55. (a) The magnitude of the magnetic field on the axis of a circular loop, a distance z from the loop center, is given by Eq. 30-28:

$$B = \frac{N\mu_0 i R^2}{2(R^2+z^2)^{3/2}} ,$$

where R is the radius of the loop, N is the number of turns, and i is the current. Both of the loops in the problem have the same radius, the same number of turns, and carry the same current. The currents are in the same sense, and the fields they produce are in the same direction in the region between them. We place the origin at the center of the left-hand loop and let x be the coordinate of a point on the axis between the loops. To calculate the field of the left-hand loop, we set $z = x$ in the equation above. The chosen point on the axis is a distance $s - x$ from the center of the right-hand loop. To calculate the field it produces, we put $z = s - x$ in the equation above. The total field at the point is therefore

$$B = \frac{N\mu_0 i R^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(R^2+x^2-2sx+s^2)^{3/2}}\right] .$$

Its derivative with respect to x is

$$\frac{dB}{dx} = -\frac{N\mu_0 i R^2}{2}\left[\frac{3x}{(R^2+x^2)^{5/2}} + \frac{3(x-s)}{(R^2+x^2-2sx+s^2)^{5/2}}\right] .$$

When this is evaluated for $x = s/2$ (the midpoint between the loops) the result is

$$\left.\frac{dB}{dx}\right|_{s/2} = -\frac{N\mu_0 i R^2}{2}\left[\frac{3s/2}{(R^2+s^2/4)^{5/2}} - \frac{3s/2}{(R^2+s^2/4-s^2+s^2)^{5/2}}\right] = 0$$

independently of the value of s.

(b) The second derivative is

$$\begin{aligned}\frac{d^2B}{dx^2} &= \frac{N\mu_0 i R^2}{2}\left[-\frac{3}{(R^2+x^2)^{5/2}} + \frac{15x^2}{(R^2+x^2)^{7/2}}\right.\\ &\qquad \left. -\frac{3}{(R^2+x^2-2sx+s^2)^{5/2}} + \frac{15(x-s)^2}{(R^2+x^2-2sx+s^2)^{7/2}}\right] .\end{aligned}$$

At $x = s/2$,

$$\begin{aligned}\left.\frac{d^2B}{dx^2}\right|_{s/2} &= \frac{N\mu_0 i R^2}{2}\left[-\frac{6}{(R^2+s^2/4)^{5/2}} + \frac{30s^2/4}{(R^2+s^2/4)^{7/2}}\right]\\ &= \frac{N\mu_0 R^2}{2}\left[\frac{-6(R^2+s^2/4)+30s^2/4}{(R^2+s^2/4)^{7/2}}\right] = 3N\mu_0 i R^2\,\frac{s^2-R^2}{(R^2+s^2/4)^{7/2}} .\end{aligned}$$

Clearly, this is zero if $s = R$.