

82. (a) If point  $P$  is infinitely far away, then the small distance  $d$  between the two sources is of no consequence (they seem effectively to be the same distance away from  $P$ ). Thus, there is no perceived phase difference.
- (b) Since the sources oscillate in phase, then the situation described in part (a) produces constructive interference.
- (c) For finite values of  $x$ , the difference in source positions becomes significant. The path lengths for waves to travel from  $S_1$  and  $S_2$  become is now different. We interpret the question as asking for the behavior of the absolute value of the phase difference  $|\Delta\phi|$ , in which case any change from zero (the answer for part (a)) is certainly an increase.
- (d) The path length difference for waves traveling from  $S_1$  and  $S_2$  is

$$\Delta\ell = \sqrt{d^2 + x^2} - x \quad \text{for } x > 0 .$$

The phase difference in “cycles” (in absolute value) is therefore

$$|\Delta\phi| = \frac{\Delta\ell}{\lambda} = \frac{\sqrt{d^2 + x^2} - x}{\lambda} .$$

Thus, in terms of  $\lambda$ , the phase difference is identical to the path length difference:  $|\Delta\phi| = \Delta\ell > 0$ . Consider  $\Delta\ell = \lambda/2$ . Then  $\sqrt{d^2 + x^2} = x + \lambda/2$ . Squaring both sides, rearranging, and solving, we find

$$x = \frac{d^2}{\lambda} - \frac{\lambda}{4} .$$

In general, if  $\Delta\ell = \xi\lambda$  for some multiplier  $\xi > 0$ , we find

$$x = \frac{d^2}{2\xi\lambda} - \frac{1}{2}\xi\lambda .$$

Using  $d = 16$  m and  $\lambda = 2.0$  m, we insert  $\xi = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$  into this expression and find the respective values (in meters)  $x = 128, 63, 41, 30, 23$ . Since whole cycle phase differences are equivalent (as far as the wave superposition goes) to zero phase difference, then the  $\xi = 1, 2$  cases give constructive interference. A shift of a half-cycle brings “troughs” of one wave in superposition with “crests” of the other, thereby canceling the waves; therefore, the  $\xi = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$  cases produce destructive interference.