

Chapter 20: Stellar Evolution

The Life and Death of Stars

Outline

- 20.1 Leaving the Main Sequence
- 20.2 Evolution of a Sun-like Star
- 20.3 The Death of a Low-Mass Star
- 20.4 Evolution of Stars More Massive than the Sun
- 20.5 Observing Stellar Evolution in Star Clusters
- 20.6 The Evolution of Binary-Star Systems

Summary

In this chapter, the stages of the evolution of both low-mass and high-mass stars are discussed. Red giants, red supergiants, white dwarfs, planetary nebulae and novae are all objects representing different stages in the life of a star that are explained and described here. The evolutionary tracks through these stages are shown on the H–R diagram. The term “evolution,” being a key term for this chapter, is used in a very different way than students may expect. In biology evolution refers to change over (many) generations. It usually takes a long time. For stellar evolution, change occurs, but within the same generation. It also takes a long time and that is why the term was borrowed from biology. Stellar evolution is the story of a star’s life, not of generations of stars. This is a commonly made error when discussing stellar evolution.

This chapter presents other opportunities for misconceptions that need to be recognized when discussing these topics. Knowing them ahead of time will allow you to address these as learning issues that will challenge your students’ thinking. They must “think out of the box” when it comes to these concepts. Stars do many things that seem contradictory to logic. They are full-size when born and never get heavier. The least massive stars take the longest to form. Those with the greatest masses (most fuel) live the shortest lives. Stars grow bigger without becoming more massive. When they run out of fuel in their cores they get brighter. Building up the most stable element destabilizes the star. Whenever the core shrinks, the outer part of the star expands... and vice versa. A planetary nebula has nothing to do with planets! 100 million years is a “short” amount of time.

The chapter concludes with a discussion of star clusters and binary star systems. The usefulness of star clusters in the study of stellar evolution is outlined and stars that are part of binary systems, the evolution of which can be drastically different than their solitary counterparts, is discussed.

Major Concepts

- Hydrostatic Equilibrium
- Evolution of low-mass stars
 - Leaving the main sequence
 - Subgiant and red giant branches
 - Helium flash
 - Asymptotic giant branch
 - Planetary nebula
 - White dwarf

- Nova
- Evolution of high-mass stars
 - Similarities to low-mass stars
 - Heavy element fusion
 - Red supergiants
 - Supernovae – type I and type II
 - Supernova remnants
- Star clusters and stellar evolution
- Evolutionary tracks on the H–R diagram

Teaching Suggestions and Demonstrations

In the last chapter, students learned about the different types of stars, including main-sequence stars, red giants, and white dwarfs. Begin this chapter with a quick review of the different types of stars and their main characteristics. Red giants, for instance, are cool and bright, and therefore they must be very large. Introduce students to the idea that the different classes are different stages in a star's life. Be careful, though, to make sure that students understand that the different types of stars *on the main sequence* do *not* represent different stages. An M star will never evolve into a G star; however, both will evolve into red giants.

Section 20.1

The entire life span of a star, including the stages of stellar formation covered in the last chapter, can be viewed as a balance (or imbalance) between **gravity** and something else. When the star is in equilibrium, the stage of evolution is stable. When something happens to upset the balance, such as depletion of fuel in the core and consequent cessation of core fusion, then the star undergoes a change. For a main-sequence star, the balance is between the inward pull of gravity and the outward push of pressure due to the fusion of hydrogen into helium.

If you constructed a timeline for the Sun in the previous chapter, bring it back out and add to it as the later stages are discussed. Students sometimes find it confusing that a total of 13 stages are distinguished in the life of a Sun-like star, but the star spends most of its lifetime in just one of them, stage 7, as a main sequence star burning hydrogen at its core.

A note on terminology: Remind your students that although astronomers use the terms “burn” and “fire” when referring to hydrogen or helium fusion, the hydrogen or helium are not really burning; they are fusing. These are different processes and can be very confusing to students learning the jargon.

Section 20.2

Go through the **stellar evolution stages 7 through 11** carefully. At each one, ask students what they think will happen next. Figure 20.6 shows a plot of the star on the H–R diagram at each of these stages. Compare the location of each stage to the data given about it in Table 20.1. You can then consider many different aspects of each stage.

