10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 **[a]** $\mathbf{V} = 100\underline{/-45^\circ}\,\text{V}, \qquad \mathbf{I} = 20\underline{/15^\circ}\,\text{A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45 - (15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\underline{/0^\circ}, \qquad \mathbf{I} = 20\underline{/120^\circ}$

$$P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

AP 10.2 pf $= \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos(-60^\circ) = 0.5$ leading

rf $= \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$

AP 10.3 From Ex. 9.4 $I_{\text{eff}} = \dfrac{I_\rho}{\sqrt{3}} = \dfrac{0.18}{\sqrt{3}}\,\text{A}$

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54\,\text{W}$$

AP 10.4 **[a]** $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

$$\text{Therefore} \quad \mathbf{I}_\ell = \frac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\,\text{A(rms)}$$

$$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\,\text{V(rms)}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}\,\text{A(rms)}$$

[b] $S_\text{L} = \mathbf{V}_L\mathbf{I}_L^* = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$$= (1129.09 + j752.73)\,\text{VA}$$

$$P_\text{L} = 1129.09\,\text{W}; \qquad Q_\text{L} = 752.73\,\text{VAR}$$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\,\text{W}; \qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\,\text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\,\text{VA}$

Therefore the source is delivering $1152.62\,\text{W}$ and absorbing 376.36 magnetizing VAR.

[e] $Q_{\text{cap}} = \dfrac{|\mathbf{V}_\text{L}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\,\text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $94.09 + 752.73 + 376.36 = 1223.18\,\text{VAR}$ and

$1129.09 + 23.52 = 1152.62\,\text{W}$

AP 10.5 Series circuit derivation:

$$S = 250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

$$\text{Therefore} \quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\,\text{A(rms)}$$

$$\mathbf{I} = 200\underline{/36.87^\circ}\,\text{A(rms)}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

$$\text{Therefore} \quad R = 1\,\Omega, \qquad X_\text{C} = -0.75\,\Omega$$

Parallel circuit derivation:

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_C}; \qquad \text{therefore} \quad X_C = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

AP 10.6 $S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\text{VA}$

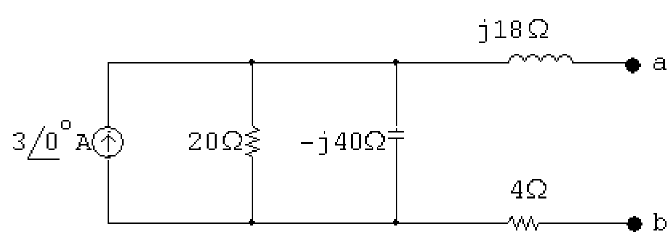
$$S_2 = 6000(0.8) + j6000(0.6) = 4800 - j3600\,\text{VA}$$

$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\text{VA}$$

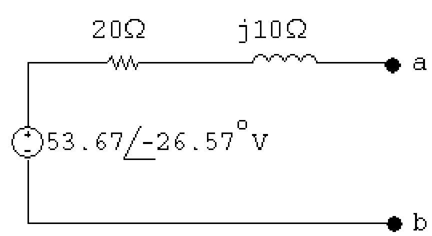
$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\text{A}$$

$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\text{V(rms)}$$

AP 10.7 **[a]** The phasor domain equivalent circuit and the Thévenin equivalent are shown below:
Phasor domain equivalent circuit:



Thévenin equivalent:



$$\mathbf{V}_{\text{Th}} = 3\frac{-j800}{20 - j40} = 48 - j24 = 53.67\underline{/-26.57^\circ}\,\text{V}$$

$$Z_{\text{Th}} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36\underline{/26.57^\circ}\,\Omega$$

For maximum power transfer, $Z_L = (20 - j10)\,\Omega$

[b] $\mathbf{I} = \dfrac{53.67\angle{-26.57^\circ}}{40} = 1.34\angle{-26.57^\circ}\,\text{A}$

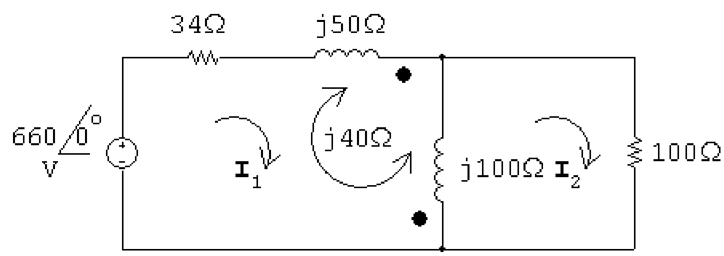
Therefore $\quad P = \left(\dfrac{1.34}{\sqrt{2}}\right)^2 20 = 18\,\text{W}$

[c] $R_\text{L} = |Z_\text{Th}| = 22.36\,\Omega$

[d] $\mathbf{I} = \dfrac{53.67\angle{-26.57^\circ}}{42.36 + j10} = 1.23\angle{-39.85^\circ}\,\text{A}$

Therefore $\quad P = \left(\dfrac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$

AP 10.8



Mesh current equations:

$$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

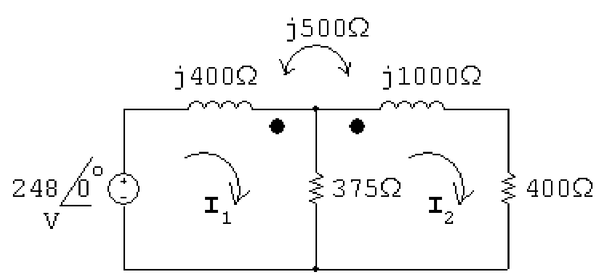
$$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$$

Solving,

$\mathbf{I}_1 = 3.536\angle{-45^\circ}$ A,

$\mathbf{I}_2 = 3.5\angle{0^\circ}$ A; $\qquad \therefore \quad P = \dfrac{1}{2}(3.5)^2(100) = 612.50\,\text{W}$

AP 10.9 **[a]**



$$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$$

Solving,

$\mathbf{I}_1 = 0.80 - j0.62\,\text{A}; \qquad \mathbf{I}_2 = 0.4 - j0.3 = 0.5\angle{-36.87^\circ}\,\text{A}$

$\therefore \quad P = \dfrac{1}{2}(0.25)(400) = 50\,\text{W}$

[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32\,\text{A}$

$$P_{375} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2(375) = 49.20\,\text{W}$$

[c] $P_g = \dfrac{1}{2}(248)(0.8) = 99.20\,\text{W}$

$$\sum P_{\text{abs}} = 50 + 49.2 = 99.20\,\text{W} \quad \text{(checks)}$$

AP 10.10 **[a]** $V_{\text{Th}} = 210\angle 0^\circ\,\text{V}; \qquad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$

Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = 14\,\text{A}; \qquad R_{\text{Th}} = \frac{210}{14} = 15\,\Omega$$

[b] $P_{\max} = \left(\dfrac{210}{30}\right)^2 15 = 735\,\text{W}$

AP 10.11 **[a]** $\mathbf{V}_{\text{Th}} = -4(146\angle 0^\circ) = -584\angle 0^\circ\,\text{V(rms)} = 584\angle 180^\circ\,\text{V(rms)}$

$$\mathbf{V}_2 = 4\mathbf{V}_1; \qquad \mathbf{I}_1 = -4\mathbf{I}_2$$

Short circuit equations:

$$146\angle 0^\circ = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = -146/365 = -0.40\,\text{A}; \qquad R_{\text{Th}} = \frac{-584}{-0.4} = 1460\,\Omega$$

[b] $P = \left(\dfrac{-584}{2920}\right)^2 1460 = 58.40\,\text{W}$

Problems

P 10.1 **[a]** $P = \frac{1}{2}(100)(10)\cos(50 - 15) = 500\cos 35^\circ = 409.58\,\text{W} \quad \text{(abs)}$

$Q = 500\sin 35^\circ = 286.79\,\text{VAR} \quad \text{(abs)}$

[b] $P = \frac{1}{2}(40)(20)\cos(-15 - 60) = 400\cos(-75^\circ) = 103.53\,\text{W} \quad \text{(abs)}$

$Q = 400\sin(-75^\circ) = -386.37\,\text{VAR} \quad \text{(del)}$

[c] $P = \frac{1}{2}(400)(10)\cos(30 - 150) = 2000\cos(-120^\circ) = -1000\,\text{W} \quad \text{(del)}$

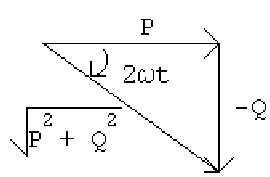
$Q = 2000\sin(-120^\circ) = -1732.05\,\text{VAR} \quad \text{(del)}$

[d] $P = \frac{1}{2}(200)(5)\cos(160 - 40) = 500\cos(120^\circ) = -250\,\text{W} \quad \text{(del)}$

$Q = 500\sin(120^\circ) = 433.01\,\text{VAR} \quad \text{(abs)}$

P 10.2 $p = P + P\cos 2\omega t - Q\sin 2\omega t; \qquad \frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$\frac{dp}{dt} = 0$ when $-2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t$ or $\tan 2\omega t = -\frac{Q}{P}$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2 + Q^2}}; \qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2 + Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

Therefore $p_{\max} = P + P\cdot\frac{P}{\sqrt{P^2+Q^2}} - \frac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$

and $p_{\min} = P - P\cdot\frac{P}{\sqrt{P^2+Q^2}} - Q\cdot\frac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$

P 10.3 **[a]** hair dryer $= 600\,\text{W}$ vacuum $= 630\,\text{W}$

sun lamp $= 279\,\text{W}$ air conditioner $= 860\,\text{W}$

television $= 240\,\text{W}$ $\sum P = 2609\,\text{W}$

Therefore $I_{\text{eff}} = \dfrac{2609}{120} = 21.74\,\text{A}$

Yes, the breaker will trip.

[b] $\sum P = 2609 - 909 = 1700\,\text{W}$; $I_{\text{eff}} = \dfrac{1700}{120} = 14.17\,\text{A}$

Yes, the breaker will not trip if the current is reduced to $14.17\,\text{A}$.

P 10.4 **[a]** $I_{\text{eff}} = 40/115 \cong 0.35\,\text{A}$; [b] $I_{\text{eff}} = 130/115 \cong 1.13\,\text{A}$

P 10.5 $W_{\text{dc}} = \dfrac{V_{\text{dc}}^2}{R}T$; $W_s = \displaystyle\int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 10.6 **[a]** Area under one cycle of v_g^2:

$$A = (5^2)(2)(30 \times 10^{-6}) + 2^2(2)(37.5 \times 10^{-6})$$
$$= 1800 \times 10^{-6}$$

Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{200 \times 10^{-6}} = \frac{1800 \times 10^{-6}}{200 \times 10^{-6}} = 9$$

$$\therefore \quad V_{\text{rms}} = \sqrt{9} = 3\,\text{V(rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{3^2}{2.25} = 4\,\text{W}$

P 10.7 $i(t) = 200t \qquad 0 \le t \le 75\,\text{ms}$

$$i(t) = 60 - 600t \qquad 75\,\text{ms} \le t \le 100\,\text{ms}$$

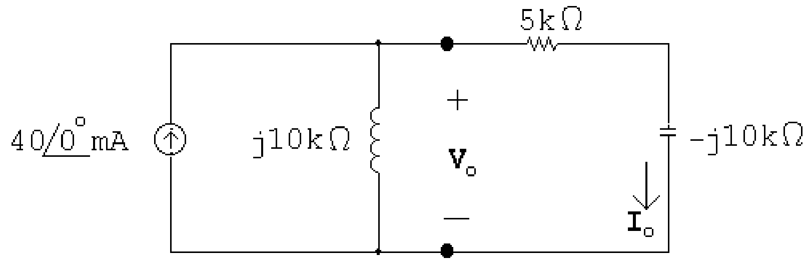
$$I_{\text{rms}} = \sqrt{\frac{1}{0.1}\left\{\int_0^{0.075} (200)^2 t^2\,dt + \int_{0.075}^{0.1} (60 - 600t)^2\,dt\right\}}$$

$$= \sqrt{10(5.625) + 10(1.875)} = \sqrt{75} = 8.66\ \text{A(rms)}$$

P 10.8 $P = I_{\rm rms}^2 R \qquad \therefore \quad R = \dfrac{3\times 10^3}{75} = 40\,\Omega$

P 10.9 $\mathbf{I}_g = 40\underline{/0^\circ}\,\text{mA}$

$$j\omega L = j10{,}000\,\Omega; \qquad \frac{1}{j\omega C} = -j10{,}000\,\Omega$$



$$\mathbf{I}_o = \frac{j10{,}000}{5000}(40\underline{/0^\circ}) = 80\underline{/90^\circ}\,\text{mA}$$

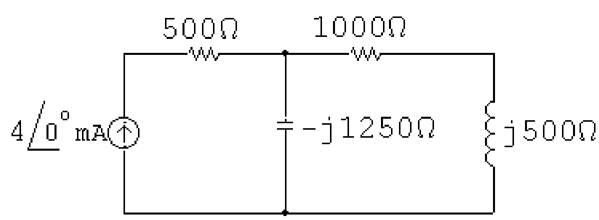
$$P = \frac{1}{2}|\mathbf{I}_o|^2(5000) = \frac{1}{2}(0.08)^2(5000) = 16\,\text{W}$$

$$Q = \frac{1}{2}|\mathbf{I}_o|^2(-10{,}000) = -32\,\text{VAR}$$

$$S = P + jQ = 16 - j32\,\text{VA}$$

$$|S| = 35.78\,\text{VA}$$

P 10.10 $\mathbf{I}_g = 4\underline{/0^\circ}\,\text{mA}; \qquad \dfrac{1}{j\omega C} = -j1250\,\Omega; \qquad j\omega L = j500\,\Omega$

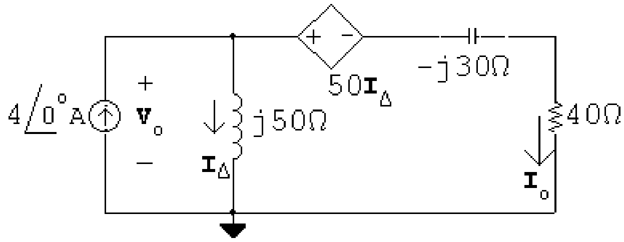


$$Z_{\rm eq} = 500 + [-j1250\|(1000 + j500)] = 1500 - j500\,\Omega$$

$$P_g = -\frac{1}{2}|I|^2\text{Re}\{Z_{\rm eq}\} = -\frac{1}{2}(0.004)^2(1500) = -12\,\text{mW}$$

The source delivers 12 mW of power to the circuit.

P 10.11 $j\omega L = j10^5(0.5\times10^{-3}) = j50\,\Omega;\qquad \frac{1}{j\omega C} = \frac{1}{j10^5[(1/3)\times10^{-6}]} = -j30\,\Omega$



$$-4 + \frac{\mathbf{V}_o}{j50} + \frac{\mathbf{V}_o - 50\mathbf{I}_\Delta}{40 - j30} = 0$$

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{j50}$$

Place the equations in standard form:

$$\mathbf{V}_o\left(\frac{1}{j50} + \frac{1}{40 - j30}\right) + \mathbf{I}_\Delta\left(\frac{-50}{40 - j30}\right) = 4$$

$$\mathbf{V}_o\left(\frac{1}{j50}\right) + \mathbf{I}_\Delta(-1) = 0$$

Solving,

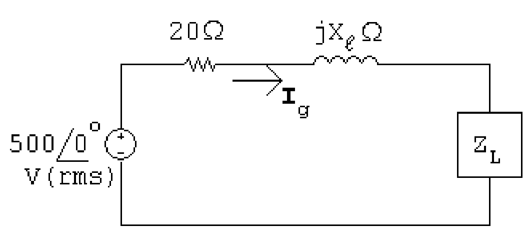
$$\mathbf{V}_o = 200 - j400\text{ V};\qquad \mathbf{I}_\Delta = -8 - j4\text{ A}$$

$$\mathbf{I}_o = 4 - (-8 - j4) = 12 + j4\text{ A}$$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_o|^2(40) = \frac{1}{2}(160)(40) = 3200\,\text{W}$$

P 10.12 **[a]** line loss $= 7500 - 2500 = 5\,\text{kW}$

line loss $= |\mathbf{I}_g|^2 20 \qquad \therefore\quad |\mathbf{I}_g|^2 = 250$

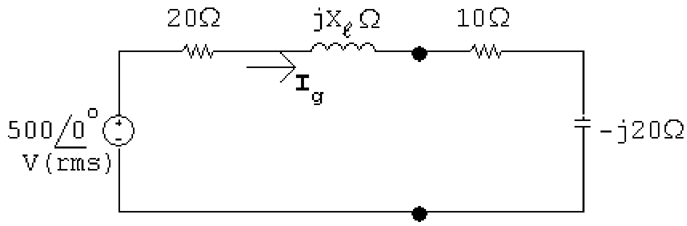


$$|\mathbf{I}_g| = \sqrt{250}\,\text{A}$$

$|\mathbf{I}_g|^2 R_{\mathrm{L}} = 2500 \qquad \therefore \quad R_{\mathrm{L}} = 10\,\Omega$

$|\mathbf{I}_g|^2 X_{\mathrm{L}} = -5000 \qquad \therefore \quad X_{\mathrm{L}} = -20\,\Omega$

Thus,



$$|Z| = \sqrt{(30)^2 + (X_\ell - 20)^2} \qquad |\mathbf{I}_g| = \frac{500}{\sqrt{900 + (X_\ell - 20)^2}}$$

$$\therefore \quad 900 + (X_\ell - 20)^2 = \frac{25 \times 10^4}{250} = 1000$$

Solving, $\quad (X_\ell - 20) = \pm 10.$

Thus, $\quad X_\ell = 10\,\Omega \qquad$ or $\qquad X_\ell = 30\,\Omega$

[b] If $X_\ell = 30\,\Omega$:

$$\mathbf{I}_g = \frac{500}{30 + j10} = 15 - j5\,\mathrm{A}$$

$S_g = -500\mathbf{I}_g^* = -7500 - j2500\,\mathrm{VA}$

Thus, the voltage source is delivering 7500 W and 2500 magnetizing vars.

