21. (a) and (b) Schrödinger's equation for the region $x > L$ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi = 0 \ ,$$

where $E - U_0 < 0$. If $\psi^2(x) = Ce^{-2kx}$, then $\psi(x) = C'e^{-kx}$, where C' is another constant satisfying $C'^2 = C$. Thus $d^2\psi/dx^2 = 4k^2 C' e^{-kx} = 4k^2\psi$ and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi = k^2\psi + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi \ .$$

This is zero provided that

$$k^2 = \frac{8\pi^2 m}{h^2}\left[U_0 - E\right] \ .$$

The quantity on the right-hand side is positive, so k is real and the proposed function satisfies Schrödinger's equation. If k is negative, however, the proposed function would be physically unrealistic. It would increase exponentially with x. Since the integral of the probability density over the entire x axis must be finite, ψ diverging as $x \to \infty$ would be unacceptable. Therefore, we choose

$$k = \frac{2\pi}{h}\sqrt{2m\left(U_0 - E\right)} > 0 \ .$$