52. The y axis is arranged so that ground level is $y = 0$ and $+y$ is upward.

(a) At the point where its fuel gets exhausted, the rocket has reached a height of

$$y' = \frac{1}{2}at^2 = \frac{(4.00)(6.00)^2}{2} = 72.0 \text{ m} .$$

From Eq. 2-11, the speed of the rocket (which had started at rest) at this instant is

$$v' = at = (4.00)(6.00) = 24.0 \text{ m/s} .$$

The additional height Δy_1 the rocket can attain (beyond y') is given by Eq. 2-16 with vanishing final speed: $0 = v'^{\,2} - 2g\Delta y_1$. This gives

$$\Delta y_1 = \frac{v'^{\,2}}{2g} = \frac{(24.0)^2}{2(9.8)} = 29.4 \text{ m} .$$

Recalling our value for y', the total height the rocket attains is seen to be $72.0 + 29.4 = 101$ m.

(b) The time of free-fall flight (from y' until it returns to $y = 0$) after the fuel gets exhausted is found from Eq. 2-15:

$$-y' = v't - \frac{1}{2}gt^2 \implies -72.0 = (24.0)t - \frac{9.80}{2}t^2 .$$

Solving for t (using the quadratic formula) we obtain $t = 7.00$ s. Recalling the upward acceleration time used in part (a), we see the total time of flight is $7.00 + 6.00 = 13.0$ s.