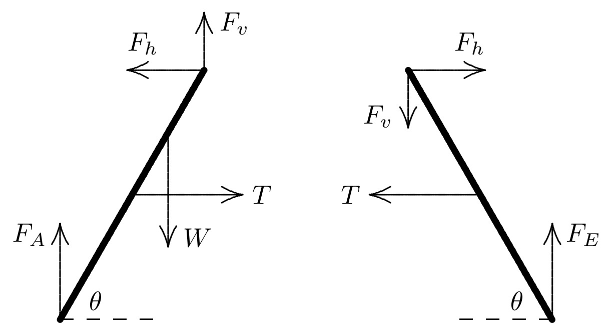
31. The diagrams to the right show the forces on the two sides of the ladder, separated. F_A and F_E are the forces of the floor on the two feet, T is the tension force of the tie rod, W is the force of the man (equal to his weight), F_h is the horizontal component of the force exerted by one side of the ladder on the other, and F_v is the vertical component of that force. Note that the forces exerted by the floor are normal to the floor since the floor is frictionless. Also note that the force of the left side on the right and the force of the right side on the left are equal in magnitude and opposite in direction.



Since the ladder is in equilibrium, the vertical components of the forces on the left side of the ladder must sum to zero: $F_v + F_A - W = 0$. The horizontal components must sum to zero: $T - F_h = 0$. The torques must also sum to zero. We take the origin to be at the hinge and let L be the length of a ladder side. Then $F_A L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0$. Here we recognize that the man is one-fourth the length of the ladder side from the top and the tie rod is at the midpoint of the side.

The analogous equations for the right side are $F_E - F_v = 0$, $F_h - T = 0$, and $F_E L\cos\theta - T(L/2)\sin\theta = 0$.

There are 5 different equations:

$$\begin{gathered} F_v + F_A - W = 0\,, \\ T - F_h = 0 \\ F_A L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0 \\ F_E - F_v = 0 \\ F_E L\cos\theta - T(L/2)\sin\theta = 0\ . \end{gathered}$$

The unknown quantities are F_A, F_E, F_v, F_h, and T.

(a) First we solve for T by systematically eliminating the other unknowns. The first equation gives $F_A = W - F_v$ and the fourth gives $F_v = F_E$. We use these to substitute into the remaining three equations to obtain

$$\begin{gathered} T - F_h = 0 \\ WL\cos\theta - F_E L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0 \\ F_E L\cos\theta - T(L/2)\sin\theta = 0\ . \end{gathered}$$

The last of these gives $F_E = T\sin\theta/2\cos\theta = (T/2)\tan\theta$. We substitute this expression into the second equation and solve for T. The result is

$$T = \frac{3W}{4\tan\theta}\ .$$

To find $\tan\theta$, we consider the right triangle formed by the upper half of one side of the ladder, half the tie rod, and the vertical line from the hinge to the tie rod. The lower side of the triangle has a length of 0.381 m, the hypotenuse has a length of 1.22 m, and the vertical side has a length of $\sqrt{(1.22\,\mathrm{m})^2 - (0.381\,\mathrm{m})^2} = 1.16\,\mathrm{m}$. This means $\tan\theta = (1.16\,\mathrm{m})/(0.381\,\mathrm{m}) = 3.04$. Thus,

$$T = \frac{3(854\,\mathrm{N})}{4(3.04)} = 211\ \mathrm{N}\ .$$

(b) We now solve for F_A. Since $F_v = F_E$ and $F_E = T\sin\theta/2\cos\theta$, $F_v = 3W/8$. We substitute this into $F_v + F_A - W = 0$ and solve for F_A. We find

$$F_A = W - F_v = W - 3W/8 = 5W/8 = 5(884\,\mathrm{N})/8 = 534\ \mathrm{N}\ .$$

(c) We have already obtained an expression for F_E: $F_E = 3W/8$. Evaluating it, we get $F_E = 320$ N.