98. We establish coordinates with $\hat{\mathrm{i}}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\mathrm{j}}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}| = u = 6.4$ km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = 3.2\,\hat{\mathrm{j}}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{bg} = v\,\hat{\mathrm{i}}$ where $v > 0$ is unknown. Thus, all $\hat{\mathrm{j}}$ components must cancel in the vector sum

$$\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$$

which means the $u\sin\theta = -3.2$, so $\theta = \sin^{-1}(-3.2/6.4) = -30°$.

(b) Using the result from part (a), we find $v = u\cos\theta = 5.5$ km/h. Thus, traveling a distance of $\ell = 6.4$ km requires a time of $6.4/5.5 = 1.15$ h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with $v_w = 3.2$ km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{u + v_w} + \frac{D}{u - v_w} = 1.33 \text{ h}$$

where $D = 3.2$ km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{u + v_w} + \frac{D}{u - v_w} = \frac{D}{u - v_w} + \frac{D}{u + v_w}$$

the answer is the same as in the previous part.

(e) The case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = u\cos\theta\,\hat{\mathrm{i}} + (u\sin\theta + v_w)\,\hat{\mathrm{j}}$$

where the x component of $\vec{v}_{bg}$ must equal ell/t. Thus,

$$t = \frac{\ell}{u\cos\theta}$$

which can be minimized using $dt/d\theta = 0$ (though, of course, an easier way is to appeal to either physical or mathematical intuition – concluding that the shortest-time path should have $\theta = 0$). Then $t = 6.4/6.4 = 1.0$ h, or 60 min.