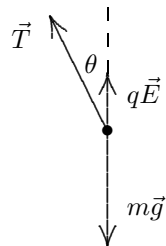


60. (a) Suppose the pendulum is at the angle  $\theta$  with the vertical. The force diagram

is shown to the right.  $\vec{T}$  is the tension in the thread,  $mg$  is the magnitude of the force of gravity, and  $qE$  is the magnitude of the electric force. The field points upward and the charge is positive, so the force is upward. Taking the angle shown to be positive, then the torque on the sphere about the point where the thread is attached to the upper plate is  $\tau = -(mg - qE)\ell \sin \theta$ . If  $mg > qE$  then the torque is a restoring torque; it tends to pull the pendulum back to its equilibrium position.



If the amplitude of the oscillation is small,  $\sin \theta$  can be replaced by  $\theta$  in radians and the torque is  $\tau = -(mg - qE)\ell \theta$ . The torque is proportional to the angular displacement and the pendulum moves in simple harmonic motion. Its angular frequency is  $\omega = \sqrt{(mg - qE)\ell / I}$ , where  $I$  is the rotational inertia of the pendulum. Since  $I = m\ell^2$  for a simple pendulum,

$$\omega = \sqrt{\frac{(mg - qE)\ell}{m\ell^2}} = \sqrt{\frac{g - qE/m}{\ell}}$$

and the period is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{\ell}{g - qE/m}} .$$

If  $qE > mg$  the torque is not a restoring torque and the pendulum does not oscillate.

- (b) The force of the electric field is now downward and the torque on the pendulum is  $\tau = -(mg + qE)\ell \theta$  if the angular displacement is small. The period of oscillation is

$$T = 2\pi \sqrt{\frac{\ell}{g + qE/m}} .$$