CHAPTER 25

Quick Quizzes

1. We would like to reduce the minimum angular separation for two objects below the angle subtended by the two stars in the binary system. We can do that by reducing the wavelength of the light—this in essence makes the aperture larger, relative to the light wavelength, increasing the resolving power. Thus, we would choose a blue filter.

Problem Solutions

25.1 Using the thin lens equation, the image distance is

$$q = \frac{pf}{p-f} = \frac{(150\text{ cm})(25.0\text{ cm})}{150\text{ cm} - 25.0\text{ cm}} = 30.0\text{ cm}$$

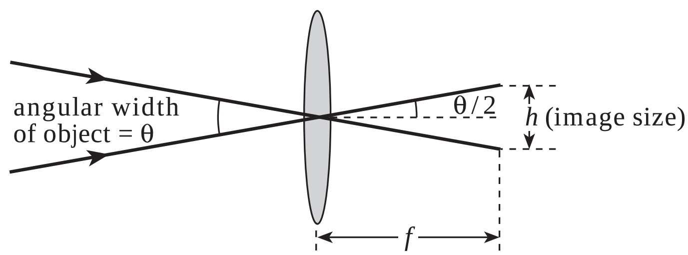
so the image is located $\boxed{30.0\text{ cm beyond the lens}}$. The lateral magnification is

$$M = -\frac{q}{p} = -\frac{30.0\text{ cm}}{150\text{ cm}} = \boxed{-\frac{1}{5}}$$

25.2 The *f*-number of a camera lens is defined as $f\text{-number} = \text{focal length}/\text{diameter}$.

Therefore, the diameter is $D = \dfrac{f}{f\text{-number}} = \dfrac{55\text{ mm}}{1.8} = \boxed{31\text{ mm}}$

25.3 Consider rays coming from opposite edges of the object and passing undeviated through the center of the lens as shown at the right. For a very distant object, the image distance equals the focal length of the lens. If the angular width of the object is θ, the full image width on the film is



$$h = 2\left[f\tan(\theta/2)\right] = 2(55.0\text{ mm})\tan\left(\frac{20^\circ}{2}\right) = \boxed{19\text{ mm}}$$

so the image easily fits within a 23.5 mm by 35.0 mm area.

25.4 The image distance is $q \approx f$ since the object is so far away. Therefore, the lateral magnification is $M = h'/h = -q/p \approx -f/p$ and the diameter of the Moon's image is

$$h' = |M|h = \left(\frac{f}{p}\right)(2R_{moon}) = \left(\frac{120\text{ mm}}{3.84\times 10^{8}\text{ m}}\right)\left[2\left(1.74\times 10^{6}\text{ m}\right)\right] = \boxed{1.09\text{ mm}}$$

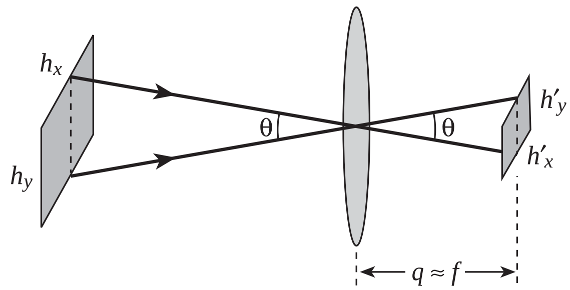
25.5 The exposure time is being reduced by a factor of $\dfrac{t_2}{t_1} = \dfrac{1/256 \text{ s}}{1/32 \text{ s}} = \dfrac{1}{8}$

Thus, to maintain correct exposure, the intensity of the light reaching the film should be increased by a factor of 8. This is done by increasing the area of the aperture by a factor of 8, so in terms of the diameter, $\pi D_2^2/4 = 8\left(\pi D_1^2/4\right)$ or $D_2 = \sqrt{8}D_1$.

The new *f*-number will be

$$(f\text{-number})_2 = \frac{f}{D_2} = \frac{f}{\sqrt{8}D_1} = \frac{(f\text{-number})_1}{\sqrt{8}} = \frac{4.0}{\sqrt{8}} = 1.4 \text{ or } \boxed{f/1.4}$$

25.6 (a) The intensity is a measure of the rate at which energy is received by the film *per unit area of the image*, or $I \propto 1/A_{image}$. Consider an object with horizontal and vertical dimensions h_x and h_y as shown at the right. If the vertical dimension intercepts angle θ, the vertical dimension of the image is $h'_y = q\theta$, or $h'_y \propto q$. Similarly for the horizontal dimension, $h'_x \propto q$, and the area of the image is $A_{image} = h'_x h'_y \propto q^2$. Assuming a very distant object, $q \approx f$, so $A_{image} \propto f^2$ and we conclude that $I \propto 1/f^2$.



The intensity of the light reaching the film is also proportional to the area of the lens, and hence to the square of the diameter of that lens, or $I \propto D^2$. Combining this with our earlier conclusion gives

$$I \propto \frac{D^2}{f^2} = \frac{1}{(f/D)^2} \text{ or } \boxed{I \propto \frac{1}{(f\text{-number})^2}}$$

(b) The total light energy hitting the film is proportional to the product of intensity and exposure time, *It*. Thus, to maintain correct exposure, this product must be kept constant, or $I_2 t_2 = I_1 t_1$ giving

$$t_2 = \left(\frac{I_1}{I_2}\right)t_1 = \left[\frac{(f_2\text{-number})^2}{(f_1\text{-number})^2}\right]t_1 = \left(\frac{4.0}{1.8}\right)^2\left(\frac{1}{500}\text{ s}\right) \approx \boxed{\frac{1}{100}\text{ s}}$$

25.7 Since the exposure time is unchanged, the intensity of the light reaching the film should be doubled so the energy delivered will be doubled. Using the result of Problem 6 (part a), we obtain

$$(f_2\text{-number})^2 = \left(\frac{I_1}{I_2}\right)(f_1\text{-number})^2 = \left(\frac{1}{2}\right)(11)^2 = 61, \text{ or } f_2\text{-number} = \sqrt{61} = 7.8$$

Thus, you should use the $\boxed{f/8.0}$ setting on the camera.

