

27. (a) Let ΔT be the change in temperature and κ be the coefficient of linear expansion for copper. Then $\Delta L = \kappa L \Delta T$ and

$$\frac{\Delta L}{L} = \kappa \Delta T = (1.7 \times 10^{-5}/\text{K})(1.0^\circ\text{C}) = 1.7 \times 10^{-5} .$$

This is equivalent to 0.0017%. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of κ used in this calculation is not inconsistent with the other units involved. Incorporating a factor of 2 for the two-dimensional nature of A , the fractional change in area is

$$\frac{\Delta A}{A} = 2\kappa \Delta T = 2(1.7 \times 10^{-5}/\text{K})(1.0^\circ\text{C}) = 3.4 \times 10^{-5}$$

which is 0.0034%. For small changes in the resistivity ρ , length L , and area A of a wire, the change in the resistance is given by

$$\Delta R = \frac{\partial R}{\partial \rho} \Delta \rho + \frac{\partial R}{\partial L} \Delta L + \frac{\partial R}{\partial A} \Delta A .$$

Since $R = \rho L/A$, $\partial R/\partial \rho = L/A = R/\rho$, $\partial R/\partial L = \rho/A = R/L$, and $\partial R/\partial A = -\rho L/A^2 = -R/A$. Furthermore, $\Delta \rho/\rho = \alpha \Delta T$, where α is the temperature coefficient of resistivity for copper ($4.3 \times 10^{-3}/\text{K} = 4.3 \times 10^{-3}/\text{C}^\circ$, according to Table 27-1). Thus,

$$\begin{aligned} \frac{\Delta R}{R} &= \frac{\Delta \rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} \\ &= (\alpha + \kappa - 2\kappa) \Delta T = (\alpha - \kappa) \Delta T \\ &= (4.3 \times 10^{-3}/\text{C}^\circ - 1.7 \times 10^{-5}/\text{C}^\circ) (1.0 \text{ C}^\circ) = 4.3 \times 10^{-3} . \end{aligned}$$

This is 0.43%, which we note (for the purposes of the next part) is primarily determined by the $\Delta \rho/\rho$ term in the above calculation.

- (b) The fractional change in resistivity is much larger than the fractional change in length and area. Changes in length and area affect the resistance much less than changes in resistivity.