

75. We use momentum conservation choosing $+x$ forward and recognizing that the initial momentum is zero. We analyze this from the point of view of an observer at rest on the ice.

- (a) If $v_{1 \text{ and } 2}$ is the speed of the stones, then the speeds are related by $v_{1 \text{ and } 2} + v_{\text{boat}} = v_{\text{rel}}$. Thus, with $m_1 = 2m_2$ and $M = 12m_2$, we obtain

$$\begin{aligned} 0 &= (m_1 + m_2)(-v_{1 \text{ and } 2}) + Mv_{\text{boat}} \\ &= (2m_2 + m_2)(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2v_{\text{boat}} \\ &= -3m_2v_{\text{rel}} + 15m_2v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{1}{5} v_{\text{rel}} = 0.2000v_{\text{rel}}$.

- (b) Using $v_1 + v'_{\text{boat}} = v_{\text{rel}}$, we find – as a result of the first throw – the boat's speed:

$$\begin{aligned} 0 &= m_1(-v_1) + (M + m_2)v'_{\text{boat}} \\ &= 2m_2(-v_{\text{rel}} + v'_{\text{boat}}) + (12m_2 + m_2)v'_{\text{boat}} \\ &= -2m_2v_{\text{rel}} + 15m_2v'_{\text{boat}} \end{aligned}$$

which yields $v'_{\text{boat}} = \frac{2}{15} v_{\text{rel}} \approx 0.133v_{\text{rel}}$. Then, using $v_2 + v_{\text{boat}} = v_{\text{rel}}$, we consider the second throw:

$$\begin{aligned} (M + m_2)v'_{\text{boat}} &= m_2(-v_2) + Mv_{\text{boat}} \\ (12m_2 + m_2)\left(\frac{2}{15}v_{\text{rel}}\right) &= m_2(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2v_{\text{boat}} \\ \frac{26}{15}m_2v_{\text{rel}} &= -m_2v_{\text{rel}} + 13m_2v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{41}{195} v_{\text{rel}} \approx 0.2103v_{\text{rel}}$.

- (c) Finally, using $v_2 + v'_{\text{boat}} = v_{\text{rel}}$, we find – as a result of the first throw – the boat's speed:

$$\begin{aligned} 0 &= m_2(-v_2) + (M + m_1)v'_{\text{boat}} \\ &= m_2(-v_{\text{rel}} + v'_{\text{boat}}) + (12m_2 + 2m_2)v'_{\text{boat}} \\ &= -m_2v_{\text{rel}} + 15m_2v'_{\text{boat}} \end{aligned}$$

which yields $v'_{\text{boat}} = \frac{1}{15} v_{\text{rel}} \approx 0.0673v_{\text{rel}}$. Then, using $v_1 + v_{\text{boat}} = v_{\text{rel}}$, we consider the second throw:

$$\begin{aligned} (M + m_1)v'_{\text{boat}} &= m_1(-v_1) + Mv_{\text{boat}} \\ (12m_2 + 2m_2)\left(\frac{1}{15}v_{\text{rel}}\right) &= 2m_2(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2v_{\text{boat}} \\ \frac{14}{15}m_2v_{\text{rel}} &= -2m_2v_{\text{rel}} + 14m_2v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{22}{105} v_{\text{rel}} \approx 0.2095v_{\text{rel}}$.