49. (a) We use the notation $P(\mu)$ for the probability of a dipole being parallel to $\vec{B}$, and $P(-\mu)$ for the probability of a dipole being antiparallel to the field. The magnetization may be thought of as a "weighted average" in terms of these probabilities:

$$M = \frac{N\mu P(\mu) - N\mu P(-\mu)}{P(\mu) + P(-\mu)} = \frac{N\mu\left(e^{\mu B/KT} - e^{-\mu B/KT}\right)}{e^{\mu B/KT} + e^{-\mu B/KT}} = N\mu \tanh\left(\frac{\mu B}{kT}\right) .$$

(b) For $\mu B \ll kT$ (that is, $\mu B/kT \ll 1$) we have $e^{\pm\mu B/kT} \approx 1 \pm \mu B/kT$, so

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx \frac{N\mu[(1+\mu B/kT) - (1-\mu B/kT)]}{(1+\mu B/kT) + (1-\mu B/kT)} = \frac{N\mu^2 B}{kT} .$$

(c) For $\mu B \gg kT$ we have $\tanh(\mu B/kT) \approx 1$, so

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx N\mu .$$

(d) One can easily plot the tanh function using, for instance, a graphical calculator. One can then note the resemblance between such a plot and Fig. 32-9. By adjusting the parameters used in one's plot, the curve in Fig. 32-9 can reliably be fit with a tanh function.