$Q_{j30} = |\mathbf{I}_g|^2 X_\ell = 250(30) = 7500\,\mathrm{VAR}$

Therefore the line reactance is absorbing 7500 magnetizing vars.

$Q_{-j20} = |\mathbf{I}_g|^2 X_{\mathrm{L}} = 250(-20) = -5000\,\mathrm{VAR}$

Therefore the load reactance is generating 5000 magnetizing vars.

$\sum Q_{\mathrm{gen}} = 7500\,\mathrm{VAR} = \sum Q_{\mathrm{abs}}$

If $X_\ell = 10\,\Omega$:

$$\mathbf{I}_g = \frac{500}{30 - j10} = 15 + j5\,\mathrm{A}$$

$S_g = -500\mathbf{I}_g^* = -7500 + j2500\,\mathrm{VA}$

Thus, the voltage source is delivering 7500 W and absorbing 2500 magnetizing vars.

$Q_{j10} = |\mathbf{I}_g|^2(10) = 250(10) = 2500\,\mathrm{VAR}$

Therefore the line reactance is absorbing 2500 magnetizing vars. The load continues to generate 5000 magnetizing vars.

$\sum Q_{\mathrm{gen}} = 5000\,\mathrm{VAR} = \sum Q_{\mathrm{abs}}$

P 10.13 $Z_{\rm f} = -j10{,}000 \| 20{,}000 = 4000 - j8000\,\Omega$

$Z_{\rm i} = 2000 - j2000\,\Omega$

$$\therefore \quad \frac{Z_{\rm f}}{Z_{\rm i}} = \frac{4000 - j8000}{2000 - j2000} = 3 - j1$$

$$\mathbf{V}_o = -\frac{Z_{\rm f}}{Z_{\rm i}}\mathbf{V}_g; \qquad \mathbf{V}_g = 1\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{V}_o = (3 - j1)(1) = 3 - j1 = 3.16\underline{/-18.43^\circ}\,\mathrm{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(10)}{1000} = 5 \times 10^{-3} = 5\,\mathrm{mW}$$

P 10.14 **[a]** $P = \dfrac{1}{2}\dfrac{(240)^2}{480} = 60\,\mathrm{W}$

$$-\frac{1}{\omega C} = \frac{-9 \times 10^6}{(5000)(5)} = -360\,\Omega$$

$$Q = \frac{1}{2}\frac{(240)^2}{(-360)} = -80\,\mathrm{VAR}$$

$$p_{\max} = P + \sqrt{P^2 + Q^2} = 60 + \sqrt{(60)^2 + (80)^2} = 160\,\mathrm{W(del)}$$

[b] $p_{\min} = 60 - \sqrt{60^2 + 80^2} = -40\,\mathrm{W(abs)}$

[c] $P = 60\,\mathrm{W}$ from (a)

[d] $Q = -80\,\mathrm{VAR}$ from (a)

[e] generate, because $Q < 0$

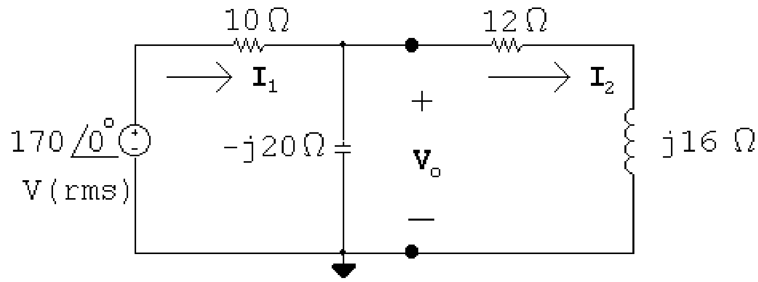
[f] pf $= \cos(\theta_v - \theta_i)$

$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\,\mathrm{A}$$

$\therefore$ pf $= \cos(0 - 53.13^\circ) = 0.6$ leading

[g] rf $= \sin(-53.13^\circ) = -0.8$

P 10.15 **[a]**



The mesh equations are:

$(10 - j20)\mathbf{I}_1 + (j20)\mathbf{I}_2 = 170$

$(j20)\mathbf{I}_1 + (12 - j4)\mathbf{I}_2 = 0$

Solving,

$\mathbf{I}_1 = 4 + j1\,\text{A}; \qquad \mathbf{I}_2 = 3.5 - j5.5\,\text{A}$

$S = -\mathbf{V}_g\mathbf{I}_1^* = -(170)(4 - j1) = -680 + j170\,\text{VA}$

[b] Source is delivering 680 W.

[c] Source is absorbing 170 magnetizing VAR.

[d] $P_{10\Omega} = (\sqrt{17})^2(10) = 170\,\text{W}$

$P_{12\Omega} = (\sqrt{42.5})^2(12) = 510\,\text{W} \qquad (\mathbf{I}_1 - \mathbf{I}_2) = 0.5 + j6.5\ \text{A}$

$Q_{-j20\Omega} = (\sqrt{42.5})^2(20) = -850\,\text{VAR} \qquad |\mathbf{I}_1 - \mathbf{I}_2| = \sqrt{42.5}$

$Q_{j16\Omega} = (\sqrt{42.5})^2(16) = 680\,\text{VAR}$

[e] $\sum P_{\text{del}} = 680\,\text{W}$

$\sum P_{\text{diss}} = 170 + 510 = 680\,\text{W}$

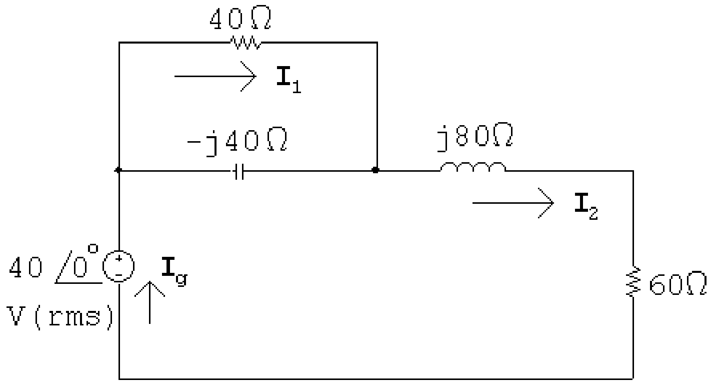
$\therefore \quad \sum P_{\text{del}} = \sum P_{\text{diss}} = 680\,\text{W}$

[f] $\sum Q_{\text{abs}} = 170 + 680 = 850\,\text{VAR}$

$\sum Q_{\text{dev}} = 850\,\text{VAR}$

$\therefore \quad \sum \text{mag VAR dev} = \sum \text{mag VAR abs} = 850$

P 10.16 **[a]** $\dfrac{1}{j\omega C} = -j40\,\Omega; \qquad j\omega L = j80\,\Omega$



$$Z_{\text{eq}} = 40\| - j40 + j80 + 60 = 80 + j60\,\Omega$$

$$\mathbf{I}_g = \frac{40/0^\circ}{80 + j60} = 0.32 - j0.24\,\text{A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = -\frac{1}{2}40(0.32 + j0.24) = -6.4 - j4.8\,\text{VA}$$

$$P = 6.4\,\text{W(del)}; \qquad Q = 4.8\,\text{VAR(del)}$$

$$|S| = |S_g| = 8\,\text{VA}$$

[b] $\mathbf{I}_1 = \dfrac{-j40}{40 - j40}\mathbf{I}_g = 0.04 - j0.28\,\text{A}$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(40) = 1.6\,\text{W}$$

$$P_{60\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(60) = 4.8\,\text{W}$$

$$\sum P_{\text{diss}} = 1.6 + 4.8 = 6.4\,\text{W} = \sum P_{\text{dev}}$$

[c] $\mathbf{I}_{-j40\Omega} = \mathbf{I}_g - \mathbf{I}_1 = 0.28 + j0.04\,\text{A}$

$$Q_{-j40\Omega} = \frac{1}{2}|\mathbf{I}_{-j40\Omega}|^2(-40) = -1.6\,\text{VAR(del)}$$

$$Q_{j80\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(80) = 6.4\,\text{VAR(abs)}$$

$$\sum Q_{\text{abs}} = 6.4 - 1.6 = 4.8\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.17 **[a]** $Z_1 = 240 + j70 = 250/16.26^\circ\,\Omega$

pf $= \cos(16.26^\circ) = 0.96$ lagging

rf $= \sin(16.26^\circ) = 0.28$

$$Z_2 = 160 - j120 = 200\underline{/-36.87^\circ}\,\Omega$$

$$\text{pf} = \cos(-36.87^\circ) = 0.80 \text{ leading}$$

$$\text{rf} = \sin(-36.87^\circ) = -0.60$$

$$Z_3 = 30 - j40 = 50\underline{/-53.13^\circ}\,\Omega$$

$$\text{pf} = \cos(-53.13^\circ) = 0.6 \text{ leading}$$

$$\text{rf} = \sin(-53.13^\circ) = -0.8$$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250\underline{/16.26^\circ}}; \qquad Y_2 = \frac{1}{200\underline{/-36.87^\circ}}; \qquad Y_3 = \frac{1}{50\underline{/-53.13^\circ}}$$

$$Y = 19.84 + j17.88\,\text{mS}$$

$$Z = \frac{1}{Y} = 37.44\underline{/-42.03^\circ}\,\Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 10.18 **[a]** $S_1 = 16 + j18\,\text{kVA}; \qquad S_2 = 6 - j8\,\text{kVA}; \qquad S_3 = 8 + j0\,\text{kVA}$

$$S_T = S_1 + S_2 + S_3 = 30 + j10\,\text{kVA}$$

$$250\mathbf{I}^* = (30 + j10) \times 10^3; \qquad \therefore \quad \mathbf{I} = 120 - j40\,\text{A}$$

$$Z = \frac{250}{120 - j40} = 1.875 + j0.625\,\Omega = 1.98\underline{/18.43^\circ}\,\Omega$$

[b] $\text{pf} = \cos(18.43^\circ) = 0.9487$ lagging

P 10.19 **[a]** From the solution to Problem 10.18 we have

$$\mathbf{I}_L = 120 - j40\,\text{A(rms)}$$

$$\therefore \quad \mathbf{V}_s = 250\underline{/0^\circ} + (120 - j40)(0.01 + j0.08) = 254.4 + j9.2$$
$$= 254.57\underline{/2.07^\circ}\,\text{V(rms)}$$

[b] $|\mathbf{I}_L| = \sqrt{16{,}000}$

$$P_\ell = (16{,}000)(0.01) = 160\,\text{W} \qquad Q_\ell = (16{,}000)(0.08) = 1280\,\text{VAR}$$

[c] $P_s = 30{,}000 + 160 = 30.16\,\text{kW} \qquad Q_s = 10{,}000 + 1280 = 11.28\,\text{kVAR}$

[d] $\eta = \dfrac{30}{30.16}(100) = 99.47\%$

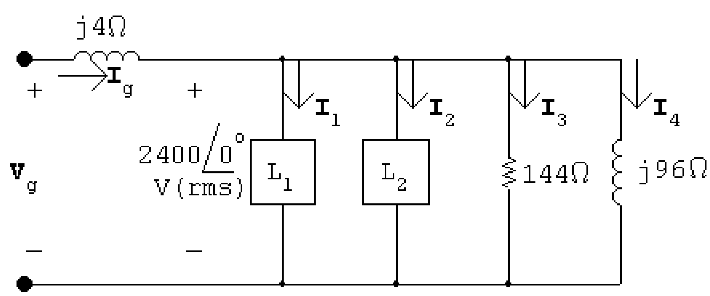
P 10.20 $S_{\rm T} = 4500 - j\frac{4500}{0.96}(0.28) = 4500 - j1312.5\,\text{VA}$

$$S_1 = \frac{2700}{0.8}(0.8 + j0.6) = 2700 + j2025\,\text{VA}$$

$$S_2 = S_{\rm T} - S_1 = 1800 - j3337.5 = 3791.95\underline{/-61.66^\circ}\,\text{VA}$$

$$\text{pf} = \cos(-61.66^\circ) = 0.4747 \text{ leading}$$

P 10.21



$$2400\mathbf{I}_1^* = 60{,}000 + j40{,}000$$

$$\mathbf{I}_1^* = 25 + j16.67; \qquad \therefore \quad \mathbf{I}_1 = 25 - j16.67\,\text{A(rms)}$$

$$2400\mathbf{I}_2^* = 20{,}000 - j10{,}000$$

$$\mathbf{I}_2^* = 8.33 - j4,167; \qquad \therefore \quad \mathbf{I}_2 = 8.33 + j4.167\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{2400\underline{/0^\circ}}{144} = 16.67 + j0\,\text{A}; \qquad \mathbf{I}_4 = \frac{2400\underline{/0^\circ}}{j96} = 0 - j25\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 50 - j37.5\,\text{A}$$

$$\mathbf{V}_g = 2400 + (j4)(50 - j37.5) = 2550 + j200 = 2557.83\underline{/4.48^\circ}\,\text{V(rms)}$$

P 10.22 **[a]** $S_1 = 60{,}000 - j70{,}000\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_{\rm L}|^2}{Z_2^*} = \frac{(2500)^2}{24 - j7} = 240{,}000 + j70{,}000\,\text{VA}$$

$$S_1 + S_2 = 300{,}000\,\text{VA}$$

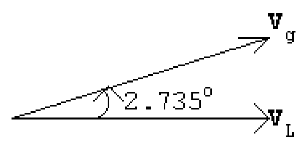
$$2500\mathbf{I}_{\rm L}^* = 300{,}000; \qquad \therefore \quad \mathbf{I}_{\rm L} = 120\underline{/0^\circ}\,\text{A(rms)}$$

$$\mathbf{V}_g = \mathbf{V}_{\rm L} + \mathbf{I}_{\rm L}(0.1 + j1) = 2500 + (120)(0.1 + j1)$$
$$= 2512 + j120 = 2514.86\underline{/2.735^\circ}\,\text{V(rms)}$$

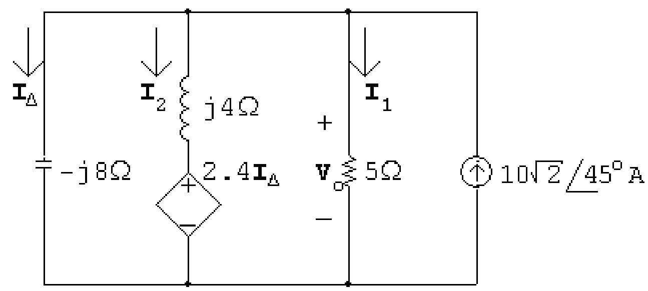
[b] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

$$\frac{2.735^\circ}{360^\circ} = \frac{t}{16.67\text{ ms}}; \qquad \therefore \quad t = 126.62\,\mu\text{s}$$

[c] $\mathbf{V}_{\text{L}}$ lags $\mathbf{V}_g$ by 2.735° or $126.62\,\mu$s



P 10.23 **[a]** From the solution to Problem 9.56 we have:



$$\mathbf{V}_o = j80 = 80\angle 90^\circ\,\text{V}$$

$$S_g = -\frac{1}{2}\mathbf{V}_o\mathbf{I}_g^* = -\frac{1}{2}(j80)(10 - j10) = -400 - j400\,\text{VA}$$

Therefore, the independent current source is delivering 400 W and 400 magnetizing vars.

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{5} = j16\,\text{A}$$

$$P_{5\Omega} = \frac{1}{2}(16)^2(5) = 640\,\text{W}$$

Therefore, the $8\,\Omega$ resistor is absorbing 640 W.

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8} = -10\,\text{A}$$

$$Q_{\text{cap}} = \frac{1}{2}(10)^2(-8) = -400\,\text{VAR}$$

Therefore, the $-j8\,\Omega$ capacitor is developing 400 magnetizing vars.

$$2.4\mathbf{I}_\Delta = -24\,\text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} = \frac{j80 + 24}{j4}$$

$$= 20 - j6\,\text{A} = 20.88\angle -16.7^\circ\,\text{A}$$

$$Q_{j4} = \frac{1}{2}|\mathbf{I}_2|^2(4) = 872\,\text{VAR}$$

Therefore, the $j4\,\Omega$ inductor is absorbing 872 magnetizing vars.

