

45. Since the slit width is much less than the wavelength of the light, the central peak of the single-slit diffraction pattern is spread across the screen and the diffraction envelope can be ignored. Consider three waves, one from each slit. Since the slits are evenly spaced, the phase difference for waves from the first and second slits is the same as the phase difference for waves from the second and third slits. The electric fields of the waves at the screen can be written $E_1 = E_0 \sin(\omega t)$, $E_2 = E_0 \sin(\omega t + \phi)$, and $E_3 = E_0 \sin(\omega t + 2\phi)$, where $\phi = (2\pi d/\lambda) \sin \theta$. Here d is the separation of adjacent slits and λ is the wavelength. The phasor diagram is shown below. It yields

$$E = E_0 \cos \phi + E_0 + E_0 \cos \phi = E_0(1 + 2 \cos \phi)$$

for the amplitude of the resultant wave. Since the intensity of a wave is proportional to the square of the electric field, we may write $I = AE_0^2(1 + 2 \cos \phi)^2$, where A is a constant of proportionality. If I_m is the intensity at the center of the pattern, for which $\phi = 0$, then $I_m = 9AE_0^2$. We take A to be $I_m/9E_0^2$ and obtain

$$I = \frac{I_m}{9} (1 + 2 \cos \phi)^2 = \frac{I_m}{9} (1 + 4 \cos \phi + 4 \cos^2 \phi) .$$

