52. (a) The transition is from $n = 2$ to $n = 1$, so Eq. 41-26 combined with Eq. 41-24 yields

$$f = \left(\frac{m_e e^4}{8\varepsilon_0^2 h^3}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(Z-1)^2$$

so that the constant in Eq. 41-27 is

$$C = \sqrt{\frac{3m_e e^4}{32\varepsilon_0^2 h^3}} = 4.9673 \times 10^7\ \mathrm{Hz}^{1/2}$$

using the values in the next-to-last column in the Table in Appendix B (but note that the power of ten is given in the middle column).

(b) We are asked to compare the results of Eq. 41-27 (squared, then multiplied by the accurate values of h/e found in Appendix B to convert to x ray energies) with those in the table of K_α energies (in eV) given at the end of the problem. We look up the corresponding atomic numbers in Appendix F. An example is shown below (for Nitrogen):

$$E_{\text{theory}} = \frac{h}{e}\,C^2(Z-1)^2 = \frac{6.6260688 \times 10^{-34}\ \mathrm{J{\cdot}s}}{1.6021765 \times 10^{-19}\ \mathrm{J/eV}}\left(4.9673 \times 10^7\ \mathrm{Hz}^{1/2}\right)^2 (7-1)^2 = 367.35\ \mathrm{eV}$$

which is 6.4% lower than the experimental value of 392.4 eV. Progressing through the list, from Lithium to Magnesium , we find all the theoretical values are lower than the experimental ones by these percentages: 24.8%, 15.4%, 10.9%, 7.9%, 6.4%, 4.7%, 3.5%, 2.6%, 2.0%, and 1.5%.

(c) The trend is clear from the list given above: the agreement between theory and experiment becomes better as Z increases. One might argue that the most questionable step in §41-10 is the replacement $e^4 \to (Z-1)^2 e^4$ and ask why this could not equally well be $e^4 \to (Z-.9)^2 e^4$ or $e^4 \to (Z-.8)^2 e^4$? For large Z, these subtleties would not matter so much as they do for small Z, since $Z - \xi \approx Z$ for $Z \gg \xi$.