34. (a) Consider an infinitesimal segment of the rod from x to $x + dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{\lambda(x)dx}{\sqrt{x^2 + y^2}} = \frac{1}{4\pi\varepsilon_0} \frac{cx}{\sqrt{x^2 + y^2}}\, dx \ .$$

Thus,

$$V = \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0} \int_0^L \frac{x}{\sqrt{x^2 + y^2}}\, dx = \frac{c}{4\pi\varepsilon_0}(\sqrt{L^2 + y^2} - y) \ .$$

(b) The y component of the field there is

$$E_y = -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0} \frac{d}{dy}(\sqrt{L^2 + y^2} - y) = \frac{c}{4\pi\varepsilon_0} \left(1 - \frac{y}{\sqrt{L^2 + y^2}}\right) \ .$$

(c) We obtained above the value of the potential at any point P strictly on the y-axis. In order to obtain $E_x(x, y)$ we need to first calculate $V(x, y)$. That is, we must find the potential for an arbitrary point located at $(x,\, y)$. Then $E_x(x,\, y)$ can be obtained from $E_x(x,\, y) = -\partial V(x,\, y)/\partial x$.