

13. If the lead sphere were not hollowed the magnitude of the force it exerts on m would be $F_1 = GMm/d^2$. Part of this force is due to material that is removed. We calculate the force exerted on m by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere. The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere. That is $M_c/r^3 = M/R^3$, where M_c is the mass that fills the cavity. The common factor $4\pi/3$ has been canceled. Thus,

$$M_c = \left(\frac{r^3}{R^3}\right) M = \left(\frac{R^3}{8R^3}\right) M = \frac{M}{8} .$$

The center of the cavity is $d - r = d - R/2$ from m , so the force it exerts on m is

$$F_2 = \frac{G(M/8)m}{(d - R/2)^2} .$$

The force of the hollowed sphere on m is

$$F = F_1 - F_2 = GMm \left(\frac{1}{d^2} - \frac{1}{8(d - R/2)^2} \right) = \frac{GMm}{d^2} \left(1 - \frac{1}{8(1 - R/2d)^2} \right) .$$