31. Let t be the present time and $t = 0$ be the time when the ratio of ^{235}U to ^{238}U was 3.0%. Let N_{235} be the number of ^{235}U nuclei present in a sample now and $N_{235,\,0}$ be the number present at $t = 0$. Let N_{238} be the number of ^{238}U nuclei present in the sample now and $N_{238,\,0}$ be the number present at $t = 0$. The law of radioactive decay holds for each specie, so

$$N_{235} = N_{235,\,0}\, e^{-\lambda_{235} t}$$

and

$$N_{238} = N_{238,\,0}\, e^{-\lambda_{238} t} \ .$$

Dividing the first equation by the second, we obtain

$$r = r_0\, e^{-(\lambda_{235} - \lambda_{238})t}$$

where $r = N_{235}/N_{238}$ $(= 0.0072)$ and $r_0 = N_{235,\,0}/N_{238,\,0}$ $(= 0.030)$. We solve for t:

$$t = -\frac{1}{\lambda_{235} - \lambda_{238}} \ln \frac{r}{r_0} \ .$$

Now we use $\lambda_{235} = (\ln 2)/T_{1/2_{235}}$ and $\lambda_{238} = (\ln 2)/T_{1/2_{238}}$ to obtain

$$t = -\frac{T_{1/2_{235}} T_{1/2_{238}}}{(T_{1/2_{238}} - T_{1/2_{235}}) \ln 2} \ln \frac{r}{r_0} = -\frac{(7.0 \times 10^8\,\text{y})(4.5 \times 10^9\,\text{y})}{(4.5 \times 10^9\,\text{y} - 7.0 \times 10^8\,\text{y}) \ln 2} \ln \frac{0.0072}{0.030} = 1.71 \times 10^9\ \text{y} \ .$$