

16. Our y axis is along the wire with the origin at the top endpoint, and the current is in the positive y direction. All segments of the wire produce magnetic fields at P that are into the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P is given by

$$dB = \frac{\mu_0 i}{4\pi} \frac{\sin \theta}{r^2} dy$$

where θ (the angle between the segment and a line drawn from the segment to P) and r (the length of that line) are functions of y . Replacing r with $\sqrt{y^2 + a^2}$ and $\sin \theta$ with $a/r = a/\sqrt{y^2 + a^2}$, we integrate from $y = -a$ to $y = 0$. The total field is

$$B = \frac{\mu_0 i a}{4\pi} \int_{-a}^0 \frac{dy}{(y^2 + a^2)^{3/2}} = \frac{\mu_0 i a}{4\pi} \frac{1}{a^2} \frac{y}{(y^2 + a^2)^{1/2}} \bigg|_{-a}^0 = \frac{\mu_0 i}{4\pi a} \frac{a}{\sqrt{a^2 + a^2}}$$

which simplifies to the desired result (noting that $\frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$).