

25. (a) The magnitude of the force on a particle with mass m at the surface of Earth is given by $F = GMm/R^2$, where M is the total mass of Earth and R is Earth's radius. The acceleration due to gravity is

$$a_g = \frac{F}{m} = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m})^2} = 9.83 \text{ m/s}^2 .$$

- (b) Now $a_g = GM/R^2$, where M is the total mass contained in the core and mantle together and R is the outer radius of the mantle ($6.345 \times 10^6 \text{ m}$, according to Fig. 14-36). The total mass is $M = 1.93 \times 10^{24} \text{ kg} + 4.01 \times 10^{24} \text{ kg} = 5.94 \times 10^{24} \text{ kg}$. The first term is the mass of the core and the second is the mass of the mantle. Thus,

$$a_g = \frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.94 \times 10^{24} \text{ kg})}{(6.345 \times 10^6 \text{ m})^2} = 9.84 \text{ m/s}^2 .$$

- (c) A point 25 km below the surface is at the mantle-crust interface and is on the surface of a sphere with a radius of $R = 6.345 \times 10^6 \text{ m}$. Since the mass is now assumed to be uniformly distributed the mass within this sphere can be found by multiplying the mass per unit volume by the volume of the sphere: $M = (R^3/R_e^3)M_e$, where M_e is the total mass of Earth and R_e is the radius of Earth. Thus,

$$M = \left(\frac{6.345 \times 10^6 \text{ m}}{6.37 \times 10^6 \text{ m}} \right)^3 (5.98 \times 10^{24} \text{ kg}) = 5.91 \times 10^{24} \text{ kg} .$$

The acceleration due to gravity is

$$a_g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.91 \times 10^{24} \text{ kg})}{(6.345 \times 10^6 \text{ m})^2} = 9.79 \text{ m/s}^2 .$$