73. (a) The wave function is now given by

$$\Psi(x,t) = \psi_0 \left[e^{i(kx-\omega t)} + e^{-i(kx+\omega t)}\right] = \psi_0\, e^{-i\omega t}\left(e^{ikx} + e^{-ikx}\right) \ .$$

Thus

$$\begin{aligned} |\Psi(x,t)|^2 &= \left|\psi_0\, e^{-i\omega t}\left(e^{ikx}+e^{-ikx}\right)\right|^2 \\ &= \left|\psi_0\, e^{-i\omega t}\right|^2 \left|e^{ikx}+e^{-ikx}\right|^2 \\ &= \psi_0^2 \left|e^{ikx}+e^{-ikx}\right|^2 \\ &= \psi_0^2 \left|(\cos kx + i\sin kx) + (\cos kx - i\sin kx)\right|^2 \\ &= 4\psi_0^2(\cos kx)^2 \\ &= 2\psi_0^2(1+\cos 2kx) \ . \end{aligned}$$

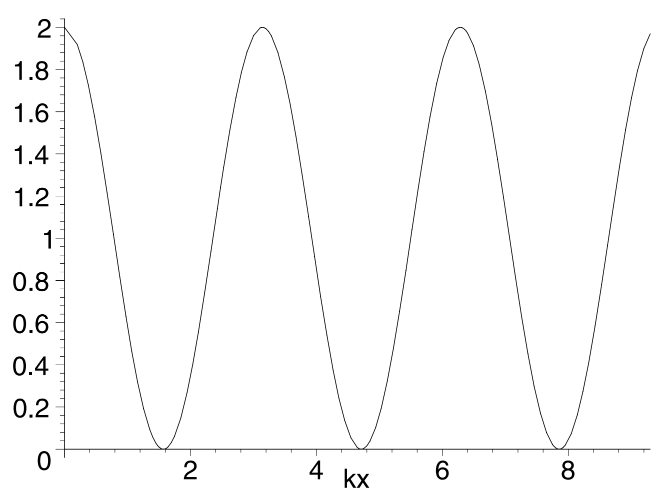
(b) Consider two plane matter waves, each with the same amplitude $\psi_0/\sqrt{2}$ and traveling in opposite directions along the x axis. The combined wave Ψ is a standing wave:

$$\begin{aligned} \Psi(x,t) &= \psi_0\, e^{i(kx-\omega t)} + \psi_0\, e^{-i(kx+\omega t)} = \psi_0\left(e^{ikx}+e^{-ikx}\right)e^{-i\omega t} \\ &= (2\psi_0 \cos kx)\, e^{-i\omega t} \ . \end{aligned}$$

Thus, the squared amplitude of the matter wave is

$$|\Psi(x,t)|^2 = (2\psi_0\cos kx)^2 \left|e^{-i\omega t}\right|^2 = 2\psi_0^2(1+\cos 2kx) \ ,$$

which is shown below.



(c) We set $|\Psi(x,t)|^2 = 2\psi_0^2(1+\cos 2kx) = 0$ to obtain $\cos(2kx) = -1$. This gives

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = (2n+1)\pi \ , \qquad (n = 0, 1, 2, 3, \ldots)$$

We solve for x:

$$x = \frac{1}{4}(2n+1)\lambda \ .$$

(d) The most probable positions for finding the particle are where $|\Psi(x,t)| \propto (1+\cos 2kx)$ reaches its maximum. Thus $\cos 2kx = 1$, or

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = 2n\pi \ , \qquad (n = 0, 1, 2, 3, \ldots)$$