

1. The speed (assumed constant) is  $(90 \text{ km/h})(1000 \text{ m/km}) / (3600 \text{ s/h}) = 25 \text{ m/s}$ . Thus, during 0.50 s, the car travels  $(0.50)(25) \approx 13 \text{ m}$ .

2. Huber's speed is

$$v_0 = (200 \text{ m}) / (6.509 \text{ s}) = 30.72 \text{ m/s} = 110.6 \text{ km/h},$$

where we have used the conversion factor  $1 \text{ m/s} = 3.6 \text{ km/h}$ . Since Whittingham beat Huber by  $19.0 \text{ km/h}$ , his speed is  $v_1 = (110.6 + 19.0) = 129.6 \text{ km/h}$ , or  $36 \text{ m/s}$  ( $1 \text{ km/h} = 0.2778 \text{ m/s}$ ). Thus, the time through a distance of  $200 \text{ m}$  for Whittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200 \text{ m}}{36 \text{ m/s}} = 5.554 \text{ s}.$$

3. We use Eq. 2-2 and Eq. 2-3. During a time  $t_c$  when the velocity remains a positive constant, speed is equivalent to velocity, and distance is equivalent to displacement, with  $\Delta x = v t_c$ .

(a) During the first part of the motion, the displacement is  $\Delta x_1 = 40$  km and the time interval is

$$t_1 = \frac{(40 \text{ km})}{(30 \text{ km/h})} = 1.33 \text{ h.}$$

During the second part the displacement is  $\Delta x_2 = 40$  km and the time interval is

$$t_2 = \frac{(40 \text{ km})}{(60 \text{ km/h})} = 0.67 \text{ h.}$$

Both displacements are in the same direction, so the total displacement is

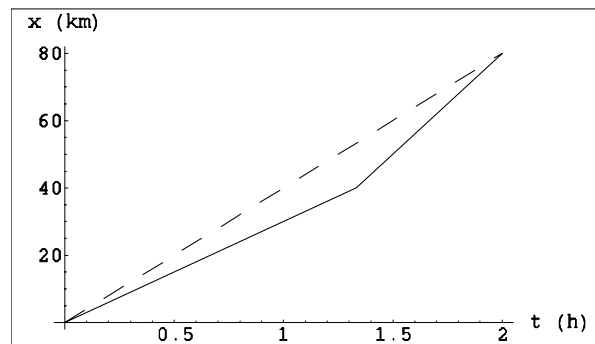
$$\Delta x = \Delta x_1 + \Delta x_2 = 40 \text{ km} + 40 \text{ km} = 80 \text{ km.}$$

The total time for the trip is  $t = t_1 + t_2 = 2.00$  h. Consequently, the average velocity is

$$v_{\text{avg}} = \frac{(80 \text{ km})}{(2.0 \text{ h})} = 40 \text{ km/h.}$$

(b) In this example, the numerical result for the average speed is the same as the average velocity 40 km/h.

(c) As shown below, the graph consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to  $(t_1, x_1) = (1.33 \text{ h}, 40 \text{ km})$  and the second having a slope of 60 km/h and connecting  $(t_1, x_1)$  to  $(t, x) = (2.00 \text{ h}, 80 \text{ km})$ . From the graphical point of view, the slope of the dashed line drawn from the origin to  $(t, x)$  represents the average velocity.



4. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance  $D$  up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have  $\text{speed} = D/t$ . Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}}$$

which, after canceling  $D$  and plugging in  $v_{\text{up}} = 40$  km/h and  $v_{\text{down}} = 60$  km/h, yields 48 km/h for the average speed.

5. Using  $x = 3t - 4t^2 + t^3$  with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write  $x = (3 \text{ m/s})t - (4 \text{ m/s}^2)t^2 + (1 \text{ m/s}^3)t^3$ . We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

(a) Plugging in  $t = 1 \text{ s}$  yields  $x = 3 - 4 + 1 = 0$ .

(b) With  $t = 2 \text{ s}$  we get  $x = 3(2) - 4(2)^2 + (2)^3 = -2 \text{ m}$ .

(c) With  $t = 3 \text{ s}$  we have  $x = 0 \text{ m}$ .

(d) Plugging in  $t = 4 \text{ s}$  gives  $x = 12 \text{ m}$ .

For later reference, we also note that the position at  $t = 0$  is  $x = 0$ .

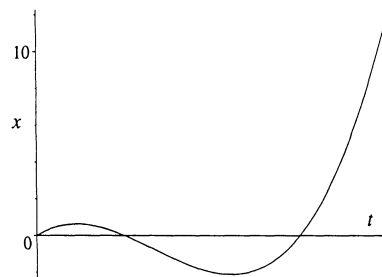
(e) The position at  $t = 0$  is subtracted from the position at  $t = 4 \text{ s}$  to find the displacement  $\Delta x = 12 \text{ m}$ .

(f) The position at  $t = 2 \text{ s}$  is subtracted from the position at  $t = 4 \text{ s}$  to give the displacement  $\Delta x = 14 \text{ m}$ . Eq. 2-2, then, leads to

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14}{2} = 7 \text{ m/s}.$$

(g) The horizontal axis is  $0 \leq t \leq 4$  with SI units understood.

Not shown is a straight line drawn from the point at  $(t, x) = (2, -2)$  to the highest point shown (at  $t = 4 \text{ s}$ ) which would represent the answer for part (f).



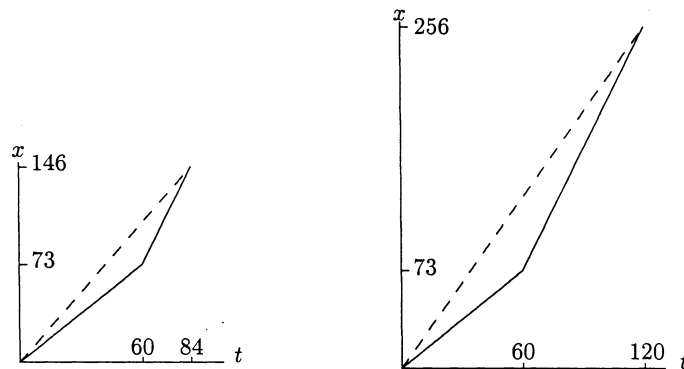
6. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

$$v_{\text{avg}} = \frac{73.2 \text{ m} + 73.2 \text{ m}}{\frac{73.2 \text{ m}}{1.22 \text{ m/s}} + \frac{73.2 \text{ m}}{3.05 \text{ m/s}}} = 1.74 \text{ m/s}.$$

(b) Using the fact that distance =  $vt$  while the velocity  $v$  is constant, we find

$$v_{\text{avg}} = \frac{(1.22 \text{ m/s})(60 \text{ s}) + (3.05 \text{ m/s})(60 \text{ s})}{120 \text{ s}} = 2.14 \text{ m/s}.$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before — the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.



7. We use the functional notation  $x(t)$ ,  $v(t)$  and  $a(t)$  in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

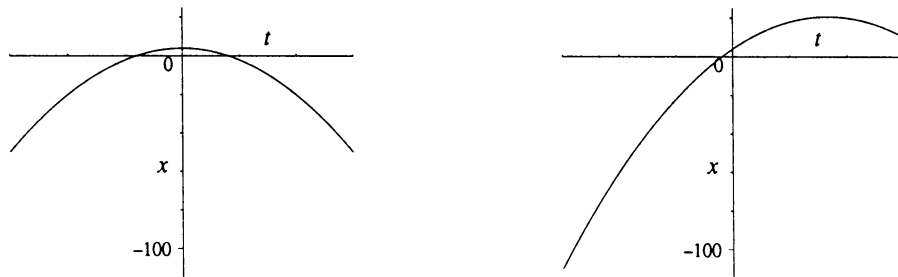
with SI units understood.

(a) From  $v(t) = 0$  we find it is (momentarily) at rest at  $t = 0$ .

(b) We obtain  $x(0) = 4.0$  m

(c) and (d) Requiring  $x(t) = 0$  in the expression  $x(t) = 4.0 - 6.0t^2$  leads to  $t = \pm 0.82$  s for the times when the particle can be found passing through the origin.

(e) We show both the asked-for graph (on the left) as well as the “shifted” graph which is relevant to part (f). In both cases, the time axis is given by  $-3 \leq t \leq 3$  (SI units understood).



(f) We arrived at the graph on the right (shown above) by adding  $20t$  to the  $x(t)$  expression.

(g) Examining where the slopes of the graphs become zero, it is clear that the shift causes the  $v = 0$  point to correspond to a larger value of  $x$  (the top of the second curve shown in part (e) is higher than that of the first).

8. The values used in the problem statement make it easy to see that the first part of the trip (at 100 km/h) takes 1 hour, and the second part (at 40 km/h) also takes 1 hour. Expressed in decimal form, the time left is 1.25 hour, and the distance that remains is 160 km. Thus, a speed of  $160/1.25 = 128$  km/h is needed.



9. Converting to seconds, the running times are  $t_1 = 147.95$  s and  $t_2 = 148.15$  s, respectively. If the runners were equally fast, then

$$s_{\text{avg}1} = s_{\text{avg}2} \Rightarrow \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

From this we obtain

$$L_2 - L_1 = \left( \frac{t_2}{t_1} - 1 \right) L_1 = \left( \frac{148.15}{147.95} - 1 \right) L_1 = 0.00135 L_1 \approx 1.4 \text{ m}$$

where we set  $L_1 \approx 1000$  m in the last step. Thus, if  $L_1$  and  $L_2$  are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if  $L_1$  is shorter than  $L_2$  by more than 1.4 m, then runner 2 would actually be faster.

10. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time which elapses before they crash is  $t = (60 \text{ km}) / (60 \text{ km/h}) = 1.0 \text{ h}$ . During this time, the bird travels a distance of  $x = vt = (60 \text{ km/h})(1.0 \text{ h}) = 60 \text{ km}$ .

11. (a) Denoting the travel time and distance from San Antonio to Houston as  $T$  and  $D$ , respectively, the average speed is

$$s_{\text{avg}1} = \frac{D}{T} = \frac{(55 \text{ km/h}) \frac{T}{2} + (90 \text{ km/h}) \frac{T}{2}}{T} = 72.5 \text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg}2} = \frac{D}{T} = \frac{D}{\frac{D/2}{55 \text{ km/h}} + \frac{D/2}{90 \text{ km/h}}} = 68.3 \text{ km/h}$$

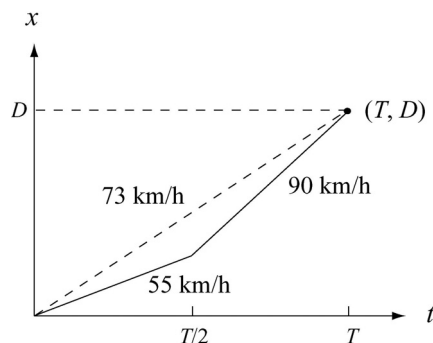
which should be rounded to 68 km/h.

(c) The total distance traveled ( $2D$ ) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5 \text{ km/h}} + \frac{D}{68.3 \text{ km/h}}} = 70 \text{ km/h}.$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance  $D$  (the intent is *not* to make the student go to an Atlas to look it up); the student can just as easily arbitrarily set  $T$  instead of  $D$ , as will be clear in the following discussion. In the interest of saving space, we briefly describe the graph (with kilometers-per-hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to  $(t_1, x_1) = (T/2, 55T/2)$  and the second having a slope of 90 and connecting  $(t_1, x_1)$  to  $(T, D)$  where  $D = (55 + 90)T/2$ . The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to  $(T, D)$ . The graph (not drawn to scale) is depicted below:



12. We use Eq. 2-4. to solve the problem.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt} (4 - 12t + 3t^2) = -12 + 6t.$$

Thus, at  $t = 1$  s, the velocity is  $v = (-12 + (6)(1)) = -6$  m/s.

(b) Since  $v < 0$ , it is moving in the negative  $x$  direction at  $t = 1$  s.

(c) At  $t = 1$  s, the *speed* is  $|v| = 6$  m/s.

(d) For  $0 < t < 2$  s,  $|v|$  decreases until it vanishes. For  $2 < t < 3$  s,  $|v|$  increases from zero to the value it had in part (c). Then,  $|v|$  is larger than that value for  $t > 3$  s.

(e) Yes, since  $v$  smoothly changes from negative values (consider the  $t = 1$  result) to positive (note that as  $t \rightarrow +\infty$ , we have  $v \rightarrow +\infty$ ). One can check that  $v = 0$  when  $t = 2$  s.

(f) No. In fact, from  $v = -12 + 6t$ , we know that  $v > 0$  for  $t > 2$  s.

13. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for  $x$  for  $t = 2.00$  s and  $t = 3.00$  s and obtain  $x_2 = 21.75$  cm and  $x_3 = 50.25$  cm, respectively. The average velocity during the time interval  $2.00 \leq t \leq 3.00$  s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25 \text{ cm} - 21.75 \text{ cm}}{3.00 \text{ s} - 2.00 \text{ s}}$$

which yields  $v_{\text{avg}} = 28.5$  cm/s.

(b) The instantaneous velocity is  $v = \frac{dx}{dt} = 4.5t^2$ , which, at time  $t = 2.00$  s, yields  $v = (4.5)(2.00)^2 = 18.0$  cm/s.

(c) At  $t = 3.00$  s, the instantaneous velocity is  $v = (4.5)(3.00)^2 = 40.5$  cm/s.

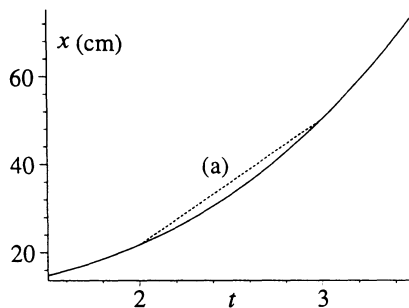
(d) At  $t = 2.50$  s, the instantaneous velocity is  $v = (4.5)(2.50)^2 = 28.1$  cm/s.

(e) Let  $t_m$  stand for the moment when the particle is midway between  $x_2$  and  $x_3$  (that is, when the particle is at  $x_m = (x_2 + x_3)/2 = 36$  cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \Rightarrow t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is  $v = 4.5(2.596)^2 = 30.3$  cm/s.

(f) The answer to part (a) is given by the slope of the straight line between  $t = 2$  and  $t = 3$  in this  $x$ -vs- $t$  plot. The answers to parts (b), (c), (d) and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



14. We use the functional notation  $x(t)$ ,  $v(t)$  and  $a(t)$  and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{dt} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

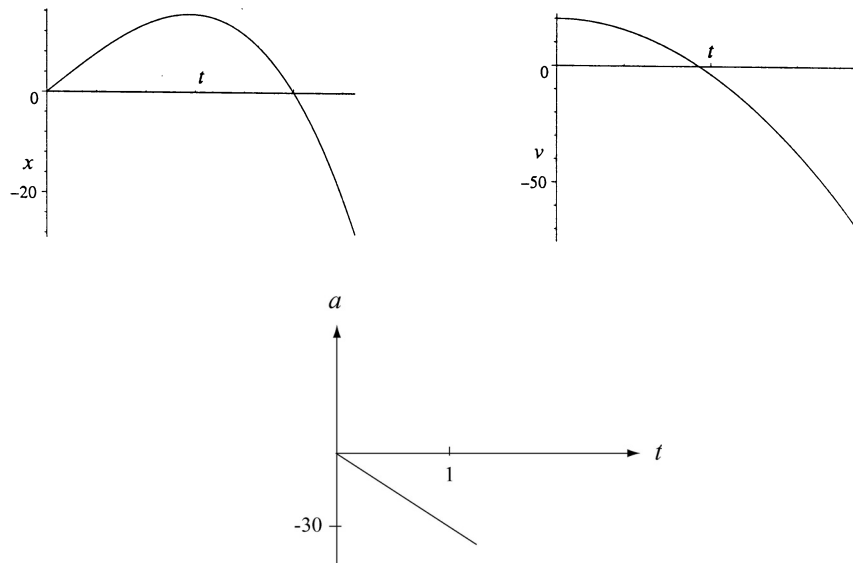
(a) From  $0 = -15t^2 + 20$ , we see that the only positive value of  $t$  for which the particle is (momentarily) stopped is  $t = \sqrt{20/15} = 1.2$  s.

(b) From  $0 = -30t$ , we find  $a(0) = 0$  (that is, it vanishes at  $t = 0$ ).

(c) It is clear that  $a(t) = -30t$  is negative for  $t > 0$

(d) The acceleration  $a(t) = -30t$  is positive for  $t < 0$ .

(e) The graphs are shown below. SI units are understood.



15. We represent its initial direction of motion as the  $+x$  direction, so that  $v_0 = +18 \text{ m/s}$  and  $v = -30 \text{ m/s}$  (when  $t = 2.4 \text{ s}$ ). Using Eq. 2-7 (or Eq. 2-11, suitably interpreted) we find

$$a_{\text{avg}} = \frac{(-30) - (+18)}{2.4} = -20 \text{ m/s}^2$$

which indicates that the average acceleration has magnitude  $20 \text{ m/s}^2$  and is in the opposite direction to the particle's initial velocity.

