

59. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \implies v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = (I_{\text{rod}} + md^2)\omega$$

where I_{rod} is found using Table 11-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2.$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{\frac{1}{3}Md^2 + md^2}$$

which means the system has kinetic energy $\frac{1}{2}(I_{\text{rod}} + md^2)\omega^2$ which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\begin{aligned} \frac{1}{2}(I_{\text{rod}} + md^2)\omega^2 &= mgH + Mg\frac{H}{2} \\ \frac{1}{2}\frac{m^2d^2(2gh)}{\frac{1}{3}Md^2 + md^2} &= \left(m + \frac{M}{2}\right)gd(1 - \cos\theta) \end{aligned}$$

from which we obtain

$$\theta = \cos^{-1}\left(1 - \frac{m^2h}{(m + \frac{1}{2}M)(m + \frac{1}{3}M)}\right).$$