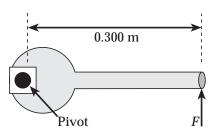
# **CHAPTER 8**

#### **Quick Quizzes**

- **1.** (d).
- **2.** (b).
- **3.** (b). The hollow cylinder has the higher moment of inertial, so it will be given the smaller acceleration and take longer to stop.
- **4.** (a). The hollow sphere has the higher moment of inertia.
- 5. (a). Nathan is correct. When one of the children curls up inside the tire, the rolling system closely resembles a solid cylinder or disk. In a race between a solid disk and a ring (or hollow cylinder), the disk always wins, independent of masses or radii. See the solution to problem 8.63.
- 6. (c). The box. All objects have the same potential energy associated with them before they are released. As the objects move down the inclines, this potential energy is transformed to kinetic energy. For the ball and cylinder, the transformation is into both rotational and translational kinetic energy. The box has only translational kinetic energy. Because the kinetic energies of the ball and cylinder are split into two types, their translational kinetic energy is necessarily less than that of the box. Consequently, their translational speeds are less than that of the box, so the ball and cylinder will lag behind.
- 7. (c). Apply conservation of angular momentum to the system (the two disks) before and after the second disk is added to get the result:  $I_1\omega_0 = (I_1 + I_2)\omega_f$ .
- 8. (a). The Earth already bulges slightly at the Equator, and is slightly flat at the poles. If more mass moved towards the Equator, it would essentially move the mass to a greater distance from the axis of rotation, and increase the moment of inertia. Because conservation of angular momentum requires that  $\omega_z I_z = \text{const}$ , an increase in the moment of inertia would decrease the angular velocity, and slow down the spinning of the Earth. Thus, the length of each day would increase.

### **Problem Solutions**

8.1 To exert a given torque using minimum force, the lever arm should be as large as possible. In this case, the maximum lever arm is used when the force is applied at the end of the wrench and perpendicular to the handle.



Then, 
$$F_{min} = \frac{\tau}{d_{max}} = \frac{40.0 \text{ N} \cdot \text{m}}{0.300 \text{ m}} = \boxed{133 \text{ N}}$$

**8.2** The lever arm is  $d = (1.20 \times 10^{-2} \text{ m})\cos 48.0^{\circ} = 8.03 \times 10^{-3} \text{ m}$ , and the torque is

$$\tau = Fd = (80.0 \text{ N})(8.03 \times 10^{-3} \text{ m}) = 0.642 \text{ N} \cdot \text{m} \text{ counterclockwise}$$

8.3 
$$\tau_A = -\lceil (100 \text{ N}) \sin 20.0^\circ \rceil (0.600 \text{ m}) - \lceil (900 \text{ N}) \sin 15.0^\circ \rceil (0.800 \text{ m}) = \lceil -207 \text{ N} \cdot \text{m} \rceil$$

$$\tau_B = -(800 \text{ N})(0.600 \text{ m}) - [(900 \text{ N})\sin 15.0^\circ](0.800 \text{ m})$$

$$+[(900 \text{ N})\cos 15.0^{\circ}](0.600 \text{ m}) = \boxed{-145 \text{ N} \cdot \text{m}}$$

$$\tau_C = -\lceil (100 \text{ N}) \sin 20.0^\circ \rceil (0.600 \text{ m}) - \lceil (100 \text{ N}) \cos 20.0^\circ \rceil (0.800 \text{ m}) = \boxed{-95.7 \text{ N} \cdot \text{m}}$$

8.4 In the 0° position,  $\tau = (mg)(0) = \boxed{0}$ 

At 30°, 
$$\tau = -(10 \text{ kg})(9.8 \text{ m/s}^2)[(0.400 \text{ m})\sin 30^\circ] = \boxed{-20 \text{ N} \cdot \text{m}}$$

At 60°, 
$$\tau = -(10 \text{ kg})(9.8 \text{ m/s}^2)[(0.400 \text{ m})\sin 60^\circ] = \boxed{-34 \text{ N} \cdot \text{m}}$$

At 90°, 
$$\tau = -(10 \text{ kg})(9.8 \text{ m/s}^2)(0.400 \text{ m}) = \boxed{-39 \text{ N} \cdot \text{m}}$$

8.5 
$$|\tau| = F \cdot (lever \ arm) = (mg) \cdot [L\sin\theta]$$

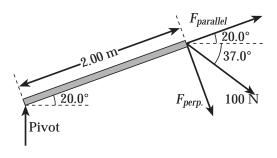
= 
$$(3.0 \text{ kg})(9.8 \text{ m/s}^2) \cdot [(2.0 \text{ m}) \sin 5.0^\circ] = 5.1 \text{ N} \cdot \text{m}$$

8.6 Resolve the 100-N force into components parallel to and perpendicular to the rod, as

$$F_{parallel} = (100 \text{ N}) \cos (20.0^{\circ} + 37.0^{\circ}) = 54.5 \text{ N}$$

and

$$F_{perp.} = (100 \text{ N}) \sin (20.0^{\circ} + 37.0^{\circ}) = 83.9 \text{ N}$$

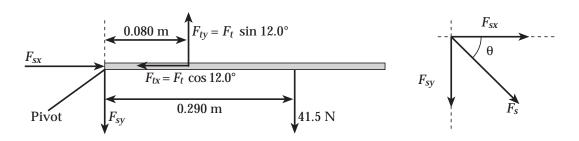


The torque due to the 100-N force is equal to the sum of the torques of its components.

Thus,

$$\tau = (54.5 \text{ N})(0) - (83.9 \text{ N})(2.00 \text{ m}) = \boxed{-168 \text{ N} \cdot \text{m}}$$

8.7



Requiring that  $\Sigma \tau = 0$ , using the shoulder joint at point O as a pivot, gives

$$\Sigma \tau = (F_t \sin 12.0^\circ)(0.080 \text{ m}) - (41.5 \text{ N})(0.290 \text{ m}) = 0$$
, or  $F_t = 724 \text{ N}$ 

Then 
$$\Sigma F_y = 0 \implies -F_{sy} + (724 \text{ N}) \sin 12.0^{\circ} - 41.5 \text{ N} = 0$$
,

yielding  $F_{sy} = 109 \text{ N}$ 

$$\Sigma F_x = 0$$
 gives  $F_{sx} - (724 \text{ N})\cos 12.0^{\circ} = 0$ , or  $F_{sx} = 708 \text{ N}$ 

Therefore, 
$$F_s = \sqrt{F_{sx}^2 + F_{sy}^2} = \sqrt{(708 \text{ N})^2 + (109 \text{ N})^2} = \boxed{716 \text{ N}}$$

8.8 If the mass of a hydrogen atom is 1.00 u (i.e., 1 unit), then the mass of the oxygen atom is 16.0 u.

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(16.0 \text{ u})(0) + 2(1.00 \text{ u})[(0.100 \text{ nm})\cos 53.0^\circ]}{(16.0 + 1.00 + 1.00) \text{ u}} = \boxed{6.69 \times 10^{-3} \text{ nm}}$$

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i}$$

$$= \frac{(16.0)(0) + (1.00)[(0.100)\sin 53.0^{\circ}] + (1.00)[-(0.100)\sin 53.0^{\circ}] u \cdot nm}{(16.0 + 1.00 + 1.00) u} = \boxed{0}$$

8.9 Require that  $\Sigma \tau = 0$  about an axis through the elbow and perpendicular to the page. This gives

$$\Sigma \tau = + [(2.00 \text{ kg})(9.80 \text{ m/s}^2)](25.0 \text{ cm} + 8.00 \text{ cm}) - (F_B \cos 75.0^\circ)(8.00 \text{ cm}) = 0$$

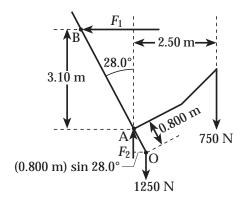
or 
$$F_B = \frac{(19.6 \text{ N})(33.0 \text{ cm})}{(8.00 \text{ cm})\cos 75.0^\circ} = \boxed{312 \text{ N}}$$

**8.10** We assume the boat is in equilibrium. The wind exerts a forward force on the sail to counterbalance water friction on the boat.

Require that the sum of the torques be zero about an axis parallel to the length of the boat and passing through point A. This gives

+(750 N)(2.50 m)+(1250 N)[(0.800 N)sin 28.0°]  
+
$$F_2(0)-F_1(3.10 m)=0$$
,

or the wind force on the sail is  $F_1 = 756 \text{ N}$ 



8.11 Let T = the tension force in the back muscles (represented by wire W in the model) and F = the compression force in the spine. The sketches below represent free-body diagrams of point P for each of the two positions.

For Position (a):

$$\Sigma F_x = 0 \implies -F \sin 20.0^\circ + T \sin 30.0^\circ = 0$$
,

or 
$$F = \left(\frac{\sin 30.0^{\circ}}{\sin 20.0^{\circ}}\right) T$$
.

$$\Sigma F_y = 0 \implies F \cos 20.0^{\circ} - T \cos 30.0^{\circ} - 250 \text{ N} = 0$$
, or

$$\[ \left( \frac{\sin 30.0^{\circ}}{\sin 20.0^{\circ}} \right) \cos 20.0^{\circ} - \cos 30.0^{\circ} \] T = 250 \text{ N, giving } T = \boxed{492 \text{ N}}.$$



Position (a)

For Position (b):

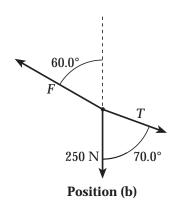
$$\Sigma F_x = 0 \implies -F \sin 60.0^\circ + T \sin 70.0^\circ = 0$$

or 
$$F = \left(\frac{\sin 70.0^{\circ}}{\sin 60.0^{\circ}}\right)T$$

$$\Sigma F_y = 0 \implies F \cos 60.0^{\circ} - T \cos 70.0^{\circ} - 250 \text{ N} = 0$$
, or

$$\left[ \left( \frac{\sin 70.0^{\circ}}{\sin 60.0^{\circ}} \right) \cos 60.0^{\circ} - \cos 70.0^{\circ} \right] T = 250 \text{ N} ,$$

giving 
$$T = 1.25 \times 10^3 \text{ N} = 1.25 \text{ kN}$$



Thus, the tension force in the back muscles is much greater when lifting a load in position (b) than when lifting the same load in position (a).

**8.12** Requiring that 
$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = 0$$
 gives

$$\frac{(5.0 \text{ kg})(0) + (3.0 \text{ kg})(0) + (4.0 \text{ kg})(3.0 \text{ m}) + (8.0 \text{ kg})x}{(5.0 + 3.0 + 4.0 + 8.0) \text{ kg}} = 0,$$

or 8.0x + 12 m = 0 which yields x = -1.5 m.

