

43. (a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\epsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\epsilon_0 A/C$ and

$$E = \frac{VC}{\kappa\epsilon_0 A} = \frac{(50 \text{ V})(100 \times 10^{-12} \text{ F})}{5.4(8.85 \times 10^{-12} \text{ F/m})(100 \times 10^{-4} \text{ m}^2)} = 1.0 \times 10^4 \text{ V/m} .$$

- (b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12} \text{ F})(50 \text{ V}) = 5.0 \times 10^{-9} \text{ C}$.
 (c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\epsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\epsilon_0 A} + \frac{q_f}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} ,$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \epsilon_0 A E \\ &= 5.0 \times 10^{-9} \text{ C} - (8.85 \times 10^{-12} \text{ F/m})(100 \times 10^{-4} \text{ m}^2)(1.0 \times 10^4 \text{ V/m}) \\ &= 4.1 \times 10^{-9} \text{ C} = 4.1 \text{ nC} . \end{aligned}$$