16. Using the general property  $\frac{d}{dx} \exp(bx) = b \exp(bx)$ , we write

$$v = \frac{dx}{dt} = \left( \frac{d(19t)}{dt} \right) \cdot e^{-t} + (19t) \cdot \left( \frac{de^{-t}}{dt} \right) .$$

If a concern develops about the appearance of an argument of the exponential ( $-t$ ) apparently having units, then an explicit factor of  $1/T$  where  $T = 1$  second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1 - t)e^{-t}$$

with  $t$  and  $v$  in SI units (s and m/s, respectively). We see that this function is zero when  $t = 1$  s. Now that we know *when* it stops, we find out *where* it stops by plugging our result  $t = 1$  into the given function  $x = 16te^{-t}$  with  $x$  in meters. Therefore, we find  $x = 5.9$  m.



17. (a) Taking derivatives of  $x(t) = 12t^2 - 2t^3$  we obtain the velocity and the acceleration functions:

$$v(t) = 24t - 6t^2 \quad \text{and} \quad a(t) = 24 - 12t$$

with length in meters and time in seconds. Plugging in the value  $t = 3$  yields  $x(3) = 54$  m .

(b) Similarly, plugging in the value  $t = 3$  yields  $v(3) = 18$  m/s.

(c) For  $t = 3$ ,  $a(3) = -12$  m/s<sup>2</sup>.

(d) At the maximum  $x$ , we must have  $v = 0$ ; eliminating the  $t = 0$  root, the velocity equation reveals  $t = 24/6 = 4$  s for the time of maximum  $x$ . Plugging  $t = 4$  into the equation for  $x$  leads to  $x = 64$  m for the largest  $x$  value reached by the particle.

(e) From (d), we see that the  $x$  reaches its maximum at  $t = 4.0$  s.

(f) A maximum  $v$  requires  $a = 0$ , which occurs when  $t = 24/12 = 2.0$  s. This, inserted into the velocity equation, gives  $v_{\max} = 24$  m/s.

(g) From (f), we see that the maximum of  $v$  occurs at  $t = 24/12 = 2.0$  s.

(h) In part (e), the particle was (momentarily) motionless at  $t = 4$  s. The acceleration at that time is readily found to be  $24 - 12(4) = -24$  m/s<sup>2</sup>.

(i) The *average velocity* is defined by Eq. 2-2, so we see that the values of  $x$  at  $t = 0$  and  $t = 3$  s are needed; these are, respectively,  $x = 0$  and  $x = 54$  m (found in part (a)). Thus,

$$v_{\text{avg}} = \frac{54-0}{3-0} = 18 \text{ m/s} \quad .$$

18. We use Eq. 2-2 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during  $5 \text{ min} \leq t \leq 10 \text{ min}$  is taken to be the positive  $x$  direction. We also use the fact that  $\Delta x = v\Delta t'$  when the velocity is constant during a time interval  $\Delta t'$ .

(a) The entire interval considered is  $\Delta t = 8 - 2 = 6 \text{ min}$  which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only  $\Delta t' = 8 - 5 = 3 \text{ min} = 180 \text{ s}$ . His position at  $t = 2 \text{ min}$  is  $x = 0$  and his position at  $t = 8 \text{ min}$  is  $x = v\Delta t' = (2.2)(180) = 396 \text{ m}$ . Therefore,

$$v_{\text{avg}} = \frac{396 \text{ m} - 0}{360 \text{ s}} = 1.10 \text{ m/s}.$$

(b) The man is at rest at  $t = 2 \text{ min}$  and has velocity  $v = +2.2 \text{ m/s}$  at  $t = 8 \text{ min}$ . Thus, keeping the answer to 3 significant figures,

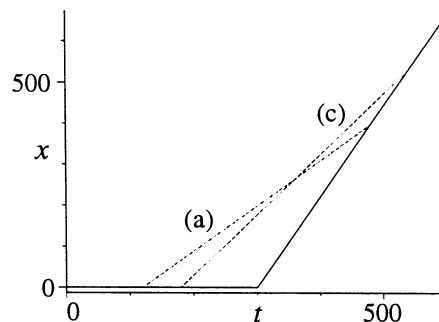
$$a_{\text{avg}} = \frac{2.2 \text{ m/s} - 0}{360 \text{ s}} = 0.00611 \text{ m/s}^2.$$

(c) Now, the entire interval considered is  $\Delta t = 9 - 3 = 6 \text{ min}$  (360 s again), whereas the sub-interval in which he is moving is  $\Delta t' = 9 - 5 = 4 \text{ min} = 240 \text{ s}$ . His position at  $t = 3 \text{ min}$  is  $x = 0$  and his position at  $t = 9 \text{ min}$  is  $x = v\Delta t' = (2.2)(240) = 528 \text{ m}$ . Therefore,

$$v_{\text{avg}} = \frac{528 \text{ m} - 0}{360 \text{ s}} = 1.47 \text{ m/s}.$$

(d) The man is at rest at  $t = 3 \text{ min}$  and has velocity  $v = +2.2 \text{ m/s}$  at  $t = 9 \text{ min}$ . Consequently,  $a_{\text{avg}} = 2.2/360 = 0.00611 \text{ m/s}^2$  just as in part (b).

(e) The horizontal line near the bottom of this  $x$ -vs- $t$  graph represents the man standing at  $x = 0$  for  $0 \leq t < 300 \text{ s}$  and the linearly rising line for  $300 \leq t \leq 600 \text{ s}$  represents his constant-velocity motion. The dotted lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of  $v$ -vs- $t$  is not shown here, but would consist of two horizontal “steps” (one at  $v = 0$  for  $0 \leq t < 300$  s and the next at  $v = 2.2$  m/s for  $300 \leq t \leq 600$  s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the “steps” at the appropriate  $t$  values (the slopes of the dotted lines representing the values of  $a_{\text{avg}}$ ).

19. In this solution, we make use of the notation  $x(t)$  for the value of  $x$  at a particular  $t$ . The notations  $v(t)$  and  $a(t)$  have similar meanings.

(a) Since the unit of  $ct^2$  is that of length, the unit of  $c$  must be that of length/time<sup>2</sup>, or m/s<sup>2</sup> in the SI system.

(b) Since  $bt^3$  has a unit of length,  $b$  must have a unit of length/time<sup>3</sup>, or m/s<sup>3</sup>.

(c) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by  $v = dx/dt = 2ct - 3bt^2$ ,  $v = 0$  occurs for  $t = 0$  and for

$$t = \frac{2c}{3b} = \frac{2(3.0 \text{ m/s}^2)}{3(2.0 \text{ m/s}^3)} = 1.0 \text{ s}.$$

For  $t = 0$ ,  $x = x_0 = 0$  and for  $t = 1.0 \text{ s}$ ,  $x = 1.0 \text{ m} > x_0$ . Since we seek the maximum, we reject the first root ( $t = 0$ ) and accept the second ( $t = 1 \text{ s}$ ).

(d) In the first 4 s the particle moves from the origin to  $x = 1.0 \text{ m}$ , turns around, and goes back to

$$x(4 \text{ s}) = (3.0 \text{ m/s}^2)(4.0 \text{ s})^2 - (2.0 \text{ m/s}^3)(4.0 \text{ s})^3 = -80 \text{ m}.$$

The total path length it travels is  $1.0 \text{ m} + 1.0 \text{ m} + 80 \text{ m} = 82 \text{ m}$ .

(e) Its displacement is  $\Delta x = x_2 - x_1$ , where  $x_1 = 0$  and  $x_2 = -80 \text{ m}$ . Thus,  $\Delta x = -80 \text{ m}$ .

The velocity is given by  $v = 2ct - 3bt^2 = (6.0 \text{ m/s}^2)t - (6.0 \text{ m/s}^3)t^2$ .

(f) Plugging in  $t = 1 \text{ s}$ , we obtain

$$v(1 \text{ s}) = (6.0 \text{ m/s}^2)(1.0 \text{ s}) - (6.0 \text{ m/s}^3)(1.0 \text{ s})^2 = 0.$$

(g) Similarly,  $v(2 \text{ s}) = (6.0 \text{ m/s}^2)(2.0 \text{ s}) - (6.0 \text{ m/s}^3)(2.0 \text{ s})^2 = -12 \text{ m/s}$ .

(h)  $v(3 \text{ s}) = (6.0 \text{ m/s}^2)(3.0 \text{ s}) - (6.0 \text{ m/s}^3)(3.0 \text{ s})^2 = -36.0 \text{ m/s}$ .

(i)  $v(4 \text{ s}) = (6.0 \text{ m/s}^2)(4.0 \text{ s}) - (6.0 \text{ m/s}^3)(4.0 \text{ s})^2 = -72 \text{ m/s}$ .

The acceleration is given by  $a = dv/dt = 2c - 6bt = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)t$ .

(j) Plugging in  $t = 1 \text{ s}$ , we obtain

$$a(1 \text{ s}) = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(1.0 \text{ s}) = -6.0 \text{ m/s}^2.$$

(k)  $a(2\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(2.0\text{ s}) = -18\text{ m/s}^2.$

(l)  $a(3\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(3.0\text{ s}) = -30\text{ m/s}^2.$

(m)  $a(4\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(4.0\text{ s}) = -42\text{ m/s}^2.$

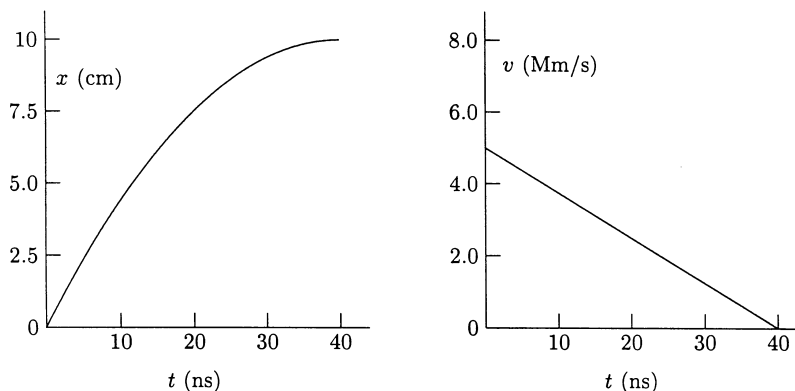
20. The constant-acceleration condition permits the use of Table 2-1.

(a) Setting  $v = 0$  and  $x_0 = 0$  in  $v^2 = v_0^2 + 2a(x - x_0)$ , we find

$$x = -\frac{1}{2} \frac{v_0^2}{a} = -\frac{1}{2} \left( \frac{5.00 \times 10^6}{-1.25 \times 10^{14}} \right) = 0.100 \text{ m} .$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time-plots of the position  $x$  and velocity  $v$  of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to  $t$ , so that other equations from Table 2-1 (such as  $v = v_0 + at$  and  $x = v_0 t + \frac{1}{2} at^2$ ) are used in making these plots.



21. We use  $v = v_0 + at$ , with  $t = 0$  as the instant when the velocity equals  $+9.6 \text{ m/s}$ .

(a) Since we wish to calculate the velocity for a time *before*  $t = 0$ , we set  $t = -2.5 \text{ s}$ . Thus, Eq. 2-11 gives

$$v = (9.6 \text{ m/s}) + (3.2 \text{ m/s}^2) (-2.5 \text{ s}) = 1.6 \text{ m/s}.$$

(b) Now,  $t = +2.5 \text{ s}$  and we find

$$v = (9.6 \text{ m/s}) + (3.2 \text{ m/s}^2) (2.5 \text{ s}) = 18 \text{ m/s}.$$

22. We take  $+x$  in the direction of motion, so  $v_0 = +24.6 \text{ m/s}$  and  $a = -4.92 \text{ m/s}^2$ . We also take  $x_0 = 0$ .

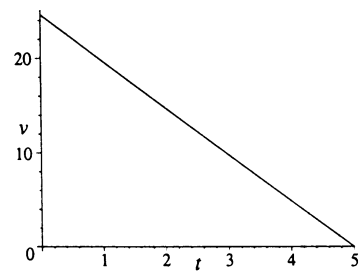
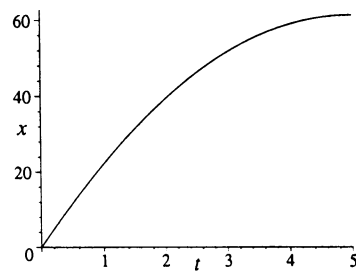
(a) The time to come to a halt is found using Eq. 2-11:

$$0 = v_0 + at \Rightarrow t = \frac{24.6}{-4.92} = 5.00 \text{ s}.$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \Rightarrow x = -\frac{24.6^2}{2(-4.92)} = 61.5 \text{ m}.$$

(c) Using these results, we plot  $v_0t + \frac{1}{2}at^2$  (the  $x$  graph, shown next, on the left) and  $v_0 + at$  (the  $v$  graph, on the right) over  $0 \leq t \leq 5 \text{ s}$ , with SI units understood.





23. The constant acceleration stated in the problem permits the use of the equations in Table 2-1.

(a) We solve  $v = v_0 + at$  for the time:

$$t = \frac{v - v_0}{a} = \frac{\frac{1}{10}(3.0 \times 10^8 \text{ m/s})}{9.8 \text{ m/s}^2} = 3.1 \times 10^6 \text{ s}$$

which is equivalent to 1.2 months.

(b) We evaluate  $x = x_0 + v_0 t + \frac{1}{2} at^2$ , with  $x_0 = 0$ . The result is

$$x = \frac{1}{2} (9.8 \text{ m/s}^2) (3.1 \times 10^6 \text{ s})^2 = 4.6 \times 10^{13} \text{ m}.$$

24. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given  $v_0 = 0$ ;  $v = 20$  m/s and  $a = 2.0$  m/s<sup>2</sup>. In part 2, the vehicle decelerates from its highest speed to a halt; we are given  $v_0 = 20$  m/s;  $v = 0$  and  $a = -1.0$  m/s<sup>2</sup> (negative because the acceleration vector points opposite to the direction of motion).

(a) From Table 2-1, we find  $t_1$  (the duration of part 1) from  $v = v_0 + at$ . In this way,  $20 = 0 + 2.0t_1$  yields  $t_1 = 10$  s. We obtain the duration  $t_2$  of part 2 from the same equation. Thus,  $0 = 20 + (-1.0)t_2$  leads to  $t_2 = 20$  s, and the total is  $t = t_1 + t_2 = 30$  s.

(b) For part 1, taking  $x_0 = 0$ , we use the equation  $v^2 = v_0^2 + 2a(x - x_0)$  from Table 2-1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(20)^2 - (0)^2}{2(2.0)} = 100 \text{ m}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$x - 100 = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (20)^2}{2(-1.0)}.$$

Thus, the final position is  $x = 300$  m. That this is also the total distance traveled should be evident (the vehicle did not “backtrack” or reverse its direction of motion).

25. Assuming constant acceleration permits the use of the equations in Table 2-1. We solve  $v^2 = v_0^2 + 2a(x - x_0)$  with  $x_0 = 0$  and  $x = 0.010$  m. Thus,

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(5.7 \times 10^5)^2 - (1.5 \times 10^5)^2}{2(0.01)} = 1.62 \times 10^{15} \text{ m/s}^2.$$

26. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020 \text{ km/h}) \left( \frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right)}{1.4 \text{ s}} = 202.4 \text{ m/s}^2 .$$

In terms of the gravitational acceleration  $g$ , this is expressed as a multiple of  $9.8 \text{ m/s}^2$  as follows:

$$a = \frac{202.4}{9.8} g = 21g .$$

27. The problem statement (see part (a)) indicates that  $a = \text{constant}$ , which allows us to use Table 2-1.

(a) We take  $x_0 = 0$ , and solve  $x = v_0 t + \frac{1}{2} a t^2$  (Eq. 2-15) for the acceleration:  $a = 2(x - v_0 t)/t^2$ . Substituting  $x = 24.0 \text{ m}$ ,  $v_0 = 56.0 \text{ km/h} = 15.55 \text{ m/s}$  and  $t = 2.00 \text{ s}$ , we find

$$a = \frac{2 \left( 24.0 \text{ m} - (15.55 \text{ m/s}) (2.00 \text{ s}) \right)}{(2.00 \text{ s})^2} = -3.56 \text{ m/s}^2,$$

or  $|a| = 3.56 \text{ m/s}^2$ . The negative sign indicates that the acceleration is opposite to the direction of motion of the car. The car is slowing down.

(b) We evaluate  $v = v_0 + at$  as follows:

$$v = 15.55 \text{ m/s} - (3.56 \text{ m/s}^2) (2.00 \text{ s}) = 8.43 \text{ m/s}$$

which can also be converted to  $30.3 \text{ km/h}$ .

28. We choose the positive direction to be that of the initial velocity of the car (implying that  $a < 0$  since it is slowing down). We assume the acceleration is constant and use Table 2-1.

