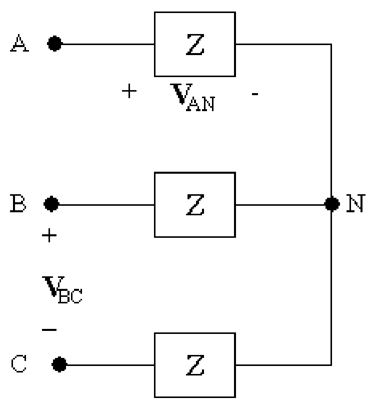
11

Balanced Three-Phase Circuits

Assessment Problems

AP 11.1 Make a sketch:



We know $\mathbf{V}_{\rm AN}$ and wish to find $\mathbf{V}_{\rm BC}$. To do this, write a KVL equation to find $\mathbf{V}_{\rm AB}$, and use the known phase angle relationship between $\mathbf{V}_{\rm AB}$ and $\mathbf{V}_{\rm BC}$ to find $\mathbf{V}_{\rm BC}$.

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} + \mathbf{V}_{\rm NB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN}$$

Since $\mathbf{V}_{\rm AN}$, $\mathbf{V}_{\rm BN}$, and $\mathbf{V}_{\rm CN}$ form a balanced set, and $\mathbf{V}_{\rm AN} = 240\angle -30^\circ$V, and the phase sequence is positive,

$$\mathbf{V}_{\rm BN} = |\mathbf{V}_{\rm AN}|\angle(\angle\mathbf{V}_{\rm AN} - 120^\circ) = 240\angle -30^\circ - 120^\circ = 240\angle -150^\circ \text{ V}$$

Then,

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN} = (240\angle -30^\circ) - (240\angle -150^\circ) = 415.46\angle 0^\circ \text{ V}$$

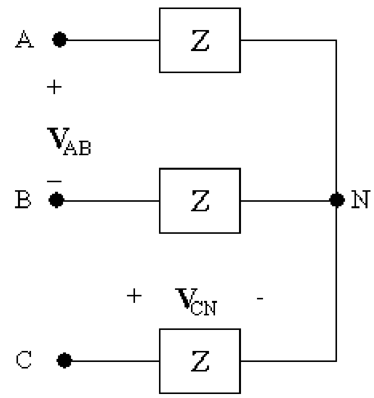
Since $\mathbf{V}_{\rm AB}$, $\mathbf{V}_{\rm BC}$, and $\mathbf{V}_{\rm CA}$ form a balanced set with a positive phase sequence, we can find $\mathbf{V}_{\rm BC}$ from $\mathbf{V}_{\rm AB}$:

$$\mathbf{V}_{\rm BC} = |\mathbf{V}_{\rm AB}|\angle(\angle\mathbf{V}_{\rm AB} - 120^\circ) = 415.69\angle 0^\circ - 120^\circ = 415.69\angle -120^\circ \text{ V}$$

Thus,

$$\mathbf{V}_{\rm BC} = 415.69\angle -120^\circ \text{ V}$$

AP 11.2 Make a sketch:



We know $\mathbf{V}_{\rm CN}$ and wish to find $\mathbf{V}_{\rm AB}$. To do this, write a KVL equation to find $\mathbf{V}_{\rm BC}$, and use the known phase angle relationship between $\mathbf{V}_{\rm AB}$ and $\mathbf{V}_{\rm BC}$ to find $\mathbf{V}_{\rm AB}$.

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} + \mathbf{V}_{\rm NC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN}$$

Since $\mathbf{V}_{\rm AN}$, $\mathbf{V}_{\rm BN}$, and $\mathbf{V}_{\rm CN}$ form a balanced set, and $\mathbf{V}_{\rm CN} = 450\angle{-25^\circ}$ V, and the phase sequence is negative,

$$\mathbf{V}_{\rm BN} = |\mathbf{V}_{\rm CN}|\angle{/\mathbf{V}_{\rm CN} - 120^\circ} = 450\angle{-23^\circ - 120^\circ} = 450\angle{-145^\circ}\ \text{V}$$

Then,

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = (450\angle{-145^\circ}) - (450\angle{-25^\circ}) = 779.42\angle{-175^\circ}\ \text{V}$$

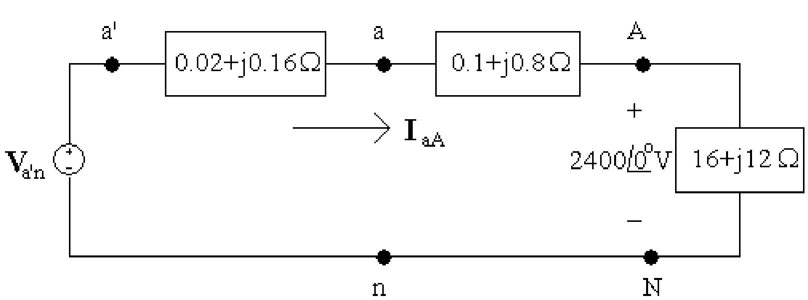
Since $\mathbf{V}_{\rm AB}$, $\mathbf{V}_{\rm BC}$, and $\mathbf{V}_{\rm CA}$ form a balanced set with a negative phase sequence, we can find $\mathbf{V}_{\rm AB}$ from $\mathbf{V}_{\rm BC}$:

$$\mathbf{V}_{\rm AB} = |\mathbf{V}_{\rm BC}|\angle{/\mathbf{V}_{\rm BC} - 120^\circ} = 779.42\angle{-295^\circ}\ \text{V}$$

But we normally want phase angle values between $+180^\circ$ and -180°. We add 360° to the phase angle computed above. Thus,

$$\mathbf{V}_{\rm AB} = 779.42\angle{65^\circ}\ \text{V}$$

AP 11.3 Sketch the a-phase circuit:



[a] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that $\mathbf{I}_{\mathrm{aA}}$, $\mathbf{I}_{\mathrm{bB}}$, and $\mathbf{I}_{\mathrm{cC}}$ form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, $\mathbf{V}_{\mathrm{AN}}$, has a phase angle of $0°$.

$$2400\underline{/0°} = \mathbf{I}_{\mathrm{aA}}(16 + j12)$$

so

$$\mathbf{I}_{\mathrm{aA}} = \frac{2400\underline{/0°}}{16 + j12} = 96 - j72 = 120\underline{/-36.87°}\,\mathrm{A}$$

With an acb phase sequence,

$$\underline{/\mathbf{I}_{\mathrm{bB}}} = \underline{/\mathbf{I}_{\mathrm{aA}}} + 120° \quad \text{and} \quad \underline{/\mathbf{I}_{\mathrm{cC}}} = \underline{/\mathbf{I}_{\mathrm{aA}}} - 120°$$

so

$$\mathbf{I}_{\mathrm{aA}} = 120\underline{/-36.87°}\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{bB}} = 120\underline{/83.13°}\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{cC}} = 120\underline{/-156.87°}\,\mathrm{A}$$

[b] The line voltages at the source are $\mathbf{V}_{\mathrm{ab}}$ $\mathbf{V}_{\mathrm{bc}}$, and $\mathbf{V}_{\mathrm{ca}}$. They form a balanced set. To find $\mathbf{V}_{\mathrm{ab}}$, use the a-phase circuit to find $\mathbf{V}_{\mathrm{AN}}$, and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\mathbf{V}_{\mathrm{an}} = \mathbf{V}_{\mathrm{aA}} + \mathbf{V}_{\mathrm{AN}} = (0.1 + j0.8)\mathbf{I}_{\mathrm{aA}} + 2400\underline{/0°}$$

$$= (0.1 + j0.8)(96 - j72) + 2400\underline{/0°} = 2467.2 + j69.6$$

$$2468.18\underline{/1.62°}\,\mathrm{V}$$

From Fig. 11.9(b),

$$\mathbf{V}_{\mathrm{ab}} = \mathbf{V}_{\mathrm{an}}(\sqrt{3}\underline{/-30°}) = 4275.02\underline{/-28.38°}\,\mathrm{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\mathrm{bc}}} = \underline{/\mathbf{V}_{\mathrm{ab}}} + 120° \quad \text{and} \quad \underline{/\mathbf{V}_{\mathrm{ca}}} = \underline{/\mathbf{V}_{\mathrm{ab}}} - 120°$$

so

$$\mathbf{V}_{\mathrm{ab}} = 4275.02\underline{/-28.38°}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{bc}} = 4275.02\underline{/91.62°}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{ca}} = 4275.02\underline{/-148.38°}\,\mathrm{V}$$

[c] Using KVL on the a-phase circuit

$$\mathbf{V}_{\mathrm{a'n}} = \mathbf{V}_{\mathrm{a'a}} + \mathbf{V}_{\mathrm{an}} = (0.2 + j0.16)\mathbf{I}_{\mathrm{aA}} + \mathbf{V}_{\mathrm{an}}$$

$$= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9)$$

$$= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\,\mathrm{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\mathrm{b'n}}} = \underline{/\mathbf{V}_{\mathrm{a'n}}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\mathrm{c'n}}} = \underline{/\mathbf{V}_{\mathrm{a'n}}} - 120^\circ$$

so

$$\mathbf{V}_{\mathrm{a'n}} = 2482.05\underline{/1.93^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{b'n}} = 2482.05\underline{/121.93^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{c'n}} = 2482.05\underline{/-118.07^\circ}\,\mathrm{V}$$

AP 11.4 $\mathbf{I}_{\mathrm{cC}} = (\sqrt{3}\underline{/-30^\circ})\mathbf{I}_{\mathrm{CA}} = (\sqrt{3}\underline{/-30^\circ}) \cdot 8\underline{/-15^\circ} = 13.86\underline{/-45^\circ}\,\mathrm{A}$

AP 11.5 $\mathbf{I}_{\mathrm{aA}} = 12\underline{/(65^\circ - 120^\circ)} = 12\underline{/-55^\circ}$

$$\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right]\mathbf{I}_{\mathrm{aA}} = \left(\frac{\underline{/-30^\circ}}{\sqrt{3}}\right) \cdot 12\underline{/-55^\circ}$$

$$= 6.93\underline{/-85^\circ}\,\mathrm{A}$$

AP 11.6 **[a]** $\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/20^\circ}\,\mathrm{A}$

Therefore $\quad Z_\phi = \dfrac{4160\underline{/0^\circ}}{40\underline{/20^\circ}} = 104\underline{/-20^\circ}\,\Omega$

[b] $\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/-40^\circ}\,\mathrm{A}$

Therefore $\quad Z_\phi = 104\underline{/40^\circ}\,\Omega$

AP 11.7 $\mathbf{I}_\phi = \dfrac{110}{3.667} + \dfrac{110}{j2.75} = 30 - j40 = 50\underline{/-53.13^\circ}\,\mathrm{A}$

Therefore $\quad |\mathbf{I}_{\mathrm{aA}}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60\,\mathrm{A}$

AP 11.8 **[a]** $|S| = \sqrt{3}(208)(73.8) = 26{,}587.67\,\text{VA}$

$$Q = \sqrt{(26{,}587.67)^2 - (22{,}659)^2} = 13{,}909.50\,\text{VAR}$$

[b] $\text{pf} = \dfrac{22{,}659}{26{,}587.67} = 0.8522 \quad \text{lagging}$

AP 11.9 **[a]** $\mathbf{V}_{\text{AN}} = \left(\dfrac{2450}{\sqrt{3}}\right)\angle 0^\circ\,\text{V}; \qquad \mathbf{V}_{\text{AN}}\mathbf{I}^*_{\text{aA}} = S_\phi = 144 + j192\,\text{kVA}$

