

47. The magnitude of the fractional energy change for the photon is given by

$$\left| \frac{\Delta E_{\text{ph}}}{E_{\text{ph}}} \right| = \left| \frac{\Delta(hc/\lambda)}{hc/\lambda} \right| = \left| \lambda \Delta \left( \frac{1}{\lambda} \right) \right| = \lambda \left( \frac{1}{\lambda} - \frac{1}{\lambda + \Delta\lambda} \right) = \frac{\Delta\lambda}{\lambda + \Delta\lambda} = \beta$$

where  $\beta = 0.10$ . Thus  $\Delta\lambda = \lambda\beta/(1 - \beta)$ . We substitute this expression for  $\Delta\lambda$  in Eq. 39-11 and solve for  $\cos \phi$ :

$$\begin{aligned} \cos \phi &= 1 - \frac{mc}{h} \Delta\lambda = 1 - \frac{mc\lambda\beta}{h(1 - \beta)} = 1 - \frac{\beta(mc^2)}{(1 - \beta)E_{\text{ph}}} \\ &= 1 - \frac{(0.10)(511 \text{ keV})}{(1 - 0.10)(200 \text{ keV})} = 0.716 . \end{aligned}$$

This leads to an angle of  $\phi = 44^\circ$ .