

# CHAPTER 22

## Quick Quizzes

1. (a). In part (a), you can see clear reflections of the headlights and the lights on the top of the truck. The reflection is specular. In part (b), although bright areas appear on the roadway in front of the headlights, the reflection is not as clear and no separate reflection of the lights from the top of the truck is visible. The reflection in part (b) is mostly diffuse.
2. Beams 2 and 4 are reflected; beams 3 and 5 are refracted.
3. (b). When light goes from one material into one having a higher index of refraction, it refracts toward the normal line of the boundary between the two materials. If, as the light travels through the new material, the index of refraction continues to increase, the light ray will refract more and more toward the normal line.
4. (c). Both the wave speed and the wavelength decrease as the index of refraction increases. The frequency is unchanged.

## Problem Solutions

22.1 The total distance the light travels is

$$\begin{aligned}\Delta d &= 2 \left( D_{\text{center to center}} - R_{\text{Earth}} - R_{\text{Moon}} \right) \\ &= 2 \left( 3.84 \times 10^8 - 6.38 \times 10^6 - 1.76 \times 10^6 \right) \text{ m} = 7.52 \times 10^8 \text{ m}\end{aligned}$$

Therefore, 
$$v = \frac{\Delta d}{\Delta t} = \frac{7.52 \times 10^8 \text{ m}}{2.51 \text{ s}} = \boxed{3.00 \times 10^8 \text{ m/s}}$$

22.2 If the wheel has 360 teeth, it turns through an angle of  $1/720 \text{ rev}$  in the time it takes the light to make its round trip. From the definition of angular velocity, we see that the time is

$$t = \frac{\theta}{\omega} = \frac{(1/720) \text{ rev}}{27.5 \text{ rev/s}} = 5.05 \times 10^{-5} \text{ s}$$

Hence, the speed of light is 
$$c = \frac{2d}{t} = \frac{2(7500 \text{ m})}{5.05 \times 10^{-5} \text{ s}} = \boxed{2.97 \times 10^8 \text{ m/s}}$$

22.3 The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next. Then,

$$\Delta \theta = \left( \frac{1}{720} \text{ rev} \right) (2\pi \text{ rad/rev}) = \frac{2\pi}{720} \text{ rad},$$

and 
$$\omega = \frac{\Delta \theta}{\Delta t} = \frac{\Delta \theta}{2d/c} = \frac{c(\Delta \theta)}{2d} = \frac{(2.998 \times 10^8 \text{ m/s})}{2(11.45 \times 10^3 \text{ m})} \left( \frac{2\pi}{720} \text{ rad} \right) = \boxed{114 \text{ rad/s}}$$

- 22.4 (a) The time for the light to travel to the stationary mirror and back is

$$\Delta t = \frac{2d}{c} = \frac{2(35.0 \times 10^3 \text{ m})}{3.00 \times 10^8 \text{ m/s}} = 2.33 \times 10^{-4} \text{ s}$$

At the lowest angular speed, the octagonal mirror will have rotated  $1/8 \text{ rev}$  in this time, so

$$\omega_{\min} = \frac{\Delta \theta}{\Delta t} = \frac{1/8 \text{ rev}}{2.33 \times 10^{-4} \text{ s}} = \boxed{536 \text{ rev/s}}$$

- (b) At the next higher angular speed, the mirror will have rotated  $2/8 \text{ rev}$  in the elapsed time, or

$$\omega_2 = 2\omega_{\min} = 2(536 \text{ rev/s}) = \boxed{1.07 \times 10^3 \text{ rev/s}}$$

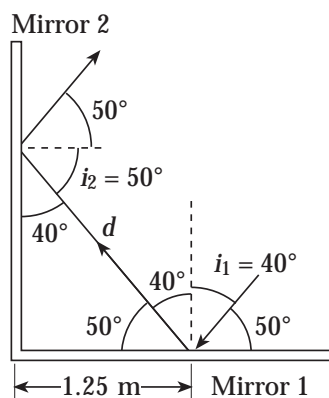
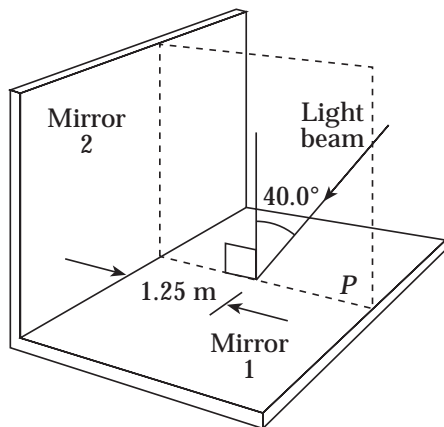
- 22.5 (a) For the light beam to make it through both slots, the time for the light to travel distance  $d$  must equal the time for the disks to rotate through angle  $\theta$ . Therefore, if  $c$  is the speed of light,

$$t = \frac{d}{c} = \frac{\theta}{\omega}, \text{ or } \boxed{c = \frac{\omega d}{\theta}}$$

- (b) If  $d = 2.500 \text{ m}$ ,  $\theta = \left(\frac{1}{60} \text{ degree}\right) \left(\frac{1 \text{ rev}}{360 \text{ degree}}\right) = \frac{1 \text{ rev}}{(60)(360)}$ , and  $\omega = 5555 \text{ rev/s}$ ,

$$c = (2.500 \text{ m}) \left(5555 \frac{\text{rev}}{\text{s}}\right) \left(\frac{(60)(360)}{1 \text{ rev}}\right) = \boxed{3.000 \times 10^8 \text{ m/s}}$$

## 22.6



(a) From geometry,  $1.25 \text{ m} = d \sin 40.0^\circ$ , so  $d = \boxed{1.94 \text{ m}}$

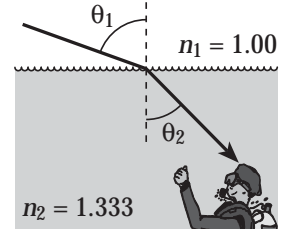
(b)  $\boxed{50.0^\circ \text{ above horizontal}}$ , or parallel to the incident ray.

22.7  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$$\sin \theta_1 = 1.333 \sin 45.0^\circ$$

$$\sin \theta_1 = (1.333)(0.707) = 0.943$$

$$\theta_1 = 70.5^\circ \rightarrow \boxed{19.5^\circ \text{ above the horizontal}}$$



22.8 The speed of light in water is  $v_{\text{water}} = \frac{c}{n_{\text{water}}}$ , and in Lucite  $v_{\text{Lucite}} = \frac{c}{n_{\text{Lucite}}}$ . Thus, the total time required to transverse the double layer is

$$\Delta t_1 = \frac{d_{\text{water}}}{v_{\text{water}}} + \frac{d_{\text{Lucite}}}{v_{\text{Lucite}}} = \frac{d_{\text{water}} n_{\text{water}} + d_{\text{Lucite}} n_{\text{Lucite}}}{c}$$

The time to travel the same distance in air is  $\Delta t_2 = \frac{d_{\text{water}} + d_{\text{Lucite}}}{c}$ , so the additional time required for the double layer is

$$\begin{aligned} \Delta t &= \Delta t_1 - \Delta t_2 = \frac{d_{\text{water}}(n_{\text{water}} - 1) + d_{\text{Lucite}}(n_{\text{Lucite}} - 1)}{c} \\ &= \frac{(1.00 \times 10^{-2} \text{ m})(1.333 - 1) + (0.500 \times 10^{-2} \text{ m})(1.59 - 1)}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.09 \times 10^{-11} \text{ s}} \end{aligned}$$

22.9 (a) From Snell's law,  $n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2} = \frac{(1.00) \sin 30.0^\circ}{\sin 19.24^\circ} = \boxed{1.52}$

(b)  $\lambda_2 = \frac{\lambda_0}{n_2} = \frac{632.8 \text{ nm}}{1.52} = \boxed{417 \text{ nm}}$

(c)  $f = \frac{c}{\lambda_0} = \frac{3.00 \times 10^8 \text{ m/s}}{632.8 \times 10^{-9} \text{ m}} = \boxed{4.74 \times 10^{14} \text{ Hz}}$  in air and in syrup