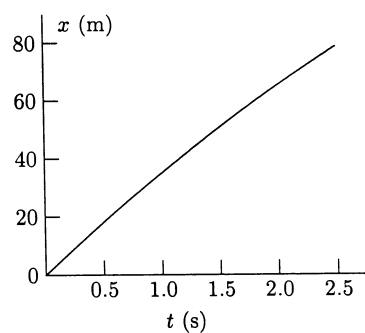
(a) Substituting  $v_0 = 137 \text{ km/h} = 38.1 \text{ m/s}$ ,  $v = 90 \text{ km/h} = 25 \text{ m/s}$ , and  $a = -5.2 \text{ m/s}^2$  into  $v = v_0 + at$ , we obtain

$$t = \frac{25 \text{ m/s} - 38 \text{ m/s}}{-5.2 \text{ m/s}^2} = 2.5 \text{ s} .$$

(b) We take the car to be at  $x = 0$  when the brakes are applied (at time  $t = 0$ ). Thus, the coordinate of the car as a function of time is given by

$$x = (38)t + \frac{1}{2}(-5.2)t^2$$

in SI units. This function is plotted from  $t = 0$  to  $t = 2.5 \text{ s}$  on the graph below. We have not shown the  $v$ -vs- $t$  graph here; it is a descending straight line from  $v_0$  to  $v$ .



29. We assume the periods of acceleration (duration  $t_1$ ) and deceleration (duration  $t_2$ ) are periods of constant  $a$  so that Table 2-1 can be used. Taking the direction of motion to be  $+x$  then  $a_1 = +1.22 \text{ m/s}^2$  and  $a_2 = -1.22 \text{ m/s}^2$ . We use SI units so the velocity at  $t = t_1$  is  $v = 305/60 = 5.08 \text{ m/s}$ .

(a) We denote  $\Delta x$  as the distance moved during  $t_1$ , and use Eq. 2-16:

$$v^2 = v_0^2 + 2a_1\Delta x \Rightarrow \Delta x = \frac{5.08^2}{2(1.22)} = 10.59 \approx 10.6 \text{ m}.$$

(b) Using Eq. 2-11, we have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08}{1.22} = 4.17 \text{ s}.$$

The deceleration time  $t_2$  turns out to be the same so that  $t_1 + t_2 = 8.33 \text{ s}$ . The distances traveled during  $t_1$  and  $t_2$  are the same so that they total to  $2(10.59) = 21.18 \text{ m}$ . This implies that for a distance of  $190 - 21.18 = 168.82 \text{ m}$ , the elevator is traveling at constant velocity. This time of constant velocity motion is

$$t_3 = \frac{168.82 \text{ m}}{5.08 \text{ m/s}} = 33.21 \text{ s}.$$

Therefore, the total time is  $8.33 + 33.21 \approx 41.5 \text{ s}$ .

30. (a) Eq. 2-15 is used for part 1 of the trip and Eq. 2-18 is used for part 2:

$$\begin{aligned}\Delta x_1 &= v_{o1} t_1 + \frac{1}{2} a_1 t_1^2 & \text{where } a_1 &= 2.25 \text{ m/s}^2 \text{ and } \Delta x_1 = \frac{900}{4} \text{ m} \\ \Delta x_2 &= v_2 t_2 - \frac{1}{2} a_2 t_2^2 & \text{where } a_2 &= -0.75 \text{ m/s}^2 \text{ and } \Delta x_2 = \frac{3(900)}{4} \text{ m}\end{aligned}$$

In addition,  $v_{o1} = v_2 = 0$ . Solving these equations for the times and adding the results gives  $t = t_1 + t_2 = 56.6 \text{ s}$ .

(b) Eq. 2-16 is used for part 1 of the trip:

$$v^2 = (v_{o1})^2 + 2a_1\Delta x_1 = 0 + 2(2.25)\left(\frac{900}{4}\right) = 1013 \text{ m}^2/\text{s}^2$$

which leads to  $v = 31.8 \text{ m/s}$  for the maximum speed.



31. (a) From the figure, we see that  $x_0 = -2.0$  m. From Table 2-1, we can apply  $x - x_0 = v_0 t + \frac{1}{2} a t^2$  with  $t = 1.0$  s, and then again with  $t = 2.0$  s. This yields two equations for the two unknowns,  $v_0$  and  $a$ . SI units are understood.

$$0.0 - (-2.0) = v_0 (1.0) + \frac{1}{2} a (1.0)^2$$

$$6.0 - (-2.0) = v_0 (2.0) + \frac{1}{2} a (2.0)^2 .$$

Solving these simultaneous equations yields the results  $v_0 = 0.0$  and  $a = 4.0 \text{ m/s}^2$ .

(b) The fact that the answer is positive tells us that the acceleration vector points in the  $+x$  direction.

32. (a) Note that 110 km/h is equivalent to 30.56 m/s. During a two second interval, you travel 61.11 m. The decelerating police car travels (using Eq. 2-15) 51.11 m. In light of the fact that the initial “gap” between cars was 25 m, this means the gap has narrowed by 10.0 m – that is, to a distance of 15.0 meters between cars.

(b) First, we add 0.4 s to the considerations of part (a). During a 2.4 s interval, you travel 73.33 m. The decelerating police car travels (using Eq. 2-15) 58.93 m during that time. The initial distance between cars of 25 m has therefore narrowed by 14.4 m. Thus, at the start of your braking (call it  $t_0$ ) the gap between the cars is 10.6 m. The speed of the police car at  $t_0$  is  $30.56 - 5(2.4) = 18.56$  m/s. Collision occurs at time  $t$  when  $x_{\text{you}} = x_{\text{police}}$  (we choose coordinates such that your position is  $x = 0$  and the police car’s position is  $x = 10.6$  m at  $t_0$ ). Eq. 2-15 becomes, for each car:

$$x_{\text{police}} - 10.6 = 18.56(t - t_0) - \frac{1}{2} (5)(t - t_0)^2$$

$$x_{\text{you}} = 30.56(t - t_0) - \frac{1}{2} (5)(t - t_0)^2 \quad .$$

Subtracting equations, we find  $10.6 = (30.56 - 18.56)(t - t_0) \Rightarrow 0.883 \text{ s} = t - t_0$ . At that time your speed is  $30.56 + a(t - t_0) = 30.56 - 5(0.883) \approx 26 \text{ m/s}$  (or 94 km/h).

33. (a) We note that  $v_A = 12/6 = 2$  m/s (with two significant figures understood). Therefore, with an initial  $x$  value of 20 m, car A will be at  $x = 28$  m when  $t = 4$  s. This must be the value of  $x$  for car B at that time; we use Eq. 2-15:

$$28 \text{ m} = (12 \text{ m/s})t + \frac{1}{2} a_B t^2 \quad \text{where } t = 4.0 \text{ s}.$$

This yields  $a_B = -\frac{5}{2} = -2.5 \text{ m/s}^2$ .

(b) The question is: using the value obtained for  $a_B$  in part (a), are there other values of  $t$  (besides  $t = 4$  s) such that  $x_A = x_B$ ? The requirement is

$$20 + 2t = 12t + \frac{1}{2} a_B t^2$$

where  $a_B = -5/2$ . There are two distinct roots unless the discriminant  $\sqrt{10^2 - 2(-20)(a_B)}$  is zero. In our case, it is zero – which means there is only one root. The cars are side by side only once at  $t = 4$  s.

(c) A sketch is not shown here, but briefly – it would consist of a straight line tangent to a parabola at  $t = 4$ .

(d) We only care about real roots, which means  $10^2 - 2(-20)(a_B) \geq 0$ . If  $|a_B| > 5/2$  then there are no (real) solutions to the equation; the cars are never side by side.

(e) Here we have  $10^2 - 2(-20)(a_B) > 0 \Rightarrow$  two real roots. The cars are side by side at two different times.

34. We assume the train accelerates from rest ( $v_0 = 0$  and  $x_0 = 0$ ) at  $a_1 = +1.34 \text{ m/s}^2$  until the midway point and then decelerates at  $a_2 = -1.34 \text{ m/s}^2$  until it comes to a stop ( $v_2 = 0$ ) at the next station. The velocity at the midpoint is  $v_1$  which occurs at  $x_1 = 806/2 = 403 \text{ m}$ .

(a) Eq. 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \Rightarrow v_1 = \sqrt{2(1.34)(403)} = 32.9 \text{ m/s}.$$

(b) The time  $t_1$  for the accelerating stage is (using Eq. 2-15)

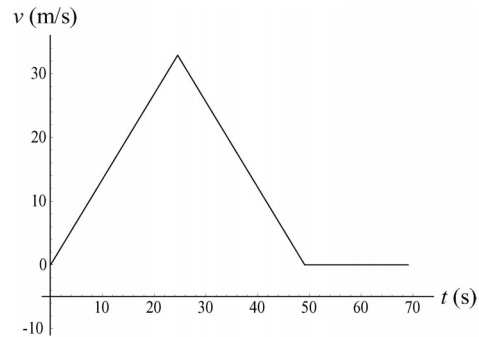
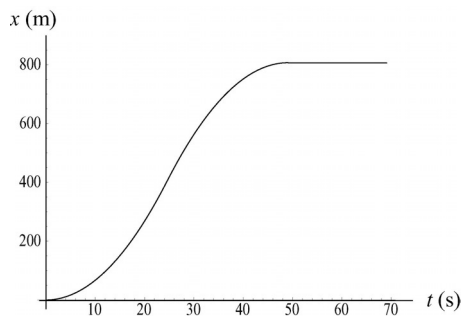
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \Rightarrow t_1 = \sqrt{\frac{2(403)}{1.34}}$$

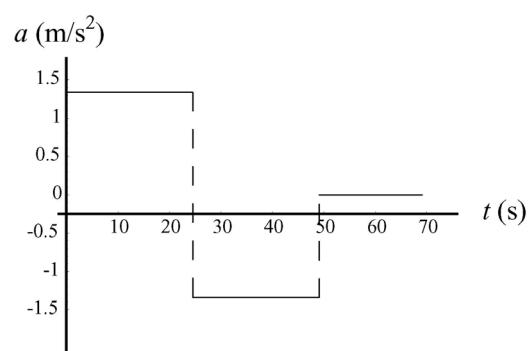
which yields  $t_1 = 24.53 \text{ s}$ . Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain  $t = 49.1 \text{ s}$  for the travel time between stations.

(c) With a “dead time” of 20 s, we have  $T = t + 20 = 69.1 \text{ s}$  for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{806 \text{ m}}{69.1 \text{ s}} = 11.7 \text{ m/s}.$$

(d) The graphs for  $x$ ,  $v$  and  $a$  as a function of  $t$  are shown below. SI units are understood. The third graph,  $a(t)$ , consists of three horizontal “steps” — one at 1.34 during  $0 < t < 24.53$  and the next at  $-1.34$  during  $24.53 < t < 49.1$  and the last at zero during the “dead time”  $49.1 < t < 69.1$ .





35. The displacement ( $\Delta x$ ) for each train is the “area” in the graph (since the displacement is the integral of the velocity). Each area is triangular, and the area of a triangle is  $\frac{1}{2}(\text{base}) \times (\text{height})$ . Thus, the (absolute value of the) displacement for one train  $(\frac{1}{2})(40 \text{ m/s})(5 \text{ s}) = 100 \text{ m}$ , and that of the other train is  $(\frac{1}{2})(30 \text{ m/s})(4 \text{ s}) = 60 \text{ m}$ . The initial “gap” between the trains was 200 m, and according to our displacement computations, the gap has narrowed by 160 m. Thus, the answer is  $200 - 160 = 40 \text{ m}$ .

36. Let  $d$  be the 220 m distance between the cars at  $t = 0$ , and  $v_1$  be the 20 km/h = 50/9 m/s speed (corresponding to a passing point of  $x_1 = 44.5$  m) and  $v_2$  be the 40 km/h = 100/9 m/s speed (corresponding to passing point of  $x_2 = 76.6$  m) of the red car. We have two equations (based on Eq. 2-17):

$$d - x_1 = v_o t_1 + \frac{1}{2} a t_1^2 \quad \text{where } t_1 = x_1 / v_1$$

$$d - x_2 = v_o t_2 + \frac{1}{2} a t_2^2 \quad \text{where } t_2 = x_2 / v_2$$

We simultaneously solve these equations and obtain the following results:

(a)  $v_o = 13.9$  m/s (or roughly 50 km/h) along the  $-x$  direction.

(b)  $a = 2.0$  m/s<sup>2</sup> along the  $-x$  direction.

37. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2-1 is permitted. We start with Eq. 2-17 and denote the train's initial velocity as  $v_t$  and the locomotive's velocity as  $v_\ell$  (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance  $\Delta x$  consists of the original gap between them  $D$  as well as the forward distance traveled during this time by the locomotive  $v_\ell t$ . Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell.$$

We now use Eq. 2-11 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell$$

leads to

$$a = \left( \frac{v_t + v_\ell}{2} - v_\ell \right) \left( \frac{v_\ell - v_t}{D} \right) = -\frac{1}{2D} (v_\ell - v_t)^2.$$

Hence,

$$a = -\frac{1}{2(0.676 \text{ km})} \left( 29 \frac{\text{km}}{\text{h}} - 161 \frac{\text{km}}{\text{h}} \right)^2 = -12888 \text{ km/h}^2$$

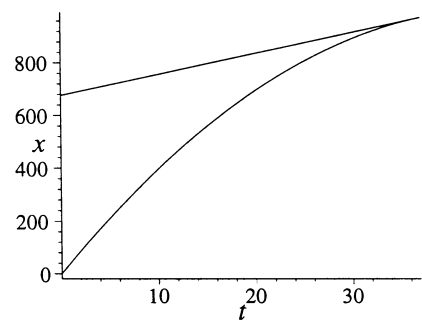
which we convert as follows:

$$a = (-12888 \text{ km/h}^2) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)^2 = -0.994 \text{ m/s}^2$$

so that its *magnitude* is  $|a| = 0.994 \text{ m/s}^2$ . A graph is shown below for the case where a collision is just avoided ( $x$  along the vertical axis is in meters and  $t$  along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.

The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.





38. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ).

(a) Noting that  $\Delta y = y - y_0 = -30 \text{ m}$ , we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute  $t$ :

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with  $v_0 = -12 \text{ m/s}$  since it is downward) leads, upon choosing the positive root (so that  $t > 0$ ), to the result:

$$t = \frac{-12 + \sqrt{(-12)^2 - 2(9.8)(-30)}}{9.8} = 1.54 \text{ s.}$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain  $v$ ; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1 \text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

39. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ).

(a) Starting the clock at the moment the wrench is dropped ( $v_0 = 0$ ), then  $v^2 = v_0^2 - 2g\Delta y$  leads to

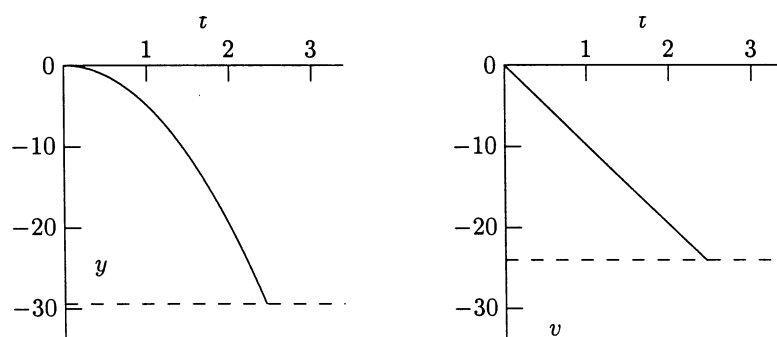
$$\Delta y = -\frac{(-24)^2}{2(9.8)} = -29.4 \text{ m}$$

so that it fell through a height of 29.4 m.

(b) Solving  $v = v_0 - gt$  for time, we find:

$$t = \frac{v_0 - v}{g} = \frac{0 - (-24)}{9.8} = 2.45 \text{ s.}$$

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. In the interest of saving space, we do not show the acceleration graph, which is a horizontal line at  $-9.8 \text{ m/s}^2$ .



40. Neglect of air resistance justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (where *down* is our  $-y$  direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ).

(a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8)(-1700)} = -183$$

in SI units. Its magnitude is therefore 183 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

41. We neglect air resistance for the duration of the motion (between “launching” and “landing”), so  $a = -g = -9.8 \text{ m/s}^2$  (we take downward to be the  $-y$  direction). We use the equations in Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because this is  $a = \text{constant}$  motion.

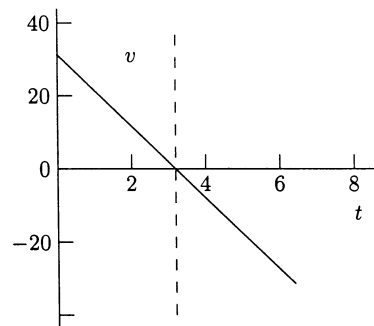
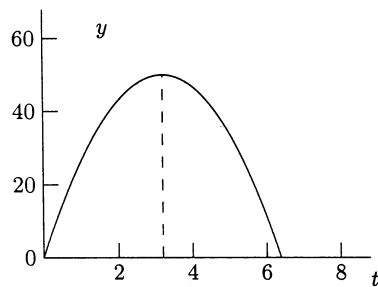
(a) At the highest point the velocity of the ball vanishes. Taking  $y_0 = 0$ , we set  $v = 0$  in  $v^2 = v_0^2 - 2gy$ , and solve for the initial velocity:  $v_0 = \sqrt{2gy}$ . Since  $y = 50 \text{ m}$  we find  $v_0 = 31 \text{ m/s}$ .

