

25. When displaced from equilibrium, the magnitude of the net force exerted by the springs is $|k_1x + k_2x|$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m \frac{d^2x}{dt^2} = -k_1x - k_2x .$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{k_1 + k_2}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}} .$$

The single springs each acting alone would produce simple harmonic motions of frequency

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m}} \quad \text{and} \quad f_2 = \frac{1}{2\pi} \sqrt{\frac{k_2}{m}} ,$$

respectively. Comparing these expressions, it is clear that $f = \sqrt{f_1^2 + f_2^2}$.