

19. The electric field at a point on the axis of a uniformly charged ring, a distance z from the ring center, is given by

$$E = \frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}$$

where q is the charge on the ring and R is the radius of the ring (see Eq. 23-16). For q positive, the field points upward at points above the ring and downward at points below the ring. We take the positive direction to be upward. Then, the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} .$$

For small amplitude oscillations $z \ll R$ and z can be neglected in the denominator. Thus,

$$F = -\frac{eqz}{4\pi\epsilon_0 R^3} .$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to z , just as if the electron were attached to a spring with spring constant $k = eq/4\pi\epsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\epsilon_0 m R^3}}$$

where m is the mass of the electron.