

81. (Second problem in **Cluster**)

It is clear by symmetry that $x_{\text{com}} = B/2$ for the system, but the value of y_{com} is not obvious. If the thickness is Δz and the density is ρ , then the relation between the mass element dm and a height element dy is

$$dm = \rho \Delta z \ell_y dy = \frac{M}{A_{\Delta}} \ell_y dy$$

where the area of the triangle is $A_{\Delta} = \frac{1}{2}BH$ and the length of each horizontal “strip” at height y is $\ell_y = B(1 - y/H)$. Therefore, using Eq. 9-9, we have

$$\begin{aligned} y_{\text{com}} &= \frac{1}{M} \int_0^H y \frac{M}{A_{\Delta}} B \left(1 - \frac{y}{H}\right) dy \\ &= \frac{B}{\frac{1}{2}BH} \int_0^H y \left(1 - \frac{y}{H}\right) dy \\ &= \frac{2}{H} \left(\frac{H^2}{2} - \frac{H^3}{3H}\right) \\ &= \frac{H}{3} . \end{aligned}$$