

29. (a) The current i is shown in Fig. 27-22 entering the truncated cone at the left end and leaving at the right. This is our choice of positive x direction. We make the assumption that the current density J at each value of x may be found by taking the ratio i/A where $A = \pi r^2$ is the cone's cross-section area at that particular value of x . The direction of \vec{J} is identical to that shown in the figure for i (our $+x$ direction). Using Eq. 27-11, we then find an expression for the electric field at each value of x , and next find the potential difference V by integrating the field along the x axis, in accordance with the ideas of Chapter 25. Finally, the resistance of the cone is given by $R = V/i$. Thus,

$$J = \frac{i}{\pi r^2} = \frac{E}{\rho}$$

where we must deduce how r depends on x in order to proceed. We note that the radius increases linearly with x , so (with c_1 and c_2 to be determined later) we may write

$$r = c_1 + c_2 x .$$

Choosing the origin at the left end of the truncated cone, the coefficient c_1 is chosen so that $r = a$ (when $x = 0$); therefore, $c_1 = a$. Also, the coefficient c_2 must be chosen so that (at the right end of the truncated cone) we have $r = b$ (when $x = L$); therefore, $c_2 = (b - a)/L$. Our expression, then, becomes

$$r = a + \left(\frac{b - a}{L} \right) x .$$

Substituting this into our previous statement and solving for the field, we find

$$E = \frac{i\rho}{\pi} \left(a + \frac{b - a}{L} x \right)^{-2} .$$

Consequently, the potential difference between the faces of the cone is

$$\begin{aligned} V &= - \int_0^L E dx = - \frac{i\rho}{\pi} \int_0^L \left(a + \frac{b - a}{L} x \right)^{-2} dx \\ &= \frac{i\rho}{\pi} \frac{L}{b - a} \left(a + \frac{b - a}{L} x \right)^{-1} \Big|_0^L = \frac{i\rho}{\pi} \frac{L}{b - a} \left(\frac{1}{a} - \frac{1}{b} \right) \\ &= \frac{i\rho}{\pi} \frac{L}{b - a} \frac{b - a}{ab} = \frac{i\rho L}{\pi ab} . \end{aligned}$$

The resistance is therefore

$$R = \frac{V}{i} = \frac{\rho L}{\pi ab} .$$

- (b) If $b = a$, then $R = \rho L / \pi a^2 = \rho L / A$, where $A = \pi a^2$ is the cross-sectional area of the cylinder.