

## **Chapter 22: Neutron Stars and Black Holes**

### *Strange States of Matter*

#### **Outline**

- 22.1 Neutron Stars
- 22.2 Pulsars
- 22.3 Neutron-Star Binaries
- 22.4 Gamma-Ray Bursts
- 22.5 Black Holes
- 22.6 The Theory of Relativity
- 22.7 Space Travel Near Black Holes
- 22.8 Observational Evidence for Black Holes

#### **Summary**

By now, your students might feel like they are losing their grip on reality. The scales in size, mass and time discussed in class no longer have any comparable counterpart in their everyday lives. If they are searching for a foothold, they will definitely not find it here. The end-products of massive stars are among the most bizarre objects that most of your students will ever be introduced to.

Chapter 22 explores the final stages in the evolution of massive stars – neutron stars and black holes. The theory of each and observational evidence for each are presented. Pulsars, neutron stars that are visible to us by virtue of the orientation of their radiation beams, are also discussed. The importance of binary systems in detecting neutron stars and black holes is noted. Einstein's special and general theories of relativity are introduced in this chapter and applied to black holes and their effects on space-time.

#### **Major Concepts**

- Neutron stars
  - Mass
  - Density
  - Size
  - Formation
  - Pulsars
  - Neutron stars in binaries
- Black holes
  - Theory
  - Escape speed and light
  - Event horizon
  - Schwarzschild radius
  - Tidal forces
  - The singularity
  - Observational evidence
- Einstein's theories of relativity
  - Special – speed of light, time dilation, length contraction
  - General – gravity and space-time

## Teaching Suggestions and Demonstrations

### Section 22.1

Begin your discussion of **neutron stars** by picking up the stellar evolution story from the end of the last chapter. A core-collapse supernova has just occurred... so what's left behind? Go through the extreme characteristics of a neutron star, including its size (show Figure 22.1), density, and rotation. ➡ **DEMO** Pull out the marble or cube you used in discussing densities in the last chapter and calculate the mass it would have if it were made of neutron star material. The result is possibly unimaginable – about 500 *billion* kg. This marble would have an acceleration (due to gravity on its surface) of about 100,000 times that on the Earth. Not only would you gravitationally “stick” to this marble, you would weigh 100,000 times more on this marble than you do on Earth!

➡ **DEMO** A good analogy of what happens in a massive stellar remnant is to use ping-pong balls or marshmallows to represent electron degeneracy pressure and hold a bunch of them loosely so they make contact. Discuss that gravity in a massive stellar remnant will be sufficient to squeeze the balls or marshmallows hard enough to collapse them. You will be able to squeeze the marshmallows easily, but for ping-pong balls, you could drop a few on the floor and step on them for effect.

### Section 22.2

The lighthouse model of **pulsars** is illustrated in Figure 22.3. Point out that Earth's magnetic and rotational poles do not coincide, either. (Students will have probably heard of Earth's north magnetic pole and how navigators using compasses have to compensate for the fact that it is not located at the north geographic pole.) Possibly the most remarkable feature of pulsars is their rapid rotation.

How does the neutron star get rotating so fast? As the text notes, it is the **conservation of angular momentum**. The period of rotation depends on the radius squared. A typical rotation period for a star is a few days, let's say 5 days. There are 430,000 seconds in 5 days. The iron core is about the size of the Earth and shrinks to about 10 km. This is a reduction in size by a factor of 640. Squaring this to see how the period is reduced gives 410,000. The original period in seconds is about the same as this number, and so the new period will be about 1 second. This is a little slow for a new pulsar but not far off either. If the core is much bigger than the Earth, this will produce a significantly shorter period.

Very few pulsars emit sufficient visible light to make them detectable. The Crab Nebula pulsar (show Figure 22.4) is the only one bright enough to be visible to the eye using a telescope. However, it flashes at a rate of about 30 flashes/s and is too fast for the eye to see. It is seen as a constant light source like any star.

➡ **DEMO** A simple demonstration of the flashing of a pulsar is made by using a strobe light with a variable flash rate. (Some people are quite bothered by strobe lights, so you should warn your students about what you are about to demonstrate.) Set the rate at about 1 flash/s. Increase the rate until your students agree they cannot make out the individual flashes. Typically, most people cannot see beyond about 10 flashes/s. Set the rate at 30 flashes/s to show the Crab Nebula pulsar; there is no way to detect the flashing with the naked eye.

