71. The *Hint* given in the problem would make the computation in part (a) very straightforward (without doing the integration as we show here), but we present this further level of detail in case that hint is not obvious or – simply – in case one wishes to see how the calculus supports our intuition.

(a) The (centripetal) force exerted on an infinitesimal portion of the blade with mass dm located a distance r from the rotational axis is (Newton's second law) $dF = (dm)\omega^2 r$, where dm can be written as $(M/L)dr$ and the angular speed is $\omega = (320)(2\pi/60) = 33.5$ rad/s. Thus for the entire blade of mass M and length L the total force is given by

$$\begin{aligned} F &= \int dF = \int \omega^2 \, r \, dm \\ &= \frac{M}{L}\int_0^L \omega^2 \, r \, dr \\ &= \left.\frac{M\omega^2 r^2}{2L}\right|_0^L = \frac{M\omega^2 L}{2} \\ &= \frac{(110\,\text{kg})(33.5\,\text{rad/s})^2(7.80\,\text{m})}{2} \\ &= 4.8\times 10^5 \text{ N} . \end{aligned}$$

(b) About its center of mass, the blade has $I = ML^2/12$ according to Table 11-2(e), and using the parallel-axis theorem to "move" the axis of rotation to its end-point, we find the rotational inertia becomes $I = ML^2/3$. Using Eq. 11-37, the torque (assumed constant) is

$$\begin{aligned} \tau &= I\alpha \\ &= \left(\frac{1}{3}ML^2\right)\left(\frac{\Delta\omega}{\Delta t}\right) \\ &= \frac{1}{3}(110\,\text{kg})(7.8\,\text{m})^2\left(\frac{33.5\,\text{rad/s}}{6.7\,\text{s}}\right) \\ &= 1.1\times 10^4 \text{ N}\cdot\text{m} . \end{aligned}$$

(c) Using Eq. 11-44, the work done is

$$\begin{aligned} W &= \Delta K = \frac{1}{2}I\omega^2 - 0 \\ &= \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \\ &= \frac{1}{6}(110\,\text{kg})(7.80\,\text{m})^2(33.5\,\text{rad/s})^2 \\ &= 1.3\times 10^6 \text{ J} . \end{aligned}$$