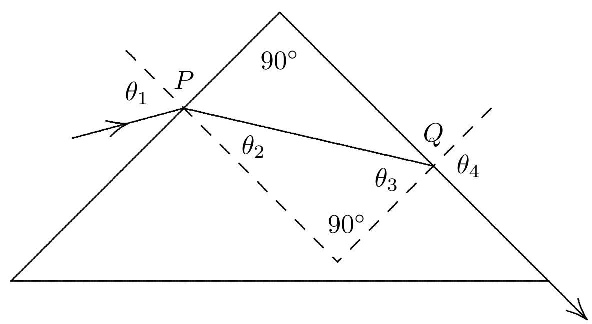
59. (a) A ray diagram is shown below. Let θ_1 be the angle of incidence and θ_2 be the angle of refraction at the first surface. Let θ_3 be the angle of incidence at the second surface. The angle of refraction there is $\theta_4 = 90°$. The law of refraction, applied to the second surface, yields $n \sin\theta_3 = \sin\theta_4 = 1$. As shown in the diagram, the normals to the surfaces at P and Q are perpendicular to each other. The interior angles of the triangle formed by the ray and the two normals must sum to 180°, so $\theta_3 = 90° - \theta_2$ and $\sin\theta_3 = \sin(90° - \theta_2) = \cos\theta_2 = \sqrt{1-\sin^2\theta_2}$. According to the law of refraction, applied at Q, $n\sqrt{1-\sin^2\theta_2} = 1$. The law of refraction, applied to point P, yields $\sin\theta_1 = n\sin\theta_2$, so $\sin\theta_2 = (\sin\theta_1)/n$ and

$$n\sqrt{1-\frac{\sin^2\theta_1}{n^2}} = 1\ .$$

Squaring both sides and solving for n, we get

$$n = \sqrt{1+\sin^2\theta_1}\ .$$



(b) The greatest possible value of $\sin^2\theta_1$ is 1, so the greatest possible value of n is $n_{\max} = \sqrt{2} = 1.41$.

(c) For a given value of n, if the angle of incidence at the first surface is greater than θ_1, the angle of refraction there is greater than θ_2 and the angle of incidence at the second face is less than θ_3 $(= 90° - \theta_2)$. That is, it is less than the critical angle for total internal reflection, so light leaves the second surface and emerges into the air.

(d) If the angle of incidence at the first surface is less than θ_1, the angle of refraction there is less than θ_2 and the angle of incidence at the second surface is greater than θ_3. This is greater than the critical angle for total internal reflection, so all the light is reflected at Q.