

46. We assume that from the moment of grabbing the stick onward, they maintain rigid postures so that the system can be analyzed as a symmetrical rigid body with center of mass midway between the skaters.

- (a) The total linear momentum is zero (the skaters have the same mass and equal-and-opposite velocities). Thus, their center of mass (the middle of the 3.0 m long stick) remains fixed and they execute circular motion (of radius  $r = 1.5$  m) about it. Using Eq. 11-18, their angular velocity (counterclockwise as seen in Fig. 12-41) is

$$\omega = \frac{v}{r} = \frac{1.4}{1.5} = 0.93 \text{ rad/s} .$$

- (b) Their rotational inertia is that of two particles in circular motion at  $r = 1.5$  m, so Eq. 11-26 yields

$$I = \sum mr^2 = 2(50)(1.5)^2 = 225 \text{ kg}\cdot\text{m}^2 .$$

Therefore, Eq. 11-27 leads to

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(225)(0.93)^2 = 98 \text{ J} .$$

- (c) Angular momentum is conserved in this process. If we label the angular velocity found in part (a)  $\omega_i$  and the rotational inertia of part (b) as  $I_i$ , we have

$$I_i\omega_i = (225)(0.93) = I_f\omega_f .$$

The final rotational inertia is  $\sum mr_f^2$  where  $r_f = 0.5$  m so  $I_f = 25 \text{ kg}\cdot\text{m}^2$ . Using this value, the above expression gives  $\omega_f = 8.4 \text{ rad/s}$ .

- (d) We find

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(25)(8.4)^2 = 8.8 \times 10^2 \text{ J} .$$

- (e) We account for the large increase in kinetic energy (part (d) minus part (b)) by noting that the skaters do a great deal of work (converting their internal energy into mechanical energy) as they pull themselves closer – “fighting” what appears to them to be large “centrifugal forces” trying to keep them apart.