5. (a) Suppose that at time t_1, the moon is starting a revolution (on the verge of going behind Jupiter, say) and that at this instant, the distance between Jupiter and Earth is ℓ_1. The time of the start of the revolution as seen on Earth is $t_1^* = t_1 + \ell_1/c$. Suppose the moon starts the next revolution at time t_2 and at that instant, the Earth-Jupiter distance is ℓ_2. The start of the revolution as seen on Earth is $t_2^* = t_2 + \ell_2/c$. Now, the actual period of the moon is given by $T = t_2 - t_1$ and the period as measured on Earth is

$$T^* = t_2^* - t_1^* = t_2 - t_1 + \frac{\ell_2}{c} - \frac{\ell_1}{c} = T + \frac{\ell_2 - \ell_1}{c} \, .$$

The period as measured on Earth is longer than the actual period. This is due to the fact that Earth moves during a revolution, and light takes a finite time to travel from Jupiter to Earth. For the situation depicted in Fig. 34-38, light emitted at the end of a revolution travels a longer distance to get to Earth than light emitted at the beginning. Suppose the position of Earth is given by the angle θ, measured from x. Let R be the radius of Earth's orbit and d be the distance from the Sun to Jupiter. The law of cosines, applied to the triangle with the Sun, Earth, and Jupiter at the vertices, yields $\ell^2 = d^2 + R^2 - 2dR\cos\theta$. This expression can be used to calculate ℓ_1 and ℓ_2. Since Earth does not move very far during one revolution of the moon, we may approximate $\ell_2 - \ell_1$ by $(d\ell/dt)T$ and T^* by $T + (d\ell/dt)(T/c)$. Now

$$\frac{d\ell}{dt} = \frac{2Rd\sin\theta}{\sqrt{d^2 + R^2 - 2dR\cos\theta}}\frac{d\theta}{dt} = \frac{2vd\sin\theta}{\sqrt{d^2 + R^2 - 2dR\cos\theta}} \, ,$$

where $v = R\,(d\theta/dt)$ is the speed of Earth in its orbit. For $\theta = 0$, $(d\ell/dt) = 0$ and $T^* = T$. Since Earth is then moving perpendicularly to the line from the Sun to Jupiter, its distance from the planet does not change appreciably during one revolution of the moon. On the other hand, when $\theta = 90°$, $d\ell/dt = vd/\sqrt{d^2 + R^2}$ and

$$T^* = T\left(1 + \frac{vd}{c\sqrt{d^2 + R^2}}\right) \, .$$

The Earth is now moving parallel to the line from the Sun to Jupiter, and its distance from the planet changes during a revolution of the moon.

(b) Our notation is as follows: t is the actual time for the moon to make N revolutions, and t^* is the time for N revolutions to be observed on Earth. Then,

$$t^* = t + \frac{\ell_2 - \ell_1}{c} \, ,$$

where ℓ_1 is the Earth-Jupiter distance at the beginning of the interval and ℓ_2 is the Earth-Jupiter distance at the end. Suppose Earth is at position x at the beginning of the interval, and at y at the end. Then, $\ell_1 = d - R$ and $\ell_2 = \sqrt{d^2 + R^2}$. Thus,

$$t^* = t + \frac{\sqrt{d^2 + R^2} - (d - R)}{c} \, .$$

A value can be found for t by measuring the observed period of revolution when Earth is at x and multiplying by N. We note that the observed period is the true period when Earth is at x. The time interval as Earth moves from x to y is t^*. The difference is

$$t^* - t = \frac{\sqrt{d^2 + R^2} - (d - R)}{c} \, .$$

If the radii of the orbits of Jupiter and Earth are known, the value for $t^* - t$ can be used to compute c. Since Jupiter is much further from the Sun than Earth, $\sqrt{d^2 + R^2}$ may be approximated by d and $t^* - t$ may be approximated by R/c. In this approximation, only the radius of Earth's orbit need be known.