47. If the ring diameter at 0.000°C is D_{r0} then its diameter when the ring and sphere are in thermal equilibrium is

$$D_r = D_{r0}(1 + \alpha_c T_f)\,,$$

where T_f is the final temperature and α_c is the coefficient of linear expansion for copper. Similarly, if the sphere diameter at T_i (= 100.0°C) is D_{s0} then its diameter at the final temperature is

$$D_s = D_{s0}[1 + \alpha_a(T_f - T_i)]\,,$$

where α_a is the coefficient of linear expansion for aluminum. At equilibrium the two diameters are equal, so

$$D_{r0}(1 + \alpha_c T_f) = D_{s0}[1 + \alpha_a(T_f - T_i)]\ .$$

The solution for the final temperature is

$$\begin{aligned} T_f &= \frac{D_{r0} - D_{s0} + D_{s0}\alpha_a T_i}{D_{s0}\alpha_a - D_{r0}\alpha_c} \\ &= \frac{2.54000\,\text{cm} - 2.54508\,\text{cm} + (2.54508\,\text{cm})\left(23\times10^{-6}/\text{C}^\circ\right)(100^\circ\text{C})}{(2.54508\,\text{cm})\,(23\times10^{-6}/\text{C}^\circ) - (2.54000\,\text{cm})\,(17\times10^{-6}/\text{C}^\circ)} \\ &= 50.38^\circ\text{C}\ . \end{aligned}$$

The expansion coefficients are from Table 19-2 of the text. Since the initial temperature of the ring is 0°C, the heat it absorbs is

$$Q = c_c m_r T_f\ ,$$

where c_c is the specific heat of copper and m_r is the mass of the ring. The heat rejected up by the sphere is

$$|Q| = c_a m_s\,(T_i - T_f)$$

where c_a is the specific heat of aluminum and m_s is the mass of the sphere. Since these two heats are equal,

$$c_c m_r T_f = c_a m_s\,(T_i - T_f)\ \ ,$$

we use specific heat capacities from the textbook to obtain

$$m_s = \frac{c_c m_r T_f}{c_a\,(T_i - T_f)} = \frac{(386\,\text{J/kg}\cdot\text{K})(0.0200\,\text{kg})(50.38^\circ\text{C})}{(900\,\text{J/kg}\cdot\text{K})(100^\circ\text{C} - 50.38^\circ\text{C})} = 8.71\times10^{-3}\ \text{kg}\ .$$