19. (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$\begin{aligned} J &= \int_0^{3.0\times10^{-3}} F\,dt \\ &= \int_0^{3.0\times10^{-3}} \left(6.0\times10^6\right)t - \left(2.0\times10^9\right)t^2\ dt \\ &= \left[\frac{1}{2}(6.0\times10^6)t^2 - \frac{1}{3}(2.0\times10^9)t^3\right]_0^{3.0\times10^{-3}} \\ &= 9.0\ \text{N}\cdot\text{s}\ . \end{aligned}$$

(b) Since $J = F_{\text{avg}}\,\Delta t$, we find

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.0\,\text{N}\cdot\text{s}}{3.0\times10^{-3}\,\text{s}} = 3.0\times10^3\ \text{N}\ .$$

(c) To find the time at which the maximum force occurs, we set the derivative of F with respect to time equal to zero – and solve for t. The result is $t = 1.5\times10^{-3}\,$s. At that time the force is

$$F_{\text{max}} = (6.0\times10^6)(1.5\times10^{-3}) - (2.0\times10^9)(1.5\times10^{-3})^2 = 4.5\times10^3\,\text{N}\ .$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let m be the mass of the ball and v be its speed as it leaves the foot. Then,

$$v = \frac{p}{m} = \frac{J}{m} = \frac{9.0\,\text{N}\cdot\text{s}}{0.45\,\text{kg}} = 20\ \text{m/s}\ .$$