CHAPTER 3

Quick Quizzes

1. (c). The largest possible magnitude of the resultant occurs when the two vectors are in the same direction. In this case, the magnitude of the resultant is the sum of the magnitudes of **A** and **B**: $R = A + B = 20$ units. The smallest possible magnitude of the resultant occurs when the two vectors are in opposite directions, and the magnitude is the difference of the magnitudes of **A** and **B**: $R = |A - B| = 4$ units.

2. (b). The resultant has magnitude $A + B$ when **A** is oriented in the same direction as **B**.

3. (b). The distance traveled, unless you have a very unusual commute, *must* be larger than the magnitude of the displacement vector. The distance includes all of the twists and turns that you made in following the roads from home to work or school. However, the magnitude of the displacement vector is the length of a straight line from your home to work or school. The only way that the distance could be the same as the magnitude of the displacement vector is if your commute is a perfect straight line, which is unlikely!

4.

Vector	x component	y component
A	–	+
B	+	–
A + B	–	–

5. (a) False. As a particle moves in a circle with constant speed, the direction of its velocity changes continuously, so it undergoes an acceleration.

(b) False. If the velocity is constant, the magnitude of the instantaneous velocity, which is the speed, is constant.

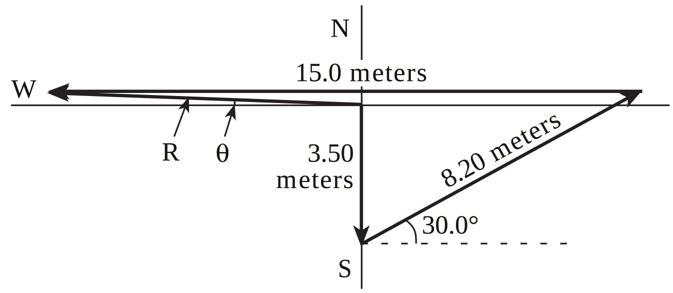
6. (c).

7. (b).

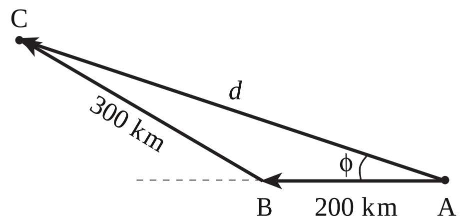
8. The answers to all three parts are the same—the acceleration is that due to gravity, 9.8 m/s^2, because the force of gravity is pulling downward on the ball during the entire motion. During the rising part of the trajectory, the downward acceleration results in the decreasing positive values of the vertical component of the velocity of the ball. During the falling part of the trajectory, the downward acceleration results in the increasing negative values of the vertical component of the velocity.

Problem Solutions

3.1 Your sketch should be drawn to scale, and be similar to that pictured at the right. The angle from the westward direction, θ, can be measured as 4°, and the length of **R** found to be 7.9 m. The resultant displacement is then $\boxed{7.9 \text{ m at } 4° \text{ N of W}}$.



3.2 (a) The distance d from **A** to **C** is



$$d = \sqrt{x^2 + y^2}$$

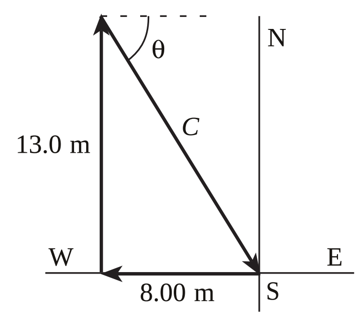
where $x = 200 \text{ km} + (300 \text{ km})\cos 30.0° = 460 \text{ km}$

and $y = 0 + (300 \text{ km})\sin 30.0° = 150 \text{ km}$.

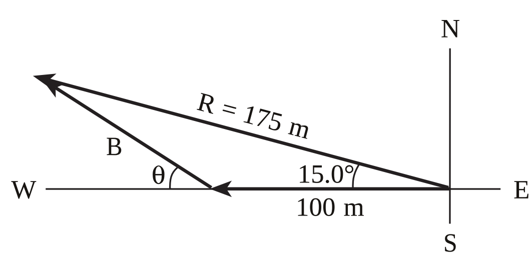
$$\therefore \quad d = \sqrt{(460 \text{ km})^2 + (150 \text{ km})^2} = \boxed{484 \text{ km}}$$

(b) $\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{150 \text{ km}}{460 \text{ km}}\right) = \boxed{18.1° \text{ N of W}}$

3.3 The displacement vectors $\mathbf{A} = 8.00 \text{ m westward}$ and $\mathbf{B} = 13.0 \text{ m north}$ can be drawn to scale as at the right. The vector **C** represents the displacement that the man in the maze must undergo to return to his starting point. The scale used to draw the sketch can be used to find **C** to be $\boxed{15 \text{ m at } 58° \text{ S of E}}$.

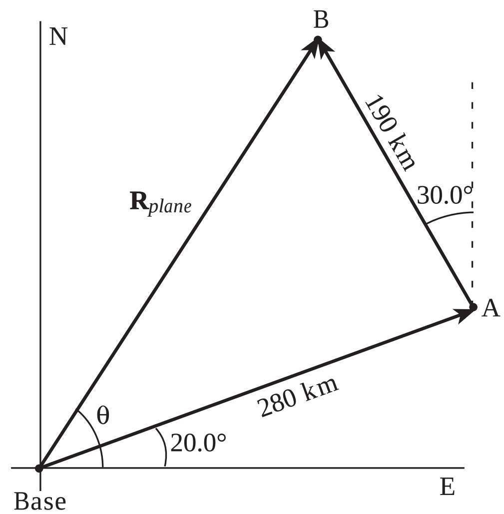


3.4 Your vector diagram should look like the one shown at the right. The initial displacement $\mathbf{A} = 100 \text{ m due west}$ and the resultant $\mathbf{R} = 175 \text{ m at } 15.0° \text{ N of W}$ are both known. In order to reach the end point of the run following the initial displacement, the jogger must follow the path shown as **B**. The length of **B** and the angle θ can be measured. The results should be $\boxed{83 \text{ m at } 33° \text{ N of W}}$.



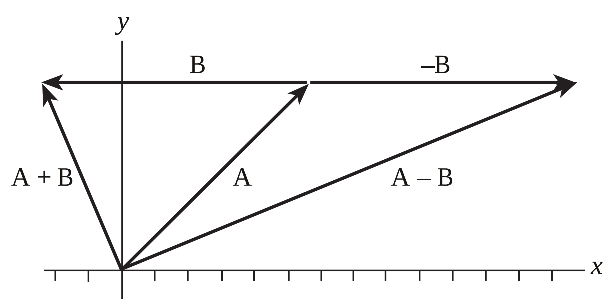
3.5 Using a vector diagram, drawn to scale, like that shown at the right, the final displacement of the plane can be found to be $\mathbf{R}_{plane} = 310\text{ km at }\theta = 57°\text{ N of E}$. The requested displacement of the base from point B is $-\mathbf{R}_{plane}$, which has the same magnitude but the opposite direction. Thus, the answer is

$$-\mathbf{R}_{plane} = \boxed{310\text{ km at }\theta = 57°\text{ S of W}}.$$



3.6 (a) Using graphical methods, place the tail of vector **B** at the head of vector **A**. The new vector $\mathbf{A}+\mathbf{B}$ has a magnitude of $\boxed{6.1\text{ units at }113°}$ from the positive x-axis.

(b) The vector difference $\mathbf{A}-\mathbf{B}$ is found by placing the negative of vector **B** at the head of vector **A**. The resultant vector $\mathbf{A}-\mathbf{B}$ has magnitude $\boxed{15\text{ units at }23°}$ from the positive x-axis.

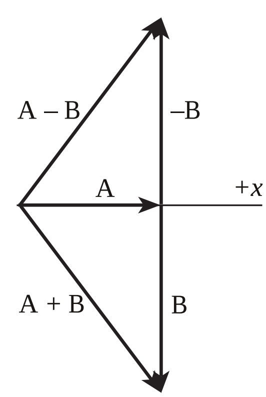


3.7 (a) In your vector diagram, place the tail of Vector **B** at the tip of Vector **A**. The vector sum, $\mathbf{A}+\mathbf{B}$, is then found as shown in the vector diagram and should be

$$\boxed{\mathbf{A}+\mathbf{B} = 5.0\text{ units at } -53°}.$$

(b) To find the vector difference, form the vector –**B** (same magnitude, opposite direction) and add it to vector **A** as shown in the diagram. You should find that

$$\boxed{\mathbf{A}-\mathbf{B} = 5.0\text{ units at } +53°}.$$



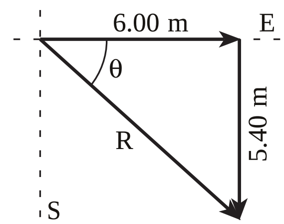
3.8 (a) Drawing these vectors to scale and maintaining their respective directions yields a resultant of $\boxed{5.2 \text{ m at } +60°}$.

(b) Maintain the direction of **A**, but reverse the direction of **B** by 180°. The resultant is $\boxed{3.0 \text{ m at } -30°}$.

(c) Maintain the direction of **B**, but reverse the direction of **A**. The resultant is $\boxed{3.0 \text{ m at } +150°}$.

(d) Maintain the direction of **A**, reverse the direction of **B**, and multiply its magnitude by two. The resultant is $\boxed{5.2 \text{ m at } -60°}$.

3.9 Using the vector diagram given at the right, we find



$$R = \sqrt{(6.00 \text{ m})^2 + (5.40 \text{ m})^2} = 8.07 \text{ m}$$

and $\theta = \tan^{-1}\left(\frac{5.40 \text{ m}}{6.00 \text{ m}}\right) = \tan^{-1}(0.900) = 42.0^\circ$.

Thus, the required displacement is $8.07 \text{ m at } 42.0° \text{ S of E}$.

3.10 The total displacement is $\mathbf{D} = 3.10 \text{ km at } 25.0° \text{ N of E}$. The north and east components of this displacement are:

$$D_y = (3.10 \text{ km})\sin 25.0° = \boxed{1.31 \text{ km north}},$$

and $$D_x = (3.10 \text{ km})\cos 25.0° = \boxed{2.81 \text{ km east}}.$$

3.11 (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00 \text{ blocks}$, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00 \text{ blocks}$. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{net})^2 + (y_{net})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00 \text{ blocks},$$

and the angle the resultant makes with the x-axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°$$

The resultant displacement is then $\boxed{5.00 \text{ blocks at } 53.1° \text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0 \text{ blocks}}$.

3.12 $+x = \text{eastward}$, $+y = \text{northward}$

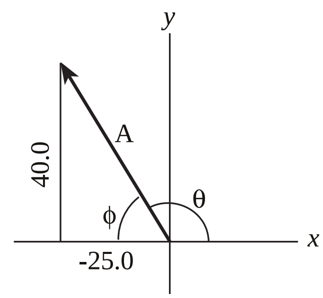
$$\Sigma x = 250 \text{ m} + (125 \text{ m})\cos 30.0° = 358 \text{ m}$$

$$\Sigma y = 75.0 \text{ m} + (125 \text{ m})\sin 30.0° - 150 \text{ m} = -12.5 \text{ m}$$

$$d = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(358 \text{ m})^2 + (-12.5 \text{ m})^2} = 358 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(-\frac{12.5}{358}\right) = -2.00° \quad \boxed{\mathbf{d} = 358 \text{ m at } 2.00° \text{ S of E}}$$

3.13 $A_x = -25.0$ $A_y = 40.0$



$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2 \text{ units}}$$

From the triangle, we find that $\phi = 58.0°$,
so that $\boxed{\theta = 122°}$.

3.14 Let **A** be the vector corresponding to the 10.0 yd run, **B** to the 15.0 yd run, and **C** to the 50.0 yd pass. Also, we choose a coordinate system with the +*y* direction downfield, and the **+** *x* direction toward the sideline to which the player runs.

The components of the vectors are then

$A_x = 0$ $A_y = -10.0 \text{ yds}$

$B_x = 15.0 \text{ yds}$ $B_y = 0$

$C_x = 0$ $C_y = +50.0 \text{ yds}$

From these, $R_x = \Sigma F_x = 15.0 \text{ yds}$, and $R_y = \Sigma F_y = 40.0 \text{ yds}$,

and $$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(15.0 \text{ yds})^2 + (40.0 \text{ yds})^2} = \boxed{42.7 \text{ yards}}.$$

3.15 After 3.00 h moving at 41.0 km/h, the hurricane is 123 km at 60.0° N of W from the island. In the next 1.50 h, it travels 37.5 km due north. The components of these two displacements are:

Displacement	x-component (eastward)	y-component (northward)
123 km	-61.5 km	+107 km
37.5 km	0	+37.5 km
Resultant	-61.5 km	144 km

Therefore, the eye of the hurricane is now

$$R = \sqrt{(-61.5\text{ km})^2 + (144\text{ km})^2} = \boxed{157\text{ km from the island}}.$$

3.16 Choose the positive x direction to be eastward and positive y as northward. Then, the components of the resultant displacement from Dallas to Chicago are:

$$R_x = \Sigma x = (730\text{ mi})\cos 5.00° - (560\text{ mi})\sin 21.0° = 527\text{ mi},$$

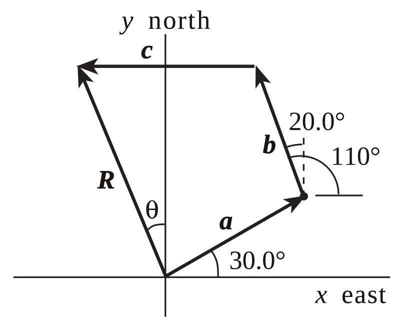
and $$R_y = \Sigma y = (730\text{ mi})\sin 5.00° + (560\text{ mi})\cos 21.0° = 586\text{ mi}.$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(527\text{ mi})^2 + (586\text{ mi})^2} = 788\text{ mi}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}(1.11) = 48.1°$$

Thus, the displacement from Dallas to Chicago is $\mathbf{R} = \boxed{788\text{ mi at }48.1°\text{ N of E}}$.

3.17 The components of the displacements **a**, **b**, and **c** are:



$$a_x = a\cdot\cos 30.0° = +152 \text{ km},$$
$$b_x = b\cdot\cos 110° = -51.3 \text{ km},$$
$$c_x = c\cdot\cos 180° = -190 \text{ km},$$

and

$$a_y = a\cdot\sin 30.0° = +87.5 \text{ km},$$
$$b_y = b\cdot\sin 110° = +141 \text{ km},$$
$$c_y = c\cdot\sin 180° = 0.$$

Thus, $R_x = a_x + b_x + c_x = -89.7 \text{ km}$, and $R_y = a_y + b_y + c_y = +228 \text{ km}$,

so $R = \sqrt{R_x^2 + R_y^2} = 245 \text{ km}$, and $\theta = \tan^{-1}\left(\frac{|R_x|}{R_y}\right) = \tan^{-1}(1.11) = 21.4°$

City C is $\boxed{\text{245 km at 21.4° W of N}}$ from the starting point.

3.18 (a) $F_1 = 120 \text{ N}$ $F_{1x} = 60.0 \text{ N}$ $F_{1y} = 104 \text{ N}$

$F_2 = 80.0 \text{ N}$ $F_{2x} = -20.7 \text{ N}$ $F_{2y} = 77.3 \text{ N}$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(39.3 \text{ N})^2 + (181 \text{ N})^2} = 185 \text{ N}$$

and $\theta = \tan^{-1}\left(\frac{181 \text{ N}}{39.3 \text{ N}}\right) = \tan^{-1}(4.61) = 77.8°$

The resultant is $\mathbf{R} = \boxed{\text{185 N at 77.8° from the x-axis}}$.

(b) To have zero net force on the mule, the resultant above must be cancelled by a force equal in magnitude and oppositely directed. Thus, the required force is $\boxed{\text{185 N at 258° from the x-axis}}$.

3.19 The resultant displacement is $\mathbf{R} = \mathbf{A} + \mathbf{B}$, where **A** is the 150 cm displacement at 120° and **B** is the required second displacement. Solving for **B**: $\mathbf{B} = \mathbf{R} - \mathbf{A} = \mathbf{R} + (-\mathbf{A})$.

The components of **B** are

$$B_x = R_x - A_x = +190 \text{ cm} \;\text{ and }\; B_y = R_y - A_y = -49.6 \text{ cm} .$$

Hence, $B = \sqrt{B_x^2 + B_y^2} = 196 \text{ cm}$ and $\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}(-0.262) = -14.7°$.

$\mathbf{B} = \boxed{\text{196 cm at 14.7° below the x-axis}}$.

3.20 Let $+x$ = East, $+y$ = North,

Displacement	x (km)	y (km)
300 km, due E	300	0
350 km, 30° W of N	–175	303
150 km, due N	0	150
Resultant	$\Sigma x = 125$	$\Sigma y = 453$

(a) $\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \boxed{74.6° \text{ N of E}}$ (b) $|\mathbf{R}| = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \boxed{470 \text{ km}}$

3.21 (a) Your first displacement takes you to point A, so $\mathbf{r}_1 = \mathbf{r}_A$. In the second displacement, you go one-half the distance from A toward B, so $\Delta\mathbf{r} = \frac{1}{2}(\mathbf{r}_B - \mathbf{r}_A)$ and your current position vector is $\mathbf{r}_2 = \mathbf{r}_1 + \Delta\mathbf{r} = \mathbf{r}_A + \frac{1}{2}(\mathbf{r}_B - \mathbf{r}_A) = \frac{\mathbf{r}_A + \mathbf{r}_B}{2}$. On the next leg of the hunt, your displacement is $\Delta\mathbf{r} = \mathbf{r}_A + \frac{1}{3}(\mathbf{r}_C - \mathbf{r}_2)$ and your new position vector becomes $\mathbf{r}_3 = \mathbf{r}_2 + \Delta\mathbf{r} = \mathbf{r}_2 + \frac{1}{3}(\mathbf{r}_C - \mathbf{r}_2) = \frac{2}{3}\mathbf{r}_2 + \frac{1}{3}\mathbf{r}_C = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C}{3}$. The next displacement is $\Delta\mathbf{r} = \frac{1}{4}(\mathbf{r}_D - \mathbf{r}_3)$ and your position vector changes to

$\mathbf{r}_4 = \mathbf{r}_3 + \Delta\mathbf{r} = \mathbf{r}_3 + \frac{1}{4}(\mathbf{r}_D - \mathbf{r}_3) = \frac{3}{4}\mathbf{r}_3 + \frac{1}{4}\mathbf{r}_D = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D}{4}$. On the final leg of the hunt, the displacement is $\Delta\mathbf{r} = \frac{1}{5}(\mathbf{r}_E - \mathbf{r}_4)$. Therefore, the position vector of the treasure is

$$\mathbf{r}_5 = \mathbf{r}_4 + \Delta\mathbf{r} = \mathbf{r}_4 + \frac{1}{5}(\mathbf{r}_E - \mathbf{r}_4).$$

$$= \frac{4}{5}\mathbf{r}_4 + \frac{1}{5}\mathbf{r}_E = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}.$$

To determine the coordinates of this location, we consider

$$\mathbf{R} = \mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E.$$

Vector	x component (m)	y component (m)
$\mathbf{r}_A$	30.0	–20.0
$\mathbf{r}_B$	60.0	80.0
$\mathbf{r}_C$	–10.0	–10.0
$\mathbf{r}_D$	40.0	–30.0
$\mathbf{r}_E$	–70.0	60.0
$\mathbf{R}$	$\Sigma x = 50.0$	$\Sigma y = 80.0$

The position vector of the treasure is $\mathbf{r}_5 = \frac{\mathbf{R}}{5}$ and its coordinates are then seen to be $x = \frac{1}{5}R_x = \boxed{+10.0\text{ m}}$ and $y = \frac{1}{5}R_y = \boxed{+16.0\text{ m}}$.

(b) From the solution of part (a), the position vector of the treasure's location is seen to depend on the sum of the position vectors of the individual trees: $\mathbf{r}_5 = \frac{\mathbf{R}}{5} = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}$. Interchanging the trees would only change the order of the vectors in this sum. Since a vector sum is independent of the order in which the vectors are added, the answer found in part (a) does not depend on the order of the trees.

3.22 $v_{ix} = 100.8\ \text{mi/h} = 45.06\ \text{m/s}$ and $\Delta x = 60.0\ \text{ft} = 18.3\ \text{m}$

The time to reach home plate is $t = \dfrac{\Delta x}{v_{ix}} = \dfrac{18.3\ \text{m}}{45.06\ \text{m/s}} = 0.406\ \text{s}$.

In this time interval, the vertical displacement is

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left(0.406\ \text{s}\right)^2 = -0.807\ \text{m}.$$

Thus, the ball drops vertically $0.807\ \text{m}\left(\dfrac{3.281\ \text{ft}}{1\ \text{m}}\right) = \boxed{2.65\ \text{ft}}$.

3.23 The constant horizontal speed of the falcon is

$$v_x = 200\ \frac{\text{mi}}{\text{h}}\left(\frac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right) = 89.4\ \text{m/s}.$$

The time required to travel 100 m horizontally is $t = \dfrac{\Delta x}{v_x} = \dfrac{100\ \text{m}}{89.4\ \text{m/s}} = 1.12\ \text{s}$. The vertical displacement during this time is

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left(1.12\ \text{s}\right)^2 = -6.13\ \text{m},$$

or the falcon has a vertical fall of $\boxed{6.13\ \text{m}}$.

3.24 We find the time of fall from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ with $v_{iy} = 0$:

$$t = \sqrt{\frac{2\left(\Delta y\right)}{a}} = \sqrt{\frac{2\left(-50.0\ \text{m}\right)}{-9.80\ \text{m/s}^2}} = \boxed{3.19\ \text{s}}.$$

At impact, $v_x = v_{ix} = 18.0\ \text{m/s}$, and the vertical component is

$$v_y = v_{iy} + a_y t = 0 + \left(-9.80\ \text{m/s}^2\right)\left(3.19\ \text{s}\right) = -31.3\ \text{m/s}.$$

Thus, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(18.0\ \text{m/s})^2 + (-31.3\ \text{m/s})^2} = 36.1\ \text{m/s}$

and $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-31.3}{18.0}\right) = -60.1°$,

or $\mathbf{v} = \boxed{36.1\ \text{m/s at } 60.1° \text{ below the horizontal}}$.

3.25 At the maximum height $v_y = 0$, and the time to reach this height is found from $v_y = v_{iy} + a_y t$ as $t = \frac{v_y - v_{iy}}{a_y} = \frac{0 - v_{iy}}{-g} = \frac{v_{iy}}{g}$.

The vertical displacement that has occurred during this time is

$$(\Delta y)_{max} = \bar{v}_y t = \left(\frac{v_y + v_{iy}}{2}\right)t = \left(\frac{0 + v_{iy}}{2}\right)\left(\frac{v_{iy}}{g}\right) = \frac{v_{iy}^2}{2g}.$$

Thus, if $(\Delta y)_{max} = 12\ \text{ft}\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right) = 3.7\ \text{m}$, then

$$v_{iy} = \sqrt{2g(\Delta y)_{max}} = \sqrt{2(9.80\ \text{m/s}^2)(3.7\ \text{m})} = 8.5\ \text{m/s},$$

and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_i = \frac{v_{iy}}{\sin\theta} = \frac{8.5\ \text{m/s}}{\sin 45°} = \boxed{12\ \text{m/s}}.$$

3.26 The time of flight for Tom is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ with $v_{iy} = 0$:

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-1.5\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.55\ \text{s}.$$

The horizontal displacement during this time is

$$\Delta x = v_{ix}t = (5.0\ \text{m/s})(0.55\ \text{s}) = 2.8\ \text{m}.$$

Thus, he lands $\boxed{2.8\ \text{m from the base of the table}}$.

The horizontal component of velocity does not change during the flight, so $v_x = v_{ix} = \boxed{5.0 \text{ m/s}}$. The vertical component of velocity is found as

$$v_y = v_{iy} + a_y t = 0 - \left(-9.80 \text{ m/s}^2\right)(0.55 \text{ s}) = \boxed{-5.4 \text{ m/s}}.$$

3.27 When $\Delta y = (\Delta y)_{max}$, $v_y = 0$.

Thus, $v_y = v_{iy} + a_y t$ yields $0 = v_i \sin 3.00° - gt$ or $t = \dfrac{v_i \sin 3.00°}{g}$.

The vertical displacement is $\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$. At the maximum height, this becomes

$$(\Delta y)_{max} = (v_i \sin 3.00°)\left(\frac{v_i \sin 3.00°}{g}\right) - \frac{1}{2} g \left(\frac{v_i \sin 3.00°}{g}\right)^2 = \frac{v_i^2 \sin^2 3.00°}{2g}.$$

If $(\Delta y)_{max} = 0.330 \text{ m}$, the initial speed is

$$v_i = \frac{\sqrt{2g(\Delta y)_{max}}}{\sin 3.00°} = \frac{\sqrt{2\left(9.80 \text{ m/s}^2\right)(0.330 \text{ m})}}{\sin 3.00°} = \boxed{48.6 \text{ m/s}}.$$

Note that it was unnecessary to use the horizontal distance of 12.6 m in this solution.

3.28 The horizontal displacement at $t = 42.0 \text{ s}$ is

$$x = v_{ix} t = (v_i \cos\theta) t = (300 \text{ m/s})(\cos 55.0°)(42.0 \text{ s}) = \boxed{7.23 \times 10^3 \text{ m}}.$$

The vertical displacement at $t = 42.0 \text{ s}$ is

$$y = v_{iy} t + \frac{1}{2} a_y t^2 = (v_i \sin\theta) t - \frac{1}{2} g t^2$$

$$= (300 \text{ m/s})(\sin 55.0°)(42.0 \text{ s}) + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)(42.0 \text{ s})^2 = \boxed{1.68 \times 10^3 \text{ m}}$$

3.29 We choose our origin at the initial position of the projectile. After 3.00 s, it is at ground level, so the vertical displacement is $\Delta y = -H$.

To find *H*, we use $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$, which becomes

$$-H = (15\ \text{m/s})(\sin 25°)(3.0\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(3.0\ \text{s})^2,\ \text{or}\ H = \boxed{25\ \text{m}}.$$

3.30 The components of the initial velocity are:

$$v_{ix} = v_i \cos 53.0° = 12.0\ \text{m/s},\ \text{and}\ v_{iy} = v_i \sin 53.0° = 16.0\ \text{m/s}.$$

(a) The time required for the ball to reach the position of the crossbar is

$$t = \frac{\Delta x}{v_{ix}} = \frac{36.0\ \text{m}}{12.0\ \text{m/s}} = 3.00\ \text{s}.$$

At this time, the height of the football above the ground is

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 = \left(16.0\ \frac{\text{m}}{\text{s}}\right)(3.00\ \text{s}) + \frac{1}{2}\left(-9.80\ \frac{\text{m}}{\text{s}^2}\right)(3.00\ \text{s})^2 = 3.90\ \text{m}$$

Thus, the ball $\boxed{\text{clears the crossbar by } 3.90\ \text{m} - 3.05\ \text{m} = 0.85\text{m}}$.

(b) The vertical component of the velocity of the ball as it moves over the crossbar is $v_y = v_{iy} + a_y t = (16.0\ \text{m/s}) + (-9.80\ \text{m/s}^2)(3.00\ \text{s}) = -13.4\ \text{m/s}$. The negative sign indicates the ball is moving downward or $\boxed{\text{falling}}$.

3.31 The speed of the car when it reaches the edge of the cliff is

$$v = \sqrt{v_i^2 + 2a(\Delta x)} = \sqrt{0 + 2(4.00\ \text{m/s}^2)(50.0\ \text{m})} = 20.0\ \text{m/s}.$$

Now, consider the projectile phase of the car's motion. The vertical velocity of the car as it reaches the water is

$$v_y = -\sqrt{v_{iy}^2 + 2a_y(\Delta y)} = \sqrt{\left[-(20.0\ \text{m/s})\sin 24.0°\right]^2 + 2(-9.80\ \text{m/s}^2)(-30.0\ \text{m})},$$

or $\qquad v_y = -25.6\ \text{m/s}$.

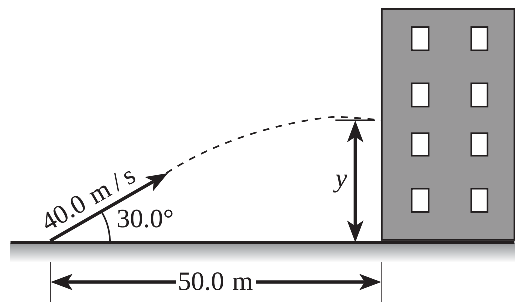
(b) The time of flight is

$$t = \frac{v_y - v_{iy}}{a_y} = \frac{-25.6\ \text{m/s} - \left[-(20.0\ \text{m/s})\sin 24.0°\right]}{-9.80\ \text{m/s}^2} = \boxed{1.78\ \text{s}}.$$

(a) The horizontal displacement of the car during this time is

$$\Delta x = v_{ix}t = \left[(20.0\ \text{m/s})\cos 24.0°\right](1.78\ \text{s}) = \boxed{32.5\ \text{m}}.$$

3.32 The components of the initial velocity are



$$v_{ix} = (40.0\ \text{m/s})\cos 30.0° = 34.6\ \text{m/s}, \text{ and}$$

$$v_{iy} = (40.0\ \text{m/s})\sin 30.0° = 20.0\ \text{m/s}.$$

The time for the water to reach the building is

$$t = \frac{\Delta x}{v_{ix}} = \frac{50.0\ \text{m}}{34.6\ \text{m}} = 1.44\ \text{s}.$$

The height of the water at this time is

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 = (20.0\ \text{m/s})(1.44\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.44\ \text{s})^2 = \boxed{18.6\ \text{m}}.$$

3.33 (a) At the highest point of the trajectory, the projectile is moving horizontally with velocity components of $v_y = 0$ and

$$v_x = v_{ix} = v_i \cos\theta = (60.0\ \text{m/ s})\cos 30.0° = \boxed{52.0\ \text{m/s}}.$$

(b) The horizontal displacement is $\Delta x = v_{ix}\, t = (52.0\ \text{m/s})(4.00\ \text{s}) = 208\ \text{m}$ and, from $\Delta y = (v_i \sin\theta)t + \frac{1}{2}a_y t^2$, the vertical displacement is

$$\Delta y = (60.0\ \text{m/s})(\sin 30.0°)(4.00\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(4.00\ \text{s})^2 = 41.6\ \text{m}.$$

The straight line distance is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(208\ \text{m})^2 + (41.6\ \text{m})^2} = \boxed{212\ \text{m}}.$$

3.34 The horizontal kick gives zero initial vertical velocity to the ball. Then, from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$, the time of flight is

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-40.0\ \text{m})}{-9.80\ \text{m/s}^2}} = \sqrt{8.16}\ \text{s}\ .$$

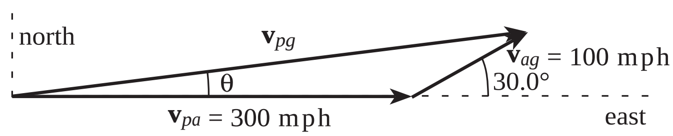
The extra time $\Delta t = 3.00\ \text{s} - \sqrt{8.16}\ \text{s} = 0.143\ \text{s}$ is the time required for the sound to travel in a straight line back to the player. The distance the sound travels is $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = v_{sound}\Delta t$ where Δx represents the horizontal displacement of the ball when it hits the water. Thus,

$$\Delta x = \sqrt{d^2 - (\Delta y)^2} = \sqrt{\left[(343\ \text{m/s})(0.143\ \text{s})\right]^2 - (-40.0\ \text{m})^2} = 28.3\ \text{m}\ .$$

The initial velocity given the ball must have been

$$v_i = v_{ix} = \frac{\Delta x}{t} = \frac{28.3\ \text{m}}{\sqrt{8.16}\ \text{s}} = \boxed{9.91\ \text{m/s}}$$

3.35 The velocity of the plane relative to the ground is the vector sum of the velocity of the plane relative to the air and the velocity of the air relative to the ground, or $\mathbf{v}_{pg} = \mathbf{v}_{pa} + \mathbf{v}_{ag}$.



The components of this velocity are

$$v_{pg}\big|_{east} = 300\ \text{mi/h} + (100\ \text{mi/h})\cos 30.0° = 387\ \text{mi/h}\ ,$$

and $$v_{pg}\big|_{north} = 0 + (100\ \text{mi/h})\sin 30.0° = 50.0\ \text{mi/h}\ .$$

Thus, $$v_{pg} = \sqrt{\left(v_{pg}\right)^2_{east} + \left(v_{pg}\right)^2_{north}} = \sqrt{(387)^2 + (50.0)^2}\ \text{mi/h} = 390\ \text{mi/h}$$

and $$\theta = \tan^{-1}\left(\frac{v_{pg}\big|_{north}}{v_{pg}\big|_{east}}\right) = \tan^{-1}\left(\frac{50.0}{387}\right) = 7.37°$$

The plane moves at $\boxed{390\ \text{mi/h at } 7.37°\ \text{N of E relative to the ground}}$.

