

15. We imagine the square loop in the yz plane (with its center at the origin) and the evaluation point for the field being along the x axis (as suggested by the notation in the problem). The origin is a distance $a/2$ from each side of the square loop, so the distance from the evaluation point to each side of the square is, by the Pythagorean theorem,

$$R = \sqrt{(a/2)^2 + x^2} = \frac{1}{2}\sqrt{a^2 + 4x^2}.$$

Only the x components of the fields (contributed by each side) will contribute to the final result (other components cancel in pairs), so a trigonometric factor of

$$\frac{a/2}{R} = \frac{a}{\sqrt{a^2 + 4x^2}}$$

multiplies the expression of the field given by the result of problem 11 (for each side of length $L = a$). Since there are four sides, we find

$$B(x) = 4 \left(\frac{\mu_0 i}{2\pi R} \right) \left(\frac{a}{\sqrt{a^2 + 4R^2}} \right) \left(\frac{a}{\sqrt{a^2 + 4x^2}} \right) = \frac{4\mu_0 i a^2}{2\pi \left(\frac{1}{2}\right) (\sqrt{a^2 + 4x^2})^2 \sqrt{a^2 + 4(a/2)^2 + 4x^2}}$$

which simplifies to the desired result. It is straightforward to set $x = 0$ and see that this reduces to the expression found in problem 12 (noting that $\frac{4}{\sqrt{2}} = 2\sqrt{2}$).