

39. (a) The rms speed of molecules in a gas is given by  $v_{\text{rms}} = \sqrt{3RT/M}$ , where  $T$  is the temperature and  $M$  is the molar mass of the gas. See Eq. 20–34. The speed required for escape from Earth's gravitational pull is  $v = \sqrt{2gr_e}$ , where  $g$  is the acceleration due to gravity at Earth's surface and  $r_e$  ( $= 6.37 \times 10^6$  m) is the radius of Earth. To derive this expression, take the zero of gravitational potential energy to be at infinity. Then, the gravitational potential energy of a particle with mass  $m$  at Earth's surface is  $U = -GMm/r_e^2 = -mgr_e$ , where  $g = GM/r_e^2$  was used. If  $v$  is the speed of the particle, then its total energy is  $E = -mgr_e + \frac{1}{2}mv^2$ . If the particle is just able to travel far away, its kinetic energy must tend toward zero as its distance from Earth becomes large without bound. This means  $E = 0$  and  $v = \sqrt{2gr_e}$ . We equate the expressions for the speeds to obtain  $\sqrt{3RT/M} = \sqrt{2gr_e}$ . The solution for  $T$  is  $T = 2gr_e M/3R$ . The molar mass of hydrogen is  $2.02 \times 10^{-3}$  kg/mol, so for that gas

$$T = \frac{2(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})(2.02 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol} \cdot \text{K})} = 1.0 \times 10^4 \text{ K} .$$

- (b) The molar mass of oxygen is  $32.0 \times 10^{-3}$  kg/mol, so for that gas

$$T = \frac{2(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})(32.0 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol} \cdot \text{K})} = 1.6 \times 10^5 \text{ K} .$$

- (c) Now,  $T = 2g_m r_m M/3R$ , where  $r_m$  ( $= 1.74 \times 10^6$  m) is the radius of the Moon and  $g_m$  ( $= 0.16g$ ) is the acceleration due to gravity at the Moon's surface. For hydrogen

$$T = \frac{2(0.16)(9.8 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})(2.02 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol} \cdot \text{K})} = 4.4 \times 10^2 \text{ K} .$$

For oxygen

$$T = \frac{2(0.16)(9.8 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})(32.0 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol} \cdot \text{K})} = 7.0 \times 10^3 \text{ K} .$$

- (d) The temperature high in Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result the atmosphere is depleted of hydrogen. On the other hand, very few oxygen atoms escape.