3.36 We use the following notation:

$\mathbf{v}_{bs} =$ velocity of boat relative to the shore

$\mathbf{v}_{bw} =$ velocity of boat relative to the water,

and $\mathbf{v}_{ws} =$ velocity of water relative to the shore.

If we take downstream as the positive direction, then $\mathbf{v}_{ws} = +1.5\ \text{m/s}$ for both parts of the trip. Also, $\mathbf{v}_{bw} = +10\ \text{m/s}$ while going downstream and $\mathbf{v}_{bw} = -10\ \text{m/s}$ for the upstream part of the trip.

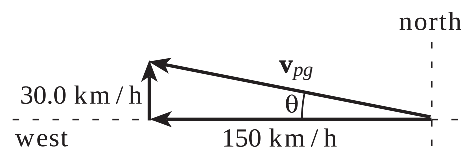
The velocity of the boat relative to the shore is given by $\mathbf{v}_{bs} = \mathbf{v}_{bw} + \mathbf{v}_{ws}$.

While going downstream, $v_{bs} = 10\ \text{m/s} + 1.5\ \text{m/s}$ and the time to go 300 m downstream is $t_{down} = \dfrac{300\ \text{m}}{(10+1.5)\ \text{m/s}} = 26\ \text{s}$.

When going upstream, $v_{bs} = -10\ \text{m/s} + 1.5\ \text{m/s} = -8.5\ \text{m/s}$ and the time required to move 300 m upstream is $t_{up} = \dfrac{-300\ \text{m}}{-8.5\ \text{m/s}} = 35\ \text{s}$.

The time for the round trip is $t = t_{down} + t_{up} = (26+35)\ \text{s} = \boxed{61\ \text{s}}$.

3.37 The velocity of the plane relative to the ground is the vector sum of the velocity of the plane relative to the air and the velocity of the air relative to the ground as shown in the diagram. Thus,

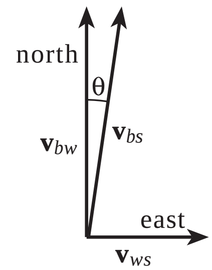


$$v_{pg} = \sqrt{(150)^2 + (30.0)^2}\ \text{km/h} = 153\ \text{km/h}$$

and $$\theta = \tan^{-1}\left(\frac{30.0}{150}\right) = 11.3°.$$

The plane has a velocity of $\boxed{153\ \text{km/h at } 11.3°\text{ N of W}}$ relative to the ground.

3.38 $\mathbf{v}_{bw} = 10\ \text{m/s}$, directed northward, is the velocity of the boat relative to the water.



$\mathbf{v}_{ws} = 1.5\ \text{m/s}$, directed eastward, is the velocity of the water relative to shore.

$\mathbf{v}_{bs}$ is the velocity of the boat relative to shore, and directed at an angle of θ, relative to the northward direction as shown.

$$\mathbf{v}_{bs} = \mathbf{v}_{bw} + \mathbf{v}_{ws}$$

The northward component of $\mathbf{v}_{bs}$ is $v_{bs}\cos\theta = v_{bw} = 10\ \text{m/s}$. (1)

The eastward component is $v_{bs}\sin\theta = v_{ws} = 1.5\ \text{m/s}$. (2)

(a) Dividing equation (2) by equation (1) gives

$$\theta = \tan^{-1}\left(\frac{v_{ws}}{v_{bw}}\right) = \tan^{-1}\left(\frac{1.50}{10.0}\right) = 8.53°\ .$$

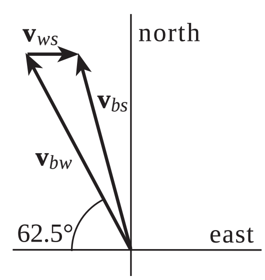
From equation (1), $v_{bs} = \dfrac{10\ \text{m/s}}{\cos 8.53°} = 10.1\ \text{m/s}$.

Therefore, $\mathbf{v}_{bs} = \boxed{10.1\ \text{m/s at } 8.53°\text{ E of N}}$.

(b) The time to cross the river is $t = \dfrac{300\ \text{m}}{v_{bs}\cos\theta} = \dfrac{300\ \text{m}}{10.0\ \text{m/s}} = 30.0\ \text{s}$ and the downstream drift of the boat during this crossing is

$$drift = \left(v_{bs}\sin\theta\right)t = \left(1.50\ \text{m/s}\right)\left(30.0\ \text{s}\right) = \boxed{45.0\ \text{m}}.$$

3.39 $\mathbf{v}_{bw} =$ velocity of boat relative to the water,



$\mathbf{v}_{ws} =$ velocity of water relative to the shore

and $\mathbf{v}_{bs} =$ velocity of boat relative to the shore.

$\mathbf{v}_{bs} = \mathbf{v}_{bw} + \mathbf{v}_{ws}$ as shown in the diagram.

The northward (i.e., cross-stream) component of $\mathbf{v}_{bs}$ is

$$\left(\mathbf{v}_{bs}\right)_{north} = \left(v_{bw}\right)\sin 62.5° + 0 = \left(3.30\ \text{mi/h}\right)\sin 62.5° + 0 = 2.93\ \text{mi/h}\ .$$

The time required to cross the stream is then $t=\dfrac{0.505\text{ mi}}{2.93\text{ mi/h}}=0.173\text{ h}$.

The eastward (i.e., downstream) component of $\mathbf{v}_{bs}$ is

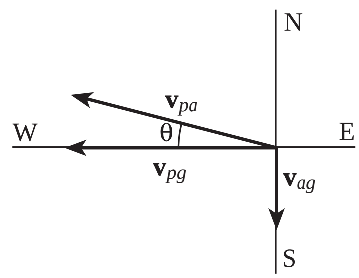
$$\left(\mathbf{v}_{bs}\right)_{east}=-\left(v_{bw}\right)\cos 62.5°+v_{ws}$$

$$=-\left(3.30\text{ mi/h}\right)\cos 62.5°+1.25\text{ mi/h}=-0.274\text{ mi/h} .$$

Since the last result is negative, it is seen that the boat moves upstream as it crosses the river. The distance it moves upstream is

$$d=\left|\left(\mathbf{v}_{bs}\right)_{east}\right|t=\left(0.274\text{ mi/h}\right)\left(0.173\text{ h}\right)=\left(4.72\times10^{-2}\text{ mi}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)=\boxed{249\text{ ft}}.$$

3.40 $\mathbf{v}_{pa}$ = the velocity of the plane relative to the air $=200\text{ km/h}$.



$\mathbf{v}_{ag}$ = the velocity of the air relative to the ground $=50.0\text{ km/h}$ (south).

$\mathbf{v}_{pg}$ = the velocity of the plane relative to the ground (to be due west).

(a) The velocity of the plane relative to the ground is given by the vector sum $\mathbf{v}_{pg}=\mathbf{v}_{pa}+\mathbf{v}_{ag}$. If $\mathbf{v}_{pg}$ is to have zero northward component as shown in the diagram, we must have $v_{pa}\sin\theta=v_{ag}$, or

$$\theta=\sin^{-1}\left(\frac{v_{ag}}{v_{pa}}\right)=\sin^{-1}\left(\frac{50.0\text{ km/h}}{200\text{ km/h}}\right)=14.5° .$$

Thus, the plane should head at $\boxed{14.5°\text{ N of W}}$.

(b) Since $\mathbf{v}_{pg}$ has zero northward component, the plane's ground speed is

$$v_{pg}=v_{pa}\cos\theta=\left(200\text{ km/h}\right)\cos 14.5°=\boxed{194\text{ km/h}}.$$

3.41 The velocity of the faster car relative to the slower car is given by $\mathbf{v}_{fs} = \mathbf{v}_{fe} + \mathbf{v}_{es}$, where $\mathbf{v}_{fe} = +60.0\ \text{km/h}$ is the velocity of the faster car relative to Earth and $\mathbf{v}_{es} = -\mathbf{v}_{se} = -40.0\ \text{km/h}$ is the velocity of Earth relative to the slower car.

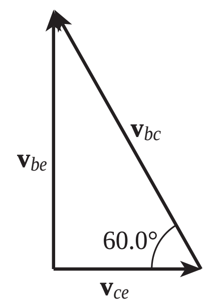
Thus, $\mathbf{v}_{fs} = +60.0\ \text{km/h} - 40.0\ \text{km/h} = +20.0\ \text{km/h}$ and the time required for the faster car to move 100 m (0.100 km) closer to the slower car is

$$t = \frac{d}{v_{fs}} = \frac{0.100\ \text{km}}{20.0\ \text{km/h}} = 5.00\times10^{-3}\ \text{h}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = \boxed{18.0\ \text{s}}$$

3.42 $\mathbf{v}_{bc} =$ the velocity of the ball relative to the car.

$\mathbf{v}_{ce} =$ velocity of the car relative to the earth $= 10\ \text{m/s}$.

$\mathbf{v}_{be} =$ the velocity of the ball relative to the earth.



These velocities are related by the equation $\mathbf{v}_{be} = \mathbf{v}_{bc} + \mathbf{v}_{ce}$ as illustrated in the diagram.

Considering the horizontal components, we see that

$$v_{bc}\cos 60.0° = v_{ce} \text{ or } v_{bc} = \frac{v_{ce}}{\cos 60.0°} = \frac{10.0\ \text{m/s}}{\cos 60.0°} = 20.0\ \text{m/s}.$$

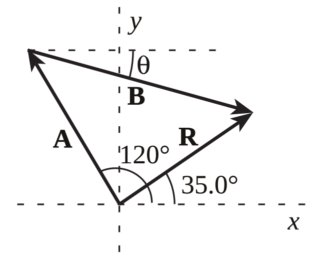
From the vertical components, the initial velocity of the ball relative to the earth is $v_{be} = v_{bc}\sin 60.0° = 17.3\ \text{m/s}$.

Using $v_y^2 = v_{iy}^2 + 2a_y(\Delta y)$, with $v_y = 0$ when the ball is at maximum height, we find

$$(\Delta y)_{max} = \frac{0 - v_{iy}^2}{2a_y} = \frac{0 - v_{be}^2}{2(-g)} = \frac{(17.3\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{15.3\ \text{m}}$$

as the maximum height the ball rises.

3.43 Since $\mathbf{R} = \mathbf{A} + \mathbf{B}$, then $\mathbf{B} = \mathbf{R} - \mathbf{A}$ and the components of the second displacement are:



$$B_x = R_x - A_x$$

$$= (140 \text{ cm})\cos 35.0° - (150 \text{ cm})\cos 120° = +190 \text{ cm}$$

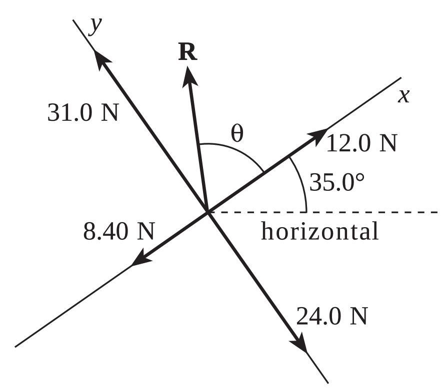
and

$$B_y = R_y - A_y = (140 \text{ cm})\sin 35.0° - (150 \text{ cm})\sin 120° = -49.6 \text{ cm}.$$

Thus, $B = \sqrt{B_x^2 + B_y^2} = 196 \text{ cm}$, and $\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}(-0.262) = -14.7°$.

The second displacement is 196 cm at 14.7° below the positive x-axis.

3.44 Observe that when one chooses the x and y axes as shown in the drawing, each of the four forces lie along one of the axes. The resultant, **R**, is easily computed as



$$R_x = \Sigma F_x = +12.0 \text{ N} - 8.40 \text{ N} = +3.60 \text{ N}$$

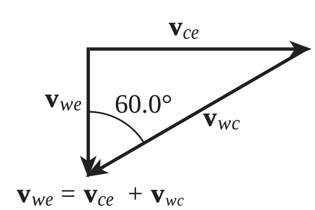
$$R_y = \Sigma F_y = +31.0 \text{ N} - 24.0 \text{ N} = +7.00 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(3.60 \text{ N})^2 + (7.00 \text{ N})^2} = 7.87 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 62.8°, \text{ or } 62.8° + 35.0° = 97.8° \text{ from the horizontal}.$$

R = 7.87 N at 97.8° counterclockwise from the horizontal line to the right.

3.45 $\mathbf{v}_{ce}$ = the velocity of the car relative to the earth.



$\mathbf{v}_{wc}$ = the velocity of the water relative to the car.

$\mathbf{v}_{we}$ = the velocity of the water relative to the earth.

These velocities are related as shown in the diagram at the right.

(a) Since $\mathbf{v}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0\ \text{km/h}$

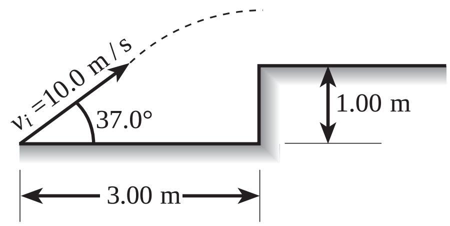
or $\mathbf{v}_{wc} = \boxed{57.7\ \text{km/h at } 60.0° \text{ west of vertical}}$.

(b) Since $\mathbf{v}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\ \text{km/h})\cos 60.0° = \boxed{28.9\ \text{km/h downward}}.$$

3.46 (a) $v_{ix} = v_i \cos\theta = (10.0\ \text{m/s})\cos 37.0° = \boxed{7.99\ \text{m/s}}$

$v_{iy} = v_i \sin\theta = (10.0\ \text{m/s})\sin 37.0° = \boxed{6.02\ \text{m/s}}$



(b) First, we make sure the stone does not hit the front edge of the dock.

When $\Delta x = 3.00\ \text{m}$, $t = \dfrac{\Delta x}{v_{ix}} = \dfrac{3.00\ \text{m}}{7.99\ \text{m/s}} = 0.376\ \text{s}$.

At this time,

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 = (6.02\ \text{m/s})(0.376\ \text{s}) - (4.90\ \text{m/s}^2)(0.376\ \text{s})^2$$

or $\Delta y = 1.57\ \text{m} > 1.00\ \text{m}$. The stone clears the front corner of the dock.

To find the maximum height reached, use $v_y^2 = v_{iy}^2 + 2a_y(\Delta y)$.

Since $v_y = 0$ when $\Delta y = (\Delta y)_{max}$, this gives

$$(\Delta y)_{max} = \frac{0 - v_{iy}^2}{2a_y} = \frac{-(6.02\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{1.85\ \text{m above the ground}}.$$

(c) When $\Delta y = 1.00\text{ m}$, $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ yields

$$1.00\text{ m} = (6.02\text{ m/s})t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

or $t^2 - (1.23\text{ s})t + (0.204\text{ s}^2) = 0$ with solutions of

t = 0.198 s and t = 1.03 s.

Since the stone hits the dock as it comes back down, use the second solution t = 1.03 s. At this time, $\Delta x = v_{ix}\,t = (7.99\text{ m/s})(1.03\text{ s}) = 8.23\text{ m}$.

Thus, the stone hits the dock **5.23 m beyond the edge**.

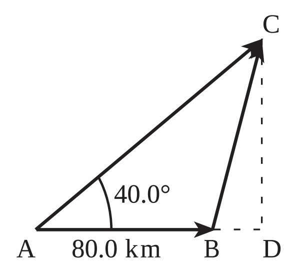
(d) At t = 1.03 s,

$$v_y = v_{iy} + a_y t = 6.02\text{ m/s} - (9.80\text{ m/s}^2)(1.03\text{ s}) = -4.08\text{ m/s}$$

and $v_x = v_{ix} = 7.99\text{ m/s}$.

Therefore, $v = \sqrt{v_x^2 + v_y^2} = \boxed{8.97\text{ m/s}}$

3.47 $\overline{AC} = v_1 t = (90.0\text{ km/h})(2.50\text{ h}) = 225\text{ km}$



$\overline{BD} = \overline{AD} - \overline{AB} = \overline{AC}\cos 40.0° - 80.0\text{ km} = 92.4\text{ km}$

From the triangle BCD,

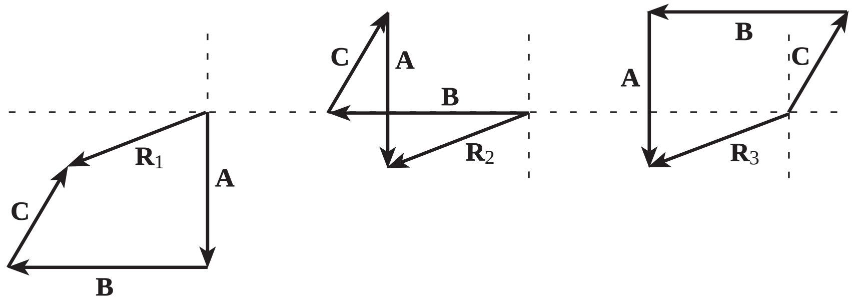
$$\overline{BC} = \sqrt{(\overline{BD})^2 + (\overline{DC})^2}$$

$$= \sqrt{(92.4\text{ km})^2 + (\overline{AC}\sin 40.0°)^2} = 172\text{ km}$$

Since Car 2 travels this distance in 2.50 h, its constant speed is

$$v_2 = \frac{172\text{ km}}{2.50\text{ h}} = \boxed{68.6\text{ km/h}}$$

3.48 The three diagrams shown below represent the graphical solutions for the three vector sums: $\mathbf{R}_1 = \mathbf{A} + \mathbf{B} + \mathbf{C}$, $\mathbf{R}_2 = \mathbf{B} + \mathbf{C} + \mathbf{A}$, and $\mathbf{R}_3 = \mathbf{C} + \mathbf{B} + \mathbf{A}$. You should observe that $\mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}_3$, illustrating that the sum of a set of vectors is not affected by the order in which the vectors are added.



3.49 The distance, *s*, moved in the first 3.00 seconds is given by

$$s = v_i t + \frac{1}{2}at^2 = (100\ \text{m/s})(3.00\ \text{s}) + \frac{1}{2}(30.0\ \text{m/s}^2)(3.00\ \text{s})^2 = 435\ \text{m}\ .$$

At the end of powered flight, the coordinates of the rocket are:

$$x_1 = s\cos 53.0° = 262\ \text{m, and } y_1 = s\sin 53.0° = 347\ \text{m}$$

The speed of the rocket at the end of powered flight is

$$v_1 = v_i + at = 100\ \text{m/s} + (30.0\ \text{m/s}^2)(3.00\ \text{s}) = 190\ \text{m/s}\ ,$$

so the initial velocity components for the free-fall phase of the flight are $v_{ix} = v_1 \cos 53.0° = 114\ \text{m/s}$ and $v_{iy} = v_1 \sin 53.0° = 152\ \text{m/s}$.

(a) When the rocket is at maximum altitude, $v_y = 0$. The rise time during the free-fall phase can be found from $v_y = v_{iy} + a_y t$ as

$$t_{rise} = \frac{0 - v_{iy}}{a_y} = \frac{0 - 152\ \text{m}}{-9.80\ \text{m/s}^2} = 15.5\ \text{s}\ .$$

The vertical displacement occurring during this time is

$$\Delta y = \left(\frac{v_{fy} + v_{iy}}{2}\right) t_{rise} = \left(\frac{0 + 152\ \text{m/s}}{2}\right)(15.5\ \text{s}) = 1.17 \times 10^3\ \text{m}\ .$$

The maximum altitude reached is then

$$H = y_1 + \Delta y = 347\ \text{m} + 1.17 \times 10^3\ \text{m} = \boxed{1.52 \times 10^3\ \text{m}}$$

(b) After reaching the top of the arc, the rocket falls 1.52×10^3 m to the ground, starting with zero vertical velocity $(v_{iy}=0)$. The time for this fall is found from $\Delta y = v_{iy}t+\frac{1}{2}a_y t^2$ as

$$t_{fall}=\sqrt{\frac{2(\Delta y)}{a_y}}=\sqrt{\frac{2(-1.52\times10^3\text{ m})}{-9.80\text{ m/s}^2}}=17.6\text{ s}.$$

The total time of flight is

$$t=t_{powered}+t_{rise}+t_{fall}=(3.00+15.5+17.6)\text{ s}=\boxed{36.1\text{ s}}.$$

(c) The free-fall phase of the flight lasts for

$$t_2=t_{rise}+t_{fall}=(15.5+17.6)\text{ s}=33.1\text{ s}.$$

The horizontal displacement occurring during this time is

$$\Delta x=v_{ix}t_2=(114\text{ m/s})(33.1\text{ s})=3.78\times10^3\text{ m}$$

and the full horizontal range is

$$R=x_1+\Delta x=262\text{ m}+3.78\times10^3\text{ m}=\boxed{4.05\times10^3\text{ m}}.$$

3.50 The velocity of a canoe relative to the shore is given by $\mathbf{v}_{cs}=\mathbf{v}_{cw}+\mathbf{v}_{ws}$, where $\mathbf{v}_{cw}$ is the velocity of the canoe relative to the water and $\mathbf{v}_{ws}$ is the velocity of the water relative to shore.

Applied to the canoe moving upstream, this gives

$$-1.2\text{ m/s}=-v_{cw}+v_{ws} \qquad (1)$$

and for the canoe going downstream

$$+2.9\text{ m/s}=+v_{cw}+v_{ws} \qquad (2)$$

(a) Adding equations (1) and (2) gives

$$2v_{ws}=1.7\text{ m/s},\text{ so } v_{ws}=\boxed{0.85\text{ m/s}}.$$

(b) Subtracting (1) from (2) yields

$$2v_{cw} = 4.1\ \text{m/s},\ \text{or}\ v_{cw} = \boxed{2.1\ \text{m/s}}.$$

3.51 The time of flight is found from $\Delta y = v_{iy}\,t + \frac{1}{2}a_y t^2$ with $\Delta y = 0$, as $t = \frac{2v_{iy}}{g}$. This gives the range as $R = v_{ix}t = \frac{2v_{ix}v_{iy}}{g}$.

