# 15

# Active Filter Circuits

## Assessment Problems

AP 15.1 $H(s) = \dfrac{-(R_2/R_1)s}{s + (1/R_1C)}$

$\dfrac{1}{R_1C} = 1\,\text{rad/s}; \qquad R_1 = 1\,\Omega, \quad \therefore \ \ C = 1\,\text{F}$

$\dfrac{R_2}{R_1} = 1, \quad \therefore \ \ R_2 = R_1 = 1\,\Omega$

$\therefore \qquad H_{\text{prototype}}(s) = \dfrac{-s}{s+1}$

AP 15.2 $H(s) = \dfrac{-(1/R_1C)}{s + (1/R_2C)} = \dfrac{-20{,}000}{s + 5000}$

$\dfrac{1}{R_1C} = 20{,}000; \quad C = 5\,\mu\text{F}$

$\therefore \ \ R_1 = \dfrac{1}{(20{,}000)(5 \times 10^{-6})} = 10\,\Omega$

$\dfrac{1}{R_2C} = 5000$

$\therefore \ \ R_2 = \dfrac{1}{(5000)(5 \times 10^{-6})} = 40\,\Omega$

AP 15.3  $\omega_c = 2\pi f_c = 2\pi \times 10^4 = 20{,}000\pi$ rad/s

$$\therefore \quad k_f = 20{,}000\pi = 62{,}831.85$$

$$C' = \frac{C}{k_f k_m} \quad \therefore \quad 0.5 \times 10^{-6} = \frac{1}{k_f k_m}$$

$$\therefore \quad k_m = \frac{1}{(0.5 \times 10^{-6})(62{,}831.85)} = 31.83$$

AP 15.4  For a 2nd order prototype Butterworth high pass filter

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

For the circuit in Fig. 15.25

$$H(s) = \frac{s^2}{s^2 + \left(\frac{2}{R_2 C}\right)s + \left(\frac{1}{R_1 R_2 C^2}\right)}$$

Equate the transfer functions. For $C = 1$F,

$$\frac{2}{R_2 C} = \sqrt{2}, \quad \therefore \quad R_2 = \sqrt{2} = 1.414\,\Omega$$

$$\frac{1}{R_1 R_2 C^2} = 1, \quad \therefore \quad R_1 = \frac{1}{\sqrt{2}} = 0.707\,\Omega$$

AP 15.5  $Q = 8, K = 5, \omega_o = 1000$ rad/s, $C = 1\,\mu$F

For the circuit in Fig 15.26

$$H(s) = \frac{-\left(\frac{1}{R_1 C}\right)s}{s^2 + \left(\frac{2}{R_3 C}\right)s + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2}\right)}$$

$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}$$

$$\beta = \frac{2}{R_3 C}, \quad \therefore \quad R_3 = \frac{2}{\beta C}$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{8} = 125\,\text{rad/s}$$

$$\therefore \quad R_3 = \frac{2 \times 10^6}{(125)(1)} = 16\,\mathrm{k\Omega}$$

$$K\beta = \frac{1}{R_1 C}$$

$$\therefore \quad R_1 = \frac{1}{K\beta C} = \frac{1}{5(125)(1 \times 10^{-6})} = 1.6\,\mathrm{k\Omega}$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}$$

$$10^6 = \frac{(1600 + R_2)}{(1600)(R_2)(16{,}000)(10^{-6})^2}$$

Solving for $R_2$,

$$R_2 = \frac{(1600 + R_2)10^6}{256 \times 10^5}, \quad 246 R_2 = 16{,}000, \quad R_2 = 65.04\,\Omega$$

**AP 15.6** $\omega_o = 1000\,\mathrm{rad/s}; \qquad Q = 4;$

$$C = 2\,\mu\mathrm{F}$$

$$
\begin{aligned}
H(s) &= \frac{s^2 + (1/R^2C^2)}{s^2 + \left[\dfrac{4(1-\sigma)}{RC}\right]s + \left(\dfrac{1}{R^2C^2}\right)} \\[2ex]
&= \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}; \qquad \omega_o = \frac{1}{RC}; \qquad \beta = \frac{4(1-\sigma)}{RC}
\end{aligned}
$$

$$R = \frac{1}{\omega_o C} = \frac{1}{(1000)(2 \times 10^{-6})} = 500\,\Omega$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{4} = 250$$

$$\therefore \quad \frac{4(1-\sigma)}{RC} = 250$$

$$4(1-\sigma) = 250RC = 250(500)(2 \times 10^{-6}) = 0.25$$

$$1 - \sigma = \frac{0.25}{4} = 0.0625; \quad \therefore \quad \sigma = 0.9375$$

## Problems

P 15.1    Summing the currents at the inverting input node yields

$$\frac{0 - V_i}{Z_i} + \frac{0 - V_o}{Z_f} = 0$$

$$\therefore \quad \frac{V_o}{Z_f} = -\frac{V_i}{Z_i}$$

$$\therefore \quad H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

P 15.2    **[a]** $Z_f = \dfrac{R_2(1/sC_2)}{[R_2 + (1/sC_2)]} = \dfrac{R_2}{R_2 C_2 s + 1}$

$$= \frac{(1/C_2)}{s + (1/R_2 C_2)}$$

Likewise

$$Z_i = \frac{(1/C_1)}{s + (1/R_1 C_1)}$$

$$\therefore \quad H(s) = \frac{-(1/C_2)[s + (1/R_1 C_1)]}{[s + (1/R_2 C_2)](1/C_1)}$$

$$= -\frac{C_1}{C_2} \frac{[s + (1/R_1 C_1)]}{[s + (1/R_2 C_2)]}$$

**[b]** $H(j\omega) = \dfrac{-C_1}{C_2} \left[ \dfrac{j\omega + (1/R_1 C_1)}{j\omega + (1/R_2 C_2)} \right]$

$$H(j0) = \frac{-C_1}{C_2} \left( \frac{R_2 C_2}{R_1 C_1} \right) = \frac{-R_2}{R_1}$$

**[c]** $H(j\infty) = -\dfrac{C_1}{C_2} \left( \dfrac{j}{j} \right) = \dfrac{-C_1}{C_2}$

**[d]** As $\omega \to 0$ the two capacitor branches become open and the circuit reduces to a resistive inverting amplifier having a gain of $-R_2/R_1$.
As $\omega \to \infty$ the two capacitor branches approach a short circuit and in this case we encounter an indeterminate situation; namely $v_n \to v_i$ but $v_n = 0$ because of the ideal op amp. At the same time the gain of the ideal op amp is infinite so we have the indeterminate form $0 \cdot \infty$. Although $\omega = \infty$ is indeterminate we can reason that for finite large values of $\omega$ $H(j\omega)$ will approach $-C_1/C_2$ in value. In other words, the circuit approaches a purely capacitive inverting amplifier with a gain of $(-1/j\omega C_2)/(1/j\omega C_1)$ or $-C_1/C_2$.

P 15.3  **[a]** $Z_f = \dfrac{(1/C_2)}{s + (1/R_2 C_2)}$

$Z_i = R_1 + \dfrac{1}{sC_1} = \dfrac{R_1}{s}[s + (1/R_1 C_1)]$

$H(s) = -\dfrac{(1/C_2)}{[s + (1/R_2 C_2)]} \cdot \dfrac{s}{R_1[s + (1/R_1 C_1)]}$

$= -\dfrac{1}{R_1 C_2} \dfrac{s}{[s + (1/R_1 C_1)][s + (1/R_2 C_2)]}$

**[b]** $H(j\omega) = -\dfrac{1}{R_1 C_2} \dfrac{j\omega}{\left(j\omega + \frac{1}{R_1 C_1}\right)\left(j\omega + \frac{1}{R_2 C_2}\right)}$

$H(j0) = 0$

**[c]** $H(j\infty) = 0$

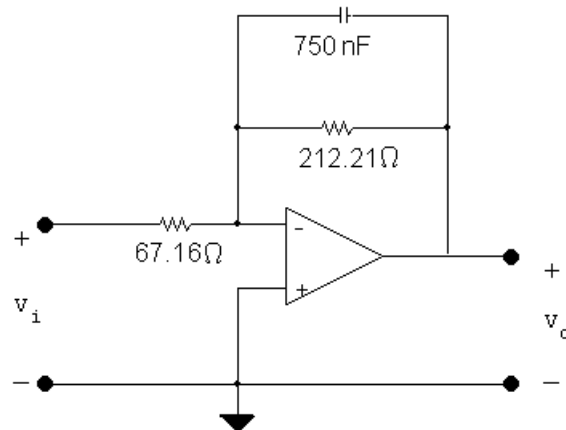**[d]** As $\omega \to 0$ the capacitor $C_1$ disconnects $v_i$ from the circuit. Therefore $v_o = v_n = 0$.
As $\omega \to \infty$ the capacitor short circuits the feedback network, thus $Z_f = 0$ and therefore $v_o = 0$.

P 15.4  **[a]** $K = 10^{(10/20)} = 3.16 = \dfrac{R_2}{R_1}$

$R_2 = \dfrac{1}{\omega_c C} = \dfrac{1}{(2\pi)(10^3)(750 \times 10^{-9})} = 212.21\,\Omega$

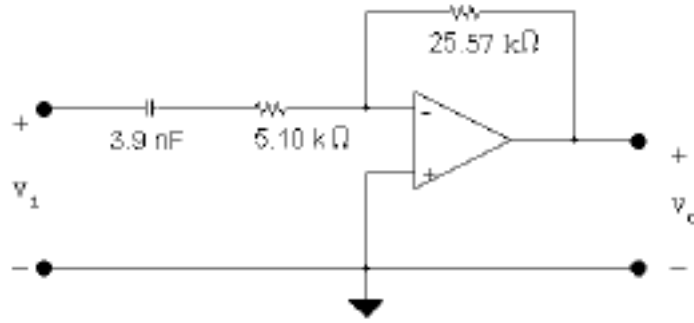$R_1 = \dfrac{R_2}{K} = \dfrac{212.21}{3.16} = 67.11\,\Omega$

**[b]**



P 15.5  **[a]** $R_1 = \dfrac{1}{\omega_c C} = \dfrac{1}{(2\pi)(8 \times 10^3)(3.9 \times 10^{-9})} = 5.10\,\text{k}\Omega$

$K = 10^{(14/20)} = 5.01 = \dfrac{R_2}{R_1}$

$\therefore\quad R_2 = 5.01 R_1 = 25.57\,\text{k}\Omega$

**[b]**



P 15.6    For the $RC$ circuit

$$H(s) = \frac{V_o}{V_i} = \frac{(1/RC)}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = k_m R \frac{C}{k_m k_f} = \frac{1}{k_f} RC = \frac{1}{k_f}$$

$$\frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{(1/R'C')}{s + (1/R'C')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

For the $RL$ circuit

$$H(s) = \frac{V_0}{V_i} = \frac{R/L}{s + (R/L)}$$

$$R' = k_m R; \qquad L' = \frac{k_m}{k_f} L$$

$$\frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left( \frac{R}{L} \right) = k_f$$

$$H'(s) = \frac{(R'/L')}{s + (R'/L')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

P 15.7   For the $RC$ circuit

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = \frac{RC}{k_f} = \frac{1}{k_f}; \qquad \frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{s}{s + (1/R'C')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

For the $RL$ circuit

$$H(s) = \frac{s}{s + (R/L)}$$

$$R' = k_m R; \qquad L' = \frac{k_m L}{k_f}$$

$$\frac{R'}{L'} = k_f \left( \frac{R}{L} \right) = k_f$$

$$H'(s) = \frac{s}{s + (R'/L')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

P 15.8   $$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \frac{\beta s}{s^2 \beta s + \omega_o^2}$$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.
For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

where $R' = k_m R;\ L' = \dfrac{k_m}{k_f} L;$ and $C' = \dfrac{C}{k_f k_m}$

$$\therefore \quad \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left( \frac{R}{L} \right) = k_f \beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f} LC} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{k_f \omega_o}{k_f \beta} = Q$$

therefore the $Q$ of the scaled circuit is the same as the $Q$ of the unscaled circuit. Also note $\beta' = k_f \beta$.

$$\therefore \quad H'(s) = \frac{\left(\frac{k_f}{Q}\right) s}{s^2 + \left(\frac{k_f}{Q}\right) s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q}\left(\frac{s}{k_f}\right) + 1\right]}$$

**P 15.9**    **[a]** $L = 1\,\mathrm{H}$;    $C = 1\,\mathrm{F}$

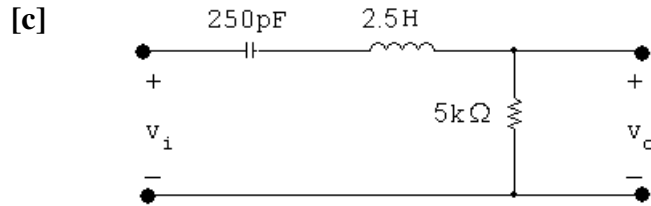$$R = \frac{1}{Q} = \frac{1}{20} = 0.05\,\Omega$$

**[b]** $k_f = \dfrac{\omega_o'}{\omega_o} = 40{,}000$;    $k_m = \dfrac{R'}{R} = \dfrac{5000}{0.05} = 100{,}000$

Thus,

$$R' = k_m R = (0.05)(100{,}000) = 5\,\mathrm{k\Omega}$$

$$L' = \frac{k_m}{k_f} L = \frac{100{,}000}{40{,}000}(1) = 2.5\,\mathrm{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(40{,}000)(100{,}000)} = 250\,\mathrm{pF}$$

**[c]**



**P 15.10**    **[a]** Since $\omega_o^2 = 1/LC$ and $\omega_o = 1$ rad/s,

$$C = \frac{1}{L} = \frac{1}{Q}$$

**[b]** $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$

$$H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$$

[c] In the prototype circuit

$$R = 1\,\Omega; \qquad L = 16\,\text{H}; \qquad C = \frac{1}{L} = 0.0625\,\text{F}$$

$$\therefore \quad k_m = \frac{R'}{R} = 10{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 25{,}000$$
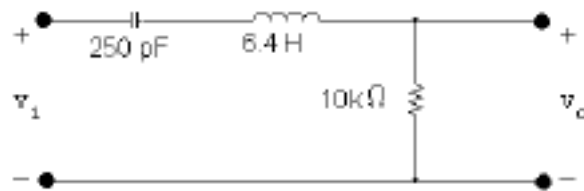
Thus

$$R' = k_m R = 10\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{10{,}000}{25{,}000}(16) = 6.4\,\text{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{0.0625}{(10{,}000)(25{,}000)} = 250\,\text{pF}$$

[d]



