

75. We take the derivative with respect to x of both sides of Eq. 34-11:

$$\frac{\partial}{\partial x} \left(\frac{\partial E}{\partial x} \right) = \frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x} \left(-\frac{\partial B}{\partial t} \right) = -\frac{\partial^2 B}{\partial x \partial t} .$$

Now we differentiate both sides of Eq. 34-18 with respect to t :

$$\frac{\partial}{\partial t} \left(-\frac{\partial B}{\partial x} \right) = -\frac{\partial^2 B}{\partial x \partial t} = \frac{\partial}{\partial t} \left(\varepsilon_0 \mu_0 \frac{\partial E}{\partial t} \right) = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} .$$

Substituting $\partial^2 E / \partial x^2 = -\partial^2 B / \partial x \partial t$ from the first equation above into the second one, we get

$$\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} ,$$

or

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2 E}{\partial x^2} .$$

Similarly, we differentiate both sides of Eq. 34-11 with respect to t

$$\frac{\partial^2 E}{\partial x \partial t} = -\frac{\partial^2 B}{\partial t^2} ,$$

and differentiate both sides of Eq. 34-18 with respect to x

$$-\frac{\partial^2 B}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial x \partial t} .$$

Combining these two equations, we get

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2} = c^2 \frac{\partial^2 B}{\partial x^2} .$$