81. (a) If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U and width L is given by

$$T = e^{-2kL} ,$$

where

$$k = \sqrt{\frac{8\pi^2 m(U-E)}{h^2}} .$$

If the change ΔU in U is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dU}\,\Delta U = -2LT\,\frac{dk}{\mathrm{d}U}\,\Delta U .$$

Now,

$$\frac{dk}{dU} = \frac{1}{2\sqrt{U-E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U-E)}\sqrt{\frac{8\pi^2 m(U-E)}{h^2}} = \frac{k}{2(U-E)} .$$

Thus,

$$\Delta T = -LTk\,\frac{\Delta U}{U-E} .$$

For the data of Sample Problem 39-7, $2kL = 10.0$, so $kL = 5.0$ and

$$\frac{\Delta T}{T} = -kL\frac{\Delta U}{U-E} = -(5.0)\frac{(0.010)(6.8\,\mathrm{eV})}{6.8\,\mathrm{eV} - 5.1\,\mathrm{eV}} = -0.20 .$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dL}\,\Delta L = -2ke^{-2kL}\,\Delta L = -2kT\,\Delta L$$

and

$$\frac{\Delta T}{T} = -2k\,\Delta L = -2(6.67\times10^{9}\,\mathrm{m}^{-1})(0.010)(750\times10^{-12}\,\mathrm{m}) = -0.10 .$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dE}\,\Delta E = -2Le^{-2kL}\,\frac{dk}{dE}\,\Delta E = -2LT\frac{dk}{dE}\,\Delta E .$$

Now, $dk/dE = -dk/dU = -k/2(U-E)$, so

$$\frac{\Delta T}{T} = kL\frac{\Delta E}{U-E} = (5.0)\frac{(0.010)(5.1\,\mathrm{eV})}{6.8\,\mathrm{eV} - 5.1\,\mathrm{eV}} = 0.15 .$$

There is a 15% increase in the transmission coefficient.