

28. We are looking for the values of the ratio

$$\frac{E_{n_x, n_y, n_z}}{\hbar^2/8mL^2} = L^2 \left( \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right) = (n_x^2 + n_y^2 + n_z^2)$$

and the corresponding differences.

- (a) For  $n_x = n_y = n_z = 1$ , the ratio becomes  $1 + 1 + 1 = 3.00$ .
- (b) For  $n_x = n_y = 2$  and  $n_z = 1$ , the ratio becomes  $4 + 4 + 1 = 9.00$ . One can check (by computing other  $(n_x, n_y, n_z)$  values) that this is the third lowest energy in the system. One can also check that this same ratio is obtained for  $(n_x, n_y, n_z) = (2, 1, 2)$  and  $(1, 2, 2)$ .
- (c) For  $n_x = n_y = 1$  and  $n_z = 3$ , the ratio becomes  $1 + 1 + 9 = 11.00$ . One can check (by computing other  $(n_x, n_y, n_z)$  values) that this is three “steps” up from the lowest energy in the system. One can also check that this same ratio is obtained for  $(n_x, n_y, n_z) = (1, 3, 1)$  and  $(3, 1, 1)$ . If we take the difference between this and the result of part (b), we obtain  $11.00 - 9.00 = 2.00$ .
- (d) For  $n_x = n_y = 1$  and  $n_z = 2$ , the ratio becomes  $1 + 1 + 4 = 6.00$ . One can check (by computing other  $(n_x, n_y, n_z)$  values) that this is the next to the lowest energy in the system. One can also check that this same ratio is obtained for  $(n_x, n_y, n_z) = (2, 1, 1)$  and  $(1, 2, 1)$ . Thus, three states (three arrangements of  $(n_x, n_y, n_z)$  values) have this energy.
- (e) For  $n_x = 1$ ,  $n_y = 2$  and  $n_z = 3$ , the ratio becomes  $1 + 4 + 9 = 14.00$ . One can check (by computing other  $(n_x, n_y, n_z)$  values) that this is five “steps” up from the lowest energy in the system. One can also check that this same ratio is obtained for  $(n_x, n_y, n_z) = (1, 3, 2), (2, 3, 1), (2, 1, 3), (3, 1, 2)$  and  $(3, 2, 1)$ . Thus, six states (six arrangements of  $(n_x, n_y, n_z)$  values) have this energy.