48. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = +60°$ for the proton (1) which is assumed to scatter into the first quadrant and $\phi = -30°$ for the target proton (2) which scatters into the fourth quadrant (recall that the problem has told us that this is perpendicular to θ). We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_1 v_1 &= m_1 v_1' \cos\theta + m_2 v_2' \cos\phi \\ 0 &= m_1 v_1' \sin\theta + m_2 v_2' \sin\phi \end{aligned}$$

We are given $v_1 = 500$ m/s, which provides us with two unknowns and two equations, which is sufficient for solving. Since $m_1 = m_2$ we can cancel the mass out of the equations entirely.

(a) Combining the above equations and solving for v_2' we obtain

$$v_2' = \frac{v_1 \sin(\theta)}{\sin(\theta - \phi)} = \frac{500 \sin(60°)}{\sin(90°)} = 433$$

in SI units (m/s). We used the identity $\sin(\theta)\cos(\phi) - \cos(\theta)\sin(\phi) = \sin(\theta - \phi)$ in simplifying our final expression.

(b) In a similar manner, we find

$$v_1' = \frac{v_1 \sin(\phi)}{\sin(\phi - \theta)} = \frac{500 \sin(-30°)}{\sin(-90°)} = 250 \text{ m/s} .$$