

50. (a) An incident ray which is normal to the water surface is not refracted, so the angle at which it strikes the first mirror is $\theta_1 = 45^\circ$. According to the law of reflection, the angle of reflection is also 45° . This means the ray is horizontal as it leaves the first mirror, and the angle of incidence at the second mirror is $\theta_2 = 45^\circ$. Since the angle of reflection at the second mirror is also 45° the ray leaves that mirror normal again to the water surface. There is no refraction at the water surface, and the emerging ray is parallel to the incident ray.
- (b) We imagine that the incident ray makes an angle θ_1 with the normal to the water surface. The angle of refraction θ_2 is found from $\sin \theta_1 = n \sin \theta_2$, where n is the index of refraction of the water. The normal to the water surface and the normal to the first mirror make an angle of 45° . If the normal to the water surface is continued downward until it meets the normal to the first mirror, the triangle formed has an interior angle of $180^\circ - 45^\circ = 135^\circ$ at the vertex formed by the normal. Since the interior angles of a triangle must sum to 180° , the angle of incidence at the first mirror satisfies $\theta_3 + \theta_2 + 135^\circ = 180^\circ$, so $\theta_3 = 45^\circ - \theta_2$. Using the law of reflection, the angle of reflection at the first mirror is also $45^\circ - \theta_2$. We note that the triangle formed by the ray and the normals to the two mirrors is a right triangle. Consequently, $\theta_3 + \theta_4 + 90^\circ = 180^\circ$ and $\theta_4 = 90^\circ - \theta_3 = 90^\circ - 45^\circ + \theta_2 = 45^\circ + \theta_2$. The angle of reflection at the second mirror is also $45^\circ + \theta_2$. Now, we continue the normal to the water surface downward from the exit point of the ray to the second mirror. It makes an angle of 45° with the mirror. Consider the triangle formed by the second mirror, the ray, and the normal to the water surface. The angle at the intersection of the normal and the mirror is $180^\circ - 45^\circ = 135^\circ$. The angle at the intersection of the ray and the mirror is $90^\circ - \theta_4 = 90^\circ - (45^\circ + \theta_2) = 45^\circ - \theta_2$. The angle at the intersection of the ray and the water surface is θ_5 . These three angles must sum to 180° , so $135^\circ + 45^\circ - \theta_2 + \theta_5 = 180^\circ$. This means $\theta_5 = \theta_2$. Finally, we use the law of refraction to find θ_6 :

$$\sin \theta_6 = n \sin \theta_5 \implies \sin \theta_6 = n \sin \theta_2 ,$$

since $\theta_5 = \theta_2$. Finally, since $\sin \theta_1 = n \sin \theta_2$, we conclude that $\sin \theta_6 = \sin \theta_1$ and $\theta_6 = \theta_1$. The exiting ray is parallel to the incident ray.