

45. The distance traveled by the pion in the frame of Earth is (using Eq. 38-12)  $d = v \Delta t$ . The proper lifetime  $\Delta t_0$  is related to  $\Delta t$  by the time-dilation formula:  $\Delta t = \gamma \Delta t_0$ . To use this equation, we must first find the Lorentz factor  $\gamma$  (using Eq. 38-45). Since the total energy of the pion is given by  $E = 1.35 \times 10^5 \text{ MeV}$  and its  $mc^2$  value is  $139.6 \text{ MeV}$ , then

$$\gamma = \frac{E}{mc^2} = \frac{1.35 \times 10^5 \text{ MeV}}{139.6 \text{ MeV}} = 967.05 .$$

Therefore, the lifetime of the moving pion as measured by Earth observers is

$$\Delta t = \gamma \Delta t_0 = (967.1)(35.0 \times 10^{-9} \text{ s}) = 3.385 \times 10^{-5} \text{ s} ,$$

and the distance it travels is

$$d \approx c \Delta t = (2.998 \times 10^8 \text{ m/s})(3.385 \times 10^{-5} \text{ s}) = 1.015 \times 10^4 \text{ m} = 10.15 \text{ km}$$

where we have approximated its speed as  $c$  (note: its speed can be found by solving Eq. 38-8, which gives  $v = 0.9999995c$ ; this more precise value for  $v$  would not significantly alter our final result). Thus, the altitude at which the pion decays is  $120 \text{ km} - 10.15 \text{ km} = 110 \text{ km}$ .