Chapter 17: Red Giants and White Dwarfs
A Field Guide to the Stars

Outline

Summary

Chapter 17 is a particularly long chapter in terms of content as it contains many important concepts. It discusses methods for determining the major stellar characteristics, including spectral type, temperature, mass, size, luminosity, and apparent brightness. The H-R (Hertzsprung-Russell) diagram and regions within it are covered. Binary stars are introduced as well. The first two main methods of distance determination (which cover distances out to about 10,000 pc) are also explained.

Major Concepts

- Measuring nearby stars
  - Parallax and distance
  - Proper motion and transverse velocity
  - Radial velocity
- Stellar characteristics
  - Apparent magnitude
  - Absolute magnitude and luminosity
  - Temperature
  - Spectral classification
  - Size
  - Mass
- The H–R diagram
  - Axes
  - Star groups: main sequence, red giants, white dwarfs
  - Lines of equal radius
- Distance determination using the H–R diagram
- Binary stars
  - Visual
  - Spectroscopic
  - Eclipsing
  - Mass determination

Teaching Suggestions and Demonstrations

This chapter is packed with fascinating and fundamental information! Make sure you budget enough time for it. Lots of stellar characteristics are introduced. Emphasize not only *what* we know about stars but also *how* we know it. The challenge faced by astronomers in studying distant objects is enormous; present the techniques used as creative and innovative ways people have come up with to meet that challenge. Spend plenty of time on the H–R diagram. It is arguably one of the most powerful tools astronomers have; it also provides a wealth of information about stars.

Section 17.1

Begin by asking students to generate of list of **stellar characteristics** that they believe are of interest to astronomers. Make sure the final list contains all the major properties to be discussed in this chapter as well as those already discussed, including mass, temperature, color, brightness or luminosity, size, chemical composition, distance, and motion. Point out that some of these characteristics (like mass, size, temperature, and composition) are intrinsic properties of the star, whereas others (such as distance) depend upon the viewpoint of the observer. Also remind students of methods of determining these characteristics already discussed. Stellar spectra are used to help determine composition; the Doppler Effect provides information about one component of a star's velocity.

Stellar motion turns out to be more complicated than many students would think at first glance. Stars appear to move very slowly, yet we find they have high relative velocities of tens of kilometers per second. How is that possible? Of course we say this is caused by their great distance. Use the following example to explain this further. We often see a high-flying jet moving slowly across the sky. The jet appears to move slowly, although it is actually moving at around 600 mph. Imagine the same jet flying a few hundred feet overhead. It would sweep by in a matter of a second or two. The only difference between these two circumstances is the distance to the jet. We are not observing its true velocity but rather its angular velocity in the sky. High in the sky the jet appears to move at 1° or 2° per second. Close overhead it will appear to move at 50° or 100° per second!

The observed motion of a star with respect to the background stars has contributions from parallax (a result of Earth's motion) and **proper motion** (actual motion of the star perpendicular to our line of sight). Observing the same star at the same time of year over the course of many years will help to eliminate the parallax contribution. Then, proper motion (which is measured in arc seconds per year) has to be translated into speed. A rather mixed set of units are used in the proper motion equation

$$v_t = 4.7\ \mu \mathrm{d}$$

The distance d is in parsecs, μ is in arc seconds per year, and v_t is in kilometers per second! The justification for this equation comes from the above-shown triangle except now, the star is actually moving a distance, which is given in kilometers. Because of the division by time, the distance is seen as a velocity and the angle is now an angular velocity given in the form of the proper motion. Try it!

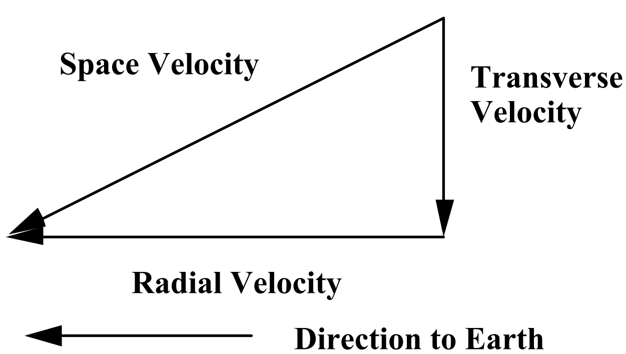
$$v_t = \frac{\text{Distance (km)}}{\text{time (s)}} = \mu \frac{\text{arc sec}}{\text{year}} \frac{\text{rad}}{206265} \frac{1 \text{ year}}{3.156\times10^{7} \text{ s}} \mathrm{d(pc)} \frac{3.09\times10^{13} \text{ km}}{\text{pc}} = 4.7$$

Ask students to imagine two stars, one twice as far from us as the other, with identical velocities perpendicular to the line of sight. Will the two stars have the same proper motion, and if not, which one will be greater? ➲ **DEMO** To demonstrate: line up three students in the front of the classroom, one on each side and one in the middle. One "end" student represents Earth. She holds an arm out straight so that the other two students are lined up with it. Then, the other students each walk the same number of paces perpendicular to the line of students. With her other arm, Earth student points to the near student and then to the far student. The angle between her arms clearly gets smaller when she points to the farther student. If the distance to a star is known, then proper motion can be converted into **transverse velocity**.

