18. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects). The reference position for computing U (and height h) is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) Careful examination of the figure leads to the trigonometric relation $h = L - L\cos\theta$ when the angle is measured from vertical as shown. Thus, the gravitational potential energy is $U = mgL(1-\cos\theta_0)$ at the position shown in Fig. 8-32 (the initial position). Thus, we have

$$\begin{aligned} K_0 + U_0 &= K_f + U_f \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

which leads to

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0)\right)} = \sqrt{v_0^2 + 2gL(1-\cos\theta_0)}\ .$$

(b) We look for the initial speed required to barely reach the horizontal position – described by $v_h = 0$ and $\theta = 90°$ (or $\theta = -90°$, if one prefers, but since $\cos(-\phi) = \cos\phi$, the sign of the angle is not a concern).

$$\begin{aligned} K_0 + U_0 &= K_h + U_h \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= 0 + mgL \end{aligned}$$

which leads to $v_0 = \sqrt{2gL\cos\theta_0}$.

(c) For the cord to remain straight, then the centripetal force (at the top) must be (at least) equal to gravitational force:

$$\frac{mv_t^2}{r} = mg \implies mv_t^2 = mgL$$

where we recognize that $r = L$. We plug this into the expression for the kinetic energy (at the top, where $\theta = 180°$).

$$\begin{aligned} K_0 + U_0 &= K_t + U_t \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= \frac{1}{2}mv_t^2 + mg\,(1-\cos 180°) \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= \frac{1}{2}(mgL) + mg(2L) \end{aligned}$$

which leads to $v_0 = \sqrt{gL(3+2\cos\theta_0)}$.

(d) The more initial potential energy there is, the less initial kinetic energy there needs to be, in order to reach the positions described in parts (b) and (c). Increasing θ_0 amounts to increasing U_0, so we see that a greater value of θ_0 leads to smaller results for v_0 in parts (b) and (c).