109. (Third problem in **Cluster 2**)

(a) Eq. 4-25, which assumes $(x_0, y_0) = (0, 0)$, gives

$$y = 5.00 = (\tan\theta_0)x - \frac{gx^2}{2\,(v_0\cos\theta_0)^2}$$

where $x = 30.0$ (lengths are in meters and time is in seconds). Using the trig identity suggested in the problem and letting u stand for $\tan\theta_0$, we have a second-degree equation for u (its two roots leading to the values $\theta_{0\,\min}$, and $\theta_{0\,\max}$) parameterized by the initial speed v_0.

$$\frac{4410}{v_0^2}\,u^2 - 30.0u + \left(\frac{4410}{v_0^2} + 5.00\right) = 0$$

where numerical simplifications have already been made. To see these steps written with the *variables* x, y, v_0 and g made explicit, see the solution to problem 111, below. Now, we solve for u using the quadratic formula, and then find the angles:

$$\theta_0 = \tan^{-1}\left(\frac{1}{294}\,v_0^2 \pm \frac{1}{294}\,\sqrt{v_0^4 - 98\,v_0^2 - 86436}\right)$$

where the plus is chosen for $\theta_{0\max}$ and the negative is chosen for $\theta_{0\min}$.

(b) These angles are plotted (in degrees) versus v_0 (in m/s) as follows. There are no (real) solutions of the above equations for $18.0 \le v_0 \le 18.6$ m/s (this is further discussed in the next problem).