

35. For a given value of the principal quantum number n , there are n possible values of the orbital quantum number ℓ , ranging from 0 to $n - 1$. For any value of ℓ , there are $2\ell + 1$ possible values of the magnetic quantum number m_ℓ , ranging from $-\ell$ to $+\ell$. Finally, for each set of values of ℓ and m_ℓ , there are two states, one corresponding to the spin quantum number $m_s = -\frac{1}{2}$ and the other corresponding to $m_s = +\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N = 2 \sum_0^{n-1} (2\ell + 1) .$$

Now

$$\sum_0^{n-1} 2\ell = 2 \sum_0^{n-1} \ell = 2 \frac{n}{2} (n - 1) = n(n - 1) ,$$

since there are n terms in the sum and the average term is $(n - 1)/2$. Furthermore,

$$\sum_0^{n-1} 1 = n .$$

Thus $N = 2 [n(n - 1) + n] = 2n^2$.