CHAPTER 13

Quick Quizzes

1. (d).

2. (c).

3. (b).

4. (d).

5. (b) and (c). An accelerating elevator is equivalent to a gravitational field. Thus, if the elevator is accelerating upward, this is equivalent to an increased effective gravitational field magnitude g, and the period will decrease. Similarly, if the elevator is accelerating downward, the effective value of g is reduced and the period increases. If the elevator moves with constant velocity, the period of the pendulum is the same as that in the stationary elevator.

6. (a). The clock will run *slow*. With a longer length, the period of the pendulum will increase. Thus, it will take longer to execute each swing, so that each second according to the clock will take longer than an actual second.

7. (b). Greater. The value of g on the Moon is about one-sixth the value of g on Earth, so the period of the pendulum on the moon will be greater than the period on Earth.

Problem Solutions

13.1 (a) The force exerted on the block by the spring is

$$F_s = -kx = -(160 \text{ N/m})(-0.15 \text{ m}) = +24 \text{ N},$$

or $F_s =$ $\boxed{\text{24 N directed toward equilibrium position}}$

(b) From Newton's second law, the acceleration is

$$a = \frac{F_s}{m} = \frac{+24 \text{ N}}{0.40 \text{ kg}} = +60 \frac{\text{m}}{\text{s}^2} = \boxed{60 \frac{\text{m}}{\text{s}^2} \text{ toward equilibrium position}}$$

13.2 (a) The spring constant is $k = \dfrac{|F_s|}{x} = \dfrac{mg}{x} = \dfrac{50 \text{ N}}{5.0 \times 10^{-2} \text{ m}} = 1.0 \times 10^3 \text{ N/m}$.

$$F = |F_s| = kx = (1.0 \times 10^3 \text{ N/m})(0.11 \text{ m}) = \boxed{1.1 \times 10^2 \text{ N}}$$

(b) The graph will be a $\boxed{\text{straight line passing through the origin}}$ with a slope equal to $k = 1.0 \times 10^3 \text{ N/m}$.

13.3 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m before coming to rest momentarily. It will then repeat this motion over and over again with a regular period.

(b) From $\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$, with $v_{yi} = 0$, the time required for the ball to reach the ground is $t = \sqrt{\dfrac{2(\Delta y)}{a_y}} = \sqrt{\dfrac{2(-4.00 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.904 \text{ s}$. This is one-half of the time for a complete cycle of the motion. Thus, the period is $T = \boxed{1.81 \text{ s}}$.

(c) $\boxed{\text{No}}$. The net force acting on the mass is a constant given by $F = -mg$ (except when it is contact with the ground). This is not in the form of Hooke's law.

13.4 The force the hand exerts on the handle is equal in magnitude and opposite in direction to the sum of the forces exerted on the handle by the springs, or

$$\mathbf{F}_h = -(\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4) = -4\mathbf{F}_1 .$$

Thus, $F_h = 4F_1 = 4(kx)$ and if $x_{max} = (0.800 - 0.435)\text{ m} = 0.365\text{ m}$, then

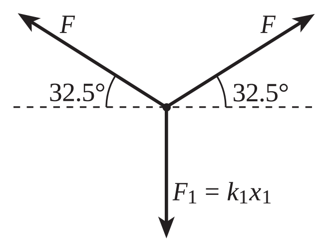
$$F_h)_{max} = 4(65.0\text{ N/m})(0.365\text{ m}) = \boxed{94.9\text{ N}}$$

13.5 Since object A is in equilibrium, the net force acting on it must be zero, giving

$$F_1 + F_2 = k_1 x + k_2 x = 80.0\text{ N}$$

Hence, $x = \dfrac{80.0\text{ N}}{k_1 + k_2} = \dfrac{80.0\text{ N}}{40.0\text{ N/cm} + 25.0\text{ N/cm}} = \boxed{1.23\text{ cm}}$

13.6 (a) The sketch at the right is a free-body diagram of the upper end of the spring shown in Figure P13.6. This point is in equilibrium, so $\Sigma F_y = 2F\sin 32.5° - k_1 x_1 = 0$.



If $F = 210\text{ N}$ and $k_1 = 5.60\times10^4\text{ N/m}$, the elongation of the spring is

$$x_1 = \frac{2F\sin 32.5°}{k_1} = \frac{2(210\text{ N})\sin 32.5°}{5.60\times10^4\text{ N/m}} = 4.03\times10^{-3}\text{ m} = \boxed{4.03\text{ mm}}$$

(b) The rope is now replaced by a pair of identical springs, lying along the original line of the rope. If the force constant of each of these springs is k and the elongation of each is $x = 2x_1$, then $\Sigma F_y = 0$ gives $2(kx)\sin 32.5° = k_1 x_1$, or

$$k = \frac{k_1 x_1}{(2\sin 32.5°)x} = \frac{k_1 x_1}{(2\sin 32.5°)(2x_1)}$$

$$= \frac{5.60\times10^4\text{ N/m}}{4\sin 32.5°} = \boxed{2.61\times10^4\text{ N/m}}$$

13.7 (a) The spring constant of each band is

$$k = \frac{F_s}{x} = \frac{15\text{ N}}{1.0\times10^{-2}\text{ m}} = 1.5\times10^{3}\text{ N/m}$$

Thus, when both bands are stretched 0.20 m, the total elastic potential energy is

$$PE_s = 2\left(\frac{1}{2}kx^2\right) = \left(1.5\times10^{3}\text{ N/m}\right)\left(0.20\text{ m}\right)^2 = \boxed{60\text{ J}}$$

(b) Conservation of mechanical energy gives $\left(KE+PE_s\right)_f = \left(KE+PE_s\right)_i$, or

$$\frac{1}{2}mv^2 + 0 = 0 + 60\text{ J},\text{ so } v = \sqrt{\frac{2(60\text{ J})}{50\times10^{-3}\text{ kg}}} = \boxed{49\text{ m/s}}$$

13.8 (a) $k = \frac{F_{max}}{x_{max}} = \frac{230\text{ N}}{0.400\text{ m}} = \boxed{575\text{ N/m}}$

(b) $\textit{work done} = PE_s = \frac{1}{2}kx^2 = \frac{1}{2}\left(575\text{ N/m}\right)\left(0.400\right)^2 = \boxed{46.0\text{ J}}$

13.9 From conservation of mechanical energy,

$$\left(KE+PE_g+PE_s\right)_f = \left(KE+PE_g+PE_s\right)_i \text{ or } 0+mgh_f+0 = 0+0+\frac{1}{2}kx_i^2,$$

giving

$$k = \frac{2mgh_f}{x_i^2} = \frac{2\left(0.100\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(0.600\text{ m}\right)}{\left(2.00\times10^{-2}\text{ m}\right)^2} = \boxed{2.94\times10^{3}\text{ N/m}}$$

13.10 Conservation of mechanical energy, $\left(KE+PE_g+PE_s\right)_f = \left(KE+PE_g+PE_s\right)_i$,

gives $\frac{1}{2}mv_i^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_f^2$,

or $$v_i = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{5.00\times10^{6}\text{ N/m}}{1000\text{ kg}}}\left(3.16\times10^{-2}\text{ m}\right) = \boxed{2.23\text{ m/s}}$$

13.11 At $x = A$, $v = 0$ and conservation of energy gives

$$E = KE + PE_s = 0 + \frac{1}{2}kA^2 \text{ or } A^2 = \frac{2E}{k}$$

(a) At $x = A/2$, the elastic potential energy is

$$PE_s = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{k}{8}A^2 = \frac{k}{8}\left(\frac{2E}{k}\right) = \boxed{\frac{E}{4}}$$

From the energy conservation equation, the kinetic energy is then

$$KE = E - PE_s = E - \frac{E}{4} = \boxed{\frac{3E}{4}}$$

(b) When $KE = PE_s$, conservation of energy yields $E = KE + PE_s = 2PE_s$ or $PE_s = E/2$. Since we also have $PE_s = kx^2/2$, this yields

$$x = \sqrt{\frac{2PE_s}{k}} = \sqrt{\frac{2(E/2)}{k}} = \sqrt{\frac{E}{k}} = \sqrt{\frac{(kA^2/2)}{k}} = \boxed{\frac{A}{\sqrt{2}}}$$

