39. (a) Assume the bullet becomes embedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet-block system is conserved during the collision. Let m be the mass of the bullet, M be the mass of the block, v_0 be the initial speed of the bullet, and v be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m+M)v$, so

$$v = \frac{mv_0}{m+M} = \frac{(0.050\,\text{kg})(150\,\text{m/s})}{0.050\,\text{kg} + 4.0\,\text{kg}} = 1.85\,\text{m/s} \ .$$

When the block is in its initial position the spring and gravitational forces balance, so the spring is elongated by Mg/k. After the collision, however, the block oscillates with simple harmonic motion about the point where the spring and gravitational forces balance with the bullet embedded. At this point the spring is elongated a distance $\ell = (M+m)g/k$, somewhat different from the initial elongation. Mechanical energy is conserved during the oscillation. At the initial position, just after the bullet is embedded, the kinetic energy is $\frac{1}{2}(M+m)v^2$ and the elastic potential energy is $\frac{1}{2}k(Mg/k)^2$. We take the gravitational potential energy to be zero at this point. When the block and bullet reach the highest point in their motion the kinetic energy is zero. The block is then a distance y_m above the position where the spring and gravitational forces balance. Note that y_m is the amplitude of the motion. The spring is compressed by $y_m - \ell$, so the elastic potential energy is $\frac{1}{2}k(y_m - \ell)^2$. The gravitational potential energy is $(M+m)gy_m$. Conservation of mechanical energy yields

$$\frac{1}{2}(M+m)v^2 + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 = \frac{1}{2}k(y_m - \ell)^2 + (M+m)gy_m \ .$$

We substitute $\ell = (M+m)g/k$. Algebraic manipulation leads to

$$\begin{aligned} y_m &= \sqrt{\frac{(m+M)v^2}{k} - \frac{mg^2}{k^2}(2M+m)} \\ &= \sqrt{\frac{(0.050\,\text{kg} + 4.0\,\text{kg})(1.85\,\text{m/s})^2}{500\,\text{N/m}} - \frac{(0.050\,\text{kg})(9.8\,\text{m/s}^2)^2}{(500\,\text{N/m})^2}\left[2(4.0\,\text{kg}) + 0.050\,\text{kg}\right]} \\ &= 0.166\ \text{m} \ . \end{aligned}$$

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050\,\text{kg})(150\,\text{m/s})^2 = 563\,\text{J}$. The kinetic energy of the bullet-block system just after the collision is

$$E = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.050\,\text{kg} + 4.0\,\text{kg})(1.85\,\text{m/s})^2 = 6.94\ \text{J} \ .$$

Since the block does not move significantly during the collision, the elastic and gravitational potential energies do not change. Thus, E is the energy that is transferred. The ratio is $E/E_0 = (6.94\,\text{J})/(563\,\text{J}) = 0.0123$ or 1.23%.