

11. Our  $x$  axis is along the wire with the origin at the midpoint. The current flows in the positive  $x$  direction. All segments of the wire produce magnetic fields at  $P_1$  that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at  $P_1$  is given by

$$dB = \frac{\mu_0 i}{4\pi} \frac{\sin \theta}{r^2} dx$$

where  $\theta$  (the angle between the segment and a line drawn from the segment to  $P_1$ ) and  $r$  (the length of that line) are functions of  $x$ . Replacing  $r$  with  $\sqrt{x^2 + R^2}$  and  $\sin \theta$  with  $R/r = R/\sqrt{x^2 + R^2}$ , we integrate from  $x = -L/2$  to  $x = L/2$ . The total field is

$$B = \frac{\mu_0 i R}{4\pi} \int_{-L/2}^{L/2} \frac{dx}{(x^2 + R^2)^{3/2}} = \frac{\mu_0 i R}{4\pi} \frac{1}{R^2} \frac{x}{(x^2 + R^2)^{1/2}} \bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R} \frac{L}{\sqrt{L^2 + 4R^2}}.$$

If  $L \gg R$ , then  $R^2$  in the denominator can be ignored and

$$B = \frac{\mu_0 i}{2\pi R}$$

is obtained. This is the field of a long straight wire. For points very close to a finite wire, the field is quite similar to that of an infinitely long wire.