First, ask students to consider the tug-of-war described above for each stage. If the stage is stable, what is supplying the **pressure** to counteract gravity? If the stage is a transition, how is the pressure changing and why? Next, consider the length of **time** the star spends at each stage. Compare to an H–R diagram of a whole cluster of stars. We find the most stars in the longest stages, and very few stars in the stages that a star passes through more quickly. Third, consider

sizes. Radii are given in Table 20.1 and Figure 20.8 is an excellent illustration of size comparisons of a G star at different points in its evolution. Fourth, use the radii and temperature data from Table 20.1 to calculate the **luminosities** of the star at different stages. Do your results compare reasonably well with the H–R diagram?

Just as the luminosity changed dramatically during pre-main sequence evolution, so it changes similarly during post-main sequence evolution. What happens to the helium core when it contracts? Obviously it heats up; gases do that when compressed. But the core also radiates off some of the gravitational energy from its contraction. When an object is dropped in a gravitational field it converts gravitational potential energy to kinetic energy. But particles in the helium core are not simply free-falling, they interact with each other. The result of this type of contraction is well-known. Exactly half of the energy of collapse goes into heating the gas (the kinetic energy of the particles) but the other half is radiated away. Thus the helium core, while contracting, radiates a substantial amount of energy. This, along with the higher temperature of the hydrogen burning shell, increases the overall energy output of the star.

Besides examining the changes in luminosity, changes in the size of the star (as compared to the Sun's size) are equally interesting. Now is the perfect time to apply hydrostatic equilibrium. Why does the core collapse? The core lacks the energy source to keep pushing outward. What must be the response by the outer part of the star to an increased energy output from the center? Expansion.

Next, consider the helium core. It was established earlier that for hydrogen to fuse into helium the temperature must be very high in order for the protons (hydrogen) to overcome the repulsion of their positive charges. This is known as a potential barrier. For helium to fuse, two protons (helium) must get equally close to two other protons (helium). Obviously the potential barrier is higher than for hydrogen fusion and so the temperature required must be higher. How much higher? This can be rather easily predicted using a little simple physics.

The average kinetic energy of atoms in a gas and the temperature of the gas are related by

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

But the kinetic energy must be equal to the work, force times distance, needed to move the particles together to a distance x .

$$\frac{1}{2}mv^2 = Fx$$

So, for a fixed distance x , the kinetic energy is proportional to the force and, from above, proportional to the temperature. So we find that

$$T \propto F$$

The force, F , is the electric force, which depends on the product of the charges. Again, remember we are keeping the distance of separation constant, for simplicity, we finally have

$$T \propto Q_1Q_2$$

The force necessary to get two helium nuclei, which have 2 protons each, together is 4 times that necessary to get 2 hydrogen nuclei together. Therefore, a temperature 4 times that of hydrogen

fusion is needed. If hydrogen fusion occurs at about 12 million K, then helium fusion should occur at about 48 million K.

☛ **DEMO** This situation can be demonstrated using some small magnets. Using small, disk magnets, orient them so they repel. Then take two in each hand and show how they repel harder. The magnets behave like protons in this analogy. The more magnets, the harder you have to push to get them close to each other.

The fusion of helium with another helium does not produce carbon, it produces beryllium, which has 4 protons. The second step of the process involves the fusion of a beryllium nucleus (4 protons) with a helium nucleus (2 protons). This is even harder to do and requires a higher temperature. The force needed is really 8 times that of hydrogen fusion or 96 million K. Thus, the well-known result that a temperature of about 100 million K is needed for helium fusion into carbon.

Finally, consider core **densities**. The range of densities given in Table 20.1 is impressive, and when neutron stars are added in the next chapter, it will become truly astounding! Help your students relate to these enormous numbers by bringing in an object with a volume of one cubic centimeter. (A die with side length of 1 cm or a marble with a diameter about 1.2 cm works well.) Find the actual mass of the object. Then, calculate what the mass would be if the object had the density listed for each stage in Table 20.1. In other words, if the marble were made of main-sequence star material, what would its mass be? For a main-sequence star, it turns out to be about 100 g. Compare this to a white dwarf; the mass would be on the order of 10,000 kg! For each stage, come up with everyday objects that have the calculated mass, and then imagine compressing them to the size of the marble. Note that the densities are given for the core of the star. The envelope of a red giant is much more diffuse. Do the calculation for the planetary nebula stage as well, for comparison. The table below lists the mass of a marble-sized chunk of a star at various stages of evolution along with masses of equivalent “typical” objects here on Earth.