25.8 To focus on a very distant object, the original distance from the lens to the film was $q_1 = f = 65.0 \text{ mm}$. To focus on an object 2.00 m away, the thin lens equation gives

$$q_2 = \frac{p_2 f}{p_2 - f} = \frac{(2.00 \times 10^3 \text{ mm})(65.0 \text{ mm})}{2.00 \times 10^3 \text{ mm} - 65.0 \text{ mm}} = 67.2 \text{ mm}$$

Thus, the lens should be moved

$$\Delta q = q_2 - q_1 = \boxed{2.2 \text{ mm farther from the film}}$$

25.9 This patient needs a lens that will form an upright, virtual image at her near point (60.0 cm) when the object distance is $p = 24.0 \text{ cm}$. From the thin lens equation, the needed focal length is

$$f = \frac{pq}{p+q} = \frac{(24.0 \text{ cm})(-60.0 \text{ cm})}{24.0 \text{ cm} - 60.0 \text{ cm}} = \boxed{+40.0 \text{ cm}}$$

25.10 For the right eye, the lens should form a virtual image of the most distant object 84.4 cm in front of the eye (i.e., $q = -84.4 \text{ cm}$ when $p = \infty$). Thus, $f_{right} = q = -84.4 \text{ cm}$ and the power is

$$P_{right} = \frac{1}{f_{right}} = \frac{1}{-0.844 \text{ m}} = \boxed{-1.18 \text{ diopters}}$$

Similarly, for the left eye $f_{left} = -122 \text{ cm}$ and

$$P_{left} = \frac{1}{f_{left}} = \frac{1}{-1.22 \text{ m}} = \boxed{-0.820 \text{ diopters}}$$

25.11 His lens must form an upright, virtual image of a very distant object ($p \approx \infty$) at his far point, 80.0 cm in front of the eye. Therefore, the focal length is $f = q = -80.0 \text{ cm}$.

If this lens is to form a virtual image at his near point ($q = -18.0 \text{ cm}$), the object distance must be

$$p = \frac{qf}{q-f} = \frac{(-18.0 \text{ cm})(-80.0 \text{ cm})}{-18.0 \text{ cm} - (-80.0 \text{ cm})} = \boxed{23.2 \text{ cm}}$$

25.12 (a) The lens should form an upright, virtual image at the near point $(q = -100 \text{ cm})$ when the object distance is $p = 25.0 \text{ cm}$. Therefore,

$$f = \frac{pq}{p+q} = \frac{(25.0 \text{ cm})(-100 \text{ cm})}{25.0 \text{ cm} - 100 \text{ cm}} = \boxed{33.3 \text{ cm}}$$

(b) The power is $P = \frac{1}{f} = \frac{1}{+0.333 \text{ m}} = \boxed{+3.00 \text{ diopters}}$

25.13 (a) The lens should form an upright, virtual image at the far point $(q = -50.0 \text{ cm})$ for very distant objects $(p \approx \infty)$. Therefore, $f = q = -50.0 \text{ cm}$ and the required power is

$$P = \frac{1}{f} = \frac{1}{-0.500 \text{ m}} = \boxed{-2.00 \text{ diopters}}$$

(b) If this lens is to form an upright, virtual image at the near point of the unaided eye $(q = -13.0 \text{ cm})$, the object distance should be

$$p = \frac{qf}{q-f} = \frac{(-13.0 \text{ cm})(-50.0 \text{ cm})}{-13.0 \text{ cm} - (-50.0 \text{ cm})} = \boxed{17.6 \text{ cm}}$$

25.14 Consider an object at infinity, imaged at the person's far point:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \text{ gives } \frac{1}{\infty}+\frac{1}{q}=-4.00\text{ m}^{-1} \text{ and } q=-25.0\text{ cm}$$

The person's far point is $25.0\text{ cm}+2.00\text{ cm}=27.0\text{ cm}$ from his eyes. For the contact lenses we want

$$P=\frac{1}{f}=\frac{1}{\infty}+\frac{1}{(-0.270\text{ m})}=\boxed{-3.70\text{ diopters}}$$

25.15 Considering the image formed by the cornea as a virtual object for the implanted lens, we have $p=-(2.80\text{ cm}+2.53\text{ cm})=-5.33\text{ cm}$ and $q=+2.80\text{ cm}$

The thin lens equation then gives the focal length of the implanted lens as

$$f=\frac{pq}{p+q}=\frac{(-5.33\text{ cm})(2.80\text{ cm})}{-5.33\text{ cm}+2.80\text{ cm}}=+5.90\text{ cm},$$

so the power is $$P=\frac{1}{f}=\frac{1}{+0.0590\text{ m}}=\boxed{+17.0\text{ diopters}}$$

25.16 (a) The upper portion of the lens should form an upright, virtual image of very distant objects $(p\approx\infty)$ at the far point of the eye $(q=-1.5\text{ m})$. The thin lens equation then gives $f=q=-1.5\text{ m}$, so the needed power is

$$P=\frac{1}{f}=\frac{1}{-1.5\text{ m}}=\boxed{-0.67\text{ diopters}}$$

(b) The lower part of the lens should form an upright, virtual image at the near point of the eye $(q=-30\text{ cm})$ when the object distance is $p=25\text{ cm}$. From the thin lens equation,

$$f=\frac{pq}{p+q}=\frac{(25\text{ cm})(-30\text{ cm})}{25\text{ cm}-30\text{ cm}}=+1.5\times10^{2}\text{ cm}=+1.5\text{ m}$$

Therefore, the power is $P=\frac{1}{f}=\frac{1}{+1.5\text{ m}}=\boxed{+0.67\text{ diopters}}$

25.17 With an erect, virtual image formed at the normal near point of the eye (25.0 cm), the angular magnification is

$$m = m_{max} = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{10.0 \text{ cm}} = \boxed{+3.50}$$

The image is at the same distance as the stamp would be held without use of the magnifier. The image is 3.50 times larger than the stamp, so it fills an angle $m = 3.50$ times larger.

