

56. The current in the circuit satisfies $i(t) = I \sin(\omega_d t - \phi)$, where

$$\begin{aligned} I &= \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \\ &= \frac{45.0 \text{ V}}{\sqrt{(16.0 \Omega)^2 + \{(3000 \text{ rad/s})(9.20 \text{ mH}) - 1/[(3000 \text{ rad/s})(31.2 \mu\text{F})]\}^2}} \\ &= 1.93 \text{ A} \end{aligned}$$

and

$$\begin{aligned} \phi &= \tan^{-1} \left(\frac{X_L - X_C}{R} \right) = \tan^{-1} \left(\frac{\omega_d L - 1/\omega_d C}{R} \right) \\ &= \tan^{-1} \left[\frac{(3000 \text{ rad/s})(9.20 \text{ mH})}{16.0 \Omega} - \frac{1}{(3000 \text{ rad/s})(31.2 \mu\text{F})} \right] \\ &= 46.5^\circ . \end{aligned}$$

(a) The power supplied by the generator is

$$\begin{aligned} P_g &= i(t)\mathcal{E}(t) = I \sin(\omega_d t - \phi) \mathcal{E}_m \sin \omega_d t \\ &= (1.93 \text{ A})(45.0 \text{ V}) \sin[(3000 \text{ rad/s})(0.442 \text{ ms})] \sin[(3000 \text{ rad/s})(0.442 \text{ ms}) - 46.5^\circ] \\ &= 41.4 \text{ W} . \end{aligned}$$

(b) The rate at which the energy in the capacitor changes is

$$\begin{aligned} P_c &= -\frac{d}{dt} \left(\frac{q^2}{2C} \right) = -i \frac{q}{C} = -iV_c \\ &= -I \sin(\omega_d t - \phi) \left(\frac{I}{\omega_d C} \right) \cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C} \sin[2(\omega_d t - \phi)] \\ &= -\frac{(1.93 \text{ A})^2}{2(3000 \text{ rad/s})(31.2 \times 10^{-6} \text{ F})} \sin[2(3000 \text{ rad/s})(0.442 \text{ ms}) - 2(46.5^\circ)] \\ &= -17.0 \text{ W} . \end{aligned}$$

(c) The rate at which the energy in the inductor changes is

$$\begin{aligned} P_i &= \frac{d}{dt} \left(\frac{1}{2} L i^2 \right) = L i \frac{di}{dt} = L I \sin(\omega_d t - \phi) \frac{d}{dt} [I \sin(\omega_d t - \phi)] \\ &= \frac{1}{2} \omega_d L I^2 \sin[2(\omega_d t - \phi)] \\ &= \frac{1}{2} (3000 \text{ rad/s})(1.93 \text{ A})^2 (9.20 \text{ mH}) \sin[2(3000 \text{ rad/s})(0.442 \text{ ms}) - 2(46.5^\circ)] \\ &= 44.1 \text{ W} . \end{aligned}$$

(d) The rate at which energy is being dissipated by the resistor is

$$\begin{aligned} P_r &= i^2 R = I^2 R \sin^2(\omega_d t - \phi) \\ &= (1.93 \text{ A})^2 (16.0 \Omega) \sin^2[(3000 \text{ rad/s})(0.442 \text{ ms}) - 46.5^\circ] \\ &= 14.4 \text{ W} . \end{aligned}$$

(e) The negative result for P_i means that energy is being taken away from the inductor at this particular time.

(f) $P_i + P_r + P_c = 44.1 \text{ W} - 17.0 \text{ W} + 14.4 \text{ W} = 41.5 \text{ W} = P_g$.