8. (a) At a distance r from the center of the Earth, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \mu}{4\pi r^3} \sqrt{1 + 3\sin^2 \lambda_m} \ ,$$

where μ is the Earth's dipole moment and λ_m is the magnetic latitude. The ratio of the field magnitudes for two different distances at the same latitude is

$$\frac{B_2}{B_1} = \frac{r_1^3}{r_2^3} \ .$$

With B_1 being the value at the surface and B_2 being half of B_1, we set r_1 equal to the radius R_e of the Earth and r_2 equal to $R_e + h$, where h is altitude at which B is half its value at the surface. Thus,

$$\frac{1}{2} = \frac{R_e^3}{(R_e + h)^3} \ .$$

Taking the cube root of both sides and solving for h, we get

$$h = \left(2^{1/3} - 1\right) R_e = \left(2^{1/3} - 1\right) (6370\,\text{km}) = 1660 \text{ km} \ .$$

(b) We use the expression for B obtained in problem 6, part (a). For maximum B, we set $\sin \lambda_m = 1$. Also, $r = 6370\,\text{km} - 2900\,\text{km} = 3470\,\text{km}$. Thus,

$$\begin{aligned} B_{\text{max}} &= \frac{\mu_0 \mu}{4\pi r^3} \sqrt{1 + 3\sin^2 \lambda_m} \\ &= \frac{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(8.00 \times 10^{22}\,\text{A}\cdot\text{m}^2\right)}{4\pi (3.47 \times 10^6\,\text{m})^3} \sqrt{1 + 3(1)^2} = 3.83 \times 10^{-4} \text{ T} \ . \end{aligned}$$

(c) The angle between the magnetic axis and the rotational axis of the Earth is $11.5°$, so $\lambda_m = 90.0° - 11.5° = 78.5°$ at Earth's geographic north pole. Also $r = R_e = 6370\,\text{km}$. Thus,

$$\begin{aligned} B &= \frac{\mu_0 \mu}{4\pi R_E^3} \sqrt{1 + 3\sin^2 \lambda_m} \\ &= \frac{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(8.0 \times 10^{22}\,\text{J/T}\right)\sqrt{1 + 3\sin^2 78.5°}}{4\pi (6.37 \times 10^6\,\text{m})^3} = 6.11 \times 10^{-5} \text{ T} \ , \end{aligned}$$

and, using the result of part (b) of problem 6,

$$\phi_i = \tan^{-1}(2 \tan 78.5°) = 84.2° \ .$$

A plausible explanation to the discrepancy between the calculated and measured values of the Earth's magnetic field is that the formulas we obtained in problem 6 are based on dipole approximation, which does not accurately represent the Earth's actual magnetic field distribution on or near its surface. (Incidentally, the dipole approximation becomes more reliable when we calculate the Earth's magnetic field far from its center.)