

33. (a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r = \sqrt{z^2 + R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P . The electric potential at P is

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{\sqrt{z^2 + R^2}} = \frac{1}{4\pi\epsilon_0} \frac{1}{\sqrt{z^2 + R^2}} \int dq = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{z^2 + R^2}} .$$

- (b) The electric field is along the axis and its component is given by

$$\begin{aligned} E &= -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\epsilon_0} \frac{\partial}{\partial z} (z^2 + R^2)^{-1/2} \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{2} \right) (z^2 + R^2)^{-3/2} (2z) = \frac{q}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} . \end{aligned}$$

This agrees with Eq. 23-16.