1. The air inside pushes outward with a force given by p_iA, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by p_oA, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$. Since 1 atm = 1.013×10^5 Pa,

$$F = (1.0 \text{ atm} - 0.96 \text{ atm})(1.013\times10^5 \text{ Pa/atm})(3.4 \text{ m})(2.1 \text{ m}) = 2.9\times10^4 \text{ N}.$$

2. We note that the container is cylindrical, the important aspect of this being that it has a uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquids. Using the fact that 1L = 1000 cm^3, we find the weight of the first liquid to be

$$\begin{aligned} W_1 &= m_1 g = \rho_1 V_1 g \\ &= (2.6\ \text{g}/\text{cm}^3)(0.50\ \text{L})(1000\ \text{cm}^3/\text{L})(980\ \text{cm/s}^2) = 1.27\times10^6\,\text{g}\cdot\text{cm/s}^2 = 12.7\ \text{N}. \end{aligned}$$

In the last step, we have converted grams to kilograms and centimeters to meters. Similarly, for the second and the third liquids, we have

$$W_2 = m_2 g = \rho_2 V_2 g = (1.0\ \text{g/cm}^3)(0.25\ \text{L})(1000\ \text{cm}^3/\text{L})(980\ \text{cm/s}^2) = 2.5\ \text{N}$$

and

$$W_3 = m_3 g = \rho_3 V_3 g = (0.80\ \text{g/cm}^3)(0.40\ \text{L})(1000\ \text{cm}^3/\text{L})(980\ \text{cm/s}^2) = 3.1\ \text{N}.$$

The total force on the bottom of the container is therefore $F = W_1 + W_2 + W_3 = 18$ N.

3. The pressure increase is the applied force divided by the area: $\Delta p = F/A = F/\pi r^2$, where r is the radius of the piston. Thus $\Delta p = (42\ \text{N})/\pi(0.011\ \text{m})^2 = 1.1 \times 10^5$ Pa. This is equivalent to 1.1 atm.

4. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\text{N/m}^2 = 1$ Pa, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0\times10^5\ \text{Pa} - \frac{480\ \text{N}}{77\times10^{-4}\ \text{m}^2} = 3.8\times10^4\ \text{Pa}.$$

5. Let the volume of the expanded air sacs be V_a and that of the fish with its air sacs collapsed be V. Then

$$\rho_{\text{fish}} = \frac{m_{\text{fish}}}{V} = 1.08 \text{ g/cm}^3 \quad \text{and} \quad \rho_w = \frac{m_{\text{fish}}}{V + V_a} = 1.00 \text{ g/cm}^3$$

where ρ_w is the density of the water. This implies $\rho_{\text{fish}} V = \rho_w (V + V_a)$ or $(V + V_a)/V = 1.08/1.00$, which gives $V_a/(V + V_a) = 7.4\%$.

6. Knowing the standard air pressure value in several units allows us to set up a variety of conversion factors:

(a) $P = \left(28\ \text{lb/in.}^2\right)\left(\dfrac{1.01\times10^5\ \text{Pa}}{14.7\ \text{lb/in}^2}\right) = 190\ \text{kPa}$

(b) $(120\ \text{mmHg})\left(\dfrac{1.01\times10^5\,\text{Pa}}{760\ \text{mmHg}}\right) = 15.9\ \text{kPa},\quad (80\ \text{mmHg})\left(\dfrac{1.01\times10^5\ \text{Pa}}{760\ \text{mmHg}}\right) = 10.6\ \text{kPa}.$

7. (a) The pressure difference results in forces applied as shown in the figure. We consider a team of horses pulling to the right. To pull the sphere apart, the team must exert a force at least as great as the horizontal component of the total force determined by "summing" (actually, integrating) these force vectors.

We consider a force vector at angle θ. Its leftward component is $\Delta p \cos \theta dA$, where dA is the area element for where the force is applied. We make use of the symmetry of the problem and let dA be that of a ring of constant θ on the surface. The radius of the ring is $r = R \sin \theta$, where R is the radius of the sphere. If the angular width of the ring is $d\theta$, in radians, then its width is $R\, d\theta$ and its area is $dA = 2\pi R^2 \sin \theta\, d\theta$. Thus the net horizontal component of the force of the air is given by

$$F_h = 2\pi R^2 \Delta p \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi R^2 \Delta p \left. \sin^2\theta \right|_0^{\pi/2} = \pi R^2 \Delta p.$$

(b) We use 1 atm = 1.01×10^5 Pa to show that Δp = 0.90 atm = 9.09×10^4 Pa. The sphere radius is R = 0.30 m, so

$$F_h = \pi(0.30 \text{ m})^2(9.09 \times 10^4 \text{ Pa}) = 2.6 \times 10^4 \text{ N}.$$

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses.

8. Note that 0.05 atm equals 5065 N/m^2. Application of Eq. 14-7 with the notation in this problem leads to

$$d_{\max} = \frac{5065}{\rho_{\text{liquid}}\, g}$$

with SI units understood. Thus the difference of this quantity between fresh water (998 kg/m^3) and Dead Sea water (1500 kg/m^3) is

$$\Delta d_{\max} = \frac{5065}{9.8}\left(\frac{1}{998} - \frac{1}{1500}\right) = 0.17 \text{ m} .$$

9. We estimate the pressure difference (specifically due to hydrostatic effects) as follows:

$$\Delta p = \rho g h = (1.06 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.83 \text{ m}) = 1.90 \times 10^4 \text{ Pa}.$$

10. Recalling that 1 atm = 1.01×10^5 Pa, Eq. 14-8 leads to

$$\rho g h = (1024 \text{ kg/m}^3)\,(9.80 \text{ m/s}^2)\,(10.9 \times 10^3 \text{ m})\left(\frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}}\right) \approx 1.08{\times}10^3 \text{ atm}.$$

11. The pressure p at the depth d of the hatch cover is $p_0 + \rho g d$, where ρ is the density of ocean water and p_0 is atmospheric pressure. The downward force of the water on the hatch cover is $(p_0 + \rho g d)A$, where A is the area of the cover. If the air in the submarine is at atmospheric pressure then it exerts an upward force of $p_0 A$. The minimum force that must be applied by the crew to open the cover has magnitude

$$F = (p_0 + \rho g d)A - p_0 A = \rho g d A = (1024 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(100 \text{ m})(1.2 \text{ m})(0.60 \text{ m})$$
$$= 7.2 \times 10^5 \text{ N}.$$

12. In this case, Bernoulli's equation reduces to Eq. 14-10. Thus,

$$p_g = \rho g(-h) = -(1800\,\text{kg/m}^3)(9.8\ \text{m/s}^2)(1.5\ \text{m}) = -2.6\times10^4\ \text{Pa}\ .$$

13. With $A = 0.000500\ \text{m}^2$ and $F = pA$ (with p given by Eq. 14-9), then we have ρghA = 9.80 N. This gives $h \approx 2.0$ m, which means $d + h = 2.80$ m.

14. Since the pressure (caused by liquid) at the bottom of the barrel is doubled due to the presence of the narrow tube, so is the hydrostatic force. The ratio is therefore equal to 2.0. The difference between the hydrostatic force and the weight is accounted for by the additional upward force exerted by water on the top of the barrel due to the increased pressure introduced by the water in the tube.

15. When the levels are the same the height of the liquid is $h = (h_1 + h_2)/2$, where h_1 and h_2 are the original heights. Suppose h_1 is greater than h_2. The final situation can then be achieved by taking liquid with volume $A(h_1 - h)$ and mass $\rho A(h_1 - h)$, in the first vessel, and lowering it a distance $h - h_2$. The work done by the force of gravity is

$$W = \rho A(h_1 - h)g(h - h_2).$$

We substitute $h = (h_1 + h_2)/2$ to obtain

$$\begin{aligned} W &= \frac{1}{4}\rho g A\left(h_1 - h_2\right)^2 = \frac{1}{4}(1.30\times10^3\,\text{kg/m}^3)(9.80\text{ m/s}^2)(4.00\times10^{-4}\,\text{m}^2)(1.56\text{ m} - 0.854\text{ m})^2 \\ &= 0.635\text{ J} \end{aligned}.$$

16. Letting $p_a = p_b$, we find

$$\rho_c g(6.0 \text{ km} + 32 \text{ km} + D) + \rho_m(y - D) = \rho_c g(32 \text{ km}) + \rho_m y$$

and obtain

$$D = \frac{(6.0\,\text{km})\,\rho_c}{\rho_m - \rho_c} = \frac{(6.0\,\text{km})\left(2.9\,\text{g/cm}^3\right)}{3.3\,\text{g/cm}^3 - 2.9\,\text{g/cm}^3} = 44\,\text{km}.$$

17. We can integrate the pressure (which varies linearly with depth according to Eq. 14-7) over the area of the wall to find out the net force on it, and the result turns out fairly intuitive (because of that linear dependence): the force is the "average" water pressure multiplied by the area of the wall (or at least the part of the wall that is exposed to the water), where "average" pressure is taken to mean $\frac{1}{2}$(pressure at surface + pressure at bottom). Assuming the pressure at the surface can be taken to be zero (in the gauge pressure sense explained in section 14-4), then this means the force on the wall is $\frac{1}{2}\rho gh$ multiplied by the appropriate area. In this problem the area is hw (where w is the 8.00 m width), so the force is $\frac{1}{2}\rho gh^2 w$, and the change in force (as h is changed) is

$$\tfrac{1}{2}\rho gw\,(h_f^{\,2} - h_i^{\,2}) = \tfrac{1}{2}(998\text{ kg/m}^3)(9.80\text{ m/s}^2)(8.00\text{ m})(4.00^2 - 2.00^2)\text{m}^2 = 4.69\times 10^5\text{ N}.$$

