V}$$

$$\mathbf{V}_\text{s} = 120 + \mathbf{V}_\ell = 418.02\underline{/42.7^\circ}\,\text{V}$$

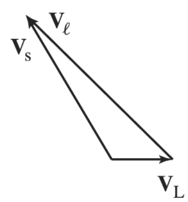


The amplitude of $\mathbf{V}_s$ must be increased from 205.43 V to 418.02 V (more than doubled) to maintain the load voltage at 120 V.

[d] $\mathbf{I}_\ell = \dfrac{120}{2.5} + \dfrac{120}{j4} + \dfrac{120}{-j2} = 48 + j30\,\text{A}$

$\mathbf{V}_\ell = (0.15 + j6)(48 + j30) = 339.73\underline{/120.57^\circ}\ \text{V}$

$\mathbf{V}_\text{s} = 120 + \mathbf{V}_\ell = 297.23\underline{/100.23^\circ}\ \text{V}$



The amplitude of $\mathbf{V}_s$ must be increased from 205.43 V to 297.23 V to maintain the load voltage at 120 V.

P 9.78 $\mathbf{V}_g = 4\underline{/0^\circ}\ \text{V}; \qquad \dfrac{1}{j\omega C} = -j20\,\text{k}\Omega$

Let $\mathbf{V}_\text{a}$ = voltage across the capacitor, positive at upper terminal
Then:

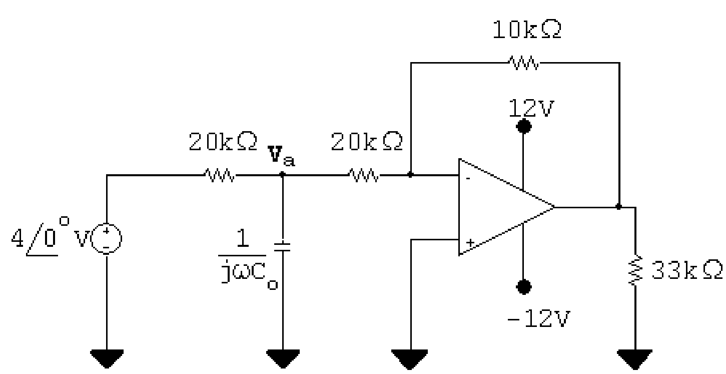
$$\frac{\mathbf{V}_\text{a} - 4\underline{/0^\circ}}{20{,}000} + \frac{\mathbf{V}_\text{a}}{-j20{,}000} + \frac{\mathbf{V}_\text{a}}{20{,}000} = 0; \qquad \therefore \quad \mathbf{V}_\text{a} = (1.6 - j0.8)\ \text{V}$$

$$\frac{0 - \mathbf{V}_\text{a}}{20{,}000} + \frac{0 - \mathbf{V}_o}{10{,}000} = 0; \qquad \mathbf{V}_o = -\frac{\mathbf{V}_\text{a}}{2}$$

$$\therefore \quad \mathbf{V}_o = -0.8 + j0.4 = 0.89\underline{/153.43^\circ}\ \text{V}$$

$$v_o = 0.89\cos(200t + 153.43^\circ)\ \text{V}$$

P 9.79 **[a]**



$$\frac{\mathbf{V}_\text{a} - 4\underline{/0^\circ}}{20{,}000} + j\omega C_o \mathbf{V}_\text{a} + \frac{\mathbf{V}_\text{a}}{20{,}000} = 0$$

$$\mathbf{V}_\text{a} = \frac{4}{2 + j20{,}000\omega C_o}$$

$\mathbf{V}_o = -\dfrac{\mathbf{V}_\text{a}}{2}$ (see solution to Prob. 9.78)

$$\mathbf{V}_o = \frac{-2}{2 + j4 \times 10^6 C_o} = \frac{2\underline{/180^\circ}}{2 + j4 \times 10^6 C_o}$$

