

25. For an object in front of a thin lens, the object distance  $p$  and the image distance  $i$  are related by  $(1/p) + (1/i) = (1/f)$ , where  $f$  is the focal length of the lens. For the situation described by the problem, all quantities are positive, so the distance  $x$  between the object and image is  $x = p + i$ . We substitute  $i = x - p$  into the thin lens equation and solve for  $x$ :

$$x = \frac{p^2}{p - f} .$$

To find the minimum value of  $x$ , we set  $dx/dp = 0$  and solve for  $p$ . Since

$$\frac{dx}{dp} = \frac{p(p - 2f)}{(p - f)^2} ,$$

the result is  $p = 2f$ . The minimum distance is

$$x_{\min} = \frac{p^2}{p - f} = \frac{(2f)^2}{2f - f} = 4f .$$

This is a minimum, rather than a maximum, since the image distance  $i$  becomes large without bound as the object approaches the focal point.