18. (a) The force on face A of area A_A due to the water pressure alone is

$$F_A = p_A A_A = \rho_w g h_A A_A = \rho_w g(2d)d^2 = 2\left(1.0\times10^3\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)\left(5.0\ \text{m}\right)^3$$
$$= 2.5\times10^6\ \text{N}.$$

Adding the contribution from the atmospheric pressure, F_0= $(1.0 \times 10^5$ Pa)(5.0 m)^2 = 2.5×10^6 N, we have

$$F_A' = F_0 + F_A = 2.5\times10^6\ \text{N} + 2.5\times10^6\ \text{N} = 5.0\times10^6\ \text{N}.$$

(b) The force on face B due to water pressure alone is

$$F_B = p_{\text{avg}B} A_B = \rho_\omega g\left(\frac{5d}{2}\right)d^2 = \frac{5}{2}\rho_w g d^3 = \frac{5}{2}\left(1.0\times10^3\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)\left(5.0\ \text{m}\right)^3$$
$$= 3.1\times10^6\ \text{N}.$$

Adding the contribution from the atmospheric pressure, F_0= $(1.0 \times 10^5$ Pa)(5.0 m)^2 = 2.5×10^6 N, we have

$$F_B' = F_0 + F_B = 2.5\times10^6\ \text{N} + 3.1\times10^6\ \text{N} = 5.6\times10^6\ \text{N}.$$

19. (a) At depth y the gauge pressure of the water is $p = \rho g y$, where ρ is the density of the water. We consider a horizontal strip of width W at depth y, with (vertical) thickness dy, across the dam. Its area is $dA = W\,dy$ and the force it exerts on the dam is $dF = p\,dA = \rho g y W\,dy$. The total force of the water on the dam is

$$F = \int_0^D \rho g y W\, dy = \frac{1}{2}\rho g W D^2$$

$$= \frac{1}{2}\left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(9.80\ \mathrm{m/s^2}\right)\left(314\ \mathrm{m}\right)\left(35.0\ \mathrm{m}\right)^2 = 1.88\times10^9\ \mathrm{N}.$$

(b) Again we consider the strip of water at depth y. Its moment arm for the torque it exerts about O is $D - y$ so the torque it exerts is $d\tau = dF(D - y) = \rho g y W\,(D - y)dy$ and the total torque of the water is

$$\tau = \int_0^D \rho g y W\left(D - y\right)dy = \rho g W\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \frac{1}{6}\rho g W D^3$$

$$= \frac{1}{6}\left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(9.80\ \mathrm{m/s^2}\right)\left(314\ \mathrm{m}\right)\left(35.0\ \mathrm{m}\right)^3 = 2.20\times10^{10}\ \mathrm{N\cdot m}.$$

(c) We write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho g W D^3}{\frac{1}{2}\rho g W D^2} = \frac{D}{3} = \frac{35.0\ \mathrm{m}}{3} = 11.7\ \mathrm{m}.$$

20. The gauge pressure you can produce is

$$p = -\rho g h = -\frac{\left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(4.0\times10^{-2}\,\text{m}\right)}{1.01\times10^5\ \text{Pa/atm}} = -3.9\times10^{-3}\,\text{atm}$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure.

21. (a) We use the expression for the variation of pressure with height in an incompressible fluid: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. For this calculation, we take the density to be uniformly 1.3 kg/m^3. Then,

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01 \times 10^5 \text{ Pa}}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 7.9 \times 10^3 \text{ m} = 7.9 \text{ km}.$$

(b) Let h be the height of the atmosphere. Now, since the density varies with altitude, we integrate

$$p_2 = p_1 - \int_0^h \rho g \, dy .$$

Assuming $\rho = \rho_0 (1 - y/h)$, where ρ_0 is the density at Earth's surface and $g = 9.8$ m/s^2 for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g \left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2}\rho_0 g h.$$

Since $p_2 = 0$, this implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01 \times 10^5 \text{ Pa})}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 16 \times 10^3 \text{ m} = 16 \text{ km}.$$

22. (a) According to Pascal's principle $F/A = f/a \rightarrow F = (A/a)f$.

(b) We obtain

$$f = \frac{a}{A} F = \frac{(3.80\text{ cm})^2}{(53.0\text{ cm})^2} (20.0 \times 10^3 \text{ N}) = 103 \text{ N}.$$

The ratio of the squares of diameters is equivalent to the ratio of the areas. We also note that the area units cancel.

23. Eq. 14-13 combined with Eq. 5-8 and Eq. 7-21 (in absolute value) gives

$$mg = kx\frac{A_1}{A_2} \ .$$

With $A_2 = 18A_1$ (and the other values given in the problem) we find m = 8.50 kg.

24. (a) Archimedes' principle makes it clear that a body, in order to float, displaces an amount of the liquid which corresponds to the weight of the body. The problem (indirectly) tells us that the weight of the boat is $W = 35.6$ kN. In salt water of density $\rho' = 1100\ \text{kg/m}^3$, it must displace an amount of liquid having weight equal to 35.6 kN.

(b) The displaced volume of salt water is equal to

$$V' = \frac{W}{\rho' g} = \frac{3.56\times10^3\ \text{N}}{(1.10\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 3.30\ \text{m}^3\ .$$

In freshwater, it displaces a volume of $V = W/\rho g = 3.63\ \text{m}^3$, where $\rho = 1000\ \text{kg/m}^3$. The difference is $V - V' = 0.330\ \text{m}^3$.

25. (a) The anchor is completely submerged in water of density ρ_w. Its effective weight is $W_{\text{eff}} = W - \rho_w gV$, where W is its actual weight (mg). Thus,

$$V = \frac{W - W_{\text{eff}}}{\rho_w g} = \frac{200 \text{ N}}{\left(1000 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)} = 2.04 \times 10^{-2} \text{ m}^3 .$$

(b) The mass of the anchor is $m = \rho V$, where ρ is the density of iron (found in Table 14-1). Its weight in air is

$$W = mg = \rho V g = \left(7870 \text{ kg/m}^3\right)(2.04 \times 10^{-2} \text{ m}^3)\left(9.80 \text{ m/s}^2\right) = 1.57 \times 10^3 \text{ N} .$$

26. (a) The pressure (including the contribution from the atmosphere) at a depth of $h_{\text{top}} = L/2$ (corresponding to the top of the block) is

$$p_{\text{top}} = p_{\text{atm}} + \rho g h_{\text{top}} = \left[1.01\times10^5 + (1030)(9.8)(0.300)\right]\text{Pa} = 1.04\times10^5\ \text{Pa}$$

where the unit Pa (Pascal) is equivalent to N/m^2. The force on the top surface (of area $A = L^2 = 0.36\ \text{m}^2$) is $F_{\text{top}} = p_{\text{top}} A = 3.75 \times 10^4$ N.

(b) The pressure at a depth of $h_{\text{bot}} = 3L/2$ (that of the bottom of the block) is

$$p_{\text{bot}} = p_{\text{atm}} + \rho g h_{\text{bot}} = \left[1.01\times10^5 + (1030)(9.8)(0.900)\right]\text{Pa} = 1.10\times10^5\ \text{Pa}$$

where we recall that the unit Pa (Pascal) is equivalent to N/m^2. The force on the bottom surface is $F_{\text{bot}} = p_{\text{bot}} A = 3.96 \times 10^4$ N.

(c) Taking the difference $F_{\text{bot}} - F_{\text{top}}$ cancels the contribution from the atmosphere (including any numerical uncertainties associated with that value) and leads to

$$F_{\text{bot}} - F_{\text{top}} = \rho g\,(h_{\text{bot}} - h_{\text{top}})\,A = \rho g L^3 = 2.18\times10^3\ \text{N}$$

which is to be expected on the basis of Archimedes' principle. Two other forces act on the block: an upward tension T and a downward pull of gravity mg. To remain stationary, the tension must be

$$T = mg - (F_{\text{bot}} - F_{\text{top}}) = (450\ \text{kg})(9.80\ \text{m/s}^2) - 2.18\times10^3\ \text{N} = 2.23\times10^3\ \text{N}.$$

(d) This has already been noted in the previous part: $F_b = 2.18\times10^3\ \text{N}$, and $T + F_b = mg$.

27. The problem intends for the children to be completely above water. The total downward pull of gravity on the system is

$$3(356\,\mathrm{N}) + N\rho_{\text{wood}}gV$$

where N is the (minimum) number of logs needed to keep them afloat and V is the volume of each log: $V = \pi(0.15\text{ m})^2\ (1.80\text{ m}) = 0.13\text{ m}^3$. The buoyant force is $F_b = \rho_{\text{water}}gV_{\text{submerged}}$ where we require $V_{\text{submerged}} \le NV$. The density of water is 1000 kg/m^3. To obtain the minimum value of N we set $V_{\text{submerged}} = NV$ and then round our "answer" for N up to the nearest integer:

$$3(356\,\mathrm{N}) + N\rho_{\text{wood}}gV = \rho_{\text{water}}gNV \ \Rightarrow\ N = \frac{3(356\,\mathrm{N})}{gV\left(\rho_{\text{water}} - \rho_{\text{wood}}\right)}$$

which yields $N = 4.28 \rightarrow 5$ logs.