Also, requiring that  $y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = 0$  gives

$$\frac{(5.0 \text{ kg})(0) + (3.0 \text{ kg})(4.0 \text{ m}) + (4.0 \text{ kg})(0) + (8.0 \text{ kg})y}{(5.0 + 3.0 + 4.0 + 8.0) \text{ kg}} = 0,$$

or 8.0y + 12 m = 0 yielding y = -1.5 m.

Thus, the 8.0-kg mass should be placed at coordinates (-1.5 m, -1.5 m)

**8.13** First, evaluate two sums involving the masses of the body parts (remembering that there are 2 of each leg or arm part). These sums for the 10-year old are

$$\Sigma m_i = [2(1.00 + 3.50 + 3.80 + 0.300 + 0.700 + 0.800) + 5.60 + 5.50 + 6.00] \text{ kg}$$
  
= 37.3 kg

and

$$\Sigma m_i y_i = \left[ 2\{(1.00)(3.50) + (3.50)(20.0) + (3.80)(43.0) + (0.300)(44.0) + (0.700)(58.0) + (0.800)(78.0) \} + (5.60)(61.0) + (5.50)(87.0) + (6.00)(110) \right] \text{ kg} \cdot \text{cm} = 2.19 \times 10^3 \text{ kg} \cdot \text{cm}$$

The location of the center of gravity is

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \frac{2.19 \times 10^3 \text{ kg} \cdot \text{cm}}{37.3 \text{ kg}} = 58.6 \text{ cm}.$$

The displacement of the center of gravity from the midpoint of the body is

$$\Delta y = y_{midpt} - y_{cg} = \left(\frac{118 \text{ cm}}{2}\right) - 58.6 \text{ cm} = 0.39 \text{ cm}$$
, and the percent

deviation is 
$$\% \text{ dev.} = \left(\frac{\Delta y}{height}\right) \cdot 100\% = \left(\frac{0.39 \text{ cm}}{118 \text{ cm}}\right) \cdot 100\% = 0.33\%$$
.

Thus, the requested shift in the position of the center of gravity of a 10-year old is 0.39 cm below midpoint of the body or 0.33% of the height.

For the 20-year old female, a similar set of calculations yield the following results:

$$\Sigma m_i = 55.4 \text{ kg}$$
,  $\Sigma m_i y_i = 4.52 \times 10^3 \text{ kg} \cdot \text{cm}$ ,  $y_{cg} = 81.6 \text{ cm}$ ,  $\Delta y = 3.4 \text{ cm}$ , and % dev. = 2.0%.

The requested shift in the position of the center of gravity of a 20-year old is

3.4 cm below midpoint of the body or 2.0% of the height

**8.14** (a) Consider the dashed line through A and D to be the *y*-axis. For stability, the center of gravity must lie on this line since it is the line of support. Thus, if  $M = \sum m_i$ , it is necessary that

$$x_{cg} = \Sigma m_i x_i / \Sigma m_i = \Sigma (m_i / M) x_i = 0$$
,

or 
$$0.65(0) + 0.25[-(40.0 \text{ cm})\tan \beta] + 0.10(10.0 \text{ cm}) = 0$$
.

This gives  $\tan \beta = 1.0 \text{ cm}/10 \text{ cm} = 0.10$ , or  $\beta = \boxed{5.7^{\circ}}$ .

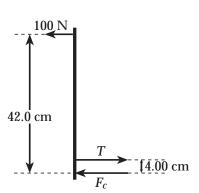
- (b)  $y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \sum \left(\frac{m_i}{M}\right) y_i = 0.65 (70.0 \text{ cm}) + 0.25 (140 \text{ cm}) + 0.10 (105 \text{ cm})$ or  $y_{cg} = \boxed{91 \text{ cm}}$ .
- 8.15 Require that  $\Sigma \tau = 0$  about an axis through the knee joint and perpendicular to the page. This gives

$$\Sigma \tau = F_c(0) - T(4.00 \text{ cm}) + (100 \text{ N})(42.0 \text{ cm}) = 0$$
,

or the tension in the muscle is

$$T = 1.05 \times 10^3 \text{ N} = 1.05 \text{ kN}$$

Then,  $\Sigma F_x = -F_c - 100 \text{ N} + 1.05 \times 10^3 \text{ N} = 0$  gives the compression force as  $F_c = 950 \text{ N}$ 

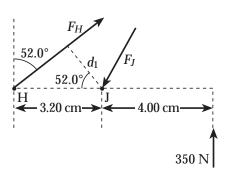


8.16 Require that  $\Sigma \tau = 0$  about an axis through point J and perpendicular to the page. This gives

$$-F_H d_1 + F_J(0) + (350 \text{ N})(4.00 \text{ cm}) = 0, \text{ or}$$

$$F_H = \frac{(350 \text{ N})(4.00 \text{ cm})}{d_1}$$

$$= \frac{(350 \text{ N})(4.00 \text{ cm})}{(3.20 \text{ cm})\cos 52.0^\circ} = \boxed{711 \text{ N}}$$

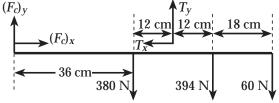


Then,  $\Sigma F_x = 0 \Rightarrow (711 \text{ N}) \sin 52.0^\circ - (F_J)_x = 0$ , or  $(F_J)_x = 560 \text{ N}$ .

Also,  $\Sigma F_y = 0 \Rightarrow (711 \text{ N})\cos 52.0^{\circ} - (F_J)_y + 350 \text{ N} = 0$ , giving  $(F_J)_y = 788 \text{ N}$ .

Hence, 
$$F_J = \sqrt{(F_J)_x^2 + (F_J)_y^2} = \sqrt{(560 \text{ N})^2 + (788 \text{ N})^2} = \boxed{966 \text{ N}}$$

**8.17** We consider the torques about an axis perpendicular to the page through the base of the spine, where the compression force  $\mathbf{F}_c$  acts.



 $\Sigma \tau = -(380 \text{ N})(36 \text{ cm}) + (T \sin 12^\circ)(48 \text{ cm}) - (394 \text{ N})(60 \text{ cm}) - (60 \text{ N})(78 \text{ cm}) = 0$ 

or 
$$T = \frac{4.2 \times 10^4 \text{ N} \cdot \text{cm}}{(48 \text{ cm}) \sin 12^\circ} = 4.2 \times 10^3 \text{ N} = \boxed{4.2 \text{ kN}}.$$

Then,  $\Sigma F_x = 0 \Rightarrow (F_c)_x - T_x = 0$ , or  $(F_c)_x = T \cos 12^\circ = (4.2 \text{ kN}) \cos 12^\circ = 4.1 \text{ kN}$ 

and 
$$\Sigma F_y = 0 \Rightarrow (F_c)_y + T_y - (380 + 394 + 60) \text{ N} = 0$$
,

or 
$$(F_c)_v = 834 \text{ N} - (4.2 \times 10^3 \text{ N}) \sin 12^\circ = -41 \text{ N}$$
.

Thus, 
$$F_c = \sqrt{(F_c)_x^2 + (F_c)_y^2} = \sqrt{(4.1 \times 10^3 \text{ N})^2 + (-41 \text{ N})^2} = 4.1 \times 10^3 \text{ N}$$

and 
$$\theta = \tan^{-1} \left[ \frac{(F_c)_y}{(F_c)_y} \right] = \tan^{-1} \left( \frac{-41 \text{ N}}{4.1 \times 10^3 \text{ N}} \right) = -0.57^{\circ}.$$

The compression force is,  $\mathbf{F}_c = \boxed{4.1 \times 10^3 \text{ N at } 0.57^\circ \text{ below the horizontal}}$ 

Consider the torques about an axis perpendicular to the page through the left end of the 8.18 scaffold.

$$\Sigma \tau = 0 \Rightarrow T_1(0) - (700 \text{ N})(1.00 \text{ m}) - (200 \text{ N})(1.50 \text{ m}) + T_2(3.00 \text{ m}) = 0.$$

From which, 
$$T_2 = 333 \text{ N}$$
.

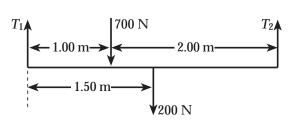
Then, from  $\Sigma F_{v} = 0$ , we have

$$T_1 + T_2 - 700 \text{ N} - 200 \text{ N} = 0$$
,

or

8.20

$$T_1 = 900 \text{ N} - T_2 = 900 \text{ N} - 333 \text{ N} = \boxed{567 \text{ N}}$$



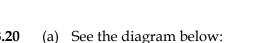
 $F_c = 50.0 \text{ N}$ 

7.50 cm

8.19 Consider the torques about an axis perpendicular to the page and through the point where the force *T* acts on the jawbone.

$$\Sigma \tau = 0 \Longrightarrow (50.0 \text{ N})(7.50 \text{ cm}) - R(3.50 \text{ cm}) = 0$$

Then, 
$$\Sigma F_{v} = 0 \Rightarrow -(50.0 \text{ N}) + T - 107 \text{ N} = 0$$
, or  $T = \boxed{157 \text{ N}}$ .



which yields R = 107 N.

80.0 N

(b) If x = 1.00 m, then

$$\Sigma \tau$$
)<sub>left end</sub> = 0  $\Rightarrow$  -(700 N)(1.00 m)-(200 N)(3.00 m) -  
(80.0 N)(6.00 m)+( $T \sin 60.0^{\circ}$ )(6.00 m)=0,

giving 
$$T = 343 \text{ N}$$

Then, 
$$\Sigma F_x = 0 \Rightarrow H - T \cos 60.0^{\circ} = 0$$
, or  $H = (343 \text{ N}) \cos 60.0^{\circ} = 171 \text{ N}$ 

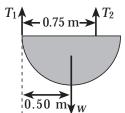
and 
$$\Sigma F_y = 0 \Rightarrow V - 980 \text{ N} + (343 \text{ N}) \sin 60.0^\circ = 0$$
, or  $V = 683 \text{ N}$ 

(c) When the wire is on the verge of breaking, T = 900 N and

$$(80.0 \text{ N})_{left\ end} = -(700 \text{ N})x_{max} - (200 \text{ N})(3.00 \text{ m}) - (80.0 \text{ N})(6.00 \text{ m}) + [(900 \text{ N})\sin 60.0^{\circ}](6.00 \text{ m}) = 0,$$

which gives 
$$x_{max} = 5.14 \text{ m}$$

**8.21** We call the tension in the cord at the left end of the sign,  $T_1$  and the tension in the cord near the right end  $T_2$ . Consider the torques about an axis perpendicular to the page and through the left end of the sign.



$$\Sigma \tau = -w (0.50 \text{ m}) + T_2 (0.75 \text{ m}) = 0$$
, so  $T_2 = \boxed{\frac{2}{3}w}$ 

From 
$$\Sigma F_y = 0$$
,  $T_1 + T_2 - w = 0$ , or  $T_1 = w - T_2 = w - \frac{2}{3}w = \boxed{\frac{1}{3}w}$ .

