52. We denote the cockroach with subscript 1 and the disk with subscript 2.

(a) Initially the angular momentum of the system consisting of the cockroach and the disk is

$$L_i = m_1 v_{1i} r_{1i} + I_2 \omega_{2i} = m_1 \omega_0 R^2 + \frac{1}{2} m_2 \omega_0 R^2 \ .$$

After the cockroach has completed its walk, its position (relative to the axis) is $r_{1f} = R/2$ so the final angular momentum of the system is

$$L_f = m_1 \omega_f \left(\frac{R}{2}\right)^2 + \frac{1}{2} m_2 \omega_f R^2 \ .$$

Then from $L_f = L_i$ we obtain

$$\omega_f \left(\frac{1}{4} m_1 R^2 + \frac{1}{2} m_2 R\right) = \omega_0 \left(m_1 R^2 + \frac{1}{2} m_2 R^2\right) \ .$$

Thus,

$$\begin{aligned} \omega_f - \omega_0 &= \omega_0 \left(\frac{m_1 R^2 + m_2 R^2/2}{m_1 R^2/4 + m_2 R^2/2}\right) - \omega_0 \\ &= \omega_0 \left(\frac{m + 10m/2}{m/4 + 10m/2} - 1\right) \\ &= \omega_0 (1.14 - 1) \end{aligned}$$

which yields $\Delta\omega = 0.14\omega_0$. For later use, we note that $\omega_f/\omega_i = 1.14$.

(b) We substitute $I = L/\omega$ into $K = \frac{1}{2} I \omega^2$ and obtain $K = \frac{1}{2} L\omega$. Since we have $L_i = L_f$, the the kinetic energy ratio becomes

$$\frac{K}{K_0} = \frac{\frac{1}{2} L_f \omega_f}{\frac{1}{2} L_i \omega_i} = \frac{\omega_f}{\omega_i} = 1.14 \ .$$

(c) The cockroach does positive work while walking toward the center of the disk, increasing the total kinetic energy of the system.