40. (a) The fractional change is

$$\begin{aligned}\frac{\Delta E}{E} &= \frac{\Delta(hc/\lambda)}{hc/\lambda} = \lambda\Delta\left(\frac{1}{\lambda}\right) = \lambda\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) = \frac{\lambda}{\lambda'} - 1 = \frac{\lambda}{\lambda + \Delta\lambda} - 1 \\ &= -\frac{1}{\lambda/\Delta\lambda + 1} = -\frac{1}{(\lambda/\lambda_C)(1-\cos\phi)^{-1} + 1} \ .\end{aligned}$$

If $\lambda = 3.0\,\text{cm} = 3.0 \times 10^{10}\,\text{pm}$ and $\phi = 90°$, the result is

$$\frac{\Delta E}{E} = -\frac{1}{(3.0 \times 10^{10}\,\text{pm}/2.43\,\text{pm})(1 - \cos 90°)^{-1} + 1} = -8.1 \times 10^{-11} \ .$$

(b) Now $\lambda = 500\,\text{nm} = 5.00 \times 10^5\,\text{pm}$ and $\phi = 90°$, so

$$\frac{\Delta E}{E} = -\frac{1}{(5.00 \times 10^5\,\text{pm}/2.43\,\text{pm})(1 - \cos 90°)^{-1} + 1} = -4.9 \times 10^{-6} \ .$$

(c) With$\lambda = 25\,\text{pm}$ and $\phi = 90°$, we find

$$\frac{\Delta E}{E} = -\frac{1}{(25\,\text{pm}/2.43\,\text{pm})(1 - \cos 90°)^{-1} + 1} = -8.9 \times 10^{-2} \ .$$

(d) In this case, $\lambda = hc/E = 1240\,\text{nm}\cdot\text{eV}/1.0\,\text{MeV} = 1.24 \times 10^{-3}\,\text{nm} = 1.24\,\text{pm}$, so

$$\frac{\Delta E}{E} = -\frac{1}{(1.24\,\text{pm}/2.43\,\text{pm})(1 - \cos 90°)^{-1} + 1} = -0.66 \ .$$

(e) From the calculation above, we see that the shorter the wavelength the greater the fractional energy change for the photon as a result of the Compton scattering. Since $\Delta E/E$ is virtually zero for microwave and visible light, the Compton effect is significant only in the x-ray to gamma ray range of the electromagnetic spectrum.