<i>Masses of One Marble (1 cm³) At Core Densities</i>			
Stage	Composition Object	Mass	Equivalent Object
7	Hydrogen/Helium Main sequence star	100 gm	1/4 Pounder meat patty
8	Helium subgiant	10 kg	Adult wiener dog
9	Helium red giant	100 kg	College football player
12	Carbon asymptotic red giant	1,000 kg	A small car
13	Carbon white dwarf	10,000 kg	100,000 meat patties or 1,000 wiener dogs or 100 football players or 10 small cars!

Section 20.3

Be careful when you introduce **black dwarfs** that students do not confuse them with black holes. Although black holes are not covered until the next chapter, students will certainly have heard of them. A black dwarf is nothing more than an old, burnt out white dwarf, analogous to the ashy coals found in the grill the day after the cookout!

Show images of **planetary nebulae**, such as those in Figures 20.10 and 20.11. Point out that the origin of the term “planetary nebula” stems from their appearance in smaller telescopes. They are fuzzy and disk shaped like planets. Emphasize that planetary nebulae are not the result of violent explosions. These will be discussed in the next chapter. In the case of planetary nebulae, the mass loss is fairly gentle; each planetary nebula itself is just the escaping outer layers of a red giant. ➡ **DEMO** To demonstrate why planetary nebulae often look like rings when they are actually shells of gas, blow up a balloon and shine a light through it. The edges of the balloon will look much darker than the center. Planetary nebulae are not usually symmetrical like balloons, however, and some may actually *be* rings of gas rather than spheres.

Expanding at a rate of 20 km/s it takes 50,000 years to move one parsec. Most planetary nebulae are smaller than 1 pc in radius. But it can be easily seen, then, that most planetary nebulae are at most a few tens of thousands of years old and by 100,000 years are completely dissipated. They typically contain about 0.2 solar masses, so by the time they have expanded out to about 1 pc they have densities indistinguishable from the interstellar matter (remembering from Chapter 18 we found that a 1 pc radius of space with a density of 10 atoms/cm³ has a total mass of one solar mass. So in the same volume of space, 0.2 solar masses will give a density of 2 atoms/cm³).

By the time you get to the discussion of the **white dwarf stage**, students will have had lots of practice answering the question “What counteracts gravity?” For the first time, the answer will not be gas pressure. To help students understand, use an analogy. Gas pressure can be visualized as a roomful of flying, bouncing, colliding, ping-pong balls (or marshmallows). Imagine sweeping all the ping-pong balls into a pile in the center of the room. With the broom, you can “contract” the “gas” so that the balls are all touching each other, but no further. (This analogy becomes useful in the next chapter, also; with a greater force than that supplied by a broom, the ping-pong balls or marshmallows would be smashed further, analogous to a neutron star.)

The luminosity and radius for a white dwarf was calculated at stage 13. Actually, this is just the beginning of the white dwarf stage; typical white dwarfs are significantly fainter than this but about the same size. Note that 0.01 solar radii is about the size of the Earth. So if white dwarfs have a size of 0.01 that of the Sun, they will have a volume $(0.01)^3 = 1$ million times smaller than the Sun. With a mass about equal to the Sun’s mass, the *average* density should be about 1 million times that of the Sun; just over 1 million gm/cm³. These numbers are easy to come by and are good examples of how a little understanding of concepts like density and volume can lead to immediate insight into the properties of a new object.

Finish these sections with an examination of Figure 20.12, which shows the **evolutionary track** of a Sun-like star all the way from the main sequence through the white dwarf stage. Project the transparency or slide and informally quiz students about each stage. What is the star called here? What is it doing? How did it get here? You can even include the stages of star formation from Chapter 19 if you wish to do a review.

As discussed briefly in the previous chapter, a very common misconception is that as stars evolve, they literally move. If you have not yet done so, focus on this misconception and clarify the idea for your students. We do say that stars “move” from the main sequence to the red giant branch and so on. This is taken literally and is combined with an earlier misconception that the

H-R diagram is actually a map of star positions in the sky. (Again, we do talk about the "position" of a star in the diagram.) Students who are making these errors are typically having difficulty understanding a graph and two dimensional classification.

Laboratory exercises in which students plot their own H–R diagrams help a lot in understanding this and other diagrams. Questions about red giants, supergiants, and white dwarfs help them understand why these stars are exceptional. Refer to Figure 20.17 for an excellent demonstration of the gradual evolution of stars to and from the main sequence. An advantage of this figure is that it shows stars plotted as dots; each one represents a single star. Relate the changes in star positions with the evolutionary tracks seen in earlier figures. Instead of talking about a star moving from this location to another in the diagram I prefer saying "The star now appears cooler but brighter and is plotted in this new location."

Additionally, you can show real star charts and compare them to the H–R diagram. Ask what is being plotted in each case. We have said that the stars actually move (proper motion), how does that affect their positions on the charts? Address the question of why stars don't gain mass with time (grow with age). Can't they pull more matter in with their strong gravity? Don't they sweep up more matter as they move through the galaxy? Don't stars collide and grow that way? All these questions have a logic to them, one based on models that work for these students. We have to help our students understand why such models do not work and to understand a new model for stellar evolution. Who knows how helpful this model may be to them under different circumstances!

Section 20.4

The **evolution of high-mass stars** is similar to the evolution of low-mass stars in the early stages. Begin with Figure 20.16 and compare the Sun and two high-mass stars. The text points out that the Sun will ascend the red giant branch almost horizontally, but a massive blue main-sequence star will move essentially horizontally to become a red supergiant. Ask students what horizontal motion on the H–R diagram represents; it means constant luminosity. Use the luminosity–temperature–radius relationship to find the radius of the red supergiant compared to the blue giant from which it evolved. (The origin of the name "supergiant" will become obvious.)

The constellation Orion has spectacular examples of both a high-mass, blue main-sequence star (Rigel) and a red supergiant (Betelgeuse). Use a star map to show your students how to find these stars; Orion is one of the easiest constellations to identify, so students should not have any trouble. The colors are noticeable to the naked eye. Aldebaran, another red giant, is nearby in Taurus, and it is sometimes considered the "red eye" of the bull. Antares, a red giant in Scorpio, is interesting because its name, which means "rival of Mars," refers to its color.

Section 20.5

Now that students understand the different stages in stellar evolution, they can return to an examination of the H–R diagram with new insights. Show again the H–R diagrams of the **globular and open star clusters** from Chapter 19 (Figures 19.18 and 19.19) and have students explain the differences, based on what they know about clusters and about stellar evolution. Figure 20.17 explains the main-sequence cutoff and its relationship with age very nicely. As discussed in the previous chapter, clusters are also important for testing our theories of stellar evolution; we do not have the opportunity to watch one star evolve through its entire lifetime (or even through a fraction of its lifetime), so instead we watch whole collections of stars.

Section 20.6

About half of all visible stars are actually multiple star systems; binaries are the most common. If their orbit around each other is small enough, then sometime during the evolution of one or both stars, they will interact. How far apart do the stars have to be in order *not* to interact? The largest supergiants are about 600 solar radii. Remembering that 200 solar radii equals 1 A.U., 600 solar radii equals 3 A.U. So we would expect stars separated by twice this amount, 6 A.U., will not interact; for stars with smaller masses and sizes this will reduce to about 2 A.U. The orbital periods, calculated from Kepler's third law, are in the range of 2 to 4 years. Any binary star with a period of a few years or less will probably experience interactions at some time; those with longer periods will probably not experience any interactions. The stars in the latter case evolve just like single stars. Visual binary systems are known to have long periods, decades to hundreds of years to thousands of years, so non-interacting binaries are not rare. When interactions do occur for the short period binaries, the effects are often not subtle!

Figure 20.21 is very helpful in showing the different types of binary and their Roche lobes. Then Figure 20.22 can be shown (both of these figures are available as transparencies) to trace the evolution of Algol-type binaries. Use a blank transparency overlay on Figure 20.22 and show the original masses of the two stars and the amount of mass transfer, about 2 solar masses, that has taken place.

