17

The Fourier Transform

Assessment Problems

AP 17.1 **[a]** $$F(\omega) = \int_{-\tau/2}^{0} (-Ae^{-j\omega t})\, dt + \int_{0}^{\tau/2} Ae^{-j\omega t}\, dt$$

$$= \frac{A}{j\omega}[2 - e^{j\omega\tau/2} - e^{-j\omega\tau/2}]$$

$$= \frac{2A}{j\omega}\left[1 - \frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2}\right]$$

$$= \frac{-j2A}{\omega}[1 - \cos\frac{\omega\tau}{2}]$$

[b] $$F(\omega) = \int_{0}^{\infty} te^{-at}e^{-j\omega t}\, dt = \int_{0}^{\infty} te^{-(a+j\omega)t}\, dt = \frac{1}{(a+j\omega)^2}$$

AP 17.2 $$f(t) = \frac{1}{2\pi}\left\{\int_{-3}^{-2} 4e^{jt\omega}\, d\omega + \int_{-2}^{2} e^{jt\omega}\, d\omega + \int_{2}^{3} 4e^{jt\omega}\, d\omega\right\}$$

$$= \frac{1}{j2\pi t}\{4e^{-j2t} - 4e^{-j3t} + e^{j2t} - e^{-j2t} + 4e^{j3t} - 4e^{j2t}\}$$

$$= \frac{1}{\pi t}\left[\frac{3e^{-j2t} - 3e^{j2t}}{j2} + \frac{4e^{j3t} - 4e^{-j3t}}{j2}\right]$$

$$= \frac{1}{\pi t}(4\sin 3t - 3\sin 2t)$$

AP 17.3 **[a]** $$F(\omega) = F(s)\mid_{s=j\omega} = \mathcal{L}\{e^{-at}\sin\omega_0 t\}_{s=j\omega}$$

$$= \frac{\omega_0}{(s+a)^2 + \omega_0^2}\Big|_{s=j\omega} = \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$$

[b] $$F(\omega) = \mathcal{L}\{f(-t)\}_{s=-j\omega} = \left[\frac{1}{(s+a)^2}\right]_{s=-j\omega} = \frac{1}{(a-j\omega)^2}$$

[c] $f^+(t) = te^{-at}, \qquad f^-(t) = te^{at}$

$$\mathcal{L}\{f^+(t)\} = \frac{1}{(s+a)^2}, \quad \mathcal{L}\{f^-(-t)\} = \frac{-1}{(s+a)^2}$$

Therefore $\quad F(\omega) = \dfrac{1}{(a+j\omega)^2} - \dfrac{1}{(a-j\omega)^2} = \dfrac{-j4a\omega}{(a^2+\omega^2)^2}$

AP 17.4 **[a]** $f'(t) = \dfrac{2A}{\tau}, \quad \dfrac{-\tau}{2} < t < 0; \qquad f'(t) = \dfrac{-2A}{\tau}, \quad 0 < t < \dfrac{\tau}{2}$

$$\therefore \qquad f'(t) = \frac{2A}{\tau}[u(t+\tau/2) - u(t)] - \frac{2A}{\tau}[u(t) - u(t-\tau/2)]$$

$$= \frac{2A}{\tau}u(t+\tau/2) - \frac{4A}{\tau}u(t) + \frac{2A}{\tau}u(t-\tau/2)$$

$$\therefore \qquad f''(t) = \frac{2A}{\tau}\delta\left(t+\frac{\tau}{2}\right) - \frac{4A}{\tau}\delta(t) + \frac{2A}{\tau}\delta\left(t-\frac{\tau}{2}\right)$$

[b] $\mathcal{F}\{f''(t)\} = \left[\dfrac{2A}{\tau}e^{j\omega\tau/2} - \dfrac{4A}{\tau} + \dfrac{2A}{\tau}e^{-j\omega\tau/2}\right]$

$$= \frac{4A}{\tau}\left[\frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2} - 1\right] = \frac{4A}{\tau}\left[\cos\left(\frac{\omega\tau}{2}\right) - 1\right]$$

[c] $\mathcal{F}\{f''(t)\} = (j\omega)^2F(\omega) = -\omega^2F(\omega); \qquad$ therefore $\quad F(\omega) = -\dfrac{1}{\omega^2}\mathcal{F}\{f''(t)\}$

Thus we have $\quad F(\omega) = -\dfrac{1}{\omega^2}\left\{\dfrac{4A}{\tau}\left[\cos\left(\dfrac{\omega\tau}{2}\right) - 1\right]\right\}$

AP 17.5 $v(t) = V_m\left[u\left(t+\dfrac{\tau}{2}\right) - u\left(t-\dfrac{\tau}{2}\right)\right]$

$$\mathcal{F}\left\{u\left(t+\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{j\omega\tau/2}$$

$$\mathcal{F}\left\{u\left(t-\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{-j\omega\tau/2}$$

Therefore $\quad V(\omega) = V_m\left[\pi\delta(\omega) + \dfrac{1}{j\omega}\right]\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$

$$= j2V_m\pi\delta(\omega)\sin\left(\frac{\omega\tau}{2}\right) + \frac{2V_m}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{(V_m\tau)\sin(\omega\tau/2)}{\omega\tau/2}$$

AP 17.6 **[a]** $I_g(\omega) = \mathcal{F}\{10\text{sgn}\,t\} = \dfrac{20}{j\omega}$

[b] $H(s) = \dfrac{V_o}{I_g}$

Using current division and Ohm's law,

$$V_o = -I_2 s = -\left[\frac{4}{4+1+s}\right](-I_g)s = \frac{4s}{5+s}I_g$$

$$H(s) = \frac{4s}{s+5}, \qquad H(\omega) = \frac{j4\omega}{5+j\omega}$$

[c] $V_o(\omega) = H(\omega)\cdot I_g(\omega) = \left(\dfrac{j4\omega}{5+j\omega}\right)\left(\dfrac{20}{j\omega}\right) = \dfrac{80}{5+j\omega}$

[d] $v_o(t) = 80e^{-5t}u(t)\,\text{V}$

[e] Using current division,

$$i_1(0^-) = \frac{1}{5}i_g = \frac{1}{5}(-10) = -2\,\text{A}$$

[f] $i_1(0^+) = i_g + i_2(0^+) = 10 + i_2(0^-) = 10 + 8 = 18\,\text{A}$

[g] Using current division,

$$i_2(0^-) = \frac{4}{5}(10) = 8\,\text{A}$$

[h] Since the current in an inductor must be continuous,

$$i_2(0^+) = i_2(0^-) = 8\,\text{A}$$

[i] Since the inductor behaves as a short circuit for $t < 0$,

$$v_o(0^-) = 0\,\text{V}$$

[j] $v_o(0^+) = 1i_2(0^+) + 4i_1(0^+) = 80\,\text{V}$

AP 17.7 **[a]** $V_g(\omega) = \dfrac{1}{1-j\omega} + \pi\delta(\omega) + \dfrac{1}{j\omega}$

$$H(s) = \frac{V_a}{V_g} = \frac{0.5\|(1/s)}{1+0.5\|(1/s)} = \frac{1}{s+3}, \qquad H(\omega) = \frac{1}{3+j\omega}$$

$$\begin{aligned}
V_a(\omega) &= H(\omega)V_g(\omega) \\
&= \frac{1}{(1-j\omega)(3+j\omega)} + \frac{1}{j\omega(3+j\omega)} + \frac{\pi\delta(\omega)}{3+j\omega} \\
&= \frac{1/4}{1-j\omega} + \frac{1/4}{3+j\omega} + \frac{1/3}{j\omega} - \frac{1/3}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\
&= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\
&= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \pi\delta(\omega)
\end{aligned}$$

Therefore $v_a(t) = \left[\frac{1}{4}e^{t}u(-t) + \frac{1}{6}\,\text{sgn}\,t - \frac{1}{12}e^{-3t}u(t) + \frac{1}{6}\right]$ V

[b] $v_a(0^-) = \frac{1}{4} - \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{4}$ V

$$v_a(0^+) = 0 + \frac{1}{6} - \frac{1}{12} + \frac{1}{6} = \frac{1}{4}\ \text{V}$$

$$v_a(\infty) = 0 + \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{3}\ \text{V}$$

AP 17.8 $v(t) = 4te^{-t}u(t);\qquad V(\omega) = \frac{4}{(1+j\omega)^2}$

Therefore $|V(\omega)| = \frac{4}{1+\omega^2}$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{\sqrt{3}} \left[\frac{4}{(1+\omega^2)}\right]^2 d\omega$$

$$= \frac{16}{\pi}\left\{\frac{1}{2}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\sqrt{3}}\right\}$$

$$= 16\left[\frac{\sqrt{3}}{8\pi} + \frac{1}{6}\right] = 3.769\ \text{J}$$

$$W_{1\Omega}(\text{total}) = \frac{8}{\pi}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\infty} = \frac{8}{\pi}\left[0 + \frac{\pi}{2}\right] = 4\ \text{J}$$

Therefore $\% = \frac{3.769}{4}(100) = 94.23\%$

AP 17.9 $|V(\omega)| = 6 - \left(\frac{6}{2000\pi}\right)\omega,\qquad 0 \le \omega \le 2000\pi$

$$|V(\omega)|^2 = 36 - \left(\frac{72}{2000\pi}\right)\omega + \left(\frac{36}{4\pi^2\times 10^6}\right)\omega^2$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{2000\pi}\left[36 - \frac{72\omega}{2000\pi} + \frac{36\times 10^{-6}}{4\pi^2}\omega^2\right] d\omega$$

$$= \frac{1}{\pi}\left[36\omega - \frac{72\omega^2}{4000\pi} + \frac{36\times 10^{-6}\omega^3}{12\pi^2}\right]_0^{2000\pi}$$

$$= \frac{1}{\pi}\left[36(2000\pi) - \frac{72}{4000\pi}(2000\pi)^2 + \frac{36\times 10^{-6}(2000\pi)^3}{12\pi^2}\right]$$

$$= 36(2000) - \frac{72(2000)^2}{4000} + \frac{36 \times 10^{-6}(2000)^3}{12}$$

$$= 24\,\mathbf{kJ}$$

$$W_{6\mathrm{k}\Omega} = \frac{24 \times 10^3}{6 \times 10^3} = 4\,\mathbf{J}$$

Problems

P 17.1 **[a]** $F(\omega) = \int_{-2}^{2} \left[A\sin\left(\frac{\pi}{2}\right)t\right] e^{-j\omega t}\,dt = \frac{-j4\pi A}{\pi^2 - 4\omega^2}\sin 2\omega$

[b] $F(\omega) = \int_{-\tau/2}^{0} \left(\frac{2A}{\tau}t + A\right) e^{-j\omega t}\,dt + \int_{0}^{\tau/2} \left(\frac{-2A}{\tau}t + A\right) e^{-j\omega t}\,dt$

$$= \frac{4A}{\omega^2\tau}\left[1 - \cos\left(\frac{\omega\tau}{2}\right)\right]$$

P 17.2 **[a]** $F(\omega) = \int_{-\tau/2}^{\tau/2} \frac{2A}{\tau} t e^{-j\omega t}\,dt$

$$= \frac{2A}{\tau}\left[\frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{2A}{\omega^2\tau}\left[e^{-j\omega\tau/2}\left(\frac{j\omega\tau}{2} + 1\right) - e^{j\omega\tau/2}\left(\frac{-j\omega\tau}{2} + 1\right)\right]$$

$$F(\omega) = \frac{2A}{\omega^2\tau}\left[e^{-j\omega\tau/2} - e^{j\omega\tau/2} + j\frac{\omega\tau}{2}\left(e^{-j\omega\tau/2} + e^{j\omega\tau/2}\right)\right]$$

$$F(\omega) = j\frac{2A}{\tau}\left[\frac{\omega\tau\cos(\omega\tau/2) - 2\sin(\omega\tau/2)}{\omega^2}\right]$$

