83. (Fourth problem in **Cluster**)
It is clear by symmetry that $x_{\rm com} = B/2$ for the system. The value of $y_{\rm com}$ is found as follows. If the thickness is Δz and the density is ρ, then the relation between the mass element dm and a height element dy is

$$dm = \rho \Delta z \ell_y \, dy = \frac{M}{A} \ell_y \, dy$$

where the area of the semicircle is $A = \frac{1}{2}\pi R^2$ and the length of each horizontal "strip" at height y is $\ell_y = 2\sqrt{R^2 - y^2}$. Therefore, using Eq. 9-9, we find

$$\begin{aligned} y_{\rm com} &= \frac{1}{M} \int_0^R y \, \frac{M}{A} \, 2\sqrt{R^2 - y^2} \, dy \\ &= \frac{2}{\frac{1}{2}\pi R^2} \int_0^R y \sqrt{R^2 - y^2} \, dy \\ &= \frac{4}{\pi R^2} \left[-\frac{1}{3} \left(R^2 - y^2 \right)^{3/2} \right]_0^R \\ &= \frac{4R}{3\pi} \ . \end{aligned}$$