29. We use conservation of mechanical energy: the mechanical energy must be the same at the top of the swing as it is initially. Newton's second law is used to find the speed, and hence the kinetic energy, at the top. There the tension force T of the string and the force of gravity are both downward, toward the center of the circle. We notice that the radius of the circle is $r = L - d$, so the law can be written $T + mg = mv^2/(L - d)$, where v is the speed and m is the mass of the ball. When the ball passes the highest point with the least possible speed, the tension is zero. Then

$$mg = m\frac{v^2}{L-d} \implies v = \sqrt{g(L-d)} \ .$$

We take the gravitational potential energy of the ball-Earth system to be zero when the ball is at the bottom of its swing. Then the initial potential energy is mgL. The initial kinetic energy is zero since the ball starts from rest. The final potential energy, at the top of the swing, is $2mg(L - d)$ and the final kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}mg(L - d)$ using the above result for v. Conservation of energy yields

$$mgL = 2mg(L-d) + \frac{1}{2}mg(L-d) \implies d = 3L/5 \ .$$

If d is greater than this value, so the highest point is lower, then the speed of the ball is greater as it reaches that point and the ball passes the point. If d is less, the ball cannot go around. Thus the value we found for d is a lower limit.