80. (a) From Eq. 33-25,

$$\frac{dq}{dt} = \frac{d}{dt}\left[Qe^{-Rt/2L}\cos(\omega' t + \phi)\right] = -\frac{RQ}{2L}e^{-Rt/2L}\cos(\omega' t + \phi) - \omega' Q e^{-Rt/2L}\sin(\omega' t + \phi)$$

and

$$\begin{aligned}\frac{d^2q}{dt^2} &= \left(\frac{R}{2L}\right)e^{-Rt/2L}\left[\left(\frac{RQ}{2L}\right)\cos(\omega' t + \phi) - \omega' Q\sin(\omega' t + \phi)\right] \\ &\quad + e^{-Rt/2L}\left[\frac{RQ\omega'}{2L}\sin(\omega' t + \phi) - \omega'^2 Q\cos(\omega' t + \phi)\right] .\end{aligned}$$

Substituting these expressions, and Eq. 33-25 itself, into Eq. 33-24, we obtain

$$Qe^{-Rt/2L}\left[-\omega'^2 L - \left(\frac{R}{2L}\right)^2 + \frac{1}{c}\right]\cos(\omega' t + \phi) = 0 .$$

Since this equation is valid at any time t, we must have

$$-\omega'^2 L - \left(\frac{R}{2L}\right)^2 + \frac{1}{C} = 0 \implies \omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2} .$$

(b) The fractional shift in frequency is

$$\begin{aligned}\frac{\Delta f}{f} = \frac{\Delta\omega}{\omega} &= \frac{\omega - \omega'}{\omega} = 1 - \frac{\sqrt{(1/LC) - (R/2L)^2}}{\sqrt{1/LC}} = 1 - \sqrt{1 - \frac{R^2C}{4L}} \\ &= 1 - \sqrt{1 - \frac{(100\,\Omega)^2(7.30 \times 10^{-6}\,\mathrm{F})}{4(4.40\,\mathrm{H})}} = 0.00210 = 0.210\% .\end{aligned}$$