58. (a) The rotational inertia of a hoop is $I = mR^2$, and the energy of the system becomes

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

and $\theta$ is in radians. We note that $r\omega = v$ (where $v = \frac{dx}{dt}$). Thus, the energy becomes

$$E = \frac{1}{2}\left(\frac{m\,R^2}{r^2}\right)v^2 + \frac{1}{2}kx^2$$

which looks like the energy of the simple harmonic oscillator discussed in §16-4 *if* we identify the mass $m$ in that section with the term $mR^2/r^2$ appearing in this problem. Making this identification, Eq. 16-12 yields

$$\omega = \sqrt{\frac{k}{mR^2/r^2}} = \frac{r}{R}\sqrt{\frac{k}{m}}\ .$$

(b) If $r = R$ the result of part (a) reduces to $\omega = \sqrt{k/m}$.

(c) And if $r = 0$ then $\omega = 0$ (the spring exerts no restoring torque on the wheel so that it is not brought back towards its equilibrium position).