

9. To find where the ball lands, we need to know its speed as it leaves the track (using conservation of energy). Its initial kinetic energy is  $K_i = 0$  and its initial potential energy is  $U_i = Mgh$ . Its final kinetic energy (as it leaves the track) is  $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$  (Eq. 12-5) and its final potential energy is  $Mgh$ . Here we use  $v$  to denote the speed of its center of mass and  $\omega$  is its angular speed – at the moment it leaves the track. Since (up to that moment) the ball rolls without sliding we can set  $\omega = v/R$ . Using  $I = \frac{2}{5}MR^2$  (Table 11-2(f)), conservation of energy leads to

$$\begin{aligned} Mgh &= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh \\ &= \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 + Mgh \\ &= \frac{7}{10}Mv^2 + Mgh . \end{aligned}$$

The mass  $M$  cancels from the equation, and we obtain

$$v = \sqrt{\frac{10}{7}g(H - h)} = \sqrt{\frac{10}{7}(9.8 \text{ m/s}^2)(6.0 \text{ m} - 2.0 \text{ m})} = 7.48 \text{ m/s} .$$

Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the “initial” position for this part of the problem) and take  $+x$  rightward and  $+y$  downward. Then (since the initial velocity is purely horizontal) the projectile motion equations become

$$x = vt \quad \text{and} \quad y = \frac{1}{2}gt^2 .$$

Solving for  $x$  at the time when  $y = h$ , the second equation gives  $t = \sqrt{2h/g}$ . Then, substituting this into the first equation, we find

$$x = v\sqrt{\frac{2h}{g}} = (7.48)\sqrt{\frac{2(2.0)}{9.8}} = 4.8 \text{ m} .$$