(d)  $v_2 = \frac{c}{n_2} = \frac{3.00 \times 10^8 \text{ m/s}}{1.52} = \boxed{1.98 \times 10^8 \text{ m/s}}$

$$22.10 \quad (a) \quad v_{\text{flintglass}} = \frac{c}{n_{\text{flintglass}}} = \frac{3.00 \times 10^8 \text{ m/s}}{1.66} = \boxed{1.81 \times 10^8 \text{ m/s}}$$

$$(b) \quad v_{\text{water}} = \frac{c}{n_{\text{water}}} = \frac{3.00 \times 10^8 \text{ m/s}}{1.333} = 2.25 \times 10^8 \text{ m/s}$$

$$(c) \quad v_{\text{zircon}} = \frac{c}{n_{\text{zircon}}} = \frac{3.00 \times 10^8 \text{ m/s}}{1.923} = 1.56 \times 10^8 \text{ m/s}$$

$$22.11 \quad (a) \quad \lambda_{\text{water}} = \frac{\lambda_0}{n_{\text{water}}}, \text{ so } \lambda_0 = n_{\text{water}} \lambda_{\text{water}} = (1.333)(438 \text{ nm}) = \boxed{584 \text{ nm}}$$

$$(b) \quad \lambda_0 = n_{\text{water}} \lambda_{\text{water}} = n_{\text{benzene}} \lambda_{\text{benzene}},$$

$$\text{and} \quad \frac{n_{\text{benzene}}}{n_{\text{water}}} = \frac{\lambda_{\text{water}}}{\lambda_{\text{benzene}}} = \frac{438 \text{ nm}}{390 \text{ nm}} = \boxed{1.12}$$

$$22.12 \quad (a) \quad \lambda_{\text{water}} = \frac{\lambda_0}{n_{\text{water}}} = \frac{436 \text{ nm}}{1.333} = \boxed{327 \text{ nm}}$$

$$(b) \quad \lambda_{\text{glass}} = \frac{\lambda_0}{n_{\text{crown glass}}} = \frac{436 \text{ nm}}{1.52} = \boxed{287 \text{ nm}}$$

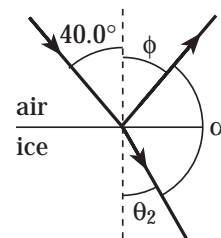
22.13 From Snell's law,

$$\theta_2 = \sin^{-1} \left[ \frac{n_1 \sin \theta_1}{n_2} \right] = \sin^{-1} \left[ \frac{(1.00) \sin 40.0^\circ}{1.309} \right] = 29.4^\circ,$$

and from the law of reflection,  $\phi = \theta_1 = 40.0^\circ$ .

Hence, the angle between the reflected and refracted rays is

$$\alpha = 180^\circ - \theta_2 - \phi = 180^\circ - 29.4^\circ - 40.0^\circ = \boxed{111^\circ}$$



$$22.14 \quad \text{From Snell's law, } \theta_2 = \sin^{-1} \left[ \frac{n_1 \sin \theta_1}{n_2} \right] = \sin^{-1} \left[ \frac{(1.00) \sin 35.0^\circ}{1.333} \right] = \boxed{25.5^\circ}$$

$$\text{The wavelength in water is } \lambda_{\text{water}} = \frac{\lambda_0}{n_{\text{water}}} = \frac{589 \text{ nm}}{1.333} = \boxed{442 \text{ nm}}$$

$$22.15 \quad n_{oil} = \frac{c}{v_{oil}} = \frac{3.00 \times 10^8 \text{ m/s}}{2.17 \times 10^8 \text{ m/s}} = 1.38$$

Then, using Snell's law

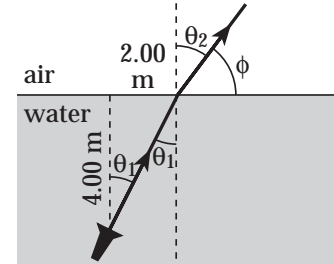
$$\theta_{oil} = \sin^{-1} \left[ \frac{n_{air} \sin \theta_{air}}{n_{oil}} \right] = \sin^{-1} \left[ \frac{(1.00) \sin 23.1^\circ}{1.38} \right] = \boxed{16.5^\circ}$$

22.16 The angle of incidence is

$$\theta_1 = \tan^{-1} \left[ \frac{2.00 \text{ m}}{4.00 \text{ m}} \right] = 26.6^\circ$$

Therefore, Snell's law gives

$$\begin{aligned} \theta_2 &= \sin^{-1} \left[ \frac{n_1 \sin \theta_1}{n_2} \right] \\ &= \sin^{-1} \left[ \frac{(1.333) \sin 26.6^\circ}{1.00} \right] = 36.6^\circ \end{aligned}$$



and the angle the refracted ray makes with the surface is

$$\phi = 90.0^\circ - \theta_2 = 90.0^\circ - 36.6^\circ = \boxed{53.4^\circ}$$

22.17 The incident light reaches the left-hand mirror at distance

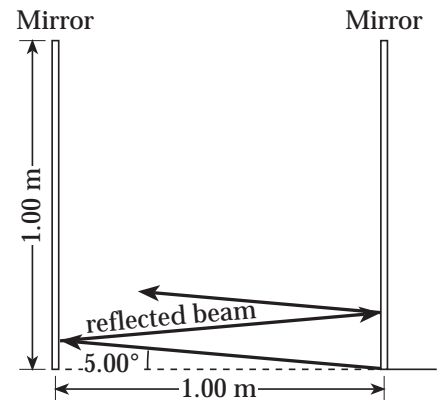
$$(1.00 \text{ m}) \tan 5.00^\circ = 0.0875 \text{ m}$$

above its bottom edge. The reflected light first reaches the right-hand mirror at height

$$2(0.0875 \text{ m}) = 0.175 \text{ m}.$$

It bounces between the mirrors with this distance between points of contact with either.

Since  $\frac{1.00 \text{ m}}{0.175 \text{ m}} = 5.72$ , the light reflects



five times from the right-hand mirror and six times from the left.

**22.18** At the first surface, the angle of incidence is  $\theta_1 = 30.0^\circ$ , and Snell's law gives

$$\theta_2 = \sin^{-1} \left[ \frac{n_{air} \sin \theta_1}{n_{glass}} \right] = \sin^{-1} \left[ \frac{(1.00) \sin 30.0^\circ}{1.50} \right] = 19.5^\circ$$

Since the second surface is parallel to the first, the angle of incidence at the second surface is  $\theta_1 = 19.5^\circ$  and the angle of refraction is

$$\theta_2 = \sin^{-1} \left[ \frac{n_{glass} \sin \theta_{glass}}{n_{air}} \right] = \sin^{-1} \left[ \frac{(1.50) \sin 19.5^\circ}{1.00} \right] = 30.0^\circ$$

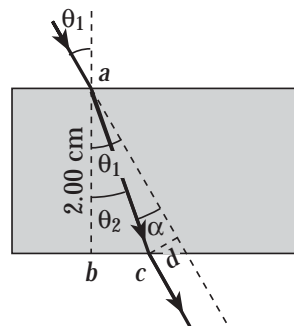
Thus, the light emerges traveling parallel to the incident beam.

**22.19** The angle of refraction at the first surface is  $\theta_2 = 19.5^\circ$  (see problem 18). Let  $h$  represent the distance from point  $a$  to  $c$  (i.e., the hypotenuse of triangle  $abc$ ). Then,

$$h = \frac{2.00 \text{ cm}}{\cos \theta_2} = \frac{2.00 \text{ cm}}{\cos 19.5^\circ} = 2.12 \text{ cm}$$

Also,  $\alpha = \theta_1 - \theta_2 = 30.0^\circ - 19.5^\circ = 10.5^\circ$ , so

$$d = h \sin \alpha = (2.12 \text{ cm}) \sin 10.5^\circ = 0.388 \text{ cm}$$



**22.20** The distance  $h$  the light travels in the glass is

$$h = \frac{2.00 \text{ cm}}{\cos 19.5^\circ} = 2.12 \text{ cm}$$

The speed of light in the glass is

$$v = \frac{c}{n_{glass}} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s}$$

$$\text{Therefore, } t = \frac{h}{v} = \frac{2.12 \times 10^{-2} \text{ m}}{2.00 \times 10^8 \text{ m/s}} = 1.06 \times 10^{-10} \text{ s}.$$

**22.21** From Snell's law, the angle of incidence at the air-oil interface is

$$\begin{aligned}\theta &= \sin^{-1} \left[ \frac{n_{oil} \sin \theta_{oil}}{n_{air}} \right] \\ &= \sin^{-1} \left[ \frac{(1.48) \sin 20.0^\circ}{1.00} \right] = \boxed{30.4^\circ},\end{aligned}$$

and the angle of refraction as the light enters the water is

$$\theta' = \sin^{-1} \left[ \frac{n_{oil} \sin \theta_{oil}}{n_{water}} \right] = \sin^{-1} \left[ \frac{(1.48) \sin 20.0^\circ}{1.333} \right] = \boxed{22.3^\circ}$$