[e] $H'(s) = \dfrac{\frac{1}{16}\left(\frac{s}{25{,}000}\right)}{\left(\frac{s}{25{,}000}\right)^2 + \frac{1}{16}\left(\frac{s}{25{,}000}\right) + 1}$

$$H'(s) = \frac{1562.5s}{s^2 + 1562.5s + 625 \times 10^6}$$

P 15.11 [a] Using the first prototype

$$\omega_o = 1\,\text{rad/s}; \qquad C = 1\,\text{F}; \qquad L = 1\,\text{H}; \qquad R = 25\,\Omega$$

$$k_m = \frac{R'}{R} = \frac{40{,}000}{25} = 1600; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{1600}{50{,}000}(1) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(1600)(50{,}000)} = 12.5\,\text{nF}$$

Using the second prototype

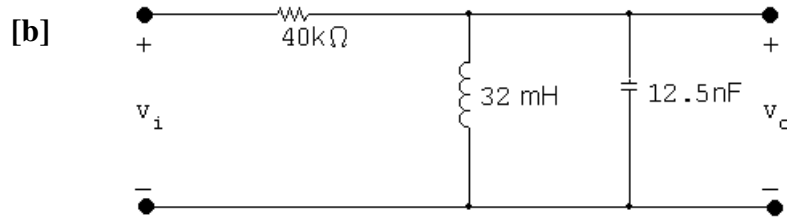$$\omega_o = 1\,\text{rad/s}; \qquad C = 25\,\text{F}$$

$$L = \frac{1}{25} = 40\,\text{mH}; \qquad R = 1\,\Omega$$

$$k_m = \frac{R'}{R} = 40{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f} L = \frac{40{,}000}{50{,}000}(0.04) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{25}{(40{,}000)(50{,}000)} = 12.5\,\text{nF}$$

**[b]**



**P 15.12**  For the scaled circuit

$$H'(s) = \frac{s^2 + \left(\frac{1}{L'C'}\right)}{s^2 + \left(\frac{R'}{L'}\right)s + \left(\frac{1}{L'C'}\right)}$$

$$L' = \frac{k_m}{k_f} L; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}; \qquad R' = k_m R$$

$$\therefore \quad \frac{R'}{L'} = k_f \left(\frac{R}{L}\right)$$

It follows then that

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{R}{L}\right) k_f s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{LC}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \left(\frac{R}{L}\right)\left(\frac{s}{k_f}\right) + \left(\frac{1}{LC}\right)\right]}$$

$$= H(s)|_{s=s/k_f}$$

**P 15.13**  For the circuit in Fig. 15.31

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \frac{s}{RC} + \left(\frac{1}{LC}\right)}$$

It follows that

$$H'(s) = \frac{s^2 + \frac{1}{L'C'}}{s^2 + \frac{s}{R'C'} + \frac{1}{L'C'}}$$

where $R' = k_m R;$     $L' = \frac{k_m}{k_f} L;$

$$C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}$$

$$\frac{1}{R'C'} = \frac{k_f}{RC}$$

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{k_f}{RC}\right)s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \frac{1}{LC}}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{RC}\right)\left(\frac{s}{k_f}\right) + \frac{1}{LC}}$$

$$= H(s)|_{s=s/k_f}$$

P 15.14  **[a]** For the circuit in Fig. P15.14(a)

$$H(s) = \frac{V_o}{V_i} = \frac{s + \frac{1}{s}}{\frac{1}{Q} + s + \frac{1}{s}} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For the circuit in Fig. P15.14(b)

$$H(s) = \frac{V_o}{V_i} = \frac{Qs + \frac{Q}{s}}{1 + Qs + \frac{Q}{s}}$$

$$= \frac{Q(s^2 + 1)}{Qs^2 + s + Q}$$

$$H(s) = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

**[b]** $k_f \overline{=} \dfrac{\omega_o'}{\omega_o} = 10^4;$     $Q = 8;$

Replace $s$ with $s/k_f$.

$$H'(s) = \dfrac{\left(\frac{s}{10^4}\right)^2 + 1}{\left(\frac{s}{10^4}\right)^2 + \frac{1}{8}\left(\frac{s}{10^4}\right) + 1}$$

$$= \dfrac{s^2 + 10^8}{s^2 + 1250s + 10^8}$$

P 15.15   For prototype circuit (a):

$$H(s) = \dfrac{V_o}{V_i} = \dfrac{Q}{Q + \frac{1}{s + \frac{1}{s}}} = \dfrac{Q}{Q + \frac{s}{s^2 + 1}}$$

$$H(s) = \dfrac{Q(s^2 + 1)}{Q(s^2 + 1) + s} = \dfrac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For prototype circuit (b):

$$H(s) = \dfrac{V_o}{V_i} = \dfrac{1}{1 + \frac{(s/Q)}{(s^2 + 1)}}$$

$$= \dfrac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

P 15.16   From the solution to Problem 14.15, $\omega_o = 100$ krad/s and $\beta = 12.5$ krad/s. Compute the two scale factors:

$$k_f = \dfrac{\omega_o'}{\omega_o} = \dfrac{2\pi(200 \times 10^3)}{100 \times 10^3} = 4\pi$$

$$k_m = \dfrac{1}{k_f}\dfrac{C}{C'} = \dfrac{1}{4\pi}\dfrac{10 \times 10^{-9}}{2.5 \times 10^{-9}} = \dfrac{1}{\pi}$$

Thus,

$$R' = k_m R = \dfrac{8000}{\pi} = 2546.48\,\Omega \qquad\qquad L' = \dfrac{k_m}{k_f}L = \dfrac{1/\pi}{4\pi}(10 \times 10^{-3}) = 253.303\,\mu\text{H}$$

Calculate the cutoff frequencies:

$$\omega_{c1}' = k_f\omega_{c1} = 4\pi(93.95 \times 10^3) = 1180.6 \text{ krad/s}$$

$$\omega_{c2}' = k_f\omega_{c2} = 4\pi(106.45 \times 10^3) = 1337.7 \text{ krad/s}$$

To check, calculate the bandwidth:

$$\beta' = \omega_{c2}' - \omega_{c1}' = 157.1 \text{ krad/s} = 4\pi\beta \text{ (Checks!)}$$

P 15.17 From the solution to Problem 14.24, $\omega_o = 10^6$ rad/s and $\beta = 2\pi(10.61)$ krad/s. Calculate the scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{50 \times 10^3}{10^6} = 0.05$$

$$k_m = \frac{k_f L'}{L} = \frac{0.05(200 \times 10^{-6})}{50 \times 10^{-6}} = 0.2$$

Thus,

$$R' = k_m R = (0.2)(750) = 150\,\Omega \qquad\qquad C' = \frac{C}{k_m k_f} = \frac{20 \times 10^{-9}}{(0.2)(0.05)} = 2\,\mu\text{F}$$

Calculate the bandwidth:

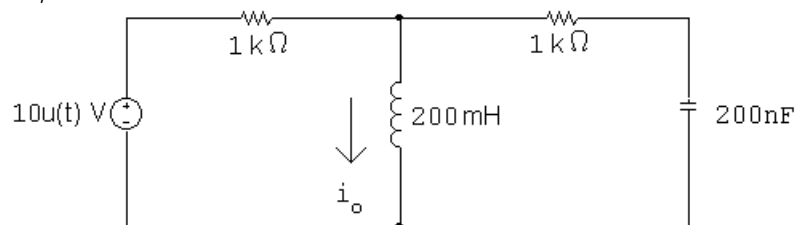$$\beta' = k_f \beta = (0.05)[2\pi(10.61 \times 10^3)] = 3333 \text{ rad/s}$$

To check, calculate the quality factor:

$$Q = \frac{\omega_o}{\beta} = \frac{10^6}{2\pi(10.61 \times 10^3)} = 15$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{50 \times 10^3}{3333} = 15 \text{ (Checks)}$$

P 15.18 **[a]** $k_m = \dfrac{R'}{R} = \dfrac{1000}{1} = 1000; \qquad k_f = \dfrac{C}{k_m C'} = \dfrac{1}{(1000)(200 \times 10^{-9})} = 5000$

$$L' = \frac{k_m}{k_f}(L) = \frac{1000}{5000}(1) = 200\,\text{mH}$$



**[b]** $\dfrac{V - 10/s}{1000} + \dfrac{V}{0.2s} + \dfrac{V}{1000 + (5 \times 10^6/s)} = 0$

$$V\left(\frac{1}{1000} + \frac{5}{s} + \frac{s}{1000s + 5 \times 10^6}\right) = \frac{1}{100s}$$

$$V = \frac{10(s + 5000)}{2s^2 + 10{,}000s + 25 \times 10^6} = \frac{5(s + 5000)}{s^2 + 5000s + 12.5 \times 10^6}$$

$$I_o = \frac{V}{0.2s} = \frac{25(s+5000)}{s(s^2 + 5000s + 12.5 \times 10^6)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 2500 - j2500} + \frac{K_2^*}{s + 2500 + j2500}$$

$$K_1 = 0.01; \qquad K_2 = -0.005$$

$$i_o(t) = 10 - 10e^{-2500t} \cos 2500t \text{ mA}$$

Since $k_m = 1000$ and the source voltage didn't change, the amplitude of the current is reduced by a factor of 1000. Since $k_f = 5000$ the coefficients of $t$ are multiplied by 5000.

**P 15.19** $\quad k_m = \dfrac{R'}{R} = \dfrac{5000}{50} = 100; \qquad k_f = \dfrac{\omega_o'}{\omega_o} = 5000$

$$C' = \frac{C}{k_m k_f} = \frac{4 \times 10^{-3}}{(100)(5000)} = 8\,\text{nF}$$

$$50\,\Omega \rightarrow 5\,\text{k}\Omega; \qquad 700\,\Omega \rightarrow 70\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{100}{5000}(20) = 0.4\,\text{H}$$

$$0.05v_\phi \rightarrow \frac{0.05}{100}v_\phi = 5 \times 10^{-4}v_\phi$$

The original expression for the current:

$$i_o(t) = 1728 + 2880e^{-20t} \cos(15t + 126.87°)\,\text{mA}$$

The frequency components will be multiplied by $k_f = 5000$:

$$20 \rightarrow 20(5000) = 10^5; \qquad 15 \rightarrow 15(5000) = 75{,}000$$

The magnitudes will be reduced by $k_m = 100$:

$$1728 \rightarrow 1728/100 = 17.28; \qquad 2880 \rightarrow 2880/100 = 28.80$$

The expression for the current in the scaled circuit is thus,

$$i_o(t) = 17.28 + 28.80e^{-10^5 t} \cos(75{,}000t + 126.87°)\,\text{mA}$$

P 15.20  **[a]** From Eq 15.1 we have

$$H(s) = \frac{-K\omega_c}{s + \omega_c}$$

where $K = \dfrac{R_2}{R_1}, \quad \omega_c = \dfrac{1}{R_2 C}$

$$\therefore \quad H'(s) = \frac{-K'\omega_c'}{s + \omega_c'}$$

where $K' = \dfrac{R_2'}{R_1'} \qquad \omega_c' = \dfrac{1}{R_2' C'}$

By hypothesis $R_1' = k_m R_1; \qquad R_2' = k_m R_2,$

and $C' = \dfrac{C}{k_f k_m}$. It follows that

$K' = K$ and $\omega_c' = k_f \omega_c$, therefore

$$H'(s) = \frac{-K k_f \omega_c}{s + k_f \omega_c} = \frac{-K\omega_c}{\left(\frac{s}{k_f}\right) + \omega_c}$$

**[b]** $H(s) = \dfrac{-K}{(s+1)}$

**[c]** $H'(s) = \dfrac{-K}{\left(\frac{s}{k_f}\right) + 1} = \dfrac{-K k_f}{s + k_f}$

P 15.21  **[a]** From Eq. 15.4

$$H(s) = \frac{-Ks}{s + \omega_c} \text{ where } K = \frac{R_2}{R_1} \text{ and}$$

$$\omega_c = \frac{1}{R_1 C}$$

$$\therefore \quad H'(s) = \frac{-K's}{s + \omega_c'} \text{ where } K' = \frac{R_2'}{R_1'}$$

and $\omega_c' = \dfrac{1}{R_1' C'}$

By hypothesis

$$R_1' = k_m R_1; \qquad R_2' = k_m R_2; \qquad C' = \frac{C}{k_m k_f}$$

It follows that

$K' = K$ and $\omega_c' = k_f \omega_c$

$$\therefore \quad H'(s) = \frac{-Ks}{s + k_f \omega_c} = \frac{-K(s/k_f)}{\left(\frac{s}{k_f}\right) + \omega_c}$$

**[b]** $H(s) = \dfrac{-Ks}{(s+1)}$

**[c]** $H'(s) = \dfrac{-K(s/k_f)}{\left(\frac{s}{k_f}+1\right)} = \dfrac{-Ks}{(s+k_f)}$

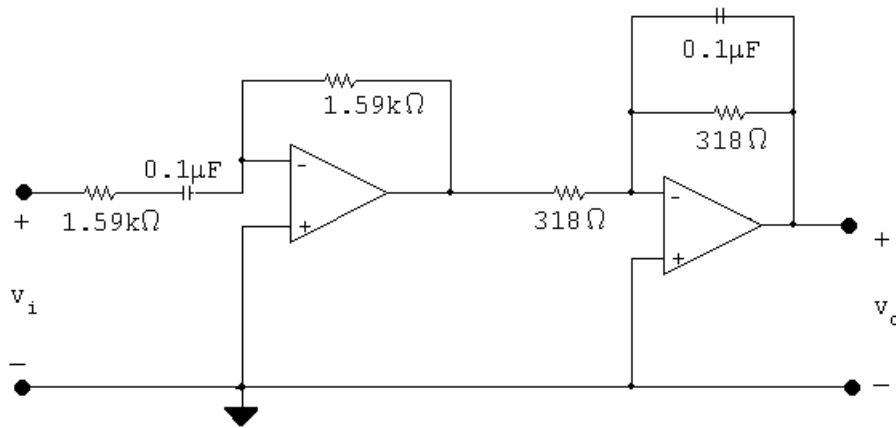**P 15.22  [a]** $H_{hp} = \dfrac{-s}{s+1};$      $k_f = \dfrac{\omega'_o}{\omega} = \dfrac{1000(2\pi)}{1} = 2000\pi$

$\therefore\ H'_{hp} = \dfrac{-s}{s+2000\pi}$

$\dfrac{1}{R_H C_H} = 2000\pi;$      $\therefore\ R_H = \dfrac{1}{(2000\pi)(0.1 \times 10^{-6})} = 1.59\,\text{k}\Omega$

$H_{lp} = \dfrac{-1}{s+1};$      $k_f = \dfrac{\omega'_o}{\omega} = \dfrac{5000(2\pi)}{1} = 10{,}000\pi$

