CHAPTER 29

Quick Quizzes

1. (c). At the end of the first half-life interval, half of the original sample has decayed and half remains. During the second half-life interval, half of the remaining portion of the sample decays. The total fraction of the sample that has decayed during the two half-lives is $\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)=\frac{3}{4}$.

2. (b). If the original activity is R_0, the activity remaining after an elapsed time *t* is $R=R_0e^{-\lambda t}=R_0e^{-(0.693/T_{1/2})t}$. Solving for the half-life yields $T_{1/2}=\left[\frac{-0.693}{\ln(R/R_0)}\right]t$. If $R=0.96R_0$ at $t=2.0\text{ hr}$, the half-life is $T_{1/2}=\left[\frac{-0.693}{\ln(0.96)}\right](2.0\text{ h})=34\text{ h}$.

3. (a). Conservation of momentum requires the momenta of the two fragments be equal in magnitude and oppositely directed. Thus, from $KE=p^2/2m$, the lighter alpha particle has more kinetic energy that the more massive daughter nucleus.

4. (a) and (b). Reactions (a) and (b) both conserve total charge and total mass number as required. Reaction (c) violates conservation of mass number with the sum of the mass numbers being 240 before reaction and being only 223 after reaction.

5. (b). In an endothermic reaction, the threshold energy exceeds the magnitude of the *Q* value by a factor of $(1+m/M)$, where *m* is the mass of the incident particle and *M* is the mass of the target nucleus.

Problem Solutions

29.1 The average nuclear radii are $r = r_0 A^{1/3}$, where $r_0 = 1.2 \times 10^{-15}\ \text{m} = 1.2\ \text{fm}$ and *A* is the mass number.

For ^2_1H, $\quad r = (1.2\ \text{fm})(2)^{1/3} = \boxed{1.5\ \text{fm}}$

For $^{60}_{27}\text{Co}$, $\quad r = (1.2\ \text{fm})(60)^{1/3} = \boxed{4.7\ \text{fm}}$

For $^{197}_{79}\text{Au}$, $\quad r = (1.2\ \text{fm})(197)^{1/3} = \boxed{7.0\ \text{fm}}$

For $^{239}_{94}\text{Pu}$, $\quad r = (1.2\ \text{fm})(239)^{1/3} = \boxed{7.4\ \text{fm}}$

29.2 An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons. So protons and neutrons are nearly equally numerous in your body, each contributing (say) 35 kg out of a total body mass of 70 kg.

$$N = 35\ \text{kg}\left(\frac{1\ \text{nucleon}}{1.67 \times 10^{-27}\ \text{kg}}\right) \boxed{\sim 10^{28}\ \text{protons}} \text{ and } \boxed{\sim 10^{28}\ \text{neutrons}}$$

The electron number is precisely equal to the proton number,

$$N_e = \boxed{\sim 10^{28}\ \text{electrons}}$$

29.3 From $M_E = \rho_n V = \rho_n\left(\frac{4}{3}\pi r^3\right)$, we find

$$r = \left(\frac{3M_E}{4\pi\rho_n}\right)^{1/3} = \left[\frac{3\left(5.98 \times 10^{24}\ \text{kg}\right)}{4\pi\left(2.3 \times 10^{17}\ \text{kg/m}^3\right)}\right]^{1/3} = \boxed{1.8 \times 10^2\ \text{m}}$$

29.4 The mass of the hydrogen atom is approximately equal to that of the proton, $1.67 \times 10^{-27}\ \text{kg}$. If the radius of the atom is $r = 0.53 \times 10^{-10}\ \text{m}$, then

$$\rho_a = \frac{m}{V} = \frac{m}{(4/3)\pi r^3} = \frac{3\left(1.67 \times 10^{-27}\ \text{kg}\right)}{4\pi\left(0.53 \times 10^{-10}\ \text{m}\right)^3} = 2.7 \times 10^3\ \text{kg/m}^3$$

The ratio of the nuclear density to this atomic density is

$$\frac{\rho_n}{\rho_a} = \frac{2.3\times 10^{17}\ \mathrm{kg/m^3}}{2.7\times 10^{3}\ \mathrm{kg/m^3}} = \boxed{8.6\times 10^{13}}$$

29.5 (a) $$F_{max} = \frac{k_e q_1 q_2}{r_{min}^2} = \frac{\left(8.99\times 10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left[(2)(6)\left(1.60\times 10^{-19}\ \mathrm{C}\right)^2\right]}{\left(1.00\times 10^{-14}\ \mathrm{m}\right)^2} = \boxed{27.6\ \mathrm{N}}$$

(b) $$a_{max} = \frac{F_{max}}{m_\alpha} = \frac{27.6\ \mathrm{N}}{6.64\times 10^{-27}\ \mathrm{kg}} = \boxed{4.16\times 10^{27}\ \mathrm{m/s^2}}$$

(c) $$PE_{max} = \frac{k_e q_1 q_2}{r_{min}} = F_{max}\cdot r_{min} = \left(27.6\ \mathrm{N}\right)\left(1.00\times 10^{-14}\ \mathrm{m}\right)$$

$$= 2.76\times 10^{-13}\ \mathrm{J}\left(\frac{1\ \mathrm{MeV}}{1.60\times 10^{-13}\ \mathrm{J}}\right) = \boxed{1.73\ \mathrm{MeV}}$$

29.6 (a) From conservation of energy, $\Delta KE = -\Delta PE$, or $\frac{1}{2}mv^2 = q(\Delta V)$. Also, the centripetal acceleration is supplied by the magnetic force,

so $\frac{mv^2}{r} = qvB$, or $v = qBr/m$

The energy equation then yields $r = \sqrt{2m(\Delta V)/qB^2}$

For $^{12}\mathrm{C}$, $m = 12\mathrm{u}$

And $$r = \sqrt{\frac{2\left[12\left(1.66\times 10^{-27}\ \mathrm{kg}\right)\right](1000\ \mathrm{V})}{\left(1.60\times 10^{-19}\ \mathrm{C}\right)(0.200\ \mathrm{T})^2}} = \boxed{7.89\ \mathrm{cm}}$$

For $^{13}\mathrm{C}$, $m = 13\mathrm{u}$

and $$r = \sqrt{\frac{2\left[13\left(1.66\times 10^{-27}\ \mathrm{kg}\right)\right](1000\ \mathrm{V})}{\left(1.60\times 10^{-19}\ \mathrm{C}\right)(0.200\ \mathrm{T})^2}} = \boxed{8.21\ \mathrm{cm}}$$

(b) $\dfrac{r_1}{r_2} = \dfrac{\sqrt{2m_1(\Delta V)/qB^2}}{\sqrt{2m_2(\Delta V)/qB^2}} = \sqrt{\dfrac{m_1}{m_2}}$

$\boxed{\dfrac{r_{12}}{r_{13}} = \dfrac{7.89 \text{ cm}}{8.21 \text{ cm}} = 0.961 \text{ and } \sqrt{\dfrac{12\text{u}}{13\text{u}}} = 0.961}$, so they do agree.

29.7 (a) At the point of closest approach, $PE_f = KE_i$, so $\dfrac{k_e(2e)(79e)}{r_{min}} = \dfrac{1}{2}m_\alpha v^2$

or $v = \sqrt{\dfrac{2k_e(2e)(79e)}{m_\alpha r_{min}}}$

$$= \sqrt{\frac{316\left(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-19}\text{ C}\right)^2}{\left(6.64\times 10^{-27}\text{ kg}\right)\left(3.2\times 10^{-14}\text{ m}\right)}} = \boxed{1.9\times 10^7 \text{ m/s}}$$

(b) $KE_i = \dfrac{1}{2}m_\alpha v^2$

$$= \frac{1}{2}\left(6.64\times 10^{-27}\text{ kg}\right)\left(1.85\times 10^7 \text{ m/s}\right)^2\left(\frac{1 \text{ MeV}}{1.60\times 10^{-13}\text{ J}}\right) = \boxed{7.1 \text{ MeV}}$$

29.8 Using $r = r_0 A^{1/3}$, with $r_2 = r_1/2$,

gives $A_2^{1/3} = A_1^{1/3}/2$ or $A_2 = A_1/2^3 = A_1/8$.