(b) It will be in the air from the time it leaves the ground until the time it returns to the ground ( $y = 0$ ). Applying Eq. 2-15 to the entire motion (the rise and the fall, of total time  $t > 0$ ) we have

$$y = v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{2v_0}{g}$$

which (using our result from part (a)) produces  $t = 6.4 \text{ s}$ . It is possible to obtain this without using part (a)’s result; one can find the time just for the rise (from ground to highest point) from Eq. 2-16 and then double it.

(c) SI units are understood in the  $x$  and  $v$  graphs shown. In the interest of saving space, we do not show the graph of  $a$ , which is a horizontal line at  $-9.8 \text{ m/s}^2$ .



42. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the motion. We are allowed to use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the  $y$  axis.

(a) Using  $y = v_0 t - \frac{1}{2} g t^2$ , with  $y = 0.544 \text{ m}$  and  $t = 0.200 \text{ s}$ , we find

$$v_0 = \frac{y + \frac{1}{2} g t^2}{t} = \frac{0.544 + \frac{1}{2} (9.8) (0.200)^2}{0.200} = 3.70 \text{ m/s} .$$

(b) The velocity at  $y = 0.544 \text{ m}$  is

$$v = v_0 - g t = 3.70 - (9.8) (0.200) = 1.74 \text{ m/s} .$$

(c) Using  $v^2 = v_0^2 - 2 g y$  (with different values for  $y$  and  $v$  than before), we solve for the value of  $y$  corresponding to maximum height (where  $v = 0$ ).

$$y = \frac{v_0^2}{2g} = \frac{3.7^2}{2(9.8)} = 0.698 \text{ m} .$$

Thus, the armadillo goes  $0.698 - 0.544 = 0.154 \text{ m}$  higher.

43. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the motion. We are allowed to use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because this is constant acceleration motion. We are placing the coordinate origin on the ground. We note that the initial velocity of the package is the same as the velocity of the balloon,  $v_0 = +12 \text{ m/s}$  and that its initial coordinate is  $y_0 = +80 \text{ m}$ .

(a) We solve  $y = y_0 + v_0 t - \frac{1}{2} g t^2$  for time, with  $y = 0$ , using the quadratic formula (choosing the positive root to yield a positive value for  $t$ ).

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 + \sqrt{12^2 + 2(9.8)(80)}}{9.8} = 5.4 \text{ s}$$

(b) If we wish to avoid using the result from part (a), we could use Eq. 2-16, but if that is not a concern, then a variety of formulas from Table 2-1 can be used. For instance, Eq. 2-11 leads to

$$v = v_0 - gt = 12 - (9.8)(5.4) = -41 \text{ m/s}.$$

Its final *speed* is  $41 \text{ m/s}$ .

44. The full extent of the bolt's fall is given by  $y - y_0 = -\frac{1}{2} g t^2$  where  $y - y_0 = -90$  m (if upwards is chosen as the positive  $y$  direction). Thus the time for the full fall is found to be  $t = 4.29$  s. The first 80% of its free fall distance is given by  $-72 = -g \tau^2/2$ , which requires time  $\tau = 3.83$  s.

(a) Thus, the final 20% of its fall takes  $t - \tau = 0.45$  s.

(b) We can find that speed using  $v = -g\tau$ . Therefore,  $|v| = 38$  m/s, approximately.

(c) Similarly,  $v_{final} = -g t \Rightarrow |v_{final}| = 42$  m/s.



45. The  $y$  coordinate of Apple 1 obeys  $y - y_{o1} = -\frac{1}{2} g t^2$  where  $y = 0$  when  $t = 2.0$  s. This allows us to solve for  $y_{o1}$ , and we find  $y_{o1} = 19.6$  m.

The graph for the coordinate of Apple 2 (which is thrown apparently at  $t = 1.0$  s with velocity  $v_2$ ) is

$$y - y_{o2} = v_2(t-1.0) - \frac{1}{2} g (t-1.0)^2$$

where  $y_{o2} = y_{o1} = 19.6$  m and where  $y = 0$  when  $t = 2.25$  s. Thus we obtain  $|v_2| = 9.6$  m/s, approximately.

46. We use Eq. 2-16,  $v_B^2 = v_A^2 + 2a(y_B - y_A)$ , with  $a = -9.8 \text{ m/s}^2$ ,  $y_B - y_A = 0.40 \text{ m}$ , and  $v_B = \frac{1}{3} v_A$ . It is then straightforward to solve:  $v_A = 3.0 \text{ m/s}$ , approximately.

47. The speed of the boat is constant, given by  $v_b = d/t$ . Here,  $d$  is the distance of the boat from the bridge when the key is dropped (12 m) and  $t$  is the time the key takes in falling. To calculate  $t$ , we put the origin of the coordinate system at the point where the key is dropped and take the  $y$  axis to be positive in the *downward* direction. Taking the time to be zero at the instant the key is dropped, we compute the time  $t$  when  $y = 45$  m. Since the initial velocity of the key is zero, the coordinate of the key is given by  $y = \frac{1}{2}gt^2$ . Thus

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(45 \text{ m})}{9.8 \text{ m/s}^2}} = 3.03 \text{ s}.$$

Therefore, the speed of the boat is

$$v_b = \frac{12 \text{ m}}{3.03 \text{ s}} = 4.0 \text{ m/s}.$$

48. (a) With upward chosen as the +y direction, we use Eq. 2-11 to find the initial velocity of the package:

$$v = v_o + at \Rightarrow 0 = v_o - (9.8 \text{ m/s}^2)(2.0 \text{ s})$$

which leads to  $v_o = 19.6 \text{ m/s}$ . Now we use Eq. 2-15:

$$\Delta y = (19.6 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(2.0 \text{ s})^2 \approx 20 \text{ m} .$$

We note that the “2.0 s” in this second computation refers to the time interval  $2 < t < 4$  in the graph (whereas the “2.0 s” in the first computation referred to the  $0 < t < 2$  time interval shown in the graph).

(b) In our computation for part (b), the time interval (“6.0 s”) refers to the  $2 < t < 8$  portion of the graph:

$$\Delta y = (19.6 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(6.0 \text{ s})^2 \approx -59 \text{ m} ,$$

or  $|\Delta y| = 59 \text{ m} .$

49. (a) We first find the velocity of the ball just before it hits the ground. During contact with the ground its average acceleration is given by

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

where  $\Delta v$  is the change in its velocity during contact with the ground and  $\Delta t = 20.0 \times 10^{-3}$  s is the duration of contact. Now, to find the velocity just *before* contact, we put the origin at the point where the ball is dropped (and take +y upward) and take  $t = 0$  to be when it is dropped. The ball strikes the ground at  $y = -15.0$  m. Its velocity there is found from Eq. 2-16:  $v^2 = -2gy$ . Therefore,

$$v = -\sqrt{-2gy} = -\sqrt{-2(9.8)(-15.0)} = -17.1 \text{ m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{0 - (-17.1)}{20.0 \times 10^{-3}} = 857 \text{ m/s}^2.$$

(b) The fact that the result is positive indicates that this acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

50. To find the “launch” velocity of the rock, we apply Eq. 2-11 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \Rightarrow 0 = v_0 - (9.8)(2.5)$$

so that  $v_0 = 24.5$  m/s (with +y up). Now we use Eq. 2-15 to find the height of the tower (taking  $y_0 = 0$  at the ground level)

$$y - y_0 = v_0 t + \frac{1}{2} a t^2 \Rightarrow y - 0 = (24.5)(1.5) - \frac{1}{2} (9.8)(1.5)^2 .$$

Thus, we obtain  $y = 26$  m.

51. The average acceleration during contact with the floor is  $a_{\text{avg}} = (v_2 - v_1) / \Delta t$ , where  $v_1$  is its velocity just before striking the floor,  $v_2$  is its velocity just as it leaves the floor, and  $\Delta t$  is the duration of contact with the floor ( $12 \times 10^{-3}$  s).

(a) Taking the  $y$  axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using  $v_1^2 = v_0^2 - 2gy$ . With  $v_0 = 0$  and  $y = -4.00$  m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8)(-4.00)} = -8.85 \text{ m/s}$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use  $v^2 = v_2^2 - 2g(y - y_0)$  with  $v = 0$ ,  $y = -2.00$  m (it ends up two meters *below* its initial drop height), and  $y_0 = -4.00$  m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8)(-2.00 + 4.00)} = 6.26 \text{ m/s}.$$

Consequently, the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26 + 8.85}{12.0 \times 10^{-3}} = 1.26 \times 10^3 \text{ m/s}^2.$$

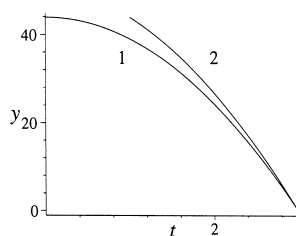
(b) The positive nature of the result indicates that the acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

52. (a) We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with  $\Delta y$  replacing  $\Delta x$ ) because this is constant acceleration motion. We use primed variables (except  $t$ ) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity  $-v_0$ , so that  $v_0$  is being used for the initial *speed*). SI units are used throughout.

$$\Delta y' = 0(t) - \frac{1}{2}gt^2$$

$$\Delta y = (-v_0)(t-1) - \frac{1}{2}g(t-1)^2$$

Since the problem indicates  $\Delta y' = \Delta y = -43.9 \text{ m}$ , we solve the first equation for  $t$  (finding  $t = 2.99 \text{ s}$ ) and use this result to solve the second equation for the initial speed of the second stone:



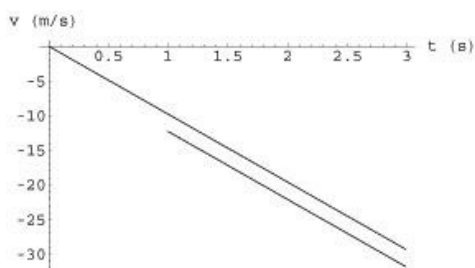
$$-43.9 = (-v_0)(1.99) - \frac{1}{2}(9.8)(1.99)^2$$

which leads to  $v_0 = 12.3 \text{ m/s}$ .

(b) The velocity of the stones are given by

$$v_y' = \frac{d(\Delta y')}{dt} = -gt, \quad v_y = \frac{d(\Delta y)}{dt} = -v_0 - g(t-1)$$

The plot is shown below:





53. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the motion. We are allowed to use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the  $y$  axis.

(a) The time drop 1 leaves the nozzle is taken as  $t = 0$  and its time of landing on the floor  $t_1$  can be computed from Eq. 2-15, with  $v_0 = 0$  and  $y_1 = -2.00 \text{ m}$ .

$$y_1 = -\frac{1}{2} g t_1^2 \Rightarrow t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00)}{9.8}} = 0.639 \text{ s}.$$

At that moment, the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at  $t = 0.639/3 = 0.213 \text{ s}$  and drop 3 leaves the nozzle at  $t = 2(0.213) = 0.426 \text{ s}$ . Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is  $t_2 = t_1 - 0.213 = 0.426 \text{ s}$ . Its position at the moment drop 1 strikes the floor is

$$y_2 = -\frac{1}{2} g t_2^2 = -\frac{1}{2} (9.8)(0.426)^2 = -0.889 \text{ m},$$

or 89 cm below the nozzle.

(b) The time in free fall (up to the moment drop 1 lands) for drop 3 is  $t_3 = t_1 - 0.426 = 0.213 \text{ s}$ . Its position at the moment drop 1 strikes the floor is

$$y_3 = -\frac{1}{2} g t_3^2 = -\frac{1}{2} (9.8)(0.213)^2 = -0.222 \text{ m},$$

or 22 cm below the nozzle.

54. We choose *down* as the +y direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as  $t - t'$  where  $t$  is the value of time when it lands and  $t'$  is one second prior to that. The corresponding distance is  $y - y' = 0.50h$ , where  $y$  denotes the location of the ground. In these terms,  $y$  is the same as  $h$ , so we have  $h - y' = 0.50h$  or  $0.50h = y'$ .

(a) We find  $t'$  and  $t$  from Eq. 2-15 (with  $v_0 = 0$ ):

$$y' = \frac{1}{2}gt'^2 \Rightarrow t' = \sqrt{\frac{2y'}{g}}$$

$$y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}}.$$

Plugging in  $y = h$  and  $y' = 0.50h$ , and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.50h)/g}{2h/g}} = \sqrt{0.50}.$$

Letting  $t' = t - 1.00$  (SI units understood) and cross-multiplying, we find

$$t - 1.00 = t\sqrt{0.50} \Rightarrow t = \frac{1.00}{1 - \sqrt{0.50}}$$

which yields  $t = 3.41$  s.

(b) Plugging this result into  $y = \frac{1}{2}gt^2$  we find  $h = 57$  m.

(c) In our approach, we did not use the quadratic formula, but we did “choose a root” when we assumed (in the last calculation in part (a)) that  $\sqrt{0.50} = +2.236$  instead of  $-2.236$ . If we had instead let  $\sqrt{0.50} = -2.236$  then our answer for  $t$  would have been roughly 0.6 s which would imply that  $t' = t - 1$  would equal a negative number (indicating a time *before* it was dropped) which certainly does not fit with the physical situation described in the problem.

55. The time  $t$  the pot spends passing in front of the window of length  $L = 2.0$  m is 0.25 s each way. We use  $v$  for its velocity as it passes the top of the window (going up). Then, with  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* to be the  $-y$  direction), Eq. 2-18 yields

$$L = vt - \frac{1}{2}gt^2 \quad \Rightarrow \quad v = \frac{L}{t} - \frac{1}{2}gt.$$

The distance  $H$  the pot goes above the top of the window is therefore (using Eq. 2-16 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{(L/t - gt/2)^2}{2g} = \frac{(2.00/0.25 - (9.80)(0.25)/2)^2}{(2)(9.80)} = 2.34 \text{ m}.$$

56. The height reached by the player is  $y = 0.76$  m (where we have taken the origin of the  $y$  axis at the floor and  $+y$  to be upward).

(a) The initial velocity  $v_0$  of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8)(0.76)} = 3.86 \text{ m/s}.$$

This is a consequence of Eq. 2-16 where velocity  $v$  vanishes. As the player reaches  $y_1 = 0.76 - 0.15 = 0.61$  m, his speed  $v_1$  satisfies  $v_0^2 - v_1^2 = 2gy_1$ , which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86)^2 - 2(9.80)(0.61)} = 1.71 \text{ m/s}.$$

The time  $t_1$  that the player spends *ascending* in the top  $\Delta y_1 = 0.15$  m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2} (v_1 + v) t_1 \Rightarrow t_1 = \frac{2(0.15)}{1.71 + 0} = 0.175 \text{ s}$$

which means that the total time spent in that top 15 cm (both ascending and descending) is  $2(0.17) = 0.35 \text{ s} = 350 \text{ ms}$ .

(b) The time  $t_2$  when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15 = v_0 t_2 - \frac{1}{2} g t_2^2 = (3.86) t_2 - \frac{9.8}{2} t_2^2,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots)  $t_2 = 0.041 \text{ s} = 41 \text{ ms}$ , which implies that the total time spent in that bottom 15 cm (both ascending and descending) is  $2(41) = 82 \text{ ms}$ .

57. We choose *down* as the +y direction and place the coordinate origin at the top of the building (which has height  $H$ ). During its fall, the ball passes (with velocity  $v_1$ ) the top of the window (which is at  $y_1$ ) at time  $t_1$ , and passes the bottom (which is at  $y_2$ ) at time  $t_2$ . We are told  $y_2 - y_1 = 1.20$  m and  $t_2 - t_1 = 0.125$  s. Using Eq. 2-15 we have

$$y_2 - y_1 = v_1(t_2 - t_1) + \frac{1}{2}g(t_2 - t_1)^2$$

which immediately yields

$$v_1 = \frac{1.20 - \frac{1}{2}(9.8)(0.125)^2}{0.125} = 8.99 \text{ m/s.}$$

From this, Eq. 2-16 (with  $v_0 = 0$ ) reveals the value of  $y_1$ :

$$v_1^2 = 2gy_1 \Rightarrow y_1 = \frac{8.99^2}{2(9.8)} = 4.12 \text{ m.}$$

It reaches the ground ( $y_3 = H$ ) at  $t_3$ . Because of the symmetry expressed in the problem (“upward flight is a reverse of the fall”) we know that  $t_3 - t_2 = 2.00/2 = 1.00$  s. And this means  $t_3 - t_1 = 1.00 + 0.125 = 1.125$  s. Now Eq. 2-15 produces

$$\begin{aligned} y_3 - y_1 &= v_1(t_3 - t_1) + \frac{1}{2}g(t_3 - t_1)^2 \\ y_3 - 4.12 &= (8.99)(1.125) + \frac{1}{2}(9.8)(1.125)^2 \end{aligned}$$

which yields  $y_3 = H = 20.4$  m.

58. The graph shows  $y = 25$  m to be the highest point (where the speed momentarily vanishes). The neglect of “air friction” (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

(a) To find the acceleration due to gravity  $g_p$  on that planet, we use Eq. 2-15 (with +y up)

$$y - y_0 = vt + \frac{1}{2} g_p t^2 \quad \Rightarrow \quad 25 - 0 = (0)(2.5) + \frac{1}{2} g_p (2.5)^2$$

so that  $g_p = 8.0 \text{ m/s}^2$ .

