

21. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3} .$$

Thus, $C_{\text{eq}} = C_2 C_3 / (C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by q_2 / C_{eq} . The potential difference across capacitor 1 is q_1 / C_1 , where q_1 is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1 / C_1 = q_2 / C_{\text{eq}}$. Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where V_0 is the original potential difference across capacitor 1. Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{\text{eq}}} \quad \text{and} \quad q_1 + q_2 = C_1 V_0$$

for q_1 and q_2 , we find

$$q_2 = C_1 V_0 - q_1 \quad \text{and} \quad q_1 = \frac{C_1^2 V_0}{C_{\text{eq}} + C_1} = \frac{C_1^2 V_0}{\frac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3} .$$

The charges on capacitors 2 and 3 are

$$q_2 = q_3 = C_1 V_0 - q_1 = C_1 V_0 - \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3} = \frac{C_1 C_2 C_3 V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3} .$$