Thus, $A_2 = \dfrac{238}{8} = 29.8 \approx 30$, or $\boxed{\text{any nucleus with } A = 30}$ will suffice.

29.9 For ${}^{93}_{41}\text{Nb}$,

$\Delta m = 41m_\text{H} + 52m_n - m_\text{Nb}$

$= 41(1.007\,825\text{u}) + 52(1.008\,665\text{u}) - (92.906\,376\,8\text{u}) = 0.865\,028\text{u}$

Thus, $\dfrac{E_b}{A} = \dfrac{(\Delta m)c^2}{A} = \dfrac{(0.865\,028\text{u})(931.5 \text{ MeV/u})}{93} = \boxed{8.66 \text{ MeV/nucleon}}$

For $^{197}_{79}\mathrm{Au}$,

$$\Delta m = 79(1.007\,825\,\mathrm{u}) + 118(1.008\,665\,\mathrm{u}) - (196.966\,543\,\mathrm{u}) = 1.674\,102\,\mathrm{u}$$

and, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(1.674\,102\,\mathrm{u})(931.5\ \mathrm{MeV/u})}{197} = \boxed{7.92\ \mathrm{MeV/nucleon}}$$

29.10 (a) For $^{2}_{1}\mathrm{H}$,

$$\Delta m = 1(1.007\,825\,\mathrm{u}) + 1(1.008\,665\,\mathrm{u}) - (2.014\,102\,\mathrm{u}) = 0.002\,388\,\mathrm{u}$$

and, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.002\,388\,\mathrm{u})(931.5\ \mathrm{MeV/u})}{2} = \boxed{1.11\ \mathrm{MeV/nucleon}}$$

(b) For $^{4}_{2}\mathrm{He}$,

$$\Delta m = 2(1.007\,825\,\mathrm{u}) + 2(1.008\,665\,\mathrm{u}) - (4.002\,602\,\mathrm{u}) = 0.030\,378\,\mathrm{u}$$

and, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.030\,378\,\mathrm{u})(931.5\ \mathrm{MeV/u})}{4} = \boxed{7.07\ \mathrm{MeV/nucleon}}$$

(c) For $^{56}_{26}\mathrm{Fe}$,

$$\Delta m = 26(1.007\,825\,\mathrm{u}) + 30(1.008\,665\,\mathrm{u}) - (55.934\,940) = 0.528\,460\,\mathrm{u}$$

and, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.528\,460\,\mathrm{u})(931.5\ \mathrm{MeV/u})}{56} = \boxed{8.79\ \mathrm{MeV/nucleon}}$$

(d) For $^{238}_{92}\mathrm{U}$,

$$\Delta m = 92(1.007\,825\,\mathrm{u}) + 146(1.008\,665\,\mathrm{u}) - (238.050\,784) = 1.934\,206\,\mathrm{u}$$

and, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(1.934\,206\,\mathrm{u})(931.5\ \mathrm{MeV/u})}{238} = \boxed{7.57\ \mathrm{MeV/nucleon}}$$

29.11 For $^{15}_{8}\text{O}$,

$$\Delta m = 8(1.007\,825\text{ u}) + 7(1.008\,665\text{ u}) - (15.003\,065) = 0.120\,190\text{ u}$$

and $$E_b\big|_{^{15}\text{O}} = (\Delta m)c^2 = (0.120\,190\text{ u})(931.5\text{ MeV/u}) = 111.957\text{ MeV}$$

For $^{15}_{7}\text{N}$,

$$\Delta m = 7(1.007\,825\text{ u}) + 8(1.008\,665\text{ u}) - (15.000\,108) = 0.123\,987\text{ u}$$

and $$E_b\big|_{^{15}\text{N}} = (\Delta m)c^2 = (0.123\,987\text{ u})(931.5\text{ MeV/u}) = 115.494\text{ MeV}$$

Therefore, $$\Delta E_b = E_b\big|_{^{15}\text{N}} - E_b\big|_{^{15}\text{O}} = \boxed{3.54\text{ MeV}}$$

29.12 $\Delta m = Zm_{\text{H}} + (A - Z)m_n - m$ and $E_b/A = \Delta m(931.5\text{ MeV/u})/A$

Nucleus	Z	$(A - Z)$	m (in u)	Δm (in u)	E_b/A (in MeV)
$^{55}_{25}\text{Mn}$	25	30	54.938 048	0.517 527	8.765
$^{56}_{26}\text{Fe}$	26	30	55.934 940	0.528 460	8.786
$^{59}_{27}\text{Co}$	27	32	58.933 198	0.555 357	8.768

Therefore, $^{56}_{26}\text{Fe}$ has a greater binding energy per nucleon than its neighbors. This gives us finer detail than is shown in Figure 29.4.

29.13 For $^{23}_{11}\text{Na}$,

$$\Delta m = 11(1.007\,825\text{ u}) + 12(1.008\,665\text{ u}) - (22.989\,770\text{ u}) = 0.200\,285\text{ u}$$

and, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.200\,285\text{ u})(931.5\text{ MeV/u})}{23} = 8.111\text{ MeV/nucleon}$$

For $^{23}_{12}\text{Mg}$,

$$\Delta m = 12(1.007\,825\text{ u}) + 11(1.008\,665\text{ u}) - (22.994\,127\text{ u}) = 0.195\,088\text{ u}$$

so, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.195\,088\text{ u})(931.5\text{ MeV/u})}{23} = 7.901\text{ MeV/nucleon}$$

The binding energy per nucleon is

greater for $^{23}_{11}\mathrm{Na}$ by $\boxed{0.210\ \mathrm{MeV/nucleon}}$.

This is attributable to $\boxed{\text{less proton repulsion in } ^{23}_{11}\mathrm{Na}}$.

29.14 For $^{24}_{12}\mathrm{Mg}$,

$$\Delta m = 12(1.007\,825\,\mathrm{u}) + 12(1.008\,665\,\mathrm{u}) - (23.985\,042\,\mathrm{u}) = 0.212\,838\,\mathrm{u}$$

and, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.212\,838\,\mathrm{u})(931.5\ \mathrm{MeV/u})}{24} = \boxed{8.26\ \mathrm{MeV/nucleon}}$$

For $^{85}_{37}\mathrm{Rb}$,

$$\Delta m = 37(1.007\,825\,\mathrm{u}) + 48(1.008\,665\,\mathrm{u}) - (84.911\,793\,\mathrm{u}) = 0.793\,652\,\mathrm{u}$$

so, $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.793\,652\,\mathrm{u})(931.5\ \mathrm{MeV/u})}{85} = \boxed{8.70\ \mathrm{MeV/nucleon}}$$

29.15 The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}}$, so the activity is

$$R = \lambda N = \frac{N\ln 2}{T_{1/2}} = \frac{(3.0\times 10^{16})\ln 2}{(14\ \mathrm{d})(8.64\times 10^{4}\ \mathrm{s/d})} = 1.7\times 10^{10}\ \mathrm{decays/s},$$

or $$R = (1.7\times 10^{10}\ \mathrm{decays/s})\left(\frac{1\ \mathrm{Ci}}{3.7\times 10^{10}\ \mathrm{decays/s}}\right) = \boxed{0.46\ \mathrm{Ci}}$$

29.16 The activity is $R = R_0 e^{-\lambda t}$ where $\lambda = \frac{\ln 2}{T_{1/2}}$. Thus,

$$R = R_0 e^{-(t\ln 2/T_{1/2})} = (1.1\times 10^{4}\ \mathrm{Bq})\,e^{-\frac{(2.0\ \mathrm{h})\ln 2}{6.05\ \mathrm{h}}} = \boxed{8.7\times 10^{3}\ \mathrm{Bq}}$$

29.17 (a) The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(8.04\text{ d})(8.64\times 10^4\text{ s/d})} = \boxed{9.98\times 10^{-7}\text{ s}^{-1}}$$