25.18 (a) With the image at the normal near point $(q = -25 \text{ cm})$, the angular magnification is

$$m = 1 + \frac{25 \text{ cm}}{f} = 1 + \frac{25 \text{ cm}}{25 \text{ cm}} = \boxed{+2.0}$$

(b) When the eye is relaxed, parallel rays enter the eye and

$$m = \frac{25 \text{ cm}}{f} = \frac{25 \text{ cm}}{25 \text{ cm}} = \boxed{+1.0}$$

25.19 (a) From the thin lens equation,

$$f = \frac{pq}{p+q} = \frac{(3.50 \text{ cm})(-25.0 \text{ cm})}{3.50 \text{ cm} - 25.0 \text{ cm}} = \boxed{+4.07 \text{ cm}}$$

(b) With the image at the normal near point, the angular magnification is

$$m = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{4.07 \text{ cm}} = \boxed{+7.14}$$

25.20 (a) For maximum magnification, the image should be at the normal near point $(q = -25.0 \text{ cm})$ of the eye. Then, from the thin lens equation,

$$p = \frac{qf}{q - f} = \frac{(-25.0 \text{ cm})(5.00 \text{ cm})}{-25.0 \text{ cm} - 5.00 \text{ cm}} = \boxed{+4.17 \text{ cm}}$$

(b) The magnification is $m = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{5.00 \text{ cm}} = \boxed{+6.00}$

25.21 (a) From the thin lens equation, a real inverted image is formed at an image distance of

$$q = \frac{pf}{p - f} = \frac{(71.0 \text{ cm})(39.0 \text{ cm})}{71.0 \text{ cm} - 39.0 \text{ cm}} = +86.5 \text{ cm}$$

so the lateral magnification produced by the lens is

$$M = \frac{h'}{h} = -\frac{q}{p} = -\frac{86.5 \text{ cm}}{71.0 \text{ cm}} = \boxed{-1.22}$$

(b) If $|h|$ is the actual length of the leaf, the small angle approximation gives the angular width of the leaf when viewed by the unaided eye from a distance of $d = 126 \text{ cm} + 71.0 \text{ cm} = 197 \text{ cm}$ as

$$\theta_0 \approx \frac{|h|}{d} = \frac{|h|}{197 \text{ cm}}$$

The length of the image formed by the lens is $|h'| = |M\,h| = 1.22|h|$, and its angular width when viewed from a distance of $d' = 126 \text{ cm} - q = 39.5 \text{ cm}$ is

$$\theta \approx \frac{|h'|}{d'} = \frac{1.22|h|}{39.5 \text{ cm}}$$

The angular magnification achieved by viewing the image instead of viewing the leaf directly is

$$\frac{\theta}{\theta_0} \approx \frac{1.22|h|/39.5 \text{ cm}}{|h|/197 \text{ cm}} = \frac{1.22(197 \text{ cm})}{39.5 \text{ cm}} = \boxed{6.08}$$

25.22 Using Equation 25.7, the overall magnification is

$$m = M_1 m_e = -\frac{L}{f_o}\left(\frac{25.0 \text{ cm}}{f_e}\right) = -\left(\frac{23.0 \text{ cm}}{0.400 \text{ cm}}\right)\left(\frac{25.0 \text{ cm}}{2.50 \text{ cm}}\right) = \boxed{-575}$$

25.23 The overall magnification is $m = M_1 m_e = M_1\left(\frac{25\text{ cm}}{f_e}\right)$,

where M_1 is the magnification produced by the objective lens. Therefore, the required focal length for the eye piece is

$$f_e = \frac{M_1(25\text{ cm})}{m} = \frac{(-12)(25\text{ cm})}{-140} = \boxed{2.1\text{ cm}}$$

25.24 Note: Some approximations made in arriving at Equation 25.7 do not work well in this case. Therefore, we start with the final image and work backwards to find the overall magnification of the microscope.

Using the thin lens equation, the object distance for the eyepiece is found to be

$$p_e = \frac{q_e f_e}{q_e - f_e} = \frac{(-29.0 \text{ cm})(0.950 \text{ cm})}{-29.0 \text{ cm} - 0.950 \text{ cm}} = 0.920 \text{ cm} \text{ ,}$$

and the magnification produced by the eyepiece is

$$M_e = -\frac{q_e}{p_e} = -\frac{(-29.0 \text{ cm})}{0.920 \text{ cm}} = +31.5$$

The image distance for the objective lens is then

$$q_1 = L - p_e = 29.0 \text{ cm} - 0.920 \text{ cm} = 28.1 \text{ cm} \text{ ,}$$

and the object distance for this lens is

$$p_1 = \frac{q_1 f_o}{q_1 - f_o} = \frac{(28.1 \text{ cm})(1.622 \text{ cm})}{28.1 \text{ cm} - 1.622 \text{ cm}} = 1.72 \text{ cm}$$

The magnification by the objective lens is given by

$$M_1 = -\frac{q_1}{p_1} = -\frac{(28.1 \text{ cm})}{1.72 \text{ cm}} = -16.3$$

and the overall magnification is $m = M_1 M_e = (-16.3)(+31.5) = -514$

The lateral size of the final image is

$$h'_e = |q_e| \cdot \theta = (29.0 \text{ cm})(1.43 \times 10^{-3} \text{ rad}) = 4.15 \times 10^{-2} \text{ cm}$$

and the size of the red blood cell serving as the original object is

$$h = \frac{h'_e}{|m|} = \frac{4.15 \times 10^{-4} \text{ m}}{514} = 8.06 \times 10^{-7} \text{ m} = \boxed{0.806 \ \mu\text{m}}$$

25.25 Approximations made in obtaining Equation 25.7 are not valid in this case. Therefore, we start with the eyepiece and work backwards to determine the overall magnification.