On Earth this becomes $R_{Earth} = \frac{2v_{ix}v_{iy}}{g_{Earth}}$, and on the moon, $R_{Moon} = \frac{2v_{ix}v_{iy}}{g_{Moon}}$.

Dividing R_{Moon} by R_{Earth}, we find $R_{Moon} = \left(\frac{g_{Earth}}{g_{Moon}}\right)R_{Earth}$. With $g_{Moon} = \left(\frac{1}{6}\right)g_{Earth}$, this gives $R_{Moon} = 6R_{Earth} = 6(3.0\ \text{m}) = \boxed{18\ \text{m}}$.

Similarly, $R_{Mars} = \left(\frac{g_{Earth}}{g_{Mars}}\right)R_{Earth} = \frac{3.0\ \text{m}}{0.38} = \boxed{7.9\ \text{m}}$.

3.52 The time to reach the opposite side is $t = \frac{\Delta x}{v_{ix}} = \frac{10\ \text{m}}{v_i \cos 15^\circ}$.

When the motorcycle returns to the original level, the vertical displacement is $\Delta y = 0$. Using this in the relation $\Delta y = v_{iy}\,t + \frac{1}{2}a_y t^2$ gives a second relation between the takeoff speed and the time of flight as:

$$0 = (v_i \sin 15^\circ)t + \frac{1}{2}(-g)t^2\ \text{or}\ v_i = \left(\frac{g}{2\sin 15^\circ}\right)t.$$

Substituting the time found earlier into this result yields the required takeoff speed as

$$v_i = \sqrt{\frac{(9.80\ \text{m/s}^2)(10\ \text{m})}{2(\sin 15^\circ)(\cos 15^\circ)}} = \boxed{14\ \text{m/s}}.$$

3.53 The time to fall 36.0 m, starting with $v_{iy} = 0$, is found from $\Delta y = v_{iy}\, t + \frac{1}{2} a_y t^2$ as $-36.0 \text{ m} = 0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right) t^2$, yielding $t = 2.71 \text{ s}$. Hence, the minimum initial horizontal speed is $v_{ix}\big|_{min} = \frac{\Delta x}{t} = \frac{6.00 \text{ m}}{2.71 \text{ s}} = \boxed{2.21 \text{ m/s}}$.

3.54 For a projectile fired with initial speed v_i at angle θ above the horizontal, the time required to achieve a horizontal displacement of Δx is $t = \frac{\Delta x}{v_{ix}} = \frac{\Delta x}{v_i \cos\theta}$. The vertical displacement of the projectile at this time is

$$\Delta y = v_{iy}\, t + \frac{1}{2} a_y t^2 = \left(v_i \sin\theta\right)\left(\frac{\Delta x}{v_i \cos\theta}\right) - \frac{g}{2}\left(\frac{\Delta x}{v_i \cos\theta}\right)^2 = (\Delta x)\tan\theta - \frac{g(\Delta x)^2}{\left(2\cos^2\theta\right) v_i^2}$$

Solving for the initial speed gives $v_i = \sqrt{\frac{g(\Delta x)^2}{\left(2\cos^2\theta\right)\left[(\Delta x)\tan\theta - \Delta y\right]}}$.

With $\Delta x = 50$ m, $\Delta y = 30$ m, θ=55°, and $g = 9.80 \text{ m/s}^2$,

we find the required initial speed to be $v_i = \boxed{30 \text{ m/s}}$.

3.55 (a) The time to reach the fence is $t = \frac{\Delta x}{v_{ix}} = \frac{130 \text{ m}}{v_i \cos 35°} = \frac{159 \text{ m}}{v_i}$.

At this time, the ball must be 20 m above its launch position.

$\Delta y = v_{iy}\, t + \frac{1}{2} a_y t^2$ gives

$$20 \text{ m} = \left(v_i \sin 35°\right)\left(\frac{159 \text{ m}}{v_i}\right) - \left(4.90 \text{ m/s}^2\right)\left(\frac{159 \text{ m}}{v_i}\right)^2.$$

From which, $v_i = \boxed{42 \text{ m/s}}$.

(b) From above, $t = \frac{159 \text{ m}}{v_i} = \frac{159 \text{ m}}{42 \text{ m/s}} = \boxed{3.8 \text{ s}}$

(c) $v_x = v_{ix} = (42\ \text{m/s})\cos 35° = \boxed{34\ \text{m/s}}$

$$v_y = v_{iy} + a_y t = (42\ \text{m/s})\sin 35° - (9.80\ \text{m/s}^2)(3.8\ \text{s}) = \boxed{-13\ \text{m/s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(34.1\ \text{m/s})^2 + (-13.4\ \text{m/s})^2} = \boxed{37\ \text{m/s}}$$

3.56 We shall first find the initial velocity of the ball thrown vertically upward. At its maximum height, $v_y = 0$ and $t = 1.50\ \text{s}$. Hence, $v_y = v_{iy} + a_y t$ gives

$$0 = v_{iy} - (9.80\ \text{m/s}^2)(1.50\ \text{s}),\ \text{or}\ v_{iy} = 14.7\ \text{m/s}.$$

In order for the second ball to reach the same vertical height as the first, the second must have the same initial vertical velocity. Thus, we find v_i as

$$v_i = \frac{v_{iy}}{\sin 30.0°} = \frac{14.7\ \text{m/s}}{0.500} = \boxed{29.4\ \text{m/s}}.$$

3.57 The time of flight of the ball is given by $\Delta y = v_{iy}\, t + \frac{1}{2} a_y t^2$, with $\Delta y = 0$, as

$$0 = \left[(20\ \text{m/s})\sin 30°\right]t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2\ \text{or}\ t = 2.0\ \text{s}.$$

The horizontal distance the football moves in this time is

$$\Delta x = v_{ix} t = \left[(20\ \text{m/s})\cos 30°\right](2.0\ \text{s}) = 35\ \text{m}.$$

Therefore, the receiver must run a distance of (35 m – 20 m) = 15 m away from the quarterback, $\boxed{\text{in the direction the ball was thrown}}$ to catch the ball. He has a time of 2.0 s to do this, so the required speed is

$$v = \frac{\Delta x}{t} = \frac{15\ \text{m}}{2.0\ \text{s}} = \boxed{7.5\ \text{m/s}}$$

3.58 The components of the initial velocity are

$$v_{ix} = v_i \cos 45° = \frac{v_i}{\sqrt{2}}, \text{ and } v_{iy} = v_i \sin 45° = \frac{v_i}{\sqrt{2}}.$$

The time for the ball to move 10.0 m horizontally is $t = \frac{\Delta x}{v_{ix}} = \frac{(10.0 \text{ m})\sqrt{2}}{v_i}$.

At this time, the vertical displacement of the ball must be

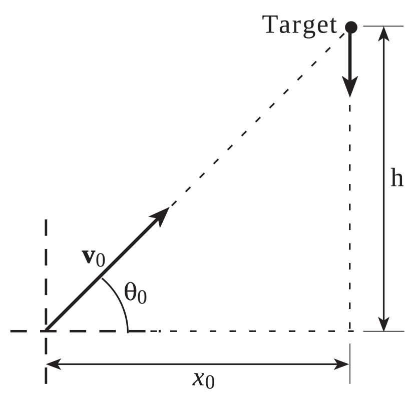
$$\Delta y = (3.05 - 2.00) \text{ m} = 1.05 \text{ m}.$$

Thus, $\Delta y = v_{iy}\,t + \frac{1}{2}a_y t^2$ becomes

$$1.05 \text{ m} = \left(\frac{v_i}{\sqrt{2}}\right)\frac{(10.0 \text{ m})\sqrt{2}}{v_i} + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)\frac{(10.0 \text{ m})^2(2)}{v_i^2},$$

which yields $v_i = \boxed{10.5 \text{ m/s}}$

3.59 Choose an origin where the projectile leaves the gun and let the y-coordinates of the projectile and the target at time t be labeled y_p and y_T, respectively.



Then, $(\Delta y)_p = y_p - 0 = (v_0 \sin\theta_o)t - \frac{g}{2}t^2$, and

$$(\Delta y)_T = y_T - h = 0 - \frac{g}{2}t^2 \text{ or } y_T = h - \frac{g}{2}t^2.$$

The time when the projectile will have the same x-coordinate as the target is $t = \frac{\Delta x}{v_{ix}} = \frac{x_0}{v_o \cos\theta_0}$.

For a collision to occur, it is necessary that $y_p = y_T$ at this time, or

$$(v_0 \sin\theta_o)\left(\frac{x_0}{v_o \cos\theta_0}\right) - \frac{g}{2}t^2 = h - \frac{g}{2}t^2 \text{ which reduces to } \tan\theta_o = \frac{h}{x_0}.$$

This requirement is satisfied provided that the gun is aimed at the initial location of the target. Thus, a collision is guaranteed if the shooter aims the gun in this manner.

3.60 (a) The components of the vectors are

Vector	x-component (cm)	y-component (cm)
$\mathbf{d}_{1m}$	0	104
$\mathbf{d}_{2m}$	46.0	19.5
$\mathbf{d}_{1f}$	0	84.0
$\mathbf{d}_{2f}$	38.0	20.2

The sums $\mathbf{d}_m = \mathbf{d}_{1m} + \mathbf{d}_{2m}$ and $\mathbf{d}_f = \mathbf{d}_{1f} + \mathbf{d}_{2f}$ are computed as:

$$d_m = \sqrt{(0+46.0)^2 + (104+19.5)^2} = 132 \text{ cm} \text{ and } \theta = \tan^{-1}\left(\frac{104+19.5}{0+46.0}\right) = 69.6°$$

$$d_f = \sqrt{(0+38.0)^2 + (84.0+20.2)^2} = 111 \text{ cm and } \theta = \tan^{-1}\left(\frac{84.0+20.2}{0+38.0}\right) = 70.0°$$

or $\boxed{\mathbf{d}_m = 132 \text{ cm at } 69.6° \text{ and } \mathbf{d}_f = 111 \text{ cm at } 70.0°}$.

(b) To normalize, multiply each component in the above calculation by the appropriate scale factor. The scale factor required for the components of $\mathbf{d}_{1m}$ and $\mathbf{d}_{2m}$ is $s_m = \frac{200 \text{ cm}}{180 \text{ cm}} = 1.11$, and the scale factor needed for components of $\mathbf{d}_{1f}$ and $\mathbf{d}_{2f}$ is $s_f = \frac{200 \text{ cm}}{168 \text{ cm}} = 1.19$. After using these scale factors and recomputing the vector sums, the results are:

$\boxed{\mathbf{d}'_m = 146 \text{ cm at } 69.6° \text{ and } \mathbf{d}'_f = 132 \text{ cm at } 70.0°}$.

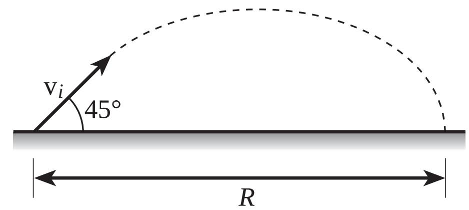
The difference in the normalized vector sums is $\Delta\mathbf{d}' = \mathbf{d}'_m - \mathbf{d}'_f$.

vector	**x-component (cm)**	**y-component (cm)**
$\mathbf{d}'_m$	50.9	137
$-\mathbf{d}'_f$	–45.1	–124
$\Delta\mathbf{d}'$	$\Sigma x = 5.74$	$\Sigma y = 12.8$

Therefore, $\Delta d' = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(5.74)^2 + (12.8)^2} \text{ cm} = 14.0 \text{ cm}$, and

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{12.8}{5.74}\right) = 65.8°, \text{ or } \boxed{\Delta\mathbf{d}' = 14.0 \text{ cm at } 65.8°}.$$

3.61 To achieve maximum range, the projectile should be launched at 45° above the horizontal. In this case, the initial velocity components are:



$$v_{ix} = v_{iy} = \frac{v_i}{\sqrt{2}}.$$

The time of flight may be found from $v_y = v_{iy} - gt$ by recognizing that when the projectile returns to the original level, $v_y = -v_{iy}$. Thus, the time of flight is

$t = \frac{-v_{iy} - v_{iy}}{-g} = \frac{2v_{iy}}{g} = \frac{2}{g}\left(\frac{v_i}{\sqrt{2}}\right) = \frac{v_i\sqrt{2}}{g}$. The maximum horizontal range is then

$$R = v_{ix}t = \left(\frac{v_i}{\sqrt{2}}\right)\left(\frac{v_i\sqrt{2}}{g}\right) = \frac{v_i^2}{g}. \qquad (1)$$

Now, consider throwing the projectile straight upward at speed v_i. At maximum height, $v_y = 0$, and the time required to reach this height is found from $v_y = v_{iy} - gt$ as $0 = v_i - gt$ which yields $t = \frac{v_i}{g}$. Therefore, the maximum height the projectile will reach is

$$(\Delta y)_{max} = \bar{v}_y t = \left(\frac{0 + v_i}{2}\right)\left(\frac{v_i}{g}\right) = \frac{v_i^2}{2g}.$$

Comparing this result with the maximum range found in equation (1) above reveals that $(\Delta y)_{max} = \boxed{\frac{R}{2}}$ provided the projectile is given the same initial speed in the two tosses.

If the boy takes a step when he makes the horizontal throw, he can likely give a higher initial speed for that throw than for the vertical throw.

3.62 (a) At the top of the arc $v_y = 0$, and from $v_y = v_{iy} - gt$, we find the time to reach the top of the arc to be $t = \frac{v_y - v_{iy}}{-g} = \frac{0 - v_0 \sin\theta_o}{g} = \left(\frac{v_0}{g}\right)\sin\theta_o$.

The vertical height, *h*, reached in this time is found from

$$\Delta y = \bar{v}_y t = \left(\frac{v_y + v_{iy}}{2}\right)t \text{ to be } h = \left(\frac{0 + v_0 \sin\theta_0}{2}\right)\left(\frac{v_0 \sin\theta_0}{g}\right) = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

(b) The total time of flight, t_f, is double the time required to reach the top of the arc, or $t_f = \left(\frac{2v_0}{g}\right)\sin\theta_o$. The horizontal range is given by

$$R = v_{ix}\,t_f = \left(v_0\cos\theta_0\right)\left(\frac{2v_0}{g}\right)\sin\theta_o = \frac{v_0^2\left(2\sin\theta_0\cos\theta_0\right)}{g} = \frac{v_0^2\sin\left(2\theta_0\right)}{g}.$$

3.63 The velocity of the boat relative to the shore is $\mathbf{v}_{bs} = \mathbf{v}_{bw} + \mathbf{v}_{ws}$, where $\mathbf{v}_{bw}$ is the velocity of the boat relative to the water and $\mathbf{v}_{ws}$ is the velocity of the water relative to shore.

In order to cross the river (flowing parallel to the banks) in minimum time, the velocity of the boat relative to the water must be perpendicular to the banks. That is, $\mathbf{v}_{bw}$ must be perpendicular to $\mathbf{v}_{ws}$. Hence, the velocity of the boat relative to the shore must be

$$v_{bs} = \sqrt{v_{bw}^2 + v_{ws}^2} = \sqrt{\left(12\ \text{km/h}\right)^2 + \left(5.0\ \text{km/h}\right)^2} = 13\ \text{km/h},$$

at $\theta = \tan^{-1}\left(\frac{v_{bw}}{v_{ws}}\right) = \tan^{-1}\left(\frac{12\ \text{km/h}}{5.0\ \text{km/h}}\right) = 67°$ to the direction of the current in the river (which is the same as the line of the riverbank).

The minimum time to cross the river is

$$t = \frac{\text{width of river}}{v_{bw}} = \frac{1.5\ \text{km}}{12\ \text{km/h}}\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = \boxed{7.5\ \text{min}}.$$

During this time, the boat drifts downstream a distance of

$$d = v_{ws}\,t = \left(5.0\ \text{km/h}\right)\left(7.5\ \text{min}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right) = 6.3\times10^2\ \text{m}.$$

3.64 Taking upstream as positive, the velocity of the water relative to the ground is $\mathbf{v}_{wg} = -0.500\ \text{m/s}$. The velocity of the skater relative to shore is $\mathbf{v}_{sg} = \frac{+0.560\ \text{m}}{0.800\ \text{s}} = +0.700\ \text{m/s}$ while moving upstream, and $\mathbf{v}_{sg} = \mathbf{v}_{wg} = -0.500\ \text{m/s}$ while drifting back downstream.

(a) At any time, $\mathbf{v}_{sg} = \mathbf{v}_{sw} + \mathbf{v}_{wg}$, or the velocity of the skater relative to the water is $\mathbf{v}_{sw} = \mathbf{v}_{sg} - \mathbf{v}_{wg}$.

(i) While going upstream, $\mathbf{v}_{sw} = +0.700\ \text{m/s} - \left(-0.500\ \text{m/s}\right) = \boxed{1.20\ \text{m/s}}$.

(ii) While drifting down stream, $\mathbf{v}_{sw} = -0.500\ \text{m/s} - (-0.500\ \text{m/s}) = \boxed{0}$.

(b) $d_{sw} = v_{sw}\, t = (1.20\ \text{m/s})(0.800\ \text{s}) = \boxed{0.960\ \text{m}}$

(c) The time to go upstream $t_{up} = 0.800\ \text{s}$ and the time to drift back downstream is

$t_{down} = \dfrac{0.560\ \text{m}}{0.500\ \text{m/s}} = 1.12\ \text{s}$, giving the cycle time as 1.92 s.

Therefore, $\bar{v}_{sw} = \dfrac{d_{sw}}{t_{cycle}} = \dfrac{0.960\ \text{m}}{1.92\ \text{s}} = \boxed{0.500\ \text{m/s}}$.

3.65 The initial velocity components for the daredevil are

$$v_{ix} = v_{iy} = \frac{v_i}{\sqrt{2}} = \frac{25.0\ \text{m/s}}{\sqrt{2}}.$$

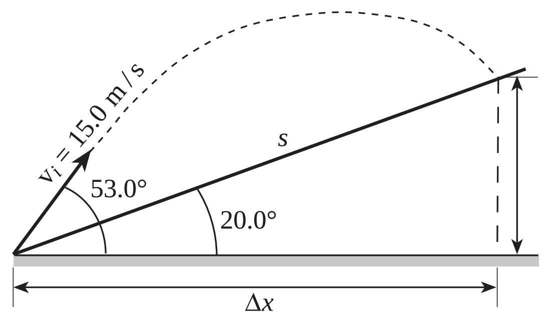
The time required to travel 50.0 m horizontally is

$$t = \frac{\Delta x}{v_{ix}} = \frac{(50.0\ \text{m})\sqrt{2}}{25.0\ \text{m/s}} = 2\sqrt{2}\ \text{s}.$$

The vertical displacement of the daredevil at this time, and the proper height above the level of the cannon to place the net, is

$$\Delta y = v_{iy}\, t + \frac{1}{2}a_y t^2 = \left(\frac{25.0\ \text{m/s}}{\sqrt{2}}\right)\left(2\sqrt{2}\ \text{s}\right) - \left(9.80\ \text{m/s}^2\right)\left(2\sqrt{2}\ \text{s}\right)^2 = \boxed{10.8\ \text{m}}.$$

3.66 At any time t, the horizontal and vertical displacements of the projectile are:



$$\Delta x = v_{ix} t = (v_i \cos 53.0°)t \text{, and}$$

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2 = (v_i \sin 53.0°)t - \frac{1}{2} g t^2$$

Observe from the diagram that at the time of impact, $\Delta y = (\Delta x)\tan 20.0°$.

Therefore, at impact, we have $(v_i \sin 53.0°)t - \frac{g}{2} t^2 = \left[(v_i \cos 53.0°)t\right]\tan 20.0°$.

Ignoring the $t = 0$ solution of this equation, this gives the time of impact as

$$t = \frac{2v_i}{g}\left[\sin 53.0° - (\cos 53.0°)\tan 20.0°\right]$$

$$= \frac{2(15.0 \text{ m/s})\left[\sin 53.0° - (\cos 53.0°)\tan 20.0°\right]}{9.80 \text{ m/s}^2} = 1.77 \text{ s}$$

At this time, the horizontal and vertical displacements are

$$\Delta x = (v_i \cos 53.0°)t = (15.0 \text{ m/s})(\cos 53.0°)(1.77 \text{ s}) = 16.0 \text{ m} \text{, and}$$

$$\Delta y = (v_i \sin 53.0°)t - \frac{1}{2} g t^2$$

$$= (15.0 \text{ m/s})(\sin 53.0°)(1.77 \text{ s}) - \frac{(9.80 \text{ m/s}^2)}{2}(1.77 \text{ s})^2 = 5.83 \text{ m}$$

The straight-line distance from the bottom of the ramp to the point of impact is then

$$s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(16.0 \text{ m})^2 + (5.83 \text{ m})^2} = \boxed{17.0 \text{ m}}.$$

3.67 (a) and (b)

Since the shot leaves the gun horizontally, the time it takes to reach the target is $t = \frac{\Delta x}{v_{ix}} = \frac{x}{v_i}$. The vertical displacement occurring in this time is

$\Delta y = 0 - y = v_{iy}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}g\left(\frac{x}{v_i}\right)^2$, which gives the drop as

$$\boxed{y = \frac{1}{2}g\left(\frac{x}{v_i}\right)^2 = Ax^2 \text{ with } A = \frac{g}{2v_i^2}, \text{ where } v_i \text{ is the muzzle velocity}}.$$

(c) If x = 3.00 m, and y = 0.210 m, then $A = \frac{y}{x^2} = \frac{0.210\text{ m}}{(3.00\text{ m})^2} = 2.33 \times 10^{-2}\text{ m}^{-1}$

and $v_i = \sqrt{\frac{g}{2A}} = \sqrt{\frac{9.80\text{ m/s}^2}{2(2.33 \times 10^{-2}\text{ m}^{-1})}} = \boxed{14.5\text{ m/s}}$.

3.68 Taking the positive x direction to be the direction of the first displacement, the components of the three successive displacements are:

Displacement	x-component (m)	y-component (m)
1	+10.0	0
2	0	–5.00
3	–7.00	0
	$\Sigma x = +3.00$ m	$\Sigma y = -5.00$ m

The resultant displacement is then

$$R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(3.00\text{ m})^2 + (-5.00\text{ m})^2} = 5.83\text{ m}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{-5.00}{3.00}\right) = -59.0°$$

or $\boxed{\mathbf{R} = 5.83\text{ m at } 59.0° \text{ to the right of the original direction}}$.

3.69 The components of the three displacements are:

Displacement	x-component (paces)	y-component (paces)
75.0 paces @ 240°	–37.5	–65.0
125 paces @ 135°	–88.4	+88.4
100 paces @ 160°	–94.0	+34.2
	$\Sigma x = -220$ paces	$\Sigma y = +57.6$ paces

The resultant displacement is then

$$R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(-220 \text{ paces})^2 + (+57.6 \text{ paces})^2} = 227 \text{ paces}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{+57.6}{-220}\right) = 165°$$

or $\boxed{\mathbf{R} = 227 \text{ paces at } 165° \text{ from the positive } x \text{ axis}}$.

3.70 For the ball thrown at 45.0°, the time of flight is found from

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 \text{ as } 0 = \left(\frac{v_i}{\sqrt{2}}\right)t_1 - \frac{g}{2}t_1^2,$$

which has the single non-zero solution of $t_1 = \dfrac{v_i\sqrt{2}}{g}$.

The horizontal range of this ball is $R_1 = v_{ix}\, t_1 = \left(\dfrac{v_i}{\sqrt{2}}\right)\left(\dfrac{v_i\sqrt{2}}{g}\right) = \dfrac{v_i^2}{g}$.

Now consider the first arc in the motion of the second ball, started at angle θ with initial speed v_i. Applied to this arc, $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ becomes

$$0 = (v_i \sin\theta)t_{21} - \frac{g}{2}t_{21}^2,$$

with non-zero solution $t_{21} = \dfrac{2v_i \sin\theta}{g}$.

Similarly, the time of flight for the second arc (started at angle θ with initial speed $v_i/2$) of this ball's motion is found to be

$$t_{22} = \frac{2(v_i/2)\sin\theta}{g} = \frac{v_i \sin\theta}{g}.$$

The horizontal displacement of the second ball during the first arc of its motion is

$$R_{21} = v_{ix}\, t_{21} = (v_i \cos\theta)\left(\frac{2v_i \sin\theta}{g}\right) = \frac{v_i^2 (2\sin\theta\cos\theta)}{g} = \frac{v_i^2 \sin(2\theta)}{g}.$$

Similarly, the horizontal displacement during the second arc of this motion is

$$R_{22} = \frac{(v_i/2)^2 \sin(2\theta)}{g} = \frac{1}{4}\frac{v_i^2 \sin(2\theta)}{g}.$$

The total horizontal distance traveled in the two arcs is then

$$R_2 = R_{21} + R_{22} = \frac{5}{4}\frac{v_i^2 \sin(2\theta)}{g}.$$

(a) Requiring that the two balls cover the same horizontal distance (i.e., requiring that $R_2 = R_1$) gives

$$\frac{5}{4}\frac{v_i^2 \sin(2\theta)}{g} = \frac{v_i^2}{g}.$$

This reduces to $\sin(2\theta) = \frac{4}{5}$ which yields $2\theta = 53.1°$, so $\theta = \boxed{26.6°}$ is the required projection angle for the second ball.

(b) The total time of flight for the second ball is

$$t_2 = t_{21} + t_{22} = \frac{2v_i \sin\theta}{g} + \frac{v_i \sin\theta}{g} = \frac{3v_i \sin\theta}{g}.$$

Therefore, the ratio of the times of flight for the two balls is

$$\frac{t_2}{t_1} = \frac{(3v_i \sin\theta)/g}{(v_i\sqrt{2})/g} = \frac{3}{\sqrt{2}}\sin\theta.$$

With θ = 26.6° as found in (a), this becomes

$$\frac{t_2}{t_1} = \frac{3}{\sqrt{2}}\sin(26.6°) = \boxed{0.950}.$$

3.71 (a) Applying $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ to the vertical motion of the first snowball gives $0 = \left[(25.0 \text{ m/s})\sin 70.0°\right]t_1 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t_1^2$ which has the non-zero solution of $t_1 = \frac{2(25.0 \text{ m/s})\sin 70.0°}{9.80 \text{ m/s}^2} = 4.79 \text{ s}$ as the time of flight for this snowball.

The horizontal displacement this snowball achieves is

$$\Delta x = v_{ix}t_1 = \left[(25.0 \text{ m/s})\cos 70.0°\right](4.79 \text{ s}) = 41.0 \text{ m}.$$

Now consider the second snowball, also given an initial speed of $v_i = 25.0 \text{ m/s}$, thrown at angle θ, and is in the air for time t_2. Applying $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ to its vertical motion yields

$$0 = \left[(25.0 \text{ m/s})\sin\theta\right]t_2 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t_2^2$$

which has a non-zero solution of

$$t_2 = \frac{2(25.0 \text{ m/s})\sin\theta}{9.80 \text{ m/s}^2} = (5.10 \text{ s})\sin\theta.$$

We require the horizontal range of this snowball be the same as that of the first ball, namely $\Delta x = v_{ix}t_2 = \left[(25.0 \text{ m/s})\cos\theta\right]\left[(5.10 \text{ s})\sin\theta\right] = 41.0 \text{ m}$. This yields the equation

$$\sin\theta\cos\theta = \frac{41.0 \text{ m}}{(25.0 \text{ m/s})(5.10 \text{ s})} = 0.321.$$

From the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, this result becomes

$$\sin 2\theta = 2(0.321) = 0.642, \text{ so } 2\theta = 40.0°$$

and the required angle of projection for the second snowball is

$$\theta = \boxed{20.0° \text{ above the horizontal}}.$$

(b) From above, the time of flight for the first snowball is $t_1 = 4.79\text{ s}$ and that for the second snowball is $t_2 = (5.10\text{ s})\sin\theta = (5.10\text{ s})\sin 20.0° = 1.74\text{ s}$.

Thus, if they are to arrive simultaneously, the time delay between the first and second snowballs should be

$$\Delta t = t_1 - t_2 = 4.79\text{ s} - 1.74\text{ s} = \boxed{3.05\text{ s}}.$$

Answers to Even Numbered Conceptual Questions

2. The magnitudes add when **A** and **B** are in the same direction. The resultant will be zero when the two vectors are equal in magnitude and opposite in direction.

4. The minimum sum for two vectors occurs when the two vectors are opposite in direction. If they are unequal, their sum cannot add to zero.

6. The component of a vector can only be equal to or less than the vector itself. It can never be greater than the vector.

8. The components of a vector will be equal in magnitude if the vector lies at a 45° angle with the two axes along which the components lie.

10. They both start from rest in the downward direction and accelerate alike in the vertical direction. Thus, they reach the ground with the same vertical speed. However, the ball thrown horizontally had an initial horizontal component of velocity which is maintained throughout the motion. Thus, the ball thrown horizontally moves with the greater speed.

12. The car can round a turn at a constant *speed* of 90 miles per hour. Its velocity will be changing, however, because it is changing in direction.

14. The balls will be closest at the instant the second ball is projected. The first ball will always be going faster than the second ball. There will be a one second time interval between their collisions with the ground. The two move with the same acceleration in the vertical direction. Thus, changing their horizontal velocity can never make them hit at the same time.

16. Let v_x and v_y represent its original velocity components. We know that the vertical component of velocity is zero at the top of the trajectory. Thus, $0 = v_y - gt$ and the time at the top of the trajectory is $t = v_y / g$.

(a) $x = v_x \left(\frac{v_y}{g} \right)$ and $y = \frac{v_y^2}{2g}$

(b) Its velocity is horizontal and equal to v_x.

(c) Its acceleration is vertically downward, $-g$.

With air resistance, the answers to (a) and (b) would be smaller. As for (c) the magnitude would be somewhat larger because the total acceleration would have a component horizontally backward in addition to the vertical component of $-g$.

18. The equations of projectile motion are only valid for objects moving freely under the influence of gravity. The only acceleration such an object has is the acceleration due to gravity, g, directed vertically downward. Of the objects listed, only (a) and (d) meet this requirement.

20. The passenger sees the ball go into the air and come back in the same way he would if he were at rest on the Earth. An observer by the tracks would see the ball follow the path of a projectile. If the train were accelerating, the ball would fall behind the position it would reach in the absence of the acceleration.

Answers to Even Numbered Problems

2. (a) 484 km (b) 18.1° N of W

4. 83 m at 33° N of W

6. (a) 6.1 units at 113° (b) 15 units at 23°

8. (a) 5.2 m at + 60° (b) 3.0 m at -30°
(c) 3.0 m at + 150° (d) 5.20 m at -60°

10. 1.31 km north, 2.81 km east

12. 358 m at 2.00° S of E

14. 42.7 yards

16. 788 mi at 48.1° N of E

18. (a) 185 N at 77.8° (b) 185 N at 258°

20. (a) 74.6° N of E (b) 470 km

22. 2.65 ft (0.807 m)

24. 3.19 s, 36.1 m/s at 60.1° below the horizontal

26. 2.8 m from base of table; $v_x = 5.0\text{ m/s}$, $v_y = -5.4\text{ m/s}$

28. $x = 7.23\times10^3\text{ m}$, $y = 1.68\times10^3\text{ m}$

30. (a) clears the bar by 0.85 m (b) falling, $v_y = -13.4\text{ m/s}$

32. 18.6 m

34. 9.91 m/s

36. 61 s

38. (a) 10.1 m/s at 8.53° E of N (b) 45.0 m

40. (a) 14.5° N of W (b) 194 km/h

42. 15.3 m

44. 7.87 N at 97.8° counterclockwise from a horizontal line to the right

46. (a) $v_{ix} = 7.99$ m/s, $v_{iy} = 6.02$ m/s (b) 1.85 m above the ground
(c) 5.23 beyond the edge (d) 8.97 m/s

50. (a) 0.85 m/s (b) 2.1 m/s

52. 14 m/s

54. 30 m/s

56. 29.4 m/s

58. 10.5 m/s

60. (a) $\mathbf{d}_m = 132$ cm at $69.6°$, $\mathbf{d}_f = 111$ cm at $70.0°$
(b) $\mathbf{d}'_m = 146$ cm at $69.6°$, $\mathbf{d}'_f = 132$ cm at $70.0°$
$\Delta\mathbf{d}' = \mathbf{d}'_m - \mathbf{d}'_f = 14.0$ cm at $65.8°$

64. (a) 1.20 m/s, 0 (b) 0.960 m (c) 0.500 m/s

66. 17.0 m

68. 5.83 m at 59.0° to the right of the original direction

70. (a) 26.6° (b) 0.950