**8.22** (a) Consider the torques about an axis perpendicular to the page and through the left end of the horizontal beam.

$$H$$
 30.0° 196 N

$$\Sigma \tau = + (T \sin 30.0^{\circ}) d - (196 \text{ N}) d = 0,$$

giving 
$$T = 392 \text{ N}$$

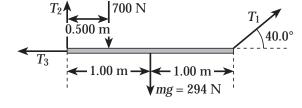
(b) From 
$$\Sigma F_{x} = 0$$
,  $H - T \cos 30.0^{\circ} = 0$ , or

$$H = (392 \text{ N})\cos 30.0^{\circ} = 339 \text{ N to the right}$$

From 
$$\Sigma F_{v} = 0$$
,  $V + T \sin 30.0^{\circ} - 196 \text{ N} = 0$ ,

or 
$$V = 196 \text{ N} - (392 \text{ N}) \sin 30.0^{\circ} = \boxed{0}$$

**8.23** Consider the torques about an axis perpendicular to the page and through the left end of the plank.



$$\Sigma \tau = 0$$
 gives

$$-(700 \text{ N})(0.500 \text{ m}) - (294 \text{ N})(1.00 \text{ m}) + (T_1 \sin 40.0^\circ)(2.00 \text{ m}) = 0$$

or 
$$T_1 = 501 \text{ N}$$

Then, 
$$\Sigma F_x = 0$$
 gives  $-T_3 + T_1 \cos 40.0^{\circ} = 0$ , or

$$T_3 = (501 \text{ N})\cos 40.0^\circ = 384 \text{ N}$$

From 
$$\Sigma F_y = 0$$
,  $T_2 - 994 \text{ N} + T_1 \sin 40.0^\circ = 0$ ,

or 
$$T_2 = 994 \text{ N} - (501 \text{ N}) \sin 40.0^\circ = 672 \text{ N}$$

**8.24** First, we compute some needed dimensions:

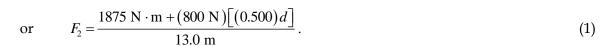
$$d_1 = (7.50 \text{ m})\cos 60.0^\circ = 3.75 \text{ m}$$

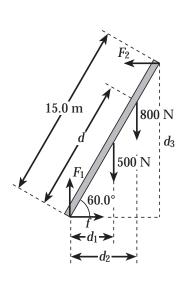
$$d_2 = d\cos 60.0^{\circ} = (0.500) d$$

$$d_3 = (15.0 \text{ m}) \sin 60.0^\circ = 13.0 \text{ m}$$

Using an axis perpendicular to the page and through the lower end of the ladder,  $\Sigma \tau = 0$  gives

$$-(500 \text{ N})d_1 - (800 \text{ N})d_2 + F_2 d_3 = 0$$
,





(a) When d = 4.00 m, equation (1) gives  $F_2 = 267$  N to the left .

Then, 
$$\Sigma F_x = 0$$
 gives  $f - 267$  N = 0, or  $f = 267$  N to the right, and  $\Sigma F_y = 0$  yields  $F_1 - 500$  N  $- 800$  N = 0, or  $F_1 = 1.30$  kN upward.

(b) When d = 9.00 m, equation (1) gives  $F_2 = 421$  N to the left .

Then, 
$$\Sigma F_x = 0$$
 gives  $f = 421 \text{ N}$  to the right, while

$$\Sigma F_y = 0$$
 yields  $F_1 = 1.30 \times 10^3$  N = 1.30 kN as before .

If the ladder is ready to slip under these conditions, then  $f = (f_s)_{max}$ ,

and 
$$\mu_s = \frac{(f_s)_{max}}{n} = \frac{(f_s)_{max}}{F_1} = \frac{421 \text{ N}}{1.30 \times 10^3 \text{ N}} = \boxed{0.324}.$$

**8.25** The required dimensions are:

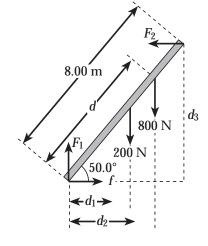
$$d_1 = (4.00 \text{ m})\cos 50.0^\circ = 2.57 \text{ m}$$

$$d_2 = d\cos 50.0^\circ = (0.643) d$$

$$d_3 = (8.00 \text{ m}) \sin 50.0^\circ = 6.13 \text{ m}$$

$$\Sigma F_y = 0$$
 yields  $F_1 - 200 \text{ N} - 800 \text{ N} = 0$ ,

or 
$$F_1 = 1.00 \times 10^3 \text{ N}$$
.



When the ladder is on the verge of slipping,

$$f = (f_s)_{max} = \mu_s n = \mu_s F_1$$
, or  $f = (0.600)(1.00 \times 10^3 \text{ N}) = 600 \text{ N}$ .

Then,  $\Sigma F_x = 0$  gives  $F_2 = 600$  N to the left.

Finally, using an axis perpendicular to the page and through the lower end of the ladder,  $\Sigma \tau = 0$  gives

$$-(200 \text{ N})(2.57 \text{ m}) - (800 \text{ N})(0.643)d + (600 \text{ N})(6.13 \text{ m}) = 0$$

or 
$$d = \frac{(3.68 \times 10^3 - 550) \text{ N} \cdot \text{m}}{0.643(800 \text{ N})} = 6.15 \text{ m}$$
 when the ladder is ready to slip.

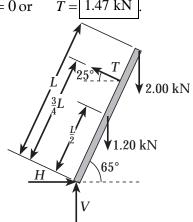
8.26 Observe that the cable is perpendicular to the boom. Then, using  $\Sigma \tau = 0$  for an axis perpendicular to the page and through the lower end of the boom gives

$$-(1.20 \text{ kN})\left(\frac{L}{2}\cos 65^{\circ}\right) + T\left(\frac{3}{4}L\right) - (2.00 \text{ kN})(L\cos 65^{\circ}) = 0 \text{ or}$$

From 
$$\Sigma F_x = 0$$
,  $H = T \cos 25^\circ = 1.33 \text{ kN to the right}$ 

and 
$$\Sigma F_{v} = 0$$
 gives,

$$V = 3.20 \text{ kN} - T \sin 25^\circ = 2.58 \text{ kN upward}$$



**8.27** First, we resolve all forces into components parallel to and perpendicular to the tibia, as shown. Note that

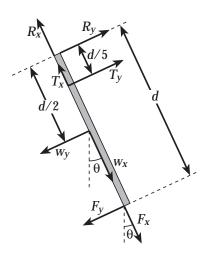
$$w_y = (30.0 \text{ N}) \sin 40.0^\circ = 19.3 \text{ N}$$
,

$$F_y = (12.5 \text{ N}) \sin 40.0^\circ = 8.03 \text{ N}$$
,

and 
$$T_y = T \sin 25.0^\circ$$
.

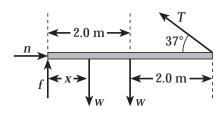
Using  $\Sigma \tau = 0$  for an axis perpendicular to the page and through the upper end of the tibia gives

$$(T \sin 25.0^{\circ}) \frac{d}{5} - (19.3 \text{ N}) \frac{d}{2} - (8.03 \text{ N}) d = 0$$
, or



$$T = 209 \text{ N}$$

**8.28** When  $x = x_{min}$ , the rod is on the verge of slipping, so  $f = (f_s)_{max} = \mu_s n = 0.50 n$ .



From  $\Sigma F_x = 0$ ,

$$n - T\cos 37^{\circ} = 0$$
, or  $n = 0.80T$ 

Thus, 
$$f = 0.50(0.80T) = 0.40T$$

From 
$$\Sigma F_v = 0$$
,  $f + T \sin 37^\circ - 2w = 0$ , or  $0.40T + 0.60T - 2w = 0$ ,

giving T = 2w.

Using  $\Sigma \tau = 0$  for an axis perpendicular to the page and through the left end of the beam gives  $-w \cdot x_{min} - w(2.0 \text{ m}) + [(2w)\sin 37^{\circ}](4.0 \text{ m}) = 0$ ,

which reduces to  $x_{min} = 2.8 \text{ m}$ 

- 8.29 The moment of inertia for rotations about an axis is  $I = \sum m_i r_i^2$ , where  $r_i$  is the distance mass  $m_i$  is from that axis.
  - (a) For rotation about the *x*-axis,

$$I_x = (3.00 \text{ kg})(3.00 \text{ m})^2 + (2.00 \text{ kg})(3.00 \text{ m})^2 +$$
  
 $(2.00 \text{ kg})(3.00 \text{ m})^2 + (4.00 \text{ kg})(3.00 \text{ m})^2 = \boxed{99.0 \text{ kg} \cdot \text{m}^2}$ 

(b) When rotating about the *y*-axis,

$$I_y = (3.00 \text{ kg})(2.00 \text{ m})^2 + (2.00 \text{ kg})(2.00 \text{ m})^2 +$$

$$(2.00 \text{ kg})(2.00 \text{ m})^2 + (4.00 \text{ kg})(2.00 \text{ m})^2 = \boxed{44.0 \text{ kg} \cdot \text{m}^2}$$

(c) For rotations about an axis perpendicular to the page through point O, the distance  $r_i$  for each mass is  $r_i = \sqrt{(2.00 \text{ m})^2 + (3.00 \text{ m})^2} = \sqrt{13.0} \text{ m}$ .

Thus, 
$$I_o = [(3.00 + 2.00 + 2.00 + 4.00) \text{ kg}] (13.0 \text{ m}^2) = 143 \text{ kg} \cdot \text{m}^2].$$

**8.30** The required torque in each case is  $\tau = I\alpha$ .

Thus, 
$$\tau_x = I_x \alpha = (99.0 \text{ kg} \cdot \text{m}^2)(1.50 \text{ rad/s}^2) = \boxed{149 \text{ N} \cdot \text{m}},$$

$$\tau_y = I_y \alpha = (44.0 \text{ kg} \cdot \text{m}^2)(1.50 \text{ rad/s}^2) = \boxed{66.0 \text{ N} \cdot \text{m}},$$
and  $\tau_o = I_o \alpha = (143 \text{ kg} \cdot \text{m}^2)(1.50 \text{ rad/s}^2) = \boxed{215 \text{ N} \cdot \text{m}}.$ 

**8.31** (a) 
$$\tau = F \cdot r = (0.800 \text{ N})(30.0 \text{ m}) = 24.0 \text{ N} \cdot \text{m}$$

(b) 
$$\alpha = \frac{\tau}{I} = \frac{\tau}{mr^2} = \frac{24.0 \text{ N} \cdot \text{m}}{(0.750 \text{ kg})(30.0 \text{ m})^2} = \boxed{0.0356 \text{ rad/s}^2}$$

(c) 
$$a_t = r\alpha = (30.0 \text{ m})(0.0356 \text{ rad/s}^2) = 1.07 \text{ m/s}^2$$

**8.32** The angular acceleration is 
$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = -\left(\frac{\omega_i}{\Delta t}\right)$$
 since  $\omega_f = 0$ .