$\therefore\ H'_{lp} = \dfrac{-10{,}000\pi}{s+10{,}000\pi}$

$\dfrac{1}{R_L C_L} = 10{,}000\pi;$      $\therefore\ R_L = \dfrac{1}{(10{,}000\pi)(0.1 \times 10^{-6})} = 318.3\,\Omega$
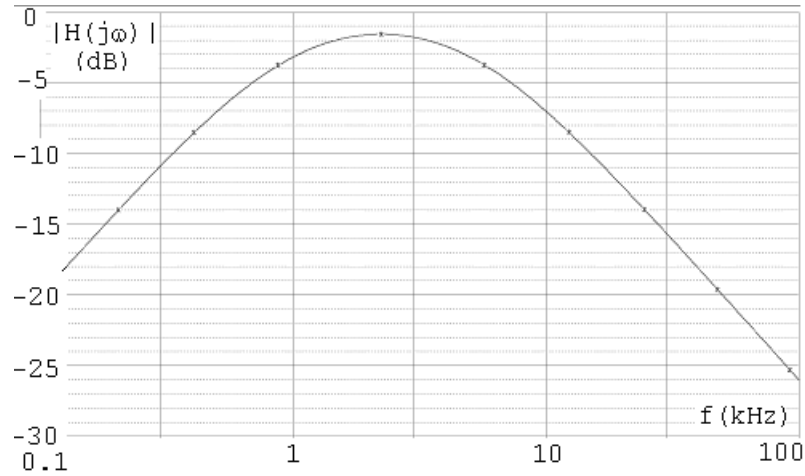


**[b]** $H'(s) = \dfrac{-s}{s+2000\pi} \cdot \dfrac{-10{,}000\pi}{s+10{,}000\pi}$

$= \dfrac{10{,}000\pi s}{(s+2000\pi)(s+10{,}000\pi)}$

**[c]** $\omega_o = \sqrt{\omega_{c1}\omega_{c1}} = \sqrt{(2000\pi)(10{,}000\pi)} = 1000\pi\sqrt{20}$ rad/s

$H'(j\omega_o) = \dfrac{(10{,}000\pi)(j1000\pi\sqrt{20})}{(2000\pi + j1000\pi\sqrt{20})(10{,}000\pi + j1000\pi\sqrt{20})}$

$= \dfrac{j10\sqrt{20}}{(2+j\sqrt{20})(10+j\sqrt{20})} = 0.8333\underline{/0°}$

**[d]** $G = 20 \log_{10}(0.8333) = -1.58$ dB

**[e]**



P 15.23  **[a]**  For the high-pass section:

$$k_f = \frac{\omega'_o}{\omega} = \frac{4000(2\pi)}{1} = 8000\pi$$

$$H'(s) = \frac{-s}{s + 8000\pi}$$

$$\therefore \quad \frac{1}{R_1(10 \times 10^{-9})} = 8000\pi; \qquad R_1 = 3.98 \, \text{k}\Omega \quad \therefore \quad R_2 = 3.98 \, \text{k}\Omega$$
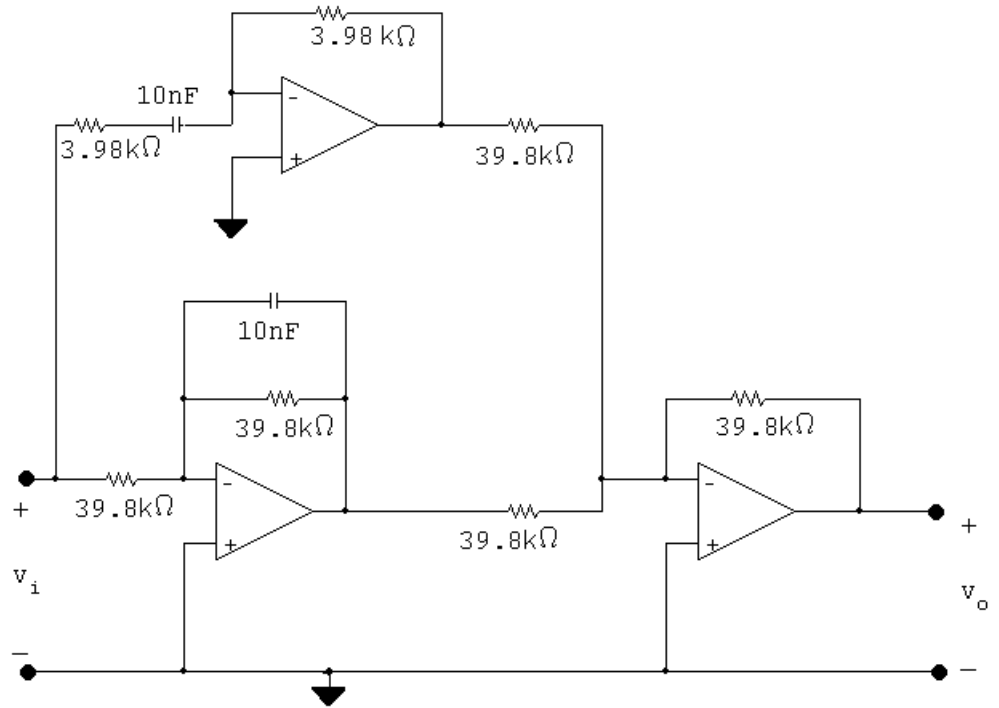
For the low-pass section:

$$k_f = \frac{\omega'_o}{\omega} = \frac{400(2\pi)}{1} = 800\pi$$

$$H'(s) = \frac{-800\pi}{s + 800\pi}$$

$$\therefore \quad \frac{1}{R_2(10 \times 10^{-9})} = 800\pi; \qquad R_2 = 39.8 \, \text{k}\Omega \quad \therefore \quad R_1 = 39.8 \, \text{k}\Omega$$

0 dB gain corresponds to $K = 1$. In the summing amplifier we are free to choose $R_f$ and $R_i$ so long as $R_f/R_i = 1$. To keep from having many different resistance values in the circuit we opt for $R_f = R_i = 39.8 \, \text{k}\Omega$.
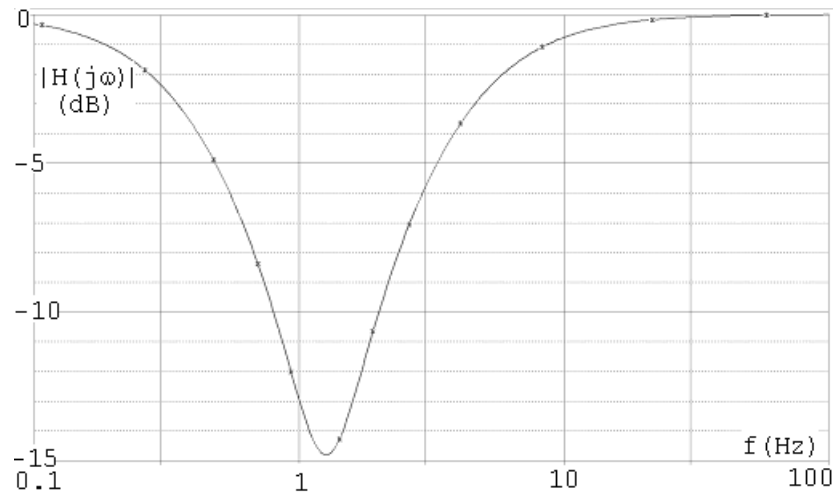
[b]



[c]  $H'(s) = \dfrac{s}{s + 8000\pi} + \dfrac{800\pi}{s + 800\pi}$

$= \dfrac{s^2 + 1600\pi s + 64 \times 10^5 \pi^2}{(s + 800\pi)(s + 8000\pi)}$

[d]  $\omega_o = \sqrt{(8000\pi)(800\pi)} = 800\pi\sqrt{10}$

$H'(j800\pi\sqrt{10}) = \dfrac{-(800\pi\sqrt{10})^2 + 1600\pi(j800\pi\sqrt{10}) + 64 \times 10^5 \pi^2}{(800\pi + j800\pi\sqrt{10})(8000\pi + j800\pi\sqrt{10})}$

$= \dfrac{j128 \times 10^4\sqrt{10}\,\pi^2}{(800\pi)^2(1 + j\sqrt{10})(10 + j\sqrt{10})}$

$= \dfrac{j2\sqrt{10}}{(1 + j\sqrt{10})(10 + j\sqrt{10})}$

$= 0.1818\underline{/0°}$

[e]  $G = 20\log_{10} 0.1818 = -14.81$ dB

**[f]**



P 15.24 **[a]** $H(s) = \dfrac{(1/sC)}{R + (1/sC)} = \dfrac{(1/RC)}{s + (1/RC)}$

$H(j\omega) = \dfrac{(1/RC)}{j\omega + (1/RC)}$

$|H(j\omega)| = \dfrac{(1/RC)}{\sqrt{\omega^2 + (1/RC)^2}}$

$|H(j\omega)|^2 = \dfrac{(1/RC)^2}{\omega^2 + (1/RC)^2}$

**[b]** Let $V_a$ be the voltage across the capacitor, positive at the upper terminal. Then

$\dfrac{V_a - V_i}{R_1} + sCV_a + \dfrac{V_a}{R_2 + sL} = 0$

Solving for $V_a$ yields

$V_a = \dfrac{(R_2 + sL)V_i}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$

But

$V_o = \dfrac{sLV_a}{R_2 + sL}$

Therefore

$V_o = \dfrac{sLV_i}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$

$H(s) = \dfrac{sL}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$

$H(j\omega) = \dfrac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)}$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R_1 + R_2 - R_1 L C \omega^2]^2 + \omega^2 (L + R_1 R_2 C)^2}}$$

$$|H(j\omega)|^2 = \frac{\omega^2 L^2}{(R_1 + R_2 - R_1 L C \omega^2)^2 + \omega^2 (L + R_1 R_2 C)^2}$$

$$= \frac{\omega^2 L^2}{R_1^2 L^2 C^2 \omega^4 + (L^2 + R_1^2 R_2^2 C^2 - 2R_1^2 LC + 2R_1 R_2 LC)\omega^2 + (R_1 + R_2)^2}$$

**[c]** Let $V_a$ be the voltage across $R_2$ positive at the upper terminal. Then

$$\frac{V_a - V_i}{R_1} + \frac{V_a}{R_2} + V_a sC + V_a sC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore \quad V_a = \frac{R_2 V_i}{2R_1 R_2 Cs + R_1 + R_2}$$

and $V_a = -\dfrac{V_o}{2R_3 Cs}$

It follows directly that

$$H(s) = \frac{V_o}{V_i} = \frac{-2R_2 R_3 Cs}{2R_1 R_2 Cs + (R_1 + R_2)}$$

$$\therefore \quad H(j\omega) = \frac{-2R_2 R_3 C(j\omega)}{(R_1 + R_2) + j\omega(2R_1 R_2 C)}$$

$$|H(j\omega)| = \frac{2R_2 R_3 C \omega}{\sqrt{(R_1 + R_2)^2 + \omega^2 4R_1^2 R_2^2 C^2}}$$

$$|H(j\omega)|^2 = \frac{4R_2^2 R_3^2 C^2 \omega^2}{(R_1 + R_2)^2 + 4R_1^2 R_2^2 C^2 \omega^2}$$

**P 15.25**   $\omega_o = 2\pi f_o = 400\pi$ rad/s

$\beta = 2\pi(1000) = 2000\pi$ rad/s

$\therefore \quad \omega_{c_2} - \omega_{c_1} = 2000\pi$

$\sqrt{\omega_{c_1} \omega_{c_2}} = \omega_o = 400\pi$

Solve for the cutoff frequencies:

$\omega_{c_1} \omega_{c_2} = 16 \times 10^4 \pi^2$

$$\omega_{c_2} = \frac{16 \times 10^4 \pi^2}{\omega_{c_1}}$$

$$\therefore \quad \frac{16 \times 10^4 \pi^2}{\omega_{c_1}} - \omega_{c_1} = 2000\pi$$

or $\omega_{c_1}^2 + 2000\pi\omega_{c_1} - 16 \times 10^4\pi^2 = 0$

$$\omega_{c_1} = -1000\pi \pm \sqrt{10^6\pi^2 + 0.16 \times 10^6\pi^2}$$

$$\omega_{c_1} = 1000\pi(-1 \pm \sqrt{1.16}) = 242.01\,\text{rad/s}$$

$$\therefore \quad \omega_{c_2} = 2000\pi + 242.01 = 6525.19\,\text{rad/s}$$

Thus, $f_{c1} = 38.52\,\text{Hz}$     and     $f_{c2} = 1038.52\,\text{Hz}$

Check:     $\beta = f_{c2} - f_{c1} = 1000\text{Hz}$

$$\omega_{c2} = \frac{1}{R_L C_L} = 6525.19$$

$$R_L = \frac{1}{(6525.19)(5 \times 10^{-6})} = 30.65\,\Omega$$

$$\omega_{c1} = \frac{1}{R_H C_H} = 242.01$$

$$R_H = \frac{1}{(242.01)(5 \times 10^{-6})} = 826.43\,\Omega$$

P 15.26   $\omega_o = 1000\,\text{rad/s};$        GAIN $= 6$

$\beta = 4000\,\text{rad/s};$        $C = 0.2\,\mu\text{F}$

$\beta = \omega_{c_2} - \omega_{c_1} = 4000$

$\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}} = 1000$

Solve for the cutoff frequencies:

$$\therefore \quad \omega_{c_1}^2 + 4000\omega_{c_1} - 10^6 = 0$$

$$\omega_{c_1} = -2000 \pm 1000\sqrt{5} = 236.07\,\text{rad/s}$$

$$\omega_{c_2} = 4000 + \omega_{c_1} = 4236.07 \text{ rad/s}$$

Check:    $\beta = \omega_{c2} - \omega_{c1} = 4000 \text{ rad/s}$

$$\omega_{c_1} = \frac{1}{R_L C_L}$$

$$\therefore \quad R_L = \frac{1}{(0.2 \times 10^{-6})(236.07)} = 21.81 \text{ k}\Omega$$

$$\frac{1}{R_H C_H} = 4236.07$$

$$R_H = \frac{1}{(0.2 \times 10^{-6})(4236.07)} = 1.18 \text{ k}\Omega$$

$$\frac{R_f}{R_i} = 6$$

If $R_i = 1 \text{ k}\Omega$       $R_f = 6 R_i = 6 \text{ k}\Omega$

**P 15.27  [a]** $y = 20 \log_{10} \dfrac{1}{\sqrt{1 + \omega^{2n}}} = -10 \log_{10}(1 + \omega^{2n})$

From the laws of logarithms we have

$$y = \left(\frac{-10}{\ln 10}\right) \ln(1 + \omega^{2n})$$

Thus

$$\frac{dy}{d\omega} = \left(\frac{-10}{\ln 10}\right) \frac{2n\omega^{2n-1}}{(1 + \omega^{2n})}$$

$$x = \log_{10} \omega = \frac{\ln \omega}{\ln 10}$$

$$\therefore \quad \ln \omega = x \ln 10$$

$$\frac{1}{\omega}\frac{d\omega}{dx} = \ln 10, \quad \frac{d\omega}{dx} = \omega \ln 10$$

$$\frac{dy}{dx} = \left(\frac{dy}{d\omega}\right)\left(\frac{d\omega}{dx}\right) = \frac{-20n\omega^{2n}}{1 + \omega^{2n}} \text{ dB/decade}$$

at $\omega = \omega_c = 1 \text{ rad/s}$

$$\frac{dy}{dx} = -10n \text{ dB/decade.}$$

**[b]** $y = 20\log_{10}\dfrac{1}{[\sqrt{1+\omega^2}]^n} = -10n\log_{10}(1+\omega^2)$

$$= \dfrac{-10n}{\ln 10}\ln(1+\omega^2)$$

$$\dfrac{dy}{d\omega} = \dfrac{-10n}{\ln 10}\left(\dfrac{1}{1+\omega^2}\right)2\omega = \dfrac{-20n\omega}{(\ln 10)(1+\omega^2)}$$

