37. The description in the problem statement implies that an atom is at the centerpoint C of the regular tetrahedron, since its four *neighbors* are at the four vertices. The side length for the tetrahedron is given as $a = 388$ pm. Since each face is an equilateral triangle, the "altitude" of each of those triangles (which is not to be confused with the altitude of the tetrahedron itself) is $h' = \frac{1}{2}a\sqrt{3}$ (this is generally referred to as the "slant height" in the solid geometry literature). At a certain location along the line segment representing "slant height" of each face is the center C' of the face. Imagine this line segment starting at atom A and ending at the midpoint of one of the sides. Knowing that this line segment bisects the $60°$ angle of the equilateral face, then it is easy to see that C' is a distance $AC' = a/\sqrt{3}$. If we draw a line from C' all the way to farthest point on the tetrahedron (this will land on an atom we label B), then this new line is the altitude h of the tetrahedron. Using the Pythagorean theorem,

$$h = \sqrt{a^2 - (AC')^2} = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = a\sqrt{\frac{2}{3}} \ .$$

Now we include coordinates: imagine atom B is on the $+y$ axis at $y_b = h = a\sqrt{2/3}$, and atom A is on the $+x$ axis at $x_a = AC' = a/\sqrt{3}$. Then point C' is the origin. The tetrahedron centerpoint C is on the y axis at some value y_c which we find as follows: C must be equidistant from A and B, so

$$\begin{aligned} y_b - y_c &= \sqrt{x_a^2 + y_c^2} \\ a\sqrt{\frac{2}{3}} - y_c &= \sqrt{\left(\frac{a}{\sqrt{3}}\right)^2 + y_c^2} \end{aligned}$$

which yields $y_c = a/2\sqrt{6}$.

(a) In unit vector notation, using the information found above, we express the vector starting at C and going to A as

$$\vec{r}_{ac} = x_a\,\hat{\mathrm{i}} + (-y_c)\,\hat{\mathrm{j}} = \frac{a}{\sqrt{3}}\,\hat{\mathrm{i}} - \frac{a}{2\sqrt{6}}\,\hat{\mathrm{j}} \ .$$

Similarly, the vector starting at C and going to B is $\vec{r}_{bc} = (y_b - y_c)\hat{\mathrm{j}} = \frac{a}{2}\sqrt{3/2}\,\hat{\mathrm{j}}$. Therefore, using Eq. 3-20,

$$\theta = \cos^{-1}\left(\frac{\vec{r}_{ac} \cdot \vec{r}_{bc}}{|\vec{r}_{ac}|\ |\vec{r}_{bc}|}\right) = \cos^{-1}\left(-\frac{1}{3}\right)$$

which yields $\theta = 109.5°$ for the angle between adjacent bonds.

(b) The length of vector $\vec{r}_{bc}$ (which is, of course, the same as the length of $\vec{r}_{ac}$) is

$$|\vec{r}_{bc}| = \frac{a}{2}\sqrt{\frac{3}{2}} = \frac{388\,\mathrm{pm}}{2}\sqrt{\frac{3}{2}} = 237.6 \text{ pm} \ .$$

We note that in the solid geometry literature, the distance $\frac{a}{2}\sqrt{\frac{3}{2}}$ is known as the circumradius of the regular tetrahedron.