51. (a) Before looking at our solution to part (a) (which uses momentum conservation), it might be advisable to look at our solution (and accompanying remarks) for part (b) (where a very different approach is used). Since momentum is a vector, its conservation involves two equations (along the original direction of alpha particle motion, the x direction, as well as along the final proton direction of motion, the y direction). The problem states that all speeds are much less than the speed of light, which allows us to use the classical formulas for kinetic energy and momentum ($K = \frac{1}{2}mv^2$ and $\vec{p} = m\vec{v}$, respectively). Along the x and y axes, momentum conservation gives (for the components of $\vec{v}_{\text{oxy}}$):

$$\begin{aligned} m_\alpha v_\alpha &= m_{\text{oxy}} v_{\text{oxy},x} \implies v_{\text{oxy},x} = \frac{m_\alpha}{m_{\text{oxy}}} v_\alpha \approx \frac{4}{17} v_\alpha \\ 0 &= m_{\text{oxy}} v_{\text{oxy},y} + m_p v_p \implies v_{\text{oxy},y} = -\frac{m_p}{m_{\text{oxy}}} v_p \approx -\frac{1}{17} v_p \ . \end{aligned}$$

To complete these determinations, we need values (inferred from the kinetic energies given in the problem) for the initial speed of the alpha particle (v_α) and the final speed of the proton (v_p). One way to do this is to rewrite the classical kinetic energy expression as $K = \frac{1}{2}(mc^2)\beta^2$ and solve for β (using Table 38-3 and/or Eq. 38-43). Thus, for the proton, we obtain

$$\beta_p = \sqrt{\frac{2K_p}{m_p c^2}} = \sqrt{\frac{2(4.44\,\text{MeV})}{938\,\text{MeV}}} = 0.0973 \ .$$

This is almost 10% the speed of light, so one might worry that the relativistic expression (Eq. 38-49) should be used. If one does so, one finds $\beta_p = 0.969$, which is reasonably close to our previous result based on the classical formula. For the alpha particle, we write $m_\alpha c^2 = (4.0026\,\text{u})(931.5\,\text{MeV/u}) = 3728$ MeV (which is actually an overestimate due to the use of the "atomic mass" value in our calculation, but this does not cause significant error in our result), and obtain

$$\beta_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha c^2}} = \sqrt{\frac{2(7.70\,\text{MeV})}{3728\,\text{MeV}}} = 0.064 \ .$$

Returning to our oxygen nucleus velocity components, we are now able to conclude:

$$\begin{aligned} v_{\text{oxy},x} \approx \frac{4}{17} v_\alpha &\implies \beta_{\text{oxy},x} \approx \frac{4}{17}\beta_\alpha = \frac{4}{17}(0.064) = 0.015 \\ |v_{\text{oxy},y}| \approx \frac{1}{17} v_p &\implies \beta_{\text{oxy},y} \approx \frac{1}{17}\beta_p = \frac{1}{17}(0.097) = 0.0057 \end{aligned}$$

Consequently, with $m_{\text{oxy}}c^2 \approx (17\,\text{u})(931.5\,\text{MeV/u}) = 1.58 \times 10^4$ MeV, we obtain

$$K_{\text{oxy}} = \frac{1}{2}\left(m_{\text{oxy}}c^2\right)\left(\beta_{\text{oxy},x}^2 + \beta_{\text{oxy},y}^2\right) = \frac{1}{2}\left(1.58 \times 10^4\,\text{MeV}\right)\left(0.015^2 + 0.0057^2\right) \approx 2.0 \text{ MeV} \ .$$

(b) Using Eq. 38-47 and Eq. 38-43,

$$Q = -(1.007825\,\text{u} + 16.99914\,\text{u} - 4.00260\,\text{u} - 14.00307\,\text{u})c^2 = -(0.001295\,\text{u})(931.5\,\text{MeV/u})$$

which yields $Q = -1.206$ MeV. Incidentally, this provides an alternate way to obtain the answer (and a more accurate one at that!) to part (a). Eq. 38-46 leads to

$$K_{\text{oxy}} = K_\alpha + Q - K_p = 7.70\,\text{MeV} - 1.206\,\text{MeV} - 4.44\,\text{MeV} = 2.05 \text{ MeV} \ .$$

This approach to finding K_{oxy} avoids the many computational steps and approximations made in part (a).