As before

$$\dfrac{d\omega}{dx} = \omega(\ln 10); \qquad \therefore \quad \dfrac{dy}{dx} = \dfrac{-20n\omega^2}{(1+\omega^2)}$$

At the corner $\omega_c = \sqrt{2^{1/n}-1}$ $\qquad \therefore \quad \omega_c^2 = 2^{1/n}-1$

$$\dfrac{dy}{dx} = \dfrac{-20n[2^{1/n}-1]}{2^{1/n}}\ \text{dB/decade}.$$

**[c]** For the Butterworth Filter

| $n$ | $dy/dx$ (dB/decade) |
|---|---|
| 1 | $-10$ |
| 2 | $-20$ |
| 3 | $-30$ |
| 4 | $-40$ |
| $\infty$ | $-\infty$ |

For the cascade of identical sections

| $n$ | $dy/dx$ (dB/decade) |
|---|---|
| 1 | $-10$ |
| 2 | $-11.72$ |
| 3 | $-12.38$ |
| 4 | $-12.73$ |
| $\infty$ | $-13.86$ |

**[d]** It is apparent from the calculations in part (c) that as $n$ increases the amplitude characteristic at the cutoff frequency decreases at a much faster rate for the Butterworth filter.

Hence the transition region of the Butterworth filter will be much narrower than that of the cascaded sections.

**P 15.28** **[a]** $n \cong \dfrac{(-0.05)(-30)}{\log_{10}(7000/2000)} \cong 2.76$

$$\therefore \quad n = 3$$

**[b]** Gain $= 20\log_{10}\dfrac{1}{\sqrt{1+(7000/2000)^6}} = -32.65\ \text{dB}$

**P 15.29** **[a]** For the scaled circuit

$$H'(s) = \dfrac{1/(R')^2 C_1' C_2'}{s^2 + \frac{2}{R'C_1'}s + \frac{1}{(R')^2 C_1' C_2'}}$$

where

$$R' = k_m R; \qquad C_1' = C_1/k_f k_m; \qquad C_2' = C_2/k_f k_m$$

It follows that

$$\frac{1}{(R')^2 C_1' C_2'} = \frac{k_f^2}{R^2 C_1 C_2}$$

$$\frac{2}{R'C_1'} = \frac{2k_f}{RC_1}$$

$$\therefore \quad H'(s) = \frac{k_f^2/R^2 C_1 C_2}{s^2 + \frac{2k_f}{RC_1}s + \frac{k_f^2}{R^2 C_1 C_2}}$$

$$= \frac{1/R^2 C_1 C_2}{\left(\frac{s}{k_f}\right)^2 + \frac{2}{RC_1}\left(\frac{s}{k_f}\right) + \frac{1}{R^2 C_1 C_2}}$$

**P 15.30**  **[a]** $H(s) = \dfrac{1}{(s+1)(s^2 + s + 1)}$

**[b]** $f_c = 2000\,\text{Hz};$ $\qquad \omega_c = 4000\pi\,\text{rad/s};$ $\qquad k_f = 4000\pi$

$$H'(s) = \frac{1}{\left(\frac{s}{k_f} + 1\right)\left[\left(\frac{s}{k_f}\right)^2 + \frac{s}{k_f} + 1\right]}$$

$$= \frac{k_f^3}{(s + k_f)(s^2 + k_f s + k_f^2)}$$

$$= \frac{(4000\pi)^3}{(s + 4000\pi)[s^2 + 4000\pi s + (4000\pi)^2]}$$

**[c]** $H'(j14{,}000\pi) = \dfrac{64}{(4 + j14)(-180 + j56)}$

$$= 0.02332 \underline{/-236.77°}$$

Gain $= 20 \log_{10}(0.02332) = -32.65\,\text{dB}$

**P 15.31**  **[a]** In the first-order circuit $R = 1\,\Omega$ and $C = 1\,\text{F}$.
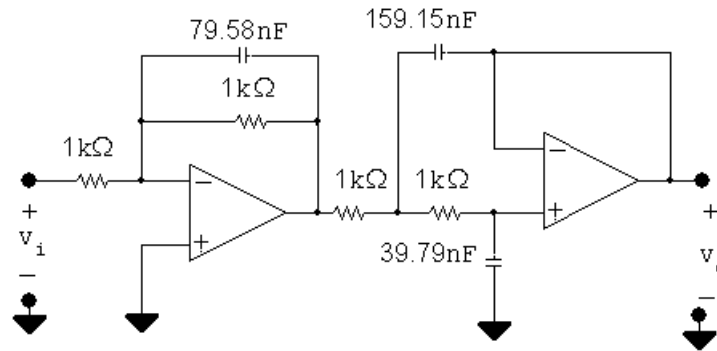
$$k_m = \frac{R'}{R} = \frac{1000}{1} = 1000; \qquad k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

$$R' = k_m R = 1000\,\Omega; \qquad C' = \frac{C}{k_m k_f} = \frac{1}{(1000)(4000\pi)} = 79.58\,\text{nF}$$

In the second-order circuit $R = 1\,\Omega$, $2/C_1 = 1$ so $C_1 = 2\,\text{F}$, and $C_2 = 1/C_1 = 0.5\,\text{F}$. Therefore in the scaled second-order circuit

$$R' = k_m R = 1000\,\Omega; \qquad C_1' = \frac{C_1}{k_m k_f} = \frac{2}{(1000)(4000\pi)} = 159.15\,\text{nF}$$

$$C_2' = \frac{C_2}{k_m k_f} = \frac{0.5}{(1000)(4000\pi)} = 39.79\,\text{nF}$$

**[b]**



**P 15.32** **[a]** $n = \dfrac{(-0.05)(-48)}{\log_{10}(2000/500)} = 3.99$     $\therefore$   $n = 4$

From Table 15.1 the transfer function of the first section is

$$H_1(s) = \frac{s^2}{s^2 + 0.765s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 0.765; \qquad R_2 = 2.61\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.383\,\Omega$$

The transfer function of the second section is

$$H_2(s) = \frac{s^2}{s^2 + 1.848s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 1.848; \qquad R_2 = 1.082\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.9240\,\Omega$$

The scaling factors are:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

$$C' = \frac{C}{k_m k_f} \qquad \therefore \qquad 10 \times 10^{-9} = \frac{1}{4000\pi k_m}$$

$$\therefore \qquad k_m = \frac{1}{4000\pi(10 \times 10^{-9})} = 7957.75$$
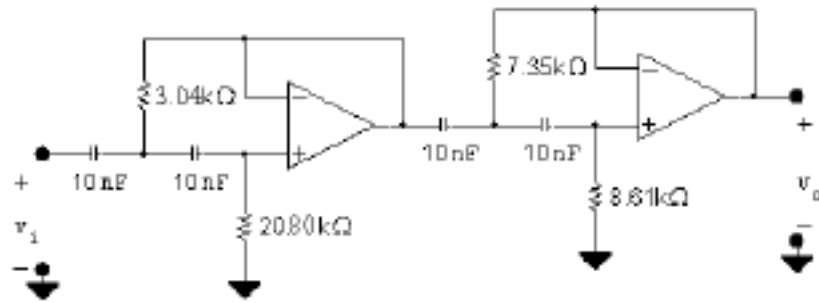
Therefore in the first section

$$R_1' = k_m R_1 = 3.04\,\text{k}\Omega; \qquad R_2' = k_m R_2 = 20.80\,\text{k}\Omega$$

In the second section

$$R_1' = k_m R_1 = 7.35\,\text{k}\Omega; \qquad R_2' = k_m R_2 = 8.61\,\text{k}\Omega$$
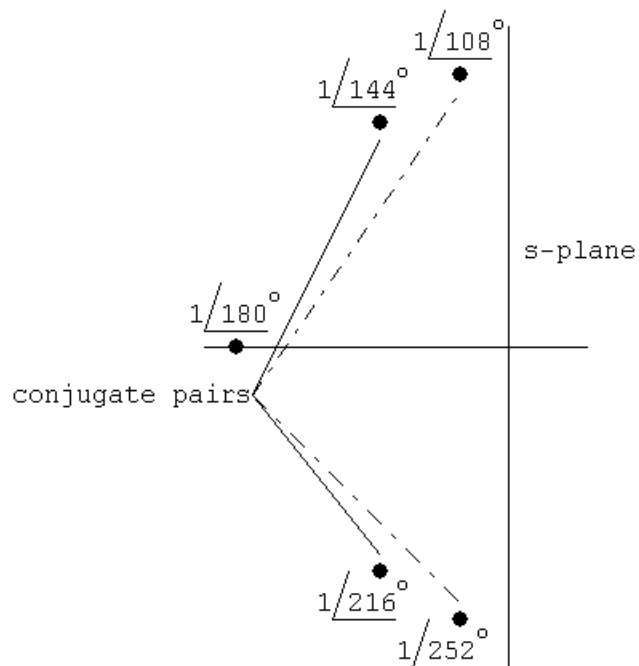
**[b]**



P 15.33  $n = 5$: $1 + (-1)^5 s^{10} = 0$;     $s^{10} = 1$

$s^{10} = 1\underline{/0 + 36°k}$

| $k$ | $s_{k+1}$ |
|---|---|
| 0 | $1\underline{/0°}$ |
| 1 | $1\underline{/36°}$ |
| 2 | $1\underline{/72°}$ |
| 3 | $1\underline{/108°}$ |
| 4 | $1\underline{/144°}$ |
| 5 | $1\underline{/180°}$ |
| 6 | $1\underline{/216°}$ |
| 7 | $1\underline{/252°}$ |
| 8 | $1\underline{/288°}$ |
| 9 | $1\underline{/324°}$ |



Group by conjugate pairs to form denominator polynomial.

$(s + 1)[s - (\cos 108° + j \sin 108°)][s - (\cos 252° + j \sin 252°)]$

$\quad \cdot [s - (\cos 144° + j \sin 144°)][s - (\cos 216° + j \sin 216°)]$

$= (s + 1)(s + 0.309 - j0.951)(s + 0.309 + j0.951)\cdot$

$\quad (s + 0.809 - j0.588)(s + 0.809 + j0.588)$

which reduces to

$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$

$$n = 6: 1 + (-1)^6 s^{12} = 0 \qquad s^{12} = -1$$

$$s^{12} = 1\underline{/15° + 36°k}$$

| $k$ | $s_{k+1}$ |
|---|---|
| 0 | $1\underline{/15°}$ |
| 1 | $1\underline{/45°}$ |
| 2 | $1\underline{/75°}$ |
| 3 | $1\underline{/105°}$ |
| 4 | $1\underline{/135°}$ |
| 5 | $1\underline{/165°}$ |
| 6 | $1\underline{/195°}$ |
| 7 | $1\underline{/225°}$ |
| 8 | $1\underline{/255°}$ |
| 9 | $1\underline{/285°}$ |
| 10 | $1\underline{/315°}$ |
| 11 | $1\underline{/345°}$ |
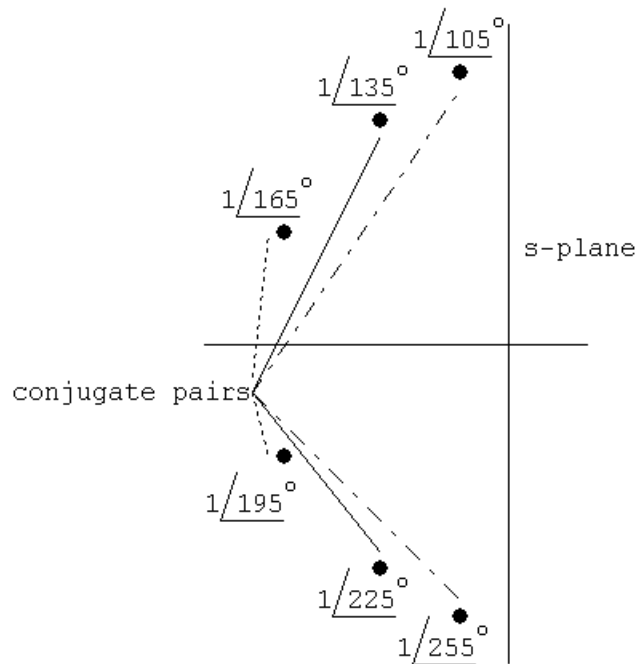
Grouping by conjugate pairs yields

$$(s + 0.2588 - j0.9659)(s + 0.2588 + j0.9659)\times$$

$$(s + 0.7071 - j0.7071)(s + 0.7071 + j0.7071)\times$$

$$(s + 0.9659 - j0.2588)(s + 0.9659 + j0.2588)$$

or $(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$

P 15.34  $$H'(s) = \dfrac{s^2}{s^2 + \frac{2}{k_m R_2(C/k_m k_f)}s + \frac{1}{k_m R_1 k_m R_2(C^2/k_m^2 k_f^2)}}$$

$$H'(s) = \dfrac{s^2}{s^2 + \frac{2k_f}{R_2 C}s + \frac{k_f^2}{R_1 R_2 C^2}}$$

$$= \dfrac{(s/k_f)^2}{(s/k_f)^2 + \frac{2}{R_2 C}\left(\frac{s}{k_f}\right) + \frac{1}{R_1 R_2 C^2}}$$

**P 15.35  [a]** $n = \dfrac{(-0.05)(-48)}{\log_{10}(32/8)} = 3.99$    $\therefore$    $n = 4$

From Table 15.1 the transfer function is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The capacitor values for the first stage prototype circuit are

$$\frac{2}{C_1} = 0.765 \quad \therefore \quad C_1 = 2.61\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.38\,\text{F}$$

