28. We use the results of problem 28 in Chapter 40. The Pauli principle requires that no more than two electrons be in the lowest energy level (at $E_{1,1,1} = 3(h^2/8mL^2)$), but – due to their degeneracies – as many as six electrons can be in the next three levels ($E' = E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2)$, $E'' = E_{1,2,2} = E_{2,2,1} = E_{2,1,2} = 9(h^2/8mL^2)$, and $E''' = E_{1,1,3} = E_{1,3,1} = E_{3,1,1} = 11(h^2/8mL^2)$). Using Eq. 40-21, the level above those can only hold two electrons: $E_{2,2,2} = (2^2 + 2^2 + 2^2)(h^2/8mL^2) = 12(h^2/8mL^2)$. And the next higher level can hold as much as twelve electrons (see part (e) of problem 28 in Chapter 40) and has energy $E'''' = 14(h^2/8mL^2)$.

(a) The configuration which provides the lowest system energy higher than that of the ground state has the first level filled, the second one with one vacancy, and the third one with one occupant:

$$E_{\text{first excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 9$$

which means (putting the "unit" factor back in) the energy of the first excited state is $E_{\text{first excited}} = 45(h^2/8mL^2)$.

(b) The configuration which provides the next higher system energy has the first level filled, the second one with one vacancy, the third one empty, and the fourth one with one occupant:

$$E_{\text{second excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 11$$

which means (putting the "unit" factor back in) the energy of the second excited state is $E_{\text{second excited}} = 47(h^2/8mL^2)$.

(c) Now, there are a couple of configurations which provides the *next* higher system energy. One has the first level filled, the second one with one vacancy, the third and fourth ones empty, and the fifth one with one occupant:

$$E_{\text{third excited}} = 2E_{1,1,1} + 5E' + E''' = 2(3) + 5(6) + 12$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 48(h^2/8mL^2)$. The other configuration with this same total energy has the first level filled, the second one with two vacancies, and the third one with one occupant.

(d) The energy states of this problem and problem 27 are suggested in the sketch below: