

12. (a) The distance between q_1 and q_2 is

$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-0.020 - 0.035)^2 + (0.015 - 0.005)^2} = 0.0559 \text{ m} .$$

The magnitude of the force exerted by q_1 on q_2 is

$$F_{21} = k \frac{|q_1 q_2|}{r_{12}^2} = \frac{(8.99 \times 10^9) (3.0 \times 10^{-6}) (4.0 \times 10^{-6})}{0.0559^2} = 34.5 \text{ N} .$$

The vector \vec{F}_{21} is directed towards q_1 and makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1} \left(\frac{y_2 - y_1}{x_2 - x_1} \right) = \tan^{-1} \left(\frac{1.5 - 0.5}{-2.0 - 3.5} \right) = -10.3^\circ .$$

- (b) Let the third charge be located at (x_3, y_3) , a distance r from q_2 . We note that q_1 , q_2 and q_3 must be colinear; otherwise, an equilibrium position for any one of them would be impossible to find. Furthermore, we cannot place q_3 on the same side of q_2 where we also find q_1 , since in that region both forces (exerted on q_2 by q_3 and q_1) would be in the same direction (since q_2 is attracted to both of them). Thus, in terms of the angle found in part (a), we have $x_3 = x_2 - r \cos \theta$ and $y_3 = y_2 - r \sin \theta$ (which means $y_3 > y_2$ since θ is negative). The magnitude of force exerted on q_2 by q_3 is $F_{23} = k|q_2 q_3|/r^2$, which must equal that of the force exerted on it by q_1 (found in part (a)). Therefore,

$$k \frac{|q_2 q_3|}{r^2} = k \frac{|q_1 q_2|}{r_{12}^2} \implies r = r_{12} \sqrt{\frac{q_3}{q_1}} = 0.0645 \text{ cm} .$$

Consequently, $x_3 = x_2 - r \cos \theta = -2.0 \text{ cm} - (6.45 \text{ cm}) \cos(-10.3^\circ) = -8.4 \text{ cm}$ and $y_3 = y_2 - r \sin \theta = 1.5 \text{ cm} - (6.45 \text{ cm}) \sin(-10.3^\circ) = 2.7 \text{ cm}$.