31. (a) Letting x be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 30-19, the field is $B = \mu_0 i/2\pi r$, where r is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length x and width dr, parallel to the wire and a distance r from it; it has area $A = x\,dr$ and the flux $d\Phi_B = (\mu_0 ix/2\pi r)dr$. By Eq. 31-3, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right) .$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}
\mathcal{E} &= \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+L}{a}\right) \\
&= \frac{(4\pi\times 10^{-7}\mathrm{T\cdot m/A})(100\mathrm{A})(5.00\,\mathrm{m/s})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right) \\
&= 2.40\times 10^{-4}\ \mathrm{V} .
\end{aligned}$$

(b) By Ohm's law, the induced current is $i_\ell = \mathcal{E}/R = (2.40\times 10^{-4}\,\mathrm{V})/(0.400\,\Omega) = 6.00\times 10^{-4}\,\mathrm{A}$. Since the flux is increasing the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate $P = i_\ell^2 R = (6.00\times 10^{-4}\mathrm{A})^2(0.400\,\Omega) = 1.44\times 10^{-7}\mathrm{W}$.

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length dr at a distance r from the long straight wire, is $dF_B = i_\ell B\,dr = (\mu_0 i_\ell i/2\pi r)\,dr$. We integrate to find the magnitude of the total magnetic force on the rod:

$$\begin{aligned}
F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right) \\
&= \frac{(4\pi\times 10^{-7}\mathrm{T\cdot m/A})(6.00\times 10^{-4}\,\mathrm{A})(100\mathrm{A})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right) \\
&= 2.87\times 10^{-8}\ \mathrm{N} .
\end{aligned}$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of $2.87\times 10^{-8}\,\mathrm{N}$, to the left.

(e) By Eq. 7-48, the external agent does work at the rate $P = Fv = (2.87\times 10^{-8}\,\mathrm{N})(5.00\,\mathrm{m/s}) = 1.44\times 10^{-7}\,\mathrm{W}$. This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.