45. (a) For a given amplitude $(E)_m$ of the generator emf, the current amplitude is given by

$$I = \frac{(E)_m}{Z} = \frac{(E)_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} .$$

We find the maximum by setting the derivative with respect to ω_d equal to zero:

$$\frac{dI}{d\omega_d} = -(E)_m \left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]^{-3/2} \left[\omega_d L - \frac{1}{\omega_d C}\right] \left[L + \frac{1}{\omega_d^2 C}\right] .$$

The only factor that can equal zero is $\omega_d L - (1/\omega_d C)$; it does so for $\omega_d = 1/\sqrt{LC} = \omega$. For this circuit,

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}} = 224 \text{ rad/s} .$$

(b) When $\omega_d = \omega$, the impedance is $Z = R$, and the current amplitude is

$$I = \frac{(E)_m}{R} = \frac{30.0\,\mathrm{V}}{5.00\,\Omega} = 6.00 \text{ A} .$$

(c) We want to find the (positive) values of ω_d for which $I = \frac{(E)_m}{2R}$:

$$\frac{(E)_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{(E)_m}{2R} .$$

This may be rearranged to yield

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2 .$$

Taking the square root of both sides (acknowledging the two $\pm$ roots) and multiplying by $\omega_d C$, we obtain

$$\omega_d^2 (LC) \pm \omega_d \left(\sqrt{3} CR\right) - 1 = 0 .$$

Using the quadratic formula, we find the smallest positive solution

$$\begin{aligned}
\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{-\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 219 \text{ rad/s} ,
\end{aligned}$$

and the largest positive solution

$$\begin{aligned}
\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{+\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 228 \text{ rad/s} .
\end{aligned}$$

(d) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega_0} = \frac{228\,\mathrm{rad/s} - 219\,\mathrm{rad/s}}{224\,\mathrm{rad/s}} = 0.04 .$$