18. (a) The rotational frequency (in revolutions per second) is identical to the time-dependent voltage frequency (in cycles per second, or Hertz). This conclusion should not be considered obvious, and the calculation shown in part (b) should serve to reinforce it.

(b) First, we define angle relative to the plane of Fig. 31-41, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0 which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position). Since the area of the semicircle is $\pi a^2/2$ then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ, is

$$A = A_0 + \frac{\pi a^2}{2} \cos\theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi f t$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(A_0 + \frac{\pi a^2}{2}\cos\theta\right)}{dt} = -B\frac{\pi a^2}{2}\,\frac{d\cos(2\pi f t)}{dt}$$

which yields $\mathcal{E} = B\pi^2 a^2 f \sin(2\pi f t)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude: $\mathcal{E}_{\max} = B\pi^2 a^2 f$.