

41. (a) We integrate the volume charge density over the volume and require the result be equal to the total charge:

$$\int dx \int dy \int dz \rho = 4\pi \int_0^R dr r^2 \rho = Q .$$

Substituting the expression $\rho = \rho_s r/R$ and performing the integration leads to

$$4\pi \left(\frac{\rho_s}{R} \right) \left(\frac{R^4}{4} \right) = Q \implies Q = \pi \rho_s R^3 .$$

- (b) At a certain point within the sphere, at some distance r_o from the center, the field (see Eq. 24-8 through Eq. 24-10) is given by Gauss' law:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_{\text{enc}}}{r_o^2}$$

where q_{enc} is given by an integral similar to that worked in part (a):

$$q_{\text{enc}} = 4\pi \int_0^{r_o} dr r^2 \rho = 4\pi \left(\frac{\rho_s}{R} \right) \left(\frac{r_o^4}{4} \right) .$$

Therefore,

$$E = \frac{1}{4\pi\epsilon_0} \frac{\pi \rho_s r_o^4}{R r_o^2}$$

which (using the relation between ρ_s and Q derived in part (a)) becomes

$$E = \frac{1}{4\pi\epsilon_0} \frac{\pi \left(\frac{Q}{\pi R^3} \right) r_o^2}{R}$$

and simplifies to the desired result (shown in the problem statement) if we change notation $r_o \rightarrow r$.