82. (Second problem in **Cluster 1**)

(a) If we interpret this "one-wheel cart" which has a wheel that is a "long cylinder" as simply the cylinder itself, then an appropriate picture for this problem is Fig. 12-30 in the textbook. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocity in this problem is positive; we choose *downhill* positive for the x axis (which is parallel to the incline surface) so that $a_{\text{com}} = R\alpha$ holds. We can combine the rotational (about the center of mass) and linear forms of Newton's second law, or we can more simply adopt the view of pure rotation (see, for example, Eq. 12-3) and examine torques about the bottom-most point P:

$$MgR\sin\theta = I_P\,\alpha = I_P\,\frac{a_{\text{com}}}{R}$$

We have assumed that the center of mass of the cart-wheel system is at the center of the wheel (the axle), although this is not stated in the problem. Now, $\theta = 30.0°$, $R = 0.200$ m, $M = 50.0$ kg, and $I_P = 0.667\ \text{kg}\cdot\text{m}^2 + MR^2 = 2.67\ \text{kg}\cdot\text{m}^2$ (using the parallel-axis theorem and the result of the previous problem). Thus, we find $a_{\text{com}} = 3.68\,\text{m/s}^2$.

(b) If we apply the linear form of Newton's law, we have

$$\begin{aligned} \sum F_x &= Mg\sin\theta - f_{s,\,\text{max}} = Ma_{\text{com}} \\ \sum F_y &= N - Mg\sin\theta = 0 \end{aligned}$$

Solving for $f_{s,\,\text{max}}$ and N and dividing, we obtain

$$\mu_s = \frac{f_{s,\,\text{max}}}{N} = 0.14 \quad .$$