39. Let I_0 be the intensity of the incident beam and f be the fraction that is polarized. Thus, the intensity of the polarized portion is fI_0. After transmission, this portion contributes $fI_0 \cos^2\theta$ to the intensity of the transmitted beam. Here θ is the angle between the direction of polarization of the radiation and the polarizing direction of the filter. The intensity of the unpolarized portion of the incident beam is $(1-f)I_0$ and after transmission, this portion contributes $(1-f)I_0/2$ to the transmitted intensity. Consequently, the transmitted intensity is

$$I = fI_0 \cos^2\theta + \frac{1}{2}(1-f)I_0 \; .$$

As the filter is rotated, $\cos^2\theta$ varies from a minimum of 0 to a maximum of 1, so the transmitted intensity varies from a minimum of

$$I_{\text{min}} = \frac{1}{2}(1-f)I_0$$

to a maximum of

$$I_{\text{max}} = fI_0 + \frac{1}{2}(1-f)I_0 = \frac{1}{2}(1+f)I_0 \; .$$

The ratio of I_{max} to I_{min} is

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{1+f}{1-f} \; .$$

Setting the ratio equal to 5.0 and solving for f, we get $f = 0.67$.