53. We refer to the points where the rope is attached as A and B, respectively. When A and B are not displaced horizontal, the rope is in its initial state (neither stretched (under tension) nor slack). If they are displaced away from each other, the rope is clearly stretched. When A and B are displaced in the same direction, by amounts (in absolute value) $|\xi_A|$ and $|\xi_B|$, then if $|\xi_A| < |\xi_B|$ then the rope is stretched, and if $|\xi_A| > |\xi_B|$ the rope is slack. We must be careful about the case where one is displaced but the other is not, as will be seen below.

(a) The standing wave solution for the shorter cable, appropriate for the initial condition $\xi = 0$ at $t = 0$, and the boundary conditions $\xi = 0$ at $x = 0$ and $x = L$ (the x axis runs vertically here), is $\xi_A = \xi_m \sin(k_A x)\sin(\omega_A t)$. The angular frequency is $\omega_A = 2\pi/T_A$, and the wave number is $k_A = 2\pi/\lambda_A$ where $\lambda_A = 2L$ (it begins oscillating in its fundamental mode) where the point of attachment is $x = L/2$. The displacement of what we are calling point A at time $t = \eta T_A$ (where η is a pure number) is

$$\xi_A = \xi_m \sin\left(\frac{2\pi}{2L}\frac{L}{2}\right)\sin\left(\frac{2\pi}{T_A}\eta T_A\right) = \xi_m \sin(2\pi\eta) \ .$$

The fundamental mode for the longer cable has wavelength $\lambda_B = 2\lambda_A = 2(2L) = 4L$, which implies (by $v = f\lambda$ and the fact that both cables support the same wave speed v) that $f_B = \frac{1}{2}f_A$ or $\omega_B = \frac{1}{2}\omega_A$. Thus, the displacement for point B is

$$\xi_B = \xi_m \sin\left(\frac{2\pi}{4L}\frac{L}{2}\right)\sin\left(\frac{1}{2}\left(\frac{2\pi}{T_A}\right)\eta T_A\right) = \frac{\xi_m}{\sqrt{2}}\sin(\pi\eta) \ .$$

Running through the possibilities ($\eta = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}$, and 2) we find the rope is under tension in the following cases. The first case is one we must be very careful in our reasoning, since A is not displaced but B is displaced in the positive direction; we interpret that as the direction *away from* A (rightwards in the figure) – thus making the rope stretch.

$$\begin{array}{lll} \eta = \frac{1}{2} & \xi_A = 0 & \xi_B = \frac{\xi_m}{\sqrt{2}} > 0 \\ \eta = \frac{3}{4} & \xi_A = -\xi_m < 0 & \xi_B = \frac{\xi_m}{2} > 0 \\ \eta = \frac{7}{4} & \xi_A = -\xi_m < 0 & \xi_B = -\frac{\xi_m}{2} < 0 \end{array}$$

where in the last case they are both displaced leftward but A more so than B so that the rope is indeed stretched.

(b) The values of η (where we have defined $\eta = t/T_A$) which reproduce the initial state are

$$\begin{array}{llll} \eta = 1 & \xi_A = 0 & \xi_B = 0 & \text{and} \\ \eta = 2 & \xi_A = 0 & \xi_B = 0 & . \end{array}$$

(c) The values of η for which the rope is slack are given below. In the first case, both displacements are to the right, but point A is farther to the right than B. In the second case, they are displaced towards each other.

$$\begin{array}{lll} \eta = \frac{1}{4} & \xi_A = x_m > 0 & \xi_B = \frac{\xi_m}{\sqrt{2}} > 0 \\ \eta = \frac{5}{4} & \xi_A = \xi_m > 0 & \xi_B = -\frac{\xi_m}{2} < 0 \\ \eta = \frac{3}{2} & \xi_A = 0 & \xi_B = -\frac{\xi_m}{\sqrt{2}} < 0 \end{array}$$

where in the third case B is displaced leftward toward the undisplaced point A.

(d) The first design works effectively to damp fundamental modes of vibration in the two cables (especially in the shorter one which would have an antinode at that point), whereas the second one only damps the fundamental mode in the longer cable.