51. (a) If we take the logarithm of Kepler's law of periods, we obtain

$$2\log(T) = \log(4\pi^2/GM) + 3\log(a) \quad \Longrightarrow \quad \log(a) = \frac{2}{3}\log(T) - \frac{1}{3}\log(4\pi^2/GM)$$

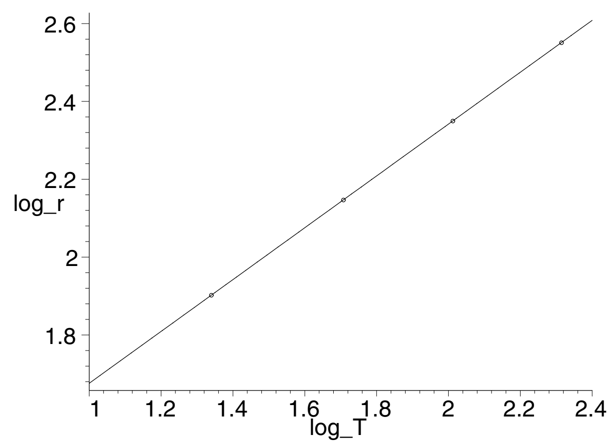
where we are ignoring an important subtlety about units (the arguments of logarithms cannot have units, since they are transcendental functions). Although the problem can be continued in this way, we prefer to set it up without units, which requires taking a ratio. If we divide Kepler's law (applied to the Jupiter-moon system, where $M$ is mass of Jupiter) by the law applied to Earth orbiting the Sun (of mass $M_o$), we obtain

$$(T/T_E)^2 = \left(\frac{M_o}{M}\right)\left(\frac{a}{r_E}\right)^3$$

where $T_E = 365.25$ days is Earth's orbital period and $r_E = 1.50 \times 10^{11}$ m is its mean distance from the Sun. In this case, it is perfectly legitimate to take logarithms and obtain

$$\log\left(\frac{r_E}{a}\right) = \frac{2}{3}\log\left(\frac{T_E}{T}\right) + \frac{1}{3}\log\left(\frac{M_o}{M}\right)$$

(written to make each term positive) which is the way we plot the data ($\log(r_E/a)$ on the vertical axis and $\log(T_E/T)$ on the horizontal axis).




(b) When we perform a least-squares fit to the data, we obtain $\log(r_E/a) = 0.666\log(T_E/T) + 1.01$, which confirms the expectation of slope$= 2/3$ based on the above equation.

(c) And the 1.01 intercept corresponds to the term $\frac{1}{3}\log\left(\frac{M_o}{M}\right)$ which implies

$$\frac{M_o}{M} = 10^{3.03} \quad \Longrightarrow \quad M = \frac{M_o}{1.07 \times 10^3} .$$

Plugging in $M_o = 1.99 \times 10^{30}$ kg (see Appendix C), we obtain $M = 1.86 \times 10^{27}$ kg for Jupiter's mass. This is reasonably consistent with the value $1.90 \times 10^{27}$ kg found in Appendix C.