Thus, the torque is  $\tau = I\alpha = -\left(\frac{I\omega_i}{\Delta t}\right)$ . But, the torque is also  $\tau = -fr$ , so the magnitude of the required friction force is

$$f = \frac{I\omega_i}{r(\Delta t)} = \frac{(12 \text{ kg} \cdot \text{m}^2)(50 \text{ rev/min})}{(0.50 \text{ m})(6.0 \text{ s})} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 21 \text{ N}$$

Therefore, the coefficient of friction is  $\mu_k = \frac{f}{n} = \frac{21 \text{ N}}{70 \text{ N}} = \boxed{0.30}$ 

**8.33** (a) 
$$\tau_{net} = I\alpha = (6.8 \times 10^{-4} \text{ kg} \cdot \text{m}^2)(66 \text{ rad/s}^2) = 4.5 \times 10^{-2} \text{ N} \cdot \text{m}^2$$

The torque exerted by the fish is  $\tau_{\mathit{fish}} = F \cdot r$ , and this also equals

$$\tau_{fish} = \tau_{net} + \tau_{friction} = (4.5 \times 10^{-2} + 1.3) \text{ N} \cdot \text{m}$$

Thus, 
$$F = \frac{\tau_{fish}}{r} = \frac{(4.5 \times 10^{-2} + 1.3) \text{ N} \cdot \text{m}}{4.0 \times 10^{-2} \text{ m}} = \boxed{34 \text{ N}}$$

(b) 
$$\theta = \omega_i t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} (66 \text{ rad/s}^2) (0.50 \text{ s})^2 = \left(\frac{33}{4}\right) \text{ rad}$$
,  
so  $s = r\theta = (4.0 \times 10^{-2} \text{ m}) \left(\frac{33}{4}\right) \text{ rad} = 0.33 \text{ m} = \boxed{33 \text{ cm}}$ 

**8.34** 
$$I = MR^2 = (1.80 \text{ kg})(0.320 \text{ m})^2 = 0.184 \text{ kg} \cdot \text{m}^2$$

$$au_{net} = au_{applied} - au_{resistive} = I \alpha$$
 , or  $F \cdot r - f \cdot R = I \alpha$ 

yielding 
$$F = \frac{I\alpha + f \cdot R}{r}$$

(a) 
$$F = \frac{(0.184 \text{ kg} \cdot \text{m}^2)(4.50 \text{ rad/s}^2) + (120 \text{ N})(0.320 \text{ m})}{4.50 \times 10^{-2} \text{ m}} = \boxed{872 \text{ N}}$$

(b) 
$$F = \frac{(0.184 \text{ kg} \cdot \text{m}^2)(4.50 \text{ rad/s}^2) + (120 \text{ N})(0.320 \text{ m})}{2.80 \times 10^{-2} \text{ m}} = \boxed{1.40 \text{ kN}}$$

8.35 
$$I = \frac{1}{2}MR^2 = \frac{1}{2}(150 \text{ kg})(1.50 \text{ m})^2 = 169 \text{ kg} \cdot \text{m}^2$$
,

and 
$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(0.500 \text{ rev/s} - 0)}{2.00 \text{ s}} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = \frac{\pi}{2} \text{ rad/s}^2$$

Thus,  $\tau = F \cdot r = I\alpha$  gives

$$F = \frac{I\alpha}{r} = \frac{(169 \text{ kg} \cdot \text{m}^2)(\frac{\pi}{2} \text{ rad/s}^2)}{1.50 \text{ m}} = \boxed{177 \text{ N}}.$$

**8.36** The moment of inertia of the reel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(5.00 \text{ kg})(0.600 \text{ m})^2 = 0.900 \text{ kg} \cdot \text{m}^2$$

Applying Newton's second law to the falling bucket gives

$$29.4 \text{ N} - T = (3.00 \text{ kg}) a_t \tag{1}$$

Then, Newton's second law for the reel gives

$$\tau = TR = I\alpha = I\left(\frac{a_t}{R}\right),$$

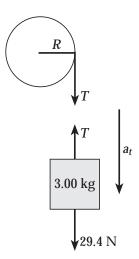
or 
$$T = \frac{I a_t}{R^2} = \frac{(0.900 \text{ kg} \cdot \text{m}^2)}{(0.600 \text{ m})^2} a_t = (2.50 \text{ kg}) a_t.$$
 (2)

(a) Solving equations (1) and (2) simultaneously gives

$$a_t = 5.35 \text{ m/s}^2 \text{ downward}$$

(b) 
$$\Delta y = v_i t + \frac{1}{2} a_t t^2 = 0 + \frac{1}{2} (5.35 \text{ m/s}^2) (4.00 \text{ s})^2 = 42.8 \text{ m}$$

(c) 
$$\alpha = \frac{a_t}{R} = \frac{5.35 \text{ m/s}^2}{0.600 \text{ m}} = \boxed{8.91 \text{ rad/s}^2}$$



8.37 The initial angular velocity of the wheel is zero, and the final angular velocity is

$$\omega_f = \frac{v}{r} = \frac{50.0 \text{ m/s}}{1.25 \text{ m}} = 40.0 \text{ rad/s}$$

Hence, the angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{40.0 \text{ rad/s} - 0}{0.480 \text{ s}} = 83.3 \text{ rad/s}^2$$

The torque acting on the wheel is  $\tau = f_k \cdot r$ , so  $\tau = I\alpha$  gives

$$f_k = \frac{I\alpha}{r} = \frac{(110 \text{ kg} \cdot \text{m}^2)(83.3 \text{ rad/s}^2)}{1.25 \text{ m}} = 7.33 \times 10^3 \text{ N}$$

Thus, the coefficient of friction is  $\mu_k = \frac{f_k}{n} = \frac{7.33 \times 10^3 \text{ N}}{1.40 \times 10^4 \text{ N}} = \boxed{0.524}$ 

**8.38** The work done on the grindstone is  $W_{net} = F \cdot s = F \cdot (r\theta) = (F \cdot r)\theta = \tau \cdot \theta$ .

Thus, 
$$W_{net} = \tau \cdot \theta = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$
,

or 
$$(25.0 \text{ N} \cdot \text{m})(15.0 \text{ rev}) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = \frac{1}{2} (0.130 \text{ kg} \cdot \text{m}^2) \omega_f^2 - 0$$

This yields 
$$\omega_f = \left(190 \frac{\text{rad}}{\text{s}}\right) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{30.3 \text{ rev/s}}.$$

**8.39** (a) 
$$KE_{trans} = \frac{1}{2}mv_t^2 = \frac{1}{2}(10.0 \text{ kg})(10.0 \text{ m/s})^2 = \boxed{500 \text{ J}}$$

(b) 
$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_t^2}{R^2}\right)$$

$$= \frac{1}{4}mv_t^2 = \frac{1}{4}(10.0 \text{ kg})(10.0 \text{ m/s})^2 = \boxed{250 \text{ J}}$$

(c) 
$$KE_{total} = KE_{trans} + KE_{rot} = \boxed{750 \text{ J}}$$

**8.40** Using 
$$W_{net} = KE_f - KE_i = \frac{1}{2}I\omega_f^2 - 0$$
,

gives 
$$I = \frac{2W_{net}}{\omega_f^2} = \frac{2(3000 \text{ J})}{(200 \text{ rad/s})^2} = \boxed{0.150 \text{ kg} \cdot \text{m}^2}.$$

**8.41** The moment of inertia of the cylinder is

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \left(\frac{w}{g}\right) R^2 = \frac{1}{2} \left(\frac{800 \text{ N}}{9.80 \text{ m/s}^2}\right) (1.50 \text{ m})^2 = 91.8 \text{ kg} \cdot \text{m}^2.$$

The angular acceleration is given by

$$\alpha = \frac{\tau}{I} = \frac{F \cdot R}{I} = \frac{(50.0 \text{ N})(1.50 \text{ m})}{91.8 \text{ kg} \cdot \text{m}^2} = 0.817 \text{ rad/s}^2.$$

At t = 3.00 s, the angular velocity is

$$\omega = \omega_i + \alpha t = 0 + (0.817 \text{ rad/s}^2)(3.00 \text{ s}) = 2.45 \text{ rad/s},$$

and the kinetic energy is

$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(91.8 \text{ kg} \cdot \text{m}^2)(2.45 \text{ rad/s})^2 = \boxed{276 \text{ J}}$$

**8.42** (a) The moment of inertial of the flywheel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(500 \text{ kg})(2.00 \text{ m})^2 = 1.00 \times 10^3 \text{ kg} \cdot \text{m}^2$$
,

and the angular velocity is

$$\omega = \left(5000 \frac{\text{rev}}{\text{min}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 524 \text{ rad/s}$$

Therefore, the stored kinetic energy is

$$KE_{stored} = \frac{1}{2}I\omega^2 = \frac{1}{2}(1.00 \times 10^3 \text{ kg} \cdot \text{m}^2)(524 \text{ rad/s})^2 = \boxed{1.37 \times 10^8 \text{ J}}$$

(b) A 10.0-hp motor supplies energy at the rate of

$$\wp = (10.0 \text{ hp}) \left( \frac{746 \text{ W}}{1 \text{ hp}} \right) = 7.46 \times 10^3 \text{ J/s}.$$

The time the flywheel could supply energy at this rate is

$$t = \frac{KE_{stored}}{\wp} = \frac{1.37 \times 10^8 \text{ J}}{7.46 \times 10^3 \text{ J/s}} = 1.84 \times 10^4 \text{ s} = \boxed{5.10 \text{ h}}.$$

**8.43** Using  $W_{net} = KE_f - KE_i = \frac{1}{2}I\omega_f^2 - 0$ , we have

$$\omega_f = \sqrt{\frac{2W_{net}}{I}} = \sqrt{\frac{2F \cdot s}{I}} = \sqrt{\frac{2(5.57 \text{ N})(0.800 \text{ m})}{4.00 \times 10^{-4} \text{ kg} \cdot \text{m}^2}} = \boxed{149 \text{ rad/s}}$$

8.44 Using conservation of mechanical energy,

$$\left(KE_{trans} + KE_{rot} + PE_{g}\right)_{f} = \left(KE_{trans} + KE_{rot} + PE_{g}\right)_{i}$$

or 
$$\frac{1}{2}Mv_t^2 + \frac{1}{2}I\omega^2 + 0 = 0 + 0 + Mg(L\sin\theta)$$
.

