47. (a) It is useful to read the whole problem before considering the sketch here in part (a) (which we do not show, but briefly describe). We find in part (d) and part (f), below, that $J_A > J_B$ which suggests that the streamlines should be closer together in region $A$ than in $B$ (at least for portions of those regions which lie close to the pipe). Associated with this (see part (g)) the sketch of the streamlines should reflect that fact that some of the conduction charge-carriers are entering the pipe walls during the transition from region $A$ to region $B$.

(b) Eq. 27-16 yields

$$\rho_{\text{pipe}} = R\frac{A}{L} = (6.0\,\Omega)\left(\frac{0.010\,\text{m}^2}{1.0\times10^6\,\text{m}}\right) = 6.0\times10^{-8}\ \Omega\cdot\text{m} .$$

(c) If the resistance of 1000 km of pipe is $6.0\,\Omega$ then the resistance of $L = 1.0$ km of pipe is $R = 6.0\,\text{m}\Omega$. Thus in region $A$, Ohm's law leads to

$$i_{\text{pipe}} = \frac{V_{ab}}{R} = \frac{8.0\,\text{mV}}{6.0\,\text{m}\Omega} = 1.3\ \text{A} .$$

(d) Using Eq. 27-11 and Eq. 25-42 (in absolute value), we find the magnitude of the current density vector in region $A$:

$$\left|\vec{J}_{\text{ground}}\right| = \frac{V_{ab}}{\rho_{\text{ground}}L} = \frac{0.0080\,\text{V}}{(500\,\Omega\cdot\text{m})(1000\,\text{m})} = 1.6\times10^{-8}\ \text{A/m}^2 .$$

(e) Similarly, in region $B$ we obtain

$$i_{\text{pipe}} = \frac{V_{cd}}{R} = \frac{9.5\,\text{mV}}{6.0\,\text{m}\Omega} = 1.6\ \text{A} ,$$

(f) and

$$\left|\vec{J}_{\text{ground}}\right| = \frac{V_{cd}}{\rho_{\text{ground}}L} = \frac{0.0095\,\text{V}}{(1000\,\Omega\cdot\text{m})(1000\,\text{m})} = 9.5\times10^{-9}\ \text{A/m}^2 .$$

(g) These results suggest that the pipe walls, in leaving region $A$ and entering region $B$, have "absorbed" some of the current, leaving the current density in the nearby ground somewhat "depleted" of the telluric flows.

(h) We assume the transition $B \to A$ is the reverse of that discussed in part (g). Here, some current leaves the pipe walls and joins in the ground-supported telluric flows.

(i) There is no current here, because there is no potential difference along this section of pipe. The reason $V_{gh} = 0$ is best seen using Eq. 27-11 and Eq. 25-18 (and remembering that the scalar dot product gives zero for perpendicular vectors). The arrows shown in the figure for current actually refer, in the technical sense, to the direction of $\vec{J}$. We refer to this as the $x$ direction. The pipe section $gh$ is oriented in what we will refer to as the $y$ direction. Eq. 27-11 implies that $\vec{J}$ and $\vec{E}$ must be in the same direction ($x$). But a nonzero voltage difference here would require (by Eq. 25-18) $\int \vec{E}\cdot d\vec{s} \neq 0$. But since $d\vec{s} = dy$ for this section of pipe, then $\vec{E}\cdot d\vec{s}$ vanishes identically.

(j) Our discussion in part (j) serves also to motivate the fact that the current in section $fg$ is less than that in section $ef$ by a factor of $\cos 45° = 1/\sqrt{2}$. To see this, one may consider the component of the electric field which would "drive" the current (in the sense of Eq. 27-11) along section $fg$; it is less than the field responsible for the current in section $ef$ by exactly the factor just mentioned. Thus,

$$i_{fg} = i_{ef}\cos 45° = \frac{1.0\,\text{A}}{\sqrt{2}} = 0.71\ \text{A} .$$

(k) The answers to the previous parts indicate that current leaves the pipe at point $f$ and

(l) at point $g$.