1. (a) The motion from maximum displacement to zero is one-fourth of a cycle so 0.170 s is one-fourth of a period. The period is $T = 4(0.170 \text{ s}) = 0.680$ s.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\,\text{Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\,\text{m/s}.$$

2. (a) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.80\,\text{m}} = 3.49\,\text{m}^{-1}.$$

(b) The speed of the wave is

$$v = \lambda f = \frac{\lambda\omega}{2\pi} = \frac{(1.80\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\,\text{m/s}.$$

3. Let $y_1 = 2.0$ mm (corresponding to time t_1) and $y_2 = -2.0$ mm (corresponding to time t_2). Then we find

$$kx + 600t_1 + \phi = \sin^{-1}(2.0/6.0)$$

and

$$kx + 600t_2 + \phi = \sin^{-1}(-2.0/6.0) \ .$$

Subtracting equations gives $600(t_1 - t_2) = \sin^{-1}(2.0/6.0) - \sin^{-1}(-2.0/6.0)$. Thus we find $t_1 - t_2 = 0.011$ s (or 1.1 ms).

4. Setting $x = 0$ in $u = -\omega y_m \cos(kx - \omega t + \phi)$ (see Eq. 16-21 or Eq. 16-28) gives $u = -\omega y_m \cos(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a positive "slope" (referring to its t-derivative) at $t = 0$:

$$\frac{du}{dt} = \frac{d(-\omega y_m \cos(-\omega t + \phi))}{dt} = -y_m \omega^2 \sin(-\omega t + \phi) > 0 \text{ at } t = 0.$$

This implies that $-\sin\phi > 0$ and consequently that ϕ is in either the third or fourth quadrant. The graph shows (at t = 0) $u = -4$ m/s, and (at some later t) $u_{max} = 5$ m/s. We note that $u_{max} = y_m \omega$. Therefore,

$$u = -u_{max} \cos(-\omega t + \phi)\Big|_{t=0} \Rightarrow \phi = \cos^{-1}\left(\tfrac{4}{5}\right) = \pm 0.6435 \text{ rad}$$

(bear in mind that $\cos\theta = \cos(-\theta)$), and we must choose $\phi = -0.64$ rad (since this is about $-37°$ and is in fourth quadrant). Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = -0.64 + 2\pi = 5.64$ rad is also an acceptable result.

5. Using $v = f\lambda$, we find the length of one cycle of the wave is $\lambda = 350/500 = 0.700$ m = 700 mm. From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}$ s = 2.00 ms.

(a) A cycle is equivalent to 2π radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6 = 700/6 = 117$ mm.

(b) The interval 1.00 ms is half of T and thus corresponds to half of one cycle, or half of 2π rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

6. (a) The amplitude is y_m = 6.0 cm.

(b) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain f = 2.0 Hz.

(d) The wave speed is $v = \lambda f$ = (100 cm) (2.0 Hz) = 2.0×10^2 cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of y) is

$$u_{\text{max}} = 2\pi f y_m = \left(4.0\pi\,\text{s}^{-1}\right)\left(6.0\,\text{cm}\right) = 75\,\text{cm/s}.$$

(g) y(3.5 cm, 0.26 s) = (6.0 cm) sin[0.020π(3.5) + 4.0π(0.26)] = –2.0 cm.

7. (a) Recalling from Ch. 12 the simple harmonic motion relation $u_m = y_m\omega$, we have

$$\omega = \frac{16}{0.040} = 400\,\text{rad/s}.$$

Since $\omega = 2\pi f$, we obtain $f = 64$ Hz.

(b) Using $v = f\lambda$, we find $\lambda = 80/64 = 1.26$ m ≈ 1.3 m.

(c) The amplitude of the transverse displacement is $y_m = 4.0\text{ cm} = 4.0\times10^{-2}\,\text{m}$.

(d) The wave number is $k = 2\pi/\lambda = 5.0$ rad/m.

(e) The angular frequency, as obtained in part (a), is $\omega = 16/0.040 = 4.0\times10^{2}$ rad/s.

(f) The function describing the wave can be written as

$$y = 0.040\sin(5x - 400t + \phi)$$

where distances are in meters and time is in seconds. We adjust the phase constant ϕ to satisfy the condition $y = 0.040$ at $x = t = 0$. Therefore, sin $\phi = 1$, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.040\sin\left(5x - 400t + \frac{\pi}{2}\right).$$

(g) The sign in front of ω is minus.

8. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = 225\pi \sin(\pi x - 15\pi t) \quad .$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi\sqrt{15^2 - y^2} \quad .$$

Therefore, where $y = 12$, u must be $\pm 135\pi$. Consequently, the *speed* there is 424 cm/s = 4.24 m/s.

9. (a) The amplitude y_m is half of the 6.00 mm vertical range shown in the figure, i.e., $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t$ = 15 m/s, where d = 0.060 m and t = 0.0040 s. The angular wave number is $k = 2\pi/\lambda$ where λ = 0.40 m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m} .$$

(c) The angular frequency is found from

$$\omega = kv = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4\times10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between kx and ωt) in the argument of the sine function because the wave is shown traveling to the right [in the $+x$ direction] – see section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16x - 2.4\times10^2 t) .$$

10. The slope that they are plotting is the physical slope of sinusoidal waveshape (not to be confused with the more abstract "slope" of its time development; the physical slope is an x-derivative whereas the more abstract "slope" would be the t-derivative). Thus, where the figure shows a maximum slope equal to 0.2 (with no unit), it refers to the maximum of the following function:

$$\frac{dy}{dx} = \frac{d\, y_m \sin(kx - \omega t)}{dx} = y_m\, k \cos(kx - \omega t)\ .$$

The problem additionally gives $t = 0$, which we can substitute into the above expression if desired. In any case, the maximum of the above expression is $y_m k$, where

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.40\ \text{m}} = 15.7\ \text{rad/m}\,.$$

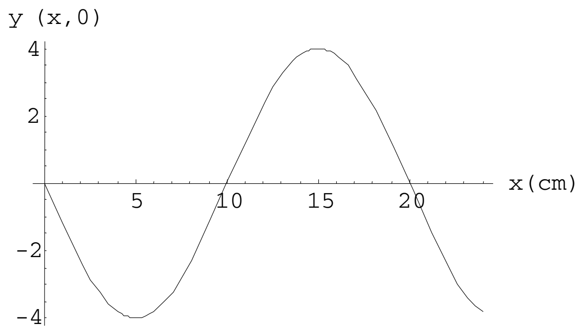
Therefore, setting $y_m k$ equal to 0.20 allows us to solve for the amplitude y_m. We find

$$y_m = \frac{0.20}{15.7\ \text{rad/m}} = 0.0127\ \text{m} \approx 1.3\ \text{cm}\,.$$

11. From Eq. (16.10), a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

(a) Figure 16.34 shows that at x = 0, $y(0,t) = y_m \sin(-\omega t + \phi)$ is a positive sine function, i.e., $y(0,t) = +y_m \sin \omega t$. Therefore, the phase constant must be $\phi = \pi$. At t =0, we then have $y(x,0) = y_m \sin(kx + \pi) = -y_m \sin kx$ which is a negative sine function. A plot of y(x,0) is depicted below.



(b) From the figure we see that the amplitude is y_m = 4.0 cm.

(c) The angular wave number is given by $k = 2\pi/\lambda = \pi/10 = 0.31$ rad/cm.

(d) The angular frequency is $\omega = 2\pi/T = \pi/5 = 0.63$ rad/s.

(e) As found in part (a), the phase is $\phi = \pi$.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

(g) Since the frequency is $f = 1/T = 0.10$ s, the speed of the wave is $v = f\lambda = 2.0$ cm/s.

(h) From the results above, the wave may be expressed as

$$y(x,t) = 4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5} + \pi\right) = -4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right).$$

Taking the derivative of y with respect to t, we find

$$u(x,t) = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{t}\right)\cos\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

which yields $u(0,5.0) = -2.5$ cm/s.

12. The volume of a cylinder of height ℓ is $V = \pi r^2 \ell = \pi d^2 \ell/4$. The strings are long, narrow cylinders, one of diameter d_1 and the other of diameter d_2 (and corresponding linear densities μ_1 and μ_2). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho \pi d^2 \ell/4}{\ell} = \frac{\pi \rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho\, d_1^2/4}{\pi\rho\, d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2.$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2.$$

13. The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope: $\mu = m/L$ = (0.0600 kg)/(2.00 m) = 0.0300 kg/m. Thus

$$v = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129\,\text{m/s}.$$

14. From $v = \sqrt{\tau/\mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}} = \sqrt{2}.$$

15. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency ($2\pi/T$), and k is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\ \text{m}^{-1}$ and $\omega = 30$ rad/s. Thus

$$v = (30\ \text{rad/s})/(2.0\ \text{m}^{-1}) = 15\ \text{m/s}.$$

(b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6\times10^{-4}\ \text{kg/m}\right)\left(15\ \text{m/s}\right)^2 = 0.036\ \text{N}.$$

16. We use $v = \sqrt{\tau / \mu} \propto \sqrt{\tau}$ to obtain

$$\tau_2 = \tau_1 \left(\frac{v_2}{v_1} \right)^2 = (120\,\text{N}) \left(\frac{180\,\text{m/s}}{170\,\text{m/s}} \right)^2 = 135\text{N}.$$

17. (a) The amplitude of the wave is y_m=0.120 mm.