13.12 (a) From the work-kinetic energy theorem,

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i,$$

or $F \cdot x_f = \frac{1}{2}mv_f^2 + 0 + \frac{1}{2}kx_f^2$

This yields

$$v_f = \sqrt{\frac{2F \cdot x - kx_f^2}{m}}$$

$$= \sqrt{\frac{2(20.0 \text{ N})(0.300 \text{ m}) - 19.6 \text{ N/m}(0.300 \text{ m})^2}{1.50 \text{ kg}}} = \boxed{2.61 \text{ m/s}}$$

(b) The work-kinetic theorem now contains one more nonzero term, giving

$$v_f = \sqrt{\frac{2\left(F-\mu_k mg\right)\cdot x - kx_f^2}{m}}$$

$$= \sqrt{\frac{2\left[20.0\text{ N}-(0.200)(1.50\text{ kg})\left(9.80\text{ m/s}^2\right)\right](0.300\text{ m})-19.6\text{ N/m}(0.300\text{ m})^2}{1.50\text{ kg}}}$$

$$v_f = \boxed{2.38\text{ m/s}}$$

13.13 An unknown quantity of mechanical energy is converted into internal energy during the collision. Thus, we apply conservation of momentum from just before to just after the collision and obtain $mv_i + M(0) = (M+m)V$, or the speed of the block and embedded bullet just after collision is

$$V = \left(\frac{m}{M+m}\right)v_i = \left(\frac{10.0\times10^{-3}\text{ kg}}{2.01\text{ kg}}\right)(300\text{ m/s}) = 1.49\text{ m/s}.$$

Now, we use conservation of mechanical energy from just after collision until the block comes to rest. This gives $0+\frac{1}{2}kx_f^2 = \frac{1}{2}(M+m)V^2$, or

$$x_f = V\sqrt{\frac{M+m}{k}} = (1.49\text{ m/s})\sqrt{\frac{2.01\text{ kg}}{19.6\text{ N/m}}} = \boxed{0.478\text{ m}}$$

13.14 (a) In the absence of friction, $\left(KE+PE_g+PE_s\right)_f = \left(KE+PE_g+PE_s\right)_i$ gives

$$\frac{1}{2}mv_f^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_i^2,$$

or $v_f = x_i\sqrt{\frac{k}{m}} = (0.30\text{ cm})\sqrt{\frac{2000\text{ N/m}}{1.5\text{ kg}}} = \boxed{11\text{ cm/s}}$

(b) When friction is present, $W_{nc} = (KE + PE_g + PE_s)_f - (KE + PE_g + PE_s)_i$ gives

$$-f \cdot x_i = \left(\frac{1}{2}mv_f^2 + 0 + 0\right) - \left(0 + 0 + \frac{1}{2}kx_i^2\right),$$

or

$$v_f = \sqrt{\frac{kx_i^2 - 2f \cdot x_i}{m}}$$
$$= \sqrt{\frac{(2000\ \text{N/m})(3.0\times10^{-3}\ \text{m})^2 - 2(2.0\ \text{N})(3.0\times10^{-3}\ \text{m})}{1.5\ \text{kg}}}$$

$$v_f = 0.063\ \text{m/s} = \boxed{6.3\ \text{cm/s}}$$

(c) If $v_f = 0$ at $x = 0$, then $W_{nc} = (KE + PE_g + PE_s)_f - (KE + PE_g + PE_s)_i$

becomes $-f \cdot x_i = (0) - \left(0 + 0 + \frac{1}{2}kx_i^2\right)$,

or $f = \frac{kx_i}{2} = \frac{(2000\ \text{N/m})(3.0\times10^{-3}\ \text{m})}{2} = \boxed{3.0\ \text{N}}$

13.15 From conservation of mechanical energy,

$$(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i,$$

we have $\frac{1}{2}mv^2 + 0 + \frac{1}{2}kx^2 = 0 + 0 + \frac{1}{2}kA^2$, or $v = \sqrt{\frac{k}{m}(A^2 - x^2)}$

(a) The speed is a maximum at the equilibrium position, $x = 0$.

$$v_{max} = \sqrt{\frac{k}{m}A^2} = \sqrt{\frac{(19.6\ \text{N/m})}{(0.40\ \text{kg})}(0.040\ \text{m})^2} = \boxed{0.28\ \text{m/s}}$$

(b) When $x = -0.015$ m,

$$v = \sqrt{\frac{(19.6\ \text{N/m})}{(0.40\ \text{kg})}\left[(0.040\ \text{m})^2 - (-0.015\ \text{m})^2\right]} = \boxed{0.26\ \text{m/s}}$$

(c) When $x = +0.015\text{ m}$,

$$v = \sqrt{\frac{(19.6\text{ N/m})}{(0.40\text{ kg})}\left[(0.040\text{ m})^2 - (+0.015\text{ m})^2\right]} = \boxed{0.26\text{ m/s}}$$

(d) If $v = \frac{1}{2}v_{max}$, then $\sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \frac{1}{2}\sqrt{\frac{k}{m}A^2}$

This gives $A^2 - x^2 = \frac{A^2}{4}$, or $x = A\frac{\sqrt{3}}{2} = (4.0\text{ cm})\frac{\sqrt{3}}{2} = \boxed{3.5\text{ cm}}$

13.16 (a) $KE = 0$ at $x = A$, so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$, or the total energy is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(250\text{ N/m})(0.035\text{ m})^2 = \boxed{0.15\text{ J}}$$

(b) The maximum speed occurs at the equilibrium position where $PE_s = 0$. Thus, $E = \frac{1}{2}mv_{max}^2$, or

$$v_{max} = \sqrt{\frac{2E}{m}} = A\sqrt{\frac{k}{m}} = (0.035\text{ m})\sqrt{\frac{250\text{ N/m}}{0.50\text{ kg}}} = \boxed{0.78\text{ m/s}}$$

(c) The acceleration is $a = \frac{\Sigma F}{m} = \frac{-kx}{m}$. Thus, $a = a_{max}$ at $x = -x_{max} = -A$.

$$a_{max} = \frac{-k(-A)}{m} = \left(\frac{k}{m}\right)A = \left(\frac{250\text{ N/m}}{0.50\text{ kg}}\right)(0.035\text{ m}) = \boxed{18\text{ m/s}^2}$$

13.17 The maximum speed occurs at the equilibrium position and is

$$v_{max} = \sqrt{\frac{k}{m}}A \text{ . Thus, } m = \frac{kA^2}{v_{max}^2} = \frac{(16.0\text{ N/m})(0.200\text{ m})^2}{(0.400\text{ m/s})^2} = 4.00\text{ kg}\text{ , and}$$

$$F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{39.2\text{ N}}$$

13.18 $$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\left(\frac{10.0\text{ N/m}}{50.0\times 10^{-3}\text{ kg}}\right)\left[(0.250\text{ m})^2 - (0.125\text{ m})^2\right]} = \boxed{3.06\text{ m/s}}$$

13.19 (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the motion of the "bump" projected on a plane perpendicular to the tire.

