28. (a) Consider a segment of the projectile between y and $y + dy$. We use Eq. 30-14 to find the magnetic force on the segment, and Eq. 30-9 for the magnetic field of each semi-infinite wire (the top rail referred to as wire 1 and the bottom as wire 2). The current in rail 1 is in the $+\hat{\imath}$ direction, and the current in the rail 2 is in the $-\hat{\imath}$ direction. The field (in the region between the wires) set up by wire 1 is into the paper (the $-\hat{k}$ direction) and that set up by wire 2 is also into the paper. The force element (a function of y) acting on the segment of the projectile (in which the current flows in the $-\hat{\jmath}$ direction) is given below. The coordinate origin is at the bottom of the projectile.

$$\begin{aligned} d\vec{F} &= d\vec{F}_1 + d\vec{F}_2 \\ &= i\,dy(-\hat{\jmath}) \times \vec{B}_1 + dy(-\hat{\jmath}) \times \vec{B}_2 \\ &= i[B_1 + B_2]\,\hat{\imath}\,dy \\ &= i\left[\frac{\mu_0 i}{4\pi(2R + w - y)} + \frac{\mu_0 i}{4\pi y}\right]\hat{\imath}\,dy\ . \end{aligned}$$

Thus, the force on the projectile is

$$\vec{F} = \int d\vec{F} = \frac{i^2\mu_0}{4\pi}\int_R^{R+w}\left(\frac{1}{2R + w - y} + \frac{1}{y}\right)dy\,\hat{\imath} = \frac{\mu_0 i^2}{2\pi}\ln\left(1 + \frac{w}{R}\right)\hat{\imath}\ .$$

(b) Using the work-energy theorem, we have $\Delta K = \frac{1}{2}mv_f^2 = W_{\text{ext}} = \int \vec{F}\cdot d\vec{s} = FL$. Thus, the final speed of the projectile is

$$\begin{aligned} v_f &= \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\frac{\mu_0 i^2}{2\pi}\ln\left(1 + \frac{w}{R}\right)L\right]^{1/2} \\ &= \left[\frac{2(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(450 \times 10^3\,\text{A})^2\,\ln(1 + 1.2\,\text{cm}/6.7\,\text{cm})(4.0\,\text{m})}{2\pi(10 \times 10^{-3}\,\text{kg})}\right]^{1/2} \\ &= 2.3 \times 10^3\ \text{m/s}\ . \end{aligned}$$