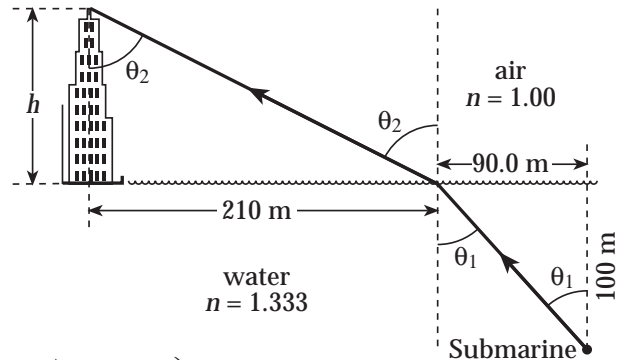
**22.22** The angle of incidence at the water surface is

$$\theta_1 = \tan^{-1} \left( \frac{90.0 \text{ m}}{100 \text{ m}} \right) = 42.0^\circ$$

Then, Snell's law gives the angle of refraction as

$$\theta_2 = \sin^{-1} \left( \frac{n_{water} \sin \theta_1}{n_{air}} \right) = \sin^{-1} \left( \frac{(1.333) \sin 42.0^\circ}{1.00} \right) = 63.1^\circ,$$

$$\text{so the height of the building is } h = \frac{210 \text{ m}}{\tan \theta_2} = \frac{210 \text{ m}}{\tan 63.1^\circ} = \boxed{107 \text{ m}}$$



**22.23**  $\Delta t = (\text{time to travel } 6.20 \text{ m in ice}) - (\text{time to travel } 6.20 \text{ m in air})$

$$\Delta t = \frac{6.20 \text{ m}}{v_{ice}} - \frac{6.20 \text{ m}}{c}$$

Since the speed of light in a medium of refractive index  $n$  is  $v = \frac{c}{n}$ ,

$$\Delta t = (6.20 \text{ m}) \left( \frac{1.309}{c} - \frac{1}{c} \right) = \frac{(6.20 \text{ m})(0.309)}{3.00 \times 10^8 \text{ m/s}} = 6.39 \times 10^{-9} \text{ s} = \boxed{6.39 \text{ ns}}$$



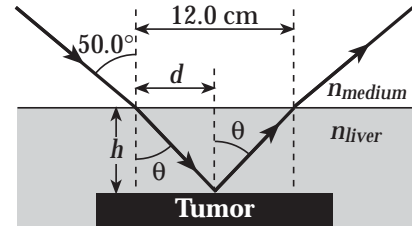
**22.24** From Snell's law,  $\sin \theta = \left( \frac{n_{\text{medium}}}{n_{\text{liver}}} \right) \sin 50.0^\circ$ .

$$\text{But, } \frac{n_{\text{medium}}}{n_{\text{liver}}} = \frac{c/v_{\text{medium}}}{c/v_{\text{liver}}} = \frac{v_{\text{liver}}}{v_{\text{medium}}} = 0.900,$$

$$\text{so, } \theta = \sin^{-1}[(0.900) \sin 50.0^\circ] = 43.6^\circ.$$

From the law of reflection,

$$d = \frac{12.0 \text{ cm}}{2} = 6.00 \text{ cm}, \text{ and } h = \frac{d}{\tan \theta} = \frac{6.00 \text{ cm}}{\tan(43.6^\circ)} = \boxed{6.30 \text{ cm}}$$



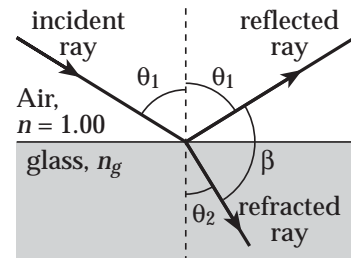
**22.25** As shown at the right,  $\theta_1 + \beta + \theta_2 = 180^\circ$ .

When  $\beta = 90^\circ$ , this gives  $\theta_2 = 90^\circ - \theta_1$ .

Then, from Snell's law

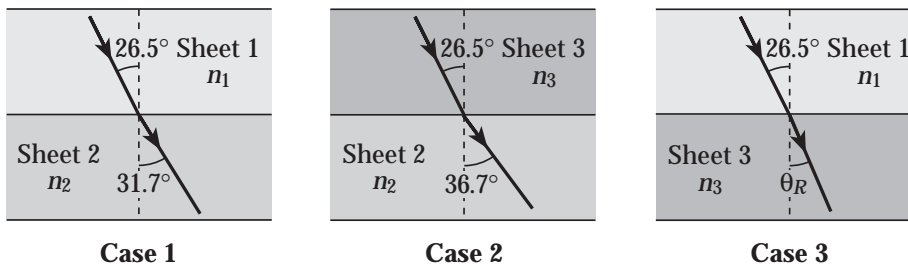
$$\begin{aligned} \sin \theta_1 &= \frac{n_g \sin \theta_2}{n_{\text{air}}} \\ &= n_g \sin(90^\circ - \theta_1) = n_g \cos \theta_1 \end{aligned}$$

$$\text{Thus, when } \beta = 90^\circ, \frac{\sin \theta_1}{\cos \theta_1} = \tan \theta_1 = n_g \text{ or } \boxed{\theta_1 = \tan^{-1}(n_g)}$$



**22.26**

**Given Conditions and Observed Results**



$$\text{For the first placement, Snell's law gives, } n_2 = \frac{n_1 \sin 26.5^\circ}{\sin 31.7^\circ}$$

In the second placement, application of Snell's law yields

$$n_3 \sin 26.5^\circ = n_2 \sin 36.7^\circ = \left( \frac{n_1 \sin 26.5^\circ}{\sin 31.7^\circ} \right) \sin 36.7^\circ, \text{ or } n_3 = \frac{n_1 \sin 36.7^\circ}{\sin 31.7^\circ}$$

Finally, using Snell's law in the third placement gives

$$\sin \theta_R = \frac{n_1 \sin 26.5^\circ}{n_3} = (n_1 \sin 26.5^\circ) \left( \frac{\sin 31.7^\circ}{n_1 \sin 36.7^\circ} \right) = 0.392,$$

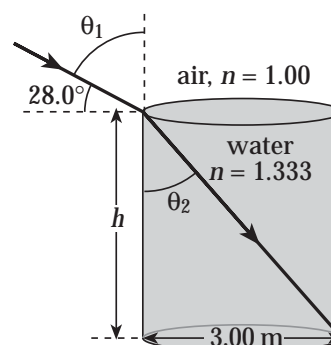
and  $\theta_R = \boxed{23.1^\circ}$

**22.27** When the Sun is  $28.0^\circ$  above the horizon, the angle of incidence for sunlight at the air-water boundary is

$$\theta_1 = 90.0^\circ - 28.0^\circ = 62.0^\circ$$

Thus, the angle of refraction is

$$\begin{aligned} \theta_2 &= \sin^{-1} \left[ \frac{n_{\text{air}} \sin \theta_1}{n_{\text{water}}} \right] \\ &= \sin^{-1} \left[ \frac{(1.00) \sin 62.0^\circ}{1.333} \right] = 41.5^\circ \end{aligned}$$



The depth of the tank is then  $h = \frac{3.00 \text{ m}}{\tan \theta_2} = \frac{3.00 \text{ m}}{\tan(41.5^\circ)} = \boxed{3.39 \text{ m}}$

**22.28** From the drawing, observe that

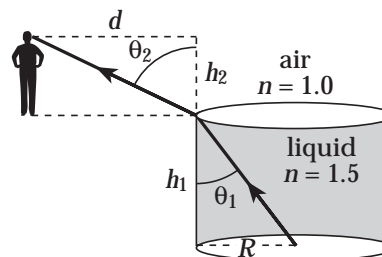
$$\theta_1 = \tan^{-1} \left( \frac{R}{h_1} \right) = \tan^{-1} \left( \frac{1.5 \text{ m}}{2.0 \text{ m}} \right) = 37^\circ$$

Applying Snell's law to the ray shown gives

$$\theta_2 = \sin^{-1} \left( \frac{n_{\text{liquid}} \sin \theta_1}{n_{\text{air}}} \right) = \sin^{-1} \left( \frac{1.5 \sin 37^\circ}{1.0} \right) = 64^\circ$$

