

37. Suppose cylinder 1 exerts a uniform force of magnitude F on cylinder 2, tangent to the cylinder's surface at the point of contact. The torque applied to cylinder 2 is $\tau_2 = R_2 F$ and the angular acceleration of that cylinder is $\alpha_2 = \tau_2/I_2 = R_2 F/I_2$. As a function of time its angular velocity is

$$\omega_2 = \alpha_2 t = \frac{R_2 F t}{I_2} .$$

The forces of the cylinders on each other obey Newton's third law, so the magnitude of the force of cylinder 2 on cylinder 1 is also F . The torque exerted by cylinder 2 on cylinder 1 is $\tau_1 = R_1 F$ and the angular acceleration of cylinder 1 is $\alpha_1 = \tau_1/I_1 = R_1 F/I_1$. This torque slows the cylinder. As a function of time, its angular velocity is $\omega_1 = \omega_0 - R_1 F t/I_1$. The force ceases and the cylinders continue rotating with constant angular speeds when the speeds of points on their rims are the same ($R_1 \omega_1 = R_2 \omega_2$). Thus,

$$R_1 \omega_0 - \frac{R_1^2 F t}{I_1} = \frac{R_2^2 F t}{I_2} .$$

When this equation is solved for the product of force and time, the result is

$$F t = \frac{R_1 I_1 I_2}{I_1 R_2^2 + I_2 R_1^2} \omega_0 .$$

Substituting this expression for $F t$ in the ω_2 equation above, we obtain

$$\omega_2 = \frac{R_1 R_2 I_1}{I_1 R_2^2 + I_2 R_1^2} \omega_0 .$$