15. The Fermi-Dirac occupation probability is given by $P_{\text{FD}} = 1/\left(e^{\Delta E/kT} + 1\right)$, and the Boltzmann occupation probability is given by $P_{\text{B}} = e^{-\Delta E/kT}$. Let f be the fractional difference. Then

$$f = \frac{P_{\text{B}} - P_{\text{FD}}}{P_{\text{B}}} = \frac{e^{-\Delta E/kT} - \dfrac{1}{e^{\Delta E/kT} + 1}}{e^{-\Delta E/kT}} .$$

Using a common denominator and a little algebra yields

$$f = \frac{e^{-\Delta E/kT}}{e^{-\Delta E/kT} + 1} .$$

The solution for $e^{-\Delta E/kT}$ is

$$e^{-\Delta E/kT} = \frac{f}{1 - f} .$$

We take the natural logarithm of both sides and solve for T. The result is

$$T = \frac{\Delta E}{k \ln\left(\dfrac{f}{1 - f}\right)} .$$

(a) Letting f equal 0.01, we evaluate the expression for T:

$$T = \frac{(1.00\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})}{(1.38 \times 10^{-23}\,\text{J/K}) \ln\left(\dfrac{0.010}{1 - 0.010}\right)} = 2.5 \times 10^3 \text{ K} .$$

(b) We set f equal to 0.10 and evaluate the expression for T:

$$T = \frac{(1.00\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})}{(1.38 \times 10^{-23}\,\text{J/K}) \ln\left(\dfrac{0.10}{1 - 0.10}\right)} = 5.3 \times 10^3 \text{ K} .$$