The values for the second stage prototype circuit are

$$\frac{2}{C_1} = 1.848 \quad \therefore \quad C_1 = 1.08\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.92\,\text{F}$$

The scaling factors are

$$k_m = \frac{R'}{R} = 1000; \qquad k_f = \frac{\omega'_o}{\omega_o} = 16{,}000\pi$$

Therefore the scaled values for the components in the first stage are

$$R_1 = R_2 = R = 1000\,\Omega$$

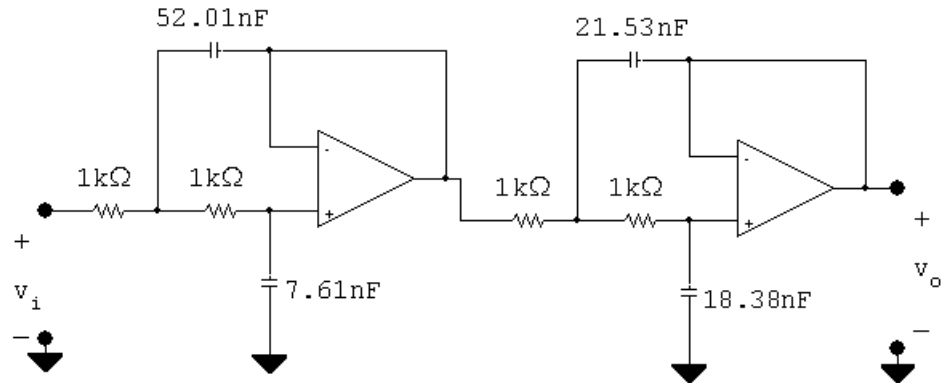$$C_1 = \frac{2.61}{(16{,}000\pi)(1000)} = 52.01\,\text{nF}$$

$$C_2 = \frac{0.38}{(16{,}000\pi)(1000)} = 7.61\,\text{nF}$$

The scaled values for the second stage are

$$R_1 = R_2 = R = 1000\,\Omega$$

$$C_1 = \frac{1.08}{(16{,}000\pi)(1000)} = 21.53\,\text{nF}$$

$$C_2 = \frac{0.92}{(16{,}000\pi)(1000)} = 18.38\,\text{nF}$$

**[b]**



P 15.36  **[a]** The cascade connection is a bandpass filter.

**[b]** The cutoff frequencies are 2 kHz and 8 kHz.
The center frequency is $\sqrt{(2)(8)} = 4$ kHz.
The $Q$ is $4/(8-2) = 2/3 = 0.67$

**[c]** For the high pass section $k_f = 4000\pi$. The prototype transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$$\therefore \quad H'_{hp}(s) = \frac{(s/4000\pi)^4}{[(s/4000\pi)^2 + 0.765(s/4000\pi) + 1]}$$

$$\cdot \frac{1}{[(s/4000\pi)^2 + 1.848(s/4000\pi) + 1]}$$

$$= \frac{s^4}{(s^2 + 3060\pi s + 16 \times 10^6\pi^2)(s^2 + 7392\pi s + 16 \times 10^6\pi^2)}$$

For the low pass section $k_f = 16{,}000\pi$

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$$\therefore \quad H'_{lp}(s) = \frac{1}{[(s/16{,}000\pi)^2 + 0.765(s/16{,}000\pi) + 1]}$$

$$\cdot \frac{1}{[(s/16{,}000\pi)^2 + 1.848(s/16{,}000\pi) + 1]}$$

$$= \frac{(16{,}000\pi)^4}{([s^2 + 12{,}240\pi s + (16{,}000\pi)^2)][s^2 + 29{,}568\pi s + (16{,}000\pi)^2]}$$

The cascaded transfer function is

$$H'(s) = H'_{hp}(s)H'_{lp}(s)$$

For convenience let

$$D_1 = s^2 + 3060\pi s + 16 \times 10^6 \pi^2$$

$$D_2 = s^2 + 7392\pi s + 16 \times 10^6 \pi^2$$

$$D_3 = s^2 + 12{,}240\pi s + 256 \times 10^6 \pi^2$$

$$D_4 = s^2 + 29{,}568\pi s + 256 \times 10^6 \pi^2$$

Then

$$H'(s) = \frac{65{,}536 \times 10^{12}\pi^4 s^4}{D_1 D_2 D_3 D_4}$$

[d] $\omega_o = 2\pi(4000) = 8000\pi$ rad/s

$$s = j8000\pi$$

$$s^4 = 4096 \times 10^{12}\pi^4$$

$$D_1 = (16 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(3060\pi)$$

$$= 10^6\pi^2(-48 + j24.48) = 10^6\pi^2(53.88\underline{/152.98°})$$

$$D_2 = (16 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(7392\pi)$$

$$= 10^6\pi^2(-48 + j59.136) = 10^6\pi^2(76.16\underline{/129.07°})$$

$$D_1 = (256 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(12{,}240\pi)$$

$$= 10^6\pi^2(192 + j97.92) = 10^6\pi^2(215.53\underline{/27.02°})$$

$$D_1 = (256 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(29{,}568\pi)$$

$$= 10^6\pi^2(192 + j236.544) = 10^6\pi^2(304.66\underline{/50.93°})$$

$$H'(j\omega_o) = \frac{(65{,}536)(4096)\pi^8 \times 10^{24}}{(\pi^8 \times 10^{24})[(53.88)(76.16)(215.53)(304.66)\underline{/360°}]}$$

$$= 0.996\underline{/-360°} = 0.996\underline{/0°}$$

P 15.37 [a] From the statement of the problem, $K = 10 \ (= 20$ dB). Therefore for the prototype bandpass circuit

$$R_1 = \frac{Q}{K} = \frac{16}{10} = 1.6 \ \Omega$$

$$R_2 = \frac{Q}{2Q^2 - K} = \frac{16}{502} \ \Omega$$

$$R_3 = 2Q = 32 \ \Omega$$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(6400) = 12{,}800\pi$$

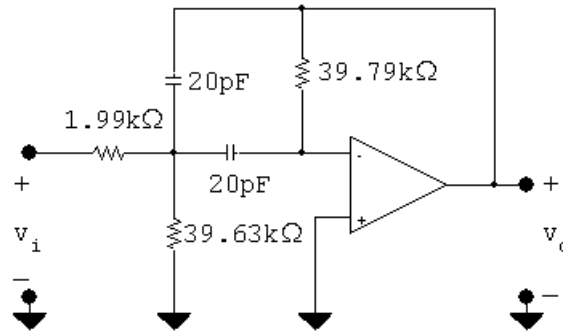$$k_m = \frac{C}{C'k_f} = \frac{1}{(20 \times 10^{-9})(12{,}800\pi)} = 1243.40$$

Therefore,

$$R_1' = k_m R_1 = (1.6)(1243.40) = 1.99\,\text{k}\Omega$$

$$R_2' = k_m R_2 = (16/502)(1243.40) = 39.63\,\Omega$$

$$R_3' = k_m R_3 = 32(1243.40) = 39.79\,\text{k}\Omega$$

**[b]**



P 15.38    From Eq 15.58 we can write

$$H(s) = \frac{-\left(\frac{2}{R_3 C}\right)\left(\frac{R_3 C}{2}\right)\left(\frac{1}{R_1 C}\right)s}{s^2 + \frac{2}{R_3 C}s + \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}}$$

or

$$H(s) = \frac{-\left(\frac{R_3}{2R_1}\right)\left(\frac{2}{R_3 C}s\right)}{s^2 + \frac{2}{R_3 C}s + \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}}$$

Therefore

$$\frac{2}{R_3 C} = \beta = \frac{\omega_o}{Q}; \qquad \frac{R_1 + R_2}{R_1 R_2 R_3 C^2} = \omega_o^2;$$

and $K = \dfrac{R_3}{2R_1}$

By hypothesis $C = 1\,\text{F}$ and $\omega_o = 1\,\text{rad/s}$

$$\therefore \quad \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1 + R_2}{R_1 R_2 R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore \ R_2 = \frac{Q}{2Q^2 - K}$$

P 15.39 **[a]** First we will design a unity gain filter and then provide the passband gain with an inverting amplifier. For the high pass section the cut-off frequency is $500$ Hz. The order of the Butterworth is

$$n = \frac{(-0.05)(-20)}{\log_{10}(500/200)} = 2.51$$

$$\therefore \ n = 3$$

$$H_{\text{hp}}(s) = \frac{s^3}{(s+1)(s^2 + s + 1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$$

For the prototype second-order section

$$R_1 = 0.5\,\Omega, \quad R_2 = 2\,\Omega, \quad C = 1\,\text{F}$$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(500) = 1000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(15 \times 10^{-9})(1000\pi)} = \frac{10^6}{15\pi}$$

In the scaled first-order section

$$R_1' = R_2' = k_m R_1 = \frac{10^6}{15\pi}(1) = 21.22\,\text{k}\Omega$$

$$C' = 15\ \text{nF}$$

In the scaled second-order section

$$R_1' = 0.5k_m = 10.61\,\text{k}\Omega$$

$$R_2' = 2k_m = 42.44\,\text{k}\Omega$$

$C' = 15\,\text{nF}$

For the low-pass section the cut-off frequency is $4500\,\text{Hz}$. The order of the Butterworth filter is

$$n = \frac{(-0.05)(-20)}{\log_{10}(11{,}250/4500)} = 2.51; \qquad \therefore \quad n = 3$$

$$H_{\text{lp}}(s) = \frac{1}{(s+1)(s^2 + s + 1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$$

For the prototype second-order section

$$R_1 = R_2 = 1\,\Omega; \qquad C_1 = 2\,\text{F}; \qquad C_2 = 0.5\,\text{F}$$

The low-pass scaling factors are

$$k_m = \frac{R'}{R} = 10^4; \qquad k_f = \frac{\omega_o'}{\omega_o} = (4500)(2\pi) = 9000\pi$$

For the scaled first-order section

$$R_1' = R_2' = 10\,\text{k}\Omega; \qquad C' = \frac{C}{k_f k_m} = \frac{1}{(9000\pi)(10^4)} = 3.54\,\text{nF}$$

For the scaled second-order section

$$R_1' = R_2' = 10\,\text{k}\Omega$$

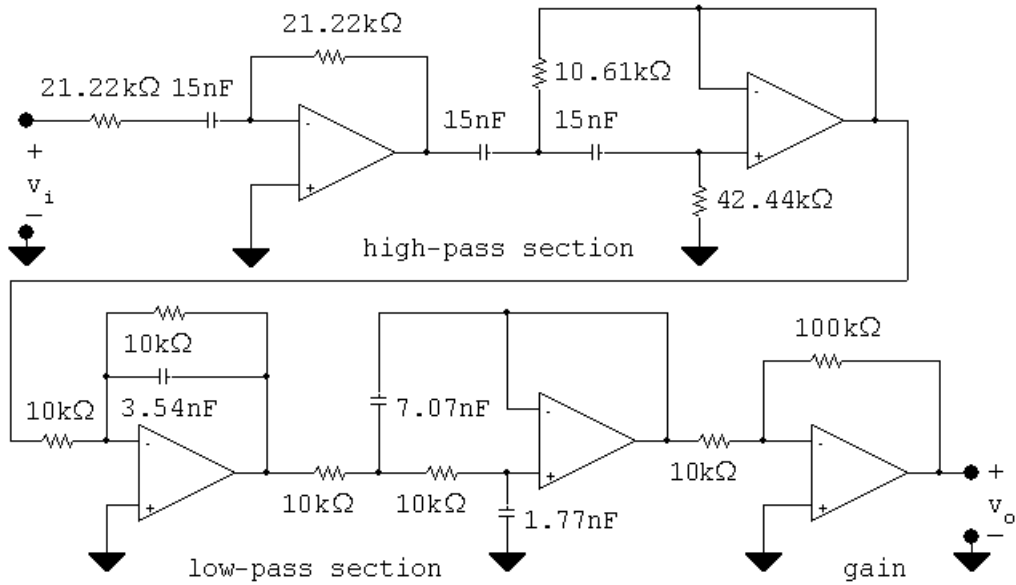$$C_1' = \frac{C_1}{k_f k_m} = \frac{2}{(9000\pi)(10^4)} = 7.07\,\text{nF}$$

$$C_2' = \frac{C_2}{k_f k_m} = \frac{0.5}{(9000\pi)(10^4)} = 1.77\,\text{nF}$$

GAIN AMPLIFIER

$$20 \log_{10} K = 20\,\text{dB}, \qquad \therefore \quad K = 10$$

Since we are using $10\,\text{k}\Omega$ resistors in the low-pass stage, we will use $R_f = 100\,\text{k}\Omega$ and $R_i = 10\,\text{k}\Omega$ in the inverting amplifier stage.

**[b]**



P 15.40 **[a]** Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 1000\pi$. Therefore the scaled transfer function is

$$H_{hp}'(s) = \frac{(s/1000\pi)^3}{\left(\frac{s}{1000\pi}+1\right)\left[\left(\frac{s}{1000\pi}\right)^2 + \frac{s}{1000\pi}+1\right]}$$

$$= \frac{s^3}{(s+1000\pi)[s^2+1000\pi s+10^6\pi^2]}$$

Unscaled low-pass stage

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 9000\pi$. Therefore the scaled transfer function is

$$H_{lp}'(s) = \frac{1}{\left(\frac{s}{9000\pi}+1\right)\left[\left(\frac{s}{9000\pi}\right)^2 + \left(\frac{s}{9000\pi}\right)+1\right]}$$

$$= \frac{(9000\pi)^3}{(s+9000\pi)(s^2+9000\pi s+81\times10^6\pi^2)}$$

Thus the transfer function for the filter is

$$H'(s) = 10H_{hp}'(s)H_{lp}'(s) = \frac{729\times10^{10}\pi^3 s^3}{D_1D_2D_3D_4}$$

where

$$D_1 = s + 1000\pi$$

$$D_2 = s + 9000\pi$$

$$D_3 = s^2 + 1000\pi s + 10^6\pi^2$$

$$D_4 = s^2 + 9000\pi s + 81 \times 10^6\pi^2$$

**[b]** At $f = 200\,\text{Hz}$    $\omega = 400\pi\,\text{rad/s}$

$$D_1(j400\pi) = 400\pi(2.5 + j1)$$

$$D_2(j400\pi) = 400\pi(22.5 + j1)$$

$$D_3(j400\pi) = 4 \times 10^5\pi^2(2.1 + j1.0)$$

$$D_4(j400\pi) = 4 \times 10^5\pi^2(202.1 + j9)$$

Therefore

$$D_1 D_2 D_3 D_4(j400\pi) = 256\pi^6 10^{14}(28{,}534.82\underline{/52.36°})$$

$$H'(j400\pi) = \frac{(729\pi^3 \times 10^{10})(64 \times 10^6\pi^3)}{256\pi^6 \times 10^{14}(28{,}534.82\underline{/52.36°})}$$

$$= 0.639\underline{/-52.36°}$$

$$\therefore \quad 20\log_{10}|H'(j400\pi)| = 20\log_{10}(0.639) = -3.89 \text{ dB}$$

At $f = 1500\,\text{Hz},$    $\omega = 3000\pi\,\text{rad/s}$

Then

$$D_1(j3000\pi) = 1000\pi(1 + j3)$$

$$D_2(j3000\pi) = 3000\pi(3 + j1)$$

$$D_3(j3000\pi) = 10^6\pi^2(-8 + j3)$$

$$D_4(j3000\pi) = 9 \times 10^6\pi^2(8 + j3)$$

$$H'(j3000\pi) = \frac{(729 \times \pi^3 \times 10^{10})(27 \times 10^9\pi^3)}{27 \times 10^{18}\pi^6(730\underline{/270°})}$$

$$= 9.99\underline{/90°}$$

$$\therefore \quad 20\log_{10}|H'(j3000\pi)| = 19.99 \text{ dB}$$

**[c]** From the transfer function the gain is down $19.99 + 3.89$ or $23.88$ dB at 200 Hz. Because the upper cut-off frequency is nine times the lower cut-off frequency we would expect the high-pass stage of the filter to predict the loss in gain at 200 Hz. For a 3nd order Butterworth

$$\text{GAIN} = 20 \log_{10} \frac{1}{\sqrt{1 + (500/200)^6}} = -23.89 \text{ dB}.$$

1500 Hz is in the passband for this bandpass filter, and is in fact the center frequency. Hence we expect the gain at 1500 Hz to equal, or nearly equal, 20 dB as specified in Problem 15.39. Thus our scaled transfer function confirms that the filter meets the specifications.