$\therefore$ denominator angle $= 45^\circ$

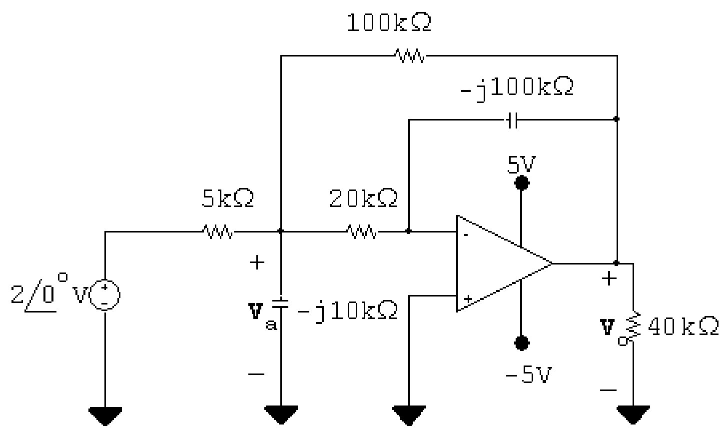
so $4 \times 10^6 C_o = 2 \quad \therefore \quad C_o = 0.5\,\mu\text{F}$

[b] $\mathbf{V}_o = \dfrac{2\underline{/180^\circ}}{2 + j2} = 0.707\underline{/135^\circ}\text{ V}$

$v_o = 0.707\cos(200t + 135^\circ)\text{ V}$

P 9.80 $\dfrac{1}{j\omega C_1} = -j10\,\text{k}\Omega$

$\dfrac{1}{j\omega C_2} = -j100\,\text{k}\Omega$



$$\frac{\mathbf{V}_\text{a} - 2}{5000} + \frac{\mathbf{V}_\text{a}}{-j10{,}000} + \frac{\mathbf{V}_\text{a}}{20{,}000} + \frac{\mathbf{V}_\text{a} - \mathbf{V}_o}{100{,}000} = 0$$

$$20\mathbf{V}_\text{a} - 40 + j10\mathbf{V}_\text{a} + 5\mathbf{V}_\text{a} + \mathbf{V}_\text{a} - \mathbf{V}_o = 0$$

$$\therefore \qquad (26 + j10)\mathbf{V}_\text{a} - \mathbf{V}_o = 40$$

$$\frac{0 - \mathbf{V}_\text{a}}{20{,}000} + \frac{0 - \mathbf{V}_o}{-j100{,}000} = 0$$

$$j5\mathbf{V}_\text{a} - \mathbf{V}_o = 0$$

Solving,

$$\mathbf{V}_o = 1.43 + j7.42 = 7.55\underline{/79.11^\circ}\text{ V}$$

$$v_o(t) = 7.55\cos(10^6 t + 79.11^\circ)\text{ V}$$

P 9.81 **[a]** $\mathbf{V}_g = 25\underline{/0^\circ}\,\text{V}$

$$\mathbf{V}_\text{p} = \frac{20}{100}\mathbf{V}_g = 5\underline{/0^\circ}; \qquad \mathbf{V}_\text{n} = \mathbf{V}_\text{p} = 5\underline{/0^\circ}\,\text{V}$$

$$\frac{5}{80{,}000} + \frac{5 - \mathbf{V}_o}{Z_\text{p}} = 0$$

$$Z_\text{p} = -j80{,}000\|40{,}000 = 32{,}000 - j16{,}000\,\Omega$$

$$\mathbf{V}_o = \frac{5Z_\text{p}}{80{,}000} + 5 = 7 - j1 = 7.07\underline{/-8.13^\circ}\ \text{V}$$

$$v_o = 7.07\cos(50{,}000t - 8.13^\circ)\,\text{V}$$

[b] $\mathbf{V}_\text{p} = 0.2V_m\underline{/0^\circ}; \qquad \mathbf{V}_\text{n} = \mathbf{V}_\text{p} = 0.2V_m\underline{/0^\circ}$

$$\frac{0.2V_m}{80{,}000} + \frac{0.2V_m - \mathbf{V}_o}{32{,}000 - j16{,}000} = 0$$

$$\therefore \quad \mathbf{V}_o = 0.2V_m + \frac{32{,}000 - j16{,}000}{80{,}000}V_m(0.2) = 0.2V_m(1.4 - j0.2)$$

