27. (a) Setting $E = E_F$ (see Eq. 42-9), Eq. 42-5 becomes

$$N(E_F) = \frac{8\pi m\sqrt{2m}}{h^3}\left(\frac{3}{16\pi\sqrt{2}}\right)^{1/3}\frac{h}{\sqrt{m}}\,n^{1/3}\ .$$

Noting that $16\sqrt{2} = 2^4 2^{1/2} = 2^{9/2}$ so that the cube root of this is $2^{3/2} = 2\sqrt{2}$, we are able to simplify the above expression and obtain

$$N(E_F) = \frac{4m}{h^2}\sqrt[3]{3\pi^2 n}$$

which is equivalent to the result shown in the problem statement. Since the desired numerical answer uses eV units, we multiply numerator and denominator of our result by c^2 and make use of the mc^2 value for an electron in Table 38-3 as well as the hc value found in problem 3 of Chapter 39:

$$N(E_F) = \left(\frac{4mc^2}{(hc)^2}\sqrt[3]{3\pi^2}\right)n^{1/3} = \left(\frac{4(511\times 10^3\,\text{eV})}{(1240\,\text{eV}\cdot\text{nm})^2}\sqrt[3]{3\pi^2}\right)n^{1/3} = \left(4.11\ \text{nm}^{-2}\cdot\text{eV}^{-1}\right)n^{1/3}$$

which is equivalent to the value indicated in the problem statement.

(b) Since there are 10^{27} cubic nanometers in a cubic meter, then the result of problem 1 may be written

$$n = 8.49\times 10^{28}\,\text{m}^{-3} = 84.9\ \text{nm}^{-3}\ .$$

The cube root of this is $n^{1/3} \approx 4.4/\text{nm}$. Hence, the expression in part (a) leads to

$$N(E_F) = \left(4.11\,\text{nm}^{-2}\cdot\text{eV}^{-1}\right)\left(4.4\,\text{nm}^{-1}\right) = 18\ \text{nm}^{-3}\cdot\text{eV}^{-1}\ .$$

If we multiply this by $10^{27}\,\text{m}^3/\text{nm}^3$, we see this compares very well with the curve in Fig. 42-5 evaluated at 7.0 eV.