(b) Since the car is moving with speed $v = 3.00\ \text{m/s}$, and its radius is 0.300 m, the angular velocity of the tire is

$$\omega = \frac{v}{r} = \frac{3.00\ \text{m/s}}{0.300\ \text{m}} = 10.0\ \text{rad/s}$$

Therefore, the period of the motion is

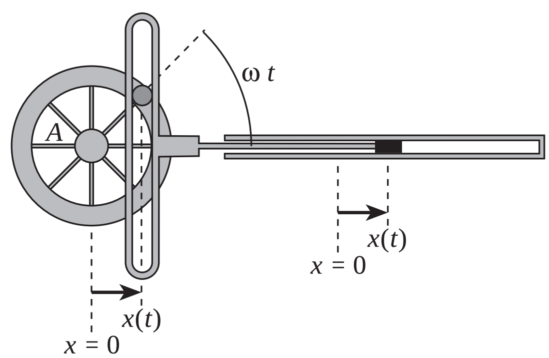
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10.0\ \text{rad/s}} = \boxed{0.628\ \text{s}}$$

13.20 (a) $v = \frac{2\pi r}{T} = \frac{2\pi(0.200\ \text{m})}{2.00\ \text{s}} = \boxed{0.628\ \text{m/s}}$

(b) $f = \frac{1}{T} = \frac{1}{2.00\ \text{s}} = \boxed{0.500\ \text{Hz}}$

(c) $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.00\ \text{s}} = \boxed{3.14\ \text{rad/s}}$

13.21 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is $x = A\cos\theta = A\cos\omega t$ where A is the distance from the center of the wheel to the crank pin. This is of the correct form to describe simple harmonic motion. Hence, one must conclude that the motion is indeed simple harmonic.



13.22 (a) From, $T = 2\pi\sqrt{\frac{m}{k}}$, we have

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.200\ \text{kg})}{(0.250\ \text{s})^2} = \boxed{126\ \text{N/m}}$$

(b) At $x = A$, the object is momentarily at rest and

$$E = KE + PE_s = 0 + \frac{1}{2}kA^2$$

Thus, the amplitude is $A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(2.00 \text{ J})}{126 \text{ N/m}}} = 0.178 \text{ m} = \boxed{17.8 \text{ cm}}$

13.23 The spring constant is found from

$$k = \frac{F_s}{x} = \frac{mg}{x} = \frac{(0.010 \text{ kg})(9.80 \text{ m/s}^2)}{3.9 \times 10^{-2} \text{ m}} = 2.5 \text{ N/m} .$$

When the object attached to the spring has mass $m = 25$ g, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025 \text{ kg}}{2.5 \text{ N/m}}} = \boxed{0.63 \text{ s}}$$

13.24 The springs compress 0.80 cm when supporting an additional load of $m = 320$ kg. Thus, the spring constant is

$$k = \frac{mg}{x} = \frac{(320 \text{ kg})(9.80 \text{ m/s}^2)}{0.80 \times 10^{-2} \text{ m}} = 3.9 \times 10^5 \text{ N/m}$$

When the empty car, $M = 2.0 \times 10^3$ kg, oscillates on the springs, the frequency will be

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{3.9 \times 10^5 \text{ N/m}}{2.0 \times 10^3 \text{ kg}}} = \boxed{2.2 \text{ Hz}}$$

13.25 The spring constant of the collagen is found from $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$ to be

$$k = 4\pi^2 f^2 m = 4\pi^2 \left(36.0 \text{ s}^{-1}\right)^2 \left(5.50\times10^{-3} \text{ kg}\right) = 281 \text{ N/m}$$

From the definition of Young's modulus, $Stress = Y \cdot Strain$, or $\frac{F}{A} = Y\left(\frac{\Delta L}{L}\right)$. This may be written as $F = \left(AY/L\right)\Delta L$, which is in the form of Hooke's law with the spring constant given by $k = AY/L$. Thus, Young's modulus for collagen is given by

$$Y = \frac{kL}{A} = \frac{\left(281 \text{ N/m}\right)\left(3.50\times10^{-2} \text{ m}\right)}{\pi\left(1.00\times10^{-3} \text{ m}\right)^2} = \boxed{3.14\times10^{6} \text{ Pa}}$$

13.26 (a) At $t = 0$, $x = \left(0.30 \text{ m}\right)\cos\left(0\right) = \boxed{0.30 \text{ m}}$, and at $t = 0.60$ s,

$$x = \left(0.30 \text{ m}\right)\cos\left[\left(\frac{\pi}{3} \text{ rad/s}\right)\left(0.60 \text{ s}\right)\right] = \left(0.30 \text{ m}\right)\cos\left(0.20\pi \text{ rad}\right) = \boxed{0.24 \text{ m}}$$

(b) $A = x_{max} = \left(0.30 \text{ m}\right)\left(1\right) = \boxed{0.30 \text{ m}}$

(c) $x = \left(0.30 \text{ m}\right)\cos\left(\frac{\pi}{3}t\right)$ is of the form $x = A\cos\left(\omega t\right)$ with an angular frequency of $\omega = \frac{\pi}{3}$ rad/s. Thus, $f = \frac{\omega}{2\pi} = \frac{\pi/3}{2\pi} = \boxed{\frac{1}{6} \text{ Hz}}$

(d) The period is $T = \frac{1}{f} = \boxed{6.0 \text{ s}}$

13.27 (a) At $t = 3.50$ s,

$$F = -kx = -\left(5.00 \ \frac{\text{N}}{\text{m}}\right)\left(3.00 \text{ m}\right)\cos\left[\left(1.58 \ \frac{\text{rad}}{\text{s}}\right)\left(3.50 \text{ s}\right)\right] = -11.0 \text{ N},$$

or $F = \boxed{\text{11.0 N directed to the left}}$

(b) The angular frequency is $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\ \text{N/m}}{2.00\ \text{kg}}} = 1.58\ \text{rad/s}$ and the period of oscillation is $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.58\ \text{rad/s}} = 3.97\ \text{s}$. Hence the number of oscillations made in 3.50 s is $N = \frac{\Delta t}{T} = \frac{3.50\ \text{s}}{3.97\ \text{s}} = \boxed{0.881}$

13.28 (a) $k = \frac{F}{x} = \frac{7.50\ \text{N}}{3.00 \times 10^{-2}\ \text{m}} = \boxed{250\ \text{N/m}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250\ \text{N/m}}{0.500\ \text{kg}}} = \boxed{22.4\ \text{rad/s}}$, $f = \frac{\omega}{2\pi} = \frac{22.4\ \text{rad/s}}{2\pi} = \boxed{3.56\ \text{Hz}}$,

and $T = \frac{1}{f} = \frac{1}{3.56\ \text{Hz}} = \boxed{0.281\ \text{s}}$

(c) At $t = 0$, $v = 0$ and $x = 5.00 \times 10^{-2}\ \text{m}$, so the total energy of the oscillator is

$$E = KE + PE_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= 0 + \frac{1}{2}(250\ \text{N/m})(5.00 \times 10^{-2}\ \text{m})^2 = \boxed{0.313\ \text{J}}$$

(d) When $x = A$, $v = 0$ so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$.

Thus, $A = \sqrt{\frac{2E}{k}} = \boxed{5.00\ \text{cm}}$

(e) At $x = 0$, $KE = \frac{1}{2}mv_{max}^2 = E$,

or $v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.313\ \text{J})}{0.500\ \text{kg}}} = \boxed{1.12\ \text{m/s}}$

$$a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \frac{(250\ \text{N/m})(5.00 \times 10^{-2}\ \text{m})}{0.500\ \text{kg}} = \boxed{25.0\ \text{m/s}^2}$$

(f) At $t = 0.500\ \text{s}$,

$$x = A\cos(\omega t) = (5.00\ \text{cm})\cos[(22.4\ \text{rad/s})(0.500\ \text{s})] = \boxed{0.919\ \text{cm}}$$

13.29 From Equation 13.6, $v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \pm\sqrt{\omega^2\left(A^2 - x^2\right)}$

Hence, $v = \pm\omega\sqrt{A^2 - A^2\cos^2(\omega t)} = \pm\omega A\sqrt{1-\cos^2(\omega t)} = \boxed{\pm\omega A\sin(\omega t)}$

From Equation 13.2, $a = -\frac{k}{m}x = -\omega^2\left[A\cos(\omega t)\right] = \boxed{-\omega^2 A\cos(\omega t)}$

13.30 (a) The height of the tower is almost the same as the length of the pendulum. From $T = 2\pi\sqrt{L/g}$, we obtain

$$L = \frac{gT^2}{4\pi^2} = \frac{\left(9.80\ \text{m/s}^2\right)\left(15.5\ \text{s}\right)^2}{4\pi^2} = \boxed{59.6\ \text{m}}$$

(b) On the Moon, where $g = 1.67\ \text{m/s}^2$, the period will be

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{59.6\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{37.5\ \text{s}}$$

13.31 The period in Tokyo is $T_T = 2\pi\sqrt{\frac{L_T}{g_T}}$

and the period in Cambridge is $T_C = 2\pi\sqrt{\frac{L_C}{g_C}}$.