(b) $R = \lambda N$, so the required number of nuclei is

$$N = \frac{R}{\lambda} = \frac{0.50\times 10^{-6}\text{ Ci}}{9.98\times 10^{-7}\text{ s}^{-1}}\left(\frac{3.7\times 10^{10}\text{ decays/s}}{1\text{ Ci}}\right) = \boxed{1.9\times 10^{10}\text{ nuclei}}$$

29.18 The activity is $R = R_0 e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

Thus, $R = R_0 e^{-(t\ln 2/T_{1/2})} = (6.40\text{ mCi})e^{-\frac{(40.2\text{ d})\ln 2}{8.04\text{ d}}} = \boxed{0.200\text{ mCi}}$

29.19 (a) One mole of ^{11}C has a mass of 11.0 g. Therefore,

$$n = \frac{m}{M} = \frac{3.50\times 10^{-6}\text{ g}}{11.0\text{ g/mol}} = \boxed{3.18\times 10^{-7}\text{ mol}}$$

(b) $N_0 = nN_A$

$$= (3.18\times 10^{-7}\text{ mol})(6.02\times 10^{23}\text{ nuclei/mol}) = \boxed{1.92\times 10^{17}\text{ nuclei}}$$

(c) $R_0 = \lambda N_0 = \dfrac{N_0 \ln 2}{T_{1/2}} = \dfrac{(1.92\times 10^{17})\ln 2}{(20.4\text{ min})(60.0\text{ s/1 min})} = \boxed{1.08\times 10^{14}\text{ Bq}}$

(d) $R = R_0 e^{-(t\ln 2/T_{1/2})} = (1.08\times 10^{14}\text{ Bq})e^{-\frac{(8.00\text{ h})\ln 2}{(20.4\text{ min})(1\text{ h}/60\text{ min})}} = \boxed{8.96\times 10^{6}\text{ Bq}}$

29.20 From $R = R_0 e^{-\lambda t}$, with $R = 0.100R_0$, we find $e^{-\lambda t} = \dfrac{R}{R_0}$

and $t = -\dfrac{\ln(R/R_0)}{\lambda} = -T_{1/2}\left[\dfrac{\ln(R/R_0)}{\ln 2}\right] = -(26\text{ h})\left[\dfrac{\ln(0.100)}{\ln 2}\right] = 86\text{ h}$

29.21 From $R = R_0 e^{-\lambda t}$, with $R = \left(1.00\times 10^{-3}\right)R_0$, we find $e^{-\lambda t} = \dfrac{R}{R_0}$

and $$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -T_{1/2}\left[\frac{\ln\left(R/R_0\right)}{\ln 2}\right]$$

$$= -\left(432\text{ yr}\right)\left[\frac{\ln\left(1.00\times 10^{-3}\right)}{\ln 2}\right] = \boxed{4.31\times 10^{3}\text{ yr}}$$

29.22 Using $R = R_0 e^{-\lambda t}$, with $R/R_0 = 0.125$, gives $\lambda t = -\ln\left(R/R_0\right)$

or $$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -T_{1/2}\left[\frac{\ln\left(R/R_0\right)}{\ln 2}\right] = -\left(5730\text{ yr}\right)\left[\frac{\ln\left(0.125\right)}{\ln 2}\right] = \boxed{1.72\times 10^{4}\text{ yr}}$$

29.23 (a) The initial activity is $R_0 = 10.0\text{ mCi}$, and at $t = 4.00\text{ h}$, $R = 8.00\text{ mCi}$. Then, from $R = R_0 e^{-\lambda t}$, the decay constant is

$$\lambda = -\frac{\ln\left(R/R_0\right)}{t} = -\frac{\ln\left(0.800\right)}{4.00\text{ h}} = \boxed{5.58\times 10^{-2}\text{ h}^{-1}}$$

and the half-life is $T_{1/2} = \dfrac{\ln 2}{\lambda} = \dfrac{\ln 2}{5.58\times 10^{-2}\text{ h}^{-1}} = \boxed{12.4\text{ h}}$

(b) $$N_0 = \frac{R_0}{\lambda} = \frac{\left(10.0\times 10^{-3}\text{ Ci}\right)\left(3.70\times 10^{10}\text{ s}^{-1}/1\text{ Ci}\right)}{\left(5.58\times 10^{-2}\text{ h}^{-1}\right)\left(1\text{ h}/3600\text{ s}\right)} = \boxed{2.39\times 10^{13}\text{ nuclei}}$$

(c) $$R = R_0 e^{-\lambda t} = \left(10.0\text{ mCi}\right)e^{-\left(5.58\times 10^{-2}\text{ h}^{-1}\right)\left(30\text{ h}\right)} = \boxed{1.9\text{ mCi}}$$

29.24 The number of ${}^{90}_{38}\text{Sr}$ nuclei initially present is

$$N_0 = \frac{\text{total mass}}{\text{mass per nucleus}} = \frac{5.00\text{ kg}}{\left(89.9077\text{ u}\right)\left(1.66\times 10^{-27}\text{ kg/u}\right)} = 3.35\times 10^{25}$$

The half-life of ${}^{90}_{38}\text{Sr}$ is $T_{1/2} = 29.1\text{ yr}$ (Appendix B), so the initial activity is

$$R_0 = \lambda N_0 = \frac{N_0 \ln 2}{T_{1/2}} = \frac{\left(3.35\times 10^{25}\right)\ln 2}{\left(29.1\text{ yr}\right)\left(3.156\times 10^{7}\text{ s/yr}\right)} = 2.53\times 10^{16}\text{ counts/s}$$

The time when the activity will be $R = 10.0\ \text{counts/min}$ is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -\left(T_{1/2}\right)\frac{\ln(R/R_0)}{\ln 2}$$

$$= -(29.1\ \text{yr})\frac{\ln\left[\dfrac{10.0\ \text{min}^{-1}}{2.53\times10^{16}\ \text{s}^{-1}}\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\right]}{\ln 2} = \boxed{1.66\times10^{3}\ \text{yr}}$$

29.25 $^{212}_{83}\text{Bi} \rightarrow \boxed{^{208}_{81}\text{Tl}} + {}^{4}_{2}\text{He}$

$^{95}_{36}\text{Kr} \rightarrow \boxed{^{95}_{37}\text{Rb}} + {}^{0}_{-1}\text{e}$

$\boxed{^{144}_{60}\text{Nd}} \rightarrow {}^{4}_{2}\text{He} + {}^{140}_{58}\text{Ce}$

29.26 $^{12}_{5}\text{B} \rightarrow \boxed{^{12}_{6}\text{C}} + {}^{0}_{-1}\text{e}$

$^{234}_{90}\text{Th} \rightarrow {}^{230}_{88}\text{Ra} + \boxed{^{4}_{2}\text{He}}$

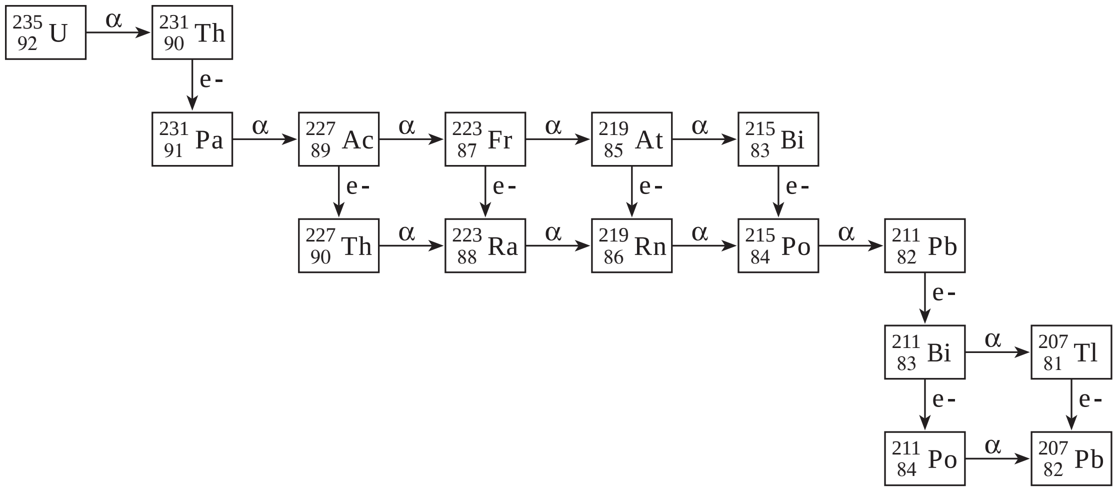
$\boxed{^{14}_{6}\text{C}} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e}$

