45. No external torques act on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero. Let $I = MR^2$ be the rotational inertia of the wheel. Its final angular momentum is $= I\omega\hat{\mathrm{k}} = -MR^2|\omega|\hat{\mathrm{k}}$, where $\hat{\mathrm{k}}$ is *up* in Fig. 12-40 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is ωR and the speed of the train (going counterclockwise in Fig. 12-40 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $m(v - |\omega|R)R\,\hat{\mathrm{k}}$. Conservation of angular momentum yields

$$0 = -MR^2|\omega|\,\hat{\mathrm{k}} + m\,(v - |\omega|R)\,R\,\hat{\mathrm{k}}\ .$$

When this equation is solved for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{mv}{(M+m)R}\ .$$