

23. We note that if the pendulum shortens, its frequency of oscillation will increase, thereby causing it to record more units of time (“ticks”) than have actually passed during an interval. Thus, as the pendulum contracts (this problem involves cooling the brass wire), the pendulum will “run fast.” Since the “direction” of the error has now been discussed, the remaining calculations are understood to be in absolute value. The differential of the equation for the pendulum period in Chapter 16 is

$$dT = \frac{1}{2}(2\pi) \frac{dL}{\sqrt{gL}}$$

which we divide by the period equation  $T = 2\pi\sqrt{L/g}$  (and replace differentials with  $|\Delta|$ 's) to obtain

$$\frac{|\Delta T|}{T} = \frac{1}{2} \frac{|\Delta L|}{L} = \frac{1}{2} \alpha |\Delta T|$$

where we use Eq. 19-9 (in absolute value) in the last step. Thus, the (unitless) fractional change in period is

$$\frac{|\Delta T|}{T} = \frac{1}{2} (19 \times 10^{-6}/\text{C}^\circ) (20 \text{ C}^\circ) = 1.9 \times 10^{-4}$$

using Table 19-2. We can express this in “mixed units” fashion by recalling that there are 3600 s in an hour. Thus,  $(3600 \text{ s/h})(1.9 \times 10^{-4}) = 0.68 \text{ s/h}$ .