[b] Using L'Hopital's rule,

$$F(0) = \lim_{\omega\to 0} j2A\left[\frac{\omega\tau(\tau/2)(-\sin\omega\tau/2) + \tau\cos\omega(\tau/2) - 2(\tau/2)\cos(\omega\tau/2)}{2\omega\tau}\right]$$

$$= \lim_{\omega\to 0} j2A\left[\frac{-\omega\tau(\tau/2)\sin(\omega\tau/2)}{2\omega\tau}\right]$$

$$= \lim_{\omega\to 0} j2A\left[\frac{-\tau\sin(\omega\tau/2)}{4}\right] = 0$$

$$\therefore \quad F(0) = 0$$

[c] When $A = 1$ and $\tau = 1$

$$F(\omega) = j2\left[\frac{\omega\cos(\omega/2) - 2\sin(\omega/2)}{\omega^2}\right]$$

$$|F(\omega)| = \left|\frac{2\omega\cos(\omega/2) - 4\sin(\omega/2)}{\omega^2}\right|$$

$$F(0) = 0$$

$$|F(2)| = \left|\frac{4\cos 1 - 4\sin 1}{4}\right| = 0.30$$

$$|F(4)| = \left|\frac{8\cos 2 - 4\sin 2}{16}\right| = 0.44$$

$$|F(6)| = \left|\frac{12\cos 3 - 4\sin 3}{36}\right| = 0.35$$

$$|F(8)| = \left|\frac{16\cos 4 - 4\sin 4}{64}\right| = 0.12$$

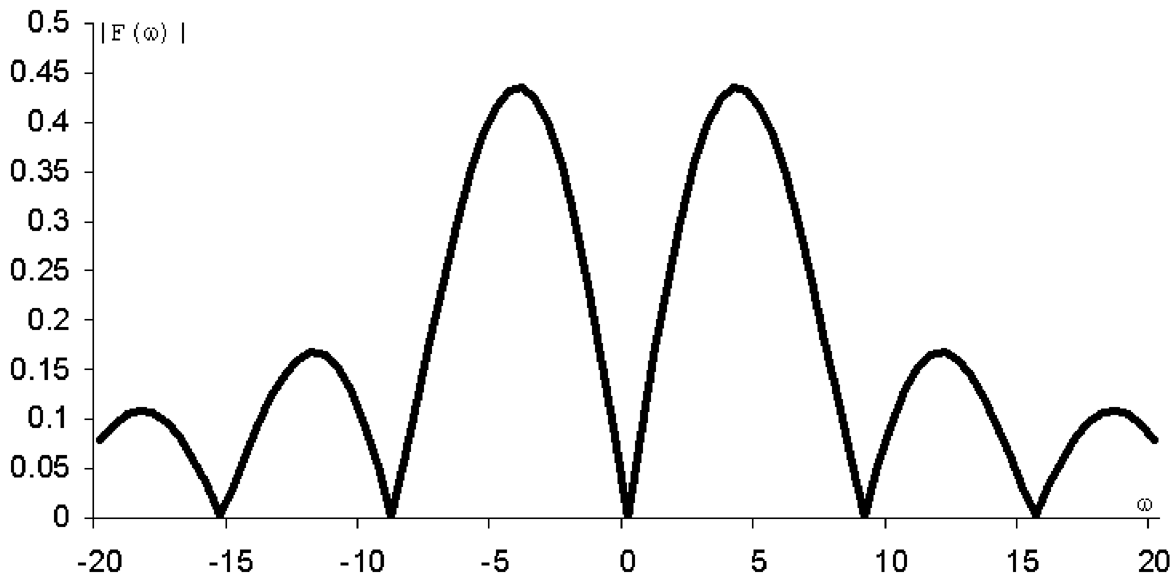
$$|F(9)| = \left|\frac{18\cos 4.5 - 4\sin 4.5}{81}\right| \cong 0$$

$$|F(10)| = \left|\frac{20\cos 5 - 4\sin 5}{100}\right| = 0.10$$

$$|F(12)| = \left|\frac{24\cos 6 - 4\sin 6}{144}\right| = 0.17$$

$$|F(14)| = \left|\frac{28\cos 7 - 4\sin 7}{196}\right| = 0.09$$

$$|F(15.5)| = \left|\frac{31\cos 7.75 - 4\sin 7.75}{240.25}\right| \cong 0$$



P 17.3 **[a]** $F(\omega) = A + \frac{2A}{\omega_o}\omega, \quad -\omega_o/2 \leq \omega \leq 0$

$$F(\omega) = A - \frac{2A}{\omega_o}\omega, \quad 0 \leq \omega \leq \omega_o/2$$

$F(\omega) = 0$ elsewhere

$$f(t) = \frac{1}{2\pi}\int_{-\omega_o/2}^{0}\left(A + \frac{2A}{\omega_o}\omega\right)e^{jt\omega}\,d\omega$$

$$+\frac{1}{2\pi}\int_{0}^{\omega_o/2}\left(A - \frac{2A}{\omega_o}\omega\right)e^{jt\omega}\,d\omega$$

$$f(t) = \frac{1}{2\pi}\left[\int_{-\omega_o/2}^{0} Ae^{jt\omega}\,d\omega + \int_{-\omega_o/2}^{0}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega\right.$$

$$\left.+\int_{0}^{\omega_o/2} Ae^{jt\omega}\,d\omega - \int_{0}^{\omega_o/2}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega\right]$$

$$= \frac{1}{2\pi}[\,\text{Int1} \;+\; \text{Int2} \;+\; \text{Int3} \;-\; \text{Int4}\,]$$

$$\text{Int1} = \int_{-\omega_o/2}^{0} Ae^{jt\omega}\,d\omega = \frac{A}{jt}(1 - e^{-jt\omega_o/2})$$

$$\text{Int2} = \int_{-\omega_o/2}^{0}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega = \frac{2A}{\omega_o t^2}(1 - j\frac{t\omega_o}{2}e^{-jt\omega_o/2} - e^{-jt\omega_o/2})$$

$$\text{Int3} = \int_{0}^{\omega_o/2} Ae^{jt\omega}\,d\omega = \frac{A}{jt}(e^{jt\omega_o/2} - 1)$$

$$\text{Int4} = \int_{0}^{\omega_o/2}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega = \frac{2A}{\omega_o t^2}(-j\frac{t\omega_o}{2}e^{jt\omega_o/2} + e^{jt\omega_o/2} - 1)$$

$$\text{Int1} \;+\; \text{Int3} = \frac{2A}{t}\sin(\omega_o t/2)$$

$$\text{Int2} \;-\; \text{Int4} = \frac{4A}{\omega_o t^2}[1 - \cos(\omega_o t/2)] - \frac{2A}{t}\sin(\omega_o t/2)$$

$$\therefore \qquad f(t) = \frac{1}{2\pi}\left[\frac{4A}{\omega_o t^2}\left(1 - \cos(\omega_o t/2)\right)\right]$$

$$= \frac{2A}{\pi\omega_o t^2}\left[2\sin^2(\omega_o t/4)\right]$$

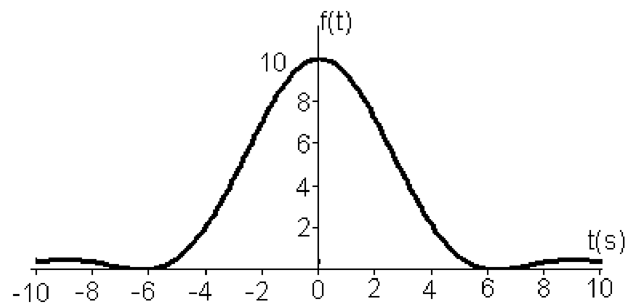
$$= \frac{4\omega_o A}{\pi\omega_o^2 t^2}\sin^2(\omega_o t/4)$$

$$= \frac{\omega_o A}{4\pi}\left[\frac{\sin(\omega_o t/4)}{(\omega_o t/4)}\right]^2$$

[b] $f(0) = \frac{\omega_o A}{4\pi}(1)^2 = 79.58 \times 10^{-3}\omega_o A$

[c] $A = 20\pi; \qquad \omega_o = 2$ rad/s

$$f(t) = 10\left[\frac{\sin(t/2)}{(t/2)}\right]^2$$



P 17.4 **[a]** $F(s) = \mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \left[\frac{1}{(a+j\omega)^2}\right] + \left[\frac{1}{(a-j\omega)^2}\right]$$

$$= \frac{2(a^2-\omega^2)}{(a^2-\omega^2)^2+4a^2\omega^2} = \frac{2(a^2-\omega^2)}{(a^2+\omega^2)^2}$$

[b] $F(s) = \mathcal{L}\{t^3e^{-at}\} = \dfrac{6}{(s+a)^4}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{6}{(a+j\omega)^4} - \frac{6}{(a-j\omega)^4} = -j48a\omega\frac{a^2-\omega^2}{(a^2+\omega^2)^4}$$

[c] $F(s) = \mathcal{L}\{e^{-at}\cos\omega_0 t\} = \dfrac{s+a}{(s+a)^2+\omega_0^2} = \dfrac{0.5}{(s+a)-j\omega_0} + \dfrac{0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{0.5}{(a+j\omega)-j\omega_0} + \frac{0.5}{(a+j\omega)+j\omega_0}$$

$$+ \frac{0.5}{(a-j\omega)-j\omega_0} + \frac{0.5}{(a-j\omega)+j\omega_0}$$

$$= \frac{a}{a^2+(\omega-\omega_0)^2} + \frac{a}{a^2+(\omega+\omega_0)^2}$$

[d] $F(s) = \mathcal{L}\{e^{-at}\sin\omega_0 t\} = \dfrac{\omega_0}{(s+a)^2+\omega_0^2} = \dfrac{-j0.5}{(s+a)-j\omega_0} + \dfrac{j0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} - F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-ja}{a^2+(\omega-\omega_0)^2} + \frac{ja}{a^2+(\omega+\omega_0)^2}$$

[e] $F(\omega) = \int_{-\infty}^{\infty} \delta(t-t_o)e^{-j\omega t}\,dt = e^{-j\omega t_o}$

(Use the sifting property of the Dirac delta function.)

P 17.5 $$\mathcal{F}\{\sin\omega_0 t\} = \mathcal{F}\left\{\frac{e^{j\omega_0 t}}{2j}\right\} - \mathcal{F}\left\{\frac{e^{-j\omega_0 t}}{2j}\right\}$$

$$= \frac{1}{2j}[2\pi\delta(\omega-\omega_0) - 2\pi\delta(\omega+\omega_0)]$$

$$= j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$$

P 17.6 $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)+jB(\omega)][\cos t\omega + j\sin t\omega]\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega - B(\omega)\sin t\omega]\,d\omega$$

$$+ \frac{j}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\sin t\omega + B(\omega)\cos t\omega]\,d\omega$$

But $f(t)$ is real, therefore the second integral in the sum is zero.

P 17.7 By hypothesis, $f(t) = -f(-t)$. From Problem 17.6, we have

$$f(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega + B(\omega)\sin t\omega]\,d\omega$$

For $f(t) = -f(-t)$, the integral $\int_{-\infty}^{\infty} A(\omega)\cos t\omega\,d\omega$ must be zero. Therefore, if $f(t)$ is real and odd, we have

$$f(t) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} B(\omega)\sin t\omega\,d\omega$$

P 17.8 $F(\omega) = \dfrac{-j2}{\omega}$; therefore $B(\omega) = \dfrac{-2}{\omega}$; thus we have

$$f(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{-2}{\omega}\right)\sin t\omega\,d\omega = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin t\omega}{\omega}\,d\omega$$

But $\dfrac{\sin t\omega}{\omega}$ is even; therefore $f(t) = \dfrac{2}{\pi}\int_0^{\infty}\dfrac{\sin t\omega}{\omega}\,d\omega$

Therefore,

$$\left.\begin{array}{ll} f(t) = \dfrac{2}{\pi}\cdot\dfrac{\pi}{2} = 1 & t > 0 \\ f(t) = \dfrac{2}{\pi}\cdot\left(\dfrac{-\pi}{2}\right) = -1 & t < 0 \end{array}\right\}\text{from a table of definite integrals}$$

Therefore $\quad f(t) = \operatorname{sgn} t$

P 17.9 From Problem 17.4[c] we have

$$F(\omega) = \frac{\epsilon}{\epsilon^2 + (\omega - \omega_0)^2} + \frac{\epsilon}{\epsilon^2 + (\omega + \omega_0)^2}$$

Note that as $\epsilon \to 0$, $F(\omega) \to 0$ everywhere except at $\omega = \pm\omega_0$. At $\omega = \pm\omega_0$, $F(\omega) = 1/\epsilon$, therefore $F(\omega) \to \infty$ at $\omega = \pm\omega_0$ as $\epsilon \to 0$. The area under each bell-shaped curve is independent of ϵ, that is

$$\int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega - \omega_0)^2} = \int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega + \omega_0)^2} = \pi$$

Therefore as $\epsilon \to 0$, $\quad F(\omega) \to \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

P 17.10 $A(\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t\, dt$

$$= \int_{-\infty}^{0} f(t)\cos\omega t\, dt + \int_{0}^{\infty} f(t)\cos\omega t\, dt$$

$$= 2\int_{0}^{\infty} f(t)\cos\omega t\, dt, \quad \text{since } f(t)\cos\omega t \text{ is also even.}$$

$B(\omega) = 0$, since $f(t)\sin\omega t$ is an odd function and

$$\int_{-\infty}^{0} f(t)\sin\omega t\, dt = -\int_{0}^{\infty} f(t)\sin\omega t\, dt$$

P 17.11 $A(\omega) = \int_{-\infty}^{0} f(t)\cos\omega t\, dt + \int_{0}^{\infty} f(t)\cos\omega t\, dt = 0$

since $f(t)\cos\omega t$ is an odd function.

$$B(\omega) = -2\int_{0}^{\infty} f(t)\sin\omega t\, dt, \quad \text{since } f(t)\sin\omega t \text{ is an even function.}$$

P 17.12 **[a]** $\mathcal{F}\left\{\dfrac{df(t)}{dt}\right\} = \int_{-\infty}^{\infty} \dfrac{df(t)}{dt} e^{-j\omega t}\, dt$

Let $u = e^{-j\omega t}$, then $du = -j\omega e^{-j\omega t}$; let $dv = [df(t)/dt]\, dt$, then $v = f(t)$.

