92. (Second problem in **Cluster 2**)

As explained in the first solution in this cluster, we take both angles θ_1 and θ_2 to be positive-valued.

(a) We first examine conservation of the y components of momentum.

$$\begin{aligned} 0 &= -m_1\, v_{1f}\, \sin\theta_1 + m_2\, v_{2f}\, \sin\theta_2 \\ 0 &= -m_1 v_{1f}\, \sin 30^\circ + 2m_1\, v_{2f}\, \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1\, v_{1i} &= m_1\, v_{1f}\, \cos\theta_1 + m_2\, v_{2f}\, \cos\theta_2 \\ m_1(10.0\ \text{m/s}) &= m_1 v_{1f}\, \cos 30^\circ + 2m_1\, v_{2f}\, \cos\theta_2 \end{aligned}$$

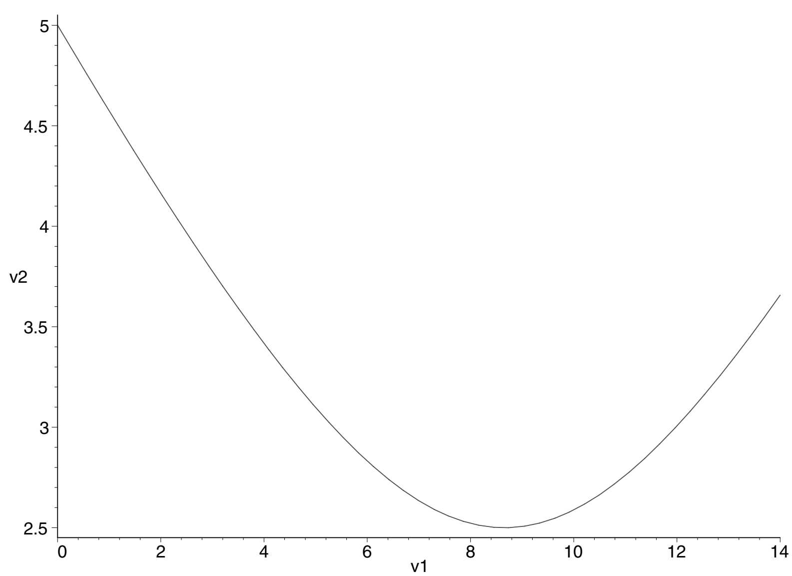
From the y equation, we obtain $v_{1f} = 4\, v_{2f}\, \sin\theta_2$; similarly, the x equation yields $20 - v_{1f}\sqrt{3} = 4v_{2f}\, \cos\theta_2$ with SI units understood (and the fact that $\cos 30^\circ = \sqrt{3}/2$ has been used). Squaring these two relations and adding them leads to

$$v_{1f}^2\,(1+3) - 40v_{1f}\sqrt{3} + 400 = 16\, v_{2f}^2\, \left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and thus to

$$v_{2f}^2 = v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25\ .$$

(b) The plot (v_{2f} versus v_{1f}) is shown below. The units for both axes are meters/second.



(c) Simply from the total kinetic energy requirement that $K_i \geq K_f$ we see immediately that $v_{1f} \leq v_{1i} = 10.0$ m/s (where the upper bound represents the trivial case where it passes m_2 by completely with $K_i = K_f$), and with the more stringent requirement that it does strike m_2 and scatters at $\theta_1 = 30^\circ$ we again find that it is bounded by the $K_i = K_f$ case. The elastic collision scenario was worked in the previous problem with the result $v_{1f} = 9.34$ m/s.

(d) And we also found the result $v_{2f} = 2.52$ m/s.

(e) As mentioned, this is an elastic collision.