

60. (a) The ideal gas is diatomic, so $f = 5$ (see Table 20-3). Since this is an isobaric (constant pressure) process, with no change in the number of moles, then the ideal gas in ratio form (see Sample Problem 20-1) leads to

$$\frac{V_f}{V_i} = \frac{T_f}{T_i} = \frac{8}{3}.$$

With $C_V = \frac{f}{2}R$, Eq. 21-4 gives

$$\Delta S_{\text{gas}} = nR \ln\left(\frac{8}{3}\right) + n\left(\frac{5}{2}R\right) \ln\left(\frac{8}{3}\right)$$

where n is the number of moles (25 mol), not to be confused with the number of reservoirs (also denoted “ n ” in the later parts of this problem). Consequently, we obtain

$$\Delta S_{\text{gas}} = \frac{7}{2}(25 \text{ mol}) \left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) \ln\left(\frac{8}{3}\right) = 713 \text{ J/K}.$$

Since $Q = nC_p\Delta T$ for this process, the entropy change of the reservoir (which transfers energy Q to the gas, so it (the heat) is negative-valued in this context) is (using Eq. 21-2)

$$\Delta S_{\text{res}} = \frac{-Q}{T} = -\frac{n\left(\frac{7}{2}R\right)(800 \text{ K} - 300 \text{ K})}{800 \text{ K}} = -454 \text{ J/K}.$$

Therefore, $\Delta S_{\text{system}} = \Delta S_{\text{gas}} + \Delta S_{\text{res}} = 259 \text{ J/K}$.

- (b) The change in entropy of the gas is the same regardless of the number of intermediate reservoirs, so long as the beginning state and final state of the gas is unchanged. The difference (relative to part (a)) is that the sum of these *two* reservoirs’ entropy changes is not equivalent to that of the one reservoir in the previous part:

$$\begin{aligned} \Delta S_{\text{res1}} + \Delta S_{\text{res2}} &= \frac{-Q_1}{T_1} + \frac{-Q_2}{T_2} \\ &= -\frac{(25 \text{ mol})\left(\frac{7}{2}R\right)(550 \text{ K} - 300 \text{ K})}{550 \text{ K}} - \frac{(25 \text{ mol})\left(\frac{7}{2}R\right)(800 \text{ K} - 550 \text{ K})}{800 \text{ K}} \\ &= -(25 \text{ mol})\left(\frac{7}{2}R\right)(250 \text{ K})\left(\frac{1}{550 \text{ K}} + \frac{1}{800 \text{ K}}\right) \end{aligned}$$

which yields -558 J/K for the total loss of entropy from the reservoirs. The entire system change in entropy is therefore $713 - 558 = 155 \text{ J/K}$.

- (c) Towards the end of the calculation in part (b), a pattern emerges in the computation of the total entropy loss from the original high-temperature reservoir plus the n intermediate reservoirs:

$$\Delta S_{\text{res total}} = -(25 \text{ mol})\left(\frac{7}{2}R\right)\left(\frac{500 \text{ K}}{n+1}\right)\left(\sum_{\text{reservoirs}}^{n+1} \frac{1}{T}\right)$$

where the temperature of a particular reservoir (the j^{th} reservoir, where $1 \leq j \leq n+1$) is $T = 300 + \left(\frac{500}{n+1}\right)$ (in Kelvins). For $n = 10$, this leads to $\Delta S_{\text{res total}} = -680 \text{ J/K}$ and therefore $713 - 680 = 33 \text{ J/K}$ for the entire system (including the gas) entropy change.

- (d) For $n = 50$, this leads to $\Delta S_{\text{res total}} = -705.82 \text{ J/K}$ and therefore $713.19 - 705.82 = 7.37 \text{ J/K}$ for the entire system (including the gas) entropy change.
(e) For $n = 100$, this leads to $\Delta S_{\text{res total}} = -709.45 \text{ J/K}$ and therefore $713.19 - 709.45 = 3.74 \text{ J/K}$ for the entire system (including the gas) entropy change.