27. (a) Let v_{ni} be the initial velocity of the neutron, v_{nf} be its final velocity, and v_f be the final velocity of the target nucleus. Then, since the target nucleus is initially at rest, conservation of momentum yields $m_n v_{ni} = m_n v_{nf} + m v_f$ and conservation of energy yields $\frac{1}{2}m_n v_{ni}^2 = \frac{1}{2}m_n v_{nf}^2 + \frac{1}{2}m v_f^2$. We solve these two equations simultaneously for v_f. This can be done, for example, by using the conservation of momentum equation to obtain an expression for v_{nf} in terms of v_f and substituting the expression into the conservation of energy equation. We solve the resulting equation for v_f. We obtain $v_f = 2m_n v_{ni}/(m + m_n)$. The energy lost by the neutron is the same as the energy gained by the target nucleus, so

$$\Delta K = \frac{1}{2}mv_f^2 = \frac{1}{2}\frac{4m_n^2 m}{(m+m_n)^2}v_{ni}^2 .$$

The initial kinetic energy of the neutron is $K = \frac{1}{2}m_n v_{ni}^2$, so

$$\frac{\Delta K}{K} = \frac{4m_n m}{(m+m_n)^2} .$$

(b) The mass of a neutron is 1.0 u and the mass of a hydrogen atom is also 1.0 u. (Atomic masses can be found in Appendix G.) Thus,

$$\frac{\Delta K}{K} = \frac{4(1.0\,\mathrm{u})(1.0\,\mathrm{u})}{(1.0\,\mathrm{u}+1.0\,\mathrm{u})^2} = 1.0 .$$

Similarly, the mass of a deuterium atom is 2.0 u, so $(\Delta K)/K = 4(1.0\,\mathrm{u})(2.0\,\mathrm{u})/(2.0\,\mathrm{u}+1.0\,\mathrm{u})^2 = 0.89$. The mass of a carbon atom is 12 u, so $(\Delta K)/K = 4(1.0\,\mathrm{u})(12\,\mathrm{u})/(12\,\mathrm{u}+1.0\,\mathrm{u})^2 = 0.28$. The mass of a lead atom is 207 u, so $(\Delta K)/K = 4(1.0\,\mathrm{u})(207\,\mathrm{u})/(207\,\mathrm{u}+1.0\,\mathrm{u})^2 = 0.019$.

(c) During each collision, the energy of the neutron is reduced by the factor $1 - 0.89 = 0.11$. If E_i is the initial energy, then the energy after n collisions is given by $E = (0.11)^n E_i$. We take the natural logarithm of both sides and solve for n. The result is

$$n = \frac{\ln(E/E_i)}{\ln 0.11} = \frac{\ln(0.025\,\mathrm{eV}/1.00\,\mathrm{eV})}{\ln 0.11} = 7.9 .$$

The energy first falls below 0.025 eV on the eighth collision.