45. (a) We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 19-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. There are three possibilities:

- None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.
- The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.
- All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

First, we suppose that no ice melts. The temperature of the water decreases from $T_{Wi} = 25°\mathrm{C}$ to some final temperature T_f and the temperature of the ice increases from $T_{Ii} = -15°\mathrm{C}$ to T_f. If m_W is the mass of the water and c_W is its specific heat then the water rejects heat

$$|Q| = c_W m_W \left(T_{Wi} - T_f\right) \ .$$

If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{Ii}) \ .$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_W m_W \left(T_{Wi} - T_f\right) = c_I m_I \left(T_f - T_{Ii}\right) \ .$$

The solution for the equilibrium temperature is

$$\begin{aligned} T_f &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{c_W m_W + c_I m_I} \\ &= \frac{(4190\,\mathrm{J/kg\cdot K})(0.200\,\mathrm{kg})(25°\mathrm{C}) + (2220\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})(-15°\mathrm{C})}{(4190\,\mathrm{J/kg\cdot K})(0.200\,\mathrm{kg}) + (2220\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})} \\ &= 16.6°\mathrm{C} \ . \end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at $T_f = 0°\mathrm{C}$, with mass m ($< m_I$) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_W m_W T_{Wi} \ ,$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{Ii}) + m L_F \ ,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + m L_F \ .$$

This equation can be solved for the mass m of ice melted:

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F} \\ &= \frac{(4190\,\mathrm{J/kg\cdot K})(0.200\,\mathrm{kg})(25°\mathrm{C}) + (2220\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})(-15°\mathrm{C})}{333 \times 10^3\,\mathrm{J/kg}} \\ &= 5.3 \times 10^{-2}\,\mathrm{kg} = 53\ \mathrm{g} \ . \end{aligned}$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C. This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.