CHAPTER 27

Quick Quizzes

1. (b). Some energy is transferred to the electron in the scattering process. Therefore, the scattered photon must have less energy (and hence, lower frequency) than the incident photon.

2. (c). Conservation of energy requires the kinetic energy given to the electron be equal to the difference between the energy of the incident photon and that of the scattered photon.

3. (c). Conservation of momentum requires the momentum of the incident photon equal the vector sum of the momenta of the electron and the scattered photon. Since the scattered photon moves in the direction opposite that of the electron, the magnitude of the electron's momentum must exceed that of the incident photon.

4. (c). Two particles with the same de Broglie wavelength will have the same momentum $p = mv$. If the electron and proton have the same momentum, they cannot have the same speed because of the difference in their masses. For the same reason, remembering that $KE = p^2/2m$, they cannot have the same kinetic energy. Because the kinetic energy is the only type of energy an isolated particle can have, and we have argued that the particles have different energies, Equation 27.15 tells us that the particles do not have the same frequency.

5. (b). The Compton wavelength, $\lambda_C = h/m_e c$, is a combination of constants and has no relation to the motion of the electron. The de Broglie wavelength, $\lambda = h/m_e v$, is associated with the motion of the electron through its momentum.

Problem Solutions

27.1 From Wien's displacement law,

(a) $T = \dfrac{0.2898 \times 10^{-2}\ \text{m}\cdot\text{K}}{\lambda_{max}} = \dfrac{0.2898 \times 10^{-2}\ \text{m}\cdot\text{K}}{970 \times 10^{-9}\ \text{m}}$

$= 2.99 \times 10^{3}\ \text{K}$, or $\boxed{\approx 3\,000\ \text{K}}$

(b) $T = \dfrac{0.2898 \times 10^{-2}\ \text{m}\cdot\text{K}}{\lambda_{max}} = \dfrac{0.2898 \times 10^{-2}\ \text{m}\cdot\text{K}}{145 \times 10^{-9}\ \text{m}}$

$= 2.00 \times 10^{4}\ \text{K}$, or $\boxed{\approx 20\,000\ \text{K}}$

27.2 Using Wien's displacement law,

(a) $\lambda_{max} = \dfrac{0.2898 \times 10^{-2}\ \text{m}\cdot\text{K}}{10^{4}\ \text{K}}$

$= 2.898 \times 10^{-7}\ \text{m}$ $\boxed{\sim 100\ \text{nm}}$ $\boxed{\text{Ultraviolet}}$

(b) $\lambda_{max} = \dfrac{0.2898 \times 10^{-2}\ \text{m}\cdot\text{K}}{10^{7}\ \text{K}} = 2.898 \times 10^{-10}\ \text{m}$ $\boxed{\sim 10^{-1}\ \text{nm}}$ $\boxed{\gamma\text{-rays}}$

27.3 (a) The wavelength of maximum radiation is given by

$$\lambda_{max} = \frac{0.2898 \times 10^{-2}\ \text{m}\cdot\text{K}}{2900\ \text{K}} = 9.99 \times 10^{-7}\ \text{m} = \boxed{999\ \text{nm}}$$

(b) The peak wavelength is $\boxed{\text{in the infrared}}$, far from the visible region of the electromagnetic spectrum.

27.4 $E = hf = \left(6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(\dfrac{1.00\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) f = \left(4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}\right) f$

(a) $E = \left(4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}\right)\left(620 \times 10^{12}\ \text{s}^{-1}\right) = \boxed{2.57\ \text{eV}}$

(b) $E = \left(4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}\right)\left(3.10 \times 10^{9}\ \text{s}^{-1}\right) = \boxed{1.28 \times 10^{-5}\ \text{eV}}$

(c) $E = \left(4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}\right)\left(46.0 \times 10^{6}\ \text{s}^{-1}\right) = \boxed{1.91 \times 10^{-7}\ \text{eV}}$

(d) $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{620 \times 10^{12} \text{ Hz}} = 4.84 \times 10^{-7} \text{ m} = \boxed{484 \text{ nm} \quad (\text{visible light, blue})}$

$\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{3.10 \times 10^9 \text{ Hz}} = 9.68 \times 10^{-2} \text{ m} = \boxed{9.67 \text{ cm} \quad (\text{microwaves})}$

$\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{46.0 \times 10^6 \text{ Hz}} = \boxed{6.52 \text{ m} \quad (\text{radio waves})}$

27.5 $E = hf = \dfrac{hc}{\lambda} = \dfrac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{\lambda}\left(\dfrac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) f,$

which yields $E = \dfrac{1.24 \times 10^{-6} \text{ m}\cdot\text{eV}}{\lambda}$

(a) $E = \dfrac{1.24 \times 10^{-6} \text{ m}\cdot\text{eV}}{5.00 \times 10^{-2} \text{ m}} = \boxed{2.49 \times 10^{-5} \text{ eV}}$

(b) $E = \dfrac{1.24 \times 10^{-6} \text{ m}\cdot\text{eV}}{500 \times 10^{-9} \text{ m}} = \boxed{2.49 \text{ eV}}$

(c) $E = \dfrac{1.24 \times 10^{-6} \text{ m}\cdot\text{eV}}{5.00 \times 10^{-9} \text{ m}} = \boxed{249 \text{ eV}}$

27.6 The energy of a single photon is

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{589.3 \times 10^{-9} \text{ m}} = 3.38 \times 10^{-19} \text{ J}$$

The number of photons emitted by the lamp in $\Delta t = 1.00 \text{ s}$ is

$$N = \frac{\Delta E}{E_\gamma} = \frac{\wp \cdot (\Delta t)}{E_\gamma} = \frac{(1000 \text{ J/s})(1.00 \text{ s})}{3.38 \times 10^{-19} \text{ J}} = \boxed{2.96 \times 10^{21}}$$

27.7 The energy of a single photon is

$$E_\gamma = hf = \left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(99.7\times10^{6}\text{ s}^{-1}\right) = 6.61\times10^{-26}\text{ J}$$

The number of photons emitted in $\Delta t = 1.00\text{ s}$ is

$$N = \frac{\Delta E}{E_\gamma} = \frac{\wp\cdot(\Delta t)}{E_\gamma} = \frac{\left(150\times10^{3}\text{ J/s}\right)(1.00\text{ s})}{6.61\times10^{-26}\text{ J}} = \boxed{2.27\times10^{30}}$$

27.8 The energy entering the eye each second is

$$\wp = I\cdot A = \left(4.0\times10^{-11}\text{ W/m}^2\right)\left[\frac{\pi}{4}\left(8.5\times10^{-3}\text{ m}\right)^2\right] = 2.3\times10^{-15}\text{ W}$$

The energy of a single photon is

$$E_\gamma = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{500\times10^{-9}\text{ m}} = 3.98\times10^{-19}\text{ J},$$

so the number of photons entering the eye in $\Delta t = 1.00\text{ s}$ is

$$N = \frac{\Delta E}{E_\gamma} = \frac{\wp\cdot(\Delta t)}{E_\gamma} = \frac{\left(2.3\times10^{-15}\text{ J/s}\right)(1.00\text{ s})}{3.98\times10^{-19}\text{ J}} = \boxed{5.7\times10^{3}}$$