Therefore $\quad \mathcal{F}\left\{\dfrac{df(t)}{dt}\right\} = f(t)e^{-j\omega t}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(t)[-j\omega e^{-j\omega t}\, dt]$

$$= 0 + j\omega F(\omega)$$

[b] Fourier transform of $f(t)$ exists, i.e., $f(\infty) = f(-\infty) = 0$.

[c] To find $\mathcal{F}\left\{\frac{d^2 f(t)}{dt^2}\right\}$, let $g(t) = \frac{df(t)}{dt}$

Then $\mathcal{F}\left\{\frac{d^2 f(t)}{dt^2}\right\} = \mathcal{F}\left\{\frac{dg(t)}{dt}\right\} = j\omega G(\omega)$

But $G(\omega) = \mathcal{F}\left\{\frac{df(t)}{dt}\right\} = j\omega F(\omega)$

Therefore we have $\mathcal{F}\left\{\frac{d^2 f(t)}{dt^2}\right\} = (j\omega)^2 F(\omega)$

Repeated application of this thought process gives

$\mathcal{F}\left\{\frac{d^n f(t)}{dt^n}\right\} = (j\omega)^n F(\omega).$

P 17.13 **[a]** $\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{t} f(x)\,dx\right] e^{-j\omega t}\,dt$

Now let $u = \int_{-\infty}^{t} f(x)\,dx,$ then $du = f(t)dt$

Let $dv = e^{-j\omega t}\,dt,$ then $v = \frac{e^{-j\omega t}}{-j\omega}$

Therefore,

$$\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \frac{e^{-j\omega t}}{-j\omega}\int_{-\infty}^{t} f(x)\,dx\,\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\left[\frac{e^{-j\omega t}}{-j\omega}\right] f(t)\,dt$$

$$= 0 + \frac{F(\omega)}{j\omega}$$

[b] We require $\int_{-\infty}^{\infty} f(x)\,dx = 0$

[c] No, because $\int_{-\infty}^{\infty} e^{-ax}u(x)\,dx = \frac{1}{a} \neq 0$

P 17.14 **[a]** $\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at)e^{-j\omega t}\,dt$

Let $u = at,$ $du = a\,dt,$ $u = \pm\infty$ when $t = \pm\infty$

Therefore,

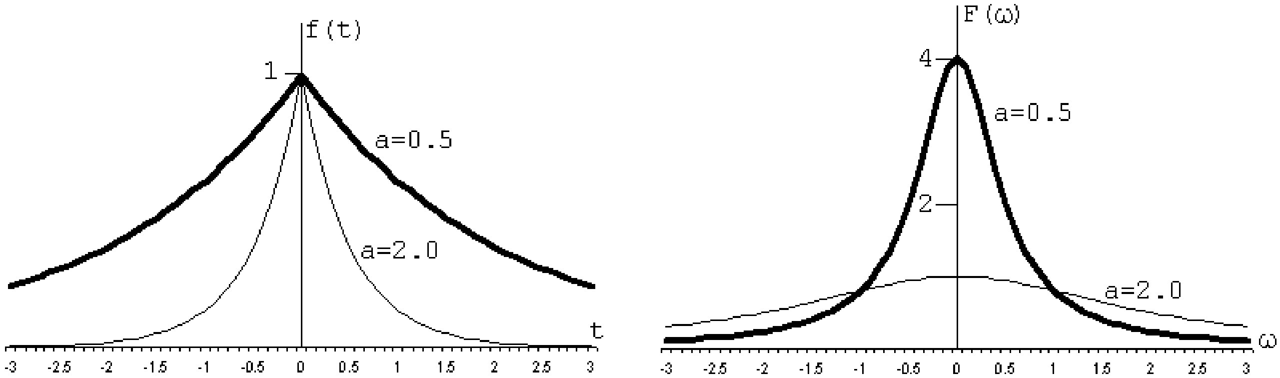
$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega u/a}\left(\frac{du}{a}\right) = \frac{1}{a}F\left(\frac{\omega}{a}\right), \qquad a > 0$$

[b] $\mathcal{F}\{e^{-|t|}\} = \dfrac{1}{1+j\omega} + \dfrac{1}{1-j\omega} = \dfrac{2}{1+\omega^2}$

Therefore $\quad \mathcal{F}\{e^{-a|t|}\} = \dfrac{(1/a)2}{(\omega/a)^2+1}$

Therefore $\quad \mathcal{F}\{e^{-0.5|t|}\} = \dfrac{4}{4\omega^2+1}, \qquad \mathcal{F}\{e^{-|t|}\} = \dfrac{2}{\omega^2+1}$

$\mathcal{F}\{e^{-2|t|}\} = 1/[0.25\omega^2+1]$, yes as "*a*" increases, the sketches show that $f(t)$ approaches zero faster and $F(\omega)$ flattens out over the frequency spectrum.



P 17.15 **[a]** $\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(t-a)e^{-j\omega t}\,dt$

Let $u = t-a$, then $du = dt$, $t = u+a$, and $u = \pm\infty$ when $t = \pm\infty$. Therefore,

$$\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega(u+a)}\,du$$

$$= e^{-j\omega a}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}\,du = e^{-j\omega a}F(\omega)$$

[b] $\mathcal{F}\{e^{j\omega_0 t}f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}\,dt = F(\omega-\omega_0)$

[c] $\mathcal{F}\{f(t)\cos\omega_0 t\} = \mathcal{F}\left\{f(t)\left[\dfrac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right]\right\}$

$$= \frac{1}{2}F(\omega-\omega_0) + \frac{1}{2}F(\omega+\omega_0)$$

P 17.16 $Y(\omega) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda\right]e^{-j\omega t}\,dt$

$$= \int_{-\infty}^{\infty} x(\lambda)\left[\int_{-\infty}^{\infty} h(t-\lambda)e^{-j\omega t}\,dt\right]d\lambda$$

Let $u = t - \lambda$, $du = dt$, and $u = \pm\infty$, when $t = \pm\infty$.

$$\text{Therefore}\quad Y(\omega) = \int_{-\infty}^{\infty} x(\lambda)\left[\int_{-\infty}^{\infty} h(u)e^{-j\omega(u+\lambda)}\,du\right]d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)\left[e^{-j\omega\lambda}\int_{-\infty}^{\infty} h(u)e^{-j\omega u}\,du\right]d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)e^{-j\omega\lambda}H(\omega)\,d\lambda = H(\omega)X(\omega)$$

P 17.17 $\mathcal{F}\{f_1(t)f_2(t)\} = \int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}F_1(u)e^{jtu}du\right]f_2(t)e^{-j\omega t}\,dt$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}F_1(u)f_2(t)e^{-j\omega t}e^{jtu}\,du\right]dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[F_1(u)\int_{-\infty}^{\infty}f_2(t)e^{-j(\omega-u)t}\,dt\right]du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}F_1(u)F_2(\omega-u)\,du$$

P 17.18 **[a]** $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$$\frac{dF}{d\omega} = \int_{-\infty}^{\infty}\frac{d}{d\omega}\left[f(t)e^{-j\omega t}\right]dt = -j\int_{-\infty}^{\infty}tf(t)e^{-j\omega t}\,dt = -j\mathcal{F}\{tf(t)\}$$

$$\text{Therefore}\quad j\frac{dF(\omega)}{d\omega} = \mathcal{F}\{tf(t)\}$$

$$\frac{d^2F(\omega)}{d\omega^2} = \int_{-\infty}^{\infty}(-jt)(-jt)f(t)e^{-j\omega t}\,dt = (-j)^2\mathcal{F}\{t^2f(t)\}$$

$$\text{Note that}\quad (-j)^n = \frac{1}{j^n}$$

$$\text{Thus we have}\quad j^n\left[\frac{d^nF(\omega)}{d\omega^n}\right] = \mathcal{F}\{t^nf(t)\}$$

[b] (i) $\mathcal{F}\{e^{-at}u(t)\} = \dfrac{1}{a+j\omega} = F(\omega);\qquad \dfrac{dF(\omega)}{d\omega} = \dfrac{-j}{(a+j\omega)^2}$

$$\text{Therefore}\quad j\left[\frac{dF(\omega)}{d\omega}\right] = \frac{1}{(a+j\omega)^2}$$

$$\text{Therefore}\quad \mathcal{F}\{te^{-at}u(t)\} = \frac{1}{(a+j\omega)^2}$$

(ii) $\mathcal{F}\{|t|e^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} - \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} - j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} + \frac{1}{(a-j\omega)^2}$$

(iii) $\mathcal{F}\{te^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} + \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} + j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} - \frac{1}{(a-j\omega)^2}$$

P 17.19 **[a]** $f_1(t) = \cos\omega_0 t, \qquad F_1(u) = \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]$

$f_2(t) = 1, \quad -\tau/2 < t < \tau/2, \qquad$ and $f_2(t) = 0$ elsewhere

Thus $\quad F_2(u) = \dfrac{\tau\sin(u\tau/2)}{u\tau/2}$

Using convolution,

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]\tau\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\int_{-\infty}^{\infty}\delta(u+\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$+ \frac{\tau}{2}\int_{-\infty}^{\infty}\delta(u-\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\cdot\frac{\sin[(\omega+\omega_0)\tau/2]}{(\omega+\omega_0)(\tau/2)} + \frac{\tau}{2}\cdot\frac{\sin[(\omega-\omega_0)\tau/2]}{(\omega-\omega_0)\tau/2}$$

[b] As τ increases, the amplitude of $F(\omega)$ increases at $\omega = \pm\omega_0$ and at the same time the duration of $F(\omega)$ approaches zero as ω deviates from $\pm\omega_0$. The area under the $[\sin x]/x$ function is independent of τ, that is

$$\frac{\tau}{2}\int_{-\infty}^{\infty}\frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}\,d\omega = \int_{-\infty}^{\infty}\frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}[(\tau/2)\,d\omega] = \pi$$

Therefore as $t \to \infty$,

$f_1(t)f_2(t) \to \cos\omega_0 t \quad$ and $\quad F(\omega) \to \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$

P 17.20 **[a]** $v_g = 100u(t)$

$$V_g(\omega) = 100\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

$$H(s) = \frac{10}{5s + 10} = \frac{2}{s + 2}$$

$$H(\omega) = \frac{2}{j\omega + 2}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{200\pi\delta(\omega)}{j\omega + 2} + \frac{200}{j\omega(j\omega + 2)}$$

$$= V_1(\omega) + V_2(\omega)$$

$$v_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{200\pi e^{jt\omega}}{j\omega + 2}\delta(\omega)\,d\omega = \frac{1}{2\pi}\left(\frac{200\pi}{2}\right) = 50 \text{ (sifting property)}$$

$$V_2(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 2} = \frac{100}{j\omega} - \frac{100}{j\omega + 2}$$

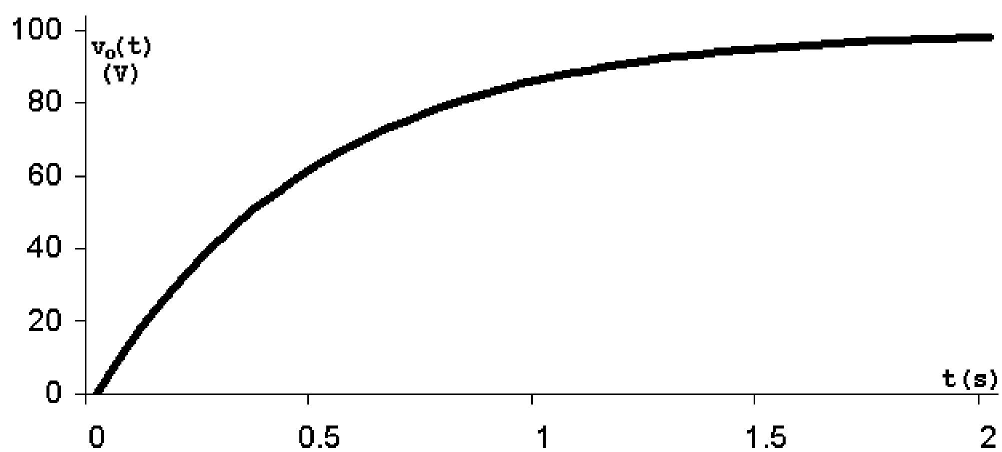
$$v_2(t) = 50\text{sgn}(t) - 100e^{-2t}u(t)$$

$$v_o(t) = v_1(t) + v_2(t) = 50 + 50\text{sgn}(t) - 100e^{-2t}u(t)$$

$$= 100u(t) - 100e^{-2t}u(t)$$

$$v_o(t) = 100(1 - e^{-2t})u(t)\text{ V}$$

[b]



