89. (Third problem in **Cluster 1**)

We note that the problem has implicitly chosen the initial direction of motion (of m_1) as the positive direction. The questions to find "greatest" and "least" values are understood in terms of that axis choice (*greatest* = largest positive value, and *least* = the negative value of greatest magnitude or the smallest non-negative value). In addition to the assumptions mentioned in the problem, we also assume that m_1 cannot pass through m_2 (like a bullet might be able to). We are only able to use momentum conservation, since no assumptions about the total kinetic energy can be made.

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

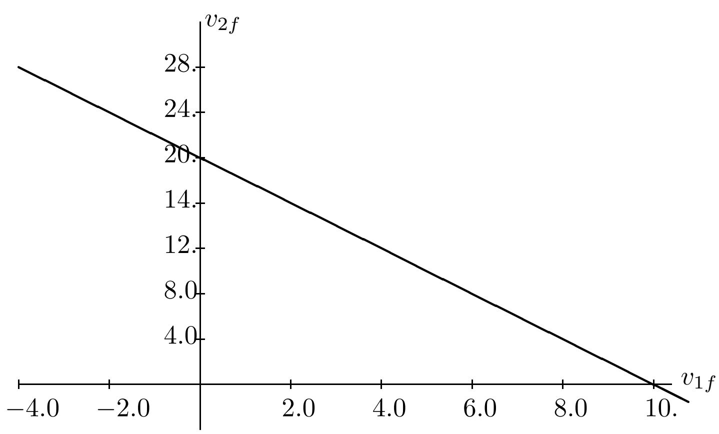
This (since $m_2 = 0.500 m_1$) simplifies to

$$v_{1i} = v_{1f} + 0.500 v_{2f} \ .$$

(a) Using $v_{1i} = 10.0$ m/s, we have

$$v_{2f} = (20.0\,\text{m/s}) - 2.00 v_{1f} \ .$$

(b) Ignoring physics considerations, our function is a line of infinite extent with negative slope.



(c) The greatest possible value of v_{1f} occurs in the completely inelastic case (reasons mentioned in the next several parts) where (see solution to part (a) of previous problem) its value would be $(10.0)(2/3) \approx 6.67$ m/s.

(d) Clearly, this is also the value of v_{2f} in this case.

(e) They stick together (completely inelastic collision).

(f) As mentioned above, we assume m_1 does not pass through m_2 and the problem states that there's no energy production so that $K_{1f} \le K_{1i}$ which implies $v_{1f} \le v_{1i}$.

(g) The plot is shown below, in part (ℓ).

(h) With energy production not a possibility, then the "hardest rebound" m_1 can suffer is in an elastic collision, in which its final velocity (see part (b) of the previous problem) is $(10.0)(2-1)/3 \approx 3.33$ m/s.

(i) Eq. 10-31 gives the velocity of m_2 as $(10.0)(4/3) \approx 13.3$ m/s (see also part (b) of previous problem).

(j) As mentioned, this is an elastic collision (no "loss" of kinetic energy).

(k) The problem states that there's no energy production so that $K_{1i} - K_{1f} = K_{2f}$ and any greater value of $|v_{2f}|$ would violate this condition.

(l) The above graph is redrawn here, with the dark part representing the physically allowed region; the small circles bounding the dark segment correspond to the values calculated in the previous parts of this problem.