

33. (a) We use the principle of conservation of energy. Initially the rocket is at Earth's surface and the potential energy is $U_i = -GMm/R_e = -mgR_e$, where M is the mass of Earth, m is the mass of the rocket, and R_e is the radius of Earth. The relationship $g = GM/R_e^2$ was used. The initial kinetic energy is $\frac{1}{2}mv^2 = 2mgR_e$, where the substitution $v = 2\sqrt{gR_e}$ was made. If the rocket can escape then conservation of energy must lead to a positive kinetic energy no matter how far from Earth it gets. We take the final potential energy to be zero and let K_f be the final kinetic energy. Then, $U_i + K_i = U_f + K_f$ leads to $K_f = U_i + K_i = -mgR_e + 2mgR_e = mgR_e$. The result is positive and the rocket has enough kinetic energy to escape the gravitational pull of Earth.
- (b) We write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Then, $\frac{1}{2}mv_f^2 = mgR_e$ and $v_f = \sqrt{2gR_e}$.