(b) That same (max) point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2} (v_0 + v)t \quad \Rightarrow \quad 25 - 0 = \frac{1}{2} (v_0 + 0)(2.5)$$

Therefore,  $v_0 = 20 \text{ m/s}$ .

59. We follow the procedures outlined in Sample Problem 2-8. The key idea here is that the speed of the head (and the torso as well) at any given time can be calculated by finding the area on the graph of the head's acceleration versus time, as shown in Eq. 2-26:

$$v_1 - v_0 = \left( \begin{array}{l} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{array} \right)$$

(a) From Fig. 2.13a, we see that the head begins to accelerate from rest ( $v_0 = 0$ ) at  $t_0 = 110$  ms and reaches a maximum value of  $90 \text{ m/s}^2$  at  $t_1 = 160$  ms. The area of this region is

$$\text{area} = \frac{1}{2}(160 - 110) \times 10^{-3} \text{ s} \cdot (90 \text{ m/s}^2) = 2.25 \text{ m/s}$$

which is equal to  $v_1$ , the speed at  $t_1$ .

(b) To compute the speed of the torso at  $t_1 = 160$  ms, we divide the area into 4 regions: From 0 to 40 ms, region A has zero area. From 40 ms to 100 ms, region B has the shape of a triangle with area

$$\text{area}_B = \frac{1}{2}(0.0600 \text{ s})(50.0 \text{ m/s}^2) = 1.50 \text{ m/s}.$$

From 100 to 120 ms, region C has the shape of a rectangle with area

$$\text{area}_C = (0.0200 \text{ s})(50.0 \text{ m/s}^2) = 1.00 \text{ m/s}.$$

From 120 to 160 ms, region D has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.0400 \text{ s})(50.0 + 20.0) \text{ m/s}^2 = 1.40 \text{ m/s}.$$

Substituting these values into Eq. 2-26, with  $v_0 = 0$  then gives

$$v_1 - 0 = 0 + 1.50 \text{ m/s} + 1.00 \text{ m/s} + 1.40 \text{ m/s} = 3.90 \text{ m/s},$$

or  $v_1 = 3.90 \text{ m/s}$ .

60. The key idea here is that the position of an object at any given time can be calculated by finding the area on the graph of the object's velocity versus time, as shown in Eq. 2-25:

$$x_1 - x_0 = \left( \begin{array}{l} \text{area between the velocity curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{array} \right).$$

(a) To compute the position of the fist at  $t = 50$  ms, we divide the area in Fig. 2-29 into two regions. From 0 to 10 ms, region *A* has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.010 \text{ s})(2 \text{ m/s}) = 0.01 \text{ m}.$$

From 10 to 50 ms, region *B* has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.040 \text{ s})(2 + 4) \text{ m/s} = 0.12 \text{ m}.$$

Substituting these values into Eq. 2-25, with  $x_0=0$  then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} = 0.13 \text{ m},$$

or  $x_1 = 0.13 \text{ m}$ .

(b) The speed of the fist reaches a maximum at  $t_1 = 120$  ms. From 50 to 90 ms, region *C* has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.040 \text{ s})(4 + 5) \text{ m/s} = 0.18 \text{ m}.$$

From 90 to 120 ms, region *D* has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.030 \text{ s})(5 + 7.5) \text{ m/s} = 0.19 \text{ m}.$$

Substituting these values into Eq. 2-25, with  $x_0=0$  then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} + 0.18 \text{ m} + 0.19 \text{ m} = 0.50 \text{ m},$$

or  $x_1 = 0.50 \text{ m}$ .



61. Since  $v = \frac{dx}{dt}$  (Eq. 2-4), then  $\Delta x = \int v dt$ , which corresponds to the area under the  $v$  vs  $t$  graph. Dividing the total area  $A$  into rectangular (base  $\times$  height) and triangular ( $\frac{1}{2}$  base  $\times$  height) areas, we have

$$\begin{aligned} A &= A_{0 < t < 2} + A_{2 < t < 10} + A_{10 < t < 12} + A_{12 < t < 16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left( (2)(4) + \frac{1}{2}(2)(4) \right) + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain  $\Delta x = 100$  m.

62. The problem is solved using Eq. 2-26:

$$v_1 - v_0 = \left( \begin{array}{l} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{array} \right)$$

To compute the speed of the helmeted head at  $t_1=7.0$  ms, we divide the area under the  $a$  vs.  $t$  graph into 4 regions: From 0 to 2 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0020 \text{ s}) (120 \text{ m/s}^2) = 0.12 \text{ m/s}.$$

From 2 ms to 4 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.0020 \text{ s}) (120 + 140) \text{ m/s}^2 = 0.26 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s}) (140 + 200) \text{ m/s}^2 = 0.34 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2}(0.0010 \text{ s}) (200 \text{ m/s}^2) = 0.10 \text{ m/s}.$$

Substituting these values into Eq. 2-26, with  $v_0=0$  then gives

$$v_{\text{helmeted}} = 0.12 \text{ m/s} + 0.26 \text{ m/s} + 0.34 \text{ m/s} + 0.10 \text{ m/s} = 0.82 \text{ m/s}.$$

Carrying out similar calculations for the unhelmeted, bare head, we have the following results: From 0 to 3 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0030 \text{ s}) (40 \text{ m/s}^2) = 0.060 \text{ m/s}.$$

From 3 ms to 4 ms, region B has the shape of a rectangle with area

$$\text{area}_B = (0.0010 \text{ s}) (40 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2} (0.0020 \text{ s}) (40 + 80) \text{ m/s}^2 = 0.12 \text{ m/s}.$$

From 6 to 7 ms, region  $D$  has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2} (0.0010 \text{ s}) (80 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

Substituting these values into Eq. 2-26, with  $v_0=0$  then gives

$$v_{\text{unhelmeted}} = 0.060 \text{ m/s} + 0.040 \text{ m/s} + 0.12 \text{ m/s} + 0.040 \text{ m/s} = 0.26 \text{ m/s}.$$

Thus, the difference in the speed is

$$\Delta v = v_{\text{helmeted}} - v_{\text{unhelmeted}} = 0.82 \text{ m/s} - 0.26 \text{ m/s} = 0.56 \text{ m/s}.$$

63. We denote the required time as  $t$ , assuming the light turns green when the clock reads zero. By this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as  $a$  and the (constant) speed of the truck as  $v$  then

$$\Delta x = \left( \frac{1}{2} a t^2 \right)_{\text{car}} = (v t)_{\text{truck}}$$

which leads to

$$t = \frac{2v}{a} = \frac{2(9.5)}{2.2} = 8.6 \text{ s} .$$

Therefore,

$$\Delta x = v t = (9.5)(8.6) = 82 \text{ m} .$$

(b) The speed of the car at that moment is

$$v_{\text{car}} = a t = (2.2)(8.6) = 19 \text{ m/s} .$$

64. We take the moment of applying brakes to be  $t = 0$ . The deceleration is constant so that Table 2-1 can be used. Our primed variables (such as  $v'_0 = 72 \text{ km/h} = 20 \text{ m/s}$ ) refer to one train (moving in the  $+x$  direction and located at the origin when  $t = 0$ ) and unprimed variables refer to the other (moving in the  $-x$  direction and located at  $x_0 = +950 \text{ m}$  when  $t = 0$ ). We note that the acceleration vector of the unprimed train points in the *positive* direction, even though the train is slowing down; its initial velocity is  $v_0 = -144 \text{ km/h} = -40 \text{ m/s}$ . Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using Eq 2-16, it should stop (meaning  $v' = 0$ ) at

$$x' = \frac{(v')^2 - (v'_0)^2}{2a'} = \frac{0 - 20^2}{-2} = 200 \text{ m} .$$

The speed of the other train, when it reaches that location, is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{(-40)^2 + 2(1.0)(200 - 950)} = \sqrt{100} = 10 \text{ m/s}$$

using Eq 2-16 again. Specifically, its velocity at that moment would be  $-10 \text{ m/s}$  since it is still traveling in the  $-x$  direction when it crashes. If the computation of  $v$  had failed (meaning that a negative number would have been inside the square root) then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the time it stops (Eq. 2-11 yields  $t = 20 \text{ s}$ ) and seeing where the unprimed train is at that moment (Eq. 2-18 yields  $x = 350 \text{ m}$ , still a good distance away from contact).

65. The  $y$  coordinate of Piton 1 obeys  $y - y_{o1} = -\frac{1}{2} g t^2$  where  $y = 0$  when  $t = 3.0$  s. This allows us to solve for  $y_{o1}$ , and we find  $y_{o1} = 44.1$  m. The graph for the coordinate of Piton 2 (which is thrown apparently at  $t = 1.0$  s with velocity  $v_1$ ) is

$$y - y_{o2} = v_1(t-1.0) - \frac{1}{2} g (t-1.0)^2$$

where  $y_{o2} = y_{o1} + 10 = 54.1$  m and where (again)  $y = 0$  when  $t = 3.0$  s. Thus we obtain  $|v_1| = 17$  m/s, approximately.

66. (a) The derivative (with respect to time) of the given expression for  $x$  yields the “velocity” of the spot:

$$v(t) = 9 - \frac{9}{4} t^2$$

with 3 significant figures understood. It is easy to see that  $v = 0$  when  $t = 2.00$  s.

(b) At  $t = 2$  s,  $x = 9(2) - \frac{3}{4}(2)^3 = 12$ . Thus, the location of the spot when  $v = 0$  is 12.0 cm from left edge of screen.

(c) The derivative of the velocity is  $a = -\frac{9}{2} t$  which gives an acceleration (leftward) of magnitude  $9.00 \text{ m/s}^2$  when the spot is 12 cm from left edge of screen.

(d) Since  $v > 0$  for times less than  $t = 2$  s, then the spot had been moving rightwards.

(e) As implied by our answer to part (c), it moves leftward for times immediately after  $t = 2$  s. In fact, the expression found in part (a) guarantees that for all  $t > 2$ ,  $v < 0$  (that is, until the clock is “reset” by reaching an edge).

(f) As the discussion in part (e) shows, the edge that it reaches at some  $t > 2$  s cannot be the right edge; it is the left edge ( $x = 0$ ). Solving the expression given in the problem statement (with  $x = 0$ ) for positive  $t$  yields the answer: the spot reaches the left edge at  $t = \sqrt{12} \approx 3.46$  s.

67. We adopt the convention frequently used in the text: that “up” is the positive  $y$  direction.

(a) At the highest point in the trajectory  $v = 0$ . Thus, with  $t = 1.60$  s, the equation  $v = v_0 - gt$  yields  $v_0 = 15.7$  m/s.

(b) One equation that is not dependent on our result from part (a) is  $y - y_0 = v_0t + \frac{1}{2}gt^2$ ; this readily gives  $y_{\max} - y_0 = 12.5$  m for the highest (“max”) point measured relative to where it started (the top of the building).

(c) Now we use our result from part (a) and plug into  $y - y_0 = v_0t + \frac{1}{2}gt^2$  with  $t = 6.00$  s and  $y = 0$  (the ground level). Thus, we have

$$0 - y_0 = (15.68 \text{ m/s})(6.00 \text{ s}) - \frac{1}{2} (9.8 \text{ m/s}^2)(6.00 \text{ s})^2 \quad .$$

Therefore,  $y_0$  (the height of the building) is equal to 82.3 meters.



68. The acceleration is constant and we may use the equations in Table 2-1.

(a) Taking the first point as coordinate origin and time to be zero when the car is there, we apply Eq. 2-17 (with SI units understood):

$$x = \frac{1}{2} (v + v_0) t = \frac{1}{2} (15.0 + v_0) (6.00).$$

With  $x = 60.0$  m (which takes the direction of motion as the  $+x$  direction) we solve for the initial velocity:  $v_0 = 5.00$  m/s.

(b) Substituting  $v = 15.0$  m/s,  $v_0 = 5.00$  m/s and  $t = 6.00$  s into  $a = (v - v_0)/t$  (Eq. 2-11), we find  $a = 1.67$  m/s<sup>2</sup>.

(c) Substituting  $v = 0$  in  $v^2 = v_0^2 + 2ax$  and solving for  $x$ , we obtain

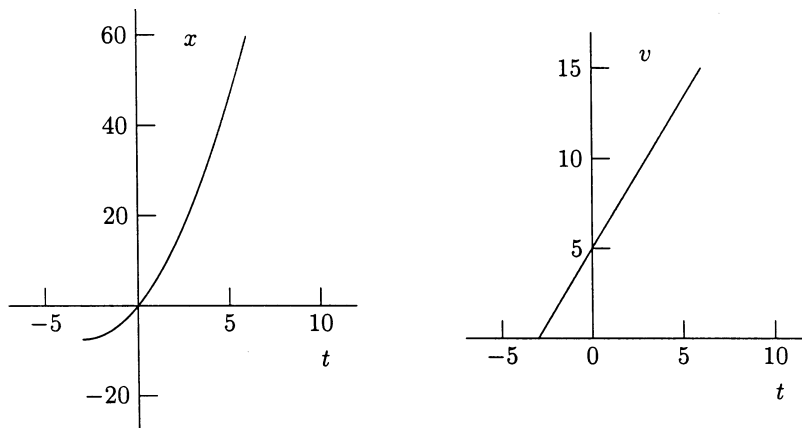
$$x = -\frac{v_0^2}{2a} = -\frac{(5.00)^2}{2(1.67)} = -7.50 \text{ m},$$

or  $|x| = 7.50$  m.

(d) The graphs require computing the time when  $v = 0$ , in which case, we use  $v = v_0 + at'$ . Thus,

$$t' = \frac{-v_0}{a} = \frac{-5.00}{1.67} = -3.0 \text{ s}$$

indicates the moment the car was at rest. SI units are assumed.



69. (a) The wording of the problem makes it clear that the equations of Table 2-1 apply, the challenge being that  $v_0$ ,  $v$ , and  $a$  are not explicitly given. We can, however, apply  $x - x_0 = v_0 t + \frac{1}{2} a t^2$  to a variety of points on the graph and solve for the unknowns from the simultaneous equations. For instance,

$$\begin{aligned} 16 - 0 &= v_0(2.0) + \frac{1}{2} a(2.0)^2 \\ 27 - 0 &= v_0(3.0) + \frac{1}{2} a(3.0)^2 \end{aligned}$$

lead to the values  $v_0 = 6.0 \text{ m/s}$  and  $a = 2.0 \text{ m/s}^2$ .

(b) From Table 2-1,

$$x - x_0 = vt - \frac{1}{2} a t^2 \quad \Rightarrow \quad 27 - 0 = v(3.0) - \frac{1}{2} (2.0)(3.0)^2$$

which leads to  $v = 12 \text{ m/s}$ .

(c) Assuming the wind continues during  $3.0 \leq t \leq 6.0$ , we apply  $x - x_0 = v_0 t + \frac{1}{2} a t^2$  to this interval (where  $v_0 = 12.0 \text{ m/s}$  from part (b)) to obtain

$$\Delta x = (12.0)(3.0) + \frac{1}{2} (2.0)(3.0)^2 = 45 \text{ m} \quad .$$

70. Taking the  $+y$  direction *downward* and  $y_0 = 0$ , we have  $y = v_0 t + \frac{1}{2} g t^2$  which (with  $v_0 = 0$ ) yields  $t = \sqrt{2y / g}$ .

(a) For this part of the motion,  $y = 50$  m so that

$$t = \sqrt{\frac{2(50)}{9.8}} = 3.2 \text{ s} .$$

(b) For this next part of the motion, we note that the total displacement is  $y = 100$  m. Therefore, the total time is

$$t = \sqrt{\frac{2(100)}{9.8}} = 4.5 \text{ s} .$$

The different between this and the answer to part (a) is the time required to fall through that second 50 m distance:  $4.5 - 3.2 = 1.3$  s.

71. We take the direction of motion as  $+x$ , so  $a = -5.18 \text{ m/s}^2$ , and we use SI units, so  $v_0 = 55(1000/3600) = 15.28 \text{ m/s}$ .

(a) The velocity is constant during the reaction time  $T$ , so the distance traveled during it is

$$d_r = v_0 T = (15.28)(0.75) = 11.46 \text{ m}.$$

We use Eq. 2-16 (with  $v = 0$ ) to find the distance  $d_b$  traveled during braking:

$$v^2 = v_0^2 + 2ad_b \Rightarrow d_b = -\frac{15.28^2}{2(-5.18)}$$

which yields  $d_b = 22.53 \text{ m}$ . Thus, the total distance is  $d_r + d_b = 34.0 \text{ m}$ , which means that the driver *is* able to stop in time. And if the driver were to continue at  $v_0$ , the car would enter the intersection in  $t = (40 \text{ m})/(15.28 \text{ m/s}) = 2.6 \text{ s}$  which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be  $34 \text{ m}$ ) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is  $32/15.28 = 2.1 \text{ s}$ , which is too long (the light turns in  $1.8 \text{ s}$ ). The driver is caught between a rock and a hard place.

72. Direction of  $+x$  is implicit in the problem statement. The initial position (when the clock starts) is  $x_0 = 0$  (where  $v_0 = 0$ ), the end of the speeding-up motion occurs at  $x_1 = 1100/2 = 550$  m, and the subway comes to a halt ( $v_2 = 0$ ) at  $x_2 = 1100$  m.

