52. (a) The binomial theorem (Appendix E) allows us to write

$$\sqrt{k(1+x)} = \sqrt{k}\left(1 + \frac{x}{2} + \frac{x^2}{8} + \frac{3x^3}{48} + \cdots\right) \quad \approx \quad \sqrt{k} + \frac{x}{2}\sqrt{k}$$

for $x \ll 1$. Thus, the end result from the solution of problem 49 yields

$$r_m = \sqrt{R\lambda m\left(1 + \frac{1}{2m}\right)} \quad \approx \quad \sqrt{R\lambda m} + \frac{1}{4m}\sqrt{R\lambda m}$$

and

$$r_{m+1} = \sqrt{R\lambda m\left(1 + \frac{3}{2m}\right)} \quad \approx \quad \sqrt{R\lambda m} + \frac{3}{4m}\sqrt{R\lambda m}$$

for very large values of m. Subtracting these, we obtain

$$\Delta r = \frac{3}{4m}\sqrt{R\lambda m} - \frac{1}{4m}\sqrt{R\lambda m} = \frac{1}{2}\sqrt{\frac{R\lambda}{m}} \ .$$

(b) We take the differential of the area: $dA = d(\pi r^2) = 2\pi r\, dr$, and replace dr with Δr in anticipation of using the result from part (a). Thus, the area between adjacent rings for large values of m is

$$2\pi r_m(\Delta r) \approx 2\pi\left(\sqrt{R\lambda m} + \frac{1}{4m}\sqrt{R\lambda m}\right)\left(\frac{1}{2}\sqrt{\frac{R\lambda}{m}}\right) \approx 2\pi\left(\sqrt{R\lambda m}\right)\left(\frac{1}{2}\sqrt{\frac{R\lambda}{m}}\right)$$

which simplifies to the desired result ($\pi\lambda R$).