42. We refer to the discussion in the textbook (Sample Problem 10-4, which uses the same notation that we use here) for some important details in the reasoning. We choose rightward in Fig. 10-15 as our $+x$ direction. We use the notation $\vec{v}$ when we refer to velocities and v when we refer to speeds (which are necessarily positive). Since the algebra is fairly involved, we find it convenient to introduce the notation $\Delta m = m_2 - m_1$ (which, we note for later reference, is a positive-valued quantity).

(a) Since $\vec{v}_{1i} = +\sqrt{2gh_1}$ where $h_1 = 9.0$ cm, we have

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\, v_{1i} = -\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}$$

which is to say that the *speed* of sphere 1 immediately after the collision is $v_{1f} = (\Delta m/(m_1 + m_2))\sqrt{2gh_1}$ and that $\vec{v}_{1f}$ points in the $-x$ direction. This leads (by energy conservation $m_1 g h_{1f} = \frac{1}{2}m_1 v_{1f}^2$) to

$$h_{1f} = \frac{v_{1f}^2}{2g} = \left(\frac{\Delta m}{m_1 + m_2}\right)^2 h_1 \,.$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{1f} \approx 0.6$ cm.

(b) Eq. 10-31 gives

$$v_{2f} = \frac{2m_1}{m_1 + m_2}\, v_{1i} = \frac{2m_1}{m_1 + m_2}\sqrt{2gh_1}$$

which leads (by energy conservation $m_2 g h_{2f} = \frac{1}{2}m_2 v_{2f}^2$) to

$$h_{2f} = \frac{v_{2f}^2}{2g} = \left(\frac{2m_1}{m_1 + m_2}\right)^2 h_1 \,.$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{2f} \approx 4.9$ cm.

(c) Fortunately, they hit again at the lowest point (as long as their amplitude of swing was "small" – this is further discussed in Chapter 16). At the risk of using cumbersome notation, we refer to the *next* set of heights as h_{1ff} and h_{2ff}. At the lowest point (before this second collision) sphere 1 has velocity $+\sqrt{2gh_{1f}}$ (rightward in Fig. 10-15) and sphere 2 has velocity $-\sqrt{2gh_{1f}}$ (that is, it points in the $-x$ direction). Thus, the velocity of sphere 1 immediately after the second collision is, using Eq. 10-38,

$$\begin{aligned}
\vec{v}_{1ff} &= \frac{m_1 - m_2}{m_1 + m_2}\sqrt{2gh_{1f}} + \frac{2m_2}{m_1 + m_2}\left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{-\Delta m}{m_1 + m_2}\left(\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}\right) - \frac{2m_2}{m_1 + m_2}\left(\frac{2m_1}{m_1 + m_2}\sqrt{2gh_1}\right) \\
&= -\frac{(\Delta m)^2 + 4m_1 m_2}{(m_1 + m_2)^2}\sqrt{2gh_1} \,.
\end{aligned}$$

This can be greatly simplified (by expanding $(\Delta m)^2$ and $(m_1 + m_2)^2$) to arrive at the conclusion that the speed of sphere 1 immediately after the second collision is simply $v_{1ff} = \sqrt{2gh_1}$ and that $\vec{v}_{1ff}$ points in the $-x$ direction. Energy conservation ($m_1 g h_{1ff} = \frac{1}{2}m_1 v_{1ff}^2$) leads to

$$h_{1ff} = \frac{v_{1ff}^2}{2g} = h_1 = 9.0 \text{ cm} \,.$$

(d) One can reason (energy-wise) that $h_{1ff} = 0$ simply based on what we found in part (c). Still, it might be useful to see how this shakes out of the algebra. Eq. 10-39 gives the velocity of sphere 2 immediately after the second collision:

$$\begin{aligned}
v_{2ff} &= \frac{2m_1}{m_1 + m_2}\sqrt{2gh_{1f}} + \frac{m_2 - m_1}{m_1 + m_2}\left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{2m_1}{m_1 + m_2}\left(\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}\right) + \frac{\Delta m}{m_1 + m_2}\left(\frac{-2m_1}{m_1 + m_2}\sqrt{2gh_1}\right)
\end{aligned}$$