29.27 $\boxed{^{40}_{20}\text{Ca}} \rightarrow {}^{0}_{+1}\text{e} + {}^{40}_{19}\text{K}$

$^{98}_{44}\text{Ru} \rightarrow {}^{4}_{2}\text{He} + \boxed{^{94}_{42}\text{Mo}}$

$^{144}_{60}\text{Nd} \rightarrow \boxed{^{4}_{2}\text{He}} + {}^{140}_{58}\text{Ce}$

29.28



29.29 The more massive ${}^{56}_{27}\mathrm{Co}$ decays into the less massive ${}^{56}_{26}\mathrm{Fe}$. To conserve charge, the charge of the emitted particle must be +1e. Since the parent and the daughter have the same mass number, the emitted particle must have essentially zero mass. Thus, the decay must be positron emission or $\boxed{e^{+}\ \mathrm{decay}}$. The decay equation is

$$\boxed{{}^{56}_{27}\mathrm{Co} \rightarrow {}^{56}_{26}\mathrm{Fe} + e^{+} + \nu}$$

29.30 The energy released in the decay ${}^{238}_{92}\mathrm{U} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{234}_{90}\mathrm{Th}$ is

$$Q = (\Delta m)c^2 = \left[m_{{}^{238}\mathrm{U}} - \left(m_{{}^{4}\mathrm{He}} + m_{{}^{234}\mathrm{Th}}\right)\right]c^2$$

$$= \left[238.050\,784\ \mathrm{u} - \left(4.002\,602\ \mathrm{u} + 234.043\,583\ \mathrm{u}\right)\right]\left(931.5\ \mathrm{MeV/u}\right)$$

$$= \boxed{4.28\ \mathrm{MeV}}$$

29.31 The mass of the alpha particle is $m_\alpha = 4.00\ \mathrm{u}$ and that of the daughter nucleus is $m_D = 224\ \mathrm{u}$. Since the total momentum was zero before the decay, the alpha particle and the daughter nucleus must recoil in opposite directions with equal magnitude momenta (i.e., $p_D = p_\alpha$). Therefore, the kinetic energy is

$$KE_D = \frac{p_D^2}{2m_D} = \frac{p_\alpha^2}{2m_D} = \left(\frac{m_\alpha}{m_D}\right)\frac{p_\alpha^2}{2m_\alpha} = \left(\frac{m_\alpha}{m_D}\right)KE_\alpha,$$

or $$KE_D = \left(\frac{4.00\ \mathrm{u}}{224\ \mathrm{u}}\right)(4.00\ \mathrm{MeV}) = 0.0714\ \mathrm{MeV} = \boxed{71.4\ \mathrm{keV}}$$

29.32 (a) $\boxed{{}^{66}_{28}\mathrm{Ni} \rightarrow {}^{66}_{29}\mathrm{Cu} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}}$

(b) Because of the mass differences, neglect the kinetic energy of the recoiling daughter nucleus in comparison to that of the other decay products. Then, the maximum kinetic energy of the beta particle occurs when the neutrino is given zero energy. That maximum is

$$KE_{max} = \left(m_{^{66}\mathrm{Ni}} - m_{^{66}\mathrm{Cu}}\right)c^2 = (65.9291\ \mathrm{u} - 65.9289\ \mathrm{u})(931.5\ \mathrm{MeV/u})$$

$$= 0.186\ \mathrm{MeV} = \boxed{186\ \mathrm{keV}}$$

29.33 In the decay ${}^{3}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$, the anti-neutrino is massless and the mass of the electron is included in the atomic masses of the parent and daughter. Therefore, the energy released is

$$E = (\Delta m)c^2 = \left(m_{^{3}\mathrm{H}} - m_{^{3}\mathrm{He}}\right)c^2 = (3.016\,049\ \mathrm{u} - 3.016\,029\ \mathrm{u})(931.5\ \mathrm{MeV/u})$$

$$= 0.0186\ \mathrm{MeV} = \boxed{18.6\ \mathrm{keV}}$$

29.34 The initial activity of the 1.00-kg carbon sample would have been

$$R_0 = \left(1.00 \times 10^3\ \mathrm{g}\right)\left(\frac{15.0\ \mathrm{counts/min}}{1.00\ \mathrm{g}}\right) = 1.50 \times 10^4\ \mathrm{min}^{-1}$$

From $R = R_0 e^{-\lambda t}$, and $T_{1/2} = 5730\ \mathrm{yr}$ for ${}^{14}\mathrm{C}$ (Appendix B), the age of the sample is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -T_{1/2}\frac{\ln(R/R_0)}{\ln 2}$$

$$= -(5730\ \mathrm{yr})\frac{\ln\left(\dfrac{2.00 \times 10^3\ \mathrm{min}^{-1}}{1.50 \times 10^4\ \mathrm{min}^{-1}}\right)}{\ln 2} = \boxed{1.67 \times 10^4\ \mathrm{yr}}$$

29.35 From $R = R_0 e^{-\lambda t}$, and $T_{1/2} = 5730\ \mathrm{y}$ for ${}^{14}\mathrm{C}$ (Appendix B), the age of the sample is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -T_{1/2}\frac{\ln(R/R_0)}{\ln 2} = -(5730\ \mathrm{yr})\frac{\ln(0.600)}{\ln 2} = \boxed{4.22 \times 10^3\ \mathrm{yr}}$$

29.36 The total number of carbon nuclei in the sample is

$$N=\left(\frac{21.0\times10^{-3}\ \text{g}}{12.0\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{nuclei/mol}\right)=1.05\times10^{21}$$

Of these, one in every 7.70×10^{11} was originally ^{14}C. Hence, the initial number of ^{14}C nuclei present was $N_0=1.05\times10^{21}/7.70\times10^{11}=1.37\times10^{9}$, and the initial activity was

$$R_0=\lambda N_0=\frac{N_0\ln 2}{T_{1/2}}=\frac{\left(1.37\times10^{9}\right)\ln 2}{(5730\ \text{yr})}\left(\frac{1\ \text{yr}}{365.24\ \text{d}}\right)\left(\frac{7\ \text{d}}{1\ \text{week}}\right)=3.17\times10^{3}\ \text{week}^{-1}$$

The current activity, accounting for counter efficiency, is

$$R=\frac{\text{count rate}}{\text{efficiency}}=\frac{837\ \text{week}^{-1}}{0.880}=951\ \text{week}^{-1}$$

Therefore, from $R=R_0e^{-\lambda t}$, the age of the sample is

$$t=-\frac{\ln\left(R/R_0\right)}{\lambda}=-T_{1/2}\frac{\ln\left(R/R_0\right)}{\ln 2}$$

$$=-(5730\ \text{yr})\frac{\ln\left(\dfrac{951\ \text{week}^{-1}}{3.17\times10^{3}\ \text{week}^{-1}}\right)}{\ln 2}=\boxed{9.96\times10^{3}\ \text{yr}}$$

29.37 (a) $^{27}_{13}\text{Al}+{}^{4}_{2}\text{He}\rightarrow{}^{1}_{0}\text{n}+\boxed{^{30}_{15}\text{P}}$

(b) $Q=(\Delta m)c^2=\left(m_{^{27}\text{Al}}+m_{^{4}\text{He}}-m_{\text{n}}-m_{^{30}\text{P}}\right)c^2$

$Q=\left[26.981\,538\ \text{u}+4.002\,602\ \text{u}-1.008\,665\ \text{u}-29.978\,310\ \text{u}\right]\left(931.5\ \text{MeV/u}\right)$

