

99. (a) In our solution here, we assume the reader has looked at our solution for problem 98. A light ray traveling directly along the central axis reaches the end in time

$$t_{\text{direct}} = \frac{L}{v_1} = \frac{n_1 L}{c} .$$

For the ray taking the critical zig-zag path, only its velocity component along the core axis direction contributes to reaching the other end of the fiber. That component is  $v_1 \cos \theta'$ , so the time of travel for this ray is

$$t_{\text{zig zag}} = \frac{L}{v_1 \cos \theta'} = \frac{n_1 L}{c \sqrt{1 - \left(\frac{1}{n_1} \sin \theta\right)^2}}$$

using results from the previous solution. Plugging in  $\sin \theta = \sqrt{n_1^2 - n_2^2}$  and simplifying, we obtain

$$t_{\text{zig zag}} = \frac{n_1 L}{c(n_2/n_1)} = \frac{n_1^2 L}{n_2 c} .$$

The difference  $t_{\text{zig zag}} - t_{\text{direct}}$  readily yields the result shown in the problem statement.

- (b) With  $n_1 = 1.58$ ,  $n_2 = 1.53$  and  $L = 300$  m, we obtain  $\Delta t = 52$  ns.