

25. (a) If the magnetization of the sphere is saturated, the total dipole moment is  $\mu_{\text{total}} = N\mu$ , where  $N$  is the number of iron atoms in the sphere and  $\mu$  is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with  $N$  iron atoms. The mass of such a sphere is  $Nm$ , where  $m$  is the mass of an iron atom. It is also given by  $4\pi\rho R^3/3$ , where  $\rho$  is the density of iron and  $R$  is the radius of the sphere. Thus  $Nm = 4\pi\rho R^3/3$  and

$$N = \frac{4\pi\rho R^3}{3m} .$$

We substitute this into  $\mu_{\text{total}} = N\mu$  to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3\mu}{3m} .$$

We solve for  $R$  and obtain

$$R = \left( \frac{3m\mu_{\text{total}}}{4\pi\rho\mu} \right)^{1/3} .$$

The mass of an iron atom is

$$m = 56 \text{ u} = (56 \text{ u})(1.66 \times 10^{-27} \text{ kg/u}) = 9.30 \times 10^{-26} \text{ kg} .$$

Therefore,

$$R = \left[ \frac{3(9.30 \times 10^{-26} \text{ kg})(8.0 \times 10^{22} \text{ J/T})}{4\pi(14 \times 10^3 \text{ kg/m}^3)(2.1 \times 10^{-23} \text{ J/T})} \right]^{1/3} = 1.8 \times 10^5 \text{ m} .$$

- (b) The volume of the sphere is

$$V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82 \times 10^5 \text{ m})^3 = 2.53 \times 10^{16} \text{ m}^3$$

and the volume of the Earth is

$$V_e = \frac{4\pi}{3}(6.37 \times 10^6 \text{ m})^3 = 1.08 \times 10^{21} \text{ m}^3 ,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{2.53 \times 10^{16} \text{ m}^3}{1.08 \times 10^{21} \text{ m}^3} = 2.3 \times 10^{-5} .$$