To further develop why the mass transfer takes place, bring up the concept of hydrostatic equilibrium mentioned at the beginning of this chapter. The red giant is expanding and filling its lobe. It loses mass and, therefore, its gravitational force is reduced. But remember, it is expanding because its core is overwhelming gravity with additional pressure. If gravity is reduced by mass loss, the star becomes even more out of equilibrium and the expansion will be increased. To make matters worse, the Roche lobe also shrinks because of the reduction in gravity. All of this produces further mass loss. Two solar masses of material are transferred to the other star; this is a lot of material and has essentially reversed the roles of the two stars.

About the same amount of mass will be transferred back to the first star when star 2 starts to evolve. Can your students guess what may occur when a hot white dwarf is suddenly loaded with lots of hydrogen gas? They will find out in the next chapter, but it is interesting for them to speculate what the consequences will be!

Student Writing Questions

1. Describe the 100-million-year period of time *on Earth* when the Sun evolves from the main sequence to the top of the red giant branch. Include how the Earth might change and what the Sun will look like as the changes take place.
2. When the Sun is a red giant, describe how this will affect each of the planets. You may need to speculate a bit; just try to make sure there is reasonable scientific merit to your descriptions.
3. Hydrogen, carbon, oxygen, and nitrogen make up 99% of the composition of life-forms on Earth. Trace the path by which each of these elements have come to you. Begin with their origin and end with how they came to make up your body.

4. In this chapter you have seen many new, exotic types of evolved stars and other related objects. If you could choose only one type of object to visit up close, which would it be? Why do you choose this object? Describe what you might see.
5. Make up four different binary star systems, each having different masses and separations. Describe the evolution of each of the systems as far as you can with the information from this chapter. How does the evolution of the systems compare? What are the similarities? What are the differences? If you change the properties of one of the systems by just a little bit, will it significantly affect how the system evolves?

Answers to End of Chapter Exercises

Review and Discussion

1. The lowest mass main sequence stars last the longest because they have the least amount of fuel. Although this sounds contradictory, their low mass makes them fuse hydrogen very slowly. They have about 10 times less mass than the Sun but use it 10,000 times more slowly. Their hydrogen fuel lasts them a very long time.
2. Hydrostatic equilibrium is a balance between the force of gravity inward and the pressure of the hot gases pushing outward. A balance or equilibrium must be attained in order for a star to have a stable size. Main sequence stars have reached such a balance.
3. As a main sequence star, stars like the Sun fuse hydrogen into helium for about 10 billion years. As the hydrogen is depleted in the core, hydrogen fusion continues in a shell around the core for about another billion years.
4. Without the fusion of hydrogen into helium occurring in the core, the core no longer has an energy source. Gravity is then able to collapse the core, forcing major changes in the entire structure of the star.
5. When a star runs out of hydrogen in its core, the core collapses. As a result, the core's temperature increases and additional energy is radiated away. With a higher temperature, the fusion in the hydrogen shell around the core becomes more efficient. So the core puts out even more energy than it did as a main sequence star. The increased gas pressure pushes on the outer part of the star, expanding it into a red giant.
6. A star like the Sun will evolve into a red giant with a size about 100 times its current size. This is equivalent to about half an A.U.
7. It takes a star like the Sun about 100 million years to evolve from the main sequence to the top of the red giant branch.
8. Stars with about one-quarter the mass of the Sun never fuse helium.
9. By the time the helium core has formed, the core has a high density of electrons that produce a pressure unlike that of normal gas. This electron pressure is not influenced by temperature. When the core temperature finally reaches about 100 million K, helium begins to fuse into carbon. Normally, the increase in temperature would expand the core and help cool it off. Because of the electron pressure, this does not happen. The fusion of helium raises the core temperature, producing more and more helium fusion, so that over a few hours a large quantity of helium fuses into carbon. This rapid fusion of helium is known as the helium flash.