Thus, the distance of the girl from the cistern is

$$d = h_2 \tan \theta_2 = (1.2 \text{ m}) \tan 64^\circ = \boxed{2.5 \text{ m}}$$



22.29 Using Snell's law gives

$$\theta_{red} = \sin^{-1} \left( \frac{n_{air} \sin \theta_i}{n_{red}} \right) = \sin^{-1} \left( \frac{(1.000) \sin 83.00^\circ}{1.331} \right) = \boxed{48.22^\circ},$$

$$\text{and } \theta_{blue} = \sin^{-1} \left( \frac{n_{air} \sin \theta_i}{n_{blue}} \right) = \sin^{-1} \left( \frac{(1.000) \sin 83.00^\circ}{1.340} \right) = \boxed{47.79^\circ}$$

22.30 The angles of refraction for the two wavelengths are

$$\theta_{red} = \sin^{-1} \left( \frac{n_{air} \sin \theta_i}{n_{red}} \right) = \sin^{-1} \left( \frac{(1.000) \sin 30.00^\circ}{1.615} \right) = 18.04^\circ,$$

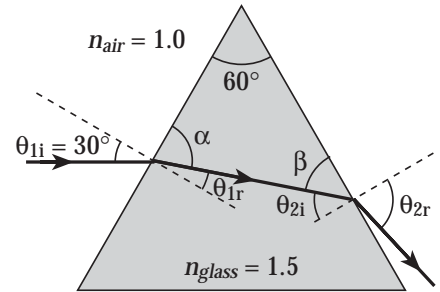
$$\text{and } \theta_{blue} = \sin^{-1} \left( \frac{n_{air} \sin \theta_i}{n_{blue}} \right) = \sin^{-1} \left( \frac{(1.000) \sin 30.00^\circ}{1.650} \right) = 17.64^\circ$$

Thus, the angle between the two refracted rays is

$$\Delta\theta = \theta_{red} - \theta_{blue} = 18.04^\circ - 17.64^\circ = \boxed{0.40^\circ}$$

22.31 (a) The angle of incidence at the first surface is  $\theta_{1i} = 30^\circ$ , and the angle of refraction is

$$\begin{aligned} \theta_{1r} &= \sin^{-1} \left( \frac{n_{air} \sin \theta_{1i}}{n_{glass}} \right) \\ &= \sin^{-1} \left( \frac{1.0 \sin 30^\circ}{1.5} \right) = \boxed{19^\circ} \end{aligned}$$



Also,  $\alpha = 90^\circ - \theta_{1r} = 71^\circ$  and  $\beta = 180^\circ - 60^\circ - \alpha = 49^\circ$

Therefore, the angle of incidence at the second surface is  $\theta_{2i} = 90^\circ - \beta = \boxed{41^\circ}$ . The angle of refraction at this surface is

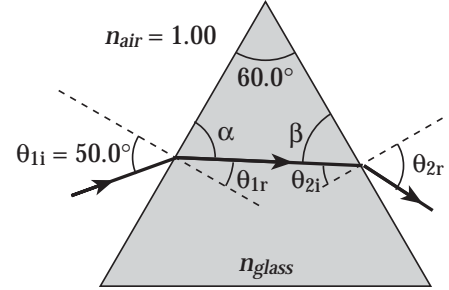
$$\theta_{2r} = \sin^{-1} \left( \frac{n_{glass} \sin \theta_{2i}}{n_{air}} \right) = \sin^{-1} \left( \frac{1.5 \sin 41^\circ}{1.0} \right) = \boxed{77^\circ}$$

- (b) The angle of reflection at each surface equals the angle of incidence at that surface. Thus,

$$(\theta_1)_{\text{reflection}} = \theta_{1i} = 30^\circ, \text{ and } (\theta_2)_{\text{reflection}} = \theta_{2i} = \boxed{41^\circ}$$

**22.32** For the violet light,  $n_{\text{glass}} = 1.66$ , and

$$\begin{aligned} \theta_{1r} &= \sin^{-1} \left( \frac{n_{\text{air}} \sin \theta_{1i}}{n_{\text{glass}}} \right) \\ &= \sin^{-1} \left( \frac{1.00 \sin 50.0^\circ}{1.66} \right) = 27.5^\circ \end{aligned}$$



$$\alpha = 90^\circ - \theta_{1r} = 62.5^\circ, \beta = 180.0^\circ - 60.0^\circ - \alpha = 57.5^\circ,$$

and  $\theta_{2i} = 90.0^\circ - \beta = 32.5^\circ$ . The final angle of refraction of the violet light is

$$\theta_{2r} = \sin^{-1} \left( \frac{n_{\text{glass}} \sin \theta_{2i}}{n_{\text{air}}} \right) = \sin^{-1} \left( \frac{1.66 \sin 32.5^\circ}{1.00} \right) = 63.2^\circ$$

Following the same steps for the red light ( $n_{\text{glass}} = 1.62$ ) gives

$$\theta_{1r} = 28.2^\circ, \alpha = 61.8^\circ, \beta = 58.2^\circ, \theta_{2i} = 31.8^\circ, \text{ and } \theta_{2r} = 58.6^\circ$$

Thus, the angular dispersion of the emerging light is

$$D_{\text{dispersion}} = \theta_{2r}|_{\text{violet}} - \theta_{2r}|_{\text{red}} = 63.2^\circ - 58.6^\circ = \boxed{4.6^\circ}$$

**22.33** When surrounded by air ( $n_2 = 1.00$ ), the critical angle of a material is

$$\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) = \sin^{-1} \left( \frac{1}{n_{\text{material}}} \right)$$

(a) For Zircon,  $n = 1.923$ , and  $\theta_c = \sin^{-1} \left( \frac{1}{1.923} \right) = \boxed{31.3^\circ}$

(b) For fluorite,  $n = 1.434$ , and  $\theta_c = \sin^{-1} \left( \frac{1}{1.434} \right) = \boxed{44.2^\circ}$

(c) For ice,  $n = 1.309$ , and  $\theta_c = \sin^{-1} \left( \frac{1}{1.309} \right) = \boxed{49.8^\circ}$

**22.34** When surrounded by air ( $n_2 = 1.00$ ), the critical angle of a material is

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1}{n_{\text{material}}}\right)$$

(a) For diamond,  $\theta_c = \sin^{-1}\left(\frac{1}{2.419}\right) = \boxed{24.4^\circ}$

(b) For flint glass,  $\theta_c = \sin^{-1}\left(\frac{1}{1.66}\right) = \boxed{37.0^\circ}$

**22.35** When surrounded by water ( $n_2 = 1.333$ ), the critical angle of a material is

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.333}{n_{\text{material}}}\right)$$

(a) For diamond,  $\theta_c = \sin^{-1}\left(\frac{1.333}{2.419}\right) = \boxed{33.4^\circ}$

(b) For flint glass,  $\theta_c = \sin^{-1}\left(\frac{1.333}{1.66}\right) = \boxed{53.4^\circ}$

**22.36** Using Snell's law, the index of refraction of the liquid is found to be

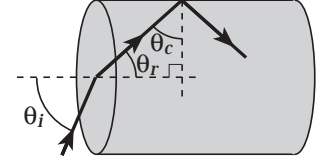
$$n_{\text{liquid}} = \frac{n_{\text{air}} \sin \theta_i}{\sin \theta_r} = \frac{(1.00) \sin 30.0^\circ}{\sin 22.0^\circ} = 1.33$$

Thus,  $\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{liquid}}}\right) = \sin^{-1}\left(\frac{1.00}{1.33}\right) = \boxed{48.5^\circ}$

**22.37** From  $\sin \theta_c = \frac{n_2}{n_1}$ , we find  $n_2 = n_1 \sin \theta_c = (1.60) \sin(59.5^\circ) = \boxed{1.38}$

**22.38** The critical angle for this material in air is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{pipe}}}\right) = \sin^{-1}\left(\frac{1.00}{1.36}\right) = 47.3^\circ$$



Thus,  $\theta_r = 90.0^\circ - \theta_c = 42.7^\circ$  and from Snell's law,

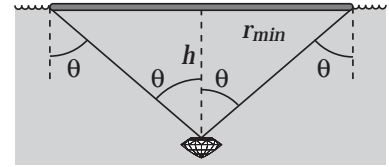
$$\theta_i = \sin^{-1}\left(\frac{n_{\text{pipe}} \sin \theta_r}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{(1.36) \sin 42.7^\circ}{1.00}\right) = \boxed{67.2^\circ}$$

**22.39** The light must be totally reflecting from the surface of a hot air layer just above the road surface. The angle of reflection, and hence the critical angle, is  $\theta_c = 90.0^\circ - 1.20^\circ = 88.8^\circ$ .

