

60. (a) We use the result of problem 3:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ nm} \cdot \text{eV}}{1.00 \text{ nm}} = 1.24 \text{ keV}$$

and for the electron

$$K = \frac{p^2}{2m_e} = \frac{(h/\lambda)^2}{2m_e} = \frac{(hc/\lambda)^2}{2m_e c^2} = \frac{1}{2(0.511 \text{ MeV})} \left(\frac{1240 \text{ eV} \cdot \text{nm}}{1.00 \text{ nm}} \right)^2 = 1.50 \text{ eV} .$$

(b) In this case, we find

$$E_{\text{photon}} = \frac{1240 \text{ nm} \cdot \text{eV}}{1.00 \times 10^{-6} \text{ nm}} = 1.24 \times 10^9 \text{ eV} = 1.24 \text{ GeV} ,$$

and for the electron (recognizing that $1240 \text{ eV} \cdot \text{nm} = 1240 \text{ MeV} \cdot \text{fm}$)

$$\begin{aligned} K &= \sqrt{p^2 c^2 + (m_e c^2)^2} - m_e c^2 = \sqrt{(hc/\lambda)^2 + (m_e c^2)^2} - m_e c^2 \\ &= \sqrt{\left(\frac{1240 \text{ MeV} \cdot \text{fm}}{1.00 \text{ fm}} \right)^2 + (0.511 \text{ MeV})^2} - 0.511 \text{ MeV} \\ &= 1.24 \times 10^3 \text{ MeV} = 1.24 \text{ GeV} . \end{aligned}$$

We note that at short λ (large K) the kinetic energy of the electron, calculated with the relativistic formula, is about the same as that of the photon. This is expected since now $K \approx E \approx pc$ for the electron, which is the same as $E = pc$ for the photon.