7. We assume the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges. We choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Then, the force on q_2 is

$$F_a = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = -k\,\frac{q_1 q_2}{r^2}$$

where $r = 0.500$ m. The negative sign indicates that the spheres attract each other. After the wire is connected, the spheres, being identical, acquire the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1 + q_2)/2$. The force is now one of repulsion and is given by

$$F_b = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{r^2} = k\,\frac{(q_1+q_2)^2}{4r^2} .$$

We solve the two force equations simultaneously for q_1 and q_2. The first gives the product

$$q_1 q_2 = -\frac{r^2 F_a}{k} = -\frac{(0.500\,\mathrm{m})^2(0.108\,\mathrm{N})}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}} = -3.00\times10^{-12}\ \mathrm{C}^2 ,$$

and the second gives the sum

$$q_1 + q_2 = 2r\sqrt{\frac{F_b}{k}} = 2(0.500\,\mathrm{m})\sqrt{\frac{0.0360\,\mathrm{N}}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}}} = 2.00\times10^{-6}\ \mathrm{C}$$

where we have taken the positive root (which amounts to assuming $q_1 + q_2 \geq 0$). Thus, the product result provides the relation

$$q_2 = \frac{-(3.00\times10^{-12}\,\mathrm{C}^2)}{q_1}$$

which we substitute into the sum result, producing

$$q_1 - \frac{3.00\times10^{-12}\,\mathrm{C}^2}{q_1} = 2.00\times10^{-6}\ \mathrm{C} .$$

Multiplying by q_1 and rearranging, we obtain a quadratic equation

$$q_1^2 - (2.00\times10^{-6}\,\mathrm{C})q_1 - 3.00\times10^{-12}\,\mathrm{C}^2 = 0 .$$

The solutions are

$$q_1 = \frac{2.00\times10^{-6}\,\mathrm{C} \pm \sqrt{(-2.00\times10^{-6}\,\mathrm{C})^2 - 4(-3.00\times10^{-12}\,\mathrm{C}^2)}}{2} .$$

If the positive sign is used, $q_1 = 3.00\times10^{-6}\,\mathrm{C}$, and if the negative sign is used, $q_1 = -1.00\times10^{-6}\,\mathrm{C}$. Using $q_2 = (-3.00\times10^{-12})/q_1$ with $q_1 = 3.00\times10^{-6}\,\mathrm{C}$, we get $q_2 = -1.00\times10^{-6}\,\mathrm{C}$. If we instead work with the $q_1 = -1.00\times10^{-6}\,\mathrm{C}$ root, then we find $q_2 = 3.00\times10^{-6}\,\mathrm{C}$. Since the spheres are identical, the solutions are essentially the same: one sphere originally had charge $-1.00\times10^{-6}\,\mathrm{C}$ and the other had charge $+3.00\times10^{-6}\,\mathrm{C}$. What if we had not made the assumption, above, that $q_1 + q_2 \geq 0$? If the signs of the charges were reversed (so $q_1 + q_2 < 0$), then the forces remain the same, so a charge of $+1.00\times10^{-6}\,\mathrm{C}$ on one sphere and a charge of $-3.00\times10^{-6}\,\mathrm{C}$ on the other also satisfies the conditions of the problem.