43. (a) During the time interval Δt, the light emitted from galaxy A has traveled a distance $c\Delta t$. Meanwhile, the distance between Earth and the galaxy has expanded from r to $r' = r + r\alpha\Delta t$. Let $c\Delta t = r' = r + r\alpha\Delta t$, which leads to

$$\Delta t = \frac{r}{c - r\alpha} \; .$$

(b) The detected wavelength λ' is longer then λ by $\lambda\alpha\Delta t$ due to the expansion of the universe: $\lambda' = \lambda + \lambda\alpha\Delta t$. Thus,

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \alpha\Delta t = \frac{\alpha r}{c - \alpha r} \; .$$

(c) We use the binomial expansion formula (see Appendix E):

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \qquad (x^2 < 1)$$

to obtain

$$\begin{aligned}
\frac{\Delta\lambda}{\lambda} &= \frac{\alpha r}{c - \alpha r} = \frac{\alpha r}{c}\left(1 - \frac{\alpha r}{c}\right)^{-1} \\
&= \frac{\alpha r}{c}\left[1 + \frac{-1}{1!}\left(-\frac{\alpha r}{c}\right) + \frac{(-1)(-2)}{2!}\left(-\frac{\alpha r}{c}\right)^2 + \cdots\right] \\
&\approx \frac{\alpha r}{c} + \left(\frac{\alpha r}{c}\right)^2 + \left(\frac{\alpha r}{c}\right)^3 \; .
\end{aligned}$$

(d) When only the first term in the expansion for $\Delta\lambda/\lambda$ is retained we have

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\alpha r}{c} \; .$$

(e) We set

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = \frac{Hr}{c}$$

and compare with the result of part (d) to obtain $\alpha = H$.

(f) We use the formula $\Delta\lambda/\lambda = \alpha r/(c - \alpha r)$ to solve for r:

$$r = \frac{c(\Delta\lambda/\lambda)}{\alpha(1 + \Delta\lambda/\lambda)} = \frac{(2.998 \times 10^8\,\mathrm{m/s})(0.050)}{(0.0193\,\mathrm{m/s{\cdot}ly})(1 + 0.050)} = 7.4 \times 10^8 \text{ ly} \; .$$

(g) From the result of part (a),

$$\Delta t = \frac{r}{c - \alpha r} = \frac{(7.4 \times 10^8\,\mathrm{ly})(9.46 \times 10^{15}\,\mathrm{m/ly})}{2.998 \times 10^8\,\mathrm{m/s} - (0.0193\,\mathrm{m/s{\cdot}ly})(7.4 \times 10^8\,\mathrm{ly})} = 2.5 \times 10^{16} \text{ s} \; ,$$

which is equivalent to 7.8×10^8 y.

(h) Letting $r = c\Delta t$, we solve for Δt:

$$\Delta t = \frac{r}{c} = \frac{7.4 \times 10^8\,\mathrm{ly}}{c} = 7.4 \times 10^8 \text{ y} \; .$$

(i) The distance is given by

$$r = c\Delta t = c(7.8 \times 10^8\,\mathrm{y}) = 7.8 \times 10^8 \text{ ly} \; .$$

(j) From the result of part (f),

$$r_{\mathrm{B}} = \frac{c(\Delta\lambda/\lambda)}{\alpha(1 + \Delta\lambda/\lambda)} = \frac{(2.998 \times 10^8\,\mathrm{m/s})(0.080)}{(0.0193\,\mathrm{mm/s{\cdot}ly})(1 + 0.080)} = 1.15 \times 10^9 \text{ ly} \; .$$