27.9 The frequency of the oscillator is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{20\text{ N/m}}{1.5\text{ kg}}} = 0.58\text{ Hz}$,

and its total energy is $E = \frac{1}{2}kA^2 = \frac{(20\text{ N/m})\left(3.0\times10^{-2}\text{ m}\right)^2}{2} = 9.0\times10^{-3}\text{ J}$

(a) From $E = nhf$, the quantum number is

$$n = \frac{E}{hf} = \frac{9.0\times10^{-3}\text{ J}}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(0.58\text{ s}^{-1}\right)} = \boxed{2.3\times10^{31}}$$

(b) $\Delta E = (\Delta n)hf$, so the fractional change in energy is

$$\frac{\Delta E}{E} = \frac{(1)\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(0.58\text{ s}^{-1}\right)}{9.0\times10^{-3}\text{ J}} = \boxed{4.3\times10^{-32}}$$

27.10 The energy to be given up by the mass is

$$\Delta E = mg\left|\Delta y\right| = (4.9\text{ N})(3.0\text{ m}) = 15\text{ J}$$

The energy of each photon is

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{5.0\times10^{-7}\text{ m}} = 4.0\times10^{-19}\text{ J},$$

so the number of photons that would be generated is

$$N = \frac{\Delta E}{E_\gamma} = \frac{15\text{ J/s}}{4.0\times10^{-19}\text{ J}} = \boxed{3.7\times10^{19}}$$

27.11 (a) From the photoelectric effect equation, the work function is

$$\phi = \frac{hc}{\lambda} - KE_{max}\text{, or}$$

$$\phi = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{350\times10^{-9}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) - 1.31\text{ eV}$$

$$\phi = \boxed{2.24\text{ eV}}$$

(b) $$\lambda_c = \frac{hc}{\phi} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{2.24\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{555\text{ nm}}$$

(c) $$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^{8}\text{ m/s}}{555\times10^{-9}\text{ m}} = \boxed{5.41\times10^{14}\text{ Hz}}$$

27.12 (a) The maximum kinetic energy of the ejected electrons is

$$KE_{max} = \frac{1}{2}m_e v_{max}^2 = \frac{1}{2}\left(9.11\times 10^{-31}\text{ kg}\right)\left(4.6\times 10^{5}\text{ m/s}\right)^2 = 9.6\times 10^{-20}\text{ J}$$

The work function of the surface is

$$\phi = \frac{hc}{\lambda} - KE_{max} = \frac{\left(6.63\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times 10^{8}\text{ m/s}\right)}{625\times 10^{-9}\text{ m}} - 9.6\times 10^{-20}\text{ J},$$

or $\phi = 2.2\times 10^{-19}\text{ J}\left(\frac{1\text{ eV}}{1.60\times 10^{-19}\text{ J}}\right) = \boxed{1.4\text{ eV}}$

(b) $f_c = \frac{\phi}{h} = \frac{2.2\times 10^{-19}\text{ J}}{6.63\times 10^{-34}\text{ J}\cdot\text{s}} = \boxed{3.3\times 10^{14}\text{ Hz}}$

27.13 (a) $\lambda_c = \frac{hc}{\phi} = \frac{\left(6.63\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times 10^{8}\text{ m/s}\right)}{\left(4.20\text{ eV}\right)\left(1.60\times 10^{-19}\text{ J/eV}\right)} = 2.96\times 10^{-7}\text{ m} = \boxed{296\text{ nm}}$

$$f_c = \frac{c}{\lambda_c} = \frac{3.00\times 10^{8}\text{ m/s}}{2.96\times 10^{-7}\text{ m}} = \boxed{1.01\times 10^{15}\text{ Hz}}$$

(b) $e\left(\Delta V_s\right) = KE_{max} = \frac{hc}{\lambda} - \phi$, so $\Delta V_s = \frac{hc}{\lambda e} - \frac{\phi}{e}$ or

$$\Delta V_s = \frac{\left(6.63\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times 10^{8}\text{ m/s}\right)}{\left(180\times 10^{-9}\text{ m}\right)\left(1.60\times 10^{-19}\text{ C}\right)} - \frac{4.20\text{ eV}\left(1.60\times 10^{-19}\text{ J/eV}\right)}{1.60\times 10^{-19}\text{ C}} = \boxed{2.71\text{ V}}$$

27.14 (a) The energy of the incident photons is

$$E_\gamma = \frac{hc}{\lambda} = \frac{\left(6.63\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times 10^{8}\text{ m/s}\right)}{400\times 10^{-9}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times 10^{-19}\text{ J}}\right) = 3.11\text{ eV}$$

For photo-electric emission to occur, it is necessary that $E_\gamma \geq \phi$. Thus, of the three metals given, $\boxed{\text{only lithium}}$ will exhibit the photo-electric effect.

(b) For lithium, $KE_{max} = \frac{hc}{\lambda} - \phi = 3.11\text{ eV} - 2.30\text{ eV} = \boxed{0.81\text{ eV}}$

27.15 The energy absorbed each second is

$$\wp = I \cdot A = \left(500\ \text{W/m}^2\right)\left[\pi\left(2.82 \times 10^{-15}\ \text{m}\right)^2\right] = 1.25 \times 10^{-26}\ \text{W}$$

The time required to absorb $E = 1.00\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$ is

$$t = \frac{E}{\wp} = \frac{1.60 \times 10^{-19}\ \text{J}}{1.25 \times 10^{-26}\ \text{J/s}} = 1.28 \times 10^{7}\ \text{s}\left(\frac{1\ \text{d}}{8.64 \times 10^{4}\ \text{s}}\right) = \boxed{148\ \text{d}}$$

This prediction, based on classical theory, is $\boxed{\text{incompatible with observation}}$

27.16 Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $KE_{max} = \dfrac{hc}{\lambda} - \phi$, or

$$KE_{max} = \frac{\left(6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{200 \times 10^{-9}\ \text{m}}\left(\frac{1.00\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) - 4.70\ \text{eV} = 1.52\ \text{eV}$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.52 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

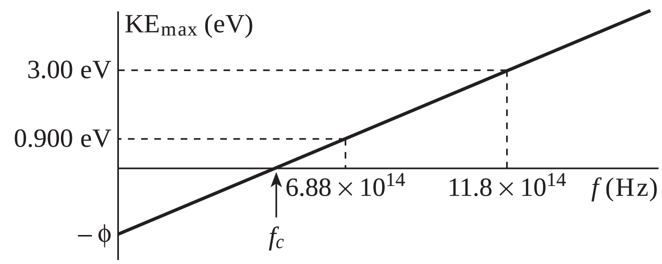
$$V = \frac{k_e Q}{r} \qquad \text{or} \qquad Q = \frac{rV}{k_e} = \frac{\left(5.00 \times 10^{-2}\ \text{m}\right)\left(1.52\ \text{N}\cdot\text{m/C}\right)}{8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.43 \times 10^{-12}\ \text{C}}$$

27.17 The two light frequencies allowed to strike the surface are

$$f_1 = \frac{c}{\lambda_1} = \frac{3.00 \times 10^{8}\ \text{m/s}}{254 \times 10^{-9}\ \text{m}} = 11.8 \times 10^{14}\ \text{Hz},$$

and $$f_2 = \frac{3.00 \times 10^{8}\ \text{m/s}}{436 \times 10^{-9}\ \text{m}} = 6.88 \times 10^{14}\ \text{Hz}$$

The graph you draw should look somewhat like that given at the right.