Therefore

$$\mathbf{I}^*_{\text{aA}} = \frac{(144 + j192)1000}{2450/\sqrt{3}} = (101.8 + j135.7)\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 101.8 - j135.7 = 169.67\angle -53.13^\circ\,\text{A}$$

$$|\mathbf{I}_{\text{aA}}| = 169.67\,\text{A}$$

[b] $P = \dfrac{(2450)^2}{R}; \qquad \text{therefore} \quad R = \dfrac{(2450)^2}{144{,}000} = 41.68\,\Omega$

$$Q = \frac{(2450)^2}{X}; \qquad \text{therefore} \quad X = \frac{(2450)^2}{192{,}000} = 31.26\,\Omega$$

[c] $Z_\phi = \dfrac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}} = \dfrac{2450/\sqrt{3}}{169.67\angle -53.13^\circ} = 8.34\angle 53.13^\circ = (5 + j6.67)\,\Omega$

$$\therefore \quad R = 5\,\Omega, \qquad X = 6.67\,\Omega$$

Problems

P 11.1 **[a]** First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 208\angle 27^\circ; \qquad \mathbf{V}_b = 208\angle 147^\circ; \qquad \mathbf{V}_c = 208\angle -93^\circ$

Subtract the phase angle of the a-phase from all phase angles:

$\angle \mathbf{V}'_a = 27^\circ - 27^\circ = 0^\circ$

$\angle \mathbf{V}'_b = 147^\circ - 27^\circ = 120^\circ$

$\angle \mathbf{V}'_c = -93^\circ - 27^\circ = -120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

[b] First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 4160\angle -18^\circ; \qquad \mathbf{V}_b = 4160\angle -138^\circ; \qquad \mathbf{V}_c = 4160\angle +102^\circ$

Subtract the phase angle of the a-phase from all phase angles:

$\angle \mathbf{V}'_a = -18^\circ + 18^\circ = 0^\circ$

$\angle \mathbf{V}'_b = -138^\circ + 18^\circ = -120^\circ$

$\angle \mathbf{V}'_c = 102^\circ + 18^\circ = 120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

P 11.2 **[a]** $\mathbf{V}_a = 180\angle 0^\circ$ V

$\mathbf{V}_b = 180\angle -120^\circ$ V

$\mathbf{V}_c = 180\angle -240^\circ = 180\angle 120^\circ$ V

Balanced, positive phase sequence

[b] $\mathbf{V}_a = 180\angle -90^\circ$ V

$\mathbf{V}_b = 180\angle 30^\circ$ V

$\mathbf{V}_c = 180\angle -210^\circ \text{ V} = 180\angle 150^\circ$ V

Balanced, negative phase sequence

[c] $\mathbf{V}_a = 400\angle -270^\circ \text{ V} = 400\angle 90^\circ$ V

$\mathbf{V}_b = 400\angle 120^\circ$ V

$\mathbf{V}_c = 400\angle -30^\circ$ V

Unbalanced, phase angle in b-phase

[d] $\mathbf{V}_a = 200\angle 30^\circ$ V

$\mathbf{V}_b = 201\angle 150^\circ$ V

$\mathbf{V}_c = 200\angle 270^\circ \text{ V} = 200\angle -90^\circ$ V

Unbalanced, unequal amplitude in the b-phase

[e] $\mathbf{V}_a = 208\angle 42^\circ$ V

$\mathbf{V}_b = 208\angle -78^\circ$ V

$\mathbf{V}_c = 208\angle -201^\circ \text{ V} = 208\angle 159^\circ$ V

Unbalanced, phase angle in the c-phase

[f] Unbalanced; the frequencies of the waveforms are not the same for the positive sequence of Eq. 11.1

P 11.3 $\mathbf{V}_a = V_m\angle 0^\circ = V_m + j0$

$$\mathbf{V}_b = V_m\angle -120^\circ = -V_m(0.5 + j0.866)$$

$$\mathbf{V}_c = V_m\angle 120^\circ = V_m(-0.5 + j0.866)$$

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = (V_m)(1 + j0 - 0.5 - j0.866 - 0.5 + j0.866)$$
$$= V_m(0) = 0$$

For the negative sequences of Eq. 11.2, $\mathbf{V}_b$ and $\mathbf{V}_c$ are interchanged, but the sum is still zero.

P 11.4 $\mathbf{I} = \dfrac{\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c}{3(R_W + jX_W)} = 0$

P 11.5 **[a]** $\mathbf{I}_{aA} = \dfrac{200}{25} = 8\angle 0^\circ$ A

$$\mathbf{I}_{bB} = \frac{200\angle -120^\circ}{30 - j40} = 4\angle -66.87^\circ \text{ A}$$

$$\mathbf{I}_{cC} = \frac{200\angle 120^\circ}{80 + j60} = 2\angle 83.13^\circ \text{ A}$$

The magnitudes are unequal and the phase angles are not 120° apart.

b] $\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 9.96\angle -9.79^\circ$ A

P 11.6 **[a]** $\mathbf{I}_{\mathrm{aA}} = \dfrac{277\angle 0^\circ}{80 + j60} = 2.77\angle -36.87^\circ\,\mathrm{A}$

$$\mathbf{I}_{\mathrm{bB}} = \frac{277\angle -120^\circ}{80 + j60} = 2.77\angle -156.87^\circ\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{cC}} = \frac{277\angle 120^\circ}{80 + j60} = 2.77\angle 83.13^\circ\,\mathrm{A}$$

$$\mathbf{I}_o = \mathbf{I}_{\mathrm{aA}} + \mathbf{I}_{\mathrm{bB}} + \mathbf{I}_{\mathrm{cC}} = 0$$

[b] $\mathbf{V}_{\mathrm{AN}} = (78 + j54)\mathbf{I}_{\mathrm{aA}} = 262.79\angle -2.17^\circ\,\mathrm{V}$

[c] $\mathbf{V}_{\mathrm{AB}} = \mathbf{V}_{\mathrm{AN}} - \mathbf{V}_{\mathrm{BN}}$

$$\mathbf{V}_{\mathrm{BN}} = (77 + j56)\mathbf{I}_{\mathrm{bB}} = 263.73\angle -120.84^\circ\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{AB}} = 262.79\angle -2.17^\circ - 263.73\angle -120.84^\circ = 452.89\angle 28.55^\circ\,\mathrm{V}$$

[d] Unbalanced — see conditions for a balanced circuit on p. 504 of the text!

P 11.7 $Z_{ga} + Z_{la} + Z_{La} = 60 + j80\,\Omega$

$$Z_{gb} + Z_{lb} + Z_{Lb} = 40 + j30\Omega$$

$$Z_{gc} + Z_{lc} + Z_{Lc} = 20 + j15\Omega$$

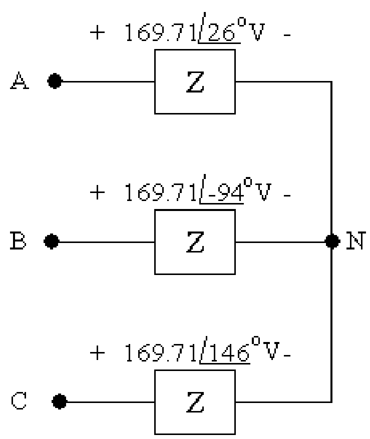
$$\frac{\mathbf{V}_N - 240}{60 + j80} + \frac{\mathbf{V}_N - 240\angle 120^\circ}{40 + j30} + \frac{\mathbf{V}_N - 240\angle -120^\circ}{20 + j15} + \frac{\mathbf{V}_N}{10} = 0$$

Solving for $\mathbf{V}_N$ yields

$$\mathbf{V}_N = 42.94\angle -156.32^\circ\,\mathrm{V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_N}{10} = 4.29\angle -156.32^\circ\,\mathrm{A}$$

P 11.8 Make a sketch of the load in the frequency domain. Note that we convert the time domain line-to-neutral voltages to phasors:



Note that these three voltages form a balanced set with an abc phase sequence. First, use KVL to find $\mathbf{V}_{\rm AB}$:

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} + \mathbf{V}_{\rm NB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN}$$

$$= (169.71\underline{/26^\circ}) - (169.71\underline{/-94^\circ}) = 293.95\underline{/56^\circ}\,\text{V}$$

With an abc phase sequence,

$$\underline{/\mathbf{V}_{\rm BC}} = \underline{/\mathbf{V}_{\rm AB}} - 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\rm CA}} = \underline{/\mathbf{V}_{\rm AB}} + 120^\circ$$

so

$$\mathbf{V}_{\rm AB} = 293.95\underline{/56^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm BC} = 293.95\underline{/-64^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm CA} = 293.95\underline{/176^\circ}\,\text{V}$$

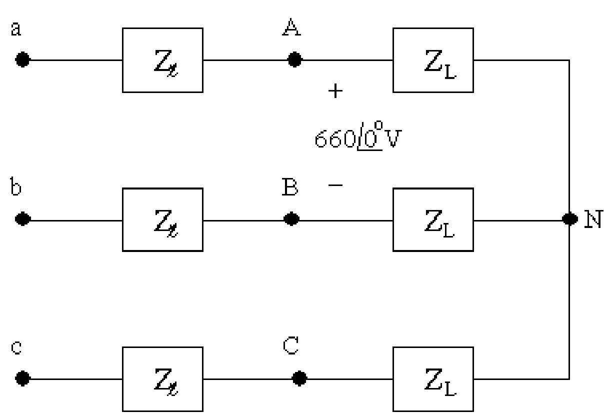
To get back to the time domain, perform an inverse phasor transform of the three line voltages, using a frequency of ω:

$$v_{\rm AB}(t) = 293.95\cos(\omega t + 56^\circ)\,\text{V}$$

$$v_{\rm BC}(t) = 293.95\cos(\omega t - 64^\circ)\,\text{V}$$

$$v_{\rm CA}(t) = 293.95\cos(\omega t + 176^\circ)\,\text{V}$$

P 11.9 Make a sketch of the three-phase line and load:



$$Z_\ell = 0.25 + j2\,\Omega/\phi$$

$$Z_{\rm L} = 30.48 + j22.86\,\Omega/\phi$$

[a] The line currents are $\mathbf{I}_{\rm aA}$, $\mathbf{I}_{\rm bB}$, and $\mathbf{I}_{\rm cC}$. To find $\mathbf{I}_{\rm aA}$, first find $\mathbf{V}_{\rm AN}$ and use Ohm's law for the a-phase load impedance. Since we are only concerned with finding voltage and current magnitudes, the phase sequence doesn't matter and we arbitrarily assume a positive phase sequence. Since we are not given any phase angles in the problem statement, we can assume the angle of $\mathbf{V}_{\rm AB}$ is $0°$. Use Fig. 11.9(a) to find $\mathbf{V}_{\rm AN}$ from $\mathbf{V}_{\rm AB}$.

$$\mathbf{V}_{\rm AN} = \frac{660}{\sqrt{3}}\underline{/(0-30°)} = 381.05\underline{/-30°}\,\text{V}$$

Now find $\mathbf{I}_{\rm aA}$ using Ohm's law:

$$\mathbf{I}_{\rm aA} = \frac{\mathbf{V}_{\rm AN}}{Z_{\rm L}} = \frac{381.05\underline{/-30°}}{30.48 + j22.86} = 3.993 - j9.20 = 10\underline{/-66.87°}\,\text{V}$$

Thus, the magnitude of the line current is

$$|\mathbf{I}_{\rm aA}| = 10\,\text{A}$$

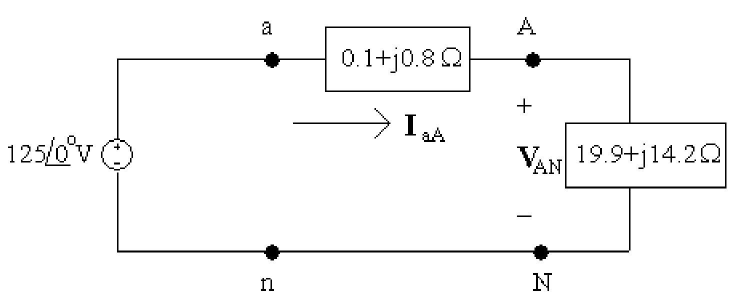
[b] The line voltage at the source is $\mathbf{V}_{\rm ab}$. From KVL on the top loop of the three-phase circuit,

$$\begin{aligned}
\mathbf{V}_{\rm ab} &= \mathbf{V}_{\rm aA} + \mathbf{V}_{\rm AB} + \mathbf{V}_{\rm Bb} \\
&= Z_\ell \mathbf{I}_{\rm aA} + \mathbf{V}_{\rm AB} + Z_\ell \mathbf{I}_{\rm Bb} \\
&= Z_\ell \mathbf{I}_{\rm aA} + \mathbf{V}_{\rm AB} - Z_\ell \mathbf{I}_{\rm bB} \\
&= (0.25 + j2)(10\underline{/-66.87°}) + 660\underline{/0°} - (0.25 + j2)(10\underline{/-173.13°}) \\
&= 684.71\underline{/2.10°}\,\text{V}
\end{aligned}$$

Thus, the magnitude of the line voltage at the source is

$$|\mathbf{V}_{\rm ab}| = 684.71\,\text{V}$$

P 11.10 Make a sketch of the a-phase:



[a] Find the a-phase line current from the a-phase circuit:

$$\mathbf{I}_{\rm aA} = \frac{125\underline{/0°}}{0.1 + j0.8 + 19.9 + j14.2} = \frac{125\underline{/0°}}{20 + j15}$$

$$= 4 - j3 = 4\underline{/-36.87°}\,\text{A}$$

Find the other line currents using the acb phase sequence:

$$\mathbf{I}_{\rm bB} = 5\underline{/-36.87^\circ + 120^\circ} = 5\underline{/83.13^\circ}\ \text{A}$$

$$\mathbf{I}_{\rm cC} = 5\underline{/-36.87^\circ - 120^\circ} = 5\underline{/-156.87^\circ}\ \text{A}$$

[b] The phase voltage at the source is $\mathbf{V}_{\rm an} = 125\underline{/0^\circ}$ V. Use Fig. 11.9(b) to find the line voltage, $\mathbf{V}_{\rm an}$, from the phase voltage:

$$\mathbf{V}_{\rm ab} = \mathbf{V}_{\rm an}(\sqrt{3}\underline{/-30^\circ}) = 216.51\underline{/-30^\circ}\ \text{V}$$

Find the other line voltages using the acb phase sequence:

$$\mathbf{V}_{\rm bc} = 216.51\underline{/-30^\circ + 120^\circ} = 216.51\underline{/90^\circ}\ \text{V}$$

$$\mathbf{V}_{\rm ca} = 216.51\underline{/-30^\circ - 120^\circ} = 216.51\underline{/-150^\circ}\ \text{V}$$

[c] The phase voltage at the load in the a-phase is $\mathbf{V}_{\rm AN}$. Calculate its value using $\mathbf{I}_{\rm aA}$ and the load impedance:

$$\mathbf{V}_{\rm AN} = \mathbf{I}_{\rm aA}Z_{\rm L} = (4 - j3)(19.9 + j14.2) = 122.2 - j2.9 = 122.23\underline{/-1.36^\circ}\ \text{V}$$

Find the phase voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{\rm BN} = 122.23\underline{/-1.36^\circ + 120^\circ} = 122.23\underline{/118.64^\circ}\ \text{V}$$

$$\mathbf{V}_{\rm CN} = 122.23\underline{/-1.36^\circ - 120^\circ} = 122.23\underline{/-121.36^\circ}\ \text{V}$$

[d] The line voltage at the load in the a-phase is $\mathbf{V}_{\rm AB}$. Find this line voltage from the phase voltage at the load in the a-phase, $\mathbf{V}_{\rm AN}$, using Fig, 11.9(b):

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN}(\sqrt{3}\underline{/-30^\circ}) = 211.71\underline{/-31.36^\circ}\ \text{V}$$

Find the line voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{\rm BC} = 211.71\underline{/-31.36^\circ + 120^\circ} = 211.71\underline{/88.69^\circ}\ \text{V}$$

$$\mathbf{V}_{\rm CA} = 211.71\underline{/-31.36^\circ - 120^\circ} = 211.71\underline{/-151.36^\circ}\ \text{V}$$

P 11.11 **[a]** $\mathbf{I}_{\rm AB} = \dfrac{480}{60 + j45} = 6.4\underline{/-36.87^\circ}\ \text{A}$

$$\mathbf{I}_{\rm BC} = 6.4\underline{/-156.87^\circ}\ \text{A}$$

$$\mathbf{I}_{\rm CA} = 6.4\underline{/83.13^\circ}\ \text{A}$$

[b] $\mathbf{I}_{\rm aA} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{\rm AB} = 11.09\underline{/-66.87^\circ}\ \text{A}$

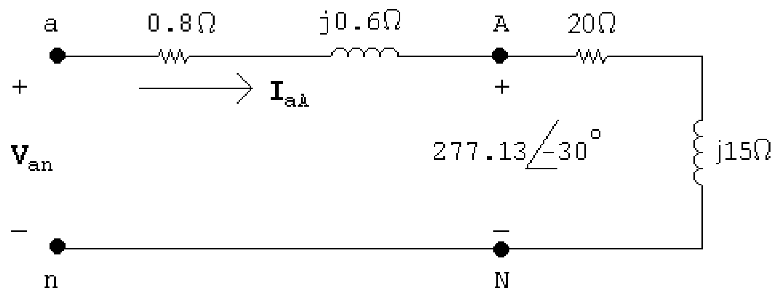
$$\mathbf{I}_{\rm bB} = 11.09\underline{/173.13^\circ}\ \text{A}$$

$$\mathbf{I}_{\rm cC} = 11.09\underline{/53.13^\circ}\ \text{A}$$

[c] Transform the Δ-connected load to a Y-connected load:

$$Z_Y = \frac{Z_\Delta}{3} = \frac{60 + j45}{3} = 20 + j15\,\Omega$$

The single-phase equivalent circuit is:



$$\mathbf{V}_{\rm an} = 277.13\underline{/-30^\circ} + (0.8 + j0.6)(11.09\underline{/-66.87^\circ})$$

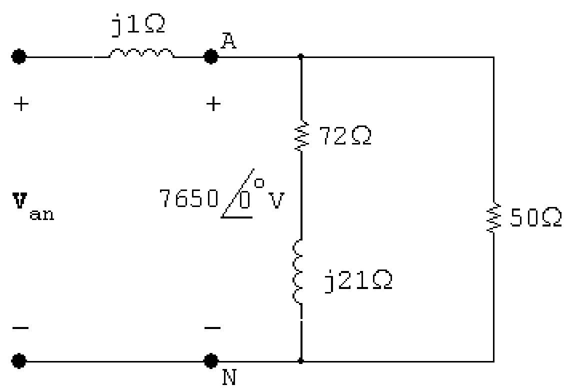
$$= 288.21\underline{/-30^\circ}\text{ V}$$

$$\mathbf{V}_{\rm ab} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\rm an} = 499.20\underline{/0^\circ}\text{ V}$$

$$\mathbf{V}_{\rm bc} = 499.20\underline{/-120^\circ}\text{ V}$$

$$\mathbf{V}_{\rm ca} = 499.20\underline{/120^\circ}\text{ V}$$

P 11.12 **[a]**



$$\mathbf{I}_{\rm aA} = \frac{7650}{72 + j21} + \frac{7650}{50} = 252.54\underline{/-6.49^\circ}\text{ A}$$

$$|\mathbf{I}_{\rm aA}| = 252.54\text{ A}$$

[b] $$\mathbf{I}_{\rm AB} = \frac{7650\sqrt{3}\underline{/30^\circ}}{150} = 88.33\underline{/30^\circ}\text{ A}$$

$$|\mathbf{I}_{\rm AB}| = 88.33\text{ A}$$

[c] $\mathbf{I}_{\rm AN} = \dfrac{7650\underline{/0^\circ}}{72 + j21} = 102\underline{/-16.26^\circ}\,\text{A}$

$|\mathbf{I}_{\rm AN}| = 102\,\text{A}$

[d] $\mathbf{V}_{\rm an} = (252.54\underline{/-6.49^\circ})(j1) + 7650\underline{/0^\circ} = 7682.66\underline{/1.87^\circ}\,\text{V}$

$|\mathbf{V}_{\rm ab}| = \sqrt{3}(7682.66) = 13{,}306.76\,\text{V}$

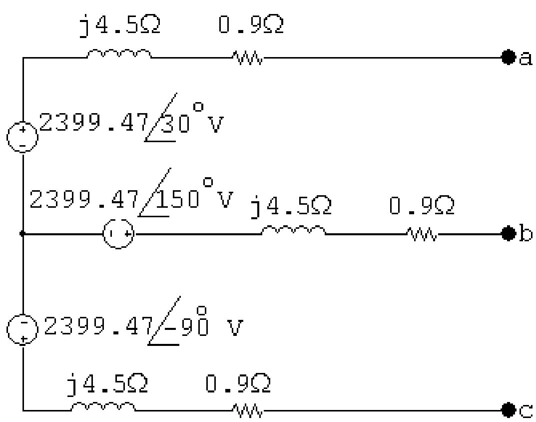
P 11.13 **[a]** Since the phase sequence is acb (negative) we have:

$\mathbf{V}_{\rm an} = 2399.47\underline{/30^\circ}\,\text{V}$

$\mathbf{V}_{\rm bn} = 2399.47\underline{/150^\circ}\,\text{V}$

$\mathbf{V}_{\rm cn} = 2399.47\underline{/-90^\circ}\,\text{V}$

$Z_Y = \frac{1}{3}Z_\Delta = 0.9 + j4.5\,\Omega/\phi$



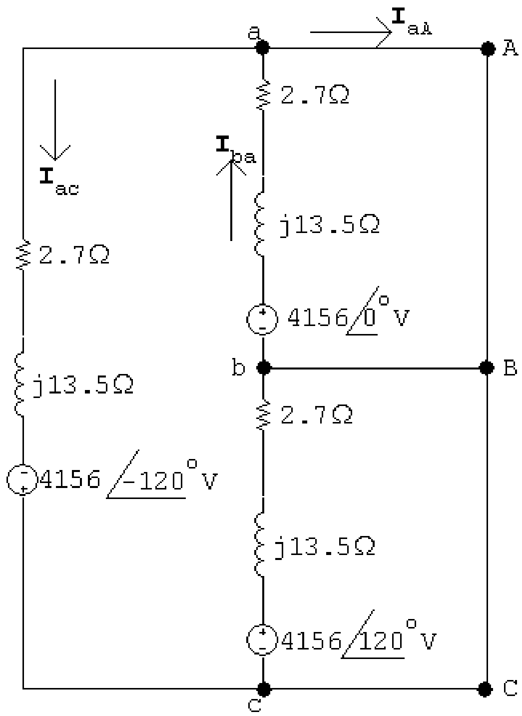
[b] $\mathbf{V}_{\rm ab} = 2399.47\underline{/30^\circ} - 2399.47\underline{/150^\circ} = 2399.47\sqrt{3}\underline{/0^\circ} = 4156\underline{/0^\circ}\,\text{V}$

