

56. For simple harmonic motion, Eq. 16-24 must reduce to

$$\tau = -L (F_g \sin \theta) \longrightarrow -L (F_g \theta)$$

where θ is in radians. We take the percent difference (in absolute value)

$$\left| \frac{(-LF_g \sin \theta) - (-LF_g \theta)}{-LF_g \sin \theta} \right| = \left| 1 - \frac{\theta}{\sin \theta} \right|$$

and set this equal to 0.010 (corresponding to 1.0%). In order to solve for θ (since this is not possible “in closed form”), several approaches are available. Some calculators have built-in numerical routines to facilitate this, and most math software packages have this capability. Alternatively, we could expand $\sin \theta \approx \theta - \theta^3/6$ (valid for small θ) and thereby find an approximate solution (which, in turn, might provide a seed value for a numerical search). Here we show the latter approach:

$$\left| 1 - \frac{\theta}{\theta - \theta^3/6} \right| \approx 0.010 \implies \frac{1}{1 - \theta^2/6} \approx 1.010$$

which leads to $\theta \approx \sqrt{6(0.01/1.01)} = 0.24 \text{ rad} = 14^\circ$. A more accurate value (found numerically) for the θ value which results in a 1.0% deviation is 13.986° .