

27. (a) Let  $P$  be the power output of the source. This is the rate at which energy crosses the surface of any sphere centered at the source and is therefore equal to the product of the intensity  $I$  at the sphere surface and the area of the sphere. For a sphere of radius  $r$ ,  $P = 4\pi r^2 I$  and  $I = P/4\pi r^2$ . The intensity is proportional to the square of the displacement amplitude  $s_m$ . If we write  $I = Cs_m^2$ , where  $C$  is a constant of proportionality, then  $Cs_m^2 = P/4\pi r^2$ . Thus  $s_m = \sqrt{P/4\pi r^2 C} = \left(\sqrt{P/4\pi C}\right)(1/r)$ . The displacement amplitude is proportional to the reciprocal of the distance from the source. We take the wave to be sinusoidal. It travels radially outward from the source, with points on a sphere of radius  $r$  in phase. If  $\omega$  is the angular frequency and  $k$  is the angular wave number then the time dependence is  $\sin(kr - \omega t)$ . Letting  $b = \sqrt{P/4\pi C}$ , the displacement wave is then given by

$$s(r, t) = \sqrt{\frac{P}{4\pi C}} \frac{1}{r} \sin(kr - \omega t) = \frac{b}{r} \sin(kr - \omega t) .$$

- (b) Since  $s$  and  $r$  both have dimensions of length and the trigonometric function is dimensionless, the dimensions of  $b$  must be length squared.