

19. (a) A charge  $e$  traveling with uniform speed  $v$  around a circular path of radius  $r$  takes time  $T = 2\pi r/v$  to complete one orbit, so the average current is

$$i = \frac{e}{T} = \frac{ev}{2\pi r} .$$

The magnitude of the dipole moment is this multiplied by the area of the orbit:

$$\mu = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2} .$$

Since the magnetic force of with magnitude  $evB$  is centripetal, Newton's law yields  $evB = m_e v^2/r$ , so

$$r = \frac{m_e v}{eB} .$$

Thus,

$$\mu = \frac{1}{2}(ev) \left( \frac{m_e v}{eB} \right) = \left( \frac{1}{B} \right) \left( \frac{1}{2} m_e v^2 \right) = \frac{K_e}{B} .$$

The magnetic force  $-e\vec{v} \times \vec{B}$  must point toward the center of the circular path. If the magnetic field is directed into the page, for example, the electron will travel clockwise around the circle. Since the electron is negative, the current is in the opposite direction, counterclockwise and, by the right-hand rule for dipole moments, the dipole moment is out of the page. That is, the dipole moment is directed opposite to the magnetic field vector.

- (b) We note that the charge canceled in the derivation of  $\mu = K_e/B$ . Thus, the relation  $\mu = K_i/B$  holds for a positive ion. If the magnetic field is directed into the page, the ion travels counterclockwise around a circular orbit and the current is in the same direction. Therefore, the dipole moment is again out of the page, opposite to the magnetic field.
- (c) The magnetization is given by  $M = \mu_e n_e + \mu_i n_i$ , where  $\mu_e$  is the dipole moment of an electron,  $n_e$  is the electron concentration,  $\mu_i$  is the dipole moment of an ion, and  $n_i$  is the ion concentration. Since  $n_e = n_i$ , we may write  $n$  for both concentrations. We substitute  $\mu_e = K_e/B$  and  $\mu_i = K_i/B$  to obtain

$$M = \frac{n}{B} (K_e + K_i) = \frac{5.3 \times 10^{21} \text{ m}^{-3}}{1.2 \text{ T}} (6.2 \times 10^{-20} \text{ J} + 7.6 \times 10^{-21} \text{ J}) = 310 \text{ A/m} .$$