75. A generalized formation reaction can be written X + x → Y, where X is the target nucleus, x is the incident light particle, and Y is the excited compound nucleus (^{20}Ne). We assume X is initially at rest. Then, conservation of energy yields

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + K_Y + E_Y$$

where m_X, m_x, and m_Y are masses, K_x and K_Y are kinetic energies, and E_Y is the excitation energy of Y. Conservation of momentum yields

$$p_x = p_Y \ .$$

Now, $K_Y = p_Y^2/2m_Y = p_x^2/2m_Y = (m_x/m_Y)K_x$, so

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + (m_x/m_Y)K_x + E_Y$$

and

$$K_x = \frac{m_Y}{m_Y - m_x}\left[(m_Y - m_X - m_x)c^2 + E_Y\right] \ .$$

(a) Let x represent the alpha particle and X represent the ^{16}O nucleus. Then, $(m_Y - m_X - m_x)c^2 = (19.99244\,\mathrm{u} - 15.99491\,\mathrm{u} - 4.00260\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = -4.722\,\mathrm{MeV}$ and

$$K_\alpha = \frac{19.99244\,\mathrm{u}}{19.99244\,\mathrm{u} - 4.00260\,\mathrm{u}}(-4.722\,\mathrm{MeV} + 25.0\,\mathrm{MeV}) = 25.35\ \mathrm{MeV} \ .$$

(b) Let x represent the proton and X represent the ^{19}F nucleus. Then, $(m_Y - m_X - m_x)c^2 = (19.99244\,\mathrm{u} - 18.99841\,\mathrm{u} - 1.00783\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = -12.85\,\mathrm{MeV}$ and

$$K_\alpha = \frac{19.99244\,\mathrm{u}}{19.99244\,\mathrm{u} - 1.00783\,\mathrm{u}}(-12.85\,\mathrm{MeV} + 25.0\,\mathrm{MeV}) = 12.80\ \mathrm{MeV} \ .$$

(c) Let x represent the photon and X represent the ^{20}Ne nucleus. Since the mass of the photon is zero, we must rewrite the conservation of energy equation: if E_γ is the energy of the photon, then $E_\gamma + m_X c^2 = m_Y c^2 + K_Y + E_Y$. Since $m_X = m_Y$, this equation becomes $E_\gamma = K_Y + E_Y$. Since the momentum and energy of a photon are related by $p_\gamma = E_\gamma/c$, the conservation of momentum equation becomes $E_\gamma/c = p_Y$. The kinetic energy of the compound nucleus is $K_Y = p_Y^2/2m_Y = E_\gamma^2/2m_Y c^2$. We substitute this result into the conservation of energy equation to obtain

$$E_\gamma = \frac{E_\gamma^2}{2m_Y c^2} + E_Y \ .$$

This quadratic equation has the solutions

$$E_\gamma = m_Y c^2 \pm \sqrt{(m_Y c^2)^2 - 2m_Y c^2 E_Y} \ .$$

If the problem is solved using the relativistic relationship between the energy and momentum of the compound nucleus, only one solution would be obtained, the one corresponding to the negative sign above. Since $m_Y c^2 = (19.99244\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = 1.862 \times 10^4\,\mathrm{MeV}$,

$$\begin{aligned} E_\gamma &= (1.862 \times 10^4\,\mathrm{MeV}) - \sqrt{(1.862 \times 10^4\,\mathrm{MeV})^2 - 2(1.862 \times 10^4\,\mathrm{MeV})(25.0\,\mathrm{MeV})} \\ &= 25.0\ \mathrm{MeV} \ . \end{aligned}$$

The kinetic energy of the compound nucleus is very small; essentially all of the photon energy goes to excite the nucleus.