

39. We refer to the (very large) wire length as ℓ and seek to compute the flux per meter: Φ_B/ℓ . Using the right-hand rule discussed in Chapter 30, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 30-19 and Eq. 30-22. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at $x = d/2$); the net field at any point $0 < x < d/2$ is the same at its “mirror image” point $d - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = d$. We make use of the symmetry by integrating over $0 < x < d/2$ and then multiplying by 2:

$$\Phi_B = 2 \int_0^{d/2} B \, dA = 2 \int_0^a B(\ell \, dx) + 2 \int_a^{d/2} B(\ell \, dx)$$

where $d = 0.0025$ m is diameter of each wire. We will r instead of x in the following steps. Thus, using the equations from Ch. 30 referred to above, we find

$$\begin{aligned} \frac{\Phi_B}{\ell} &= 2 \int_0^a \left(\frac{\mu_0 i}{2\pi a^2} r + \frac{\mu_0 i}{2\pi(d-r)} \right) dr + 2 \int_a^{d/2} \left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(d-r)} \right) dr \\ &= \frac{\mu_0 i}{2\pi} \left(1 - 2 \ln \left(\frac{d-a}{d} \right) \right) + \frac{\mu_0 i}{\pi} \ln \left(\frac{d-a}{a} \right) \end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately $\Phi_B \approx \mu_0 i \ell / \pi \ln((d-a)/a)$. Now, we use Eq. 31-35 (with $N = 1$) to obtain the inductance:

$$L = \frac{\Phi_B}{i} = \frac{\mu_0 \ell}{\pi} \ln \left(\frac{d-a}{a} \right) .$$