Thus, from  $\sin \theta_c = \frac{n_2}{n_1}$ , we find

$$n_2 = n_1 \sin \theta_c = (1.0003) \sin 88.8^\circ = \boxed{1.00008}$$

**22.40** The circular raft must cover the area of the surface through which light from the diamond could emerge. Thus, it must form the base of a cone (with apex at the diamond) whose half angle is  $\theta$ , where  $\theta$  is greater than or equal to the critical angle.



The critical angle at the water-air boundary is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{1.00}{1.333}\right) = 48.6^\circ$$

Thus, the minimum diameter of the raft is

$$2r_{\text{min}} = 2h \tan \theta_c = 2(2.00 \text{ m}) \tan 48.6^\circ = \boxed{4.54 \text{ m}}$$

**22.41** (a) Snell's law can be written as  $\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$ . At the critical angle of incidence ( $\theta_1 = \theta_c$ ),

the angle of refraction is  $90^\circ$  and Snell's law becomes  $\sin \theta_c = \frac{v_1}{v_2}$ . At the concrete-air boundary,

$$\theta_c = \sin^{-1}\left(\frac{v_1}{v_2}\right) = \sin^{-1}\left(\frac{343 \text{ m/s}}{1850 \text{ m/s}}\right) = \boxed{10.7^\circ}$$

(b) Sound can be totally reflected only if it is initially traveling in the slower medium. Hence, at the concrete-air boundary, the sound must be traveling in air.

(c) Sound in air falling on the wall from most directions is 100% reflected, so the wall is a good mirror.

**22.42** For total internal reflection to occur at each of the short faces of the prism as shown, it is necessary that  $\theta_c \leq \theta_1 = 45^\circ$ . This gives

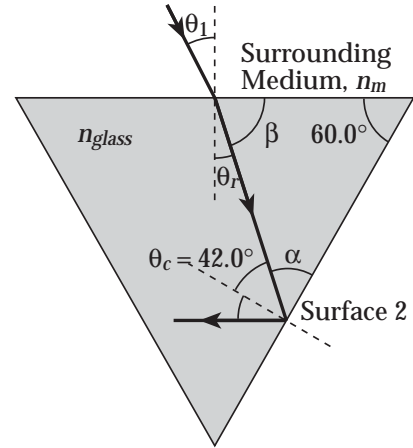
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{glass}}}\right) \leq 45^\circ,$$

or 
$$n_{\text{glass}} \geq \frac{n_{\text{air}}}{\sin 45^\circ} = \frac{1.00}{(\sqrt{2}/2)} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$$

- 22.43** If  $\theta_c = 42.0^\circ$  at the boundary between the prism glass and the surrounding medium, then  $\sin \theta_c = \frac{n_2}{n_1}$  gives

$$\frac{n_m}{n_{\text{glass}}} = \sin 42.0^\circ,$$

or 
$$\frac{n_{\text{glass}}}{n_m} = \frac{1}{\sin 42.0^\circ} = 1.494$$



From the geometry shown in the above figure,

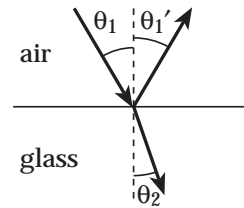
$$\alpha = 90.0^\circ - 42.0^\circ = 48.0^\circ, \beta = 180^\circ - 60.0^\circ - \alpha = 72.0^\circ,$$

and  $\theta_r = 90.0^\circ - \beta = 18.0^\circ$ . Thus, applying Snell's law at the first surface gives

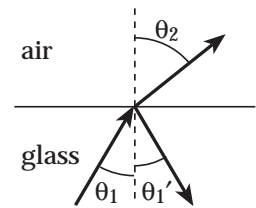
$$\theta_1 = \sin^{-1} \left( \frac{n_{\text{glass}} \sin \theta_r}{n_m} \right) = \sin^{-1} (1.494 \sin 18.0^\circ) = \boxed{27.5^\circ}$$

**22.44** (a)  $\theta'_1 = \theta_1 = \boxed{30.0^\circ}$

$$\begin{aligned} \theta_2 &= \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_2} \right) \\ &= \sin^{-1} \left( \frac{(1.00) \sin 30.0^\circ}{1.55} \right) \\ &= \boxed{18.8^\circ} \end{aligned}$$



Parts (a) and (c)



Parts (b) and (d)

(b)  $\theta'_1 = \theta_1 = \boxed{30.0^\circ}$ ,  $\theta_2 = \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_2} \right) = \sin^{-1} \left( \frac{(1.55) \sin 30.0^\circ}{1.00} \right) = \boxed{50.8^\circ}$

(c) and (d) The other entries are computed similarly, and are shown in the table below.

(c) air into glass, angles in degrees			(d) glass into air, angles in degrees		
incidence	reflection	refraction	incidence	reflection	refraction
0	0	0	0	0	0
10.0	10.0	6.43	10.0	10.0	15.6
20.0	20.0	12.7	20.0	20.0	32.0
30.0	30.0	18.8	30.0	30.0	50.8
40.0	40.0	24.5	40.0	40.0	85.1
50.0	50.0	29.6	50.0	50.0	none*



60.0	60.0	34.0	60.0	60.0	none*
70.0	70.0	37.3	70.0	70.0	none*
80.0	80.0	39.4	80.0	80.0	none*
90.0	90.0	40.2	90.0	90.0	none*

\*total internal reflection

**22.45** At the air-ice boundary, Snell's law gives the angle of refraction in the ice as

$$\theta_{1r} = \sin^{-1} \left( \frac{n_{\text{air}} \sin \theta_{1i}}{n_{\text{ice}}} \right) = \sin^{-1} \left( \frac{(1.00) \sin 30.0^\circ}{1.309} \right) = 22.5^\circ$$

Since the sides of the ice layer are parallel, the angle of incidence at the ice-water boundary is  $\theta_{2i} = \theta_{1r} = 22.5^\circ$ . Then, from Snell's law, the angle of refraction in the water is

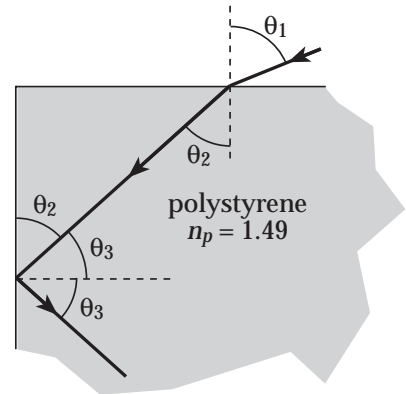
$$\theta_{2r} = \sin^{-1} \left( \frac{n_{\text{ice}} \sin \theta_{2i}}{n_{\text{water}}} \right) = \sin^{-1} \left( \frac{(1.309) \sin 22.5^\circ}{1.333} \right) = \boxed{22.0^\circ}$$

**22.46** (a) For polystyrene surrounded by air, total internal reflection at the left vertical face requires that

$$\theta_3 \geq \theta_c = \sin^{-1} \left( \frac{n_{\text{air}}}{n_p} \right) = \sin^{-1} \left( \frac{1.00}{1.49} \right) = 42.2^\circ$$

From the geometry shown in the figure at the right,

$$\theta_2 = 90.0^\circ - \theta_3 \leq 90.0^\circ - 42.2^\circ = 47.8^\circ$$



Thus, use of Snell's law at the upper surface gives

$$\sin \theta_1 = \frac{n_p \sin \theta_2}{n_{\text{air}}} \leq \frac{(1.49) \sin 47.8^\circ}{1.00} = 1.10,$$

so it is seen that any angle of incidence  $\leq 90^\circ$  at the upper surface will yield total internal reflection at the left vertical face.

(b) Repeating the steps of part (a) with the index of refraction of air replaced by that of water yields  $\theta_3 \geq 63.5^\circ$ ,  $\theta_2 \leq 26.5^\circ$ ,  $\sin \theta_1 \leq 0.499$ , and  $\theta_1 \leq \boxed{30.0^\circ}$ .

