

60. This can be worked entirely by the methods of Chapters 2-6, but we will use energy methods in as many steps as possible.

- (a) By a force analysis of the style done in Ch. 6, we find the normal force has magnitude  $N = mg \cos \theta$  (where  $\theta = 40^\circ$ ) which means  $f_k = \mu_k mg \cos \theta$  where  $\mu_k = 0.15$ . Thus, Eq. 8-29 yields  $\Delta E_{\text{th}} = f_k d = \mu_k mgd \cos \theta$ . Also, elementary trigonometry leads us to conclude that  $\Delta U = mgd \sin \theta$ . Eq. 8-31 (with  $W = 0$  and  $K_f = 0$ ) provides an equation for determining  $d$ :

$$\begin{aligned} K_i &= \Delta U + \Delta E_{\text{th}} \\ \frac{1}{2}mv_i^2 &= mgd(\sin \theta + \mu_k \cos \theta) \end{aligned}$$

where  $v_i = 1.4$  m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2g(\sin \theta + \mu_k \cos \theta)} = 0.13 \text{ m} .$$

- (b) Now that we know where on the incline it stops ( $d' = 0.13 + 0.55 = 0.68$  m from the bottom), we can use Eq. 8-31 again (with  $W = 0$  and now with  $K_i = 0$ .) to describe the final kinetic energy (at the bottom):

$$\begin{aligned} K_f &= -\Delta U - \Delta E_{\text{th}} \\ \frac{1}{2}mv^2 &= mgd'(\sin \theta - \mu_k \cos \theta) \end{aligned}$$

which – after dividing by the mass and rearranging – yields

$$v = \sqrt{2gd'(\sin \theta - \mu_k \cos \theta)} = 2.7 \text{ m/s} .$$

- (c) In part (a) it is clear that  $d$  increases if  $\mu_k$  decreases – both mathematically (since it is a positive term in the denominator) and intuitively (less friction – less energy “lost”). In part (b), there are two terms in the expression for  $v$  which imply that it should increase if  $\mu_k$  were smaller: the increased value of  $d' = d_0 + d$  and that last factor  $\sin \theta - \mu_k \cos \theta$  which indicates that less is being subtracted from  $\sin \theta$  when  $\mu_k$  is less (so the factor itself increases in value).