Since  $I = \frac{2}{5}MR^2$  for a solid sphere and  $v_t = R\omega$  when rolling without slipping, this

becomes  $\frac{1}{2}MR^2\omega^2 + \frac{1}{5}MR^2\omega^2 = Mg(L\sin\theta)$  and reduces to

$$\omega = \sqrt{\frac{10gL\sin\theta}{7R^2}} = \sqrt{\frac{10(9.8 \text{ m/s}^2)(6.0 \text{ m})\sin 37^\circ}{7(0.20 \text{ m})^2}} = \boxed{36 \text{ rad/s}}$$

8.45 The angular velocity of the Earth due to spinning on its axis is

$$\omega_{spin} = \left(\frac{2\pi \text{ rad}}{1 \text{ d}}\right) \left(\frac{1 \text{ d}}{86400 \text{ s}}\right) = 7.27 \times 10^{-5} \text{ rad/s},$$

and the angular velocity due to its orbital motion around the Sun is

$$\omega_{orb} = \left(\frac{2\pi \text{ rad}}{1 \text{ yr}}\right) \left(\frac{1 \text{ yr}}{365.24 \text{ d}}\right) \left(\frac{1 \text{ d}}{86400 \text{ s}}\right) = 1.99 \times 10^{-7} \text{ rad/s}.$$

(a) Treating the Earth as a uniform solid sphere,  $I = \frac{2}{5}M_E R_E^2$  and

$$L_{spin} = I\omega_{spin} = \frac{2}{5} (5.98 \times 10^{24} \text{ kg}) (6.38 \times 10^6 \text{ m})^2 (7.27 \times 10^{-5} \text{ rad/s}),$$
or  $L_{spin} = \boxed{7.08 \times 10^{33} \text{ J} \cdot \text{s}}.$ 

(b) Considering the orbital motion, we consider the Earth to be a point mass, so  $I = M_E r^2$ , where r is the radius of the orbit. Then, using Table C. 4 from Appendix C in the textbook,

$$L_{orb} = I\omega_{orb} = (5.98 \times 10^{24} \text{ kg})(1.496 \times 10^{11} \text{ m})^{2} (1.99 \times 10^{-7} \text{ rad/s}),$$
or  $L_{orb} = 2.66 \times 10^{40} \text{ J} \cdot \text{s}$ .

**8.46** Using conservation of angular momentum,  $L_{aphelion} = L_{perihelion}$ 

Thus,  $(mr_a^2)\omega_a = (mr_p^2)\omega_p$ . Since  $\omega = \frac{v}{r}$  at both aphelion and perihelion, this is equivalent to  $(mr_a^2)\frac{v_a}{r_a} = (mr_p^2)\frac{v_p}{r_p}$ , giving

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left(\frac{0.59 \text{ A.U.}}{35 \text{ A.U.}}\right) (54 \text{ km/s}) = \boxed{0.91 \text{ km/s}}.$$

**8.47** The initial moment of inertia of the system is

$$I_i = \sum m_i r_i^2 = 4 \lceil M (1.0 \text{ m})^2 \rceil = M (4.0 \text{ m}^2).$$

The moment of inertia of the system after the spokes are shortened is

$$I_f = \sum m_f r_f^2 = 4 \lceil M(0.50 \text{ m})^2 \rceil = M(1.0 \text{ m}^2).$$

From conservation of angular momentum,  $I_f \omega_f = I_i \omega_i$ ,

or 
$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = (4)(2.0 \text{ rev/s}) = \boxed{8.0 \text{ rev/s}}.$$

**8.48** From conservation of angular momentum,  $I_f \omega_f = I_i \omega_i$ .

Treating the child as a point mass, this becomes  $\omega_f = (I_i/I_f)\omega_i$ 

or 
$$\omega_f = \left[ \frac{250 \text{ kg} \cdot \text{m}^2}{250 \text{ kg} \cdot \text{m}^2 + (25.0 \text{ kg})(2.00 \text{ m})^2} \right] (10.0 \text{ rev/min}) = \boxed{7.14 \text{ rev/min}}$$

8.49 The moment of inertia of the cylinder before the putty arrives is

$$I_i = \frac{1}{2}MR^2 = \frac{1}{2}(10.0 \text{ kg})(1.00 \text{ m})^2 = 5.00 \text{ kg} \cdot \text{m}^2$$

After the putty sticks to the cylinder, the moment of inertia is

$$I_f = I_i + mr^2 = 5.00 \text{ kg} \cdot \text{m}^2 + (0.250 \text{ kg})(0.900 \text{ m})^2 = 5.20 \text{ kg} \cdot \text{m}^2$$

Conservation of angular momentum gives  $I_f \omega_f = I_i \omega_i$ ,

or 
$$\omega_f = \left(\frac{I_i}{I_f}\right) \omega_i = \left(\frac{5.00 \text{ kg} \cdot \text{m}^2}{5.20 \text{ kg} \cdot \text{m}^2}\right) (7.00 \text{ rad/s}) = \boxed{6.73 \text{ rad/s}}$$

**8.50** The total angular momentum of the system is

$$I_{total} = I_{masses} + I_{student} = 2(mr^2) + 3.0 \text{ kg} \cdot \text{m}^2$$

Initially, r = 1.0 m, and  $I_i = 2[(3.0 \text{ kg})(1.0 \text{ m})^2] + 3.0 \text{ kg} \cdot \text{m}^2 = 9.0 \text{ kg} \cdot \text{m}^2$ 

Afterward, r = 0.30 m, so

$$I_f = 2 \left[ (3.0 \text{ kg})(0.30 \text{ m})^2 \right] + 3.0 \text{ kg} \cdot \text{m}^2 = 3.5 \text{ kg} \cdot \text{m}^2$$

(a) From conservation of angular momentum,  $I_f \omega_f = I_i \omega_i$ , or

$$\omega_f = \left(\frac{I_i}{I_f}\right) \omega_i = \left(\frac{9.0 \text{ kg} \cdot \text{m}^2}{3.5 \text{ kg} \cdot \text{m}^2}\right) (0.75 \text{ rad/s}) = \boxed{1.9 \text{ rad/s}}$$

(b) 
$$KE_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(9.0 \text{ kg} \cdot \text{m}^2)(0.75 \text{ rad/s})^2 = \boxed{2.5 \text{ J}}$$

$$KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(3.5 \text{ kg} \cdot \text{m}^2)(1.9 \text{ rad/s})^2 = \boxed{6.4 \text{ J}}$$

**8.51** The initial angular velocity of the puck is  $\omega_i = \frac{v_i}{r_i} = \frac{0.800 \text{ m/s}}{0.400 \text{ m}} = 2.00 \frac{\text{rad}}{\text{s}}$ .

Since the tension in the string does not exert a torque about the axis of revolution, the angular momentum of the puck is conserved, or  $I_f \omega_f = I_i \omega_i$ .

Thus, 
$$\omega_f = \left(\frac{I_i}{I_f}\right) \omega_i = \left(\frac{mr_i^2}{mr_f^2}\right) \omega_i = \left(\frac{0.400 \text{ m}}{0.250 \text{ m}}\right)^2 (2.00 \text{ rad/s}) = 5.12 \text{ rad/s}$$

The net work done on the puck is

$$W_{net} = KE_f - KE_i = \frac{1}{2}I_f \omega_f^2 - \frac{1}{2}I_i \omega_i^2 = \frac{1}{2} \left[ \left( mr_f^2 \right) \omega_f^2 - \left( mr_i^2 \right) \omega_i^2 \right] = \frac{m}{2} \left[ r_f^2 \omega_f^2 - r_i^2 \omega_i^2 \right],$$

or 
$$W_{net} = \frac{(0.120 \text{ kg})}{2} [(0.250 \text{ m})^2 (5.12 \text{ rad/s})^2 - (0.400 \text{ m})^2 (2.00 \text{ rad/s})^2]$$

This yields 
$$W_{net} = 5.99 \times 10^{-2} \text{ J}$$

**8.52** The initial angular velocity of the system is

$$\omega_i = \left(0.20 \frac{\text{rev}}{\text{s}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 0.40\pi \text{ rad/s}$$

The total moment of inertia is given by

$$I = I_{man} + I_{cylinder} = mr^2 + \frac{1}{2}MR^2 = (80 \text{ kg})r^2 + \frac{1}{2}(25 \text{ kg})(2.0 \text{ m})^2.$$

Initially, the man is at r = 2.0 m from the axis, and this gives  $I_i = 3.7 \times 10^2$  kg·m<sup>2</sup>. At the end, when r = 1.0 m, the moment of inertia is  $I_f = 1.3 \times 10^2$  kg·m<sup>2</sup>.

(a) From conservation of angular momentum,  $I_f \omega_f = I_i \omega_i$ , or

$$\omega_f = \left(\frac{I_i}{I_f}\right) \omega_i = \left(\frac{3.7 \times 10^2 \text{ kg} \cdot \text{m}^2}{1.3 \times 10^2 \text{ kg} \cdot \text{m}^2}\right) (0.40\pi \text{ rad/s}) = 1.14\pi \text{ rad/s} = \boxed{3.6 \text{ rad/s}}$$

(b) The change in kinetic energy is  $\Delta KE = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_f\omega_i^2$ , or

$$\Delta KE = \frac{1}{2} \left( 1.3 \times 10^2 \text{ kg} \cdot \text{m}^2 \right) \left( 1.14 \pi \frac{\text{rad}}{\text{s}} \right)^2 - \frac{1}{2} \left( 3.7 \times 10^2 \text{ kg} \cdot \text{m}^2 \right) \left( 0.40 \pi \frac{\text{rad}}{\text{s}} \right)^2,$$

or  $\Delta KE = \boxed{5.4 \times 10^2 \text{ J}}$ . The difference is the work done by the man as he walks inward.

**8.53** (a) The table turns counterclockwise, opposite to the way the woman walks. Its angular momentum cancels that of the woman so the total angular momentum maintains a constant value of  $L_{total} = L_{woman} + L_{table} = 0$ .

Since the final angular momentum is  $L_{total} = I_w \omega_w + I_t \omega_t = 0$ , we have

$$\omega_t = -\left(\frac{I_w}{I_t}\right)\omega_w = -\left(\frac{m_w r^2}{I_t}\right)\left(\frac{v_w}{r}\right) = -\left(\frac{m_w r}{I_t}\right)v_w,$$

or 
$$\omega_t = -\left[\frac{(60.0 \text{ kg})(2.00 \text{ m})}{500 \text{ kg} \cdot \text{m}^2}\right] (-1.50 \text{ m/s}) = 0.360 \text{ rad/s}.$$

Hence 
$$\omega_{table} = 0.360 \text{ rad/s counterclockwise}$$

(b) 
$$W_{net} = \Delta KE = KE_f - 0 = \frac{1}{2}mv_w^2 + \frac{1}{2}I_t\omega_t^2$$
  

$$W_{net} = \frac{1}{2}(60.0 \text{ kg})(1.50 \text{ m/s})^2 + \frac{1}{2}(500 \text{ kg} \cdot \text{m}^2)(0.360 \text{ rad/s})^2 = \boxed{99.9 \text{ J}}$$

**8.54** For one of the crew,  $\Sigma F_c = m a_c$  becomes  $n = m \left( \frac{v_t^2}{r} \right) = m r \omega_i^2$ .

We require n = mg, so the initial angular velocity must be  $\omega_i = \sqrt{\frac{g}{r}}$ .

From conservation of angular momentum,  $I_f \omega_f = I_i \omega_i$ , or  $\omega_f = \left(\frac{I_i}{I_f}\right) \omega_i$ .

