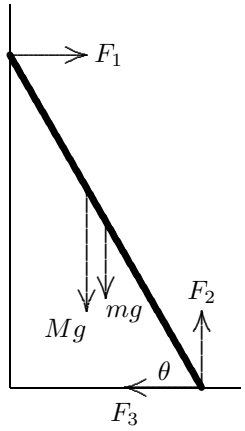
13. The forces on the ladder are shown in the diagram below.

F_1 is the force of the window, horizontal because the window is frictionless. F_2 and F_3 are components of the force of the ground on the ladder. M is the mass of the window cleaner and m is the mass of the ladder. The force of gravity on the man acts at a point 3.0 m up the ladder and the force of gravity on the ladder acts at the center of the ladder. Let θ be the angle between the ladder and the ground. We use $\cos\theta = d/L$ or $\sin\theta = \sqrt{L^2 - d^2}/L$ to find $\theta = 60°$. Here L is the length of the ladder (5.0 m) and d is the distance from the wall to the foot of the ladder (2.5 m).



(a) Since the ladder is in equilibrium the sum of the torques about its foot (or any other point) vanishes. Let ℓ be the distance from the foot of the ladder to the position of the window cleaner. Then, $Mg\ell\cos\theta + mg(L/2)\cos\theta - F_1 L\sin\theta = 0$, and

$$\begin{aligned} F_1 &= \frac{(M\ell + mL/2)g\cos\theta}{L\sin\theta} \\ &= \frac{((75\,\text{kg})(3.0\,\text{m}) + (10\,\text{kg})(2.5\,\text{m}))\,(9.8\,\text{m/s}^2)\cos 60°}{(5.0\,\text{m})\sin 60°} = 2.8\times 10^2\ \text{N}\ . \end{aligned}$$

This force is outward, away from the wall. The force of the ladder on the window has the same magnitude but is in the opposite direction: it is approximately 280 N, inward.

(b) The sum of the horizontal forces and the sum of the vertical forces also vanish:

$$\begin{aligned} F_1 - F_3 &= 0 \\ F_2 - Mg - mg &= 0 \end{aligned}$$

The first of these equations gives $F_3 = F_1 = 2.8\times 10^2$ N and the second gives

$$F_2 = (M+m)g = (75\,\text{kg} + 10\,\text{kg})(9.8\,\text{m/s}^2) = 8.3\times 10^2\ \text{N}$$

The magnitude of the force of the ground on the ladder is given by the square root of the sum of the squares of its components:

$$F = \sqrt{F_2^2 + F_3^2} = \sqrt{(2.8\times 10^2\,\text{N})^2 + (8.3\times 10^2\,\text{N})^2} = 8.8\times 10^2\ \text{N}\ .$$

The angle ϕ between the force and the horizontal is given by $\tan\phi = F_3/F_2 = 830/280 = 2.94$, so $\phi = 71°$. The force points to the left and upward, 71° above the horizontal. We note that this force is not directed along the ladder.