

20. (a) The final pressure is

$$p_f = (5.00 \text{ kPa})e^{(V_i - V_f)/a} = (5.00 \text{ kPa})e^{(1.00 \text{ m}^3 - 2.00 \text{ m}^3)/1.00 \text{ m}^3} = 1.84 \text{ kPa} .$$

- (b) We use the ratio form of the gas law (see Sample Problem 20-1) to find the final temperature of the gas:

$$T_f = T_i \left(\frac{p_f V_f}{p_i V_i} \right) = (600 \text{ K}) \frac{(1.84 \text{ kPa})(2.00 \text{ m}^3)}{(5.00 \text{ kPa})(1.00 \text{ m}^3)} = 441 \text{ K} .$$

For later purposes, we note that this result can be written “exactly” as $T_f = T_i (2e^{-1})$. In our solution, we are avoiding using the “one mole” datum since it is not clear how precise it is.

- (c) The work done by the gas is

$$\begin{aligned} W &= \int_i^f p dV = \int_{V_i}^{V_f} (5.00 \text{ kPa})e^{(V_i - V)/a} dV \\ &= (5.00 \text{ kPa})e^{V_i/a} \cdot \left[-ae^{-V/a} \right]_{V_i}^{V_f} \\ &= (5.00 \text{ kPa})e^{1.00}(1.00 \text{ m}^3) (e^{-1.00} - e^{-2.00}) \\ &= 3.16 \text{ kJ} . \end{aligned}$$

- (d) Consideration of a two-stage process as suggested in the hint, brings us simply to Eq. 21-4. Consequently, with $C_V = \frac{3}{2}R$ (see Eq. 20-43), we find

$$\begin{aligned} \Delta S &= nR \ln \left(\frac{V_f}{V_i} \right) + n \left(\frac{3}{2}R \right) \ln \left(\frac{T_f}{T_i} \right) \\ &= nR \left(\ln 2 + \frac{3}{2} \ln(2e^{-1}) \right) \\ &= \frac{p_i V_i}{T_i} \left(\ln 2 + \frac{3}{2} \ln 2 + \frac{3}{2} \ln e^{-1} \right) \\ &= \frac{(5000 \text{ Pa})(1.00 \text{ m}^3)}{600 \text{ K}} \left(\frac{5}{2} \ln 2 - \frac{3}{2} \right) \\ &= 1.94 \text{ J/K} . \end{aligned}$$