Since the phase sequence is negative, it follows that

$\mathbf{V}_{\rm bc} = 4156\underline{/120^\circ}\,\text{V}$

$\mathbf{V}_{\rm ca} = 4156\underline{/-120^\circ}\,\text{V}$

[c]



$$\mathbf{I}_{\rm ba} = \frac{4156}{2.7 + j13.5} = 301.87\underline{/-78.69^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm ac} = 301.87\underline{/-198.69^\circ}\,\text{A}$$

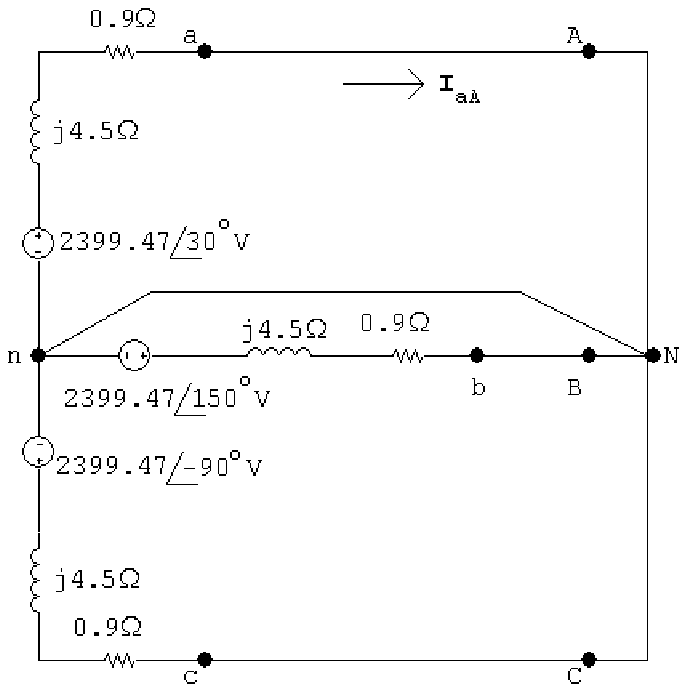
$$\mathbf{I}_{\rm aA} = \mathbf{I}_{\rm ba} - \mathbf{I}_{\rm ac} = 522.86\underline{/-48.69^\circ}\,\text{A}$$

Since we have a balanced three-phase circuit and a negative phase sequence we have:

$$\mathbf{I}_{\rm bB} = 522.86\underline{/71.31^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm cC} = 522.86\underline{/-168.69^\circ}\,\text{A}$$

[d]



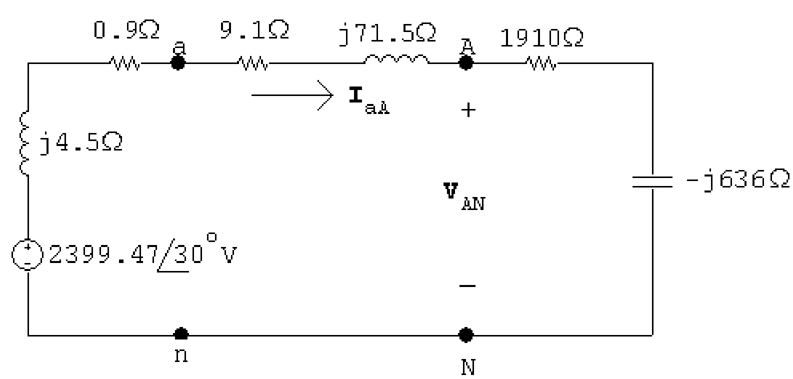
$$\mathbf{I}_{\rm aA} = \frac{2399.47\angle 30^\circ}{0.9 + j4.5} = 522.86\angle -48.69^\circ\,\text{A}$$

Since we have a balanced three-phase circuit and a negative phase sequence we have:

$$\mathbf{I}_{\rm bB} = 522.86\angle 71.31^\circ\,\text{A}$$

$$\mathbf{I}_{\rm cC} = 522.86\angle -168.69^\circ\,\text{A}$$

P 11.14 **[a]**



[b] $\mathbf{I}_{\rm aA} = \dfrac{2399.47\angle 30^\circ}{1920 - j560} = 1.2\angle 46.26^\circ\,\text{A}$

$$\mathbf{V}_{\rm AN} = (1910 - j636)(1.2\angle 46.26^\circ) = 2415.19\angle 27.84^\circ\,\text{V}$$

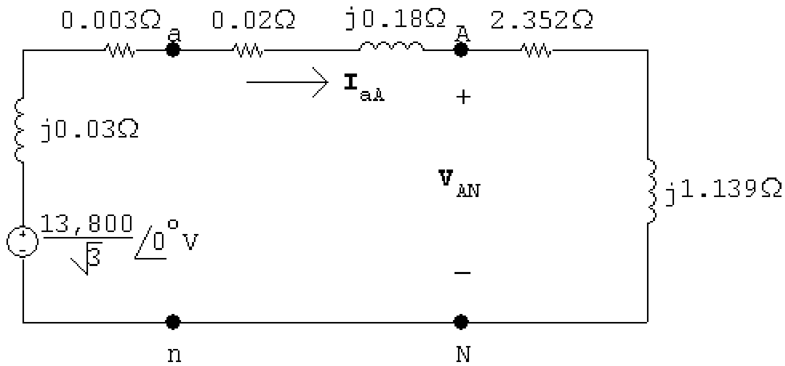
$$|\mathbf{V}_{\rm AB}| = \sqrt{3}(2415.19) = 4183.23\,\text{V}$$

[c] $|\mathbf{I}_{ab}| = \dfrac{1.2}{\sqrt{3}} = 0.69\,\text{A}$

[d] $\mathbf{V}_{an} = (1919.1 - j564.5)(1.2\underline{/46.26^\circ}) = 2400\underline{/29.87^\circ}\,\text{V}$

$|\mathbf{V}_{ab}| = \sqrt{3}(2400) = 4156.92\,\text{V}$

P 11.15 **[a]**



[b] $\mathbf{I}_{aA} = \dfrac{13{,}800}{\sqrt{3}(2.375 + j1.349)} = 2917\underline{/-29.6^\circ}\,\text{A}$

$|\mathbf{I}_{aA}| = 2917\,\text{A}$

[c] $\mathbf{V}_{AN} = (2.352 + j1.139)(2917\underline{/-29.6^\circ}) = 7622.94\underline{/-3.76^\circ}\,\text{V}$

$|\mathbf{V}_{AB}| = \sqrt{3}|\mathbf{V}_{AN}| = 13{,}203.31\,\text{V}$

[d] $\mathbf{V}_{an} = (2.372 + j1.319)(2917\underline{/-29.6^\circ}) = 7616.93\underline{/-0.52^\circ}\,\text{V}$

$|\mathbf{V}_{ab}| = \sqrt{3}|\mathbf{V}_{an}| = 13{,}712.52\,\text{V}$

[e] $|\mathbf{I}_{AB}| = \dfrac{|\mathbf{I}_{aA}|}{\sqrt{3}} = 1684.13\,\text{A}$

[f] $|\mathbf{I}_{ab}| = |\mathbf{I}_{AB}| = 1684.13\,\text{A}$

P 11.16 **[a]** $\mathbf{I}_{AB} = \dfrac{4160\underline{/0^\circ}}{160 + j120} = 20.8\underline{/-36.87^\circ}\,\text{A}$

$\mathbf{I}_{BC} = 20.8\underline{/83.13^\circ}\,\text{A}$

$\mathbf{I}_{CA} = 20.8\underline{/-156.87^\circ}\,\text{A}$

[b] $\mathbf{I}_{aA} = \sqrt{3}\underline{/30^\circ}\mathbf{I}_{AB} = 36.03\underline{/-6.87^\circ}\,\text{A}$

$\mathbf{I}_{bB} = 36.03\underline{/113.13^\circ}\,\text{A}$

$\mathbf{I}_{cC} = 36.03\underline{/-126.87^\circ}\,\text{A}$

[c] $\mathbf{I}_{ba} = \mathbf{I}_{AB} = 20.8\underline{/-36.87^\circ}\,\text{A};$

$\mathbf{I}_{cb} = \mathbf{I}_{BC} = 20.8\underline{/83.13^\circ}\,\text{A};$

$\mathbf{I}_{ac} = \mathbf{I}_{CA} = 20.8\underline{/-156.87^\circ}\,\text{A};$

P 11.17 **[a]** $\mathbf{I}_{AB} = \dfrac{480\underline{/0^\circ}}{2.4 - j0.7} = 192\underline{/16.26^\circ}\,\text{A}$

$$\mathbf{I}_{BC} = \frac{480\underline{/120^\circ}}{8 + j6} = 48\underline{/83.13^\circ}\,\text{A}$$

$$\mathbf{I}_{CA} = \frac{480\underline{/-120^\circ}}{20} = 24\underline{/-120^\circ}\,\text{A}$$

[b] $\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$

$= 210\underline{/20.79^\circ}\,\text{A}$

$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$

$= 178.68\underline{/-178.04^\circ}\,\text{A}$

$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$

$= 70.7\underline{/-104.53^\circ}\,\text{A}$

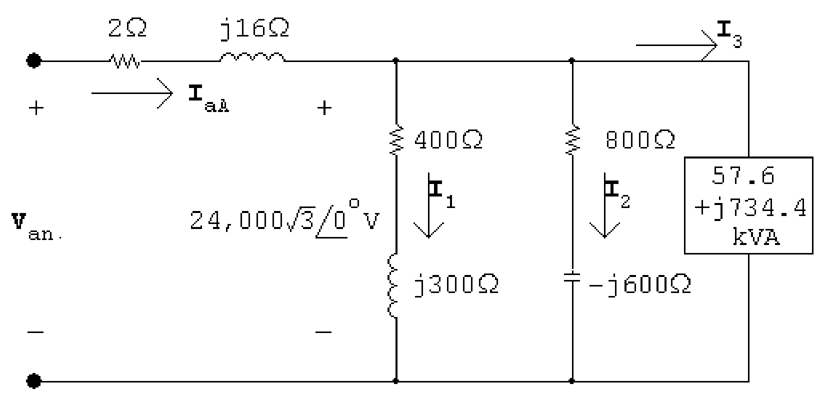
P 11.18 From the solution to Problem 11.17 we have:

$$S_{AB} = (480\underline{/0^\circ})(192\underline{/-16.26^\circ}) = 88{,}473.6 - j25{,}804.8\,\text{VA}$$

$$S_{BC} = (480\underline{/120^\circ})(48\underline{/-83.13^\circ}) = 18{,}432.0 + j13{,}824.0\,\text{VA}$$

$$S_{CA} = (480\underline{/-120^\circ})(24\underline{/120^\circ}) = 11{,}520 + j0\,\text{VA}$$

