

37. The field is radially outward and takes on equal magnitude-values over the surface of any sphere centered at the atom's center. We take the Gaussian surface to be such a sphere (of radius r). If E is the magnitude of the field, then the total flux through the Gaussian sphere is $\Phi = 4\pi r^2 E$. The charge enclosed by the Gaussian surface is the positive charge at the center of the atom plus that portion of the negative charge within the surface. Since the negative charge is uniformly distributed throughout the large sphere of radius R , we can compute the charge inside the Gaussian sphere using a ratio of volumes. That is, the negative charge inside is $-Zer^3/R^3$. Thus, the total charge enclosed is $Ze - Zer^3/R^3$ for $r \leq R$. Gauss' law now leads to

$$4\pi\epsilon_0 r^2 E = Ze \left(1 - \frac{r^3}{R^3}\right) \implies E = \frac{Ze}{4\pi\epsilon_0} \left(\frac{1}{r^2} - \frac{r}{R^3}\right).$$