

79. We refer to the particle in the first sentence of the problem statement as particle 2. Since the total momentum of the two particles is zero in S' , it must be that the velocities of these two particles are equal in magnitude and opposite in direction in S' . Letting the velocity of the S' frame be v relative to S , then the particle which is at rest in S must have a velocity of $u'_1 = -v$ as measured in S' , while the velocity of the other particle is given by solving Eq. 38-28 for u' :

$$u'_2 = \frac{u_2 - v}{1 - u_2 v / c^2} = \frac{\left(\frac{c}{2}\right) - v}{1 - \left(\frac{c}{2}\right) \left(\frac{v}{c^2}\right)} .$$

Letting $u'_2 = -u'_1 = v$, we obtain

$$\frac{\left(\frac{c}{2}\right) - v}{1 - \left(\frac{c}{2}\right) \left(\frac{v}{c^2}\right)} = v \implies v = c(2 \pm \sqrt{3}) \approx 0.27c$$

where the quadratic formula has been used (with the smaller of the two roots chosen so that $v \leq c$).