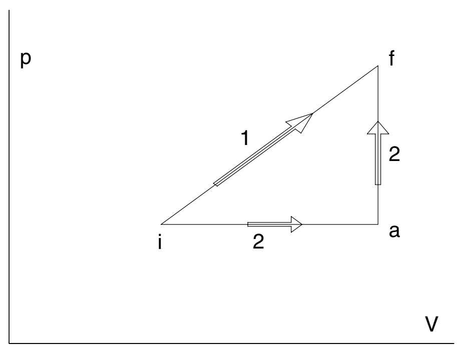
65. First we show that $\int dQ$ is path-dependent. To do this all we need is to show that $\int dQ$ is different for at least two separate paths, say path 1 and 2, as depicted in the figure below. We write $\int dQ = \int pdV + \int nC_V dT$. The second term on the right, $\int nC_V dT$, yields $nC_V \Delta T$ upon integration and is obviously path-independent. The first term, $\int pdV$, however, is different for the two paths. In fact $\int_i^f pdV$ along path 1 is greater than that along path 2, by the area of the shaded triangle enclosed by the two paths. Therefore, $\int dQ$ is indeed path-dependent.



Now we consider $\int TdQ = \int pTdV + \int nC_V TdT$. Once again the second term on the right, $\int nC_V TdT$, yields $\frac{1}{2} nC_V \Delta T^2$ upon integration and is path-independent. The first term, $\int pTdV$, however, yields a higher value along path 1 than path 2. To see that, note that

$$\int_2 pT\,dV = \int_{i\to a} pT\,dV + \int_{a\to f} pT\,dV = \int_{i\to a} pT\,dV \ .$$

Now, if we compare the two integrals, $\int_1 pTdV$ and $\int_{i\to a} pT\,dV$, we realize that the average values of both T and p along path 1 are greater than their respective corresponding values along the $i \to a$ segment of path 2. Hence, the integrand $f(p,T) = pT$ is always greater along path 1. Thus, the two integrals over V, which have the same upper and lower limits, are not equal to each other:

$$\int_1 pT\,dV > \int_{i\to a} pT\,dV = \int_2 pT\,dV \ .$$

We see then that $\int TdQ$ is greater along path 1 than path 2 and is therefore path-dependent. Similarly, one can show that for $\int dQ/T^2 = \int pdV/T^2 + \int nC_V dT/T^2$, the second term on the right is path-independent, while for the first term

$$\int pdV/T^2 = nR \int \frac{dV}{TV} \ ,$$

we have

$$nR \int_2 \frac{dV}{TV} = nR \int_{i\to a} \frac{dV}{TV} > nR \int_1 \frac{dV}{TV} \ ,$$

since the average value of $1/T$ is greater along along the $i \to a$ segment of path 2 than on path 1. Consequently, $\int dQ/T^2$ is less along path 1 than path 2 and is therefore path-dependent.