87. (First problem of **Cluster**)

(a) The field between the plates is uniform; we apply Eq. 25-42 to find the magnitude of the (horizontal) field: $|\vec{E}| = \Delta V/D$ (assuming $\Delta V > 0$). This produces a horizontal acceleration from Eq. 23-1 and Newton's second law (applied along the x axis):

$$a_x = \frac{|\vec{F}_x|}{m} = \frac{q|\vec{E}|}{m} = \frac{q\Delta V}{m\,D}$$

where $q > 0$ has been assumed; the problem indicates that the acceleration is rightward, which constitutes our choice for the $+x$ direction. If we choose upward as the $+y$ direction then $a_y = -g$, and we apply the free-fall equations of Chapter 2 to the y motion while applying the constant (a_x) acceleration equations of Table 2-1 to the x motion. The displacement is defined by $\Delta x = +D/2$ and $\Delta y = -d$, and the initial velocity is zero. Simultaneous solution of

$$\begin{aligned} \Delta x &= v_{0\text{x}}\,t + \frac{1}{2}\,a_x\,t^2 \qquad \text{and} \\ \Delta y &= v_{0\text{y}}\,t + \frac{1}{2}\,a_y\,t^2 \quad , \end{aligned}$$

leads to

$$d = \frac{gD}{2a_x} = \frac{gmD^2}{2q\Delta V} \; .$$

(b) We can continue along the same lines as in part (a) (using Table 2-1) to find v, or we can use energy conservation – which we feel is more instructive. The gain in kinetic energy derives from two potential energy changes: from gravity comes mgd and from electric potential energy comes $q|\vec{E}|\Delta x = q\Delta V/2$. Consequently,

$$\frac{1}{2}mv^2 = mgd + \frac{1}{2}q\Delta V$$

which (upon using the expression for d above) yields

$$v = \sqrt{\frac{mg^2D^2}{q\Delta V} + \frac{q\Delta V}{m}} \; .$$

(c) and (d) Using SI units (so $q = 1.0 \times 10^{-10}$ C, $m = 1.0 \times 10^{-9}$ kg) we plug into our results to obtain $d = 0.049$ m and $v = 1.4$ m/s.