65. The amplifier is connected across the primary windings of a transformer and the resistor R is connected across the secondary windings. If I_s is the rms current in the secondary coil then the average power delivered to R is $P_{\text{avg}} = I_s^2 R$. Using $I_s = (N_p/N_s)I_p$, we obtain

$$P_{\text{avg}} = \left(\frac{I_p N_p}{N_s}\right)^2 R \, .$$

Next, we find the current in the primary circuit. This is effectively a circuit consisting of a generator and two resistors in series. One resistance is that of the amplifier (r), and the other is the equivalent resistance R_{eq} of the secondary circuit. Therefore,

$$I_p = \frac{\mathcal{E}_{\text{rms}}}{r + R_{\text{eq}}} = \frac{\mathcal{E}_{\text{rms}}}{r + (N_p/N_s)^2 R}$$

where Eq. 33-82 is used for R_{eq}. Consequently,

$$P_{\text{avg}} = \frac{\mathcal{E}^2 (N_p/N_s)^2 R}{[r + (N_p/N_s)^2 R]^2} \, .$$

Now, we wish to find the value of N_p/N_s such that P_{avg} is a maximum. For brevity, let $x = (N_p/N_s)^2$. Then

$$P_{\text{avg}} = \frac{\mathcal{E}^2 R x}{(r + xR)^2} \, ,$$

and the derivative with respect to x is

$$\frac{dP_{\text{avg}}}{dx} = \frac{\mathcal{E}^2 R(r - xR)}{(r + xR)^3} \, .$$

This is zero for $x = r/R = (1000\,\Omega)/(10\,\Omega) = 100$. We note that for small x, P_{avg} increases linearly with x, and for large x it decreases in proportion to $1/x$. Thus $x = r/R$ is indeed a maximum, not a minimum. Recalling $x = (N_p/N_s)^2$, we conclude that the maximum power is achieved for $N_p/N_s = \sqrt{x} = 10$. The diagram below is a schematic of a transformer with a ten to one turns ratio. An actual transformer would have many more turns in both the primary and secondary coils.