

16. (a) The acceleration is given by Eq. 12-13:

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where upward is the positive translational direction. Taking the coordinate origin at the initial position, Eq. 2-15 leads to

$$y_{\text{com}} = v_{\text{com},0}t + \frac{1}{2}a_{\text{com}}t^2 = v_{\text{com},0}t - \frac{\frac{1}{2}gt^2}{1 + I_{\text{com}}/MR_0^2}$$

where  $y_{\text{com}} = -1.2 \text{ m}$  and  $v_{\text{com},0} = -1.3 \text{ m/s}$ . Substituting  $I_{\text{com}} = 0.000095 \text{ kg} \cdot \text{m}^2$ ,  $M = 0.12 \text{ kg}$ ,  $R_0 = 0.0032 \text{ m}$  and  $g = 9.8 \text{ m/s}^2$ , we use the quadratic formula and find

$$\begin{aligned} t &= \frac{\left(1 + \frac{I_{\text{com}}}{MR_0^2}\right) \left(v_{\text{com},0} \mp \sqrt{v_{\text{com},0}^2 - \frac{2gy_{\text{com}}}{1 + I_{\text{com}}/MR_0^2}}\right)}{g} \\ &= \frac{\left(1 + \frac{0.000095}{(0.12)(0.0032)^2}\right) \left(-1.3 \mp \sqrt{1.3^2 - \frac{2(9.8)(-1.2)}{1 + 0.000095/(0.12)(0.0032)^2}}\right)}{9.8} \\ &= -21.7 \quad \text{or} \quad 0.885 \end{aligned}$$

where we choose  $t = 0.89 \text{ s}$  as the answer.

- (b) We note that the initial potential energy is  $U_i = Mgh$  and  $h = 1.2 \text{ m}$  (using the bottom as the reference level for computing  $U$ ). The initial kinetic energy is as shown in Eq. 12-5, where the initial angular and linear speeds are related by Eq. 12-2. Energy conservation leads to

$$\begin{aligned} K_f &= K_i + U_i \\ &= \frac{1}{2}mv_{\text{com},0}^2 + \frac{1}{2}I\left(\frac{v_{\text{com},0}}{R_0}\right)^2 + Mgh \\ &= \frac{1}{2}(0.12)(1.3)^2 + \frac{1}{2}(9.5 \times 10^{-5})\left(\frac{1.3}{0.0032}\right)^2 + (0.12)(9.8)(1.2) \\ &= 9.4 \text{ J} . \end{aligned}$$

- (c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} + a_{\text{com}}t = v_{\text{com},0} - \frac{gt}{1 + I_{\text{com}}/MR_0^2} .$$

Thus, we obtain

$$v_{\text{com}} = -1.3 - \frac{(9.8)(0.885)}{1 + \frac{0.000095}{(0.12)(0.0032)^2}} = -1.41 \text{ m/s}$$

so its linear speed at that moment is approximately  $1.4 \text{ m/s}$ .

- (d) The translational kinetic energy is  $\frac{1}{2}mv_{\text{com}}^2 = \frac{1}{2}(0.12)(1.41)^2 = 0.12 \text{ J}$ .  
(e) The angular velocity at that moment is given by

$$\omega = -\frac{v_{\text{com}}}{R_0} = -\frac{-1.41}{0.0032} = 441$$

or approximately  $440 \text{ rad/s}$ .

- (f) And the rotational kinetic energy is

$$\frac{1}{2}I_{\text{com}}\omega^2 = \frac{1}{2}(9.50 \times 10^{-5} \text{ kg} \cdot \text{m}^2)(441 \text{ rad/s})^2 = 9.2 \text{ J} .$$