67. We may approximate the planets and their motions as particles in circular orbits, and use Eq. 12-26

$$L = \sum_{i=1}^{9} \ell_i = \sum_{i=1}^{9} m_i r_i^2 \omega_i$$

to compute the total angular momentum. Since we assume the angular speed of each one is constant, we have (in rad/s) $\omega_i = 2\pi/T_i$ where T_i is the time for that planet to go around the Sun (this and related information is found in Appendix C but there, the T_i are expressed in years and we'll need to convert with 3.156×10^7 s/y, and the M_i are expressed as multiples of M_{earth} which we'll convert by multiplying by 5.98×10^{24} kg).

(a) Using SI units, we find (with $i = 1$ designating Mercury)

$$\begin{aligned} L &= \sum_{i=1}^{9} m_i r_i^2 \left(\frac{2\pi}{T_i}\right) \\ &= 2\pi \frac{3.34 \times 10^{23}}{7.61 \times 10^6} \left(57.9 \times 10^9\right)^2 + 2\pi \frac{4.87 \times 10^{24}}{19.4 \times 10^7} \left(108 \times 10^9\right)^2 + \\ &\quad 2\pi \frac{5.98 \times 10^{24}}{3.156 \times 10^7} \left(150 \times 10^9\right)^2 + 2\pi \frac{6.40 \times 10^{23}}{5.93 \times 10^7} \left(228 \times 10^9\right)^2 + \\ &\quad 2\pi \frac{1.9 \times 10^{27}}{3.76 \times 10^8} \left(778 \times 10^9\right)^2 + 2\pi \frac{5.69 \times 10^{26}}{9.31 \times 10^8} \left(1430 \times 10^9\right)^2 + \\ &\quad 2\pi \frac{8.67 \times 10^{25}}{2.65 \times 10^9} \left(2870 \times 10^9\right)^2 + 2\pi \frac{1.03 \times 10^{26}}{5.21 \times 10^9} \left(4500 \times 10^9\right)^2 + \\ &\quad 2\pi \frac{1.2 \times 10^{22}}{7.83 \times 10^9} \left(5900 \times 10^9\right)^2 \\ &= 3.14 \times 10^{43}\ \text{kg}\cdot\text{m}^2/\text{s}\ . \end{aligned}$$

(b) The fractional contribution of Jupiter is

$$\frac{\ell_5}{L} = \frac{2\pi \frac{1.9 \times 10^{27}}{3.76 \times 10^8} \left(778 \times 10^9\right)^2}{3.14 \times 10^{43}} = 0.61\ .$$