An instrument can be built to look at an object at a specific rate. Working at a frequency of about 30 Hz, the Crab pulsar will either appear “on” or “off”, depending on the synchronization of the

instrument with the pulsar. Such an instrument was used in 1969 on the Steward Observatory 36-inch telescope on Kitt Peak to examine some of the stars in the middle of the Crab Nebula and to see if any were flashing. On the very first star examined, the flashing was discovered and the optical pulsar confirmed.

Why can't pulsars be explained using rapidly rotating white dwarfs? The answer to this uses simple, classical physics. Due to its rotation, gravity provides the centripetal force necessary to hold it together. If the object rotates too fast, it flies apart. The gravitational acceleration is  $GM/R^2$  and the centripetal acceleration is  $v^2/R$ , where  $M$  and  $R$  are the mass and radius of the rotating object. The equatorial velocity,  $v$ , may be written as  $2\pi R/P$ , where  $P$  is the period of rotation. How do the gravitational and centripetal accelerations compare for a rapidly rotating white dwarf?

$$M = 1 \text{ solar mass} = 2 \times 10^{30} \text{ kg}$$

$$R = 5000 \text{ km} = 5 \times 10^6 \text{ m}$$

$$P = 0.1 \text{ s (typical for a pulsar)}$$

$$GM/R^2 = 5 \times 10^6 \text{ m/s}^2 \quad \text{and} \quad v^2/R = 2 \times 10^{10} \text{ m/s}^2$$

The white dwarf would fly apart rotating this fast! The problem could be turned around and the question asked "What size object is just stable under these conditions?" The radius must be about 300 km, so anything smaller will be stable against rotation. The same calculation done for a typical neutron star shows how stable they are.

$$M = 2 \text{ solar mass} = 4 \times 10^{30} \text{ kg}$$

$$R = 10 \text{ km} = 1 \times 10^4 \text{ m}$$

$$P = 0.1 \text{ s}$$

$$GM/R^2 = 3 \times 10^{12} \text{ m/s}^2 \quad \text{and} \quad v^2/R = 4 \times 10^7 \text{ m/s}^2$$

For the millisecond pulsars, this situation is not quite so stable. The first such pulsar, PSR 1937+214, whose period is 0.001558 s (642 rotations and flashes per second!), gives  $v^2/R = 2 \times 10^{11} \text{ m/s}^2$ ; it is stable but not by much.

The Crab Nebula gives off a lot of energy at all wavelengths. After over 900 years of expansion a question arose as to how it still manages to have a luminosity of  $10^5$  times that of the Sun. The Crab pulsar is known to be slowing down at a rate of  $10^{-5}$  s/year. The loss in rotational energy is about  $5 \times 10^5$  times the solar luminosity. Thus the rotational energy is transferred to the nebula and allows it to continue to shine.

### Section 22.3

What about **millisecond pulsars**? Ask students what could explain the extremely rapid rotations of these pulsars. ➡ **DEMO** To give them some ideas, demonstrate an analogy: Have a student sit on a rotating stool with his arms out. Spin him around, and instruct him to pull his arms in; he will noticeably speed up. (Holding weights makes the change in rotational velocity even more dramatic.) This process is what results in normal pulsar rapid rotation. Next, repeat the demonstration, but this time play the role of a **binary companion** feeding mass into the pulsar

and give the student an extra spin after he pulls his arms in as well. Follow-up with a view of Figure 22.10, which shows where the extra angular momentum comes from.

## Section 22.5

This is a good time to review the concept of **escape velocity**. You may find that using escape velocity to explain black holes is conceptually straightforward for most students (their knowledge of general relativity usually being somewhat limited!). It is important to emphasize the dependence of escape velocity on both the mass and the radius.

Surprisingly, the **size of a black hole** can be calculated from classical physics even though general relativity is required to describe its details. Very simply, when the escape velocity of an object equals the speed of light, the object is a black hole. From previous chapters, we know that the escape velocity is given by  $v_{esc} = \sqrt{2GM/r}$ . If  $v_{esc}$  is set equal to the speed of light,  $c$ , then the equation can be solved for the radius. The result gives the Schwarzschild radius  $= 2GM/c^2$ . This gives a radius of 3 km for a one solar mass object; thus the well-known result of 3 km for each solar mass of a black hole.

Black holes are really tiny when compared to ordinary stars. This is not always understood. The Sun would have a radius of 3 km but it currently has a radius of about  $7 \times 10^5$  km. Taking the ratio of these two numbers, and multiplying by the angular size of the Sun, will give its apparent size if turned into a black hole (which I remind you is impossible for the Sun). The Sun is about half a degree or 1800 arc seconds in diameter. The answer is the Sun would appear 0.008 arc seconds in diameter. This is 100 times smaller than the atmospheric turbulence in the Earth's atmosphere will allow us to see. Not only would it not give off light, its size would make it much too small to see (if there were anything to see). And this is at the distance of the Sun.

If the Sun magically turned into a black hole, what would be the effect on the Earth's orbit? A typical response from most students is that the Earth would be "sucked into it." Point out that the gravitational force on the Earth is still the same as before, since the mass of the Sun (now black hole) and our distance from it have not changed. And even if the Earth somehow encountered the Sun, the Earth is much larger (but not more massive). In fact, the Earth would be about 4,000 times larger than the black hole Sun. This is comparable to a person standing next to the period at the end of the sentence.

**DEMO** The **rubber sheet model** of a black hole really works well as a demonstration. Stretch the sheet on an open frame just enough so there are no wrinkles and little sag. Put a mass at the center to warp it. Roll small marbles, ball bearings, or coins tangentially to the center. They will "orbit" around the center, moving faster as the center is approached, and finally spiraling into the center because of friction. Try different speeds and angles to change the trajectories. Small coins often work the best because of the reduced friction. Students will want to play with this demo after class. There are also some commercially available plastic models. Small ones are very popular for collecting coins for charities. (It would be best not to compare the charity with a black hole, however!)

## Section 22.6

Either before or while you are introducing black holes you will find yourself discussing the basics of **Einstein's theories of relativity** as it is difficult to separate the two subjects. The general concepts can be understood without lots of math, and students typically find the topic fascinating. For **special relativity**, emphasize that Einstein began with two postulates, and remind students just what a postulate is. Einstein did not need to prove them; he accepted them and then

determined the consequences that would result from them. The example of the car and bullet versus the spaceship and light beam that is discussed in the text is a good illustration of the postulates and their consequences. Time dilation and length contraction follow. After all, if  $\text{distance} = \text{speed} \times \text{time}$  and you keep speed constant, then weird things are going to happen to distance and time!

For **general relativity**, emphasize the equivalence principle. If you wake up in a small room with no windows, how do you know if you are on the surface of Earth (gravity) or out in space in an elevator accelerating at  $9.8 \text{ m/s}^2$  (acceleration)? You don't.

To bring the discussion out of the realm of theory and analogy, discuss some of the **tests of the general theory of relativity**. The bending of starlight as it passes the Sun during a solar eclipse is an excellent example. (See *More Precisely* 22-2.) Make sure your students do not think that a solar eclipse is necessary to bend starlight. Starlight bends as it travels by the Sun all the time; we just do not see it because the sky is too bright.

## Section 22.7

It is important for the students to understand that black holes are not swallowing up the universe. From distances beyond their (relatively close) event horizon, black holes can be treated no differently than if one were in orbit around a star of similar mass. Once within the event horizon, though, things change.

No discussion of black holes would be complete without imagining the **tidal forces** encountered when traveling near one. Walk the students through the process of tidal force stretching and what kind of effect this will have on the space traveler unlucky enough to reach the point of no return. The charming term “spagettification” to describe what would happen to a person heading feet-first or head-first into a black hole creates an image your students are not likely to forget! Do some calculations to demonstrate. Take the example of a five solar-mass black hole. What would be the tidal force over a distance of one meter if you are 100 km away from the black hole? (Note, the black hole's radius is 15 km, so you are still some distance from the event horizon.) Say you fall feet-first toward the black hole. It is easy to calculate the gravitational force on a 1-kg mass at 100 km and 100 km plus one meter and then simply compare the two. Each kilogram of your body separated by one meter from another kilogram of you would experience a stretching force of 1.3 million Newtons or about 300,000 lbs! The tidal forces will stretch the traveler on increasingly smaller scales as the travelers “body” gets closer and closer since the tidal forces increase dramatically until eventually the individual atoms that constituted the traveler are torn apart.