P 17.21 **[a]** From the solution to Problem 17.20

$$H(\omega) = \frac{2}{j\omega + 2}$$

Now,

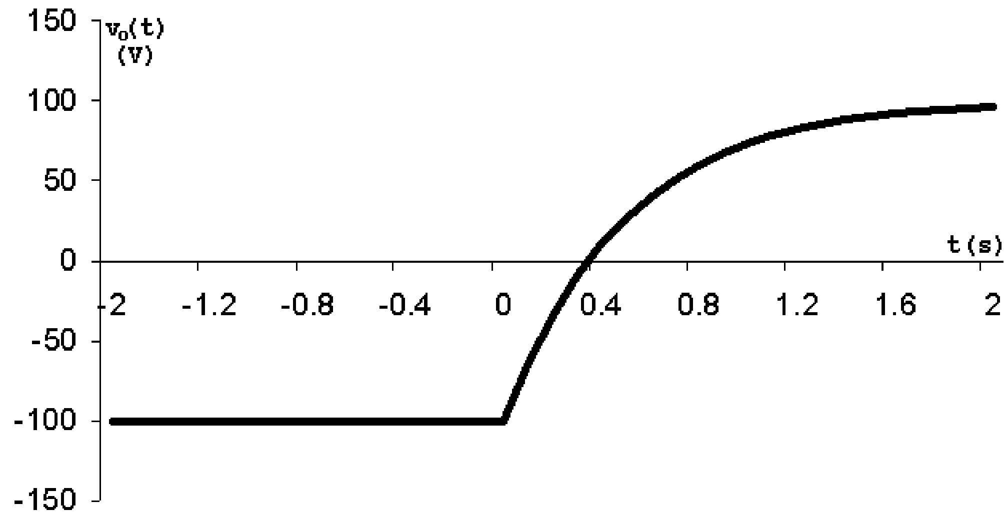
$$V_g(\omega) = \frac{200}{j\omega}$$

Then,

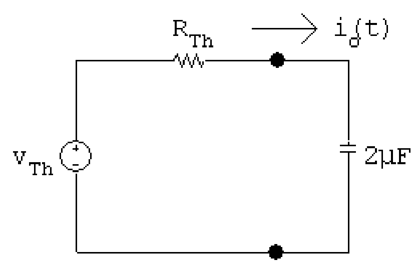
$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{400}{j\omega(j\omega + 2)} = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 2} = \frac{200}{j\omega} - \frac{200}{j\omega + 2}$$

$$\therefore \qquad v_o(t) = 100\text{sgn}(t) - 200e^{-2t}u(t)\,\text{V}$$

[b]



P 17.22 **[a]** Find the Thévenin equivalent with respect to the terminals of the capacitor:



$$v_{\text{Th}} = \frac{5}{6}v_g; \qquad R_{\text{Th}} = 60\|12 = 10\,\text{k}\Omega$$

$$I_o = \frac{V_{\text{Th}}}{10{,}000 + 10^6/2s} = \frac{2sV_{\text{Th}}}{20{,}000s + 10^6}$$

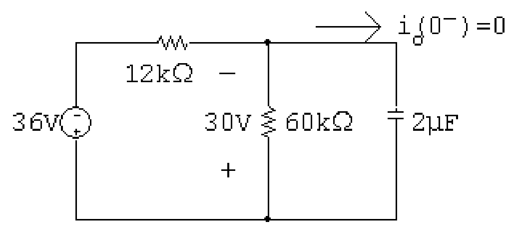
$$H(s) = \frac{I_o}{V_{\text{Th}}} = \frac{10^{-4}s}{s + 50}; \qquad H(\omega) = \frac{j\omega \times 10^{-4}}{j\omega + 50}$$

$$v_{\mathrm{Th}} = \frac{5}{6}v_g = 30\,\mathrm{sgn}(t); \qquad V_{\mathrm{Th}} = \frac{60}{j\omega}$$

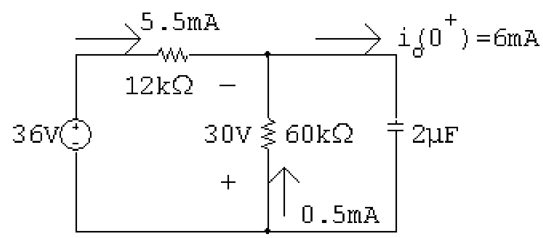
$$I_o = H(\omega)V_{\mathrm{Th}}(\omega) = \left(\frac{60}{j\omega}\right)\left(\frac{j\omega\times 10^{-4}}{j\omega+50}\right) = \frac{6\times 10^{-3}}{j\omega+50}$$

$$i_o(t) = 6e^{-50t}u(t)\,\mathrm{mA}$$

[b] At $t = 0^-$ the circuit is



At $t = 0^+$ the circuit is



$$i_g(0^+) = \frac{30+36}{12} = 5.5\,\mathrm{mA}$$

$$i_{60\mathrm{k}}(0^+) = \frac{30}{60} = 0.5\,\mathrm{mA}$$

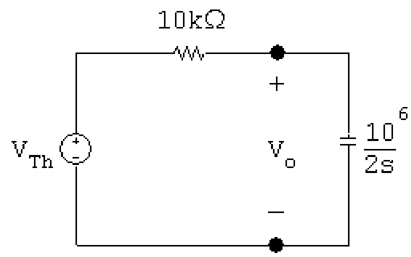
$$i_o(0^+) = 5.5 + 0.5 = 6\,\mathrm{mA}$$

which agrees with our solution.
We also know $i_o(\infty) = 0$, which agrees with our solution.
The time constant with respect to the terminals of the capacitor is $R_{\mathrm{Th}}C$ Thus,

$$\tau = (10{,}000)(2\times 10^{-6}) = 20\,\mathrm{ms}; \qquad \therefore\ \frac{1}{\tau} = 50,$$

which also agrees with our solution.
Thus our solution makes sense in terms of known circuit behavior.

P 17.23 **[a]** From the solution of Problem 17.22 we have



$$V_o = \frac{V_{\text{Th}}}{10^4 + (10^6/2s)} \cdot \frac{10^6}{2s}$$

$$H(s) = \frac{V_o}{V_{\text{Th}}} = \frac{50}{s + 50}$$

$$H(j\omega) = \frac{50}{j\omega + 50}$$

$$V_{\text{Th}}(\omega) = \frac{60}{j\omega}$$

$$V_o(\omega) = H(j\omega)V_{\text{Th}}(\omega) = \left(\frac{60}{j\omega}\right)\frac{50}{j\omega + 50}$$

$$= \frac{3000}{(j\omega)(j\omega + 50)} = \frac{60}{j\omega} - \frac{60}{j\omega + 50}$$

$$v_o(t) = 30\text{sgn}(t) - 60e^{-50t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -30\,\text{V}$

$$v_o(0^+) = 30 - 60 = -30\,\text{V}$$

This makes sense because there cannot be an instantaneous change in the voltage across a capacitor.

$$v_o(\infty) = 30\,\text{V}$$

This agrees with $v_{\text{Th}}(\infty) = 30$ V.
As in Problem 17.22 we know the time constant is 20 ms.

P 17.24 **[a]** $\dfrac{V_o}{V_g} = H(s) = \dfrac{4/s}{0.5 + 0.01s + 4/s}$

$$H(s) = \frac{400}{s^2 + 50s + 400} = \frac{400}{(s + 10)(s + 40)}$$

$$H(j\omega) = \frac{400}{(j\omega + 10)(j\omega + 40)}$$

$$V_g(\omega) = \frac{6}{j\omega}$$

$$V_o(\omega) = V_g(\omega)H(j\omega) = \frac{2400}{j\omega(j\omega+10)(j\omega+40)}$$

$$V_o(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega+10} + \frac{K_3}{j\omega+40}$$

$$K_1 = \frac{2400}{400} = 6; \qquad K_2 = \frac{2400}{(-10)(30)} = -8$$

$$K_3 = \frac{2400}{(-40)(-30)} = 2$$

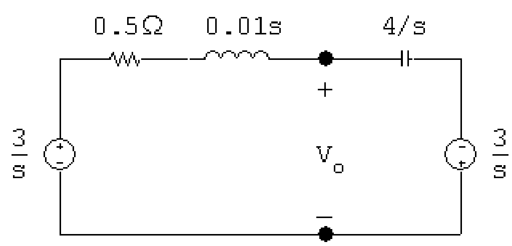
$$V_o(\omega) = \frac{6}{j\omega} - \frac{8}{j\omega+10} + \frac{2}{j\omega+40}$$

$$v_o(t) = 3\text{sgn}(t) - 8e^{-10t}u(t) + 2e^{-40t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -3\,\text{V}$

[c] $v_o(0^+) = 3 - 8 + 2 = -3\,\text{V}$

[d] For $t \geq 0^+$:



$$\frac{V_o - 3/s}{0.5 + 0.01s} + \frac{(V_o + 3/s)s}{4} = 0$$

$$V_o\left[\frac{100}{s+50} + \frac{s}{4}\right] = \frac{300}{s(s+50)} - 0.75$$

$$V_o = \frac{1200 - 3s^2 - 150s}{s(s+10)(s+40)} = \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+40}$$

$$K_1 = \frac{1200}{400} = 3; \qquad K_2 = \frac{1200 - 300 + 1500}{(-10)(30)} = -8$$

$$K_3 = \frac{1200 - 4800 + 6000}{(-40)(-30)} = 2$$

$$v_o(t) = (3 - 8e^{-10t} + 2e^{-40t})u(t)\,\text{V}$$

[e] Yes.

P 17.25 **[a]** $I_o = \dfrac{V_g}{0.5 + 0.01s + 4/s}$

$$H(s) = \frac{I_o}{V_g} = \frac{100s}{s^2 + 50s + 400} = \frac{100s}{(s+10)(s+40)}$$

$$H(\omega) = \frac{100(j\omega)}{(j\omega + 10)(j\omega + 40)}$$

$$V_g(\omega) = \frac{6}{j\omega}$$

$$I_o(\omega) = H(\omega)V_g(\omega) = \frac{600}{(j\omega + 10)(j\omega + 40)}$$

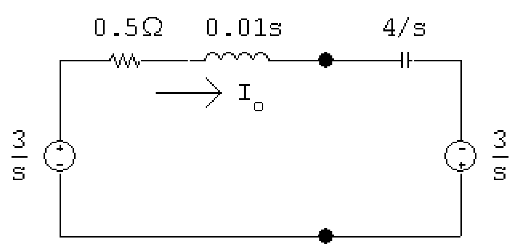
$$= \frac{20}{j\omega + 10} - \frac{20}{j\omega + 40}$$

$$i_o(t) = (20e^{-10t} - 20e^{-40t})u(t)\,\text{A}$$

[b] $i_o(0^-) = 0$

[c] $i_o(0^+) = 0$

[d]



$$I_o = \frac{6/s}{0.5 + 0.01s + 4/s} = \frac{600}{s^2 + 50s + 400}$$

$$= \frac{600}{(s+10)(s+40)} = \frac{20}{s+10} - \frac{20}{s+40}$$

$$i_o(t) = (20e^{-10t} - 20e^{-40t})u(t)\,\text{A}$$

[e] Yes.

