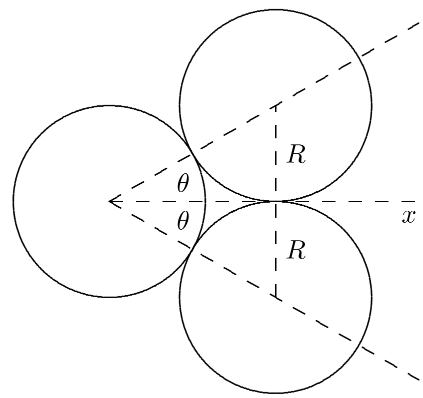
53. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two. It exerts an impulse of the same magnitude on each ball, along the line that joins the centers of the incident ball and the target ball. The target balls leave the collision along those lines, while the incident ball leaves the collision along the x axis. The three dotted lines that join the centers of the balls in contact form an equilateral triangle, so both of the angles marked θ are 30°. Let v_0 be the velocity of the incident ball before the collision and V be its velocity afterward. The two target balls leave the collision with the same speed. Let v represent that speed. Each ball has mass m.



Since the x component of the total momentum of the three-ball system is conserved,

$$mv_0 = mV + 2mv\cos\theta$$

and since the total kinetic energy is conserved,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mV^2 + 2\left(\frac{1}{2}mv^2\right) .$$

We know the directions in which the target balls leave the collision so we first eliminate V and solve for v. The momentum equation gives $V = v_0 - 2v\cos\theta$, so $V^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta$ and the energy equation becomes $v_0^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2v_0\cos\theta}{1 + 2\cos^2\theta} = \frac{2(10\,\text{m/s})\cos 30^\circ}{1 + 2\cos^2 30^\circ} = 6.93 \text{ m/s} .$$

(a) The discussion and computation above determines the final velocity of ball 2 (as labeled in Fig. 10-41) to be 6.9 m/s at 30° counterclockwise from the $+x$ axis.

(b) Similarly, the final velocity of ball 3 is 6.9 m/s at 30° clockwise from the $+x$ axis.

(c) Now we use the momentum equation to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10\,\text{m/s} - 2(6.93\,\text{m/s})\cos 30^\circ = -2.0\,\text{m/s} .$$

The minus sign indicates that it bounces back in the $-x$ direction.