53. Let $\theta_1 = 45°$ be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is $n \sin\theta_3 \geq 1$. We want to find the smallest value of the index of refraction n for which this inequality holds. The law of refraction, applied to the first surface, yields $n \sin\theta_2 = \sin\theta_1$. Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus, the condition for total internal reflection becomes $1 \leq n \sin(90° - \theta_2) = n \cos\theta_2$. Squaring this equation and using $\sin^2\theta_2 + \cos^2\theta_2 = 1$, we obtain $1 \leq n^2(1 - \sin^2\theta_2)$. Substituting $\sin\theta_2 = (1/n)\sin\theta_1$ now leads to

$$1 \leq n^2\left(1 - \frac{\sin^2\theta_1}{n^2}\right) = n^2 - \sin^2\theta_1 \ .$$

The largest value of n for which this equation is true is the value for which $1 = n^2 - \sin^2\theta_1$. We solve for n:

$$n = \sqrt{1 + \sin^2\theta_1} = \sqrt{1 + \sin^2 45°} = 1.22 \ .$$