The desired quantities, read from the axis intercepts of the graph line, should agree within their uncertainties with

$$f_c = \boxed{4.8 \times 10^{14}\ \text{Hz}} \text{ and } \phi = \boxed{2.0\ \text{eV}}$$

27.18 The total energy absorbed by an electron is

$$E_\gamma = \phi + KE_{max} = 3.44\ \text{eV}\left(\frac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right) + \frac{1}{2}\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(4.2 \times 10^5\ \text{m/s}\right)^2,$$

or $E_\gamma = 6.3 \times 10^{-19}\ \text{J}$

The energy absorbed by a square centimeter of surface in one second is

$$E = \wp \cdot t = (I \cdot A) \cdot t = \left(0.055\ \text{W/m}^2\right)\left(1.00 \times 10^{-4}\ \text{m}^2\right)\left(1.00\ \text{s}\right) = 5.5 \times 10^{-6}\ \text{J},$$

so the number of electrons released per second is

$$N = \frac{E}{E_\gamma} = \frac{5.5 \times 10^{-6}\ \text{J}}{6.3 \times 10^{-19}\ \text{J}} = \boxed{8.7 \times 10^{12}}$$

27.19 Assuming the electron produces a single photon as it comes to rest, the energy of that photon is $E_\gamma = (KE)_i = eV$. The accelerating voltage is then

$$V = \frac{E_\gamma}{e} = \frac{hc}{e\lambda} = \frac{\left(6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^8\ \text{m/s}\right)}{\left(1.60 \times 10^{-19}\ \text{C}\right)\lambda} = \frac{1.24 \times 10^{-6}\ \text{V}\cdot\text{m}}{\lambda}$$

For $\lambda = 1.0 \times 10^{-8}\ \text{m}$, $V = \dfrac{1.24 \times 10^{-6}\ \text{V}\cdot\text{m}}{1.0 \times 10^{-8}\ \text{m}} = \boxed{1.2 \times 10^2\ \text{V}}$,

and for $\lambda = 1.0 \times 10^{-13}\ \text{m}$, $V = \dfrac{1.24 \times 10^{-6}\ \text{V}\cdot\text{m}}{1.0 \times 10^{-13}\ \text{m}} = \boxed{1.2 \times 10^7\ \text{V}}$

27.20 A photon of maximum energy and minimum wavelength is produced when the electron gives up all its kinetic energy in a single collision.

$$\lambda_{min} = \frac{hc}{(E_\gamma)_{max}} = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})V} = \frac{1.24\times10^{-6}\text{ V}\cdot\text{m}}{V}$$

(a) If $V = 15.0\text{ kV}$, $\lambda_{min} = \dfrac{1.24\times10^{-6}\text{ V}\cdot\text{m}}{15.0\times10^{3}\text{ V}} = \boxed{8.29\times10^{-11}\text{ m}}$

(b) If $V = 100\text{ kV}$, $\lambda_{min} = \dfrac{1.24\times10^{-6}\text{ V}\cdot\text{m}}{100\times10^{3}\text{ V}} = \boxed{1.24\times10^{-11}\text{ m}}$

27.21 $V = \dfrac{KE}{e} = \dfrac{E_\gamma}{e} = \dfrac{hc}{e\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.0300\times10^{-9}\text{ m})}$

$= 4.14\times10^{4}\text{ V} = \boxed{41.4\text{ kV}}$

27.22 From Bragg's law, the wavelength of the reflected x-rays is

$$\lambda = \frac{2d\sin\theta}{m} = \frac{2(0.353\text{ nm})\sin 20.5^\circ}{2} = \boxed{0.124\text{ nm}}$$

27.23 Using Bragg's law, the wavelength is found to be

$$\lambda = \frac{2d\sin\theta}{m} = \frac{2(0.296\text{ nm})\sin 7.6^\circ}{1} = \boxed{0.078\text{ nm}}$$

27.24 The first-order constructive interference occurs at the smallest grazing angle. From Bragg's law, this angle is

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left[\frac{(1)(0.070\text{ nm})}{2(0.30)}\right] = \boxed{6.7^\circ}$$

27.25 The interplanar spacing in the crystal is given by Bragg's law as

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.140\text{ nm})}{2\sin 14.4^\circ} = \boxed{0.281\text{ nm}}$$

27.26 The scattering angle is given by the Compton shift formula as

$$\theta = \cos^{-1}\left(1 - \frac{\Delta\lambda}{\lambda_C}\right)$$ where the Compton wavelength is

$$\lambda_C = \frac{h}{m_e c^2} = 0.00243 \text{ nm}$$

Thus, $$\theta = \cos^{-1}\left(1 - \frac{1.50\times10^{-3}\text{ nm}}{2.43\times10^{-3}\text{ nm}}\right) = \boxed{67.5^\circ}$$

27.27 $$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{700\times10^{-9}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{1.78\text{ eV}}$$

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{700\times10^{-9}\text{ m}} = \boxed{9.47\times10^{-28}\text{ kg}\cdot\text{m/s}}$$

27.28 Using the Compton shift formula, the wavelength is found to be

$$\lambda = \lambda_0 + \Delta\lambda = \lambda_0 + \lambda_C(1-\cos\theta)$$
$$= 0.68\text{ nm} + (0.00243\text{ nm})(1-\cos45^\circ) = 0.6807\text{ nm}$$

Therefore,

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.6807\times10^{-9}\text{ m}}\left(\frac{1\text{ keV}}{1.60\times10^{-16}\text{ J}}\right) = \boxed{1.8\text{ keV}},$$

and $$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{0.6807\times10^{-9}\text{ m}} = \boxed{9.7\times10^{-25}\text{ kg}\cdot\text{m/s}}$$

27.29 If the scattered photon has energy equal to the kinetic energy of the recoiling electron, the energy of the incident photon is divided equally between them. Thus,

$$E_\gamma = \frac{(E_\gamma)_0}{2} \Rightarrow \frac{hc}{\lambda} = \frac{hc}{2\lambda_0}\text{, so } \lambda = 2\lambda_0 \text{ and } \Delta\lambda = 2\lambda_0 - \lambda_0 = 0.0016 \text{ nm}$$

The Compton scattering formula then gives the scattering angle as

$$\theta = \cos^{-1}\left(1 - \frac{\Delta\lambda}{\lambda_C}\right) = \cos^{-1}\left(1 - \frac{0.0016 \text{ nm}}{0.00243 \text{ nm}}\right) = \boxed{70°}$$