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}.$$

(c) The frequency is f = 100 Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100\text{ Hz}) = 628\text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction. In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[\left(141\,\text{m}^{-1}\right)x + \left(628\,\text{s}^{-1}\right)t\right].$$

18. (a) Comparing with Eq. 16-2, we see that k = 20/m and ω = 600/s. Therefore, the speed of the wave is (see Eq. 16-13) $v = \omega/k$ = 30 m/s.

(b) From Eq. 16–26, we find

$$\mu = \frac{\tau}{v^2} = \frac{15}{30^2} = 0.017\,\text{kg/m} = 17\,\text{g/m}.$$

19. (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15$ cm and again with the same slope at about $x = 55$ cm, so

$$\lambda = (55\text{ cm} - 15\text{ cm}) = 40\text{ cm} = 0.40\text{ m}.$$

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6\,\text{N}}{25\times10^{-3}\,\text{kg/m}}} = 12\text{ m/s}.$$

(d) The frequency is $f = v/\lambda = (12\text{ m/s})/(0.40\text{ m}) = 30$ Hz and the period is

$$T = 1/f = 1/(30\text{ Hz}) = 0.033\text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30\text{ Hz})\,(5.0\text{ cm}) = 940\text{ cm/s} = 9.4\text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40\text{ m}) = 16\text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30\text{ Hz}) = 1.9\times10^2$ rad/s

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is 4.0×10^{-2} m. The formula for the displacement gives $y(0, 0) = y_m \sin\phi$. We wish to select ϕ so that $5.0 \times 10^{-2} \sin\phi = 4.0 \times 10^{-2}$. The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. Using the results obtained above, the expression for the displacement is

$$y(x,t) = \left(5.0\times10^{-2}\,\text{m}\right)\sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right].$$

20. (a) The general expression for $y(x, t)$ for the wave is $y(x, t) = y_m \sin(kx - \omega t)$, which, at $x = 10$ cm, becomes $y(x = 10\text{ cm}, t) = y_m \sin[k(10\text{ cm} - \omega t)]$. Comparing this with the expression given, we find $\omega = 4.0$ rad/s, or $f = \omega/2\pi = 0.64$ Hz.

(b) Since $k(10\text{ cm}) = 1.0$, the wave number is $k = 0.10$/cm. Consequently, the wavelength is $\lambda = 2\pi/k = 63$ cm.

(c) The amplitude is $y_m = 5.0$ cm.

(d) In part (b), we have shown that the angular wave number is $k = 0.10$/cm.

(e) The angular frequency is $\omega = 4.0$ rad/s.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

Summarizing the results obtained above by substituting the values of k and ω into the general expression for $y(x, t)$, with centimeters and seconds understood, we obtain

$$y(x,t) = 5.0 \sin(0.10x - 4.0t).$$

(g) Since $v = \omega/k = \sqrt{\tau/\mu}$, the tension is

$$\tau = \frac{\omega^2 \mu}{k^2} = \frac{(4.0\,\text{g/cm})(4.0\,\text{s}^{-1})^2}{(0.10\,\text{cm}^{-1})^2} = 6400\,\text{g}\cdot\text{cm/s}^2 = 0.064\,\text{N}.$$

21. The pulses have the same speed v. Suppose one pulse starts from the left end of the wire at time $t = 0$. Its coordinate at time t is $x_1 = vt$. The other pulse starts from the right end, at $x = L$, where L is the length of the wire, at time $t = 30$ ms. If this time is denoted by t_0 then the coordinate of this wave at time t is $x_2 = L - v(t - t_0)$. They meet when $x_1 = x_2$, or, what is the same, when $vt = L - v(t - t_0)$. We solve for the time they meet: $t = (L + vt_0)/2v$ and the coordinate of the meeting point is $x = vt = (L + vt_0)/2$. Now, we calculate the wave speed:

$$v = \sqrt{\frac{\tau L}{m}} = \sqrt{\frac{(250\,\text{N})(10.0\,\text{m})}{0.100\,\text{kg}}} = 158\,\text{m/s}.$$

Here τ is the tension in the wire and L/m is the linear mass density of the wire. The coordinate of the meeting point is

$$x = \frac{10.0\,\text{m} + (158\,\text{m/s})(30.0\times10^{-3}\,\text{s})}{2} = 7.37\,\text{m}.$$

This is the distance from the left end of the wire. The distance from the right end is $L - x = (10.0\text{ m} - 7.37\text{ m}) = 2.63$ m.

