111. (Fifth problem in **Cluster 2**)

(a) This builds directly on the solutions of the previous two problems. If we return to the solution of problem 109 without plugging in the data for x, y, and g, we obtain the following expression for the θ_0 roots.

$$\theta_0 = \tan^{-1}\left(\frac{v_0^2}{gx}\left(1 \pm \sqrt{1 - \frac{g}{v_0^2}\left(2y + \frac{gx^2}{v_0^2}\right)}\right)\right)$$

And for the "critical case" of maximum distance for a given launch-speed, we set the square root expression to zero (as in the previous problem) and solve for x_{max}.

$$x_{\text{max}} = \frac{v_0^2}{g}\sqrt{1 - \frac{2gy}{v_0^2}}$$

which one might wish to check for the "straight-up" case (where $x = 0$, and the familiar result $y_{\text{max}} = \frac{1}{2}v_0^2/g$ is obtained) and for the "range" case (where $y = 0$ and this then agrees with Eq. 4-26 where $\theta_0 = 45°$). In the problem at hand, we have $y = 5.00$ m, and $v_0 = 15.0$ m/s. This leads to $x_{\text{max}} = 17.2$ m.

(b) When the square root term vanishes, the expression for θ_0 becomes

$$\theta_0 = \tan^{-1}\left(\frac{v_0^2}{gx}\right) = 53.1°$$

using $x = x_{\text{max}}$ from part (a).