27.30 (a) $\Delta\lambda = \lambda_C(1-\cos\theta) = (0.00243 \text{ nm})(1-\cos 37.0°) = \boxed{4.89\times10^{-4} \text{ nm}}$

(b) The wavelength of the incident x-rays is

$$\lambda_0 = \frac{hc}{(E_\gamma)_0} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{(300 \text{ keV})(1.60\times10^{-16} \text{ J/keV})}\left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = 4.14\times10^{-3} \text{ nm}$$

so the scattered wavelength is $\lambda = \lambda_0 + \Delta\lambda = 4.63\times10^{-3} \text{ nm}$

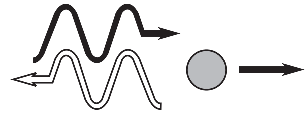
The energy of the scattered photons is then

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{(4.63\times10^{-3} \text{ nm})(10^{-9} \text{ m/1 nm})}\left(\frac{1 \text{ keV}}{1.60\times10^{-16} \text{ J}}\right) = \boxed{268 \text{ keV}}$$

(c) The kinetic energy of the recoiling electrons is

$$KE = (E_\gamma)_0 - E_\gamma = 300 \text{ keV} - 268 \text{ keV} = \boxed{32 \text{ keV}}$$

27.31 This is Compton scattering with $\theta = 180°$, so the Compton shift is $\Delta\lambda = \lambda_C(1-\cos 180°) = 2\lambda_C = 0.00486 \text{ nm}$ and the scattered wavelength is



$$\lambda = \lambda_0 + \Delta\lambda = (0.110 + 0.00486) \text{ nm} = 0.115 \text{ nm}$$

The kinetic energy of the recoiling electron is then

$$KE = (E_\gamma)_0 - E_\gamma = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) = \frac{hc(\Delta\lambda)}{\lambda_0 \lambda}$$

$$= \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})(0.00486 \text{ nm})}{(0.110\times10^{-9} \text{ m})(0.115 \text{ nm})} = 7.65\times10^{-17} \text{ J}$$

or $\quad KE = (7.65\times10^{-17} \text{ J})(1 \text{ eV}/1.60\times10^{-19} \text{ J}) = \boxed{478 \text{ eV}}$

The momentum of the recoiling electron (non-relativistic) is

$$p_e = \sqrt{2m_e(KE)} = \sqrt{2(9.11\times10^{-31} \text{ kg})(7.64\times10^{-17} \text{ J})} = \boxed{1.18\times10^{-23} \text{ kg}\cdot\text{m/s}}$$

27.32 The kinetic energy of the recoiling electron (non-relativistic) is

$$KE = \frac{1}{2}m_e v^2 = \frac{1}{2}(9.11\times10^{-31} \text{ kg})(1.40\times10^{6} \text{ m/s})^2 = 8.93\times10^{-19} \text{ J}$$

Also, $KE = (E_\gamma)_0 - E_\gamma = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) = \frac{hc(\Delta\lambda)}{\lambda_0\lambda} \approx \frac{hc(\Delta\lambda)}{\lambda_0^2}$

(a) The Compton shift is then

$$\Delta\lambda = \frac{\lambda_0^2(KE)}{hc} = \frac{(0.800\times10^{-9} \text{ m})^2(8.93\times10^{-19} \text{ J})}{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}$$

$$= 2.87\times10^{-12} \text{ m} = \boxed{0.00287 \text{ nm}}$$

(b) From the Compton shift formula,

$$\theta = \cos^{-1}\left(1 - \frac{\Delta\lambda}{\lambda_C}\right) = \cos^{-1}\left(1 - \frac{0.00287 \text{ nm}}{0.00243 \text{ nm}}\right) = \boxed{100°}$$

27.33 (a) The kinetic energy of the recoiling electron is

$$KE = \left(E_\gamma\right)_0 - E_\gamma = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) = \frac{hc(\Delta\lambda)}{\lambda_0\lambda} \approx \frac{hc(\Delta\lambda)}{\lambda_0^2} = \frac{hc\left[\lambda_C\left(1-\cos\theta\right)\right]}{\lambda_0^2}$$

$$= \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)\left(0.00243\text{ nm}\right)\left(1-\cos 23°\right)}{\left(0.45\text{ nm}\right)^2\left(10^{-9}\text{ m/1 nm}\right)}$$

$$KE = 1.9\times10^{-19}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{1.2\text{ eV}}$$

(b) $$v = \sqrt{\frac{2(KE)}{m_e}} = \sqrt{\frac{2\left(1.9\times10^{-19}\text{ J}\right)}{9.11\times10^{-31}\text{ kg}}} = \boxed{6.5\times10^{5}\text{ m/s}}$$

27.34 The de Broglie wavelength is $\lambda = h/p$, where p is the linear momentum.

If relativistic effects are ignored, $p = \sqrt{2m\left(KE\right)}$

(a) If $KE = 50.0\text{ eV}$ and the particle is an electron,

$$\lambda = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left(50.0\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}}$$

$$= 1.74\times10^{-10}\text{ m} = \boxed{0.174\text{ nm}}$$

(b) For 50.0 keV electrons,

$$\lambda = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left(50.0\times10^{3}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}} = \boxed{5.49\times10^{-12}\text{ m}}$$

27.35 (a) From $\lambda = h/p = h/mv$, the speed is

$$v = \frac{h}{m_e\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{\left(9.11\times10^{-31}\text{ kg}\right)\left(5.00\times10^{-7}\text{ m}\right)} = 1.46\times10^{3}\text{ m/s} = \boxed{1.46\text{ km/s}}$$

(b) $$\lambda = \frac{h}{m_e v} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{\left(9.11\times10^{-31}\text{ kg}\right)\left(1.00\times10^{7}\text{ m/s}\right)} = \boxed{7.28\times10^{-11}\text{ m}}$$

27.36 For non-relativistic particles, the de Broglie wavelength is $\lambda = h/p = h/\sqrt{2m(KE)}$. Since the kinetic energy equals the loss of potential energy, $KE = e(\Delta V)$, we find

$$\Delta V = \frac{h^2}{2m_e e\lambda^2} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(1.0\times10^{-10}\ \text{m}\right)^2} = \boxed{1.5\times10^2\ \text{V}}$$

27.37 (a) The momentum of the electron would be

$$p = \frac{h}{\lambda} \geq \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} \sim 7\times10^{-20}\ \text{kg}\cdot\text{m/s}$$

If the electron is nonrelativistic, then its speed would be

$$v = \frac{p}{m_e} \sim \frac{7\times10^{-20}\ \text{kg}\cdot\text{m/s}}{9.11\times10^{-31}\ \text{kg}} \sim 8\times10^{10}\ \text{m/s} >> c$$

which is impossible. Thus, a relativistic calculation is required.

