37. (a) The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -2GM^2/r_i$ since the final separation is $r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2 .$$

The solution for v is

$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2\cdot kg})(10^{30}\,\mathrm{kg})}{10^{10}\,\mathrm{m}}} = 8.2 \times 10^4\,\mathrm{m/s} .$$

(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5$ m, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes $-GM^2/r_i = -GM^2/r_f + Mv^2$. The solution for v is

$$\begin{aligned} v &= \sqrt{GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} \\ &= \sqrt{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2\cdot kg})(10^{30}\,\mathrm{kg})\left(\frac{1}{2 \times 10^5\,\mathrm{m}} - \frac{1}{10^{10}\,\mathrm{m}}\right)} \\ &= 1.8 \times 10^7\ \mathrm{m/s} . \end{aligned}$$