To complete the story remind students of the discussion of the Doppler Effect from Section 3.5. The change in wavelength (or frequency) can be used to determine a star's **radial velocity**. The star's total velocity can be found from these two perpendicular components, radial velocity and transverse velocity. Now, consider the fact that Earth itself is moving, carrying its observers with it. If the star's motion with respect to, say, the center of the galaxy is desired, then the motion of our planet must also be taken into consideration.

Finally, the **space motion** is a velocity *relative to the motion of the Sun*. Determining the space motion of a star is similar to finding the relative speed of a car next to you while driving down the freeway; it will not be the car's actual speed, just how much faster or slower it is moving than you.

The proper motion gives the transverse velocity (also known as the tangential velocity). This motion is perpendicular to the radial velocity. For any particular star, the radial velocity will be directed towards or away from the Earth. The space motion is given by solving the following triangle using the Pythagorean Theorem.



Returning to the example of cars on a freeway, if you find the car next to you is traveling 5 mph faster than you, and you are going 65 mph, then the other car's true speed is 65 + 5 = 70 mph. The true velocity of a star can be determined by adding (or subtracting) its space velocity to the Sun's velocity in space (about 220 km/s). Whereas the space velocities calculated above are typically a few tens of kilometers per second, the true velocities are typically a couple of hundred kilometers per second.

Section 17.2

The connection among the three quantities, **luminosity, temperature,** and **size** is discussed in this and the following sections. You may wish to go ahead and present this relationship at the beginning of your discussion so that students can keep it in mind as you examine each property individually.

Ask your students if they can name any bright stars. Then, look them up in Appendix 3 and compare their distances. Sirius, in the constellation Canis Major, is only 2.7 pc away, but Rigel, in Orion, is 240 pc away. Given this information, ask them to make a guess as to how the two stars would compare if they were the same distance from us. This exercise will help students understand the importance of distance in determining the **absolute brightness** of a star when the **apparent brightness** is the observed quantity. If you can darken the room sufficiently, hold identical light sources at different distances and have students qualitatively compare apparent brightness. ➲ **DEMO** The **Inverse-Square law** can be easily demonstrated using a standard frosted light bulb, turned on edge, and any simple phototube and meter. Many student optics experiments in physics have these components available. Do the demo in a darkened room and show how the meter readings decrease inversely with the square of the distance. Replace the bulb with a low power laser and repeat the demonstration. This time, however, the law is not followed. Explain that this law is only correct for spherical luminous bodies. Beamed radiation will get fainter with distance, but not as quickly, and will not necessarily follow this law.

The **magnitude scale** is very useful and also often difficult for students to understand. First, it is confusing because brighter stars have lower numbers for their magnitudes. Second, it is not linear; a difference of 5 magnitudes represents a factor of 100 times in brightness. (Briefly discuss the history of the magnitude scale and the nature of our "light-detectors," otherwise known as eyes.) Distinguish between **apparent magnitude** and **absolute magnitude**, and compare to the apparent and absolute brightness discussed above. Before introducing the equation in *More Precisely 17-1*, work through some examples that can be done with the "5 magnitudes corresponds to a factor of 100 in brightness" rule. For example, find the absolute magnitude of a star that is 100 pc away (10 times the distance of the standard 10 pc) and has an apparent magnitude of +6. Point out that 10 times farther away means an object is 100 times dimmer (inverse-square law) and 100 times dimmer in brightness corresponds to 5 magnitudes. So, the star has an absolute magnitude of +6 – 5 = +1. (Note that the 5 is subtracted because the star's absolute magnitude is brighter than its apparent magnitude because it is farther away than 10 pc.) Once you have done some examples like this one, check them with the equation in *More Precisely 17-1*. For the example just given, the equation reads

$$\text{apparent magnitude} - \text{absolute magnitude} = 6 - 1 = 5 \log_{10} (\text{distance}/10 \text{ pc}),$$

which is indeed correct for a distance of 100 pc. Reasoning through simple examples first can often help students understand formulas that may otherwise be intimidating. Having convinced students of the validity of the equation, apply it to problems that are harder to do in one's head to demonstrate the equation's usefulness.

Finally, before leaving the discussion of magnitudes, return to Appendix 3 and examine the data for some of the stars. For instance, students can compare apparent magnitude to absolute magnitude for a star closer than 10 pc and for another farther than 10 pc. While you are there, ask students to explain the difference in proper motions in the two tables: The proper motions of the brightest stars are mostly all less than 1 arc second/year, but the proper motions of the nearest stars are mostly all greater than 1 arc second/year. For practice, you can even have students use the distance and proper motion to calculate the transverse velocity and compare to the value given in Appendix 3.

Section 17.3

Stellar spectra are also related to temperature. Touch on the history of the classification scheme in order to explain the odd order of the letters. Be sure to include Annie Jump Canon (1863–1941), who became the supervisor of a team of women working in astronomy at Harvard

University and known as “computers.” Her version of the classification scheme is the one we use today, and she is responsible for the classification of some 250,000 stellar spectra!

Examine Figure 17.10, which shows **stellar spectra** for each spectral class. Point out the hydrogen Balmer lines and address some of the other differences among spectral classes. Spectral lines from molecules are only apparent at the lower temperatures; molecules would not be able to stay together at the higher temperatures!