## Section 22.8

Because no light can escape a black hole, its detection can be somewhat problematic. Before giving **evidence for black holes**, ask your students how black holes could possibly be detected. Refer back to binary systems and to the detection of extrasolar planets for hints regarding one detection method. Show Figure 22.25, an artist's conception of the accretion disk surrounding a black hole. Figure 22.26 shows an actual image of what might be signs of a black hole. Discuss each of these figures in the context that black-hole research represents one of the frontiers in scientific enquiry. The bizarre circumstances of their existence combined with their illusive nature makes them extremely difficult to detect and study.

Finally, give a preview of chapters ahead by mentioning massive black holes and where they are found.

## Student Writing Questions

1. Imagine an environment where the effects of General Relativity occur strongly for even weak gravitational fields. What would living in this environment be like?
2. So you just bought a new planet and now you find out it's orbiting a pulsar? What is your planet like? What do you get to see and experience that the rest of us on ordinary planets don't? Was it a good buy?
3. Isolated black holes must exist in the Galaxy. What if one of these moved through our solar system? What effect would it have on the solar system? On Earth and its life forms? Could we capture it? If so, to what use could we put it?
4. Try to trace all the steps in determining distances that finally lead to using supernovae as distance indicators. Start back here on Earth and work your way through the solar system, nearest stars, etc. How accurate do you think supernovae are in calculating distances?
5. Science fiction author Larry Niven wrote a short story titled *Neutron Star*. Find a copy of this and read it. How does this story relate to this chapter? What was the mysterious force encountered around the neutron star? From what you have learned in this chapter, is the physics in the story realistic?

## Answers to End of Chapter Exercises

### Review and Discussion

1. Some of the basic properties of a neutron star include its high density, rotation, and magnetic field. How these come to be this way is a direct result of the core collapse that forms the neutron star. The collapse and shock wave that propagates into the core give the high density. Although the star is rotating initially at a normal rate, the collapse must conserve angular momentum and this results in the core "spinning up" as it gets smaller. The magnetic field is confined by the gas in the core and is therefore concentrated as the core shrinks.
2. The gravity at the surface of a body depends on the mass of that body and inversely on the square of the distance to the center, i.e. its radius. A neutron star has both a high mass, 1-2 solar masses, and a small size, 10 km radius. Both of these properties result in a very high pull of gravity on its surface. An average human would weigh about 1 million tons on its surface. A person would be flattened out by this huge force.
3. All neutron stars are not seen as pulsars because their orientation does not allow their beam of radiation to pass in the direction of the Earth. The beam can be only a few degrees across, so the alignment must be close to pointing towards the Earth in order for us to see it. A neutron star that is not observed as a pulsar by us could be viewed by others in a different direction as being a pulsar.
4. Apparently during the supernova, the explosion is not symmetric around the star, resulting in the core being "kicked" in one direction.
5. X-ray bursters are found in mass-transfer binary stars where one of the stars is a neutron star and the other is a normal star. The mass transfer forms a disc around the neutron star and

slowly accumulates on its surface. Finally, the gas undergoes fusion and there is a burst of X-rays released as a result. This can occur again and again as the mass continues to be transferred to the neutron star.

6. In a mass-transfer binary, something other than just mass is transferred to the neutron star; angular momentum can be transferred too. This results in making the neutron star spin faster and faster. The angular momentum is taken from the orbital motions of the two stars.
7. A supernova explosion should completely destroy any planetary system. However, if the star that goes supernova is in a binary system, the effects of the explosion may destroy most, but not all, of the companion. What is left behind could form a disc of material that eventually accretes to form planetary-sized objects in orbit around the pulsar. This theory is still unproved, though.
8. The distribution of gamma-ray bursts is isotropic across the sky, implying they are not associated with the Galaxy. Recently, an Italian-Dutch satellite observed a burst whose position was well-enough determined that optical observations could be made. A redshift was measured and the object was at a cosmological distance. Bursts are very energetic because, at cosmological distances, they must be more powerful than a supernova in order to be observed and have all this energy emitted in just a few seconds.
9. The first model has a high mass binary evolve to the point where there are two neutron stars. If gravitational radiation continues to be emitted, these two neutron stars will finally merge in a rather violent event. The second model has a very massive star going supernova. But the black hole is able to prevent the escape of the outer portion of the star. The matter forms an accretion disk around the black hole and generates a relativistic jet.
10. If an object moves towards me at 10 mph and I move towards it at 5 mph, then the object's speed, relative to me, is 15 mph. If, instead, I am moving 5 mph away from this same object, its speed relative to me is only 5 mph.

