

39. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n . Then, since there are no resonant frequencies between, the higher frequency is associated with $n+1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n+1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n} .$$

The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315 \text{ Hz}}{420 \text{ Hz} - 315 \text{ Hz}} = 3 .$$

The lowest possible resonant frequency is $f = v/2L = f_1/n = (315 \text{ Hz})/3 = 105 \text{ Hz}$.

- (b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then $v = \lambda f = 2Lf = 2(0.75 \text{ m})(105 \text{ Hz}) = 158 \text{ m/s}$.