$$\therefore \quad |0.2V_m(1.4 - j0.2)| \leq 10$$

$$\therefore \quad V_m \leq 35.36\,\text{V}$$

P 9.82 **[a]** $\dfrac{1}{j\omega C} = -j20\,\Omega$

$$\frac{\mathbf{V}_\text{n}}{20} + \frac{\mathbf{V}_\text{n} - \mathbf{V}_o}{-j20} = 0$$

$$\frac{\mathbf{V}_o}{-j20} = \frac{\mathbf{V}_\text{n}}{20} + \frac{\mathbf{V}_\text{n}}{-j20}$$

$$\mathbf{V}_o = -j1\mathbf{V}_\text{n} + \mathbf{V}_\text{n} = (1 - j1)\mathbf{V}_\text{n}$$

$$\mathbf{V}_\text{p} = \frac{\mathbf{V}_g(1/j\omega C_o)}{5 + (1/j\omega C_o)} = \frac{\mathbf{V}_g}{1 + j(5)(10^5)C_o}$$

$$\mathbf{V}_g = 6\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_\text{p} = \frac{6\underline{/0^\circ}}{1 + j5\times 10^5 C_o} = \mathbf{V}_\text{n}$$

$$\therefore \quad \mathbf{V}_o = \frac{(1 - j1)6\underline{/0^\circ}}{1 + j5\times 10^5 C_o}$$

$$|\mathbf{V}_o| = \frac{\sqrt{2}(6)}{\sqrt{1 + 25\times 10^{10}C_o^2}} = 6$$

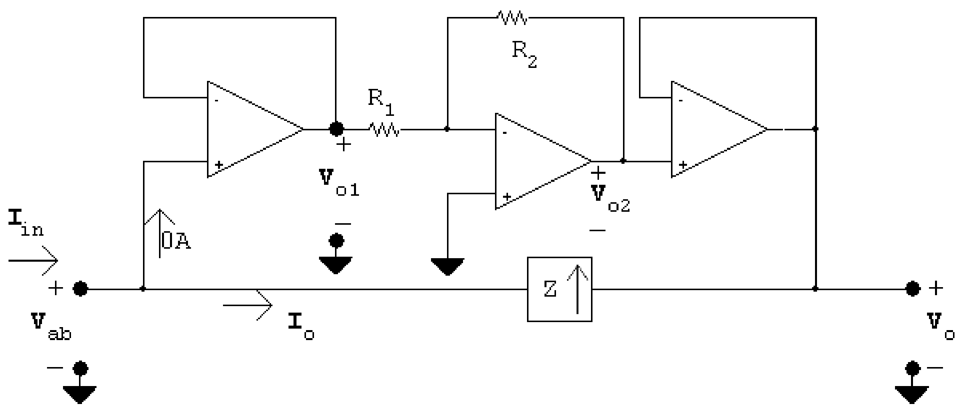
Solving,

$$C_o = 2\,\mu\text{F}$$

[b] $\mathbf{V}_o = \frac{6(1-j1)}{1+j1} = -j6\,\text{V}$

$v_o = 6\cos(10^5 t - 90^\circ)\,\text{V}$

P 9.83 **[a]**



Because the op-amps are ideal $\mathbf{I}_{\text{in}} = \mathbf{I}_o$, thus

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_{\text{in}}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_o}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - \mathbf{V}_o}{Z}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{\text{ab}}; \qquad \mathbf{V}_{o2} = -\left(\frac{R_2}{R_1}\right)\mathbf{V}_{o1} = -K\mathbf{V}_{o1} = -K\mathbf{V}_{\text{ab}}$$

$$\mathbf{V}_o = \mathbf{V}_{o2} = -K\mathbf{V}_{\text{ab}}$$

$$\therefore\quad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - (-K\mathbf{V}_{\text{ab}})}{Z} = \frac{(1+K)\mathbf{V}_{\text{ab}}}{Z}$$

$$\therefore\quad Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{(1+K)\mathbf{V}_{\text{ab}}}Z = \frac{Z}{(1+K)}$$

[b] $Z = \frac{1}{j\omega C}; \qquad Z_{\text{ab}} = \frac{1}{j\omega C(1+K)}; \qquad \therefore\quad C_{\text{ab}} = C(1+K)$

