

72. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed  $v_f$ . The corresponding increase in its kinetic energy,  $\frac{1}{2}mv_f^2$ , is equal to the work done on it by Earth's gravity:  $\int F dr = \int (-Kr)dr$  (using the notation of that Sample Problem referred to in the problem statement). Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F dr = \int_R^0 (-Kr) dr = \frac{1}{2}KR^2$$

where  $R$  is the radius of Earth. Solving for the final speed, we obtain  $v_f = R\sqrt{K/m}$ . We note that the acceleration of gravity  $a_g = g = 9.8 \text{ m/s}^2$  on the surface of Earth is given by  $a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2$ , where  $\rho$  is Earth's average density. This permits us to write  $K/m = 4\pi G\rho/3 = g/R$ . Consequently,

$$\begin{aligned} v_f &= R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} \\ &= \sqrt{(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})} = 7.9 \times 10^3 \text{ m/s} . \end{aligned}$$