

74. Let v and V be the speeds of particles m and M , respectively. These are measured in the frame of reference described in the problem (where the particles are seen as initially at rest). Now, momentum conservation demands

$$mv = MV \implies v + V = v \left(1 + \frac{m}{M}\right)$$

where $v + V$ is their relative speed (the instantaneous rate at which the gap between them is shrinking). Energy conservation applied to the two-particle system leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 - \frac{GmM}{r} &= \frac{1}{2}mv^2 + \frac{1}{2}MV^2 - \frac{GmM}{d} \\ -\frac{GmM}{r} &= \frac{1}{2}mv^2 \left(1 + \frac{m}{M}\right) - \frac{GmM}{d} . \end{aligned}$$

If we take the initial separation r to be large enough that GmM/r is approximately zero, then this yields a solution for the speed of particle m :

$$v = \sqrt{\frac{2GM}{d \left(1 + \frac{m}{M}\right)}} .$$

Therefore, the relative speed is

$$v + V = \sqrt{\frac{2GM}{d \left(1 + \frac{m}{M}\right)}} \left(1 + \frac{m}{M}\right) = \sqrt{\frac{2G(M+m)}{d}} .$$