23. (a) Let

$$x_1 = \frac{A}{2} \cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2} \cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A, the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period. We want the coordinates of the particles 0.50 s later; that is, at $t = 0.50\,\mathrm{s}$,

$$x_1 = \frac{A}{2} \cos\left(\frac{2\pi \times 0.50\,\mathrm{s}}{1.5\,\mathrm{s}}\right) = -0.250A$$

and

$$x_2 = \frac{A}{2} \cos\left(\frac{2\pi \times 0.50\,\mathrm{s}}{1.5\,\mathrm{s}} + \frac{\pi}{6}\right) = -0.433A\ .$$

Their separation at that time is $x_1 - x_2 = -0.250A + 0.433A = 0.183A$.

(b) The velocities of the particles are given by

$$v_1 = \frac{\mathrm{d}x_1}{\mathrm{d}t} = \frac{\pi A}{T} \sin\left(\frac{2\pi t}{T}\right)$$

and

$$v_2 = \frac{\mathrm{d}x_2}{\mathrm{d}t} = \frac{\pi A}{T} \sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)\ .$$

We evaluate these expressions for $t = 0.50\,\mathrm{s}$ and find they are both negative-valued, indicating that the particles are moving in the same direction.