51. (a) We know from Eq. 26-7 that the magnitude of the electric field is directly proportional to the surface charge density:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{15 \times 10^{-6}\ \text{C/m}^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.7 \times 10^6\ \text{V/m} \ .$$

Regarding the units, it is worth noting that a Volt is equivalent to a N·m/C.

(b) Eq. 26-23 yields

$$u = \frac{1}{2}\varepsilon_0 E^2 = 13\ \text{J/m}^3 \ .$$

(c) The energy U is the energy-per-unit-volume multiplied by the (variable) volume of the region between the layers of plastic food wrap. Since the distance between the layers is x, and we use A for the area over which the (say, positive) charge is spread, then that volume is Ax. Thus,

$$U = uAx \qquad \text{where} \quad u = 13\ \text{J/m}^3 \ .$$

(d) The magnitude of force is

$$\left|\vec{F}\right| = \frac{dU}{dx} = uA \ .$$

(e) The force per unit area is

$$\frac{\left|\vec{F}\right|}{A} = u = 13\ \text{N/m}^2 \ .$$

Regarding units, it is worth noting that a Joule is equivalent to a N·m, which explains how J/m^3 may be set equal to N/m^2 in the above manipulation. We note, too, that the pressure unit N/m^2 is generally known as a Pascal (Pa).

(f) Combining our steps in parts (a) through (e), we have

$$\begin{aligned} \frac{\left|\vec{F}\right|}{A} &= u = \frac{1}{2}\varepsilon_0 E^2 \\ 6.0\ \text{N/m}^2 &= \frac{1}{2}\varepsilon_0 \left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{\sigma^2}{2\varepsilon_0} \end{aligned}$$

which leads to $\sigma = \sqrt{2(8.85 \times 10^{-12})(6.0)} = 1.0 \times 10^{-5}\ \text{C/m}^2$.