59. We use the result of problem 54:

$$P_{\text{avg}} = \frac{(E)_m^2 R}{2Z^2} = \frac{(E)_m^2 R}{2\left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]} \; .$$

We use the expression $Z = \sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}$ for the impedance in terms of the angular frequency.

(a) Considered as a function of C, P_{avg} has its largest value when the factor $R^2 + (\omega_d L - 1/\omega_d C)^2$ has the smallest possible value. This occurs for $\omega_d L = 1/\omega_d C$, or

$$C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi)^2 (60.0\,\text{Hz})^2 (60.0 \times 10^{-3}\,\text{H})} = 1.17 \times 10^{-4}\ \text{F} \; .$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as C becomes very small. Thus, the smallest average power occurs for $C = 0$ (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}} = \frac{(E)_m^2}{2R} \; ,$$

so the maximum average power is in the resonant case and is equal to

$$P_{\text{avg}} = \frac{(30.0\,\text{V})^2}{2(5.00\,\Omega)} = 90.0\ \text{W} \; .$$

On the other hand, the minimum average power is $P_{\text{avg}} = 0$ (as it would be for an open switch).

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0 \; ,$$

which implies $\phi = 0$. On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that $\phi = -90°$.

(e) At maximum power, the power factor is $\cos\phi = \cos 0° = 1$, and at minimum power, it is $\cos\phi = \cos(-90°) = 0$.