

7. The potential difference between the wire and cylinder is given, not the linear charge density on the wire. We use Gauss' law to find an expression for the electric field a distance r from the center of the wire, between the wire and the cylinder, in terms of the linear charge density. Then integrate with respect to r to find an expression for the potential difference between the wire and cylinder in terms of the linear charge density. We use this result to obtain an expression for the linear charge density in terms of the potential difference and substitute the result into the equation for the electric field. This will give the electric field in terms of the potential difference and will allow you to compute numerical values for the field at the wire and at the cylinder. For the Gaussian surface use a cylinder of radius r and length ℓ , concentric with the wire and cylinder. The electric field is normal to the rounded portion of the cylinder's surface and its magnitude is uniform over that surface. This means the electric flux through the Gaussian surface is given by $2\pi r\ell E$, where E is the magnitude of the electric field. The charge enclosed by the Gaussian surface is $q = \lambda\ell$, where λ is the linear charge density on the wire. Gauss' law yields $2\pi\epsilon_0 r\ell E = \lambda\ell$. Thus,

$$E = \frac{\lambda}{2\pi\epsilon_0 r} .$$

Since the field is radial, the difference in the potential V_c of the cylinder and the potential V_w of the wire is

$$\Delta V = V_w - V_c = - \int_{r_c}^{r_w} E \, dr = \int_{r_w}^{r_c} \frac{\lambda}{2\pi\epsilon_0 r} \, dr = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_c}{r_w} ,$$

where r_w is the radius of the wire and r_c is the radius of the cylinder. This means that

$$\lambda = \frac{2\pi\epsilon_0 \Delta V}{\ln(r_c/r_w)}$$

and

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{\Delta V}{r \ln(r_c/r_w)} .$$

- (a) We substitute r_c for r to obtain the field at the surface of the wire:

$$\begin{aligned} E &= \frac{\Delta V}{r_w \ln(r_c/r_w)} = \frac{850 \text{ V}}{(0.65 \times 10^{-6} \text{ m}) \ln [(1.0 \times 10^{-2} \text{ m})/(0.65 \times 10^{-6} \text{ m})]} \\ &= 1.36 \times 10^8 \text{ V/m} . \end{aligned}$$

- (b) We substitute r_c for r to find the field at the surface of the cylinder:

$$\begin{aligned} E &= \frac{\Delta V}{r_c \ln(r_c/r_w)} = \frac{850 \text{ V}}{(1.0 \times 10^{-2} \text{ m}) \ln [(1.0 \times 10^{-2} \text{ m})/(0.65 \times 10^{-6} \text{ m})]} \\ &= 8.82 \times 10^3 \text{ V/m} . \end{aligned}$$