22. (a) The tension in each string is given by $\tau = Mg/2$. Thus, the wave speed in string 1 is

$$v_1 = \sqrt{\frac{\tau}{\mu_1}} = \sqrt{\frac{Mg}{2\mu_1}} = \sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(3.00\,\text{g/m})}} = 28.6\,\text{m/s}.$$

(b) And the wave speed in string 2 is

$$v_2 = \sqrt{\frac{Mg}{2\mu_2}} = \sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(5.00\,\text{g/m})}} = 22.1\,\text{m/s}.$$

(c) Let $v_1 = \sqrt{M_1 g/(2\mu_1)} = v_2 = \sqrt{M_2 g/(2\mu_2)}$ and $M_1 + M_2 = M$. We solve for M_1 and obtain

$$M_1 = \frac{M}{1 + \mu_2/\mu_1} = \frac{500\,\text{g}}{1 + 5.00/3.00} = 187.5\,\text{g} \approx 188\,\text{g}.$$

(d) And we solve for the second mass: $M_2 = M - M_1 = (500\text{ g} - 187.5\text{ g}) \approx 313\text{ g}$.

23. (a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where τ is the tension at that point and μ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance y from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by $\mu g y$, so the tension is $\tau = \mu g y$. The wave speed is $v = \sqrt{\mu g y/\mu} = \sqrt{gy}$.

(b) The time dt for the wave to move past a length dy, a distance y from the bottom end, is $dt = dy/v = dy\big/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\Bigg|_0^L = 2\sqrt{\frac{L}{g}}.$$

24. Using Eq. 16–33 for the average power and Eq. 16–26 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$f = \frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}} = \frac{1}{2\pi(7.70\times10^{-3}\,\text{m})}\sqrt{\frac{2(85.0\,\text{W})}{\sqrt{(36.0\,\text{N})(0.260\,\text{kg}/2.70\,\text{m})}}} = 198\ \text{Hz}.$$

25. We note from the graph (and from the fact that we are dealing with a cosine-squared, see Eq. 16-30) that the wave frequency is $f = \frac{1}{2\text{ ms}} = 500$ Hz, and that the wavelength $\lambda = 0.20$ m. We also note from the graph that the maximum value of *dK/dt* is 10 W. Setting this equal to the maximum value of Eq. 16-29 (where we just set that cosine term equal to 1) we find

$$\frac{1}{2}\mu v \omega^2 y_m^2 = 10$$

with SI units understood. Substituting in $\mu = 0.002$ kg/m, $\omega = 2\pi f$ and $v = f\lambda$, we solve for the wave amplitude:

$$y_m = \sqrt{\frac{10}{2\pi^2 \mu\lambda f^3}} = 0.0032 \text{ m} .$$

26. Comparing $y(x,t) = (3.00 \text{ mm})\sin[(4.00 \text{ m}^{-1})x - (7.00 \text{ s}^{-1})t]$ to the general expression $y(x,t) = y_m \sin(kx - \omega t)$, we see that $k = 4.00 \text{ m}^{-1}$ and $\omega = 7.00 \text{ rad/s}$. The speed of the wave is $v = \omega / k = (7.00 \text{ rad/s})/(4.00 \text{ m}^{-1}) = 1.75 \text{ m/s}$.

27. The wave $y(x,t) = (2.00\text{ mm})[(20\text{ m}^{-1})x - (4.0\text{ s}^{-1})t]^{1/2}$ is of the form $h(kx - \omega t)$ with angular wave number $k = 20\text{ m}^{-1}$ and angular frequency $\omega = 4.0\text{ rad/s}$. Thus, the speed of the wave is $v = \omega / k = (4.0\text{ rad/s})/(20\text{ m}^{-1}) = 0.20\text{ m/s}$.