Now, if the object is actually a beam of light moving at the speed of light, the previous example of the addition or subtraction of speeds no longer applies. I will always measure the speed of light to be a constant, independent of my own speed. This was discovered by Michelson and Morley, much to their own amazement, because they believed the speeds would add together. It was a discovery that led Einstein to the special theory of relativity.

11. The escape velocity from the surface of a body depends on the mass and the radius of that object. It is quite possible for an object to have sufficient mass and small size to have an escape velocity equal to the speed of light. This occurs for a solar mass at a radius of 3 km, or any multiple thereof. What this means is that light itself cannot escape from this object. By definition, this is what is meant by a black hole.
12. According to special relativity, the speed of light is a maximum speed beyond which matter cannot move. The speed of light is a limit; to reach this speed, matter requires infinite energy.
13. General relativistic effects, although always present, are not obvious unless a gravitational field is very strong. Most of the gravitational fields we encounter in the solar system, for instance, are fairly weak. But the theory can still be tested under these conditions; it just requires very careful and sensitive measurements to be made.

The two classical tests of general relativity are the deflection of starlight by the Sun's gravity and the precession of the orbit of Mercury. During a total solar eclipse, stars can be seen near the Sun. However, their positions are slightly altered due to the fact that their light has skimmed past the Sun, encountering its gravitational field. This bends the path of the light and we see the stars displaced slightly. Because Mercury has such an elliptical orbit, it moves through varying strengths of the Sun's gravitational field. This causes the orbit to slowly turn or precess. In both cases, general relativity accurately predicts the observed effects.

14. Before entering the black hole, tidal forces would pull them apart. Gravity is so strong near the black hole that the difference in the force on the two sides of the person is sufficient to him or her apart. This would be true of any type of matter that would venture close to the black hole.
15. An event horizon, in some ways, defines the physical space of a black hole. Inside the event horizon, nothing can escape. Outside, it is still possible to escape because the escape velocity is less than the speed of light.
16. Principle of Cosmic Censorship says that nature always hides any singularity inside an event horizon because it is a violation of physical law. We, in a sense, are protected from this physical flaw by having no contact with it.

The fact that singularities appear to violate known physical law is likely due to the fact that we still do not understand all physical laws, especially those that apply to such extreme conditions as found at a singularity.

17. Cygnus X-1 is a good black hole candidate because it is in a binary system; its mass has been determined to be in the range of 5 to 10 solar masses. Mass transfer is occurring and produces X-rays from the black hole candidate. The X-rays vary at a rate suggesting the candidate is small, less than 300 km.
18. X-ray images from *Chandra* show a number of sources in M82 that are likely candidates for black holes with masses in the range of 100 to 1000 solar masses. How these have formed is still not understood.
19. About the only way to discover a neutron star from Earth is if it is a pulsar. Traveling through space would allow you to see neutron stars from other directions, and thus as pulsars. One might also encounter neutron stars that are no longer pulsars because they have lost so much energy. Neutron stars are the result of the evolution of massive stars. Wherever the most stars are, there should be many neutron stars. Globular clusters might be a particularly good place to look.
20. Objects are often classified by their mass, such as various types of stars, brown dwarfs, and planets. Because these objects have masses equivalent to planets, the name is appropriate. But they almost certainly are not planets similar to those in the solar system. But then again, the newly discovered planets around other stars (large jovian masses but very near their star) seem to be rather different from our own, too. What is likely needed here are new names for new types of objects. These might be called planetoids, pulnets, or plansars!



