

15. We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If d is the depth of the lake and ρ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho g d$, where p_0 is atmospheric pressure. Since $p_1 V_1 = n R T_1$, the number of moles of gas in the bubble is $n = p_1 V_1 / R T_1 = (p_0 + \rho g d) V_1 / R T_1$, where V_1 is the volume of the bubble at the bottom of the lake and T_1 is the temperature there. At the surface of the lake the pressure is p_0 and the volume of the bubble is $V_2 = n R T_2 / p_0$. We substitute for n to obtain

$$\begin{aligned}
 V_2 &= \frac{T_2}{T_1} \frac{p_0 + \rho g d}{p_0} V_1 \\
 &= \left(\frac{293 \text{ K}}{277 \text{ K}} \right) \left(\frac{1.013 \times 10^5 \text{ Pa} + (0.998 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(40 \text{ m})}{1.013 \times 10^5 \text{ Pa}} \right) (20 \text{ cm}^3) \\
 &= 100 \text{ cm}^3 .
 \end{aligned}$$