We know that $T_T = T_C = 2.000\ \text{s}$, from which, we see that

$$\frac{L_T}{g_T} = \frac{L_C}{g_C}, \text{ or } \frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.9942}{0.9927} = \boxed{1.0015}$$

13.32 (a) The lower temperature will cause the pendulum to contract. The shorter length will produce a smaller period, so the clock will run faster or $\boxed{\text{gain time}}$.

(b) The period of the pendulum is $T_0 = 2\pi\sqrt{\frac{L_0}{g}}$ at 20°C,

and at –5.0°C it is $T = 2\pi\sqrt{\frac{L}{g}}$. The ratio of these periods is $\frac{T_0}{T} = \sqrt{\frac{L_0}{L}}$.

From Chapter 10, the length at –5.0°C is $L = L_0 + \alpha_{\text{Al}} L_0 (\Delta T)$, so

$$\frac{L_0}{L} = \frac{1}{1+\alpha_{\text{Al}}(\Delta T)} = \frac{1}{1+\left[24\times10^{-6}\ (°\text{C})^{-1}\right][-5.0°\text{C}-20°\text{C}]} = \frac{1}{0.9994} = 1.0006$$

This gives $\frac{T_0}{T} = \sqrt{\frac{L_0}{L}} = \sqrt{1.0006} = 1.0003$. Thus in one hour (3600 s), the chilled pendulum will gain $(1.0003-1)(3600\text{ s}) = \boxed{1.1\text{ s}}$.

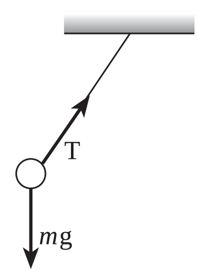
13.33 (a) The period of the pendulum is $T = 2\pi\sqrt{L/g}$. Thus, on the Moon where the acceleration of gravity is smaller, the period will be longer and the clock will run $\boxed{\text{slow}}$.

(b) The ratio of the pendulum's period on the Moon to that on Earth is

$$\frac{T_{Moon}}{T_{Earth}} = \frac{2\pi\sqrt{L/g_{Moon}}}{2\pi\sqrt{L/g_{Earth}}} = \sqrt{\frac{g_{Earth}}{g_{Moon}}} = \sqrt{\frac{9.80}{1.63}} = 2.45$$

Hence, the pendulum of the clock on Earth makes 2.45 "ticks" while the clock on the Moon is making 1.00 "tick". After the Earth clock has ticked off 24.0 h and again reads 12:00 midnight, the Moon clock will have ticked off $\frac{24.0\text{ h}}{2.45} = 9.79\text{ h}$ and will read $\boxed{9:47\text{ AM}}$.

13.34 The apparent acceleration of gravity is the vector sum of the actual acceleration of gravity and the negative of the elevator's acceleration. To see this, consider an object that is suspended by a string in the elevator and that appears to be at rest to the elevator passengers. These passengers believe the tension in the string is the negative of the object's weight, or $\mathbf{T} = -m\mathbf{g}_{app}$ where $\mathbf{g}_{app}$ is the apparent acceleration of gravity in the elevator.



An observer located outside the elevator applies Newton's second law to this object by writing $\Sigma\mathbf{F} = \mathbf{T} + m\mathbf{g} = m\mathbf{a}_e$ where $\mathbf{a}_e$ is the acceleration of the elevator and all its contents. Thus, $\mathbf{T} = m\mathbf{a}_e - m\mathbf{g} = -m\mathbf{g}_{app}$, which gives $\mathbf{g}_{app} = \mathbf{g} - \mathbf{a}_e$.

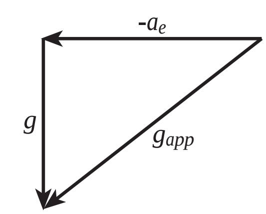
(a) When $\mathbf{a}_e = 5.00\ \text{m/s}^2$ upward, then $-\mathbf{a}_e = 5.00\ \text{m/s}^2$ downward. Thus, $\mathbf{g}_{app} = (9.80 + 5.00)\ \text{m/s}^2 = 14.8\ \text{m/s}^2$ downward and the period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{14.8\ \text{m/s}^2}} = \boxed{3.65\ \text{s}}$$

(b) If $\mathbf{a}_e = 5.00\ \text{m/s}^2$ downward, then $-\mathbf{a}_e = 5.00\ \text{m/s}^2$ upward and $\mathbf{g}_{app} = (9.80 - 5.00)\ \text{m/s}^2 = 4.80\ \text{m/s}^2$ downward. In this case, the period is given by

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{4.80\ \text{m/s}^2}} = \boxed{6.41\ \text{s}}$$

(c) If $\mathbf{a}_e = 5.00\ \text{m/s}^2$ horizontally, the vector sum $\mathbf{g}_{app} = \mathbf{g} - \mathbf{a}_e$ is as shown in the sketch at the right. The magnitude is



$$g_{app} = \sqrt{(5.00\ \text{m/s}^2)^2 + (9.80\ \text{m/s}^2)^2} = 11.0\ \text{m/s}^2,$$

and the period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{11.0\ \text{m/s}^2}} = \boxed{4.24\ \text{s}}$$

13.35 (a) From $T = 2\pi\sqrt{L/g}$, the length of a pendulum with period T is $L = \frac{gT^2}{4\pi^2}$.

On Earth, with $T = 1.0\ \text{s}$, $L = \frac{(9.80\ \text{m/s}^2)(1.0\ \text{s})^2}{4\pi^2} = 0.25\ \text{m} = \boxed{25\ \text{cm}}$

If $T = 1.0\ \text{s}$ on Mars, $L = \frac{(3.7\ \text{m/s}^2)(1.0\ \text{s})^2}{4\pi^2} = 0.094\ \text{m} = \boxed{9.4\ \text{cm}}$

(b) The period of a mass on a spring is $T = 2\pi\sqrt{m/k}$, which is independent of the local acceleration of gravity. Thus, the same mass will work on Earth and on Mars. This mass is

$$m = \frac{kT^2}{4\pi^2} = \frac{(10\ \text{N/m})(1.0\ \text{s})^2}{4\pi^2} = \boxed{0.25\ \text{kg}}$$

13.36 The length of a pendulum with period T is $L = \frac{gT^2}{4\pi^2}$. Thus, the ratio of the lengths of the two swings is $\frac{L_1}{L_2} = \left(\frac{T_1}{T_2}\right)^2$. If the elapsed time was Δt, the periods are $T_1 = \frac{\Delta t}{10}$ and $T_2 = \frac{\Delta t}{10.5}$, which gives the ratio of lengths as $\frac{L_1}{L_2} = (1.05)^2$. The percentage difference in the lengths is

$$\%\ \text{diff.} = \left[\frac{L_1 - L_2}{(L_1 + L_2)/2}\right] \cdot 100\% = \left[\frac{(L_1/L_2) - 1}{(L_1/L_2) + 1}\right] \cdot 200\%$$

$$= \left[\frac{(1.05)^2 - 1}{(1.05)^2 + 1}\right] \cdot 200\% = \boxed{9.8\%}$$

13.37 (a) The amplitude, A, is the maximum displacement from equilibrium. Thus, from Figure P13.37, $A = \frac{1}{2}(18.0\ \text{cm}) = \boxed{9.00\ \text{cm}}$

(b) The wavelength, λ, is the distance between successive crests (or successive troughs). From Figure P13.37, $\lambda = 2(10.0\ \text{cm}) = \boxed{20.0\ \text{cm}}$

(c) The period is $T = \frac{1}{f} = \frac{1}{25.0\ \text{Hz}} = 4.00 \times 10^{-2}\ \text{s} = \boxed{40.0\ \text{ms}}$

