

15. The ice warms to 0° C, then melts, and the resulting water warms to the temperature of the lake water, which is 15° C. As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I dT$, where m is the mass of the ice and c_I is the specific heat of ice. If T_i (= 263 K) is the initial temperature and T_f (= 273 K) is the final temperature, then the change in its entropy is

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln \frac{T_f}{T_i} \\ &= (0.010 \text{ kg})(2220 \text{ J/kg}\cdot\text{K}) \ln \left(\frac{273 \text{ K}}{263 \text{ K}} \right) = 0.828 \text{ J/K} .\end{aligned}$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F , where L_F is the heat of fusion for ice. Thus, $\Delta S = Q/T = mL_F/T = (0.010 \text{ kg})(333 \times 10^3 \text{ J/kg})/(273 \text{ K}) = 12.20 \text{ J/K}$. For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln \frac{T_f}{T_i} ,$$

where c_w is the specific heat of water (4190 J/kg · K). Thus,

$$\Delta S = (0.010 \text{ kg})(4190 \text{ J/kg}\cdot\text{K}) \ln \left(\frac{288 \text{ K}}{273 \text{ K}} \right) = 2.24 \text{ J/K} .$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828 \text{ J/K} + 12.20 \text{ J/K} + 2.24 \text{ J/K} = 15.27 \text{ J/K} .$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is $\Delta S = Q/T$, where Q is the energy it receives as heat (the negative of the energy it supplies the ice) and T is its temperature. When the ice warms to 0° C,

$$Q = -mc_I(T_f - T_i) = -(0.010 \text{ kg})(2220 \text{ J/kg}\cdot\text{K})(10 \text{ K}) = -222 \text{ J} .$$

When the ice melts,

$$Q = -mL_F = -(0.010 \text{ kg})(333 \times 10^3 \text{ J/kg}) = -3.33 \times 10^3 \text{ J} .$$

When the water from the ice warms,

$$Q = -mc_w(T_f - T_i) = -(0.010 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(15 \text{ K}) = -629 \text{ J} .$$

The total energy leaving the lake water is $Q = -222 \text{ J} - 3.33 \times 10^3 \text{ J} - 6.29 \times 10^2 \text{ J} = -4.18 \times 10^3 \text{ J}$. The change in entropy is

$$\Delta S = \frac{-4.18 \times 10^3 \text{ J}}{288 \text{ K}} = -14.51 \text{ J/K} .$$

The change in the entropy of the ice-lake system is $\Delta S = (15.27 - 14.51) \text{ J/K} = 0.76 \text{ J/K}$.