

45. (a) We use conservation of mechanical energy to find the speed of either ball after it has fallen a distance h . The initial kinetic energy is zero, the initial gravitational potential energy is Mgh , the final kinetic energy is $\frac{1}{2}Mv^2$, and the final potential energy is zero. Thus $Mgh = \frac{1}{2}Mv^2$ and $v = \sqrt{2gh}$. The collision of the ball of M with the floor is an elastic collision of a light object with a stationary massive object. The velocity of the light object reverses direction without change in magnitude. After the collision, the ball is traveling upward with a speed of $\sqrt{2gh}$. The ball of mass m is traveling downward with the same speed. We use Eq. 10-38 to find an expression for the velocity of the ball of mass M after the collision:

$$\begin{aligned} v_{Mf} &= \frac{M-m}{M+m} v_{Mi} + \frac{2m}{M+m} v_{mi} \\ &= \frac{M-m}{M+m} \sqrt{2gh} - \frac{2m}{M+m} \sqrt{2gh} \\ &= \frac{M-3m}{M+m} \sqrt{2gh} . \end{aligned}$$

For this to be zero, $M = 3m$.

- (b) We use the same equation to find the velocity of the ball of mass m after the collision:

$$v_{mf} = -\frac{m-M}{M+m} \sqrt{2gh} + \frac{2M}{M+m} \sqrt{2gh} = \frac{3M-m}{M+m} \sqrt{2gh}$$

which becomes (upon substituting $M = 3m$) $v_{mf} = 2\sqrt{2gh}$. We next use conservation of mechanical energy to find the height h' to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is mgh' . Thus

$$\frac{1}{2}mv_{mf}^2 = mgh' \implies h' = \frac{v_{mf}^2}{2g} = 4h$$

where $2\sqrt{2gh}$ is substituted for v_{mf} .