

9. (a) The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a (T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w (T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a (T_{ai} - T_f) = m_w c_w (T_f - T_{wi}) \implies T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}.$$

The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$\begin{aligned} T_f &= \frac{(0.200 \text{ kg})(900 \text{ J/kg}\cdot\text{K})(100^\circ\text{C}) + (0.0500 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(20^\circ\text{C})}{(0.200 \text{ kg})(900 \text{ J/kg}\cdot\text{K}) + (0.0500 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})} \\ &= 57.0^\circ\text{C} \quad \text{or} \quad 330 \text{ K} . \end{aligned}$$

- (b) Now temperatures must be given in Kelvins: $T_{ai} = 393 \text{ K}$, $T_{wi} = 293 \text{ K}$, and $T_f = 330 \text{ K}$. For the aluminum, $dQ = m_a c_a dT$ and the change in entropy is

$$\begin{aligned} \Delta S_a &= \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln \frac{T_f}{T_{ai}} \\ &= (0.200 \text{ kg})(900 \text{ J/kg}\cdot\text{K}) \ln \left(\frac{330 \text{ K}}{373 \text{ K}} \right) = -22.1 \text{ J/K} . \end{aligned}$$

- (c) The entropy change for the water is

$$\begin{aligned} \Delta S_w &= \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln \frac{T_f}{T_{wi}} \\ &= (0.0500 \text{ kg})(4190 \text{ J/kg}\cdot\text{K}) \ln \left(\frac{330 \text{ K}}{293 \text{ K}} \right) = +24.9 \text{ J/K} . \end{aligned}$$

- (d) The change in the total entropy of the aluminum-water system is $\Delta S = \Delta S_a + \Delta S_w = -22.1 \text{ J/K} + 24.9 \text{ J/K} = +2.8 \text{ J/K}$.