**P 15.41**  **[a]** From Table 15.1

$$H_{lp}(s) = \frac{1}{(s^2 + 0.518s + 1)(s^2 + \sqrt{2}s + 1)(s^2 + 1.932s + 1)}$$

$$H_{hp}(s) = \frac{1}{\left(\frac{1}{s^2} + 0.518\left(\frac{1}{s}\right) + 1\right)\left(\frac{1}{s^2} + \sqrt{2}\left(\frac{1}{s}\right) + 1\right)\left(\frac{1}{s^2} + 1.932\left(\frac{1}{s}\right) + 1\right)}$$

$$H_{hp}(s) = \frac{s^6}{(s^2 + 0.518s + 1)(s^2 + \sqrt{2}s + 1)(s^2 + 1.932s + 1)}$$

**P 15.42**  **[a]** $k_f = 25{,}000$

$$H'_{hp}(s) = \frac{(s/25{,}000)^6}{[(s/25{,}000)^2 + 0.518(s/25{,}000) + 1]}$$

$$\cdot \frac{1}{[(s/25{,}000)^2 + 1.414s/25{,}000 + 1][(s/25{,}000)^2 + 1.932s/25{,}000 + 1]}$$

$$= \frac{s^6}{(s^2 + 12{,}950s + 625 \times 10^6)(s^2 + 35{,}350s + 625 \times 10^6)}$$

$$\cdot \frac{1}{(s^2 + 48{,}300s + 625 \times 10^6)}$$

**[b]** $H'(j25{,}000) = \dfrac{-(25{,}000)^6}{[12{,}950(j25{,}000)][35{,}350(j25{,}000)][48{,}300(j25{,}000)]}$

$$= \frac{-(25{,}000)^3}{(12{,}950)(25{,}350)(48{,}300)j^3}$$

$$= 0.7067\underline{/-90^\circ}$$

$$20 \log_{10} |H'(j25{,}000)| = -3.02 \text{ dB}$$

P 15.43  **[a]** At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in $R_3$ is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of $R_2/R_1$. At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

**[b]** Let the node where $R_1$, $R_2$, $R_3$, and $C_2$ join be denoted as $a$, then

$$(V_a - V_i)G_1 + V_a s C_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o s C_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

$$V_a = \frac{-sC_1}{G_3}V_o$$

Solving for $V_o/V_i$ yields

$$H(s) = \frac{-G_1 G_3}{(G_1 + G_2 + G_3 + sC_2)sC_1 + G_2 G_3}$$

$$= \frac{-G_1 G_3}{s^2 C_1 C_2 + (G_1 + G_2 + G_3)C_1 s + G_2 G_3}$$

$$= \frac{-G_1 G_3/C_1 C_2}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right]s + \frac{G_2 G_3}{C_1 C_2}}$$

$$= \frac{-\frac{G_1 G_2 G_3}{G_2 C_1 C_2}}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right]s + \frac{G_2 G_3}{C_1 C_2}}$$

$$= \frac{-Kb_o}{s^2 + b_1 s + b_o}$$

where $K = \dfrac{G_1}{G_2};$   $b_o = \dfrac{G_2 G_3}{C_1 C_2}$

and $b_1 = \dfrac{G_1 + G_2 + G_3}{C_2}$

**[c]** Equating coefficients we see that

$$G_1 = KG_2$$

$$G_3 = \frac{b_o C_1 C_2}{G_2} = \frac{b_o C_1}{G_2}$$

since by hypothesis $C_2 = 1\,\mathrm{F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore\ b_1 = KG_2 + G_2 + \frac{b_oC_1}{G_2}$$

$$b_1 = G_2(1+K) + \frac{b_oC_1}{G_2}$$

Solving this quadratic equation for $G_2$ we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - b_oC_14(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}$$

For $G_2$ to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

**[d]**   1. Select $C_2 = 1\,\text{F}$

2. Select $C_1$ such that $C_1 < \dfrac{b_1^2}{4b_o(1+K)}$

3. Calculate $G_2\ (R_2)$

4. Calculate $G_1\ (R_1)$; $G_1 = KG_2$

5. Calculate $G_3\ (R_3)$; $G_3 = b_oC_1/G_2$

**P 15.44** **[a]** In the second order section of a third order Butterworth filter $b_o = b_1 = 1$
Therefore,

$$C_1 \le \frac{b_1^2}{4b_o(1+K)} = \frac{1}{(4)(1)(5)} = 0.05\,\text{F}$$

$$\therefore\qquad C_1 = 0.05\,\text{F}\quad\text{(limiting value)}$$

**[b]** $G_2 = \dfrac{1}{2(1+4)} = 0.1\,\text{S}$

$$G_3 = \frac{1}{0.1}(0.05) = 0.5\,\text{S}$$

$$G_1 = 4(0.1) = 0.4\,\text{S}$$

Therefore,

$$R_1 = \frac{1}{G_1} = 2.5\,\Omega;\qquad R_2 = \frac{1}{G_2} = 10\,\Omega;\qquad R_3 = \frac{1}{G_3} = 2\,\Omega$$

**[c]** $k_f = \dfrac{\omega_o'}{\omega_o} = 2\pi(2500) = 5000\pi$

$$k_m = \dfrac{C_2}{C_2' k_f} = \dfrac{1}{(10 \times 10^{-9})k_f} = 6366.2$$

$$C_1' = \dfrac{0.05}{k_f k_m} = 0.5 \times 10^{-9} = 500\,\text{pF}$$

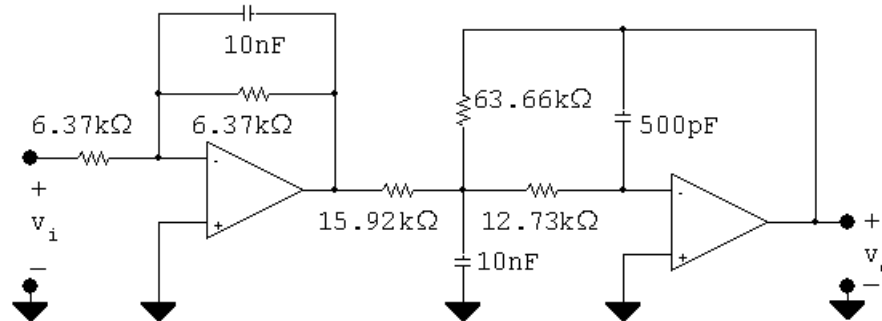$$R_1' = (2.5)(6366.2) = 15.92\,\text{k}\Omega$$

$$R_2' = (10)(6366.2) = 63.66\,\text{k}\Omega$$

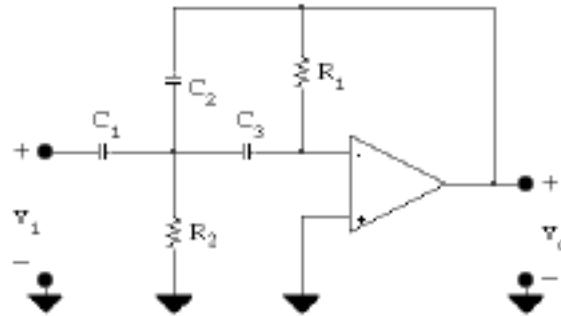$$R_3' = (2)(6366.2) = 12.73\,\text{k}\Omega$$

**[d]** $R_1' = R_2' = (6366.2)(1) = 6.37\,\text{k}\Omega$

$$C' = \dfrac{C}{k_f k_m} = \dfrac{1}{10^8} = 10\,\text{nF}$$

**[e]**



P 15.45 **[a]** By hypothesis the circuit becomes:



For very small frequencies the capacitors behave as open circuits and therefore $v_o$ is zero. As the frequency increases, the capacitive branch impedances become small compared to the resistive branches. When this happens the circuit becomes an inverting amplifier with the capacitor $C_2$ dominating the feedback path. Hence the gain of the amplifier approaches $(1/j\omega C_2)/(1/j\omega C_1)$ or $C_1/C_2$. Therefore the circuit behaves like a high-pass filter with a passband gain of $C_1/C_2$.

**[b]** Summing the currents away from the upper terminal of $R_2$ yields

$$V_a G_2 + (V_a - V_i)sC_1 + (V_a - V_o)sC_2 + V_a sC_3 = 0$$

or

$$V_a[G_2 + s(C_1 + C_2 + C_3)] - V_o sC_2 = sC_1 V_i$$

Summing the currents away from the inverting input terminal gives

$$(0 - V_a)sC_3 + (0 - V_o)G_1 = 0$$

or

$$sC_3 V_a = -G_1 V_o; \qquad V_a = \frac{-G_1 V_o}{sC_3}$$

Therefore we can write

$$\frac{-G_1 V_o}{sC_3}[G_2 + s(C_1 + C_2 + C_3)] - sC_2 V_o = sC_1 V_i$$

Solving for $V_o/V_i$ gives

$$H(s) = \frac{V_o}{V_i} = \frac{-C_1 C_3 s^2}{[C_2 C_3 s^2 + G_1(C_1 + C_2 + C_3)s + G_1 G_2]}$$

$$= \frac{\frac{-C_1}{C_2} s^2}{\left[s^2 + \frac{G_1}{C_2 C_3}(C_1 + C_2 + C_3)s + \frac{G_1 G_2}{C_2 C_3}\right]}$$

$$= \frac{-Ks^2}{s^2 + b_1 s + b_o}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of $C_1/C_2$.

**[c]** $C_1 = K$:

$$b_1 = \frac{G_1}{(1)(1)}(K + 2) = G_1(K + 2)$$

$$\therefore \quad G_1 = \frac{b_1}{K + 2}; \qquad R_1 = \left(\frac{K + 2}{b_1}\right)$$

$$b_o = \frac{G_1 G_2}{(1)(1)} = G_1 G_2$$

$$\therefore \quad G_2 = \frac{b_o}{G_1} = \frac{b_o}{b_1}(K + 2)$$

$$\therefore \quad R_2 = \frac{b_1}{b_o(K + 2)}$$

[d] From Table 15.1 the transfer function of the second-order section of a third-order high-pass Butterworth filter is

$$H(s) = \frac{Ks^2}{s^2 + s + 1}$$

Therefore $b_1 = b_o = 1$

Thus

$$C_1 = K = 8\,\text{F}$$

$$R_1 = \frac{8 + 2}{1} = 10\,\Omega$$

$$R_2 = \frac{1}{1(8 + 2)} = 0.10\,\Omega$$

P 15.46  [a] Low-pass filter:

$$n = \frac{(-0.05)(-30)}{\log_{10}(1000/400)} = 3.77; \qquad \therefore \quad n = 4$$

In the first prototype second-order section: $b_1 = 0.765$, $b_o = 1$, $C_2 = 1\,\text{F}$

$$C_1 \leq \frac{b_1^2}{4b_o(1 + K)} \leq \frac{(0.765)^2}{(4)(2)} \leq 0.0732$$

choose $C_1 = 0.03\,\text{F}$

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.03)}}{4} = \frac{0.765 \pm 0.588}{4}$$

Arbitrarily select the larger value for $G_2$, then

$$G_2 = 0.338 \text{ S}; \quad \therefore \qquad R_2 = \frac{1}{G_2} = 2.96\,\Omega$$

$$G_1 = KG_2 = 0.338 \text{ S}; \quad \therefore \qquad R_1 = \frac{1}{G_1} = 2.96\,\Omega$$

$$G_3 = \frac{b_o C_1}{G_2} = \frac{(1)(0.03)}{0.338} = 0.089 \quad \therefore \qquad R_3 = 1/G_3 = 11.3\,\Omega$$

Therefore in the first second-order prototype circuit

$$R_1 = R_2 = 2.96\,\Omega; \qquad R_3 = 11.3\,\Omega$$

$$C_1 = 0.03\,\text{F}; \qquad C_2 = 1\,\text{F}$$

In the second second-order prototype circuit: $b_1 = 1.848$, $b_0 = 1$, $C_2 = 1\,\text{F}$

$$\therefore \quad C_1 \leq \frac{(1.848)^2}{8} \leq 0.427$$

choose $C_1 = 0.30\,\text{F}$

$$G_2 = \frac{1.848 \pm \sqrt{(1.848)^2 - 8(0.3)}}{4} = \frac{1.848 \pm 1.008}{4}$$

Arbitrarily select the larger value, then

$$G_2 = 0.7139 \text{ S}; \quad \therefore \quad R_2 = \frac{1}{G_2} = 1.4008\,\Omega$$

$$G_1 = KG_2 = 0.7139 \text{ S}; \quad \therefore \quad R_1 = \frac{1}{G_1} = 1.4008\,\Omega$$

$$G_3 = \frac{b_o C_1}{G_2} = \frac{(1)(0.30)}{0.7139} = 0.4202 \text{ S} \quad \therefore \quad R_3 = 1/G_3 = 2.3796\,\Omega$$

In the low-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(400) = 800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(10 \times 10^{-9})k_f} = \frac{125{,}000}{\pi}$$

Therefore in the first scaled second-order section

$$R_1' = R_2' = 2.96k_m = 118\,\text{k}\Omega$$

$$R_3' = 11.3k_m = 450\,\text{k}\Omega$$

$$C_1' = \frac{0.03}{k_f k_m} = 300\,\text{pF}$$

$$C_2' = 10\,\text{nF}$$

In the second scaled second-order section

$$R_1' = R_2' = 1.4008k_m = 55.74\,\text{k}\Omega$$

$$R_3' = 2.3796k_m = 94.68\,\text{k}\Omega$$

$$C_1' = \frac{0.3}{k_f k_m} = 3\,\text{nF}$$

$$C_2' = 10\,\text{nF}$$

High-pass filter section

$$n = \frac{(-0.05)(-30)}{\log_{10}(6400/2560)} = 3.77; \quad n = 4.$$

In the first prototype second-order section: $b_1 = 0.765$; $b_o = 1$; $C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{0.765}{3} = 0.255\,\Omega$$

In the second prototype second-order section: $b_1 = 1.848$; $b_o = 1$;
$C_2 = C_3 = 1\,\text{F}$

$C_1 = K = 1\,\text{F}$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.623\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{1.848}{3} = 0.616\,\Omega$$

In the high-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(6400) = 12{,}800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(10 \times 10^{-9})(12{,}800\pi)} = \frac{7812.5}{\pi}$$

In the first scaled second-order section

$$R_1' = 3.92k_m = 9.75\,\text{k}\Omega$$

$$R_2' = 0.255k_m = 634\,\Omega$$

$$C_1' = C_2' = C_3' = 10\,\text{nF}$$

In the second scaled second-order section

$$R_1' = 1.623k_m = 4.04\,\text{k}\Omega$$

$$R_2' = 0.616k_m = 1.53\,\text{k}\Omega$$

$$C_1' = C_2' = C_3' = 10\,\text{nF}$$

In the gain section, let $R_i = 10\,\text{k}\Omega$ and $R_f = 10\,\text{k}\Omega$.