With a rest energy of $E_R = 0.511\ \text{MeV} \approx 8\times10^{-14}\ \text{J}$, its kinetic energy is

$$KE = E - E_R = \sqrt{p^2c^2 + E_R^2} - E_R$$

Thus, $KE \sim \sqrt{\left(7\times10^{-20}\ \text{kg}\cdot\text{m/s}\right)^2\left(3\times10^8\ \text{m/s}\right)^2 + \left(8\times10^{-14}\ \text{J}\right)^2} - 8\times10^{-14}\ \text{J}$

or $KE \sim 2\times10^{-11}\ \text{J}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) \to \boxed{\sim10^2\ \text{MeV}}$ or more

(b) The negative electrical potential energy of the electron (i.e., binding energy) would be

$$V = \frac{k_e q_1 q_2}{r} \sim \frac{\left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10^{-19}\ \text{C}\right)(-e)}{10^{-14}\ \text{m}} = -10^5\ \text{eV} = -10^{-1}\ \text{MeV}$$

With its kinetic energy much greater than the magnitude of its negative potential energy, the electron $\boxed{\text{would immediately escape}}$ from the nucleus.

27.38 (a) The de Broglie wavelength is $\lambda = h/p = h/mv$, so $v = h/m\lambda$.

With $\lambda = w/10 = 0.075\text{ m} \sim 10^{-1}\text{ m}$ and $m = 80\text{ kg} \sim 10^{2}\text{ kg}$, we find

$$v \sim \frac{6\times10^{-34}\text{ J}\cdot\text{s}}{\left(10^{2}\text{ kg}\right)\left(10^{-1}\text{ m}\right)} = 6\times10^{-35}\text{ m/s or } \boxed{\sim 10^{-34}\text{ m/s}}$$

(b) With $d = 0.15\text{ m} \sim 10^{-1}\text{ m}$, the maximum time to pass through the door in order to avoid significant diffraction is

$$t_{min} = \frac{d}{v} \sim \frac{10^{-1}\text{ m}}{10^{-34}\text{ m/s}} \text{ or } \boxed{\sim 10^{33}\text{ s}}$$

(c) $\boxed{\text{No.}}$ The maximum time to pass through the door is $\sim 10^{15}$ times the age of the Universe.

27.39 For relativistic particles, $p = \dfrac{\sqrt{E^2 - E_R^2}}{c}$ and $\lambda = \dfrac{h}{p} = \dfrac{hc}{\sqrt{E^2 - E_R^2}}$.

For 3.00 MeV electrons, $E = KE + E_R = 3.00\text{ MeV} + 0.511\text{ MeV} = 3.51\text{ MeV}$, so

$$\lambda = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\sqrt{\left(3.51\text{ MeV}\right)^2 - \left(0.511\text{ MeV}\right)^2}}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = \boxed{3.58\times10^{-13}\text{ m}}$$

27.40 If $\lambda = h/p = \lambda_C$, where $\lambda_C = h/m_e c$, the momentum must be $p = m_e c$.

The total energy is $E = \gamma\left(m_e c^2\right) = \sqrt{p^2c^2 + \left(m_e c^2\right)^2}$.

With $p = m_e c$, this reduces to $\gamma = \sqrt{2}$. Since $\gamma = \dfrac{1}{\sqrt{1-\left(v/c\right)^2}}$, we have

$$v = c\sqrt{1 - \frac{1}{\gamma^2}} = c\sqrt{1 - \frac{1}{2}} = \boxed{\frac{c}{\sqrt{2}}}$$

27.41 (a) The required electron momentum is

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{1.0\times10^{-11}\text{ m}}\left(\frac{1\text{ keV}}{1.60\times10^{-16}\text{ J}}\right)=4.1\times10^{-7}\ \frac{\text{keV}\cdot\text{s}}{\text{m}},$$

and the total energy is

$$E=\sqrt{p^2c^2+E_R^2}$$

$$=\sqrt{\left(4.1\times10^{-7}\ \frac{\text{keV}\cdot\text{s}}{\text{m}}\right)^2\left(3.00\times10^8\text{ m/s}\right)^2+\left(511\text{ keV}\right)^2}=526\text{ keV}$$

The kinetic energy is then,

$$KE=E-E_R=526\text{ keV}-511\text{ keV}=\boxed{15\text{ keV}}$$

(b) $E_\gamma=\dfrac{hc}{\lambda}$

$$=\frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{1.0\times10^{-11}\text{ m}}\left(\frac{1\text{ keV}}{1.60\times10^{-16}\text{ J}}\right)=\boxed{1.2\times10^2\text{ keV}}$$

27.42 $p=mv$, and $\Delta p=m(\Delta v)$ assuming m is without uncertainty.

Since $\Delta v=1.0\times10^{-3}\,v=3.0\times10^{-2}\text{ m/s}$, we have

$$\Delta p=\left(50.0\times10^{-3}\text{ kg}\right)\left(3.0\times10^{-2}\text{ m/s}\right)=1.5\times10^{-3}\text{ kg}\cdot\text{m/s},$$

and $$\Delta x\geq\frac{h}{4\pi(\Delta p)}=\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(1.5\times10^{-3}\text{ kg}\cdot\text{m/s}\right)}=\boxed{3.5\times10^{-32}\text{ m}}$$

27.43 With an uncertainty of Δx in position, the minimum uncertainty in the speed is

$$\Delta v=\frac{\Delta p}{m}\geq\frac{h}{4\pi m(\Delta x)}=\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(0.50\text{ kg}\right)\left(0.50\times10^{-2}\text{ m}\right)}=2.1\times10^{-32}\ \frac{\text{m}}{\text{s}}$$

Since the speed must be at least as large as its own uncertainty,

$$v_{min}=\boxed{2.1\times10^{-32}\text{ m/s}}$$

27.44 (a) With uncertainty Δx in position, the minimum uncertainty in the speed is

$$\Delta v = \frac{\Delta p}{m} \geq \frac{h}{4\pi m(\Delta x)} = \frac{2\pi \text{ J}\cdot\text{s}}{4\pi(2.00 \text{ kg})(1.00 \text{ m})} = \boxed{0.250 \text{ m/s}}$$

(b) Fuzzy might move by $(0.250 \text{ m/s})(5.00 \text{ s}) = 1.25 \text{ m}$. With original uncertainty of 1.00 m, we can think of Δx growing to

$$1.00 \text{ m} + 1.25 \text{ m} = \boxed{2.25 \text{ m}}$$

27.45 With $\Delta x = 5.00 \times 10^{-7} \text{ m/s}$, the minimum uncertainty in the speed is

$$\Delta v = \frac{\Delta p}{m_e} \geq \frac{h}{4\pi m_e(\Delta x)} = \frac{6.63\times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(9.11\times 10^{-31} \text{ kg})(5.00\times 10^{-7} \text{ m})} = \boxed{116 \text{ m/s}}$$

27.46 (a) For a non-relativistic particle, $KE = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \boxed{\frac{p^2}{2m}}$

(b) From the uncertainty principle,

$$\Delta p \geq \frac{h}{4\pi(\Delta x)} = \frac{6.63\times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(1.0\times 10^{-15} \text{ m})} = 5.3\times 10^{-20} \text{ kg}\cdot\text{m/s}$$

