

14. The “coincidence” of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 38-20 to apply without additional terms. In part (a), we apply these equations directly with $v = +0.400c = 1.199 \times 10^8 \text{ m/s}$, and in part (b) we simply change $v \rightarrow -v$ and recalculate the primed values.

(a) The position coordinate measured in the S' frame is

$$\begin{aligned} x' &= \gamma(x - vt) = \frac{x - vt}{\sqrt{1 - \beta^2}} \\ &= \frac{3.00 \times 10^8 \text{ m} - (1.199 \times 10^8 \text{ m/s})(2.50 \text{ s})}{\sqrt{1 - (0.400)^2}} \\ &= 2.7 \times 10^5 \text{ m/s} \approx 0 , \end{aligned}$$

where we conclude that the numerical result (2.7×10^5 or 2.3×10^5 depending on how precise a value of v is used) is not meaningful (in the significant figures sense) and should be set equal to zero (that is, it is “consistent with zero” in view of the statistical uncertainties involved). The time coordinate measured in the S' frame is

$$\begin{aligned} t' &= \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t - \frac{\beta x}{c}}{\sqrt{1 - \beta^2}} \\ &= \frac{2.50 \text{ s} - \frac{(0.400)(3.00 \times 10^8 \text{ m})}{2.998 \times 10^8 \text{ m/s}}}{\sqrt{1 - (0.400)^2}} \\ &= 2.29 \text{ s} . \end{aligned}$$

(b) Now, we obtain

$$x' = \frac{x + vt}{\sqrt{1 - \beta^2}} = \frac{3.00 \times 10^8 \text{ m} + (1.199 \times 10^8 \text{ m/s})(2.50 \text{ s})}{\sqrt{1 - (0.400)^2}} = 6.54 \times 10^8 \text{ m} ,$$

and

$$t' = \gamma\left(t + \frac{vx}{c^2}\right) = \frac{2.50 \text{ s} + \frac{(0.400)(3.00 \times 10^8 \text{ m})}{2.998 \times 10^8 \text{ m/s}}}{\sqrt{1 - (0.400)^2}} = 3.16 \text{ s} .$$