38. (a) The average and rms speeds are as follows:

$$v_{\text{avg}} = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{10}[4(200\,\text{m/s}) + 2(500\,\text{m/s}) + 4(600\,\text{m/s})] = 420\,\text{m/s}\,,$$

$$v_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200\,\text{m/s})^2 + 2(500\,\text{m/s})^2 + 4(600\,\text{m/s})^2]} = 458\ \text{m/s}\ .$$

From these results, we see that $v_{\text{rms}} > v_{\text{avg}}$.

(b) One may check the validity of the inequality $v_{\text{rms}} \geq v_{\text{avg}}$ for any speed distribution. For example, we consider a set of ten particles divided into two groups of five particles each, with the first group of particles moving at speed v_1 and the second group at v_2 where both v_1 and v_2 are positive-valued (by the definition of speed). In this case, $v_{\text{avg}} = (v_1 + v_2)/2$ and

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2}{2}}\ .$$

To show this must be greater than (or equal to) v_{avg} we examine the difference in the squares of the quantities:

$$\begin{aligned} v_{\text{rms}}^2 - v_{\text{avg}}^2 &= \frac{v_1^2 + v_2^2}{2} - \frac{1}{4}\left(v_1^2 + v_2^2 + 2v_1v_2\right) \\ &= \frac{v_1^2 + v_2^2 - 2v_1v_2}{4} \\ &= \frac{1}{4}\left(v_1 - v_2\right)^2 \quad \geq\ 0 \end{aligned}$$

which demonstrates that $v_{\text{rms}} \geq v_{\text{avg}}$ in this situation.

(c) As one can infer from our manipulation in the previous part, we will obtain $v_{\text{rms}} = v_{\text{avg}}$ if all speeds are the same (if $v_1 = v_2$ in the previous part).