31. We use a right-handed coordinate system with $+\hat{\mathrm{k}}$ directed out of the xy plane so as to be consistent with counterclockwise rotation (and the right-hand rule). Thus, all the angular momenta being considered are along the $-\hat{\mathrm{k}}$ direction; for example, in part (b) $\vec{\ell} = -4.0t^2\,\hat{\mathrm{k}}$ in SI units. We use Eq. 12-23.

(a) The angular momentum is constant so its derivative is zero. There is no torque in this instance.

(b) Taking the derivative with respect to time, we obtain the torque:

$$\vec{\tau} = \frac{d\vec{\ell}}{dt} = (-4.0\,\hat{\mathrm{k}})\,\frac{dt^2}{dt} = -8.0t\;\hat{\mathrm{k}}$$

in SI units (N·m). This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$.

(c) With $\vec{\ell} = -4.0\sqrt{t}\,\hat{\mathrm{k}}$ in SI units, the torque is

$$\vec{\tau} = \left(-4.0\hat{\mathrm{k}}\right)\frac{d\sqrt{t}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\left(\frac{1}{2\sqrt{t}}\right)$$

which yields $\vec{\tau} = -2.0/\sqrt{t}\,\hat{\mathrm{k}}$ in SI units. This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$ (and it is undefined for $t < 0$).

(d) Finally, we have

$$\vec{\tau} = \left(-4.0\hat{\mathrm{k}}\right)\frac{dt^{-2}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\left(\frac{-2}{t^3}\right)$$

which yields $\vec{\tau} = 8.0/t^3\,\hat{\mathrm{k}}$ in SI units. This vector points in the $+\hat{\mathrm{k}}$ direction (causing the initially clockwise motion to slow down) for all $t > 0$.