

45. (a) The potential energy is

$$U = \frac{q^2}{4\pi\epsilon_0 d} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(5.0 \times 10^{-6} \text{ C})^2}{1.00 \text{ m}} = 0.225 \text{ J}$$

relative to the potential energy at infinite separation.

- (b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\epsilon_0 d^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(5.0 \times 10^{-6} \text{ C})^2}{(1.00 \text{ m})^2} = 0.225 \text{ N} .$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let  $m_A$  and  $m_B$  be the masses of the spheres. The acceleration of sphere  $A$  is

$$a_A = \frac{F}{m_A} = \frac{0.225 \text{ N}}{5.0 \times 10^{-3} \text{ kg}} = 45.0 \text{ m/s}^2$$

and the acceleration of sphere  $B$  is

$$a_B = \frac{F}{m_B} = \frac{0.225 \text{ N}}{10 \times 10^{-3} \text{ kg}} = 22.5 \text{ m/s}^2 .$$

- (c) Energy is conserved. The initial potential energy is  $U = 0.225 \text{ J}$ , as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is  $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$ , where  $v_A$  and  $v_B$  are the final velocities. Thus,

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 .$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B .$$

These equations may be solved simultaneously for  $v_A$  and  $v_B$ . Substituting  $v_B = -(m_A/m_B)v_A$ , from the momentum equation into the energy equation, and collecting terms, we obtain  $U = \frac{1}{2}(m_A/m_B)(m_A + m_B)v_A^2$ . Thus,

$$\begin{aligned} v_A &= \sqrt{\frac{2Um_B}{m_A(m_A + m_B)}} \\ &= \sqrt{\frac{2(0.225 \text{ J})(10 \times 10^{-3} \text{ kg})}{(5.0 \times 10^{-3} \text{ kg})(5.0 \times 10^{-3} \text{ kg} + 10 \times 10^{-3} \text{ kg})}} = 7.75 \text{ m/s} . \end{aligned}$$

We thus obtain

$$v_B = -\frac{m_A}{m_B}v_A = -\left(\frac{5.0 \times 10^{-3} \text{ kg}}{10 \times 10^{-3} \text{ kg}}\right)(7.75 \text{ m/s}) = -3.87 \text{ m/s} .$$