

31. (a) The angular positions θ of the bright interference fringes are given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a \sin \theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so we should count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or, equivalently, the number of values of m for which $-\sin \theta_1 < \sin \theta < +\sin \theta_1$. This means $-1/a < m/d < 1/a$ or $-d/a < m < +d/a$. Now $d/a = (0.150 \times 10^{-3} \text{ m}) / (30.0 \times 10^{-6} \text{ m}) = 5.00$, so the values of m are $m = -4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. There are nine fringes.
- (b) The intensity at the screen is given by

$$I = I_m (\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha} \right)^2$$

where $\alpha = (\pi a / \lambda) \sin \theta$, $\beta = (\pi d / \lambda) \sin \theta$, and I_m is the intensity at the center of the pattern. For the third bright interference fringe, $d \sin \theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2 \beta = 1$. Similarly, $\alpha = 3\pi a / d = 3\pi / 5.00 = 0.600\pi$ rad and

$$\left(\frac{\sin \alpha}{\alpha} \right)^2 = \left(\frac{\sin 0.600\pi}{0.600\pi} \right)^2 = 0.255 .$$

The intensity ratio is $I/I_m = 0.255$.