Have a contest in your class to come up with a mnemonic to remember the order of the classes. An example other than the one given in the text is “Oh Bother, Astronomers Frequently Give Killer Midterms!” The 10 subdivisions of each spectral type is sometimes a bit confusing to the uninitiated. Be certain to show some of the entire sequence, such as .... A8, A9, F0, F1, F2, F3, F4,... F9, G0,.... This continues towards lower temperatures.

One of the most important concepts regarding **stellar temperature** is that surface temperature determines a star’s **color**. Review the blackbody curves introduced in Chapter 3 and discuss Table 17.1. Contrary to the everyday phrase “red-hot,” red stars are a lot *cooler* than blue stars. (Remind students of a match or candle flame.)

There is one additional specification needed when referring to any type of magnitude; where in the spectrum is the magnitude being observed? The region of the spectrum is specifically defined by use of filters that pass a band of wavelengths a few hundred Å wide. Instead of referring to apparent magnitudes as *m*, astronomers use the symbol for the filter, e.g. U, B, or V. In the case of absolute magnitudes, the capital *M* is used but is subscripted with the appropriate symbol, e.g. M_V. Figure 17.9 shows how sampling the spectrum through just two filters, B and V, allow the temperature to be determined. This is a very efficient process, since the measurement at the telescope may take only a few seconds to a few minutes. Recording the entire spectrum can take much longer.

Astronomers always express the color index of stars as the difference B - V. In the example given in the text, for a 10,000 K star, the B and V intensities are the same. The magnitude equation gives B - V = 0. This relates back to the zero-point of the magnitude system which is set at zero for all filters for an A0 star (which has a temperature of 10,000 K). For the 3,000 K star the B to V ratio is 1/5. The resulting B - V is 1.75. The B - V scale runs from about -0.5 for the hottest stars to about 2.5 for the coolest stars. Besides the speed in observation, another advantage of the color index over the spectral types for determining temperature is that the color index is a continuous numerical scale, whereas there are only 70 discreet spectral types and there can be significant temperature differences between some of the types.

Section 17.4

The **Radius-Luminosity-Temperature Relationship**, given as a proportionality in the text, is a fundamental radiation law used extensively in astronomy. It comes simply from Stefan’s law (see Chapter 3; also known as the Stefan-Boltzmann law) applied to a spherical body such as a star. Stefan’s law gives the energy emitted per square meter per second. If this is multiplied by $4\pi R^2$, the surface area of a sphere of radius *R*, then the total luminosity of the star is determined. $L = \pi R^2 \sigma T^4$, $\sigma = 5.670\text{x}10^{-8}$ J/m^{2}K^{4}s. This relationship can also be put in solar units, where *L*, *R*, and *T* are compared to the Sun’s values. This is commonly done for convenience and they are regularly used units in astronomy. This simplifies the relationship (all in solar units) to

$$L = R^2 T^4$$

For example, Betelgeuse, with a radius of 300 times the Sun's radius and 0.59 of its temperature, the luminosity is calculated to be $L = (300)^2(0.59)^4$ or 11,000 solar luminosities. Betelgeuse is obviously an intrinsically bright star and at a distance of 120 pc it is not surprising that it is one of the brightest stars in the sky.

Although this radiation law is very useful, it does have some limitations. The luminosity depends strongly on the temperature; any significant error in the temperature will be raised to the fourth power and give a poor result. The radius depends only on the square of the temperature, so the situation is not as bad, but still the temperature needs to be known as accurately as possible. Lastly, astronomers must be very careful not to use this equation when the object is not spherical.

Finally, consider the **sizes** of stars. Figure 17.12 is an excellent one to use to illustrate sizes. The range of sizes is quite impressive. Preview coming attractions by asking students to guess what factors might account for the variety of sizes of stars.

As large as stars are, they are all too far away to make any kind of direct measurements of their diameters. The difficulty of resolving the image sizes of the nearest stars is realized by asking what the angular size of the Sun would be at one parsec. The diameter of the Sun is 1.34×10^{11} cm. One parsec is 3.09×10^{18} cm. Taking the ratio of these two numbers and multiplying by 206,265 to convert the answer to arc seconds gives 0.0093" or about 0.01". The atmosphere of the Earth blurs stellar images to 0.5" or more.

A star like the Sun 100 times farther away will appear 100 times smaller than this, or 0.0001". A star 100 times larger than the Sun, at one parsec, will appear 100 times larger or 1". Of course, no star is as close as one parsec and large stars are rare and generally much farther away. Betelgeuse, shown in Figure 17.11, is a good example. At a distance of 150 parsecs, Betelgeuse would appear as 0.000067" if it were the same size as the Sun. However, it is 300 times the size of the Sun and therefore appears to have a diameter of 0.02". Speckle interferometry is able to resolve detail a little smaller than this, so Betelgeuse and some of its detail can be seen. But relatively nearby stars that are this large are certainly the exception and most stars cannot be resolved this way.

A useful and convenient comparison is that of the radius of the Sun to the astronomical unit. This ratio is 1/215. A star about 200 times the size of the Sun has a size the same as the orbit of the Earth. At 300 times the size of the Sun, like Betelgeuse, the size is that of the orbit of Mars (1.5 A.U.). So the equivalence of 1 A.U. ≈ 200 solar radii is helpful to remember.

