

42. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2 , the portion of the xz plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\begin{aligned}\Phi_B(S_1) &= \Phi_B(S_2) = \int_{-r}^r B(x)L \, dx \\ &= 2 \int_{-r}^r B_{\text{left}}(x)L \, dx \\ &= 2 \int_{-r}^r \frac{\mu_0 i}{2\pi} \frac{1}{2r-x} L \, dx = \frac{\mu_0 i L}{\pi} \ln 3 .\end{aligned}$$