100. (a) Since the speed of sound is lower in air than in water, the speed of sound in the air-water mixture is lower than in pure water (see Table 18-1). Frequency is proportional to the speed of sound (see Eq. 18-39 and Eq. 18-41), so the decrease in speed is "heard" due to the accompanying decrease in frequency.

(b) This follows from Eq. 18-3 and Eq. 18-2 (with Δ's replaced by derivatives). Thus,

$$\frac{1}{v^2} = \frac{\rho}{B} = \frac{\rho}{V\left|\frac{dp}{dV}\right|} = \frac{\rho}{V}\left|\frac{dV}{dp}\right| .$$

(c) Returning to the Δ notation, and letting the absolute values be "understood," we write $\Delta V = \Delta V_w + \Delta V_a$ as indicated in the problem. Subject to the approximations mentioned in the problem, our equation becomes

$$\frac{1}{v^2} = \frac{\rho_w}{V_w}\left(\frac{\Delta V_w}{\Delta p} + \frac{\Delta V_a}{\Delta p}\right) = \frac{\rho_w}{V_w}\frac{\Delta V_w}{\Delta p} + \frac{\rho_w}{\rho_a}\frac{V_a}{V_w}\left(\frac{\rho_a}{V_a}\frac{\Delta V_a}{\Delta p}\right) .$$

In a pure water system or a pure air system, we would have

$$\frac{1}{v_w^2} = \frac{\rho_w}{V_w}\frac{\Delta V_w}{\Delta p} \qquad \text{or} \qquad \frac{1}{v_a^2} = \frac{\rho_a}{V_a}\frac{\Delta V_a}{\Delta p} .$$

Substituting these into the above equation, and using the notation $r = V_a/V_w$, we arrive at

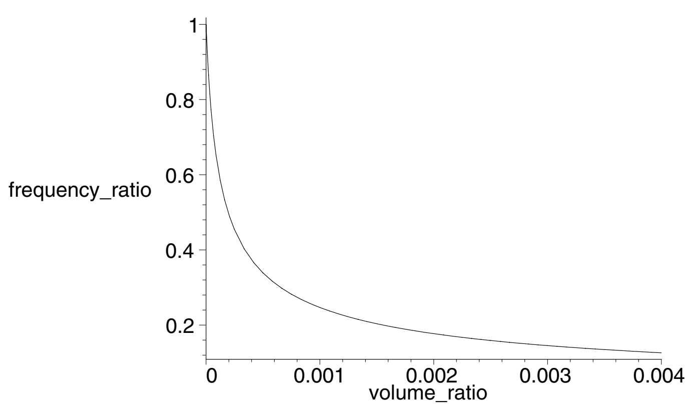
$$\frac{1}{v^2} = \frac{1}{v_w^2} + \frac{\rho_w}{\rho_a}\frac{r}{v_a^2} \quad \Longrightarrow \quad v = \frac{1}{\sqrt{1/v_w^2 + r(\rho_w/\rho_a)/v_a^2}} .$$

(d) Dividing our result in the previous part by v_w and using the fact that the wave speed is proportional to the frequency, we find

$$\frac{v}{v_w} = \frac{f_{\text{shift}}}{f} = \frac{1}{v_w\sqrt{1/v_w^2 + r(\rho_w/\rho_a)/v_a^2}} = \frac{1}{\sqrt{1 + r(\rho_w/\rho_a)(v_w/v_a)^2}}$$

which becomes the expression shown in the problem when we plug in $\rho_w = 1000\,\text{kg/m}^3$, $\rho_a = 1.21\,\text{kg/m}^3$, $v_w = 1482$ m/s and $v_a = 343$ m/s, and round to three significant figures.

(e) The graph of f_{shift}/f versus r is shown below.



(f) From the graph (or more accurately by solving the equation itself) we find $r = 5.2\times10^{-4}$ corresponds to $f_{\text{shift}}/f = 1/3$.