12. We can use the mc^2 value for an electron from Table 38-3 (511×10^3 eV) and the hc value developed in problem 3 of Chapter 39 by rewriting Eq. 40-4 as

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (hc)^2}{8(mc^2)L^2} \ .$$

(a) The first excited state is characterized by $n = 2$, and the third by $n' = 4$. Thus,

$$\begin{aligned} \Delta E &= \frac{(hc)^2}{8(mc^2)L^2}\left(n'^2 - n^2\right) \\ &= \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8(511 \times 10^3\,\text{eV})(0.250\,\text{nm})^2}\left(4^2 - 2^2\right) \\ &= (6.02\,\text{eV})(16 - 4) \end{aligned}$$

which yields $\Delta E = 72.2\,\text{eV}$.

(b) Now that the electron is in the $n' = 4$ level, it can "drop" to a lower level (n'') in a variety of ways. Each of these drops is presumed to cause a photon to be emitted of wavelength

$$\lambda = \frac{hc}{E_{n'} - E_{n''}} = \frac{8(mc^2)L^2}{hc\left(n'^2 - n''^2\right)} \ .$$

For example, for the transition $n' = 4$ to $n'' = 3$, the photon emitted would have wavelength

$$\lambda = \frac{8(511 \times 10^3\,\text{eV})(0.250\,\text{nm})^2}{(1240\,\text{eV}\cdot\text{nm})\left(4^2 - 3^2\right)} = 29.4 \text{ nm} \ ,$$

and once it is then in level $n'' = 3$ it might fall to level $n''' = 2$ emitting another photon. Calculating in this way all the possible photons emitted during the de-excitation of this system, we find $\lambda_{4\to1} = 13.7\,\text{nm}$, $\lambda_{4\to2} = 17.2\,\text{nm}$, $\lambda_{3\to1} = 25.8\,\text{nm}$, $\lambda_{4\to3} = 29.4\,\text{nm}$, $\lambda_{3\to2} = 41.2\,\text{nm}$, and $\lambda_{2\to1} = 68.7\,\text{nm}$.

(c) A system making the $4 \to 1$ transition will make no further transitions unless it is re-excited. If it makes the $4 \to 2$ transition, then that is likely to followed by the $2 \to 1$ transition. However, if it makes the $4 \to 3$ transition, then it could make either the $3 \to 1$ transition or the pair of transitions: $3 \to 2$ and $2 \to 1$.

(d) The possible transitions are shown below. The energy levels are not drawn to scale.