45. To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance. We use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center. The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho \, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2 \, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 \, dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 \, dr = 4\pi A \int_a^{r_g} r \, dr = 2\pi A(r_g^2 - a^2) \; .$$

The total charge inside the Gaussian surface is $q + q_s = q + 2\pi A(r_g^2 - a^2)$. The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields

$$4\pi\varepsilon_0 E r_g^2 = q + 2\pi A(r_g^2 - a^2) \; .$$

We solve for E:

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2}\right] \; .$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$.