

39. (a) The number of electrons in the valence band is

$$N_{\text{ev}} = N_v P(E_v) = \frac{N_v}{e^{(E_v - E_F)/kT} + 1} .$$

Since there are a total of  $N_v$  states in the valence band, the number of holes in the valence band is

$$\begin{aligned} N_{\text{hv}} &= N_v - N_{\text{ev}} = N_v \left[ 1 - \frac{1}{e^{(E_v - E_F)/kT} + 1} \right] \\ &= \frac{N_v}{e^{-(E_v - E_F)/kT} + 1} . \end{aligned}$$

Now, the number of electrons in the conduction band is

$$N_{\text{ec}} = N_c P(E_c) = \frac{N_c}{e^{(E_c - E_F)/kT} + 1} ,$$

Hence, from  $N_{\text{ev}} = N_{\text{hc}}$ , we get

$$\frac{N_v}{e^{-(E_v - E_F)/kT} + 1} = \frac{N_c}{e^{(E_c - E_F)/kT} + 1} .$$

(b) In this case,  $e^{(E_c - E_F)/kT} \gg 1$  and  $e^{-(E_v - E_F)/kT} \gg 1$ . Thus, from the result of part (a),

$$\frac{N_c}{e^{(E_c - E_F)/kT}} \approx \frac{N_v}{e^{-(E_v - E_F)/kT}} ,$$

or  $e^{(E_v - E_c + 2E_F)/kT} \approx N_v/N_c$ . We solve for  $E_F$ :

$$E_F \approx \frac{1}{2}(E_c + E_v) + \frac{1}{2}kT \ln\left(\frac{N_v}{N_c}\right) .$$