(d) The speed of the wave is $v = \lambda f = (0.200\ \text{m})(25.0\ \text{Hz}) = \boxed{5.00\ \text{m/s}}$

13.38 From $v = \lambda f$, the wavelength (and size of smallest detectable insect) is

$$\lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{60.0 \times 10^3\ \text{Hz}} = 5.67 \times 10^{-3}\ \text{m} = \boxed{5.67\ \text{mm}}$$

13.39 (a) $T = \frac{1}{f} = \frac{1}{88.0 \times 10^6 \text{ Hz}} = 1.14 \times 10^{-8} \text{ s} = \boxed{11.4 \text{ ns}}$

(b) $\lambda = \frac{v}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{88.0 \times 10^6 \text{ Hz}} = \boxed{3.41 \text{ m}}$

13.40 The longest emitted wavelength is $\lambda_{long} = \frac{v}{f_{low}} = \frac{343 \text{ m/s}}{28 \text{ Hz}} = 12 \text{ m}$,

and the shortest is $\lambda_{short} = \frac{v}{f_{high}} = \frac{343 \text{ m/s}}{4200 \text{ Hz}} = 0.082 \text{ m} = 8.2 \text{ cm}$

Thus, the range of wavelengths produced is $\boxed{8.2 \text{ cm to } 12 \text{ m}}$

13.41 The speed of the wave is $v = \frac{\Delta x}{\Delta t} = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$,

and the frequency is $f = \frac{40.0 \text{ vib}}{30.0 \text{ s}} = 1.33 \text{ Hz}$

Thus, $\lambda = \frac{v}{f} = \frac{42.5 \text{ cm/s}}{1.33 \text{ Hz}} = \boxed{31.9 \text{ cm}}$

13.42 (a) When the boat is at rest in the water, the speed of the wave relative to the boat is the same as the speed of the wave relative to the water, $v = 4.0 \text{ m/s}$. The frequency detected in this case is

$$f = \frac{v}{\lambda} = \frac{4.0 \text{ m/s}}{20 \text{ m}} = \boxed{0.20 \text{ Hz}}$$

(b) Taking westward as positive, $\mathbf{v}_{boat,water} = \mathbf{v}_{boat,wave} + \mathbf{v}_{wave,water}$ gives

$$\mathbf{v}_{boat,wave} = \mathbf{v}_{boat,water} - \mathbf{v}_{wave,water} = +1.0 \text{ m/s} - (-4.0 \text{ m/s}) = +5.0 \text{ m/s}$$

Thus, $f = \frac{v_{boat,wave}}{\lambda} = \frac{5.0 \text{ m/s}}{20 \text{ m}} = \boxed{0.25 \text{ Hz}}$

13.43 The down and back distance is $4.00 \text{ m} + 4.00 \text{ m} = 8.00 \text{ m}$.

The speed is then $v = \frac{d_{total}}{t} = \frac{4(8.00 \text{ m})}{0.800 \text{ s}} = 40.0 \text{ m/s} = \sqrt{F/\mu}$

Now, $\mu = \frac{m}{L} = \frac{0.200 \text{ kg}}{4.00 \text{ m}} = 5.00 \times 10^{-2} \text{ kg/m}$, so

$$F = \mu v^2 = (5.00 \times 10^{-2} \text{ kg/m})(40.0 \text{ m/s})^2 = \boxed{80.0 \text{ N}}$$

13.44 The speed of the wave is $v = \frac{\Delta x}{\Delta t} = \frac{20.0 \text{ m}}{0.800 \text{ s}} = 25.0 \text{ m/s}$, and the mass per unit length of the rope is $\mu = \frac{m}{L} = 0.350 \text{ kg/m}$. Thus, from $v = \sqrt{F/\mu}$, we obtain

$$F = v^2\mu = (25.0 \text{ m/s})^2(0.350 \text{ kg/m}) = \boxed{219 \text{ N}}$$

13.45 (a) The mass per unit length is $\mu = \frac{m}{L} = \frac{0.0600 \text{ kg}}{5.00 \text{ m}} = 0.0120 \text{ kg/m}$

From $v = \sqrt{F/\mu}$, the required tension in the string is

$$F = v^2\mu = (50.0 \text{ m/s})^2(0.0120 \text{ kg/m}) = \boxed{30.0 \text{ N}},$$

(b) $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{8.00 \text{ N}}{0.0120 \text{ kg/m}}} = \boxed{25.8 \text{ m/s}}$

13.46 The mass per unit length of the wire is

$$\mu = \frac{m}{L} = \frac{4.00 \times 10^{-3} \text{ kg}}{1.60 \text{ m}} = 2.50 \times 10^{-3} \text{ kg/m},$$

and the speed of the pulse is $v = \frac{L}{\Delta t} = \frac{1.60 \text{ m}}{0.0361 \text{ s}} = 44.3 \text{ m/s}$.

Thus, the tension in the wire is

$$F = v^2 \mu = (44.3 \text{ m/s})^2 \left(2.50 \times 10^{-3} \text{ kg/m}\right) = 4.91 \text{ N}$$

But, the tension in the wire is the weight of a 3.00-kg object on the Moon. Hence, the local acceleration of gravity is

$$g = \frac{F}{m} = \frac{4.91 \text{ N}}{3.00 \text{ kg}} = \boxed{1.64 \text{ m/s}^2}$$

13.47 The period of the pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$, so the length of the string is

$$L = \frac{gT^2}{4\pi^2} = \frac{\left(9.80 \text{ m/s}^2\right)(2.00 \text{ s})^2}{4\pi^2} = 0.993 \text{ m}$$

Then mass per unit length of the string is then

$$\mu = \frac{m}{L} = \frac{0.0600 \text{ kg}}{0.993 \text{ m}} = 0.0604 \frac{\text{kg}}{\text{m}}$$

When the pendulum is vertical and stationary, the tension in the string is

$$F = M_{ball} g = (5.00 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 49.0 \text{ N},$$

and the speed of transverse waves in it is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{49.0 \text{ N}}{0.0604 \text{ kg/m}}} = \boxed{28.5 \text{ m/s}}$$

13.48 If $\mu_1 = m_1/L$ is the mass per unit length for the first string, then $\mu_2 = m_2/L = m_1/2L = \mu_1/2$ is that of the second string. Thus, with $F_2 = F_1 = F$, the speed of waves in the second string is

$$v_2 = \sqrt{\frac{F}{\mu_2}} = \sqrt{\frac{2F}{\mu_1}} = \sqrt{2}\left(\sqrt{\frac{F}{\mu_1}}\right) = \sqrt{2}\,v_1 = \sqrt{2}(5.00 \text{ m/s}) = \boxed{7.07 \text{ m/s}}$$

13.49 (a) The tension in the string is $F = mg = (3.0 \text{ kg})(9.80 \text{ m/s}^2) = 29 \text{ N}$. Then, from $v = \sqrt{F/\mu}$, the mass per unit length is

$$\mu = \frac{F}{v^2} = \frac{29 \text{ N}}{(24 \text{ m/s})^2} = \boxed{0.051 \text{ kg/m}}$$

(b) When $m = 2.00 \text{ kg}$, the tension is

$$F = mg = (2.0 \text{ kg})(9.80 \text{ m/s}^2) = 20 \text{ N}$$

and the speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20 \text{ N}}{0.051 \text{ kg/m}}} = \boxed{20 \text{ m/s}}$$

13.50 If the tension in the wire is F, the tensile stress is $Stress = F/A$, so the speed of transverse waves in the wire may be written as

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{A \cdot Stress}{m/L}} = \sqrt{\frac{Stress}{m/(A \cdot L)}}$$

But, $A \cdot L = V = \text{volume}$, so $m/(A \cdot L) = \rho = \text{density}$. Thus, $v = \sqrt{\dfrac{Stress}{\rho}}$.