P 11.19 **[a]**



$$\mathbf{I}_1 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{400 + j300} = 66.5 - j49.9\,\text{A}$$

$$\mathbf{I}_2 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{800 - j600} = 33.3 + j24.9\,\text{A}$$

$$\mathbf{I}_3^* = \frac{57{,}600 + j734{,}400}{24{,}000\sqrt{3}} = 1.4 + j17.7$$

$$\mathbf{I}_3 = 1.4 - j17.7\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 101.2 - j42.7\,\text{A} = 109.8\underline{/-22.8^\circ}\,\text{A}$$

$$\mathbf{V}_{\text{an}} = (2 + j16)(101.2 - j42.7) + 24{,}000\sqrt{3} = 42{,}456.2 + j1533.2\,\text{V}$$

$$S_\phi = \mathbf{V}_{\text{an}}\mathbf{I}^*_{\text{aA}} = (42{,}456.2 + j1533.8)(101.2 + j42.7)$$

$$= 4{,}229.2 + j1964.0\,\text{kVA}$$

$$S_T = 3S_\phi = 12{,}687.7 + j9892.1\,\text{kVA}$$

[b] $S_{1/\phi} = 24{,}000\sqrt{3}(66.5 + j49.9) = 2765.0 + j2073.8\,\text{kVA}$

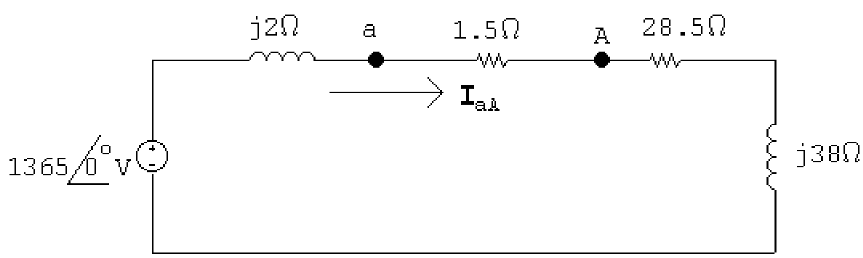
$$S_{2/\phi} = 24{,}000\sqrt{3}(33.3 - j24.9) = 1382.5 - j1036.9\,\text{kVA}$$

$$S_{3/\phi} = 57.6 + j734.4\,\text{kVA}$$

$$S_\phi(\text{load}) = 4205.1 + j1771.3\,\text{kVA}$$

$$\%\text{ delivered } = \left(\frac{4205.1}{4229.2}\right)(100) = 99.4\%$$

P 11.20 **[a]**



$$\mathbf{I}_{\text{aA}} = \frac{1365\underline{/0^\circ}}{30 + j40} = 27.3\underline{/-53.13^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{CA}} = \frac{\mathbf{I}_{\text{aA}}}{\sqrt{3}}\underline{/150^\circ} = 15.76\underline{/96.87^\circ}\,\text{A}$$

[b] $S_{g/\phi} = -1365\mathbf{I}^*_{\text{aA}} = -22{,}358.7 - j29{,}811.6\,\text{VA}$

$$\therefore \quad P_{\text{developed/phase}} = 22.359\,\text{kW}$$

$$P_{\text{absorbed/phase}} = |\mathbf{I}_{\text{aA}}|^2 28.5 = 21.241\,\text{kW}$$

$$\%\text{ delivered } = \frac{21.241}{22.359}(100) = 95\%$$

P 11.21 Let $p_{\rm a}$, $p_{\rm b}$, and $p_{\rm c}$ represent the instantaneous power of phases a, b, and c, respectively. Then assuming a positive phase sequence, we have

$$p_{\rm a} = v_{\rm an}i_{\rm aA} = [V_m \cos \omega t][I_m \cos(\omega t - \theta_\phi)]$$

$$p_{\rm b} = v_{\rm bn}i_{\rm bB} = [V_m \cos(\omega t - 120^\circ)][I_m \cos(\omega t - \theta_\phi - 120^\circ)]$$

$$p_{\rm c} = v_{\rm cn}i_{\rm cC} = [V_m \cos(\omega t + 120^\circ)][I_m \cos(\omega t - \theta_\phi + 120^\circ)]$$

The total instantaneous power is $p_T = p_{\rm a} + p_{\rm b} + p_{\rm c}$, so

$$p_T = V_m I_m[\cos \omega t \cos(\omega t - \theta_\phi) + \cos(\omega t - 120^\circ) \cos(\omega t - \theta_\phi - 120^\circ)$$

$$+ \cos(\omega t + 120^\circ) \cos(\omega t - \theta_\phi + 120^\circ)]$$

Now simplify using trigonometric identities. In simplifying, collect the coefficients of $\cos(\omega t - \theta_\phi)$ and $\sin(\omega t - \theta_\phi)$. We get

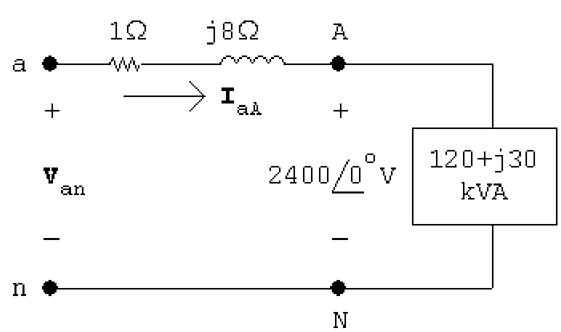
$$\begin{aligned} p_T &= V_m I_m[\cos \omega t(1 + 2\cos^2 120^\circ)\cos(\omega t - \theta_\phi) \\ &\quad +2\sin \omega t \sin^2 120^\circ \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m[\cos \omega t \cos(\omega t - \theta_\phi) + \sin \omega t \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m \cos \theta_\phi \end{aligned}$$

P 11.22 **[a]** $S_{1/\phi} = 40{,}000(0.96) - j40{,}000(0.28) = 38{,}400 - j11{,}200$ VA

$S_{2/\phi} = 60{,}000(0.8) + j60{,}000(0.6) = 48{,}000 + j36{,}000$ VA

$S_{3/\phi} = 33{,}600 + j5200$ VA

$S_{T/\phi} = S_1 + S_2 + S_3 = 120{,}000 + j30{,}000$ VA



$$\therefore \quad \mathbf{I}^*_{\rm aA} = \frac{120{,}000 + j30{,}000}{2400} = 50 + j12.5$$

$$\therefore \quad \mathbf{I}_{\rm aA} = 50 - j12.5\,\text{A}$$

$$\mathbf{V}_{\rm an} = 2400 + (50 - j12.5)(1 + j8) = 2550 + j387.5 = 2579.27\underline{/8.64^\circ}\,\text{V}$$

$$|\mathbf{V}_{\rm ab}| = \sqrt{3}(2579.27) = 4467.43\,\text{V}$$

[b] $S_{g/\phi} = (2550 + j387.5)(50 + j12.5) = 122{,}656.25 + j51{,}250\,\text{VA}$

$$\%\ \text{efficiency} = \frac{120{,}000}{122{,}656.25}(100) = 97.83\%$$

P 11.23 **[a]** $S_1 = (4.864 + j3.775)\,\text{kVA}$

$$S_2 = 17.636(0.96) + j17.636(0.28) = (16.931 + j4.938)\,\text{kVA}$$

$$\sqrt{3}V_{\rm L}I_{\rm L}\sin\theta_3 = 13{,}853; \qquad \sin\theta_3 = \frac{13{,}853}{\sqrt{3}(208)(73.8)} = 0.521$$

Therefore $\cos\theta_3 = 0.854$

Therefore

$$P_3 = \frac{13{,}853}{0.521} \times 0.854 = 22{,}693.58\,\text{W}$$

$$S_3 = 22.694 + j13.853\,\text{kVA}$$

$$S_T = S_1 + S_2 + S_3 = 44.49 + j22.57\,\text{kVA}$$

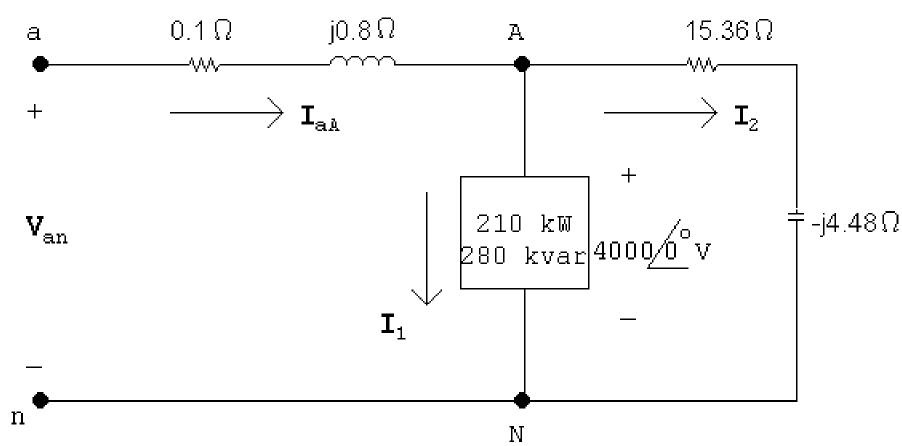
$$S_{T/\phi} = \frac{1}{3}S_T = 14.83 + j7.52\,\text{kVA}$$

$$\frac{208}{\sqrt{3}}\mathbf{I}^*_{\rm aA} = (14.83 + j7.52)10^3; \qquad \mathbf{I}^*_{\rm aA} = 123.49 + j62.64\,\text{A}$$

$$\mathbf{I}_{\rm aA} = 123.49 - j62.64 = 138.46\underline{/-26.90^\circ}\,\text{A} \quad \text{(rms)}$$

[b] pf $= \cos(-26.90^\circ) = 0.892$ lagging

P 11.24



$$4000\mathbf{I}^*_1 = (210 + j280)10^3$$

$$\mathbf{I}_1^* = \frac{210}{4} + j\frac{280}{4} = 52.5 + j70\,\text{A}$$

$$\mathbf{I}_1 = 52.5 - j70\,\text{A}$$

$$\mathbf{I}_2 = \frac{4000\underline{/0^\circ}}{15.36 - j4.48} = 240 + j70\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 = 292.5 + j0\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 4000 + j0 + 292.5(0.1 + j0.8) = 4036.04\underline{/3.32^\circ}\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 6990.62\,\text{V}$$

P 11.25 **[a]** $P_{\text{OUT}} = 746 \times 100 = 74{,}600\,\text{W}$

$$P_{\text{IN}} = 74{,}600/(0.97) = 76{,}907.22\,\text{W}$$

$$\sqrt{3}V_L I_L \cos\theta = 76{,}907.22$$

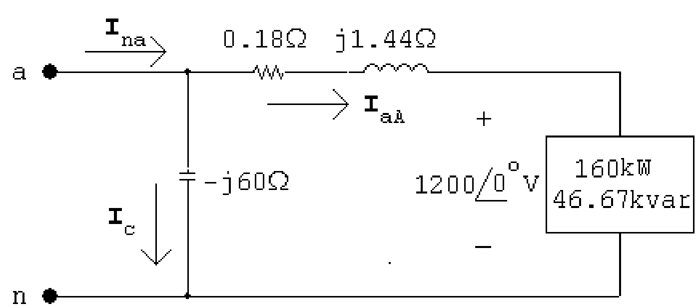
$$I_L = \frac{76{,}907.22}{\sqrt{3}(208)(0.88)} = 242.58\,\text{A}$$

[b] $Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3}(208)(242.58)(0.475) = 41{,}510.12\,\text{VAR}$

P 11.26 **[a]** $\mathbf{I}_{\text{aA}}^* = \dfrac{(160 + j46.67)10^3}{1200} = 133.3 + j38.9$

$$\mathbf{I}_{\text{aA}} = 133.3 - j38.9\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 1200 + (133.3 - j38.9)(0.18 + j1.44) = 1280 + j185\,\text{V}$$



$$\mathbf{I}_{\text{C}} = \frac{1280 + j185}{-j60} = -3.1 + j21.3\,\text{A}$$

$$\mathbf{I}_{\text{na}} = (\mathbf{I}_{\text{aA}} + \mathbf{I}_{\text{C}}) = -130.3 - j17.6 = 131.4\underline{/7.7^\circ}\,\text{A}$$

[b] $S_{g/\phi} = (1280 + j185)(-130.3 - j17.6) = -163{,}472 - j46{,}567.4\,\text{VA}$

$S_{gT} = 3S_{g/\phi} = -490.4 - j139.7\,\text{kVA}$

Therefore, the source is delivering 490.4 kW and 139.7 kvars.

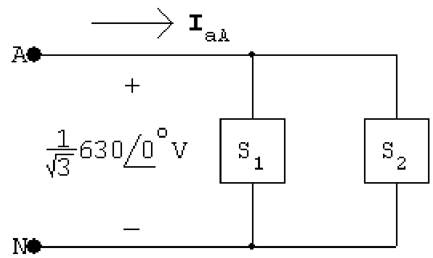
[c] $P_{\text{del}} = 490.4\,\text{kW}$

$$P_{\text{abs}} = 3(160{,}000) + 3|\mathbf{I}_{\text{aA}}|^2(0.18)$$
$$= 490.4\,\text{kW} = P_{\text{del}}$$

[d] $Q_{\text{del}} = 3|\mathbf{I}_{\text{C}}|^2(60) + 139.7 \times 10^3 = 223.3\,\text{kVAR}$

$$Q_{\text{abs}} = 3(46{,}666) + 3|\mathbf{I}_{\text{aA}}|^2(1.44)$$
$$= 223.3\,\text{kVAR} = Q_{\text{del}}$$

P 11.27 **[a]**



$$S_{s/\phi} = \frac{1}{3}(60)(0.96 - j0.28) \times 10^3 = 19.2 - j5.6\,\text{kVA}$$

$$S_{1/\phi} = 15\,\text{kVA}$$

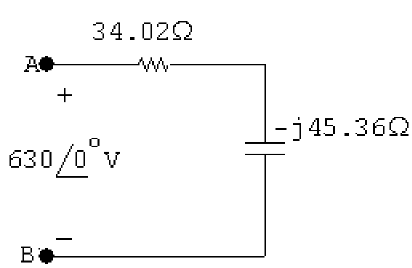
$$S_{2/\phi} = S_{s/\phi} - S_{1/\phi} = 4.2 - j5.6\,\text{kVA}$$

$$\therefore \quad \mathbf{I}_2^* = \frac{4200 - j5600}{630/\sqrt{3}} = 11.547 - j15.396\,\text{A}$$

$$\mathbf{I}_2 = 11.547 + j15.396\,\text{A}$$

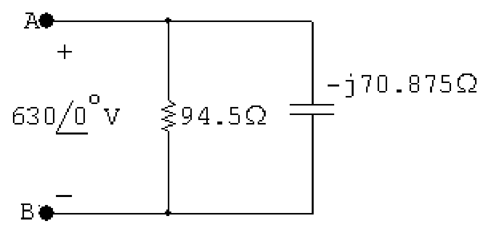
$$Z_y = \frac{630\underline{/0^\circ}/\sqrt{3}}{\mathbf{I}_2} = 11.34 - j15.12\,\Omega$$

$$Z_\Delta = 3Z_y = 34.02 - j45.36\,\Omega$$



[b] $R = \dfrac{(630/\sqrt{3})^2}{4200} = 31.5\,\Omega; \qquad R_\Delta = 3R = 94.5\,\Omega$

$$X_L = \frac{(630/\sqrt{3})^2}{-5600} = -23.625\,\Omega; \qquad X_\Delta = 3X_L = -70.875\,\Omega$$

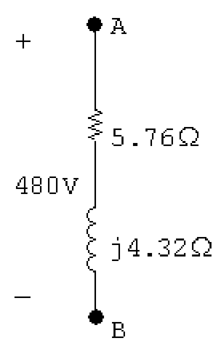


P 11.28 Assume a Δ-connect load (series):

$$S_\phi = \frac{1}{3}(96 \times 10^3)(0.8 + j0.6) = 25{,}600 + j19{,}200\,\text{VA}$$

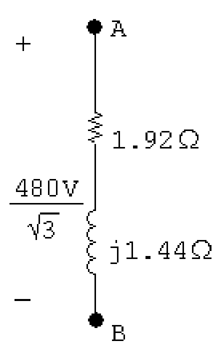
$$Z^*_{\Delta\phi} = \frac{|480|^2}{25{,}600 + j19{,}200} = 5.76 - j4.32\,\Omega$$

$$Z_{\Delta\phi} = 5.76 + 4.32\,\Omega$$



Now assume a Y-connected load (series):

$$Z_{Y\phi} = \frac{1}{3}Z_{\Delta\phi} = 1.92 + j1.44\,\Omega$$



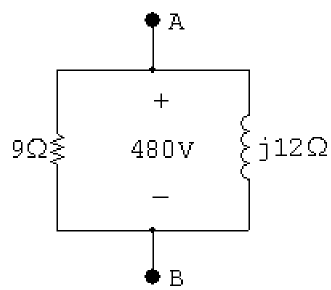
Now assume a Δ-connected load (parallel):

$$P_\phi = \frac{|480|^2}{R_\Delta}$$

$$R_{\Delta\phi} = \frac{|480|^2}{25{,}600} = 9\,\Omega$$

$$Q_\phi = \frac{|480|^2}{X_\Delta}$$

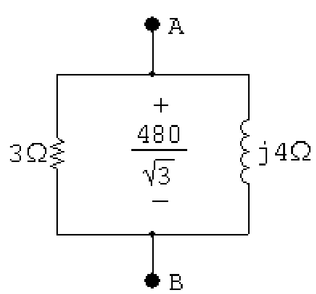
$$X_\Delta\phi = \frac{|480|^2}{19{,}200} = 12\,\Omega$$



Now assume a Y-connected load (parallel):

$$R_{Y\phi} = \frac{1}{3}R_{\Delta\phi} = 3\,\Omega$$

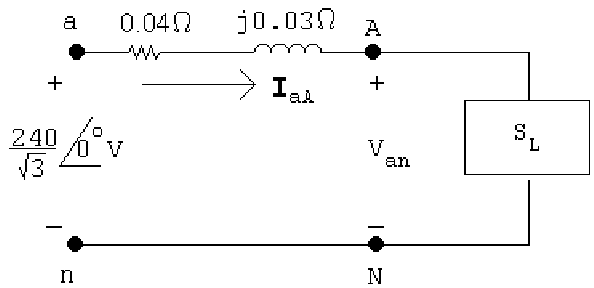
$$X_{Y\phi} = \frac{1}{3}X_{\Delta\phi} = 4\,\Omega$$



P 11.29 $S_{g/\phi} = \frac{1}{3}(41.6)(0.707 + j0.707) \times 10^3 = 9803.73 + j9806.69\,\text{VA}$

$$\mathbf{I}_{\text{aA}}^* = \frac{9803.73 + j9803.73}{240/\sqrt{3}} = 70.75 + j70.77\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 70.75 - j70.77\,\text{A}$$



$$\mathbf{V}_{\text{AN}} = \frac{240}{\sqrt{3}} - (0.04 + j0.03)(70.75 - j70.77)$$
$$= 133.61 + j0.71 = 133.61\angle 0.30^\circ\,\text{V}$$

$$|\mathbf{V}_{\text{AB}}| = \sqrt{3}(133.61) = 231.42\,\text{V}$$

[b] $S_{L/\phi} = (133.61 + j0.71)(70.76 + j70.76) = 9403.1 + j9506.3\,\text{VA}$

$$S_L = 3S_{L/\phi} = 28{,}209 + j28{,}519\,\text{VA}$$

Check:

$$S_g = 41{,}600(0.707 + j0.707) = 29{,}411 + j29{,}420\,\text{VA}$$

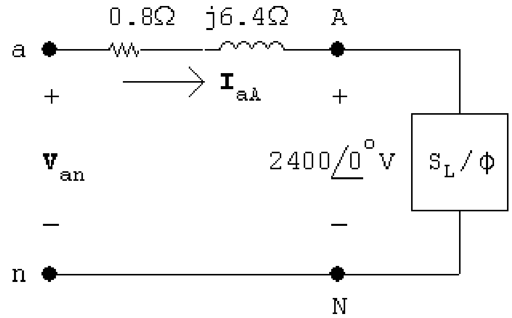
$$P_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.04) = 1202\,\text{W}$$

$$P_g = P_L + P_\ell = 28{,}209 + 1202 = 29{,}411\,\text{W} \quad \text{(checks)}$$

$$Q_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.03) = 901\,\text{VAR}$$

$$Q_g = Q_L + Q_\ell = 28{,}519 + 901 = 29{,}420\,\text{VAR} \quad \text{(checks)}$$

P 11.30 **[a]**



$$S_{L/\phi} = \frac{1}{3}\left[720 + j\frac{720}{0.8}(0.6)\right]10^3 = 240{,}000 + j180{,}000\,\text{VA}$$

$$\mathbf{I}^*_{\text{aA}} = \frac{240{,}000 + j180{,}000}{2400} = 100 + j75\,\text{A}$$

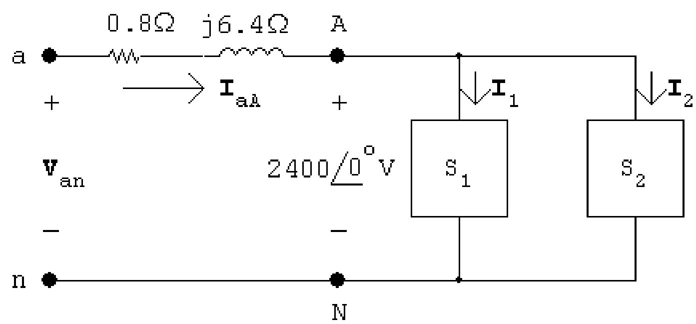
$$\mathbf{I}_{\text{aA}} = 100 - j75\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 2400 + (0.8 + j6.4)(100 - j75)$$

$$= 2960 + j580 = 3016.29\angle 11.09^\circ\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(3016.29) = 5224.37\,\text{V}$$

