39. The current in R_2 is i. Let i_1 be the current in R_1 and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. We apply the loop rule to the left-hand loop to obtain

$$\mathcal{E} - iR_2 - i_1 R_1 - ir = 0 \; .$$

We apply the loop rule to the right-hand loop to obtain

$$i_1 R_1 - (i - i_1) R_V = 0 \; .$$

The second equation yields

$$i = \frac{R_1 + R_V}{R_V} \, i_1 \; .$$

We substitute this into the first equation to obtain

$$\mathcal{E} - \frac{(R_2 + r)(R_1 + R_V)}{R_V} \, i_1 + R_1 i_1 = 0 \; .$$

This has the solution

$$i_1 = \frac{\mathcal{E} R_V}{(R_2 + r)(R_1 + R_V) + R_1 R_V} \; .$$

The reading on the voltmeter is

$$\begin{aligned} i_1 R_1 &= \frac{\mathcal{E} R_V R_1}{(R_2 + r)(R_1 + R_V) + R_1 R_V} \\ &= \frac{(3.0\,\mathrm{V})(5.0 \times 10^3\,\Omega)(250\,\Omega)}{(300\,\Omega + 100\,\Omega)(250\,\Omega + 5.0 \times 10^3\,\Omega) + (250\,\Omega)(5.0 \times 10^3\,\Omega)} = 1.12\,\mathrm{V} \; . \end{aligned}$$

The current in the absence of the voltmeter can be obtained by taking the limit as R_V becomes infinitely large. Then

$$i_1 R_1 = \frac{\mathcal{E} R_1}{R_1 + R_2 + r} = \frac{(3.0\,\mathrm{V})(250\,\Omega)}{250\,\Omega + 300\,\Omega + 100\,\Omega} = 1.15\,\mathrm{V} \; .$$

The fractional error is $(1.12 - 1.15)/(1.15) = -0.030$, or -3.0%.