When the stress is at its maximum, the speed of waves in the wire is

$$v_{max} = \sqrt{\frac{(Stress)_{max}}{\rho}} = \sqrt{\frac{2.70 \times 10^9 \text{ Pa}}{7.86 \times 10^3 \text{ kg/m}^3}} = \boxed{586 \text{ m/s}}$$

13.51 From $v=\sqrt{F/\mu}$, the tension in the string is $F=v^2\mu$. Thus, the ratio of the new tension to the original is

$$\frac{F_2}{F_1}=\frac{v_2^2}{v_1^2}\text{, giving } F_2=\left(\frac{v_2}{v_1}\right)^2 F_1=\left(\frac{30.0\ \text{m/s}}{20.0\ \text{m/s}}\right)^2(6.00\ \text{N})=\boxed{13.5\ \text{N}}$$

13.52 (a) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is $\boxed{\text{zero}}$.

(b) If the end is free there is no inversion on reflection. When they meet the amplitude is $A'=2A=2(0.15\ \text{m})=\boxed{0.30\ \text{m}}$.

13.53 (a) $\boxed{\text{Constructive interference}}$ produces the maximum amplitude

$$A'_{max}=A_1+A_2=\boxed{0.50\ \text{m}}$$

(b) $\boxed{\text{Destructive interference}}$ produces the minimum amplitude

$$A'_{\min}=A_1-A_2=\boxed{0.10\ \text{m}}$$

13.54 We are given that $x=A\cos(\omega t)=(0.25\ \text{m})\cos(0.4\pi t)$.

(a) By inspection, the amplitude is seen to be $A=\boxed{0.25\ \text{m}}$

(b) The angular frequency is $\omega=0.4\pi\ \text{rad/s}$. But $\omega=\sqrt{k/m}$, so the spring constant is

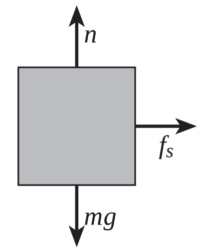
$$k=m\omega^2=(0.30\ \text{kg})(0.4\pi\ \text{rad/s})^2=\boxed{0.47\ \text{N/m}}$$

(c) At $t=0.30\ \text{s}$, $x=(0.25\ \text{m})\cos\left[(0.4\pi\ \text{rad/ s})(0.30\ \text{s})\right]=\boxed{0.23\ \text{m}}$

(d) From conservation of mechanical energy, the speed at displacement x is given by $v=\omega\sqrt{A^2-x^2}$. Thus, at $t=0.30\ \text{s}$, when $x=0.23\ \text{m}$, the speed is

$$v=(0.4\pi\ \text{rad/s})\sqrt{(0.25\ \text{m})^2-(0.23\ \text{m})^2}=\boxed{0.12\ \text{m/s}}$$

13.55 The maximum acceleration of the oscillating system is



$$a_{max} = \omega^2 A = (2\pi f)^2 A$$

The friction force exerted between the two blocks must be capable of accelerating block *B* at this rate. When block *B* is on the verge of slipping, $f_s = (f_s)_{max} = \mu_s n = \mu_s mg = ma_{max}$ and we must have $a_{max} = (2\pi f)^2 A = \mu_s g$.

Thus, $A = \dfrac{\mu_s g}{(2\pi f)^2} = \dfrac{(0.600)(9.80\ \text{m/s}^2)}{[2\pi(1.50\ \text{Hz})]^2} = 6.62 \times 10^{-2}\ \text{m} = \boxed{6.62\ \text{cm}}$

13.56 Since the spring is "light", we neglect any small amount of energy lost in the collision with the spring, and apply conservation of mechanical energy from when the block first starts until it comes to rest again. This gives

$$(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i, \text{ or } 0 + 0 + \frac{1}{2}kx_{max}^2 = 0 + 0 + mgh_i$$

Thus, $x_{max} = \sqrt{\dfrac{2mgh_i}{k}} = \sqrt{\dfrac{2(0.500\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})}{20.0\ \text{N/m}}} = \boxed{0.990\ \text{m}}$

13.57 Choosing $PE_g = 0$ at the initial height of the block, conservation of mechanical energy gives $(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$, or

$$\frac{1}{2}mv^2 + mg(-x) + \frac{1}{2}kx^2 = 0,$$

where *v* is the speed of the block after falling distance *x*.

(a) When $v = 0$, the non-zero solution to the energy equation from above gives

$$\frac{1}{2}kx_{max}^2 = mgx_{max},$$

or $k = \dfrac{2mg}{x_{max}} = \dfrac{2(3.00\ \text{kg})(9.80\ \text{m/s}^2)}{0.100\ \text{m}} = \boxed{588\ \text{N/m}}$

(b) When $x = 5.00\text{ m} = 0.0500\text{ m}$, the energy equation gives

$$v = \sqrt{2gx - \frac{kx^2}{m}}\text{, or}$$

$$v = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(0.0500\text{ m}\right) - \frac{\left(588\text{ N/m}\right)\left(0.0500\text{ m}\right)^2}{3.00\text{ kg}}} = \boxed{0.700\text{ m/s}}$$

13.58 (a) We apply conservation of mechanical energy from <u>*just after*</u> the collision until the block comes to rest.

$$\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i \text{ gives } 0 + \frac{1}{2}kx_f^2 = \frac{1}{2}MV^2 + 0$$

or the speed of the block just after the collision is

$$V = \sqrt{\frac{kx_f^2}{M}} = \sqrt{\frac{\left(900\text{ N/m}\right)\left(0.0500\text{ m}\right)^2}{1.00\text{ kg}}} = 1.50\text{ m/s}$$

Now, we apply conservation of momentum from just before impact to immediately after the collision. This gives

$$m\left(v_{bullet}\right)_i + 0 = m\left(v_{bullet}\right)_f + MV\text{ ,}$$

or $\left(v_{bullet}\right)_f = \left(v_{bullet}\right)_i - \left(\dfrac{M}{m}\right)V$

$$= 400\text{ m/s} - \left(\frac{1.00\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)\left(1.5\text{ m/s}\right) = \boxed{100\text{ m/s}}$$

(b) The mechanical energy converted into internal energy during the collision is

$$\Delta E = KE_i - \Sigma KE_f = \frac{1}{2}m\left(v_{bullet}\right)_i^2 - \frac{1}{2}m\left(v_{bullet}\right)_f^2 - \frac{1}{2}MV^2\text{ ,}$$

or

$$\Delta E = \frac{1}{2}\left(5.00\times10^{-3}\text{ kg}\right)\left[\left(400\text{ m/s}\right)^2 - \left(100\text{ m/s}\right)^2\right] - \frac{1}{2}\left(1.00\text{ kg}\right)\left(1.50\text{ m/s}\right)^2$$

$$\Delta E = \boxed{374\text{ J}}$$

13.59 Choose $PE_g = 0$ when the blocks start from rest. Then, using conservation of mechanical energy from when the blocks are released until the spring returns to its unstretched length gives $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$, or

$$\frac{1}{2}(m_1 + m_2)v_f^2 + \left(m_1 g x \sin 40^\circ - m_2 g x\right) + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$\frac{1}{2}\left[(25+30)\text{ kg}\right]v_f^2 + (25\text{ kg})\left(9.80\text{ m/s}^2\right)\left[(0.200\text{ m})\sin 40^\circ\right]$$
$$-(30\text{ kg})\left(9.80\text{ m/s}^2\right)(0.200\text{ m}) = \frac{1}{2}(200\text{ N/m})(0.200\text{ m})^2$$

yielding $v_f = \boxed{1.1\text{ m/s}}$

13.60 The total time is the sum of the two times.

In each wire $t = \dfrac{L}{v} = \dfrac{L}{\sqrt{F/\mu}} = L\sqrt{\dfrac{\mu}{F}}$,

where $\mu = \dfrac{m}{L} = \dfrac{\rho\left[\left(\pi d^2/4\right)L\right]}{L} = \dfrac{\rho\pi d^2}{4}$. Thus, $t = L\sqrt{\dfrac{\rho\pi d^2}{4F}}$

