33. The charge is held constant while the plates are being separated, so we write the expression for the stored energy as $U = q^2/2C$, where q is the charge and C is the capacitance. The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/x$, where A is the plate area and x is the plate separation, so

$$U = \frac{q^2 x}{2\varepsilon_0 A} .$$

If the plate separation increases by dx, the energy increases by $dU = (q^2/2\varepsilon_0 A)\,dx$. Suppose the agent pulling the plate apart exerts force F. Then the agent does work $F\,dx$ and if the plates begin and end at rest, this must equal the increase in stored energy. Thus,

$$F\,dx = \left(\frac{q^2}{2\varepsilon_0 A}\right) dx$$

and

$$F = \frac{q^2}{2\varepsilon_0 A} .$$

The net force on a plate is zero, so this must also be the magnitude of the force one plate exerts on the other. The force can also be computed as the product of the charge q on one plate and the electric field E_1 due to the charge on the other plate. Recall that the field produced by a uniform plane surface of charge is $E_1 = q/2\varepsilon_0 A$. Thus, $F = q^2/2\varepsilon_0 A$.