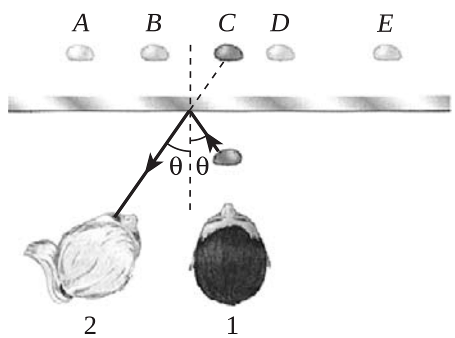
CHAPTER 23

Quick Quizzes

1. At C.



2. (c). Since $n_{water} > n_{air}$, the virtual image of the fish formed by refraction at the flat water surface is closer to the surface than is the fish. See Equation 23.9.

3. (a) False. A concave mirror forms an inverted image when the object distance is greater than the focal length.

(b) False. The magnitude of the magnification produced by a concave mirror is greater than 1 if the object distance is less than the radius of curvature.

(c) True

4. (b). In this case, the index of refraction of the lens material is less than that of the surrounding medium. Under these conditions, a biconvex lens will be divergent.

5. Although a ray diagram only uses 2 or 3 rays (those whose direction is easily determined using only a straight edge), an infinite number of rays leaving the object will always pass through the lens.

6. (a) False. A virtual image is formed on the left side of the lens if $p < f$.

(b) True. An upright, virtual image is formed when $p < f$, while an inverted, real image is formed when $p > f$.

(c) False. A magnified, real image is formed if $2f > p > f$, and a magnified, virtual image is formed if $p < f$.

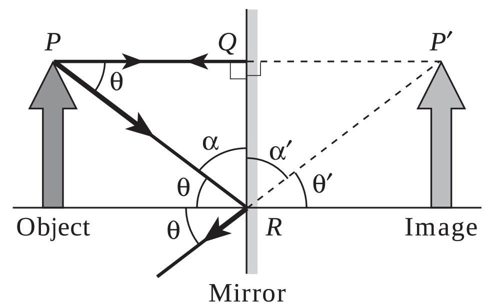
Problem Solutions

23.1 If you stand 40 cm in front of the mirror, the time required for light scattered from your face to travel to the mirror and back to your eye is

$$\Delta t = \frac{2d}{c} = \frac{2(0.40 \text{ m})}{3.0 \times 10^8 \text{ m/s}} = 2.7 \times 10^{-9} \text{ s}.$$

Thus, the image you observe shows you $\boxed{\sim 10^{-9} \text{ s younger}}$ than your current age.

23.2 In the figure at the right, $\theta' = \theta$ since they are vertical angles formed by two intersecting straight lines. Their complementary angles are also equal or $\alpha' = \alpha$. The right triangles PQR and $P'QR$ have the common side QR and are then congruent by the angle-side-angle theorem. Thus, the corresponding sides PQ and $P'Q$ are equal, or the image is as far behind the mirror as the object is in front of it.

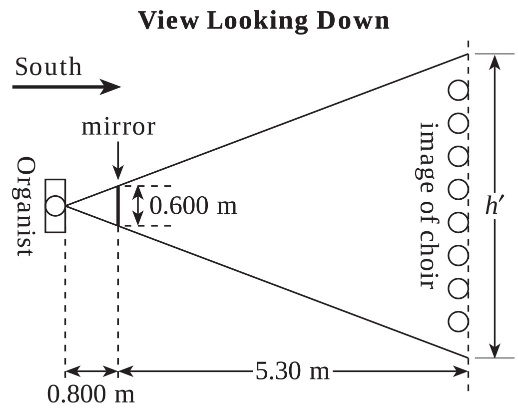


23.3 (1) The first image in the left-hand mirror is 5.00 ft behind the mirror, or $\boxed{10.0 \text{ ft from the person}}$.

(2) The first image in the right-hand mirror serves as an object for the left-hand mirror. It is located 10.0 ft behind the right-hand mirror, which is 25.0 ft from the left-hand mirror. Thus, the second image in the left-hand mirror is 25.0 ft behind the mirror, or $\boxed{30.0 \text{ ft from the person}}$.

(3) The first image in the left-hand mirror serves as an object for the right-hand mirror. It is located 20.0 ft in front of the right-hand mirror and forms an image 20.0 ft behind that mirror. This image then serves as an object for the left-hand mirror. The distance from this object to the left-hand mirror is 35.0 ft. Thus, the third image in the left-hand mirror is 35.0 ft behind the mirror, or $\boxed{40.0 \text{ ft from the person}}$.

23.4 The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror.



The image of the choir is

$$0.800 \text{ m} + 5.30 \text{ m} = 6.10 \text{ m}$$

from the organist. Using similar triangles, gives

$$\frac{h'}{0.600 \text{ m}} = \frac{6.10 \text{ m}}{0.800 \text{ m}}$$

or $$h' = (0.600 \text{ m})\left(\frac{6.10 \text{ m}}{0.800 \text{ m}}\right) = \boxed{4.58 \text{ m}}$$

23.5 Since the mirror is convex, $R < 0$. Thus, $R = -0.550 \text{ m}$. With a real object, $p > 0$, so $p = +10.0 \text{ m}$. The mirror equation then gives the image distance as

$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2}{-0.550 \text{ m}} - \frac{1}{10.0 \text{ m}}, \text{ or } q = -0.268 \text{ m}$$

Thus, the image is $\boxed{\text{virtual}}$ and located $\boxed{0.268 \text{ m behind the mirror}}$.

The magnification is $M = -\frac{q}{p} = -\frac{-0.268 \text{ m}}{10.0 \text{ m}} = \boxed{0.0268}$

Therefore, the image is $\boxed{\text{upright}}$ (since $M > 0$) and $\boxed{\text{diminished in size}}$ (since $|M| < 1$).

23.6 The ornament is a convex mirror, so $R = \frac{1}{2}(-6.00 \text{ cm}) = -3.00 \text{ cm}$. The object distance is positive when the object is in front of the mirror. Therefore, $p = +10.0 \text{ cm}$ and the mirror equation gives

$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2}{-3.00 \text{ cm}} - \frac{1}{10.0 \text{ cm}}, \text{ or } q = -1.30 \text{ cm}$$

The magnification is then $M = -\frac{q}{p} = -\frac{-1.30 \text{ cm}}{10.0 \text{ cm}} = \boxed{0.130}$

23.7 The radius of curvature of a concave mirror is positive, so $R = +20.0 \text{ cm}$. The mirror equation then gives

$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{p} = \frac{p - 10.0 \text{ cm}}{(10.0 \text{ cm})p}, \text{ or } q = \frac{(10.0 \text{ cm})p}{p - 10.0 \text{ cm}}.$$

(a) If $p = 40.0 \text{ cm}$, $q = +13.3 \text{ cm}$ and $M = -\frac{q}{p} = -\frac{13.3 \text{ cm}}{40.0 \text{ cm}} = \boxed{-0.333}$

The image is $\boxed{13.3 \text{ cm in front of the mirror, real, and inverted}}$

(b) When $p = 20.0 \text{ cm}$, $q = +20.0 \text{ cm}$ and $M = -\frac{q}{p} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = \boxed{-1.00}$

The image is $\boxed{20.0 \text{ cm in front of the mirror, real, and inverted}}$

(c) If $p = 10.0 \text{ cm}$, $q = \frac{(10.0 \text{ cm})(10.0 \text{ cm})}{10.0 \text{ cm} - 10.0 \text{ cm}} \to \infty$,

and $\boxed{\text{no image is formed. Parallel rays leave the mirror}}$

23.8 (a) Since the object is in front of the mirror, $p > 0$. With the image behind the mirror, $q < 0$. The mirror equation gives the radius of curvature as

$$\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.00 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = \frac{10-1}{10.0 \text{ cm}},$$

or $R = 2\left(\frac{10.0 \text{ cm}}{9}\right) = \boxed{+2.22 \text{ cm}}$

(b) The magnification is $M = -\frac{q}{p} = -\frac{(-10.0 \text{ cm})}{1.00 \text{ cm}} = \boxed{+10.0}$

23.9 The cylindrical wall is a highly efficient mirror for sound, with radius of curvature, $R = 2.50 \text{ m}$.

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2}{2.50 \text{ m}} - \frac{1}{2.00 \text{ m}}, \text{ which gives } q = \boxed{3.33 \text{ m}}$$

23.10 A convex mirror will form only upright, virtual images of real objects.

Thus, $M = -\frac{q}{p} = \frac{h'}{h} = +\frac{1}{2}$, giving $q = -\frac{p}{2}$

The mirror equation, $\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{f}$, then gives

$$\frac{1}{p} - \frac{2}{p} = \frac{1}{f} \text{ or } p = -f = -(-20.0 \text{ cm}) = \boxed{+20.0 \text{ cm}}$$

23.11 The *magnified*, *virtual* images formed by a concave mirror are upright, so $M > 0$.

Thus, $M = -\frac{q}{p} = \frac{h'}{h} = \frac{5.00 \text{ cm}}{2.00 \text{ cm}} = +2.50$, giving

$$q = -2.50p = -2.50(+3.00 \text{ cm}) = -7.50 \text{ cm}$$

The mirror equation then gives,

$$\frac{1}{f} = \frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{3.00 \text{ cm}} - \frac{1}{7.50 \text{ cm}} = \frac{2.50 - 1}{7.50 \text{ cm}},$$

or $$f = \frac{7.50 \text{ cm}}{1.50} = \boxed{5.00 \text{ cm}}$$

23.12 The object is in front of the mirror, so $p = +10.0 \text{ cm}$. Since the image is upright, the magnification is positive, giving

$$M = -\frac{q}{p} = \frac{h'}{h} = \frac{+4.00 \text{ cm}}{2.00 \text{ cm}} = +2.00$$

or $$q = -2.00p = -2.00(10.0 \text{ cm}) = -20.0 \text{ cm}$$

Then, the mirror equation gives the radius of curvature as

$$\frac{2}{R}=\frac{1}{p}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}-\frac{1}{20.0\text{ cm}}=\frac{2-1}{20.0\text{ cm}},$$

or $$R=2\left(\frac{20.0\text{ cm}}{+1}\right)=\boxed{40.0\text{ cm}}$$

Since $R>0$, the mirror is $\boxed{\text{concave}}$

23.13 The image is upright, so $M>0$ and we have

$$M=-\frac{q}{p}=+2.0\text{, or } q=-2.0p=-2.0(25\text{ cm})=-50\text{ cm}$$

The radius of curvature is then found to be

$$\frac{2}{R}=\frac{1}{p}+\frac{1}{q}=\frac{1}{25\text{ cm}}-\frac{1}{50\text{ cm}}=\frac{2-1}{50\text{ cm}}\text{, or } R=2\left(\frac{0.50\text{ m}}{+1}\right)=\boxed{1.0\text{ m}}$$

23.14 Since the image is upright, $M>0$ giving

$$M=-\frac{q}{p}=+4.00\text{, or } q=-4.00p,$$

Then, from the mirror equation,

$$\frac{1}{f}=\frac{2}{R}=\frac{1}{p}+\frac{1}{q}=\frac{1}{p}-\frac{1}{4.00p}=\frac{4-1}{4.00p},$$

or $$p=\frac{3f}{4.00}=\frac{3(40.0\text{ cm})}{4.00}=\boxed{30.0\text{ cm}}$$

23.15 The focal length of the mirror may be found from the given object and image distances as $1/f = 1/p + 1/q$, or

$$f = \frac{pq}{p+q} = \frac{(152\text{ cm})(18.0\text{ cm})}{152\text{ cm} + 18.0\text{ cm}} = +16.1\text{ cm}$$

For an upright image twice the size of the object, the magnification is

$$M = -\frac{q}{p} = +2.00 \text{ giving } q = -2.00p$$

Then, using the mirror equation again, $1/p + 1/q = 1/f$ becomes

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{p} - \frac{1}{2.00p} = \frac{2-1}{2.00p} = \frac{1}{f},$$

or $$p = \frac{f}{2.00} = \frac{16.1\text{ cm}}{2.00} = \boxed{8.05\text{ cm}}$$

23.16 A convex mirror $(R < 0)$ produces upright, virtual images of real objects.

Thus, $M > 0$ giving $M = -\frac{q}{p} = +\frac{1}{3}$, or $q = -\frac{p}{3}$

Then, $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ becomes $\frac{1}{p} - \frac{3}{p} = -\frac{2}{10.0\text{ cm}}$, and yields $p = +10.0\text{ cm}$

The object is $\boxed{10.0\text{ cm in front of the mirror}}$

23.17 A convex mirror forms upright, virtual images of objects that are in front of it. Therefore, $M > 0$ and we have

$$M = -\frac{q}{p} = +\frac{1}{2}, \text{ or } q = -\frac{p}{2}$$

Then, $\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{p} - \frac{2}{p} = -\frac{1}{p}$ or $R = -2p = -2(10.0\text{ cm}) = \boxed{-20.0\text{ cm}}$

23.18 The magnified, *real* images formed by concave mirrors are inverted. Thus, $M < 0$ giving

$$M = -\frac{q}{p} = -4, \text{ or } q = 4p$$

Then, $\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{p} + \frac{1}{4p} = \frac{5}{4p}$ or $R = \frac{8}{5}p = \frac{8}{5}(30.0 \text{ cm}) = \boxed{48.0 \text{ cm}}$

23.19 (a) An image formed on a screen is a real image. Thus, the mirror must be $\boxed{\text{concave}}$ since, of mirrors, only concave mirrors can form real images of real objects.

(b) The magnified, real images formed by concave mirrors are inverted, so $M < 0$ and

$$M = -\frac{q}{p} = -5, \text{ or } p = \frac{q}{5} = \frac{5.0 \text{ m}}{5} = 1.0 \text{ m}$$

The object should be $\boxed{1.0 \text{ m in front of the mirror}}$

(a – revisited) The focal length of the mirror is

$$\frac{1}{f} = \frac{1}{1.0 \text{ m}} + \frac{1}{5.0 \text{ m}} = \frac{6}{5.0 \text{ m}}, \text{ or } f = \frac{5.0 \text{ m}}{6} = 0.83 \text{ m}$$

23.20 (a) From $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$, we find $q = \frac{Rp}{2p - R} = \frac{(1.00 \text{ m})p}{2p - 1.00 \text{ m}}$

The table gives the image position at a few critical points in the motion. Between $p = 3.00 \text{ m}$ and $p = 0.500 \text{ m}$, the real image moves from 0.600 m to infinity. From $p = 5.00 \text{ m}$ to $p = 0$, the virtual image moves from negative infinity to 0.

p	q
3.00 m	0.600 m
0.500 m	$\pm\infty$
0	0

Note the "jump" in the image position as the ball passes through the focal point of the mirror.

(b) The ball and its image coincide when $p = 0$ and when

$$1/p + 1/p = 2/p = 2/R \text{, or } p = R = 1.00 \text{ m}$$

From $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$, with $v_{iy} = 0$, the times for the ball to fall from $p = 3.00 \text{ m}$ to these positions are found to be

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-2.00 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{0.639 \text{ s}} \text{ and}$$

$$t = \sqrt{\frac{2(-3.00 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{0.782 \text{ s}}$$

23.21 From $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$, with $R \to \infty$, the image position is found to be

$$q = -\frac{n_2}{n_1}p = -\left(\frac{1.00}{1.309}\right)(50.0 \text{ cm}) = -38.2 \text{ cm},$$

or the virtual image is $\boxed{38.2 \text{ cm below the upper surface of the ice}}$

23.22 For a plane refracting surface $(R \to \infty)$,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \text{ becomes } q = -\frac{n_2}{n_1}p$$

(a) When the pool is full, $p = 2.00 \text{ m}$ and

$$q = -\left(\frac{1.00}{1.333}\right)(2.00 \text{ m}) = -1.50 \text{ m},$$

or the pool appears to be $\boxed{1.50 \text{ m}}$ deep.

(b) If the pool is half filled, then $p = 1.00 \text{ m}$ and $q = -0.750 \text{ m}$. Thus, the bottom of the pool appears to be 0.75. m below the water surface or $\boxed{1.75 \text{ m}}$ below ground level.

