

23. Schrödinger's equation for the region $x > L$ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} [E - U_0] \psi = 0 .$$

If $\psi = De^{2kx}$, then $d^2\psi/dx^2 = 4k^2De^{2kx} = 4k^2\psi$ and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} [E - U_0] \psi = 4k^2\psi + \frac{8\pi^2m}{h^2} [E - U_0] \psi .$$

This is zero provided

$$k = \frac{\pi}{h} \sqrt{2m(U_0 - E)} .$$

The proposed function satisfies Schrödinger's equation provided k has this value. Since U_0 is greater than E in the region $x > L$, the quantity under the radical is positive. This means k is real. If k is positive, however, the proposed function is physically unrealistic. It increases exponentially with x and becomes large without bound. The integral of the probability density over the entire x axis must be unity. This is impossible if ψ is the proposed function.