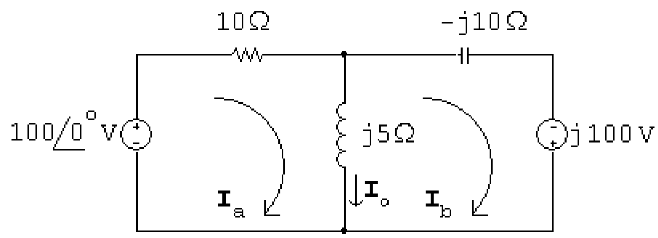
$$S_{\text{d.s.}} = \tfrac{1}{2}(2.4\mathbf{I}_\Delta)\mathbf{I}_2^* = \tfrac{1}{2}(-24)(20 + j6)$$
$$= -240 - j72\,\text{VA}$$

Thus the dependent source is delivering 240 W and 72 magnetizing vars.

[b] $\sum P_{\text{gen}} = 400 + 240 = 640\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{gen}} = 400 + 400 + 72 = 872\,\text{VAR} = \sum Q_{\text{abs}}$

P 10.24 **[a]** From the solution to Problem 9.58 we have



$\mathbf{I}_{\text{a}} = -j10\,\text{A}$; $\quad \mathbf{I}_{\text{b}} = -20 + j10\,\text{A}$; $\quad \mathbf{I}_o = 20 - j20$ A

$$S_{100V} = -\frac{1}{2}(100)\mathbf{I}_{\text{a}}^* = -50(j10) = -j500\,\text{VA}$$

Thus, the 100 V source is developing 500 magnetizing vars.

$$S_{j100V} = -\tfrac{1}{2}(j100)\mathbf{I}_{\text{b}}^* = -j50(-20 - j10)$$
$$= -500 + j1000\,\text{VA}$$

Thus, the $j100$ V source is developing 500 W and absorbing 1000 magnetizing vars.

$$P_{10\Omega} = \frac{1}{2}|\mathbf{I}_{\text{a}}|^2(10) = 500\,\text{W}$$

Thus the $10\,\Omega$ resistor is absorbing 500 W.

$$Q_{-j10\Omega} = \frac{1}{2}|\mathbf{I}_{\text{b}}|^2(-10) = -2500\,\text{VAR}$$

Thus the $-j10\,\Omega$ capacitor is developing 2500 magnetizing vars.

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_o|^2(5) = 2000\,\text{VAR}$$

Thus the $j5\,\Omega$ inductor is absorbing 2000 magnetizing vars.

[b] $\sum P_{\text{dev}} = 500\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{dev}} = 500 + 2500 = 3000\,\text{VAR}$

$$\sum Q_{\text{abs}} = 1000 + 2000 = 3000\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.25 **[a]** $\mathbf{I} = \dfrac{465\underline{/0^\circ}}{124 + j93} = 2.4 - j1.8 = 3\underline{/-36.87^\circ}\,\text{A(rms)}$

$$P = (3)^2(4) = 36\,\text{W}$$

[b] $Y_{\text{L}} = \dfrac{1}{120 + j90} = 5.33 - j4\text{ mS}$

$$\therefore \quad X_{\text{C}} = \frac{1}{-4 \times 10^{-3}} = -250\,\Omega$$

[c] $Z_{\text{L}} = \dfrac{1}{5.33 \times 10^{-3}} = 187.5\,\Omega$

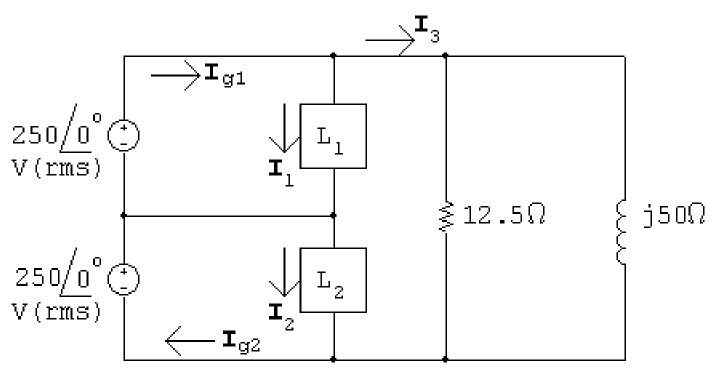
[d] $\mathbf{I} = \dfrac{465\underline{/0^\circ}}{191.5 + j3} = 2.43\underline{/-0.9^\circ}\,\text{A}$

$$P = (2.43)^2(4) = 23.58\,\text{W}$$

[e] $\% = \dfrac{23.58}{36}(100) = 65.5\%$

Thus the power loss after the capacitor is added is 65.6% of the power loss before the capacitor is added.

P 10.26 **[a]**



$$250\mathbf{I}_1^* = 7500 + j2500; \qquad \therefore \quad \mathbf{I}_1 = 30 - j10\,\text{A(rms)}$$

$$250\mathbf{I}_2^* = 2800 - j9600; \qquad \therefore \quad \mathbf{I}_2 = 11.2 + j38.4\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{500}{12.5} + \frac{500}{j50} = 40 - j10\,\text{A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 70 - j20\,\text{A}$$

$$S_{g1} = 250(70 + j20) = 17{,}500 + j5000\,\text{VA}$$

Thus the $\mathbf{V}_{g1}$ source is delivering 17.5 kW and 5000 magnetizing vars.

$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 51.2 + j28.4\,\text{A(rms)}$

$S_{g2} = 250(51.2 - j28.4) = 12{,}800 - j7100\,\text{VA}$

Thus the $\mathbf{V}_{g2}$ source is delivering 12.8 kW and absorbing 7100 magnetizing vars.

[b] $\sum P_{\text{gen}} = 17.5 + 12.8 = 30.3\,\text{kW}$

$$\sum P_{\text{abs}} = 7500 + 2800 + \frac{(500)^2}{12.5} = 30.3\text{kW} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 9600 + 5000 = 14.6\,\text{kVAR}$$

$$\sum Q_{\text{abs}} = 2500 + 7100 + \frac{(500)^2}{50} = 14.6\,\text{kVAR} = \sum Q_{\text{del}}$$

P 10.27 $S_1 = 1200 + 1196 = 2396 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_1 = \frac{2396}{120} = 19.97\,\text{A}$$

$S_2 = 860 + 600 + 240 = 1700 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_2 = \frac{1700}{120} = 14.167\,\text{A}$$

$S_3 = 4474 + 12{,}200 = 16{,}674 + j0\,\text{VA}$

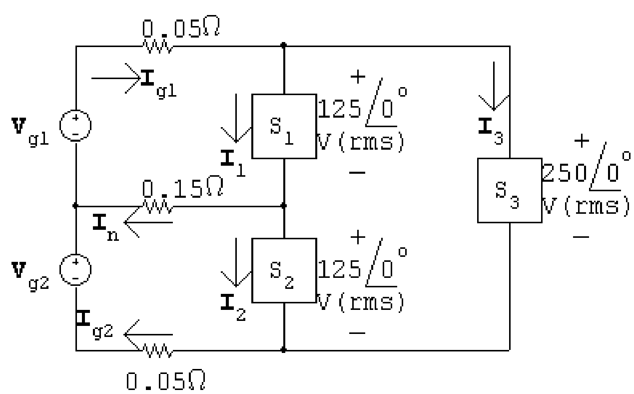
$$\therefore \quad \mathbf{I}_3 = \frac{16{,}674}{240} = 69.48\,\text{A}$$

$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 89.44\,\text{A}$

$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 83.64\,\text{A}$

Breakers will not trip since both feeder currents are less than 100 A.

P 10.28 **[a]**



$$\mathbf{I}_1 = \frac{4000 - j1000}{125} = 32 - j8\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{5000 - j2000}{125} = 40 - j16\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{10{,}000 + j0}{250} = 40 + j0\,\text{A (rms)}$$

$$\therefore\quad \mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 72 - j8\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = -8 + j8\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 80 - j16\,\text{A(rms)}$$

$$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + 0.15\mathbf{I}_n = 127.4 + j0.8\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.15\mathbf{I}_n + 125 + 0.05\mathbf{I}_{g2} = 130.2 - j2\,\text{V(rms)}$$

$$S_{g1} = [(127.4 + j0.8)(72 + j8)] = [9166.4 + j1076.8]\,\text{VA}$$

$$S_{g2} = [(130.2 - j2)(80 + j16)] = [10{,}448 + j1923.2]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b] $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 262.4\,\text{W}$

$$P_{0.15} = |\mathbf{I}_n|^2(0.15) = 19.2\,\text{W}$$

$$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 332.8\,\text{W}$$

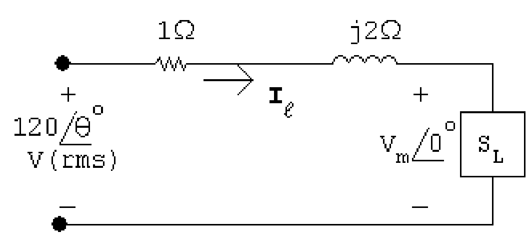
$$\sum P_{\text{dis}} = 262.4 + 19.2 + 332.8 + 4000 + 5000 + 10{,}000 = 19{,}614.4\,\text{W}$$

$$\sum P_{\text{dev}} = 9166.4 + 10{,}448 = 19{,}614.4\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 1000 + 2000 = 3000\,\text{VAR}$$

$$\sum Q_{\text{del}} = 1076.8 + 1923.2 = 3000\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.29 **[a]** Let $\mathbf{V}_\text{L} = V_m\angle 0^\circ$:



$$S_\text{L} = 600(0.8 + j0.6) = 480 + j360\,\text{VA}$$

$$\mathbf{I}_\ell^* = \frac{480}{V_m} + j\frac{360}{V_m}; \qquad \mathbf{I}_\ell = \frac{480}{V_m} - j\frac{360}{V_m}$$

$$120\underline{/\theta} = V_m + \left(\frac{480}{V_m} - j\frac{360}{V_m}\right)(1 + j2)$$

$$120V_m\underline{/\theta} = V_m^2 + (480 - j360)(1 + j2) = V_m^2 + 1200 + j600$$

$$120V_m \cos\theta = V_m^2 + 1200; \qquad 120V_m \sin\theta = 600$$

$$(120)^2V_m^2 = (V_m^2 + 1200)^2 + 600^2$$

$$14{,}400V_m^2 = V_m^4 + 2400V_m^2 + 18 \times 10^5$$

or

$$V_m^4 - 12{,}000V_m^2 + 18 \times 10^5 = 0$$

Solving,

$V_m = 108.85$ V and $V_m = 12.326$ V

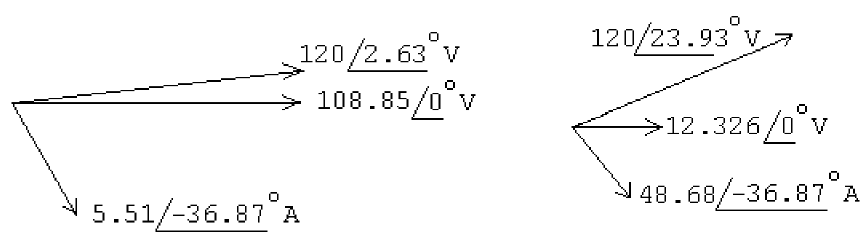
If $V_m = 108.85$ V:

$$\sin\theta = \frac{600}{(108.85)(120)} = 0.045935; \qquad \therefore \quad \theta = 2.63^\circ$$

If $V_m = 12.326$ V:

$$\sin\theta = \frac{600}{(12.326)(120)} = 0.405647; \qquad \therefore \quad \theta = 23.93^\circ$$

[b]



P 10.30 **[a]** $S_L = 20{,}000(0.85 + j0.53) = 17{,}000 + j10{,}535.65$ VA

$$125\mathbf{I}_L^* = (17{,}000 + j10{,}535.65); \quad \mathbf{I}_L^* = 136 + j84.29 \text{ A(rms)}$$

$$\therefore \quad \mathbf{I}_L = 136 - j84.29 \text{ A(rms)}$$

$$\mathbf{V}_s = 125 + (136 - j84.29)(0.01 + j0.08) = 133.10 + j10.04$$

$$= 133.48\underline{/4.31^\circ} \text{ V(rms)}$$

$$|\mathbf{V}_s| = 133.48 \text{ V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.01) = (160)^2(0.01) = 256$ W

[c] $\dfrac{(125)^2}{X_C} = -10{,}535.65; \qquad X_C = -1.483\,\Omega$

$$-\frac{1}{\omega C} = -1.48; \qquad C = \frac{1}{(1.48)(120\pi)} = 1788.59\,\mu\text{F}$$

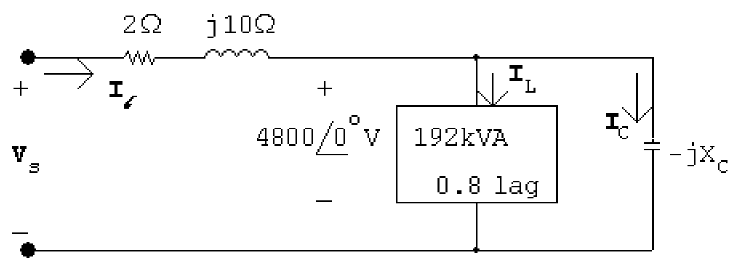
[d] $\mathbf{I}_\ell = 136 + j0\,\text{A(rms)}$

$$\mathbf{V}_s = 125 + 136(0.01 + j0.08) = 126.36 + j10.88$$
$$= 126.83\underline{/4.92^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 126.83\,\text{V(rms)}$$

[e] $P_\ell = (136)^2(0.01) = 184.96\,\text{W}$

P 10.31



$$\mathbf{I}_L = \frac{153{,}600 - j115{,}200}{4800} = 32 - j24\,\text{A(rms)}$$

$$\mathbf{I}_C = \frac{4800}{-jX_C} = j\frac{4800}{X_C} = jI_C$$

$$\mathbf{I}_\ell = 32 - j24 + jI_C = 32 + j(I_C - 24)$$

$$\mathbf{V}_s = 4800 + (2 + j10)[32 + j(I_C - 24)]$$
$$= (5104 - 10I_C) + j(272 + 2I_C)$$

$$|\mathbf{V}_s|^2 = (5104 - 10I_C)^2 + (272 + 2I_C)^2 = (4800)^2$$

$$\therefore \quad 104I_C^2 - 100{,}992I_C + 3{,}084{,}800 = 0$$

Solving, $\quad \mathbf{I}_C = 31.57\,\text{A(rms)}; \quad \mathbf{I}_C = 939.51\,\text{A(rms)}$

*Select the smaller value of I_C to minimize the magnitude of I_ℓ.

$$\therefore \quad X_C = -\frac{4800}{31.57} = -152.04$$

$$\therefore \quad C = \frac{1}{(152.04)(120\pi)} = 17.45\,\mu\text{F}$$

P 10.32 $Z_\text{L} = |Z_\text{L}|\underline{/\theta^\circ} = |Z_\text{L}|\cos\theta^\circ + j|Z_\text{L}|\sin\theta^\circ$

Thus $$|\mathbf{I}| = \frac{|\mathbf{V}_\text{Th}|}{\sqrt{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}}$$

Therefore $$P = \frac{0.5|\mathbf{V}_\text{Th}|^2|Z_\text{L}|\cos\theta}{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}$$

Let D = demoninator in the expression for P, then

$$\frac{dP}{d|Z_\text{L}|} = \frac{(0.5|\mathbf{V}_\text{Th}|^2\cos\theta)(D\cdot 1 - |Z_\text{L}|dD/d|Z_\text{L}|)}{D^2}$$

$$\frac{dD}{d|Z_\text{L}|} = 2(R_\text{Th} + |Z_\text{L}|\cos\theta)\cos\theta + 2(X_\text{Th} + |Z_\text{L}|\sin\theta)\sin\theta$$

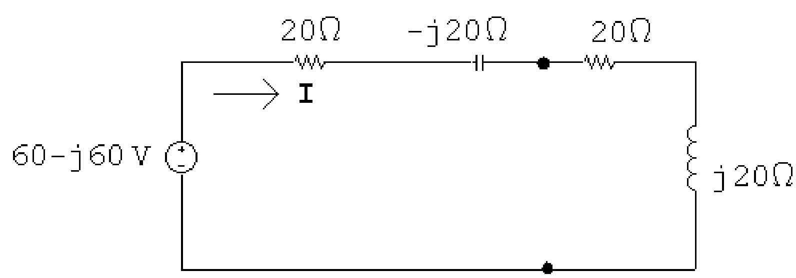
$$\frac{dP}{d|Z_\text{L}|} = 0 \quad \text{when} \quad D = |Z_\text{L}|\left(\frac{dD}{d|Z_\text{L}|}\right)$$

Substituting the expressions for D and $(dD/d|Z_\text{L}|)$ into this equation gives us the relationship $R^2_\text{Th} + X^2_\text{Th} = |Z_\text{L}|^2$ or $|Z_\text{Th}| = |Z_\text{L}|$.

P 10.33 **[a]** $Z_\text{Th} = j40\|40 - j40 = 20 - j20$

$$\therefore \quad Z_\text{L} = Z^*_\text{Th} = 20 + j20\,\Omega$$

[b] $$\mathbf{V}_\text{Th} = \frac{40}{40 + j40}(120) = 60 - j60\,\text{V}$$



$$\mathbf{I} = \frac{60 - j60}{40} = 1.5 - j1.5\,\text{A}$$

$$P_\text{load} = \frac{1}{2}|\mathbf{I}|^2(20) = 45\,\text{W}$$

P 10.34 **[a]** $$\frac{115.2 + j33.6 - 240}{Z_\text{Th}} + \frac{115.2 + j33.6}{80 - j60} = 0$$

$$\therefore \quad Z_\text{Th} = 40 - j100\,\Omega$$

$$\therefore \quad Z_\text{L} = 40 + j100\,\Omega$$

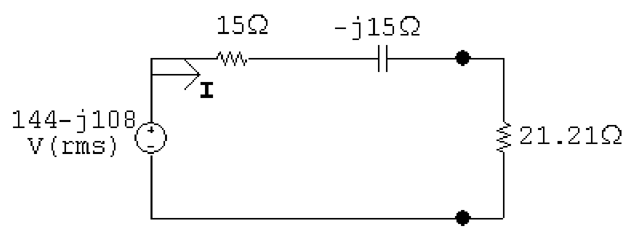
[b] $\mathbf{I} = \dfrac{240}{80} = 3\,\text{A(rms)}$

$$P = (3)^2(40) = 360\,\text{W}$$

P 10.35 **[a]** $Z_{\text{Th}} = [(3 + j4) \| - j8] + 7.32 - j17.24 = 15 - j15\,\Omega$

$$\therefore \quad R = |Z_{\text{Th}}| = 21.21\,\Omega$$

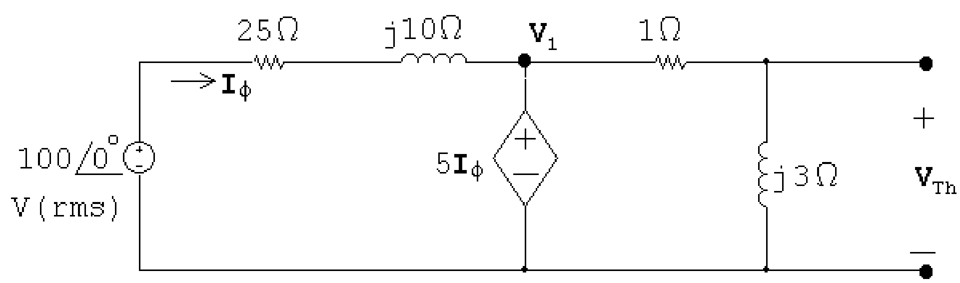
[b] $\mathbf{V}_{\text{Th}} = \dfrac{-j8}{3 - j4}(112.5) = 144 - j108\,\text{V(rms)}$



$$\mathbf{I} = \frac{144 - j108}{35.21 - j15} = 4.45 - j1.14$$

$$P = |\mathbf{I}|^2(21.21) = 447.35\,\text{W}$$

P 10.36 **[a]** Open circuit voltage:



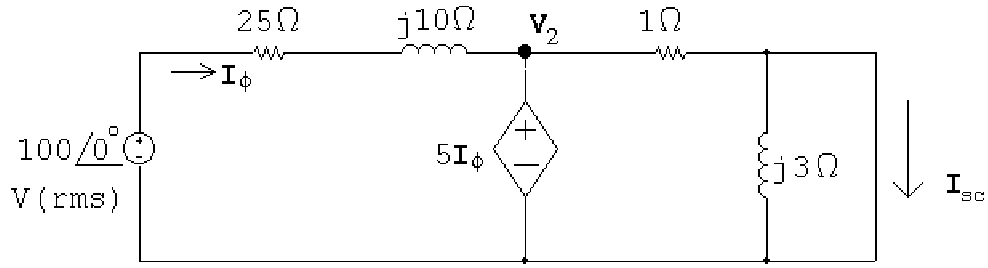
$$\mathbf{V}_1 = 5\mathbf{I}\phi = 5\frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$(25 + j10)\mathbf{I}_\phi = 100 - 5\mathbf{I}\phi$$

$$\mathbf{I}_\phi = \frac{100}{30 + j10} = 3 - j1\,\text{A}$$

$$\mathbf{V}_{\text{Th}} = \frac{j3}{1 + j3}(5\mathbf{I}_\phi) = 15\underline{/0^\circ}\,\text{V}$$

Short circuit current:



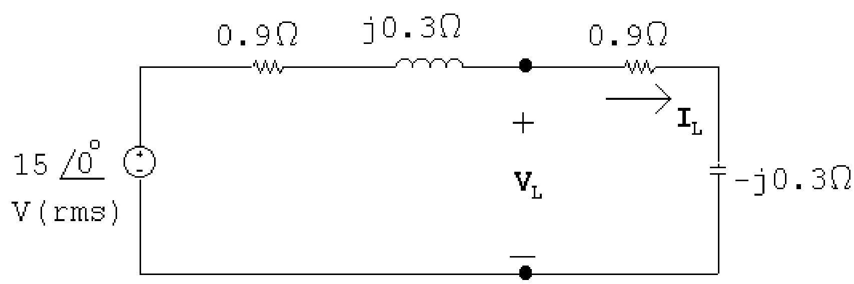
$$\mathbf{V}_2 = 5\mathbf{I}_\phi = \frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$\mathbf{I}_\phi = 3 - j1\,\text{A}$$

$$\mathbf{I}_{\text{sc}} = \frac{5\mathbf{I}_\phi}{1} = 15 - j5\,\text{A}$$

$$Z_{\text{Th}} = \frac{15}{15 - j5} = 0.9 + j0.3\,\Omega$$

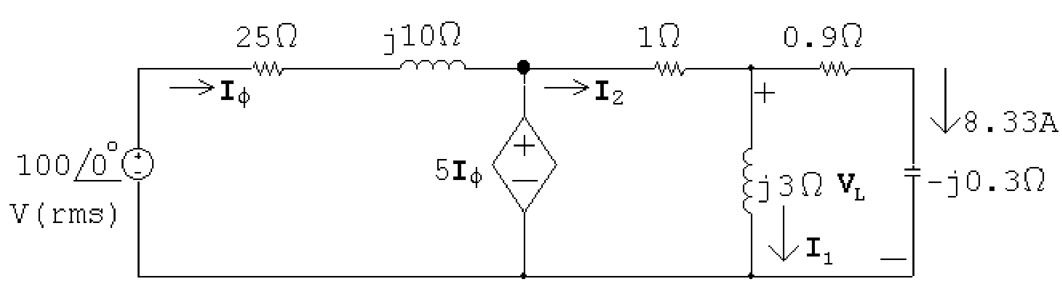
$$Z_L = Z_{\text{Th}}^* = 0.9 - j0.3\,\Omega$$



$$\mathbf{I}_{\text{L}} = \frac{15}{1.8} = 8.33\,\text{A(rms)}$$

$$P = |\mathbf{I}_L|^2(0.9) = 62.5\,\text{W}$$

[b] $\mathbf{V}_{\text{L}} = (0.9 - j0.3)(8.33) = 7.5 - j2.5\,\text{V(rms)}$



$$\mathbf{I}_1 = \frac{\mathbf{V}_{\text{L}}}{j3} = -0.833 - j2.5\,\text{A(rms)}$$

$$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_L = 7.5 - j2.5\,\mathrm{A(rms)}$$

$$5\mathbf{I}_\phi = \mathbf{I}_2 + \mathbf{V}_L \qquad \therefore \qquad \mathbf{I}_\phi = 3 - j1\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{d.s.}} = \mathbf{I}_\phi - \mathbf{I}_2 = -4.5 + j1.5\,\mathrm{A}$$

$$S_g = -100(3 + j1) = -300 - j100\,\mathrm{VA}$$

$$S_{\mathrm{d.s.}} = 5(3 - j1)(-4.5 - j1.5) = -75 + j0\,\mathrm{VA}$$

$$P_{\mathrm{dev}} = 300 + 75 = 375\,\mathrm{W}$$

$$\%\ \text{developed} = \frac{62.5}{375}(100) = 16.67\%$$

Checks:

$$P_{25\Omega} = (10)(25) = 250\,\mathrm{W}$$

$$P_{1\Omega} = (62.5)(1) = 62.5\,\mathrm{W}$$

$$P_{0.9\Omega} = 62.5\,\mathrm{W}$$

$$\sum P_{\mathrm{abs}} = 250 + 62.5 + 62.5 = 375\ \mathrm{W} = \sum P_{\mathrm{dev}}$$

$$Q_{j10} = (10)(10) = 100\,\mathrm{VAR}$$

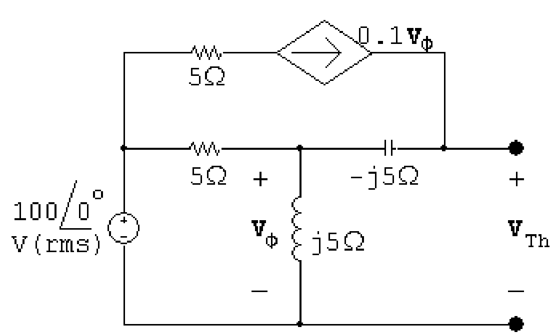
$$Q_{j3} = (6.94)(3) = 20.83\,\mathrm{VAR}$$

$$Q_{-j0.3} = (69.4)(-0.3) = -20.83\,\mathrm{VAR}$$

$$Q_{\mathrm{source}} = -100\,\mathrm{VAR}$$

$$\sum Q = 100 + 20.83 - 20.83 - 100 = 0$$

P 10.37 **[a]** Open circuit voltage:

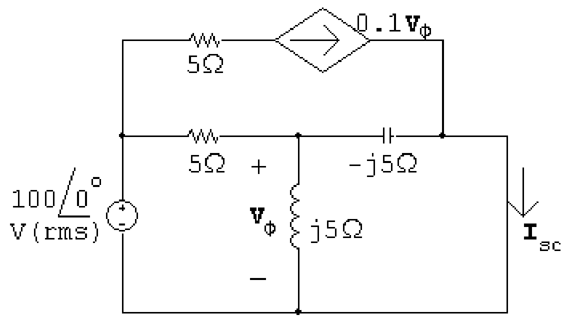


$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$$\therefore \quad \mathbf{V}_\phi = 40 + j80\,\mathrm{V(rms)}$$

$$\mathbf{V}_{\rm Th} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60\,\text{V(rms)}$$

Short circuit current:



$$\mathbf{I}_{\rm sc} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

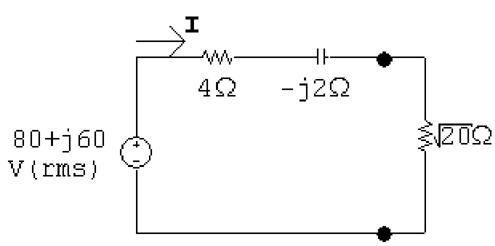
$$\therefore \quad \mathbf{V}_\phi = 100\,\text{V(rms)}$$

$$\mathbf{I}_{\rm sc} = (0.1 + j0.2)(100) = 10 + j20\,\text{A(rms)}$$

$$Z_{\rm Th} = \frac{\mathbf{V}_{\rm Th}}{\mathbf{I}_{\rm sc}} = \frac{80 + j60}{10 + j20} = 4 - j2\,\Omega$$

$$\therefore \quad R_o = |Z_{\rm Th}| = 4.47\,\Omega$$

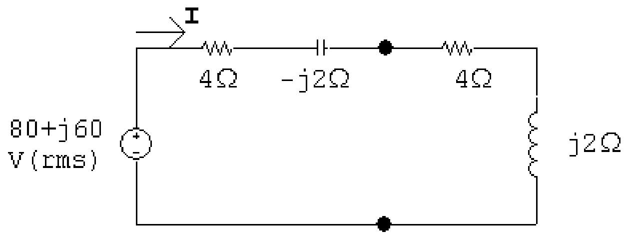
[b]



$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82\,\text{A(rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17\,\text{W}$$

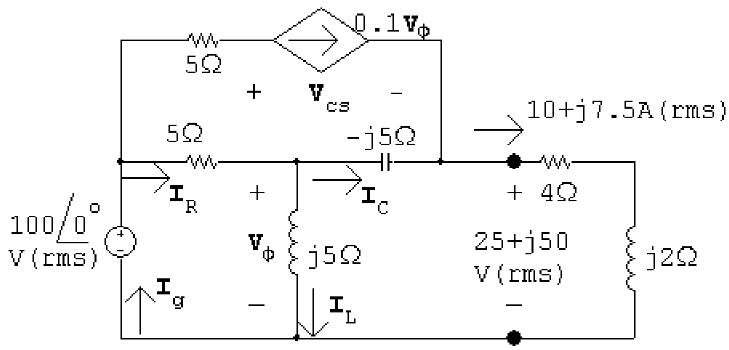
[c]



$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5\,\text{A(rms)}$$

$$P = (10^2 + 7.5^2)(4) = 625\,\text{W}$$

[d]



$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi - (25 + j50)}{-j5} = 0$$

$$\mathbf{V}_\phi = 50 + j25\,\text{V(rms)}$$

$$0.1\mathbf{V}_\phi = 5 + j2.5$$

$$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$$

$$\mathbf{I}_C = 5 + j5\,\text{A(rms)}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_\phi}{j5} = 5 - j10\,\text{A(rms)}$$

$$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5\,\text{A(rms)}$$

$$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5\,\text{A(rms)}$$

$$S_g = -100\mathbf{I}_g^* = -1500 - j250\,\text{VA}$$

$$100 = 5(5 + j2.5) + \mathbf{V}_{\text{cs}} + 25 + j50 \quad \therefore \quad \mathbf{V}_{\text{cs}} = 50 - j62.5\,\text{V(rms)}$$

$$S_{\text{cs}} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5\,\text{VA}$$

Thus,

$$\sum P_{\text{dev}} = 1500$$

$$\%\text{ delivered to } R_o = \frac{625}{1500}(100) = 41.67\%$$

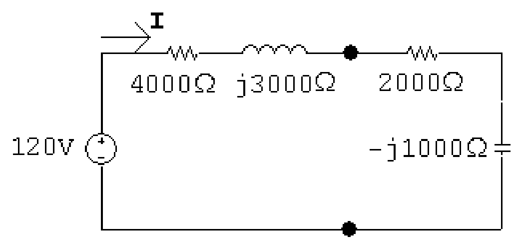
P 10.38 **[a]** First find the Thévenin equivalent:

$$j\omega L = j3000\,\Omega$$

$$Z_{\text{Th}} = 6000\|12{,}000 + j3000 = 4000 + j3000\,\Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{12{,}000}{6000 + 12{,}000}(180) = 120\underline{/0^\circ}\text{ V}$$

$$\frac{-j}{\omega C} = -j1000\,\Omega$$



$$\mathbf{I} = \frac{120}{6000 + j2000} = 18 - j6\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(2000) = 360\,\text{mW}$$

[b] Set $C_o = 0.1\,\mu$F so $-j/\omega C = -j2000\,\Omega$ $\quad j3000 - j2000 = j1000\,\Omega$
Set R_o as close as possible to

$$R_o = \sqrt{4000^2 + 1000^2} = 4123.1\,\Omega$$

$$\therefore \quad R_o = 4000\,\Omega$$

[c] $\mathbf{I} = \dfrac{120}{8000 + j1000} = 14.77 - j1.85\text{ mA}$

$$P = \frac{1}{2}|\mathbf{I}|^2(4000) = 443.1\,\text{mW}$$

Yes; $\quad 443.1\,\text{mW} > 360\,\text{mW}$

[d] $\mathbf{I} = \dfrac{120}{8000} = 15\,\text{mA}$

$$P = \frac{1}{2}(0.015)^2(4000) = 450\,\text{mW}$$

[e] $R_o = 4000\,\Omega$; $\quad C_o = 66.67\,\text{nF}$

[f] Yes; $\quad 450\,\text{mW} > 443.1\,\text{mW}$

P 10.39 **[a]** Set $C_o = 0.1\,\mu$F, so $-j/\omega C = -j2000\,\Omega$; also set $R_o = 4123.1\,\Omega$

$$\mathbf{I} = \frac{120}{8123.1 + j1000} = 14.55 - j1.79\text{ mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4123.1) = 443.18\,\text{mW}$$

[b] Yes; $443.18\,\text{mW} > 360\,\text{mW}$

[c] Yes; $443.18\,\text{mW} < 450\,\text{mW}$

P 10.40 **[a]** $\dfrac{1}{\omega C} = 100\,\Omega;\qquad C = \dfrac{1}{(100)(120\pi)} = 26.53\,\mu\text{F}$

[b] $\mathbf{I}_{\text{wo}} = \dfrac{13{,}800}{300} + \dfrac{13{,}800}{j100} = 46 - j138\,\text{A(rms)}$

$$\mathbf{V}_{\text{swo}} = 13{,}800 + (46 - j138)(1.5 + j12) = 15{,}525 + j345$$
$$= 15{,}528.83\underline{/1.27^\circ}\,\text{V(rms)}$$

$$\mathbf{I}_{\text{w}} = \frac{13{,}800}{300} = 46\,\text{A(rms)}$$

$$\mathbf{V}_{\text{sw}} = 13{,}800 + 46(1.5 + j12) = 13{,}869 + j552 = 13{,}879.98\underline{/2.28^\circ}\,\text{V(rms)}$$

$$\%\text{ increase } = \left(\frac{15{,}528.82}{13{,}879.98} - 1\right)(100) = 11.88\%$$

[c] $P_{\ell\text{wo}} = |46 - j138|^2 1.5 = 31.74\,\text{kW}$

$$P_{\ell\text{w}} = 46^2(1.5) = 3174\,\text{W}$$

$$\%\text{ increase } = \left(\frac{31{,}740}{3174} - 1\right)(100) = 900\%$$

P 10.41 **[a]** $S_o = \text{ original load } = 1600 + j\dfrac{1600}{0.8}(0.6) = 1600 + j1200\,\text{kVA}$

$$S_f = \text{ final load } = 1920 + j\frac{1920}{0.96}(0.28) = 1920 + j560\,\text{kVA}$$

$$\therefore\quad Q_{\text{added}} = 560 - 1200 = -640\,\text{kVAR}$$

[b] deliver

[c] $S_{\text{a}} = \text{ added load } = 320 - j640 = 715.54\underline{/-63.43^\circ}\,\text{kVA}$

$$\text{pf } = \cos(-63.43) = 0.4472\text{ leading}$$

[d] $\mathbf{I}_\mathrm{L}^* = \dfrac{(1600 + j1200) \times 10^3}{2400} = 666.67 + j500\,\mathrm{A}$

$\mathbf{I}_\mathrm{L} = 666.67 - j500 = 833.33\underline{/-36.87^\circ}\,\mathrm{A(rms)}$

$|\mathbf{I}_\mathrm{L}| = 833.33\,\mathrm{A(rms)}$

[e] $\mathbf{I}_\mathrm{L}^* = \dfrac{(1920 + j560) \times 10^3}{2400} = 800 + j233.33$

$\mathbf{I}_\mathrm{L} = 800 - j233.33 = 833.33\underline{/-16.26^\circ}\,\mathrm{A(rms)}$

$|\mathbf{I}_\mathrm{L}| = 833.33\,\mathrm{A(rms)}$

P 10.42 **[a]** $P_\text{before} = P_\text{after} = (833.33)^2(0.05) = 34{,}722.22\,\mathrm{W}$

[b] $\mathbf{V}_s(\text{before}) = 2400 + (666.67 - j500)(0.05 + j0.4)$

$= 2633.33 + j241.67 = 2644.4\underline{/5.24^\circ}\,\mathrm{V(rms)}$

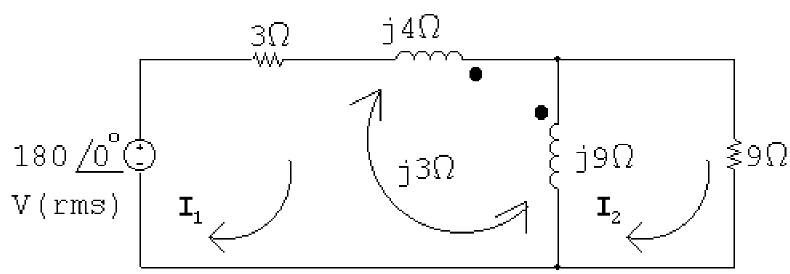
$|\mathbf{V}_s(\text{before})| = 2644.4\,\mathrm{V(rms)}$

$\mathbf{V}_s(\text{after}) = 2400 + (800 + j233.33)(0.05 + j0.4)$

$= 2346.67 + j331.67 = 2369.99\underline{/8.04^\circ}\,\mathrm{V(rms)}$

$|\mathbf{V}_s(\text{after})| = 2369.99\,\mathrm{V(rms)}$

P 10.43 **[a]**



$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 + j3(\mathbf{I}_2 - \mathbf{I}_1) + j9(\mathbf{I}_1 - \mathbf{I}_2) - j3\mathbf{I}_1$

$0 = 9\mathbf{I}_2 + j9(\mathbf{I}_2 - \mathbf{I}_1) + j3\mathbf{I}_1$

Solving,

$\mathbf{I}_1 = 18 - j18\,\mathrm{A(rms)}; \qquad \mathbf{I}_2 = 12\underline{/0^\circ}\,\mathrm{A(rms)}$

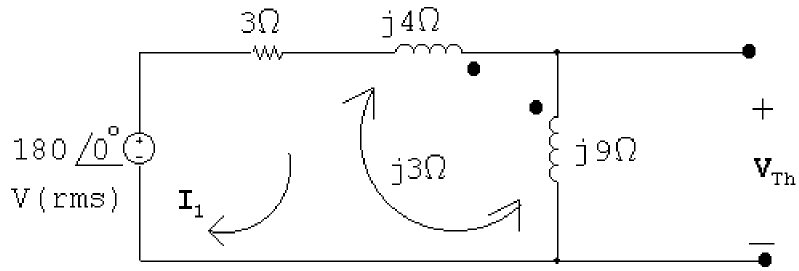
$\therefore \quad \mathbf{V}_o = (12)(9) = 108\underline{/0^\circ}\,\mathrm{V(rms)}$

[b] $P = (12)^2(9) = 1296\,\mathrm{W}$

[c] $S_g = -(180)(18 + j18) = -3240 - j3240\,\mathrm{VA} \quad \therefore \quad P_g = -3240\,\mathrm{W}$

$\%\ \text{delivered} = \dfrac{1296}{3240}(100) = 40\%$

P 10.44 **[a]** Open circuit voltage:

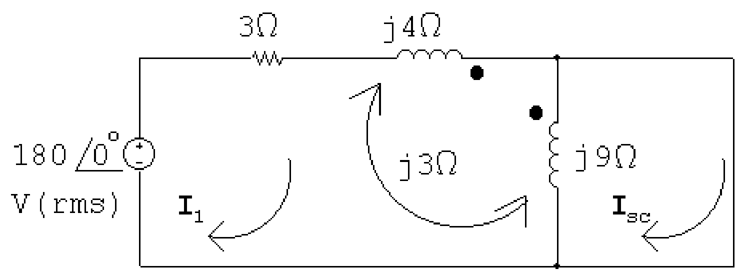


$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 - j3\mathbf{I}_1 + j9\mathbf{I}_1 - j3\mathbf{I}_1$$

$$\therefore \quad \mathbf{I}_1 = \frac{180}{3 + j7} = 9.31 - j21.72\,\mathrm{A(rms)}$$

$$\mathbf{V}_{\mathrm{Th}} = j9\mathbf{I}_1 - j3\mathbf{I}_1 = j6\mathbf{I}_1 = 130.34 + j55.86\,\mathrm{V} = 141.81\underline{/23.20^\circ}\ \mathrm{V(rms)}$$

Short circuit current:



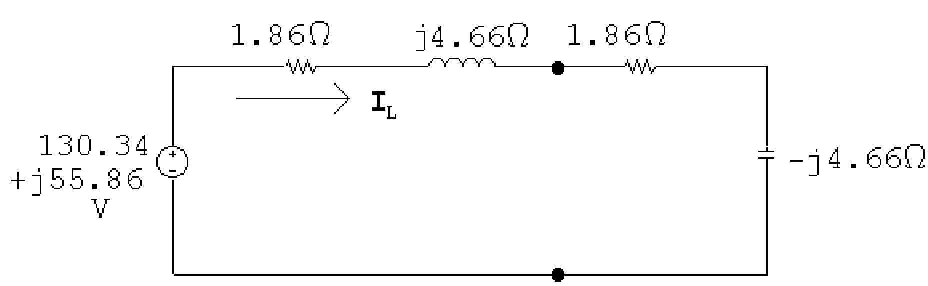
$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 + j3(\mathbf{I}_{\mathrm{sc}} - \mathbf{I}_1) + j9(\mathbf{I}_1 - \mathbf{I}_{\mathrm{sc}}) - j3\mathbf{I}_1$$

$$0 = j9(\mathbf{I}_{\mathrm{sc}} - \mathbf{I}_1) + j3\mathbf{I}_1$$

Solving,

$$\mathbf{I}_{\mathrm{sc}} = 20 - j20\,\mathrm{A} \quad \mathbf{I}_1 = 30 - j20\ \mathrm{A}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{130.34 + j55.86}{20 - j20} = 1.86 + j4.66\,\Omega$$



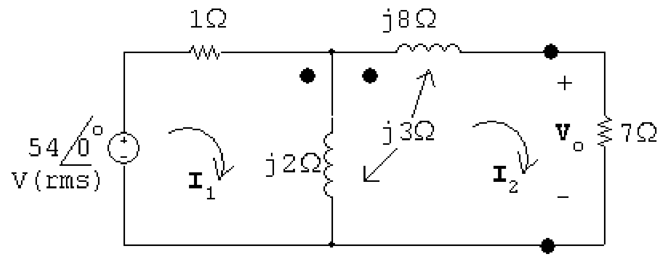
$$\mathbf{I}_{\mathrm{L}} = \frac{130.34 + j55.86}{3.72} = 35 + j15 = 38.08\underline{/23.20^\circ}\ \mathrm{A}$$

$$P_{\rm L} = (38.12)^2(1.86) = 2700\,\text{W}$$

[b] $\mathbf{I}_1 = \dfrac{Z_o + j9}{j6}\mathbf{I}_2 = \dfrac{1.86 - j4.66 + j9}{j6}(35 + j15) = 30\underline{/0^\circ}\,\text{A(rms)}$

$$P_{\rm dev} = (180)(30) = 5400\,\text{W}$$

P 10.45 **[a]**



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j3\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j3\mathbf{I}_2 + j8\mathbf{I}_2 + j3(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$\mathbf{I}_1 = 12 - j21\,\text{A(rms)}; \qquad \mathbf{I}_2 = -3\,\text{A(rms)}$

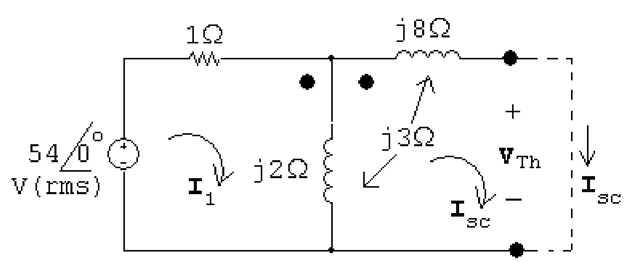
$\mathbf{V}_o = 7\mathbf{I}_2 = -21\underline{/180^\circ}\,\text{V(rms)}$

[b] $P = |\mathbf{I}_2|^2(7) = 63\,\text{W}$

[c] $P_g = (54)(12) = 648\,\text{W}$

$$\%\text{ delivered } = \frac{63}{648}(100) = 9.72\%$$

P 10.46 **[a]**



Open circuit:

$$\mathbf{V}_{\rm Th} = -j3\mathbf{I}_1 + j_2\mathbf{I}_1 = -j\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{54}{1 + j2} = 10.8 - j21.6\ \text{A}$$

$$\mathbf{V}_{\rm Th} = -21.6 - j10.8\,\text{V}$$

Short circuit:

$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_{\rm sc}) + j3\mathbf{I}_{\rm sc}$$

$$0 = j2(\mathbf{I}_{\rm sc} - \mathbf{I}_1) - j3\mathbf{I}_{\rm sc} + j8\mathbf{I}_{\rm sc} + j3(\mathbf{I}_1 - \mathbf{I}_{\rm sc})$$

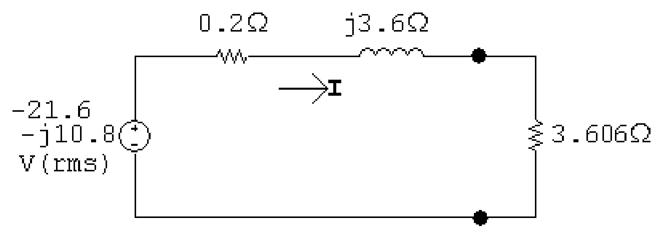
Solving,

$$\mathbf{I}_{\rm sc} = -3.32 + j5.82$$

$$Z_{\rm Th} = \frac{\mathbf{V}_{\rm Th}}{\mathbf{I}_{\rm sc}} = \frac{-21.6 - j10.8}{-3.32 + j5.82} = 0.2 + 3.6j = 3.6\underline{/86.82^\circ}\,\Omega$$

$$\therefore \quad R_{\rm L} = |Z_{\rm Th}| = 3.606\,\Omega$$

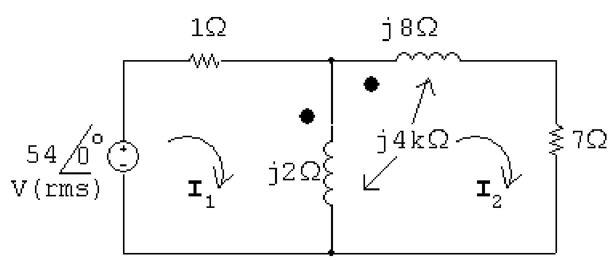
[b]



$$\mathbf{I} = \frac{-21.6 - j10.8}{3.806 + j3.6} = 4.610\underline{/163.2^\circ}\text{ A}$$

$P = |\mathbf{I}|^2(3.6) = 76.6\,\text{W}$, which is greater than when $R_L = 7\,\Omega$

P 10.47 **[a]**



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j4k\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j4k\mathbf{I}_2 + j8\mathbf{I}_2 + j4k(\mathbf{I}_1 - \mathbf{I}_2)$$

Place the equations in standard form:

$$54 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$$

$$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{54 - \mathbf{I}_2 j(4k - 2)}{(1 + j2)}$$

Substituting,

$$\mathbf{I}_2 = -\frac{j54(4k - 2)}{[7 + j(10 - 8k)](1 + j2) + (4k - 2)^2}$$

For $\mathbf{V}_o = 0, \mathbf{I}_2 = 0$, so if $4k - 2 = 0$, then $k = 0.5$.

[b] When $\mathbf{I}_2 = 0$

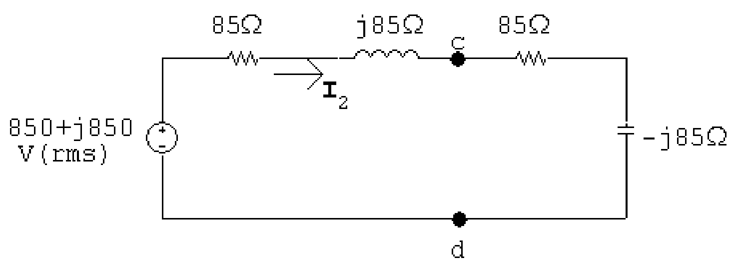
$$\mathbf{I}_1 = \frac{54}{1+j2} = 10.8 - j21.6\,\text{A(rms)}$$

$$P_g = (54)(10.8) = 583.2\,\text{W}$$

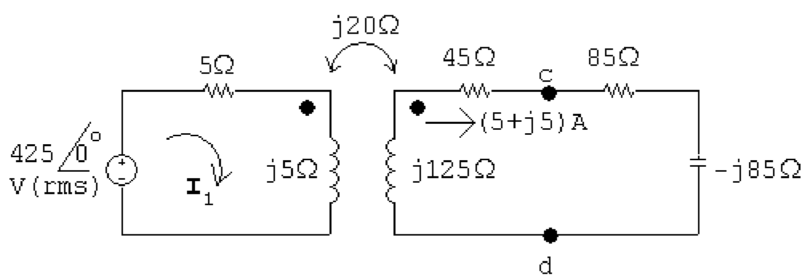
Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(1) = 583.2\,\text{W}$$

P 10.48 **[a]** From Problem 9.67, $Z_{\text{Th}} = 85 + j85\,\Omega$ and $\mathbf{V}_{\text{Th}} = 850 + j850\,\text{V}$. Thus, for maximum power transfer, $Z_{\text{L}} = Z_{\text{Th}}^* = 85 - j85\,\Omega$:



$$\mathbf{I}_2 = \frac{850 + j850}{170} = 5 + j5\,\text{A}$$



$$425\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - j20(5 + j5)$$

$$\therefore \quad \mathbf{I}_1 = \frac{325 + j100}{5 + j5} = 42.5 - j22.5\,\text{A}$$

$$S_g(\text{del}) = 425(42.5 + j22.5) = 18{,}062.5 + j9562.5\,\text{VA}$$

$$P_g = 18{,}062.5\,\text{W}$$

[b] $P_{\text{loss}} = |\mathbf{I}_1|^2(5) + |\mathbf{I}_2|^2(45) = 11{,}562.5 + 2250 = 13{,}812.5\,\text{W}$

$$\%\text{ loss in transformer} = \frac{18{,}062.5 - 13{,}812.5}{18{,}062.5}(100) = 23.53\%$$

