

22. (a) First, we observe that a large portion of the figure contributes flux which “cancels out.” The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is  $b - a$ , then a strip below the wire (where the strip borders the long wire, and extends a distance  $b - a$  away from it) has exactly the equal-but-opposite flux which cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B dA = \int_{b-a}^a \left( \frac{\mu_0 i}{2\pi r} \right) (b dr) = \frac{\mu_0 i b}{2\pi} \ln \left( \frac{a}{b-a} \right) .$$

Faraday’s law, then, (with SI units and 3 significant figures understood) leads to

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \left[ \frac{\mu_0 i b}{2\pi} \ln \left( \frac{a}{b-a} \right) \right] \\ &= -\frac{\mu_0 b}{2\pi} \ln \left( \frac{a}{b-a} \right) \frac{di}{dt} = -\frac{\mu_0 b}{2\pi} \ln \left( \frac{a}{b-a} \right) \frac{d}{dt} \left( \frac{9}{2} t^2 - 10t \right) \\ &= \frac{-\mu_0 b (9t - 10)}{2\pi} \ln \left( \frac{a}{b-a} \right) . \end{aligned}$$

With  $a = 0.120$  m and  $b = 0.160$  m, then, at  $t = 3.00$  s, the magnitude of the emf induced in the rectangular loop is

$$|\mathcal{E}| = \frac{(4\pi \times 10^{-7}) (0.16)(9(3) - 10)}{2\pi} \ln \left( \frac{0.12}{0.16 - 0.12} \right) = 5.98 \times 10^{-7} \text{ V} .$$

- (b) We note that  $\frac{di}{dt} > 0$  at  $t = 3$  s. The situation is roughly analogous to that shown in Fig. 31-5(c). From Lenz’s law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.