$=\boxed{-2.64\ \text{MeV}}$

29.38 $\boxed{^{4}_{2}\text{He}}+{}^{14}_{7}\text{N}\rightarrow{}^{1}_{1}\text{H}+{}^{17}_{8}\text{O}$

$^{7}_{3}\text{Li}+{}^{1}_{1}\text{H}\rightarrow{}^{4}_{2}\text{He}+\boxed{^{4}_{2}\text{He}}$

29.39 (a) $\boxed{{}^{21}_{10}\text{Ne}} + {}^{4}_{2}\text{He} \to {}^{24}_{12}\text{Mn} + {}^{1}_{0}\text{n}$

(b) ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \to {}^{90}_{38}\text{Sr} + \boxed{{}^{144}_{54}\text{Xe}} + 2{}^{1}_{0}\text{n}$

(c) $2{}^{1}_{1}\text{H} \to {}^{2}_{1}\text{H} + \boxed{{}^{0}_{+1}\text{e}} + \boxed{{}^{0}_{0}\nu}$

29.40 $Q = (\Delta m)c^2 = \left(m_{{}^1\text{H}} + m_{{}^7\text{Li}} - 2m_{{}^4\text{He}}\right)c^2$

$$= \left[1.007\,825\text{ u} + 7.016\,003\text{ u} - 2\left(4.002\,602\text{ u}\right)\right]\left(931.5\text{ MeV/u}\right) = \boxed{17.3\text{ MeV}}$$

29.41 (a) ${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \to {}^{1}_{1}\text{H} + \boxed{{}^{13}_{6}\text{C}}$

(b) ${}^{13}_{6}\text{C} + {}^{1}_{1}\text{H} \to {}^{4}_{2}\text{He} + \boxed{{}^{10}_{5}\text{B}}$

29.42 (a) ${}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He} \to \boxed{{}^{10}_{5}\text{B}} + {}^{1}_{0}\text{n}$

(b) $Q = (\Delta m)c^2 = \left(m_{{}^7\text{Li}} + m_{{}^4\text{He}} - m_{{}^{10}\text{B}} - m_{\text{n}}\right)c^2$

$$= \left[7.016\,003\text{ u} + 4.002\,602\text{ u} - 10.012\,936\text{ u} - 1.008\,665\text{ u}\right]\left(931.5\text{ MeV/u}\right)$$

$$= \boxed{-2.79\text{ MeV}}$$

29.43 Neglecting the kinetic energy of the product nucleus in the reaction ${}^{1}_{1}\text{H} + {}^{27}_{13}\text{Al} \to {}^{27}_{14}\text{Si} + {}^{1}_{0}\text{n}$, the kinetic energy of the emerging neutron is $KE_{\text{n}} = KE_{{}^1\text{H}} + Q$, where

$$Q = (\Delta m)c^2 = \left(m_{{}^1\text{H}} + m_{{}^{27}\text{Al}} - m_{{}^{27}\text{Si}} - m_{\text{n}}\right)c^2$$

$$= \left[1.007\,825\text{ u} + 26.981\,538\text{ u} - 26.986\,721\text{ u} - 1.008\,665\text{ u}\right]\left(931.5\text{ MeV/u}\right)$$

$$= -5.61\text{ MeV}.$$

Therefore, $KE_{\text{n}} = 6.61\text{ MeV} - 5.61\text{ MeV} = \boxed{1.00\text{ MeV}}$

29.44 The energy released in the reaction ${}^{1}_{0}\mathrm{n} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H}$ is

$$Q = (\Delta m)c^2 = \left(m_{\mathrm{n}} + m_{{}^4\mathrm{He}} - m_{{}^2\mathrm{H}} - m_{{}^3\mathrm{H}}\right)c^2$$

$$= \left[1.008\,665\text{ u} + 4.002\,602\text{ u} - 2.014\,102\text{ u} - 3.016\,049\text{ u}\right](931.5\text{ MeV/u})$$

$$= -17.6\text{ MeV}$$

The threshold energy of the incident neutron is then

$$KE_{min} = \left(1 + \frac{m_{\mathrm{n}}}{m_{{}^4\mathrm{He}}}\right)|Q| = \left(1 + \frac{1.008\,665\text{ u}}{4.002\,602\text{ u}}\right)\left|-17.6\text{ MeV}\right| = \boxed{22.0\text{ MeV}}$$

29.45 (a) ${}^{18}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{18}_{9}\mathrm{F} + \boxed{{}^{1}_{0}\mathrm{n}}$

(b) The energy released in this reaction is

$$Q = (\Delta m)c^2 = \left(m_{{}^{18}\mathrm{O}} + m_{{}^{1}\mathrm{H}} - m_{{}^{18}\mathrm{F}} - m_{\mathrm{n}}\right)c^2$$

$$= \left[17.999\,160\text{ u} + 1.007\,825\text{ u} - m_{{}^{18}\mathrm{F}} - 1.008\,665\text{ u}\right](931.5\text{ MeV/u})$$

Since we know that $Q = -2.453\text{ MeV}$, we find that $m_{{}^{18}\mathrm{F}} = \boxed{18.000\,953\text{ u}}$

29.46 For equivalent doses, it is necessary that

$$(\text{heavy ion dose in rad}) \times \text{RBE}_{\text{heavy ions}} = (\text{x-ray dose in rad}) \times \text{RBE}_{\text{x-rays}},$$

or

$$\text{ion dose in rad} = \frac{(\text{x-ray dose in rad}) \times \text{RBE}_{\text{x-rays}}}{\text{RBE}_{\text{heavy ions}}} = \frac{(100\text{ rad})(1.0)}{20} = \boxed{5.0\text{ rad}}$$

29.47 For each rad of radiation, 10^{-2} J of energy is delivered to each kilogram of absorbing material. Thus, the total energy delivered in this whole body dose to a 75.0-kg person is

$$E = (25.0\text{ rad})\left(10^{-2}\ \frac{\text{J/kg}}{\text{rad}}\right)(75.0\text{ kg}) = \boxed{18.8\text{ J}}$$

29.48 (a) Each rad of radiation delivers 10^{-2} J of energy to each kilogram of absorbing material. Thus, the energy delivered per unit mass with this dose is

$$\frac{E}{m} = (200 \text{ rad})\left(10^{-2} \frac{\text{J/kg}}{\text{rad}}\right) = \boxed{2.00 \text{ J/kg}}$$

(b) From $E = Q = mc(\Delta T)$, the expected temperature rise with this dosage is

$$\Delta T = \frac{E/m}{c} = \frac{2.00 \text{ J/kg}}{4186 \text{ J/kg}\cdot°\text{C}} = \boxed{4.78 \times 10^{-4} \text{ °C}}$$

29.49 The rate of delivering energy to each kilogram of absorbing material is

$$\left(\frac{E/m}{\Delta t}\right) = (10 \text{ rad/s})\left(10^{-2} \frac{\text{J/kg}}{\text{rad}}\right) = 0.10 \frac{\text{J/kg}}{\text{s}}$$

The total energy needed per unit mass is

$$E/m = c(\Delta T) = \left(4186 \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)(50°\text{C}) = 2.1 \times 10^5 \text{ J/kg},$$

so the required time will be

$$\Delta t = \frac{\text{energy needed}}{\text{delivery rate}} = \frac{2.1 \times 10^5 \text{ J/kg}}{0.10 \text{ J/kg}\cdot\text{s}} = 2.1 \times 10^6 \text{ s}\left(\frac{1 \text{ d}}{8.64 \times 10^4 \text{ s}}\right) = \boxed{24 \text{ d}}$$

29.50 (a) The number of x-rays taken per year is

$$production = (8 \text{ x-ray/d})(5 \text{ d/week})(50 \text{ weeks/yr}) = 2.0 \times 10^3 \text{ x-ray/yr},$$

so the exposure per x-ray taken is

$$exposure\ rate = \frac{exposure}{production} = \frac{5.0 \text{ rem/yr}}{2.0 \times 10^3 \text{ x-ray/yr}} = \boxed{2.5 \times 10^{-3} \text{ rem/x-ray}}$$

(b) The exposure due to background radiation is 0.13 rem/yr. Thus, the work-related exposure of 5.0 rem/yr is

$$\frac{5.0 \text{ rem/yr}}{0.13 \text{ rem/yr}} \approx \boxed{38 \text{ times background levels}}$$

29.51 (a) From $N = \frac{R}{\lambda} = \frac{R_0 e^{-\lambda t}}{\lambda} = \left(\frac{T_{1/2} R_0}{\ln 2}\right) e^{-t \ln 2/T_{1/2}}$, the number of decays occurring during the 10-day period is

$$\Delta N = N_0 - N = \left(\frac{T_{1/2} R_0}{\ln 2}\right)\left(1 - e^{-t \ln 2/T_{1/2}}\right)$$

$$= \left[\frac{(14.3 \text{ d})(1.31\times 10^6 \text{ decay/s})}{\ln 2}\left(\frac{8.64\times 10^4 \text{ s}}{1 \text{ d}}\right)\left(1 - e^{-(10.0 \text{ d})\ln 2/14.3 \text{ d}}\right)\right]$$

$= \boxed{8.97\times 10^{11} \text{ decays}}$, and one electron is emitted per decay.

