

46. The  $q$  in the denominator is to be interpreted as  $|q|$  (so that the orbital radius  $r$  is a positive number). We interpret the given 10.0 MeV to be the kinetic energy of the electron. In order to make use of the  $mc^2$  value for the electron given in Table 38-3 (511 keV = 0.511 MeV) we write the classical kinetic energy formula as

$$K_{\text{classical}} = \frac{1}{2}mv^2 = \frac{1}{2}(mc^2) \left( \frac{v^2}{c^2} \right) = \frac{1}{2}(mc^2) \beta^2 .$$

- (a) If  $K_{\text{classical}} = 10.0 \text{ MeV}$ , then

$$\beta = \sqrt{\frac{2K_{\text{classical}}}{mc^2}} = \sqrt{\frac{2(10.0 \text{ MeV})}{0.511 \text{ MeV}}} = 6.256 ,$$

which, of course, is impossible (see the Ultimate Speed subsection of §38-2). If we use this value anyway, then the classical orbital radius formula yields

$$\begin{aligned} r &= \frac{mv}{|q|B} = \frac{m\beta c}{eB} \\ &= \frac{(9.11 \times 10^{-31} \text{ kg})(6.256)(2.998 \times 10^8 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(2.20 \text{ T})} \\ &= 4.85 \times 10^{-3} \text{ m} . \end{aligned}$$

If, however, we use the correct value for  $\beta$  (calculated in the next part) then the classical radius formula would give about 0.77 mm.

- (b) Before using the relativistically correct orbital radius formula, we must compute  $\beta$  in a relativistically correct way:

$$K = mc^2(\gamma - 1) \implies \gamma = \frac{10.0 \text{ MeV}}{0.511 \text{ MeV}} + 1 = 20.57$$

which implies (from Eq. 38-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99882 .$$

Therefore,

$$\begin{aligned} r &= \frac{\gamma mv}{|q|B} = \frac{\gamma m\beta c}{eB} \\ &= \frac{(20.57)(9.11 \times 10^{-31} \text{ kg})(0.99882)(2.998 \times 10^8 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(2.20 \text{ T})} \\ &= 1.59 \times 10^{-2} \text{ m} . \end{aligned}$$

- (c) The period is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(0.0159 \text{ m})}{(0.99882)(2.998 \times 10^8 \text{ m/s})} = 3.34 \times 10^{-10} \text{ s} .$$

Whereas the purely classical result gives a period which is independent of speed, this is no longer true in the relativistic case (due to the  $\gamma$  factor in the equation).