P 10.49 **[a]** From Problem 9.70,

$$Z_{\text{ab}} = 100 + j136.26 \quad \text{so}$$

$$\mathbf{I}_1 = \frac{50}{100 + j13.74 + 100 + 136.26} = \frac{50}{200 + j150} = 160 - j120\,\text{mA}$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{22}}\mathbf{I}_1 = \frac{j270}{800 + j600}(0.16 - j0.12) = 51.84 + j15.12\,\text{mA}$$

$$\mathbf{V}_L = (300 + j100)(51.84 + j15.12)10^3 = 14.04 + j9.72\text{ V}$$

$$|\mathbf{V}_L| = 17.08\,\text{V}$$

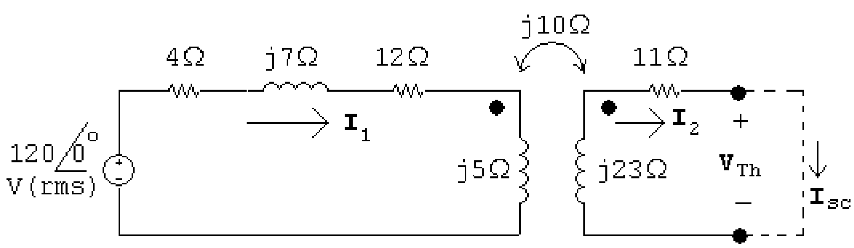
[b] $P_g(\text{ideal}) = 50(0.16) = 8\,\text{W}$

$$P_g(\text{practical}) = 8 - |\mathbf{I}_1|^2(100) = 4\,\text{W}$$

$$P_{\text{L}} = |\mathbf{I}_2|^2(300) = 874.8\text{ mW}$$

$$\%\text{ delivered} = \frac{0.8748}{4}(100) = 21.87\%$$

P 10.50 **[a]**



Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{120}{16 + j12}(j10) = 36 + j48\,\text{V}$$

Short circuit:

$$(16 + j12)\mathbf{I}_1 - j10\mathbf{I}_{\text{sc}} = 120$$

$$-j10\mathbf{I}_1 + (11 + j23)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 2.4\underline{/0^\circ}\text{ A}$$

$$Z_{\text{Th}} = \frac{36 + j48}{2.4} = 15 + j20\,\Omega$$

$$\therefore\quad Z_{\text{L}} = Z_{\text{Th}}^* = 15 - j20\,\Omega$$

$$\mathbf{I}_{\text{L}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}} + Z_L} = \frac{36 + j48}{30} = 1.2 + j1.6\,\text{A(rms)} = 2.0\underline{/53.13^\circ}\text{ A(rms)}$$

$$P_{\text{L}} = |\mathbf{I}_{\text{L}}|^2(15) = 60\,\text{W}$$

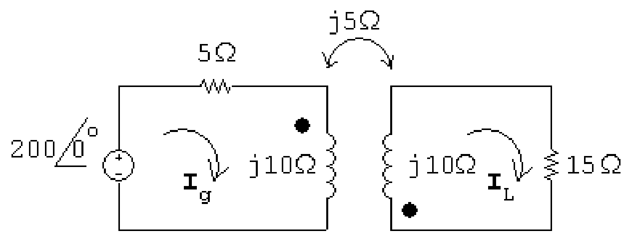
[b] $\mathbf{I}_1 = \dfrac{Z_{22}\mathbf{I}_2}{j\omega M} = \dfrac{26 + j3}{j10}(1.2 + j1.6) = 5.23\underline{/-30.29^\circ}\,\text{A)rms)}$

$$P_{\text{transformer}} = (120)(5.23)\cos(-30.29^\circ) - (5.23)^2(4) = 432.8\,\text{W}$$

$$\%\text{ delivered } = \frac{60}{432.8}(100) = 13.86\%$$

P 10.51 **[a]** $j\omega L_1 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$$



$$200 = (5 + j10)\mathbf{I}_g + j5\mathbf{I}_{\text{L}}$$

$$0 = j5\mathbf{I}_g + (15 + j10)\mathbf{I}_{\text{L}}$$

Solving,

$\mathbf{I}_g = 10 - j15\,\text{A}; \qquad \mathbf{I}_{\text{L}} = -5\,\text{A}$

Thus,

$$i_g = 18.03\cos(10{,}000t - 56.31^\circ)\,\text{A}$$

$$i_{\text{L}} = 5\cos(10{,}000t - 180^\circ)\,\text{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{0.5}{\sqrt{1}} = 0.5$

[c] When $t = 50\pi\,\mu\text{s}$:

$$10{,}000t = (10{,}000)(50\pi) \times 10^{-6} = 0.5\pi\text{ rad } = 90^\circ$$

$$i_g(50\pi\,\mu\text{s}) = 18.03\cos(90^\circ - 56.31^\circ) = 15\,\text{A}$$

$$i_{\text{L}}(50\pi\,\mu\text{s}) = 5\cos(90^\circ - 180^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(10^{-3})(15)^2 + 0 + 0 = 112.5\,\text{mJ}$$

When $t = 100\pi\,\mu\text{s}$:

$$10{,}000t = (10^4)(100\pi) \times 10^{-6} = \pi = 180^\circ$$

$$i_g(100\pi\,\mu\text{s}) = 18.03\cos(180 - 56.31^\circ) = -10\,\text{A}$$

$$i_{\text{L}}(100\pi\,\mu\text{s}) = 5\cos(180 - 180^\circ) = 5\,\text{A}$$

$$w = \frac{1}{2}(10^{-3})(10)^2 + \frac{1}{2}(10^{-3})(5)^2 + 0.5 \times 10^{-3}(-10)(5) = 37.5\,\text{mJ}$$

[d] From (a), $I_m = 5$ A,

$$\therefore \quad P = \frac{1}{2}(5)^2(15) = 187.5\,\text{W}$$

[e] Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{200}{5 + j10}(-j5) = -80 - j40\,\text{V}$$

Short circuit:

$$200 = (5 + j10)\mathbf{I}_1 + j5\mathbf{I}_{\text{sc}}$$

$$0 = j10\mathbf{I}_{\text{sc}} + j5\mathbf{I}_1$$

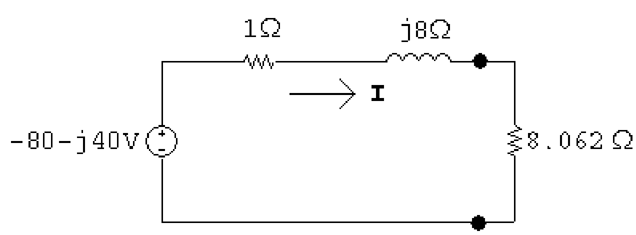
Solving,

$$\mathbf{I}_{\text{sc}} = -11.094\underline{/123.69^\circ}\text{ A}; \quad \mathbf{I}_1 = 22.188\underline{/-56.31^\circ}\text{ A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-80 - j40}{11.094\underline{/123.69^\circ}} = 1 + j8\,\Omega$$

$$\therefore \quad R_{\text{L}} = 8.962\,\Omega$$

[f]



$$\mathbf{I} = \frac{-80 - j40}{9.062 + j8} = 7.399\underline{/165.13^\circ}\text{ A}$$

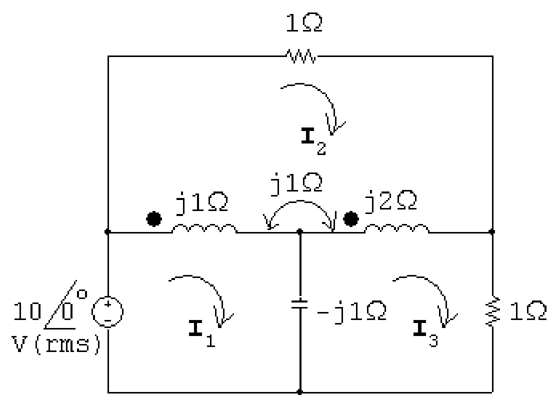
$$P = \frac{1}{2}(7.399)^2(8.062) = 220.70\text{ W}$$

[g] $Z_{\text{L}} = Z_{\text{Th}}^* = 1 - j8\,\Omega$

[h] $\mathbf{I} = \dfrac{-80 - j40}{2} = 44.72\underline{/-153.43^\circ}\text{ A}$

$$P = \frac{1}{2}(44.72)^2(1) = 1000\,\text{W}$$

P 10.52 **[a]**



$$10 = j1(\mathbf{I}_1 - \mathbf{I}_2) + j1(\mathbf{I}_3 - \mathbf{I}_2) - j1(\mathbf{I}_1 - \mathbf{I}_3)$$

$$0 = 1\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_3) + j1(\mathbf{I}_2 - \mathbf{I}_1) + j1(\mathbf{I}_2 - \mathbf{I}_1) + j1(\mathbf{I}_2 - \mathbf{I}_3)$$

$$0 = 1\mathbf{I}_3 - j1(\mathbf{I}_3 - \mathbf{I}_1) + j2(\mathbf{I}_3 - \mathbf{I}_2) + j1(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

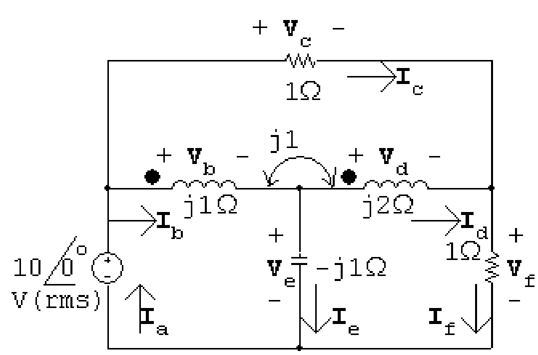
$\mathbf{I}_1 = 6.25 + j7.5\,\mathrm{A(rms)}$; $\quad \mathbf{I}_2 = 5 + j2.5\,\mathrm{A(rms)}$; $\quad \mathbf{I}_3 = 5 - j2.5\,\mathrm{A(rms)}$

$\mathbf{I}_\mathrm{a} = \mathbf{I}_1 = 6.25 + j7.5\,\mathrm{A}$ $\qquad \mathbf{I}_\mathrm{b} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25 + j5\,\mathrm{A}$

$\mathbf{I}_\mathrm{c} = \mathbf{I}_2 = 5 + j2.5\,\mathrm{A}$ $\qquad \mathbf{I}_\mathrm{d} = \mathbf{I}_3 - \mathbf{I}_2 = -j5\,\mathrm{A}$

$\mathbf{I}_\mathrm{e} = \mathbf{I}_1 - \mathbf{I}_3 = 1.25 + j10\,\mathrm{A}$ $\qquad \mathbf{I}_\mathrm{f} = \mathbf{I}_3 = 5 - j2.5\,\mathrm{A}$

[b]



$\mathbf{V}_\mathrm{a} = 10\,\mathrm{V}$ $\qquad \mathbf{V}_\mathrm{b} = j1\mathbf{I}_\mathrm{b} + j1\mathbf{I}_\mathrm{d} = j1.25\,\mathrm{V}$

$\mathbf{V}_\mathrm{c} = 1\mathbf{I}_\mathrm{c} = 5 + j2.5\,\mathrm{V}$ $\qquad \mathbf{V}_\mathrm{d} = j2\mathbf{I}_\mathrm{d} + j1\mathbf{I}_\mathrm{b} = 5 + j1.25\,\mathrm{V}$

$\mathbf{V}_\mathrm{e} = -j1\mathbf{I}_\mathrm{e} = 10 - j1.25\,\mathrm{V}$ $\qquad \mathbf{V}_\mathrm{f} = 1\mathbf{I}_\mathrm{f} = 5 - j2.5\,\mathrm{V}$

$$S_\mathrm{a} = -10\mathbf{I}_\mathrm{a}^* = -62.5 + j75\,\mathrm{VA}$$

$$S_\mathrm{b} = \mathbf{V}_\mathrm{b}\mathbf{I}_\mathrm{b}^* = 6.25 + j1.5625\,\mathrm{VA}$$

$$S_c = \mathbf{V}_c\mathbf{I}_c^* = 31.25 + j0\,\text{VA}$$

$$S_d = \mathbf{V}_d\mathbf{I}_d^* = -6.25 + j25\,\text{VA}$$

$$S_e = \mathbf{V}_e\mathbf{I}_e^* = 0 - j101.5625\,\text{VA}$$

$$S_f = \mathbf{V}_f\mathbf{I}_f^* = 31.25\,\text{VA}$$

[c] $\sum P_{\text{dev}} = 62.5\,\text{W}$

$$\sum P_{\text{abs}} = 6.25 + 31.25 - 6.25 + 31.25 = 62.5\,\text{W}$$

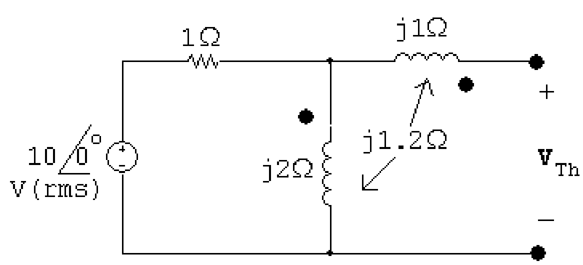
Note that the total power absorbed by the coupled coils is zero:
$6.25 - 6.25 = 0 = P_b + P_d$

[d] $\sum Q_{\text{dev}} = 101.5625\,\text{VAR}$
The capacitor is developing magnetizing vars.

$$\sum Q_{\text{abs}} = 75 + 1.5625 + 25 = 101.5625\,\text{VAR}$$

$\sum Q$ absorbed by the coupled coils is $Q_b + Q_d = 26.5625\,\text{VAR}$

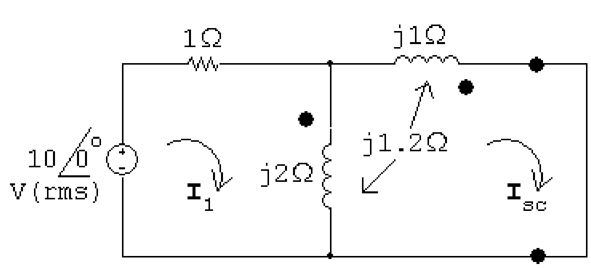
P 10.53 Open circuit voltage:



$$\mathbf{I}_1 = \frac{10/0^\circ}{1 + j2} = 2 - j4\,\text{A}$$

$$\mathbf{V}_{\text{Th}} = j2\mathbf{I}_1 + j1.2\mathbf{I}_1 = j3.2\mathbf{I}_1 = 12.8 + j6.4 = 14.31/26.57^\circ\ \text{V}$$

Short circuit current:



$$10/0^\circ = (1 + j2)\mathbf{I}_1 - j3.2\mathbf{I}_{\text{sc}}$$

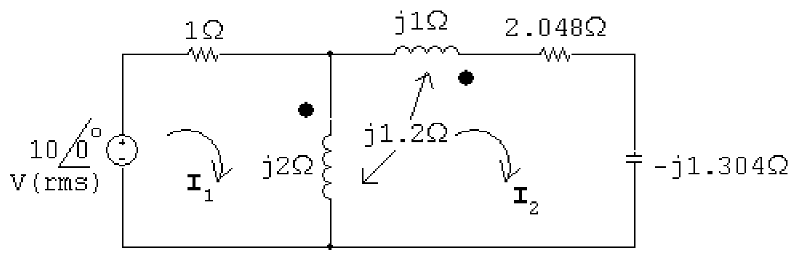
$0 = -j3.2\mathbf{I}_1 + j5.4\mathbf{I}_{sc}$

Solving,

$\mathbf{I}_{sc} = 5.89\underline{/-5.92^\circ}\text{ A}$

$$Z_{\text{Th}} = \frac{14.31\underline{/26.57^\circ}}{5.89\underline{/-5.92^\circ}} = 2.43\underline{/32.49^\circ} = 2.048 + j1.304\,\Omega$$

$$\therefore\quad \mathbf{I}_2 = \frac{14.31\underline{/26.57^\circ}}{4.096} = 3.49\underline{/26.57^\circ}\text{ A}$$

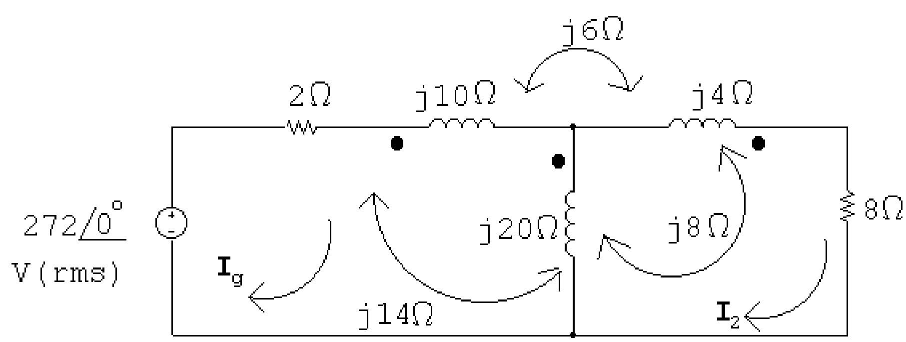


$10\underline{/0^\circ} = (1 + j2)\mathbf{I}_1 - j3.2\mathbf{I}_2$

$$\therefore\qquad \mathbf{I}_1 = \frac{10 + j3.2\mathbf{I}_2}{1 + j2} = \frac{10 + j3.2(3.49\underline{/26.57^\circ})}{1 + j2} = 5\underline{/0^\circ}\text{ A}$$

$$Z_g = \frac{10\underline{/0^\circ}}{5\underline{/0^\circ}} = 2 + j0 = 2\underline{/0^\circ}\,\Omega$$

