23. (a) We refer to the (very large) wire length as L and seek to compute the flux per meter: Φ_B/L. Using the right-hand rule discussed in Chapter 30, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 30-19 and Eq. 30-22. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at what we will call $x = \ell/2$, where $\ell = 20\,\text{mm} = 0.020\,\text{m}$); the net field at any point $0 < x < \ell/2$ is the same at its "mirror image" point $\ell - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = \ell$. We make use of the symmetry by integrating over $0 < x < \ell/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{\ell/2} B\,dA = 2\int_0^{d/2} B\,(L\,dx) + 2\int_{d/2}^{\ell/2} B\,(L\,dx)$$

where $d = 0.0025$ m is the diameter of each wire. We will use $R = d/2$, and r instead of x in the following steps. Thus, using the equations from Ch. 30 referred to above, we find

$$\begin{aligned}
\frac{\Phi_B}{L} &= 2\int_0^R \left(\frac{\mu_0 i}{2\pi R^2}r + \frac{\mu_0 i}{2\pi(\ell - r)}\right) dr + 2\int_R^{\ell/2}\left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(\ell - r)}\right) dr \\
&= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{\ell - R}{\ell}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{\ell - R}{R}\right) \\
&= 0.23\times 10^{-5}\,\text{T·m} + 1.08\times 10^{-5}\,\text{T·m}
\end{aligned}$$

which yields $\Phi_B/L = 1.3\times 10^{-5}\,\text{T·m}$ or $1.3\times 10^{-5}\,\text{Wb/m}$.

(b) The flux (per meter) existing within the regions of space occupied by one or the other wires was computed above to be $0.23\times 10^{-5}\,\text{T·m}$. Thus,

$$\frac{0.23\times 10^{-5}\,\text{T·m}}{1.3\times 10^{-5}\,\text{T·m}} = 0.17 = 17\%\ .$$

(c) What was described in part (a) as a symmetry plane at $x = \ell/2$ is now (in the case of parallel currents) a plane of vanishing field (the fields subtract from each other in the region between them, as the right-hand rule shows). The flux in the $0 < x < \ell/2$ region is now of opposite sign of the flux in the $\ell/2 < x < \ell$ region which causes the total flux (or, in this case, flux per meter) to be zero.