(b) The total energy deposited is found to be

$$E = \left(700 \frac{\text{keV}}{\text{decay}}\right)(8.97\times 10^{11} \text{ decays})\left(\frac{1.60\times 10^{-16} \text{ J}}{1 \text{ keV}}\right) = \boxed{0.100 \text{ J}}$$

(c) The total absorbed dose (measured in rad) is given by

$$dose = \frac{\text{energy deposited per unit mass}}{\text{energy deposition per rad}}$$

$$= \frac{(0.100 \text{ J}/0.100 \text{ kg})}{\left(10^{-2} \frac{\text{J/kg}}{\text{rad}}\right)} = \boxed{100 \text{ rad}}$$

29.52 (a) The dose (in rem) received in time Δt is given by

$$dose = (dose \text{ in } rad)\times RBE$$

$$= \left[\left(100\times 10^{-3} \frac{\text{rad}}{\text{h}}\right)\Delta t\right]\times(1.00) = \left(0.100 \frac{\text{rem}}{\text{h}}\right)\Delta t$$

If this dose is to be 1.0 rem, the required time is

$$\Delta t = \frac{1.0 \text{ rem}}{0.100 \text{ rem/h}} = \boxed{10 \text{ h}}$$

(b) Assuming the radiation is emitted uniformly in all directions, the intensity of the radiation is given by $I = I_0/4\pi r^2$.

Therefore, $\dfrac{I_r}{I_1} = \dfrac{I_0/4\pi r^2}{I_0/4\pi(1.0\text{ m})^2} = \dfrac{(1.0\text{ m})^2}{r^2}$,

and $r = (1.0\text{ m})\sqrt{\dfrac{I_1}{I_r}} = (1.0\text{ m})\sqrt{\dfrac{100\text{ mrad/h}}{10\text{ mrad/h}}} = \boxed{3.2\text{ m}}$

29.53 From $R = R_0 e^{-\lambda t}$, the elapsed time is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -T_{1/2}\frac{\ln(R/R_0)}{\ln 2} = -(14.0\text{ d})\frac{\ln(20.0\text{ mCi}/200\text{ mCi})}{\ln 2} = \boxed{46.5\text{ d}}$$

29.54 The energy released in the reaction ${}^1_1\text{H} + {}^7_3\text{Li} \rightarrow {}^7_4\text{Be} + {}^1_0\text{n}$ is

$$Q = (\Delta m)c^2 = \left(m_{{}^1\text{H}} + m_{{}^7\text{Li}} - m_{{}^7\text{Be}} - m_{\text{n}}\right)c^2$$

$$= [1.007\,825\text{ u} + 7.016\,003\text{ u} - 7.016\,928\text{ u} - 1.008\,665\text{ u}](931.5\text{ MeV/u})$$

$$= -1.64\text{ MeV}$$

The threshold energy for the incident protons is then

$$KE_{min} = \left(1 + \frac{m_{{}^1\text{H}}}{m_{{}^7\text{Li}}}\right)|Q| = \left(1 + \frac{1.007\,825\text{ u}}{7.016\,003\text{ u}}\right)\left|-1.64\text{ MeV}\right| = \boxed{1.88\text{ MeV}}$$

29.55 The energy released in the reaction ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_0\text{n}$ is

$$Q = (\Delta m)c^2 = \left(2m_{{}^2\text{H}} - m_{{}^3\text{He}} - m_{\text{n}}\right)c^2$$

$$= [2(2.014\,102\text{ u}) - 3.016\,029\text{ u} - 1.008\,665\text{ u}](931.5\text{ MeV/u})$$

$$= \boxed{+3.27\text{ MeV}}$$

Since $Q > 0$, $\boxed{\text{no threshold energy is required}}$

29.56 From $N = N_0 e^{-\lambda t}$, the desired ratio is

$$\frac{\Delta N_1}{\Delta N_2} = \frac{N_0 - N_0 e^{-\lambda\left(\frac{T_{1/2}}{2}\right)}}{N_0 e^{-\lambda\left(\frac{T_{1/2}}{2}\right)} - N_0 e^{-\lambda\left(T_{1/2}\right)}}$$

Since $\lambda T_{1/2} = \ln 2$, this reduces to $\frac{\Delta N_1}{\Delta N_2} = \frac{1 - e^{-\frac{\ln 2}{2}}}{e^{-\frac{\ln 2}{2}} - e^{-\ln 2}} = \frac{1 - \sqrt{e^{-\ln 2}}}{\sqrt{e^{-\ln 2}} - e^{-\ln 2}}$

But $e^{\ln x} = x$, so $e^{-\ln 2} = \frac{1}{e^{\ln 2}} = \frac{1}{2}$ and we have

$$\frac{\Delta N_1}{\Delta N_2} = \frac{1 - 1/\sqrt{2}}{1/\sqrt{2} - 1/2} = \frac{2 - \sqrt{2}}{\sqrt{2} - 1} = \frac{\sqrt{2}\left(\sqrt{2} - 1\right)}{\sqrt{2} - 1} = \boxed{\sqrt{2}}$$

29.57 (a) $N_0 = \frac{\text{mass of sample}}{\text{mass per atom}} = \frac{1.00\text{ kg}}{(239.05\text{ u})\left(1.66 \times 10^{-27}\text{ kg/u}\right)} = \boxed{2.52 \times 10^{24}}$

(b) $R_0 = \lambda N_0 = \frac{N_0 \ln 2}{T_{1/2}} = \frac{\left(2.52 \times 10^{24}\right)\ln 2}{(24\,120\text{ yr})\left(3.156 \times 10^7\text{ s/yr}\right)} = \boxed{2.29 \times 10^{12}\text{ Bq}}$

(c) From $R = R_0 e^{-\lambda t}$, the required time is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -\frac{T_{1/2}\ln\left(R/R_0\right)}{\ln 2}$$

$$= -\frac{(24\,120\text{ yr})\ln\left(0.100/2.29 \times 10^{12}\right)}{\ln 2} = \boxed{1.07 \times 10^6\text{ yr}}$$

29.58 (a) $r = r_0 A^{1/3} = \left(1.2 \times 10^{-15}\text{ m}\right)(12)^{1/3} = 2.7 \times 10^{-15}\text{ m} = \boxed{2.7\text{ fm}}$

(b) With $Z = 6$,

$$F = \frac{k_e e\left[(Z-1)e\right]}{r^2} = \frac{5\left(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\text{ C}\right)^2}{\left(2.7 \times 10^{-15}\text{ m}\right)^2}$$

or $F = \boxed{1.5 \times 10^2\text{ N}}$

(c) The work done is the increase in the electrical potential energy, or

$$W = PE|_r - PE|_{r=\infty} = \frac{k_e e[(Z-1)e]}{r} - 0 = \frac{5(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{2.7\times10^{-15}\ \text{m}}$$

$$= 4.2\times10^{-13}\ \text{J}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{2.6\ \text{MeV}}$$