### Conceptual Self-Test

1. T
2. T
3. T
4. T
5. F
6. T
7. F
8. F
9. T
10. F
11. B
12. C
13. A
14. B
15. C
16. D
17. B
18. B
19. C
20. B

### Problems

1. Since angular momentum is conserved, the “before” and “after” values must be equal. Thus we have  $1 \text{ rev}/1 \text{ day} \times 10,000^2 = 1 \text{ rev}/P \times 10^2$ ,  $P = 10^{-6} \text{ day} = 0.086 \text{ s}$  or 11.6 rev/s.
2. Mass is volume times density, so your mass will be  $3 \times 10^{14}$  greater than what it is now. A 100-pound person normally has a mass of 45 kg. At neutron-star density, the mass would be  $1.3 \times 10^{16} \text{ kg}$ . An asteroid’s mass, assuming a density of  $3000 \text{ kg/m}^3$ , will be  $4/3\pi(5000)^3 \times 3000 = 1.6 \times 10^{15} \text{ kg}$ . Your mass is about 10 times greater than this.
3. The acceleration due to gravity,  $g$ , is given by  $g = GM/R^2$ . Putting in the numbers gives  $g = 6.7 \times 10^{-11} \times 1.4 \times 2 \times 10^{30} / (10,000)^2 = 1.9 \times 10^{12} \text{ m/s}^2$ . The escape velocity will be  $v_{\text{esc}} = \sqrt{(2 \times 6.7 \times 10^{-11} \times 1.4 \times 2 \times 10^{30} / 10,000)} = 1.9 \times 10^8 \text{ m/s}$  (0.63c)
4. Determine the luminosity for the first example, at  $T = 10^5 \text{ K}$ . Remember,  $T$  must be put into solar units, as will be  $R$  and  $L$ .  $L = (10 / 7 \times 10^5)^2 \times (10^5 / 6000)^4 = 1.6 \times 10^{-5}$ . For  $10^7 \text{ K}$ ,  $L = 1600$ . For  $10^9 \text{ K}$ ,  $L = 1.6 \times 10^{11}$ .

In the first case, the luminosity is so low that it could not be plotted on the H–R diagram. In the second and third cases, the temperatures are so high, again, neither could be plotted on the H–R diagram. The luminosities in the second case suggest that this neutron star might be visible; the temperatures, though, would place the peak radiation in the X-ray region of the spectrum. The third case is much too bright and hot to be plotted.

5. Find the total surface area of a sphere with a radius of 1000 Mpc.  $\text{Area} = 4\pi \times (10^9 \times 3 \times 10^{16})^2 = 1 \times 10^{52} \text{ m}^2$ . Multiply this area by the energy per area of the detector to get the total energy.  $\text{Energy} = (10^{-8} \text{ J} / 0.5 \text{ m}^2) \times 1 \times 10^{52} \text{ m}^2 = 2 \times 10^{44} \text{ J}$ .

At 10,000 pc, this energy will reduce by the ratio of the square of the distances,  $(10^4 / 10^9)^2 = 10^{-10}$ . Energy =  $2 \times 10^{34}$  J.

At 50,000 A.U. = 0.24 pc this energy will reduce by the ratio of the square of the distances,  $(0.24 / 10^9)^2 = 6 \times 10^{-20}$ . Energy =  $1 \times 10^{25}$  J.

6. This is just the opposite calculation of the previous question. Determine the area of a sphere of 5000 Mpc radius. Area =  $4\pi \times (5 \times 10^9 \times 3 \times 10^{16})^2 = 2.8 \times 10^{53} \text{ m}^2$ . Each gamma ray has an energy of  $250,000 \times 1.6 \times 10^{-19} = 4 \times 10^{-14}$  J.  $10^{45}$  J of energy will contain  $10^{45} \text{ J} / 4 \times 10^{-14} \text{ J} = 2.5 \times 10^{58}$  photons of gamma rays. The number passing through the detector will be  $0.75 \times 2.5 \times 10^{58} / 2.8 \times 10^{53} \text{ m}^2 = 67,000$
7. A point on the equator travels 1000 circumferences each second.  $1000 \times 2\pi \times 10 = 63,000$  km/s. But the speed of light is 300,000 km/s, so the speed is  $63,000 / 300,000 = 0.21$  or 21% speed of light.

The orbit will be 10 km in radius; converted to A.U. gives  $6.7 \times 10^{-8}$  A.U.  $v = \sqrt{(6.710^{-11} \times 1.4 \times 2 \times 10^{30} / 10,000)} = 1.4 \times 10^8 \text{ m/s} = 0.46c$

8. The Schwarzschild radius is the mass of an object, given in solar masses, times 3 km. For the 1-million-solar-mass black hole,  $10^6 \times 3 \text{ km} = 3 \times 10^6 \text{ km}$ , dividing by the radius of the Sun gives  $3 \times 10^6 \text{ km} / 6.96 \times 10^5 \text{ km} = 4.3$  solar radii.

