39. The "current per unit x-length" may be viewed as current density multiplied by the thickness Δy of the sheet; thus, $\lambda = J\Delta y$. Ampere's law may be (and often is) expressed in terms of the current density vector as follows:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

where the area integral is over the region enclosed by the path relevant to the line integral (and $\vec{J}$ is in the $+z$ direction, out of the paper). With J uniform throughout the sheet, then it clear that the right-hand side of this version of Ampere's law should reduce, in this problem, to $\mu_0 JA = \mu_0 J\Delta y\Delta x = \mu_0\lambda\Delta x$.

(a) Figure 30-52 certainly has the horizontal components of $\vec{B}$ drawn correctly at points P and P' (as reference to Fig. 30-4 will confirm [consider the current elements nearest each of those points]), so the question becomes: is it possible for $\vec{B}$ to have vertical components in the figure? Our focus is on point P. Fig. 30-4 suggests that the current element just to the right of the nearest one (the one directly under point P) will contribute a downward component, but by the same reasoning the current element just to the left of the nearest one should contribute an upward component to the field at P. The current elements are all equivalent, as is reflected in the horizontal-translational symmetry built into this problem; therefore, all vertical components should cancel in pairs. The field at P must be purely horizontal, as drawn.

(b) The path used in evaluating $\oint \vec{B} \cdot d\vec{s}$ is rectangular, of horizontal length Δx (the horizontal sides passing through points P and P' respectively) and vertical size $\delta y > \Delta y$. The vertical sides have no contribution to the integral since $\vec{B}$ is purely horizontal (so the scalar dot product produces zero for those sides), and the horizontal sides contribute two equal terms, as shown below. Ampere's law yields

$$2B\Delta x = \mu_0\lambda\Delta x \implies B = \frac{1}{2}\mu_0\lambda \ .$$