

36. We integrate Eq. 12-29 (for a single torque) over the time interval (where the angular speed at the beginning is ω_i and at the end is ω_f)

$$\int \tau dt = \int \frac{dL}{dt} dt = L_f - L_i = I (\omega_f - \omega_i)$$

and if we use the calculus-based notion of the average of a function f

$$f_{\text{avg}} = \frac{1}{\Delta t} \int f dt$$

then (using Eq. 12-16) we obtain

$$\int \tau dt = \tau_{\text{avg}} \Delta t = F_{\text{avg}} R \Delta t .$$

Inserting this into the top line proves the relationship shown in the problem.