23.23 Since the center of curvature of the surface is on the side the light comes from, $R<0$ giving $R=-4.0\text{ cm}$. Then, $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes

$$\frac{1.00}{q}=\frac{1.00-1.50}{-4.0\text{ cm}}-\frac{1.50}{4.0\text{ cm}}\text{, or } q=-4.0\text{ cm}$$

Thus, the magnification $M=\frac{h'}{h}=-\left(\frac{n_1}{n_2}\right)\frac{q}{p}$, gives

$$h'=-\left(\frac{n_1 q}{n_2 p}\right)h=-\frac{1.50(-4.0\text{ cm})}{1.00(4.0\text{ cm})}(2.5\text{ mm})=\boxed{3.8\text{ mm}}$$

23.24 Light scattered from the bottom of the plate undergoes two refractions, once at the top of the plate and once at the top of the water. All surfaces are planes $(R\to\infty)$, so the image distance for each refraction is $q=-(n_2/n_1)p$. At the top of the plate,

$$q_{1B}=-\left(\frac{n_{water}}{n_{glass}}\right)p_{1B}=-\left(\frac{1.333}{1.66}\right)(8.00\text{ cm})=-6.42\text{ cm},$$

or the first image is 6.42 cm below the top of the plate. This image serves as a real object for the refraction at the top of the water, so the final image of the bottom of the plate is formed at

$$q_{2B}=-\left(\frac{n_{air}}{n_{water}}\right)p_{2B}=-\left(\frac{n_{air}}{n_{water}}\right)\left(12.0\text{ cm}+|q_{1B}|\right)$$

$$=-\left(\frac{1.00}{1.333}\right)(18.4\text{ cm})=-13.8\text{ cm}$$ or 13.8 cm below the water surface.

Now, consider light scattered from the top of the plate. It undergoes a single refraction, at the top of the water. This refraction forms an image of the top of the plate at

$$q_T=-\left(\frac{n_{air}}{n_{water}}\right)p_T=-\left(\frac{1.00}{1.333}\right)(12.0\text{ cm})=-9.00\text{ cm},$$

or 9.00 cm below the water surface.

The apparent thickness of the plate is then

$$\Delta y=|q_{2B}|-|q_T|=13.8\text{ cm}-9.00\text{ cm}=\boxed{4.8\text{ cm}}$$

23.25 Since the center of curvature is on the side of the surface the light is going to (i.e., in back of the surface), $R = +8.00 \text{ cm}$. Then, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ yields

$$\frac{1.50}{q} = \frac{1.50 - 1.00}{8.00 \text{ cm}} - \frac{1.00}{p} = \frac{1}{16.0 \text{ cm}} - \frac{1}{p}, \text{ and reduces to}$$

$$q = \frac{(24.0 \text{ cm})p}{p - 16.0 \text{ cm}}$$

Using this result, the computed image distance for each given object distance is:

(a) $p = 20.0 \text{ cm}$ gives $q = \boxed{120 \text{ cm}}$ (b) $p = 8.00 \text{ cm}$ gives $q = \boxed{-24.0 \text{ cm}}$

(c) $p = 4.00 \text{ cm}$ gives $q = \boxed{-8.00 \text{ cm}}$ (d) $p = 2.00 \text{ cm}$ gives $q = \boxed{-3.43 \text{ cm}}$

23.26 The wall of the aquarium is a plane $(R \to \infty)$ refracting surface separating water $(n_1 = 1.333)$ and air $(n_2 = 1.00)$. Thus, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ gives the image position as $q = -\left(\frac{n_2}{n_1}\right)p = -\frac{p}{1.333}$. When the object position changes by Δp, the change in the image position is $\Delta q = -\frac{\Delta p}{1.333}$. The apparent *speed* of the fish is then given by

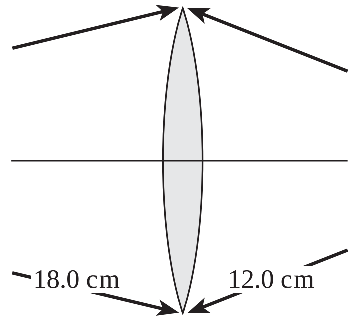
$$v_{image} = \left|\frac{\Delta q}{\Delta t}\right| = \frac{(\Delta p/\Delta t)}{1.333} = \frac{2.00 \text{ cm/s}}{1.333} = \boxed{1.50 \text{ cm/s}}$$

23.27 With $R_1 = +2.00 \text{ cm}$ and $R_2 = +2.50 \text{ cm}$, the lens maker's equation gives the focal length as

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50 - 1)\left(\frac{1}{2.00 \text{ cm}} - \frac{1}{2.50 \text{ cm}}\right) = 0.0500 \text{ cm}^{-1}$$

or $$f = \frac{1}{0.0500 \text{ cm}^{-1}} = \boxed{20.0 \text{ cm}}$$

23.28 The lens maker's equation is used to compute the focal length in each case.



(a) $\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

$$f=(1.44-1)\left[\frac{1}{12.0\text{ cm}}-\frac{1}{(-18.0\text{ cm})}\right]=\boxed{16.4\text{ cm}}$$

(b) $\frac{1}{f}=(1.44-1)\left[\frac{1}{18.0\text{ cm}}-\frac{1}{(-12.0\text{ cm})}\right]$ $\quad f=\boxed{16.4\text{ cm}}$

23.29 From the thin lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, the image distance is found to be

$$q=\frac{fp}{p-f}=\frac{(20.0\text{ cm})p}{p-20.0\text{ cm}}$$

(a) If $p=40.0\text{ cm}$, then $q=40.0\text{ cm}$ and $M=-\frac{q}{p}=-\frac{40.0\text{ cm}}{40.0\text{ cm}}=\boxed{-1.00}$

The image is $\boxed{\text{real, inverted, and 40.0 cm beyond the lens}}$

(b) If $p=20.0\text{ cm}$, $q\to\infty$ $\boxed{\text{No image formed. Parallel rays leave the lens.}}$

(c) When $p=10.0\text{ cm}$, $q=-20.0\text{ cm}$ and

$$M=-\frac{q}{p}=-\frac{(-20.0\text{ cm})}{10.0\text{ cm}}=\boxed{+2.00}$$

The image is $\boxed{\text{virtual, upright, and 20.0 cm in front of the lens}}$

23.30 Since $M=-\frac{q}{p}$, the image distance is $q=-Mp$ and the thin lens equation becomes

$\frac{1}{p}-\frac{1}{Mp}=\frac{1}{f}$ and reduces to $p=\left(1-\frac{1}{M}\right)f$

(a) If $f = 12.0\text{ cm}$, then $p = \left(1 - \frac{1}{M}\right)(+12.0\text{ cm})$,

$M = -1.00 \Rightarrow \boxed{p = +24.0\text{ cm}}$

$M = +1.00 \Rightarrow \boxed{p = 0 \text{ (object is against the lens)}}$

(b) If $f = -12.0\text{ cm}$, then $p = \left(1 - \frac{1}{M}\right)(-12.0\text{ cm})$,

$M = -1.00 \Rightarrow \boxed{p = -24.0\text{ cm}}$

$M = +1.00 \Rightarrow \boxed{p = 0 \text{ (object is against the lens)}}$

Note that in both cases, M approaches +1.00 only in the limit as p approaches zero. Note also in part (b), the object must be virtual to obtain $M = -1.00$.

