65. We can find the age t of the rock from the masses of ^{238}U and ^{206}Pb. The initial mass of ^{238}U is

$$m_{\mathrm{U}_0} = m_{\mathrm{U}} + \frac{238}{206} m_{\mathrm{Pb}} \ .$$

Therefore, $m_{\mathrm{U}} = m_{\mathrm{U}_0} e^{-\lambda_{\mathrm{U}} t} = (m_{\mathrm{U}} + m_{^{238}\mathrm{Pb}}/206) e^{-(t \ln 2)/T_{1/2\mathrm{U}}}$. We solve for t:

$$\begin{aligned} t &= \frac{T_{1/2\mathrm{U}}}{\ln 2} \ln\left(\frac{m_{\mathrm{U}} + (238/206) m_{\mathrm{Pb}}}{m_{\mathrm{U}}} \right) \\ &= \frac{4.47 \times 10^9 \text{ y}}{\ln 2} \ln\left[1 + \left(\frac{238}{206} \right) \left(\frac{0.15 \text{ mg}}{0.86 \text{ mg}} \right) \right] \\ &= 1.18 \times 10^9 \text{ y} \ . \end{aligned}$$

For the β decay of ^{40}K, the initial mass of ^{40}K is

$$m_{\mathrm{K}_0} = m_{\mathrm{K}} + (40/40) m_{\mathrm{Ar}} = m_{\mathrm{K}} + m_{\mathrm{Ar}} \ ,$$

so

$$m_{\mathrm{K}} = m_{\mathrm{K}_0} e^{-\lambda_{\mathrm{K}} t} = (m_{\mathrm{K}} + m_{\mathrm{Ar}}) e^{-\lambda_{\mathrm{K}} t} \ .$$

We solve for m_{K}:

$$\begin{aligned} m_{\mathrm{K}} &= \frac{m_{\mathrm{Ar}} e^{-\lambda_{\mathrm{K}} t}}{1 - e^{-\lambda_{\mathrm{K}} t}} = \frac{m_{\mathrm{Ar}}}{e^{\lambda_{\mathrm{K}} t} - 1} \\ &= \frac{1.6 \text{ mg}}{e^{(\ln 2)(1.18 \times 10^9 \text{ y})/(1.25 \times 10^9 \text{ y})} - 1} = 1.7 \text{ mg} \ . \end{aligned}$$