For copper, $t = (20.0\text{ m})\sqrt{\dfrac{\left(8920\text{ kg/m}^3\right)\pi\left(1.00\times 10^{-3}\text{ m}\right)^2}{4(150\text{ N})}} = 0.137\text{ s}$

For steel, $t = (30.0\text{ m})\sqrt{\dfrac{\left(7860\text{ kg/m}^3\right)\pi\left(1.00\times 10^{-3}\text{ m}\right)^2}{4(150\text{ N})}} = 0.192\text{ s}$

The total time is $t_{total} = 0.137\text{ s} + 0.192\text{ s} = \boxed{0.329\text{ s}}$

13.61 (a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{500\text{ N/m}}{2.00\text{ kg}}} = \boxed{15.8\text{ rad/s}}$

(b) Apply Newton's second law to the block while the elevator is accelerating:

$$\Sigma F_y = F_s - mg = ma_y$$

With $F_s = kx$ and $a_y = g/3$, this gives $kx = m(g + g/3)$, or

$$x = \frac{4mg}{3k} = \frac{4(2.00\text{ kg})(9.80\text{ m/s}^2)}{3(500\text{ N/m})} = 5.23\times10^{-2}\text{ m} = \boxed{5.23\text{ cm}}$$

13.62 (a) When the block is given some small upward displacement, the net restoring force exerted on it by the rubber bands is

$$F_{net} = \Sigma F_y = -2F\sin\theta\text{, where } \tan\theta = \frac{y}{L}$$

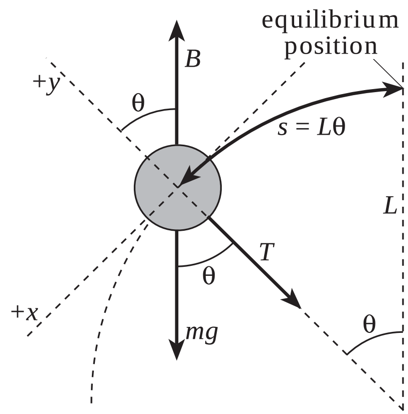
For small displacements, the angle θ will be very small. Then $\sin\theta \approx \tan\theta = \frac{y}{L}$, and the net restoring force is

$$F_{net} = -2F\left(\frac{y}{L}\right) = \boxed{-\left(\frac{2F}{L}\right)y}$$

(b) The net restoring force found in part (a) is in the form of Hooke's law $F = -ky$, with $k = \frac{2F}{L}$. Thus, the motion will be simple harmonic, and the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{2F}{mL}}}$$

13.63 The free-body diagram at the right shows the forces acting on the balloon when it is displaced distance $s = L\theta$ along the circular arc it follows. The net force tangential to this path is



$$F_{net} = \Sigma F_x = -B\sin\theta + mg\sin\theta = -(B - mg)\sin\theta$$

For small angles, $\sin\theta \approx \theta = \dfrac{s}{L}$

Also, $mg = (\rho_{\text{He}} V)g$

and the buoyant force is $B = (\rho_{\text{air}} V)g$. Thus, the net restoring force

acting on the balloon is $\boxed{F_{net} \approx -\left[\dfrac{(\rho_{air} - \rho_{\text{He}})Vg}{L}\right]s}$

Observe that this is in the form of Hooke's law, $F = -ks$,

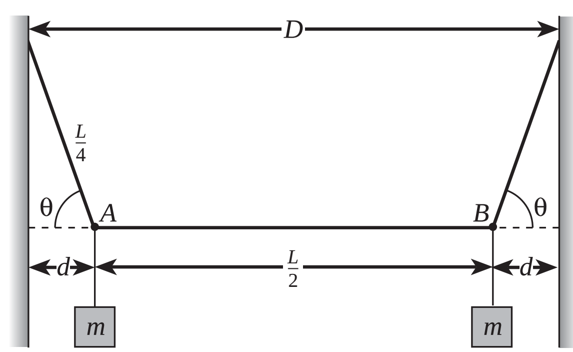
with $k = (\rho_{air} - \rho_{\text{He}})Vg/L$

Thus, the motion will be $\boxed{\text{simple harmonic}}$ and the period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\rho_{\text{He}}V}{(\rho_{air} - \rho_{\text{He}})Vg/L}} = 2\pi\sqrt{\left(\frac{\rho_{\text{He}}}{\rho_{air} - \rho_{\text{He}}}\right)\frac{L}{g}}$$

This yields $T = 2\pi\sqrt{\left(\dfrac{0.180}{1.29 - 0.180}\right)\dfrac{(3.00\text{ m})}{(9.80\text{ m/s}^2)}} = \boxed{1.40\text{ s}}$

13.64 Observe in the sketch at the right that $2d + L/2 = D$, or

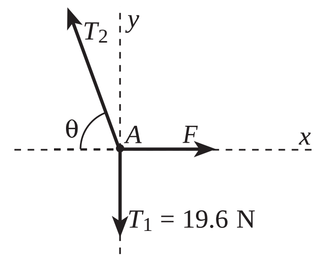


$$d = \frac{D - L/2}{2} = \frac{2.00\text{ m} - 1.50\text{ m}}{2} = 0.250\text{ m}$$

Thus,

$$\theta = \cos^{-1}\left(\frac{d}{L/4}\right) = \cos^{-1}\left(\frac{0.250\text{ m}}{0.750\text{ m}}\right) = 70.5°$$

Now, consider a free body diagram of point A:



$$\Sigma F_x = 0 \Rightarrow F = T_2 \cos(70.5°),$$

and $\Sigma F_y = 0 \Rightarrow T_2 \sin(70.5°) = 19.6 \text{ N}$

Hence, the tension in the section between A and B is

$$F = \frac{19.6 \text{ N}}{\tan(70.5°)} = 6.93 \text{ N}$$

The mass per unit length of the string is

$$\mu = \frac{10.0 \times 10^{-3} \text{ kg}}{3.00 \text{ m}} = 3.33 \times 10^{-3} \text{ kg/m},$$

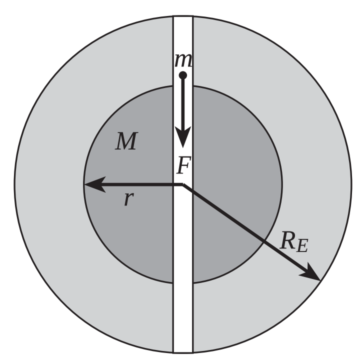
so the speed of transverse waves in the string between points A and B is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{6.93 \text{ N}}{3.33 \times 10^{-3} \text{ kg/m}}} = 45.6 \text{ m/s}$$

The time for the pulse to travel from A to B is

$$t = \frac{L/2}{v} = \frac{1.50 \text{ m}}{45.6 \text{ m/s}} = 3.29 \times 10^{-2} \text{ s} = \boxed{32.9 \text{ ms}}$$

13.65 Newton's law of gravitation is



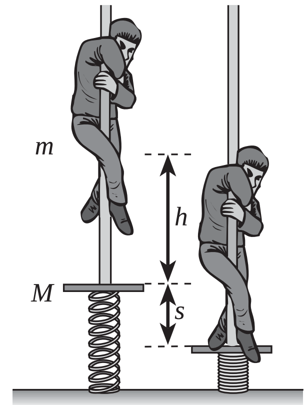
$$F = -\frac{GMm}{r^2}, \text{ where } M = \rho\left(\frac{4}{3}\pi r^3\right)$$

Thus, $F = -\left(\frac{4}{3}\pi\rho G m\right) r$,

which is of Hooke's law form, $F = -kr$, with

$$k = \frac{4}{3}\pi\rho G m$$

13.66 (a) Apply the work-kinetic energy theorem from the instant the firefighter starts from rest until just before contact with the platform.



$$W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i \text{ gives}$$

$$-f \cdot h = \left(\frac{1}{2}mv^2 + 0\right) - \left(0 + mgh\right), \text{ or } v = \sqrt{2\left(g - \frac{f}{m}\right)h}$$

$$v = \sqrt{2\left(9.80 \text{ m/s}^2 - \frac{300 \text{ N}}{60 \text{ kg}}\right)(5.00 \text{ m})} = \boxed{6.93 \text{ m/s}}$$

(b) Next, apply conservation of momentum to find the speed V of the firefighter and platform immediately after the perfectly inelastic collision. This gives

$$(m + M)V = mv + M(0),$$

$$\text{or } V = \left(\frac{m}{m+M}\right)v = \left(\frac{60.0}{60.0+20.0}\right)(6.93 \text{ m/s}) = 5.20 \text{ m/s}$$

Finally, apply the work-kinetic energy theorem from just after the collision until the firefighter comes to rest.