(d) Repeating the previous calculations for $^{238}_{92}\text{U}$ (with $Z = 92$ and $A = 238$) gives

$$r = r_0(238)^{1/3} = \boxed{7.4\ \text{fm}},\ F = \frac{k_e e[(91)e]}{r^2} = \boxed{3.8\times10^2\ \text{N}},$$

and $$W = \frac{k_e e[(91)e]}{r} = \boxed{18\ \text{MeV}}.$$

29.59 (a) If we assume all the ^{87}Sr nuclei came from the decay of ^{87}Rb nuclei, the original number of ^{87}Rb nuclei was

$$N_0 = 1.82\times10^{10} + 1.07\times10^9 = 1.93\times10^{10}$$

Then, from $N = N_0 e^{-\lambda t}$, the elapsed time is

$$t = -\frac{\ln(N/N_0)}{\lambda} = -\frac{T_{1/2}\ln(N/N_0)}{\ln 2}$$

$$= -\frac{(4.8\times10^{10}\ \text{yr})\ln\left(\frac{1.82\times10^{10}}{1.93\times10^{10}}\right)}{\ln 2} = \boxed{4.0\times10^9\ \text{yr}}$$

(b) It could be $\boxed{\text{no older}}$. It could be younger

$\boxed{\text{if some } ^{87}\text{Sr were initially present}}$

29.60 From $R = \lambda N$, the number of $^{60}\mathrm{Co}$ nuclei present in a $10\ \mathrm{Ci}$ source is

$$N = \frac{R}{\lambda} = \frac{R\left(T_{1/2}\right)}{\ln 2}$$

$$= \left(10\ \mathrm{Ci}\right)\left(3.7 \times 10^{10}\ \frac{\mathrm{decay/s}}{\mathrm{Ci}}\right)\frac{\left(5.2\ \mathrm{yr}\right)}{\ln 2}\left(\frac{3.156 \times 10^{7}\ \mathrm{s}}{1\ \mathrm{yr}}\right) = 8.8 \times 10^{19}$$

The number that was present 30 months (2.5 years) ago must have been

$$N_0 = \frac{N}{e^{-\lambda t}} = N\,e^{t\ln 2/T_{1/2}} = \left(8.8 \times 10^{19}\right)e^{(2.5\ \mathrm{yr})\ln 2/5.2\ \mathrm{yr}} = 1.2 \times 10^{20},$$

and the initial mass of $^{60}\mathrm{Co}$ was

$$m = N_0 m_{atom} = \left(1.2 \times 10^{20}\right)\left[\left(59.93\ \mathrm{u}\right)\left(1.66 \times 10^{-27}\ \mathrm{kg/u}\right)\right]$$

$$= 1.2 \times 10^{-5}\ \mathrm{kg} = \boxed{12\ \mathrm{mg}}$$

29.61 The total activity of the working solution at $t = 0$ was

$$\left(R_0\right)_{total} = \left(2.5\ \mathrm{mCi/mL}\right)\left(10\ \mathrm{mL}\right) = 25\ \mathrm{mCi}$$

Therefore, the initial activity of the 5.0-mL sample which will be drawn from the 250-mL working solution was

$$\left(R_0\right)_{sample} = \left(R_0\right)_{total}\left(\frac{5.0\ \mathrm{mL}}{250\ \mathrm{mL}}\right) = \left(25\ \mathrm{mCi}\right)\left(\frac{5.0\ \mathrm{mL}}{250\ \mathrm{mL}}\right) = 0.50\ \mathrm{mCi} = 5.0 \times 10^{-4}\ \mathrm{Ci}$$

With a half-life of 14.96 h for $^{24}\mathrm{Na}$ (Appendix B), the activity of the sample after 48 h is

$$R = R_0 e^{-\lambda t} = R_0 e^{-t\ln 2/T_{1/2}} = \left(5.0 \times 10^{-4}\ \mathrm{Ci}\right)e^{-(48\ \mathrm{h})\ln 2/(14.96\ \mathrm{h})}$$

$$= 5.4 \times 10^{-5}\ \mathrm{Ci} = \boxed{54\ \mu\mathrm{Ci}}$$

29.62 The decay constants for the two uranium isotopes are

$$\lambda_1 = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{0.70 \times 10^9 \text{ yr}} = 9.9 \times 10^{-10} \text{ yr}^{-1} \quad \left(\text{for } {}^{235}_{92}\text{U}\right)$$

and $$\lambda_2 = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47 \times 10^9 \text{ yr}} = 1.55 \times 10^{-10} \text{ yr}^{-1} \quad \left(\text{for } {}^{238}_{92}\text{U}\right)$$

If there were N_0 nuclei of each isotope present at $t = 0$, the ratio of the number remaining would be

$$\frac{N_{235}}{N_{238}} = \frac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}} = e^{(\lambda_2 - \lambda_1)t}\text{, and the elapsed time is } t = \frac{\ln\left(N_{235}/N_{238}\right)}{\lambda_2 - \lambda_1}$$

With a measured value of 0.007 for this ratio, the estimated age is

$$t = \frac{\ln(0.007)}{(1.55 - 9.9) \times 10^{-10} \text{ yr}^{-1}} = \boxed{6 \times 10^9 \text{ yr}}$$

29.63 The original activity per unit area is

$$R_0 = \frac{5.0 \times 10^6 \text{ Ci}}{1.0 \times 10^4 \text{ km}^2}\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right)^2 = 5.0 \times 10^{-4} \text{ Ci/m}^2 = 5.0 \times 10^2 \ \mu\text{Ci/m}^2$$

From $R = R_0 e^{-\lambda t}$, the required time is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -\frac{T_{1/2} \ln\left(R/R_0\right)}{\ln 2}$$

$$= -\frac{(28.7 \text{ yr}) \ln\left(2.0 \ \mu\text{Ci}/5.0 \times 10^2 \ \mu\text{Ci}\right)}{\ln 2} = \boxed{2.3 \times 10^2 \text{ yr}}$$

29.64 (a) The mass of a single $^{40}\mathrm{K}$ atom is

$$m = (39.964\ \mathrm{u})\left(1.66\times10^{-27}\ \mathrm{kg/u}\right) = 6.63\times10^{-26}\ \mathrm{kg} = 6.63\times10^{-23}\ \mathrm{g}$$

Therefore, the number of $^{40}\mathrm{K}$ nuclei in a liter of milk is

$$N = \frac{\text{total mass of } ^{40}\mathrm{K} \text{ present}}{\text{mass per atom}} = \frac{(2.00\ \mathrm{g/L})(0.0117/100)}{6.63\times10^{-23}\ \mathrm{g}} = 3.53\times10^{18}/\mathrm{L}$$

and the activity due to potassium is

$$R = \lambda N = \frac{N\ln 2}{T_{1/2}} = \frac{\left(3.53\times10^{18}/\mathrm{L}\right)\ln 2}{\left(1.28\times10^{9}\ \mathrm{yr}\right)\left(3.156\times10^{7}\ \mathrm{s/yr}\right)} = \boxed{60.5\ \mathrm{Bq/L}}$$

(b) Using $R = R_0 e^{-\lambda t}$, the time required for the $^{131}\mathrm{I}$ activity to decrease to the level of the potassium is given by

$$t = -\frac{\ln(R/R_0)}{\lambda} = -\frac{T_{1/2}\ln(R/R_0)}{\ln 2} = -\frac{(8.04\ \mathrm{d})\ln(60.5/2000)}{\ln 2} = \boxed{40.6\ \mathrm{d}}$$

29.65 The total activity due to $^{59}\mathrm{Fe}$ at the end of the 1000-h test will be

$$R = R_0 e^{-\lambda t} = R_0 e^{-t\ln 2/T_{1/2}} = (20.0\ \mu\mathrm{Ci})\, e^{-\left[(10^3\ \mathrm{h})\cdot\ln 2/(45.1\ \mathrm{d})(24\ \mathrm{h/d})\right]} = 10.5\ \mu\mathrm{Ci}$$

