

24. Since the volume of a sphere is $4\pi R^3/3$, the density is

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{3M_{\text{total}}}{4\pi R^3} .$$

When we test for gravitational acceleration (caused by the sphere, or by parts of it) at radius r (measured from the center of the sphere), the mass M which is at radius less than r is what contributes to the reading (GM/r^2). Since $M = \rho(4\pi r^3/3)$ for $r \leq R$ then we can write this result as

$$\frac{G \left(\frac{3M_{\text{total}}}{4\pi R^3} \right) \left(\frac{4\pi r^3}{3} \right)}{r^2} = \frac{GM_{\text{total}} r}{R^3}$$

when we are considering points on or inside the sphere. Thus, the value a_g referred to in the problem is the case where $r = R$:

$$a_g = \frac{GM_{\text{total}}}{R^2} ,$$

and we solve for the case where the acceleration equals $a_g/3$:

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}} r}{R^3} \implies r = \frac{R}{3} .$$

Now we treat the case of an external test point. For points with $r > R$ the acceleration is GM_{total}/r^2 , so the requirement that it equal $a_g/3$ leads to

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}}{r^2} \implies r = R\sqrt{3} .$$