47. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = +64°$ for the alpha (α) particle (after collision) and $\phi = -51°$ for the oxygen nucleus (o) (which is going into the fourth quadrant, in our scenario). We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_\alpha v_\alpha &= m_\alpha v'_\alpha \cos\theta + m_o v'_o \cos\phi \\ 0 &= m_\alpha v'_\alpha \sin\theta + m_o v'_o \sin\phi \end{aligned}$$

We are given $v'_o = 1.2 \times 10^5$ m/s, which leaves us two unknowns and two equations, which is sufficient for solving.

(a) We solve for the final alpha particle speed using the y-momentum equation:

$$v'_\alpha = -\frac{m_\alpha v'_\alpha \sin\theta}{m_o \sin\phi} = -\frac{(16)\left(1.2 \times 10^5\right)\sin(-51°)}{(4)\sin(64°)}$$

which yields $v'_\alpha = 4.15 \times 10^5$ m/s.

(b) Plugging our result from part (a) into the x-momentum equation produces the initial alpha particle speed:

$$\begin{aligned} m_\alpha v_\alpha &= \frac{m_\alpha v'_\alpha \cos\theta + m_o v'_o \cos\phi}{m_\alpha} \\ &= \frac{(4)\left(4.15 \times 10^5\right)\cos(64°) + (16)\left(1.2 \times 10^5\right)\cos(-51°)}{4} \\ &= 4.84 \times 10^5 \text{ m/s} . \end{aligned}$$