82. (Third problem in **Cluster**)
It is clear by symmetry that $x_{\rm com} = B/2$ for the system, but the value of $y_{\rm com}$ is not obvious. If the cross-section area of the wire is A and the density is ρ, then in one quadrant the relation between the mass element dm and height element dy is

$$dm = \rho A \frac{R}{\sqrt{R^2 - y^2}}\, dy = \frac{M}{\ell_\cap} \frac{R}{\sqrt{R^2 - y^2}}\, dy$$

where the length of the semicircle is $\ell_\cap = \pi R$. To include the contributions from both quadrants shown, we multiply by 2, and Eq. 9-9 becomes

$$\begin{aligned} y_{\rm com} &= \frac{2}{M} \int_0^R y \frac{M}{\ell_\cap} \frac{R}{\sqrt{R^2 - y^2}}\, dy \\ &= \frac{2}{\pi} \int_0^R \frac{y}{\sqrt{R^2 - y^2}}\, dy \\ &= \frac{2}{\pi} \left[-\sqrt{R^2 - y^2} \right]_0^R \\ &= \frac{2R}{\pi} \ . \end{aligned}$$