28. Work is the integral of the force (over distance – see Eq. 7-32), and referring to the equation immediately preceding Eq. 14-7, we see the work can be written as

$$W = \int \rho_{\text{water}}\, gA(-y)\, dy$$

where we are using $y = 0$ to refer to the water surface (and the $+y$ direction is upward). Let $h = 0.500$ m. Then, the integral has a lower limit of $-h$ and an upper limit of y_f, which can be determined by the condition described in Sample Problem 14-4 (which implies that $y_f / h = -\rho_{\text{cylinder}} / \rho_{\text{water}} = -0.400$). The integral leads to

$$W = \tfrac{1}{2}\, \rho_{\text{water}} gAh^2(1 - 0.4^2) \;=\; 4.11 \text{ kJ} .$$

29. (a) Let V be the volume of the block. Then, the submerged volume is $V_s = 2V/3$. Since the block is floating, the weight of the displaced water is equal to the weight of the block, so $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block. We substitute $V_s = 2V/3$ to obtain

$$\rho_b = 2\rho_w/3 = 2(1000 \text{ kg/m}^3)/3 \approx 6.7 \times 10^2 \text{ kg/m}^3.$$

(b) If ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s = \rho_b V$. We substitute $V_s = 0.90V$ to obtain $\rho_o = \rho_b/0.90 = 7.4 \times 10^2 \text{ kg/m}^3$.

30. Taking "down" as the positive direction, then using Eq. 14-16 in Newton's second law, we have $5g - 3g = 5a$ (where "5" = 5.00 kg, and "3" = 3.00 kg and g = 9.8 m/s^2). This gives $a = \frac{2}{5}g$. Then (see Eq. 2-15) $\frac{1}{2}at^2$ = 0.0784 m (in the downward direction).

31. (a) The downward force of gravity mg is balanced by the upward buoyant force of the liquid: $mg = \rho g\, V_s$. Here m is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3)\, r_o^3$, where r_o is the outer radius. Therefore,

$$m = \frac{2\pi}{3}\rho r_o^3 = \left(\frac{2\pi}{3}\right)\left(800 \text{ kg/m}^3\right)(0.090 \text{ m})^3 = 1.22 \text{ kg}.$$

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where m is the mass of the sphere and V is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3}(r_o^3 - r_i^3) = \frac{4\pi}{3}\left((0.090 \text{ m})^3 - (0.080 \text{ m})^3\right) = 9.09\times 10^{-4} \text{ m}^3 .$$

The density is

$$\rho_m = \frac{1.22 \text{ kg}}{9.09\times 10^{-4} \text{ m}^3} = 1.3\times 10^3 \text{ kg/m}^3 .$$

32. (a) An object of the same density as the surrounding liquid (in which case the "object" could just be a packet of the liquid itself) is not going to accelerate up or down (and thus won't gain any kinetic energy). Thus, the point corresponding to zero K in the graph must correspond to the case where the density of the object equals ρ_{liquid}. Therefore, ρ_{ball} = 1.5 g/cm^3 (or 1500 kg/m^3).

(b) Consider the $\rho_{\text{liquid}} = 0$ point (where K_{gained} = 1.6 J). In this case, the ball is falling through perfect vacuum, so that $v^2 = 2gh$ (see Eq. 2-16) which means that $K = \frac{1}{2}mv^2$ = 1.6 J can be used to solve for the mass. We obtain m_{ball} = 4.082 kg. The volume of the ball is then given by $m_{\text{ball}}/\rho_{\text{ball}} = 2.72 \times 10^{-3}$ m^3.

33. For our estimate of $V_{\text{submerged}}$ we interpret "almost completely submerged" to mean

$$V_{\text{submerged}} \approx \frac{4}{3}\pi r_o^3 \quad \text{where } r_o = 60 \text{ cm} .$$

Thus, equilibrium of forces (on the iron sphere) leads to

$$F_b = m_{\text{iron}}\, g \quad \Rightarrow \quad \rho_{\text{water}}\, g V_{\text{submerged}} = \rho_{\text{iron}}\, g \left(\frac{4}{3}\pi r_o^3 - \frac{4}{3}\pi r_i^3 \right)$$

where r_i is the inner radius (half the inner diameter). Plugging in our estimate for $V_{\text{submerged}}$ as well as the densities of water (1.0 g/cm^3) and iron (7.87 g/cm^3), we obtain the inner diameter:

$$2r_i = 2r_o \left(1 - \frac{1}{7.87}\right)^{1/3} = 57.3 \text{ cm}.$$

34. From the "kink" in the graph it is clear that d = 1.5 cm. Also, the h = 0 point makes it clear that the (true) weight is 0.25 N. We now use Eq. 14-19 at $h = d$ = 1.5 cm to obtain F_b = (0.25 N – 0.10 N) = 0.15 N. Thus, $\rho_{\text{liquid}}\, g\, V = 0.15$, where V = (1.5 cm)(5.67 cm^2) = 8.5×10^{-6} m^3. Thus, ρ_{liquid} = 1800 kg/m^3 = 1.8 g/cm^3.

35. The volume V_{cav} of the cavities is the difference between the volume V_{cast} of the casting as a whole and the volume V_{iron} contained: $V_{cav} = V_{cast} - V_{iron}$. The volume of the iron is given by $V_{iron} = W/g\rho_{iron}$, where W is the weight of the casting and ρ_{iron} is the density of iron. The effective weight in water (of density ρ_w) is $W_{eff} = W - g\rho_w V_{cast}$. Thus, $V_{cast} = (W - W_{eff})/g\rho_w$ and

$$V_{cav} = \frac{W - W_{eff}}{g\rho_w} - \frac{W}{g\rho_{iron}} = \frac{6000\text{ N} - 4000\text{ N}}{(9.8\text{ m/s}^2)(1000\,\text{kg/m}^3)} - \frac{6000\text{ N}}{(9.8\text{ m/s}^2)(7.87\times10^3\text{ kg/m}^3)}$$
$$= 0.126\text{ m}^3 .$$

36. Due to the buoyant force, the ball accelerates upward (while in the water) at rate a given by Newton's second law:

$$\rho_{\text{water}}Vg - \rho_{\text{ball}}Vg = \rho_{\text{ball}}Va \quad \Rightarrow \quad \rho_{\text{ball}} = \rho_{\text{water}}\,(1 + \text{"}a\text{"})$$

where – for simplicity – we are using in that last expression an acceleration "a" measured in "gees" (so that "a" = 2, for example, means that a = 2(9.80) = 19.6 m/s^2). In this problem, with $\rho_{\text{ball}} = 0.300\ \rho_{\text{water}}$, we find therefore that "a" = 7/3. Using Eq. 2-16, then the speed of the ball as it emerges from the water is

$$v = \sqrt{2a\Delta y}\ ,$$

were $a = (7/3)g$ and Δy = 0.600 m. This causes the ball to reach a maximum height $h_{\max}$ (measured above the water surface) given by $h_{\max} = v^2/2g$ (see Eq. 2-16 again). Thus, $h_{\max} = (7/3)\Delta y$ = 1.40 m.

37. (a) If the volume of the car below water is V_1 then $F_b = \rho_w V_1 g = W_{\text{car}}$, which leads to

$$V_1 = \frac{W_{\text{car}}}{\rho_w g} = \frac{(1800\,\text{kg})(9.8\,\text{m/s}^2)}{(1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.80\ \text{m}^3.$$

(b) We denote the total volume of the car as V and that of the water in it as V_2. Then

$$F_b = \rho_w V g = W_{\text{car}} + \rho_w V_2 g$$

which gives

$$V_2 = V - \frac{W_{\text{car}}}{\rho_w g} = \left(0.750\,\text{m}^3 + 5.00\,\text{m}^3 + 0.800\,\text{m}^3\right) - \frac{1800\,\text{kg}}{1000\,\text{kg/m}^3} = 4.75\ \text{m}^3.$$

38. (a) Since the lead is not displacing any water (of density ρ_w), the lead's volume is not contributing to the buoyant force F_b. If the immersed volume of wood is V_i, then

$$F_b = \rho_w V_i g = 0.900\,\rho_w V_{\text{wood}}\, g = 0.900\,\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right),$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.900\,\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) = (m_{\text{wood}} + m_{\text{lead}})g\,.$$

Thus,

$$m_{\text{lead}} = 0.900\rho_w\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) - m_{\text{wood}} = \frac{(0.900)(1000\,\text{kg/m}^3)(3.67\,\text{kg})}{600\ \text{kg/m}^3} - 3.67\ \text{kg} = 1.84\ \text{kg}\ .$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to F_b. Consequently,

$$F_b = 0.900\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) + \left(\frac{\rho_w}{\rho_{\text{lead}}}\right) m_{\text{lead}}\, g = (m_{\text{wood}} + m_{\text{lead}})g,$$

which leads to

$$\begin{aligned} m_{\text{lead}} &= \frac{0.900(\rho_w/\rho_{\text{wood}})m_{\text{wood}} - m_{\text{wood}}}{1-\rho_w/\rho_{\text{lead}}} = \frac{1.84\ \text{kg}}{1-(1.00\times10^3\ \text{kg/m}^3/1.13\times10^4\ \text{kg/m}^3)} \\ &= 2.01\ \text{kg}. \end{aligned}$$

39. (a) When the model is suspended (in air) the reading is F_g (its true weight, neglecting any buoyant effects caused by the air). When the model is submerged in water, the reading is lessened because of the buoyant force: $F_g - F_b$. We denote the difference in readings as Δm. Thus,

$$F_g - (F_g - F_b) = \Delta m g$$

which leads to $F_b = \Delta m g$. Since $F_b = \rho_w g V_m$ (the weight of water displaced by the model) we obtain

$$V_m = \frac{\Delta m}{\rho_w} = \frac{0.63776\,\text{kg}}{1000\ \text{kg/m}} \approx 6.378 \times 10^{-4}\ \text{m}^3 .$$

(b) The $\frac{1}{20}$ scaling factor is discussed in the problem (and for purposes of significant figures is treated as exact). The actual volume of the dinosaur is

$$V_{\text{dino}} = 20^3\ V_m = 5.102\ \text{m}^3 .$$

(c) Using $\rho \approx \rho_w = 1000$ kg/m^3, we find

$$\rho = \frac{m_{\text{dino}}}{V_{\text{dino}}} \Rightarrow m_{\text{dino}} = (1000\,\text{kg/m}^3)\ (5.102\ \text{m}^3)$$

which yields 5.102×10^3 kg for the *T. rex* mass.

40. Let ρ be the density of the cylinder (0.30 g/cm^3 or 300 kg/m^3) and ρ_{Fe} be the density of the iron (7.9 g/cm^3 or 7900 kg/m^3). The volume of the cylinder is $V_c = (6\times12)$ cm^3 = 72 cm^3 (or 0.000072 m^3), and that of the ball is denoted V_b. The part of the cylinder that is submerged has volume $V_s = (4 \times 12)$ cm^3 = 48 cm^3 (or 0.000048 m^3). Using the ideas of section 14-7, we write the equilibrium of forces as

$$\rho g V_c + \rho_{Fe}\, g V_b = \rho_w\, g V_s + \rho_w\, g V_b \quad \Rightarrow \quad V_b = 3.8 \text{ cm}^3$$

where we have used ρ_w = 998 kg/m^3 (for water, see Table 14-1). Using $V_b = \frac{4}{3}\pi r^3$ we find $r = 9.7$ mm.

41. We use the equation of continuity. Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. $A_1 = \pi R^2$ is the cross-sectional area of the hose. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2 \left(N A_2 \right) \quad \Rightarrow \quad v_2 = \frac{A_1}{N A_2} v_1 = \frac{R^2}{N r^2} v_1$$

where R is the radius of the hose and r is the radius of a hole. Noting that $R/r = D/d$ (the ratio of diameters) we find

$$v_2 = \frac{D^2}{N d^2} v_1 = \frac{(1.9\,\text{cm})^2}{24(0.13\,\text{cm})^2}(0.91\,\text{m/s}) = 8.1\,\text{m/s}.$$

42. We use the equation of continuity and denote the depth of the river as h. Then,

$$(8.2\,\text{m})(3.4\,\text{m})(2.3\,\text{m/s})+(6.8\,\text{m})(3.2\,\text{m})(2.6\,\text{m/s})=h(10.5\,\text{m})(2.9\,\text{m/s})$$

which leads to $h = 4.0$ m.

43. Suppose that a mass Δm of water is pumped in time Δt. The pump increases the potential energy of the water by Δmgh, where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\frac{1}{2}\Delta mv^2$, where v is its final speed. The work it does is $\Delta W = \Delta mgh + \frac{1}{2}\Delta mv^2$ and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right).$$

Now the rate of mass flow is $\Delta m/\Delta t = \rho_w Av$, where ρ_w is the density of water and A is the area of the hose. The area of the hose is $A = \pi r^2 = \pi(0.010\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$ and

$$\rho_w Av = (1000\text{ kg/m}^3)\,(3.14 \times 10^{-4}\text{ m}^2)\,(5.00\text{ m/s}) = 1.57\text{ kg/s}.$$

Thus,

$$P = \rho Av\left(gh + \frac{1}{2}v^2\right) = (1.57\text{ kg/s})\left((9.8\text{ m/s}^2)(3.0\text{ m}) + \frac{(5.0\text{ m/s})^2}{2}\right) = 66\text{ W}.$$

44. (a) The equation of continuity provides (26 + 19 + 11) L/min = 56 L/min for the flow rate in the main (1.9 cm diameter) pipe.

(b) Using $v = R/A$ and $A = \pi d^2/4$, we set up ratios:

$$\frac{v_{56}}{v_{26}} = \frac{56/\pi(1.9)^2/4}{26/\pi(1.3)^2/4} \approx 1.0.$$

45. (a) We use the equation of continuity: $A_1 v_1 = A_2 v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. Thus $v_2 = (A_1/A_2)v_1 = [(4.0\ \text{cm}^2)/(8.0\ \text{cm}^2)]\ (5.0\ \text{m/s}) =$ 2.5m/s.

(b) We use the Bernoulli equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$, where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude. Thus

$$
\begin{aligned}
p_2 &= p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right) \\
&= 1.5\times10^5\ \text{Pa} + \frac{1}{2}(1000\ \text{kg/m}^3)\left[(5.0\ \text{m/s})^2 - (2.5\ \text{m/s})^2\right] + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(10\ \text{m}) \\
&= 2.6\times10^5\ \text{Pa}.
\end{aligned}
$$

46. We use Bernoulli's equation:

$$p_2 - p_i = \rho g D + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

where ρ = 1000 kg/m^3, D = 180 m, v_1 = 0.40 m/s and v_2 = 9.5 m/s. Therefore, we find Δp = 1.7 × 10^6 Pa, or 1.7 MPa. The SI unit for pressure is the Pascal (Pa) and is equivalent to N/m^2.

47. (a) The equation of continuity leads to

$$v_2 A_2 = v_1 A_1 \quad \Rightarrow \quad v_2 = v_1 \left(\frac{r_1^2}{r_2^2} \right)$$

which gives $v_2 = 3.9$ m/s.

(b) With $h = 7.6$ m and $p_1 = 1.7 \times 10^5$ Pa, Bernoulli's equation reduces to

$$p_2 = p_1 - \rho g h + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 8.8 \times 10^4 \,\text{Pa}.$$

48. (a) We use Av = const. The speed of water is

$$v = \frac{(25.0\,\text{cm})^2 - (5.00\,\text{cm})^2}{(25.0\,\text{cm})^2}(2.50\,\text{m/s}) = 2.40\,\text{m/s}.$$

(b) Since $p + \frac{1}{2}\rho v^2 = \text{const.},$ the pressure difference is

$$\Delta p = \frac{1}{2}\rho\Delta v^2 = \frac{1}{2}(1000\,\text{kg/m}^3)\left[(2.50\,\text{m/s})^2 - (2.40\,\text{m/s})^2\right] = 245\,\text{Pa}.$$

49. (a) We use the Bernoulli equation: $p_1+\frac{1}{2}\rho v_1^2+\rho g h_1 = p_2+\frac{1}{2}\rho v_2^2+\rho g h_2$, where h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. ρ is the density of water. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then becomes $\rho g h_1 = \frac{1}{2}\rho v_2^2+\rho g h_2$ and

$$v_2=\sqrt{2g\left(h_1-h_2\right)}=\sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.30\,\text{m}\right)}=2.42\,\text{m/s}.$$

The flow rate is $A_2v_2 = (6.5\times10^{-4}\text{ m}^2)(2.42\text{ m/s}) = 1.6\times10^{-3}\text{ m}^3\text{/s}$.

(b) We use the equation of continuity: $A_2v_2 = A_3v_3$, where $A_3=\frac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole. Thus $v_3 = (A_2/A_3)v_2 = 2v_2 =$ 4.84 m/s. The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\frac{1}{2}\rho v_2^2+\rho g h_2=\frac{1}{2}\rho v_3^2+\rho g h_3$. Thus

$$h_2-h_3=\frac{v_3^2-v_2^2}{2g}=\frac{\left(4.84\,\text{m/s}\right)^2-\left(2.42\,\text{m/s}\right)^2}{2\left(9.8\,\text{m/s}^2\right)}=0.90\text{ m}.$$

50. The left and right sections have a total length of 60.0 m, so (with a speed of 2.50 m/s) it takes 60.0/2.50 = 24.0 seconds to travel through those sections. Thus it takes (88.8 – 24.0) s = 64.8 s to travel through the middle section. This implies that the speed in the middle section is v_{mid} = (110 m)/(64.8 s) = 0.772 m/s. Now Eq. 14-23 (plus that fact that $A = \pi r^2$) implies $r_{mid} = r_A\sqrt{(2.5 \text{ m/s})/(0.772 \text{ m/s})}$ where r_A = 2.00 cm. Therefore, r_{mid} = 3.60 cm.

