51. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are n_1 loops in the aluminum section of the wire. Then, $L_1 = n_1\lambda_1/2 = n_1 v_1/2f$, where λ_1 is the wavelength and v_1 is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where f is the frequency. Thus $f = n_1 v_1/2L_1$. A similar expression holds for the steel section: $f = n_2 v_2/2L_2$. Since the frequency is the same for the two sections, $n_1 v_1/L_1 = n_2 v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\tau/\mu_1}$, where μ_1 is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1 A L_1$, where ρ_1 is the mass density (mass per unit volume) for aluminum and A is the cross-sectional area of the wire. Thus $\mu_1 = \rho_1 A L_1/L_1 = \rho_1 A$ and $v_1 = \sqrt{\tau/\rho_1 A}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\tau/\rho_2 A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$, where A has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\text{m})\sqrt{7.80\times 10^3\,\text{kg/m}^3}}{(0.600\,\text{m})\sqrt{2.60\times 10^3\,\text{kg/m}^3}} = 2.5\ .$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is $f = n_1 v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1 A}$. The tension is provided by the hanging block and is $\tau = mg$, where m is the mass of the block. Thus

$$f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{(10.0\,\text{kg})(9.8\,\text{m/s}^2)}{(2.60\times 10^3\,\text{kg/m}^3)(1.00\times 10^{-6}\,\text{m}^2)}} = 324\,\text{Hz}\ .$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.