**[b]**



P 15.47  **[a]** The prototype low-pass transfer function is

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The low-pass frequency scaling factor is

$$k_{f_{lp}} = 2\pi(400) = 800\pi$$

The scaled transfer function for the low-pass filter is

$$H'_{lp}(s) = \frac{1}{\left[\left(\frac{s}{800\pi}\right)^2 + \frac{0.765s}{800\pi} + 1\right]\left[\left(\frac{s}{800\pi}\right)^2 + \frac{1.848s}{800\pi} + 1\right]}$$

$$= \frac{4096 \times 10^8 \pi^4}{[s^2 + 612\pi s + (800\pi)^2][s^2 + 1478.4\pi s + (800\pi)^2]}$$

The prototype high-pass transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The high-pass frequency scaling factor is

$$k_{f_{hp}} = 2\pi(6400) = 12{,}800\pi$$

The scaled transfer function for the high-pass filter is

$$H'_{hp}(s) = \frac{(s/12{,}800\pi)^4}{\left[\left(\frac{s}{12{,}800\pi}\right)^2 + \frac{0.765s}{12{,}800\pi} + 1\right]\left[\left(\frac{s}{12{,}800\pi}\right)^2 + \frac{1.848s}{12{,}800\pi} + 1\right]}$$

$$= \frac{s^4}{[s^2 + 9792\pi s + (12{,}800\pi)^2][s^2 + 23{,}654.4\pi s + (12{,}800\pi)^2]}$$

The transfer function for the filter is

$$H'(s) = \left[H'_{lp}(s) + H'_{hp}(s)\right]$$

**[b]** $f_o = \sqrt{f_{c1}f_{c2}} = \sqrt{(400)(6400)} = 1600\,\text{Hz}$

$\omega_o = 2\pi f_o = 3200\pi\,\text{rad/s}$

$(j\omega_o)^2 = -1024 \times 10^4\pi^2$

$(j\omega_o)^4 = 1{,}048{,}576 \times 10^8\pi^4$

$$H'_{lp}(j\omega_o) = \frac{4096 \times 10^8\pi^4}{[-960 \times 10^4\pi^2 + j612(3200\pi^2)]} \times$$

$$\frac{1}{[-960 \times 10^4\pi^2 + j1478.4(3200\pi^2)]}$$

$$= \frac{40{,}000}{(-3000 + j612)(-3000 + j1478.4)}$$

$$= 3906.2 \times 10^{-6}\underline{/37.76^\circ}$$

$$H'_{hp}(j\omega_o) = \frac{1{,}048{,}576 \times 10^8\pi^4}{[15{,}360 \times 10^4\pi^2 + j9792(3200\pi^2)]}$$

$$\frac{1}{[15{,}360 \times 10^4\pi^2 + j23{,}654.4(3200\pi^2)]}$$

$$= \frac{10.24 \times 10^6}{(48{,}000 + j9792)(48{,}000 + j23{,}654.4)}$$

$$= 3906.2 \times 10^{-6}\underline{/-37.76^\circ}$$

$\therefore\quad H'(j\omega_o) = -3906.2 \times 10^{-6}(1\underline{/37.76^\circ} + 1\underline{/-37.76^\circ})$

$= -3906.2 \times 10^{-6}(1.58\underline{/0^\circ}) = -6176.35 \times 10^{-6}\underline{/0^\circ}$

$G = 20\log_{10}|H'(j\omega_o)| = 20\log_{10}(6176.35 \times 10^{-6}) = -44.19\,\text{dB}$

P 15.48 **[a]** At low frequencies the capacitor branches are open; $v_o = v_i$. At high frequencies the capacitor branches are short circuits and the output voltage is zero. Hence the circuit behaves like a unity-gain low-pass filter.

**[b]** Let $v_a$ represent the voltage-to-ground at the right-hand terminal of $R_1$. Observe this will also be the voltage at the left-hand terminal of $R_2$. The $s$-domain equations are

$$(V_a - V_i)G_1 + (V_a - V_o)sC_1 = 0$$

$$(V_o - V_a)G_2 + sC_2V_o = 0$$

or

$$(G_1 + sC_1)V_a - sC_1V_o = G_1V_i$$

$$-G_2V_a + (G_2 + sC_2)V_o = 0$$

$$\therefore \quad V_a = \frac{G_2 + sC_2V_o}{G_2}$$

$$\therefore \quad \left[(G_1 + sC_1)\frac{(G_2 + sC_2)}{G_2} - sC_1\right]V_o = G_1V_i$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{G_1G_2}{(G_1 + sC_1)(G_2 + sC_2) - C_1G_2s}$$

which reduces to

$$\frac{V_o}{V_i} = \frac{G_1G_2/C_1C_2}{s^2 + \frac{G_1}{C_1}s + \frac{G_1G_2}{C_1C_2}} = \frac{b_o}{s^2 + b_1s + b_o}$$

**[c]** There are four circuit components and two restraints imposed by $H(s)$; therefore there are two free choices.

**[d]** $b_1 = \dfrac{G_1}{C_1} \quad \therefore \quad G_1 = b_1C_1$

$$b_o = \frac{G_1G_2}{C_1C_2} \quad \therefore \quad G_2 = \frac{b_o}{b_1}C_2$$

**[e]** No, all physically realizeable capacitors will yield physically realizeable resistors.

**[f]** From Table 15.1 we know the transfer function of the prototype 4th order Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

In the first section $b_o = 1, \quad b_1 = 0.765$

$$\therefore \quad G_1 = (0.765)(1) = 0.765 \text{ S}$$

$$R_1 = 1/G_1 = 1.307 \, \Omega$$

$$G_2 = \frac{1}{0.765}(1) = 1.307 \text{ S}$$
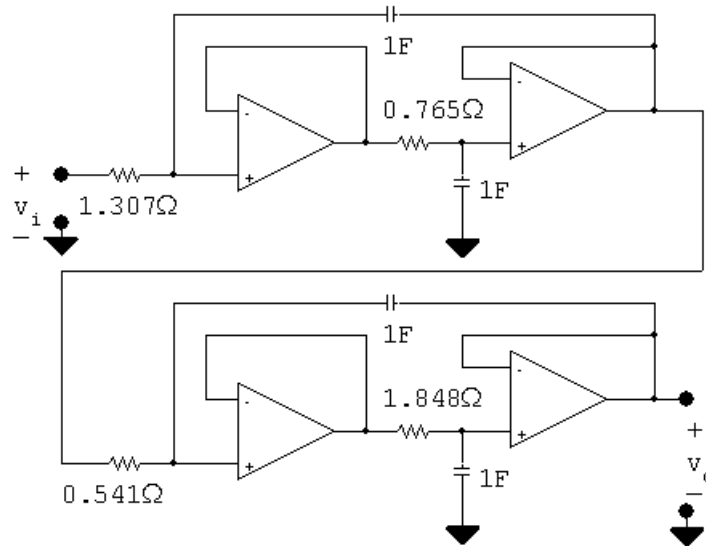
$$R_2 = 1/G_2 = 0.765\,\Omega$$

In the second section $b_o = 1, \quad b_1 = 1.848$

$$\therefore \quad G_1 = 1.848\,\text{S}$$

$$R_1 = 1/G_1 = 0.541\,\Omega$$

$$G_2 = \left(\frac{1}{1.848}\right)(1) = 0.541\,\text{S}$$

$$R_2 = 1/G_2 = 1.848\,\Omega$$



**P 15.49** **[a]** $k_f = \dfrac{\omega'_o}{\omega_o} = 2\pi(3000) = 6000\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(4.7 \times 10^{-9})(6000\pi)} = \frac{10^6}{28.2\pi}$$
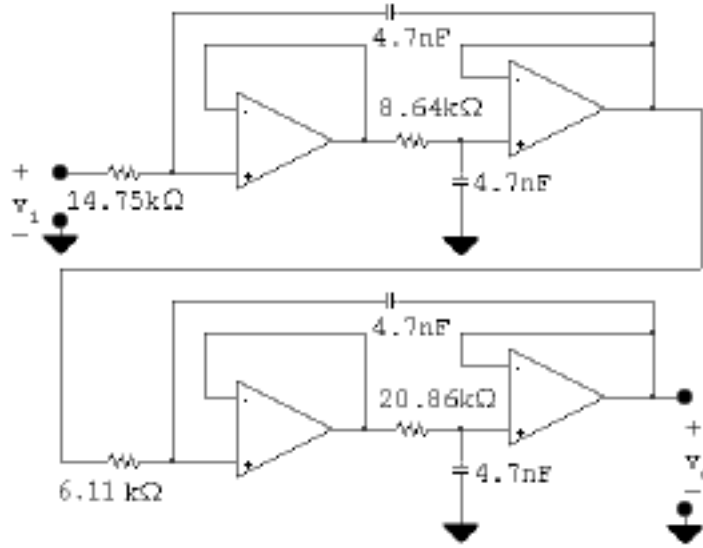
In the first section

$$R'_1 = 1.307k_m = 14.75\,\text{k}\Omega$$

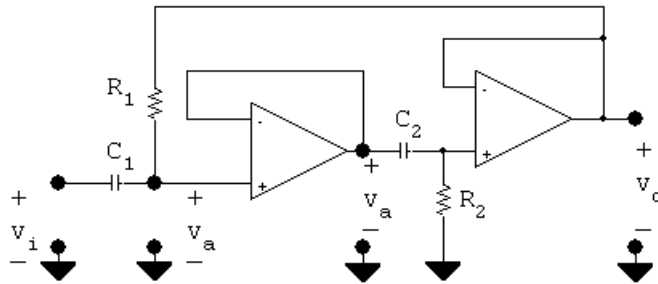$$R'_2 = 0.765k_m = 8.64\,\text{k}\Omega$$

In the second section

$$R'_1 = 0.541k_m = 6.11\,\text{k}\Omega$$

$$R'_2 = 1.848k_m = 20.86\,\text{k}\Omega$$

**[b]**



P 15.50  **[a]** Interchanging the $R$s and $C$s yields the following circuit.



At low frequencies the capacitors appear as open circuits and hence the output voltage is zero. As the frequency increases the capacitor branches approach short circuits and $v_a = v_i = v_o$. Thus the circuit is a unity-gain, high-pass filter.

**[b]** The $s$-domain equations are

$$(V_a - V_i)sC_1 + (V_a - V_o)G_1 = 0$$

$$(V_o - V_a)sC_2 + V_oG_2 = 0$$

It follows that

$$V_a(G_1 + sC_1) - G_1V_o = sC_1V_i$$

and $V_a = \dfrac{(G_2 + sC_2)V_o}{sC_2}$

Thus

$$\left\{\left[\frac{(G_2 + sC_2)}{sC_2}\right](G_1 + sC_1) - G_1\right\}V_o = sC_1V_i$$

$$V_o\{s^2C_1C_2 + sC_1G_2 + G_1G_2\} = s^2C_1C_2V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{s^2}{\left(s^2 + \dfrac{G_2}{C_2}s + \dfrac{G_1 G_2}{C_1 C_2}\right)}$$

$$= \frac{V_o}{V_i} = \frac{s^2}{s^2 + b_1 s + b_o}$$

**[c]** There are 4 circuit components: $R_1$, $R_2$, $C_1$ and $C_2$.

There are two transfer function constraints: $b_1$ and $b_o$.

Therefore there are two free choices.

