67. The distance from Q to P is $5a$, and the distance from q to P is $3a$. Therefore, the magnitudes of the individual electric fields are, using Eq. 23-3 (writing $1/4\pi\varepsilon_0 = k$),

$$|\vec{E}_Q| = \frac{k\,|Q|}{25\,a^2}\ , \qquad\qquad |\vec{E}_q| = \frac{k\,|q|}{9\,a^2}\ .$$

We note that $\vec{E}_q$ is along the y axis (directed towards $\pm y$ in accordance with the sign of q), and $\vec{E}_Q$ has x and y components, with $\vec{E}_{Q\,x} = \pm\frac{4}{5}|\vec{E}_Q|$ and $\vec{E}_{Q\,y} = \pm\frac{3}{5}|\vec{E}_Q|$ (signs corresponding to the sign of Q). Consequently, we can write the addition of components in a simple way (basically, by dropping the absolute values):

$$\begin{aligned}\vec{E}_{\text{net}\,x} &= \frac{4\,k\,Q}{125\,a^2}\\ \vec{E}_{\text{net}\,y} &= \frac{3\,k\,Q}{125\,a^2} + \frac{k\,q}{9\,a^2}\end{aligned}$$

(a) Equating $\vec{E}_{\text{net}\,x}$ and $\vec{E}_{\text{net}\,y}$, it is straightforward to solve for the relation between Q and q. We obtain $Q = \frac{125}{9}q \approx 14q$.

(b) We set $\vec{E}_{\text{net}\,y} = 0$ and find the necessary relation between Q and q. We obtain $Q = -\frac{125}{27}q \approx -4.6q$.