65. We label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with $V_2 = 4\,\mathrm{m}^3$; then continues onto state 3 isothermally, with $V_3 = 10\,\mathrm{m}^3$; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process $1 \to 2$ $p_1 V_1^\gamma = p_2 V_2^\gamma$, for the isothermal process $2 \to 3$ $p_2 V_2 = p_3 V_3$, and finally for the adiabatic process $3 \to 4$ $p_3 V_3^\gamma = p_4 V_4^\gamma$. These equations yield

$$p_4 = p_3 \left(\frac{V_3}{V_4}\right)^\gamma = p_2 \left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma = p_1 \left(\frac{V_1}{V_2}\right)^\gamma \left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma .$$

We substitute this expression for p_4 into the equation $p_1 V_1 = p_4 V_4$ (since $T_1 = T_4$) to obtain $V_1 V_3 = V_2 V_4$. Solving for V_4 we obtain

$$V_4 = \frac{V_1 V_3}{V_2} = \frac{(2\,\mathrm{m}^3)(10\,\mathrm{m}^3)}{4\,\mathrm{m}^3} = 5\ \mathrm{m}^3 .$$