If the eye is relaxed, the eyepiece image is at infinity $(q_e \to -\infty)$, so the object distance is $p_e = f_e = 2.50 \text{ cm}$ and the angular magnification by the eyepiece is

$$m_e = \frac{25.0 \text{ cm}}{f_e} = \frac{25.0 \text{ cm}}{2.50 \text{ cm}} = 10.0$$

The image distance for the objective lens is then,

$$q_1 = L - p_e = 15.0 \text{ cm} - 2.50 \text{ cm} = 12.5 \text{ cm}$$

and the object distance is $p_1 = \frac{q_1 f_o}{q_1 - f_o} = \frac{(12.5 \text{ cm})(1.00 \text{ cm})}{12.5 \text{ cm} - 1.00 \text{ cm}} = 1.09 \text{ cm}$

The magnification by the objective lens is $M_1 = -\frac{q_1}{p_1} = -\frac{(12.5 \text{ cm})}{1.09 \text{ cm}} = -11.5$, and the overall magnification of the microscope is

$$m = M_1 m_e = (-11.5)(10.0) = \boxed{-115}$$

25.26 (a) The angular magnification is $m = \frac{f_o}{f_e} = \frac{20.0 \text{ m}}{(2.50 \times 10^{-2} \text{ m})} = \boxed{800}$

(b) The images formed by astronomical telescopes are always $\boxed{\text{upside down}}$

25.27 The length of the telescope is $L = f_o + f_e = 92 \text{ cm}$,

and the angular magnification is $m = \frac{f_o}{f_e} = 45$

Therefore, $f_o = 45 f_e$ and $L = f_o + f_e = 45 f_e + f_e = 46 f_e = 92 \text{ cm}$, giving

$\boxed{f_e = 2.0 \text{ cm}}$ and $f_o = 92 \text{ cm} - f_e$ or $\boxed{f_o = 90 \text{ cm}}$

25.28 Use the larger focal length (lowest power) lens as the objective element and the shorter focal length (largest power) lens for the eye piece. The focal lengths are

$$f_o = \frac{1}{+1.20 \text{ diopters}} = +0.833 \text{ m}, \text{ and } f_e = \frac{1}{+9.00 \text{ diopters}} = +0.111 \text{ m}$$

(a) The angular magnification (or magnifying power) of the telescope is then

$$m = \frac{f_o}{f_e} = \frac{+0.833 \text{ m}}{+0.111 \text{ m}} = \boxed{7.50}$$

(b) The length of the telescope is

$$L = f_o + f_e = 0.833 \text{ m} + 0.111 \text{ m} = \boxed{0.944 \text{ m}}$$

25.29 (a) From the thin lens equation, $q = \frac{pf}{p - f}$, so the lateral magnification by the objective lens is $M = h'/h = -q/p = -f/(p - f)$. Therefore, the image size will be

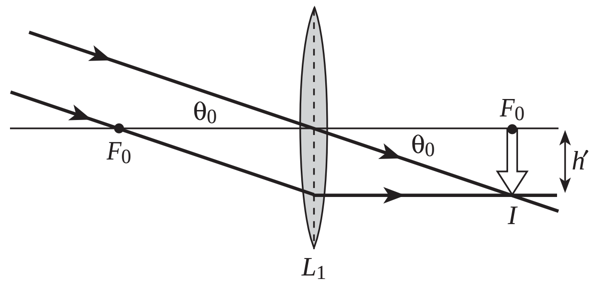
$$h' = M h = -\frac{fh}{p - f} = \boxed{\frac{fh}{f - p}}$$

(b) If $p >> f$, then $f - p \approx -p$ and $h' \approx \boxed{-\frac{fh}{p}}$

(c) Suppose the telescope observes the space station at the zenith.

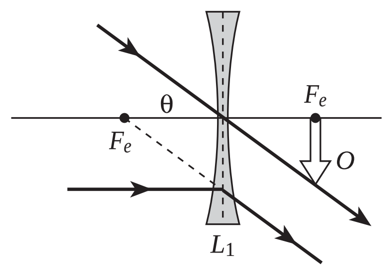
Then, $$h' \approx -\frac{fh}{p} = -\frac{(4.00 \text{ m})(108.6 \text{ m})}{407 \times 10^3 \text{ m}} = -1.07 \times 10^{-3} \text{ m} = \boxed{-1.07 \text{ mm}}$$

25.30 (b) The objective forms a real, diminished, inverted image of a very distant object at $q_1 = f_o$. This image is a virtual object for the eyepiece at $p_e = -|f_e|$, giving



$$\frac{1}{q_e} = \frac{1}{p_e} - \frac{1}{f_e} = \frac{1}{-|f_e|} + \frac{1}{|f_e|} = 0$$

and $\boxed{q_e \to \infty}$



(a) Parallel rays emerge from the eyepiece, so the eye observes a $\boxed{\text{virtual image}}$

(c) The angular magnification is $m = \frac{f_o}{|f_e|} = 3.00$, giving $f_o = 3.00|f_e|$. Also, the length of the telescope is $L = f_o + f_e = 3.00|f_e| - |f_e| = 10.0 \text{ cm}$, giving

$$f_e = -|f_e| = -\frac{10.0 \text{ cm}}{2.00} = \boxed{-5.00 \text{ cm}} \text{ and } f_o = 3.00|f_e| = \boxed{15.0 \text{ cm}}$$

25.31 The lens for the left eye forms an upright, virtual image at $q_L = -50.0 \text{ cm}$ when the object distance is $p_L = 25.0 \text{ cm}$, so the thin lens equation gives its focal length as

$$f_L = \frac{p_L q_L}{p_L + q_L} = \frac{(25.0 \text{ cm})(-50.0 \text{ cm})}{25.0 \text{ cm} - 50.0 \text{ cm}} = 50.0 \text{ cm}$$

Similarly for the other lens, $q_R = -100 \text{ cm}$ when $p_R = 25.0 \text{ cm}$, and $f_R = 33.3 \text{ cm}$.

(a) Using the lens for the left eye as the objective,

$$m = \frac{f_o}{f_e} = \frac{f_L}{f_R} = \frac{50.0 \text{ cm}}{33.3 \text{ cm}} = \boxed{1.50}$$

(b) Using the lens for the right eye as the eyepiece and, for maximum magnification, requiring that the final image be formed at the normal near point $(q_e = -25.0 \text{ cm})$ gives

$$p_e = \frac{q_e f_e}{q_e - f_e} = \frac{(-25.0 \text{ cm})(33.3 \text{ cm})}{-25.0 \text{ cm} - 33.3 \text{ cm}} = +14.3 \text{ cm}$$

The maximum magnification by the eyepiece is then

$$m_e = 1 + \frac{25.0 \text{ cm}}{f_e} = 1 + \frac{25.0 \text{ cm}}{33.3 \text{ cm}} = +1.75,$$

and the image distance for the objective is

$$q_1 = L - p_e = 10.0 \text{ cm} - 14.3 \text{ cm} = -4.30 \text{ cm}$$

The thin lens equation then gives the object distance for the objective as

$$p_1 = \frac{q_1 f_1}{q_1 - f_1} = \frac{(-4.30 \text{ cm})(50.0 \text{ cm})}{-4.30 \text{ cm} - 50.0 \text{ cm}} = +3.94 \text{ cm}$$

The magnification by the objective is then

$$M_1 = -\frac{q_1}{p_1} = -\frac{(-4.30 \text{ cm})}{3.94 \text{ cm}} = +1.09,$$

and the overall magnification is $m = M_1 m_e = (+1.09)(+1.75) = \boxed{1.90}$

25.32 The angular resolution needed is

$$\theta_m = \frac{s}{r} = \frac{300 \text{ m}}{3.8 \times 10^8 \text{ m}} = 7.9 \times 10^{-7} \text{ rad}$$

For a circular aperture $\theta_m = 1.22 \dfrac{\lambda}{D}$,

so $$D = 1.22 \frac{\lambda}{\theta_m} = 1.22 \left(\frac{500 \times 10^{-9} \text{ m}}{7.9 \times 10^{-7} \text{ rad}} \right) = \boxed{0.77 \text{ m}}$$ (about 30 inches)

25.33 If just resolved, the angular separation is

$$\theta = \theta_m = 1.22\frac{\lambda}{D} = 1.22\left(\frac{500\times 10^{-9}\text{ m}}{0.300\text{ m}}\right) = 2.03\times 10^{-6}\text{ rad}$$

Thus, the altitude is $h = \frac{d}{\theta} = \frac{1.00\text{ m}}{2.03\times 10^{-6}\text{ rad}} = 4.92\times 10^{5}\text{ m} = \boxed{492\text{ km}}$

25.34 For a narrow slit, Rayleigh's criterion gives

$$\theta_m = \frac{\lambda}{a} = \frac{500\times 10^{-9}\text{ m}}{0.500\times 10^{-3}\text{ m}} = 1.00\times 10^{-3} = \boxed{1.00\text{ m rad}}$$

25.35 The limit of resolution in air is $\theta_m\big|_{air} = 1.22\frac{\lambda_0}{D} = 0.60\ \mu\text{rad}$

In oil, the limiting angle of resolution will be

$$\theta_m\big|_{oil} = 1.22\frac{\lambda_{oil}}{D} = 1.22\frac{\left(\lambda_0/n_{oil}\right)}{D} = \left(1.22\frac{\lambda_0}{D}\right)\frac{1}{n_{oil}},$$

or $$\theta_m\big|_{oil} = \frac{\theta_m\big|_{air}}{n_{oil}} = \frac{0.60\ \mu\text{rad}}{1.5} = \boxed{0.40\ \mu\text{rad}}$$

25.36 The angular separation of the headlights is

$$\theta = \frac{s}{r} = \frac{2.00\text{ m}}{10.0\times 10^{3}\text{ m}} = 2.00\times 10^{-4}\text{ rad}$$

If the lamps are barely resolved, then $\theta = \theta_m = 1.22\frac{\lambda}{D}$ and the diameter is

$$D = 1.22\frac{\lambda}{\theta} = 1.22\left(\frac{885\times 10^{-9}\text{ m}}{2.00\times 10^{-4}\text{ rad}}\right) = 5.40\times 10^{-3}\text{ m} = \boxed{5.40\text{ mm}}$$

25.37 When the beam passes through the circular opening, it diffracts into a cone with the apex at the opening. The angle intercepted by the radius of the bright central disk (i.e., the distance from the center of the beam to the first dark ring) equals the limiting angle of resolution for the aperture, or

$$\theta = \frac{r}{d} = \theta_m = 1.22\frac{\lambda}{D} = 1.22\left(\frac{632.8\times10^{-9}\text{ m}}{0.500\times10^{-2}\text{ m}}\right) = 1.54\times10^{-4}\text{ rad}$$

The diameter of the beam at $d = 10.0\text{ km}$ is then

$$diameter = 2r = 2\theta_m\cdot d = 2\left(1.54\times10^{-4}\text{ rad}\right)\left(10.0\times10^{3}\text{ m}\right) = \boxed{3.09\text{ m}}$$

25.38 If just resolved, the angular size of the object is $\theta = \theta_m = 1.22\dfrac{\lambda}{D}$

and $s = r\theta = \left(200\times10^{3}\text{ m}\right)\left[1.22\left(\dfrac{550\times10^{-9}\text{ m}}{0.35\text{ m}}\right)\right] = 0.38\text{ m} = \boxed{38\text{ cm}}$

25.39 If just resolved, the angular size of the object is $\theta = \theta_m = 1.22\dfrac{\lambda}{D}$

and $s = r\theta = \left(8.0\times10^{7}\text{ km}\right)\left[1.22\left(\dfrac{500\times10^{-9}\text{ m}}{5.00\text{ m}}\right)\right] = \boxed{9.8\text{ km}}$

25.40 A fringe shift occurs when the mirror moves distance $\lambda/4$. Thus, if the mirror moves distance $\Delta L = 0.180\text{ mm}$, the number of fringe shifts observed is

$$N_{shifts} = \frac{\Delta L}{\lambda/4} = \frac{4(\Delta L)}{\lambda} = \frac{4\left(0.180\times10^{-3}\text{ m}\right)}{550\times10^{-9}\text{ m}} = \boxed{1.31\times10^{3}\text{ fringe shifts}}$$

25.41 A fringe shift occurs when the mirror moves distance $\lambda/4$. Thus, the distance moved (length of the bacterium) as 310 shifts occur is

$$\Delta L = N_{shifts}\left(\frac{\lambda}{4}\right) = 310\left(\frac{650\times10^{-9}\text{ m}}{4}\right) = 5.04\times10^{-5}\text{ m} = \boxed{50.4\ \mu\text{m}}$$

25.42 A fringe shift occurs when the mirror moves distance $\lambda/4$. Thus, the distance the mirror moves as 250 fringe shifts are counted is

$$\Delta L = N_{shifts}\left(\frac{\lambda}{4}\right) = 250\left(\frac{632.8\times10^{-9}\text{ m}}{4}\right) = 3.96\times10^{-5}\text{ m} = \boxed{39.6\ \mu\text{m}}$$

25.43 Two fringe shifts occur each time the central spot changes from bright to dark and back to bright, so 1700 "reproductions" equals 3400 fringe shifts. Since each fringe shift represents a mirror movement of $\lambda/4$, the total distance the mirror moves is $\Delta L = N_{shifts}(\lambda/4)$. Thus, we have

$$\lambda = \frac{4(\Delta L)}{N_{shifts}} = \frac{4\left(0.382\times10^{-3}\text{ m}\right)}{3400} = 4.49\times10^{-7}\text{ m} = \boxed{449\text{ nm}}$$

The light is $\boxed{\text{blue}}$ in color.

25.44 A fringe shift will occur each time the effective length of the tube changes by a quarter of a wavelength (i.e., for each additional wavelength fitted into the length of the tube, 4 fringe shifts occur). If *L* is the length of the tube, the number of fringe shifts observed as the tube is filled with gas is

$$N_{shifts} = 4\left[\frac{L}{\lambda_n} - \frac{L}{\lambda_0}\right] = 4\left[\frac{L}{\lambda_0/n_{gas}} - \frac{L}{\lambda_0}\right] = \frac{4L}{\lambda_0}\left(n_{gas}-1\right)$$

Hence, $n_{gas} = 1 + \left(\frac{\lambda_0}{4L}\right)N_{shifts} = 1 + \left[\frac{600\times10^{-9}\text{ m}}{4\left(5.00\times10^{-2}\text{ m}\right)}\right](160) = \boxed{1.0005}$

25.45 Removing air from the cell alters the wavelength of the light passing through the cell. Four fringe shifts will occur for each additional wavelength fitted into the length of the cell. Therefore, the number of fringe shifts that occur as the cell is evacuated will be

$$N_{shifts} = 4\left[\frac{L}{\lambda_n} - \frac{L}{\lambda_0}\right] = 4\left[\frac{L}{\lambda_0/n_{air}} - \frac{L}{\lambda_0}\right] = \frac{4L}{\lambda_0}\left(n_{air}-1\right)$$

or $$N_{shifts} = \frac{4\left(5.00\times10^{-2}\text{ m}\right)}{590\times10^{-9}\text{ m}}\left(1.000\,29-1\right) = 98.3\ \boxed{\text{98 complete shifts}}$$

25.46 The resolving power of a diffraction grating is $R = \frac{\lambda}{\Delta\lambda} = Nm$.

(a) The number of lines the grating must have to resolve the H_α line in the first order is

$$N = \frac{R}{m} = \frac{\lambda/\Delta\lambda}{(1)} = \frac{656.2 \text{ nm}}{0.18 \text{ nm}} = \boxed{3.6 \times 10^3 \text{ lines}}$$

(b) In the second order $(m = 2)$, $N = \frac{R}{2} = \frac{656.2 \text{ nm}}{2(0.18 \text{ nm})} = \boxed{1.8 \times 10^3 \text{ lines}}$

25.47 The grating spacing is $d = \frac{1 \text{ cm}}{6000} = 1.67 \times 10^{-4} \text{ cm} = 1.67 \times 10^{-6} \text{ m}$, and the highest order of 600 nm light that can be observed is

$$m_{max} = \frac{d \sin 90°}{\lambda} = \frac{(1.67 \times 10^{-6} \text{ m})(1)}{600 \times 10^{-9} \text{ m}} = 2.78 \rightarrow 2 \text{ orders}$$

The total number of slits is $N = (15.0 \text{ cm})(6000 \text{ slits/cm}) = 9.00 \times 10^4$ and the resolving power of the grating in the second order is

$$R_{available} = Nm = (9.00 \times 10^4)2 = \boxed{1.80 \times 10^5}$$

The resolving power required to separate the given spectral lines is

$$R_{needed} = \frac{\lambda}{\Delta\lambda} = \frac{600.000 \text{ nm}}{0.003 \text{ nm}} = \boxed{2.0 \times 10^5}$$

These lines $\boxed{\text{cannot be separated}}$ with this grating.

25.48 (a) Since this eye can already focus on objects located at the near point of a normal eye (25 cm), no correction is needed for near objects. To correct the distant vision, a corrective lens (located 2.0 cm from the eye) should form virtual images of very distant objects at 23 cm in front of the lens (or at the far point of the eye). Thus, we must require that $q = -23 \text{ cm}$ when $p \rightarrow \infty$. This gives

$$P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-0.23 \text{ m}} = \boxed{-4.3 \text{ diopters}}$$

(b) A corrective lens in contact with the cornea should form virtual images of very distant objects at the far point of the eye. Therefore, we require that that $q = -25\text{ cm}$ when $p \to \infty$, giving

$$P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-0.25\text{ m}} = \boxed{-4.0\text{ diopters}}$$

When the contact lens $\left(f = \frac{1}{P} = -25\text{ cm} \right)$ is in place, the object distance which yields a virtual image at the near point of the eye (i.e., $q = -16\text{ cm}$) is given by

$$p = \frac{qf}{q - f} = \frac{(-16\text{ cm})(-25\text{ cm})}{-16\text{ cm} - (-25\text{ cm})} = \boxed{44\text{ cm}}$$

25.49 (a) The lens should form an upright, virtual image at the near point of the eye $q = -75.0\text{ cm}$ when the object distance is $p = 25.0\text{ cm}$. The thin lens equation then gives

$$f = \frac{pq}{p + q} = \frac{(25.0\text{ cm})(-75.0\text{ cm})}{25.0\text{ cm} - 75.0\text{ cm}} = 37.5\text{ cm} = 0.375\text{ m},$$

so the needed power is $P = \frac{1}{f} = \frac{1}{0.375\text{ m}} = \boxed{+2.67\text{ diopters}}$

(b) If the object distance must be $p = 26.0\text{ cm}$ to position the image at $q = -75.0\text{ cm}$, the actual focal length is

$$f = \frac{pq}{p + q} = \frac{(26.0\text{ cm})(-75.0\text{ cm})}{26.0\text{ cm} - 75.0\text{ cm}} = 0.398\text{ m}$$

and $P = \frac{1}{f} = \frac{1}{0.398\text{ m}} = +2.51\text{ diopters}$

The error in the power is

$$\Delta P = (2.67 - 2.51)\text{ diopters} = \boxed{0.16\text{ diopters too low}}$$

25.50 For the objective, the thin lens equation gives the image distance as

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(3.40\text{ mm})(3.00\text{ mm})}{3.40\text{ mm} - 3.00\text{ mm}} = 25.5\text{ mm}$$

The magnification by the objective lens is then

$$M_1 = -\frac{q_1}{p_1} = -\frac{25.5\text{ mm}}{3.40\text{ mm}} = -7.50$$

The eyepiece serves as a simple magnifier. Assuming the microscope is adjusted for most comfortable viewing $(q_e \to \infty)$, the magnification by the eyepiece is

$$m_e = \frac{25.0\text{ cm}}{f_e} = \frac{25.0\text{ cm}}{2.50\text{ cm}} = 10.0$$

and the overall magnification is $m = M_1 m_e = (-7.50)(10.0) = \boxed{-75.0}$

25.51 (a) The implanted lens should give an image distance of $q = 22.4\text{ mm}$ for distant $(p \to \infty)$ objects. The thin lens equation then gives the focal length as $f = q = 22.4\text{ mm}$, so the power of the implanted lens should be

$$P_{implant} = \frac{1}{f} = \frac{1}{22.4 \times 10^{-3}\text{ m}} = \boxed{+44.6\text{ diopters}}$$

(b) When the object distance is $p = 33.0\text{ cm}$, the corrective lens should produce parallel rays $(q \to \infty)$. Then the implanted lens will focus the final image on the retina. From the thin lens equation, the required focal length is $f = p = 33.0\text{ cm}$ and the power of this lens should be

$$P_{corrective} = \frac{1}{f} = \frac{1}{0.330\text{ m}} = \boxed{+3.03\text{ diopters}}$$

25.52 When viewed from a distance of 50 meters, the angular length of a mouse (assumed to have an actual length of $\approx 10\text{ cm}$) is

$$\theta = \frac{s}{r} = \frac{0.10\text{ m}}{50\text{ m}} = 2.0\times 10^{-3}\text{ radians}$$

Thus, the limiting angle of resolution of the eye of the hawk must be

$$\theta_m \le \theta = \boxed{2.0\times 10^{-3}\text{ rad}}$$

25.53 The resolving power of the grating is $R = \lambda/\Delta\lambda = Nm$. Thus, the total number of lines needed on the grating to resolve the wavelengths in order *m* is

$$N = \frac{R}{m} = \frac{\lambda}{m(\Delta\lambda)}$$

(a) For the sodium doublet in the first order,

$$N = \frac{589.30\text{ nm}}{(1)(0.59\text{ nm})} = \boxed{1.0\times 10^{3}}$$

(b) In the third order, we need $N = \frac{589.30\text{ nm}}{(3)(0.59\text{ nm})} = \boxed{3.3\times 10^{2}}$

25.54 (a) The image distance for the objective lens is

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(40.0\text{ m})(8.00\times 10^{-2}\text{ m})}{40.0\text{ m} - 8.00\times 10^{-2}\text{m}} = 8.02\times 10^{-2}\text{ m} = 8.02\text{ cm}$$

The magnification by the objective is $M_1 = h'/h = -q_1/p_1$, so the size of the image formed by this lens is

