37. We label the two isotopes with subscripts 1 (for ^{32}P) and 2 (for ^{33}P). Initially, 10% of the decays come from ^{33}P, which implies that the initial rate $R_{02} = 9R_{01}$. Using Eq. 43-16, this means

$$R_{01} = \lambda_1 N_{01} = \frac{1}{9} R_{02} = \frac{1}{9} \lambda_2 N_{02} \ .$$

At time t, we have $R_1 = R_{01} e^{-\lambda_1 t}$ and $R_2 = R_{02} e^{-\lambda_2 t}$. We seek the value of t for which $R_1 = 9R_2$ (which means 90% of the decays arise from ^{33}P). We divide equations to obtain $(R_{01}/R_{02}) e^{-(\lambda_1 - \lambda_2)t} = 9$, and solve for t:

$$\begin{aligned} t &= \frac{1}{\lambda_1 - \lambda_2} \ln\left(\frac{R_{01}}{9R_{02}}\right) = \frac{\ln(R_{01}/9R_{02})}{\ln 2/T_{1/2\,1} - \ln 2/T_{1/2\,2}} \\ &= \frac{\ln[(1/9)^2]}{\ln 2[(14.3\,\mathrm{d})^{-1} - (25.3\,\mathrm{d})^{-1}]} = 209 \text{ d} \ . \end{aligned}$$