23.31 From the thin lens equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, the image distance is found to be

$$q = \frac{fp}{p - f} = \frac{(-20.0\text{ cm})p}{p - (-20.0\text{ cm})} = -\frac{(20.0\text{ cm})p}{p + 20.0\text{ cm}}$$

(a) If $p = 40.0\text{ cm}$, then $q = -13.3\text{ cm}$ and $M = -\frac{q}{p} = -\frac{(-13.3\text{ cm})}{40.0\text{ cm}} = \boxed{+1/3}$

The image is $\boxed{\text{virtual, upright, and } 13.3\text{ cm in front of the lens}}$

(b) If $p = 20.0\text{ cm}$, then $q = -10.0\text{ cm}$ and

$$M = -\frac{q}{p} = -\frac{(-10.0\text{ cm})}{20.0\text{ cm}} = \boxed{+1/2}$$

The image is $\boxed{\text{virtual, upright, and } 10.0\text{ cm in front of the lens}}$

(c) When $p = 10.0\text{ cm}$, $q = -6.67\text{ cm}$ and $M = -\frac{q}{p} = -\frac{(-6.67\text{ cm})}{10.0\text{ cm}} = +2/3$

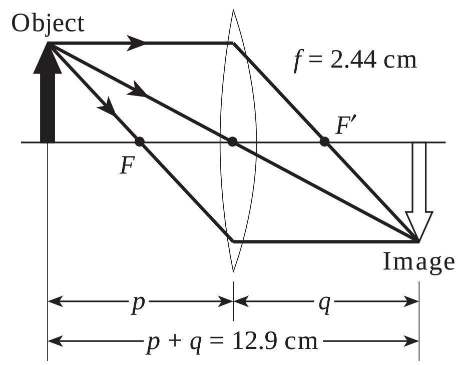
The image is $\boxed{\text{virtual, upright, and } 6.67\text{ cm in front of the lens}}$

23.32 If $M = -\dfrac{q}{p} = +2.00$, then $q = -2.00p$ and the thin lens equation gives

$$\frac{1}{p} - \frac{1}{2.00p} = +\frac{1}{2.00p} = \frac{1}{f}, \text{ or } p = \frac{f}{2.00} = \frac{15.0\text{ cm}}{2.00} = 7.50\text{ cm}$$

so the object should be $\boxed{7.50\text{ cm in front of the lens}}$

23.33 (a) The real image case is shown in the ray diagram. Notice that $p + q = 12.9\text{ cm}$, or $q = 12.9\text{ cm} - p$. The thin lens equation, with $f = 2.44\text{ cm}$, then gives



$$\frac{1}{p} + \frac{1}{12.9\text{ cm} - p} = \frac{1}{2.44\text{ cm}},$$

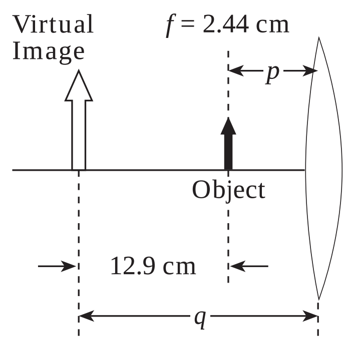
or $p^2 - (12.9\text{ cm})p + 31.5\text{ cm}^2 = 0$

Using the quadratic formula to solve gives

$\boxed{p = 9.63\text{ cm or } p = 3.27\text{ cm}}$

Both are valid solutions for the real image case.

(b) The virtual image case is shown in the second ray diagram. Note that in this case, $q = -(12.9\text{ cm} + p)$, so the thin lens equation gives



$$\frac{1}{p} - \frac{1}{12.9\text{ cm} + p} = \frac{1}{2.44\text{ cm}},$$

or $p^2 + (12.9\text{ cm})p - 31.5\text{ cm}^2 = 0$

The quadratic formula then gives $p = 2.10\text{ cm or } p = -15.0\text{ cm}$.

Since the object is real, the negative solution must be rejected leaving $\boxed{p = 2.10\text{ cm}}$.

23.34 We must first realize that we are looking at an upright, magnified, virtual image. Thus, we have a real object located between a converging lens and its front-side focal point, so $q < 0, p > 0,$ and $f > 0$.

The magnification is $M = -\frac{q}{p} = +2$, giving $q = -2p$. Then, from the thin lens equation,

$\frac{1}{p} - \frac{1}{2p} = +\frac{1}{2p} = \frac{1}{f}$ or $f = 2p = 2(2.84\text{ cm}) = \boxed{5.68\text{ cm}}$

23.35 It is desired to form a magnified, real image on the screen using a single thin lens. To do this, a converging lens must be used and the image will be inverted. The magnification then gives

$$M = \frac{h'}{h} = \frac{-1.80\text{ m}}{24.0 \times 10^{-3}\text{ m}} = -\frac{q}{p}, \text{ or } q = 75.0p$$

Also, we know that $p + q = 3.00\text{ m}$. Therefore, $p + 75.0p = 3.00\text{ m}$ giving

(b) $p = \frac{3.00\text{ m}}{76.0} = 3.95 \times 10^{-2}\text{ m} = \boxed{39.5\text{ mm}}$

(a) The thin lens equation then gives $\frac{1}{p} + \frac{1}{75.0p} = \frac{76.0}{75.0p} = \frac{1}{f}$,

or $f = \left(\frac{75.0}{76.0}\right)p = \left(\frac{75.0}{76.0}\right)(39.5\text{ mm}) = \boxed{39.0\text{ mm}}$

23.36 We are given that $p = 10f$. Then, the thin lens equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, gives

$$q = \frac{pf}{p - f} = \frac{(10f)f}{10f - f} = \left(\frac{10}{9}\right)f$$

Thus, the image is located distance

$$\Delta x = q - f = \left(\frac{10}{9}\right)f - f = \boxed{f/9 \text{ outside the focal point}}$$

23.37 All virtual images formed by diverging lenses are upright images. Thus, $M > 0$ and the magnification gives

$$M = -\frac{q}{p} = +\frac{1}{3}, \text{ or } q = -\frac{p}{3}$$

Then, from the thin lens equation, $\frac{1}{p} - \frac{3}{p} = -\frac{2}{p} = \frac{1}{f}$ or $p = -2f = 2|f|$

The object should be placed at distance $\boxed{2|f| \text{ in front of the lens}}$

23.38 (a) The total distance from the object to the real image is the object-to-screen distance, so $p + q = 5.00 \text{ m}$ or $q = 5.00 \text{ m} - p$. The thin lens equation then becomes

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{5.00 \text{ m} - p} = \frac{5.00 \text{ m}}{p(5.00 \text{ m} - p)},$$

or $p^2 - (5.00 \text{ m})p + (5.00 \text{ m})f = 0$

With $f = 0.800 \text{ m}$, this gives $p^2 - (5.00 \text{ m})p + 4.00 \text{ m}^2 = 0$ which factors as $(p - 4.00 \text{ m})(p - 1.00 \text{ m}) = 0$ with two solutions:

$$\boxed{p = 4.00 \text{ m and } p = 1.00 \text{ m}}$$

(b) If $p = 4.00 \text{ m}$, then $q = 1.00 \text{ m}$ and $M = -\frac{q}{p} = -\frac{1.00 \text{ m}}{4.00 \text{ m}} = -\frac{1}{4}$

In this case, the image is $\boxed{\text{real, inverted, and one-quarter the size of the object}}$

If $p = 1.00 \text{ m}$, then $q = 4.00 \text{ m}$ and $M = -\frac{q}{p} = -\frac{4.00 \text{ m}}{1.00 \text{ m}} = -4$. In this case, the image is $\boxed{\text{real, inverted, and four times the size of the object}}$

23.39 Since the light incident to the first lens is parallel, $p_1 = \infty$ and the thin lens equation gives $q_1 = f_1 = -10.0 \text{ cm}$.

The virtual image formed by the first lens serves as the object for the second lens, so $p_2 = 30.0 \text{ cm} + |q_1| = 40.0 \text{ cm}$. If the light leaving the second lens is parallel, then $q_2 = \infty$ and the thin lens equation gives $f_2 = p_2 = \boxed{40.0 \text{ cm}}$.

