31. The connection between angle θ (measured from vertical – see Fig. 8-29) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy mgh) is given by $h = L(1 - \cos\theta)$ where L is the length of the pendulum.

(a) Using this formula (or simply using intuition) we see the initial height is $h_1 = 2L$, and of course $h_2 = 0$. We use energy conservation in the form of Eq. 8-17.

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mg\,(2L) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

This leads to $v = 2\sqrt{gL}$.

(b) The ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2/r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m\,\frac{v^2}{r} \implies T = m\left(g + \frac{4gL}{L}\right) = 5mg\ .$$

(c) The pendulum is now started (with zero speed) at $\theta_i = 90°$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ, then Newton's second law applied to the axis along the rod yields

$$T - mg\cos\theta = m\,\frac{v^2}{r}$$

which (since $r = L$) implies $v^2 = gL(1 - \cos\theta)$ at the position we are looking for. Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 + mgL &= \frac{1}{2}mv^2 + mgL\,(1 - \cos\theta) \\ gL &= \frac{1}{2}\,(gL(1 - \cos\theta)) + gL\,(1 - \cos\theta) \end{aligned}$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5°\ .$$