

27. At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + R^2}} \right]$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 23-26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of z such that $E/E_c = 1/2$. This means

$$1 - \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2} \implies \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2} .$$

Squaring both sides, then multiplying them by $z^2 + R^2$, we obtain $z^2 = (z^2/4) + (R^2/4)$. Thus, $z^2 = R^2/3$ and $z = R/\sqrt{3}$.