

60. (a) In a reference frame fixed on Earth, the ether travels leftward with speed  $v$ . Thus, the speed of the light beam in this reference frame is  $c - v$  as the beam travels rightward from  $M$  to  $M_1$ , and  $c + v$  as it travels back from  $M_1$  to  $M$ . The total time for the round trip is therefore given by

$$t_1 = \frac{d_1}{c - v} + \frac{d_1}{c + v} = \frac{2cd_1}{c^2 - v^2} .$$

- (b) In a reference frame fixed on the ether, the mirrors travel rightward with speed  $v$ , while the speed of the light beam remains  $c$ . Thus, in this reference frame, the total distance the beam has to travel is given by

$$d_2' = 2\sqrt{d_2^2 + \left[v\left(\frac{t_2}{2}\right)\right]^2}$$

[see Fig. 36-37(h)-(j)]. Thus,

$$t_2 = \frac{d_2'}{c} = \frac{2}{c} \sqrt{d_2^2 + \left[v\left(\frac{t_2}{2}\right)\right]^2} ,$$

which we solve for  $t_2$ :

$$t_2 = \frac{2d_2}{\sqrt{c^2 - v^2}} .$$

- (c) We use the binomial expansion (Appendix E)

$$(1 + x)^n = 1 + nx + \cdots \approx 1 + nx \quad (|x| \ll 1) .$$

In our case let  $x = v/c \ll 1$ , then

$$L_1 = \frac{2c^2d_1}{c^2 - v^2} = 2d_1 \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1} \approx 2d_1 \left[1 + \left(\frac{v}{c}\right)^2\right] ,$$

and

$$L_2 = \frac{2cd_2}{\sqrt{c^2 - v^2}} = 2d_2 \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} \approx 2d_2 \left[1 + \frac{1}{2} \left(\frac{v}{c}\right)^2\right] .$$

Thus, if  $d_1 = d_2 = d$  then

$$\Delta L = L_1 - L_2 \approx 2d \left[1 + \left(\frac{v}{c}\right)^2\right] - 2d \left[1 + \frac{1}{2} \left(\frac{v}{c}\right)^2\right] = \frac{dv^2}{c^2} .$$

- (d) In terms of the wavelength, the phase difference is given by

$$\frac{\Delta L}{\lambda} = \frac{dv^2}{\lambda c^2} .$$

- (e) We now must reverse the indices 1 and 2, so the new phase difference is

$$\frac{-\Delta L}{\lambda} = -\frac{dv^2}{\lambda c^2} .$$

The shift in phase difference between these two cases is

$$\text{shift} = \left(\frac{\Delta L}{\lambda}\right) - \left(-\frac{\Delta L}{\lambda}\right) = \frac{2dv^2}{\lambda c^2} .$$

- (f) Assume that  $v$  is about the same as the orbital speed of the Earth, so that  $v \approx 29.8 \text{ km/s}$  (see Appendix C). Thus,

$$\text{shift} = \frac{2dv^2}{\lambda c^2} = \frac{2(10 \text{ m})(29.8 \times 10^3 \text{ m/s})^2}{(500 \times 10^{-9} \text{ m})(2.998 \times 10^8 \text{ m/s})^2} = 0.40 .$$