

79. (a) The energy density at any point is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field. The magnitude of the field inside a toroid, a distance r from the center, is given by Eq. 30-26: $B = \mu_0 i N / 2\pi r$, where N is the number of turns and i is the current. Thus

$$u_B = \frac{1}{2\mu_0} \left(\frac{\mu_0 i N}{2\pi r} \right)^2 = \frac{\mu_0 i^2 N^2}{8\pi^2 r^2} .$$

- (b) We evaluate the integral $U_B = \int u_B dV$ over the volume of the toroid. A circular strip with radius r , height h , and thickness dr has volume $dV = 2\pi r h dr$, so

$$U_B = \frac{\mu_0 i^2 N^2}{8\pi^2} 2\pi h \int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 N^2 h}{4\pi} \ln \left(\frac{b}{a} \right) .$$

Substituting in the given values, we find

$$\begin{aligned} U_B &= \frac{(4\pi \times 10^{-7} \text{T}\cdot\text{m/A})(0.500 \text{A})^2(1250)^2(13 \times 10^{-3} \text{m})}{4\pi} \ln \left(\frac{95 \text{mm}}{52 \text{mm}} \right) \\ &= 3.06 \times 10^{-4} \text{ J} . \end{aligned}$$

- (c) The inductance is given in Sample Problem 31-11:

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln \left(\frac{b}{a} \right)$$

so the energy is given by

$$U_B = \frac{1}{2} L i^2 = \frac{\mu_0 N^2 i^2 h}{4\pi} \ln \left(\frac{b}{a} \right) .$$

This is exactly the same as the expression found in part (b) and yields the same numerical result.