

89. (a) At -40°F the tuning fork is shorter and takes less time to execute a “tick.” The record of the clock assumes every “tick” corresponds to some standard unit of time – the net effect being that its time-record is “fast” or “ahead” of the correct time. We write the (absolute value of) relative error as

$$\left| \frac{t_{\text{fork}} - t_{\text{correct}}}{t_{\text{correct}}} \right| = \frac{T_{\text{fork}}}{T_{\text{correct}}} - 1 \quad .$$

Using Eq. 16-28, this becomes

$$\left| \frac{\Delta t}{t_{\text{correct}}} \right| = \sqrt{\frac{L_{-40}}{L_{25}}} - 1 \quad ,$$

where we have used the fact that the tuning fork would be accurate if the temperature were 25°F . Now, Eq. 19-9 tells us that $L_{-40} = L_{25}(1 + \alpha\Delta T)$, where $\Delta T = -65^\circ\text{F}$. Also, $\alpha = 5 \times 10^{-7}/^\circ\text{C}$ according to Table 19-2, which we convert to $\alpha = 2.8 \times 10^{-7}/^\circ\text{F}$ for the needed computations. Now, the above equation becomes

$$\left| \frac{\Delta t}{t_{\text{correct}}} \right| = \sqrt{1 + \alpha\Delta T} - 1 \quad .$$

We can expand this with the binomial theorem (Appendix E) or compute it the “brute force” way; in any case we find $|\Delta t/t_{\text{correct}}| = 9 \times 10^{-6}$. Since the clock, as mentioned above, is “fast” we say the relative *gain* in time is 9×10^{-6} . *Note:* a more elegant approach to this problem in terms of differentials is as follows (with k some constant of proportionality).

$$\begin{aligned} t_{\text{fork}} &= k\sqrt{L} & k &= \text{constant} \\ dt_{\text{fork}} &= \frac{1}{2}kL^{-1/2} dL \\ dt_{\text{fork}} &= \frac{\alpha}{2}t_{\text{fork}} dT \\ \frac{dt_{\text{fork}}}{t_{\text{fork}}} &= \frac{\alpha}{2} dT \end{aligned}$$

At this point $dT \rightarrow \Delta T$ and the previous results are obtained.

- (b) This proceeds very similarly to part (a), but with the tuning fork longer – and thus ticking more slowly, and with $\Delta T = 95^\circ\text{F}$. The result is a relative *loss* in time of magnitude 13×10^{-6} .