35. With no loss of generality, we assume the electron is initially at rest (which simply means we are analyzing the collision from its initial rest frame). If the photon gave all its momentum and energy to the (free) electron, then the momentum and the kinetic energy of the electron would become

$$p = \frac{hf}{c} \quad \text{and} \quad K = hf \ ,$$

respectively. Plugging these expressions into Eq. 38-51 (with m referring to the mass of the electron) leads to

$$\begin{aligned} (pc)^2 &= K^2 + 2Kmc^2 \\ (hf)^2 &= (hf)^2 + 2hfmc^2 \end{aligned}$$

which is clearly impossible, since the last term ($2hfmc^2$) is not zero. We have shown that considering total momentum and energy absorption of a photon by a free electron leads to an inconsistency in the mathematics, and thus cannot be expected to happen in nature.