7. (a) Changes in temperature take place by means of radiation, conduction, and convection. The constant A can be reduced by placing the object in isolation, by surrounding it with a vacuum jacket, for example. This reduces conduction and convection. Absorption of radiation can be reduced by polishing the surface to a mirror finish. We note that A depends on the condition of the surface and on the ability of the environment to conduct or convect energy to or from the object. A has the dimensions of reciprocal time.

(b) We rearrange the equation to obtain

$$\frac{1}{\Delta T}\,\frac{d\Delta T}{dt} = -A\ .$$

Now, we integrate with respect to time and recognize that

$$\int \frac{1}{\Delta T}\,\frac{d\,\Delta T}{dt}\,dt = \int \frac{1}{\Delta T}\,d(\Delta T)\ .$$

Thus,

$$\int_{\Delta T_0}^{\Delta T} \frac{1}{\Delta T}\,d(\Delta T) = -\int_0^t A\,dt\ .$$

The integral on the right side yields $-At$ and the integral on the left yields $\ln \Delta T|_{\Delta T_0}^{\Delta T} = \ln(\Delta T) - \ln(\Delta T_0) = \ln(\Delta T/\Delta T_0)$, so

$$\ln \frac{\Delta T}{\Delta T_0} = -At\ .$$

We use each side as the exponent of e, the base of the natural logarithms, to obtain

$$\frac{\Delta T}{\Delta T_0} = e^{-At}$$

or

$$\Delta T = \Delta T_0\, e^{-At}\ .$$