[b]



$$\mathbf{I}_1 = 100 - j75\,\text{A} \qquad \text{(from part [a])}$$

$$S_2 = 0 - j\frac{1}{3}(576) \times 10^3 = -j192{,}000\,\text{VAR}$$

$$\mathbf{I}^*_2 = \frac{-j192{,}000}{2400} = -j80\,\text{A}$$

$$\therefore \quad \mathbf{I}_2 = j80\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 100 - j75 + j80 = 100 + j5\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 2400 + (100 + j5)(0.8 + j6.4)$$

$$= 2448 + j644 = 2531.29\angle 14.74^\circ\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2531.29) = 4384.33\,\text{V}$$

[c] $|\mathbf{I}_{aA}| = 125\,\text{A}$

$$P_{\text{loss}/\phi} = (125)^2(0.8) = 12{,}500\,\text{W}$$

$$P_{g/\phi} = 240{,}000 + 12{,}500 = 252.5\,\text{kW}$$

$$\%\,\eta = \frac{240}{252.5}(100) = 95.05\%$$

[d] $|\mathbf{I}_{aA}| = 100.125\,\text{A}$

$$P_{\ell/\phi} = (100.125)^2(0.8) = 8020\,\text{W}$$

$$\%\,\eta = \frac{240{,}000}{248{,}200}(100) = 96.77\%$$

[e] $Z_{\text{cap/Y}} = -j\dfrac{2400^2}{-192{,}000} = -j30\,\Omega$

$$Z_{\text{cap}/\Delta} = 3Z_{\text{cap/Y}} = -j90\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 90; \qquad C = \frac{1}{(90)(120\pi)} = 29.47\,\mu\text{F}$$

P 11.31 **[a]** From Assessment Problem 11.9, $\mathbf{I}_{aA} = (101.8 - j135.7)\,\text{A}$

Therefore $\mathbf{I}_{\text{cap}} = j135.7\,\text{A}$

Therefore $Z_{CY} = \dfrac{2450/\sqrt{3}}{j135.7} = -j10.42\,\Omega$

Therefore $C_Y = \dfrac{1}{(10.42)(2\pi)(60)} = 254.5\,\mu\text{F}$

$$Z_{C\Delta} = (-j10.42)(3) = -j31.26\,\Omega$$

Therefore $C_\Delta = \dfrac{254.5}{3} = 84.84\,\mu\text{F}$

[b] $C_Y = 254.5\,\mu\text{F}$

[c] $|\mathbf{I}_{aA}| = 101.8\,\text{A}$

P 11.32 $Z_\phi = |Z|\underline{/\theta} = \frac{\mathbf{V}_{\rm AN}}{\mathbf{I}_{\rm aA}}$

$\theta = \underline{/\mathbf{V}_{\rm AN}} - \underline{/\mathbf{I}_{\rm aA}}$

$\theta_1 = \underline{/\mathbf{V}_{\rm AB}} - \underline{/\mathbf{I}_{\rm aA}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{\rm AB}} = \underline{/\mathbf{V}_{\rm AN}} + 30^\circ$

Thus,

$\theta_1 = \underline{/\mathbf{V}_{\rm AN}} + 30^\circ - \underline{/\mathbf{I}_{\rm aA}} = \theta + 30^\circ$

Similarly,

$Z_\phi = |Z|\underline{/\theta} = \frac{\mathbf{V}_{\rm CN}}{\mathbf{I}_{\rm cC}}$

$\theta = \underline{/\mathbf{V}_{\rm CN}} - \underline{/\mathbf{I}_{\rm cC}}$

$\theta_2 = \underline{/\mathbf{V}_{\rm CB}} - \underline{/\mathbf{I}_{\rm cC}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{\rm CB}} = \underline{/\mathbf{V}_{\rm BA}} - 120^\circ = \underline{/\mathbf{V}_{\rm AB}} + 60^\circ$

$\underline{/\mathbf{I}_{\rm cC}} = \underline{/\mathbf{I}_{\rm aA}} + 120^\circ$

Thus,

$$\begin{aligned}\theta_2 &= \underline{/\mathbf{V}_{\rm AB}} + 60^\circ - \underline{/\mathbf{I}_{\rm aA}} - 120^\circ = \theta_1 - 60^\circ \\ &= \theta + 30^\circ - 60^\circ = \theta - 30^\circ\end{aligned}$$

P 11.33 Use values from the negative sequence part of Example 11.1 — part (g):

$\mathbf{V}_{\rm AB} = 199.58\underline{/-31.19^\circ}\ \mathrm{V}$

$\mathbf{I}_{\rm aA} = 2.5\underline{/-36.87^\circ}\ \mathrm{A}$

$W_{m1} = |\mathbf{V}_{\rm AB}||\mathbf{I}_{\rm aA}|\cos(\underline{/\mathbf{V}_{\rm AB}} - \underline{/\mathbf{I}_{\rm aA}}) = (199.58)(2.4)\cos(5.68^\circ) = 476.63\,W$

$W_{m2} = |\mathbf{V}_{\rm CB}||\mathbf{I}_{\rm cC}|\cos(\underline{/\mathbf{V}_{\rm CB}} - \underline{/\mathbf{I}_{\rm cC}}) = (199.58)(2.4)\cos(65.68^\circ) = 197.29\,W$

CHECK: $W_1 + W_2 = 673.9 = (2.4)^2(39)(3) = 673.9\,W$

P 11.34 **[a]** $W_2 - W_1 = V_L I_L[\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)]$

$$= V_L I_L[\cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ - \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ]$$

$$= 2V_L I_L \sin\theta\sin 30^\circ = V_L I_L \sin\theta,$$

therefore $\sqrt{3}(W_2 - W_1) = \sqrt{3}V_L I_L \sin\theta = Q_T$

[b] $Z_\phi = (8 + j6)\,\Omega$

$Q_T = \sqrt{3}[2476.25 - 979.75] = 2592\,\text{VAR},$

$Q_T = 3(12)^2(6) = 2592\,\text{VAR};$

$Z_\phi = (8 - j6)\,\Omega$

$Q_T = \sqrt{3}[979.75 - 2476.25] = -2592\,\text{VAR},$

$Q_T = 3(12)^2(-6) = -2592\,\text{VAR};$

$Z_\phi = 5(1 + j\sqrt{3})\,\Omega$

$Q_T = \sqrt{3}[2160 - 0] = 3741.23\,\text{VAR},$

$Q_T = 3(12)^2(5\sqrt{3}) = 3741.23\,\text{VAR};$

$Z_\phi = 10\underline{/-75^\circ}\,\Omega$

$Q_T = \sqrt{3}[-645.53 - 1763.63] = -4172.80\,\text{VAR},$

$Q_T = 3(12)^2[-10\sin 75^\circ] = -4172.80\,\text{VAR}$

P 11.35 $\mathbf{I}_{\text{aA}} = (\mathbf{V}_{\text{AN}}/Z_\phi) = |\mathbf{I}_L|\underline{/-\theta_\phi}\,\text{A},$

$Z_\phi = |Z|\underline{/\theta_\phi}, \qquad \mathbf{V}_{\text{BC}} = |\mathbf{V}_L|\underline{/-90^\circ}\,\text{V},$

$$W_m = |\mathbf{V}_L|\,|\mathbf{I}_L|\cos[-90^\circ - (-\theta_\phi)]$$

$$= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos(\theta_\phi - 90^\circ)$$

$$= |\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi,$$

therefore $\sqrt{3}W_m = \sqrt{3}|\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi = Q_{\text{total}}$

P 11.36 **[a]** $Z = 16 - j12 = 20\underline{/-36.87^\circ}\,\Omega$

$\mathbf{V}_{\rm AN} = 680\underline{/0^\circ}\,\text{V}; \qquad \therefore \quad \mathbf{I}_{\rm aA} = 34\underline{/36.87^\circ}\,\text{A}$

$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = 680\sqrt{3}\underline{/-90^\circ}\,\text{V}$

$W_m = (680\sqrt{3})(34)\cos(-90 - 36.87^\circ) = -24{,}027.0\,\text{W}$

$\sqrt{3}W_m = -41{,}616.0\,\text{VAR}$

[b] $Q_\phi = (34^2)(-12) = -13{,}872\,\text{VAR}$

$Q_T = 3Q_\phi = -41{,}616\,\text{VAR} = \sqrt{3}W_m$

P 11.37 **[a]** $Z_\phi = 160 + j120 = 200\underline{/36.87^\circ}\,\Omega$

$$S_\phi = \frac{4160^2}{160 - j120} = 69{,}222.4 + j51{,}916.8\,\text{VA}$$

$S_T = 3S_\phi = 207{,}667.2 + j155{,}750.4\,\text{VA}$

[b] $W_{m1} = (4160)(36.03)\cos(0 + 6.87^\circ) = 148{,}808.64\,\text{W}$

$W_{m2} = (4160)(36.03)\cos(-60^\circ + 126.87^\circ) = 58{,}877.55\,\text{W}$

Check: $\qquad P_T = 207.7\,\text{kW} = W_{m1} + W_{m2}.$

P 11.38 **[a]** $\mathbf{I}^*_{\rm aA} = \dfrac{144(0.96 - j0.28)10^3}{7200} = 20\underline{/-16.26^\circ}\,\text{A}$

$\mathbf{V}_{\rm BN} = 7200\underline{/-120^\circ}\,\text{V}; \qquad \mathbf{V}_{\rm CN} = 7200\underline{/120^\circ}\,\text{V}$

$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = 7200\sqrt{3}\underline{/-90^\circ}\,\text{V}$

$\mathbf{I}_{\rm bB} = 20\underline{/-103.74^\circ}\,\text{A}$

$W_{\rm m1} = (7200\sqrt{3})(20)\cos(-90^\circ + 103.74^\circ) = 242{,}278.14\,\text{W}$

[b] Current coil in line aA, measure $\mathbf{I}_{\rm aA}$.
Voltage coil across AC, measure $\mathbf{V}_{\rm AC}$.