P 9.84 **[a]** $\mathbf{I}_1 = \frac{120}{24} + \frac{240}{8.4+j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\,\text{A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4+j6} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\,\text{A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_6 = \frac{240}{8.4+j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\,\text{A}$$

[b]

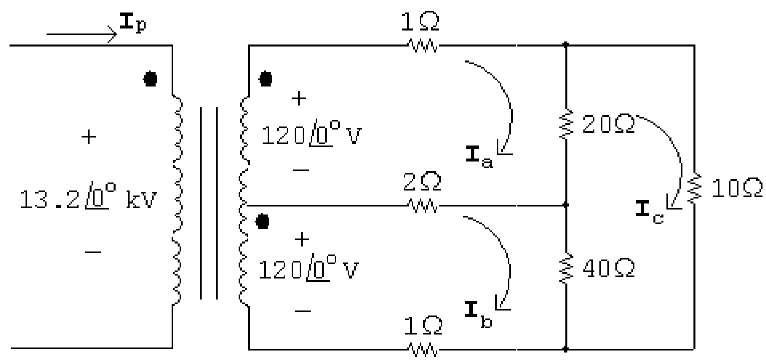
$\mathbf{I}_1 = 0$	$\mathbf{I}_3 = 15\,\text{A}$	$\mathbf{I}_5 = 10\,\text{A}$
$\mathbf{I}_2 = 10 + 5 = 15\,\text{A}$	$\mathbf{I}_4 = -5\,\text{A}$	$\mathbf{I}_6 = 5\,\text{A}$

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the $12\,\Omega$ load includes the clock and the TV set.

[d] No, the motor current drops to $5\,\text{A}$, well below its normal running value of $22.86\,\text{A}$.

[e] After fuse A opens, the current in fuse B is only 15 A.

P 9.85 [a] The circuit is redrawn, with mesh currents identified:



The mesh current equations are:

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$\mathbf{I}_a = 24\underline{/0^\circ}\,\text{A}$ $\quad\mathbf{I}_b = 21.96\underline{/0^\circ}\,\text{A}$ $\quad\mathbf{I}_c = 19.40\underline{/0^\circ}\,\text{A}$

The branch currents are:

$$\mathbf{I}_1 = \mathbf{I}_a = 24\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \mathbf{I}_b = 21.96\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_4 = \mathbf{I}_c = 19.40\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55\underline{/0^\circ}\,\text{A}$$

[b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13{,}200}{N_1} = \frac{240}{2N_2} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{1}{110}$$

The ampere turn balance requires

$$N_1\mathbf{I}_p = N_2\mathbf{I}_1 + N_2\mathbf{I}_3$$

Therefore,

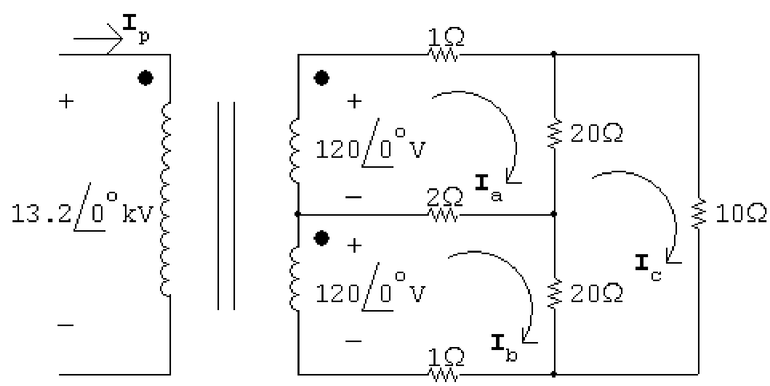
$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42\underline{/0^\circ}\,\text{A}$$

Check voltages —

$$\mathbf{V}_4 = 10\text{I}_4 = 194\underline{/0^\circ}\text{ V}$$
$$\mathbf{V}_5 = 20\text{I}_5 = 92\underline{/0^\circ}\text{ V}$$
$$\mathbf{V}_6 = 40\text{I}_6 = 102\underline{/0^\circ}\text{ V}$$

All of these voltages are low for a reasonable distribution circuit.

