31. We first need to find an expression for the energy stored in a cylinder of radius R and length L, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where E is the magnitude of the electric field at that point. If q is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance r from the cylinder axis is given by

$$E = \frac{q}{2\pi\varepsilon_0 L r}$$

(see Eq. 26-12), and the energy density at that point is given by

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2} .$$

The energy in the cylinder is the volume integral

$$U_R = \int u \, d\mathcal{V} .$$

Now, $d\mathcal{V} = 2\pi r L \, dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2} \, 2\pi r L \, dr = \frac{q^2}{4\pi\varepsilon_0 L} \int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L} \ln \frac{R}{a} .$$

To find an expression for the total energy stored in the capacitor, we replace R with b:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L} \ln \frac{b}{a} .$$

We want the ratio U_R/U_b to be $1/2$, so

$$\ln \frac{R}{a} = \frac{1}{2} \ln \frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln(\sqrt{b/a})$, $\ln(R/a) = \ln(\sqrt{b/a})$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.