

23. (a) As the body moves along the x axis from $x_i = 3.0$ m to $x_f = 4.0$ m the work done by the force is

$$\begin{aligned} W &= \int_{x_i}^{x_f} F_x dx \\ &= \int_{x_i}^{x_f} -6x dx = -3(x_f^2 - x_i^2) \\ &= -3(4.0^2 - 3.0^2) = -21 \text{ J} . \end{aligned}$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21)}{2.0} + 8.0^2} = 6.6 \text{ m/s} .$$

- (b) The velocity of the particle is $v_f = 5.0$ m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f . The net work done on the particle is $W = -3(x_f^2 - x_i^2)$, so the theorem leads to

$$-3(x_f^2 - x_i^2) = \frac{1}{2}m(v_f^2 - v_i^2) .$$

Thus,

$$\begin{aligned} x_f &= \sqrt{-\frac{m}{6}(v_f^2 - v_i^2) + x_i^2} \\ &= \sqrt{-\frac{2.0 \text{ kg}}{6 \text{ N/m}}((5.0 \text{ m/s})^2 - (8.0 \text{ m/s})^2) + (3.0 \text{ m})^2} \\ &= 4.7 \text{ m} . \end{aligned}$$