Section 17.5

So far you have been discussing individual properties of stars. Now it is time to look at relationships between properties. Before introducing the **Hertzsprung-Russell (H-R) diagram**, give an example of how a relationship between two properties of a population could be examined. For instance, ask students to consider all the children in an elementary school and to imagine finding the height and age of each one and then plotting each one as a dot on a height vs. age diagram. Call a student to the board to sketch his predication of what the diagram might look like. Briefly discuss the exercise and the outcome, and note that the collection of dots may follow a general trend (older is taller), but the dots probably won't fall exactly on a line. Are there exceptions to the general trend?

Next, present students with the axes of the **H-R diagram** and ask them to make a guess at the outcome if a large number of stars were plotted. Non-science students often are quite inexperienced with two dimensional classification, as is done in the H-R diagram. They often confuse brightness and temperature as being one and the same. Project a large H-R diagram during lecture and review the general properties of stars in the 4 quadrants of the diagram; bright and hot, bright and cool, faint and hot, faint and cool.

Be sure to emphasize that on the temperature (horizontal) axis, hotter is on the left and cooler is on the right. This will probably seem backwards to them, but it is important to emphasize so that they can compare their guesses with an actual H-R diagram. On the luminosity (vertical) axis, brighter is at the top and dimmer is at the bottom. Once students have sketched a guess, show an actual diagram. Try overlaying the transparences of Figures 17.14 and 17.15, which show H-R diagrams of the nearest and the brightest stars, respectively. (You can return to these later and discuss why these two selected populations have different diagrams.) Your students will very likely have predicted the trend of the **main sequence**, though they may not have the shape right. The relationship shown is that hotter stars are brighter. Next, introduce the other two main regions, the white dwarfs and the red giants, and discuss how they are exceptions to the "brighter = hotter" trend. Engage your students in speculation about why this might be. What could make a cool star appear bright, or a hot star appear dim? The answer is size.

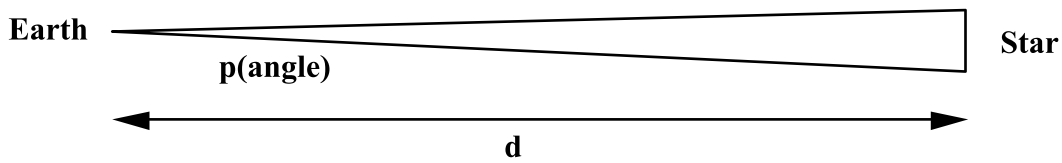
Adding a third characteristic, **stellar size**, to a two-dimensional plot is very confusing, so begin with an example. Ask students to consider, for instance, an A or a B star that is the *same size* as the Sun. Each square meter of the A star surface would be brighter than a corresponding square meter of a Sun surface. So, if the two were the same size, the A or B star would clearly be brighter. (Compare the luminosities of the Sun and Procyon A or Altair on Figure 17.14.) Next, imagine two stars with the same luminosities but different colors, such as Sirius A and Arcturus in Figure 17.15. Arcturus is cooler, and therefore each square meter of its surface is dimmer than a square meter of the surface of Sirius A. Therefore, if it has the same luminosity, it must have many more square meters of surface to make up for each one being dimmer. Indeed, it lies above the 10 solar-mass line on the H-R diagram and Sirius A is just above the 1 solar-mass line. Compare an H-R diagram (*luminosity* vs. *temperature*) with the lines of constant *radius* drawn in to the *radius-luminosity-temperature relationship* given in Section 17.4.

Section 17.6

Determining **distance** is clearly a challenge in astronomy. We have no depth perception when we look up at the night sky; all the stars appear as points of light on the celestial sphere. This chapter begins with a review of stellar parallax and presents it as a rung on the "cosmic distance ladder" (see Figure 17.17). Later chapters add more rungs to the ladder, until the final rung is discussed in Chapter 24. Point out to students now that different methods are necessary for measuring different distances, just as different methods and tools are used to find the thickness of a piece of paper, the length of a table, the height of a building, the distance across town, and the distance across the country. You might want to mention the term **standard candle** in this chapter even though it is discussed in more detail in chapter 24.

Review the demonstrations of parallax from Chapter 1. That was 16 chapters ago and students will get a little rusty; parallax is a very important concept in the study of stars. Have students repeat the demonstration, in which they hold a finger out and sight beyond it to the far wall, first with one eye closed and then the other. Then show Figure 17.1 and use it to define the related unit of distance, the **parsec** (pc), and to derive the formula relating distance and parallactic angle. Using geometry (arc length/radius equals central angle in radians) or trigonometry (for small angles, $\sin \theta \approx \theta$) derive the distance to an imaginary object with a parallax of 1 arc second in kilometers and compare to the conversion given in Appendix 3. Student should also try to remember the relationship between parsecs and light-years, as these two units are often used with the same objects. Parallax is useful for finding the distances to nearby stars; show Figure 17.2 to give some examples of stars in our neighborhood.

Another way of looking at how the parsec and the parallax equation for distance is derived, consider for a moment the "skinny" triangle made by the apparent motion of the star, the distance to the star, and the Earth at the apex of the smallest angle.



