31. (a) If the object distance is x, then the image distance is $D - x$ and the thin lens equation becomes

$$\frac{1}{x} + \frac{1}{D-x} = \frac{1}{f} .$$

We multiply each term in the equation by $fx(D-x)$ and obtain $x^2 - Dx + Df = 0$. Solving for x, we find that the two object distances for which images are formed on the screen are

$$x_1 = \frac{D - \sqrt{D(D-4f)}}{2} \quad \text{and} \quad x_2 = \frac{D + \sqrt{D(D-4f)}}{2} .$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)} .$$

(b) The ratio of the image sizes is the same as the ratio of the lateral magnifications. If the object is at $p = x_1$, the magnitude of the lateral magnification is

$$|m_1| = \frac{i_1}{p_1} = \frac{D - x_1}{x_1} .$$

Now $x_1 = \frac{1}{2}(D-d)$, where $d = \sqrt{D(D-f)}$, so

$$|m_1| = \frac{D - (D-d)/2}{(D-d)/2} = \frac{D+d}{D-d} .$$

Similarly, when the object is at x_2, the magnitude of the lateral magnification is

$$|m_2| = \frac{I_2}{p_2} = \frac{D - x_2}{x_2} = \frac{D - (D+d)/2}{(D+d)/2} = \frac{D-d}{D+d} .$$

The ratio of the magnifications is

$$\frac{m_2}{m_1} = \frac{(D-d)/(D+d)}{(D+d)/(D-d)} = \left(\frac{D-d}{D+d}\right)^2 .$$