Thus, the angular velocity of the station during the union meeting is

$$\omega_f = \left(\frac{I_i}{I_f}\right) \sqrt{\frac{g}{r}} = \left[\frac{5.00 \times 10^8 \text{ kg} \cdot \text{m}^2 + 150(65.0 \text{ kg})(100 \text{ m})^2}{5.00 \times 10^8 \text{ kg} \cdot \text{m}^2 + 50(65.0 \text{ kg})(100 \text{ m})^2}\right] \sqrt{\frac{g}{r}} = 1.12 \sqrt{\frac{g}{r}}.$$

The centripetal acceleration experienced by the managers still on the rim is

$$a_c = r\omega_f^2 = r(1.12)^2 \frac{g}{r} = (1.12)^2 (9.80 \text{ m/s}^2) = \boxed{12.3 \text{ m/s}^2}$$

**8.55** (a) From conservation of angular momentum,  $I_f \omega_f = I_i \omega_i$ ,

so 
$$\omega_f = \left(\frac{I_i}{I_f}\right) \omega_i = \left(\frac{I_1}{I_1 + I_2}\right) \omega_o$$
.

(b) 
$$KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(I_1 + I_2)\left(\frac{I_1}{I_1 + I_2}\right)^2\omega_o^2 = \left(\frac{I_1}{I_1 + I_2}\right)\left[\frac{1}{2}I_1\omega_o^2\right] = \left(\frac{I_1}{I_1 + I_2}\right)KE_i$$

or  $\frac{KE_f}{KE_i} = \boxed{\frac{I_1}{I_1 + I_2}}$ . Since this is less than 1.0, kinetic energy was lost.

**8.56** (a) To determine *m*, we require that  $\Sigma F_v = 0$ , giving

$$+19.6 \text{ N} - (m + 0.700 \text{ kg} + 0.100 \text{ kg})g = 0$$

or 
$$m = \frac{19.6 \text{ N}}{9.80 \text{ m/s}^2} - 0.700 \text{ kg} - 0.100 \text{ kg} = \boxed{1.20 \text{kg}}.$$

(b) Require that  $\Sigma \tau = 0$  for a horizontal axis perpendicular to the meter stick and passing through the "zero end". This gives

$$-[(0.700 \text{ kg})g](0.0500 \text{ m}) + (19.6 \text{ N})(0.400 \text{ m}) - [(0.100 \text{ kg})g](0.500 \text{ m}) - [(1.20 \text{ kg})g]x = 0$$

or

$$x = \frac{1}{1.20 \text{ kg}} \left[ \frac{(19.6 \text{ N})(0.400 \text{ m})}{9.80 \text{ m/s}^2} - (0.700 \text{ kg})(0.0500 \text{ m}) - (0.100 \text{ kg})(0.500 \text{ m}) \right]$$

$$x = 0.596 \text{ m}$$

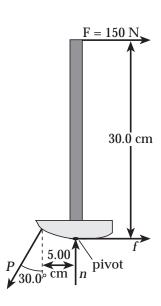
or the 1.20-kg mass should be hung at the 59.6 cm mark

8.57 
$$KE = \frac{1}{2}I\omega^2 = \frac{\left(I\omega\right)^2}{2I} = \boxed{\frac{L^2}{2I}}$$

8.58 (a) Choose an axis perpendicular to the page and passing through the indicated pivot. Then,  $\Sigma \tau = 0$  gives

$$+(P \cdot \cos 30.0^{\circ})(5.00 \text{ cm}) - (150 \text{ N})(30.0 \text{ cm}) = 0$$

so 
$$P = \frac{(150 \text{ N})(30.0 \text{ cm})}{(5.00 \text{ cm})\cos 30.0^{\circ}} = \boxed{1.04 \text{ kN}}$$



(b) 
$$\Sigma F_v = 0 \Rightarrow n - P \cos 30.0^\circ = 0$$
, giving

$$n = P\cos 30.0^{\circ} = (1.04 \times 10^{3} \text{ N})\cos 30.0^{\circ} = 900 \text{ N}$$

$$\Sigma F_x = 0 \Rightarrow f + F - P \sin 30.0^\circ = 0$$
, or

$$f = P \sin 30.0^{\circ} - F = (1.04 \times 10^{3} \text{ N}) \sin 30.0^{\circ} - 150 \text{ N} = 370 \text{ N}$$

The resultant force exerted on the hammer at the pivot is

$$R = \sqrt{f^2 + n^2} = \sqrt{(370)^2 + (900 \text{ N})^2} = 973 \text{ N}$$

at 
$$\theta = \tan^{-1} \left( \frac{n}{f} \right) = \tan^{-1} \left( \frac{900 \text{ N}}{370 \text{ N}} \right) = 67.7^{\circ}$$
,

or  $\mathbf{R} = 973 \text{ N}$  at  $67.7^{\circ}$  above the horizontal to the right.

8.59 Refer to the sketch of the flywheel and pulley.

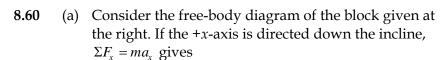
$$R = \frac{1.25 \text{ m}}{2} = 0.625 \text{m}$$
,  $r = 0.230 \text{ m}$ 

The rotation axis is through the center of the flywheel and perpendicular to the page.

$$\Sigma \tau = I\alpha$$
 gives  $(T_1 - T_2) \cdot r = \left[\frac{1}{2}MR^2\right]\alpha$ ,

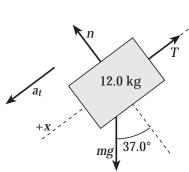
$$\Sigma \tau = I\alpha \text{ gives } \left(T_1 - T_2\right) \cdot r = \left\lfloor \frac{1}{2}MR^2 \right\rfloor \alpha,$$

or 
$$T_2 = T_1 - \left(\frac{MR^2}{2r}\right) \alpha = 135 \text{ N} - \frac{(80.0 \text{ kg})(0.625 \text{ m})^2}{2(0.230 \text{ m})} (1.67 \text{ rad/s}^2) = \boxed{21.5 \text{ N}}$$



$$mg \sin 37.0^{\circ} - T = m a_t$$
, or  $T = m(g \sin 37.0^{\circ} - a_t)$ 

$$T = (12.0 \text{ kg})[(9.80 \text{ m/s}^2)\sin 37.0^\circ - 2.00 \text{ m/s}^2]$$



 $T_1 = 135$  N

(b) Now, consider the free-body diagram of the pulley. Choose an axis perpendicular to the page and passing through the center of the pulley,

$$\Sigma \tau = I\alpha$$
 gives  $T \cdot r = I\left(\frac{a_t}{r}\right)$ , or

$$I = \frac{T \cdot r^2}{a_t} = \frac{(46.8 \text{ N})(0.100 \text{ m})^2}{2.00 \text{ m/s}^2} = \boxed{0.234 \text{ kg} \cdot \text{m}^2}$$

(c) 
$$\omega = \omega_i + \alpha t = 0 + \left(\frac{a_i}{r}\right)t = \left(\frac{2.00 \text{ m/s}^2}{0.100 \text{ m}}\right)(2.00 \text{ s}) = \boxed{40.0 \text{ rad/s}}$$

**8.61** If the ladder is on the verge of slipping,  $f = (f_s)_{max} = \mu_s n$  at both the floor and the wall.

From  $\Sigma F_x = 0$ , we find  $f_1 - n_2 = 0$ ,

or 
$$n_2 = \mu_s n_1 \tag{1}$$

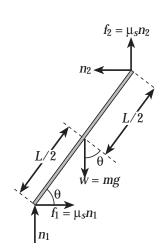
Also,  $\Sigma F_v = 0$  gives  $n_1 - w + \mu_s n_2 = 0$ .

Using equation (1), this becomes

$$n_1 - w + \mu_s \left( \mu_s n_1 \right) = 0$$

or 
$$n_1 = \frac{w}{1 + \mu_s^2} = \frac{w}{1.25} = 0.800 w$$
 (2)

Thus, equation (1) gives  $n_2 = 0.500(0.800 w) = 0.400 w$  (3)



Choose an axis perpendicular to the page and passing through the lower end of the ladder. Then,  $\Sigma \tau = 0$  yields

$$-w\left(\frac{L}{2}\cos\theta\right)+n_2\left(L\sin\theta\right)+f_2\left(L\cos\theta\right)=0.$$

Making the substitutions  $n_2 = 0.400w$  and  $f_2 = \mu_s n_2 = 0.200w$ , this becomes

$$-w\left(\frac{L}{2}\cos\theta\right) + (0.400w)(L\sin\theta) + (0.200w)(L\cos\theta) = 0$$

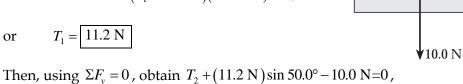
and reduces to 
$$\sin \theta = \left(\frac{0.500 - 0.200}{0.400}\right) \cos \theta$$

Hence,  $\tan \theta = 0.750$  and  $\theta = 36.9^{\circ}$ 

8.62 Use an axis perpendicular to the page and passing through the lower left corner of the frame. Then,  $\Sigma \tau = 0$  gives

$$-(10.0 \text{ N})(0.150 \text{ m}) - (T_1 \cos 50.0^\circ)(0.150 \text{ m}) + (T_1 \sin 50.0^\circ)(0.300 \text{ m}) = 0,$$

or 
$$T_1 = 11.2 \text{ N}$$



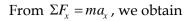
 $T_1 \cos 50.0^\circ$ 

\_0.150 m\_

or 
$$T_2 = 1.39 \text{ N}$$

Finally, 
$$\Sigma F_x = 0$$
 gives  $F - T_1 \cos 50.0^\circ = 0$ , or  $F = (11.2 \text{ N}) \cos 50.0^\circ = \boxed{7.23 \text{ N}}$ 

8.63 Consider the free-body diagram of an object rolling down the incline. If it rolls without slipping,  $a_i = r\alpha$ .



$$mg\sin\theta - f = ma_t$$
. (1)

Now, consider an axis perpendicular to the page and passing through the center of the object.

$$\Sigma \tau = I\alpha$$
 becomes  $f \cdot r = I\alpha = I\left(\frac{a_t}{r}\right)$ , or  $f = \left(\frac{I}{r^2}\right)a_t$ .

Substitute this result into equation (1) and simplify to obtain

$$a_t = \frac{g \sin \theta}{\left(1 + \frac{I}{mr^2}\right)}$$

as the linear acceleration of the center of gravity of the object.

For a sphere, 
$$I = \frac{2}{5}mr^2$$
, so  $a_{sphere} = \frac{g\sin\theta}{1.4}$ . For a disk,  $I = \frac{1}{2}mr^2$ , and

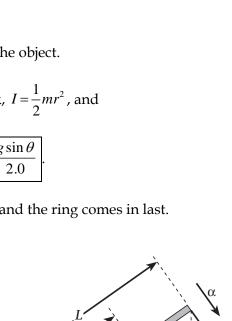
$$a_{disk} = \frac{g \sin \theta}{1.5}$$
. Finally, for a ring,  $I = mr^2$ , so  $a_{ring} = \frac{g \sin \theta}{2.0}$ .

