59. (a) ψ_{210} is real. Squaring it, we obtain the probability density:

$$|\psi_{210}|^2 = \frac{r^2}{32\pi a^5} e^{-r/a} \cos^2\theta \ .$$

Each of the other functions is multiplied by its complex conjugate, obtained by replacing i with $-i$ in the function. Since $e^{i\phi}e^{-i\phi} = e^0 = 1$, the result is the square of the function without the exponential factor:

$$|\psi_{21+1}|^2 = \frac{r^2}{64\pi a^5} e^{-r/a} \sin^2\theta$$

and

$$|\psi_{21-1}|^2 = \frac{r^2}{64\pi a^5} e^{-r/a} \sin^2\theta \ .$$

The last two functions lead to the same probability density.

(b) The total probability density for the three states is the sum:

$$\begin{aligned} |\psi_{210}|^2 + |\psi_{21+1}|^2 + |\psi_{21-1}|^2 &= \frac{r^2}{32\pi a^5} e^{-r/a} \left[\cos^2\theta + \frac{1}{2}\sin^2\theta + \frac{1}{2}\sin^2\theta\right] \\ &= \frac{r^2}{32\pi a^5} e^{-r/a} \ . \end{aligned}$$

The trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. We note that the total probability density does not depend on θ or ϕ; it is spherically symmetric.