

54. We use $\Phi = \int \vec{E} \cdot d\vec{A}$. We note that the side length of the cube is $3.0 \text{ m} - 1.0 \text{ m} = 2.0 \text{ m}$.

- (a) On the top face of the cube $y = 2.0 \text{ m}$ and $d\vec{A} = (dA)\hat{j}$. So $\vec{E} = 4\hat{i} - 3((2.0)^2 + 2)\hat{j} = 4\hat{i} - 18\hat{j}$. Thus the flux is

$$\begin{aligned}\Phi &= \int_{\text{top}} \vec{E} \cdot d\vec{A} = \int_{\text{top}} (4\hat{i} - 18\hat{j}) \cdot (dA)\hat{j} \\ &= -18 \int_{\text{top}} dA = (-18)(2.0)^2 \text{ N}\cdot\text{m}^2/\text{C} = -72 \text{ N}\cdot\text{m}^2/\text{C} .\end{aligned}$$

- (b) On the bottom face of the cube $y = 0$ and $d\vec{A} = (dA)(-\hat{j})$. So $\vec{E} = 4\hat{i} - 3(0^2 + 2)\hat{j} = 4\hat{i} - 6\hat{j}$. Thus, the flux is

$$\begin{aligned}\Phi &= \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = \int_{\text{bottom}} (4\hat{i} - 6\hat{j}) \cdot (dA)(-\hat{j}) \\ &= 6 \int_{\text{bottom}} dA = 6(2.0)^2 \text{ N}\cdot\text{m}^2/\text{C} = +24 \text{ N}\cdot\text{m}^2/\text{C} .\end{aligned}$$

- (c) On the left face of the cube $d\vec{A} = (dA)(-\hat{i})$. So

$$\begin{aligned}\Phi &= \int_{\text{left}} \vec{E} \cdot d\vec{A} = \int_{\text{left}} (4\hat{i} + E_y\hat{j}) \cdot (dA)(-\hat{i}) \\ &= -4 \int_{\text{bottom}} dA = -4(2.0)^2 \text{ N}\cdot\text{m}^2/\text{C} = -16 \text{ N}\cdot\text{m}^2/\text{C} .\end{aligned}$$

- (d) On the back face of the cube $d\vec{A} = (dA)(-\hat{k})$. But since \vec{E} has no z component $\vec{E} \cdot d\vec{A} = 0$. Thus, $\Phi = 0$.

- (e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or $+16 \text{ N}\cdot\text{m}^2/\text{C}$. Thus the net flux through the cube is $\Phi = (-72 + 24 - 16 + 0 + 0 + 16) \text{ N}\cdot\text{m}^2/\text{C} = -48 \text{ N}\cdot\text{m}^2/\text{C}$.