49. (a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}MR^2$, where M is its mass and R is its radius. The disk of this problem rotates about a point that is displaced from its center by $R + L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}MR^2 + M(L+R)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is

$$\begin{aligned} I &= \frac{1}{2}MR^2 + M(L+R)^2 + \frac{1}{3}mL^2 \\ &= \frac{1}{2}(0.500\,\mathrm{kg})(0.100\,\mathrm{m})^2 + (0.500\,\mathrm{kg})(0.500\,\mathrm{m} + 0.100\,\mathrm{m})^2 + \frac{1}{3}(0.270\,\mathrm{kg})(0.500\,\mathrm{m})^2 \\ &= 0.205\ \mathrm{kg}\cdot\mathrm{m}^2\ . \end{aligned}$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + R = 0.500\,\mathrm{m} + 0.100\,\mathrm{m} = 0.600\ \mathrm{m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\,\mathrm{m})/2 = 0.250\,\mathrm{m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M + m} = \frac{(0.500\,\mathrm{kg})(0.600\,\mathrm{m}) + (0.270\,\mathrm{kg})(0.250\,\mathrm{m})}{0.500\,\mathrm{kg} + 0.270\,\mathrm{kg}} = 0.477\ \mathrm{m}\ .$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\,\mathrm{kg}\cdot\mathrm{m}^2}{(0.500\,\mathrm{kg} + 0.270\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.447\,\mathrm{m})}} = 1.50\ \mathrm{s}\ .$$