

44. (a) From Eq. 39-11

$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos \phi) = (2.43 \text{ pm})(1 - \cos 90^\circ) = 2.43 \text{ pm} .$$

(b) The fractional shift should be interpreted as  $\Delta\lambda$  divided by the original wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{2.425 \text{ pm}}{590 \text{ nm}} = 4.11 \times 10^{-6} .$$

(c) The change in energy for a photon with  $\lambda = 590 \text{ nm}$  is given by

$$\begin{aligned} \Delta E_{\text{ph}} &= \Delta \left( \frac{hc}{\lambda} \right) \approx -\frac{hc\Delta\lambda}{\lambda^2} \\ &= -\frac{(4.14 \times 10^{-15} \text{ eV}\cdot\text{s})(2.998 \times 10^8 \text{ m/s})(2.43 \text{ pm})}{(590 \text{ nm})^2} \\ &= -8.67 \times 10^{-6} \text{ eV} . \end{aligned}$$

For an x ray photon of energy  $E_{\text{ph}} = 50 \text{ keV}$ ,  $\Delta\lambda$  remains the same (2.43 pm), since it is independent of  $E_{\text{ph}}$ . The fractional change in wavelength is now

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\lambda}{hc/E_{\text{ph}}} = \frac{(50 \times 10^3 \text{ eV})(2.43 \text{ pm})}{(4.14 \times 10^{-15} \text{ eV}\cdot\text{s})(2.998 \times 10^8 \text{ m/s})} = 9.78 \times 10^{-2} ,$$

and the change in photon energy is now

$$\Delta E_{\text{ph}} = hc \left( \frac{1}{\lambda + \Delta\lambda} - \frac{1}{\lambda} \right) = -\left( \frac{hc}{\lambda} \right) \frac{\Delta\lambda}{\lambda + \Delta\lambda} = -E_{\text{ph}} \left( \frac{\alpha}{1 + \alpha} \right)$$

where  $\alpha = \Delta\lambda/\lambda$ . We substitute  $E_{\text{ph}} = 50 \text{ keV}$  and  $\alpha = 9.78 \times 10^{-2}$  to obtain  $\Delta E_{\text{ph}} = -4.45 \text{ keV}$ . (Note that in this case  $\alpha \approx 0.1$  is not close enough to zero so the approximation  $\Delta E_{\text{ph}} \approx hc\Delta\lambda/\lambda^2$  is not as accurate as in the first case, in which  $\alpha = 4.12 \times 10^{-6}$ . In fact if one were to use this approximation here, one would get  $\Delta E_{\text{ph}} \approx -4.89 \text{ keV}$ , which does not amount to a satisfactory approximation.)