41. The difference between the electron-photon scattering process in this problem and the one studied in the text (the Compton shift, see Eq. 39-11) is that the electron is in motion relative with speed v to the laboratory frame. To utilize the result in Eq. 39-11, shift to a new reference frame in which the electron is at rest before the scattering. Denote the quantities measured in this new frame with a prime ($'$), and apply Eq. 39-11 to yield

$$\Delta\lambda' = \lambda' - \lambda_0' = \frac{h}{m_e c}(1 - \cos\pi) = \frac{2h}{m_e c} ,$$

where we note that $\phi = \pi$ (since the photon is scattered back in the direction of incidence). Now, from the Doppler shift formula (Eq. 38-25) the frequency f_0' of the photon prior to the scattering in the new reference frame satisfies

$$f_0' = \frac{c}{\lambda_0'} = f_0\sqrt{\frac{1+\beta}{1-\beta}} ,$$

where $\beta = v/c$. Also, as we switch back from the new reference frame to the original one after the scattering

$$f = f'\sqrt{\frac{1-\beta}{1+\beta}} = \frac{c}{\lambda'}\sqrt{\frac{1-\beta}{1+\beta}} .$$

We solve the two Doppler-shift equations above for λ' and λ_0' and substitute the results into the Compton shift formula for $\Delta\lambda'$:

$$\Delta\lambda' = \frac{1}{f}\sqrt{\frac{1-\beta}{1+\beta}} - \frac{1}{f_0}\sqrt{\frac{1-\beta}{1+\beta}} = \frac{2h}{m_e c^2} .$$

Some simple algebra then leads to

$$E = hf = hf_0\left(1 + \frac{2h}{m_e c^2}\sqrt{\frac{1+\beta}{1-\beta}}\right)^{-1} .$$