

10. (a) When the small sphere is released at the edge of the large “bowl” (the hemisphere of radius  $R$ ), its center of mass is at the same height at that edge, but when it is at the bottom of the “bowl” its center of mass is a distance  $r$  above the the bottom surface of the hemisphere. Since the small sphere descends by  $R - r$ , its loss in gravitational potential energy is  $mg(R - r)$ , which, by conservation of mechanical energy, is equal to its kinetic energy at the bottom of the track.
- (b) Using Eq. 12-5 for  $K$ , the asked-for fraction becomes

$$\frac{K_{\text{rot}}}{K} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{\text{com}}^2} = \frac{1}{1 + \left(\frac{M}{I}\right) \left(\frac{v_{\text{com}}}{\omega}\right)^2} .$$

Substituting  $v_{\text{com}} = R\omega$  (Eq. 12-2) and  $I = \frac{2}{5}MR^2$  (Table 11-2(f)), we obtain

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1 + \left(\frac{5}{2R^2}\right) R^2} = \frac{2}{7} .$$

- (c) The small sphere is executing circular motion so that when it reaches the bottom, it experiences a radial acceleration upward (in the direction of the normal force which the “bowl” exerts on it). From Newton’s second law along the vertical axis, the normal force  $N$  satisfies  $N - mg = ma_{\text{com}}$  where  $a_{\text{com}} = v_{\text{com}}^2/(R - r)$ . Therefore,

$$N = mg + \frac{mv_{\text{com}}^2}{R - r} = \frac{mg(R - r) + mv_{\text{com}}^2}{R - r} .$$

But from part (a),  $mg(R - r) = K$ , and from Eq. 12-5,  $\frac{1}{2}mv_{\text{com}}^2 = K - K_{\text{rot}}$ . Thus,

$$N = \frac{K + 2(K - K_{\text{rot}})}{R - r} = 3\frac{K}{R - r} - 2\frac{K_{\text{rot}}}{R - r} .$$

We now plug in  $R - r = K/mg$  and use the result of part (b):

$$N = 3mg - 2mg \left(\frac{2}{7}\right) = \frac{17}{7} mg .$$