31. (a) We first find the currents. Let i_1 be the current in R_1 and take it to be positive if it is upward. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is to the right. The junction rule produces

$$i_1 + i_2 + i_3 = 0 \ .$$

The loop rule applied to the left-hand loop produces

$$\mathcal{E}_1 - i_3 R_3 + i_1 R_1 = 0$$

and applied to the right-hand loop produces

$$\mathcal{E}_2 - i_2 R_2 + i_1 R_1 = 0 \ .$$

We substitute $i_1 = -i_2 - i_3$, from the first equation, into the other two to obtain

$$\mathcal{E}_1 - i_3 R_3 - i_2 R_1 - i_3 R_1 = 0$$

and

$$\mathcal{E}_2 - i_2 R_2 - i_2 R_1 - i_3 R_1 = 0 \ .$$

The first of these yields

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} \ .$$

Substituting this into the second equation and solving for i_2, we obtain

$$\begin{aligned} i_2 &= \frac{\mathcal{E}_2(R_1 + R_3) - \mathcal{E}_1 R_1}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\ &= \frac{(1.00\,\mathrm{V})(5.00\,\Omega + 4.00\,\Omega) - (3.00\,\mathrm{V})(5.00\,\Omega)}{(5.00\,\Omega)(2.00\,\Omega) + (5.00\,\Omega)(4.00\,\Omega) + (2.00\,\Omega)(4.00\,\Omega)} = -0.158\,\mathrm{A} \ . \end{aligned}$$

We substitute into the expression for i_3 to obtain

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} = \frac{3.00\,\mathrm{V} - (-0.158\,\mathrm{A})(5.00\,\Omega)}{5.00\,\Omega + 4.00\,\Omega} = 0.421\,\mathrm{A} \ .$$

Finally,

$$i_1 = -i_2 - i_3 = -(-0.158\,\mathrm{A}) - (0.421\,\mathrm{A}) = -0.263\,\mathrm{A} \ .$$

Note that the current in R_1 is actually downward and the current in R_2 is to the right. The current in R_3 is also to the right. The power dissipated in R_1 is $P_1 = i_1^2 R_1 = (-0.263\,\mathrm{A})^2(5.00\,\Omega) = 0.346\,\mathrm{W}$.

(b) The power dissipated in R_2 is $P_2 = i_2^2 R_2 = (-0.158\,\mathrm{A})^2(2.00\,\Omega) = 0.0499\,\mathrm{W}$.

(c) The power dissipated in R_3 is $P_3 = i_3^2 R_3 = (0.421\,\mathrm{A})^2(4.00\,\Omega) = 0.709\,\mathrm{W}$.

(d) The power supplied by $\mathcal{E}_1$ is $i_3 \mathcal{E}_1 = (0.421\,\mathrm{A})(3.00\,\mathrm{V}) = 1.26\,\mathrm{W}$.

(e) The power "supplied" by $\mathcal{E}_2$ is $i_2 \mathcal{E}_2 = (-0.158\,\mathrm{A})(1.00\,\mathrm{V}) = -0.158\,\mathrm{W}$. The negative sign indicates that $\mathcal{E}_2$ is actually absorbing energy from the circuit.