57. Careful consideration of how the angle θ relates to height h (measured from the lowest position) gives $h = R(1 - \cos\theta)$. The energy at the amplitude point is equal to the energy as it swings through the lowest position:

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 \\ gR\,(1 - \cos\theta_m) &= \frac{1}{2}v^2 \end{aligned}$$

where the mass has been canceled in the last step. The tension (acting upward on the bob when it swings through the lowest position) is related to the bob's weight mg and the centripetal acceleration using Newton's second law:

$$T - mg = m\,\frac{v^2}{R}\ .$$

From the above, we substitute for v^2:

$$T - mg = m\,\frac{2gR\,(1 - \cos\theta_m)}{R} = 2mg\,(1 - \cos\theta_m)\ .$$

(a) This provides an "exact" answer for the tension, but the problem directs us to examine the small angle behavior: $\cos\theta \approx 1 - \theta^2/2$ (where θ is in radians). Solving for T and using this approximation, we find

$$T \approx mg + 2mg\left(\frac{\theta_m^2}{2}\right) = mg\left(1 + \theta_m^2\right)\ .$$

(b) At other values of θ (other than the lowest position, where $\theta = 0$), Newton's second law yields

$$T' - mg\cos\theta = m\,\frac{v^2}{R} \quad \text{or} \quad T' - mg\left(1 - \frac{\theta^2}{2}\right) \approx m\,\frac{v^2}{R}\ .$$

Making the same substitutions as before, we obtain

$$T' \approx mg\left(1 + \theta_m^2 - \theta^2\right)$$

which is clearly smaller than the result of part (a).