10. Red giants put out a strong stellar wind of gas. Apparently this gas cools sufficiently to form dust particles around the star. As a result, these stars contribute a significant amount of gas and dust to the interstellar medium. They are returning that which they once received.
11. Low mass stars eventually form a carbon core that collapses but is unable to attain a high enough temperature to allow the fusion of carbon. The outer part of the star continues to expand and as the final shells of hydrogen and helium fusion die out, this outer part of the star is ejected into space. This cloud of gas is known as a planetary nebula. The core of the star remains, continues to cool, and is known as a white dwarf.

High mass stars also form a carbon core that collapses and fuses into still heavier elements. This happens again and again, very quickly. With the formation of the last core, the star suddenly explodes.

12. The star has a carbon core in which a small amount fuses with helium to form oxygen. Around the core, helium continues to fuse into carbon, and outside this region, hydrogen fuses into helium. The temperature of the core is about 300 million K, too cool to fuse carbon.
13. A planetary nebula is the ejected shell of a giant star. It is in the shape of a spherical shell and is composed of relatively cool, thin gas. It was once the outer part of the star. This shell often appears as a ring; the thickest parts are seen in cross-section and look like a ring and the parts toward the core are thin and emit little light.
14. The remnant of a star at the center of its planetary nebula is the carbon core. As it cools and shrinks in size it becomes a white dwarf. Its size is about that of the Earth, its density about one million times that of the Sun, its luminosity about one thousand times less than the Sun. Although initially rather hot, the white dwarf will cool and fade until it becomes a black dwarf.
15. A helium white dwarf could exist today if it was a member of a binary in which its outer layers were stripped away by its companion just as it finished building up a helium core.
16. White dwarfs are difficult to observe because they are so faint. As they age, they get cooler and fainter and so become even harder to observe.
17. A black dwarf is a white dwarf that has cooled so much that it produces very little light. A black dwarf will continue to cool off forever. Its size will be maintained by the electron pressure it initially had. Black dwarfs will ultimately be just cold, dark embers of very dense matter. Such objects will always produce some light (infrared is more likely) because they will never have a temperature of 0 K. This takes a very long time, so long, that the universe may not be old enough to have produced any black dwarfs as yet.
18. The age of a star cluster is determined by the cut-off point of its main sequence. As stars evolve, more and more of the main sequence stars become red giants and die out, leaving only the fainter main sequence stars.
19. In a binary star system, the region around each star in which its gravity dominates is known as its Roche lobe. The region is teardrop-shaped; seen around both stars, it appears like a three-dimensional figure-8.
20. If two stars in a binary are sufficiently close to each other, during stellar evolution, mass-transfer may occur. Since the more massive star would be the first to evolve, it would

transfer mass over to the lower mass star. The mass-transfer can be so extensive that the lower mass star becomes the more massive of the two. This is what happened to the Algol system.

Conceptual Self-Test

1. F
2. T
3. T
4. T
5. F
6. F
7. F
8. F
9. F
10. T
11. C
12. B
13. A
14. B
15. A
16. C
17. A
18. B
19. B
20. D

Problems

1. First, calculate the amount of hydrogen that is fused into helium. From Chapter 16, the Sun is 71% hydrogen; 10% of that will be fused into helium. $2 \times 10^{30} \text{ kg} \times 0.71 \times 0.10 = 1.4 \times 10^{29} \text{ kg}$.

In the fusion process, 99.3% of the mass goes into helium; the remaining 0.7% is converted into energy. This mass is $0.007 \times 1.4 \times 10^{29} \text{ kg} = 9.9 \times 10^{26} \text{ kg}$.

Using $E = mc^2$ the total energy released can be calculated. $E = 9.9 \times 10^{26} \times (3 \times 10^8)^2$, $E = 8.9 \times 10^{43} \text{ J}$.

2. $10,000 = R^2(3000 / 6000)^4$, $R = 400$ solar radii. This star's radius is about 2 A.U. and would engulf the four inner planets of our solar system: Mercury, Venus, Earth, and Mars.
3. Refer to *More Precisely 17-2*:
 - (a) 1 A.U. = 215 solar radii. $L = 215^2(3000 / 5800)^4$, $L = 3300$ solar luminosities.
 - (b) The radius is five times bigger than in (a) and so L is 25 times bigger or 82,500 solar luminosities.
4. $0.0004 = R^2(12000 / 6000)^4$, $R = 0.005$ solar radii = 3480 km.
5. It looks like about a factor of 90 ± 10 . If C, N, and O were 10 times less abundant, the CNO energy production would decline by a factor of 10.

