

19. We assume the charge density of both the conducting cylinder and the shell are uniform, and we neglect fringing. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

- (a) We take the Gaussian surface to be a cylinder of length L , coaxial with the given cylinders and of larger radius r than either of them. The flux through this surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Now, the charge enclosed by the Gaussian surface is $q - 2q = -q$. Consequently, Gauss' law yields $2\pi r\epsilon_0 LE = -q$, so

$$E = -\frac{q}{2\pi\epsilon_0 Lr} .$$

The negative sign indicates that the field points inward.

- (b) Next, we consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod is known to have charge q , then the inner surface of the shell must have charge $-q$. And since the shell is known to have total charge $-2q$, it must therefore have charge $-q$ on its outer surface.
- (c) Finally, we consider a cylindrical Gaussian surface whose radius places it between the outside of conducting rod and inside of the shell. Similarly to part (a), the flux through the Gaussian surface is $\Phi = 2\pi rLE$, where E is the field at this Gaussian surface, in the region between the rod and the shell. The charge enclosed by the Gaussian surface is only the charge q on the rod. Therefore, Gauss' law yields

$$2\pi\epsilon_0 rLE = q \implies E = \frac{q}{2\pi\epsilon_0 Lr} .$$

The positive sign indicates that the field points outward.