93. Since the aim of this problem is to invite student creativity (and possibly some research), we "invent a problem" (and give its solution) somewhat along the lines of part (b) (in fact, the student might consider running our example "in reverse"). Consider a block of mass M that falls from rest a distance H to a vertical spring of spring constant k. The spring compresses by x_c in order to halt the block, but on the rebound (due to the fact that the block is stuck on the end of the spring) the spring stretches (relative to its original relaxed length) an amount x_s before the block is momentarily at rest again. Take both values of x to be positive. Find x_c and x_s and their difference.

Solution: The height to which the spring reaches when it is relaxed is our $y = 0$ reference level. We relate the initial situation (when the block is dropped) to the situation of maximum compression using energy conservation.

$$K_0 + U_0 = K_c + U_c \implies 0 + MgH = 0 + Mg(-x_c) + \frac{1}{2}kx_c^2$$

The positive root stemming from a quadratic formula solution for x_c yields

$$x_c = \frac{Mg}{k}\left(1 + \sqrt{1 + \frac{2kH}{Mg}}\right) .$$

Next, we relate the initial situation to the final situation (of maximal stretch) using energy conservation.

$$K_0 + U_0 = K_s + U_s \implies 0 + MgH = 0 + Mgx_s + \frac{1}{2}kx_s^2$$

The positive root stemming from a quadratic formula solution for x_s yields

$$x_s = \frac{Mg}{k}\left(-1 + \sqrt{1 + \frac{2kH}{Mg}}\right) .$$

Finally, we note that $x_c > x_s$ with the difference being $x_c - x_s = 2Mg/k$.