Since the momentum must be at least as large as its own uncertainty, the minimum kinetic energy is

$$KE_{min} = \frac{p_{min}^2}{2m} = \frac{(5.3\times 10^{-20} \text{ kg}\cdot\text{m/s})^2}{2(1.67\times 10^{-27} \text{ kg})}\left(\frac{1 \text{ MeV}}{1.60\times 10^{-13} \text{ J}}\right) = \boxed{5.2 \text{ MeV}}$$

27.47 The peak radiation occurs at approximately 560 nm wavelength. From Wien's displacement law,

$$T = \frac{0.2898\times 10^{-2} \text{ m}\cdot\text{K}}{\lambda_{max}} = \frac{0.2898\times 10^{-2} \text{ m}\cdot\text{K}}{560\times 10^{-9} \text{ m}} \approx \boxed{5200 \text{ K}}$$

Clearly, a firefly is not at this temperature,

so $\boxed{\text{this is not blackbody radiation}}$

27.48 (a) Minimum wavelength photons are produced when an electron gives up all its kinetic energy in a single collision. Then, $E_\gamma = 50\,000 \text{ eV}$ and

$$\lambda_{min} = \frac{hc}{E_\gamma} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(5.00\times10^{4}\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = \boxed{2.49\times10^{-11}\text{ m}}$$

(b) From Bragg's law, the interplanar spacing is

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(2.49\times10^{-11}\text{ m})}{2\sin(2.50°)} = 2.85\times10^{-10}\text{ m} = \boxed{0.285\text{ nm}}$$

27.49 The x-ray wavelength is $\lambda = hc/E_\gamma$, so Bragg's law yields

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{m\,hc}{2dE_\gamma}\right),$$

or $$\theta = \sin^{-1}\left[\frac{2(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{2(0.352\times10^{-9}\text{ m})(11.3\text{ keV})(1.60\times10^{-16}\text{ J/keV})}\right] = \boxed{18.2°}$$

27.50 (a) From $v^2 = v_i^2 + 2a_y(\Delta y)$, Johnny's speed just before impact is

$$v = \sqrt{2g|\Delta y|} = \sqrt{2(9.80\text{ m/s})(50.0\text{ m})} = 31.3\text{ m/s},$$ and

his de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(75.0\text{ kg})(31.3\text{ m/s})} = \boxed{2.82\times10^{-37}\text{ m}}$$

(b) The energy uncertainty is

$$\Delta E \geq \frac{h}{4\pi(\Delta t)} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(5.00\times10^{-3}\text{ s})} = \boxed{1.06\times10^{-32}\text{ J}}$$

(c) $$\%\text{ error} = \frac{\Delta E}{mg|\Delta y|}(100\%)$$

$$\geq \frac{(1.06\times10^{-32}\text{ J})(100\%)}{(75.0\text{ kg})(9.80\text{ m/s}^2)(50.0\text{ m})} \geq \boxed{2.87\times10^{-35}\%}$$

27.51 The magnetic force supplies the centripetal acceleration for the electrons, so $m\dfrac{v^2}{r} = qvB$, or $p = mv = qrB$

The maximum kinetic energy is then $KE_{max} = \dfrac{p^2}{2m} = \dfrac{q^2 r^2 B^2}{2m}$, or

$$KE_{max} = \frac{\left(1.60\times 10^{-19}\text{ J}\right)^2 (0.200\text{ m})^2 \left(2.00\times 10^{-5}\text{ T}\right)^2}{2\left(9.11\times 10^{-31}\text{ kg}\right)} = 2.25\times 10^{-19}\text{ J}$$

The work function of the surface is given by $\phi = E_\gamma - KE_{max} = hc/\lambda - KE_{max}$, or

$$\phi = \frac{\left(6.63\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times 10^{8}\text{ m/s}\right)}{450\times 10^{-9}\text{ m}} - 2.25\times 10^{-19}\text{ J}$$

$$= 2.17\times 10^{-19}\text{ J}\left(\frac{1\text{ eV}}{1.60\times 10^{-19}\text{ J}}\right) = \boxed{1.36\text{ eV}}$$

27.52 The incident wavelength is $\lambda_0 = hc/E_\gamma$, and the Compton shift is

$$\Delta\lambda = \frac{h}{m_p c}(1-\cos\theta) = \frac{hc}{m_p c^2}(1-\cos\theta) = \frac{hc}{\left(E_R\right)_{proton}}(1-\cos\theta)$$

The scattered wavelength is $\lambda = \lambda_0 + \Delta\lambda = hc\left[\dfrac{1}{E_\gamma} + \dfrac{1-\cos\theta}{\left(E_R\right)_{proton}}\right]$, or

$$\lambda = \frac{\left(6.63\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times 10^{8}\text{ m/s}\right)}{1.60\times 10^{-13}\text{ J/MeV}}\left[\frac{1}{200\text{ MeV}} + \frac{1-\cos(40.0^\circ)}{939\text{ MeV}}\right] = 6.53\times 10^{-15}\text{ m}$$

The energy of the scattered photon is then

$$E'_\gamma = \frac{hc}{\lambda} = \frac{\left(6.63\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times 10^{8}\text{ m/s}\right)}{6.53\times 10^{-15}\text{ m}}\left(\frac{1\text{ MeV}}{1.60\times 10^{-13}\text{ J}}\right) = \boxed{191\text{ MeV}}$$

27.53 From the photoelectric effect equation, $KE_{max} = E_\gamma - \phi = \dfrac{hc}{\lambda} - \phi$.

For $\lambda = \lambda_0$, $KE_{max} = 1.00\text{ eV}$ so $\quad 1.00\text{ eV} = \dfrac{hc}{\lambda_0} - \phi \qquad (1)$

For $\lambda = \dfrac{\lambda_0}{2}$, $KE_{max} = 4.00\text{ eV}$ giving $\quad 4.00\text{ eV} = \dfrac{2hc}{\lambda_0} - \phi \qquad (2)$

Multiplying equation (1) by a factor of 2 and subtracting

the result from equation (2) gives the work function as $\phi = \boxed{2.00\text{ eV}}$

27.54 From the photoelectric effect equation, $KE_{max} = E_\gamma - \phi = \dfrac{hc}{\lambda} - \phi$.

For $\lambda = 670\text{ nm}$, $KE_{max} = E_1$ so $\quad E_1 = \dfrac{hc}{670\text{ nm}} - \phi \qquad (1)$

For $\lambda = 520\text{ nm}$, $KE_{max} = 1.50E_1$ giving $\quad 1.50E_1 = \dfrac{hc}{520\text{ nm}} - \phi \qquad (2)$

Multiplying equation (1) by a factor of 1.50 and subtracting the result from equation (2) gives the work function as

$$\phi = hc\left(\frac{3.00}{670\text{ nm}} - \frac{2.00}{520\text{ nm}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)$$

$$\phi = 1.26 \times 10^{-19}\text{ J}\left(\frac{1\text{ eV}}{1.60 \times 10^{-19}\text{ J}}\right) = \boxed{0.785\text{ eV}}$$

