

7. Let t_f be the time for the stone to fall to the water and t_s be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If d is the depth of the well, then the kinematics of free fall gives $d = \frac{1}{2}gt_f^2$, or $t_f = \sqrt{2d/g}$. The sound travels at a constant speed v_s , so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = \sqrt{2d/g} + d/v_s$. This equation is to be solved for d . Rewrite it as $\sqrt{2d/g} = t - d/v_s$ and square both sides to obtain $2d/g = t^2 - 2(t/v_s)d + (1/v_s^2)d^2$. Now multiply by gv_s^2 and rearrange to get $gd^2 - 2v_s(gt + v_s)d + gv_s^2t^2 = 0$. This is a quadratic equation for d . Its solutions are

$$d = \frac{2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2v_s^2t^2}}{2g}.$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7$ m is obtained.