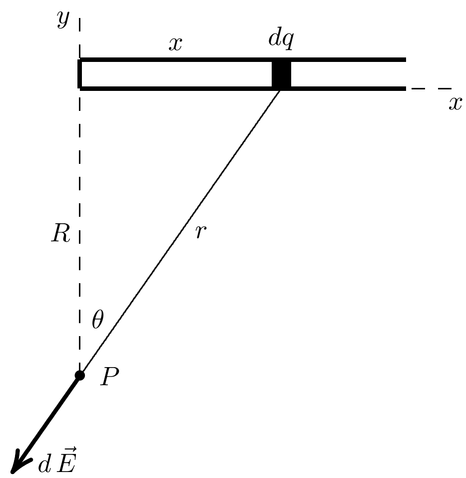
25. Consider an infinitesimal section of the rod of length dx, a distance x from the left end, as shown in the diagram below. It contains charge $dq = \lambda\, dx$ and is a distance r from P. The magnitude of the field it produces at P is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\ .$$

The x component is $$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\sin\theta$$

and the y component is $$dE_y = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\cos\theta\ .$$



We use θ as the variable of integration and substitute $r = R/\cos\theta$, $x = R\tan\theta$ and $dx = (R/\cos^2\theta)\,d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus,

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\sin\theta\,d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\cos\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

and

$$E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta\,d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\ .$$

We notice that $E_x = E_y$ no matter what the value of R. Thus, $\vec{E}$ makes an angle of 45° with the rod for all values of R.