

68. (a) Placing a wire (of resistance r) with current i running directly from point a to point b in Fig. 28-41 divides the top of the picture into a left and a right triangle. If we label the currents through each resistor with the corresponding subscripts (for instance, i_s goes toward the lower right through R_s and i_x goes toward the upper right through R_x), then the currents must be related as follows:

$$\begin{array}{lll} i_0 = i_1 + i_s & \text{and} & i_1 = i + i_2 \\ i_s + i = i_x & \text{and} & i_2 + i_x = i_0 \end{array}$$

where the last relation is not independent of the previous three. The loop equations for the two triangles and also for the bottom loop (containing the battery and point b) lead to

$$\begin{aligned} i_s R_s - i_1 R_1 - i r &= 0 \\ i_2 R_2 - i_x R_x - i r &= 0 \\ \mathcal{E} - i_0 R_0 - i_s R_s - i_x R_x &= 0 . \end{aligned}$$

We incorporate the current relations from above into these loop equations in order to obtain three well-posed “simultaneous” equations, for three unknown currents (i_s , i_1 and i):

$$\begin{aligned} i_s R_s - i_1 R_1 - i r &= 0 \\ i_1 R_2 - i_s R_x - i (r + R_x + R_2) &= 0 \\ \mathcal{E} - i_s (R_0 + R_s + R_x) - i_1 R_0 - i R_x &= 0 \end{aligned}$$

The problem statement further specifies $R_1 = R_2 = R$ and $R_0 = 0$, which causes our solution for i to simplify significantly. It becomes

$$i = \frac{\mathcal{E} (R_s - R_x)}{2rR_s + 2R_xR_s + R_sR + 2rR_x + R_xR}$$

which is equivalent to the result shown in the problem statement.

- (b) Examining the numerator of our final result in part (a), we see that the condition for $i = 0$ is $R_s = R_x$. Since $R_1 = R_2 = R$, this is equivalent to $R_x = R_s R_2 / R_1$, consistent with the result of Problem 43.