28. The wave $y(x,t) = (4.00\text{ mm})\, h[(30\text{ m}^{-1})x + (6.0\text{ s}^{-1})t]$ is of the form $h(kx - \omega t)$ with angular wave number $k = 30\text{ m}^{-1}$ and angular frequency $\omega = 6.0\text{ rad/s}$. Thus, the speed of the wave is $v = \omega / k = (6.0\text{ rad/s})/(30\text{ m}^{-1}) = 0.20\text{ m/s}$.

29. The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \;\; = 2y_m \cos\left(\tfrac{1}{2}\phi\right) \sin\left(kx - \omega t + \tfrac{1}{2}\phi\right),$$

where $\phi = \pi/2$. The amplitude is

$$A = 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41 y_m .$$

30. (a) Let the phase difference be ϕ. Then from Eq. 16–52, $2y_m \cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8°.$$

(b) Converting to radians, we have $\phi = 1.45$ rad.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to 2π rad), this is equivalent to $1.45 \text{ rad}/2\pi = 0.230$ wavelength.

31. (a) The amplitude of the second wave is $y_m = 9.00$ mm, as stated in the problem.

(b) The figure indicates that λ = 40 cm = 0.40 m, which implies that the angular wave number is $k = 2\pi/0.40 = 16$ rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is $v = dx/t$ = (56.0 cm)/(8.0 ms) = 70 m/s. This, in turn, implies that the angular frequency is $\omega = kv = 1100$ rad/s $= 1.1\times10^3$ rad/s.

(d) We observe that Figure 16-38 depicts two traveling waves (both going in the $-x$ direction) of equal amplitude y_m. The amplitude of their resultant wave, as shown in the figure, is $y'_m = 4.00$ mm. Eq. 16-52 applies:

$$y'_m = 2\, y_m \cos(\tfrac{1}{2}\phi_2) \quad\Rightarrow\quad \phi_2 = 2\cos^{-1}(2.00/9.00) = 2.69 \text{ rad.}$$

(e) In making the plus-or-minus sign choice in $y = y_m \sin(kx \pm \omega t + \phi)$, we recall the discussion in section 16-5, where it shown that sinusoidal waves traveling in the $-x$ direction are of the form $y = y_m \sin(kx + \omega t + \phi)$. Here, φ should be thought of as the phase *difference* between the two waves (that is, $\phi_1 = 0$ for wave 1 and $\phi_2 = 2.69$ rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16x + 1100\,t) \quad\text{and}\quad y_2 = (0.00900)\sin(16x + 1100\,t + 2.7)\,.$$

32. (a) We use Eq. 16-26 and Eq. 16-33 with μ = 0.00200 kg/m and y_m = 0.00300 m. These give $v = \sqrt{\tau / \mu}$ = 775 m/s and

$$P_{\text{avg}} = \tfrac{1}{2}\, \mu v\, \omega^2 y_m^{\,2} = 10 \text{ W}.$$

(b) In this situation, the waves are two separate string (no superposition occurs). The answer is clearly twice that of part (a); P = 20 W.

(c) Now they are on the same string. If they are interfering constructively (as in Fig. 16-16(a)) then the amplitude y_m is doubled which means its square $y_m^{\,2}$ increases by a factor of 4. Thus, the answer now is four times that of part (a); P = 40 W.

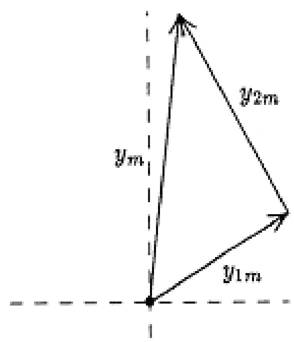
(d) Eq. 16-52 indicates in this case that the amplitude (for their superposition) is $2\,y_m\cos(0.2\pi)$ = 1.618 times the original amplitude y_m. Squared, this results in an increase in the power by a factor of 2.618. Thus, P = 26 W in this case.

(e) Now the situation depicted in Fig. 16-16(b) applies, so P = 0.

33. The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

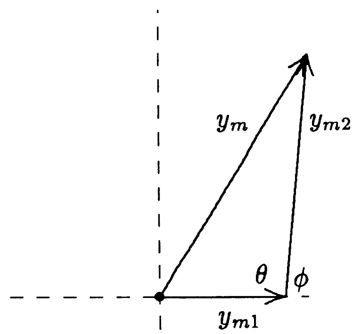
Thus y_m = 5.0 cm.