(a) Using Eq. 2-15, the subway reaches  $x_1$  at

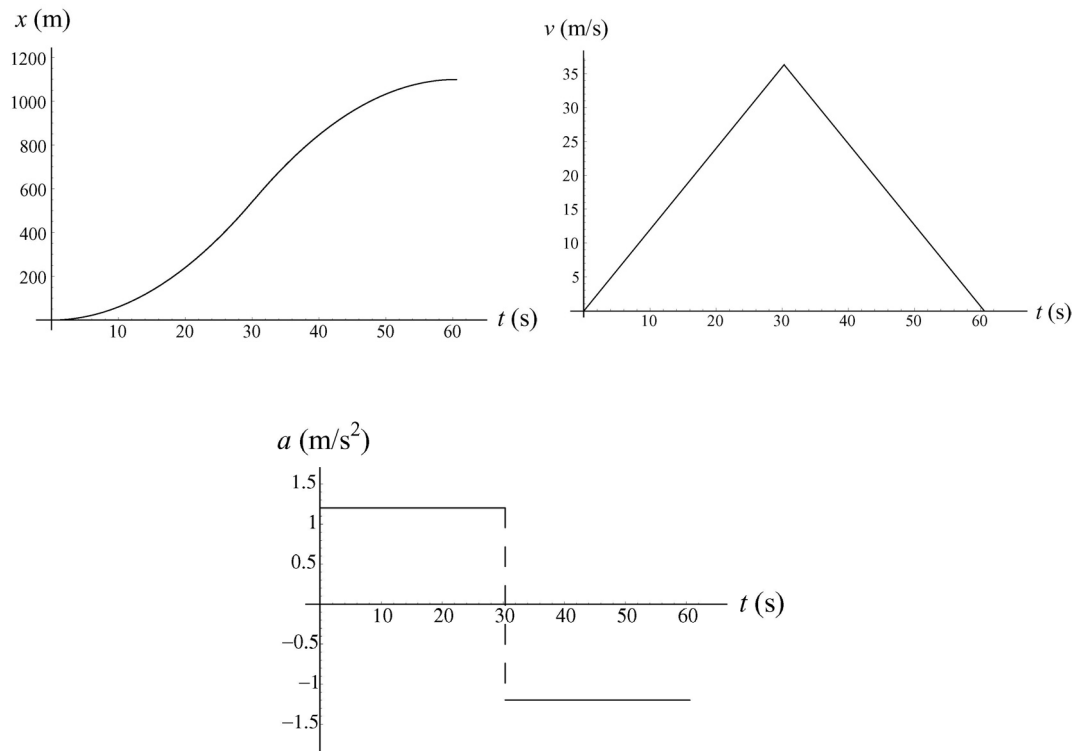
$$t_1 = \sqrt{\frac{2x_1}{a_1}} = \sqrt{\frac{2(550)}{1.2}} = 30.3 \text{ s}.$$

The time interval  $t_2 - t_1$  turns out to be the same value (most easily seen using Eq. 2-18 so the total time is  $t_2 = 2(30.3) = 60.6$  s.

(b) Its maximum speed occurs at  $t_1$  and equals

$$v_1 = v_0 + a_1 t_1 = 36.3 \text{ m/s}.$$

(c) The graphs are shown below:



73. During  $T_r$  the velocity  $v_0$  is constant (in the direction we choose as  $+x$ ) and obeys  $v_0 = D_r/T_r$  where we note that in SI units the velocity is  $v_0 = 200(1000/3600) = 55.6$  m/s. During  $T_b$  the acceleration is opposite to the direction of  $v_0$  (hence, for us,  $a < 0$ ) until the car is stopped ( $v = 0$ ).

(a) Using Eq. 2-16 (with  $\Delta x_b = 170$  m) we find

$$v^2 = v_0^2 + 2a\Delta x_b \Rightarrow a = -\frac{v_0^2}{2\Delta x_b}$$

which yields  $|a| = 9.08 \text{ m/s}^2$ .

(b) We express this as a multiple of  $g$  by setting up a ratio:

$$a = \left( \frac{9.08}{9.8} \right) g = 0.926g .$$

(c) We use Eq. 2-17 to obtain the braking time:

$$\Delta x_b = \frac{1}{2}(v_0 + v)T_b \Rightarrow T_b = \frac{2(170)}{55.6} = 6.12 \text{ s} .$$

(d) We express our result for  $T_b$  as a multiple of the reaction time  $T_r$  by setting up a ratio:

$$T_b = \left( \frac{6.12}{400 \times 10^{-3}} \right) T_r = 15.3T_r .$$

(e) Since  $T_b > T_r$ , most of the full time required to stop is spent in braking.

(f) We are only asked what the *increase* in distance  $D$  is, due to  $\Delta T_r = 0.100$  s, so we simply have

$$\Delta D = v_0 \Delta T_r = (55.6)(0.100) = 5.56 \text{ m} .$$

74. We assume constant velocity motion and use Eq. 2-2 (with  $v_{\text{avg}} = v > 0$ ). Therefore,

$$\Delta x = v\Delta t = \left( 303 \frac{\text{km}}{\text{h}} \left( \frac{1000 \text{ m} / \text{km}}{3600 \text{ s} / \text{h}} \right) \right) (100 \times 10^{-3} \text{ s}) = 8.4 \text{ m}.$$

75. Integrating (from  $t = 2$  s to variable  $t = 4$  s) the acceleration to get the velocity (and using the velocity datum mentioned in the problem, leads to

$$v = 17 + \frac{1}{2} (5)(4^2 - 2^2) = 47 \text{ m/s.}$$



76. The statement that the stoneflies have “constant speed along a straight path” means we are dealing with constant velocity motion (Eq. 2-2 with  $v_{\text{avg}}$  replaced with  $v_s$  or  $v_{\text{ns}}$ , as the case may be).

(a) We set up the ratio and simplify (using  $d$  for the common distance).

$$\frac{v_s}{v_{\text{ns}}} = \frac{d/t_s}{d/t_{\text{ns}}} = \frac{t_{\text{ns}}}{t_s} = \frac{25.0}{7.1} = 3.52 \approx 3.5.$$

(b) We examine  $\Delta t$  and simplify until we are left with an expression having numbers and no variables other than  $v_s$ . Distances are understood to be in meters.

$$t_{\text{ns}} - t_s = \frac{2}{v_{\text{ns}}} - \frac{2}{v_s} = \frac{2}{(v_s/3.52)} - \frac{2}{v_s} = \frac{2}{v_s} (3.52 - 1) \approx \frac{5.0 \text{ m}}{v_s}$$

77. We orient + along the direction of motion (so  $a$  will be negative-valued, since it is a deceleration), and we use Eq. 2-7 with

$$a_{\text{avg}} = -3400g = -3400(9.8) = -3.33 \times 10^4 \text{ m/s}^2$$

and  $v = 0$  (since the recorder finally comes to a stop).

$$a_{\text{avg}} = \frac{v - v_0}{\Delta t} \Rightarrow v_0 = (3.33 \times 10^4 \text{ m/s}^2) (6.5 \times 10^{-3} \text{ s})$$

which leads to  $v_0 = 217 \text{ m/s}$ .

78. (a) We estimate  $x \approx 2$  m at  $t = 0.5$  s, and  $x \approx 12$  m at  $t = 4.5$  s. Hence, using the definition of average velocity Eq. 2-2, we find

$$v_{\text{avg}} = \frac{12 - 2}{4.5 - 0.5} = 2.5 \text{ m/s}.$$

(b) In the region  $4.0 \leq t \leq 5.0$ , the graph depicts a straight line, so its slope represents the instantaneous velocity for any point in that interval. Its slope is the average velocity between  $t = 4.0$  s and  $t = 5.0$  s:

$$v_{\text{avg}} = \frac{16.0 - 8.0}{5.0 - 4.0} = 8.0 \text{ m/s}.$$

Thus, the instantaneous velocity at  $t = 4.5$  s is 8.0 m/s. (Note: similar reasoning leads to a value needed in the next part: the slope of the  $0 \leq t \leq 1$  region indicates that the instantaneous velocity at  $t = 0.5$  s is 4.0 m/s.)

(c) The average acceleration is defined by Eq. 2-7:

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{8.0 - 4.0}{4.5 - 0.5} = 1.0 \text{ m/s}^2$$

(d) The instantaneous acceleration is the instantaneous rate-of-change of the velocity, and the constant  $x$  vs.  $t$  slope in the interval  $4.0 \leq t \leq 5.0$  indicates that the velocity is constant during that interval. Therefore,  $a = 0$  at  $t = 4.5$  s.

79. We use the functional notation  $x(t)$ ,  $v(t)$  and  $a(t)$  and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{dt} = 6.0t^2 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = 12t$$

with SI units understood. These expressions are used in the parts that follow.

(a) Using the definition of average velocity, Eq. 2-2, we find

$$v_{\text{avg}} = \frac{x(2) - x(1)}{2.0 - 1.0} = \frac{2(2)^3 - 2(1)^3}{1.0} = 14 \text{ m/s} .$$

(b) The average acceleration is defined by Eq. 2-7:

$$a_{\text{avg}} = \frac{v(2) - v(1)}{2.0 - 1.0} = \frac{6(2)^2 - 6(1)^2}{1.0} = 18 \text{ m/s}^2 .$$

(c) The value of  $v(t)$  when  $t = 1.0$  s is  $v(1) = 6(1)^2 = 6.0$  m/s.

(d) The value of  $a(t)$  when  $t = 1.0$  s is  $a(1) = 12(1) = 12$  m/s<sup>2</sup>.

(e) The value of  $v(t)$  when  $t = 2.0$  s is  $v(2) = 6(2)^2 = 24$  m/s.

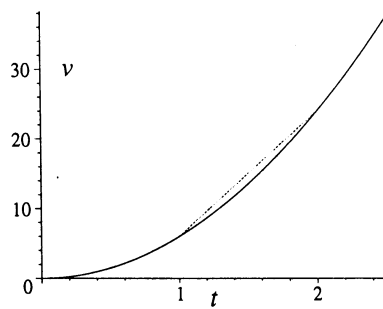
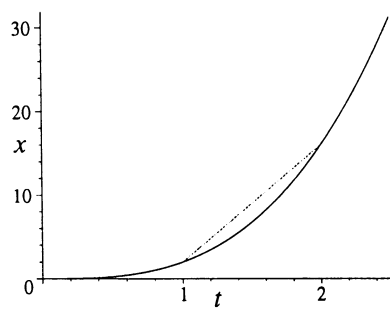
(f) The value of  $a(t)$  when  $t = 2.0$  s is  $a(2) = 12(2) = 24$  m/s<sup>2</sup>.

(g) We don't expect average values of a quantity, say, heights of trees, to equal any particular height for a specific tree, but we are sometimes surprised at the different kinds of averaging that can be performed. Now, the acceleration is a linear function (of time) so its average as defined by Eq. 2-7 is, not surprisingly, equal to the arithmetic average of its  $a(1)$  and  $a(2)$  values. The velocity is not a linear function so the result of part (a) is not equal to the arithmetic average of parts (c) and (e) (although it is fairly close). This reminds us that the calculus-based definition of the average a function (equivalent to Eq. 2-2 for  $v_{\text{avg}}$ ) is not the same as the simple idea of an arithmetic average of two numbers; in other words,

$$\frac{1}{t' - t} \int_t^{t'} f(\tau) d\tau \neq \frac{f(t') - f(t)}{2}$$

except in very special cases (like with linear functions).

(h) The graphs are shown below,  $x(t)$  on the left and  $v(t)$  on the right. SI units are understood. We do not show the tangent lines (representing instantaneous slope values) at  $t = 1$  and  $t = 2$ , but we do show line segments representing the average quantities computed in parts (a) and (b).



80. (a) Let the height of the diving board be  $h$ . We choose *down* as the  $+y$  direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). Thus,  $y = h$  designates the location where the ball strikes the water. Let the depth of the lake be  $D$ , and the total time for the ball to descend be  $T$ . The speed of the ball as it reaches the surface of the lake is then  $v = \sqrt{2gh}$  (from Eq. 2-16), and the time for the ball to fall from the board to the lake surface is  $t_1 = \sqrt{2h/g}$  (from Eq. 2-15). Now, the time it spends descending in the lake (at constant velocity  $v$ ) is

$$t_2 = \frac{D}{v} = \frac{D}{\sqrt{2gh}} .$$

Thus,  $T = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{D}{\sqrt{2gh}}$ , which gives

$$D = T \sqrt{2gh} - 2h = (4.80) \sqrt{(2)(9.80)(5.20)} - (2)(5.20) = 38.1 \text{ m} .$$

(b) Using Eq. 2-2, the magnitude of the average velocity is

$$v_{\text{avg}} = \frac{D + h}{T} = \frac{38.1 + 5.20}{4.80} = 9.02 \text{ m/s}$$

(c) In our coordinate choices, a positive sign for  $v_{\text{avg}}$  means that the ball is going downward. If, however, upwards had been chosen as the positive direction, then this answer in (b) would turn out negative-valued.

(d) We find  $v_0$  from  $\Delta y = v_0 t + \frac{1}{2} g t^2$  with  $t = T$  and  $\Delta y = h + D$ . Thus,

$$v_0 = \frac{h + D}{T} - \frac{gT}{2} = \frac{5.20 + 38.1}{4.80} - \frac{(9.8)(4.80)}{2} = 14.5 \text{ m/s}$$

(e) Here in our coordinate choices the negative sign means that the ball is being thrown upward.

81. The time being considered is 6 years and roughly 235 days, which is approximately  $\Delta t = 2.1 \times 10^7$  s. Using Eq. 2-3, we find the average speed to be

$$\frac{30600 \times 10^3 \text{ m}}{2.1 \times 10^8 \text{ s}} = 0.15 \text{ m/s}.$$

82. (a) It follows from Eq. 2-8 that  $v - v_0 = \int a \, dt$ , which has the geometric interpretation of being the area under the graph. Thus, with  $v_0 = 2.0 \text{ m/s}$  and that area amounting to  $3.0 \text{ m/s}$  (adding that of a triangle to that of a square, over the interval  $0 \leq t \leq 2 \text{ s}$ ), we find  $v = 2.0 + 3.0 = 5.0 \text{ m/s}$  (which we will denote as  $v_2$  in the next part). The information given that  $x_0 = 4.0 \text{ m}$  is not used in this solution.

(b) During  $2 < t \leq 4 \text{ s}$ , the graph of  $a$  is a straight line with slope  $1.0 \text{ m/s}^3$ . Extrapolating, we see that the intercept of this line with the  $a$  axis is zero. Thus, with SI units understood,

$$v = v_2 + \int_{2.0}^t a \, d\tau = 5.0 + \int_{2.0}^t (1.0)\tau \, d\tau = 5.0 + \frac{(1.0)t^2 - (1.0)(2.0)^2}{2}$$

which yield  $v = 3.0 + 0.50t^2$  in m/s.



83. We take  $+x$  in the direction of motion. We use subscripts 1 and 2 for the data. Thus,  $v_1 = +30$  m/s,  $v_2 = +50$  m/s and  $x_2 - x_1 = +160$  m.

(a) Using these subscripts, Eq. 2-16 leads to

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{50^2 - 30^2}{2(160)} = 5.0 \text{ m/s}^2 .$$

(b) We find the time interval corresponding to the displacement  $x_2 - x_1$  using Eq. 2-17:

$$t_2 - t_1 = \frac{2(x_2 - x_1)}{v_1 + v_2} = \frac{2(160)}{30 + 50} = 4.0 \text{ s} .$$

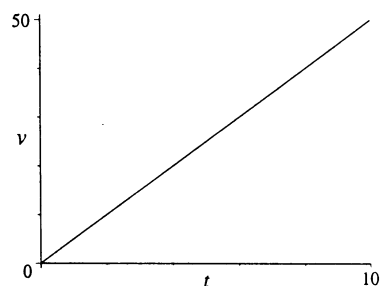
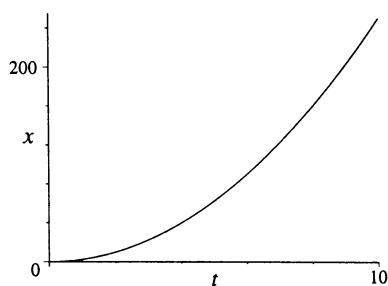
(c) Since the train is at rest ( $v_0 = 0$ ) when the clock starts, we find the value of  $t_1$  from Eq. 2-11:

$$v_1 = v_0 + at_1 \Rightarrow t_1 = \frac{30}{5.0} = 6.0 \text{ s} .$$

(d) The coordinate origin is taken to be the location at which the train was initially at rest (so  $x_0 = 0$ ). Thus, we are asked to find the value of  $x_1$ . Although any of several equations could be used, we choose Eq. 2-17:

$$x_1 = \frac{1}{2}(v_0 + v_1)t_1 = \frac{1}{2}(30)(6.0) = 90 \text{ m} .$$

(e) The graphs are shown below, with SI units assumed.



84. We choose *down* as the +y direction and use the equations of Table 2-1 (replacing  $x$  with  $y$ ) with  $a = +g$ ,  $v_0 = 0$  and  $y_0 = 0$ . We use subscript 2 for the elevator reaching the ground and 1 for the halfway point.