6. $L = (100)^2(1/3)^2$, $L = 123$. Because of the inverse square law, this star will appear of equal brightness at a distance that is $\sqrt{123}$ times the original distance, $11 \times 20 = 220$ pc.
7. $2.5 \log_{10}(100) = 5$, so a luminosity change of 100 corresponds to an absolute magnitude change of 5 over a given the time span of 10^5 years. This is a change in magnitude of 5×10^{-5} per year, which would not be noticeable to the naked eye, but possibly with modern instrumentation.
8. Density is mass divided by volume. For the core:

$$\text{Density} = \frac{0.25 \times 2 \times 10^{30}}{\frac{4}{3} \pi (1.5 \times 10^7)^3}$$

$$\text{Density} = 3.5 \times 10^7 \text{ kg/m}^3$$

The core density of the Sun, as stated in Chapter 16, is $1.5 \times 10^5 \text{ kg/m}^3$ so this is 230 times higher.

For the giant's envelope:

$$\text{Density} = \frac{0.5 \times 2 \times 10^{30}}{\frac{4}{3} \pi (7.5 \times 10^{10})^3}$$

$$\text{Density} = 5.6 \times 10^{-4} \text{ kg/m}^3$$

This 3.7×10^{-9} times lower than the core density of the Sun.

9. Neptune is at 30.1 A.U. This distance is 4.5×10^9 km. At 50 km/s, the time it takes to travel this distance will be 9×10^7 s or about 2.9 years.

Proxima Centauri, the nearest star, is 1.30 pc distant, which is 4×10^{13} km. This will take 8.1×10^{11} seconds or 26,000 years.

- 10.

$$v_{esc} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 1.1 \times 2 \times 10^{30}}{0.008 \times 7 \times 10^8}}$$

$$v_{esc} = 7.2 \times 10^6 \text{ m/s}$$

$$\text{Surface gravity} = \frac{6.67 \times 10^{-11} \times 1.1 \times 2 \times 10^{30}}{(0.008 \times 7 \times 10^8)^2}$$

$$\text{Surface gravity} = 4.7 \times 10^6 \text{ m/s}^2$$

Divided by 9.8 m/s^2 gives 480,000 times the Earth's gravity

11. $R_1^2 20,000^4 = R_2^2 4000^4$, $R_2/R_1 = 25$ times.
12. $L_1/1^2 = L_2/1.6^2$, $L_2/L_1 = 2.56$. This is almost exactly a factor of one magnitude.

13. In solar units $L = M^4$. We also know the main sequence lifetime of a star (t) is M/L times the lifetime of the Sun. That is $t = 10^{10} M/L$. Substituting for L gives $t = 10^{10}/M^3$.
- (a) For a value of $t = 400$ million years gives $4 \times 10^8 = 10^{10}/M^3$, $M = 2.9$ solar masses.
- (b) $2 \times 10^9 = 10^{10}/M^3$, $M = 1.7$ solar masses.
14. (a) Angular momentum is mvr but $v = \sqrt{2GM/r}$. Here m is Jupiter's mass, M the Sun's mass and r is the radius of the orbit. Substituting for v gives angular momentum of $m\sqrt{2GM}r$. If angular momentum is conserved then $M_1 r_1 = M_2 r_2$, $1 \times 5.2 = 0.8 \times r_2$, $r_2 = 6.5$ A.U. The period will be $P^2 = 6.5^3 / 0.8$, $P = 18.5$ yrs.
- (b) The new semi-major axis is $6.5 \times 2.5 = 16.25$. The new mass is 0.5. $P^2 = 16.25^3 / 0.5$, $P = 93$ yrs.
15. Using Kepler's third law, $2^2 = a^3 / (1 + 2)$, $a = 2.3$ A.U. Using conservation of angular momentum, $1 \times 2 \times \sqrt{2.3} = 1.2 \times 1.8 \times \sqrt{R}$, $R = 1.97$ A.U. The new period is determined from $P^2 = 1.97^3 / (1.2 + 1.8)$, $P = 1.6$ yrs.

