15. We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The $y$ component of the velocity is given by $v = v_{0\,y} - gt$ and this is zero at time $t = v_{0\,y}/g = (v_0/g)\sin\theta_0$, where $v_0$ is the initial speed and $\theta_0$ is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\,\mathrm{m/s})^2}{9.8\,\mathrm{m/s}^2}\sin 60^\circ\cos 60^\circ = 17.7\ \mathrm{m}$$

and

$$y = v_{0\,y}\,t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\,\mathrm{m/s})^2}{9.8\,\mathrm{m/s}^2}\sin^2 60^\circ = 15.3\ \mathrm{m}\ .$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive $x$ direction. Let $M$ be the mass of the shell and let $V_0$ be the velocity of the fragment. Then $Mv_0\cos\theta_0 = MV_0/2$, since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\,\mathrm{m/s})\cos 60^\circ = 20\ \mathrm{m/s}\ .$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of $20\,\mathrm{m/s}$ from a location having coordinates $x_0 = 17.7\,\mathrm{m}$, $y_0 = 15.3\,\mathrm{m}$. Its $y$ coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the $x$ coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\,\mathrm{m} + (20\,\mathrm{m/s})\sqrt{\frac{2(15.3\,\mathrm{m})}{9.8\,\mathrm{m/s}^2}} = 53\ \mathrm{m}\ .$$