The two angles at the star end are one arc second or less than being 90°. The angle at the Earth end, *p*, will be one arc second or less. In reality, though, it is not the star that has moved, but rather the Earth. The Earth has moved 1 A.U. (It actually moves 2 A.U. but the equation uses a baseline of 1 A.U. by convention.) Simple trigonometry tells us that the small side will equal the distance times the angle,

$$1\text{ A.U.} = d \times p$$

where the angle is measured in radians. One radian is equal to about 57.3° or 206,265 seconds of arc. We must divide *p* by 206,265 if we wish to use seconds of arc instead of radians. Solving for *d*:

$$d = \frac{1\text{ A.U.}}{p(\text{rad}) \div 206265}$$

This gives the distance to the star in A.U. By defining the parsec to be equal to 206,265 A.U., we have,

$$d(\text{pc}) = \frac{1}{p(\text{arc sec})}$$

The smallest parallax measured is about 0.005". This gives a distance of 200 pc. However, what if the uncertainties in that angle are ± 0.002"? The parallax might be as low as 0.003" or as high as 0.007". So the uncertainty actually gives the distance in the *range* of 140 to 330 pc! Although astronomers can measure parallaxes accurately, there remain very large uncertainties for measured distances at 200 pc and beyond. Knowing uncertainties is a very important part of any measurement. Without knowledge of the uncertainty we have no knowledge of the accuracy of the measurement. We may deceive ourselves into thinking we know a distance is, say, 200 pc when in fact our measurements have only succeeded in determining the distance to within some range of values. The true value may not be 200 pc at all. *Hipparcos* data has allowed a vast improvement in the uncertainties of parallax measurements. Within an uncertainty of 5%, ground-based astronomers had measured only about 100 stars. *Hipparcos* has now provided over 7000 stars at this accuracy. Before, distances over 50 pc were very uncertain. Now distances out to 150 pc are very well known.

The H-R diagram forms the next rung in the cosmic distance ladder with a method called **spectroscopic parallax**. Project an H-R diagram and demonstrate how knowing a star's spectral class leads to a determination of its luminosity or absolute magnitude, which, combined with its apparent brightness or magnitude, yields the distance. Ask students if they can pick out the difficulty with this method. What about a K star? Its luminosity will be very different if is determined to be a main sequence star or a red giant. If you have access to a set of standard stellar spectra, show students the spectra of stars with the same spectral type (for instance, K2) but different luminosity classifications (such as V vs. I.) They will be able to tell a difference in line widths.

Section 17.7 and 17.8

The final sections of this chapter discuss **stellar masses**. Because **binary star systems** are so useful in determining stellar masses, binaries are also introduced in this section. Mass determinations from binary star systems provide an excellent opportunity to review and practice Kepler's third law. In discussing the types of binaries (visual, spectroscopic, and eclipsing), be sure to emphasize that the different classifications refer to how we perceive the stars, not to any differences in intrinsic characteristics.

Binaries are **eclipsing** if their orbital plane happens to lie edge-on to our line of sight. Examine the binary light curve in Figure 17.21 and discuss each dip in the light intensity. Then, draw some different examples and see whether students can figure out why the curves are different. For instance, if the eclipse is not total, the dips will be pointed rather than flat at the bottom. If a main sequence star is orbiting a larger red giant, the dip corresponding to the smaller star being in front will be greater than that corresponding to the smaller star in back, unlike Figure 10.21.

Because **lifetime** is related to mass, this property is also considered. The connections among mass, luminosity, and lifetime will become more apparent in chapters on stellar evolution.

As you wrap up this chapter, return to the list of stellar properties generated by students in the beginning. Briefly touch on each one. Can each property be determined for at least some stars? How? Many of these properties are given in terms of the Sun. For instance, mass is often given as a certain number of solar masses instead of in kilograms. Conclude the chapter by listing all the properties of the Sun, as they are the basis of measurements of other stars as well.

Student Writing Questions

1. Imagine what it would be like for the Earth to orbit one of the typical stars found near the Sun, say the star named Ross 154. Refer to information provided in this chapter and in the tables at the back of the text to find the properties of this star. Pay close attention to its luminosity and spectral type. Describe what the environment of the Earth would be like orbiting this star.

2. About half of all stars in the sky are binary. Imagine the Earth orbiting the binary star named 61 Cygni. Refer to information provided in this chapter and in the tables at the back of the text to find the properties of this star. Pay close attention to its luminosity and spectral type. Describe what the environment of the Earth would be like orbiting this star. In particular, give details about the day-night cycle and how the two "suns" would appear.

3. If you had the choice of visiting any type of star or star system studied in this chapter, which would it be? What is it about this object that fascinates you? Do you think you would have to travel far to find one of these?

4. Astronomers used to measure stellar brightnesses one at a time at the telescope using a photometer. Now, CCDs allow this to be done for whole fields of stars in about the same amount of time. Discuss how these changes have increased the amount of data astronomers now deal with. How many stars might there be on one CCD image? CCDs are also more

sensitive to light. How many more stars can astronomers now observe? What will be possible with the new, large telescopes being built?

5. Give all the observations necessary to plot just one star in the H-R diagram. Give the details of the observations as they would be made from a traditional observatory on earth. How much time would it take to make all these observations? How much longer would it have taken 80 years ago when Hertzsprung and Russell did this work?

Answers to End of Chapter Exercises

Review and Discussion

1. Parallax is the apparent motion of a nearby object due to the change in the viewing position of the observer. Astronomers view a nearby star from opposite sides of the Earth's orbit. The position of this star appears to change, relative to background distant stars. The amount of this motion is inversely proportional to the distance. The inverse of this motion measured in arc seconds is equal to the distance measured in parsecs.