Thus, we find  $a_{sphere} > a_{disk} > a_{ring}$  , so the sphere wins and the ring comes in last.

8.64 Choose an axis perpendicular to the page and passing through the lower end of the board. Then, when the support rod is removed,  $\Sigma \tau = I\alpha$  gives the initial angular acceleration of the board as

$$\Sigma \tau = I\alpha \implies (mg) \left(\frac{L}{2}\cos\theta\right) = \left(\frac{1}{3}mL^2\right)\alpha$$

or 
$$\alpha = \frac{3g\cos\theta}{2L}$$



w = mg

The tangential acceleration of the upper end of the board is  $a_t = L\alpha = \frac{3g\cos\theta}{2}$  and the vertical component of this is

$$a_{y} = a_{t} \cos \theta = \frac{3 g \cos^{2} \theta}{2}$$

(a) If  $a_y > g$  (the vertical acceleration of the freely-falling ball), the board will get ahead of the ball. Thus, the criterion is

$$\frac{3g\cos^2\theta}{2} > g \text{ or } \cos^2\theta > \frac{2}{3}, \text{ which yields } \theta < \cos^{-1}\left(\sqrt{\frac{2}{3}}\right) = \boxed{35.3^{\circ}}.$$

(b) For the ball to land in the cup, the cup must strike the table directly below the initial position of the ball. Thus, when starting from the limiting angle, we must have

$$r_c = L\cos\theta = \frac{L\cos^2\theta}{\cos\theta} = \boxed{\frac{2}{3}\left(\frac{L}{\cos\theta}\right)}.$$

- 8.65 Let  $m_p$  be the mass of the pulley,  $m_1$  be the mass of the sliding block, and  $m_2$  be the mass of the counterweight.
  - (a) The moment of inertia of the pulley is  $I = \frac{1}{2} m_p R_p^2$  and its angular velocity at any time is  $\omega = \frac{v}{R_p}$ , where v is the linear speed of the other masses. The friction force retarding the sliding block is  $f_k = \mu_k n = \mu_k (m_1 g)$ .

Choose  $PE_g = 0$  at the level of the counterweight when the sliding mass reaches the second photogate. Then, from the work-kinetic energy theorem,

$$W_{nc} = \left(KE_{trans} + KE_{rot} + PE_{g}\right)_{f} - \left(KE_{trans} + KE_{rot} + PE_{g}\right)_{i}$$

$$-f_k \cdot s = \frac{1}{2} (m_1 + m_2) v_f^2 + \frac{1}{2} \left( \frac{1}{2} m_p R_p^2 \right) \left( \frac{v_f^2}{R_p^2} \right) + 0$$
$$- \frac{1}{2} (m_1 + m_2) v_i^2 - \frac{1}{2} \left( \frac{1}{2} m_p R_p^2 \right) \left( \frac{v_i^2}{R_p^2} \right) - m_2 g s,$$

or 
$$\frac{1}{2} \left( m_1 + m_2 + \frac{1}{2} m_p \right) v_f^2 = \frac{1}{2} \left( m_1 + m_2 + \frac{1}{2} m_p \right) v_i^2 + m_2 g s - \mu_k (m_1 g) \cdot s$$
.

This reduces to 
$$v_f = \sqrt{v_i^2 + \frac{2(m_2 - \mu_k m_1)gs}{m_1 + m_2 + \frac{1}{2}m_p}}$$
,

and yields

$$v_f = \sqrt{\left(0.820 \, \frac{\text{m}}{\text{s}}\right)^2 + \frac{2(0.208 \, \text{kg})(9.80 \, \text{m/s}^2)(0.700 \, \text{m})}{1.45 \, \text{kg}}} = \boxed{1.63 \, \text{m/s}}.$$

(b) 
$$\omega_f = \frac{v_f}{R_p} = \frac{1.63 \text{ m/s}}{0.0300 \text{ m}} = \boxed{54.2 \text{ rad/s}}$$

**8.66** (a) The center of each wheel moves forward at v = 3.35 m/s and each wheel also turns at angular speed  $\omega = v/R$ . The total kinetic energy of the bicycle is  $KE = KE_{trans} + KE_{rot}$ , or

$$\begin{split} \textit{KE} &= \frac{1}{2} \Big( m_{\textit{frame}} + 2 m_{\textit{wheel}} \Big) v^2 + 2 \left( \frac{1}{2} I_{\textit{wheel}} \omega^2 \right) \\ &= \frac{1}{2} \Big( m_{\textit{frame}} + 2 m_{\textit{wheel}} \Big) v^2 + \frac{1}{2} \Big( m_{\textit{wheel}} R^2 \Big) \left( \frac{v^2}{R^2} \right) \end{split}$$

This yields

$$KE = \frac{1}{2} \left( m_{frame} + 3 m_{wheel} \right) v^2$$
$$= \frac{1}{2} \left[ 8.44 \text{ kg} + 3 \left( 0.820 \text{ kg} \right) \right] \left( 3.35 \text{ m/s} \right)^2 = \boxed{61.2 \text{ J}}$$

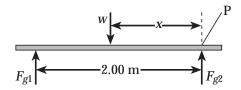
(b) In this case, the top of each roller moves forward at v = 0.335 m/s. The center of each roller moves forward at v/2 = 0.168 m/s and each roller also turns at angular speed  $\omega = \frac{v/2}{R} = \frac{v}{2R}$ . The total kinetic energy is  $KE = KE_{trans} + KE_{rot}$ , or

$$\begin{split} KE &= \frac{1}{2} m_{stone} v^2 + 2 \left[ \frac{1}{2} m_{tree} \left( \frac{v}{2} \right)^2 \right] + 2 \left( \frac{1}{2} I_{tree} \omega^2 \right) \\ &= \left( \frac{1}{2} m_{stone} + \frac{1}{4} m_{tree} \right) v^2 + \frac{1}{2} m_{tree} R^2 \left( \frac{v^2}{4R^2} \right) \end{split}$$

This gives 
$$KE = \frac{1}{2} \left( m_{stone} + \frac{3}{4} m_{tree} \right) v^2$$
, or

$$KE = \frac{1}{2} \left[ 844 \text{ kg} + \frac{3}{4} (82.0 \text{ kg}) \right] (0.335 \text{ m/s})^2 = \boxed{50.8 \text{ J}}$$

8.67 We neglect the weight of the board and assume that the woman's feet are directly above the point of support by the rightmost scale. Then, the free-body diagram for the situation is as shown at the right.



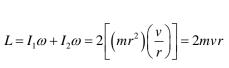
From 
$$\Sigma F_y = 0$$
, we have  $F_{g1} + F_{g2} - w = 0$ , or  $w = 380 \text{ N} + 320 \text{ N} = 700 \text{ N}$ .

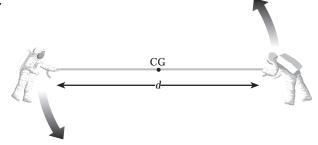
Choose an axis perpendicular to the page and passing through point P.

Then 
$$\Sigma \tau = 0$$
 gives  $w \cdot x - F_{g1}(2.00 \text{ m}) = 0$ , or

$$x = \frac{F_{g1}(2.00 \text{ m})}{w} = \frac{(380 \text{ N})(2.00 \text{ m})}{700 \text{ N}} = \boxed{1.09 \text{ m}}$$

**8.68** We treat each astronaut as a point mass, *m*, moving at speed *v* in a circle of radius *r*. Then the total angular momentum is





- (a)  $L_i = 2mv_i r_i = 2(75.0 \text{ kg})(5.00 \text{ m/s})(5.00 \text{ m})$  $L_i = 3.75 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}$
- (b)  $KE_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = 2\left(\frac{1}{2}mv_i^2\right)$  $KE_i = (75.0 \text{ kg})(5.00 \text{ m/s})^2 = 1.88 \times 10^3 \text{ J} = \boxed{1.88 \text{ kJ}}$
- (c) Angular momentum is conserved:  $L_f = L_i = 3.75 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}$

(d) 
$$v_f = \frac{L_f}{2(mr_f)} = \frac{3.75 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}}{2(75.0 \text{ kg})(2.50 \text{ m})} = \boxed{10.0 \text{ m/s}}$$

(e) 
$$KE_f = 2\left(\frac{1}{2}mv_f^2\right) = (75.0 \text{ kg})(10.0 \text{ m/s})^2 = \boxed{7.50 \text{ kJ}}$$

(f) 
$$W_{net} = KE_f - KE_i = \boxed{5.62 \text{ kJ}}$$

**8.69** (a) 
$$L_i = 2 \left[ M v \left( \frac{d}{2} \right) \right] = M v d$$

(b) 
$$KE_i = 2\left(\frac{1}{2}Mv_i^2\right) = Mv^2$$

(c) 
$$L_f = L_i = Mvd$$

(d) 
$$v_f = \frac{L_f}{2(Mr_f)} = \frac{Mvd}{2M(d/4)} = \boxed{2v}$$

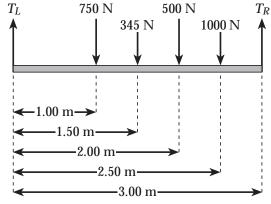
(e) 
$$KE_f = 2\left(\frac{1}{2}Mv_f^2\right) = M(2v)^2 = 4Mv^2$$

(f) 
$$W_{net} = KE_f - KE_i = 3Mv^2$$

8.70 Choose an axis that is perpendicular to the page and passing through the left end of the scaffold. Then  $\Sigma \tau = 0$  gives

$$-(750 \text{ N})(1.00 \text{ m}) - (345 \text{ N})(1.50 \text{ m})$$
$$-(500 \text{ N})(2.00 \text{ m}) - (1000 \text{ N})(2.50 \text{ m})$$
$$+ T_R(3.00 \text{ m}) = 0,$$

or 
$$T_R = 1.59 \times 10^3 \text{ N} = 1.59 \text{ kN}$$
.



Then, 
$$\Sigma F_y = 0 \Rightarrow T_L = (750 + 345 + 500 + 1000) \text{ N} - 1.59 \times 10^3 \text{ N} = \boxed{1.01 \text{ kN}}$$

**8.71** First, we define the following symbols:

 $I_P$  = moment of inertia due to mass of people on the equator

 $I_E$  = moment of inertia of the Earth alone (without people)

 $\omega$  = angular velocity of the Earth (due to rotation on its axis)

 $T = \frac{2\pi}{\omega}$  = rotational period of the Earth (length of the day)

R = radius of the Earth

The initial angular momentum of the system (before people start running) is

$$L_i = I_P \omega_i + I_E \omega_i = (I_P + I_E) \omega_i$$

When the Earth has angular speed  $\omega$ , the tangential speed of a point on the equator is  $v_t = R\omega$ . Thus, when the people run eastward along the equator at speed v relative to the surface of the Earth, their tangential speed is  $v_p = v_t + v = R\omega + v$  and their angular

speed is 
$$\omega_P = \frac{v_p}{R} = \omega + \frac{v}{R}$$
.

