67. From the previous chapter, we know that the radial field due to an infinite line-source is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

which integrates, using Eq. 25-18, to obtain

$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f}\frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right) .$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_o$ be the potential along some reference axis (which will be the z axis described in this problem) at a distance $r_f = a$ from the wire. In the "end-view" presented below, the wires and the z axis appear as points as they intersect the xy plane. The potential due to the wire on the left (intersecting the plane at $x = -a$) is

$$V_{\text{negative wire}} = V_o + \frac{(-\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right) ,$$

and the potential due to the wire on the right (intersecting the plane at $x = +a$) is

$$V_{\text{positive wire}} = V_o + \frac{(+\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right) .$$

Since potential is a scalar quantity, the net potential at point P is the addition of $V_{-\lambda}$ and $V_{+\lambda}$ which simplifies to

$$V_{\text{net}} = 2V_o + \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right) - \ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right)\right) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right)$$

where we have set the potential along the z axis equal to zero ($V_o = 0$) in the last step (which we are free to do). This is the expression used to obtain the equipotentials shown below. The center dot in the figure is the intersection of the z axis with the xy plane, and the dots on either side are the intersections of the wires with the plane.