58. (a) We refer to the entry point for the original incident ray as point A (which we take to be on the left side of the prism, as in Fig. 34-49), the prism vertex as point B, and the point where the interior ray strikes the right surface of the prism as point C. The angle between line AB and the interior ray is β (the complement of the angle of refraction at the first surface), and the angle between the line BC and the interior ray is α (the complement of its angle of incidence when it strikes the second surface). When the incident ray is at the minimum angle for which light is able to exit the prism, the light exits along the second face. That is, the angle of refraction at the second face is 90°, and the angle of incidence there for the interior ray is the critical angle for total internal reflection. Let θ_1 be the angle of incidence for the original incident ray and θ_2 be the angle of refraction at the first face, and let θ_3 be the angle of incidence at the second face. The law of refraction, applied to point C, yields $n \sin \theta_3 = 1$, so $\sin \theta_3 = 1/n = 1/1.60 = 0.625$ and $\theta_3 = 38.68°$. The interior angles of the triangle ABC must sum to 180°, so $\alpha + \beta = 120°$. Now, $\alpha = 90° - \theta_3 = 51.32°$, so $\beta = 120° - 51.32° = 69.68°$. Thus, $\theta_2 = 90° - \beta = 21.32°$. The law of refraction, applied to point A, yields $\sin \theta_1 = n \sin \theta_2 = 1.60 \sin 21.32° = 0.5817$. Thus $\theta_1 = 35.6°$.

(b) We apply the law of refraction to point C. Since the angle of refraction there is the same as the angle of incidence at A, $n \sin \theta_3 = \sin \theta_1$. Now, $\alpha + \beta = 120°$, $\alpha = 90° - \theta_3$, and $\beta = 90° - \theta_2$, as before. This means $\theta_2 + \theta_3 = 60°$. Thus, the law of refraction leads to

$$\sin \theta_1 = n \sin(60° - \theta_2) \implies \sin \theta_1 = n \sin 60° \cos \theta_2 - n \cos 60° \sin \theta_2$$

where the trigonometric identity $\sin(A - B) = \sin A \cos B - \cos A \sin B$ is used. Next, we apply the law of refraction to point A:

$$\sin \theta_1 = n \sin \theta_2 \implies \sin \theta_2 = (1/n) \sin \theta_1$$

which yields $\cos \theta_2 = \sqrt{1 - \sin^2 \theta_2} = \sqrt{1 - (1/n^2) \sin^2 \theta_1}$. Thus,

$$\sin \theta_1 = n \sin 60° \sqrt{1 - (1/n)^2 \sin^2 \theta_1} - \cos 60° \sin \theta_1$$

or

$$(1 + \cos 60°) \sin \theta_1 = \sin 60° \sqrt{n^2 - \sin^2 \theta_1} \, .$$

Squaring both sides and solving for $\sin \theta_1$, we obtain

$$\sin \theta_1 = \frac{n \sin 60°}{\sqrt{(1 + \cos 60°)^2 + \sin^2 60°}} = \frac{1.60 \sin 60°}{\sqrt{(1 + \cos 60°)^2 + \sin^2 60°}} = 0.80$$

and $\theta_1 = 53.1°$.