For the 1-billion-solar-mass black hole,  $10^9 \times 3 \text{ km} = 3 \times 10^9 \text{ km}$ , dividing by the radius of the solar system (= 40 A.U.) gives  $3 \times 10^9 \text{ km} / (40 \times 1.5 \times 10^8 \text{ km}) = 0.5$  solar system radii or 20 A.U.

9. The deflection can be calculated as  $\text{def} = 1.75'' M/R$  where  $M$  is the mass of the deflecting body and  $R$  is the distance from its center, both in solar units.
  - (a)  $\text{def} = 1.75'' (1 / 28,000,000) / (1 / 403) = .000025'' = 25 \text{ microarcsec.}$
  - (b)  $\text{def} = 1.75'' (1 / 1050) / (1 / 9.7) = 0.016''$
  - (c) From Table 20.2, Sirius B has a radius of 0.008 solar radii and a mass of 1.1 solar masses.  $\text{def} = 1.75'' (1.1) / (0.008) = 241'' = 4'1''$ .
  - (d)  $10^{-6} = 1.75'' (1) / (R) \text{ m } R = 1.75 \times 10^6 \text{ solar radii} = 8140 \text{ A.U.} = 0.039 \text{ pc.}$

10.  $gh/c^2 = 9.8 \times 500,000 / (3 \times 10^8)^2 = 5.4 \times 10^{-11}$ . This times  $10^{10} \text{ Hz} = 0.54 \text{ Hz.}$

11. Use the equation developed in *More Precisely* 7-3 for the tidal force =  $2 \times 6.7 \times 10^{11} \times 2 \times 10^{30} \times 2 / (3000)^3 = 1.9 \times 10^{10} \text{ m/s}^2 = 1.9 \times 10^9 \text{ g.}$

As the mass of the black hole increases, so does its size, so the force varies by  $M/M^3 = 1/M^2$ .  
 A one million-solar-mass black hole will have an acceleration diminished  $10^{-12}$  or  $0.0019g$ .  
 A one billion-solar-mass black hole will have an acceleration diminished by  $10^{-18}$  or  $1.9 \times 10^{-9}g$ .

12.  $2 \times 6.7 \times 10^{-11} \times 2 \times 10^{30} \times 2 / (R)^3 = 98 \text{ m/s}^2$ .  $R = 1.76$  million meters or 1760 km.
13.  $2 \times 6.7 \times 10^{-11} \times 2 \times 10^{30} M \times 2 / (3000M)^3 = 98 \text{ m/s}^2$ .  $M = 14,200$  solar masses.
14. The temperature is inversely related to the mass of the black hole. For the example given for the Sun,  $T = 10^{-6} \text{ K}$ . Set up a relationship  $1 \text{ solar mass} \times 10^{-6} = M \times 6000$ ,  $M = 1.7 \times 10^{-10}$

solar masses. The radius is 3 km times the mass given in solar masses, so  $r = 5 \times 10^{-10}$  km,  $r = 5 \times 10^{-7}$  m. Putting  $r$  and  $T$  into solar units,  $L$  can be calculated.  $L = (5 \times 10^{-7} / 7 \times 10^8)^2 \times (1)^4$ ,  $L = 5 \times 10^{-31}$  solar luminosities.  $L = 0.0002$  watts

15. The black hole mass is 10 solar masses; the primary star's mass is 25 solar masses. The orbital period is 5.6 days = 0.015 yr. Using Kepler's third law gives  $0.015^2 = a^3 / (25 + 10)$ ,  $a = 0.2$  A.U. = 30 million km.

$25M / 20^2 = 10M / 10^2$ ,  $0.0625 = 0.10$ . The gravitational force of the black hole is 1.6 times greater than the surface gravity of the companion star.