Resource Information

Student CD Media

Movies/Animations

Death of a Sun
 Death of a Sun Part 2
 Formation of the Helix Nebula
 Bi-Polar Planetary Nebula
 H-R Diagram Tracks Stellar Evolution

Interactive Student Tutorials

None

Physlet Illustrations

None

Transparencies

T-184	Figure 20.1	Hydrostatic Equilibrium	p. 517
T-185	Figure 20.3/7	Hydrogen-Shell Burning and Helium-Shell Burning	p. 519/523
T-186	Table 20.1	Evolution of a Sun-like Star	p. 521
T-187	Figure 20.6	Red Giant Branch Revisited	p. 523
T-188	Figure 20.8	G-Type Star Evolution	p. 524
T-189	Figure 20.10	Ejected Envelope	p. 526
T-190	Figure 20.11	Planetary Nebulae	p. 527
T-191	Figure 20.12	White Dwarf on the H-R Diagram	p. 527
T-192	Figure 20.16	High-Mass Evolutionary Tracks	p. 531
T-193	Figure 20.17	Cluster Evolution on the H-R Diagram	p. 534
T-194	Figures 20.18/19	Newborn and Young Cluster H-R Diagrams	p. 535
T-195	Figure 20.20	Old Cluster H-R Diagram	p. 536
T-196	Figure 20.21	Stellar Roche Lobes	p. 537

Materials

A poster-sized periodic table is helpful when discussing core fusion in high-mass stars.

Suggested Readings

Goldstein, Alan. "Touring a stellar graveyard." *Astronomy* (Dec 1997). p. 84. A guide to observing planetary nebula.

Hayden, Thomas. "Curtain call." *Astronomy* (Jan 2000). p. 44. Describes the life cycle of the Sun from the perspective of the consequences on Earth.

Iben, Icko, Jr.; Tutukov, Alexander V. "The lives of stars: from birth to death and beyond." *Sky & Telescope* (Dec 1997). p. 36. Describes the life cycle of stars.

Kaler, J. "Lighting the Nebulae." *Mercury* (July/August 2002). p.17. Excellent article discussing the theory of planetary nebula formation as well as the history of planetary nebula research. Many nice images and diagrams.

Kaler, J. "Eyewitness to stellar evolution." *Sky & Telescope* (Mar 1999). p. 40. Describes the life cycle of stars using examples of objects visible in the night time sky.

Kwok, S. "Planetary Nebulae: Shrouds of Mystery." *Mercury* (July/August 2002). p. 24. More background and many more images. This and the Kaler article mentioned above make this back-issue worth purchasing.

Kwok, Sun. "What is the real shape of the Ring Nebula?" *Sky & Telescope* (July 2000). p. 32. Describes the structure of the Ring Nebula.

Naeye, Robert. "White dwarfs by the trillions?" *Astronomy* (Apr 2000). p. 22. A brief news report on MACHO results about old white dwarfs in the Milky Way Galaxy.

Naeye, Robert. "Stars' last gasps." *Astronomy* (Apr 1998). p. 36. Describes HST images of planetary nebula.

Naeye, Robert. "The beginning and the end." *Astronomy* (Sept 1999). p. 36. Features a spectacular HST image of NGC3603 which contains examples of all stages of stellar evolution.

Naeye, R. "Stellar Flash-Bulb." *Mercury* (May/June 2003). p. 32. Nice images and discussion of a strange star, V838 Monocerotis, which has characteristics of a planetary nebula, but it isn't one. A pulse of energy from the star is seen propagating through a surrounding nebula.

Naeye, Robert. "White dwarfs by the trillions?" *Astronomy* (Apr 2000). p. 22. A brief news report on MACHO results about old white dwarfs in the Milky Way galaxy.

Sandage, Allan. "Twinkle twinkle." *Natural History* (Feb 2000). p. 64. An overview of stellar evolution and the H-R diagram.

Southwell, Karen. "Inside a star's cocoon." *Astronomy* (May 1997). p. 60. Discusses possible mechanisms for producing the structure seen in HST images of planetary nebula.

Notes and Ideas

Class time spent on material: Estimated: _____ Actual: _____

Demonstration and activity materials:

Notes for next time: