40. (a) The vector equation $\vec{r} = \vec{a} - \vec{b} - \vec{c}$ is computed as follows: $(5.0 - (-2.0) + 4.0)\hat{\mathrm{i}} + (4.0 - 2.0 + 3.0)\hat{\mathrm{j}} + ((-6.0) - 3.0 + 2.0)\hat{\mathrm{k}}$. This leads to $\vec{r} = 11\,\hat{\mathrm{i}} + 5.0\,\hat{\mathrm{j}} - 7.0\,\hat{\mathrm{k}}$.

(b) We find the angle from $+z$ by "dotting" (taking the scalar product) $\vec{r}$ with $\hat{\mathrm{k}}$. Noting that $r = |\vec{r}| = \sqrt{11^2 + 5^2 + (-7)^2} = 14$, Eq. 3-20 with Eq. 3-23 leads to

$$\vec{r} \cdot \hat{\mathrm{k}} = -7.0 = (14)(1)\cos\phi \quad \Longrightarrow \quad \phi = 120^\circ \ .$$

(c) To find the component of a vector in a certain direction, it is efficient to "dot" it (take the scalar product of it) with a unit-vector in that direction. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{-2\,\hat{\mathrm{i}} + 2\,\hat{\mathrm{j}} + 3\,\hat{\mathrm{k}}}{\sqrt{(-2)^2 + 2^2 + 3^2}} \ .$$

We therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{(5)(-2) + (4)(2) + (-6)(3)}{\sqrt{(-2)^2 + 2^2 + 3^2}} = -4.9 \ .$$

(d) One approach (if we all we require is the magnitude) is to use the vector cross product, as the problem suggests; another (which supplies more information) is to subtract the result in part (c) (multiplied by $\hat{b}$) from $\vec{a}$. We briefly illustrate both methods. We note that if $a\cos\theta$ (where θ is the angle between $\vec{a}$ and $\vec{b}$) gives a_b (the component along $\hat{b}$) then we expect $a\sin\theta$ to yield the orthogonal component:

$$a\sin\theta = \frac{|\vec{a} \times \vec{b}|}{b} = 7.3$$

(alternatively, one might compute θ form part (c) and proceed more directly). The second method proceeds as follows:

$$\begin{aligned} \vec{a} - a_b\,\hat{b} &= (5.0 - 2.35)\hat{\mathrm{i}} + (4.0 - (-2.35))\hat{\mathrm{j}} + ((-6.0) - (-3.53))\hat{\mathrm{k}} \\ &= 2.65\,\hat{\mathrm{i}} + 6.35\,\hat{\mathrm{j}} - 2.47\,\hat{\mathrm{k}} \end{aligned}$$

This describes the perpendicular part of $\vec{a}$ completely. To find the magnitude of this part, we compute

$$\sqrt{2.65^2 + 6.35^2 + (-2.47)^2} = 7.3$$

which agrees with the first method.