19. (a) The charge (as a function of time) is given by $q = Q\sin\omega t$, where Q is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that $q = 0$ at time $t = 0$. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q\cos\omega t\ ,$$

and at $t = 0$, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC}$,

$$Q = I\sqrt{LC} = (2.00\,\mathrm{A})\sqrt{(3.00\times 10^{-3}\,\mathrm{H})(2.70\times 10^{-6}\,\mathrm{F})} = 1.80\times 10^{-4}\ \mathrm{C}\ .$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2\sin^2\omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2\omega\sin\omega t\cos\omega t}{C}\ .$$

We use the trigonometric identity $\cos\omega t\sin\omega t = \frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C}\ \sin(2\omega t)\ .$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2\,\mathrm{rad}$. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi T}{4(2\pi)} = \frac{T}{8}$$

where T is the period of oscillation. The relationship $\omega = 2\pi/T$ was used.

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}\ .$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times 10^{-3}\,\mathrm{H})(2.70\times 10^{-6}\,\mathrm{F})} = 5.655\times 10^{-4}\,\mathrm{s}$, so

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{\pi(1.80\times 10^{-4}\,\mathrm{C})^2}{(5.655\times 10^{-4}\,\mathrm{s})(2.70\times 10^{-6}\,\mathrm{F})} = 66.7\ \mathrm{W}\ .$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.