

41. We assume there is charge q on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1 \varepsilon_0 A} ,$$

where A is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2 \varepsilon_0 A} .$$

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\varepsilon_0 A} \left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right] = \frac{qd}{2\varepsilon_0 A} \frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2} ,$$

so

$$C = \frac{q}{V} = \frac{2\varepsilon_0 A}{d} \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} .$$

This expression is exactly the same as the that for C_{eq} of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2 . Each has plate area A and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1 \varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area A , plate separation d , and dielectric constant κ_1 .