80. (First problem in **Cluster**)

(a) The length of each of the tall sides is $\ell = \sqrt{H^2 + (B/2)^2}$, so that the total length of the wire is $L = 2\sqrt{H^2 + (B/2)^2} + B$. If A is the cross-section area and ρ is the density, then the total mass of the wire is $M = \rho AL$ and the mass of each of the tall sides is

$$m_\ell = \rho A\sqrt{H^2 + (B/2)^2} = M\frac{\sqrt{H^2 + (B/2)^2}}{2\sqrt{H^2 + (B/2)^2} + B} .$$

It is clear by symmetry that $x_{\text{com}} = B/2$ for the system, but the value of y_{com} is not obvious. Note that the base does not contribute to this computation:

$$y_{\text{com}} = \frac{1}{M}\left(m_\ell\frac{H}{2} + m_\ell\frac{H}{2}\right)$$

which can be 'simplified' to the following form.

$$y_{\text{com}} = \frac{H}{2 + \frac{B}{\sqrt{H^2+(B/2)^2}}}$$

(b) The element of mass on the left-hand tall side is related to $d\ell = \sqrt{dx^2 + dy^2}$ and ultimately to the individual coordinate elements (since $dy = (2H/B)dx$):

$$dm_\ell = \rho A d\ell = \begin{cases} \rho A\sqrt{1 + (2H/B)^2}\, dx \\ \rho A\sqrt{(B/2H)^2 + 1}\, dy \end{cases}$$

where $\rho A = m_\ell/\sqrt{H^2 + (B/2)^2}$ (see part (a)). Therefore, using Eq. 9-9, we have

$$\begin{aligned}
x_{\text{com}} &= \frac{1}{m_\ell}\int_0^{B/2} x\,\frac{m_\ell}{\sqrt{H^2 + (B/2)^2}}\sqrt{1 + (2H/B)^2}\, dx \\
&= \frac{\sqrt{1 + (2H/B)^2}}{\sqrt{H^2 + (B/2)^2}}\int_0^{B/2} x\, dx \\
&= \frac{2}{B}\frac{\sqrt{(B/2)^2 + H^2}}{\sqrt{H^2 + (B/2)^2}}\left(\frac{B^2}{8} - 0\right) \\
&= \frac{B}{4} \quad \text{and} \\
y_{\text{com}} &= \frac{1}{m_\ell}\int_0^{H} y\,\frac{m_\ell}{\sqrt{H^2 + (B/2)^2}}\sqrt{(B/2H)^2 + 1}\, dy \\
&= \frac{\sqrt{(B/2H)^2 + 1}}{\sqrt{H^2 + (B/2)^2}}\int_0^{H} y\, dy \\
&= \frac{1}{H}\frac{\sqrt{(B/2)^2 + H^2}}{\sqrt{H^2 + (B/2)^2}}\left(\frac{H^2}{2} - 0\right) \\
&= \frac{H}{2} .
\end{aligned}$$