23.40 With $p_1 = 20.0\text{ cm}$ and $f_1 = 25.0\text{ cm}$, the thin lens equation gives the position of the image formed by the first lens as

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(20.0\text{ cm})(25.0\text{ cm})}{20.0\text{ cm} - 25.0\text{ cm}} = -100\text{ cm}$$

and the magnification by this lens is $M_1 = -\frac{q_1}{p_1} = -\frac{(-100\text{ cm})}{20.0\text{ cm}} = +5.00$

This virtual image serves as the object for the second lens, so the object distance is $p_2 = 25.0\text{ cm} + |q_1| = 125\text{ cm}$. Then, the thin lens equation gives the final image position as

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(125\text{ cm})(-10.0\text{ cm})}{125\text{ cm} - (-10.0\text{ cm})} = -9.26\text{ cm}$$

with a magnification by the second lens of

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-9.26\text{ cm})}{125\text{ cm}} = +0.0741$$

Thus, the final image is located $\boxed{9.26\text{ cm in front of the second lens}}$ and

the overall magnification is $M = M_1 M_2 = (+5.00)(+0.0741) = \boxed{+0.370}$

23.41 The thin lens equation gives the image position for the first lens as

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(30.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm} - 15.0\text{ cm}} = +30.0\text{ cm},$$

and the magnification by this lens is $M_1 = -\frac{q_1}{p_1} = -\frac{30.0\text{ cm}}{30.0\text{ cm}} = -1.00$

The real image formed by the first lens serves as the object for the second lens, so $p_2 = 40.0 \text{ cm} - q_1 = +10.0 \text{ cm}$. Then, the thin lens equation gives

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(10.0 \text{ cm})(15.0 \text{ cm})}{10.0 \text{ cm} - 15.0 \text{ cm}} = -30.0 \text{ cm}$$

and the magnification by the second lens is

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-30.0 \text{ cm})}{10.0 \text{ cm}} = +3.00$$

Thus, the final, virtual image is located $\boxed{30.0 \text{ cm in front of the second lens}}$

and the overall magnification is $M = M_1 M_2 = (-1.00)(+3.00) = \boxed{-3.00}$

23.42 (a) With $p_1 = +15.0 \text{ cm}$, the thin lens equation gives the position of the image formed by the first lens as

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(15.0 \text{ cm})(10.0 \text{ cm})}{15.0 \text{ cm} - 10.0 \text{ cm}} = +30.0 \text{ cm}$$

This image serves as the object for the second lens, with an object distance of $p_2 = 10.0 \text{ cm} - q_1 = 10.0 \text{ cm} - 30.0 \text{ cm} = -20.0 \text{ cm}$ (a virtual object). If the image formed by this lens is at the position of O_1, the image distance is

$$q_2 = -(10.0 \text{ cm} + p_1) = -(10.0 \text{ cm} + 15.0 \text{ cm}) = -25.0 \text{ cm}$$

The thin lens equation then gives the focal length of the second lens as

$$f_2 = \frac{p_2 q_2}{p_2 + q_2} = \frac{(-20.0 \text{ cm})(-25.0 \text{ cm})}{-20.0 \text{ cm} - 25.0 \text{ cm}} = \boxed{-11.1 \text{ cm}}$$

(b) The overall magnification is

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{30.0 \text{ cm}}{15.0 \text{ cm}}\right)\left[-\frac{(-25.0 \text{ cm})}{(-20.0 \text{ cm})}\right] = \boxed{+2.50}$$

(c) Since $q_2 < 0$, the final image is $\boxed{\text{virtual}}$; and since $M > 0$, it is $\boxed{\text{upright}}$.

23.43 From the thin lens equation, $q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(4.00 \text{ cm})(8.00 \text{ cm})}{4.00 \text{ cm} - 8.00 \text{ cm}} = -8.00 \text{ cm}$

The magnification by the first lens is $M_1 = -\frac{q_1}{p_1} = -\frac{(-8.00 \text{ cm})}{4.00 \text{ cm}} = +2.00$

The virtual image formed by the first lens is the object for the second lens, so $p_2 = 6.00 \text{ cm} + |q_1| = +14.0 \text{ cm}$ and the thin lens equation gives

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(14.0 \text{ cm})(-16.0 \text{ cm})}{14.0 \text{ cm} - (-16.0 \text{ cm})} = -7.47 \text{ cm}$$

The magnification by the second lens is $M_2 = -\frac{q_2}{p_2} = -\frac{(-7.47 \text{ cm})}{14.0 \text{ cm}} = +0.533$, so the overall magnification is $M = M_1 M_2 = (+2.00)(+0.533) = +1.07$

The position of the final image is $\boxed{7.47 \text{ cm in front of the second lens}}$, and its height is $h' = M h = (+1.07)(1.00 \text{ cm}) = \boxed{1.07 \text{ cm}}$

Since $M > 0$, the final image is $\boxed{\text{upright}}$; and since $q_2 < 0$, this image is $\boxed{\text{virtual}}$.

23.44 (a) We start with the final image and work backward. From Figure P22.44, observe that $q_2 = -(50.0 \text{ cm} - 31.0 \text{ cm}) = -19.0 \text{ cm}$. The thin lens equation

then gives $$p_2 = \frac{q_2 f_2}{q_2 - f_2} = \frac{(-19.0 \text{ cm})(20.0 \text{ cm})}{-19.0 \text{ cm} - 20.0 \text{ cm}} = +9.74 \text{ cm}$$

The image formed by the first lens serves as the object for the second lens and is located 9.74 cm in front of the second lens.

Thus, $q_1 = 50.0 \text{ cm} - 9.74 \text{ cm} = 40.3 \text{ cm}$ and the thin lens equation gives

$$p_1 = \frac{q_1 f_1}{q_1 - f_1} = \frac{(40.3 \text{ cm})(10.0 \text{ cm})}{40.3 \text{ cm} - 10.0 \text{ cm}} = +13.3 \text{ cm}$$

The original object should be located $\boxed{13.3 \text{ cm}}$ in front of the first lens.

(b) The overall magnification is

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{40.3\text{ cm}}{13.3\text{ cm}}\right)\left(-\frac{(-19.0\text{ cm})}{9.74\text{ cm}}\right) = \boxed{-5.90}$$

(c) Since $M < 0$, the final image is $\boxed{\text{inverted}}$;

and since $q_2 < 0$, it is $\boxed{\text{virtual}}$

23.45 Since the final image is to be real and in the film plane, $q_2 = +d$.

Then, the thin lens equation gives $p_2 = \dfrac{q_2 f_2}{q_2 - f_2} = \dfrac{d(13.0\text{ cm})}{d - 13.0\text{ cm}}$.

Note from Figure P23.45 that $d < 12.0\text{ cm}$. The above result then shows that $p_2 < 0$, so the object for the second lens will be a virtual object.

The object of the second lens (L_2) is the image formed by the first lens (L_1), so

$$q_1 = (12.0\text{ cm} - d) - p_2 = 12.0\text{ cm} - d\left(1 + \frac{13.0\text{ cm}}{d\text{-}13.0\text{ cm}}\right) = 12.0\text{ cm} - \frac{d^2}{d\text{-}13.0\text{ cm}}$$

If $d = 5.00\text{ cm}$, then $q_1 = +15.1\text{ cm}$; and when $d = 10.0\text{ cm}$, $q_1 = +45.3\text{ cm}$

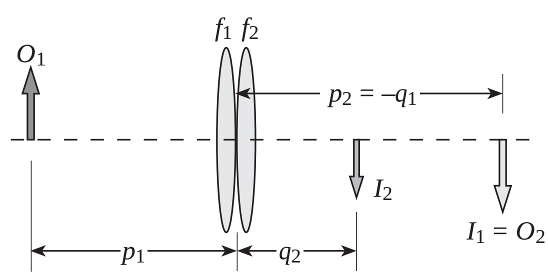
From the thin lens equation, $p_1 = \dfrac{q_1 f_1}{q_1 - f_1} = \dfrac{q_1(15.0\text{ cm})}{q_1 - 15.0\text{ cm}}$

When $q_1 = +15.1\text{ cm}$ $(d = 5.00\text{ cm})$, then $p_1 = 1.82 \times 10^3\text{ cm} = 18.2\text{ m}$

When $q_1 = +45.3\text{ cm}$ $(d = 10.0\text{ cm})$, then $p_1 = 22.4\text{ cm} = 0.224\text{ m}$

Thus, the range of focal distances for this camera is $\boxed{0.224\text{ m to } 18.2\text{ m}}$

23.46 Consider an object O_1 at distance p_1 in front of the first lens. The thin lens equation gives the image position for this lens as $\dfrac{1}{q_1} = \dfrac{1}{f_1} - \dfrac{1}{p_1}$.



The image, I_1, formed by the first lens serves as the object, O_2, for the second lens. With the lenses in contact, this will be a virtual object if I_1 is real and will be a real object if I_1 is virtual. In either case, if the thicknesses of the lenses may be ignored,

$$p_2 = -q_1 \text{ and } \frac{1}{p_2} = -\frac{1}{q_1} = -\frac{1}{f_1} + \frac{1}{p_1}$$

Applying the thin lens equation to the second lens, $\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$ becomes

$$-\frac{1}{f_1} + \frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_2} \text{ or } \frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_1} + \frac{1}{f_2}$$

Observe that this result is a thin lens type equation relating the position of the original object O_1 and the position of the final image I_2 formed by this two lens combination. Thus, we see that we may treat two thin lenses in contact as a single lens having a focal length, *f*, given by $\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$.

23.47 Since $q = +8.00 \text{ cm}$ when $p = +10.0 \text{ cm}$, we find that

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}} + \frac{1}{8.00 \text{ cm}} = \frac{18.0}{80.0 \text{ cm}}$$

Then, when $p = 20.0 \text{ cm}$,

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{18.0}{80.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = \frac{18.0 - 4.00}{80.0 \text{ cm}} = \frac{14.0}{80.0 \text{ cm}},$$

or $q = \frac{80.0 \text{ cm}}{14.0} = +5.71 \text{ cm}$

Thus, a $\boxed{\text{real}}$ image is formed $\boxed{\text{5.71 cm in front of the mirror}}$

23.48 Applying the thin lens equation to the first lens gives

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(12 \text{ cm})(-6.0 \text{ cm})}{12 \text{ cm} - (-6.0 \text{ cm})} = -4.0 \text{ cm}$$

The virtual image formed by the first lens serves as the object for the second lens. If the image formed by that converging lens is to be located at infinity, the image formed by the diverging lens must be located at the focal point in front of the converging lens.

Thus, $p_2 = d + |q_1| = f_2$ or $d = f_2 - |q_1| = 12 \text{ cm} - 4.0 \text{ cm} = \boxed{8.0 \text{ cm}}$

23.49 We are given that the image is upright, so

$$M = -\frac{q}{p} = +0.500 \text{ or } q = -0.500p$$

The distance between image and object is $p + q = p - 0.500p = 20.0 \text{ cm}$, which gives $p = 40.0 \text{ cm}$ and $q = -0.500p = -20.0 \text{ cm}$. Then, from the thin lens equation,

$$f = \frac{pq}{p+q} = \frac{(40.0 \text{ cm})(-20.0 \text{ cm})}{40.0 \text{ cm} - 20.0 \text{ cm}} = \boxed{-40.0 \text{ cm}}$$

23.50 Since the object is midway between the lens and mirror, the object distance for the mirror is $p_1 = +12.5 \text{ cm}$. The mirror equation gives the image position as

$$\frac{1}{q_1} = \frac{2}{R} - \frac{1}{p_1} = \frac{2}{20.0 \text{ cm}} - \frac{1}{12.5 \text{ cm}} = \frac{5-4}{50.0 \text{ cm}} = \frac{1}{50.0 \text{ cm}}, \text{ or } q_1 = +50.0 \text{ cm}$$

This image serves as the object for the lens, so $p_2 = 25.0 \text{ cm} - q_1 = -25.0 \text{ cm}$. Note that since $p_2 < 0$, this is a virtual object. The thin lens equation gives the image position for the lens as

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-25.0 \text{ cm})(-16.7 \text{ cm})}{-25.0 \text{ cm} - (-16.7 \text{ cm})} = -50.3 \text{ cm}$$

Since $q_2 < 0$, this is a $\boxed{\text{virtual}}$ image that is located 50.3 cm in front of the lens or $\boxed{\text{25.3 cm behind the mirror}}$. The overall magnification is

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{50.0 \text{ cm}}{12.5 \text{ cm}}\right)\left[-\frac{(-50.3 \text{ cm})}{(-25.0 \text{ cm})}\right] = \boxed{+8.05}$$

Since $M > 0$, the final image is $\boxed{\text{upright}}$

23.51 As light passes left-to-right through the lens, the image position is given by

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(100 \text{ cm})(80.0 \text{ cm})}{100 \text{ cm} - 80.0 \text{ cm}} = +400 \text{ cm}$$

This image serves as an object for the mirror with an object distance of $p_2 = 100 \text{ cm} - q_1 = -300 \text{ cm}$ (virtual object). From the mirror equation, the position of the image formed by the mirror is

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-300 \text{ cm})(-50.0 \text{ cm})}{-300 \text{ cm} - (-50.0 \text{ cm})} = -60.0 \text{ cm}$$

This image is the object for the lens as light now passes through it going right-to-left. The object distance for the lens is $p_3 = 100 \text{ cm} - q_2 = 100 \text{ cm} - (-60.0 \text{ cm})$, or $p_3 = 160 \text{ cm}$. From the thin lens equation,

$$q_3 = \frac{p_3 f_3}{p_3 - f_3} = \frac{(160 \text{ cm})(80.0 \text{ cm})}{160 \text{ cm} - 80.0 \text{ cm}} = +160 \text{ cm}$$

Thus, the final image is located $\boxed{\text{160 cm to the left of the lens}}$

The overall magnification is $M = M_1 M_2 M_3 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right)\left(-\frac{q_3}{p_3}\right)$, or

$$M = \left(-\frac{400 \text{ cm}}{100 \text{ cm}}\right)\left[-\frac{(-60.0 \text{ cm})}{(-300 \text{ cm})}\right]\left(-\frac{160 \text{ cm}}{160 \text{ cm}}\right) = \boxed{-0.800}$$

Since $M < 0$, the final image is $\boxed{\text{inverted}}$.

23.52 (a) Using the sign convention from Table 23.2, the radii of curvature of the surfaces are $R_1 = -15.0\text{ cm}$ and $R_2 = +10.0\text{ cm}$. The lens maker's equation then gives

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{-15.0\text{ cm}} - \frac{1}{10.0\text{ cm}}\right) \text{ or } f = \boxed{-12.0\text{ cm}}$$

(b) If $p \to \infty$, then $q = f = \boxed{-12.0\text{ cm}}$

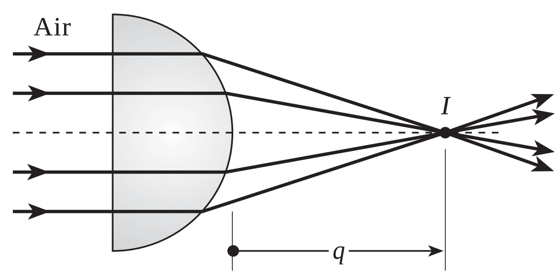
The thin lens equation gives, $q = \dfrac{pf}{p-f} = \dfrac{p(-12.0\text{ cm})}{p+12.0\text{ cm}}$ and the following results:

(c) If $p = 3|f| = +36.0\text{ cm}$, $q = \boxed{-9.00\text{ cm}}$

(d) If $p = |f| = +12.0\text{ cm}$, $q = \boxed{-6.00\text{ cm}}$

(e) If $p = |f|/2 = +6.00\text{ cm}$, $q = \boxed{-4.00\text{ cm}}$

23.53 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the curved surface, for which $R = -6.00\text{ cm}$.



The incident rays are parallel, so $p = \infty$.

Then, $\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R}$ becomes $0 + \dfrac{1.00}{q} = \dfrac{1.00 - 1.56}{-6.00\text{ cm}}$

from which $\boxed{q = 10.7\text{ cm}}$

23.54 (a) The thin lens equation gives the image distance for the first lens as

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(40.0\text{ cm})(20.0\text{ cm})}{40.0\text{ cm} - 20.0\text{ cm}} = +40.0\text{ cm},$$

The magnification by this lens is then $M_1 = -\dfrac{q_1}{p_1} = -\dfrac{40.0\text{ cm}}{40.0\text{ cm}} = -1.00$

The real image formed by the first lens is the object for the second lens. Thus, $p_2 = 50.0\text{ cm} - q_1 = +10.0\text{ cm}$ and the thin lens equation gives

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(10.0\text{ cm})(5.00\text{ cm})}{10.0\text{ cm} - 5.00\text{ cm}} = +10.0\text{ cm}$$

The final image is $\boxed{10.0\text{ cm in back of the second lens}}$

(b) The magnification by the second lens is $M_2 = -\frac{q_2}{p_2} = -\frac{10.0\text{ cm}}{10.0\text{ cm}} = -1.00$, so the overall magnification is $M = M_1 M_2 = (-1.00)(-1.00) = +1.00$. Since this magnification has a value of unity, the final image is the same size as the original object, or $h' = M\, h_1 = (+1.00)(2.00\text{ cm}) = \boxed{2.00\text{ cm}}$

The image distance for the second lens is positive, so the final image is $\boxed{\text{real}}$.