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i \text{ gives}$$

$$-f \cdot s = \left(0 + 0 + \frac{1}{2}ks^2\right) - \left(\frac{1}{2}(m+M)V^2 + (m+M)gs + 0\right),$$

$$\text{or } s^2 - \frac{2}{k}\left[(m+M)g - f\right]s - \left(\frac{m+M}{k}\right)V^2 = 0$$

Using the given data, we obtain $s^2 - (0.387 \text{ m})s - 0.865 \text{ m}^2 = 0$, and the quadratic formula gives a positive solution of $s = \boxed{1.14 \text{ m}}$

13.67 (a) Using conservation of mechanical energy, $(KE+PE_s)_f=(KE+PE_s)_i$, from the moment of release to the instant of separation gives

$$\frac{1}{2}(m_1+m_2)v^2+0=0+\frac{1}{2}kA^2,$$

or $v=A\sqrt{\frac{k}{m_1+m_2}}=(0.20\text{ m})\sqrt{\frac{100\text{ N/m}}{(9.0+7.0)\text{ kg}}}=\boxed{0.50\text{ m/s}}$

(b) After the two blocks separate, m_1 oscillates with new amplitude A' found by applying $(KE+PE_s)_f=(KE+PE_s)_i$ to the $m_1+\text{spring}$ system from the moment of separation until the spring is fully stretched the first time.

$$0+\frac{1}{2}kA'^2=\frac{1}{2}m_1v^2$$

or $A'=v\sqrt{\frac{m_1}{k}}=(0.50\text{ m/s})\sqrt{\frac{9.0\text{ kg}}{100\text{ N/m}}}=0.15\text{ m}$

The period of this oscillation is $T=2\pi\sqrt{\frac{m_1}{k}}=2\pi\sqrt{\frac{9.0\text{ kg}}{100\text{ N/m}}}=1.9\text{ s}$,

so the spring is full stretched for the first time at $t=\frac{T}{4}=0.47\text{ s}$ after separation. During this time, m_2 has moved distance $x=vt$ from the point of separation. Thus, the distance separating the two blocks at this instant is

$$D=vt-A'=(0.50\text{ m/s})(0.47\text{ s})-0.15\text{ m}=0.086\text{ m}=\boxed{8.6\text{ cm}}$$

13.68 (a) Apply the work-kinetic energy theorem from the instant before the block contacts the spring until the instant the block leaves the spring.

$$W_{nc}=KE_f-KE_i=\frac{1}{2}m\left(v_f^2-v_i^2\right)$$

$$=\frac{1}{2}(8.00\text{ kg})\left[(3.00\text{ m/s})^2-(4.00\text{ m/s})^2\right]=-28.0\text{ J}$$

or the mechanical energy lost is $|W_{nc}|=\boxed{28.0\text{ J}}$

(b) The energy spent overcoming the friction force while the block is in contact with the spring is $f \cdot s = |W_{nc}|$, where $s = 2x_{max}$ with x_{max} being the maximum distance the spring was compressed. Hence,

$$x_{max} = \frac{|W_{nc}|}{2f} = \frac{|W_{nc}|}{2\mu_k n} = \frac{|W_{nc}|}{2\mu_k mg}$$

$$= \frac{28.0 \text{ J}}{2(0.400)(8.00 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.446 \text{ m}}$$

Answers to Even Numbered Conceptual Questions

2. Each half-spring will have twice the spring constant of the full spring, as shown by the following argument. The force exerted by a spring is proportional to the separation of the coils as the spring is extended. Imagine that we extend a spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Thus, it takes twice as much force to stretch the half-spring, from which we conclude that the half-spring has a spring constant which is twice that of the complete spring.

4. To understand how we might have anticipated this similarity in speeds, consider sound as a motion of air molecules in a certain direction superimposed on the random, high speed, thermal molecular motions predicted by kinetic theory. Individual molecules experience billions of collisions per second with their neighbors, and as a result, do not travel very far in any appreciable time interval. With this interpretation, the energy of a sound wave is carried as kinetic energy of a molecule and transferred to neighboring molecules by collision. Thus, the energy transmitted by a sound wave in, say, a compression, travels from molecule to molecule at about the rms speed, or actually somewhat less, as observed, since multiple collisions slow the process a bit.

6. Friction. This includes both air-resistance and damping within the spring.

8. No. The period of vibration is $T = 2\pi\sqrt{L/g}$ and g is smaller at high altitude. Therefore, the period is longer on the mountain top and the clock will run slower.

10. Shorten the pendulum to decrease the period between ticks.

12. The speed of the pulses is $v = \sqrt{F/\mu}$, so increasing the tension F in the hose increases the speed of the pulses. Filling the hose with water increases the mass per unit length μ, and will decrease the speed of the pulses.

14. You can attach one end to a wall while holding the other end in your hand. To create a longitudinal wave oscillate the spring back and forth along the direction of the stretched spring. To create a transverse wave, shake the string perpendicular to the direction in which the spring is stretched.

16. If the tension remains the same, the speed of a wave on the string does not change. This means, from $v = \lambda f$, that if the frequency is doubled, the wavelength must decrease by a factor of two.

18. The speed of a wave on a string is given by $v = \sqrt{F/\mu}$. This says the speed is independent of the frequency of the wave. Thus, doubling the frequency leaves the speed unaffected.

20. We assume here for simplicity that the Earth's orbit is circular. The motion is not simple harmonic because the resultant force acting on the Earth is not dependent on the displacement. Also, the speed of the Earth is a constant with time, not varying with x as in simple harmonic motion.

Answers to Even Numbered Problems

2. (a) 1.1×10^{2} N
(b) The graph is a straight line passing through the origin with slope equal to $k = 1.0\times 10^{3}$ N/m .

4. 94.9 N

6. (a) 4.03 mm (b) 2.61×10^{4} N/m

8. (a) 575 N/m (b) 46.0 J

10. 2.23 m/s

12. (a) 2.61 m/s (b) 2.38 m/s

14. (a) 11 cm/s (b) 6.3 cm/s (c) 3.0 N

16. (a) 0.15 J (b) 0.78 m/s (c) 18 m/s^2

18. 3.06 m/s

20. (a) 0.628 m/s (b) 0.500 Hz (c) 3.14 rad/s

22. (a) 126 N/m (b) 17.8 cm

24. 2.2 Hz

26. (a) 0.30 m, 0.24 m (b) 0.30 m (c) 1/6 Hz (d) 6.0 s

28. (a) 250 N/m (b) $T = 0.281$ s, $f = 3.56$ Hz, $\omega = 22.4$ rad/s
(c) 0.313 J (d) 5.00 cm (e) 1.12 m/s, 25.0 m/s^2 (f) 0.919 cm

30. (a) 59.6 m (b) 37.5 s

32. (a) gain time (b) 1.1 s

34. (a) 3.65 s (b) 6.41 s (c) 4.24 s

36. 9.8% difference

38. 5.67 mm

40. 8.2 cm to 12 m

42. (a) 0.20 Hz (b) 0.25 Hz

44. 219 N

46. 1.64 m/s^2

48. 7.07 m/s

50. 586 m/s

52. (a) 0 (b) 0.30 m

54. (a) 0.25 m (b) 0.47 N/m (c) 0.23 m (d) 0.12 m/s

56. 0.990 m

58. (a) 100 m/s (b) 374 J

60. 0.329 s

64. 32.9 ms

66. (a) 6.93 m/s (b) 1.14 m

68. (a) 28.0 J (b) 0.446 m