P 10.54 **[a]**



$$\begin{aligned}272\underline{/0^\circ} &= 2\mathbf{I}_g + j10\mathbf{I}_g + j14(\mathbf{I}_g - \mathbf{I}_2) - j6\mathbf{I}_2\\ &\quad + j14\mathbf{I}_g - j8\mathbf{I}_2 + j20(\mathbf{I}_g - \mathbf{I}_2)\\ 0 &= j20(\mathbf{I}_2 - \mathbf{I}_g) - j14\mathbf{I}_g + j8\mathbf{I}_2 + j4\mathbf{I}_2\\ &\quad + j8(\mathbf{I}_2 - \mathbf{I}_g) - j6\mathbf{I}_g + 8\mathbf{I}_2\end{aligned}$$

Solving,

$\mathbf{I}_g = 20 - j4\,\mathrm{A(rms)}; \qquad \mathbf{I}_2 = 24\underline{/0^\circ}\,\mathrm{A(rms)}$

$P_{8\Omega} = (24)^2(8) = 4608\,\mathrm{W}$

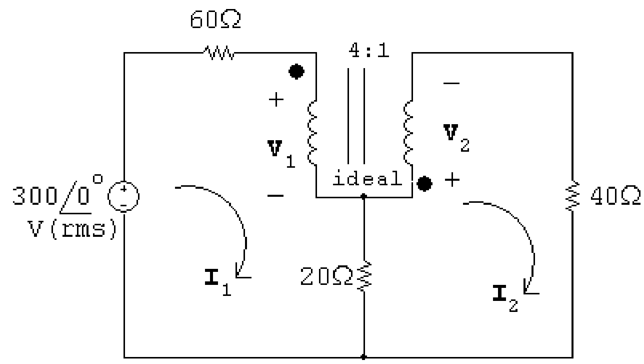
[b] $P_g(\text{developed}) = (272)(20) = 5440\,\mathrm{W}$

[c] $Z_{\mathrm{ab}} = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} - 2 = \dfrac{272}{20 - j4} - 2 = 11.08 + j2.62 = 11.38\underline{/13.28^\circ}\,\Omega$

[d] $P_{2\Omega} = |\mathbf{I}_g|^2(2) = 832\,\mathrm{W}$

$\sum P_{\mathrm{diss}} = 832 + 4608 = 5440\,\mathrm{W} = \sum P_{\mathrm{dev}}$

P 10.55 **[a]**



$300 = 60\mathbf{I}_1 + \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2)$

$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2 + 40\mathbf{I}_2$

$\mathbf{V}_2 = \dfrac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_2 = -4\mathbf{I}_1$

Solving,

$\mathbf{V}_1 = 260\,\mathrm{V(rms)}; \qquad \mathbf{V}_2 = 65\,\mathrm{V(rms)}$

$\mathbf{I}_1 = 0.25\,\mathrm{A(rms)}; \qquad \mathbf{I}_2 = -1.0\,\mathrm{A(rms)}$

$\mathbf{V}_{5\mathrm{A}} = \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2) = 285\,\mathrm{V(rms)}$

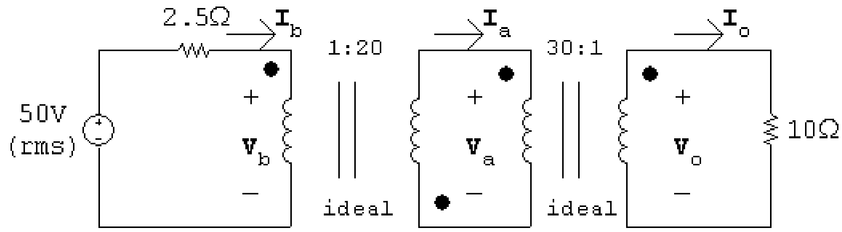
$\therefore \quad P = -(285)(5) = -1425\,\mathrm{W}$

Thus 1425 W is delivered by the current source to the circuit.

[b] $\mathbf{I}_{20\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25\,\mathrm{A(rms)}$

$\therefore \quad P_{20\Omega} = (1.25)^2(20) = 31.25\,\mathrm{W}$

P 10.56



$$30\mathbf{V}_o = \mathbf{V}_\text{a}; \qquad \frac{\mathbf{I}_o}{30} = \mathbf{I}_\text{a}; \qquad \mathbf{V}_o = 10\mathbf{I}_o \qquad \text{therefore} \quad \frac{\mathbf{V}_\text{a}}{\mathbf{I}_\text{a}} = 9\,\text{k}\Omega$$

$$\frac{\mathbf{V}_\text{b}}{1} = \frac{-\mathbf{V}_\text{a}}{20}; \qquad \mathbf{I}_\text{b} = -20\mathbf{I}_\text{a}; \qquad \text{therefore} \quad \frac{\mathbf{V}_\text{b}}{\mathbf{I}_\text{b}} = \frac{9000}{400} = 22.5\,\Omega$$

Therefore $\mathbf{I}_\text{b} = [50/(2.5 + 22.5)] = 2$ A (rms); since the ideal transformers are lossless, $P_{10\Omega} = P_{22.5\Omega}$, and the power delivered to the $22.5\,\Omega$ resistor is $2^2(22.5)$ or 90 W.

P 10.57 **[a]** $\dfrac{\mathbf{V}_\text{b}}{\mathbf{I}_\text{b}} = \dfrac{a^2 10}{400} = 2.5\,\Omega; \qquad$ therefore $\quad a^2 = 100, \qquad a = 10$

[b] $\mathbf{I}_\text{b} = \dfrac{50}{5} = 10\,\text{A}; \qquad P = (100)(2.5) = 250\,\text{W}$

P 10.58 **[a]** $Z_\text{Th} = 720 + j1500 + \left(\dfrac{200}{50}\right)^2 (40 - j30) = 1360 + j1020 = 1700\underline{/36.87^\circ}\,\Omega$

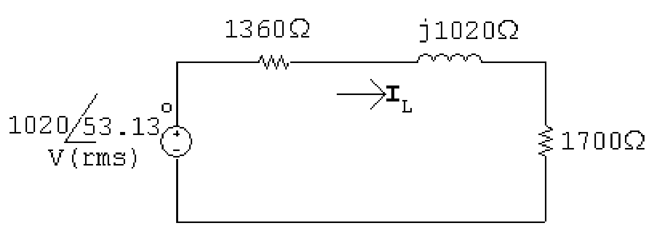
$$\therefore \quad Z_\text{ab} = 1700\,\Omega$$

$$Z_\text{ab} = \frac{Z_\text{L}}{(1 + N_1/N_2)^2}$$

$$(1 + N_1/N_2)^2 = 6800/1700 = 4$$

$$\therefore \quad N_1/N_2 = 1 \quad \text{or} \quad N_2 = N_1 = 1000 \text{ turns}$$

[b] $\mathbf{V}_\text{Th} = \dfrac{255\underline{/0^\circ}}{40 + j30}(j200) = 1020\underline{/53.13^\circ}\,\text{V}$

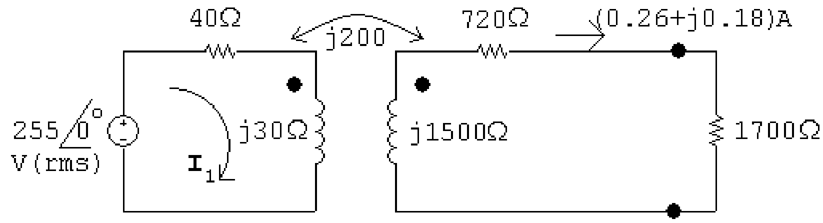


$$\mathbf{I}_L = \frac{1020\underline{/53.13^\circ}}{3060 + j1020} = 0.316\underline{/34.7^\circ}\,\text{A(rms)}$$

Since the transformer is ideal, $P_{6800} = P_{1700}$.

$$P = |\mathbf{I}_L|^2(1700) = 170\,\text{W}$$

[c]



$$255\underline{/0^\circ} = (40 + j30)\mathbf{I}_1 - j200(0.26 + j0.18)$$

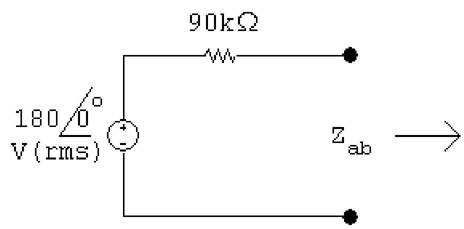
$$\therefore \quad \mathbf{I}_1 = 4.13 - j1.80\,\text{A(rms)}$$

$$P_{\text{gen}} = (255)(4.13) = 1053\,\text{W}$$

$$P_{\text{trans}} = 1053 - 170 = 883\,\text{W}$$

$$\%\ \text{transmitted} = \frac{883}{1053}(100) = 83.85\%$$

P 10.59 **[a]**



For maximum power transfer, $Z_{\text{ab}} = 90\,\text{k}\Omega$

$$Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_{\text{L}}$$

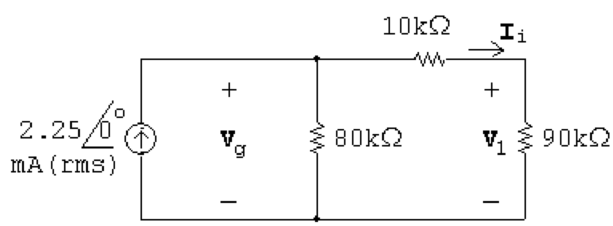
$$\therefore \quad \left(1 + \frac{N_1}{N_2}\right)^2 = \frac{90{,}000}{400} = 225$$

$$1 + \frac{N_1}{N_2} = \pm 15; \qquad \frac{N_1}{N_2} = 15 - 1 = 14$$

[b] $P = |\mathbf{I}_i|^2(90{,}000) = \left(\dfrac{180}{180{,}000}\right)^2(90{,}000) = 90\,\text{mW}$

[c] $\mathbf{V}_1 = R_i\mathbf{I}_i = (90{,}000)\left(\dfrac{180}{180{,}000}\right) = 90\,\text{V}$

[d]



$$\mathbf{V}_g = (2.25 \times 10^{-3})(100{,}000\|80{,}000) = 100\,\text{V}$$

$$P_g(\text{del}) = (2.25 \times 10^{-3})(100) = 225\,\text{mW}$$

$$\%\text{ delivered } = \frac{90}{225}(100) = 40\%$$

P 10.60 **[a]** $Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 (1 - j2) = 25 - j50\,\Omega$

$$\therefore \quad \mathbf{I}_1 = \frac{100\underline{/0^\circ}}{15 + j50 + 25 - j50} = 2.5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1 = 10\underline{/0^\circ}\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\text{L}} = \mathbf{I}_1 + \mathbf{I}_2 = 12.5\underline{/0^\circ}\,\text{A(rms)}$$

$$P_{1\Omega} = (12.5)^2(1) = 156.25\,\text{W}$$

$$P_{15\Omega} = (2.5)^2(15) = 93.75\,\text{W}$$

[b] $P_g = -100(2.5\underline{/0^\circ}) = -250\,\text{W}$

$$\sum P_{\text{abs}} = 156.25 + 93.75 = 250\,\text{W} = \sum P_{\text{dev}}$$

P 10.61 **[a]** $25a_1^2 + 4a_2^2 = 500$

$$\mathbf{I}_{25} = a_1\mathbf{I}; \qquad P_{25} = a_1^2\mathbf{I}^2(25)$$

$$\mathbf{I}_4 = a_2\mathbf{I}; \qquad P_4 = a_2^2\mathbf{I}^2(4)$$

$$P_4 = 4P_{25}; \qquad a_2^2\mathbf{I}^2 4 = 100a_1^2\mathbf{I}^2$$

$$\therefore \qquad 100a_1^2 = 4a_2^2$$

$$25a_1^2 + 100a_1^2 = 500; \qquad a_1 = 2$$

$$25(4) + 4a_2^2 = 500; \qquad a_2 = 10$$

[b] $\mathbf{I} = \frac{2000\underline{/0^\circ}}{500 + 500} = 2\underline{/0^\circ}\,\text{A(rms)}$

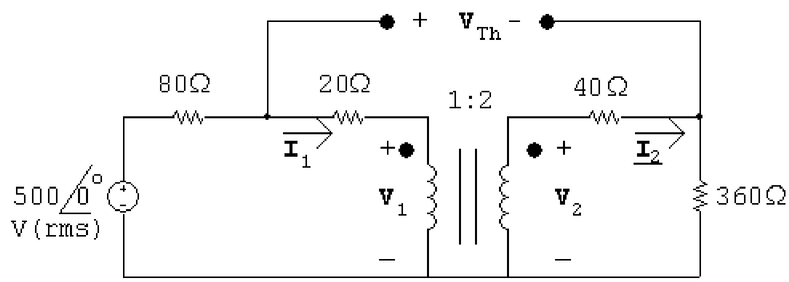
$$\mathbf{I}_{25} = a_1\mathbf{I} = 4\,\text{A}$$

$$P_{25\Omega} = (16)(25) = 400\,\text{W}$$

[c] $\mathbf{I}_4 = a_2\mathbf{I} = 10(2) = 20\,\text{A(rms)}$

$$\mathbf{V}_4 = (20)(4) = 80\underline{/0^\circ}\,\text{V(rms)}$$

P 10.62 **[a]** Open circuit voltage:



$$500 = 100\mathbf{I}_1 + \mathbf{V}_1$$

$$\mathbf{V}_2 = 400\mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{1} = \frac{\mathbf{V}_2}{2} \qquad \therefore \qquad \mathbf{V}_2 = 2\mathbf{V}_1$$

$$\mathbf{I}_1 = 2\mathbf{I}_2$$

Substitute and solve:

$$2\mathbf{V}_1 = 400\mathbf{I}_1/2 = 200\mathbf{I}_1 \qquad \therefore \qquad \mathbf{V}_1 = 100\mathbf{I}_1$$

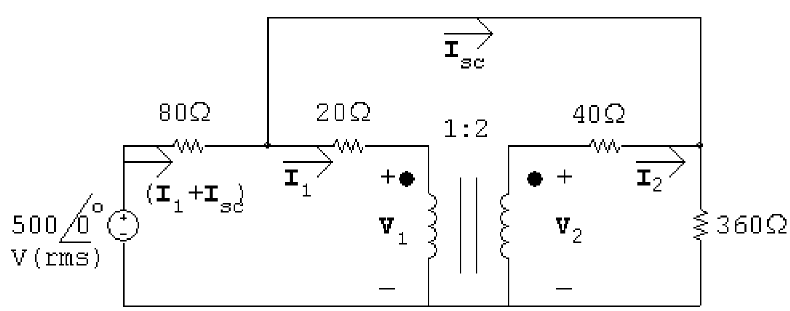
$$500 = 100\mathbf{I}_1 + 100\mathbf{I}_1 \qquad \therefore \qquad \mathbf{I}_1 = 500/200 = 2.5\,\text{A}$$

$$\therefore \qquad \mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = 1.25\,\text{A}$$

$$\mathbf{V}_1 = 100(2.5) = 250\,\text{V}; \qquad \mathbf{V}_2 = 2\mathbf{V}_1 = 500\,\text{V}$$

$$V_{\text{Th}} = 20\mathbf{I}_1 + \mathbf{V}_1 - \mathbf{V}_2 + 40\mathbf{I}_2 = -150\,\text{V(rms)}$$

Short circuit current:



$$500 = 80(\mathbf{I}_{\text{sc}} + \mathbf{I}_1) + 360(\mathbf{I}_{\text{sc}} + 0.5\mathbf{I}_1)$$

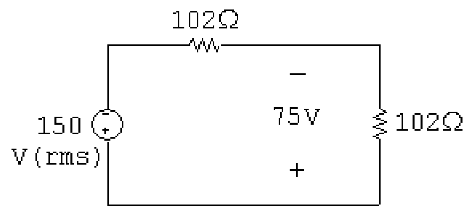
$$2\mathbf{V}_1 = 40\frac{\mathbf{I}_1}{2} + 360(\mathbf{I}_{\text{sc}} + 0.5\mathbf{I}_1)$$

$$500 = 80(\mathbf{I}_1 + \mathbf{I}_{\text{sc}}) + 20\mathbf{I}_1 + \mathbf{V}_1$$