(c) Total internal reflection is not possible since  $n_{\text{polystyrene}} < n_{\text{carbon disulfide}}$

**22.47** Applying Snell's law at points A, B, and C, gives

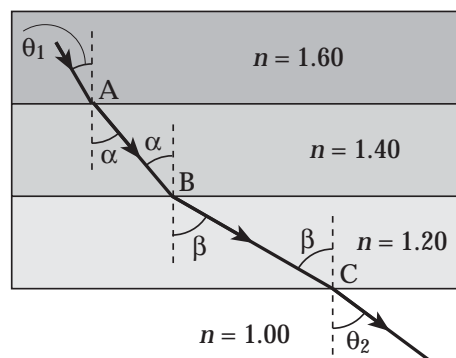
$$1.40 \sin \alpha = 1.60 \sin \theta_1, \quad (1)$$

$$1.20 \sin \beta = 1.40 \sin \alpha, \quad (2)$$

$$\text{and } 1.00 \sin \theta_2 = 1.20 \sin \beta. \quad (3)$$

Combining equations (1), (2), and (3) yields

$$\sin \theta_2 = 1.60 \sin \theta_1 \quad (4)$$



Note that equation (4) is exactly what Snell's law would yield if the second and third layers of this "sandwich" were ignored. This will always be true if the surfaces of all the layers are parallel to each other.

(a) If  $\theta_1 = 30.0^\circ$ , then equation (4) gives  $\theta_2 = \sin^{-1}(1.60 \sin 30.0^\circ) = \boxed{53.1^\circ}$

(b) At the critical angle of incidence on the lowest surface,  $\theta_2 = 90.0^\circ$ . Then, equation (4) gives

$$\theta_1 = \sin^{-1}\left(\frac{\sin \theta_2}{1.60}\right) = \sin^{-1}\left(\frac{\sin 90.0^\circ}{1.60}\right) = \boxed{38.7^\circ}$$

**22.48** (a) We see the Sun swinging around a circle in the extended plane of our parallel of latitude. Its angular speed is

$$\omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 1.72 \times 10^{-4} \text{ m/s} = \boxed{0.172 \text{ m/s}}$$

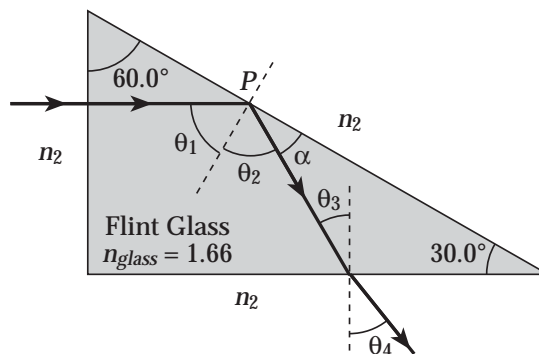
(b) The mirror folds into the cell the motion that would occur in a room twice as wide. Thus,

$$\begin{aligned} v &= (2r)\omega = [2(2.37 \text{ m})](7.27 \times 10^{-5} \text{ rad/s}) \\ &= 3.45 \times 10^{-4} \text{ m/s} = \boxed{0.345 \text{ m/s}} \end{aligned}$$

(c) and (d)

As the Sun moves southward and upward at  $50.0^\circ$ , we may regard the corner of the window as fixed, and both patches of light move  
 northward at  $50.0^\circ$  below the horizontal.

- 22.49** (a) From the geometry of the figure at the right, observe that  $\theta_1 = 60.0^\circ$ . Also, from the law of reflection,  $\theta_2 = \theta_1 = 60.0^\circ$ . Therefore,  $\alpha = 90.0^\circ - \theta_2 = 30.0^\circ$ , and  $\theta_3 + 90.0^\circ = 180 - \alpha - 30.0^\circ$  or  $\theta_3 = 30.0^\circ$ .



Then, since the prism is immersed in water ( $n_2 = 1.333$ ), Snell's law gives

$$\theta_4 = \sin^{-1}\left(\frac{n_{\text{glass}} \sin \theta_3}{n_2}\right) = \sin^{-1}\left(\frac{(1.66) \sin 30.0^\circ}{1.333}\right) = 38.5^\circ$$

- (b) For refraction to occur at point  $P$ , it is necessary that  $\theta_c > \theta_1$ .

Thus,  $\theta_c = \sin^{-1}\left(\frac{n_2}{n_{\text{glass}}}\right) > \theta_1$ , which gives

$$n_2 > n_{\text{glass}} \sin \theta_1 = (1.66) \sin 60.0^\circ = 1.44$$

- 22.50** Applying Snell's law to this refraction gives  $n_{\text{glass}} \sin \theta_2 = n_{\text{air}} \sin \theta_1$ .

If  $\theta_1 = 2\theta_2$ , this becomes

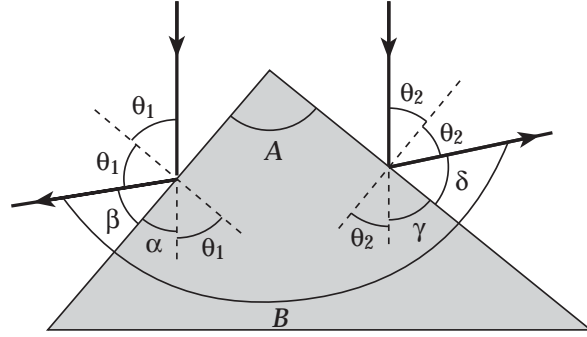
$$n_{\text{glass}} \sin \theta_2 = \sin(2\theta_2) = 2 \sin \theta_2 \cos \theta_2 \text{ or } \cos \theta_2 = \frac{n_{\text{glass}}}{2}$$

Then, the angle of incidence is

$$\theta_1 = 2\theta_2 = 2 \cos^{-1}\left(\frac{n_{\text{glass}}}{2}\right) = 2 \cos^{-1}\left(\frac{1.56}{2}\right) = 77.5^\circ$$

- 22.51** In the Figure at the right, observe that  $\beta = 90^\circ - \theta_1$  and  $\alpha = 90^\circ - \theta_1$ . Thus,  $\beta = \alpha$ .

Similarly, on the right side of the prism,  $\delta = 90^\circ - \theta_2$  and  $\gamma = 90^\circ - \theta_2$ , giving  $\delta = \gamma$ .



Next, observe that the angle between the reflected rays is  $B = (\alpha + \beta) + (\gamma + \delta)$ ,

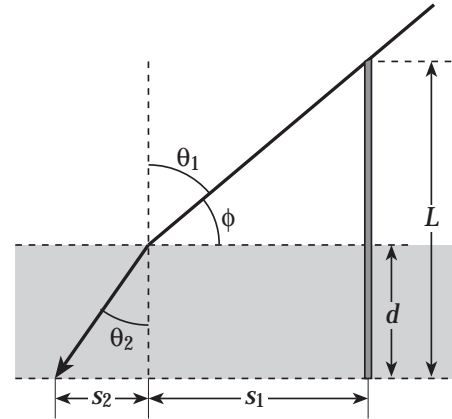
so  $B = 2(\alpha + \gamma)$ . Finally, observe that the left side of the prism is sloped at angle  $\alpha$  from the vertical and the right side is sloped at angle  $\gamma$ . Thus, the angle between the two sides is  $A = \alpha + \gamma$ , and we obtain the result  $B = 2(\alpha + \gamma) = \boxed{2A}$

- 22.52** Light passing the top of the pole strikes the surface at angle  $\phi = 40.0^\circ$  from the horizontal. The angle of incidence is then

$$\theta_1 = 90.0^\circ - \phi = 50.0^\circ$$

Snell's law gives the angle of refraction as

$$\begin{aligned} \theta_2 &= \sin^{-1} \left( \frac{n_{\text{air}} \sin \theta_1}{n_{\text{water}}} \right) \\ &= \sin^{-1} \left( \frac{(1.00) \sin 50.0^\circ}{1.333} \right) = 35.1^\circ \end{aligned}$$



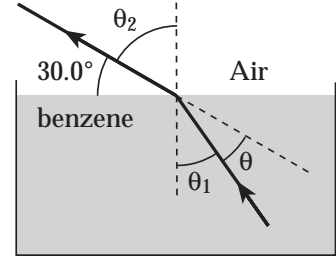
The light strikes the lake bed at distance  $x = s_1 + s_2$  from the pole, where

$$s_1 = \frac{L - d}{\tan \phi} \text{ and } s_2 = d \tan \theta_2$$

$$\text{Thus, } x = \frac{4.00 \text{ m} - 2.00 \text{ m}}{\tan 40.0^\circ} + (2.00 \text{ m}) \tan 35.1^\circ = \boxed{3.79 \text{ m}}$$

