

88. Using Eq. 16-29 and the parallel-axis formula for rotational inertia, we have

$$I = 2\pi\sqrt{\frac{I_{\text{cm}} + mh^2}{mgh}} = 2\pi\sqrt{\frac{L^2}{12gh} + \frac{h}{g}}$$

where we have used the fact (from Ch. 11) that $I_{\text{cm}} = mL^2/12$ for a uniform rod. We wish to minimize by taking the derivative and setting equal to zero, but we observe that this is done more easily if we consider I^2 (the square of the above expression) instead of I . Thus,

$$\frac{dI^2}{dh} = 0 = 4\pi^2 \left(-\frac{L^2}{12gh^2} + \frac{1}{g} \right)$$

which leads to

$$0 = -\frac{L^2}{12h^2} + 1 \implies h = \frac{L}{\sqrt{12}} \approx 0.29L .$$