**[d]** $b_o = \dfrac{G_1 G_2}{C_1 C_2};$    $b_1 = \dfrac{G_2}{C_2}$

$\therefore$   $G_2 = b_1 C_2;$    $R_2 = \dfrac{1}{b_1 C_2}$

$G_1 = \dfrac{b_o}{b_1} C_1 \therefore$   $R_1 = \dfrac{b_1}{b_o C_1}$

**[e]** No, all realizeable capacitors will produce realizeable resistors.

**[f]** The second-order section in a 3rd-order Butterworth high-pass filter is $s^2/(s^2 + s + 1)$. Therefore $b_o = b_1 = 1$ and
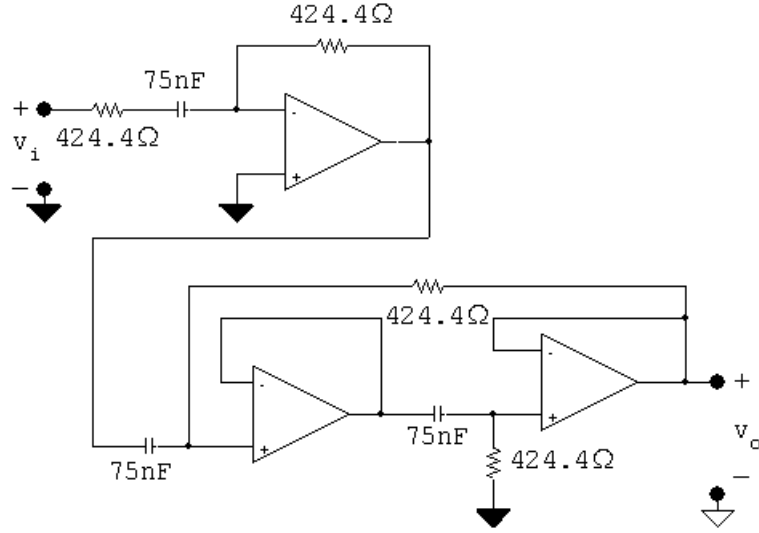
$$R_1 = \frac{1}{(1)(1)} = 1\,\Omega.$$

$$R_2 = \frac{1}{(1)(1)} = 1\,\Omega.$$

**P 15.51** **[a]** $k_f = \dfrac{\omega_o'}{\omega_o} = 10^4 \pi$

$$k_m = \frac{C}{C' k_f} = \frac{1}{(75 \times 10^{-9})(10^4 \pi)} = \frac{10^5}{75\pi}$$

$C_1 = C_2 = 75\,\text{nF};$    $R_1' = R_2' = k_m R = 424.4\,\Omega$

**[b]** $R = 424.4\,\Omega;$    $C = 75\,\text{nF}$

**[c]**



**[d]** $H_{\text{hp}}(s) = \dfrac{s^3}{(s+1)(s^2+s+1)}$

$$H'_{\text{hp}}(s) = \dfrac{(s/10^4\pi)^3}{[(s/10^4\pi)+1][(s/10^4\pi)^2+(s/10^4\pi)+1]}$$

$$= \dfrac{s^3}{(s+10^4\pi)(s^2+10^4\pi s+10^8\pi^2)}$$

**[e]** $H'_{\text{hp}}(j10^4\pi) = \dfrac{(j10^4\pi)^3}{(j10^4\pi+10^4\pi)[(j10^4\pi)^2+10^4\pi(j10^4\pi)+10^8\pi^2]} = 0.7071\underline{/135°}$

$\therefore \quad |H'_{\text{hp}}| = 0.7071 = -3\ \text{dB}$

**P 15.52 [a]** It follows directly from Eq 15.64 that

$$H(s) = \dfrac{s^2+1}{s^2+4(1-\sigma)s+1}$$

Now note from Eq 15.69 that $(1-\sigma)$ equals $1/4Q$, hence

$$H(s) = \dfrac{s^2+1}{s^2+\frac{1}{Q}s+1}$$

**[b]** For Example 15.13, $\omega_o = 5000\ \text{rad/s}$ and $Q = 5$. Therefore $k_f = 5000$ and

$$H'(s) = \dfrac{(s/5000)^2+1}{(s/5000)^2+\dfrac{1}{5}\left(\dfrac{s}{5000}\right)+1}$$

$$= \dfrac{s^2+25\times10^6}{s^2+1000s+25\times10^6}$$

**P 15.53 [a]** $\omega_o = 2000\pi\ \text{rad/s}$

$$\therefore \quad k_f = \dfrac{\omega'_o}{\omega_o} = 2000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(15 \times 10^{-9})(2000\pi)} = \frac{10^5}{3\pi}$$

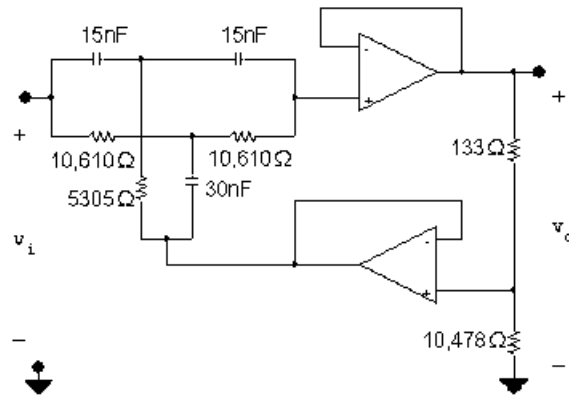$$R' = k_m R = \frac{10^5}{3\pi}(1) = 10{,}610\,\Omega$$

$$\frac{R'}{2} = 5{,}305\,\Omega$$

$$\sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(20)} = 0.9875$$

$$\sigma R' = 10{,}478\,\Omega; \qquad (1-\sigma)R' = 133\,\Omega$$

$$C' = 15\,\text{nF}$$

$$2C' = 30\,\text{nF}$$

**[b]**



**[c]** $k_f = 2000\pi$

$$H(s) = \frac{(s/2000\pi)^2 + 1}{(s/2000\pi)^2 + \frac{1}{20}(s/2000\pi) + 1}$$

$$= \frac{s^2 + 4 \times 10^6\pi^2}{s^2 + 100\pi s + 4 \times 10^6\pi^2}$$

P 15.54  To satisfy the gain specification of 20 dB at $\omega = 0$ and $\alpha = 1$ requires

$$\frac{R_1 + R_2}{R_1} = 10 \qquad \text{or} \qquad R_2 = 9R_1$$

Choose a standard resistor of $11.1\,\text{k}\Omega$ for $R_1$ and a $100\,\text{k}\Omega$ potentiometer for $R_2$. Since $(R_1 + R_2)/R_1 \gg 1$ the value of $C_1$ is

$$C_1 = \frac{1}{2\pi(40)(10^5)} = 39.79\,\text{nF}$$

Choose a standard capacitor value of 39 nF. Using the selected values of $R_1$ and $R_2$ the maximum gain for $\alpha = 1$ is

$$20 \log_{10} \left( \frac{111.1}{11.1} \right)_{\alpha=1} = 20.01 \text{ dB}$$

When $C_1 = 39$ nF the frequency $1/R_2C_1$ is

$$\frac{1}{R_2C_1} = \frac{10^9}{10^5(39)} = 256.41 \text{ rad/s} = 40.81 \text{ Hz}$$

The magnitude of the transfer function at 256.41 rad/s is

$$|H(j256.41)|_{\alpha=1} = \frac{|111.1 \times 10^3 + j256.41(11.1)(100)(39)10^{-3}|}{|11.1 \times 10^3 + j256.41(11.1)(100)(39)10^{-3}|} = 7.11$$

Therefore the gain at 40.81 Hz is

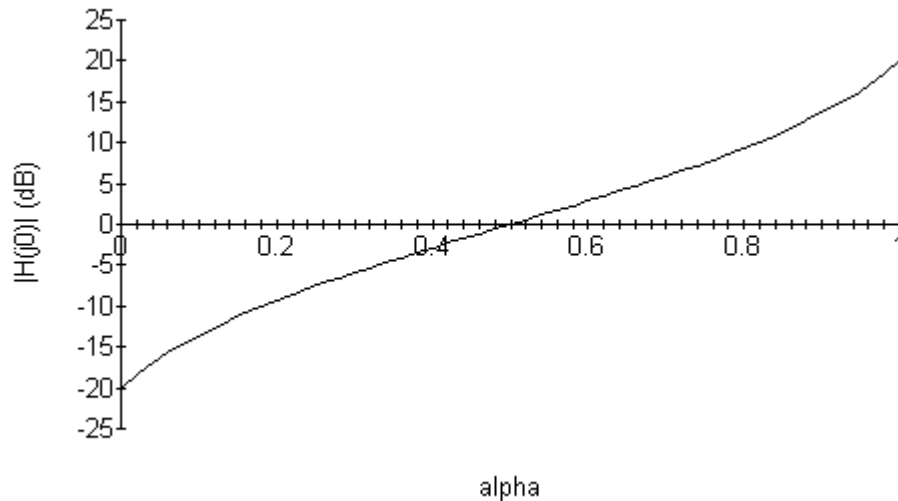$$20 \log_{10}(7.11)_{\alpha=1} = 17.04 \text{ dB}$$

**P 15.55**   $20 \log_{10} \left( \dfrac{R_1 + R_2}{R_1} \right) = 13.98$

$$\therefore \quad \frac{R_1 + R_2}{R_1} = 5; \qquad \therefore \quad R_2 = 4R_1$$

Choose    $R_1 = 100 \text{ k}\Omega$. Then    $R_2 = 400 \text{ k}\Omega$

$$\frac{1}{R_2C_1} = 100\pi \text{ rad/s}; \qquad \therefore \quad C_1 = \frac{1}{(100\pi)(400 \times 10^3)} = 7.96 \text{ nF}$$
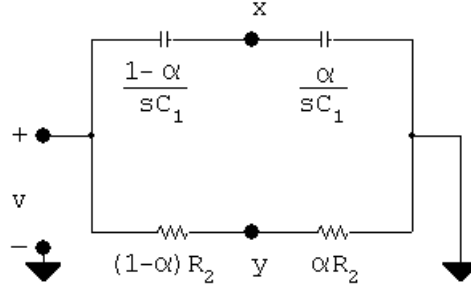
**P 15.56  [a]** $|$

P 15.57  **[a]** Combine the impedances of the capacitors in series in Fig. P15.53(b) to get

$$C_{eq} = \frac{1-\alpha}{sC_1} + \frac{\alpha}{sC_1} = \frac{1}{sC_1}$$

which is identical to the impedance of the capacitor in Fig. P15.53(a).

**[b]**



$$V_x = \frac{\alpha/sC_1}{(1-\alpha)/sC_1 + \alpha/sC_1} V = \alpha$$

$$V_y = \frac{\alpha R_2}{(1-\alpha)R_2 + \alpha R_2} = \alpha = V_x$$

**[c]** Since $x$ and $y$ are both at the same potential, they can be shorted together, and the circuit in Fig. 15.34 can thus be drawn as shown in Fig. 15.53(c).

**[d]** The feedback path between $V_o$ and $V_s$ containing the resistance $R_4 + 2R_3$ has no effect on the ratio $V_o/V_s$, as this feedback path is not involved in the nodal equation that defines the voltage ratio. Thus, the circuit in Fig. 15.53(c) can be simplified into the form of Fig. 15.2, where the input impedance is the equivalent impedance of $R_1$ in series with the parallel combination of $(1-\alpha)/sC_1$ and $(1-\alpha)R_2$, and the feedback impedance is the equivalent impedance of $R_1$ in series with the parallel combination of $\alpha/sC_1$ and $\alpha R_2$:

$$Z_i = R_1 + \frac{\frac{(1-\alpha)}{sC_1} \cdot (1-\alpha)R_2}{(1-\alpha)R_2 + \frac{(1-\alpha)}{sC_1}}$$

$$= \frac{R_1 + (1-\alpha)R_2 + R_1 R_2 C_1 s}{1 + R_2 C_1 s}$$

$$Z_f = R_1 + \frac{\frac{\alpha}{sC_1} \cdot \alpha R_2}{\alpha R_2 + \frac{\alpha}{sC_1}}$$

$$= \frac{R_1 + \alpha R_2 + R_1 R_2 C_1 s}{1 + R_2 C_1 s}$$

P 15.58  As $\omega \to 0$

$$|H(i\omega)| \to \frac{2R_3 + R_4}{2R_3 + R_4} = 1$$

Therefore the circuit would have no effect on low frequency signals. As $\omega \to \infty$

$$|H(j\omega)| \to \frac{[(1-\beta)R_4 + R_o](\beta R_4 + R_3)}{[(1-\beta)R_4 + R_3](\beta R_4 + R_o)}$$

When $\beta = 1$

$$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=1} \cong \frac{R_o}{R_3} > 1$$

Thus, when $\beta = 1$ we have amplification or "boost". When $\beta = 0$

$$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_3)}{R_o(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=0} \cong \frac{R_3}{R_o} < 1$$

Thus, when $\beta = 0$ we have attenuation or "cut".
Also note that when $\beta = 0.5$

$$|H(j\omega)|_{\beta=0.5} = \frac{(0.5R_4 + R_o)(0.5R_4 + R_3)}{(0.5R_4 + R_3)(0.5R_4 + R_o)} = 1$$

Thus, the transition from amplification to attenuation occurs at $\beta = 0.5$. If $\beta > 0.5$ we have amplification, and if $\beta < 0.5$ we have attenuation.
Also note the amplification an attenuation are symmetric about $\beta = 0.5$. i.e.

$$|H(j\omega)|_{\beta=0.6} = \frac{1}{|H(j\omega)|_{\beta=0.4}}$$

Yes, the circuit can be used as a treble volume control because

- The circuit has no effect on low frequency signals

- Depending on $\beta$ the circuit can either amplify ($\beta > 0.5$) or attenuate ($\beta < 0.5$) signals in the treble range

- The amplification (boost) and attenuation (cut) are symmetric around $\beta = 0.5$. When $\beta = 0.5$ the circuit has no effect on signals in the treble frequency range.

P 15.59 **[a]** $|H(j\infty)|_{\beta=1} = \dfrac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)} = \dfrac{(65.9)(505.9)}{(5.9)(565.9)} = 9.99$

$\therefore$   maximum boost $= 20\log_{10} 9.99 = 19.99$ dB

**[b]** $|H(j\infty)|_{\beta=0} = \dfrac{R_3(R_4 + R_3)}{R_o(R_4 + R_o)}$

$\therefore$   maximum cut $= -21.93$ dB

**[c]** $R_4 = 500\,\text{k}\Omega;$      $R_o = R_1 + R_3 + 2R_5 = 65.9\,\text{k}\Omega$

$\therefore$   $R_4 = 7.59 R_o$

Yes, $R_4$ is significantly greater than $R_o$.

**[d]** $|H(j/R_3C_2)|_{\beta=1} = \left| \dfrac{(2R_3 + R_4) + j\frac{R_o}{R_3}(R_4 + R_3)}{(2R_3 + R_4) + j(R_4 + R_o)} \right|$

$= \left| \dfrac{511.8 + j\frac{65.9}{5.9}(505.9)}{511.8 + j565.9} \right|$

$= 7.44$

$20\log_{10}|H(j/R_3C_2)|_{\beta=1} = 20\log_{10} 7.44 = 17.43$ dB

**[e]** When $\beta = 0$

$|H(j/R_3C_2)|_{\beta=0} = \dfrac{(2R_3 + R_4) + j(R_4 + R_o)}{(2R_3 + R_4) + j\dfrac{R_o}{R_3}(R_4 + R_3)}$

Note this is the reciprocal of $|H(j/R_3C_2)|_{\beta=1}$.

$\therefore$   $20\log_{10}|H(j/R_3C_2)|_{\beta+0} = -17.43$ dB

**[f]** The frequency $1/R_3C_2$ is very nearly where the gain is 3 dB off from its maximum boost or cut. Therefore for frequencies higher than $1/R_3C_2$ the circuit designer knows that gain or cut will be within 3 dB of the maximum.

P 15.60   $|H(j\infty)| = \dfrac{[(1-\beta)R_4 + R_o][\beta R_4 + R_3]}{[(1-\beta R_4 + R_3][\beta R_4 + R_o]}$