61. In the following $C_V = \frac{3}{2}R$ is the molar specific heat at constant volume, $C_p = \frac{5}{2}R$ is the molar specific heat at constant pressure, ΔT is the temperature change, and n is the number of moles.

(a) The process $1 \to 2$ takes place at constant volume. The heat added is

$$\begin{aligned} Q &= nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T \\ &= \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = 3.74 \times 10^3\ \text{J}\ . \end{aligned}$$

Since the process takes place at constant volume the work W done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74 \times 10^3\ \text{J}\ .$$

The process $2 \to 3$ is adiabatic. The heat added is zero. The change in the internal energy is

$$\begin{aligned} \Delta E_{\text{int}} &= nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T \\ &= \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K} - 600\,\text{K}) = -1.81 \times 10^3\ \text{J}\ . \end{aligned}$$

According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81 \times 10^3\ \text{J}\ .$$

The process $3 \to 1$ takes place at constant pressure. The heat added is

$$\begin{aligned} Q &= nC_p\,\Delta T = \frac{5}{2}nR\,\Delta T \\ &= \frac{5}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -3.22 \times 10^3\ \text{J}\ . \end{aligned}$$

The change in the internal energy is

$$\begin{aligned} \Delta E_{\text{int}} &= nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T \\ &= \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -1.93 \times 10^3\ \text{J}\ . \end{aligned}$$

According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22 \times 10^3\,\text{J} + 1.93 \times 10^3\,\text{J} = -1.29 \times 10^3\ \text{J}\ .$$

For the entire process the heat added is

$$Q = 3.74 \times 10^3\,\text{J} + 0 - 3.22 \times 10^3\,\text{J} = 520\ \text{J}\ ,$$

the change in the internal energy is

$$\Delta E_{\text{int}} = 3.74 \times 10^3\,\text{J} - 1.81 \times 10^3\,\text{J} - 1.93 \times 10^3\,\text{J} = 0\ ,$$

and the work done by the gas is

$$W = 0 + 1.81 \times 10^3\,\text{J} - 1.29 \times 10^3\,\text{J} = 520\ \text{J}\ .$$

(b) We first find the initial volume. Use the ideal gas law $p_1V_1 = nRT_1$ to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{(1.013 \times 10^5\,\text{Pa})} = 2.46 \times 10^{-2}\,\text{m}^3\ .$$