59. (a) To get to the detector, the wave from S_1 travels a distance x and the wave from S_2 travels a distance $\sqrt{d^2 + x^2}$. The phase difference (in terms of wavelengths) between the two waves is

$$\sqrt{d^2 + x^2} - x = m\lambda \qquad m = 0, 1, 2, \ldots$$

where we are requiring constructive interference. The solution is

$$x = \frac{d^2 - m^2\lambda^2}{2m\lambda} \ .$$

We see that setting $m = 0$ in this expression produces $x = \infty$; hence, the phase difference between the waves when P is very far away is 0.

(b) The result of part (a) implies that the waves constructively interfere at P.

(c) As is particularly evident from our results in part (d), the phase difference increases as x decreases.

(d) We can use our formula from part (a) for the 0.5λ, 1.50λ, etc differences by allowing m in our formula to take on half-integer values. The half- integer values, though, correspond to destructive interference. Using the values $\lambda = 0.500\,\mu$m and $d = 2.00\,\mu$m, we find $x = 7.88\,\mu$m for $m = \frac{1}{2}$, $x = 3.75\,\mu$m for $m = 1$, $x = 2.29\,\mu$m for $m = \frac{3}{2}$, $x = 1.50\,\mu$m for $m = 2$, and $x = 0.975\,\mu$m for $m = \frac{5}{2}$.