48. We find the "percent error" in the use of Stirling's approximation by computing

$$\frac{(N(\ln N) - N) - \ln(N!)}{\ln(N!)} = \frac{(N(\ln N) - N)}{\ln(N!)} - 1$$

which would be multiplied by 100% to be expressed as a percentage.

(a) For $N = 50$, the percent error is

$$\frac{50\ln(50) - 50}{\ln(50!)} - 1 = \frac{145.6}{\ln(3.04 \times 10^{64})} - 1 = \frac{145.6}{148.5} - 1$$

which yields -1.9%, meaning Stirling's approximation produces a value that is 1.9% lower than the correct one.

(b) For $N = 100$, this procedure gives the result -0.89%.

(c) And for $N = 250$, we obtain -0.32%.

(d) The trend is such that Stirling's approximation becomes a better estimate of $\ln(N!)$ for larger values of N.