53. If the polar cap melts, the resulting body of water will effectively increase the equatorial radius of the Earth from R_e to $R'_e = R_e + \Delta R$, thereby increasing the moment of inertia of the Earth and slowing its rotation (by conservation of angular momentum), causing the duration T of a day to increase by ΔT. We note that (in rad/s) $\omega = 2\pi/T$ so

$$\frac{\omega'}{\omega} = \frac{2\pi/T'}{2\pi/T} = \frac{T}{T'}$$

from which it follows that

$$\frac{\Delta\omega}{\omega} = \frac{\omega'}{\omega} - 1 = \frac{T}{T'} - 1 = -\frac{\Delta T}{T'} \ .$$

We can approximate that last denominator as T so that we end up with the simple relationship $|\Delta\omega|/\omega = \Delta T/T$. Now, conservation of angular momentum gives us

$$\Delta L = 0 = \Delta(I\omega) \approx I(\Delta\omega) + \omega(\Delta I)$$

so that $|\Delta\omega|/\omega = \Delta I/I$. Thus, using our expectation that rotational inertia is proportional to the equatorial radius squared (supported by Table 11-2(f) for a perfect uniform sphere, but then this isn't a perfect uniform sphere) we have

$$\begin{aligned}\frac{\Delta T}{T} &= \frac{\Delta I}{I} \\ &= \frac{\Delta(R_e^2)}{R_e^2} \approx \frac{2\Delta R_e}{R_e} \\ &= \frac{2(30\,\mathrm{m})}{6.37 \times 10^6\,\mathrm{m}}\end{aligned}$$

so with $T = 86400\,\mathrm{s}$ we find (approximately) that $\Delta T = 0.8$ s. The radius of the earth can be found in Appendix C or on the inside front cover of the textbook.