54. (a) Since Sample Problem 15-9 deals with a similar situation, we use the final equation (labeled "Answer") from it:

$$v = \sqrt{2gh} \implies v = v_o \text{ for the projectile motion.}$$

The stream of water emerges horizontally ($\theta_o = 0°$ in the notation of Chapter 4), and setting $y - y_o = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t = \sqrt{\frac{-2(H-h)}{-g}} = \sqrt{\frac{2}{g}(H-h)} \ .$$

Using this in Eq. 4-21, where $x_o = 0$ by choice of coordinate origin, we find

$$x = v_o t = \sqrt{2gh}\sqrt{\frac{2}{g}(H-h)} = 2\sqrt{h(H-h)} \ .$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H-h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2 - Hh + \frac{x^2}{4} = 0 \implies h = \frac{H \pm \sqrt{H^2 - x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H \ .$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$.

(c) We wish to maximize the function $f = x^2 = 4h(H-h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh} = 4H - 8h = 0 \implies h = \frac{H}{2}$$

as the depth from which an emerging stream of water will travel the maximum horizontal distance.