21. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v(t)$ and $a(t)$ have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system. Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

(b) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(3.0\,\mathrm{m/s^2})}{3(2.0\,\mathrm{m/s^3})} = 1.0\,\mathrm{s}\,.$$

For $t = 0$, $x = x_0 = 0$ and for $t = 1.0\,\mathrm{s}, x = 1.0\,\mathrm{m} > x_0$. Since we seek the maximum, we reject the first root ($t = 0$) and accept the second ($t = 1\,\mathrm{s}$).

(c) In the first 4 s the particle moves from the origin to $x = 1.0\,\mathrm{m}$, turns around, and goes back to

$$x(4\,\mathrm{s}) = (3.0\,\mathrm{m/s^2})(4.0\,\mathrm{s})^2 - (2.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^3 = -80\,\mathrm{m}\ .$$

The total path length it travels is $1.0\,\mathrm{m} + 1.0\,\mathrm{m} + 80\,\mathrm{m} = 82\,\mathrm{m}$.

(d) Its displacement is given by $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80\,\mathrm{m}$. Thus, $\Delta x = -80\,\mathrm{m}$.

(e) The velocity is given by $v = 2ct - 3bt^2 = (6.0\,\mathrm{m/s^2})t - (6.0\,\mathrm{m/s^3})t^2$. Thus

$$\begin{aligned}
v(1\,\mathrm{s}) &= (6.0\,\mathrm{m/s^2})(1.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(1.0\,\mathrm{s})^2 = 0 \\
v(2\,\mathrm{s}) &= (6.0\,\mathrm{m/s^2})(2.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(2.0\,\mathrm{s})^2 = -12\,\mathrm{m/s} \\
v(3\,\mathrm{s}) &= (6.0\,\mathrm{m/s^2})(3.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(3.0\,\mathrm{s})^2 = -36.0\,\mathrm{m/s} \\
v(4\,\mathrm{s}) &= (6.0\,\mathrm{m/s^2})(4.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^2 = -72\,\mathrm{m/s}\ .
\end{aligned}$$

(f) The acceleration is given by $a = dv/dt = 2c - 6b = 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})t$. Thus

$$\begin{aligned}
a(1\,\mathrm{s}) &= 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(1.0\,\mathrm{s}) = -6.0\,\mathrm{m/s^2} \\
a(2\,\mathrm{s}) &= 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(2.0\,\mathrm{s}) = -18\,\mathrm{m/s^2} \\
a(3\,\mathrm{s}) &= 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(3.0\,\mathrm{s}) = -30\,\mathrm{m/s^2} \\
a(4\,\mathrm{s}) &= 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(4.0\,\mathrm{s}) = -42\,\mathrm{m/s^2}\ .
\end{aligned}$$