29. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let i be the current in either battery and take it to be positive to the left. According to the junction rule the current in R is $2i$ and it is positive to the right. The loop rule applied to either loop containing a battery and R yields

$$\mathcal{E} - ir - 2iR = 0 \quad \Longrightarrow \quad i = \frac{\mathcal{E}}{r + 2R} \ .$$

The power dissipated in R is

$$P = (2i)^2 R = \frac{4\mathcal{E}^2 R}{(r + 2R)^2} \ .$$

We find the maximum by setting the derivative with respect to R equal to zero. The derivative is

$$\frac{dP}{dR} = \frac{4\mathcal{E}^2}{(r + 2R)^2} - \frac{16\mathcal{E}^2 R}{(r + 2R)^3} = \frac{4\mathcal{E}^2 (r - 2R)}{(r + 2R)^3} \ .$$

The derivative vanishes (and P is a maximum) if $R = r/2$.

(b) We substitute $R = r/2$ into $P = 4\mathcal{E}^2 R/(r + 2R)^2$ to obtain

$$P_{\text{max}} = \frac{4\mathcal{E}^2 (r/2)}{[r + 2(r/2)]^2} = \frac{\mathcal{E}^2}{2r} \ .$$