42. (a) The volume occupied by the sand within $r \le \frac{1}{2}r_m$ is that of a cylinder of height h' plus a cone atop that of height h. To find h, we consider

$$\tan\theta = \frac{h}{\frac{1}{2}r_m} \implies h = \frac{1.82\,\text{m}}{2}\tan 33^\circ = 0.59 \text{ m} .$$

Therefore, since $h' = H - h$, the volume V contained within that radius is

$$\pi\left(\frac{r_m}{2}\right)^2(H-h) + \frac{\pi}{3}\left(\frac{r_m}{2}\right)^2 h = \pi\left(\frac{r_m}{2}\right)^2\left(H - \frac{2}{3}h\right)$$

which yields $V = 6.78 \text{ m}^3$.

(b) Since weight W is mg, and mass m is ρV, we have

$$W = \rho V g = \left(1800\,\text{kg/m}^3\right)\left(6.78\,\text{m}^3\right)\left(9.8\,\text{m/s}^2\right) = 1.20 \times 10^5 \text{ N} .$$

(c) Since the slope is $(\sigma_m - \sigma_\text{o})/r_m$ and the y-intercept is σ_o we have

$$\sigma = \left(\frac{\sigma_m - \sigma_\text{o}}{r_m}\right) r + \sigma_\text{o} \qquad \text{for } r \le r_m$$

or (with numerical values, SI units assumed) $\sigma \approx 13r + 40000$.

(d) The length of the circle is $2\pi r$ and it's "thickness" is dr, so the infinitesimal area of the ring is $dA = 2\pi r\,dr$.

(e) The force results from the product of stress and area (if both are well-defined). Thus, with SI units understood,

$$dF = \sigma\,dA = \left(\left(\frac{\sigma_m - \sigma_\text{o}}{r_m}\right) r + \sigma_\text{o}\right)(2\pi r\,dr) \approx 83r^2dr + 2.5 \times 10^5 r dr .$$

(f) We integrate our expression (using the precise numerical values) for dF and find

$$F = \int_0^{r_m/2} \left(82.855r^2 + 251327r\right) dr = \frac{82.855}{3}\left(\frac{r_m}{2}\right)^3 + \frac{251327}{2}\left(\frac{r_m}{2}\right)^2$$

which yields $F = 104083 \approx 1.04 \times 10^5$ N for $r_m = 1.82$ m.

(g) The fractional reduction is

$$\frac{F-W}{W} = \frac{F}{W} - 1 = \frac{104083}{1.20 \times 10^5} - 1 = -0.13 .$$