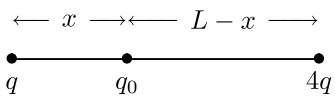
9. (a) If the system of three charges is to be in equilibrium, the force on each charge must be zero. Let the third charge be q_0. It must lie between the other two or else the forces acting on it due to the other charges would be in the same direction and q_0 could not be in equilibrium. Suppose q_0 is a distance x from q, as shown on the diagram below. The force acting on q_0 is then given by

$$F_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_0}{x^2} - \frac{4qq_0}{(L-x)^2}\right)$$

where the positive direction is rightward. We require $F_0 = 0$ and solve for x. Canceling common factors yields $1/x^2 = 4/(L-x)^2$ and taking the square root yields $1/x = 2/(L-x)$. The solution is $x = L/3$.



The force on q is

$$F_q = \frac{-1}{4\pi\varepsilon_0}\left(\frac{qq_0}{x^2} + \frac{4q^2}{L^2}\right) .$$

The signs are chosen so that a negative force value would cause q to move leftward. We require $F_q = 0$ and solve for q_0:

$$q_0 = -\frac{4qx^2}{L^2} = -\frac{4}{9}q$$

where $x = L/3$ is used. We now examine the force on $4q$:

$$\begin{aligned} F_{4q} &= \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4qq_0}{(L-x)^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2}\right) \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2}\right) \end{aligned}$$

which we see is zero. Thus, with $q_0 = -(4/9)q$ and $x = L/3$, all three charges are in equilibrium.

(b) If q_0 moves toward q the force of attraction exerted by q is greater in magnitude than the force of attraction exerted by $4q$. This causes q_0 to continue to move toward q and away from its initial position. The equilibrium is unstable.