

49. (a) The charge on the positive plate of the capacitor is given by

$$q = C\mathcal{E} (1 - e^{-t/\tau}) ,$$

where \mathcal{E} is the emf of the battery, C is the capacitance, and τ is the time constant. The value of τ is $\tau = RC = (3.00 \times 10^6 \Omega)(1.00 \times 10^{-6} \text{ F}) = 3.00 \text{ s}$. At $t = 1.00 \text{ s}$, $t/\tau = (1.00 \text{ s})/(3.00 \text{ s}) = 0.333$ and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\mathcal{E}}{\tau} e^{-t/\tau} = \frac{(1.00 \times 10^{-6})(4.00 \text{ V})}{3.00 \text{ s}} e^{-0.333} = 9.55 \times 10^{-7} \text{ C/s} .$$

- (b) The energy stored in the capacitor is given by

$$U_C = \frac{q^2}{2C} .$$

and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C} \frac{dq}{dt} .$$

Now

$$q = C\mathcal{E} (1 - e^{-t/\tau}) = (1.00 \times 10^{-6})(4.00 \text{ V})(1 - e^{-0.333}) = 1.13 \times 10^{-6} \text{ C} ,$$

so

$$\frac{dU_C}{dt} = \left(\frac{1.13 \times 10^{-6} \text{ C}}{1.00 \times 10^{-6} \text{ F}} \right) (9.55 \times 10^{-7} \text{ C/s}) = 1.08 \times 10^{-6} \text{ W} .$$

- (c) The rate at which energy is being dissipated in the resistor is given by $P = i^2 R$. The current is $9.55 \times 10^{-7} \text{ A}$, so

$$P = (9.55 \times 10^{-7} \text{ A})^2 (3.00 \times 10^6 \Omega) = 2.74 \times 10^{-6} \text{ W} .$$

- (d) The rate at which energy is delivered by the battery is

$$i\mathcal{E} = (9.55 \times 10^{-7} \text{ A})(4.00 \text{ V}) = 3.82 \times 10^{-6} \text{ W} .$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\mathcal{E} = (q/C) (dq/dt) + i^2 R$. Except for some round-off error the numerical results support the conservation principle.