

49. (a) From problem #43, we have $\mathcal{N}_{\text{total}} = 2^{100} = 1.27 \times 10^{30}$ microstates.

(b) Using Eq. 21-18, we find

$$\begin{aligned}\frac{W}{\mathcal{N}_{\text{total}}} &= \frac{\left(\frac{100!}{(50!)(50!)}\right)}{1.27 \times 10^{30}} = \frac{12611418068195524166851562157}{158456325028528675187087900672} \\ &= 0.079589 \approx 8.0\% .\end{aligned}$$

(c) Similarly, for $n_1 = 48$ and $n_2 = 52$ we obtain

$$\begin{aligned}\frac{W}{\mathcal{N}_{\text{total}}} &= \frac{\left(\frac{100!}{(48!)(52!)}\right)}{1.27 \times 10^{30}} = \frac{23301639718762469237395420275}{316912650057057350374175801344} \\ &= 0.073527 \approx 7.4\% .\end{aligned}$$

(d) With $n_1 = 52$ and $n_2 = 48$, we obtain the same result as in part (c).

(e) For $n_1 = 40$ and $n_2 = 60$ we obtain

$$\begin{aligned}\frac{W}{\mathcal{N}_{\text{total}}} &= \frac{\left(\frac{100!}{(40!)(60!)}\right)}{1.27 \times 10^{30}} = \frac{1718279268225351437658421215}{158456325028528675187087900672} \\ &= 0.010844 \approx 1.1\% .\end{aligned}$$

(f) Finally, for $n_1 = 30$ and $n_2 = 70$ we find

$$\begin{aligned}\frac{W}{\mathcal{N}_{\text{total}}} &= \frac{\left(\frac{100!}{(30!)(70!)}\right)}{1.27 \times 10^{30}} = \frac{1835771238850684051497735}{79228162514264337593543950336} \\ &= 0.00002317 \approx 0.0023\% .\end{aligned}$$