

18. (a) In an adiabatic process  $Q = 0$ . This can be done by placing the gas in a thermally insulated container whose volume can be adjusted (say, by means of a movable piston). If the volume is *slowly* increased from  $V_i$  to  $V_x$ , then the process is reversible. To realize the reversible, constant-volume process from  $x$  to  $f$ , we would place the gas in a rigid container, which has a fixed volume  $V_f$  and is in thermal contact with a heat reservoir. If we *gradually* increase the temperature of the reservoir from  $T_x$  to  $T_f$ , the gas will undergo the desired reversible process from  $x$  to  $f$ .
- (b) For the two states  $i$  and  $x$  we have  $p_i V_i / T_i = p_x V_x / T_x$  and  $p_i V_i^\gamma = p_x V_x^\gamma$ . We eliminate  $p_i$  and  $p_x$  from these equations to obtain

$$\frac{T_x}{T_i} = \left( \frac{V_i}{V_x} \right)^{\gamma-1}.$$

For monatomic ideal gases  $\gamma = 5/3$  (see §20-8 and §20-11), so  $\gamma - 1 = 2/3$ . Also  $V_x = V_f$ . Substituting these into the equation above, we obtain  $T_x = T_i (V_i / V_f)^{2/3}$ .

- (c) For an ideal gas undergoing an isothermal process, Eq. 20-45 implies  $\Delta E_{\text{int}} = 0$ . And Eq. 20-14 gives  $W = nRT \ln(V_f / V_i)$  for such a process. Therefore, the first law of thermodynamics leads to

$$Q_{\text{path I}} = \Delta E_{\text{int I}} + W_{\text{I}} = nRT_i \ln \left( \frac{V_f}{V_i} \right).$$

And for path II, we have

$$Q_{\text{path II}} = Q_{\text{adiabat}} + Q_{\text{const vol}} = 0 + nC_V \Delta T.$$

But  $C_V = \frac{3}{2}R$  (see Eq. 20-43), so we obtain

$$Q_{\text{path II}} = \frac{3}{2}nR(T_f - T_x).$$

We see that  $Q_{\text{path I}} \neq Q_{\text{path II}}$ .

- (d) Since the first part of path II is reversibly adiabatic, then the entropy changes only during the second, constant-volume, part of the path:

$$\Delta S = \int_{T_x}^{T_f} \frac{nC_V dT}{T} = nC_V \ln \left( \frac{T_f}{T_x} \right) = \frac{3}{2}nR \ln \left( \frac{T_f}{T_x} \right).$$

Entropy is a function of “where you are” on the  $pV$  diagram, not “how you got there.” Since the beginning and ending point of path I are the same as those of path II, then  $\Delta S$  is the same for both.

- (e) Using the result in part (b) with  $V_i / V_f = \frac{1}{2}$  and  $T_i = 500$  K, we find

$$T_x = (500 \text{ K}) \left( \frac{1}{2} \right)^{2/3} = 315 \text{ K}.$$

For path I, Eq. 21-2 gives  $Q_I = (\Delta S) T$  where  $T = T_i = T_f$  and  $\Delta S$  is the expression calculated in the part (d). Thus,

$$Q_I = \frac{3}{2}nRT_f \ln \left( \frac{T_f}{T_x} \right)$$

which can be alternatively derived from Eq. 20-14 and the first law of thermodynamics. With  $n = 1$  mol,  $T_f = T_i = 500$  K, we find

$$Q_I = \frac{3}{2}(1)(8.31)(500) \ln \left( \frac{500}{315} \right) = 2880 \text{ J}.$$

For path II,  $Q = Q_{\text{constant volume}} = nC_V \Delta T$  and we obtain

$$Q_{II} = (1) \left( \frac{3}{2}(8.31) \right) (500 - 315) = 2306 \text{ J}.$$