89. (Third problem of **Cluster**)

(a) By momentum conservation we see that their final speeds are the same. We use energy conservation (where the "final" subscript refers to when they are infinitely far away from each other):

$$\begin{aligned} U_i &= K_f \\ \frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{D} &= 2\left(\frac{1}{2}mv^2\right) \end{aligned}$$

which (using $k = 1/4\pi\varepsilon_0$) yields

$$v = |Q|\sqrt{\frac{2k}{m\,D}} \; .$$

(b) As noted above, this result is the same as that of part (a).

(c) We use energy conservation (where the "final" subscript refers to when their surfaces have made contact):

$$\begin{aligned} U_i &= K_f + U_f \\ \frac{1}{4\pi\varepsilon_0}\frac{-2Q^2}{D} &= 2\left(\frac{1}{2}mv^2\right) + \frac{1}{4\pi\varepsilon_0}\frac{-2Q^2}{2r} \end{aligned}$$

which (using $k = 1/4\pi\varepsilon_0$) yields

$$v = |Q|\sqrt{\frac{k}{m\,r} - \frac{2k}{m\,D}} \;\approx\; |Q|\sqrt{\frac{k}{m\,r}}$$

since $r \ll D$.

(d) As before, the speeds of the particles are equal (by momentum conservation).

(e) and (f) The collision being elastic means no kinetic energy is lost (or gained), so they are able to return to their original positions (climbing back up that potential "hill") whereupon their potential energy is again U_i and their kinetic energies (hence, speeds) are zero.