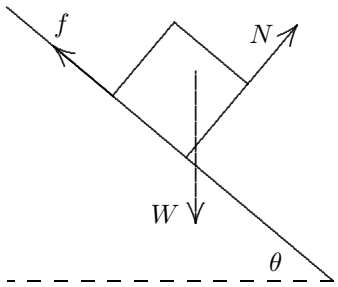
35.

The force diagram shown on the right depicts the situation just before the crate tips, when the normal force acts at the front edge. However, it may also be used to calculate the angle for which the crate begins to slide. W is the force of gravity on the crate, N is the normal force of the plane on the crate, and f is the force of friction. We take the x axis to be down the plane and the y axis to be in the direction of the normal force. We assume the acceleration is zero but the crate is on the verge of sliding.



(a) The x and y components of Newton's second law are

$$W \sin\theta - f = 0 \quad \text{and} \quad N - W\cos\theta = 0$$

respectively. The y equation gives $N = W\cos\theta$. Since the crate is about to slide $f = \mu_s N = \mu_s W\cos\theta$, where μ_s is the coefficient of static friction. We substitute into the x equation and find

$$W\sin\theta - \mu_s W\cos\theta = 0 \quad \Longrightarrow \quad \tan\theta = \mu_s \ .$$

This leads to $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.60 = 31.0°$.

In developing an expression for the total torque about the center of mass when the crate is about to tip, we find that the normal force and the force of friction act at the front edge. The torque associated with the force of friction tends to turn the crate clockwise and has magnitude fh, where h is the perpendicular distance from the bottom of the crate to the center of gravity. The torque associated with the normal force tends to turn the crate counterclockwise and has magnitude $N\ell/2$, where ℓ is the length of a edge. Since the total torque vanishes, $fh = N\ell/2$. When the crate is about to tip, the acceleration of the center of gravity vanishes, so $f = W\sin\theta$ and $N = W\cos\theta$. Substituting these expressions into the torque equation, we obtain

$$\theta = \tan^{-1}\frac{\ell}{2h} = \tan^{-1}\frac{1.2\,\text{m}}{2(0.90\,\text{m})} = 33.7° \ .$$

As θ is increased from zero the crate slides before it tips. It starts to slide when $\theta = 31.0°$.

(b) The analysis is the same. The crate begins to slide when $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.70 = 35.0°$ and begins to tip when $\theta = 33.7°$. Thus, it tips first as the angle is increased. Tipping begins at $\theta = 33.7°$.