

65. Let  $h$  be the thickness of the slab and  $A$  be its area. Then, the rate of heat flow through the slab is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{h}$$

where  $k$  is the thermal conductivity of ice,  $T_H$  is the temperature of the water ( $0^\circ\text{C}$ ), and  $T_C$  is the temperature of the air above the ice ( $-10^\circ\text{C}$ ). The heat leaving the water freezes it, the heat required to freeze mass  $m$  of water being  $Q = L_F m$ , where  $L_F$  is the heat of fusion for water. Differentiate with respect to time and recognize that  $dQ/dt = P_{\text{cond}}$  to obtain

$$P_{\text{cond}} = L_F \frac{dm}{dt} .$$

Now, the mass of the ice is given by  $m = \rho Ah$ , where  $\rho$  is the density of ice and  $h$  is the thickness of the ice slab, so  $dm/dt = \rho A(dh/dt)$  and

$$P_{\text{cond}} = L_F \rho A \frac{dh}{dt} .$$

We equate the two expressions for  $P_{\text{cond}}$  and solve for  $dh/dt$ :

$$\frac{dh}{dt} = \frac{k(T_H - T_C)}{L_F \rho h} .$$

Since  $1 \text{ cal} = 4.186 \text{ J}$  and  $1 \text{ cm} = 1 \times 10^{-2} \text{ m}$ , the thermal conductivity of ice has the SI value  $k = (0.0040 \text{ cal/s}\cdot\text{cm}\cdot\text{K})(4.186 \text{ J/cal})/(1 \times 10^{-2} \text{ m/cm}) = 1.674 \text{ W/m}\cdot\text{K}$ . The density of ice is  $\rho = 0.92 \text{ g/cm}^3 = 0.92 \times 10^3 \text{ kg/m}^3$ . Thus,

$$\frac{dh}{dt} = \frac{(1.674 \text{ W/m}\cdot\text{K})(0^\circ\text{C} + 10^\circ\text{C})}{(333 \times 10^3 \text{ J/kg})(0.92 \times 10^3 \text{ kg/m}^3)(0.050 \text{ m})} = 1.1 \times 10^{-6} \text{ m/s} = 0.40 \text{ cm/h} .$$