(c) When the two lenses are in contact, the focal length of the combination is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{20.0\text{ cm}} + \frac{1}{5.00\text{ cm}}, \text{ or } f = 4.00\text{ cm}$$

The image position is then

$$q = \frac{pf}{p - f} = \frac{(5.00\text{ cm})(4.00\text{ cm})}{5.00\text{ cm} - 4.00\text{ cm}} = \boxed{+20.0\text{ cm}}$$

23.55 With light going through the piece of glass from left to right, the radius of the first surface is positive and that of the second surface is negative according to the sign convention of Table 23.2. Thus, $R_1 = +2.00\text{ cm}$ and $R_2 = -4.00\text{ cm}$.

Applying $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ to the first surface gives

$$\frac{1.00}{1.00\text{ cm}} + \frac{1.50}{q_1} = \frac{1.50 - 1.00}{+2.00\text{ cm}},$$

which yields $q_1 = -2.00\text{ cm}$. The first surface forms a virtual image 2.00 cm to the left of that surface and 16.0 cm to the left of the second surface.

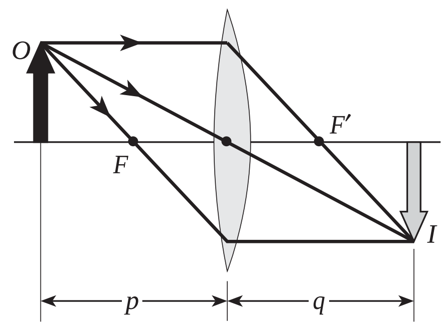
The image formed by the first surface is the object for the second surface, so $p_2 = +16.0\text{ cm}$ and $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ gives

$$\frac{1.50}{16.0\text{ cm}} + \frac{1.00}{q_2} = \frac{1.00 - 1.50}{-4.00\text{ cm}} \text{ or } q_2 = +32.0\text{ cm}$$

The final image formed by the piece of glass is a real image located

$\boxed{32.0\text{ cm to the right of the second surface}}$

23.56 In the original configuration,



$$p_1 + q_1 = 1.50\text{ m},$$

or $$q_1 = 1.50\text{ m} - p_1$$

In the final configuration, $p_2 = p_1 + 0.900\text{ m}$, and

$$q_2 = q_1 - 0.900\text{ m} = 1.50\text{ m} - p_1 - 0.900\text{ cm}$$

or $q_2 = 0.600\text{ m} - p_1$

From the thin lens equation $\frac{1}{f} = \frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{p_2} + \frac{1}{q_2}$,

or $\frac{1}{p_1} + \frac{1}{1.50\text{ m} - p_1} = \frac{1}{p_1 + 0.900\text{ m}} + \frac{1}{0.600\text{ m} - p_1}$

Combining fractions gives

$$\frac{1.50\text{ m}}{p_1(1.50\text{ m} - p_1)} = \frac{1.50\text{ m}}{(p_1 + 0.900\text{ m})(0.600\text{ m} - p_1)},$$

or $$(p_1 + 0.900\text{ m})(0.600\text{ m} - p_1) = p_1(1.50\text{ m} - p_1)$$

(a) Simplifying yields $0.540\text{ m}^2 + (0.600\text{ m} - 0.900\text{ m})p_1 = (1.50\text{ m})p_1$

or $p_1 = \frac{0.540\text{ m}^2}{1.80\text{ m}} = \boxed{0.300\text{ m}}$ and $p_2 = p_1 + 0.900\text{ m} = \boxed{1.20\text{ m}}$

(b) Then, $q_1 = 1.50 \text{ m} - p_1 = 1.20 \text{ m}$ and the thin lens equation gives

$$f = \frac{p_1 q_1}{p_1 + q_1} = \frac{(0.300 \text{ m})(1.20 \text{ m})}{1.50 \text{ m}} = \boxed{0.240 \text{ m}}$$

(c) The second image distance is $q_2 = 0.600 \text{ m} - p_1 = +0.300 \text{ m}$ and the magnification for this configuration is

$$M_2 = -\frac{q_2}{p_2} = -\frac{0.300 \text{ m}}{1.20 \text{ m}} = \boxed{-0.250}$$

Thus, the second image is $\boxed{\text{real, inverted, and diminished}}$

23.57 From the thin lens equation, the image distance for the first lens is

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(40.0 \text{ cm})(30.0 \text{ cm})}{40.0 \text{ cm} - 30.0 \text{ cm}} = +120 \text{ cm},$$

and the magnification by this lens is $M_1 = -\frac{q_1}{p_1} = -\frac{120 \text{ cm}}{40.0 \text{ cm}} = -3.00$

The real image formed by the first lens serves as the object for the second lens, with object distance of $p_2 = 110 \text{ cm} - q_1 = -10.0 \text{ cm}$ (a virtual object). The thin lens equation gives the image distance for the second lens as

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-10.0 \text{ cm}) f_2}{-10.0 \text{ cm} - f_2}$$

(a) If $f_2 = -20.0 \text{ cm}$, then $q_2 = +20.0 \text{ cm}$ and the magnification by the second lens is $M_2 = -q_2/p_2 = -(20.0 \text{ cm})/(-10.0 \text{ cm}) = +2.00$

The final image is located $\boxed{\text{20.0 cm to the right of the second lens}}$

and the overall magnification is $M = M_1 M_2 = (-3.00)(+2.00) = \boxed{-6.00}$

(b) Since $M < 0$, the final image is $\boxed{\text{inverted}}$

(c) If $f_2 = +20.0 \text{ cm}$, then $q_2 = +6.67 \text{ cm}$,

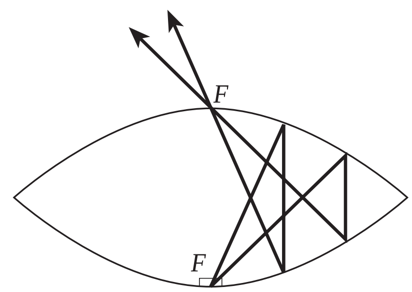
and $$M_2 = -\frac{q_2}{p_2} = -\frac{6.67 \text{ cm}}{(-10.0 \text{ cm})} = +0.667$$

The final image is $\boxed{6.67 \text{ cm to the right of the second lens}}$

and the overall magnification is $M = M_1 M_2 = (-3.00)(+0.667) = \boxed{-2.00}$

Since $M < 0$, the final image is $\boxed{\text{inverted}}$

23.58 The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror). The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror. Thus, the $\boxed{\text{image is real, inverted, and actual size}}$

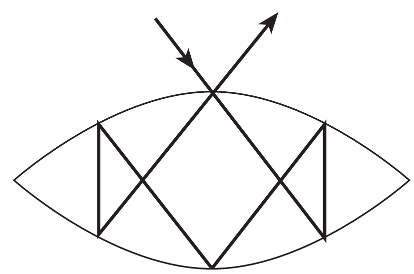


For the upper mirror:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \frac{1}{7.50 \text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50 \text{ cm}}: \qquad q_1 = \infty$$

For the lower mirror:

$$\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50 \text{ cm}}: \qquad q_2 = 7.50 \text{ cm}$$



Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

23.59 (a) The lens maker's equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, gives

$$\frac{1}{5.00 \text{ cm}} = (n-1)\left(\frac{1}{9.00 \text{ cm}} - \frac{1}{-11.0 \text{ cm}}\right)$$

which simplifies to $n = 1 + \frac{1}{5.00}\left(\frac{99.0}{11.0 + 9.00}\right) = \boxed{1.99}$