[c] $I_{\rm aA} = 20\underline{/16.76^\circ}\,\text{A}$

$\mathbf{V}_{\rm AC} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm CN} = 7200\sqrt{3}\underline{/-30^\circ}\,\text{V}$

$W_{\rm m2} = (7200\sqrt{3})(20)\cos(-30^\circ - 16.26^\circ) = 172{,}441.86\,\text{W}$

[d] $W_{\rm m1} + W_{\rm m2} = 414.72\text{kW}$

$P_T = 432{,}000(0.96) = 414.72\,\text{kW} = W_{\rm m1} + W_{\rm m2}$

P 11.39 **[a]** $W_1 = |\mathbf{V}_{\rm BA}||\mathbf{I}_{\rm bB}|\cos\theta_1$

Negative phase sequence:

$$\mathbf{V}_{\rm BA} = 240\sqrt{3}\underline{/150^\circ}\,\mathrm{V}$$

$$\mathbf{I}_{\rm aA} = \frac{240\underline{/0^\circ}}{13.33\underline{/-30^\circ}} = 18\underline{/30^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\rm bB} = 18\underline{/150^\circ}\,\mathrm{A}$$

$$W_1 = (18)(240)\sqrt{3}\cos 0^\circ = 7482.46\,\mathrm{W}$$

$$W_2 = |\mathbf{V}_{\rm CA}||\mathbf{I}_{\rm cC}|\cos\theta_2$$

$$\mathbf{V}_{\rm CA} = 240\sqrt{3}\underline{/-150^\circ}\,\mathrm{V}$$

$$\mathbf{I}_{\rm cC} = 18\underline{/-90^\circ}\,\mathrm{A}$$

$$W_2 = (18)(240)\sqrt{3}\cos(-60^\circ) = 3741.23\,\mathrm{W}$$

[b] $P_\phi = (18)^2(40/3)\cos(-30^\circ) = 3741.23\,\mathrm{W}$

$$P_T = 3P_\phi = 11{,}223.69\,\mathrm{W}$$

$$W_1 + W_2 = 7482.46 + 3741.23 = 11{,}223.69\,\mathrm{W}$$

$\therefore\quad W_1 + W_2 = P_T$ (checks)

P 11.40 **[a]** Negative phase sequence:

$$\mathbf{V}_{\rm AB} = 240\sqrt{3}\underline{/-30^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\rm BC} = 240\sqrt{3}\underline{/90^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\rm CA} = 240\sqrt{3}\underline{/-150^\circ}\,\mathrm{V}$$

$$\mathbf{I}_{\rm AB} = \frac{240\sqrt{3}\underline{/-30^\circ}}{20\underline{/30^\circ}} = 20.78\underline{/-60^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\rm BC} = \frac{240\sqrt{3}\underline{/90^\circ}}{60\underline{/0^\circ}} = 6.93\underline{/90^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\rm CA} = \frac{240\sqrt{3}\underline{/-150^\circ}}{40\underline{/-30^\circ}} = 10.39\underline{/-120^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\rm aA} = \mathbf{I}_{\rm AB} + \mathbf{I}_{\rm AC} = 18\underline{/-30^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\rm cC} = \mathbf{I}_{\rm CB} + \mathbf{I}_{\rm CA} = \mathbf{I}_{\rm CA} - \mathbf{I}_{\rm BC} = 16.75\underline{/-108.06^\circ}\,\mathrm{A}$$

$$W_{m1} = 240\sqrt{3}(18)\cos(-30 + 30^\circ) = 7482.46\,\mathrm{W}$$

$$W_{m2} = 240\sqrt{3}(16.75)\cos(-90 + 108.07^\circ) = 6621.23\,\mathrm{W}$$

[b] $W_{m1} + W_{m2} = 14{,}103.69\,\text{W}$

$$P_{\text{A}} = (12\sqrt{3})^2(20\cos 30^\circ) = 7482.46\,\text{W}$$

$$P_{\text{B}} = (4\sqrt{3})^2(60) = 2880\,\text{W}$$

$$P_{\text{C}} = (6\sqrt{3})^2[40\cos(-30^\circ)] = 3741.23\,\text{W}$$

$$P_{\text{A}} + P_{\text{B}} + P_{\text{C}} = 14{,}103.69 = W_{m1} + W_{m2}$$

P 11.41 $\tan\phi = \dfrac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = 0.7498$

$$\therefore \quad \phi = 36.86^\circ$$

$$\therefore \quad 2400|\mathbf{I}_{\text{L}}|\cos 66.87^\circ = 40{,}823.09$$

$$|\mathbf{I}_{\text{L}}| = 43.3\,\text{A}$$

$$|Z_\phi| = \frac{2400/\sqrt{3}}{43.3} = 32\,\Omega \qquad \therefore \quad Z_\phi = 32\underline{/36.87^\circ}\,\Omega$$

P 11.42 **[a]** $Z = \dfrac{1}{3}Z_\Delta = 4.48 + j15.36 = 16\underline{/73.74^\circ}\,\Omega$

$$\mathbf{I}_{\text{aA}} = \frac{600\underline{/0^\circ}}{16\underline{/73.74^\circ}} = 37.5\underline{/-73.74^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{bB}} = 37.5\underline{/-193.74^\circ}\,\text{A}$$

$$\mathbf{V}_{\text{AC}} = 600\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{\text{BC}} = 600\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$W_1 = (600\sqrt{3})(37.5)\cos(-30 + 73.74^\circ) = 28{,}156.15\,\text{W}$$

$$W_2 = (600\sqrt{3})(37.5)\cos(-90 + 193.74^\circ) = -9256.15\,\text{W}$$

[b] $W_1 + W_2 = 18{,}900\,\text{W}$

$$P_T = 3(37.5)^2(13.44/3) = 18{,}900\,\text{W}$$

[c] $\sqrt{3}(W_1 - W_2) = 64{,}800\,\text{VAR}$

$$Q_T = 3(37.5)^2(46.08/3) = 64{,}800\,\text{VAR}$$

P 11.43 From the solution to Prob. 11.17 we have

$$\mathbf{I}_{aA} = 210\underline{/20.79^\circ}\text{ A} \qquad \text{and} \qquad \mathbf{I}_{bB} = 178.68\underline{/-178.04^\circ}\text{ A}$$

[a] $W_1 = |\mathbf{V}_{ac}|\,|\mathbf{I}_{aA}|\cos(\theta_{ac} - \theta_{aA})$

$= 480(210)\cos(60^\circ - 20.79^\circ) = 78{,}103.2\text{ W}$

[b] $W_2 = |\mathbf{V}_{bc}|\,|\mathbf{I}_{bB}|\cos(\theta_{bc} - \theta_{bB})$

$= 480(178.68)\cos(120^\circ + 178.04^\circ) = 40{,}317.7\text{ W}$

[c] $W_1 + W_2 = 118{,}421\text{ W}$

$P_{AB} = (192)^2(2.4) = 88{,}473.6\text{ W}$

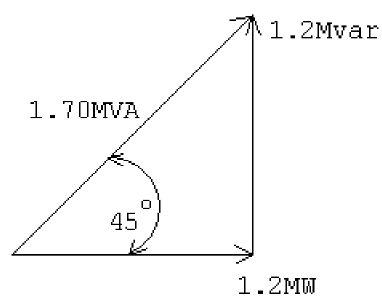
$P_{BC} = (48)^2(8) = 18{,}432\text{ W}$

$P_{CA} = (24)^2(20) = 11{,}520\text{ W}$

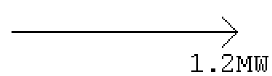
$P_{AB} + P_{BC} + P_{CA} = 118{,}425.7$

therefore $W_1 + W_2 \approx P_{\text{total}}$ (round-off differences)

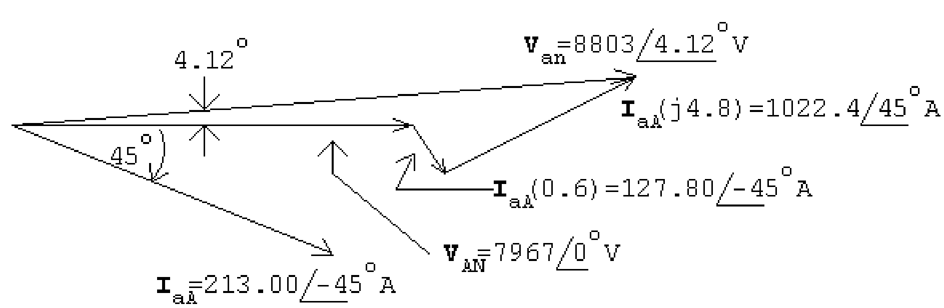
P 11.44 **[a]** For one phase,



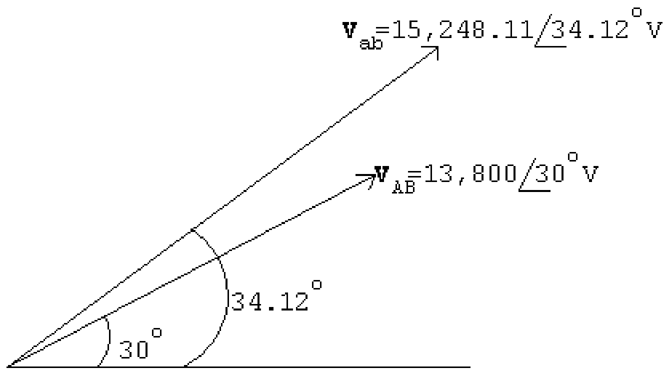
[b]



[c]



[d]



P 11.45 **[a]** $Q = \dfrac{|\mathbf{V}|^2}{X_C}$

$$\therefore \quad |X_C| = \frac{(13{,}800)^2}{1.2\times10^6} = 158.70\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 158.70; \qquad C = \frac{1}{2\pi(60)(158.70)} = 16.71\,\mu\text{F}$$

[b] $|X_C| = \dfrac{(13{,}800/\sqrt{3})^2}{1.2\times10^6} = \dfrac{1}{3}(158.70)$

$$\therefore \quad C = 3(16.71) = 50.14\,\mu\text{F}$$

P 11.46 If the capacitors remain connected when the substation drops its load, the expression for the line current becomes

$$\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{\text{aA}}^* = -j1.2\times10^6$$

or $\quad \mathbf{I}_{\text{aA}}^* = -j150.61\,\text{A}$

Hence $\quad \mathbf{I}_{\text{aA}} = j150.61\,\text{A}$

Now,

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(j150.61) = 7244.49 + j90.37 = 7245.05\underline{/0.71^\circ}\,\text{V}$$

The magnitude of the line-to-line voltage at the generating plant is

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(7245.05) = 12{,}548.80\,\text{V}.$$

This is a problem because the voltage is below the acceptable minimum of 13 kV. Thus when the load at the substation drops off, the capacitors must be switched off.

P 11.47 Before the capacitors are added the total line loss is

$$P_L = 3|150.61 + j150.61|^2(0.6) = 81.66\,\text{kW}$$

After the capacitors are added the total line loss is

$$P_L = 3|150.61|^2(0.6) = 40.83\,\text{kW}$$

Note that adding the capacitors to control the voltage level also reduces the amount of power loss in the lines, which in this example is cut in half.

P 11.48 **[a]** $\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{aA}^* = 80 \times 10^3 + j200 \times 10^3 - j1200 \times 10^3$

$$\mathbf{I}_{aA}^* = \frac{80\sqrt{3} - j1000\sqrt{3}}{13.8} = 10.04 - j125.51\,\text{A}$$

$$\therefore \quad \mathbf{I}_{aA} = 10.04 + j125.51\,\text{A}$$

$$\mathbf{V}_{an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(10.04 + j125.51)$$

$$= 7371.01 + j123.50 = 7372.04\underline{/0.96^\circ}\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{ab}| = \sqrt{3}(7372.04) = 12{,}768.75\,\text{V}$$

[b] Yes, the magnitude of the line-to-line voltage at the power plant is less than the allowable minimum of 13 kV.

P 11.49 **[a]** $\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{aA}^* = (80 + j200) \times 10^3$

$$\mathbf{I}_{aA}^* = \frac{80\sqrt{3} + j200\sqrt{3}}{13.8} = 10.04 + j25.1\,\text{A}$$

$$\therefore \quad \mathbf{I}_{aA} = 10.04 - j25.1\,\text{A}$$

$$\mathbf{V}_{an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(10.04 - j25.1)$$

$$= 8093.95 + j33.13 = 8094.02\underline{/0.23^\circ}\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{ab}| = \sqrt{3}(8094.02) = 14{,}019.25\,\text{V}$$

[b] Yes: $13\,\text{kV} < 14{,}019.25 < 14.6\,\text{kV}$

[c] $P_{loss} = 3|10.04 + j125.51|^2(0.6) = 28.54\,\text{kW}$

[d] $P_{loss} = 3|10.04 + j25.1|^2(0.6) = 1.32\,\text{kW}$

[e] Yes, the voltage at the generating plant is at an acceptable level and the line loss is greatly reduced.