51. We rewrite the formula for work W (when the force is constant in a direction parallel to the displacement d) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where V is the volume of the water being forced through, and p is to be interpreted as the pressure difference between the two ends of the pipe. Thus,

$$W = (1.0 \times 10^5 \text{ Pa})\,(1.4 \text{ m}^3) = 1.4 \times 10^5 \text{ J}.$$

52. (a) The speed v of the fluid flowing out of the hole satisfies $\frac{1}{2}\rho v^2 = \rho g h$ or $v = \sqrt{2gh}$. Thus, $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$, which leads to

$$\rho_1 \sqrt{2gh} A_1 = \rho_2 \sqrt{2gh} A_2 \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{A_2}{A_1} = 2.$$

(b) The ratio of volume flow is

$$\frac{R_1}{R_2} = \frac{v_1 A_1}{v_2 A_2} = \frac{A_1}{A_2} = \frac{1}{2}$$

(c) Letting $R_1/R_2 = 1$, we obtain $v_1/v_2 = A_2/A_1 = 2 = \sqrt{h_1/h_2}$ Thus

$$h_2 = h_1/4 = (12.0 \text{ cm})/4 = 3.00 \text{ cm}.$$

53. (a) The friction force is

$$f = A\Delta p = \rho_\omega g d A = (1.0\times10^3\ \text{kg/m}^3)\ (9.8\ \text{m/s}^2)\ (6.0\text{m})\left(\frac{\pi}{4}\right)(0.040\ \text{m})^2 = 74\ \text{N}.$$

(b) The speed of water flowing out of the hole is $v = \sqrt{2gd}.$ Thus, the volume of water flowing out of the pipe in t = 3.0 h is

$$V = Avt = \frac{\pi^2}{4}(0.040\ \text{m})^2\sqrt{2(9.8\ \text{m/s}^2)\ (6.0\ \text{m})}\ (3.0\ \text{h})\ (3600\ \text{s/h}) = 1.5\times10^2\,\text{m}^3.$$

54. (a) The volume of water (during 10 minutes) is

$$V = (v_1 t) A_1 = (15\,\text{m/s})(10\,\text{min})(60\,\text{s/min})\left(\frac{\pi}{4}\right)(0.03\,\text{m})^2 = 6.4\,\text{m}^3.$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (15\,\text{m/s})\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\,\text{m/s}.$$

(c) Since $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ and $h_1 = h_2$, $p_1 = p_0$, which is the atmospheric pressure,

$$p_2 = p_0 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 1.01\times10^5\,\text{Pa} + \frac{1}{2}\left(1.0\times10^3\,\text{kg/m}^3\right)\left[(15\,\text{m/s})^2 - (5.4\,\text{m/s})^2\right]$$
$$= 1.99\times10^5\,\text{Pa} = 1.97\,\text{atm}.$$

Thus the gauge pressure is (1.97 atm – 1.00 atm) = 0.97 atm = 9.8×10^4 Pa.

55. (a) Since Sample Problem 14-8 deals with a similar situation, we use the final equation (labeled "Answer") from it:

$$v = \sqrt{2gh} \;\Rightarrow\; v = v_0 \text{ for the projectile motion.}$$

The stream of water emerges horizontally (θ_0 = 0° in the notation of Chapter 4), and setting $y - y_0 = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t = \sqrt{\frac{-2(H-h)}{-g}} = \sqrt{\frac{2}{g}(H-h)}.$$

Using this in Eq. 4-21, where $x_0 = 0$ by choice of coordinate origin, we find

$$x = v_0 t = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)} = 2\sqrt{(10\text{ cm})(40\text{ cm} - 10\text{ cm})} = 35\text{ cm}.$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H - h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2 - Hh + \frac{x^2}{4} = 0 \Rightarrow h = \frac{H \pm \sqrt{H^2 - x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H.$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$. Its numerical value is $h' = 40\text{cm} - 10\text{ cm} = 30\text{ cm}$.

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh} = 4H - 8h = 0 \Rightarrow h = \frac{H}{2}$$

or h = (40 cm)/2 = 20 cm, as the depth from which an emerging stream of water will travel the maximum horizontal distance.

56. (a) We note (from the graph) that the pressures are equal when the value of inverse-area-squared is 16 (in SI units). This is the point at which the areas of the two pipe sections are equal. Thus, if $A_1 = 1/\sqrt{16}$ when the pressure difference is zero, then A_2 is 0.25 m^2.

(b) Using Bernoulli's equation (in the form Eq. 14-30) we find the pressure difference may be written in the form a straight line: $mx + b$ where x is inverse-area-squared (the horizontal axis in the graph), m is the slope, and b is the intercept (seen to be –300 kN/m^2). Specifically, Eq. 14-30 predicts that b should be $-\frac{1}{2}\rho\, v_2^2$. Thus, with ρ = 1000 kg/m^3 we obtain $v_2 = \sqrt{600}$ m/s. Then the volume flow rate (see Eq. 14-24) is $R = A_2\, v_2 = (0.25\ m^2)(\sqrt{600}\ m/s) = 6.12\ m^3/s$. If the more accurate value (see Table 14-1) ρ = 998 kg/m^3 is used, then the answer is 6.13 m^3/s.

57. (a) This is similar to the situation treated in Sample Problem 14-7, and we refer to some of its steps (and notation). Combining Eq. 14-35 and Eq. 14-36 in a manner very similar to that shown in the textbook, we find

$$R = A_1 A_2 \sqrt{\frac{2\Delta p}{\rho\left(A_1^2 - A_2^2\right)}}$$

for the flow rate expressed in terms of the pressure difference and the cross-sectional areas. Note that this reduces to Eq. 14-38 for the case $A_2 = A_1/2$ treated in the Sample Problem. Note that $\Delta p = p_1 - p_2 = -7.2 \times 10^3$ Pa and $A_1^2 - A_2^2 = -8.66\times10^{-3}\ \text{m}^4$, so that the square root is well defined. Therefore, we obtain $R = 0.0776\ \text{m}^3/\text{s}$.

(b) The mass rate of flow is $\rho R = 69.8$ kg/s.

58. By Eq. 14-23, we note that the speeds in the left and right sections are $\frac{1}{4}\mathrm{v}_{\mathrm{mid}}$ and $\frac{1}{9}\mathrm{v}_{\mathrm{mid}}$, respectively, where $\mathrm{v}_{\mathrm{mid}} = 0.500$ m/s. We also note that 0.400 m^3 of water has a mass of 399 kg (see Table 14-1). Then Eq. 14-31 (and the equation below it) gives

$$W = \frac{1}{2} m\, \mathrm{v}_{\mathrm{mid}}{}^{2}\left(\frac{1}{9^2} - \frac{1}{4^2}\right) = -2.50 \text{ J} .$$

59. (a) The continuity equation yields $Av = aV$, and Bernoulli's equation yields $\Delta p+\frac{1}{2}\rho v^2=\frac{1}{2}\rho V^2$, where $\Delta p = p_1 - p_2$. The first equation gives $V = (A/a)v$. We use this to substitute for V in the second equation, and obtain $\Delta p+\frac{1}{2}\rho v^2=\frac{1}{2}\rho\left(A/a\right)^2 v^2$. We solve for v. The result is

$$v=\sqrt{\frac{2\Delta p}{\rho\left(\frac{A^2}{a^2}-1\right)}}=\sqrt{\frac{2a^2\Delta p}{\rho\left(A^2-a^2\right)}}.$$

(b) We substitute values to obtain

$$v=\sqrt{\frac{2(32\times10^{-4}\,\text{m}^2)^2(55\times10^3\,\text{Pa}-41\times10^3\,\text{Pa})}{(1000\,\text{kg/m}^3)\left((64\times10^{-4}\,\text{m}^2)^2-(32\times10^{-4}\,\text{m}^2)^2\right)}}=3.06\,\text{m/s}.$$

Consequently, the flow rate is

$$Av=(64\times10^{-4}\,\text{m}^2)\,(3.06\text{ m/s})=2.0\times10^{-2}\,\text{m}^3/\text{s}.$$

60. We use the result of part (a) in the previous problem.

(a) In this case, we have $\Delta p = p_1 = 2.0$ atm. Consequently,

$$v = \sqrt{\frac{2\Delta p}{\rho((A/a)^2 - 1)}} = \sqrt{\frac{4(1.01\times 10^5 \text{ Pa})}{(1000 \text{ kg/m}^3)\,[(5a/a)^2 - 1]}} = 4.1\,\text{m/s}.$$

(b) And the equation of continuity yields $V = (A/a)v = (5a/a)v = 5v = 21$ m/s.

(c) The flow rate is given by

$$Av = \frac{\pi}{4}\,(5.0\times 10^{-4}\text{ m}^2)\,(4.1\text{ m/s}) = 8.0\times 10^{-3}\,\text{m}^3/\text{s}.$$

61. (a) Bernoulli's equation gives $p_A = p_B + \frac{1}{2}\rho_{\text{air}} v^2$. But $\Delta p = p_A - p_B = \rho g h$ in order to balance the pressure in the two arms of the U-tube. Thus $\rho g h = \frac{1}{2}\rho_{\text{air}} v^2$, or

$$v = \sqrt{\frac{2\rho g h}{\rho_{\text{air}}}}.$$

(b) The plane's speed relative to the air is

$$v = \sqrt{\frac{2\rho g h}{\rho_{\text{air}}}} = \sqrt{\frac{2\left(810\,\text{kg/m}^3\right)(9.8\,\text{m/s}^2)(0.260\,\text{m})}{1.03\,\text{kg/m}^3}} = 63.3\,\text{m/s}.$$

62. We use the formula for v obtained in the previous problem:

$$v = \sqrt{\frac{2\Delta p}{\rho_{\text{air}}}} = \sqrt{\frac{2(180\,\text{Pa})}{0.031\,\text{kg/m}^3}} = 1.1 \times 10^2\ \text{m/s}.$$

63. We use Bernoulli's equation $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$.

When the water level rises to height h_2, just on the verge of flooding, v_2, the speed of water in pipe M, is given by

$$\rho g(h_1 - h_2) = \frac{1}{2}\rho v_2^2 \quad \Rightarrow \quad v_2 = \sqrt{2g(h_1 - h_2)} = 13.86 \text{ m/s}.$$

By continuity equation, the corresponding rainfall rate is

$$v_1 = \left(\frac{A_2}{A_1}\right) v_2 = \frac{\pi(0.030 \text{ m})^2}{(30 \text{ m})(60 \text{ m})}(13.86 \text{ m/s}) = 2.177 \times 10^{-5} \text{ m/s} \approx 7.8 \text{ cm/h}.$$

64. The volume rate of flow is $R = vA$ where $A = \pi r^2$ and $r = d/2$. Solving for speed, we obtain

$$v = \frac{R}{A} = \frac{R}{\pi (d/2)^2} = \frac{4R}{\pi d^2}.$$

(a) With $R = 7.0 \times 10^{-3}$ m^3/s and $d = 14 \times 10^{-3}$ m, our formula yields $v = 45$ m/s, which is about 13% of the speed of sound (which we establish by setting up a ratio: v/v_s where v_s = 343 m/s).

