

23. (a) The wave speed at any point on the rope is given by  $v = \sqrt{\tau/\mu}$ , where  $\tau$  is the tension at that point and  $\mu$  is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance  $y$  from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by  $\mu gy$ , so the tension is  $\tau = \mu gy$ . The wave speed is  $v = \sqrt{\mu gy/\mu} = \sqrt{gy}$ .
- (b) The time  $dt$  for the wave to move past a length  $dy$ , a distance  $y$  from the bottom end, is  $dt = dy/v = dy/\sqrt{gy}$  and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}} \Big|_0^L = 2\sqrt{\frac{L}{g}} .$$