

99. (a) The potential energy at the turning point is equal (in the absence of friction) to the total kinetic energy (translational plus rotational) as it passes through the equilibrium position:

$$\begin{aligned}
 \frac{1}{2}kx_m^2 &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2 \\
 &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2 \\
 &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{4}Mv_{\text{cm}}^2 = \frac{3}{4}Mv_{\text{cm}}^2 .
 \end{aligned}$$

which leads to $Mv_{\text{cm}}^2 = 2kx_m^2/3 = 0.125$ J. The translational kinetic energy is therefore $\frac{1}{2}Mv_{\text{cm}}^2 = kx_m^2/3 = 0.0625$ J.

- (b) And the rotational kinetic energy is $\frac{1}{4}Mv_{\text{cm}}^2 = kx_m^2/6 = 0.03125$ J.
(c) In this part, we use v_{cm} to denote the speed at any instant (and not just the maximum speed as we had done in the previous parts). Since the energy is constant, then

$$\begin{aligned}
 \frac{dE}{dt} &= 0 \\
 \frac{d}{dt}\left(\frac{3}{4}Mv_{\text{cm}}^2\right) \frac{d}{dt}\left(\frac{1}{2}kx^2\right) &= 0 \\
 \frac{3}{2}Mv_{\text{cm}}a_{\text{cm}} + kxv_{\text{cm}} &= 0
 \end{aligned}$$

which leads to

$$a_{\text{cm}} = -\left(\frac{2k}{3M}\right)x .$$

Comparing with Eq. 16-8, we see that $\omega = \sqrt{2k/3M}$ for this system. Since $\omega = 2\pi/T$, we obtain the desired result: $T = 2\pi\sqrt{3M/2k}$.