- 22.53** Consider light from the bottom end of the wire that happens to be headed up along the surface of the wire before and after refraction with angle of incidence  $\theta_1$  and angle of refraction  $\theta_2 = 60.0^\circ$ . Then, from Snell's law, the angle of incidence was

$$\begin{aligned}\theta_1 &= \sin^{-1}\left(\frac{n_{\text{air}} \sin \theta_2}{n_{\text{benzene}}}\right) \\ &= \sin^{-1}\left(\frac{(1.00) \sin 60.0^\circ}{1.50}\right) = 35.3^\circ\end{aligned}$$



Thus, the wire is bent by angle  $\theta = 60.0^\circ - \theta_1 = 60.0^\circ - 35.3^\circ = \boxed{24.7^\circ}$

- 22.54** Consider glass  $d = 4.0 \text{ mm}$  thick, and with index of refraction  $n = 1.5$ . The speed of light in the glass is

$$v = \frac{c}{n} = \frac{3.0 \times 10^8 \text{ m/s}}{1.5} = 2.0 \times 10^8 \text{ m/s}$$

The extra travel time is

$$\Delta t = \frac{d}{v} - \frac{d}{c} = \frac{4.0 \times 10^{-3} \text{ m}}{2.0 \times 10^8 \text{ m/s}} - \frac{4.0 \times 10^{-3} \text{ m}}{3.0 \times 10^8 \text{ m/s}} = 6.7 \times 10^{-12} \text{ s} \quad \boxed{\sim 10^{-11} \text{ s}}$$

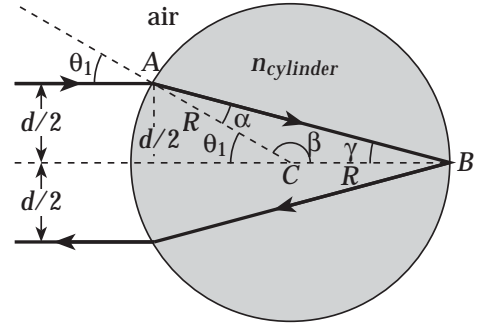
For light of wavelength  $\lambda_0 = 600 \text{ nm}$  in vacuum, the wavelength in the glass is

$$\lambda = \frac{\lambda_0}{n} = \frac{600 \text{ nm}}{1.5} = 400 \text{ nm}$$

and the extra path length as a number of wavelengths is

$$\Delta d = \frac{d}{\lambda} - \frac{d}{\lambda_0} = \frac{4.0 \times 10^{-3} \text{ m}}{400 \times 10^{-9} \text{ m}} - \frac{4.0 \times 10^{-3} \text{ m}}{600 \times 10^{-9} \text{ m}} = 3.3 \times 10^3 \quad \boxed{\sim 10^3}$$

- 22.55** The path of a light ray during a reflection and/or refraction process is always reversible. Thus, if the emerging ray is parallel to the incident ray, the path which the light follows through this cylinder must be symmetric about the center line as shown at the right.



$$\text{Thus, } \theta_1 = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00 \text{ m}}{2.00 \text{ m}}\right) = 30.0^\circ$$

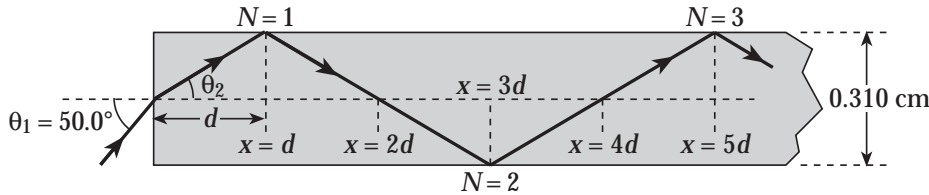
Triangle  $ABC$  is isosceles, so  $\gamma = \alpha$  and

$$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 2\alpha. \text{ Also, } \beta = 180^\circ - \theta_1$$

which gives  $\alpha = \theta_1/2 = 15.0^\circ$ . Then, from applying Snell's law at point  $A$ ,

$$n_{\text{cylinder}} = \frac{n_{\text{air}} \sin \theta_1}{\sin \alpha} = \frac{(1.00) \sin 30.0^\circ}{\sin 15.0^\circ} = \boxed{1.93}$$

**22.56**



The angle of refraction as the light enters the left end of the slab is

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{air}} \sin \theta_1}{n_{\text{slab}}}\right) = \sin^{-1}\left(\frac{(1.00) \sin 50.0^\circ}{1.48}\right) = 31.2^\circ$$

Observe from the figure that the first reflection occurs at  $x = d$ , the second reflection is at  $x = 3d$ , the third is at  $x = 5d$ , and so forth. In general, the  $N^{\text{th}}$  reflection occurs at  $x = (2N - 1)d$  where

$$d = \frac{(0.310 \text{ cm} / 2)}{\tan \theta_2} = \frac{0.310 \text{ cm}}{2 \tan 31.2^\circ} = 0.256 \text{ cm}$$

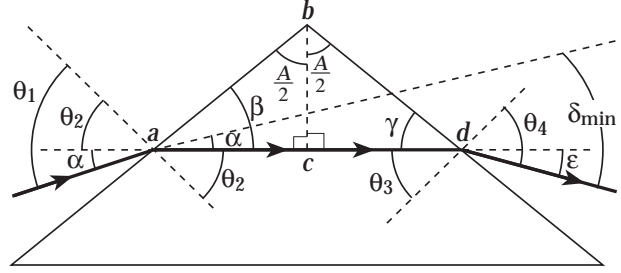
Therefore, the number of reflections made before reaching the other end of the slab at  $x = L = 42 \text{ cm}$  is found from  $L = (2N - 1)d$  to be

$$N = \frac{1}{2}\left(\frac{L}{d} + 1\right) = \frac{1}{2}\left(\frac{42 \text{ cm}}{0.256 \text{ cm}} + 1\right) = 82.5 \text{ or } \boxed{82 \text{ complete reflections}}$$

22.57 Refer to the figure given at the right.

$$\delta_{\min} = \alpha + \varepsilon = (\theta_1 - \theta_2) + (\theta_4 - \theta_3)$$

Note that triangles  $abc$  and  $bcd$  are congruent. Therefore,  $\beta = \gamma$  and their complementary angles are also equal, or  $\theta_2 = \theta_3$ .



From Snell's law at point  $a$ ,  $\sin \theta_1 = n \sin \theta_2$  (1)

and at point  $d$ ,  $\sin \theta_4 = n \sin \theta_3$

Since  $\theta_2 = \theta_3$ , comparison of these results shows that  $\theta_4 = \theta_1$ , so  $\delta_{\min} = 2(\theta_1 - \theta_2)$ .

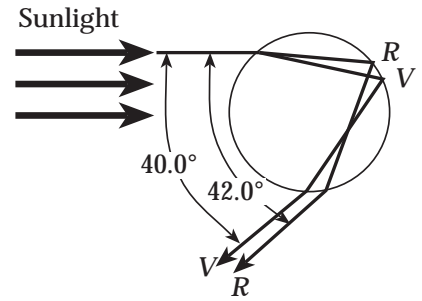
But,  $\theta_2 = 90^\circ - \beta = 90^\circ - (90^\circ - A/2)$ , or  $\theta_2 = A/2$

Thus,  $\delta_{\min} = 2\left(\theta_1 - \frac{A}{2}\right)$ , or  $\theta_1 = \frac{A + \delta_{\min}}{2}$  and equation (1) then yields

$$n = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin \left[ \frac{1}{2}(A + \delta_{\min}) \right]}{\sin \left( \frac{1}{2}A \right)}$$

22.58 Horizontal light rays from the setting Sun pass above the hiker. Those light rays that encounter raindrops at  $40.0^\circ$  to  $42.0^\circ$  from the hiker's shadow are twice refracted and once reflected, and reach the hiker as the rainbow.