P 9.86 **[a]**



The three mesh current equations are

$$120\underline{/0^\circ} = 23\mathbf{I}_\text{a} - 2\mathbf{I}_\text{b} - 20\mathbf{I}_\text{c}$$

$$120\underline{/0^\circ} = -2\mathbf{I}_\text{a} + 23\mathbf{I}_\text{b} - 20\mathbf{I}_\text{c}$$

$$0 = -20\mathbf{I}_\text{a} - 20\mathbf{I}_\text{b} + 50\mathbf{I}_\text{c}$$

Solving,

$$\mathbf{I}_\text{a} = 24\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_\text{b} = 24\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_\text{c} = 19.2\underline{/0^\circ}\,\text{A}$$

$$\therefore\quad \mathbf{I}_2 = \mathbf{I}_\text{a} - \mathbf{I}_\text{b} = 0\,\text{A}$$

[b] $\mathbf{I}_\text{p} = \dfrac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \dfrac{N_2}{N_1}(\mathbf{I}_\text{a} + \mathbf{I}_\text{b}$

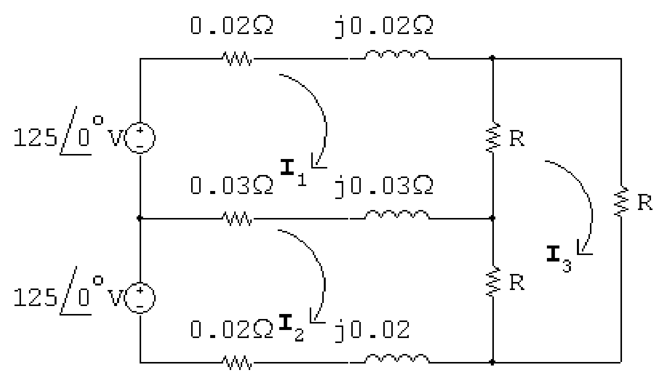
$$= \frac{1}{110}(24 + 24) = 0.436\underline{/0^\circ}\text{ A}$$

[c] Check voltages —

$$\mathbf{V}_4 = 10\mathrm{I}_4 = 10\mathrm{I}_c = 192/\!\underline{0^\circ}\ \mathrm{V}$$
$$\mathbf{V}_5 = 20\mathrm{I}_5 = 20(\mathrm{I}_a - \mathrm{I}_c) = 96/\!\underline{0^\circ}\ \mathrm{V}$$
$$\mathbf{V}_6 = 40\mathrm{I}_6 = 20(\mathrm{I}_b - \mathrm{I}_c) = 96/\!\underline{0^\circ}\ \mathrm{V}$$

Where the two loads are equal, the current in the neutral conductor (I_2) is zero, and the voltages V_5 and V_6 are equal. The voltages V_4, V_5, and V_6 are too low for a reasonable dirtribution circuit.

P 9.87 **[a]**



$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

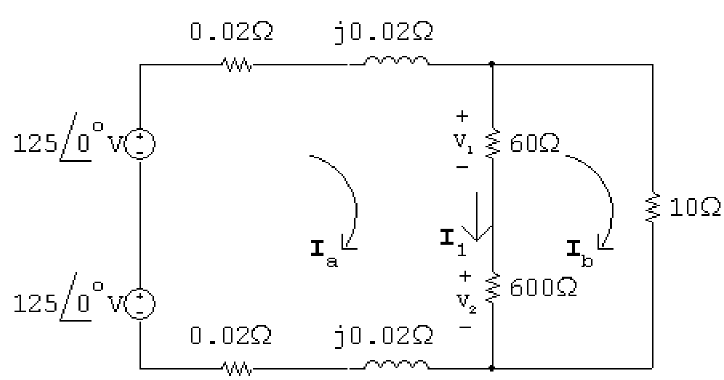
$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$$\therefore\quad \mathbf{I}_1 = \mathbf{I}_2 \quad \text{so} \quad \mathbf{I}_\mathrm{n} = \mathbf{I}_1 - \mathbf{I}_2 = 0\ \mathrm{A}$$

