

15. The water is medium 1, so  $n_1 = n_w$  which we simply write as  $n$ . The air is medium 2, for which  $n_2 \approx 1$ . We refer points where the light rays strike the water surface as  $A$  (on the left side of Fig. 35-32) and  $B$  (on the right side of the picture). The point midway between  $A$  and  $B$  (the center point in the picture) is  $C$ . The penny  $P$  is directly below  $C$ , and the location of the “apparent” or Virtual penny is  $V$ . We note that the angle  $\angle CVB$  (the same as  $\angle CVA$ ) is equal to  $\theta_2$ , and the angle  $\angle CPB$  (the same as  $\angle CPA$ ) is equal to  $\theta_1$ . The triangles  $CVB$  and  $CPB$  share a common side, the horizontal distance from  $C$  to  $B$  (which we refer to as  $x$ ). Therefore,

$$\tan \theta_2 = \frac{x}{d_a} \quad \text{and} \quad \tan \theta_1 = \frac{x}{d} \quad .$$

Using the small angle approximation (so a ratio of tangents is nearly equal to a ratio of sines) and the law of refraction, we obtain

$$\begin{aligned} \frac{\tan \theta_2}{\tan \theta_1} &\approx \frac{\sin \theta_2}{\sin \theta_1} \\ \frac{\frac{x}{d_a}}{\frac{x}{d}} &\approx \frac{n_1}{n_2} \\ \frac{d}{d_a} &\approx n \end{aligned}$$

which yields the desired relation:  $d_a = d/n$ .