

28. (a) Noting that the magnitude of the electric field (assumed uniform) is given by $E = V/d$ (where $d = 5.0$ mm), we use the result of part (a) in Sample Problem 32-3

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 r}{2d} \frac{dV}{dt} \quad (\text{for } r \leq R) .$$

We also use the fact that the time derivative of $\sin(\omega t)$ (where $\omega = 2\pi f = 2\pi(60) \approx 377/\text{s}$ in this problem) is $\omega \cos(\omega t)$. Thus, we find the magnetic field as a function of r (for $r \leq R$; note that this neglects “fringing” and related effects at the edges):

$$B = \frac{\mu_0 \varepsilon_0 r}{2d} V_{\max} \omega \cos(\omega t) \implies B_{\max} = \frac{\mu_0 \varepsilon_0 r V_{\max} \omega}{2d}$$

where $V_{\max} = 150$ V. This grows with r until reaching its highest value at $r = R = 30$ mm:

$$\begin{aligned} B_{\max} \Big|_{r=R} &= \frac{\mu_0 \varepsilon_0 R V_{\max} \omega}{2d} \\ &= \frac{(4\pi \times 10^{-7} \text{ H/m}) (8.85 \times 10^{-12} \text{ F/m}) (30 \times 10^{-3} \text{ m}) (150 \text{ V})(377/\text{s})}{2(5.0 \times 10^{-3} \text{ m})} \\ &= 1.9 \times 10^{-12} \text{ T} . \end{aligned}$$

- (b) For $r \leq 0.03$ m, we use the $B_{\max} = \frac{\mu_0 \varepsilon_0 r V_{\max} \omega}{2d}$ expression found in part (a) (note the $B \propto r$ dependence), and for $r \geq 0.03$ m we perform a similar calculation starting with the result of part (b) in Sample Problem 32-3:

$$\begin{aligned} B_{\max} &= \left(\frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} \right)_{\max} \\ &= \left(\frac{\mu_0 \varepsilon_0 R^2}{2rd} \frac{dV}{dt} \right)_{\max} \\ &= \left(\frac{\mu_0 \varepsilon_0 R^2}{2rd} V_{\max} \omega \cos(\omega t) \right)_{\max} \\ &= \frac{\mu_0 \varepsilon_0 R^2 V_{\max} \omega}{2rd} \quad (\text{for } r \geq R) \end{aligned}$$

(note the $B \propto r^{-1}$ dependence – See also Eqs. 32-40 and 32-41). The plot (with SI units understood) is shown below.