P 17.26 **[a]** $I_o = \dfrac{I_g R}{R + 1/sC} = \dfrac{RCsI_g}{RCs + 1}; \qquad H(s) = \dfrac{I_o}{I_g} = \dfrac{s}{s + 1/RC}$

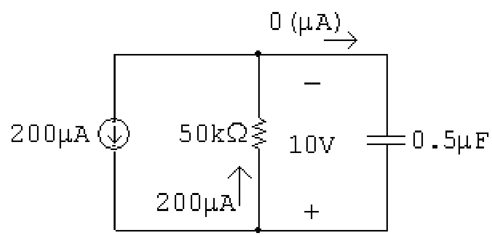
$$\frac{1}{RC} = \frac{10^6}{25 \times 10^3} = 40; \qquad H(\omega) = \frac{j\omega}{j\omega + 40}$$

$$i_g = 200\text{sgn}(t)\,\mu\text{A}; \qquad I_g = (200 \times 10^{-6})\left(\frac{2}{j\omega}\right) = \frac{400 \times 10^{-6}}{j\omega}$$

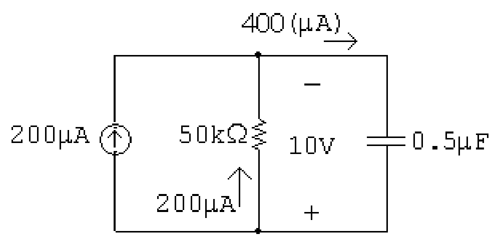
$$I_o = I_g[H(\omega)] = \frac{400 \times 10^{-6}}{j\omega} \cdot \frac{j\omega}{j\omega + 40} = \frac{400 \times 10^{-6}}{j\omega + 40}$$

$$i_o(t) = 400e^{-40t}u(t)\,\mu\text{A}$$

[b] Yes, at the time the source current jumps from $-200\,\mu$A to $+200\,\mu$A the capacitor is charged to $(200)(50) \times 10^{-3} = 10$ V, positive at the lower terminal. The circuit at $t = 0^-$ is



At $t = 0^+$ the circuit is



The time constant is $(50 \times 10^3)(0.5 \times 10^{-6}) = 25$ ms.

$$\therefore \quad \frac{1}{\tau} = 40 \quad \therefore \quad \text{for } t > 0, \quad i_o = 400e^{-40t}\,\mu\text{A}$$

P 17.27 **[a]** $V_o = \dfrac{I_g R(1/sC)}{R + (1/sC)} = \dfrac{I_g R}{RCs + 1}$

$$H(s) = \frac{V_o}{I_g} = \frac{1/C}{s + (1/RC)} = \frac{2 \times 10^6}{s + 40}$$

$$H(\omega) = \frac{2 \times 10^6}{40 + j\omega}; \qquad I_g(\omega) = \frac{400 \times 10^{-6}}{j\omega}$$

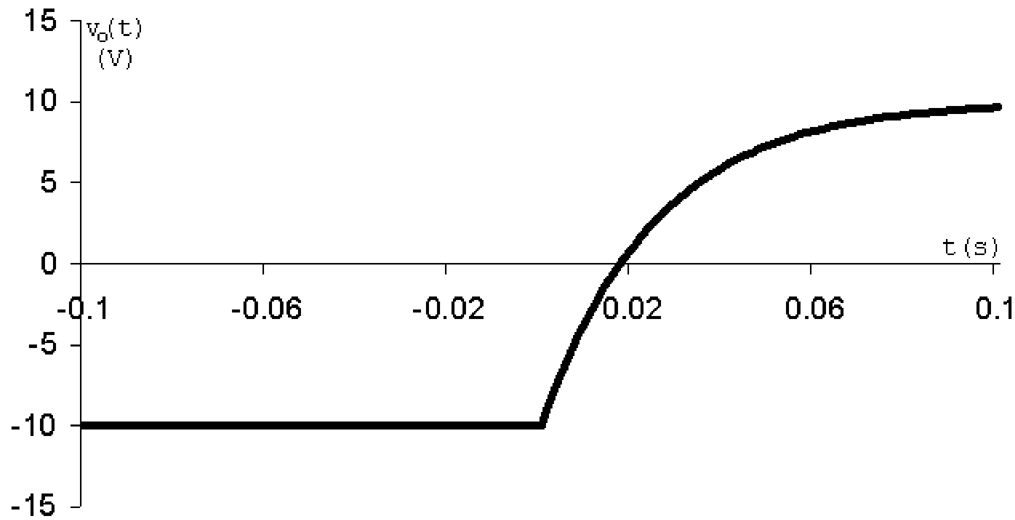
$$V_o(\omega) = H(\omega)I_g(\omega) = \left(\frac{400 \times 10^{-6}}{j\omega}\right)\left(\frac{2 \times 10^6}{40 + j\omega}\right)$$

$$= \frac{800}{j\omega(40 + j\omega)} = \frac{20}{j\omega} - \frac{20}{40 + j\omega}$$

$$v_o(t) = 10\text{sgn}(t) - 20e^{-40t}u(t)\text{ V}$$

[b] Yes, at the time the current source jumps from -200 to $+200\,\mu$A the capacitor is charged to -10 V. That is, at $t = 0^-$, $v_o(0^-) = (50 \times 10^3)(-200 \times 10^{-6}) = -10$ V.

At $t = \infty$ the capacitor will be charged to $+10$ V. That is,
$v_o(\infty) = (50 \times 10^3)(200 \times 10^{-6}) = 10$ V
The time constant of the circuit is $(50 \times 10^3)(0.5 \times 10^{-6}) = 25$ ms, so $1/\tau = 40$. The function $v_o(t)$ is plotted below:



P 17.28 **[a]** $i_g = 3e^{-5|t|}$

$$\therefore \qquad I_g(\omega) = \frac{3}{j\omega + 5} + \frac{3}{-j\omega + 5} = \frac{30}{(j\omega + 5)(-j\omega + 5)}$$

$$\frac{V_o}{10} + \frac{V_o s}{10} = I_g$$

$$\therefore \qquad \frac{V_o}{I_g} = H(s) = \frac{10}{s+1}; \qquad H(\omega) = \frac{10}{j\omega + 1}$$

$$V_o(\omega) = I_g(\omega)H(\omega) = \frac{300}{(j\omega + 1)(j\omega + 5)(-j\omega + 5)}$$

$$= \frac{K_1}{j\omega + 1} + \frac{K_2}{j\omega + 5} + \frac{K_3}{-j\omega + 5}$$

$$K_1 = \frac{300}{(4)(6)} = 12.5$$

$$K_2 = \frac{300}{(-4)(10)} = -7.5$$

$$K_3 = \frac{300}{(6)(10)} = 5$$

$$V_o(\omega) = \frac{12.5}{j\omega + 1} - \frac{7.5}{j\omega + 5} + \frac{5}{-j\omega + 5}$$

$$v_o(t) = [12.5e^{-t} - 7.5e^{-5t}]u(t) + 5e^{5t}u(-t)\ \text{V}$$

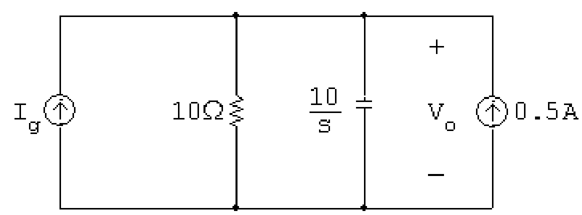
[b] $v_o(0^-) = 5\,\text{V}$

[c] $v_o(0^+) = 12.5 - 7.5 = 5\,\text{V}$

[d] $i_g = 3e^{-5t}u(t), \quad t \geq 0^+$

$$I_g = \frac{3}{s+5}; \qquad H(s) = \frac{10}{s+1}$$

$$v_o(0^+) = 5\,\text{V}; \qquad \gamma C = 0.5$$



$$\frac{V_o}{10} + \frac{V_o s}{10} = I_g + 0.5$$

$$V_o(s+1) = \frac{30}{s+5} + 5$$

$$V_o = \frac{30}{(s+5)(s+1)} + \frac{5}{s+1}$$

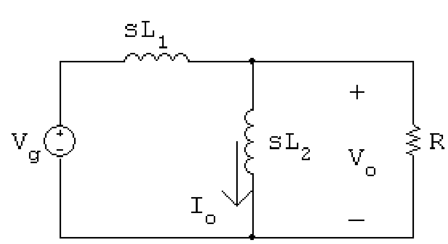
$$= \frac{-7.5}{s+5} + \frac{7.5}{s+1} + \frac{5}{s+1} = \frac{12.5}{s+1} - \frac{7.5}{s+5}$$

$$\therefore \quad v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\,\text{V}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\,\text{V}$$

P 17.29 **[a]**



$$\frac{V_o - V_g}{sL_1} + \frac{V_o}{sL_2} + \frac{V_o}{R} = 0$$

$$\therefore \quad V_o = \frac{RV_g}{L_1\left[s + R\left(\frac{1}{L_1} + \frac{1}{L_2}\right)\right]}$$

$$I_o = \frac{V_o}{sL_2}$$

$$\therefore \quad \frac{I_o}{V_g} = H(s) = \frac{R/L_1L_2}{s(s + R[(1/L_1) + (1/L_2)])}$$

$$\frac{R}{L_1L_2} = 12 \times 10^5$$

$$R\left(\frac{1}{L_1} + \frac{1}{L_2}\right) = 3 \times 10^4$$

$$\therefore \quad H(s) = \frac{12 \times 10^5}{s(s + 3 \times 10^4)}$$

$$H(\omega) = \frac{12 \times 10^5}{j\omega(j\omega + 3 \times 10^4)}$$

$$V_g(\omega) = 125\pi[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]$$

$$I_o(\omega) = H(\omega)V_g(\omega) = \frac{1500\pi \times 10^5[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]}{j\omega(j\omega + 3 \times 10^4)}$$

$$i_o(t) = \frac{1500\pi \times 10^5}{2\pi}\int_{-\infty}^{\infty} \frac{[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]e^{jt\omega}}{j\omega(j\omega + 3 \times 10^4)}\, d\omega$$

$$i_o(t) = 750 \times 10^5 \left\{\frac{e^{-j40{,}000t}}{-j40{,}000(30{,}000 - j40{,}000)} + \frac{e^{j40{,}000t}}{j40{,}000(30{,}000 + j40{,}000)}\right\}$$

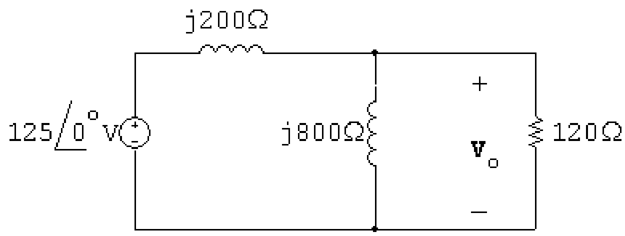
$$= \frac{75 \times 10^6}{4 \times 10^8}\left\{\frac{e^{-j40{,}000t}}{-j(3 - j4)} + \frac{e^{j40{,}000t}}{j(3 + j4)}\right\}$$

$$= \frac{75}{400}\left\{\frac{e^{-j40{,}000t}}{5\underline{/-143.13^\circ}} + \frac{e^{j40{,}000t}}{5\underline{/143.13^\circ}}\right\}$$

$$= 0.075\cos(40{,}000t - 143.13^\circ)\ \text{A}$$

$$i_o(t) = 75\cos(40{,}000t - 143.13^\circ)\ \text{mA}$$

[b] In the phasor domain:



$$\frac{\mathbf{V}_o - 125}{j200} + \frac{\mathbf{V}_o}{j800} + \frac{\mathbf{V}_o}{120} = 0$$

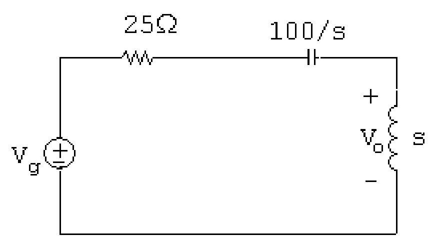
$$12\mathbf{V}_o - 1500 + 3\mathbf{V}_o + j20\mathbf{V}_o = 0$$

$$\mathbf{V}_o = \frac{1500}{15 + j20} = 60\underline{/-53.13^\circ}\text{ V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{j800} = 75 \times 10^{-3}\underline{/-143.13^\circ}\text{ A}$$

$$i_o(t) = 75\cos(40{,}000t - 143.13^\circ)\text{ mA}$$

P 17.30 **[a]**



$$V_o = \frac{V_g s}{25 + (100/s) + s} = \frac{V_g s^2}{s^2 + 25s + 100}$$

$$H(s) = \frac{V_o}{V_g} = \frac{s^2}{(s+5)(s+20)}; \qquad H(\omega) = \frac{(j\omega)^2}{(j\omega+5)(j\omega+20)}$$

$$v_g = 25i_g = -450e^{10t}u(-t) - 450e^{-10t}u(t)\text{ V}$$

$$V_g = -\frac{450}{-j\omega + 10} - \frac{450}{j\omega + 10}$$

$$V_o(\omega) = H(\omega)V_g = \frac{-450(j\omega)^2}{(-j\omega+10)(j\omega+5)(j\omega+20)}$$

$$+ \frac{-450(j\omega)^2}{(j\omega+10)(j\omega+5)(j\omega+20)}$$

$$= \frac{K_1}{-j\omega+10} + \frac{K_2}{j\omega+5} + \frac{K_3}{j\omega+20} + \frac{K_4}{j\omega+5} + \frac{K_5}{j\omega+10} + \frac{K_6}{j\omega+20}$$

$$K_1 = \frac{450(100)}{(15)(30)} = -100 \qquad K_4 = \frac{-450(25)}{(5)(15)} = -150$$

$$K_2 = \frac{450(25)}{(15)(15)} = -50 \qquad K_5 = \frac{-450(100)}{(-5)(10)} = 900$$

$$K_3 = \frac{450(400)}{(30)(-15)} = 400 \qquad K_6 = \frac{-450(400)}{(-15)(-10)} = -1200$$

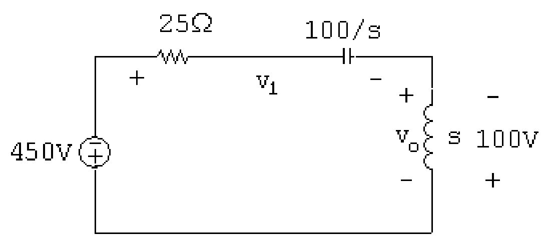
$$V_o(\omega) = \frac{-100}{-j\omega + 10} + \frac{-200}{j\omega + 5} + \frac{-800}{j\omega + 20} + \frac{900}{j\omega + 10}$$

$$v_o = -100e^{10t}u(-t) + [900e^{-10t} - 200e^{-5t} - 800e^{-20t}]u(t)\text{ V}$$

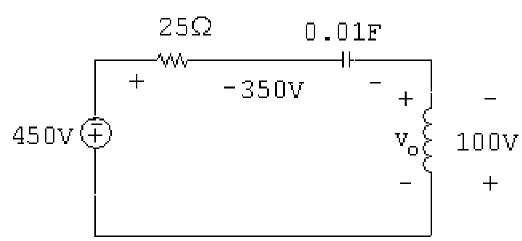
[b] $v_o(0^-) = -100\text{ V}$

[c] $v_o(0^+) = 900 - 200 - 800 = -100\text{ V}$

[d] At $t = 0^-$ the circuit is

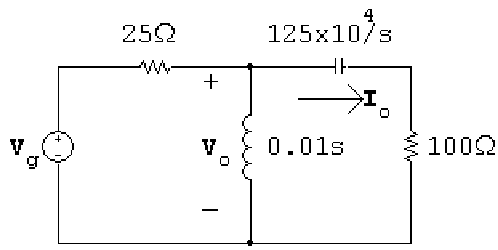


Therefore, the solution predicts $v_1(0^-)$ will be -350 V.
Now $v_1(0^+) = v_1(0^-)$ because the inductor will not let the current in the $25\,\Omega$ resistor change instantaneously, and the capacitor will not let the voltage across the 0.01 F capacitor change instantaneously.
At $t = 0^+$ the circuit is



From the circuit at $t = 0^+$ we see that v_o must be -100 V, which is consistent with the solution for v_o obtained in part (c).

