13. From $I = \frac{2}{3}MR^2$ (Table 11-2(g)) we find

$$M = \frac{3I}{2R^2} = \frac{3(0.040)}{2(0.15)^2} = 2.7 \text{ kg} .$$

It also follows from the rotational inertia expression that $\frac{1}{2}I\omega^2 = \frac{1}{3}MR^2\omega^2$. Furthermore, it rolls without slipping, $v_{\text{com}} = R\omega$, and we find

$$\frac{K_{\text{rot}}}{K_{\text{com}} + K_{\text{rot}}} = \frac{\frac{1}{3}MR^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{3}MR^2\omega^2} .$$

(a) Simplifying the above ratio, we find $K_{\text{rot}}/K = 0.4$. Thus, 40% of the kinetic energy is rotational, or $K_{\text{rot}} = (0.4)(20) = 8.0$ J.

(b) From $K_{\text{rot}} = \frac{1}{3}MR^2\omega^2 = 8.0$ J (and using the above result for M) we find

$$\omega = \frac{1}{0.15\,\text{m}}\sqrt{\frac{3(8.0\,\text{J})}{2.7\,\text{kg}}} = 20 \text{ rad/s}$$

which leads to $v_{\text{com}} = (0.15)(20) = 3.0$ m/s.

(c) We note that the inclined distance of 1.0 m corresponds to a height $h = 1.0\sin 30° = 0.50$ m. Mechanical energy conservation leads to

$$\begin{aligned} K_i &= K_f + U_f \\ 20\,\text{J} &= K_f + Mgh \end{aligned}$$

which yields (using the values of M and h found above) $K_f = 6.9$ J.

(d) We found in part (a) that 40% of this must be rotational, so

$$\frac{1}{3}MR^2\omega_f^2 = (0.40)K_f \quad \Longrightarrow \quad \omega_f = \frac{1}{0.15}\sqrt{\frac{3(0.40)(6.9)}{2.7}}$$

which yields $\omega_f = 12$ rad/s and leads to

$$v_{\text{com}\,f} = R\omega_f = (0.15)(12) = 1.8 \text{ m/s} .$$