

68. We use $nT/2$ to represent the integer number of half-periods specified in the problem. Note that $T = 2\pi/\omega$. We use the calculus-based definition of an average of a function:

$$\begin{aligned}
 [\sin^2(\omega t - \phi)]_{\text{avg}} &= \frac{1}{nT/2} \int_0^{nT/2} \sin^2(\omega t - \phi) dt \\
 &= \frac{2}{nT} \int_0^{nT/2} \frac{1 - \cos(2\omega t - 2\phi)}{2} dt \\
 &= \frac{2}{nT} \left[\frac{t}{2} - \frac{1}{4\omega} \sin(2\omega t - 2\phi) \right] \bigg|_0^{nT/2} \\
 &= \frac{1}{2} - \frac{1}{2nT\omega} \left[\sin(n\omega T - 2\phi) + \sin 2\phi \right].
 \end{aligned}$$

Since $n\omega T = n\omega(2\pi/\omega) = 2n\pi$, we have $\sin(n\omega T - 2\phi) = \sin(2n\pi - 2\phi) = -\sin 2\phi$ so $[\sin(n\omega T - 2\phi) + \sin 2\phi] = 0$. Thus,

$$[\sin^2(\omega t - \phi)]_{\text{avg}} = \frac{1}{2}.$$