P 17.31



$$\frac{V_o - V_g}{25} + \frac{100V_o}{s} + \frac{V_o s}{100s + 125 \times 10^4} = 0$$

$$\therefore \quad V_o = \frac{s(100s + 125 \times 10^4)V_g}{125(s^2 + 12{,}000s + 25 \times 10^6)}$$

$$I_o = \frac{sV_o}{100s + 125 \times 10^4}$$

$$H(s) = \frac{I_o}{V_g} = \frac{s^2}{125(s^2 + 12{,}000s + 25 \times 10^6)}$$

$$H(\omega) = \frac{-8 \times 10^{-3}\omega^2}{(25 \times 10^6 - \omega^2) + j12{,}000\omega}$$

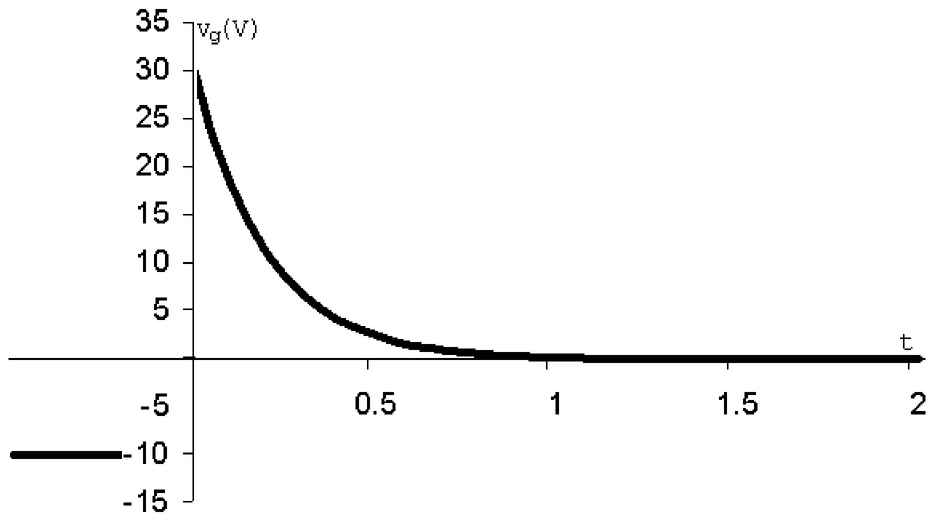
$$V_g(\omega) = 300\pi[\delta(\omega + 5000) + \delta(\omega - 5000)]$$

$$I_o(\omega) = H(\omega)V_g(\omega) = \frac{-2.4\pi\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12{,}000\omega}$$

$$\begin{aligned} i_o(t) &= \frac{-2.4\pi}{2\pi}\int_{-\infty}^{\infty} \frac{\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12{,}000\omega} e^{jt\omega}\, d\omega \\ &= -1.2\left\{\frac{25 \times 10^6 e^{-j5000t}}{-j(12{,}000)(5000)} + \frac{25 \times 10^6 e^{j5000t}}{j(12{,}000)(5000)}\right\} \\ &= \frac{6}{12}\left\{\frac{e^{-j5000t}}{-j} + \frac{e^{j5000t}}{j}\right\} \\ &= 0.5[e^{-j(5000t+90^\circ)} + e^{j(5000t+90^\circ)}] \end{aligned}$$

$$i_o(t) = 1\cos(5000t + 90^\circ)\,\text{A}$$

P 17.32 **[a]**



From the plot of v_g note that v_g is -10 V for an infinitely long time before $t = 0$. Therefore

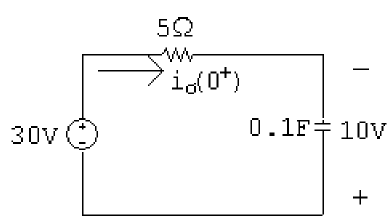
$$\therefore \qquad v_o(0^-) = -10\,\text{V}$$

There cannot be an instantaneous change in the voltage across a capacitor, so

$$v_o(0^+) = -10\,\text{V}$$

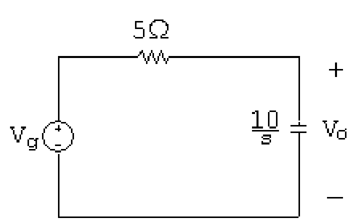
[b] $i_o(0^-) = 0\,\text{A}$

At $t = 0^+$ the circuit is



$$i_o(0^+) = \frac{30 - (-10)}{5} = \frac{40}{5} = 8\,\text{A}$$

[c] The s-domain circuit is



$$V_o = \left[\frac{V_g}{5 + (10/s)}\right]\left(\frac{10}{s}\right) = \frac{2V_g}{s+2}$$

$$\frac{V_o}{V_g} = H(s) = \frac{2}{s+2}$$

$$H(\omega) = \frac{2}{j\omega + 2}$$

$$V_g(\omega) = 5\left(\frac{2}{j\omega}\right) - 5[2\pi\delta(\omega)] + \frac{30}{j\omega + 5} = \frac{10}{j\omega} - 10\pi\delta(\omega) + \frac{30}{j\omega + 5}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{2}{j\omega + 2}\left[\frac{10}{j\omega} - 10\pi\delta(\omega) + \frac{30}{j\omega + 5}\right]$$

$$= \frac{20}{j\omega(j\omega + 2)} - \frac{20\pi\delta(\omega)}{j\omega + 2} + \frac{60}{(j\omega + 2)(j\omega + 5)}$$

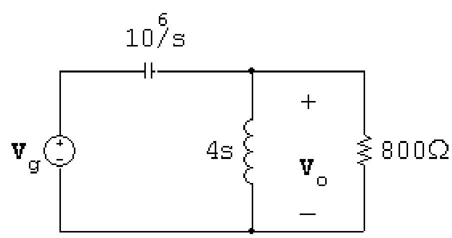
$$= \frac{K_0}{j\omega} + \frac{K_1}{j\omega + 2} + \frac{K_2}{j\omega + 2} + \frac{K_3}{j\omega + 5} - \frac{20\pi\delta(\omega)}{j\omega + 2}$$

$$K_0 = \frac{20}{2} = 10; \quad K_1 = \frac{20}{-2} = -10; \quad K_2 = \frac{60}{3} = 20; \quad K_3 = \frac{60}{-3} = -20$$

$$V_o(\omega) = \frac{10}{j\omega} + \frac{10}{j\omega + 2} - \frac{20}{j\omega + 5} - \frac{20\pi\delta(\omega)}{j\omega + 2} = \frac{10}{j\omega} + \frac{10}{j\omega + 2} + \frac{20}{j\omega + 5} - 10\pi\delta(\omega)$$

$$v_o(t) = 5\text{sgn}(t) + [10e^{-2t} - 20e^{-5t}]u(t) - 5\text{ V}$$

P 17.33 **[a]**



$$\frac{(V_o - V_g)s}{10^6} + \frac{V_o}{4s} + \frac{V_o}{800} = 0$$

$$\therefore \quad V_o = \frac{s^2 V_g}{s^2 + 1250s + 25 \times 10^4}$$

$$\frac{V_o}{V_g} = H(s) = \frac{s^2}{(s + 250)(s + 1000)}$$

$$H(\omega) = \frac{(j\omega)^2}{(j\omega + 250)(j\omega + 1000)}$$

$$v_g = 45e^{-500|t|}; \qquad V_g(\omega) = \frac{45{,}000}{(j\omega + 500)(-j\omega + 500)}$$

$$\therefore \quad V_o(\omega) = H(\omega)V_g(\omega) = \frac{45{,}000(j\omega)^2}{(j\omega + 250)(j\omega + 500)(j\omega + 1000)(-j\omega + 500)}$$

$$= \frac{K_1}{j\omega + 250} + \frac{K_2}{j\omega + 500} + \frac{K_3}{j\omega + 1000} + \frac{K_4}{-j\omega + 500}$$

$$K_1 = \frac{45{,}000(-250)^2}{(250)(750)(750)} = 20$$

$$K_2 = \frac{45{,}000(-500)^2}{(-250)(500)(1000)} = -90$$

$$K_3 = \frac{45{,}000(-1000)^2}{(-750)(-500)(1500)} = 80$$

$$K_4 = \frac{45{,}000(500)^2}{(750)(1000)(1500)} = 10$$

$$\therefore \quad v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t) + 10e^{500t}u(-t)\,\text{V}$$

[b] $v_o(0^-) = 10\,\text{V}; \qquad V_o(0^+) = 20 - 90 + 80 = 10\,\text{V}$

$$v_o(\infty) = 0\,\text{V}$$

[c] $I_L = \dfrac{V_o}{4s} = \dfrac{0.25sV_g}{(s+250)(s+1000)}$

$$H(s) = \frac{I_L}{V_g} = \frac{0.25s}{(s+250)(s+1000)}$$

$$H(\omega) = \frac{0.25(j\omega)}{(j\omega+250)(j\omega+1000)}$$

$$I_L(\omega) = \frac{0.25(j\omega)(45{,}000)}{(j\omega+250)(j\omega+500)(j\omega+1000)(-j\omega+500)}$$

$$= \frac{K_1}{j\omega+250} + \frac{K_2}{j\omega+500} + \frac{K_3}{j\omega+1000} + \frac{K_4}{-j\omega+500}$$

$$K_4 = \frac{(0.25)(500)(45{,}000)}{(750)(1000)(1500)} = 5\,\text{mA}$$

$$i_L(t) = 5e^{500t}u(-t); \qquad \therefore \quad i_L(0^-) = 5\,\text{mA}$$

$$K_1 = \frac{(0.25)(-250)(45{,}000)}{(250)(750)(750)} = -20\,\text{mA}$$

$$K_2 = \frac{(0.25)(-500)(45{,}000)}{(-250)(500)(1000)} = 45\,\text{mA}$$

$$K_3 = \frac{(0.25)(-1000)(45{,}000)}{(-750)(-500)(1500)} = -20\,\text{mA}$$

$$\therefore \quad i_L(0^+) = K_1 + K_2 + K_3 = -20 + 45 - 20 = 5\,\text{mA}$$

Checks, i.e., $\quad i_L(0^+) = i_L(0^-) = 5\,\text{mA}$

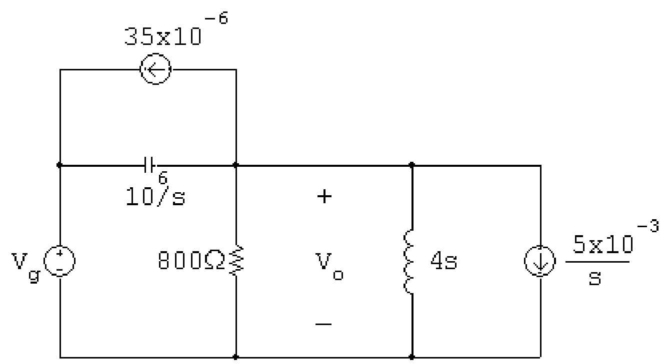
At $t = 0^-$:

$v_C(0^-) = 45 - 10 = 35\,\text{V}$

At $t = 0^+$:

$v_C(0^+) = 45 - 10 = 35\,\text{V}$

[d] We can check the correctness of our solution for $t \geq 0^+$ by using the Laplace transform. Our circuit becomes



$$\frac{V_o}{800} + \frac{V_o}{4s} + \frac{(V_o - V_g)s}{10^6} + 35 \times 10^{-6} + \frac{5 \times 10^{-3}}{s} = 0$$

$$\therefore \quad (s^2 + 1250s + 25 \times 10^4)V_o = s^2 V_g - (35s + 5000)$$

$$v_g(t) = 45e^{-500t}u(t)\,\text{V}; \qquad V_g = \frac{45}{s + 500}$$

$$\therefore \quad (s + 250)(s + 1000)V_o = \frac{45s^2 - (35s + 5000)(s + 500)}{(s + 500)}$$

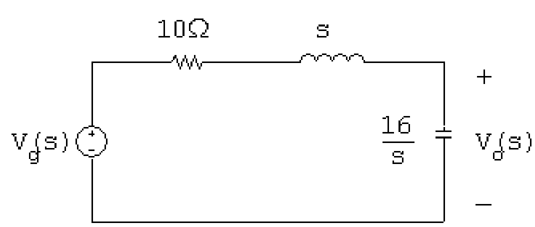
$$\therefore \quad V_o = \frac{10s^2 - 22{,}500s - 250 \times 10^4}{(s + 250)(s + 500)(s + 1000)}$$

$$= \frac{20}{s + 250} - \frac{90}{s + 500} + \frac{80}{s + 1000}$$

$$\therefore \quad v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t)\,\text{V}$$

This agrees with our solution for $v_o(t)$ for $t \geq 0^+$.

