29. The average energy of the conduction electrons is given by

$$E_{\text{avg}} = \frac{1}{n}\int_0^\infty EN(E)P(E)\,dE$$

where n is the number of free electrons per unit volume, $N(E)$ is the density of states, and $P(E)$ is the occupation probability. The density of states is proportional to $E^{1/2}$, so we may write $N(E) = CE^{1/2}$, where C is a constant of proportionality. The occupation probability is one for energies below the Fermi energy and zero for energies above. Thus,

$$E_{\text{avg}} = \frac{C}{n}\int_0^{E_F} E^{3/2}\,dE = \frac{2C}{5n}E_F^{5/2} \ .$$

Now

$$n = \int_0^\infty N(E)P(E)\,dE = C\int_0^{E_F} E^{1/2}\,dE = \frac{2C}{3}E_F^{3/2} \ .$$

We substitute this expression into the formula for the average energy and obtain

$$E_{\text{avg}} = \left(\frac{2C}{5}\right)E_F^{5/2}\left(\frac{3}{2CE_F^{3/2}}\right) = \frac{3}{5}E_F \ .$$