51. (a) If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad. The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where $d = 0.500\,\text{m}$ is the length of the rod. The angular momentum of the wad is $mvd/2$ where $m = 0.0500\,\text{kg}$ and $v = 3.00$ m/s are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$ where $I = (2M+m)(d/2)^2$ and $M = 2.00$ kg is the mass of each of the balls. We solve $mvd/2 = (2M+m)(d/2)^2\omega$ for the angular speed:

$$\omega = \frac{2mv}{(2M+m)d} = \frac{2(0.0500)(3.00)}{(2(2.00)+0.0500)(0.500)} = 0.148 \text{ rad/s} .$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is $K_f/K_i = I\omega^2/mv^2$. When $I = (2M+m)d^2/4$ and $\omega = 2mv/(2M+m)d$ are substituted, this becomes

$$\frac{K_f}{K_i} = \frac{m}{2M+m} = \frac{0.0500}{2(2.00)+0.0500} = 0.0123 .$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance h, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a $90°$ arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mgd/2$. If it swings up to the angular position θ, as measured from its lowest point, then its final height is $(d/2)(1-\cos\theta)$ above the lowest point and its final potential energy is $U_f = mg(d/2)(1-\cos\theta)$. The initial kinetic energy is the sum of that of the balls and wad: $K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M+m)(d/2)^2\omega^2$. At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$mg\,\frac{d}{2} + \frac{1}{2}(2M+m)\left(\frac{d}{2}\right)^2\omega^2 = mg\,\frac{d}{2}\,(1-\cos\theta) .$$

When this equation is solved for $\cos\theta$, the result is

$$\begin{aligned}
\cos\theta &= -\frac{1}{2}\left(\frac{2M+m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 \\
&= -\frac{1}{2}\left(\frac{2(2.00\,\text{kg})+0.0500\,\text{kg}}{(0.0500\,\text{kg})(9.8\,\text{m/s}^2)}\right)\left(\frac{0.500\,\text{m}}{2}\right)(0.148\,\text{rad/s})^2 \\
&= -0.0226 .
\end{aligned}$$

Consequently, the result for θ is $91.3°$. The total angle through which it has swung is $90° + 91.3° = 181°$.