

68. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution(s) to the overall deviation is (are) the reflection(s). Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r , and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after [each] reflection) is $\delta\theta_r = 180^\circ - 2\theta_r$. Thus, for k reflections, we have $\delta\theta_2 = k\theta_r$ to account for these contributions. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 2(\theta_i - \theta_r) + k(180^\circ - 2\theta_r) = k(180^\circ) + 2\theta_i - 2(k+1)\theta_r.$$

- (b) For $k = 2$ and $n = 1.331$ (given in problem 67), we search for the second-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 230.37° , and this occurs at $\theta_i = 71.90^\circ$.
- (c) Similarly, we find that the second-order θ_{dev} minimum for blue light (for which $n = 1.343$) is 233.48° , and this occurs at $\theta_i = 71.52^\circ$.
- (d) The difference in θ_{dev} in the previous two parts is 3.11° .
- (e) Setting $k = 3$, we search for the third-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 317.53° , and this occurs at $\theta_i = 76.88^\circ$.
- (f) Similarly, we find that the third-order θ_{dev} minimum for blue light is 321.89° , and this occurs at $\theta_i = 76.62^\circ$.
- (g) The difference in θ_{dev} in the previous two parts is 4.37° .