

51. Over a cycle, the internal energy is the same at the beginning and end, so the heat Q absorbed equals the work done: $Q = W$. Over the portion of the cycle from A to B the pressure p is a linear function of the volume V and we may write

$$p = \frac{10}{3} \text{ Pa} + \left(\frac{20}{3} \text{ Pa/m}^3 \right) V ,$$

where the coefficients were chosen so that $p = 10 \text{ Pa}$ when $V = 1.0 \text{ m}^3$ and $p = 30 \text{ Pa}$ when $V = 4.0 \text{ m}^3$. The work done by the gas during this portion of the cycle is

$$\begin{aligned} W_{AB} &= \int_1^4 p dV = \int_1^4 \left(\frac{10}{3} + \frac{20}{3} V \right) dV = \left(\frac{10}{3} V + \frac{10}{3} V^2 \right) \Big|_1^4 \\ &= \frac{40}{3} + \frac{160}{3} - \frac{10}{3} - \frac{10}{3} = 60 \text{ J} . \end{aligned}$$

The BC portion of the cycle is at constant pressure and the work done by the gas is $W_{BC} = p \Delta V = (30 \text{ Pa})(1.0 \text{ m}^3 - 4.0 \text{ m}^3) = -90 \text{ J}$. The CA portion of the cycle is at constant volume, so no work is done. The total work done by the gas is $W = W_{AB} + W_{BC} + W_{CA} = 60 \text{ J} - 90 \text{ J} + 0 = -30 \text{ J}$ and the total heat absorbed is $Q = W = -30 \text{ J}$. This means the gas loses 30 J of energy in the form of heat.