33. We assume no external forces act on the system composed of the two parts of the last stage. Hence, the total momentum of the system is conserved. Let m_c be the mass of the rocket case and m_p be the mass of the payload. At first they are traveling together with velocity v. After the clamp is released m_c has velocity v_c and m_p has velocity v_p. Conservation of momentum yields $(m_c + m_p)v = m_c v_c + m_p v_p$.

(a) After the clamp is released the payload, having the lesser mass, will be traveling at the greater speed. We write $v_p = v_c + v_{\text{rel}}$, where v_{rel} is the relative velocity. When this expression is substituted into the conservation of momentum condition, the result is

$$(m_c + m_p)\, v = m_c v_c + m_p v_c + m_p v_{\text{rel}} \ .$$

Therefore,

$$\begin{aligned} v_c &= \frac{(m_c + m_p)\, v - m_p v_{\text{rel}}}{m_c + m_p} \\ &= \frac{(290.0\,\text{kg} + 150.0\,\text{kg})(7600\,\text{m/s}) - (150.0\,\text{kg})(910.0\,\text{m/s})}{290.0\,\text{kg} + 150.0\,\text{kg}} \\ &= 7290 \text{ m/s} \ . \end{aligned}$$

(b) The final speed of the payload is $v_p = v_c + v_{\text{rel}} = 7290\,\text{m/s} + 910.0\,\text{m/s} = 8200\,\text{m/s}$.

(c) The total kinetic energy before the clamp is released is

$$K_i = \frac{1}{2}\,(m_c + m_p)v^2 = \frac{1}{2}(290.0\,\text{kg} + 150.0\,\text{kg})(7600\,\text{m/s})^2 = 1.271 \times 10^{10} \text{ J} \ .$$

(d) The total kinetic energy after the clamp is released is

$$\begin{aligned} K_f &= \frac{1}{2}\, m_c v_c^2 + \frac{1}{2}\, m_p v_p^2 \\ &= \frac{1}{2}(290.0\,\text{kg})(7290\,\text{m/s})^2 + \frac{1}{2}(150.0\,\text{kg})(8200\,\text{m/s})^2 \\ &= 1.275 \times 10^{10} \text{ J} \ . \end{aligned}$$

The total kinetic energy increased slightly. Energy originally stored in the spring is converted to kinetic energy of the rocket parts.