

62. (a) The volume rate of flow is related to speed by $R = vA$. Thus,

$$v_1 = \frac{R_1}{\pi r_{\text{stream}}^2} = \frac{7.9 \text{ cm}^3/\text{s}}{\pi(0.13 \text{ cm})^2} = 148.8 \text{ cm/s} = 1.5 \text{ m/s} .$$

- (b) The depth d of spreading water becomes smaller as r (the distance from the impact point) increases due to the equation of continuity (and the assumption that the water speed remains equal to v_1 in this region). The water that has reached radius r (with perimeter $2\pi r$) is crossing an area of $2\pi r d$. Thus, the equation of continuity gives

$$R_1 = v_1 2\pi r d \implies d = \frac{R}{2\pi r v_1} .$$

- (c) As noted above, d is a decreasing function of r .
 (d) At $r = r_J$ we apply the formula from part (b):

$$d_J = \frac{R_1}{2\pi r_J v_1} = \frac{7.9 \text{ cm}^3/\text{s}}{2\pi(2.0 \text{ cm})(148.8 \text{ cm/s})} = 0.0042 \text{ cm} .$$

- (e) We are told “the depth just after the jump is 2.0 mm” which means $d_2 = 0.20 \text{ cm}$, and we are asked to find v_2 . We use the equation of continuity:

$$R_1 = R_2 \implies 2\pi r_J v_1 d_J = 2\pi r'_J v_2 d_2$$

where r'_J is some very small amount greater than r_J (and for calculation purposes is taken to be the same numerical value, 2.0 cm). This yields

$$v_2 = v_1 \left(\frac{d_1}{d_2} \right) = (148.8 \text{ cm/s}) \left(\frac{0.0042 \text{ cm}}{0.20 \text{ cm}} \right) = 3.1 \text{ cm/s} .$$

- (f) The kinetic energy per unit volume at $r = r_J$ with $v = v_1$ is

$$\frac{1}{2} \rho_w v_1^2 = \frac{1}{2} (1000 \text{ kg/m}^3) (1.488 \text{ m/s})^2 = 1.1 \times 10^3 \text{ J/m}^3 .$$

- (g) The kinetic energy per unit volume at $r = r'_J$ with $v = v_2$ is

$$\frac{1}{2} \rho_w v_2^2 = \frac{1}{2} (1000 \text{ kg/m}^3) (0.031 \text{ m/s})^2 = 0.49 \text{ J/m}^3 .$$

- (h) The hydrostatic pressure change is due to the change in depth:

$$\Delta p = \rho_w g (d_2 - d_1) = (1000 \text{ kg/m}^3) (9.8 \text{ m/s}^2) (0.0020 \text{ m} - 0.000042 \text{ m}) = 19 \text{ Pa} .$$

- (i) Certainly, $\frac{1}{2} \rho_w v_1^2 + \rho_w g d_1 + p_1$ is greater than $\frac{1}{2} \rho_w v_2^2 + \rho_w g d_2 + p_2$ which is not unusual with “shock-like” fluids structures such as this hydraulic jump. Not only does Bernoulli’s equation not apply but the very concept of a streamline becomes difficult to define in this circumstance.