

11. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0 \sin 30^\circ = 3.0$ m (we are using the edge of the roof as our reference level for computing U). Its final kinetic energy (as it leaves the roof) is $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ (Eq. 12-5). Here we use v to denote the speed of its center of mass and ω is its angular speed – at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where $R = 0.10$ m. Using $I = \frac{1}{2}MR^2$ (Table 11-2(c)), conservation of energy leads to

$$\begin{aligned} Mgh &= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 \\ &= \frac{3}{4}MR^2\omega^2 . \end{aligned}$$

The mass M cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10\text{ m}}\sqrt{\frac{4}{3}(9.8\text{ m/s}^2)(3.0\text{ m})} = 63\text{ rad/s} .$$

- (b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the “initial” position for this part of the problem) and take $+x$ leftward and $+y$ downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$\begin{aligned} v_{0x} &= v_0 \cos 30^\circ = 5.4\text{ m/s} \quad \text{and} \\ v_{0y} &= v_0 \sin 30^\circ = 3.1\text{ m/s} . \end{aligned}$$

The projectile motion equations become

$$x = v_{0x}t \quad \text{and} \quad y = v_{0y}t + \frac{1}{2}gt^2 .$$

We first find the time when $y = 5.0$ m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gy}}{g} = 0.74\text{ s} .$$

Then we substitute this into the x equation and obtain

$$x = (5.4\text{ m/s})(0.74\text{ s}) = 4.0\text{ m} .$$