Solving,

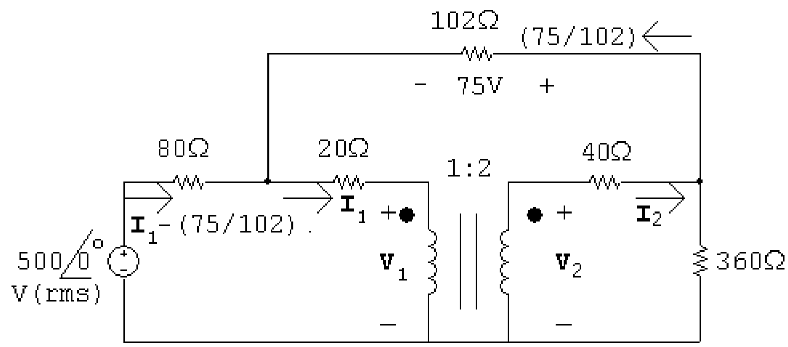
$\mathbf{I}_{sc} = -1.47\,\text{A}; \quad \mathbf{I}_1 = 4.41\ \text{A}; \quad \mathbf{V}_1 = 176.47\ \text{V}$

$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{sc}} = \frac{-150}{-1.47} = 102\,\Omega$$



$$P = \frac{75^2}{102} = 55.15\,\text{W}$$

[b]



$$500 = 80[\mathbf{I}_1 - (75/102)] - 75 + 360[\mathbf{I}_2 - (75/102)]$$

$$575 + \frac{6000}{102} + \frac{27{,}000}{102} = 80\mathbf{I}_1 + 180\mathbf{I}_1$$

$\therefore \qquad \mathbf{I}_1 = 3.456\,\text{A}$

$$P_{\text{source}} = (500)[3.456 - (75/102)] = 1360.29\,\text{W}$$

$$\%\ \text{delivered} = \frac{55.15}{1360.29}(100) = 4.05\%$$

[c] $P_{80\Omega} = 80\left(\mathbf{I}_1 - \dfrac{75}{102}\right)^2 = 592.13\,\text{W}$

$$P_{20\Omega} = 20\mathbf{I}_1^2 = 238.86\,\text{W}$$

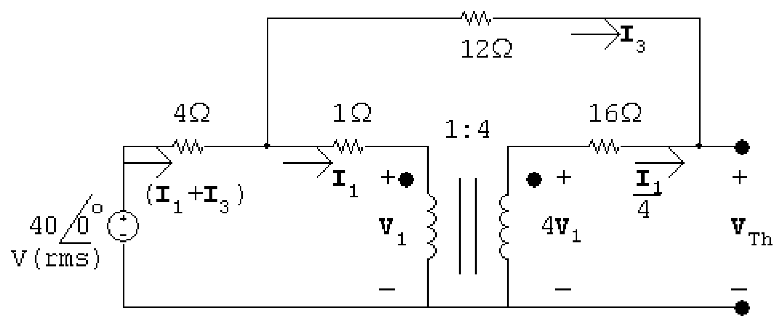
$$P_{40\Omega} = 40\mathbf{I}_2^2 = 119.43\,\text{W}$$

$$P_{102\Omega} = \frac{75^2}{102} = 55.15\,\text{W}$$

$$P_{360\Omega} = 360\left(\mathbf{I}_2 - \frac{75}{102}\right)^2 = 354.73\,\text{W}$$

$$\sum P_{\text{abs}} = 592.13 + 238.86 + 119.43 + 55.15 + 354.73 = 1360.3\,\text{W} = \sum P_{\text{dev}}$$

P 10.63 **[a]** Open circuit voltage:



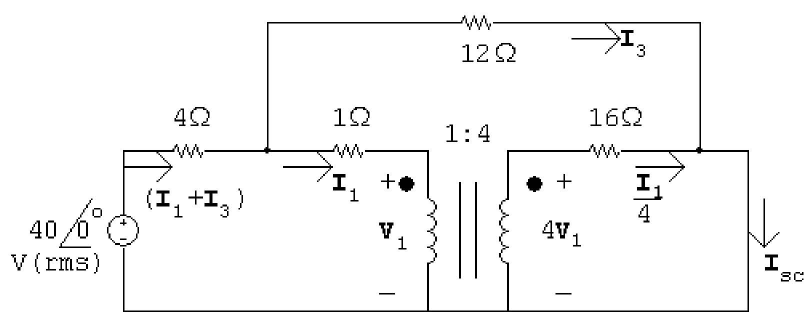
$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + \mathbf{V}_{\text{Th}}$

$\frac{\mathbf{I}_1}{4} = -\mathbf{I}_3; \qquad \mathbf{I}_1 = -4\mathbf{I}_3$

Solving,

$\mathbf{V}_{\text{Th}} = 40\underline{/0^\circ}\,\text{V}$

Short circuit current:



$40\underline{/0^\circ} = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$

$4\mathbf{V}_1 = 16(\mathbf{I}_1/4) = 4\mathbf{I}_1; \qquad \therefore\ \ \mathbf{V}_1 = \mathbf{I}_1$

$\therefore\ \ 40\underline{/0^\circ} = 6\mathbf{I}_1 + 4\mathbf{I}_3$

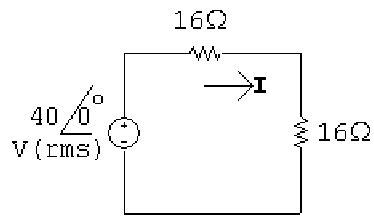
Also,

$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = 1\,\text{A}; \qquad \mathbf{I}_{\text{sc}} = \mathbf{I}_1/4 + \mathbf{I}_3 = 2.5\,\text{A}$

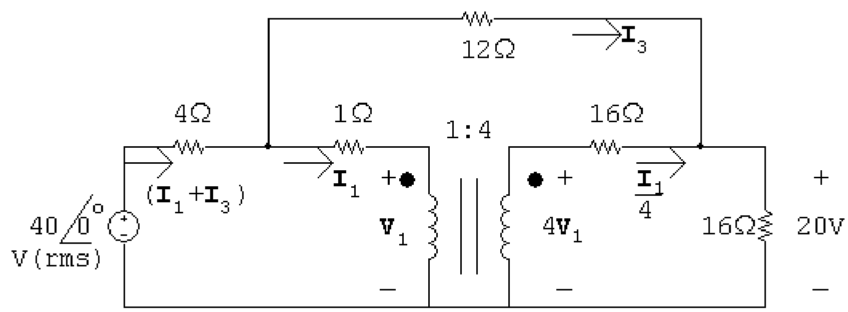
$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{40}{2.5} = 16\,\Omega$$



$$\mathbf{I} = \frac{40/0^\circ}{32} = 1.25/0^\circ\,\text{A(rms)}$$

$$P = (1.25)^2(16) = 25\,\text{W}$$

[b]



$$40 = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + 20$$

$$4\mathbf{V}_1 = 4\mathbf{I}_1 + 16(\mathbf{I}_1/4 + \mathbf{I}_3); \qquad \therefore \quad \mathbf{V}_1 = 2\mathbf{I}_1 + 4\mathbf{I}_3$$

$$40 = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$\therefore \quad \mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = -0.25\,\text{A}; \qquad \mathbf{I}_1 + \mathbf{I}_3 = 5.75/0^\circ\,\text{A}; \qquad \mathbf{V}_1 = 11/0^\circ\,\text{V}$$

$$P_{40V}(\text{developed}) = 40(5.75) = 230\,\text{W}$$

$$\therefore \quad \%\ \text{delivered} = \frac{25}{230}(100) = 10.87\%$$

[c] $P_{R_L} = 25\,\text{W}; \qquad P_{16\Omega} = (1.5)^2(16) = 36\,\text{W}$

$$P_{4\Omega} = (5.75)^2(4) = 132.25\,\text{W}; \qquad P_{1\Omega} = (6)^2(1) = 36\,\text{W}$$

$$P_{12\Omega} = (-0.25)^2(12) = 0.75\,\text{W}$$

$$\sum P_{\text{abs}} = 25 + 36 + 132.25 + 36 + 0.75 = 230\,\text{W} = \sum P_{\text{dev}}$$

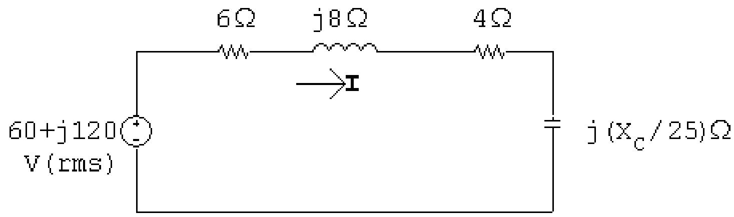
P 10.64 **[a]** Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_{\text{Th}} = (15)(20\|j10) = 60 + j120\,\text{V}$$

$$Z_{\text{Th}} = 2 + 20\|j10 = 6 + j8\,\Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{25}(100 + jX_{\rm C}) = 4 + j\frac{X_{\rm C}}{25}\,\Omega$$



Now maximize **I** by setting $(X_{\rm C}/25) = -8\,\Omega$:

$$\therefore \quad C = \frac{1}{200(20\times 10^3)} = 0.25\,\mu\text{F}$$

[b] $\mathbf{I} = \dfrac{60 + j120}{10} = 6 + j12\,\text{A}$

$$P = |\mathbf{I}|^2(4) = 720\,\text{W}$$

[c] $\dfrac{R_o}{25} = 6\,\Omega; \qquad \therefore \quad R_o = 150\,\Omega$

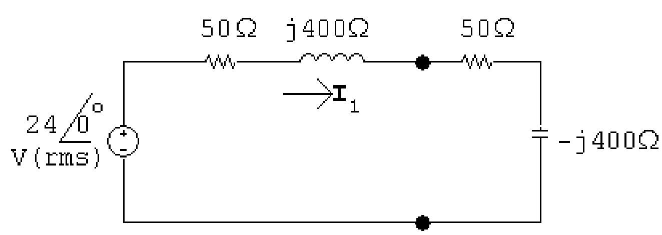
[d] $\mathbf{I} = \dfrac{60 + j120}{12} = 5 + j10\,\text{A}$

$$P = |\mathbf{I}|^2(6) = 750\,\text{W}$$

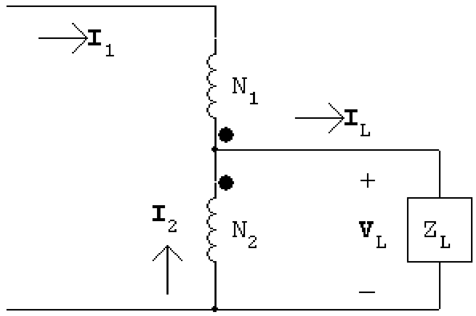
P 10.65 **[a]** $Z_{\rm ab} = 50 - j400 = \left(1 - \dfrac{N_1}{N_2}\right)^2 Z_{\rm L} = \left(1 - \dfrac{2800}{700}\right)^2 Z_L = 9Z_L$

$$\therefore \quad Z_{\rm L} = \frac{1}{9}(50 - j400) = 5.556 - j44.444\,\Omega$$

[b]



$$\mathbf{I}_1 = \frac{24}{100} = 240\underline{/0^\circ}\,\text{mA}$$



$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -4\mathbf{I}_1 = 960\underline{/180^\circ}\,\text{mA}$$

$$\mathbf{I}_\text{L} = \mathbf{I}_1 + \mathbf{I}_2 = 720\underline{/180^\circ}\,\text{mA(rms)}$$

$$\mathbf{V}_\text{L} = (5.556 - j44.444)\mathbf{I}_\text{L} = -4 + j32 = 32.25\underline{/97.13^\circ}\,\text{V(rms)}$$

P 10.66 **[a]** Begin with the MEDIUM setting, as shown in Fig. 10.31, as it involves only the resistor R_2. Then,

$$P_\text{med} = 500\,\text{W} = \frac{V^2}{R_2} = \frac{120^2}{R_2}$$

Thus,

$$R_2 = \frac{120^2}{500} = 28.8\,\Omega$$

[b] Now move to the LOW setting, as shown in Fig. 10.30, which involves the resistors R_1 and R_2 connected in series:

$$P_\text{low} = \frac{V^2}{R_1 + R_2} = \frac{V^2}{R_1 + 28.8} = 250\,\text{W}$$

Thus,

$$R_1 = \frac{120^2}{250} - 28.8 = 28.8\,\Omega$$

[c] Note that the HIGH setting has R_1 and R_2 in parallel:

$$P_\text{high} = \frac{V^2}{R_1\|R_2} = \frac{120^2}{28.8\|28.8} = 1000\,\text{W}$$

If the HIGH setting has required power other than 1000 W, this problem sould not have been solved. In other words, the HIGH power setting was chosen in such a way that it would be satisfied once the two resistor values were calculated to satisfy the LOW and MEDIUM power settings.

P 10.67 **[a]** $P_{\rm L} = \dfrac{V^2}{R_1 + R_2}; \qquad R_1 + R_2 = \dfrac{V^2}{P_{\rm L}}$

$$P_{\rm M} = \frac{V^2}{R_2}; \qquad R_2 = \frac{V^2}{P_{\rm M}}$$

$$P_{\rm H} = \frac{V^2(R_1 + R_2)}{R_1 R_2}$$

$$R_1 + R_2 = \frac{V^2}{P_{\rm L}}; \qquad R_1 = \frac{V^2}{P_{\rm L}} - \frac{V^2}{P_{\rm M}}$$

$$P_{\rm H} = \frac{V^2 V^2/P_{\rm L}}{\left(\frac{V^2}{P_{\rm L}} - \frac{V^2}{P_{\rm M}}\right)\left(\frac{V^2}{P_{\rm M}}\right)} = \frac{P_{\rm M} P_{\rm L} P_{\rm M}}{P_{\rm L}(P_{\rm M} - P_{\rm L})}$$

$$P_{\rm H} = \frac{P_{\rm M}^2}{P_{\rm M} - P_{\rm L}}$$

[b] $P_{\rm H} = \dfrac{(750)^2}{(750 - 250)} = 1125\,\text{W}$

P 10.68 First solve the expression derived in P10.67 for $P_{\rm M}$ as a function of $P_{\rm L}$ and $P_{\rm H}$. Thus

$$P_{\rm M} - P_{\rm L} = \frac{P_{\rm M}^2}{P_{\rm H}} \quad \text{or} \quad \frac{P_{\rm M}^2}{P_{\rm H}} - P_{\rm M} + P_{\rm L} = 0$$

$$P_{\rm M}^2 - P_{\rm M} P_{\rm H} + P_{\rm L} P_{\rm H} = 0$$

$$\therefore \quad P_{\rm M} = \frac{P_{\rm H}}{2} \pm \sqrt{\left(\frac{P_{\rm H}}{2}\right)^2 - P_{\rm L} P_{\rm H}}$$

$$= \frac{P_{\rm H}}{2} \pm P_{\rm H}\sqrt{\frac{1}{4} - \left(\frac{P_{\rm L}}{P_{\rm H}}\right)}$$

For the specified values of $P_{\rm L}$ and $P_{\rm H}$

$$P_{\rm M} = 500 \pm 1000\sqrt{0.25 - 0.24} = 500 \pm 100$$

$$\therefore \quad P_{M1} = 600\,\text{W}; \qquad P_{M2} = 400\,\text{W}$$

Note in this case we design for two medium power ratings
If $P_{M1} = 600\,\text{W}$

$$R_2 = \frac{(120)^2}{600} = 24\,\Omega$$

$$R_1 + R_2 = \frac{(120)^2}{240} = 60\,\Omega$$

$$R_1 = 60 - 24 = 36\,\Omega$$

CHECK: $P_{\rm H} = \dfrac{(120)^2(60)}{(36)(24)} = 1000\,$W

If $P_{M2} = 400\,$W

$$R_2 = \frac{(120)^2}{400} = 36\,\Omega$$

$R_1 + R_2 = 60\,\Omega$ (as before)

$$R_1 = 24\,\Omega$$

CHECK: $P_{\rm H} = 1000\,$W

P 10.69 $R_1 + R_2 + R_3 = \dfrac{(120)^2}{600} = 24\,\Omega$

$$R_2 + R_3 = \frac{(120)^2}{900} = 16\,\Omega$$

$$\therefore \quad R_1 = 24 - 16 = 8\,\Omega$$

$$R_3 + R_1 \| R_2 = \frac{(120)^2}{1200} = 12\,\Omega$$

$$\therefore \quad 16 - R_2 + \frac{8R_2}{8 + R_2} = 12$$

$$R_2 - \frac{8R_2}{8 + R_2} = 4$$

$$8R_2 + R_2^2 - 8R_2 = 32 + 4R_2$$

$$R_2^2 - 4R_2 - 32 = 0$$

$$R_2 = 2 \pm \sqrt{4 + 32} = 2 \pm 6$$

$$\therefore \quad R_2 = 8\,\Omega; \qquad \therefore \quad R_3 = 8\,\Omega$$

P 10.70 $R_2 = \dfrac{(220)^2}{500} = 96.8\,\Omega$

$$R_1 + R_2 = \frac{(220)^2}{250} = 193.6\,\Omega$$

$$\therefore \quad R_1 = 96.8\,\Omega$$

CHECK: $R_1 \| R_2 = 48.4\,\Omega$

$$P_{\text{H}} = \frac{(220)^2}{48.4} = 1000\,\text{W}$$