The angular momentum of the system after the people begin to run is

$$L_f = I_P \omega_P + I_E \omega = I_P \left( \omega + \frac{v}{R} \right) + I_E \omega = \left( I_P + I_E \right) \omega + \frac{I_P v}{R} .$$

Since no external torques have acted on the system, angular momentum is conserved

 $(L_f = L_i)$ , giving  $(I_P + I_E)\omega + \frac{I_P v}{R} = (I_P + I_E)\omega_i$ . Thus, the final angular velocity of the Earth is  $\omega = \omega_i - \frac{I_P v}{(I_P + I_E)R} = \omega_i (1 - x)$ , where  $x = \frac{I_P v}{(I_P + I_E)R \omega_i}$ .

The new length of the day is  $T=\frac{2\pi}{\omega}=\frac{2\pi}{\omega_i(1-x)}=\frac{T_i}{1-x}\approx T_i(1+x)$ , so the increase in the length of the day is  $\Delta T=T-T_i\approx T_ix=T_i\bigg[\frac{I_P v}{(I_P+I_E)R\,\omega_i}\bigg]$ . Since  $\omega_i=\frac{2\pi}{T_i}$ , this may be written as  $\Delta T\approx\frac{T_i^2\,I_P\,v}{2\pi\,(I_P+I_E)R}$ .

To obtain a numeric answer, we compute

$$I_P = m_p R^2 = \left[ \left( 5.5 \times 10^9 \right) \left( 70 \text{ kg} \right) \right] \left( 6.38 \times 10^6 \text{ m} \right)^2 = 1.57 \times 10^{25} \text{ kg} \cdot \text{m}^2$$

and

$$I_E = \frac{2}{5} m_E R^2 = \frac{2}{5} (5.98 \times 10^{24} \text{ kg}) (6.38 \times 10^6 \text{ m})^2 = 9.74 \times 10^{37} \text{ kg} \cdot \text{m}^2.$$

Thus,

$$\Delta T \approx \frac{\left(8.64 \times 10^4 \text{ s}\right)^2 \left(1.57 \times 10^{25} \text{ kg} \cdot \text{m}^2\right) \left(2.5 \text{ m/s}\right)}{2\pi \left[\left(1.57 \times 10^{25} + 9.74 \times 10^{37}\right) \text{ kg} \cdot \text{m}^2\right] \left(6.38 \times 10^6 \text{ m}\right)} = \boxed{7.5 \times 10^{-11} \text{ s}}$$

8.72 Choose  $PE_g = 0$  at the level of the base of the ramp. Then, conservation of mechanical energy gives

$$\left(KE_{trans} + KE_{rot} + PE_{g}\right)_{f} = \left(KE_{trans} + KE_{rot} + PE_{g}\right)_{i},$$

$$0 + 0 + (mg)(s\sin\theta) = \frac{1}{2}mv_i^2 + \frac{1}{2}(mR^2)\left(\frac{v_i}{R}\right)^2 + 0,$$

or 
$$s = \frac{v_i^2}{g\sin\theta} = \frac{R^2\omega_i^2}{g\sin\theta} = \frac{(3.0 \text{ m})^2(3.0 \text{ rad/s})^2}{(9.80 \text{ m/s}^2)\sin 20^\circ} = \boxed{24 \text{ m}}$$

8.73 Choose an axis perpendicular to the page and passing through the center of the cylinder. Then, applying  $\Sigma \tau = I\alpha$  to the cylinder gives

$$(2T) \cdot R = \left(\frac{1}{2}MR^2\right)\alpha = \left(\frac{1}{2}MR^2\right)\left(\frac{a_t}{R}\right), \text{ or } T = \frac{1}{4}Ma_t.$$
 (1)

Now apply  $\Sigma F_y = ma_y$  to the falling objects to obtain

$$(2m)g - 2T = (2m)a_t$$
, or  $a_t = g - \frac{T}{m}$ . (2)

(a) Substituting equation (2) into (1) yields

$$T = \frac{Mg}{4} - \left(\frac{M}{4m}\right)T$$
, which reduces to  $T = \boxed{\frac{Mmg}{M+4m}}$ 

(b) From equation (2) above,

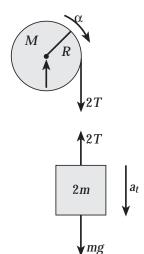
$$a_{t} = g - \frac{1}{m} \left( \frac{Mmg}{M + 4m} \right) = g - \frac{Mg}{M + 4m} = \boxed{\frac{4mg}{M + 4m}}$$

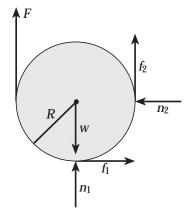
8.74 Slipping occurs simultaneously at both the bottom and side contact points. Just before slipping occurs, both static friction forces must have their maximum values. When the cylinder is about to slip,  $f_1 = \mu_s n_1 = 0.5 n_1$  and  $f_2 = \mu_s n_2 = 0.5 n_2$ . Choose an axis perpendicular to the page and passing through the center of the cylinder.

Then, 
$$\Sigma \tau = 0 \implies f_1 \cdot R + f_2 \cdot R - F \cdot R = 0$$

or 
$$F = f_1 + f_2$$
. (1)

From 
$$\Sigma F_x = 0$$
,  $f_1 = n_2 = \frac{f_2}{\mu_s} = 2 f_2$ . (2)





Combining equation (2) with equation (1) gives

$$F = 2f_2 + f_2 = 3f_2$$
, or  $f_2 = \frac{F}{3}$ . Then equation (2) yields  $f_1 = \frac{2F}{3}$ .

From 
$$\Sigma F_y = 0$$
,  $w = F + f_2 + n_1 = F + f_2 + \frac{f_1}{\mu_s} = F + f_2 + 2f_1 = F + \frac{F}{3} + 2\left(\frac{2F}{3}\right)$ ,

or 
$$w = \frac{8F}{3}$$
. Solving for the applied force,  $F = \boxed{\frac{3w}{8}}$ .

## **Answers to Even Numbered Conceptual Questions**

- 2. If the bar is, say, seven feet above the ground, a high jumper has to lift his center of gravity approximately to a height of seven feet in order to clear the bar. A tall person already has his center of gravity higher than that of a short person. Thus, the taller athlete has to raise his center of gravity through a smaller distance.
- **4.** The lever arm of a particular force is found with respect to some reference point. Thus, an origin for calculating torques must be specified. However, for an object in equilibrium, the calculation of the torque is independent of the location of the origin.
- 6. Drawing her legs up against her chest reduces her moment of inertia. Because angular momentum is conserved, a decrease in moment of inertia is accompanied by an increase in angular velocity. Thus, she rotates faster. To come out of the flip, she must increase her moment of inertia. This can be accomplished by extending her arms and/or legs.
- **8.** The object of the game in walking a tightrope is to keep the center of gravity of the walker directly above the rope. If the body becomes slightly overbalanced such as to slip off to the right, a small movement of the pole to the left will help to restore balance.
- **12.** After the head crosses the bar, the jumper should arch his back so the head and legs are lower than the midsection of the body. In this position, the center of gravity may pass under the bar while the midsection of the body is still above the bar. As the feet approach the bar, the legs should be straightened to avoid hitting the bar.
- **14.** (a) Consider two people pushing with equal magnitude forces in opposite directions and at opposite ends of a table. The net force will be zero, yet the net torque is not zero.
  - (b) Consider a falling body. The net force acting on it is its weight, yet the net torque about the center of gravity is zero.
- **16.** As the cat falls, angular momentum must be conserved. Thus, if the upper half of the body twists in one direction, something must get an equal angular momentum in the opposite direction. Rotating the lower half of the body in the opposite direction satisfies the law of conservation of angular momentum.
- **18.** The body will remain in the position it is placed, making no attempt to return to the original position nor to move farther from the original position.

#### **Answers to Even Numbered Problems**

- 2. 0.642 N m counterclockwise
- **4.** 0, -20 N m, -34 N m, -39 N m
- **6.** -168 N m
- 8.  $x_{cg} = 6.69 \times 10^{-3} \text{ nm}, y_{cg} = 0$
- **10.** 756 N
- **12.** (-1.5 m, -1.5 m)
- **14.** (a) 5.7°

- (b) 91 cm
- **16.**  $F_H = 711 \text{ N}, F_J = 966 \text{ N}$
- **18.** 567 N (left end), 333 N (right end)
- **20.** (b) T = 343 N, H = 171 N, V = 683 N
  - (c) 5.14 m

**22.** (a) 392 N

- (b) H = 339 N (to right), V = 0
- **24.** (a) 267 N (to right), 1.30 kN (upward) (b)  $\mu_s = 0.324$
- **26.** T = 1.47 kN, H = 1.33 kN (to right), V = 2.58 kN (upward)
- 28. 2.8 m
- **30.** 149 N m, 66.0 N m, 215 N m
- 32. 0.30
- **34.** (a) 872 N
- (b) 1.40 kN
- **36.** (a)  $5.35 \text{ m/s}^2 \text{ downward}$  (b) 42.8 m

(c)  $8.91 \text{ rad/s}^2$ 

- 30.3 rev/s 38.
- $0.150 \text{ kg} \cdot \text{m}^2$ **40.**
- **42.** (a)  $1.37 \times 10^8$  J
- (b) 5.10 h

36 rad/s 44.

- **46.** 0.91 km/s
- **48.** 7.14 rev/min
- **50.** (a) 1.9 rad/s
- (b)  $KE_i = 2.5 \text{ J}, KE_f = 6.4 \text{ J}$

- **52.** (a) 3.6 rad/s
- (b)  $5.4 \times 10^2$  J, work done by the man as he walks inward
- **54.**  $12.3 \text{ m/s}^2$
- **56.** (a) 1.20 kg

(b) at the 59.6 cm mark

**58.** (a) 1.04 kN

(b) 973 N at 67.7° above the horizontal to the right

**60.** (a) 46.8 N

- (b)  $0.234 \text{ kg} \cdot \text{m}^2$
- (c) 40.0 rad/s

- **62.**  $T_1 = 11.2 \text{ N}, T_2 = 1.39 \text{ N}, F = 7.23 \text{ N}$
- **66.** (a) 61.2 J

- (b) 50.8 J
- **68.** (a)  $3.75 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}$
- (b) 1.88 kJ

(c)  $3.75 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}$ 

(d) 10.0 m/s

(e) 7.50 kJ

(f) 5.62 kJ

- **70.**  $T_{right} = 1.59 \text{ kN}, T_{left} = 1.01 \text{ kN}$
- **72.** 24 m
- **74.**  $\frac{3}{8}$  w