

45. Consider a circle of radius  $r$ , inside the toroid and concentric with it (like either of the loops drawn in Fig. 30-20). The current that passes through the region between this circle and another larger radius circle (well outside the toroid) is  $Ni$ , where  $N$  is the number of turns and  $i$  is the current (note that this region includes a “slice” of the outer rim of the toroid). The current per unit length (of the circle) is  $\lambda = Ni/2\pi r$ , and  $\mu_0\lambda$  is therefore  $\mu_0 Ni/2\pi r$ , the magnitude of the magnetic field at the circle (call it  $B_1$ ). Since the field outside a toroid (call it  $B_2$ ) is zero, the above result is also the *change* in the magnitude of the field encountered as you move from the circle to the outside (say, to the larger radius circle mentioned above). The equality is not really surprising in light of Ampere’s law, particularly if the path used in  $\oint \vec{B} \cdot d\vec{s}$  is made to connect the circle in the toroid and the larger radius circle (or portions of each of them, of lengths  $\Delta s_1$  and  $\Delta s_2$ ). The connecting paths (each of size  $\Delta r$ ) between the circles can be made perpendicular to the magnetic field lines (so that  $\vec{B} \cdot \vec{s} = 0$ ). In fact, we can keep the connecting paths roughly perpendicular to  $\vec{B}$  and manage to have  $\Delta s_1 \approx \Delta s_2$  if our Amperian loop is very small (especially if  $\Delta r$  is much smaller than the outer radius of the toroid). Simplifying our notation, the current through the loop is therefore  $\Delta s\lambda$ , so Ampere’s law yields  $(B_1 - B_2)\Delta s = \mu_0\Delta s\lambda$  and  $B_2 - B_1 = \mu_0\lambda$ . What this demonstrates is that the change of the magnetic field is  $\mu_0\lambda$  when moving from one point to another (in a direction perpendicular to the field) across a current sheet (as the term is used in problem 39); this principle is useful in any discussion of boundary conditions in electrodynamics applications.