

27. We wish to find the effective spring constant for the combination of springs shown in Fig. 16–31. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx . Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r . The left-hand spring exerts a force of magnitude $k \Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k \Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k \Delta x_\ell$. Thus $k_{\text{eff}} = k \Delta x_\ell / 2\Delta x_r = k/2$. The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion replace k_{eff} in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{2m}} .$$