43. (a) The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v)\,dv = 1$. Evaluate the integral by calculating the area under the curve in Fig. 20–22. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0. Thus $\int P(v)\,dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0$, so $\frac{3}{2}av_0 = 1$ and $a = 2/3v_0$.

(b) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)\,dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is $Na(2.0v_0 - 1.5v_0) = 0.5Nav_0 = N/3$, where $2/3v_0$ was substituted for a.

(c) The average speed is given by

$$v_{\text{avg}} = \int vP(v)\,dv\ .$$

For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2\,dv = \frac{a}{3v_0}\,v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0\,,$$

where $2/3v_0$ was substituted for a. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\,dv = \frac{a}{2}\left(4v_0^2 - v_0^2\right) = \frac{3a}{2}v_0^2 = v_0\ .$$

Therefore,

$$v_{\text{avg}} = \frac{2}{9}v_0 + v_0 = 1.22v_0\ .$$

(d) The mean-square speed is given by

$$v_{\text{rms}}^2 = \int v^2 P(v)\,\mathrm{d}v\ .$$

The contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3\,dv = \frac{a}{4v_0}\,v_0^4 = \frac{1}{6}\,v_0^2\ .$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2\,dv = \frac{a}{3}\left(8v_0^3 - v_0^3\right) = \frac{7a}{3}v_0^3 = \frac{14}{9}\,v_0^2\ .$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0\ .$$