

25. Without loss of generality, we assume \vec{a} points along the $+x$ axis, and that \vec{b} is at θ measured counter-clockwise from \vec{a} . We wish to verify that

$$r^2 = a^2 + b^2 + 2ab \cos \theta$$

where $a = |\vec{a}| = a_x$ (we'll call it a for simplicity) and $b = |\vec{b}| = \sqrt{b_x^2 + b_y^2}$. Since $\vec{r} = \vec{a} + \vec{b}$ then $r = |\vec{r}| = \sqrt{(a + b_x)^2 + b_y^2}$. Thus, the above expression becomes

$$\begin{aligned} \left(\sqrt{(a + b_x)^2 + b_y^2} \right)^2 &= a^2 + \left(\sqrt{b_x^2 + b_y^2} \right)^2 + 2ab \cos \theta \\ a^2 + 2ab_x + b_x^2 + b_y^2 &= a^2 + b_x^2 + b_y^2 + 2ab \cos \theta \end{aligned}$$

which makes a valid equality since (the last term) $2ab \cos \theta$ is indeed the same as $2ab_x$ (on the left-hand side). In a *later* section, the scalar (dot) product of vectors is presented and this result can be revisited with a somewhat different-looking derivation.