(b) As light passes from left to right through the lens, the thin lens equation gives the image distance as

$$q_1 = \frac{p_1 f}{p_1 - f} = \frac{(8.00\text{ cm})(5.00\text{ cm})}{8.00\text{ cm} - 5.00\text{ cm}} = +13.3\text{ cm}$$

This image formed by the lens serves as an object for the mirror with object distance $p_2 = 20.0\text{ cm} - q_1 = +6.67\text{ cm}$. The mirror equation then gives

$$q_2 = \frac{p_2 R}{2p_2 - R} = \frac{(6.67\text{ cm})(8.00\text{ cm})}{2(6.67\text{ cm}) - 8.00\text{ cm}} = +10.0\text{ cm}$$

This real image, formed 10.0 cm to the left of the mirror, serves as an object for the lens as light passes through it from right to left. The object distance is $p_3 = 20.0\text{ cm} - q_2 = +10.0\text{ cm}$, and the thin lens equation gives

$$q_3 = \frac{p_1 f}{p_3 - f} = \frac{(10.0\text{ cm})(5.00\text{ cm})}{10.0\text{ cm} - 5.00\text{ cm}} = +10.0\text{ cm}$$

The final image is located $\boxed{10.0\text{ cm to the left of the lens}}$ and its overall magnification is

$$M = M_1 M_2 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right)\left(-\frac{q_3}{p_3}\right) = \left(-\frac{13.3}{8.00}\right)\left(-\frac{10.0}{6.67}\right)\left(-\frac{10.0}{10.0}\right) = -2.50$$

(c) Since $M < 0$, the final image is $\boxed{\text{inverted}}$

23.60 From the thin lens equation, the object distance is $p = \frac{qf}{q - f}$

(a) If $q = +4f$, then $p = \frac{(4f)f}{4f - f} = \boxed{\frac{4f}{3}}$

(b) When $q = -3f$, we find $p = \frac{(-3f)f}{-3f - f} = \boxed{\frac{3f}{4}}$

(c) In case (a), $M = -\frac{q}{p} = -\frac{4f}{4f/3} = \boxed{-3}$

and in case (b), $M = -\frac{q}{p} = -\frac{-3f}{3f/4} = \boxed{+4}$

23.61 If $R_1 = -3.00\text{ m}$ and $R_2 = -6.00\text{ m}$, the focal length is given by

$$\frac{1}{f} = \left(\frac{n_1}{n_2} - 1\right)\left(\frac{1}{-3.00\text{ m}} + \frac{1}{6.00\text{ m}}\right) = \left(\frac{n_1 - n_2}{n_2}\right)\left(\frac{-1}{6.00\text{ m}}\right)$$

or $$f = \frac{(6.00\text{ m})n_2}{n_2 - n_1} \qquad (1)$$

(a) If $n_1 = 1.50$ and $n_2 = 1.00$, then $f = \frac{(6.00\text{ m})(1.00)}{1.00 - 1.50} = -12.0\text{ m}$

The thin lens equation gives $q = \frac{pf}{p-f} = \frac{(10.0\text{ m})(-12.0\text{ m})}{10.0\text{ m} + 12.0\text{ m}} = -5.45\text{ m}$

A virtual image is formed $\boxed{5.45\text{ m to the left of the lens}}$

(b) If $n_1 = 1.50$ and $n_2 = 1.33$, the focal length is

$$f = \frac{(6.00\text{ m})(1.33)}{1.33 - 1.50} = -46.9\text{ m}$$

and $$q = \frac{pf}{p-f} = \frac{(10.0\text{ m})(-46.9\text{ m})}{10.0\text{ m} + 46.9\text{ m}} = -8.24\text{ m}$$

The image is located $\boxed{8.24\text{ m to the left of the lens}}$

(c) When $n_1 = 1.50$ and $n_2 = 2.00$, $f = \frac{(6.00\text{ m})(2.00)}{2.00 - 1.50} = +24.0\text{ m}$

and $$q = \frac{pf}{p-f} = \frac{(10.0\text{ m})(24.0\text{ m})}{10.0\text{ m} - 24.0\text{ m}} = -17.1\text{ m}$$

The image is $\boxed{17.1\text{ m to the left of the lens}}$

(d) Observe from equation (1) that $f < 0$ if $n_1 > n_2$ and $f > 0$ when $n_1 < n_2$. Thus, a diverging lens can be changed to converging by surrounding it with a medium whose index of refraction exceeds that of the lens material.

Answers to Even Numbered Conceptual Questions

2. If the finger and the image are at the same distance from you, then they will coincide regardless of what angle you view them from. However, if one is closer than the other, they will appear to coincide only when viewed along the line connecting their positions. When viewed at any angle to this line, the finger and image are seen separately.

4. Chromatic aberration is produced when light passes *through* a material, as it does when passing through the glass of a lens. A mirror, silvered on its front surface never has light passing through it, so this aberration cannot occur. This is only one of many reasons why large telescopes use mirrors rather than lenses for their primary optical elements.

6. Make the mirror an efficient reflector (shiny); use a parabolic shaped mirror so that it reflects all rays to the image point, even those far from the axis; most important, use a large-diameter mirror in order to collect more solar power.

8. A flat mirror does not reverse left and right. The image of the left hand forms on the left side and the image of the right hand forms on the right side.

10. All objects beneath the stream appear to be closer to the surface than they really are because of refraction. Thus, the pebbles on the bottom of the stream appear to be close to the surface of a shallow stream.

12. An effect similar to a mirage is produced except the “mirage” is seen hovering in the air. Ghost lighthouses in the sky have been seen over bodies of water by this effect.

14. Actually no physics is involved here. The design is chosen so your eyelashes will not brush against the glass as you blink. A reason involving a little physics is that with this design, when you direct your gaze near the outer circumference of the lens you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Then the lens minimally distorts the direction to the object you are looking at.

16. Both words are inverted. However OXIDE looks the same right side up and upside down. LEAD does not.

Answers to Even Numbered Problems

4. 4.58 m

6. 0.130

8. (a) 2.22 cm (b) $M = +10.0$

10. 20.0 cm in front of the mirror

12. concave, $R = 40.0 \text{ cm}$

14. 30.0 cm

16. 10.0 cm in front of the mirror

18. 48.0 cm

20. (a) From $p = 3.00 \text{ m}$ to $p = 0.500 \text{ m}$, the image is real and moves from $q = 0.600 \text{ m}$ to $q = +\infty$. From $p = 0.500 \text{ m}$ to $p = 0$, the image is virtual and moves from $q = -\infty$ to $q = 0$.
(b) 0.639 s and 0.782 s

22. (a) 1.50 m (b) 1.75 m

24. 4.82 cm

26. 1.50 cm/s

28. (a) 16.4 cm (b) 16.4 cm

30. (a) $M = -1.00$ for $p = +24.0 \text{ cm}$,
$M = +1.00$ only if $p = 0$ (object against lens)
(b) $M = -1.00$ for $p = -24.0 \text{ cm}$,
$M = +1.00$ only if $p = 0$ (object against lens)

32. 7.50 cm in front of the lens

34. 5.68 cm

36. $f/9$ outside the focal point

38. (a) $p = 4.00 \text{ m}$ or $p = 1.00 \text{ m}$
(b) One image is real, inverted and one-quarter the size of the object. The other image is real, inverted and four times the size of the object.

40. 9.26 cm in front of the second lens, $M = +0.370$

42. (a) –11.1 cm (b) $M = +2.50$ (c) virtual, upright

44. (a) 13.3 cm (b) $M = -5.90$ (c) inverted, virtual

48. 8.0 cm

50. 25.3 cm behind the mirror, virtual, upright, $M = +8.05$

52. (a) –12.0 cm (b) –12.0 cm (c) –9.00 cm
(d) –6.00 cm (e) –4.00 cm

54. (a) 10.0 cm in back of the second lens (b) 2.00 cm, real
(c) 20.0 cm in back of the second lens

56. (a) $p_1 = 0.300\text{ m}$, $p_2 = 1.20\text{ m}$ (b) 0.240 m
(c) real, inverted, and diminished with $M = -0.250$

58. It is real, inverted, and actual size.

60. (a) $4f/3$ (b) $3f/4$ (c) –3, +4