

54. We refer to Fig. 35-2 in the textbook. Consider the two light rays, r and r' , which are closest to and on either side of the normal ray (the ray that reverses when it reflects). Each of these rays has an angle of incidence equal to θ when they reach the mirror. Consider that these two rays reach the top and bottom edges of the pupil after they have reflected. If ray r strikes the mirror at point A and ray r' strikes the mirror at B , the distance between A and B (call it x) is

$$x = 2d_o \tan \theta$$

where d_o is the distance from the mirror to the object. We can construct a right triangle starting with the image point of the object (a distance d_o behind the mirror; see I in Fig. 35-2). One side of the triangle follows the extended normal axis (which would reach from I to the middle of the pupil), and the hypotenuse is along the extension of ray r (after reflection). The distance from the pupil to I is $d_{\text{ey}} + d_o$, and the small angle in this triangle is again θ . Thus,

$$\tan \theta = \frac{R}{d_{\text{ey}} + d_o}$$

where R is the pupil radius (2.5 mm). Combining these relations, we find

$$x = 2d_o \frac{R}{d_{\text{ey}} + d_o} = 2(100 \text{ mm}) \frac{2.5 \text{ mm}}{300 \text{ mm} + 100 \text{ mm}}$$

which yields $x = 1.67 \text{ mm}$. Now, x serves as the diameter of a circular area A on the mirror, in which all rays that reflect will reach the eye. Therefore,

$$A = \frac{1}{4} \pi x^2 = \frac{\pi}{4} (1.67 \text{ mm})^2 = 2.2 \text{ mm}^2 .$$