2. If an object had a parallactic angle of one arc second, its distance would be exactly one parsec. Conversely, one Astronomical Unit will subtend an angle of one arc second at a distance of one parsec. The parsec is equal to 206,265 A.U.

3. A star's real space motion is observed as two components; the radial velocity and the proper motion. The radial velocity is just the star's motion towards or away from us. The proper motion is an angular motion measured in seconds of arc per year. If the distance to the star is known it can be converted into the true transverse velocity. The transverse velocity and radial velocity can be combined to obtain the true space motion. The transverse velocity is determined from the star's proper motion and its distance. Once both of the velocity components are known, the space motion is calculated using the Pythagorean theorem: $(\text{space motion})^2 = (\text{radial velocity})^2 + (\text{transverse velocity})^2$.

4. Stellar luminosities depend, in part, on the size of the star. Large stars have low pressure and narrow absorption lines; small stars have high pressure and wide absorption lines. High luminosity stars are typically giants and supergiants; their lines are narrow. Main sequence stars have lower luminosity and wider lines.

5. There are several methods to measure stellar radii. First, they can be determined by measuring the effective temperature and luminosity of a star. Using the relationship between radius, temperature and luminosity, the radius is calculated. Radii can also be measured directly using eclipsing binaries. The timings of the length of the eclipses provide data on the size of the stars.

6. Giants can be tens of times, up to about 100 times, larger than the Sun. Main Sequence dwarfs are about 10 times smaller than the Sun. White dwarfs are 100 times smaller than the Sun. Giants are brighter than the Sun. Main Sequence dwarf stars are fainter and less massive than the Sun. White dwarfs may have a mass about the same as the Sun or a little more or less, but they are much fainter than the Sun.

7. The apparent brightness of a star depends on its intrinsic brightness and its distance; it is what the astronomer always measures. The absolute brightness is a measure of the star's intrinsic brightness. Using the magnitude system, the absolute magnitude is the magnitude of the star if it were at a distance of 10 parsecs. For the absolute magnitude to be calculated, the apparent magnitude and distance to the star must be known.

8. The temperatures of stars are measured photometrically by using the B and V filters. The brightnesses are compared and matched to a blackbody curve of a specific temperature.

9. The absorption spectra of stars depends strongly on temperature. The temperature determines which elements produce absorption lines in the visible spectrum. Spectra are classified as either O, B, A, F, G, K, or M. The O type is the hottest and the M type is the coolest. Within each of these types is a numerical sub-classification ranging from 0 to 9, e.g. F0, F1, F2, ... F8, F9, G0, G1, ... For a specific spectral type, the number 0 is the hottest and 9 is the coolest. To classify a star's spectrum, the absorption lines have to be identified. They are then matched to the corresponding spectral type. A spectrum with strong ionized helium lines is type O; one with strong hydrogen lines is type A, and so forth.

10. There are two conditions that make hydrogen lines weak in a stellar spectrum; the star is too hot or too cold. Too hot and most of the gas is ionized, making transitions from the second level very rare. Too cold and all the electrons are in the ground state and unable to absorb the energy.

11. The Hertzsprung-Russell (H-R) diagram is a plot of stars' absolute magnitude against spectral type. Each star to be plotted must have its spectral type determined. The apparent magnitude must be observed and the distance determined by some method such as parallax so that the absolute magnitude can be calculated. Absolute magnitude is on the vertical scale, with the brightest end of the scale at the top. Spectral types are on the horizontal scale, with O-type on the left and M-type on the right. Note that the corresponding temperature scale goes from hottest to coolest from left to right. See Figures 17.11, .12, or .13 for examples.

12. About 90% of all stars plotted in the H-R diagram are found along a narrow S-shaped band running diagonally from upper left to lower right. This is the main sequence. Stars along the main sequence all have a common source of energy, the fusion of hydrogen into helium. The main sequence exists because stars have different masses. The most massive are in the upper left end while the lowest mass stars are in the lower right end. The Sun is in about the middle of the main sequence. See Figures 12.11 and 12.12 for examples.

13. If an observation of a star can place it in the H-R diagram *without* the distance being known, then the star's absolute magnitude can be compared to its apparent magnitude and its distance calculated. The spectrum of a star will determine the spectral type, e.g. K5. But is this a main sequence star, giant, or supergiant? Closer examination of the absorption lines allows astronomers to distinguish between the luminosity classes. With this, the absolute magnitude is known and the distance is then calculated.

14. The brightest stars in the sky also happen to be intrinsically bright stars. Although seen at relatively large distances, they still appear bright. The H-R diagram of the brightest stars fills the upper parts of the diagram, especially the giants and supergiants. The H-R diagram of the nearest stars is a correct stellar census in which low mass stars dominate and the highest mass stars are rare. These low mass stars populate the lower main sequence and the region of white dwarfs.

15. The most commonly occurring stars in the H-R diagram are M-type main sequence stars. About three-quarters of all stars are of this type. However, these are not the stars that we commonly see with our eyes or even with telescopes. The most commonly seen stars are those with high intrinsic brightness, which can be seen over large distances. M-type main sequence stars are intrinsically faint and are difficult to detect.

