23. (a) The linear charge density is the charge per unit length of rod. Since the charge is uniformly distributed on the rod, $\lambda = -q/L$.

(b) We position the x axis along the rod with the origin at the left end of the rod, as shown in the diagram. Let dx be an infinitesimal length of rod at x. The charge in this segment is $dq = \lambda\, dx$. The charge dq may be considered to be a point charge. The electric field it produces at point P has only an x component and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dx}{(L+a-x)^2} \,.$$

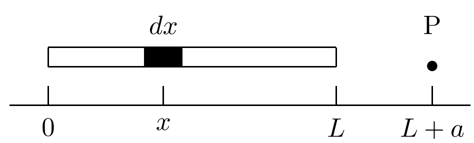
The total electric field produced at P by the whole rod is the integral

$$\begin{aligned} E_x &= \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(L+a-x)^2} \\ &= \frac{\lambda}{4\pi\varepsilon_0} \left. \frac{1}{L+a-x} \right|_0^L \\ &= \frac{\lambda}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{L+a} \right) \\ &= \frac{\lambda}{4\pi\varepsilon_0} \frac{L}{a(L+a)} \,. \end{aligned}$$

When $-q/L$ is substituted for λ the result is

$$E_x = -\frac{1}{4\pi\varepsilon_0} \frac{q}{a(L+a)} \,.$$

The negative sign indicates that the field is toward the rod.



(c) If a is much larger than L, the quantity $L + a$ in the denominator can be approximated by a and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2} \,.$$

This is the expression for the electric field of a point charge at the origin.