

24. We assume  $q > 0$ . Using the notation  $\lambda = q/L$  we note that the (infinitesimal) charge on an element  $dx$  of the rod contains charge  $dq = \lambda dx$ . By symmetry, we conclude that all horizontal field components (due to the  $dq$ 's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ( $0 \leq x \leq L/2$ ) and then simply double the result. In that regard we note that  $\sin \theta = y/r$  where  $r = \sqrt{x^2 + y^2}$ . Using Eq. 23-3 (with the 2 and  $\sin \theta$  factors just discussed) we obtain

$$\begin{aligned}
 |\vec{E}| &= 2 \int_0^{L/2} \left( \frac{dq}{4\pi\epsilon_0 r^2} \right) \sin \theta \\
 &= \frac{2}{4\pi\epsilon_0} \int_0^{L/2} \left( \frac{\lambda dx}{x^2 + y^2} \right) \left( \frac{y}{\sqrt{x^2 + y^2}} \right) \\
 &= \frac{\lambda y}{2\pi\epsilon_0} \int_0^{L/2} \frac{dx}{(x^2 + y^2)^{3/2}} \\
 &= \frac{(q/L)y}{2\pi\epsilon_0} \left[ \frac{x}{y^2 \sqrt{x^2 + y^2}} \right]_0^{L/2} \\
 &= \frac{q}{2\pi\epsilon_0 L y} \frac{L/2}{\sqrt{(L/2)^2 + y^2}} \\
 &= \frac{q}{2\pi\epsilon_0 y} \frac{1}{\sqrt{L^2 + 4y^2}}
 \end{aligned}$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals).