110. (Fourth problem in **Cluster 2**)

Following the hint in the problem (regarding *analytic* solution), we equate the square root expression, above, to zero:

$$\sqrt{v_0^4 - 98\, v_0^2 - 86436} = 0 \quad \Longrightarrow \quad v_0 = 18.6 \text{ m/s} .$$

That solution can be obtained either with the quadratic formula (by writing the equation, first, in terms of $w = v_0^2$) or with a polynomial solver built into many calculators; in the latter approach, this is straightforwardly handled as a fourth degree polynomial. Note that the other root ($v_0 = 15.8$ m/s) is dismissed since we are finding where the *real* solutions for angle disappear as one decreases the initial speed from roughly 20 m/s. In case this problem was assigned without assigning Problem 109 first, then this (the choice of root) might be a confusing point. Plugging $v_0 = 18.6$ m/s into

$$\theta_0 = \tan^{-1}\left(\frac{1}{294}\, v_0^2 \pm \frac{1}{294}\sqrt{v_0^4 - 98\, v_0^2 - 86436}\right)$$

(which is unambiguous since the square root factor is zero) provides the launch angle: $\theta_0 = 49.7°$ in this "critical" case.