

54. (a) By imagining that each of the segments bg and cf (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude (i) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.
- (b) The dipole moment of path $abcdefgha$ is

$$\begin{aligned}\vec{\mu} &= \vec{\mu}_{bcf gb} + \vec{\mu}_{abgha} + \vec{\mu}_{cdefc} = (ia^2)(\hat{j} - \hat{i} + \hat{i}) = ia^2\hat{j} \\ &= (6.0 \text{ A})(0.10 \text{ m})^2\hat{j} = 6.0 \times 10^{-2} \text{ A}\cdot\text{m}^2 \hat{j} .\end{aligned}$$

- (c) Since both points are far from the cube we can use the dipole approximation. For $(x, y, z) = (0, 5.0 \text{ m}, 0)$

$$\begin{aligned}\vec{B}(0, 5.0 \text{ m}, 0) &\approx \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{y^3} \\ &= \frac{(1.26 \times 10^{-6} \text{ T}\cdot\text{m/A})(6.0 \times 10^{-2} \text{ m}^2 \cdot \text{A})\hat{j}}{2\pi(5.0 \text{ m})^3} \\ &= 9.6 \times 10^{-11} \text{ T} \hat{j} .\end{aligned}$$

For $(x, y, z) = (5.0 \text{ m}, 0, 0)$, note that the line joining the end point of interest and the location of the dipole is perpendicular to the axis of the dipole. You can check easily that if an electric dipole is used, the field would be $E \approx (1/4\pi\epsilon_0)(p/x^3)$, which is half of the magnitude of E for a point on the y axis the same distance from the dipole. By analogy, in our case B is also half the value or $B(0, 5.0 \text{ m}, 0)$, i.e.,

$$B(5.0 \text{ m}, 0, 0) = \frac{1}{2}B(0, 5.0 \text{ m}, 0) = \frac{1}{2}(9.6 \times 10^{-11} \text{ T}) = 4.8 \times 10^{-11} \text{ T} .$$

Just like the electric dipole case, $\vec{B}(5.0 \text{ m}, 0, 0)$ points in the negative y direction.