P 17.34 **[a]**



$$V_g(\omega) = \frac{36}{4 - j\omega} - \frac{36}{4 + j\omega} = \frac{72j\omega}{(4 - j\omega)(4 + j\omega)}$$

$$V_o(s) = \frac{(16/s)}{10 + s + (16/s)} V_g(s)$$

$$H(s) = \frac{V_o(s)}{V_g(s)} = \frac{16}{s^2 + 10s + 16} = \frac{16}{(s+2)(s+8)}$$

$$H(\omega) = \frac{16}{(j\omega + 2)(j\omega + 8)}$$

$$V_o(\omega) = H(\omega) \cdot V_g(\omega) = \frac{1152 j\omega}{(4 - j\omega)(4 + j\omega)(2 + j\omega)(8 + j\omega)}$$

$$= \frac{K_1}{4 - j\omega} + \frac{K_2}{4 + j\omega} + \frac{K_3}{2 + j\omega} + \frac{K_4}{8 + j\omega}$$

$$K_1 = \frac{1152(4)}{(8)(6)(12)} = 8$$

$$K_2 = \frac{1152(-4)}{(8)(-2)(4)} = 72$$

$$K_3 = \frac{1152(-2)}{(6)(2)(6)} = -32$$

$$K_4 = \frac{1152(-8)}{(12)(-4)(-6)} = -32$$

$$\therefore \quad V_o(j\omega) = \frac{8}{4 - j\omega} + \frac{72}{4 + j\omega} - \frac{32}{2 + j\omega} - \frac{32}{8 + j\omega}$$

$$\therefore \quad v_o(t) = 8e^{4t}u(-t) + [72e^{-4t} - 32e^{-2t} - 32e^{-8t}]u(t)\text{V}$$

[b] $v_o(0^-) = 8$ V

[c] $v_o(0^+) = 72 - 32 - 32 = 8$ V

The voltages at 0^- and 0^+ must be the same since the voltage cannot change instantaneously across a capacitor.

P 17.35 $$V_o(s) = \frac{10}{s} + \frac{30}{s + 20} - \frac{40}{s + 30} = \frac{600(s + 10)}{s(s + 20)(s + 30)}$$

$$V_o(s) = H(s) \cdot \frac{15}{s}$$

$$\therefore \qquad H(s) = \frac{40(s + 10)}{(s + 20)(s + 30)}$$

$$\therefore \qquad H(\omega) = \frac{40(j\omega + 10)}{(j\omega + 20)(j\omega + 30)}$$

$$\therefore \qquad V_o(\omega) = \frac{30}{j\omega} \cdot \frac{40(j\omega+10)}{(j\omega+20)(j\omega+30)} = \frac{1200(j\omega+10)}{j\omega(j\omega+20)(j\omega+30)}$$

$$v_o(\omega) = \frac{20}{j\omega} + \frac{60}{j\omega+20} - \frac{80}{j\omega+30}$$

$$v_o(t) = 10\text{sgn}(t) + [60e^{-20t} - 80e^{-30t}]u(t)\ \text{V}$$

P 17.36 **[a]** $f(t) = \frac{1}{2\pi}\left\{\int_{-\infty}^{0} e^{\omega}e^{jt\omega}\,d\omega + \int_0^{\infty} e^{-\omega}e^{jt\omega}\,d\omega\right\} = \frac{1/\pi}{1+t^2}$

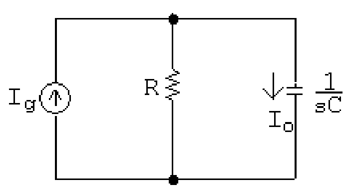
[b] $W = 2\int_0^{\infty} \frac{(1/\pi)^2}{(1+t^2)^2}\,dt = \frac{2}{\pi^2}\int_0^{\infty}\frac{dt}{(1+t^2)^2} = \frac{1}{2\pi}$ J

[c] $W = \frac{1}{\pi}\int_0^{\infty} e^{-2\omega}\,d\omega = \frac{1}{\pi}\frac{e^{-2\omega}}{-2}\Big|_0^{\infty} = \frac{1}{2\pi}$ J

[d] $\frac{1}{\pi}\int_0^{\omega_1} e^{-2\omega}\,d\omega = \frac{0.9}{2\pi}, \qquad 1 - e^{-2\omega_1} = 0.9, \qquad e^{2\omega_1} = 10$

$\omega_1 = (1/2)\ln 10 \cong 1.15$ rad/s

P 17.37



$$I_o = \frac{I_gR}{R + (1/sC)} = \frac{RCsI_g}{RCs+1}$$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s + (1/RC)}$$

$$RC = (100\times10^3)(1.25\times10^{-6}) = 125\times10^{-3}; \qquad \frac{1}{RC} = \frac{1}{0.125} = 8$$

$$H(s) = \frac{s}{s+8}; \qquad H(\omega) = \frac{j\omega}{j\omega+8}$$

$$I_g(\omega) = \frac{30\times10^{-6}}{j\omega+2}$$

$$I_o(\omega) = H(\omega)I_g(\omega) = \frac{30\times10^{-6}j\omega}{(j\omega+2)(j\omega+8)}$$

$$|I_o(\omega)| = \frac{\omega(30\times 10^{-6})}{(\sqrt{\omega^2+4})(\sqrt{\omega^2+64})}$$

$$|I_o(\omega)|^2 = \frac{900\times 10^{-12}\omega^2}{(\omega^2+4)(\omega^2+64)} = \frac{K_1}{\omega^2+4} + \frac{K_2}{\omega^2+64}$$

$$K_1 = \frac{(900\times 10^{-12})(-4)}{(60)} = -60\times 10^{-12}$$

$$K_2 = \frac{(900\times 10^{-12})(-64)}{(-60)} = 960\times 10^{-12}$$

$$|I_o(\omega)|^2 = \frac{960\times 10^{-12}}{\omega^2+64} - \frac{60\times 10^{-12}}{\omega^2+4}$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^\infty |I_o(\omega)|^2\,d\omega = \frac{960\times 10^{-12}}{\pi}\int_0^\infty \frac{d\omega}{\omega^2+64} - \frac{60\times 10^{-12}}{\pi}\int_0^\infty \frac{d\omega}{\omega^2+4}$$

$$= \frac{120\times 10^{-12}}{\pi}\tan^{-1}\frac{\omega}{8}\Big|_0^\infty - \frac{30\times 10^{-12}}{\pi}\tan^{-1}\frac{\omega}{2}\Big|_0^\infty$$

$$= \left(\frac{120}{\pi}\cdot\frac{\pi}{2} - \frac{30}{\pi}\cdot\frac{\pi}{2}\right)\times 10^{-12} = (60-15)\times 10^{-12} = 45\,\text{pJ}$$

Between 0 and 4 rad/s

$$W_{1\Omega} = \left[\frac{120}{\pi}\tan^{-1}\frac{1}{2} - \frac{30}{\pi}\tan^{-1}2\right]\times 10^{-12} = 7.14\,\text{pJ}$$

$$\% = \frac{7.14}{45}(100) = 15.86\%$$

P 17.38 **[a]** $V_g(\omega) = \dfrac{60}{(j\omega+1)(-j\omega+1)}$

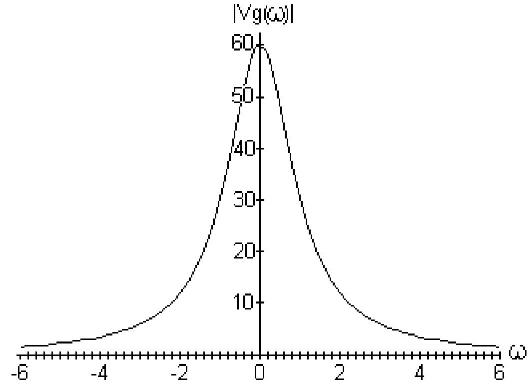
$$H(s) = \frac{V_o}{V_g} = \frac{0.4}{s+0.5}; \qquad H(\omega) = \frac{0.4}{(j\omega+0.5)}$$

$$V_o(\omega) = \frac{24}{(j\omega+1)(j\omega+0.5)(-j\omega+1)}$$

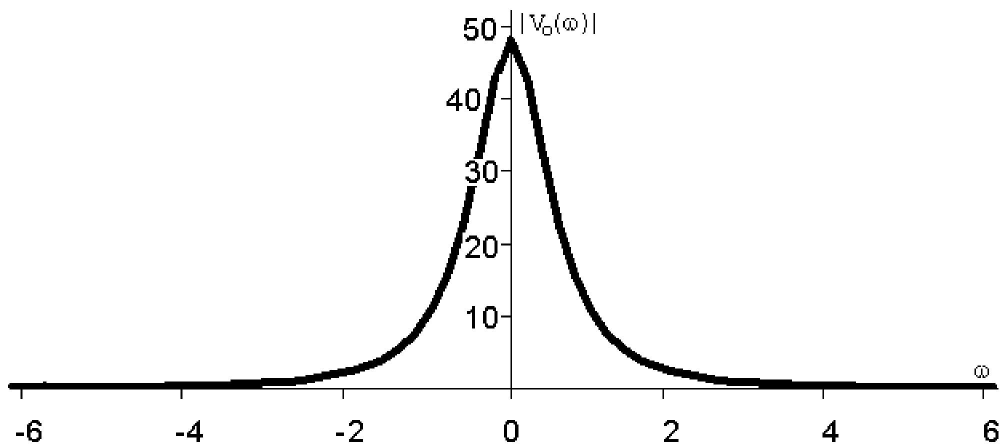
$$V_o(\omega) = \frac{-24}{j\omega+1} + \frac{32}{j\omega+0.5} + \frac{8}{-j\omega+1}$$

$$v_o(t) = [-24e^{-t} + 32e^{-t/2}]u(t) + 8e^{t}u(-t)\,\text{V}$$

[b] $|V_g(\omega)| = \dfrac{60}{(\omega^2 + 1)}$



[c] $|V_o(\omega)| = \dfrac{24}{(\omega^2 + 1)\sqrt{\omega^2 + 0.25}}$



[d] $W_i = 2\int_0^{\infty} 900e^{-2t}\,dt = 1800\,\dfrac{e^{-2t}}{-2}\Bigg|_0^{\infty} = 900\,\mathrm{J}$

