51. We label the light ray's point of entry A, the vertex of the prism B, and the light ray's exit point C. Also, the point in Fig. 34-49 where ψ is defined (at the point of intersection of the extrapolations of the incident and emergent rays) is denoted D. The angle indicated by ADC is the supplement of ψ, so we denote it $\psi_s = 180° - \psi$. The angle of refraction in the glass is $\theta_2 = \frac{1}{n}\sin\theta$. The angles between the interior ray and the nearby surfaces is the complement of θ_2, so we denote it $\theta_{2c} = 90° - \theta_2$. Now, the angles in the triangle ABC must add to 180°:

$$180° = 2\theta_{2c} + \phi \quad\Longrightarrow\quad \theta_2 = \frac{\phi}{2}\ .$$

Also, the angles in the triangle ADC must add to 180°:

$$180° = 2\,(\theta - \theta_2) + \psi_s \quad\Longrightarrow\quad \theta = 90° + \theta_2 - \frac{1}{2}\psi_s$$

which simplifies to $\theta = \theta_2 + \frac{1}{2}\psi$. Combining this with our previous result, we find $\theta = \frac{1}{2}(\phi + \psi)$. Thus, the law of refraction yields

$$n = \frac{\sin(\theta)}{\sin(\theta_2)} = \frac{\sin\left(\frac{1}{2}(\phi + \psi\right)}{\sin\left(\frac{1}{2}\,\phi\right)}\ .$$