[b] $\mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3); \qquad \mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

[c]



$250 = (660.04 + j0.04)\mathbf{I}_a - 660\mathbf{I}_b$

$0 = -660\mathbf{I}_a + 670\mathbf{I}_b$

Solving,

$\mathbf{I}_a = 25.28\underline{/-0.23^\circ} = 25.28 - j0.10\,\text{A}$

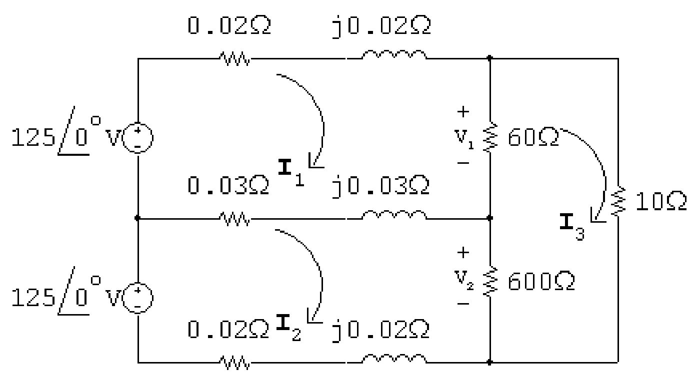
$\mathbf{I}_b = 24.90\underline{/-0.23^\circ} = 24.90 - j0.10\,\text{A}$

$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_b = 0.377 - j0.00153\,\text{A}$

$\mathbf{V}_1 = 60\mathbf{I}_1 = 22.63 - j0.0195 = 22.64\underline{/-0.23^\circ}\,\text{V}$

$\mathbf{V}_2 = 600\mathbf{I}_1 = 226.3 - j0.915 = 226.4\underline{/-0.23^\circ}\,\text{V}$

[d]



$125 = (60.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 60\mathbf{I}_3$

$125 = -(0.03 + j0.03)\mathbf{I}_1 + (600.05 + j0.05)\mathbf{I}_2 - 600\mathbf{I}_3$

$0 = -60\mathbf{I}_1 - 600\mathbf{I}_2 + 670\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 26.97\underline{/-0.24^\circ} = 26.97 - j0.113\,\text{A}$

$\mathbf{I}_2 = 25.10\underline{/-0.24^\circ} = 25,10 - j0.104\,\text{A}$

$\mathbf{I}_3 = 24.90\underline{/-0.24^\circ} = 24.90 - j0.104\,\text{A}$

$\mathbf{V}_1 = 60(\mathbf{I}_1 - \mathbf{I}_3) = 124.4\underline{/-0.27^\circ}\,\text{V}$

$\mathbf{V}_2 = 600(\mathbf{I}_2 - \mathbf{I}_3) = 124.6\underline{/-0.20^\circ}\,\text{V}$

[e] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.88 **[a]** Let N_1 = primary winding turns and $2N_2$ = secondary winding turns. Then

$$\frac{14{,}000}{N_1} = \frac{250}{2N_2}; \qquad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$$\mathbf{I}_{\text{p}} = 2a\mathbf{I}_{\text{a}}$$

$$\therefore \quad \mathbf{I}_{\text{p}} = \frac{2N_2\mathbf{I}_{\text{a}}}{N_1} = \frac{1}{56}\mathbf{I}_{\text{a}}$$

$$= \frac{1}{56}(25.28 - j0.10)$$

$$\mathbf{I}_{\text{p}} = 451.4 - j1.8\,\text{mA} = 451.4\underline{/-0.23^\circ}\ \text{mA}$$

In part d),

$$\mathbf{I}_{\text{p}}N_1 = \mathbf{I}_1N_2 + \mathbf{I}_2N_2$$

$$\therefore \quad \mathbf{I}_{\text{p}} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= \frac{1}{112}(26.97 - j0.11 + 25.10 - j0.10)$$

$$= \frac{1}{112}(52.07 - j0.22)$$

$$\mathbf{I}_{\text{p}} = 464.9 - j1.9\,\text{mA} = 464.9\underline{/-0.24^\circ}\ \text{mA}$$

[b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.