

53. (a) The initial energy stored in a capacitor is given by

$$U_C = \frac{q_0^2}{2C} ,$$

where C is the capacitance and q_0 is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2(1.0 \times 10^{-6} \text{ F})(0.50 \text{ J})} = 1.0 \times 10^{-3} \text{ C} .$$

- (b) The charge as a function of time is given by $q = q_0 e^{-t/\tau}$, where τ is the capacitive time constant. The current is the derivative of the charge

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau} ,$$

and the initial current is $i_0 = q_0/\tau$. The time constant is $\tau = RC = (1.0 \times 10^{-6} \text{ F})(1.0 \times 10^6 \Omega) = 1.0 \text{ s}$. Thus $i_0 = (1.0 \times 10^{-3} \text{ C})/(1.0 \text{ s}) = 1.0 \times 10^{-3} \text{ A}$.

- (c) We substitute $q = q_0 e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C} e^{-t/\tau} = \left(\frac{1.0 \times 10^{-3} \text{ C}}{1.0 \times 10^{-6} \text{ F}} \right) e^{-t/1.0 \text{ s}} = (1.0 \times 10^3 \text{ V}) e^{-1.0t} ,$$

where t is measured in seconds. We substitute $i = (q_0/\tau) e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0 R}{\tau} e^{-t/\tau} = \frac{(1.0 \times 10^{-3} \text{ C})(1.0 \times 10^6 \Omega)}{1.0 \text{ s}} e^{-t/1.0 \text{ s}} = (1.0 \times 10^3 \text{ V}) e^{-1.0t} ,$$

where t is measured in seconds.

- (d) We substitute $i = (q_0/\tau) e^{-t/\tau}$ into $P = i^2 R$ to obtain

$$P = \frac{q_0^2 R}{\tau^2} e^{-2t/\tau} = \frac{(1.0 \times 10^{-3} \text{ C})^2 (1.0 \times 10^6 \Omega)}{(1.0 \text{ s})^2} e^{-2t/1.0 \text{ s}} = (1.0 \text{ W}) e^{-2.0t} ,$$

where t is again measured in seconds.