75. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y axis, where there is presumed to be no acceleration, leads to

$$N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\,\mathrm{max}}$ is assumed, then Newton's second law applied to the x axis (which also has $a = 0$ even though it is "verging" on moving) yields

$$\begin{aligned} F\cos\theta - f_s &= ma \quad , \quad \text{or} \\ F\cos\theta - \mu_s(mg - F\sin\theta) &= 0 \end{aligned}$$

which we solve, for $\theta = 42°$ and $\mu_s = 0.42$, to obtain $F = 74$ N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s\sin\theta} \quad .$$

(c) We minimize the above expression for F by working through the $\frac{dF}{d\theta} = 0$ condition:

$$\frac{dF}{d\theta} = \frac{\mu_s W\,(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)^2} = 0$$

which leads to the result $\theta = \tan^{-1}\mu_s = 23°$.

(d) Plugging $\theta = 23°$ into the above result for F, with $\mu_s = 0.42$ and $W = 180$ N, yields $F = 70$ N.