15. (a) The allowed energy values are given by $E_n = n^2h^2/8mL^2$. The difference in energy between the state n and the state $n+1$ is

$$\Delta E_{\mathrm{adj}} = E_{n+1} - E_n = \left[(n+1)^2 - n^2\right] \frac{h^2}{8mL^2} = \frac{(2n+1)h^2}{8mL^2}$$

and

$$\frac{\Delta E_{\mathrm{adj}}}{E} = \left[\frac{(2n+1)h^2}{8mL^2}\right]\left(\frac{8mL^2}{n^2h^2}\right) = \frac{2n+1}{n^2} \ .$$

As n becomes large, $2n+1 \longrightarrow 2n$ and $(2n+1)/n^2 \longrightarrow 2n/n^2 = 2/n$.

(b) As $n \longrightarrow \infty, \Delta E_{\mathrm{adj}}$ and E do not approach 0, but $\Delta E_{\mathrm{adj}}/E$ does.

(c) See part (b).

(d) See part (b).

(e) $\Delta E_{\mathrm{adj}}/E$ is a better measure than either ΔE_{adj} or E alone of the extent to which the quantum result is approximated by the classical result.