

41. The particle with charge $-q$ has both potential and kinetic energy, and both of these change when the radius of the orbit is changed. We first find an expression for the total energy in terms of the orbit radius r . Q provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\epsilon_0 r^2$. The acceleration of $-q$ is v^2/r , where v is its speed. Newton's second law yields

$$\frac{Qq}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \implies mv^2 = \frac{Qq}{4\pi\epsilon_0 r} ,$$

and the kinetic energy is $K = \frac{1}{2}mv^2 = Qq/8\pi\epsilon_0 r$. The potential energy is $U = -Qq/4\pi\epsilon_0 r$, and the total energy is

$$E = K + U = \frac{Qq}{8\pi\epsilon_0 r} - \frac{Qq}{4\pi\epsilon_0 r} = -\frac{Qq}{8\pi\epsilon_0 r} .$$

When the orbit radius is r_1 the energy is $E_1 = -Qq/8\pi\epsilon_0 r_1$ and when it is r_2 the energy is $E_2 = -Qq/8\pi\epsilon_0 r_2$. The difference $E_2 - E_1$ is the work W done by an external agent to change the radius:

$$W = E_2 - E_1 = -\frac{Qq}{8\pi\epsilon_0} \left(\frac{1}{r_2} - \frac{1}{r_1} \right) = \frac{Qq}{8\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) .$$