## Resource Information

### Student CD Media

#### Movies/Animations

Black Hole Geometry

#### Interactive Student Tutorials

SuperSpaceship-Voyage to the Sun

Escape Speed and Black Hole Event Horizons

#### Physlet Illustrations

Equivalence Principle

Period of Pulsar

### Transparencies

T-205	Figure 22.1	Neutron Star	p. 568
T-206	Figure 22.3	Pulsar "Lighthouse" Model	p. 570
T-207	Figure 22.4	Crab Pulsar	p. 571
T-208	Figure 22.9	X-Ray Emission	p. 574
T-209	Figure 22.10	Millisecond Pulsar	p. 575
T-210	Figure 22.15	Gamma-Ray Burst Models	p. 578
T-211	Figure 22.17/18	Speed of Light and Einstein's Elevator	p. 582/583
T-212	Figure 22.19	Curved Space	p. 583
T-213	Figure 22.25	Black Hole Binary	p. 594
T-214	Figure 22.26	Intermediate-Mass Black Holes?	p. 595

### Suggested Readings

Charles, Philip A.; Wagner, R. Mark. "Black holes in binary stars: weighing the evidence." *Sky & Telescope* (May 1996). p. 38. Describes the observations used to determine if an object is a black hole.

Parker, Wayne J. "Anatomy of a Crab." *Sky & Telescope* (Jan 1995). p. 38. Discusses what we have learned about the Crab Nebula and pulsar from recent HST images.

Ford, Lawrence H.; Roman, Thomas A. "Negative energy, wormholes and warp drive." *Scientific American* (Jan 2000). p. 46. Discusses the phenomena associated with negative energy

Hawking, Stephen. *The Illustrated a Brief History of Time*. Bantam Books, New York, 1996. An excellent book with superb illustrations.

Helfand, D. "Way Too Cool." *Astronomy* (March 2003). p. 54. Particle physics meets astrophysics. Discusses research into exotic neutron stars that might be squeezed enough to "spill out quarks, the fundamental building blocks of nature."

Irion, Robert. "Pursuing the most extreme stars." *Astronomy* (Jan 1999). p. 48. Describes observations of pulsars.

Jayawardhana, R. "Beyond Black." *Astronomy* (June 2002). p. 28. Overview of current research into detecting black holes, primarily at the center of galaxies. Discussion of challenges faced by researchers.

Lasota, Jean Pierre. "Unmasking black holes." *Scientific American* (May 1999). p. 40. Discusses the accretion of material onto a black hole.

Leonard, Peter J. T.; Bonnell, Jerry T. "Gamma-ray bursts of doom." *Sky & Telescope* (Feb 1998). p. 28. Gives a short overview of gamma ray burst and merging neutron stars, then discusses the potential impact on Earth of a gamma ray burst from within our part of the galaxy.

Nadis, Steve. "Neutron stars with attitude: magnetars." *Astronomy* (Mar 1999). p. 52. Discusses observations of a class of neutron stars with extremely strong magnetic fields.

Olson, Steve. "Black hole hunters: L. Dressel, T. Heckman, R. van der Marel and M. Urry." *Astronomy* (May 1999). p. 48. Describes the black hole-related research of four different astronomers.

Pickover, Clifford A. "A black hole user's guide." *Sky & Telescope* (May 1996). p. 92. Describes and gives a program for using a personal computer to investigate the effects of black holes.

Rees, Martin. "To the edge of space and time." *Astronomy* (July 1998). p. 48. Gives an overview of the theory of black holes.

Reichart, D. "The Gamma-Ray Burst/Supernova Connection." *Mercury* (September/October 2003). p. 15. Detailed discussion of the current theories explaining Gamma-Ray Bursters. Many diagrams and images.

Shipman, Harry L. "How do we know its a black hole?" *Sky & Telescope* (May 1996). p. 42. Explains how astronomers observe black holes.

Talcott, Richard. "Another record-setting burst." *Astronomy* (Jan 2001). p. 28. Reports on a very distant gamma ray burst.

Talcott, Richard. "Black holes in all sizes." *Astronomy* (Dec 2000). p. 26. Describes the difference between black holes that form from the evolution of a single star and the massive black holes at the centers of galaxies.

Tyson, Neil De Grasse. "Flashes of ignorance: gamma-ray bursts." *Natural History* (June 1997). p. 78. Gives a brief discussion of the history of observations of gamma ray bursts.

Winn, Joshua N. "The life of a neutron star." *Sky & Telescope* (July 1999). p. 30. Summarizes the formation and evolution of neutron stars.

Zimmerman, Robert. "When neutron stars collide." *Astronomy* (Apr 1997). p. 52. Discusses the merging of two neutron stars as a source of gamma ray bursts.

## **Notes and Ideas**

*Class time spent on material: Estimated: \_\_\_\_\_ Actual: \_\_\_\_\_*

*Demonstration and activity materials:*

*Notes for next time:*