90. (Fourth problem of **Cluster**)

(a) At its displaced position, its potential energy (using $k = 1/4\pi\varepsilon_0$) is

$$U_i = k\frac{qQ}{d - x_0} + k\frac{qQ}{d + x_0} \quad = \quad \frac{2kqQd}{d^2 - x_0^2} \ .$$

And at A, the potential energy is

$$U_A = 2\left(k\frac{qQ}{d}\right) \ .$$

Setting this difference equal to the kinetic energy of the particle ($\frac{1}{2}mv^2$) and solving for the speed yields

$$v = \sqrt{\frac{2\,(U_i - U_A)}{m}} \quad = \quad 2\,x_0\sqrt{\frac{k\,q\,Q}{m\,d\,(d^2 - x_0^2)}} \ .$$

(b) It is straightforward to consider small x_0 (more precisely, $x_0/d \ll 1$) in the above expression (so that $d^2 - x_0^2 \approx d^2$). The result is

$$v \approx 2\,\frac{x_0}{d}\sqrt{\frac{k\,q\,Q}{m\,d}} \ .$$

(c) Plugging in the given values (converted to SI units) yields $v \approx 19$ m/s.

(d) Using the Pythagorean theorem, we now have

$$U_i = 2k\frac{-qQ}{\sqrt{d^2 + x_0^2}} \ .$$

Therefore, (with U_A in this part equal to the negative of U_A in the previous part)

$$v = \sqrt{\frac{2\,(U_i - U_A)}{m}} \quad = \quad 2\sqrt{\frac{k\,q\,Q}{m}\left(\frac{1}{d} - \frac{1}{\sqrt{d^2 + x_0^2}}\right)} \ .$$

To simplify, the binomial theorem (Appendix E) is employed:

$$\frac{1}{\sqrt{d^2 + x_0^2}} \quad \approx \quad \frac{1}{d}\left(1 - \frac{1}{2}\frac{x_0^2}{d^2}\right)$$

which leads to

$$v \quad \approx \quad \frac{x_0}{d}\sqrt{\frac{2k\,q\,Q}{m\,d}} \ .$$