22. To evaluate the field using Gauss' law, we employ a cylindrical surface of area $2\pi\, r\, L$ where L is very large (large enough that contributions from the ends of the cylinder become irrelevant to the calculation). The volume within this surface is $V = \pi\, r^2 L$, or expressed more appropriate to our needs: $dV = 2\pi\, r\, L\, dr$. The charge enclosed is, with $A = 2.5 \times 10^{-6}$ C/m^5,

$$q_{\rm enc} = \int_0^r A\, r^2\, 2\pi\, r\, L\, dr = \frac{\pi}{2}\, A\, L\, r^4 \quad .$$

By Gauss' law, we find $\Phi = |\vec{E}|(2\pi r L) = q_{\rm enc}/\varepsilon_0$; we thus obtain

$$\left|\vec{E}\right| = \frac{A\, r^3}{4\,\varepsilon_0} \quad .$$

(a) With $r = 0.030$ m, we find $|\vec{E}| = 1.9$ N/C.

(b) Once outside the cylinder, Eq. 24-12 is obeyed. To find $\lambda = q/L$ we must find the total charge q. Therefore,

$$\frac{q}{L} = \frac{1}{L}\int_0^{0.04} A\, r^2\, 2\pi\, r\, L\, dr = 1.0 \times 10^{-11}\ \text{C/m} \ .$$

And the result, for $r = 0.050$ m, is $|\vec{E}| = \lambda/2\pi\varepsilon_0\, r = 3.6$ N/C.