(a) Eq. 2-16,  $v_2^2 = v_0^2 + 2a(y_2 - y_0)$ , leads to

$$v_2 = \sqrt{2gy_2} = \sqrt{2(9.8)(120)} = 48.5 \text{ m/s}.$$

(b) The time at which it strikes the ground is (using Eq. 2-15)

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2(120)}{9.8}} = 4.95 \text{ s}.$$

(c) Now Eq. 2-16, in the form  $v_1^2 = v_0^2 + 2a(y_1 - y_0)$ , leads to

$$v_1 = \sqrt{2gy_1} = \sqrt{2(9.8)(60)} = 34.3 \text{ m/s}.$$

(d) The time at which it reaches the halfway point is (using Eq. 2-15)

$$t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(60)}{9.8}} = 3.50 \text{ s}.$$

85. We take the direction of motion as  $+x$ , take  $x_0 = 0$  and use SI units, so  $v = 1600(1000/3600) = 444 \text{ m/s}$ .

(a) Eq. 2-11 gives  $444 = a(1.8)$  or  $a = 247 \text{ m/s}^2$ . We express this as a multiple of  $g$  by setting up a ratio:

$$a = \left( \frac{247}{9.8} \right) g = 25g.$$

(b) Eq. 2-17 readily yields

$$x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(444)(1.8) = 400 \text{ m}.$$

86. This problem consists of two parts: part 1 with constant acceleration (so that the equations in Table 2-1 apply),  $v_0 = 0$ ,  $v = 11.0$  m/s,  $x = 12.0$  m, and  $x_0 = 0$  (adopting the starting line as the coordinate origin); and, part 2 with constant velocity (so that  $x - x_0 = vt$  applies) with  $v = 11.0$  m/s,  $x_0 = 12.0$ , and  $x = 100.0$  m.

(a) We obtain the time for part 1 from Eq. 2-17

$$x - x_0 = \frac{1}{2}(v_0 + v) t_1 \Rightarrow 12.0 - 0 = \frac{1}{2}(0 + 11.0)t_1$$

so that  $t_1 = 2.2$  s, and we find the time for part 2 simply from  $88.0 = (11.0)t_2 \rightarrow t_2 = 8.0$  s. Therefore, the total time is  $t_1 + t_2 = 10.2$  s.

(b) Here, the total time is required to be 10.0 s, and we are to locate the point  $x_p$  where the runner switches from accelerating to proceeding at constant speed. The equations for parts 1 and 2, used above, therefore become

$$\begin{aligned} x_p - 0 &= \frac{1}{2}(0 + 11.0)t_1 \\ 100.0 - x_p &= (11.0)(10.0 - t_1) \end{aligned}$$

where in the latter equation, we use the fact that  $t_2 = 10.0 - t_1$ . Solving the equations for the two unknowns, we find that  $t_1 = 1.8$  s and  $x_p = 10.0$  m.

87. We take  $+x$  in the direction of motion, so

$$v = (60 \text{ km/h}) \left( \frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) = +16.7 \text{ m/s}$$

and  $a > 0$ . The location where it starts from rest ( $v_0 = 0$ ) is taken to be  $x_0 = 0$ .

(a) Eq. 2-7 gives  $a_{\text{avg}} = (v - v_0)/t$  where  $t = 5.4 \text{ s}$  and the velocities are given above. Thus,  $a_{\text{avg}} = 3.1 \text{ m/s}^2$ .

(b) The assumption that  $a = \text{constant}$  permits the use of Table 2-1. From that list, we choose Eq. 2-17:

$$x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(16.7)(5.4) = 45 \text{ m}.$$

(c) We use Eq. 2-15, now with  $x = 250 \text{ m}$ :

$$x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250)}{3.1}}$$

which yields  $t = 13 \text{ s}$ .

88. (a) Using the fact that the area of a triangle is  $\frac{1}{2}$  (base) (height) (and the fact that the integral corresponds to the area under the curve) we find, from  $t = 0$  through  $t = 5$  s, the integral of  $v$  with respect to  $t$  is 15 m. Since we are told that  $x_0 = 0$  then we conclude that  $x = 15$  m when  $t = 5.0$  s.

(b) We see directly from the graph that  $v = 2.0$  m/s when  $t = 5.0$  s.

(c) Since  $a = dv/dt = \text{slope of the graph}$ , we find that the acceleration during the interval  $4 < t < 6$  is uniformly equal to  $-2.0$  m/s<sup>2</sup>.

(d) Thinking of  $x(t)$  in terms of accumulated area (on the graph), we note that  $x(1) = 1$  m; using this and the value found in part (a), Eq. 2-2 produces

$$v_{\text{avg}} = \frac{x(5) - x(1)}{5 - 1} = \frac{15 - 1}{4} = 3.5 \text{ m/s}.$$

(e) From Eq. 2-7 and the values  $v(t)$  we read directly from the graph, we find

$$a_{\text{avg}} = \frac{v(5) - v(1)}{5 - 1} = \frac{2 - 2}{4} = 0.$$

89. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the stone's motion. We are allowed to use Table 2-1 (with  $\Delta x$  replaced by  $y$ ) because the ball has constant acceleration motion (and we choose  $y_0 = 0$ ).

(a) We apply Eq. 2-16 to both measurements, with SI units understood.

$$\begin{aligned} v_B^2 &= v_0^2 - 2gy_B & \Rightarrow & \left(\frac{1}{2}v\right)^2 + 2g(y_A + 3) = v_0^2 \\ v_A^2 &= v_0^2 - 2gy_A & \Rightarrow & v^2 + 2gy_A = v_0^2 \end{aligned}$$

We equate the two expressions that each equal  $v_0^2$  and obtain

$$\frac{1}{4}v^2 + 2gy_A + 2g(3) = v^2 + 2gy_A \quad \Rightarrow \quad 2g(3) = \frac{3}{4}v^2$$

which yields  $v = \sqrt{2g(4)} = 8.85 \text{ m/s}$ .

(b) An object moving upward at  $A$  with speed  $v = 8.85 \text{ m/s}$  will reach a maximum height  $y - y_A = v^2/2g = 4.00 \text{ m}$  above point  $A$  (this is again a consequence of Eq. 2-16, now with the “final” velocity set to zero to indicate the highest point). Thus, the top of its motion is  $1.00 \text{ m}$  above point  $B$ .

90. The object, once it is dropped ( $v_0 = 0$ ) is in free-fall ( $a = -g = -9.8 \text{ m/s}^2$  if we take *down* as the  $-y$  direction), and we use Eq. 2-15 repeatedly.

(a) The (positive) distance  $D$  from the lower dot to the mark corresponding to a certain reaction time  $t$  is given by  $\Delta y = -D = -\frac{1}{2}gt^2$ , or  $D = gt^2/2$ . Thus, for  $t_1 = 50.0 \text{ ms}$ ,

$$D_1 = \frac{(9.8 \text{ m/s}^2)(50.0 \times 10^{-3} \text{ s})^2}{2} = 0.0123 \text{ m} = 1.23 \text{ cm}.$$

$$(b) \text{ For } t_2 = 100 \text{ ms, } D_2 = \frac{(9.8 \text{ m/s}^2)(100 \times 10^{-3} \text{ s})^2}{2} = 0.049 \text{ m} = 4D_1.$$

$$(c) \text{ For } t_3 = 150 \text{ ms, } D_3 = \frac{(9.8 \text{ m/s}^2)(150 \times 10^{-3} \text{ s})^2}{2} = 0.11 \text{ m} = 9D_1.$$

$$(d) \text{ For } t_4 = 200 \text{ ms, } D_4 = \frac{(9.8 \text{ m/s}^2)(200 \times 10^{-3} \text{ s})^2}{2} = 0.196 \text{ m} = 16D_1.$$

$$(e) \text{ For } t_5 = 250 \text{ ms, } D_5 = \frac{(9.8 \text{ m/s}^2)(250 \times 10^{-3} \text{ s})^2}{2} = 0.306 \text{ m} = 25D_1.$$



91. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the motion. We are allowed to use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the  $y$  axis. The total time of fall can be computed from Eq. 2-15 (using the quadratic formula).

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root chosen. With  $y = 0$ ,  $v_0 = 0$  and  $y_0 = h = 60 \text{ m}$ , we obtain

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5 \text{ s}.$$

Thus, “1.2 s earlier” means we are examining where the rock is at  $t = 2.3 \text{ s}$ :

$$y - h = v_0(2.3) - \frac{1}{2} g(2.3)^2 \Rightarrow y = 34 \text{ m}$$

where we again use the fact that  $h = 60 \text{ m}$  and  $v_0 = 0$ .

92. With +y upward, we have  $y_0 = 36.6$  m and  $y = 12.2$  m. Therefore, using Eq. 2-18 (the last equation in Table 2-1), we find

$$y - y_0 = vt + \frac{1}{2}gt^2 \Rightarrow v = -22 \text{ m/s}$$

at  $t = 2.00$  s. The term *speed* refers to the magnitude of the velocity vector, so the answer is  $|v| = 22.0$  m/s.

93. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the motion. We are allowed to use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because this is constant acceleration motion. When something is thrown straight up and is caught at the level it was thrown from (with a trajectory similar to that shown in Fig. 2-25), the time of flight  $t$  is half of its time of ascent  $t_a$ , which is given by Eq. 2-18 with  $\Delta y = H$  and  $v = 0$  (indicating the maximum point).

$$H = vt_a + \frac{1}{2}gt_a^2 \Rightarrow t_a = \sqrt{\frac{2H}{g}}$$

Writing these in terms of the total time in the air  $t = 2t_a$  we have

$$H = \frac{1}{8}gt^2 \Rightarrow t = 2\sqrt{\frac{2H}{g}}.$$

We consider two throws, one to height  $H_1$  for total time  $t_1$  and another to height  $H_2$  for total time  $t_2$ , and we set up a ratio:

$$\frac{H_2}{H_1} = \frac{\frac{1}{8}gt_2^2}{\frac{1}{8}gt_1^2} = \left(\frac{t_2}{t_1}\right)^2$$

from which we conclude that if  $t_2 = 2t_1$  (as is required by the problem) then  $H_2 = 2^2H_1 = 4H_1$ .

94. Taking +y to be upward and placing the origin at the point from which the objects are dropped, then the location of diamond 1 is given by  $y_1 = -\frac{1}{2}gt^2$  and the location of diamond 2 is given by  $y_2 = -\frac{1}{2}g(t-1)^2$ . We are starting the clock when the first object is dropped. We want the time for which  $y_2 - y_1 = 10$  m. Therefore,

$$-\frac{1}{2}g(t-1)^2 + \frac{1}{2}gt^2 = 10 \Rightarrow t = (10/g) + 0.5 = 1.5 \text{ s.}$$

95. We denote  $t_r$  as the reaction time and  $t_b$  as the braking time. The motion during  $t_r$  is of the constant-velocity (call it  $v_0$ ) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2} a t_b^2$$

where  $v_0$  is the initial velocity and  $a$  is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied the velocity of the car is given by  $v = v_0 + a t_b$ . Using this equation, with  $v = 0$ , we eliminate  $t_b$  from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2} \frac{v_0^2}{a} = v_0 t_r - \frac{1}{2} \frac{v_0^2}{a}.$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2} \frac{v_{01}^2}{a}$$

and

$$x_2 = v_{02} t_r - \frac{1}{2} \frac{v_{02}^2}{a}.$$

Solving these equations simultaneously for  $t_r$  and  $a$  we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02} (v_{02} - v_{01})}$$

and

$$a = -\frac{1}{2} \frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2}.$$

(a) Substituting  $x_1 = 56.7$  m,  $v_{01} = 80.5$  km/h = 22.4 m/s,  $x_2 = 24.4$  m and  $v_{02} = 48.3$  km/h = 13.4 m/s, we find

$$t_r = \frac{13.4^2(56.7) - 22.4^2(24.4)}{(22.4)(13.4)(13.4 - 22.4)} = 0.74 \text{ s}.$$

(b) In a similar manner, substituting  $x_1 = 56.7$  m,  $v_{01} = 80.5$  km/h = 22.4 m/s,  $x_2 = 24.4$  m and  $v_{02} = 48.3$  km/h = 13.4 m/s gives

$$a = -\frac{1}{2} \frac{(13.4)22.4^2 - (22.4)13.4^2}{(13.4)(56.7) - (22.4)(24.4)} = -6.2 \text{ m/s}^2.$$

The *magnitude* of the deceleration is therefore  $6.2 \text{ m/s}^2$ . Although rounded off values are displayed in the above substitutions, what we have input into our calculators are the “exact” values (such as  $v_{02} = \frac{161}{12} \text{ m/s}$ ).

96. Assuming the horizontal velocity of the ball is constant, the horizontal displacement is

$$\Delta x = v\Delta t$$

where  $\Delta x$  is the horizontal distance traveled,  $\Delta t$  is the time, and  $v$  is the (horizontal) velocity. Converting  $v$  to meters per second, we have  $160 \text{ km/h} = 44.4 \text{ m/s}$ . Thus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4 \text{ m}}{44.4 \text{ m/s}} = 0.414 \text{ s}.$$

The velocity-unit conversion implemented above can be figured “from basics” ( $1000 \text{ m} = 1 \text{ km}$ ,  $3600 \text{ s} = 1 \text{ h}$ ) or found in Appendix D.

97. In this solution, we make use of the notation  $x(t)$  for the value of  $x$  at a particular  $t$ . Thus,  $x(t) = 50t + 10t^2$  with SI units (meters and seconds) understood.

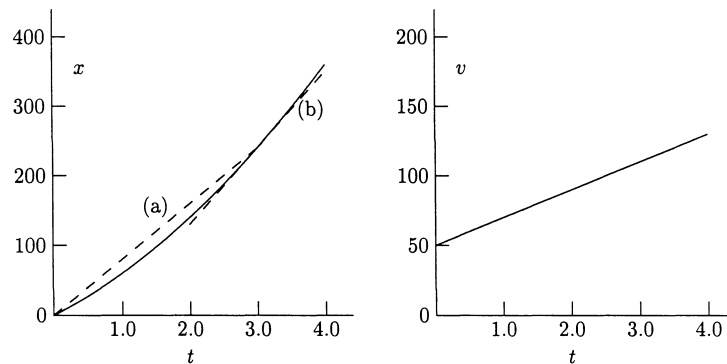
(a) The average velocity during the first 3 s is given by

$$v_{\text{avg}} = \frac{x(3) - x(0)}{\Delta t} = \frac{(50)(3) + (10)(3)^2 - 0}{3} = 80 \text{ m/s}.$$

(b) The instantaneous velocity at time  $t$  is given by  $v = dx/dt = 50 + 20t$ , in SI units. At  $t = 3.0$  s,  $v = 50 + (20)(3.0) = 110$  m/s.

(c) The instantaneous acceleration at time  $t$  is given by  $a = dv/dt = 20 \text{ m/s}^2$ . It is constant, so the acceleration at any time is  $20 \text{ m/s}^2$ .

(d) and (e) The graphs that follow show the coordinate  $x$  and velocity  $v$  as functions of time, with SI units understood. The dashed line marked (a) in the first graph runs from  $t = 0$ ,  $x = 0$  to  $t = 3.0$  s,  $x = 240$  m. Its slope is the average velocity during the first 3 s of motion. The dashed line marked (b) is tangent to the  $x$  curve at  $t = 3.0$  s. Its slope is the instantaneous velocity at  $t = 3.0$  s.





98. The bullet starts at rest ( $v_0 = 0$ ) and after traveling the length of the barrel ( $\Delta x = 1.2$  m) emerges with the given velocity ( $v = 640$  m/s), where the direction of motion is the positive direction. Turning to the constant acceleration equations in Table 2-1, we use

$$\Delta x = \frac{1}{2}(v_0 + v) t .$$

Thus, we find  $t = 0.00375$  s (about 3.8 ms).

99. The velocity  $v$  at  $t = 6$  (SI units and two significant figures understood) is  $v_{\text{given}} + \int_2^6 a dt$ . A quick way to implement this is to recall the area of a triangle ( $\frac{1}{2}$  base  $\times$  height). The result is  $v = 7 + 32 = 39$  m/s.

100. Let  $D$  be the distance up the hill. Then

$$\text{average speed} = \frac{\text{total distance traveled}}{\text{total time of travel}} = \frac{2D}{\frac{D}{20 \text{ km/h}} + \frac{D}{35 \text{ km/h}}} \approx 25 \text{ km/h} .$$

101. The time  $\Delta t$  is  $2(60) + 41 = 161$  min and the displacement  $\Delta x = 370$  cm. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{370}{161} = 2.3 \text{ cm/min} .$$

102. Converting to SI units, we have  $v = 3400(1000/3600) = 944$  m/s (presumed constant) and  $\Delta t = 0.10$  s. Thus,  $\Delta x = v\Delta t = 94$  m.

103. The (ideal) driving time before the change was  $t = \Delta x/v$ , and after the change it is  $t' = \Delta x/v'$ . The time saved by the change is therefore

$$t - t' = \Delta x \left( \frac{1}{v} - \frac{1}{v'} \right) = \Delta x \left( \frac{1}{55} - \frac{1}{65} \right) = \Delta x (0.0028 \text{ h / mi})$$

which becomes, converting  $\Delta x = 700/1.61 = 435 \text{ mi}$  (using a conversion found on the inside front cover of the textbook),  $t - t' = (435)(0.0028) = 1.2 \text{ h}$ . This is equivalent to 1 h and 13 min.

