

80. (a) The light that passes through the surface of the lake is within a cone of apex angle $2\theta_c$ making a “circle of light” there; reference to Fig. 34-24 may help in visualizing this (consider revolving that picture about a vertical axis). Since the source is point-like, its energy spreads out with perfect spherical symmetry, until reaching the surface and other boundaries of the lake. The problem asks us to assume there are no partial reflections at the surface, only the total reflections outside the “circle of light.” Thus, of the full sphere of light (of area $A_s = 4\pi R^2$) emitted by the source, only a fraction of it – coinciding with the cone of apex angle $2\theta_c$ – enters the air above. If we label the area of that portion of the sphere which reaches the air above as A , then the fraction of the total energy emitted that passes through the surface is

$$frac = \frac{A}{4\pi R^2} \quad \text{where} \quad R = \frac{h}{\cos \theta_c}$$

is the distance from the point-source to the edge of the “circle of light.” Now, the area A of the spherical cap of height H bounded by that circle is

$$A = 2\pi RH = 2\pi R(R - h)$$

may be looked up in a number of references, or can be derived from $A = 2\pi R^2 \int_0^{\theta_c} \sin \theta d\theta$. Consequently,

$$frac = \frac{2\pi R(R - h)}{4\pi R^2} = \frac{1}{2} \left(1 - \frac{h}{R} \right) = \frac{1}{2} (1 - \cos \theta_c) .$$

The critical angle is given by $\sin \theta_c = 1/n$, which implies $\cos \theta_c = \sqrt{1 - \sin^2 \theta_c} = \sqrt{1 - 1/n^2}$. When this expression is substituted into our result above, we obtain

$$frac = \frac{1}{2} \left(1 - \sqrt{1 - \frac{1}{n^2}} \right) .$$

- (b) For $n = 1.33$,

$$frac = \frac{1}{2} \left(1 - \sqrt{1 - \frac{1}{(1.33)^2}} \right) = 0.170 .$$