(b) With the contracted trachea ($d = 5.2 \times 10^{-3}$ m) we obtain $v = 330$ m/s, or 96% of the speed of sound.

65. This is very similar to Sample Problem 14-4, where the ratio of densities is shown equal to a particular ratio of volumes. With volume equal to area multiplied by height, then that result becomes $h_{\text{submerged}}/h_{\text{total}} = \rho_{\text{block}}/\rho_{\text{liquid}}$. Applying this to the first liquid, then applying it again to the second liquid, and finally dividing the two applications we arrive at another ratio: $h_{\text{submerged in liquid 2}}$ divided by $h_{\text{submerged in liquid 1}}$ is equal to ρ_{liquid2} divided by ρ_{liquid1}. Since the height *submerged* in liquid 1 is (8.00 – 6.00) cm = 2 cm, then this last ratio tells us that the height submerged in liquid 2 is twice as much (because liquid 2 is half as dense), so $h_{\text{submerged in liquid 2}}$ = 4.00 cm. Since the total height is 8 cm, then the height above the surface is also 4.00 cm.

66. The normal force $\vec{F}_N$ exerted (upward) on the glass ball of mass m has magnitude 0.0948 N. The buoyant force exerted by the milk (upward) on the ball has magnitude

$$F_b = \rho_{\text{milk}}\, g\, V$$

where $V = \frac{4}{3}\pi r^3$ is the volume of the ball. Its radius is $r = 0.0200$ m. The milk density is $\rho_{\text{milk}} = 1030$ kg/m^3. The (actual) weight of the ball is, of course, downward, and has magnitude $F_g = m_{\text{glass}}\, g$. Application of Newton's second law (in the case of zero acceleration) yields

$$F_N + \rho_{\text{milk}}\, g\, V - m_{\text{glass}}\, g = 0$$

which leads to $m_{\text{glass}} = 0.0442$ kg. We note the above equation is equivalent to Eq.14-19 in the textbook.

67. If we examine both sides of the U-tube at the level where the low-density liquid (with ρ = 0.800 g/cm^3 = 800 kg/m^3) meets the water (with ρ_w = 0.998 g/cm^3 = 998 kg/m^3), then the pressures there on either side of the tube must agree:

$$\rho g h = \rho_w g h_w$$

where h = 8.00 cm = 0.0800 m, and Eq. 14-9 has been used. Thus, the height of the water column (as measured from that level) is h_w = (800/998)(8.00 cm) = 6.41 cm. The volume of water in that column is therefore $\pi r^2 h_w = \pi(1.50 \text{ cm})^2(6.41 \text{ cm}) = 45.3 \text{ cm}^3$.

68. Since (using Eq. 5-8) $F_g = mg = \rho_{\text{skier}}\, g\, V$ and (Eq. 14-16) the buoyant force is $F_b = \rho_{\text{snow}}\, g\, V$, then their ratio is

$$\frac{F_b}{F_g} = \frac{\rho_{\text{snow}}\, g\, V}{\rho_{\text{skier}}\, g\, V} = \frac{\rho_{\text{snow}}}{\rho_{\text{skier}}} = \frac{96}{1020} = 0.094 \ \text{(or 9.4\%)}.$$

69. (a) We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B and C. Applying Bernoulli's equation to points D and C, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

which leads to

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \approx \sqrt{2g(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{\text{air}}$ and $v_D/v_C \approx 0$. Plugging in the values, we obtain

$$v_c = \sqrt{2(9.8\text{ m/s}^2)(0.40\text{ m} + 0.12\text{ m})} = 3.2\text{ m/s}.$$

(b) We now consider points B and C:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C .$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{\text{air}}$, Bernoulli's equation becomes

$$\begin{aligned} p_B &= p_C + \rho g(h_C - h_B) = p_{\text{air}} - \rho g(h_1 + h_2 + d) \\ &= 1.0\times10^5\text{ Pa} - (1.0\times10^3\text{kg/m}^3)(9.8\text{ m/s}^2)(0.25\text{ m} + 0.40\text{ m} + 0.12\text{ m}) \\ &= 9.2\times10^4\text{ Pa}. \end{aligned}$$

(c) Since $p_B \ge 0$, we must let $p_{\text{air}} - \rho g(h_1 + d + h_2) \ge 0$, which yields

$$h_1 \le h_{1,\max} = \frac{p_{\text{air}}}{\rho} - d - h_2 \le \frac{p_{\text{air}}}{\rho} = 10.3\text{ m}.$$

70. To be as general as possible, we denote the ratio of body density to water density as f (so that $f = \rho/\rho_w = 0.95$ in this problem). Floating involves an equilibrium of vertical forces acting on the body (Earth's gravity pulls down and the buoyant force pushes up). Thus,

$$F_b = F_g \Rightarrow \rho_w g V_w = \rho g V$$

where V is the total volume of the body and V_w is the portion of it which is submerged.

(a) We rearrange the above equation to yield

$$\frac{V_w}{V} = \frac{\rho}{\rho_w} = f$$

which means that 95% of the body is submerged and therefore 5% is above the water surface.

(b) We replace ρ_w with $1.6\rho_w$ in the above equilibrium of forces relationship, and find

$$\frac{V_w}{V} = \frac{\rho}{1.6\rho_w} = \frac{f}{1.6}$$

which means that 59% of the body is submerged and thus 41% is above the quicksand surface.

(c) The answer to part (b) suggests that a person in that situation is able to breathe.

71. (a) To avoid confusing weight with work, we write out the word instead of using the symbol W. Thus,

$$\text{weight} = mg = (1.85\times 10^4\ \text{kg})(9.8\,\text{m/s}^2) \approx 1.8\times 10^2\,\text{kN}.$$

(b) The buoyant force is $F_b = \rho_w g V_w$ where ρ_w = 1000 kg/m^3 is the density of water and V_w is the volume of water displaced by the dinosaur. If we use f for the fraction of the dinosaur's total volume V which is submerged, then $V_w = fV$. We can further relate V to the dinosaur's mass using the assumption that the density of the dinosaur is 90% that of water: $V = m/(0.9\rho_w)$. Therefore, the apparent weight of the dinosaur is

$$\text{weight}_{\text{app}} = \text{weight} - \rho_w g\left(f\frac{m}{0.9\rho_w}\right) = \text{weight} - g\,f\frac{m}{0.9}.$$

If f = 0.50, this yields 81 kN for the apparent weight.

(c) If f = 0.80, our formula yields 20 kN for the apparent weight.

(d) If f = 0.90, we find the apparent weight is zero (it floats).

(e) Eq. 14-8 indicates that the water pressure at that depth is greater than standard air pressure (the assumed pressure at the surface) by $\rho_w gh$ = (1000)(9.8)(8) = 7.8×10^4 Pa. If we assume the pressure of air in the dinosaur's lungs is approximately standard air pressure, then this value represents the pressure difference which the lung muscles would have to work against.

(f) Assuming the maximum pressure difference the muscles can work with is 8 kPa, then our previous result (78 kPa) spells doom to the wading Diplodocus hypothesis.

72. We note that in “gees” (where acceleration is expressed as a multiple of g) the given acceleration is 0.225/9.8 = 0.02296. Using $m = \rho V$, Newton’s second law becomes

$$\rho_{\text{wat}} V g - \rho_{\text{bub}} V g = \rho_{\text{bub}} V a \quad \Rightarrow \quad \rho_{\text{bub}} = \rho_{\text{wat}}\,(1 + \text{“}a\text{”})$$

where in the final expression “a” is to be understood to be in “gees.” Using ρ_{wat} = 998 kg/m^3 (see Table 14-1) we find ρ_{bub} = 975.6 kg/m^3. Using volume $V = \frac{4}{3}\pi r^3$ for the bubble, we then find its mass: $m_{\text{bub}} = 5.11 \times 10^{-7}$ kg.

