

9. (a) We use Gauss' law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside. Outside the charge distribution the magnitude of the field is  $E = q/4\pi\epsilon_0 r^2$  and the potential is  $V = q/4\pi\epsilon_0 r$ , where  $r$  is the distance from the center of the distribution. This is the same as the field and potential of a point charge at the center of the spherical distribution. To find an expression for the magnitude of the field inside the charge distribution, we use a Gaussian surface in the form of a sphere with radius  $r$ , concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is  $4\pi r^2 E$ . The charge enclosed is  $qr^3/R^3$ . Gauss' law becomes

$$4\pi\epsilon_0 r^2 E = \frac{qr^3}{R^3} ,$$

so

$$E = \frac{qr}{4\pi\epsilon_0 R^3} .$$

If  $V_s$  is the potential at the surface of the distribution ( $r = R$ ) then the potential at a point inside, a distance  $r$  from the center, is

$$V = V_s - \int_R^r E dr = V_s - \frac{q}{4\pi\epsilon_0 R^3} \int_R^r r dr = V_s - \frac{qr^2}{8\pi\epsilon_0 R^3} + \frac{q}{8\pi\epsilon_0 R} .$$

The potential at the surface can be found by replacing  $r$  with  $R$  in the expression for the potential at points outside the distribution. It is  $V_s = q/4\pi\epsilon_0 R$ . Thus,

$$V = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R} \right] = \frac{q}{8\pi\epsilon_0 R^3} (3R^2 - r^2) .$$

- (b) In problem 8 the electric potential was taken to be zero at the center of the sphere. In this problem it is zero at infinity. According to the expression derived in part (a) the potential at the center of the sphere is  $V_c = 3q/8\pi\epsilon_0 R$ . Thus  $V - V_c = -qr^2/8\pi\epsilon_0 R^3$ . This is the result of problem 8.
- (c) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\epsilon_0 R} - \frac{3q}{8\pi\epsilon_0 R} = -\frac{q}{8\pi\epsilon_0 R} .$$

The expression obtained in problem 8 would give this same value.

- (d) Only potential differences have physical significance, not the value of the potential at any particular point. The same value can be added to the potential at every point without changing the electric field, for example. Changing the reference point from the center of the distribution to infinity changes the value of the potential at every point but it does not change any potential differences.