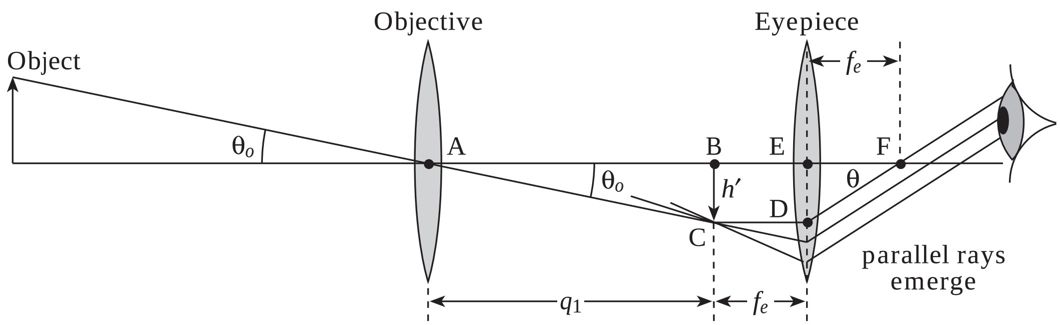
$$h' = h|M_1| = h\left(\frac{q_1}{p_1}\right) = (30.0\text{ cm})\left(\frac{8.02\times 10^{-2}\text{ m}}{40.0\text{ m}}\right) = \boxed{0.0601\text{ cm}}$$

(b) To have parallel rays emerge from the eyepiece, its virtual object must be at its focal point, or $p_e = f_e = \boxed{-2.00\text{ cm}}$.

(c) The distance between the lenses is $L = q_1 + p_e = 8.02\text{ cm} - 2.00\text{ cm} = \boxed{6.02\text{ cm}}$.

(d) The overall angular magnification is $m = \left|\frac{f_1}{f_e}\right| = \left|\frac{8.00 \text{ cm}}{-2.00 \text{ cm}}\right| = \boxed{4.00}$.

25.55 The angular magnification is $m = \theta/\theta_0$, where θ is the angle subtended by the final image and θ_0 is the angle subtended by the object as shown in the figure. When the telescope is adjusted for minimum eyestrain, the rays entering the eye are parallel. Thus, the objective lens must form its image at the focal point of the eyepiece.



From triangle ABC, $\theta_0 \approx \tan\theta_0 = h'/q_1$ and from triangle DEF, $\theta \approx \tan\theta = h'/f_e$. The angular magnification is then $m = \frac{\theta}{\theta_0} = \frac{h'/f_e}{h'/q_1} = \frac{q_1}{f_e}$.

From the thin lens equation, the image distance of the objective lens in this case is

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(300 \text{ cm})(20.0 \text{ cm})}{300 \text{ cm} - 20.0 \text{ cm}} = 21.4 \text{ cm}$$

With an eyepiece of focal length $f_e = 2.00 \text{ cm}$, the angular magnification for this telescope is

$$m = \frac{q_1}{f_e} = \frac{21.4 \text{ cm}}{2.00 \text{ cm}} = \boxed{10.7}$$

25.56 We use $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$, with $p \to \infty$ and q equal to the cornea to retina distance. Then,

$$R = q\left(\frac{n_2 - n_1}{n_2}\right) = (2.00 \text{ cm})\left(\frac{1.34 - 1.00}{1.34}\right) = 0.507 \text{ cm} = \boxed{5.07 \text{ mm}}$$

Answers to Conceptual Questions

2. The objective lens of the microscope must form a real image just inside the focal point of the eyepiece lens. In order for this to occur, the object must be located just outside the focal point of the objective lens. Since the focal length of the objective lens is typically quite short $(\sim 1\text{ cm})$, this means that the microscope can focus properly only on objects close to the end of the barrel and will be unable to focus on objects across the room.

4. The telescope is constructed so the distance between the objective and eyepiece equals the sum of the focal lengths of these lenses. For proper focusing, the objective lens forms a real image near the focal point of the eyepiece, and hence near the focal point of the objective. If this is to occur, the object distance for the objective must be very large in comparison to the focal length of that lens. Since a telescope typically contains a long focal length objective element, the telescope will be unable to focus on objects close to it.

6. The aperture of a camera is a close approximation to the iris of the eye. The retina of the eye corresponds to the film of the camera, and a close approximation to the cornea of the eye is the lens of the camera.

8. You want a real image formed at the location of the paper. To form such an image, the object distance must be greater than the focal length of the lens.

10. Under low ambient light conditions, a photoflash unit is used to insure that light entering the camera lens will deliver sufficient energy for a proper exposure to each area of the film. Thus, the most important criterion is the additional energy per unit area (product of intensity and the duration of the flash, assuming this duration is less than the shutter speed) provided by the flash unit.

Answers to Even Numbered Problems

2. 31 mm

4. 1.09 mm

6. (b) $\approx 1/100\ \text{s}$

8. 2.2 mm farther from the film

10. For the right eye, $P = -1.18\ \text{diopters}$; for the left eye, $P = -0.820\ \text{diopters}$.

12. (a) 33.3 cm (b) +3.00 diopters

14. -3.70 diopters

16. (a) -0.67 diopters (b) +0.67 diopters

18. (a) $m = +2.00$ (b) $m = +1.00$

20. (a) 4.17 cm in front of the lens (b) $m = +6.00$

22. $m = -575$

24. $0.806\ \mu\text{m}$

26. (a) $m = 800$ (b) upside down

28. (a) $m = 7.50$ (b) 0.944 m

30. (a) virtual image (b) $q_2 \to \infty$ (c) $f_o = 15.0\ \text{cm}$, $f_e = -5.00\ \text{cm}$

32. 0.77 m (≈30 inches)

34. 1.00 mrad

36. 5.40 mm

38. 38 cm

40. 1.31×10^3 fringe shifts

42. $39.6\ \mu\text{m}$

44. 1.0005

46. (a) 3.6×10^3 lines (b) 1.8×10^3 lines

48. (a) -4.3 diopters (b) -4.0 diopters, 44 cm

50. $m = -75.0$

52. $\theta_m \leq 2.0 \times 10^{-3}$ rad

54. (a) 0.0601 cm (b) -2.00 cm (c) 6.02 cm (d) $m = 4.00$

56. 5.07 mm