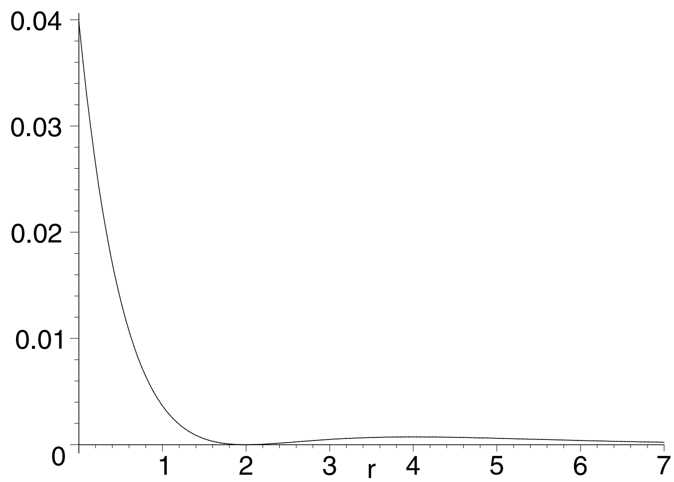
58. (a) The plot shown below for $|\psi_{200}(r)|^2$ is to be compared with the dot plot of Fig. 40-20. We note that the horizontal axis of our graph is labeled "r," but it is actually r/a (that is, it is in units of the parameter a). Now, in the plot below there is a high central peak between $r = 0$ and $r \sim 2a$, corresponding to the densely dotted region around the center of the dot plot of Fig. 40-20. Outside this peak is a region of near-zero values centered at $r = 2a$, where $\psi_{200} = 0$. This is represented in the dot plot by the empty ring surrounding the central peak. Further outside is a broader, flatter, low peak which reaches its maximum value at $r = 4a$. This corresponds to the outer ring with near-uniform dot density which is lower than that of the central peak.



(b) The extrema of $\psi^2(r)$ for $0 < r < \infty$ may be found by squaring the given function, differentiating with respect to r, and setting the result equal to zero:

$$-\frac{1}{32}\,\frac{(r-2a)(r-4a)}{a^6\pi}\,e^{-r/a} = 0$$

which has roots at $r = 2a$ and $r = 4a$. We can verify directly from the plot above that $r = 4a$ is indeed a local maximum of $\psi^2_{200}(r)$. As discussed in part (a), the other root ($r = 2a$) is a local minimum.

(c) Using Eq. 40-30 and Eq. 40-28, the radial probability is

$$P_{200}(r) = 4\pi r^2 \psi^2_{200}(r) = \frac{r^2}{8a^3}\left(2 - \frac{r}{a}\right)^2 e^{-r/a} \, .$$

(d) Let $x = r/a$. Then

$$\begin{aligned}
\int_0^\infty P_{200}(r)\,dr &= \int_0^\infty \frac{r^2}{8a^3}\left(2-\frac{r}{a}\right)^2 e^{-r/a}\,dr \\
&= \frac{1}{8}\int_0^\infty x^2(2-x)^2 e^{-x}\,dx \\
&= \int_0^\infty (x^4 - 4x^3 + 4x^2)e^{-x}\,dx \\
&= \frac{1}{8}\left[4! - 4(3!) + 4(2!)\right] \\
&= 1
\end{aligned}$$

where the integral formula

$$\int_0^\infty x^n\,e^{-x}\,dx = n!$$

is used.