52. We write our magnitude-angle results in the form $(R \angle \theta)$ with SI units for the magnitude understood (m for distances, m/s for speeds, m/s^2 for accelerations). All angles θ are measured counterclockwise from $+x$, but we will occasionally refer to angles ϕ which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see r in the figure). We note that the speed of the particle is $v = 2\pi r/T$ where $r = 3.00$ m and $T = 20.0$ s; thus, $v = 0.942$ m/s. The particle is moving counterclockwise in Fig. 4-37.

(a) At $t = 5.00$ s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00}{20.0} = \frac{1}{4}$$

of a full revolution around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \frac{1}{4}(360°) = 90°$$

measured from vertical (as explained above). Referring to Fig. 4-37, we see that this position (which is the "3 o'clock" position on the circle) corresponds to $x = 3.00$ m and $y = 3.00$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (4.24 \angle 45°)$. Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of x and y relative to coordinate origin can be gotten from the angle ϕ from the relations $x = r\sin\phi$ and $y = r - r\cos\phi$. Of course, $R = \sqrt{x^2 + y^2}$ and θ comes from choosing the appropriate possibility from $\tan^{-1}(y/x)$ (or by using particular functions of vector capable calculators).

(b) At $t = 7.50$ s, the particle has traveled a fraction of $7.50/20.0 = 3/8$ of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at $\phi = 3/8\,(360°) = 135°$ measured from vertical in the manner discussed above. Referring to Fig. 4-37, we compute that this position corresponds to $x = 3.00\sin 135° = 2.12$ m and $y = 3.00 - 3.00\cos 135° = 5.12$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (5.54 \angle 67.5°)$.

(c) At $t = 10.0$ s, the particle has traveled a fraction of $10.0/20.0 = 1/2$ of a revolution around the circle. Relative to the circle-center, the particle is at $\phi = 180°$ measured from vertical (see explanation, above). Referring to Fig. 4-37, we see that this position corresponds to $x = 0$ and $y = 6.00$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (6.00 \angle 90.0°)$.

(d) We subtract the position vector in part (a) from the position vector in part (c): $(6.00 \angle 90.0°) - (4.24 \angle 45°) = (4.24 \angle 135°)$ using magnitude-angle notation (convenient when using vector capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta\vec{R} = (0 - 3)\,\hat{\imath} + (6 - 3)\,\hat{\jmath} = -3\,\hat{\imath} + 3\,\hat{\jmath}$$

which leads to $|\Delta\vec{R}| = 4.24$ m and $\theta = 135°$.

(e) From Eq. 4-8, we have

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{R}}{\Delta t} \quad \text{where} \quad \Delta t = 5.00 \text{ s}$$

which produces $-0.6\,\hat{\imath} + 0.6\,\hat{\jmath}$ m/s in unit-vector notation or $(0.849 \angle 135°)$ in magnitude-angle notation.

(f) The speed has already been noted ($v = 0.942$ m/s), but its direction is best seen by referring again to Fig. 4-37. The velocity vector is tangent to the circle at its "3 o'clock position" (see part (a)), which means $\vec{v}$ is vertical. Thus, our result is $(0.942 \angle 90°)$.

(g) Again, the speed has been noted above ($v = 0.942$ m/s), but its direction is best seen by referring to Fig. 4-37. The velocity vector is tangent to the circle at its "12 o'clock position" (see part (c)), which means $\vec{v}$ is horizontal. Thus, our result is $(0.942 \angle 180°)$.