27.55 (a) If $KE = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.00\times10^{-8}\text{ m}} = 1.99\times10^{-17}\text{ J} = 124\text{ eV}$,

the electron is non-relativistic and

$$v = \sqrt{\frac{2(KE)}{m_e}} = \sqrt{\frac{2(1.99\times10^{-17}\text{ J})}{9.11\times10^{-31}\text{ kg}}}$$

$$= \left(6.61\times10^{6}\ \frac{\text{m}}{\text{s}}\right)\left(\frac{c}{3.00\times10^{8}\text{ m/s}}\right) = \boxed{0.0220c}$$

(b) When

$$KE = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.00\times10^{-13}\text{ m}}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = 12.4\text{ MeV},$$

the electron is highly relativistic and $KE = (\gamma - 1)E_R$, or

$$\gamma = 1 + \frac{KE}{E_R} = 1 + \frac{12.4\text{ MeV}}{0.511\text{ MeV}} = 25.3$$

Then, $v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(25.3)^2} = \boxed{0.9992c}$

27.56 $\Delta p \geq \frac{h}{4\pi(\Delta x)} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(2.0\times10^{-15}\text{ m})}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = 1.65\times10^{-7}\ \frac{\text{MeV}\cdot\text{s}}{\text{m}}$

Since the momentum must be at least as large as its own uncertainty,

$$p \geq 1.65\times10^{-7}\ \frac{\text{MeV}\cdot\text{s}}{\text{m}}$$

(a) If the confined particle is an electron, $E_R = 0.511\text{ MeV}$ and

$$E = \sqrt{(pc)^2 + E_R^2} \geq \sqrt{\left(1.65\times10^{-7}\ \frac{\text{MeV}\cdot\text{s}}{\text{m}}\right)^2(3.00\times10^{8}\text{ m/s})^2 + (0.511\text{ MeV})^2}$$

or $E \geq 49.5\text{ MeV}$. Then, $\gamma = \frac{E}{E_R} \geq \frac{49.5\text{ MeV}}{0.511\text{ MeV}} = 96.8$ and

$v = c\sqrt{1 - 1/\gamma^2} \geq c\sqrt{1 - 1/(96.8)^2} = \boxed{0.9999c}$ (highly relativistic)

(b) If the confined particle is a proton, $E_R = 939\ \mathrm{MeV}$ and

$$E = \sqrt{(pc)^2 + E_R^2} \geq \sqrt{\left(1.65\times 10^{-7}\ \frac{\mathrm{MeV\cdot s}}{\mathrm{m}}\right)^2 \left(3.00\times 10^{8}\ \mathrm{m/s}\right)^2 + \left(939\ \mathrm{MeV}\right)^2}$$

or $E \geq 940\ \mathrm{MeV}$. Then, $\gamma = \dfrac{E}{E_R} \geq \dfrac{940\ \mathrm{MeV}}{939\ \mathrm{MeV}} = 1.001$ and

$$v = c\sqrt{1 - 1/\gamma^2} \geq c\sqrt{1 - 1/(1.001)^2} = \boxed{0.053c}$$ (non-relativistic)

27.57 (a) From conservation of energy, $E + E_R = \phi + \sqrt{(pc)^2 + E_R^2}$, where E is the photon energy and $E_R = m_e c^2$.

The de Broglie wavelength of the electron is $\lambda = \dfrac{h}{p}$, giving $pc = \dfrac{hc}{\lambda}$.

If λ is also the wavelength of the incident photon, then

$E = hc/\lambda$ and $pc = E$

The energy conservation equation then becomes

$$E + E_R - \phi = \sqrt{E^2 + E_R^2}$$

Squaring both sides and simplifying yields $2E_R E - 2\phi(E + E_R) + \phi^2 = 0$, which reduces to

$$E = \frac{\phi(2E_R - \phi)}{2(E_R - \phi)} = \boxed{\frac{\phi(m_e c^2 - \phi/2)}{m_e c^2 - \phi}}$$

(b) From the photoelectric effect equation, $KE_{max} = E - \phi$. Using the result from above, and the fact that $m_e c^2 = 0.511 \text{ MeV}$, gives

$$KE_{max} = \frac{\phi\left(m_e c^2 - \phi/2\right)}{m_e c^2 - \phi} - \phi = \frac{\phi^2}{2\left(m_e c^2 - \phi\right)} = \frac{(6.35 \text{ eV})^2}{2\left(5.11\times 10^5 \text{ eV} - 6.35 \text{ eV}\right)},$$

or $$KE_{max} = 3.95\times 10^{-5} \text{ eV} = 6.31\times 10^{-24} \text{ J}$$

Therefore,

$$v \le \sqrt{\frac{2\left(KE_{max}\right)}{m_e}} = \sqrt{\frac{2\left(6.31\times 10^{-24} \text{ J}\right)}{9.11\times 10^{-31} \text{ kg}}} = 3.72\times 10^3 \ \frac{\text{m}}{\text{s}} = \boxed{3.72 \text{ km/s}}$$

27.58 (a) From conservation of energy, $\left(E_\gamma\right)_0 + E_R = E_\gamma + \left(E_R + KE\right)$ or

$$\left(E_\gamma\right)_0 = E_\gamma + KE = 120 \text{ keV} + 40.0 \text{ keV} = 160 \text{ keV},$$

Therefore, the wavelength of the incident photon is

$$\lambda_0 = \frac{hc}{\left(E_\gamma\right)_0} = \frac{\left(6.63\times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00\times 10^8 \text{ m/s}\right)}{(160 \text{ keV})\left(1.60\times 10^{-16} \text{ J/keV}\right)} = \boxed{7.77\times 10^{-12} \text{ m}}$$

(b) The wavelength of the scattered photon is

$$\lambda = \frac{hc}{E_\gamma} = \frac{\left(6.63\times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00\times 10^8 \text{ m/s}\right)}{(120 \text{ keV})\left(1.60\times 10^{-16} \text{ J/keV}\right)} = 1.04\times 10^{-11} \text{ m},$$

so the Compton shift is $\Delta\lambda = \lambda - \lambda_0 = 2.59\times 10^{-12} \text{ m}$.

The Compton shift formula then gives the photon scattering angle as

$$\theta = \cos^{-1}\left(1 - \frac{\Delta\lambda}{\lambda_C}\right) = \cos^{-1}\left(1 - \frac{2.59\times 10^{-12} \text{ m}}{2.43\times 10^{-12} \text{ m}}\right) = \boxed{93.8^\circ}$$

(c) The momentum of the scattered photon is $p_\gamma = \frac{E_\gamma}{c} = 120\ \frac{\text{keV}}{c}$.

