86. We note that for two points on a circle, separated by angle θ (in radians), the direct-line distance between them is $r = 2R\sin(\theta/2)$. Using this fact, distinguishing between the cases where $N =$ odd and $N =$ even, and counting the pair-wise interactions very carefully, we arrive at the following results for the total potential energies. We use $k = 1/4\pi\varepsilon_0$. For configuration 1 (where all N electrons are on the circle), we have

$$\begin{aligned} U_{1,N=\text{even}} &= \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta/2)} + \frac{1}{2}\right) \\ U_{1,N=\text{odd}} &= \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N-1}{2}}\frac{1}{\sin(j\theta/2)}\right) \end{aligned}$$

where $\theta = \frac{2\pi}{N}$. For configuration 2, we find

$$\begin{aligned} U_{2,N=\text{even}} &= \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta'/2)} + 2\right) \\ U_{2,N=\text{odd}} &= \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N-3}{2}}\frac{1}{\sin(j\theta'/2)} + \frac{5}{2}\right) \end{aligned}$$

where $\theta' = \frac{2\pi}{N-1}$. The results are all of the form

$$U_{1 \text{ or } 2} = \frac{ke^2}{2R} \times \text{a pure number}\,.$$

In our table, below, we have the results for those "pure numbers" as they depend on N and on which configuration we are considering. The values listed in the U rows are the potential energies divided by $ke^2/2R$.

N	4	5	6	7	8	9	10	11	12	13	14	15
U_1	3.83	6.88	10.96	16.13	22.44	29.92	38.62	48.58	59.81	72.35	86.22	101.5
U_2	4.73	7.83	11.88	16.96	23.13	30.44	39.92	48.62	59.58	71.81	85.35	100.2

We see that the potential energy for configuration 2 is greater than that for configuration 1 for $N < 12$, but for $N \geq 12$ it is configuration 1 that has the greatest potential energy.

(a) Configuration 1 has the smallest U for $2 \leq N \leq 11$, and configuration 2 has the smallest U for $12 \leq N \leq 15$.

(b) $N = 12$ is the smallest value such that $U_2 < U_1$.

(c) For $N = 12$, configuration 2 consists of 11 electrons distributed at equal distances around the circle, and one electron at the center. A specific electron e_0 on the circle is R distance from the one in the center, and is

$$r = 2R\sin\left(\frac{\pi}{11}\right) \approx 0.56R$$

distance away from its nearest neighbors on the circle (of which there are two – one on each side). Beyond the nearest neighbors, the next nearest electron on the circle is

$$r = 2R\sin\left(\frac{2\pi}{11}\right) \approx 1.1R$$

distance away from e_0. Thus, we see that there are only two electrons closer to e_0 than the one in the center.