The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius  $R$  of the circle of droplets is



$$R = (8.00 \text{ km}) \sin 42.0^\circ = 5.35 \text{ km}$$

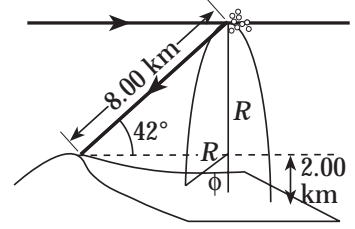
Then the angle  $\phi$ , between the vertical and the radius where the bow touches the ground, is given by

$$\cos \phi = \frac{2.00 \text{ km}}{R} = \frac{2.00 \text{ km}}{5.35 \text{ km}} = 0.374$$

or  $\phi = 68.1^\circ$

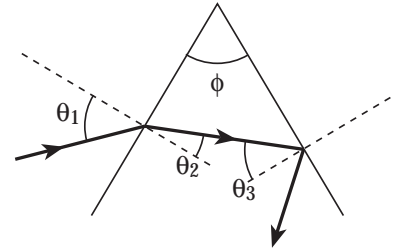
The angle filled by the visible bow is  $360^\circ - (2 \times 68.1^\circ) = 224^\circ$ , so the visible bow is

$$\frac{224^\circ}{360^\circ} = \boxed{62.2\% \text{ of a circle}}$$



**22.59** Applying Snell's law at the first surface in the figure at the right gives the angle of incidence as

$$\theta_1 = \sin^{-1} \left( \frac{n \sin \theta_2}{n_{\text{air}}} \right) = \sin^{-1} (n \sin \theta_2) \quad (1)$$



Since the sum of the interior angles of a triangle equals  $180^\circ$ , observe that  $\phi + (90^\circ - \theta_2) + (90^\circ - \theta_3) = 180^\circ$ , which reduces to  $\theta_2 = \phi - \theta_3$ . Thus, equation (1) becomes

$$\theta_1 = \sin^{-1} [n \sin(\phi - \theta_3)] = \sin^{-1} [n(\sin \phi \cos \theta_3 - \cos \phi \sin \theta_3)]$$

At the smallest allowed value for  $\theta_1$ ,  $\theta_3$  is equal to the critical angle at the second surface, or  $\sin \theta_3 = \sin \theta_c = \frac{n_{\text{air}}}{n} = \frac{1}{n}$ . Then,

$$\cos \theta_3 = \sqrt{1 - \sin^2 \theta_3} = \sqrt{1 - \frac{1}{n^2}} = \frac{\sqrt{n^2 - 1}}{n},$$

$$\text{and } \theta_1 = \sin^{-1} \left[ n \left( \sin \phi \frac{\sqrt{n^2 - 1}}{n} - \frac{1}{n} \cos \phi \right) \right] = \boxed{\sin^{-1} (\sqrt{n^2 - 1} \sin \phi - \cos \phi)}$$

Note that when  $\tan \phi < 1/\sqrt{n^2 - 1}$ , the value of  $\theta_1$  given by this result is negative. This simply means that, when  $\tan \phi < 1/\sqrt{n^2 - 1}$  and the refracted beam is striking the second surface of the prism at the critical angle, the incident beam at the first surface will be on the opposite side of the normal line from what is shown in the figure.



## Answers to Even Numbered Conceptual Questions

2. Ceilings are generally painted a light color so they will reflect more light, making the room brighter. Textured materials are often used on the ceiling to diffuse the reflected light and reduce glare (specular reflections).
4. At the altitude of the plane the surface of the Earth does not block off the lower half of the rainbow. Thus, the full circle can be seen. You can see such a rainbow by climbing on a stepladder above a garden sprinkler in the middle of a sunny day.
6. The spectrum of the light sent back to you from a drop at the top of the rainbow arrives such that the red light (deviated by an angle of  $42^\circ$ ) strikes the eye while the violet light (deviated by  $40^\circ$ ) passes over your head. Thus, the top of the rainbow looks red. At the bottom of the bow, violet light arrives at your eye and red light is deviated toward the ground. Thus, the bottom part of the bow appears violet.
8. A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction. The different indices of refraction occur because the air has different densities at different temperatures. Two images are seen; One from a direct path from the object to you, and the second arriving by rays originally heading toward the earth but refracted to your eye. On a hot day, the Sun makes the surface of blacktop hot, so the air is hot directly above it, becoming cooler as one moves higher into the sky. The “water” we see far in front of us is an image of the blue sky. Adding to the effect is the fact that the image shimmers as the air changes in temperature, giving the appearance of moving water.
10. The upright image of the hill is formed by light that has followed a direct path from the hill to the eye of the observer. The second image is a result of refraction in the atmosphere. Some light is reflected from the hill toward the water. As this light passes through warmer layers of air directly above the water, it is refracted back up toward the eye of the observer, resulting in the observation of an inverted image of the hill directly below the upright image.
12. The color traveling slowest is bent the most. Thus, X travels more slowly in the glass prism.
14. Total internal reflection occurs only when light attempts to move from a medium of high index of refraction to a medium of lower index of refraction. Thus, light moving from air ( $n = 1$ ) to water ( $n = 1.33$ ) cannot undergo total internal reflection.
16. Objects beneath the surface of water appear to be raised toward the surface by refraction. Thus, the bottom of the oar appears to be closer to the surface than it really is, and the oar looks to be bent.

18. The cross section can be visualized by considering just the two rays of light on the edges of the beam. If the beam of light enters a new medium with a higher index of refraction, the rays bend toward the normal, and the cross section of the refracted beam will be larger than that of the incident beam as suggested by Fig. CQ22.18a. If the new index of refraction is lower, the rays bend away from the normal, and the cross section of the beam is reduced, as shown in Fig. CQ22.18b.

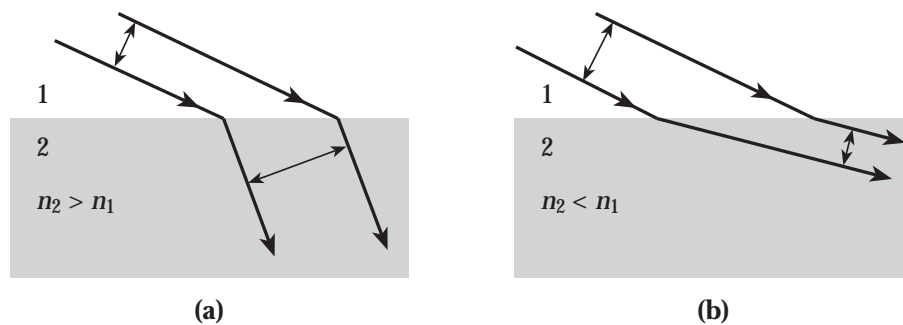


Figure CQ22.18

## Answers to Even Numbered Problems

2.  $2.97 \times 10^8 \text{ m/s}$
4. (a)  $536 \text{ rev/s}$  (b)  $1.07 \times 10^3 \text{ rev/s}$
6. (a)  $1.94 \text{ m}$  (b)  $50.0^\circ$  above horizontal (parallel to incident ray)
8.  $2.09 \times 10^{-11} \text{ s}$
10. (a)  $1.81 \times 10^8 \text{ m/s}$  (b)  $2.25 \times 10^8 \text{ m/s}$  (c)  $1.56 \times 10^8 \text{ m/s}$
12. (a)  $327 \text{ nm}$  (b)  $287 \text{ nm}$
14.  $25.5^\circ, 442 \text{ nm}$
16.  $53.4^\circ$
18. First surface:  $\theta_i = 30.0^\circ, \theta_r = 19.5^\circ$   
Second surface:  $\theta_i = 19.5^\circ, \theta_r = 30.0^\circ$
20.  $1.06 \times 10^{-10} \text{ s}$
22.  $107 \text{ m}$
24.  $6.30 \text{ cm}$
26.  $23.1^\circ$
28.  $2.5 \text{ m}$
30.  $0.40^\circ$
32.  $4.6^\circ$
34. (a)  $24.4^\circ$  (b)  $37.0^\circ$
36.  $48.5^\circ$
38.  $67.2^\circ$
40.  $4.54 \text{ m}$
42.  $\sqrt{2}$
44. (a)  $\theta'_1 = 30.0^\circ, \theta_2 = 18.8^\circ$  (b)  $\theta'_1 = 30.0^\circ, \theta_2 = 50.8^\circ$   
(c) See solution. (d) See solution.

46. (a) Any angle of incidence  $\leq 90^\circ$  (b)  $30.0^\circ$   
 (c) not possible since  $n_{\text{polystyrene}} < n_{\text{carbon disulfide}}$
48. (a)  $0.172 \text{ m m/s}$  (b)  $0.345 \text{ m m/s}$   
 (c) and (d) Northward at  $50.0^\circ$  below horizontal.
50.  $77.5^\circ$
52.  $3.79 \text{ m}$
54.  $\sim 10^{-11} \text{ s}$ ,  $\sim 10^3 \text{ wavelengths}$
56. 82
58. 62.2% of a circle