58. (a) As explained in the problem, the height of the n^{th} domino is $h_n = 1.5^{n-1}$ in centimeters. Therefore, $h_{32} = 1.5^{31} = 2.9 \times 10^5$ cm $= 2.9$ km (!).

(b) When the center of the domino is directly over the corner, the height of the center-point is

$$h_c = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{d}{2}\right)^2} = \frac{d}{2}\sqrt{101}$$

where $h = 10d$ has been used in that last step. While the domino is in its usual resting position, the height of that point is only $h_o = h/2$ which can be written as $5d$. Since the answer is requested to be in terms of U_1 then

$$U_1 = mg(5d) \implies d = \frac{U_1}{5mg} \ .$$

Therefore, the energy needed to push over the domino is

$$\Delta U = mgh_c - U_1 = mg\left(\frac{d}{2}\sqrt{101}\right) - U_1 = \frac{U_1}{10}\sqrt{101} - U_1$$

which yields approximately $0.005U_1$; the problem refers to this as $\Delta E_{1,\text{in}}$.

(c) The "loss" of potential energy equal to

$$mgh_c - mg\left(\frac{h}{2}\sin\theta\right)$$

becomes the kinetic energy (denoted $\Delta E_{1,\text{out}}$ in the problem). Therefore, we obtain

$$\Delta E_{1,\text{out}} = mg\left(\frac{d}{2}\sqrt{101}\right) - mg\left(\frac{10d}{2}\sin\theta\right)$$

which (using $\theta = 45°$) simplifies to $1.49mgd$. Since $d = U_1/5mg$ this becomes roughly $\Delta E_{1,\text{out}} = 0.30U_1$.

(d) We see from part (b) that $\Delta E_{n,\text{in}}$ is directly proportional to $m_n d_n$ and consequently (since the density is assumed the same for all of them and the volume of a domino is hdw where w is the width) is proportional to $w_n h_n d_n^2$. The width also scales like the other quantities, so $\Delta E_{n,\text{in}}$ is proportional to $1.5^{4(n-1)}$. Therefore, $\Delta E_{2,\text{in}} = 1.5^4 \Delta E_{1,\text{in}}$ which implies $\Delta E_{2,\text{in}} = 0.025U_1$.

(e) Therefore,

$$\frac{\Delta E_{1,\text{out}}}{\Delta E_{2,\text{in}}} = \frac{0.30U_1}{0.025U_1} = 12 \ .$$