

41. The efficiency of the engine is defined by $\varepsilon = W/Q_1$ and is shown in the text to be $\varepsilon = (T_1 - T_2)/T_1$, so $W/Q_1 = (T_1 - T_2)/T_1$. The coefficient of performance of the refrigerator is defined by $K = Q_4/W$ and is shown in the text to be $K = T_4/(T_3 - T_4)$, so $Q_4/W = T_4/(T_3 - T_4)$. Now $Q_4 = Q_3 - W$, so $(Q_3 - W)/W = T_4/(T_3 - T_4)$. The work done by the engine is used to drive the refrigerator, so W is the same for the two. Solve the engine equation for W and substitute the resulting expression into the refrigerator equation. The engine equation yields $W = (T_1 - T_2)Q_1/T_1$ and the substitution yields

$$\frac{T_4}{T_3 - T_4} = \frac{Q_3}{W} - 1 = \frac{Q_3 T_1}{Q_1 (T_1 - T_2)} - 1.$$

Solve for Q_3/Q_1 :

$$\frac{Q_3}{Q_1} = \left(\frac{T_4}{T_3 - T_4} + 1 \right) \left(\frac{T_1 - T_2}{T_1} \right) = \left(\frac{T_3}{T_3 - T_4} \right) \left(\frac{T_1 - T_2}{T_1} \right) = \frac{1 - (T_2/T_1)}{1 - (T_4/T_3)}.$$