47. Assuming the charge on one plate is $+q$ and the charge on the other plate is $-q$, we find an expression for the electric field in each region, in terms of q, then use the result to find an expression for the potential difference V between the plates. The capacitance is

$$C = \frac{q}{V} .$$

The electric field in the dielectric is $E_d = q/\kappa\varepsilon_0 A$, where κ is the dielectric constant and A is the plate area. Outside the dielectric (but still between the capacitor plates) the field is $E = q/\varepsilon_0 A$. The field is uniform in each region so the potential difference across the plates is

$$V = E_d b + E(d - b) = \frac{qb}{\kappa\varepsilon_0 A} + \frac{q(d-b)}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 A}\,\frac{b + \kappa(d-b)}{\kappa} .$$

The capacitance is

$$C = \frac{q}{V} = \frac{\kappa\varepsilon_0 A}{\kappa(d-b)+b} = \frac{\kappa\varepsilon_0 A}{\kappa d - b(\kappa - 1)} .$$

The result does not depend on where the dielectric is located between the plates; it might be touching one plate or it might have a vacuum gap on each side.

For the capacitor of Sample Problem 26-8, $\kappa = 2.61$, $A = 115\,\text{cm}^2 = 115 \times 10^{-4}\,\text{m}^2$, $d = 1.24\,\text{cm} = 1.24 \times 10^{-2}\,\text{m}$, and $b = 0.78\,\text{cm} = 0.78 \times 10^{-2}\,\text{m}$, so

$$\begin{aligned} C &= \frac{2.61(8.85 \times 10^{-12}\,\text{F/m})(115 \times 10^{-4}\,\text{m}^2)}{2.61(1.24 \times 10^{-2}\,\text{m}) - (0.780 \times 10^{-2}\,\text{m})(2.61 - 1)} \\ &= 1.34 \times 10^{-11}\,\text{F} = 13.4\ \text{pF} \end{aligned}$$

in agreement with the result found in the sample problem. If $b = 0$ and $\kappa = 1$, then the expression derived above yields $C = \varepsilon_0 A/d$, the correct expression for a parallel-plate capacitor with no dielectric. If $b = d$, then the derived expression yields $C = \kappa\varepsilon_0 A/d$, the correct expression for a parallel-plate capacitor completely filled with a dielectric.