16. The least common stars in the Galaxy are the massive stars. They evolve too fast.

17. The mass of stars in binary systems can be determined using Kepler's third law. If the period of the orbit and the semi-major axis can be observed, then the sum of the two stellar masses can be directly calculated. If the center of mass of the system can also be determined, then the individual masses can be calculated. Generally, the complete solution can be done with visual and eclipsing binaries. Spectroscopic binaries provide only partial information on the masses.

18. The lifetime of a star does not only depend on the amount of fuel available to it; it also depends on how fast it uses that fuel, given by its luminosity. One star may have 10 times as much mass (fuel) as another star, but it also uses that fuel 1000 times faster. The net result is the more massive star having a lifetime 100 times shorter than the low mass star. It pays to be an underweight, among stars!

19. If the star is in the upper main sequence, it must be relatively young. But in the lower parts, they can be much older and still look the same.

20. Visual binaries must be both well separated and nearby in order to be seen as two stars. Eclipsing binaries must be aligned almost exactly edge-on for eclipses to occur.

Conceptual Self-Test

1. F
2. F
3. T
4. F
5. T
6. F
7. T
8. F
9. T
10. T
11. A
12. D
13. D
14. A
15. C
16. B
17. C
18. B
19. C
20. D

Problems

1. The distance in parsecs is 1 / parallax. For Spica, its distance is 1/ 0.012 = 83 pc.

   Neptune orbits the Sun at 30.1 A.U. so the parallax would be 30.1 times larger, or 0.36".

2. Using the relationship $v_t = 4.7\mu d$, $v_t = 4.7 \times 0.5 \times 20$, $v_t = 47$ km/s.

   A redshift of 0.01 percent is a shift of $\Delta\lambda / \lambda = 0.0001$. Multiplying by the speed of light gives 30 km/s. The space velocity of the star is $v^2 = 47^2 + 30^2$. $v = 56$ km/s.

3. In *solar units*, the radius-luminosity-temperature relationship is $L = R^2T^4$. Using the values provided, $L = 3^2(10{,}000 / 5800)^4$. $L = 80$ solar luminosities.

4. As in the previous problem, using the values provided, $64 = R^2 2^4$. $R = 2$ solar radii.

5. The first question to ask is "How much brighter is star B than star A?" This is easy, it is 4.5 / 0.5 = 9 times brighter. This is how they would appear if at the same distance from us. But both stars appear to be the same brightness. Obviously star B, being intrinsically brighter than star A, must be farther away than star A. But how much farther away must it be? It must be dimmed by a factor of 9. Using the inverse square law, making a star 3 times farther away makes it 9 times fainter, so star B must be 3 times farther away than star A.

6. Since they appear the same brightness but A is intrinsically brighter, it must also be more distant. A is 5 magnitudes brighter than B, which is exactly a factor of 100 in brightness. Using the inverse square law, A must be 10 times farther away than B. (Notice, this problem is best solved using concepts rather than formal equations. This is good practice for students to do.)

7. At 1 A.U. the solar constant is 1400 Watts/m^2. One parsec is equal to 206,000 A.U. A distance of 10 pc is 2,060,000 A.U. Using the inverse square law, the solar energy flux will be reduced by the square of this distance. $1400 / (2.06\text{x}10^6)^2 = 3.3\text{x}10^{-10}$ W/m^2. It is smaller by $1 / (2.06\text{x}10^6)^2 = 2.4\text{x}10^{-13}$.

8. Using the magnitude equation given earlier, $m_1 - m_2 = 2.5\text{Log}(L_2/L_1)$, $6 - (-27) = 2.5\text{Log}(L_2/L_1)$, $(L_2/L_1) = 1.6\text{x}10^{13}$. This is a factor of 16 trillion.

9. Using the $m - M = 5\text{Log}(d) - 5$ gives $4.0 - M = 5\text{Log}(100) - 5$, $M = -1.0$.

10. Using the same relationship as the previous problem, $10.0 - 2.5 = 5\text{Log}(d) - 5$, $d = 316$ pc.

11. (a) The Sun has an absolute magnitude of 4.85. The binocular limit is apparent magnitude 10. $10 - 4.85 = 5\text{Log}(d) - 5$, $d = 100$ pc. (Note: round-off is necessary because the limits are very approximate. You could use an absolute magnitude of 5 without significant error.)

    (b) $18 - 5 = 5\text{Log}(d) - 5$, $d = 4{,}000$ pc.

    (c) $26 - 5 = 5\text{Log}(d) - 5$, $d = 160{,}000$ pc.

    (d) $30 - 5 = 5\text{Log}(d) - 5$, $d = 1{,}000{,}000$ pc.

12. L_1 is the luminosity of the brighter star and during the first eclipse $L_1 = 0.99(L_1 + L_2)$, $L_1 = 100\, L_2$. Area is proportional to R^2. $(R_2/R_1)^2$ is the fraction of the area of star 1 that is covered during the second eclipse. The light seen then is $L_1(R_2/R_1)^2 + L_2 = 0.10(L_1 + L_2)$, substituting for L_1 and solving for R_2/R_1 gives 0.33. If the masses are proportional to the radii then $m_2/m_1 = 0.33$.

13. Using Kepler's third law, $P^2 = a^3/(m_1+m_2)$. Substituting the values given:

$$(25 / 365)^2 = (0.3)^3 / (m_1+m_2)$$

$$m_1+m_2 = 5.8 \text{ solar masses}$$

But $m_2 = 1.5m_1$ so $2.5m_1 = 5.8$, $m_1 = 2.3$ and $m_2 = 3.5$ solar masses.

