

12. The connection between molar heat capacity and the degrees of freedom of a diatomic gas is given by setting $f = 5$ in Eq. 20-51. Thus, $C_V = \frac{5}{2}R$, $C_p = \frac{7}{2}R$, and $\gamma = \frac{7}{5}$. In addition to various equations from Chapter 20, we also make use of Eq. 21-4 of this chapter. We note that we are asked to use the ideal gas constant as R and not plug in its numerical value. We also recall that isothermal means constant-temperature, so $T_2 = T_1$ for the $1 \rightarrow 2$ process. The statement (at the end of the problem) regarding “per mole” may be taken to mean that n may be set identically equal to 1 wherever it appears.

- (a) The gas law in ratio form (see Sample Problem 20-1) as well as the adiabatic relations Eq. 20-54 and Eq. 20-56 are used to obtain

$$\begin{aligned} p_2 &= p_1 \left(\frac{V_1}{V_2} \right) = \frac{p_1}{3} , \\ p_3 &= p_1 \left(\frac{V_1}{V_3} \right)^\gamma = \frac{p_1}{3^{1.4}} , \\ T_3 &= T_1 \left(\frac{V_1}{V_3} \right)^{\gamma-1} = \frac{T_1}{3^{0.4}} . \end{aligned}$$

- (b) The energy and entropy contributions from all the processes are

- process $1 \rightarrow 2$

The internal energy change is $\Delta E_{\text{int}} = 0$ since this is an ideal gas process without a temperature change (see Eq. 20-45).

The work is given by Eq. 20-14: $W = nRT_1 \ln(V_2/V_1) = RT_1 \ln 3$ which is approximately $1.10RT_1$.

The energy absorbed as heat is given by the first law of thermodynamics: $Q = \Delta E_{\text{int}} + W \approx 1.10RT_1$.

The entropy change is $\Delta S = Q/T_1 = 1.10R$.

- process $2 \rightarrow 3$

The work is zero since there is no volume change.

The internal energy change is

$$\Delta E_{\text{int}} = nC_V (T_3 - T_2) = (1) \left(\frac{5}{2}R \right) \left(\frac{T_1}{3^{0.4}} - T_1 \right) \approx -0.889RT_1 .$$

This ($-0.889RT_1$) is also the value for Q (by either the first law of thermodynamics or by the definition of C_V).

For the entropy change, we obtain

$$\begin{aligned} \Delta S &= nR \ln \left(\frac{V_3}{V_1} \right) + nC_V \ln \left(\frac{T_3}{T_1} \right) \\ &= (1)R \ln(1) + (1) \left(\frac{5}{2}R \right) \ln \left(\frac{T_1/3^{0.4}}{T_1} \right) \\ &= 0 + \frac{5}{2}R \ln(3^{-0.4}) \approx -1.10R . \end{aligned}$$

- process $3 \rightarrow 1$

By definition, $Q = 0$ in an adiabatic process, which also implies an absence of entropy change (taking this to be a reversible process). The internal change must be the negative of the value obtained for it in the previous process (since all the internal energy changes must add up to zero, for an entire cycle, and its change is zero for process $1 \rightarrow 2$), so $\Delta E_{\text{int}} = +0.889RT_1$. By the first law of thermodynamics, then, $W = Q - \Delta E_{\text{int}} = 0.889RT_1$.