CHAPTER 7

Quick Quizzes

1. (c). For a rotation of more than 180°, the angular displacement must be larger than $\pi = 3.14\ \text{rad}$. The angular displacements in the three choices are (a) $6\ \text{rad} - 3\ \text{rad} = 3\ \text{rad}$, (b) $1\ \text{rad} - (-1)\ \text{rad} = 2\ \text{rad}$, (c) $5\ \text{rad} - 1\ \text{rad} = 4\ \text{rad}$.

2. (b). Because all angular displacements occurred in the same time interval, the displacement with the lowest value will be associated with the lowest average angular speed.

3. (b) and (d). Points on the spokes which are closer to the axis of rotation have smaller values of tangential speed and acceleration than do points on the rim of the wheel.

4. (e), (a), (b).

5. (c). Both change in direction, but have constant magnitudes.

6. (b) and (c). According to Newton's law of universal gravitation, the force between the ball and the Earth depends on the product of their masses, so both forces, that of the ball on the Earth, and that of the Earth on the ball, are equal in magnitude. This follows also, of course, from Newton's third law. The ball has large motion compared to the Earth because according to Newton's second law, the force gives a much greater acceleration to the small mass of the ball.

7. (e).

8. (a). From Kepler's third law and the given period, the major axis of the comet can be calculated. It is found to be $1.2 \times 10^{11}\ \text{m}$. Because this is smaller than the Earth-Sun distance, the comet cannot possibly collide with the Earth.

Problem Solutions

7.1   (a) $\theta = \dfrac{s}{r} = \dfrac{60\,000\ \text{mi}}{1.0\ \text{ft}} \left(\dfrac{5280\ \text{ft}}{1\ \text{mi}} \right) = \boxed{3.2 \times 10^8\ \text{rad}}$.

   (b) $\theta = 3.2 \times 10^8\ \text{rad} \left(\dfrac{1\ \text{rev}}{2\pi\ \text{rad}} \right) = \boxed{5.0 \times 10^7\ \text{rev}}$.

7.2   The distance traveled is $s = r\theta$, where θ is in radians.

   For 30°, $s = r\theta = (4.1\ \text{m}) \left[30° \left(\dfrac{\pi\ \text{rad}}{180°} \right) \right] = \boxed{2.1\ \text{m}}$.

   For 30 radians, $s = r\theta = (4.1\ \text{m})(30\ \text{rad}) = \boxed{1.2 \times 10^2\ \text{m}}$.

   For 30 revolutions, $s = r\theta = (4.1\ \text{m}) \left[30\ \text{rev} \left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}} \right) \right] = \boxed{7.7 \times 10^2\ \text{m}}$.

7.3   The Earth moves through 2π rad in one year $\left(3.156 \times 10^7\ \text{s} \right)$. Thus,

$$\omega = \dfrac{2\pi\ \text{rad}}{3.156 \times 10^7\ \text{s}} = \boxed{1.99 \times 10^{-7}\ \text{rad/s}}.$$

   Alternatively, the Earth moves through 360° in one year (365.242 days).

   Thus, $\omega = \dfrac{360°}{365.2\ \text{days}} = \boxed{0.986\ \text{deg/day}}$.

7.4   We use $\alpha = \dfrac{\omega_f - \omega_i}{t}$ and find

$$\alpha = \dfrac{0.20\ \text{rev/s} - 0}{30\ \text{s}} \left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}} \right) = \boxed{4.2 \times 10^{-2}\ \text{rad/s}^2}.$$

7.5   (a) $\alpha = \dfrac{\left(2.51 \times 10^4\ \text{rev/min} - 0 \right)}{3.20\ \text{s}} \left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}} \right) \left(\dfrac{1\ \text{min}}{60.0\ \text{s}} \right) = \boxed{821\ \text{rad/s}^2}$

(b) $\theta = \omega_i t + \dfrac{1}{2}\alpha t^2 = 0 + \dfrac{1}{2}\left(821\ \dfrac{\text{rad}}{\text{s}^2}\right)(3.20\ \text{s})^2 = \boxed{4.21 \times 10^3\ \text{rad}}$

7.6 $\omega_i = 100\ \dfrac{\text{rev}}{\text{min}}\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60.0\ \text{s}}\right) = 10.47\ \text{rad/s}$

(a) From $\omega_f = \omega_i + \alpha t$, $t = \dfrac{\omega_f - \omega_i}{\alpha} = \dfrac{0 - 10.47\ \text{rad/s}}{-2.00\ \text{rad/s}^2} = \boxed{5.24\ \text{s}}$.

(b) $\theta = \bar{\omega} t = \left(\dfrac{\omega_f + \omega_i}{2}\right) t = \left(\dfrac{0 + 10.47\ \text{rad/s}}{2}\right)(5.24\ \text{s}) = \boxed{27.4\ \text{rad}}$.

7.7 The final linear speed of the car is

$$v_f = v_i + at = 17.0\ \text{m/s} + \left(2.00\ \text{m/s}^2\right)(5.00\ \text{s}) = 27.0\ \text{m/s},$$

and the distance traveled in this time interval is

$$s = \bar{v}\,t = \left(\dfrac{v_f + v_i}{2}\right) t = \left(\dfrac{(27.0 + 17.0)\ \text{m/s}}{2}\right)(5.00\ \text{s}) = 110\ \text{m}.$$

The angular displacement of the wheel is

$$\theta = \dfrac{s}{r} = \dfrac{110\ \text{m}}{0.48\ \text{m}}\left(\dfrac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{36.5\ \text{rev}}.$$

7.8 The initial angular velocity is $\omega_i = \dfrac{v_i}{r} = \dfrac{24.0\ \text{m/s}}{0.480\ \text{m}} = 50.0\ \text{rad/s}$.

Then, from $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$, the angular displacement as the wheel stops is

$$\Delta\theta = \dfrac{\omega_f^2 - \omega_i^2}{2\alpha} = \dfrac{0 - (50.0\ \text{rad/s})^2}{2(-1.35\ \text{rad/s}^2)}\left(\dfrac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{147\ \text{rev}}.$$

7.9   Main Rotor: $v = r\omega = (3.80 \text{ m})\left(450 \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{179 \text{ m/s}}$

$v = \left(179 \dfrac{\text{m}}{\text{s}}\right)\left(\dfrac{v_{sound}}{343 \text{ m/s}}\right) = \boxed{0.522 v_{sound}}$

Tail Rotor: $v = r\omega = (0.510 \text{ m})\left(4138 \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{221 \text{ m/s}}$

$v = \left(221 \dfrac{\text{m}}{\text{s}}\right)\left(\dfrac{v_{sound}}{343 \text{ m/s}}\right) = \boxed{0.644 v_{sound}}$

7.10   We will break the motion into two stages: (1) an acceleration period and (2) a deceleration period.

The angular displacement during the acceleration period is

$$\theta_1 = \bar{\omega}t = \left(\dfrac{\omega_f + \omega_i}{2}\right)t = \left[\dfrac{(5.0 \text{ rev/s})(2\pi \text{ rad}/1 \text{ rev}) + 0}{2}\right](8.0 \text{ s}) = 126 \text{ rad} \ ,$$

and while decelerating,

$$\theta_2 = \left(\dfrac{\omega_f + \omega_i}{2}\right)t = \left[\dfrac{0 + (5.0 \text{ rev/s})(2\pi \text{ rad}/1 \text{ rev})}{2}\right](12 \text{ s}) = 188 \text{ rad} \ .$$

The total displacement is $\theta = \theta_1 + \theta_2 = \left[(126 + 188) \text{ rad}\right]\left(\dfrac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{50 \text{ rev}}.$

7.11   When completely rewound, the tape is a hollow cylinder with a difference between the inner and outer radii of ~1 cm. Let N represent the number of revolutions through which the driving spindle turns in 30 minutes (and hence the number of layers of tape on the spool). We can determine N from:

$$N = \dfrac{\Delta\theta}{2\pi} = \dfrac{\omega(\Delta t)}{2\pi} = \dfrac{(1 \text{ rad/s})\left[(30 \text{ min})(60 \text{ s}/1 \text{ min})\right]}{2\pi \text{ rad/rev}} = 286 \text{ rev}$$

Then, $thickness \sim \dfrac{1 \text{ cm}}{N}$ $\boxed{\sim 10^{-2} \text{ cm}}$

7.12 The angular displacement of the coin while stopping is

$$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{0 - (18.0 \text{ rad/s})^2}{2(-1.90 \text{ rad/s}^2)} = 85.3 \text{ rad} .$$

The linear displacement is $\Delta s = r(\Delta\theta)$ with $r = diameter/2 = 1.20 \text{ cm}$, or

$$\Delta s = (1.20 \text{ cm})(85.3 \text{ rad}) = 102 \text{ cm} = \boxed{1.02 \text{ m}} .$$

7.13 The pulsar must rotate with angular velocity $\omega = 15$ rev/s (i.e., one-half revolution per flash). Thus, from $v = r\omega$, the maximum radius is

$$r_{max} = \frac{v_{max}}{\omega} = \frac{3.00 \times 10^8 \text{ m/s}}{15 \text{ rev/s}} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = \boxed{3.2 \times 10^6 \text{ m}} .$$

With mass greater than 1.4 times the mass of the Sun and radius comparable to the Earth, the object is thought to be a neutron star.

7.14 The radius of the cylinder is $r = 2.5 \text{ mi} \left(\frac{1609 \text{ m}}{1 \text{ mi}} \right) = 4.0 \times 10^3 \text{ m}$. Thus, from $a_c = r\omega^2$, the required angular velocity is

$$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9.80 \text{ m/s}^2}{4.0 \times 10^3 \text{ m}}} = \boxed{4.9 \times 10^{-2} \text{ rad/s}} .$$

7.15 The rotational velocity of the Earth is

$$\omega = \left(2\pi \frac{\text{rad}}{\text{day}} \right) \left(\frac{1 \text{ day}}{86\,400 \text{ s}} \right) = 7.27 \times 10^{-5} \text{ rad/s} ,$$

and the equatorial radius of the Earth is $R_E = 6.38 \times 10^6 \text{ m}$.

(a) At the equator, the radius of the circular motion is $r = R_E$.

Thus, $a_c = r\omega^2 = (6.38 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s})^2 = \boxed{3.37 \times 10^{-2} \text{ m/s}^2}$.

(b) A point at the north pole is on the axis of rotation, so $r = 0$ and $a_c = r\omega^2 = \boxed{0}$.

7.16 The speed of the plane is $v = \dfrac{\Delta s}{\Delta t} = \dfrac{2\pi r}{\Delta t}$. Thus, the radius of the circular path is

$$r = \frac{v(\Delta t)}{2\pi} = \frac{(180 \text{ m/s})(120 \text{ s})}{2\pi} = 3.44 \times 10^3 \text{ m} = \boxed{3.44 \text{ km}},$$

and the centripetal acceleration is $a_c = \dfrac{v^2}{r} = \dfrac{(180 \text{ m/s})^2}{3.44 \times 10^3 \text{ m}} = \boxed{9.42 \text{ m/s}^2}$.

7.17 The final angular velocity is $\omega_f = 78 \dfrac{\text{rev}}{\text{min}}\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 8.17 \text{ rad/s}$,

and the radius of the disk is $r = 5.0 \text{ in}\left(\dfrac{2.54 \text{ cm}}{1 \text{ in}}\right) = 12.7 \text{ cm} = 0.127 \text{ m}$.

(a) The tangential acceleration of the bug as the disk speeds up is

$$a_t = r\alpha = r\left(\frac{\Delta\omega}{\Delta t}\right) = (0.127 \text{ m})\left(\frac{8.17 \text{ rad/s}}{3.0 \text{ s}}\right) = \boxed{0.35 \text{ m/s}^2}.$$

(b) The final tangential speed of the bug is

$$v_t = r\omega_f = (0.127 \text{ m})(8.17 \text{ rad/s}) = \boxed{1.0 \text{ m/s}}.$$

(c) At $t = 1.0$ s, $\omega = \omega_i + \alpha t = 0 + \left(\dfrac{8.17 \text{ rad/s}}{3.0 \text{ s}}\right)(1.0 \text{ s}) = 2.7 \text{ rad/s}$.

Thus, $a_t = r\alpha = \boxed{0.35 \text{ m/s}^2}$ as above, while the radial acceleration is

$$a_c = r\omega^2 = (0.127 \text{ m})(2.7 \text{ rad/s})^2 = \boxed{0.94 \text{ m/s}^2}.$$

The total acceleration is $a = \sqrt{a_c^2 + a_t^2} = \boxed{1.0 \text{ m/s}^2}$, and the angle this acceleration makes with the direction of $\mathbf{a}_c$ is

$$\theta = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{0.35}{0.94}\right) = \boxed{20°}.$$

7.18 (a) The centripetal acceleration is $a_c = \dfrac{v_t^2}{r}$. Thus, when $a_c = a_t = 0.500 \text{ m/s}^2$, we have

$$v_t = \sqrt{r\,a_c} = \sqrt{(400 \text{ m})(0.500 \text{ m/s}^2)} = \boxed{14.1 \text{ m/s}}.$$

(b) At this time, $t = \dfrac{v_t - v_i}{a_t} = \dfrac{14.1 \text{ m/s} - 0}{0.500 \text{ m/s}^2} = 28.2 \text{ s}$, and the linear displacement is

$$s = \overline{v}_t\, t = \left(\frac{v_t + v_i}{2}\right) t = \left(\frac{14.1 \text{ m/s} + 0}{2}\right)(28.2 \text{ s}) = \boxed{200 \text{ m}}.$$

(c) The time is $t = \boxed{28.2 \text{ s}}$ as found in part (b) above.

7.19 (a) From $\Sigma F_r = ma_c$, we have

$$T = m\left(\frac{v_t^2}{r}\right) = \frac{(55.0 \text{ kg})(4.00 \text{ m/s})^2}{0.800 \text{ m}} = 1.10 \times 10^3 \text{ N} = \boxed{1.10 \text{ kN}}.$$

(b) The tension is larger than her weight by a factor of

$$\frac{T}{mg} = \frac{1.10 \times 10^3 \text{ N}}{(55.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{2.04 \text{ times}}.$$

7.20 Since $F_c = m\dfrac{v_t^2}{r} = mr\omega^2$, the needed angular velocity is

$$\omega = \sqrt{\frac{F_c}{mr}} = \sqrt{\frac{4.0 \times 10^{-11} \text{ N}}{(3.0 \times 10^{-16} \text{ kg})(0.150 \text{ m})}}$$

$$= (9.4 \times 10^2 \text{ rad/s})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{1.5 \times 10^2 \text{ rev/s}}$$

7.21 Friction must supply the needed centripetal force. Hence, it is necessary that

$$F_c \le (f_s)_{max}, \text{ or } m\frac{v_t^2}{r} \le \mu_s(mg) \text{ and the maximum safe speed is}$$

$$(v_t)_{max} = \sqrt{\mu_s\, r\, g} = \sqrt{(0.70)(20 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{12 \text{ m/s}}.$$

7.22 $a_c = \dfrac{v_t^2}{r} = \dfrac{\left[\left(86.5 \ \dfrac{km}{h}\right)\left(\dfrac{1 \ h}{3600 \ s}\right)\left(\dfrac{1000 \ m}{1 \ km}\right)\right]^2}{61.0 \ m}\left(\dfrac{1 \ g}{9.80 \ m/s^2}\right) = \boxed{0.966 \ g}$

7.23   (a) $a_c = r\omega^2 = (2.00 \ \text{m})(3.00 \ \text{rad/s})^2 = \boxed{18.0 \ \text{m/s}^2}$

(b) $F_c = m a_c = (50.0 \ \text{kg})(18.0 \ \text{m/s}^2) = \boxed{900 \ \text{N}}$

(c)   We know the centripetal acceleration is produced by the force of friction. Therefore, the needed static friction force is $f_s = 900 \ \text{N}$. Also, the normal force is $n = mg = 490 \ \text{N}$. Thus, the minimum coefficient of friction required is

$\mu_s = \dfrac{(f_s)_{max}}{n} = \dfrac{900 \ \text{N}}{490 \ \text{N}} = \boxed{1.84}$.

So large a coefficient of friction is unreasonable, and he will not be able to stay on the merry-go-round.

7.24   (a)   From $\Sigma F_y = 0$, we have

$n\cos\theta = mg$ (1)

where n is the normal force exerted on the car by the ramp.

Now, require that, $F_{radial \ inward} = m a_c$, or
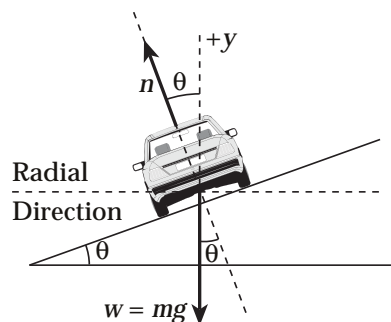
$n\sin\theta = m\dfrac{v_t^2}{r}$ (2)

Divide (2) by (1) to obtain $\boxed{\tan\theta = \dfrac{v_t^2}{rg}}$

(b)   If r = 50.0 m and $v_t = 13.4 \ \text{m/s}$, the needed bank angle is

$\theta = \tan^{-1}\left[\dfrac{(13.4 \ \text{m/s})^2}{(50.0 \ \text{m})(9.80 \ \text{m/}^2)}\right] = \boxed{20.1°}$.

7.25   (a)   Since the 1.0-kg mass is in equilibrium, the tension in the string is

$$T = m\,g = (1.0 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{9.8 \text{ N}}.$$

(b)   The tension in the string must produce the centripetal acceleration of the puck. Hence, $F_c = T = \boxed{9.8 \text{ N}}$.

(c)   From $F_c = m_{puck}\left(\dfrac{v_t^2}{r}\right)$, we find $v_t = \sqrt{\dfrac{r\,F_c}{m_{puck}}} = \sqrt{\dfrac{(1.0 \text{ m})(9.8 \text{ N})}{0.25 \text{ kg}}} = \boxed{6.3 \text{ m/s}}.$

7.26   As he passes through the bottom of his swing, the tension in the vine must equal (1) his weight plus (2) the force needed produce the centripetal acceleration. That is,

$$T = mg + m\left(\frac{v_t^2}{r}\right) = (85 \text{ kg})(9.80 \text{ m/s}^2) + (85 \text{ kg})\left[\frac{(8.0 \text{ m/s})^2}{10 \text{ m}}\right] = \boxed{1.4 \times 10^3 \text{ N}}.$$

Since $T > 1000$ N, $\boxed{\text{he will not make it}}$.

7.27   (a)   The centripetal acceleration is

$$a_c = r\omega^2 = (9.00 \text{ m})\left[\left(4.00 \ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\right]^2 = \boxed{1.58 \text{ m/s}^2}.$$

(b)   At the bottom of the circular path, the normal force exerted by the seat must support the weight and also produce the centripetal acceleration.

Thus, $n = m(g + a_c) = (40.0 \text{ kg})\left[(9.80 + 1.58) \text{ m/s}^2\right] = \boxed{455 \text{ N upward}}.$

(c)   At the top of the path, the weight must offset the normal force of the seat plus supply the needed centripetal acceleration. Therefore, $mg = n + ma_c$, or

$$n = m(g - a_c) = (40.0 \text{ kg})\left[(9.80 - 1.58) \text{ m/s}^2\right] = \boxed{329 \text{ N upward}}.$$

(d) At a point halfway up, the seat exerts an upward vertical component equal to the child's weight (392 N) and a component toward the center having magnitude $F_c = ma_c = (40.0 \text{ kg})(1.58 \text{ m/s}^2) = 63.2 \text{ N}$. The total force exerted by the seat is

$$F_R = \sqrt{(392 \text{ N})^2 + (63.2 \text{ N})^2} = 397 \text{ N} \quad \text{directed inward and at}$$

$$\theta = \tan^{-1}\left(\frac{392 \text{ N}}{63.2 \text{ N}}\right) = \boxed{80.8° \text{ above the horizontal}}.$$

7.28 (a) At A, the track supports the weight and supplies the centripetal acceleration.

Thus, $n = mg + m\dfrac{v_t^2}{r} = (500 \text{ kg})\left[9.80 \text{ m/s}^2 + \dfrac{(20.0 \text{ m/s})^2}{10 \text{ m}}\right] = \boxed{25 \text{ kN}}$.

(b) At B, the weight must offset the normal force exerted by the track and produce the needed centripetal acceleration, or $mg = n + m\dfrac{v_t^2}{r}$. If the car is on the verge of leaving the track, then $n = 0$ and $mg = m\dfrac{v_t^2}{r}$. Hence,

$$v_t = \sqrt{rg} = \sqrt{(15 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{12 \text{ m/s}}.$$

7.29 At the half-way point the spaceship is 1.92×10^8 m from both bodies. The force exerted on the ship by the Earth is directed toward the Earth and has magnitude

$$F_E = \frac{Gm_E m_s}{r^2}$$

$$= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(3.00 \times 10^4 \text{ kg})}{(1.92 \times 10^8 \text{ m})^2} = 325 \text{ N}$$

The force exerted on the ship by the Moon is directed toward the Moon and has a magnitude of

$$F_M = \frac{Gm_M m_s}{r^2}$$

$$= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.36 \times 10^{22} \text{ kg})(3.00 \times 10^4 \text{ kg})}{(1.92 \times 10^8 \text{ m})^2} = 4.00 \text{ N}$$

The resultant force is $(325 \text{ N} - 4.00 \text{ N}) = \boxed{321 \text{ N directed toward Earth}}$.

7.30   The Sun-Earth distance is $r_1 = 1.496 \times 10^{11}$ m , the Earth-Moon distance is $r_2 = 3.84 \times 10^8$ m , and the distance from the Sun to the Moon during a solar eclipse is $r_1 - r_2$.

(a)   The force exerted on the Moon by the Sun is

$$F_{MS} = \frac{Gm_M m_S}{\left(r_1 - r_2\right)^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(7.36 \times 10^{22} \ \text{kg}\right)\left(1.991 \times 10^{30} \ \text{kg}\right)}{\left(1.496 \times 10^{11} \ \text{m} - 3.84 \times 10^8 \ \text{m}\right)^2}$$

or $F_{MS} = \boxed{4.39 \times 10^{20} \ \text{N toward the Sun}}$.

(b)   The force exerted on the Moon by the Earth is

$$F_{ME} = \frac{Gm_M m_E}{r_2^2} = \frac{\left(6.67 \times 10^{-11} \ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(7.36 \times 10^{22} \ \text{kg}\right)\left(5.98 \times 10^{24} \ \text{kg}\right)}{\left(3.84 \times 10^8 \ \text{m}\right)^2}$$

$$= \boxed{1.99 \times 10^{20} \ \text{N toward the Earth}} .$$
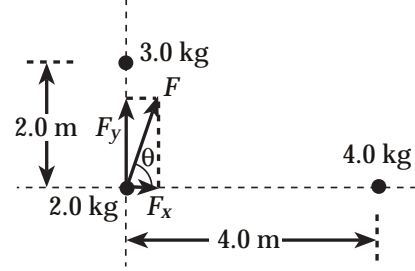
(c)   The force exerted on the Earth by the Sun is

$$F_{ES} = \frac{Gm_E m_S}{r_1^2} = \frac{\left(6.67 \times 10^{-11} \ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \ \text{kg}\right)\left(1.991 \times 10^{30} \ \text{kg}\right)}{\left(1.496 \times 10^{11} \ \text{m}\right)^2}$$

$$= \boxed{3.55 \times 10^{22} \ \text{N toward the Sun}} .$$

7.31 The forces exerted on the 2.0-kg by the other bodies are F_x and F_y as shown in the diagram at the right. The magnitudes of these forces are

$$F_x = \frac{\left(6.67 \times 10^{-11} \ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(2.0 \ \text{kg}\right)\left(4.0 \ \text{kg}\right)}{\left(4.0 \ \text{m}\right)^2}$$

$$= 3.3 \times 10^{-11} \ \text{N}$$

and $\quad F_y = \dfrac{\left(6.67 \times 10^{-11} \ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(2.0 \ \text{kg}\right)\left(3.0 \ \text{kg}\right)}{\left(2.0 \ \text{m}\right)^2} = 1.0 \times 10^{-10} \ \text{N}$.

The resultant force exerted on the 2.0-kg is $F = \sqrt{F_x^2 + F_y^2} = \boxed{1.1 \times 10^{-10} \ \text{N}}$

directed at $\theta = \tan^{-1}\left(\dfrac{F_y}{F_x}\right) = \tan^{-1}(3.0) = \boxed{72° \text{ above the } +x - \text{axis}}$.

7.32 The equilibrium position lies between the Earth and the Sun on the line connecting their centers. At this point, the gravitational forces exerted on the object by the Earth and Sun have equal magnitudes and opposite directions. Let this point be located distance r from the center of the Earth. Then, its distance from the Sun is $\left(1.496 \times 10^{11} \ \text{m} - r\right)$, and we may determine the value of r by requiring that $\dfrac{Gm_E m}{r^2} = \dfrac{Gm_S m}{\left(1.496 \times 10^{11} \ \text{m} - r\right)^2}$, where

m_E and m_S are the masses of the Earth and Sun respectively. This reduces to

$\dfrac{\left(1.496 \times 10^{11} \ \text{m} - r\right)}{r} = \sqrt{\dfrac{m_S}{m_E}} = 577$, or $1.496 \times 10^{11} \ \text{m} = 578r$, which yields

$r = \boxed{2.59 \times 10^8 \ \text{m from center of the Earth}}$.

7.33 (a) At the midpoint between the two masses, the forces exerted by the 200-kg and 500-kg masses are oppositely directed, and from $F = \dfrac{Gm_1 m_2}{r^2}$ we have

$$\Sigma F = \frac{G\left(50.0 \ \text{kg}\right)\left(500 \ \text{kg} - 200 \ \text{kg}\right)}{\left(0.200 \ \text{m}\right)^2} = \boxed{2.50 \times 10^{-5} \ \text{N}} \text{ toward the 500-kg.}$$

(b) At a point between the two masses and distance d from the 500-kg mass, the net force will be zero when

$$\frac{G\left(50.0 \ \text{kg}\right)\left(200 \ \text{kg}\right)}{\left(0.400 \ \text{m} - d\right)^2} = \frac{G\left(50.0 \ \text{kg}\right)\left(500 \ \text{kg}\right)}{d^2} \quad \text{or } d = \boxed{0.245 \ \text{m}} \ .$$

7.34   We know that $m_1 + m_2 = 5.00$ kg , or $m_2 = 5.00$ kg $- m_1$.

$$F = \frac{Gm_1 m_2}{r^2} \Rightarrow 1.00 \times 10^{-8} \text{ N} = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{m_1 (5.00 \text{ kg} - m_1)}{(0.200 \text{ m})^2}$$

$$(5.00 \text{ kg}) m_1 - m_1^2 = \frac{(1.00 \times 10^{-8} \text{ N})(0.200 \text{ m})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 6.00 \text{ kg}^2$$

Thus, $m_1^2 - (5.00 \text{ kg}) m_1 + 6.00 \text{ kg}^2 = 0$

or $(m_1 - 3.00 \text{ kg})(m_1 - 2.00 \text{ kg}) = 0$

giving $\boxed{m_1 = 3.00 \text{ kg, so } m_2 = 2.00 \text{ kg}}$.

The answer $m_1 = 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

7.35   (a)   The gravitational force must supply the required centripetal acceleration, so

$$\frac{Gm_E m}{r^2} = m\left(\frac{v_t^2}{r}\right). \text{ This reduces to } r = \frac{Gm_E}{v_t^2}, \text{ which gives}$$

$$r = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{(5.98 \times 10^{24} \text{ kg})}{(5000 \text{ m/s})^2} = 1.596 \times 10^7 \text{ m} .$$

The altitude above the surface of the Earth is then

$$h = r - R_E = 1.596 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m} = \boxed{9.58 \times 10^6 \text{ m}} .$$

(b)   The time required to complete one orbit is

$$T = \frac{\text{circumference of orbit}}{\text{orbital speed}} = \frac{2\pi (1.596 \times 10^7 \text{ m})}{5000 \text{ m/s}} = 2.00 \times 10^4 \text{ s} = \boxed{5.57 \text{ h}} .$$

7.36   (a)   The satellite moves in an orbit of radius $r = 2R_E$ and the gravitational force supplies the required centripetal acceleration. Hence, $m(v_t^2/2R_E) = Gm_E m/(2R_E)^2$, or

$$v_t = \sqrt{\frac{Gm_E}{2R_E}} = \sqrt{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{(5.98 \times 10^{24} \text{ kg})}{2(6.38 \times 10^6 \text{ m})}} = \boxed{5.59 \times 10^3 \text{ m/s}}$$

(b)   The period of the satellite's motion is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi\left[2\left(6.38\times10^6\ \text{m}\right)\right]}{5.59\times10^3\ \text{m/s}} = 1.43\times10^4\ \text{s} = \boxed{3.98\ \text{h}}.$$

(c)   The gravitational force acting on the satellite is $F = Gm_E m/r^2$, or

$$F = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(600\ \text{kg}\right)}{\left[2\left(6.38\times10^6\ \text{m}\right)\right]^2} = \boxed{1.47\times10^3\ \text{N}}.$$

7.37   The gravitational force exerted on Io by Jupiter provides the centripetal acceleration, so

$$m\left(\frac{v_t^2}{r}\right) = \frac{GMm}{r^2},\ \text{or}\ M = \frac{rv_t^2}{G}.$$

The orbital speed of Io is

$$v_t = \frac{2\pi r}{T} = \frac{2\pi\left(4.22\times10^8\ \text{m}\right)}{\left(1.77\ \text{days}\right)\left(86400\ \text{s/day}\right)} = 1.73\times10^4\ \text{m/s}.$$

Thus, $$M = \frac{\left(4.22\times10^8\ \text{m}\right)\left(1.73\times10^4\ \text{m/s}\right)^2}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = \boxed{1.90\times10^{27}\ \text{kg}}.$$

7.38   The radius of the satellite's orbit is

$$r = R_E + h = 6.38\times10^6\ \text{m} + 2.00\times10^6\ \text{m} = 8.38\times10^6\ \text{m}.$$

(a) $PE_g = -\dfrac{GM_E m}{r}$

$$= -\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{8.38\times10^6\ \text{m}} = \boxed{-4.76\times10^9\ \text{J}}$$

(b) $F = \dfrac{GM_E m}{r^2} = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{\left(8.38\times10^6\ \text{m}\right)^2} = \boxed{568\ \text{N}}$

7.39   The radius of the satellite's orbit is

$$r = R_E + h = 6.38\times10^6\ \text{m} + 200\times10^3\ \text{m} = 6.58\times10^6\ \text{m}.$$

(a)   Since the gravitational force provides the centripetal acceleration,

$$m\left(\frac{v_t^2}{r}\right) = \frac{Gm_E m}{r^2}, \text{ or}$$

$$v_t = \sqrt{\frac{Gm_E}{r}} = \sqrt{\left(6.67 \times 10^{-11} \ \frac{N \cdot m^2}{kg^2}\right)\frac{\left(5.98 \times 10^{24} \ kg\right)}{\left(6.58 \times 10^6 \ m\right)}} = 7.79 \times 10^3 \ m/s \ .$$

Hence, the period of the orbital motion is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi\left(6.58 \times 10^6 \ m\right)}{7.79 \times 10^3 \ m/s} = 5.31 \times 10^3 \ s = \boxed{1.48 \ h} \ .$$

(b)   The orbital speed is $v_t = \boxed{7.79 \times 10^3 \ m/s}$ as computed above.

(c)   Assuming the satellite is launched from a point on the equator of the Earth, its initial speed is the rotational speed of the launch point, or

$$v_i = \frac{2\pi R_E}{1 \ day} = \frac{2\pi\left(6.38 \times 10^6 \ m\right)}{86\,400 \ s} = 464 \ m/s \ .$$

The work-kinetic energy theorem gives the energy input required to place the satellite in orbit as $W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i$, or

$$W_{nc} = \left(\frac{1}{2}mv_t^2 - \frac{GM_E m}{r}\right) - \left(\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E}\right) = m\left[\frac{v_t^2 - v_i^2}{2} + GM_E\left(\frac{1}{R_E} - \frac{1}{r}\right)\right]$$

Substitution of appropriate numeric values into this result gives the minimum energy input as $W_{nc} = \boxed{6.43 \times 10^9 \ J}$.

7.40   The gravitational force on a small parcel of material at the star's equator supplies the centripetal acceleration, or $\dfrac{GM_s m}{R_s^2} = m\left(\dfrac{v_t^2}{R_s}\right) = m\left(R_s \omega^2\right)$.

Hence, $\omega = \sqrt{GM_s / R_s^3}$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \ N \cdot m^2/kg^2\right)\left[2\left(1.99 \times 10^{30} \ kg\right)\right]}{\left(10.0 \times 10^3 \ m\right)^3}} = \boxed{1.63 \times 10^4 \ rad/s} \ .$$

7.41 (a) $\omega = \dfrac{2\pi \text{ rad}}{1 \text{ day}} = \dfrac{2\pi \text{ rad}}{86\,400 \text{ s}} = \boxed{7.27 \times 10^{-5} \text{ rad/s}}$

(b) The elapsed time is the time required for the Earth to rotate 107° on its axis, or

$$\Delta t = \frac{107°}{\omega} = \frac{(107°)(\pi \text{ rad}/180°)}{(7.27 \times 10^{-5} \text{ rad/s})} = \boxed{7.13 \text{ h}}$$

7.42 (a) The gravitational force exerted on an object of mass m near the surface (i.e., at distance R from the center) of a planet of mass M is

$$F_g = \frac{GMm}{R^2}. \text{ Equating this to } F_g = mg \text{ gives } mg = \frac{GMm}{R^2}, \text{ or}$$

$$g = \frac{GM}{R^2} \text{ as the acceleration of gravity at the planet's surface.}$$

For Mars,

$$g = \left(6.67 \times 10^{-11} \text{ } \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{\left[0.1074\left(5.98 \times 10^{24} \text{ kg}\right)\right]}{\left[0.5282\left(6.38 \times 10^6 \text{ m}\right)\right]^2} = \boxed{3.77 \text{ m/s}^2}$$

(b) From $\Delta y = v_i t + \dfrac{1}{2} a_y t^2$ with $v_i = 0$ and $a_y = -g$, we find

$$t = \sqrt{\frac{2(\Delta y)}{-g}} = \sqrt{\frac{2(-20.0 \text{ m})}{-3.77 \text{ m/s}^2}} = \boxed{3.26 \text{ s}}$$

7.43 The angular velocity of the ball is $\omega = 0.500 \text{ rev/s} = \pi \text{ rad/s}$.

(a) $v_t = r\omega = (0.800 \text{ m})(\pi \text{ rad/s}) = \boxed{2.51 \text{ m/s}}$

(b) $a_c = \dfrac{v_t^2}{r} = r\omega^2 = (0.800 \text{ m})(\pi \text{ rad/s})^2 = \boxed{7.90 \text{ m/s}^2}$

(c) We imagine that the weight of the ball is supported by a frictionless platform. Then, the rope tension need only produce the centripetal acceleration. The force required to produce the needed centripetal acceleration is $F = m\left(v_t^2/r\right)$. Thus, if the maximum force the rope can exert is 100 N, the maximum tangential speed of the ball is

$$\left(v_t\right)_{max} = \sqrt{\frac{r\,F_{max}}{m}} = \sqrt{\frac{(0.800 \text{ m})(100 \text{ N})}{5.00 \text{ kg}}} = \boxed{4.00 \text{ m/s}}$$

7.44 (a) $\omega_i = \dfrac{v_t}{r_i} = \dfrac{1.30 \text{ m/s}}{2.30 \times 10^{-2} \text{ m}} = \boxed{56.5 \text{ rad/s}}$

(b) $\omega_f = \dfrac{v_t}{r_f} = \dfrac{1.30 \text{ m/s}}{5.80 \times 10^{-2} \text{ m}} = \boxed{22.4 \text{ rad/s}}$

(c) The duration of the recording is

$$\Delta t = (74 \text{ min})(60 \text{ s/min}) + 33 \text{ s} = 4473 \text{ s} .$$

Thus, $\bar{\alpha} = \dfrac{\omega_f - \omega_i}{\Delta t} = \dfrac{(22.4 - 56.8) \text{ rad/s}}{4473 \text{ s}} = \boxed{-7.63 \times 10^{-3} \text{ rad/s}^2}$

(d) $\Delta\theta = \dfrac{\omega_f^2 - \omega_i^2}{2\alpha} = \dfrac{(22.4 \text{ rad/s})^2 - (56.5 \text{ rad/s})^2}{2(-7.63 \times 10^{-3} \text{ rad/s}^2)} = \boxed{1.77 \times 10^5 \text{ rad}}$

(e) The track moves past the lens at a constant speed of $v_t = 1.30$ m/s for 4473 seconds. Therefore, the length of the spiral track is

$$\Delta s = v_t\,(\Delta t) = (1.30 \text{ m/s})(4473 \text{ s}) = 5.81 \times 10^3 \text{ m} = \boxed{5.81 \text{ km}}$$

7.45 The radius of the satellite's orbit is

$$r = R_E + h = 6.38 \times 10^6 \text{ m} + (1.50 \times 10^2 \text{ mi})(1609 \text{ m/1 mi}) = 6.62 \times 10^6 \text{ m} .$$

(a) The required centripetal acceleration is produced by the gravitational force, so
$$m\left(\frac{v_t^2}{r}\right) = \frac{GM_E m}{r^2}, \text{ which gives } v_t = \sqrt{\frac{GM_E}{r}} .$$

$$v_t = \sqrt{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{(5.98 \times 10^{24} \text{ kg})}{6.62 \times 10^6 \text{ m}}} = \boxed{7.76 \times 10^3 \text{ m/s}}$$

(b)   The time for one complete revolution is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi\left(6.62 \times 10^6 \text{ m}\right)}{7.76 \times 10^3 \text{ m/s}} = 5.36 \times 10^3 \text{ s} = \boxed{89.3 \text{ min}}$$

7.46   (a)   When the car is about to slip down the incline, the friction force, f, is directed up the incline as shown and has the magnitude $f = \mu n$. Thus,

$$\Sigma F_y = n\cos\theta + \mu n\sin\theta - mg = 0 ,$$

or $n = \dfrac{mg}{\cos\theta + \mu\sin\theta}$ (1)

Also, $\Sigma F_x = n\sin\theta - \mu n\cos\theta = m\left(\dfrac{v_{min}^2}{R}\right) ,$

or $v_{min} = \sqrt{\dfrac{nR}{m}\left(\sin\theta - \mu\cos\theta\right)} .$ (2)

Substituting equation (1) into (2) gives

$$v_{min} = \sqrt{Rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)} = \boxed{\sqrt{Rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}}$$

If the car is about to slip up the incline, $f = \mu n$ is directed down the slope (opposite to what is shown in the sketch). Then,
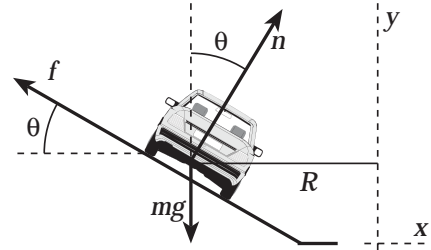
$$\Sigma F_y = n\cos\theta - \mu n\sin\theta - mg = 0, \text{ or } n = \frac{mg}{\cos\theta - \mu\sin\theta}$$ (3)

Also, $\Sigma F_x = n\sin\theta + \mu n\cos\theta = m\left(\dfrac{v_{max}^2}{R}\right) ,$ or

$$v_{max} = \sqrt{\frac{nR}{m}\left(\sin\theta + \mu\cos\theta\right)} .$$ (4)

Combining equations (3) and (4) gives

$$v_{max} = \sqrt{Rg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} = \boxed{\sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}} .$$

(b) If $R = 100$ m, $\theta = 10°$, and $\mu = 0.10$, the lower and upper limits of safe speeds are:

$$v_{min} = \sqrt{(100 \text{ m})(9.8 \text{ m/s}^2)\left(\frac{\tan 10° - 0.10}{1 + 0.10 \tan 10°}\right)} = \boxed{8.6 \text{ m/s}},$$

and $\qquad v_{max} = \sqrt{(100 \text{ m})(9.8 \text{ m/s}^2)\left(\frac{\tan 10° + 0.10}{1 - 0.10 \tan 10°}\right)} = \boxed{17 \text{ m/s}}.$

7.47 (a) When the car is at the top of the arc, the normal force is upward and the weight downward. The net force directed downward, toward the center of the circular path and hence supplying the centripetal acceleration, is $\Sigma F_{down} = mg - n = m\left(v_t^2/r\right)$.

Thus, the normal force is $\boxed{n = m\left(g - v_t^2/r\right)}$.

(b) If $r = 30.0$ m and $n \to 0$, then $g - \dfrac{v_t^2}{r} \to 0$ or the speed of the car must be

$$v_t = \sqrt{rg} = \sqrt{(30.0 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{17.1 \text{ m/s}}.$$

7.48 (a) At the lowest point on the path, the net upward force (i.e., the force directed toward the center of the path and supplying the centripetal acceleration) is

$$\Sigma F_{up} = T - mg = m\left(\frac{v_t^2}{r}\right), \text{ so the tension in the cable is}$$

$$T = m\left(g + \frac{v_t^2}{r}\right) = (0.400 \text{ kg})\left(9.80 \text{ m/s}^2 + \frac{(3.00 \text{ m/s})^2}{0.800 \text{ m}}\right) = \boxed{8.42 \text{ N}}.$$

(b) Using conservation of mechanical energy, $\left(KE + PE_g\right)_f = \left(KE + PE_g\right)_i$, as the bob goes from the lowest to the highest point on the path gives

$$0 + mg\left[L(1 - \cos\theta_{max})\right] = \frac{1}{2}mv_i^2 + 0, \text{ or } \cos\theta_{max} = 1 - \frac{v_i^2}{2gL}.$$

$$\theta_{max} = \cos^{-1}\left(1 - \frac{v_i^2}{2gL}\right) = \cos^{-1}\left(1 - \frac{(3.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(0.800 \text{ m})}\right) = \boxed{64.8°}$$

(c) At the highest point on the path, the bob is at rest and the net radial force is

$$\Sigma F_r = T - m\,g\cos\theta_{max} = m\left(\frac{v_t^2}{r}\right) = 0$$

Therefore,

$$T = m\,g\cos\theta_{max} = (0.400 \text{ kg})(9.80 \text{ m/s}^2)\cos(64.8°) = \boxed{1.67 \text{ N}}$$

7.49 The speed the person has due to the rotation of the Earth is $v_t = r\omega$ where r is the distance from the rotation axis and ω is the angular velocity of rotation.

The person's apparent weight, $\left(F_g\right)_{apparent}$, equals the magnitude of the upward normal force exerted on him by the scales. The true weight, $\left(F_g\right)_{true} = mg$, is directed downward. The net downward force produces the needed centripetal acceleration, or

$$\Sigma F_{down} = -n + \left(F_g\right)_{true} = -\left(F_g\right)_{apparent} + \left(F_g\right)_{true} = m\left(\frac{v_t^2}{r}\right) = mr\omega^2.$$

(a) At the equator, $r = R_E$, so $\boxed{\left(F_g\right)_{true} = \left(F_g\right)_{apparent} + mR_E\omega^2} > \left(F_g\right)_{apparent}$.

(b) At the equator, it is given that $r\omega^2 = 0.0340 \text{ m/s}^2$, so the apparent weight is

$$\left(F_g\right)_{apparent} = \left(F_g\right)_{true} - mr\omega^2 = (75.0 \text{ kg})\left[(9.80 - 0.0340) \text{ m/s}^2\right] = \boxed{732 \text{ N}}.$$

At either pole, $r = 0$ (the person is on the rotation axis) and

$$\left(F_g\right)_{apparent} = \left(F_g\right)_{true} = m\,g = (75.0 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{735 \text{ N}}.$$

7.50 When the rope makes angle θ with the vertical, the net force directed toward the center of the circular path is $\Sigma F_r = T - m g \cos\theta$ as shown in the sketch. This force supplies the needed centripetal acceleration, so

$$T - m g \cos\theta = m\left(\frac{v_t^2}{r}\right), \text{ or } T = m\left(g\cos\theta + \frac{v_t^2}{r}\right).$$
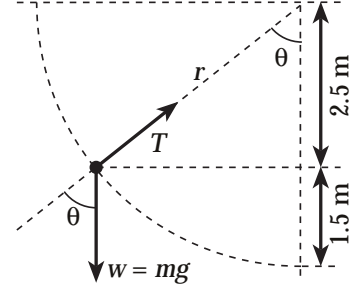
Using conservation of mechanical energy, with $KE=0$ at $\theta=90°$ and $PE_g=0$ at the bottom of the arc, the speed when the rope is at angle θ from the vertical is given by $\frac{1}{2}mv_t^2 + m g(r - r\cos\theta) = 0 + m g r$, or $v_t^2 = 2 g r\cos\theta$. The expression for the tension in the rope at angle θ then reduces to $T = 3m g\cos\theta$.

(a) At the beginning of the motion, $\theta=90°$ and $T = \boxed{0}$.

(b) At 1.5 m from the bottom of the arc, $\cos\theta = \dfrac{2.5 \text{ m}}{r} = \dfrac{2.5 \text{ m}}{4.0 \text{ m}} = 0.63$ and the tension is

$$T = 3(70 \text{ kg})(9.8 \text{ m/s}^2)(0.63) = 1.3 \times 10^3 \text{ N} = \boxed{1.3 \text{ kN}}.$$

(c) At the bottom of the arc, $\theta=0°$ and $\cos\theta=1.0$, so the tension is

$$T = 3(70 \text{ kg})(9.8 \text{ m/s}^2)(1.0) = 2.1 \times 10^3 \text{ N} = \boxed{2.1 \text{ kN}}.$$

7.51 The normal force exerted on the person by the cylindrical wall must provide the centripetal acceleration, so $n = m(r\omega^2)$.

If the minimum acceptable coefficient of friction is present, the person is on the verge of slipping and the maximum static friction force equals the person's weight, or $(f_s)_{max} = (\mu_s)_{min} n = mg$.

Thus, $(\mu_s)_{min} = \dfrac{mg}{n} = \dfrac{g}{r\omega^2} = \dfrac{9.80 \text{ m/s}^2}{(3.00 \text{ m})(5.00 \text{ rad/s})^2} = \boxed{0.131}$.

7.52 The horizontal component of the tension in the cord is the only force directed toward the center of the circular path, so it must supply the centripetal acceleration. Thus,

$$T\sin\theta = m\left(\frac{v_t^2}{r}\right) = m\left(\frac{v_t^2}{L\sin\theta}\right), \text{ or } T\sin^2\theta = \frac{mv_t^2}{L} \quad (1)$$
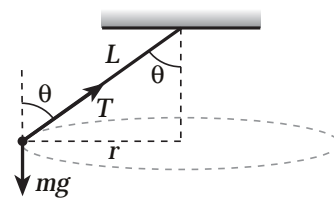
Also, the vertical component of the tension must support the weight of the ball, or

$$T\cos\theta = mg. \quad (2)$$

(a) Dividing equation (1) by (2) gives

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v_t^2}{Lg}, \text{ or } v_t = \sin\theta\sqrt{\frac{Lg}{\cos\theta}}. \quad (3)$$

With $L = 1.5$ m/s and $\theta = 30°$,

$$v_t = \sin 30°\sqrt{\frac{(1.5\text{ m})(9.8\text{ m/s}^2)}{\cos 30°}} = \boxed{2.1\text{ m/s}}.$$

(b) From equation (3), with $\sin^2\theta = 1 - \cos^2\theta$, we find

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v_t^2}{Lg} \text{ or } \cos^2\theta + \left(\frac{v_t^2}{Lg}\right)\cos\theta - 1 = 0.$$

Solving this quadratic equation for $\cos\theta$ gives

$$\cos\theta = -\left(\frac{v_t^2}{2Lg}\right) \pm \sqrt{\left(\frac{v_t^2}{2Lg}\right)^2 + 1}$$

If $L = 1.5$ m and $v_t = 4.0$ m/s, this yields solutions: $\cos\theta = -1.7$ (which is impossible), and $\cos\theta = +0.59$ (which is possible).

Thus, $\theta = \cos^{-1}(0.59) = \boxed{54°}$.

(c) From equation (2), when $T = 9.8$ N and the cord is about to break, the angle is

$$\theta = \cos^{-1}\left(\frac{m\,g}{T}\right) = \cos^{-1}\left(\frac{(0.50\text{ kg})(9.8\text{ m/s}^2)}{9.8\text{ N}}\right) = 60°.$$

Then equation (3) gives

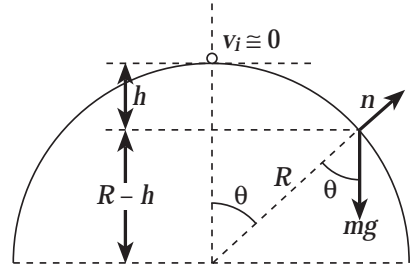$$v_t = \sin\theta\sqrt{\frac{L\,g}{\cos\theta}} = \sin 60°\sqrt{\frac{(1.5\text{ m})(9.8\text{ m/s}^2)}{\cos 60°}} = \boxed{4.7\text{ m/s}}.$$

7.53 Choosing $PE_g = 0$ at the top of the hill, the speed of the skier after dropping distance h is found using conservation of mechanical energy as

$$\frac{1}{2}m v_t^2 - m\,g\,h = 0 + 0\text{, or } v_t^2 = 2\,g\,h.$$

The net force directed toward the center of the circular path, and providing the centripetal acceleration, is

$$\Sigma F_r = m\,g\cos\theta - n = m\left(\frac{v_t^2}{R}\right).$$

Solving for the normal force, after making the substitutions $v_t^2 = 2\,g\,h$ and

$$\cos\theta = \frac{R-h}{R} = 1 - \frac{h}{R},$$

gives $\quad n = m\,g\left(1 - \frac{h}{R}\right) - m\left(\frac{2\,g\,h}{R}\right) = m\,g\left(1 - \frac{3h}{R}\right).$

The skier leaves the hill when $n \to 0$. This occurs when

$$1 - \frac{3h}{R} = 0 \text{ or } \boxed{h = \frac{R}{3}}$$

7.54 (a) $F = \dfrac{G M m}{r^2}$

$$= \left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{\left[100\left(1.99 \times 10^{30}\text{ kg}\right)\right](1000\text{ kg})}{\left[10 \times 10^3\text{ m} + 50\text{ m}\right]^2} = \boxed{1.3 \times 10^{17}\text{ N}}.$$

(b) $\Delta F = \dfrac{GM(1.0\text{ kg})}{r_{front}^2} - \dfrac{GM(1.0\text{ kg})}{r_{back}^2}$, or $\dfrac{\Delta F}{1.0\text{ kg}} = GM\left[\dfrac{1}{r_{front}^2} - \dfrac{1}{r_{back}^2}\right]$

$$\dfrac{\Delta F}{1.0\text{ kg}} = \left(6.67 \times 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{32}\text{ kg})\left[\dfrac{1}{\left(10^4\text{ m}\right)^2} - \dfrac{1}{\left(10^4\text{ m}+100\text{ m}\right)^2}\right]$$

$$= \boxed{2.6 \times 10^{12}\ \text{N/kg}}$$

7.55   Define the following symbols: M_m = mass of moon, M_e = mass of the Earth, R_m = radius of moon, R_e = radius of the Earth, and r = radius of the Moon's orbit around the Earth.

We interpret "lunar escape speed" to be the escape speed from the surface of a stationary moon alone in the universe. Then,

$$v_{launch} = 2v_{escape} = 2\sqrt{\dfrac{2GM_m}{R_m}}\ ,\ \text{or}\ \ v_{launch}^2 = \dfrac{8GM_m}{R_m}\ .$$

Applying conservation of mechanical energy from launch to impact gives

$$\frac{1}{2}mv_{impact}^2 + \left(PE_g\right)_f = \frac{1}{2}mv_{launch}^2 + \left(PE_g\right)_i \text{ , or}$$

$$v_{impact} = \sqrt{v_{launch}^2 + \frac{2}{m}\left[\left(PE_g\right)_i - \left(PE_g\right)_f\right]}$$

The needed potential energies are

$$\left(PE_g\right)_i = -\frac{G\,M_m\,m}{R_m} - \frac{GM_e\,m}{r} \text{ and } \left(PE_g\right)_f = -\frac{G\,M_e\,m}{R_e} - \frac{GM_m\,m}{r} \text{ .}$$

Using these potential energies and the expression for v_{launch}^2 from above, the equation for the impact speed reduces to

$$v_{impact} = \sqrt{2G\left(\frac{3M_m}{R_m} + \frac{M_e}{R_e} - \frac{\left(M_e - M_m\right)}{r}\right)} \text{ .}$$

With numeric values of $G = 6.67 \times 10^{-11} \dfrac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$, $M_m = 7.36 \times 10^{22}$ kg , $R_m = 1.74 \times 10^6$ m ,

$R_e = 6.38 \times 10^6$ m , and $r = 3.84 \times 10^8$ m , we find

$$v_{impact} = 1.18 \times 10^4 \text{ m/s} = \boxed{11.8 \text{ km/s}} \text{ .}$$

7.56   The escape speed from the surface of a planet of radius R and mass M is given by

$$v_e = \sqrt{\frac{2GM}{R}} \text{ .}$$

If the planet has uniform density, ρ, the mass is given by

$$M = \rho(volume) = \rho\left(4\pi R^3/3\right) = 4\pi\rho R^3/3 \text{ .}$$

The expression for the escape speed then becomes

$$v_e = \sqrt{\frac{2G}{R}\left(\frac{4\pi\rho R^3}{3}\right)} = \left(\sqrt{\frac{8\pi\rho G}{3}}\right)R = \boxed{(constant)R} \text{ ,}$$

or the escape speed is directly proportional to the radius of the planet.

7.57   If the block will just make it through the top of the loop, the force required to produce the centripetal acceleration at point C must equal the block's weight, or $m\left(v_c^2/R\right) = m\,g$.

This gives $\qquad v_c = \sqrt{R\,g} \qquad$ as the required speed of the block at point C.

We apply the work-kinetic energy theorem in the form

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$$

from when the block is first released until it reaches point C to obtain

$$f_k\left(\overline{AB}\right)\cos 180° = \frac{1}{2}mv_c^2 + mg\left(2R\right) + 0 - 0 - 0 - \frac{1}{2}kd^2\,.$$

The friction force is $f_k = u_k\left(mg\right)$, and for minimum initial compression of the spring, $v_c^2 = Rg$ as found above. Thus, the work-energy equation reduces to

$$d_{min} = \sqrt{\frac{2\,\mu_k mg\left(\overline{AB}\right) + mRg + 2mg\left(2R\right)}{k}} = \sqrt{\frac{mg\left(2\,\mu_k\,\overline{AB} + 5R\right)}{k}}\,.$$

$$d_{min} = \sqrt{\frac{\left(0.50\text{ kg}\right)\left(9.8\ \text{m/s}^2\right)\left[2\left(0.30\right)\left(2.5\text{ m}\right) + 5\left(1.5\text{ m}\right)\right]}{78.4\ \text{N/m}}} = \boxed{0.75\text{ m}}$$

7.58   Choosing $y = 0$ and $PE_g = 0$ at the level of point B, applying the work-kinetic energy theorem to the block's motion gives

$$W_{nc} = \frac{1}{2}mv^2 + mgy - \frac{1}{2}mv_0^2 - mg\left(2R\right),\ \text{or}\ v^2 = v_0^2 + \frac{2W_{nc}}{m} + 2g\left(2R - y\right)\ \ (1)$$

(a)   At point A, $y = R$ and $W_{nc} = 0$ (no non-conservative force has done work on the block yet). Thus, $v_A^2 = v_0^2 + 2gR$. The normal force exerted on the block by the track must supply the centripetal acceleration at point A, so

$$n_A = m\left(\frac{v_A^2}{R}\right) = m\left(\frac{v_0^2}{R} + 2g\right)$$

$$= \left(0.50\text{ kg}\right)\left(\frac{\left(4.0\ \text{m/s}\right)^2}{1.5\text{ m}} + 2\left(9.8\ \text{m/s}^2\right)\right) = \boxed{15\text{ N}}\,.$$

At point B, $y = 0$ and W_{nc} is still zero. Thus, $v_B^2 = v_0^2 + 4\,gR$. Here, the normal force must supply the centripetal acceleration *and* support the weight of the block. Therefore,

$$n_B = m\left(\frac{v_B^2}{R}\right) + mg = m\left(\frac{v_0^2}{R} + 5\,g\right)$$

$$= \left(0.50\text{ kg}\right)\left[\frac{\left(4.0\text{ m/s}\right)^2}{1.5\text{ m}} + 5\left(9.8\text{ m/s}^2\right)\right] = \boxed{30\text{ N}}.$$

(b) When the block reaches point C, $y = 2R$ and $W_{nc} = -f_k L = -\mu_k \left(mg\right)L$. At this point, the normal force is to be zero, so the weight alone must supply the centripetal acceleration. Thus, $m\left(v_c^2/R\right) = mg$, or the required speed at point C is $v_c^2 = R\,g$. Substituting this into equation (1) yields $R\,g = v_0^2 - 2\mu_k gL + 0$, or

$$\mu_k = \frac{v_0^2 - R\,g}{2\,gL} = \frac{\left(4.0\text{ m/s}\right)^2 - \left(1.5\text{ m}\right)\left(9.8\text{ m/s}^2\right)}{2\left(9.8\text{ m/s}^2\right)\left(0.40\text{ m}\right)} = \boxed{0.17}.$$

7.59 (a) At point A, the weight of the coaster must be just large enough to supply the centripetal acceleration. Thus, $m\left(\frac{v_A^2}{R}\right) = mg$, or $v_A^2 = R\,g$.

Applying conservation of mechanical energy from the start to point A,

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_A^2 + mg\left(\frac{2h}{3}\right),\text{ or }v_0^2 = v_A^2 - \frac{2\,gh}{3}.$$

Using the value of $v_A^2 = R\,g$, so the car barely stays on track at A, gives

$$v_0 = \sqrt{R\,g - \frac{2\,gh}{3}} = \boxed{\sqrt{g\left(R - \frac{2h}{3}\right)}}$$

(b) If the speed of the coaster is to be zero at point B, conservation of mechanical energy from the start to point B gives

$$0 + mg\,h' = \frac{1}{2}mv_0^2 + mgh = \frac{1}{2}m\left[g\left(R - \frac{2h}{3}\right)\right] + mgh,$$

or $h' = \boxed{\dfrac{R}{2} + \dfrac{2h}{3}}.$

Answers to Even Numbered Conceptual Questions

2. The scratch on the rim of the disc moves in a circular path around the axis of rotation. Whenever the disc is rotating, whether its angular speed is constant or not, the scratch has a non-zero tangential component of velocity, but has zero radial velocity. The scratch has a radial acceleration (the centripetal acceleration) any time the disc is rotating. It has a tangential component of acceleration only while the disc has an angular acceleration as it is coming up to speed.

4. The need for a large force toward the center of the circular path on objects near the equator will cause the Earth to bulge at the equator. A force toward the center of the circular path is not needed at the poles, so the radius in this direction will be smaller than at the equator.

6. To a good first approximation, your bathroom scale reading is unaffected because you, Earth, and the scale are all in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weight slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational fields is a little weaker at the center of the Earth than at the surface sub-solar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.

8. The astronaut is accelerating toward the Earth at the same rate as is the spaceship. Thus, if the astronaut drops a wrench, it will float in space next to him. Likewise, he will float in space next to a desk or with reference to the spaceship. Thus, he believes himself to be weightless.

10. Consider one end of a string connected to a spring scale and the other end connected to an object, of true weight w. The tension T in the string will be measured by the scale and construed as the apparent weight. We have $w-T=ma_c$. This gives, $T=w-ma_c$. Thus, the apparent weight is less than the actual weight by the term ma_c. At the poles the centripetal acceleration is zero. Thus, $T=w$. However, at the equator the term containing the centripetal acceleration is nonzero, and the apparent weight is less than the true weight.

12. If the acceleration is constant in magnitude and perpendicular to the velocity, the object is moving in a circular path at constant speed. If the acceleration is parallel to the velocity, the object is either speeding up, v and a in same direction, or slowing down, v and a in opposite directions.

14. When an object follows an orbital path, there is a force acting on it that produces the centripetal acceleration. This force and the centripetal acceleration are always directed toward the center of the orbit. In the case of a planet orbiting the Sun, this force is the gravitational force exerted on the planet by the Sun, and it points toward the center of the Sun. Hence, the center of the Sun must coincide with the center of the orbit and lie in the plane of the orbit.

16. Kepler's second law says that equal areas are swept out in equal times by a line drawn from the Sun to the planet. For this to be so, the planet must move fastest when it is closest to the Sun. This, surprisingly, occurs during the winter.

Answers to Even Numbered Problems

2. 2.1 m, 1.2×10^2 m , 7.7×10^2 m

4. 4.2×10^{-2} rad/s^2

6. (a) 5.24 s   (b) 27.4 rad

8. 147 rev

10. 50 rev

12. 1.02 m

14. 4.9×10^{-2} rad/s

16. 3.44 km, 9.42 m/s^2

18. (a) 14.1 m/s   (b) 200 m   (c) 28.2 s

20. 1.5×10^2 rev/s

22. $0.966\,g$

24. (b) 20.1°

26. The required tension in the vine is 1.4×10^3 N . He does not make it.

28. (a) 25 kN   (b) 12 m/s

30. (a) 4.39×10^{20} N toward the Sun   (b) 1.99×10^{20} N toward the Earth
(c) 3.55×10^{22} N toward the Sun

32. 2.59×10^8 m from the center of the Earth

34. 2.00 kg and 3.00 kg

36. (a) 5.59×10^3 m/s   (b) 3.98 h   (c) 1.47×10^3 N

38. (a) -4.76×10^9 J   (b) 568 N

40. 1.63×10^4 rad/s

42. (a) 3.77 m/s^2 (b) 3.26 s

44. (a) 56.5 rad/s   (b) 22.4 rad/s   (c) -7.63×10^{-3} rad/s^2

     (d) 1.77×10^5 rad   (e) 5.81 km

46. (a) $v_{min} = \sqrt{Rg\left(\dfrac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$, $v_{max} = \sqrt{Rg\left(\dfrac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$

     (b) 8.6 m/s to 17 m/s

48. (a) 8.42 N   (b) 64.8°   (c) 1.67 N

50. (a) 0   (b) 1.3 kN   (c) 2.1 kN

52. (a) 2.1 m/s   (b) 54°   (c) 4.7 m/s

54. (a) 1.3×10^{17} N   (b) 2.6×10^{12} N/kg

58. (a) 15 N, 30 N   (b) 0.17