73. (a) We denote a point at the top surface of the liquid *A* and a point at the opening *B*. Point *A* is a vertical distance h = 0.50 m above *B*. Bernoulli's equation yields $p_A = p_B + \frac{1}{2}\rho v_B^2 - \rho g h$. Noting that $p_A = p_B$ we obtain

$$v_B = \sqrt{2gh + \frac{2}{\rho}(p_A - p_B)} = \sqrt{2(9.8\ \text{m/s}^2)(0.50\ \text{m})} = 3.1\ \text{m/s}.$$

(b)

$$v_B = \sqrt{2gh + \frac{2}{\rho}(p_A - p_B)} = \sqrt{2(9.8\ \text{m/s}^2)(0.50\ \text{m}) + \frac{2(1.40\ \text{atm} - 1.00\ \text{atm})}{1.0\times10^3\ \text{kg/m}^3}} = 9.5\ \text{m/s}.$$

74. Since all the blood that passes through the capillaries must have also passed through the aorta, the volume flow rate through the aorta is equal to the total volume flow rate through the capillaries. Assuming that the capillaries are identical with cross-sectional area A and flow speed v, we then have $A_0 v_0 = nAv$, where n is the number of capillaries. Solving for n yields

$$n = \frac{A_0 v_0}{Av} = \frac{(3\ \text{cm}^2)(30\ \text{cm/s})}{(3\times 10^{-7}\,\text{cm}^2)(0.05\ \text{cm/s})} = 6\times 10^9$$

75. We assume the fluid in the press is incompressible. Then, the work done by the output force is the same as the work done by the input force. If the large piston moves a distance D and the small piston moves a distance d, then $fd = FD$ and

$$D = \frac{fd}{F} = \frac{(103\,\text{N})(0.85\,\text{m})}{20.0\times 10^{3}\,\text{N}} = 4.4\times 10^{-3}\,\text{m} = 4.4\ \text{mm}.$$

76. The downward force on the balloon is mg and the upward force is $F_b = \rho_{\text{out}}Vg$. Newton's second law (with $m = \rho_{\text{in}}V$) leads to

$$\rho_{\text{out}}Vg - \rho_{\text{in}}Vg = \rho_{\text{in}}Va \quad \Rightarrow \left(\frac{\rho_{\text{out}}}{\rho_{\text{in}}} - 1\right)g = a.$$

The problem specifies $\rho_{\text{out}} / \rho_{\text{in}} = 1.39$ (the outside air is cooler and thus more dense than the hot air inside the balloon). Thus, the upward acceleration is $(1.39 - 1.00)(9.80 \text{ m/s}^2) = 3.82 \text{ m/s}^2$.

77. The equation of continuity is $A_i v_i = A_f v_f$, where $A = \pi r^2$. Therefore,

$$v_f = v_i \left(\frac{r_i}{r_f} \right)^2 = (0.09 \text{ m/s}) \left(\frac{0.2}{0.6} \right)^2 .$$

Consequently, $v_f = 1.00 \times 10^{-2}$ m/s.

78. We equate the buoyant force F_b to the combined weight of the cork and sinker:

$$\rho_w V_w g = \rho_c V_c g + \rho_s V_s g$$

With $V_w = \frac{1}{2} V_c$ and ρ_w = 1.00 g/cm^3, we obtain

$$V_c = \frac{2\rho_s V_s}{\rho_w - 2\rho_c} = \frac{2(11.4)(0.400)}{1.00 - 2(0.200)} = 15.2\,\text{cm}^3.$$

Using the formula for the volume of a sphere (Appendix E), we have

$$r = \left(\frac{3V_c}{4\pi}\right)^{1/3} = 1.54\,\text{cm}.$$

79. (a) From Bernoulli equation $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$, the height of the water extended up into the standpipe for section B is related to that for section D by

$$h_B = h_D + \frac{1}{2g}\left(v_D^2 - v_B^2\right)$$

Equation of continuity further implies that $v_D A_D = v_B A_B$, or

$$v_B = \left(\frac{A_D}{A_B}\right)v_D = \left(\frac{2R_B}{R_B}\right)^2 v_D = 4v_D$$

where $v_D = R_V/(\pi R_D^2) = (2.0\times10^{-3}\,\text{m}^3/\text{s})/(\pi(0.040\text{ m})^2) = 0.40\text{ m/s}$. With $h_D = 0.50\text{ m}$, we have

$$h_B = 0.50\text{ m} + \frac{1}{2(9.8\text{ m/s}^2)}(-15)(0.40\text{ m/s})^2 = 0.38\text{ m}.$$

(b) From the above result, we see that the greater the radius of the cross-sectional area, the greater the height. Thus, $h_C > h_D > h_B > h_A$.

80. The absolute pressure is

$$
\begin{aligned}
p &= p_0 + \rho g h \\
&= 1.01\times10^5\ \text{N/m}^2 + \left(1.03\times10^3\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)\left(150\ \text{m}\right) = 1.62\times10^6\ \text{Pa}.
\end{aligned}
$$

81. We consider the can with nearly its total volume submerged, and just the rim above water. For calculation purposes, we take its submerged volume to be V = 1200 cm^3. To float, the total downward force of gravity (acting on the tin mass m_t and the lead mass m_ℓ) must be equal to the buoyant force upward:

$$(m_t + m_\ell)\, g = \rho_w V g \Rightarrow m_\ell = (1\text{g/cm}^3)\,(1200\text{ cm}^3) - 130\text{ g}$$

which yields 1.07×10^3 g for the (maximum) mass of the lead (for which the can still floats). The given density of lead is not used in the solution.

82. If the mercury level in one arm of the tube is lowered by an amount x, it will rise by x in the other arm. Thus, the net difference in mercury level between the two arms is $2x$, causing a pressure difference of $\Delta p = 2\rho_{Hg}gx$, which should be compensated for by the water pressure $p_w = \rho_w gh$, where h = 11.2 cm. In these units, ρ_w = 1.00 g/cm^3 and ρ_{Hg} = 13.6 g/cm^3 (see Table 14-1). We obtain

$$x = \frac{\rho_w gh}{2\rho_{Hg} g} = \frac{(1.00 \text{ g/cm}^3)\ (11.2 \text{ cm})}{2(13.6 \text{ g/cm}^3)} = 0.412 \text{ cm}.$$

83. Neglecting the buoyant force caused by air, then the 30 N value is interpreted as the true weight *W* of the object. The buoyant force of the water on the object is therefore (30 – 20) N = 10 N, which means

$$F_b = \rho_w V g \;\Rightarrow\; V = \frac{10\,\text{N}}{(1000\ \text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.02\times10^{-3}\ \text{m}^3$$

is the volume of the object. When the object is in the second liquid, the buoyant force is (30 – 24) N = 6.0 N, which implies

$$\rho_2 = \frac{6.0\ \text{N}}{(9.8\ \text{m/s}^2)(1.02\times10^{-3}\ \text{m}^3)} = 6.0\times10^2\ \text{kg/m}^3\ .$$

84. (a) Using Eq. 14-10, we have $p_g = \rho g h = 1.21 \times 10^7$ Pa.

(b) By definition, $p = p_g + p_{\text{atm}} = 1.22 \times 10^7$ Pa.

(c) We interpret the question as asking for the total force *compressing* the sphere's surface, and we multiply the pressure by total area:

$$p\ (4\pi r^2) = 3.82 \times 10^5\ \text{N}.$$

(d) The (upward) buoyant force exerted on the sphere by the seawater is

$$F_b = \rho_w g V \qquad\qquad \text{where } V = \frac{4}{3}\pi r^3\ .$$

Therefore, $F_b = 5.26$ N.

(e) Newton's second law applied to the sphere (of mass $m = 7.00$ kg) yields

$$F_b - mg = ma$$

which results in $a = -9.04$ m/s^2, which means the acceleration vector has a magnitude of 9.04 m/s^2.

(f) The direction is downward.

85. The volume of water that drains back into the river annually is ¾(0.48 m)(3.0×10^9 m^2) = 1.08×10^9 m^3. Dividing this (on a per unit time basis, according to Eq. 14-24) by area gives the (average) speed:

$$v = \frac{1.08 \times 10^9}{20 \times 4} = 1.35 \times 10^7 \text{ m/y} = 0.43 \text{ m/s}.$$

86. An object of mass $m = \rho V$ floating in a liquid of density ρ_{liquid} is able to float if the downward pull of gravity mg is equal to the upward buoyant force $F_b = \rho_{\text{liquid}} g V_{\text{sub}}$ where V_{sub} is the portion of the object which is submerged. This readily leads to the relation:

$$\frac{\rho}{\rho_{liquid}} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged of a floating object. When the liquid is water, as described in this problem, this relation leads to

$$\frac{\rho}{\rho_w} = 1$$

since the object "floats fully submerged" in water (thus, the object has the same density as water). We assume the block maintains an "upright" orientation in each case (which is not necessarily realistic).

(a) For liquid A,

$$\frac{\rho}{\rho_A} = \frac{1}{2}$$

so that, in view of the fact that $\rho = \rho_w$, we obtain $\rho_A/\rho_w = 2$.

(b) For liquid B, noting that two-thirds *above* means one-third *below*,

$$\frac{\rho}{\rho_B} = \frac{1}{3}$$

so that $\rho_B/\rho_w = 3$.

(c) For liquid C, noting that one-fourth *above* means three-fourths *below*,

$$\frac{\rho}{\rho_C} = \frac{3}{4}$$

so that $\rho_C/\rho_w = 4/3$.

87. The pressure (relative to standard air pressure) is given by Eq. 14-8:

$$\rho g h = (1024\ \text{kg/m}^3)\ (9.8\ \text{m/s}^2)\ (6.0 \times 10^3\ \text{m}) = 6.02 \times 10^7\ \text{Pa}\ .$$

88. Eq. 14-10 gives

$$\rho_{\text{water}}\, g\, (-0.11 \text{ m}) = -1076 \text{ N/m}^2 \text{ (or } -1076 \text{ Pa).}$$

Quoting the answer to two significant figures, we have the gauge pressure equal to -1.1×10^3 Pa.