The total activity in the oil at the end of the test is

$$R_{oil} = \left(\frac{800\ \mathrm{min}^{-1}}{\mathrm{L}}\right)(6.5\ \mathrm{L})$$

$$= 5.2\times10^{3}\ \mathrm{min}^{-1}\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)\left(\frac{1\ \mu\mathrm{Ci}}{3.7\times10^{4}\ \mathrm{s}^{-1}}\right) = 2.3\times10^{-3}\ \mu\mathrm{Ci}$$

Therefore, the fraction of the iron that was worn away during the test is

$$\mathit{fraction} = \frac{R_{oil}}{R} = \frac{2.3\times10^{-3}\ \mu\mathrm{Ci}}{10.5\ \mu\mathrm{Ci}} = 2.2\times10^{-4}$$

This represents a mass of

$$m_{\text{worn away}} = (\text{fraction})\cdot(\text{total mass of iron})$$
$$= (2.2\times10^{-4})(0.20\text{ kg}) = 4.4\times10^{-5}\text{ kg}$$

so the wear rate was $\dfrac{4.4\times10^{-5}\text{ kg}}{1000\text{ h}} = \boxed{4.4\times10^{-8}\text{ kg/h}}$

29.66 (a) The energy available from the reaction

$$4\,{}^1_1\text{H} + 2\,{}^{0}_{-1}\text{e} \rightarrow {}^4_2\text{He} + 2\nu + \gamma$$

is

$$Q = (\Delta m)c^2 = \left[4m_{^1\text{H}} - m_{^4\text{He}}\right]c^2$$
$$= \left[4(1.007\,825\text{ u}) - 4.002\,602\text{ u}\right](931.5\text{ MeV/u})$$
$$= 26.7\text{ MeV}\left(1.60\times10^{-13}\text{ J/MeV}\right) = \boxed{4.28\times10^{-12}\text{ J}}$$

(b) $N = \dfrac{m_{\text{Sun}}}{m_{^1\text{H}}} = \dfrac{1.99\times10^{30}\text{ kg}}{1.67\times10^{-27}\text{ kg}} = \boxed{1.19\times10^{57}}$

(c) With 4 hydrogen nuclei consumed per reaction, the total energy available is

$$E = \left(\frac{N}{4}\right)Q = \left(\frac{1.19\times10^{57}}{4}\right)\left(4.28\times10^{-12}\text{ J}\right) = 1.27\times10^{45}\text{ J}$$

At a power output of $\wp = 3.76\times10^{26}\text{ W}$, this supply would last for

$$t = \frac{E}{\wp} = \frac{1.27\times10^{45}\text{ J}}{3.76\times10^{26}\text{ J/s}} = 3.39\times10^{18}\text{ s}\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right) = 1.07\times10^{11}\text{ yr}$$
$$= 107\times10^{9}\text{ yr} = \boxed{107\text{ billion years}}$$

Answers to Even Numbered Conceptual Questions

2. Neutrinos interact very weakly with matter. This allows neutrinos to penetrate a great thickness of matter with very low probability of being absorbed.

4. An alpha particle is a doubly positive charged helium nucleus, is very massive and does not penetrate very well. A beta particle is a singly negative charged electron, and is very light and only slightly more difficult to shield from. A gamma ray is a high energy photon or high frequency electromagnetic wave, and has high penetrating ability.

6. Beta particles have greater penetrating ability than do alpha particles.

8. The much larger mass of the alpha particle as compared to that of the beta particle ensures that it will not deflect as much as does the beta, which has a mass about 7000 times smaller.

10. Consider the reaction ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + \bar{\nu}$. We have six positive charges before the event on the carbon-14 nucleus. After the decay, we still have a net of six positive charges, as $+7$ from the nitrogen and -1 from the electron. Thus, in order to have conservation of charge, the neutrino must be uncharged.

12. We would have to revise our age values upward for ancient materials. That is, we would conclude that the materials were older than we had thought, because the greater cosmic ray intensity would have left the samples with a larger percentage of carbon-14 when they died, and a longer time would have been necessary for it to decay to the percentage found at present.

14. The amount of carbon-14 left in very old materials is extremely small, and detection cannot be accomplished with a high degree of accuracy.

16. The mass number, A, equals the number of nucleons and N equals the number of neutrons. The atomic number, $Z = A - N$, equals the number of protons in the nucleus and also the number of electrons in the atom. The chemical properties are determined by the electronic structure, so Z is the number that determines the chemical properties of the atom.

18. Neutrons, being neutral particles, do not experience electrical repulsion as they approach a positively charged nucleus in the way alpha particles or protons do. Therefore, it is easier for neutrons to penetrate the nucleus and initiate nuclear reactions.

Answers to Even Numbered Problems

2. $\sim 10^{28}$ protons, $\sim 10^{28}$ neutrons, $\sim 10^{28}$ electrons

4. $\rho_n/\rho_a = 8.6 \times 10^{13}$

6. (a) 7.89 cm for ^{12}C, 8.21 cm for ^{13}C (b) $\frac{7.89\text{ cm}}{8.21\text{ cm}} = \sqrt{\frac{12}{13}} = 0.961$

8. any nucleus with $A = 30$

10. (a) 1.11 MeV/nucleon (b) 7.07 MeV/nucleon
(c) 8.79 MeV/nucleon (d) 7.57 MeV/nucleon

12. $E_b/A = 8.765$ MeV/nucleon for ^{55}Mn;
$E_b/A = 8.786$ MeV/nucleon for ^{56}Fe;
$E_b/A = 8.768$ MeV/nucleon for ^{59}Co

14. 8.26 MeV/nucleon for $^{24}_{12}Mg$, 8.70 MeV/nucleon for $^{85}_{37}Rb$

16. 8.7×10^3 Bq

18. 0.200 mCi

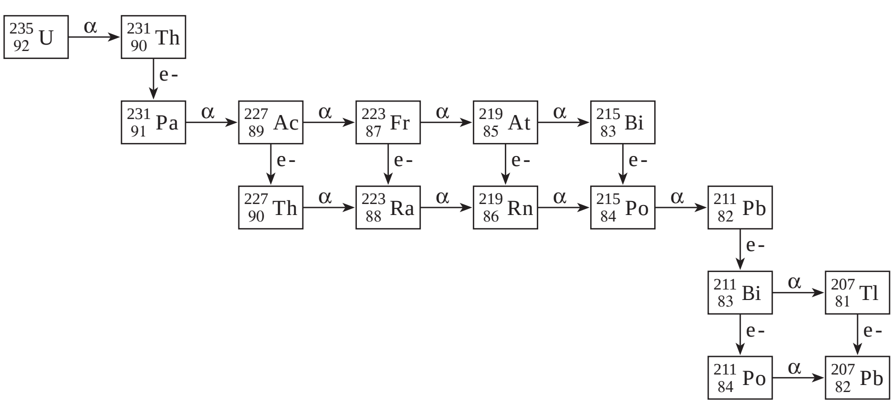
20. 86 h

22. 1.72×10^4 yr

24. 1.66×10^3 yr

26. $^{12}_{6}C$, $^{4}_{2}He$, $^{14}_{6}C$

28.



30. 4.28 MeV

32. (a) $^{66}_{28}Ni \rightarrow ^{66}_{29}Cu + ^{0}_{-1}e + \bar{\nu}$ (b) 186 keV

34. 1.67×10^{4} yr

36. 9.96×10^{3} yr

38. $^{4}_{2}He$, $^{4}_{2}He$

40. 17.3 MeV

42. (a) $^{10}_{5}B$ (b) –2.79 MeV

44. 22.0 MeV

46. 5.0 rad

48. (a) 2.00 J/kg (b) 4.78×10^{-4} °C

50. (a) 2.5×10^{-3} rem/x-ray (b) ≈ 38 times background levels

52. (a) 10 h (b) 3.2 m

54. 1.88 MeV

56. $\sqrt{2}$

58. (a) 2.7 fm (b) 1.5×10^{2} N (c) 2.6 MeV
(d) 7.4 fm, 3.8×10^{2} N, 18 MeV

60. 12 mg

62. 6×10^{9} yr

64. (a) 60.5 Bq/L (b) 40.6 d

66. (a) 4.28×10^{-12} J (b) 1.19×10^{57} (c) 107 billion years