

27. The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P , so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk. We consider a ring of charge with radius r and (infinitesimal) width dr . Its area is $2\pi r dr$ and it contains charge $dq = 2\pi\sigma r dr$. All the charge in it is a distance $\sqrt{r^2 + z^2}$ from P , so the potential it produces at P is

$$dV = \frac{1}{4\pi\epsilon_0} \frac{2\pi\sigma r dr}{\sqrt{r^2 + z^2}} = \frac{\sigma r dr}{2\epsilon_0\sqrt{r^2 + z^2}} .$$

The total potential at P is

$$V = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{r dr}{\sqrt{r^2 + z^2}} = \frac{\sigma}{2\epsilon_0} \sqrt{r^2 + z^2} \Big|_0^R = \frac{\sigma}{2\epsilon_0} \left[\sqrt{R^2 + z^2} - z \right] .$$

The potential V_{sq} at P due to a single quadrant is

$$V_{sq} = \frac{V}{4} = \frac{\sigma}{8\epsilon_0} \left[\sqrt{R^2 + z^2} - z \right] .$$