89. (a) Bernoulli's equation implies $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h$, or

$$p_2 - p_1 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g h$$

where $p_1 = 2.00$ atm , $p_2 = 1.00$ atm and $h = 9.40$ m . Using continuity equation $v_1 A_1 = v_2 A_2$, the above equation may be rewritten as

$$(p_2 - p_1) - \rho g h = \frac{1}{2}\rho v_2^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = \frac{1}{2}\rho v_2^2\left[1 - \left(\frac{R_2^2}{R_1^2}\right)^2\right]$$

With $R_2 / R_1 = 1/6$, we obtain $v_2 = 4.216$ m/s . Thus, the amount of time required to fill up a 10.0 m by 10.0 m swimming pool to a height of 2.00 m is

$$t = \frac{V}{A_2 v_2} = \frac{(10.0\text{ m})(10.0\text{ m})(2.00\text{ m})}{\pi(1.00\times10^{-2}\text{ m})^2(4.216\text{ m/s})} = 1.51\times10^5\text{ s} \approx 42\text{ h}.$$

(b) Yes, the filling time is acceptable.

90. This is analogous to the same "weighted average" idea encountered in the discussion of centers of mass (in Chapter 9), particularly due to the assumption that the volume does not change:

$$\rho_{mix} = \frac{d_1\rho_1 + d_2\rho_2}{d_1 + d_2} = \frac{(8)(1.2) + (4)(2.0)}{8 + 4} = 1.5 \text{ g/cm}^3.$$

91. Equilibrium of forces (on the floating body) is expressed as

$$F_b = m_{\text{body}}\, g \Rightarrow \rho_{\text{liquid}}\, g V_{\text{submerged}} = \rho_{\text{body}} g V_{\text{total}}$$

which leads to

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{liquid}}}.$$

We are told (indirectly) that two-thirds of the body is below the surface, so the fraction above is 2/3. Thus, with $\rho_{\text{body}} = 0.98$ g/cm^3, we find $\rho_{\text{liquid}} \approx 1.5$ g/cm^3 — certainly much more dense than normal seawater (the Dead Sea is about seven times saltier than the ocean due to the high evaporation rate and low rainfall in that region).

92. (a) We assume that the top surface of the slab is at the surface of the water and that the automobile is at the center of the ice surface. Let M be the mass of the automobile, ρ_i be the density of ice, and ρ_w be the density of water. Suppose the ice slab has area A and thickness h. Since the volume of ice is Ah, the downward force of gravity on the automobile and ice is $(M + \rho_i Ah)g$. The buoyant force of the water is $\rho_w Ahg$, so the condition of equilibrium is $(M + \rho_i Ah)g - \rho_w Ahg = 0$ and

$$A = \frac{M}{(\rho_w - \rho_i)h} = \frac{1100\,\text{kg}}{(998\,\text{kg/m}^3 - 917\,\text{kg/m}^3)(0.30\text{m})} = 45\ \text{m}^2.$$

These density values are found in Table 14-1 of the text.

(b) It does matter where the car is placed since the ice tilts if the automobile is not at the center of its surface.

93. (a) The total weight is

$$W = \rho g h A = \left(1.00\times10^{3}\ \text{kg/m}^{3}\right)\left(9.8\ \text{m/s}^{2}\right)\left(200\ \text{m}\right)\left(3000\ \text{m}^{2}\right) = 6.06\times10^{9}\ \text{N}.$$

(b) The water pressure is

$$p = \rho g h = \left(1.03\times10^{3}\ \text{kg/m}^{3}\right)\left(9.8\ \text{m/s}^{2}\right)\left(200\text{m}\right)\left(\frac{1\ \text{atm}}{1.01\times10^{5}\ \text{Pa}}\right) = 20\ \text{atm}$$

(c) No, because the pressure is too much for anybody to endure without special equipment.

94. The area facing down (and up) is A = (0.050 m)(0.040 m) = 0.0020 m^2. The submerged volume is $V = Ad$ where d = 0.015 m. In order to float, the downward pull of gravity mg must equal the upward buoyant force exerted by the seawater of density ρ:

$$mg = \rho V g \Rightarrow m = \rho V = (1025)(0.0020)(0.015) = 0.031\text{kg}.$$

95. Note that "surface area" refers to the *total* surface area of all six faces, so that the area of each (square) face is 24/6 = 4 m^2. From Archimedes' principle and the requirement that the cube (of total volume V and density ρ) floats, we find

$$\rho V g = \rho_w V_{\text{sub}} g \Rightarrow \frac{\rho}{\rho_w} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged. The assumption that the cube floats upright, as described in this problem, simplifies this relation to

$$\frac{\rho}{\rho_w} = \frac{h_{\text{sub}}}{h}$$

where h is the length of one side, and $\rho_w \; 4\rho =$ is given. With $h = \sqrt{4} = 2\,\text{m},$ we find h_{sub} = h/4 = 0.50 m.

96. The beaker is indicated by the subscript b. The volume of the glass of which the beaker walls and base are made is $V_b = m_b/\rho_b$. We consider the case where the beaker is slightly more than half full (which, for calculation purposes, will be simply set equal to half-volume) and thus remains on the bottom of the sink — as the water around it reaches its rim. At this point, the force of buoyancy exerted on it is given by $F = (V_b + V)\rho_w g$, where V is the interior volume of the beaker. Thus $F = (V_b + V)\rho_w g = \rho_w g(V/2) + m_b$, which we solve for ρ_b:

$$\rho_b = \frac{2m_b\rho_w}{2m_b - \rho_w V} = \frac{2(390\,\text{g})(1.00\ \text{g/cm}^3)}{2(390\ \text{g}) - (1.00\,\text{g/cm}^3)(500\ \text{cm}^3)} = 2.79\ \text{g/cm}^3\ .$$

97. (a) Since the pressure (due to the water) increases linearly with depth, we use its average (multiplied by the dam area) to compute the force exerted on the face of the dam, its average being simply half the pressure value near the bottom (at depth d_4 = 48 m). The maximum static friction will be μF_N where the normal force F_N (exerted upward by the portion of the bedrock directly underneath the concrete) is equal to the weight mg of the dam. Since $m = \rho_c V$ with ρ_c being the density of the concrete and V being the volume (thickness times width times height: $d_1 d_2 d_3$), we write $F_N = \rho_c d_1 d_2 d_3 g$. Thus, the safety factor is

$$\frac{\mu\rho_c d_1 d_2 d_3 g}{\frac{1}{2}\rho_w g d_4 A_{\text{face}}} = \frac{2\mu\rho_c d_1 d_2 d_3}{\rho_w d_4 (d_1 d_4)} = \frac{2\mu\rho_c d_2 d_3}{\rho_w d_4^2}$$

which (since ρ_w = 1 g/cm^3) yields 2(0.47) (3.2) (24) (71) / (48)^2 = 2.2.

(b) To compute the torque due to the water pressure, we will need to integrate Eq. 14-7 (multiplied by ($d_4 - y$) and the dam width d_1) as shown below. The countertorque due to the weight of the concrete is the weight multiplied by half the thickness d_3, since we take the center of mass of the dam at its geometric center and the axis of rotation at A. Thus, the safety factor relative to rotation is

$$\frac{mg\,(d_3/2)}{\int_0^{d_4}\rho_w g y (d_4 - y) d_1\,dy} = \frac{\rho_c d_1 d_2 d_3 g (d_3/2)}{\rho_w g d_1 d_4^3/6} = \frac{3\rho_c d_3^2 d_2}{\rho_w d_4^3}$$

which yields 3(3.2) (24)^2 (71)/(48)^3 = 3.6.

98. Let F_o be the buoyant force of air exerted on the object (of mass m and volume V), and F_{brass} be the buoyant force on the brass weights (of total mass m_{brass} and volume V_{brass}). Then we have

$$F_o = \rho_{\text{air}} V g = \rho_{\text{air}} \left(\frac{mg}{\rho} \right)$$

and

$$F_{\text{brass}} = \rho_{\text{air}} V_{\text{brass}} g = \rho_{\text{air}} \left(\frac{m_{\text{brass}}}{\rho_{\text{brass}}} \right).$$

For the two arms of the balance to be in mechanical equilibrium, we require $mg - F_o = m_{\text{brass}} g - F_{\text{brass}}$, or

$$mg - mg\left(\frac{\rho_{\text{air}}}{\rho} \right) = m_{\text{brass}} g - m_{\text{brass}} g \left(\frac{m_{\text{brass}}}{\rho_{\text{brass}}} \right),$$

which leads to

$$m_{\text{brass}} = \left(\frac{1 - \rho_{\text{air}} / \rho}{1 - \rho_{\text{air}} / \rho_{\text{brass}}} \right) m.$$

Therefore, the percent error in the measurement of m is

$$\begin{aligned} \frac{\Delta m}{m} &= \frac{m - m_{\text{brass}}}{m} = 1 - \frac{1 - \rho_{\text{air}} / \rho}{1 - \rho_{\text{air}} / \rho_{\text{brass}}} = \frac{\rho_{\text{air}} \ (1/\rho - 1/\rho_{\text{brass}}}{1 - \rho_{\text{air}} / \rho_{\text{brass}}} \\ &= \frac{0.0012(1/\rho - 1/8.0)}{1 - 0.0012/8.0)} \approx 0.0012\left(\frac{1}{\rho} - \frac{1}{8.0} \right), \end{aligned}$$

where ρ is in g/cm^3. Stating this as a *percent* error, our result is 0.12% multiplied by $(1/\rho - 1/8.0)$.