57. (a) The diagram below shows a cross section, through the center of the cube and parallel to a face. L is the length of a cube edge and S labels the spot. A portion of a ray from the source to a cube face is also shown. Light leaving the source at a small angle θ is refracted at the face and leaves the cube; light leaving at a sufficiently large angle is totally reflected. The light that passes through the cube face forms a circle, the radius r being associated with the critical angle for total internal reflection. If θ_c is that angle, then

$$\sin\theta_c = \frac{1}{n}$$

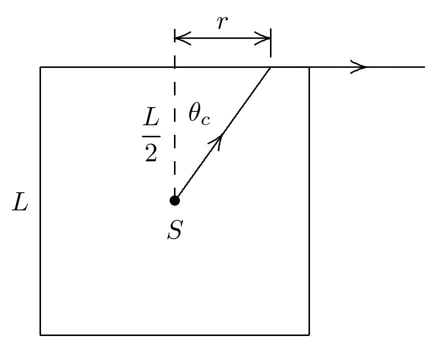
where n is the index of refraction for the glass. As the diagram shows, the radius of the circle is given by $r = (L/2)\tan\theta_c$. Now,

$$\tan\theta_c = \frac{\sin\theta_c}{\cos\theta_c} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}} = \frac{1/n}{\sqrt{1-(1/n)^2}} = \frac{1}{\sqrt{n^2-1}}$$

and the radius of the circle is

$$r = \frac{L}{2\sqrt{n^2-1}} = \frac{10\,\text{mm}}{2\sqrt{(1.5)^2-1}} = 4.47\,\text{mm}\ .$$

If an opaque circular disk with this radius is pasted at the center of each cube face, the spot will not be seen (provided internally reflected light can be ignored).



(b) There must be six opaque disks, one for each face. The total area covered by disks is $6\pi r^2$ and the total surface area of the cube is $6L^2$. The fraction of the surface area that must be covered by disks is

$$f = \frac{6\pi r^2}{6L^2} = \frac{\pi r^2}{L^2} = \frac{\pi(4.47\,\text{mm})^2}{(10\,\text{mm})^2} = 0.63\ .$$