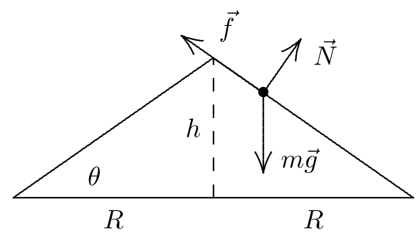
11. A cross section of the cone of sand is shown below. To pile the most sand without extending the radius, sand is added to make the height h as great as possible. Eventually, however, the sides become so steep that sand at the surface begins to slide. The goal is to find the greatest height (corresponding to greatest slope) for which the sand does not slide. A grain of sand is shown on the diagram and the forces on it are labeled. $\vec{N}$ is the normal force of the surface, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of (static) friction. We take the x axis to be down the plane and the y axis to be in the direction of the normal force. We assume the grain does not slide, so its acceleration is zero. Then the x component of Newton's second law is $mg\sin\theta - f = 0$ and the y component is $N - mg\cos\theta = 0$.

The first equation gives $f = mg\sin\theta$ and the second gives $N = mg\cos\theta$. If the grain does not slide, the condition $f < \mu_s N$ must hold. This means $mg\sin\theta < \mu_s mg\cos\theta$ or $\tan\theta < \mu_s$. The surface of the cone has the greatest slope (and the height of the cone is the greatest) if $\tan\theta = \mu_s$.



Since R and h are two sides of a right triangle, $h = R\tan\theta$. Replacing $\tan\theta$ with μ_s we obtain $h = \mu_s R$. We substitute this into the volume equation $V = \pi R^2 h/3$ to obtain the result $V = \pi\mu_s R^3/3$.