14. (a) The 0.2 solar mass means that this star has 5 times less mass or fuel than the Sun; it should last 5 times less than the Sun considering this factor alone. But it also has a luminosity of 0.01 solar luminosities which means it is putting out 100 times less energy. That also means it is using its fuel 100 times slower than the Sun, so considering this factor alone, it should last 100 times as long as the Sun. The result, 100 times longer times 1/5 as long = 20 times longer than the Sun or 200 billion years.

    (b) Using the same type of argument as in part (a), 3 solar masses means 3 times as long. 30 solar luminosity means using its fuel 30 times faster and lasting 1/30 as long as the Sun. The result is 3 x 1/30 = 1/10 as long as the Sun or 1 billion years.

    (c) Similarly to part (b), 10 x 1/1000 = 1/100 as long as the Sun. The result is 100 million years.

15. (a) A 1-meter telescope has a limiting magnitude of 18. The absolute magnitude of the faintest star at 50,000 pc by this telescope is $18 - M = 5\text{Log}(50{,}000) - 5$, $M = -0.49$. Use $m_1 - m_2 = 2.5\text{Log}(L_2/L_1)$ to calculate the luminosity (L_1 is the solar luminosity). $4.85 - (-0.49) = 2.5\text{Log}(L_2/L_1)$, $L_2/L_1 = 137$. Since $L = M^4$ in solar units, solve for M. $137 = M^4$, $M = 3.4$ solar masses.

    (b) The limiting magnitude for the *Hubble Space Telescope* is 30. Similarly, $M = 11.5$, $L_2/L_1 = 0.0022$. $M = 0.22$ solar masses.

Resource Information

Student CD Media

Movies/Animations
The Inverse-Square Law

Interactive Student Tutorials
Hertzsprung-Russell Diagram
Binary Stars - Radial Velocity Curves
Eclipsing Binary Stars - Light Curves

Physlet Illustrations
Brightness vs. 1/Distance^2 – Magnitude Scale
Blackbody and Stellar Temperature

Transparencies

T-152	Figure 17.1	Stellar Parallax	p. 438
T-153	Figure 17.2	The Solar Neighborhood	p. 440
T-154	Figure 17.5	Inverse-Square Law	p. 442
T-155	Figure 17.7	Apparent Magnitude	p. 444
T-156	Figure 17.10	Stellar Spectra	p. 447
T-157	Table 17.1	Stellar Colors and Temperatures	p. 448
T-158	Table 17.2	Stellar Spectral Classes	p. 449
T-159	Figure 17.12	Stellar Sizes	p. 450

T-160	Figure 17.14	H-R Diagram of Nearby Stars	p. 453
T-161	Figure 17.15	H-R Diagram of Bright Stars	p. 453
T-162	Figure 17.17	Stellar Distances	p. 456
T-163	Figure 17.19	Visual Binary	p. 458
T-164	Figure 17.20	Spectroscopic Binary	p. 459
T-165	Figure 17.21	Eclipsing Binary	p. 459
T-166	Table 17.5	Measuring the Stars	p. 461

Materials

Light sources, a light meter, and lasers are available from Edmund Scientific.

A Second Atlas of Objective-Prism Spectra, by Nancy Houk and Michael Newberry, provides standard spectra for the different spectral and luminosity classes.

Suggested Readings

Berman, Bob. “Magnitude cum laude.” *Astronomy* (Dec 1998). p. 92. Describes the stellar magnitude system.

Boss, Alan P. “The birth of binary stars.” *Sky & Telescope* (June 1999). p. 32. Describes our knowledge of the formation of binary star systems.

Kaler, James B. “Stars in the cellar: classes lost and found.” *Sky & Telescope* (Sept 2000). p. 38. Discusses the development of stellar spectral classes, including the recently added classes L and T.

MacRobert, Alan M. “The spectral types of stars.” *Sky & Telescope* (Oct 1996). p. 48. Discusses stellar spectra, including spectral class, luminosity class, and peculiar spectra.

Perryman, Michael. “*Hipparcos*: the stars in three dimensions.” *Sky & Telescope* (June 1999). p. 40. A summary of findings from the *Hipparcos* mission about a variety of topics including: the bending of starlight, stellar oscillations, dark matter searches, distances to the Hyades and Pleiades, and cosmology.

Schilling, Govert. “A hundred million points of light: Sloan Digital Sky Survey.” *Nature* (Oct 5, 2000). p. 557. Describes the goals of the Sloan Digital Sky Survey.

Tanguay, Ronald Charles. “Observing double stars for fun and science.” *Sky & Telescope* (Feb 1999). p. 116. Describes projects for observing binary stars.

Tomkin, Jocelyn. “Once and future celestial kings: calculating a star’s past and future brightness.” *Sky & Telescope* (Apr 1998). p. 59. Discusses the connection between distance, proper motion, and the apparent brightness of stars, including results from the *Hipparcos* mission.

Trefil, James. “Putting stars in their place.” *Astronomy* (Nov 2000). p. 62. Discusses the development of the Hertzsprung-Russell diagram.

Notes and Ideas

*Class time spent on material: Estimated:*_______ *Actual:*______________

Demonstration and activity materials:

Notes for next time: