104. (a) Since no torque is being applied to the system, the angular momentum is constant.

(b) The maximum ω occurs when the maximum speed v occurs (as it passes through vertical: $\theta = 0$). The angular momentum of the "particle" may be written as $mvr = mr^2\omega$ so that conservation of momentum (applied to the $\theta = 0$ position) leads to

$$mr^2\omega_{\max} = mr_0^2\omega_{0,\max} \implies \omega_{\max} = \left(\frac{r_0}{r}\right)^2 \omega_{0,\max}$$

which becomes (with $r_0 = 0.80$ m and $\omega_{0,\max} = 1.30$ rad/s) $\omega_{\max} = 0.832/r^2$ in SI units.

(c) The maximum kinetic energy occurs at this same position: $K_{\max} = \frac{1}{2}mv_{\max}^2$ which we write as

$$K_{\max} = \frac{1}{2}mr^2\omega_{\max}^2 = \frac{1}{2}mr^2\left(\left(\frac{r_0}{r}\right)^2 \omega_{0,\max}\right)^2 = \frac{mr_0^4\omega_{0,\max}^2}{2\,r^2} \;.$$

(d) We note from the previous result that $K_{\max}$ depends *inversely* on r^2, so it decreases as r increases.

(e) Measuring height h from the low point of the swing, consideration of the geometry leads to the relation $h = r(1-\cos\theta)$. The maximum height is therefore related to the maximum angle (measured from vertical) by

$$h_{\max} = r\,(1 - \cos\theta_{\max})$$

which means the maximum potential energy (which must equal the same numerical value as the maximum kinetic energy if we assume mechanical energy conservation) is

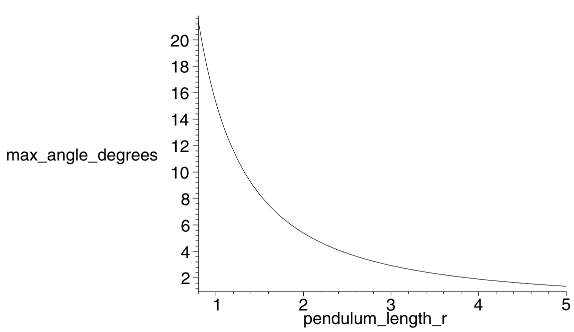
$$U_{\max} = K_{\max} = mgh_{\max} = mgr\,(1 - \cos\theta_{\max}) \;.$$

(f) Combining the results of part (c) and part (e), we obtain

$$\frac{mr_0^4\omega_{0,\max}^2}{2\,r^2} = mgr\,(1 - \cos\theta_{\max}) \implies \theta_{\max} = \cos^{-1}\left(1 - \frac{r_0^4\omega_{0,\max}^2}{2gr^3}\right)$$

which evaluates to be $\theta_{\max} = \cos^{-1}(1 - 0.0353/r^3)$ in SI units.

(g) As can be seen in the graph below, the angle of the pendulum "turning point" decreases as the pendulum lengthens (note that r is in meters).



(h) The original value of $\theta_{\max}$ is $\cos^{-1}(1 - 0.0353/r_0^3)$ where $r_0 = 0.80$ m. This gives 21.4° as the initial "turning point" angle. The question, then, asks us to solve for r in the case that $\theta_{\max} = \frac{1}{2}(21.4°) = 10.7°$. We know to look for half the initial value (as opposed to one twice as big) because the previous part shows $\theta_{\max}$ decreases with r. This value of the turning point angle occurs for

$$r = \left(\frac{0.0353}{1 - \cos 10.7°}\right)^{1/3} = 1.27 \text{ m} \;.$$