57. (a) The potential energy of the atom in association with the presence an external magnetic field $\vec{B}_{\text{ext}}$ is given by Eqs. 32-11 and 32-12:

$$U = -\mu_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}} = -m_l \mu_B B_{\text{ext}} \ .$$

For level E_1 there is no change in energy as a result of the introduction of $\vec{B}_{\text{ext}}$, so $U \propto m_l = 0$, meaning that $m_l = 0$ for this level. For level E_2 the single level splits into a triplet (i.e., three separate ones) in the presence of $\vec{B}_{\text{ext}}$, meaning that there are three different values of m_l. The middle one in the triplet is unshifted from the original value of E_2 so its m_l must be equal to 0.

(b) The other two in the triplet then correspond to $m_l = -1$ and $m_1 = +1$, respectively.

(c) For any pair of adjacent levels in the triplet $|\Delta m_l| = 1$. Thus, the spacing is given by

$$\begin{aligned} \Delta U &= |\Delta(-m_l \mu_B B)| = |\Delta m_l| \mu_B B = \mu_B B \\ &= \left(9.27 \times 10^{-24}\,\text{J/T}\right)(0.50\,\text{T}) = 4.6 \times 10^{-24}\ \text{J} \end{aligned}$$

which is equivalent to $2.9 \times 10^{-5}\,\text{eV}$.