

50. (a) The charge  $q$  on the capacitor as a function of time is  $q(t) = (\mathcal{E}C)(1 - e^{-t/RC})$ , so the charging current is  $i(t) = dq/dt = (\mathcal{E}/R)e^{-t/RC}$ . The energy supplied by the emf is then

$$U = \int_0^\infty \mathcal{E}i \, dt = \frac{\mathcal{E}^2}{R} \int_0^\infty e^{-t/RC} \, dt = C\mathcal{E}^2 = 2U_C$$

where  $U_C = \frac{1}{2}C\mathcal{E}^2$  is the energy stored in the capacitor.

- (b) By directly integrating  $i^2R$  we obtain

$$U_R = \int_0^\infty i^2 R \, dt = \frac{\mathcal{E}^2}{R} \int_0^\infty e^{-2t/RC} \, dt = \frac{1}{2}C\mathcal{E}^2 .$$