[e] $W_o = \int_{-\infty}^{0} 64e^{2t}\,dt + \int_0^{\infty} (-24e^{-t} + 32e^{-t/2})^2\,dt$

$$= 32 + \textstyle\int_0^{\infty} [576e^{-2t} - 1536e^{-3t/2} + 1024e^{-t}]\,dt$$

$$= 32 + 288 - 1024 + 1024 = 320\,\mathrm{J}$$

[f] $|V_g(\omega)| = \dfrac{60}{\omega^2+1}, \quad |V_g^2(\omega)| = \dfrac{3600}{(\omega^2+1)^2}$

$$W_g = \frac{3600}{\pi}\int_0^2 \frac{d\omega}{(\omega^2+1)^2}$$

$$= \frac{3600}{\pi}\left\{\frac{1}{2}\left(\frac{\omega}{\omega^2+1} + \tan^{-1}\omega\right)\Big|_0^2\right\}$$

$$= \frac{1800}{\pi}\left(\frac{2}{5} + \tan^{-1} 2\right) = 863.53\,\text{J}$$

$$\therefore \quad \% = \left(\frac{863.53}{900}\right) \times 100 = 95.95\%$$

[g] $|V_o(\omega)|^2 = \dfrac{576}{(\omega^2+1)^2(\omega^2+0.25)}$

$$= \frac{1024}{\omega^2+0.25} - \frac{768}{(\omega^2+1)^2} - \frac{1024}{(\omega^2+1)}$$

$$W_o = \frac{1}{\pi}\left\{1024\cdot 2\cdot \tan^{-1} 2\omega\,\Big|_0^2 - 768\left(\frac{1}{2}\right)\left(\frac{\omega}{\omega^2+1} + \tan^{-1}\omega\right)_0^2\right.$$

$$\left. -1024\tan^{-1}\omega\,\Big|_0^2\right\}$$

$$= \frac{2048}{\pi}\tan^{-1} 4 - \frac{384}{\pi}\left(\frac{2}{5} + \tan^{-1} 2\right) - \frac{1024}{\pi}\tan^{-1} 2$$

$$= 319.2\,\text{J}$$

$$\% = \frac{319.2}{320} \times 100 = 99.75\%$$

P 17.39 $I_o = \dfrac{0.5sI_g}{0.5s+25} = \dfrac{sI_g}{s+50}$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s+50}$$

$$H(\omega) = \frac{j\omega}{j\omega+50}$$

$$I(\omega) = \frac{12}{j\omega+10}$$

$$I_o(\omega) = H(\omega)I(\omega) = \frac{12(j\omega)}{(j\omega+10)(j\omega+50)}$$

$$|I_o(\omega)| = \frac{12\omega}{\sqrt{(\omega^2+100)(\omega^2+2500)}}$$

$$|I_o(\omega)|^2 = \frac{144\omega^2}{(\omega^2+100)(\omega^2+2500)}$$

$$= \frac{-6}{\omega^2+100} + \frac{150}{\omega^2+2500}$$

$$W_o(\text{total}) = \frac{1}{\pi}\int_0^\infty \frac{150d\omega}{\omega^2+2500} - \frac{1}{\pi}\int_0^\infty \frac{6d\omega}{\omega^2+100}$$

$$= \frac{3}{\pi}\tan^{-1}\left(\frac{\omega}{50}\right)\Big|_0^\infty - \frac{0.6}{\pi}\tan^{-1}\left(\frac{\omega}{10}\right)\Big|_0^\infty$$

$$= 1.5 - 0.3 = 1.2\,\text{J}$$

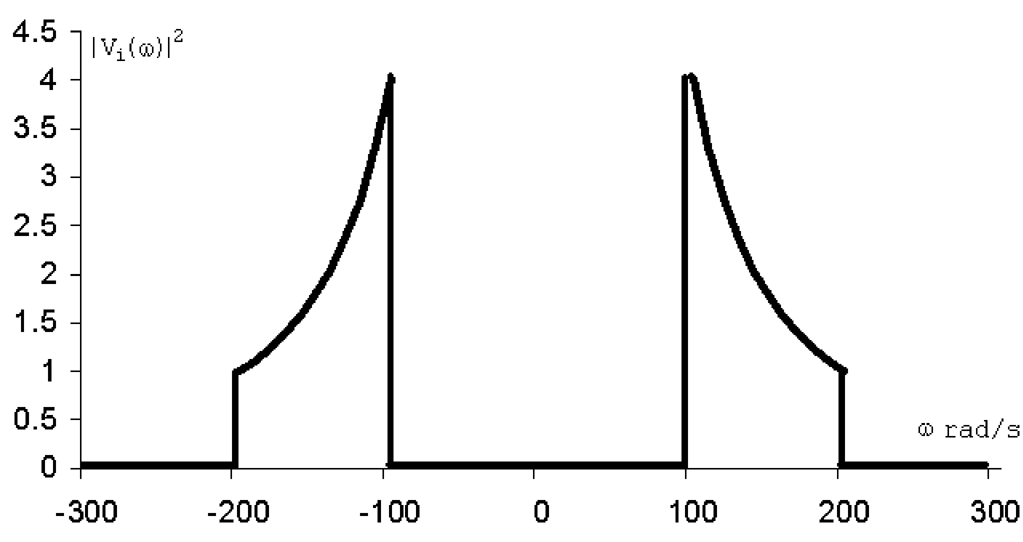
$$W_o(\text{0–100 rad/s}) = \frac{3}{\pi}\tan^{-1}(2) - \frac{0.6}{\pi}\tan^{-1}(10)$$

$$= 1.06 - 0.28 = 0.78\,\text{J}$$

Therefore, the percent between 0 and 100 rad/s is

$$\frac{0.78}{1.2}(100) = 64.69\%$$

P 17.40 **[a]** $|V_i(\omega)|^2 = \frac{4\times10^4}{\omega^2}$; $\quad |V_i(100)|^2 = \frac{4\times10^4}{100^2} = 4$; $\quad |V_i(200)|^2 = \frac{4\times10^4}{200^2} = 1$



[b] $V_o = \frac{V_i R}{R + (1/sC)} = \frac{RCV_i}{RCs + 1}$

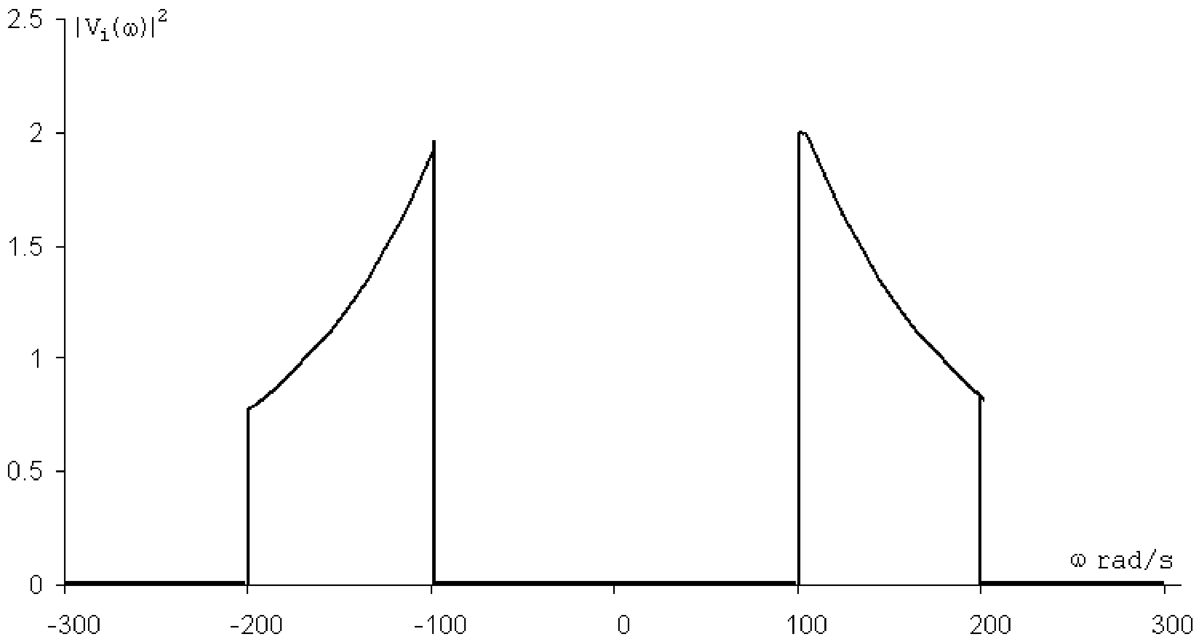
$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}; \qquad \frac{1}{RC} = \frac{10^6 10^{-3}}{(0.5)(20)} = \frac{1000}{10} = 100$$

$$H(\omega) = \frac{j\omega}{j\omega + 100}$$

$$|V_o(\omega)| = \frac{200}{|\omega|} \cdot \frac{|\omega|}{\sqrt{\omega^2 + 10^4}} = \frac{200}{\sqrt{\omega^2 + 10^4}}$$

$$|V_o(\omega)|^2 = \frac{4 \times 10^4}{\omega^2 + 10^4}, \quad 100 \leq \omega \leq 200 \text{ rad/s}; \quad |V_o(\omega)|^2 = 0, \quad \text{elsewhere}$$

$$|V_o(100)|^2 = \frac{4 \times 10^4}{10^4 + 10^4} = 2; \qquad |V_o(200)|^2 = \frac{4 \times 10^4}{5 \times 10^4} = 0.8$$



[c] $W_{1\Omega} = \frac{1}{\pi} \int_{100}^{200} \frac{4 \times 10^4}{\omega^2} \, d\omega = \frac{4 \times 10^4}{\pi} \left[-\frac{1}{\omega} \right]_{100}^{200}$

$$= \frac{4 \times 10^4}{\pi} \left[\frac{1}{100} - \frac{1}{200} \right] = \frac{200}{\pi} \cong 63.66 \text{ J}$$

[d] $W_{1\Omega} = \frac{1}{\pi} \int_{100}^{200} \frac{4 \times 10^4}{\omega^2 + 10^4} \, d\omega = \frac{4 \times 10^4}{\pi} \cdot \tan^{-1} \frac{\omega}{100} \Big|_{100}^{200}$

$$= \frac{400}{\pi} [\tan^{-1} 2 - \tan^{-1} 1] \cong 40.97 \text{ J}$$

P 17.41 **[a]** $V_i(\omega) = \dfrac{A}{a + j\omega}; \qquad |V_i(\omega)| = \dfrac{A}{\sqrt{a^2 + \omega^2}}$

$H(s) = \dfrac{s}{s + \alpha}; \qquad H(\omega) = \dfrac{j\omega}{\alpha + j\omega}; \qquad |H(\omega)| = \dfrac{\omega}{\sqrt{\alpha^2 + \omega^2}}$

Therefore $\quad |V_o(\omega)| = \dfrac{\omega A}{\sqrt{(a^2 + \omega^2)(\alpha^2 + \omega^2)}}$

Therefore $\quad |V_o(\omega)|^2 = \dfrac{\omega^2 A^2}{(a^2 + \omega^2)(\alpha^2 + \omega^2)}$

$W_{\text{IN}} = \int_0^\infty A^2 e^{-2at}\, dt = \dfrac{A^2}{2a}; \qquad$ when $\alpha = a$ we have

$$W_{\text{OUT}}(a) = \frac{A^2}{\pi}\int_0^a \frac{\omega^2\, d\omega}{(\omega^2 + a^2)^2} = \frac{A^2}{\pi}\left\{\int_0^a \frac{d\omega}{a^2 + \omega^2} - \int_0^a \frac{a^2\, d\omega}{(a^2 + \omega^2)^2}\right\}$$

$$= \frac{A^2}{4a\pi}\left(\frac{\pi}{2} - 1\right)$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi}\int_0^\infty \left[\frac{\omega^2}{(a^2 + \omega^2)^2}\right] d\omega = \frac{A^2}{4a}$$

Therefore $\quad \dfrac{W_{\text{OUT}}(a)}{W_{\text{OUT}}(\text{total})} = 0.5 - \dfrac{1}{\pi} = 0.1817 \quad$ or $\quad 18.17\%$

[b] When $\alpha \neq a$ we have

$$W_{\text{OUT}}(\alpha) = \frac{1}{\pi}\int_0^\alpha \frac{\omega^2 A^2 d\omega}{(a^2 + \omega^2)(\alpha^2 + \omega^2)}$$

$$= \frac{A^2}{\pi}\left\{\int_0^\alpha \left[\frac{K_1}{a^2 + \omega^2} + \frac{K_2}{\alpha^2 + \omega^2}\right] d\omega\right\}$$

where $\quad K_1 = \dfrac{a^2}{a^2 - \alpha^2} \quad$ and $\quad K_2 = \dfrac{-\alpha^2}{a^2 - \alpha^2}$

Therefore

$$W_{\text{OUT}}(\alpha) = \frac{A^2}{\pi(a^2 - \alpha^2)}\left[a\tan^{-1}\left(\frac{\alpha}{a}\right) - \frac{\alpha\pi}{4}\right]$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi(a^2 - \alpha^2)}\left[a\frac{\pi}{2} - \alpha\frac{\pi}{2}\right] = \frac{A^2}{2(a + \alpha)}$$

Therefore $\quad \dfrac{W_{\text{OUT}}(\alpha)}{W_{\text{OUT}}(\text{total})} = \dfrac{2}{\pi(a - \alpha)} \cdot \left[a\tan^{-1}\left(\dfrac{\alpha}{a}\right) - \dfrac{\alpha\pi}{4}\right]$

For $\alpha = a\sqrt{3}$, this ratio is 0.2723, or 27.23% of the output energy lies in the frequency band between 0 and $a\sqrt{3}$.

[c] For $\alpha = a/\sqrt{3}$, the ratio is 0.1057, or 10.57% of the output energy lies in the frequency band between 0 and $a/\sqrt{3}$.