The rest energy of an electron is $E_R = 0.511\ \text{MeV} = 511\ \text{keV}$, so the total energy of the recoiling electron is

$$E = E_R + KE = 511\ \text{keV} + 40.0\ \text{keV} = 551\ \text{keV}$$

The momentum of the electron is then

$$p_e = \frac{\sqrt{E^2 - E_R^2}}{c} = \frac{\sqrt{(551\ \text{keV})^2 - (511\ \text{keV})^2}}{c} = 206\ \frac{\text{keV}}{c}$$

Taking the direction of the incident photon to be the *x*-axis, conservation of momentum in the *y* direction requires that $p_\gamma \sin\theta = p_e \sin\phi$, where ϕ is the recoil angle of the electron. Thus,

$$\phi = \sin^{-1}\left(\frac{p_\gamma \sin\theta}{p_e}\right) = \sin^{-1}\left(\frac{(120\ \text{keV}/c)\sin 93.8^\circ}{206\ \text{keV}/c}\right) = \boxed{35.5^\circ}$$

27.59 (a) The woman tries to release the pellets from rest directly above the spot on the floor. In releasing the pellets, she will give them some average horizontal velocity Δv_x, causing an uncertainty $\Delta p_x = m(\Delta v_x)$ in the horizontal momentum of the pellets as they fall. On average, the pellets will miss the spot by a distance $\Delta x = (\Delta v_x)t = \left(\frac{\Delta p_x}{m}\right)t$ where $t = \sqrt{\frac{2H}{g}}$ is the time of fall.

From the uncertainty principle, $\Delta p_x \geq \frac{h}{4\pi(\Delta x)}$. Thus, the average-miss equation becomes $\Delta x \geq \frac{h}{4\pi m(\Delta x)}\sqrt{\frac{2H}{g}} = \frac{h}{2\pi m(\Delta x)}\sqrt{\frac{H}{2g}}$.

Solving for Δx yields $\boxed{\Delta x \geq \left(\frac{h}{2\pi m}\right)^{\frac{1}{2}}\left(\frac{H}{2g}\right)^{\frac{1}{4}}}$

(b) If $H = 2.00\ \text{m}$ and $m = 0.500\ \text{g}$, then

$$\Delta x \geq \left[\frac{6.63\times 10^{-34}\ \text{J}\cdot\text{s}}{2\pi(0.500\times 10^{-3}\ \text{kg})}\right]^{\frac{1}{2}}\left[\frac{2.00\ \text{m}}{2(9.80\ \text{m/s}^2)}\right]^{\frac{1}{4}} = \boxed{2.60\times 10^{-16}\ \text{m}}$$

27.60 The Compton wavelength is $\lambda_C = \dfrac{h}{m\,c}$, where m is the mass of the particle imagined to be scattering the photon.

The de Broglie wavelength of the particle is

$$\lambda = \frac{h}{p} = \frac{h}{\gamma\, m\, v}\text{ , where } \gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Therefore, $h = (m\,c)\lambda_C = (\gamma\, m\, v)\lambda$ which gives $\gamma(v/c) = \lambda_C/\lambda$

Squaring this result yields $\dfrac{(v/c)^2}{1-(v/c)^2} = \left(\dfrac{\lambda_C}{\lambda}\right)^2$, which simplifies to

$$\left(\frac{v}{c}\right)^2 = \frac{(\lambda_C/\lambda)^2}{(\lambda_C/\lambda)^2+1} = \frac{1}{1+(\lambda/\lambda_C)^2} \text{ or } \boxed{v = \frac{c}{\sqrt{1+(\lambda/\lambda_C)^2}}}$$

Answers to Even Numbered Conceptual Questions

2. A microscope can see details no smaller than the wavelength of the waves it uses to produce images. Electrons with kinetic energies of several electron volts have wavelengths of less than a nanometer, which is much smaller than the wavelength of visible light (having wavelengths ranging from about 400 to 700 nm). Therefore, an electron microscope can resolve details of much smaller sizes as compared to an optical microscope.

4. Measuring the position of a particle implies bouncing a photon off it. However, this will change the velocity of the particle.

6. Light has both wave and particle characteristics. In Young's double-slit experiment, light behaves as a wave. In the photoelectric effect, it behaves like a particle. Light can be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time, light can be characterized as a stream of photons, each carrying a discrete energy, hf.

8. Because of the magnitude of Planck's constant, the size of the uncertainties predicted by Heisenberg's uncertainty principle are very small. In dealing with ordinary objects, such as a baseball, these uncertainties are insignificant in comparison to the magnitudes of the relevant parameters such as altitude, range, velocity, time of flight, etc. The limitations of the uncertainty principle become important only when dealing with objects on an atomic or subatomic level.

10. Ultraviolet light has a shorter wavelength and higher photon energy than visible light.

12. Increasing the temperature of the substance increases the average kinetic energy of the electrons inside the material. This makes it slightly easier for an electron to escape from the material when it absorbs a photon.

14. Most stars radiate nearly as blackbodies. Vega has a higher surface temperature than Arcturus. Vega radiates more intensely at shorter wavelengths.

16. The electron behaves like a particle as it follows a circular orbit in a magnetic field or as it is ejected from a metal surface in the photoelectric effect. It behaves like a wave in forming an interference pattern.

18. Exposure to x-rays tends to damage bone marrow. With a significant portion of your total bone marrow located in the head, it is best to avoid any unnecessary dental x-rays or other forms of exposure to this radiation.

Answers to Even Numbered Problems

2. (a) $\sim 100\text{ nm}$, ultraviolet (b) $\sim 0.1\text{ nm}$, γ-rays

4. (a) 2.57 eV (b) 1.28×10^{-5} eV (c) 1.91×10^{-7} eV
(d) 484 nm (visible), 9.67 cm (microwaves), 6.52 m (radio waves)

6. 2.96×10^{21} photons/sec

8. 5.7×10^{3} photons/sec

10. 3.7×10^{19}

12. (a) 1.4 eV (b) 3.3×10^{14} Hz

14. (a) only lithium (b) 0.81 eV

16. 8.43×10^{-12} C

18. 8.7×10^{12} electrons/s

20. (a) 8.29×10^{-11} m (b) 1.24×10^{-11} m

22. 0.124 nm

24. 6.7°

26. 67.5°

28. 1.8 keV, $9.7\times10^{-25}\text{ kg}\cdot\text{m/s}$

30. (a) 4.89×10^{-4} nm (b) 268 keV (c) 32 keV

32. (a) 0.00287 nm (b) 100°

34. (a) 0.174 nm (b) 5.49×10^{-12} m

36. 1.5×10^{2} V

38. (a) $\sim 10^{-34}$ m/s (b) $\sim 10^{33}$ s
(c) No. The maximum time to pass through the door is $\sim 10^{15}$ times the age of the Universe.

40. $c/\sqrt{2}$

42. 3.5×10^{-32} m

44. (a) 0.250 m/s (b) 2.25 m

46. (b) 5.2 MeV

48. (a) 2.49×10^{-11} m (b) 0.285 nm

50. (a) 2.82×10^{-37} m (b) 1.06×10^{-32} J (c) 2.87×10^{-37} % or more

52. 191 MeV

54. 0.785 eV

56. (a) $v \geq 0.9999c$ (b) $v \geq 0.053c$

58. (a) 7.77×10^{-12} m (b) 93.8° (c) 35.5°