13. We can use the mc^2 value for an electron from Table 38-3 (511×10^3 eV) and the hc value developed in problem 3 of Chapter 39 by writing Eq. 40-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(mc^2)L^2} \ .$$

(a) With $L = 3.0 \times 10^9$ nm, the energy difference is

$$E_2 - E_1 = \frac{1240^2}{8(511 \times 10^3)(3.0 \times 10^9)^2}\left(2^2 - 1^2\right) = 1.3 \times 10^{-19} \text{ eV} \ .$$

(b) Since $(n+1)^2 - n^2 = 2n + 1$, we have

$$\Delta E = E_{n+1} - E_n = \frac{h^2}{8mL^2}(2n+1) = \frac{(hc)^2}{8(mc^2)L^2}(2n+1) \ .$$

Setting this equal to 1.0 eV, we solve for n:

$$\begin{aligned} n &= \frac{4(mc^2)L^2\Delta E}{(hc)^2} - \frac{1}{2} \\ &= \frac{4(511 \times 10^3 \text{ eV})(3.0 \times 10^9 \text{ nm})^2(1.0 \text{ eV})}{(1240 \text{ eV}\cdot\text{nm})^2} - \frac{1}{2} \\ &\approx 12 \times 10^{18} \ . \end{aligned}$$

(c) At this value of n, the energy is

$$E_n = \frac{1240^2}{8(511 \times 10^3)(3.0 \times 10^9)^2}\left(6 \times 10^{18}\right)^2 \approx 6 \times 10^{18} \text{ eV} \ .$$

(d) Since

$$\frac{E_n}{mc^2} = \frac{6 \times 10^{18} \text{ eV}}{511 \times 10^3 \text{ eV}} \gg 1 \ ,$$

the energy is indeed in the relativistic range.