51. The interacting potential energy between the magnetic dipole of the compass and the Earth's magnetic field is $U = -\vec{\mu}\cdot\vec{B}_e = -\mu B_e \cos\theta$, where θ is the angle between $\vec{\mu}$ and $\vec{B}_e$. For small angle θ

$$U(\theta) = -\mu B_e \cos\theta \quad \approx \quad -\mu B_e \left(1 - \frac{\theta^2}{2}\right) = \frac{1}{2}\kappa\theta^2 - \mu B_e$$

where $\kappa = \mu B_e$. Conservation of energy for the compass then gives

$$\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\kappa\theta^2 = \text{const.}\ .$$

This is to be compared with the following expression for the mechanical energy of a spring-mass system:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 = \text{const.}\ ,$$

which yields $\omega = \sqrt{k/m}$. So by analogy, in our case

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{\mu B_e}{I}} = \sqrt{\frac{\mu B_e}{ml^2/12}}\ ,$$

which leads to

$$\mu = \frac{ml^2\omega^2}{12B_e} = \frac{(0.050\,\text{kg})(4.0\times10^{-2}\,\text{m})^2(45\,\text{rad/s})^2}{12(16\times10^{-6}\,\text{T})} = 8.4\times10^2\ \text{J/T}\ .$$