11. (a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2 \alpha}{\alpha^2}$$

where $\alpha = (\pi a/\lambda) \sin\theta$, a is the slit width and λ is the wavelength. The angle θ is measured from the forward direction. We require $I = I_m/2$, so

$$\sin^2 \alpha = \frac{1}{2}\alpha^2 \ .$$

(b) We evaluate $\sin^2 \alpha$ and $\alpha^2/2$ for $\alpha = 1.39\,\text{rad}$ and compare the results. To be sure that $1.39\,\text{rad}$ is closer to the correct value for α than any other value with three significant digits, we could also try $1.385\,\text{rad}$ and $1.395\,\text{rad}$.

(c) Since $\alpha = (\pi a/\lambda) \sin\theta$,

$$\theta = \sin^{-1}\left(\frac{\alpha\lambda}{\pi a}\right) \ .$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1}\left(\frac{0.442\lambda}{a}\right) \ .$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{0.442\lambda}{a}\right) \ .$$

(d) For $a/\lambda = 1.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/1.0) = 0.916\,\text{rad} = 52.5^\circ \ ,$$

for $a/\lambda = 5.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/5.0) = 0.177\,\text{rad} = 10.1^\circ \ ,$$

and for $a/\lambda = 10$,

$$\Delta\theta = 2\sin^{-1}(0.442/10) = 0.0884\,\text{rad} = 5.06^\circ \ .$$