53. (a) To stretch the spring an external force, equal in magnitude to the force of the spring but opposite to its direction, is applied. Since a spring stretched in the positive x direction exerts a force in the negative x direction, the applied force must be $F = 52.8x + 38.4x^2$, in the $+x$ direction. The work it does is

$$W = \int_{0.50}^{1.00} (52.8x + 38.4x^2) \, dx = \left[\frac{52.8}{2} x^2 + \frac{38.4}{3} x^3 \right]_{0.50}^{1.00} = 31.0 \text{ J} .$$

(b) The spring does 31.0 J of work and this must be the increase in the kinetic energy of the particle. Its speed is then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(31.0 \text{ J})}{2.17 \text{ kg}}} = 5.35 \text{ m/s} .$$

(c) The force is conservative since the work it does as the particle goes from any point x_1 to any other point x_2 depends only on x_1 and x_2, not on details of the motion between x_1 and x_2.