

13. Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i},$$

where the ideal gas law $pV = nRT$ was used to replace p with nRT/V . Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with $p_i V_i$ to obtain

$$W = p_i V_i \ln \frac{p_i}{p_f}.$$

Since the initial gauge pressure is 1.03×10^5 Pa, $p_i = 1.03 \times 10^5$ Pa + 1.013×10^5 Pa = 2.04×10^5 Pa. The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus

$$W = (2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) \ln \frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} = 2.00 \times 10^4 \text{ J}.$$

During the constant pressure portion of the process the work done by the gas is $W = p_f(V_i - V_f)$. The gas starts in a state with pressure p_f , so this is the pressure throughout this portion of the process. We also note that the volume decreases from V_f to V_i . Now $V_f = p_i V_i / p_f$, so

$$\begin{aligned} W &= p_f \left(V_i - \frac{p_i V_i}{p_f} \right) = (p_f - p_i) V_i \\ &= (1.013 \times 10^5 \text{ Pa} - 2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) = -1.44 \times 10^4 \text{ J}. \end{aligned}$$

The total work done by the gas over the entire process is $W = 2.00 \times 10^4 \text{ J} - 1.44 \times 10^4 \text{ J} = 5.6 \times 10^3 \text{ J}$.