104. We take  $+x$  in the direction of motion, so  $v_0 = +30$  m/s,  $v_1 = +15$  m/s and  $a < 0$ . The acceleration is found from Eq. 2-11:  $a = (v_1 - v_0)/t_1$  where  $t_1 = 3.0$  s. This gives  $a = -5.0$  m/s<sup>2</sup>. The displacement (which in this situation is the same as the distance traveled) to the point it stops ( $v_2 = 0$ ) is, using Eq. 2-16,

$$v_2^2 = v_0^2 + 2a\Delta x \Rightarrow \Delta x = -\frac{30^2}{2(-5)} = 90 \text{ m}.$$

105. During free fall, we ignore the air resistance and set  $a = -g = -9.8 \text{ m/s}^2$  where we are choosing *down* to be the  $-y$  direction. The initial velocity is zero so that Eq. 2-15 becomes  $\Delta y = -\frac{1}{2}gt^2$  where  $\Delta y$  represents the *negative* of the distance  $d$  she has fallen. Thus, we can write the equation as  $d = \frac{1}{2}gt^2$  for simplicity.

(a) The time  $t_1$  during which the parachutist is in free fall is (using Eq. 2-15) given by

$$d_1 = 50 \text{ m} = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.80 \text{ m/s}^2)t_1^2$$

which yields  $t_1 = 3.2 \text{ s}$ . The *speed* of the parachutist just before he opens the parachute is given by the positive root  $v_1^2 = 2gd_1$ , or

$$v_1 = \sqrt{2gh_1} = \sqrt{(2)(9.80 \text{ m/s}^2)(50 \text{ m})} = 31 \text{ m/s}.$$

If the final speed is  $v_2$ , then the time interval  $t_2$  between the opening of the parachute and the arrival of the parachutist at the ground level is

$$t_2 = \frac{v_1 - v_2}{a} = \frac{31 \text{ m/s} - 3.0 \text{ m/s}}{2 \text{ m/s}^2} = 14 \text{ s}.$$

This is a result of Eq. 2-11 where *speeds* are used instead of the (negative-valued) velocities (so that final-velocity minus initial-velocity turns out to equal initial-speed minus final-speed); we also note that the acceleration vector for this part of the motion is positive since it points upward (opposite to the direction of motion — which makes it a deceleration). The total time of flight is therefore  $t_1 + t_2 = 17 \text{ s}$ .

(b) The distance through which the parachutist falls after the parachute is opened is given by

$$d = \frac{v_1^2 - v_2^2}{2a} = \frac{(31 \text{ m/s})^2 - (3.0 \text{ m/s})^2}{(2)(2.0 \text{ m/s}^2)} \approx 240 \text{ m}.$$

In the computation, we have used Eq. 2-16 with both sides multiplied by  $-1$  (which changes the negative-valued  $\Delta y$  into the positive  $d$  on the left-hand side, and switches the order of  $v_1$  and  $v_2$  on the right-hand side). Thus the fall begins at a height of  $h = 50 + d \approx 290 \text{ m}$ .



106. If the plane (with velocity  $v$ ) maintains its present course, and if the terrain continues its upward slope of  $4.3^\circ$ , then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan \theta} = \frac{35 \text{ m}}{\tan 4.3^\circ} = 465.5 \text{ m} \approx 0.465 \text{ km}.$$

This corresponds to a time of flight found from Eq. 2-2 (with  $v = v_{\text{avg}}$  since it is constant)

$$t = \frac{\Delta x}{v} = \frac{0.465 \text{ km}}{1300 \text{ km/h}} = 0.000358 \text{ h} \approx 1.3 \text{ s}.$$

This, then, estimates the time available to the pilot to make his correction.

107. (a) We note each reaction distance (second column in the table) is 0.75 multiplied by the values in the first column (initial speed). We conclude that a reaction time of 0.75 s is being assumed. Since we will need the assumed deceleration (during braking) in order to part (b), we point out here that the first column squared, divided by 2 and divided by the third column (see Eq. 2-16) gives  $|a| = 10 \text{ m/s}^2$ .

(b) Multiplying 25 m/s by 0.75 s gives a reaction distance of 18.75 m (where we are carrying out more figures than are meaningful, at least in these intermediate results, for the sake of not introducing round off errors into our calculations). Using Eq. 2-16 with an initial speed of 25 m/s and a deceleration of  $-10 \text{ m/s}^2$  leads to a braking distance of 31.25 m. Adding these distances gives the answer: 50 m. We note that this is close (but not exactly the same) as the value one would get if one simply interpolated using the last column in the table.

108. The problem consists of two constant-acceleration parts: part 1 with  $v_0 = 0$ ,  $v = 6.0$  m/s,  $x = 1.8$  m, and  $x_0 = 0$  (if we take its original position to be the coordinate origin); and, part 2 with  $v_0 = 6.0$  m/s,  $v = 0$ , and  $a_2 = -2.5$  m/s<sup>2</sup> (negative because we are taking the positive direction to be the direction of motion).

(a) We can use Eq. 2-17 to find the time for the first part

$$x - x_0 = \frac{1}{2} (v_0 + v) t_1 \Rightarrow 1.8 - 0 = \frac{1}{2} (0 + 6.0) t_1$$

so that  $t_1 = 0.6$  s. And Eq. 2-11 is used to obtain the time for the second part

$$v = v_0 + a_2 t_2 \Rightarrow 0 = 6.0 + (-2.5)t_2$$

from which  $t_2 = 2.4$  s is computed. Thus, the total time is  $t_1 + t_2 = 3.0$  s.

(b) We already know the distance for part 1. We could find the distance for part 2 from several of the equations, but the one that makes no use of our part (a) results is Eq. 2-16

$$v^2 = v_0^2 + 2a_2\Delta x_2 \Rightarrow 0 = (6.0)^2 + 2(-2.5)\Delta x_2$$

which leads to  $\Delta x_2 = 7.2$  m. Therefore, the total distance traveled by the shuffleboard disk is  $(1.8 + 7.2)$  m = 9.0 m.

109. We obtain the velocity by integration of the acceleration:  $v - v_0 = \int_0^t (6.1 - 1.2t') dt'$ .

Lengths are in meters and times are in seconds. The student is encouraged to look at the discussion in the textbook in §2-7 to better understand the manipulations here.

(a) The result of the above calculation is

$$v = v_0 + 6.1t - 0.6t^2 ,$$

where the problem states that  $v_0 = 2.7$  m/s. The maximum of this function is found by knowing when its derivative (the acceleration) is zero ( $a = 0$  when  $t = 6.1/1.2 = 5.1$  s) and plugging that value of  $t$  into the velocity equation above. Thus, we find  $v = 18$  m/s.

(b) We integrate again to find  $x$  as a function of  $t$ :

$$x - x_0 = \int_0^t v dt' = \int_0^t (v_0 + 6.1t' - 0.6t'^2) dt' = v_0 t + 3.05t^2 - 0.2t^3 .$$

With  $x_0 = 7.3$  m, we obtain  $x = 83$  m for  $t = 6$ . This is the correct answer, but one has the right to worry that it might not be; after all, the problem asks for the total distance traveled (and  $x - x_0$  is just the *displacement*). If the cyclist backtracked, then his total distance would be greater than his displacement. Thus, we might ask, “did he backtrack?” To do so would require that his velocity be (momentarily) zero at some point (as he reversed his direction of motion). We could solve the above quadratic equation for velocity, for a positive value of  $t$  where  $v = 0$ ; if we did, we would find that at  $t = 10.6$  s, a reversal does indeed happen. However, in the time interval concerned with in our problem ( $0 \leq t \leq 6$  s), there is no reversal and the displacement is the same as the total distance traveled.

110. The time required is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7). First, we convert the velocity change to SI units:

$$\Delta v = (100 \text{ km/h}) \left( \frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) = 27.8 \text{ m/s} .$$

Thus,  $\Delta t = \Delta v/a = 27.8/50 = 0.556 \text{ s}$ .

111. From Table 2-1,  $v^2 - v_0^2 = 2a\Delta x$  is used to solve for  $a$ . Its minimum value is

$$a_{\min} = \frac{v_2 - v_0^2}{2\Delta x_{\max}} = \frac{(360 \text{ km/h})^2}{2(1.80 \text{ km})} = 36000 \text{ km/h}^2$$

which converts to  $2.78 \text{ m/s}^2$ .

112. (a) For the automobile  $\Delta v = 55 - 25 = 30 \text{ km/h}$ , which we convert to SI units:

$$a = \frac{\Delta v}{\Delta t} = \frac{(30 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)}{(0.50 \text{ min})(60 \text{ s/min})} = 0.28 \text{ m/s}^2 .$$

(b) The change of velocity for the bicycle, for the same time, is identical to that of the car, so its acceleration is also  $0.28 \text{ m/s}^2$ .

113. For each rate, we use distance  $d = vt$  and convert to SI using  $0.0254 \text{ cm} = 1 \text{ inch}$  (from which we derive the factors appearing in the computations below).

(a) The total distance  $d$  comes from summing

$$d_1 = \left(120 \frac{\text{steps}}{\text{min}}\right) \left(\frac{0.762 \text{ m / step}}{60 \text{ s / min}}\right) (5 \text{ s}) = 7.62 \text{ m}$$

$$d_2 = \left(120 \frac{\text{steps}}{\text{min}}\right) \left(\frac{0.381 \text{ m / step}}{60 \text{ s / min}}\right) (5 \text{ s}) = 3.81 \text{ m}$$

$$d_3 = \left(180 \frac{\text{steps}}{\text{min}}\right) \left(\frac{0.914 \text{ m / step}}{60 \text{ s / min}}\right) (5 \text{ s}) = 13.72 \text{ m}$$

$$d_4 = \left(180 \frac{\text{steps}}{\text{min}}\right) \left(\frac{0.457 \text{ m / step}}{60 \text{ s / min}}\right) (5 \text{ s}) = 6.86 \text{ m}$$

so that  $d = d_1 + d_2 + d_3 + d_4 = 32 \text{ m}$ .

(b) Average velocity is computed using Eq. 2-2:  $v_{\text{avg}} = 32/20 = 1.6 \text{ m/s}$ , where we have used the fact that the total time is 20 s.

(c) The total time  $t$  comes from summing

$$t_1 = \frac{8 \text{ m}}{\left(120 \frac{\text{steps}}{\text{min}}\right) \left(\frac{0.762 \text{ m / step}}{60 \text{ s / min}}\right)} = 5.25 \text{ s}$$

$$t_2 = \frac{8 \text{ m}}{\left(120 \frac{\text{steps}}{\text{min}}\right) \left(\frac{0.381 \text{ m / step}}{60 \text{ s / min}}\right)} = 10.5 \text{ s}$$

$$t_3 = \frac{8 \text{ m}}{\left(180 \frac{\text{steps}}{\text{min}}\right) \left(\frac{0.914 \text{ m / step}}{60 \text{ s / min}}\right)} = 2.92 \text{ s}$$

$$t_4 = \frac{8 \text{ m}}{\left(180 \frac{\text{steps}}{\text{min}}\right) \left(\frac{0.457 \text{ m / step}}{60 \text{ s / min}}\right)} = 5.83 \text{ s}$$

so that  $t = t_1 + t_2 + t_3 + t_4 = 24.5 \text{ s}$ .

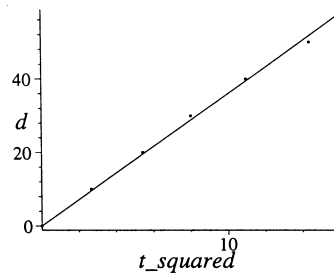
(d) Average velocity is computed using Eq. 2-2:  $v_{\text{avg}} = 32/24.5 = 1.3 \text{ m/s}$ , where we have used the fact that the total distance is  $4(8) = 32 \text{ m}$ .



114. (a) It is the intent of this problem to treat the  $v_0 = 0$  condition rigidly. In other words, we are not fitting the distance to just any second-degree polynomial in  $t$ ; rather, we require  $d = At^2$  (which meets the condition that  $d$  and its derivative is zero when  $t = 0$ ). If we perform a least squares fit with this expression, we obtain  $A = 3.587$  (SI units understood). We return to this discussion in part (c). Our expectation based on Eq. 2-15, assuming no error in starting the clock at the moment the acceleration begins, is  $d = \frac{1}{2}at^2$  (since he started at the coordinate origin, the location of which presumably is something we can be fairly certain about).

(b) The graph ( $d$  on the vertical axis, SI units understood) is shown.

The horizontal axis is  $t^2$  (as indicated by the problem statement) so that we have a straight line instead of a parabola.



(c) Comparing our two expressions for  $d$ , we see the parameter  $A$  in our fit should correspond to  $\frac{1}{2}a$ , so  $a = 2(3.587) \approx 7.2 \text{ m/s}^2$ . Now, other approaches might be considered (trying to fit the data with  $d = Ct^2 + B$  for instance, which leads to  $a = 2C = 7.0 \text{ m/s}^2$  and  $B \neq 0$ ), and it might be useful to have the class discuss the assumptions made in each approach.

115. When comparing two positions at the same elevation,  $\Delta y = 0$ , which means

$$\Delta y_U = 0 = v_U \Delta T_U - \frac{1}{2} g \Delta T_U^2 \quad (\text{Equation 1})$$

$$\Delta y_L = 0 = v_L \Delta T_L - \frac{1}{2} g \Delta T_L^2 \quad (\text{Equation 2}) .$$

Now the time between the instants where the (upward moving) ball has velocities  $v_U$  and  $v_L$  is  $\frac{1}{2} (\Delta T_L - \Delta T_U)$ , which is evident from the graph. That distance is  $H$ , so Eq. 2-17 leads to

$$H = \left( \frac{v_L + v_U}{2} \right) \frac{1}{2} (\Delta T_L - \Delta T_U) = \frac{1}{4} (v_L \Delta T_L - v_U \Delta T_U) \quad (\text{Equation 3})$$

where we have also used the fact that  $v_L / \Delta T_L = v_U / \Delta T_U$  (see Eq. 2-11, keeping in mind the acceleration is the same for both time intervals). We subtract (Equation 2) from (Equation 1), then divide through by 4, and add to (Equation 3). The result is

$$H = \frac{1}{8} g (\Delta T_L^2 - \Delta T_U^2)$$

which readily yields

$$g = \frac{8H}{\Delta T_L^2 - \Delta T_U^2}$$

116. There is no air resistance, which makes it quite accurate to set  $a = -g = -9.8 \text{ m/s}^2$  (where downward is the  $-y$  direction) for the duration of the fall. We are allowed to use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because this is constant acceleration motion; in fact, when the acceleration changes (during the process of catching the ball) we will again assume constant acceleration conditions; in this case, we have  $a_2 = +25g = 245 \text{ m/s}^2$ .

(a) The time of fall is given by Eq. 2-15 with  $v_0 = 0$  and  $y = 0$ . Thus,

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(145)}{9.8}} = 5.44 \text{ s.}$$

(b) The final velocity for its free-fall (which becomes the initial velocity during the catching process) is found from Eq. 2-16 (other equations can be used but they would use the result from part (a)).

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{2gy_0} = -53.3 \text{ m/s}$$

where the negative root is chosen since this is a downward velocity. Thus, the speed is  $|v| = 53.3 \text{ m/s}$ .

(c) For the catching process, the answer to part (b) plays the role of an *initial* velocity ( $v_0 = -53.3 \text{ m/s}$ ) and the final velocity must become zero. Using Eq. 2-16, we find

$$\Delta y_2 = \frac{v^2 - v_0^2}{2a_2} = \frac{-(-53.3)^2}{2(245)} = -5.80 \text{ m},$$

where the negative value of  $\Delta y_2$  signifies that the distance traveled while arresting its motion is downward.

117. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking down as the  $-y$  direction) for the duration of the motion. We are allowed to use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because this is constant acceleration motion. The ground level is taken to correspond to  $y = 0$ .

(a) With  $y_0 = h$  and  $v_0$  replaced with  $-v_0$ , Eq. 2-16 leads to

$$v = \sqrt{(-v_0)^2 - 2g(y - y_0)} = \sqrt{v_0^2 + 2gh}.$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity).

(b) We use the quadratic formula to solve Eq. 2-15 for  $t$ , with  $v_0$  replaced with  $-v_0$ ,

$$\Delta y = -v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{-v_0 + \sqrt{(-v_0)^2 - 2g\Delta y}}{g}$$

where the positive root is chosen to yield  $t > 0$ . With  $y = 0$  and  $y_0 = h$ , this becomes

$$t = \frac{\sqrt{v_0^2 + 2gh} - v_0}{g}.$$

(c) If it were thrown upward with that speed from height  $h$  then (in the absence of air friction) it would return to height  $h$  with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation).

(d) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having  $+v_0$  in the equation where we had put in  $-v_0$  in part (b). The details follow:

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root again chosen to yield  $t > 0$ . With  $y = 0$  and  $y_0 = h$ , we obtain

$$t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g}.$$