22. (a) The compression of the spring is $d = 0.12$ m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)(0.12\,\mathrm{m}) = 0.29\ \mathrm{J}\ .$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2}kd^2 = -\frac{1}{2}(250\,\mathrm{N/m})(0.12\,\mathrm{m})^2 = -1.8\ \mathrm{J}\ .$$

(c) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15).

$$\Delta K = 0 - \frac{1}{2}mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29 - 1.8)}{0.25}} = 3.5\ \mathrm{m/s}\ .$$

(d) If we instead had $v_i' = 7$ m/s, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$\begin{aligned} 0 - \frac{1}{2}mv_i'^{\,2} &= W_1' + W_2' \\ &= mgd' - \frac{1}{2}kd'^{\,2} \end{aligned}$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2g^2 + mkv_i'^{\,2}}}{k}$$

which yields $d' = 0.23$ m. In order to obtain this, we have used more digits in our intermediate results than are shown above (so $v_i = \sqrt{12.048} = 3.471$ m/s and $v_i' = 6.942$ m/s).