34. The phasor diagram is shown below. We use the cosine theorem:

$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos\theta = y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos\phi.$$

We solve for cos ϕ:



$$\cos\phi = \frac{y_m^2 - y_{m1}^2 - y_{m2}^2}{2y_{m1}\,y_{m2}} = \frac{(9.0\,\text{mm})^2 - (5.0\,\text{mm})^2 - (7.0\,\text{mm})^2}{2(5.0\,\text{mm})(7.0\,\text{mm})} = 0.10.$$

The phase constant is therefore $\phi = 84°$.

35. (a) As shown in Figure 16-16(b) in the textbook, the least-amplitude resultant wave is obtained when the phase difference is π rad.

(b) In this case, the amplitude is (8.0 mm – 5.0 mm) = 3.0 mm.

(c) As shown in Figure 16-16(a) in the textbook, the greatest-amplitude resultant wave is obtained when the phase difference is 0 rad.

(d) In the part (c) situation, the amplitude is (8.0 mm + 5.0 mm) = 13 mm.

(e) Using phasor terminology, the angle "between them" in this case is π/2 rad (90º), so the Pythagorean theorem applies:

$$\sqrt{(8.0\text{ mm})^2+(5.0\text{ mm})^2} = 9.4\text{ mm}.$$

36. We see that y_1 and y_3 cancel (they are 180º) out of phase, and y_2 cancels with y_4 because their phase difference is also equal to π rad (180º). There is no resultant wave in this case.

37. (a) Using the phasor technique, we think of these as two "vectors" (the first of "length" 4.6 mm and the second of "length" 5.60 mm) separated by an angle of $\phi = 0.8\pi$ radians (or 144º). Standard techniques for adding vectors then leads to a resultant vector of length 3.29 mm.

(b) The angle (relative to the first vector) is equal to 88.8º (or 1.55 rad).

(c) Clearly, it should in "in phase" with the result we just calculated, so its phase angle relative to the first phasor should be also 88.8º (or 1.55 rad).

38. The nth resonant frequency of string A is

$$f_{n,A} = \frac{v_A}{2l_A} n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}},$$

while for string B it is

$$f_{n,B} = \frac{v_B}{2l_B} n = \frac{n}{8L}\sqrt{\frac{\tau}{\mu}} = \frac{1}{4} f_{n,A}.$$

(a) Thus, we see $f_{1,A} = f_{4,B}$. That is, the fourth harmonic of B matches the frequency of A's first harmonic.

(b) Similarly, we find $f_{2,A} = f_{8,B}$.

(c) No harmonic of B would match $f_{3,A} = \frac{3v_A}{2l_A} = \frac{3}{2L}\sqrt{\frac{\tau}{\mu}}$,

39. Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250 \text{ N}}{(10.0 \text{ m})\,(0.100 \text{ kg})}} = n\ (7.91 \text{ Hz}).$$

(a) The lowest frequency is $f_1 = 7.91$ Hz.

(b) The second lowest frequency is $f_2 = 2(7.91 \text{ Hz}) = 15.8$ Hz.

(c) The third lowest frequency is $f_3 = 3(7.91 \text{ Hz}) = 23.7$ Hz.

40. (a) The wave speed is given by

$$v=\sqrt{\frac{\tau}{\mu}}=\sqrt{\frac{7.00\,\text{N}}{2.00\times10^{-3}\,\text{kg}/1.25\,\text{m}}}=66.1\,\text{m/s}.$$

(b) The wavelength of the wave with the lowest resonant frequency f_1 is $\lambda_1 = 2L$, where L = 125 cm. Thus,

$$f_1=\frac{v}{\lambda_1}=\frac{66.1\text{ m/s}}{2(1.25\text{ m})}=26.4\text{ Hz}.$$

41. (a) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0\ \text{N})\ (8.40\ \text{m})}{0.120\ \text{kg}}} = 82.0\ \text{m/s}.$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L = 2(8.40\ \text{m}) = 16.8\ \text{m}$.

(c) The frequency is $f = v/\lambda = (82.0\ \text{m/s})/(16.8\ \text{m}) = 4.88\ \text{Hz}$.

42. The string is flat each time the particles passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes *half* of one complete cycle, so we conclude T = 2(0.50 s) = 1.0 s. Thus, $f = 1/T$ = 1.0 Hz, and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\,\text{cm/s}}{1.0\,\text{Hz}} = 10\ \text{cm}.$$

43. (a) Eq. 16–26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\,\text{N}}{7.20\times10^{-3}\ \text{kg/m}}} = 144.34\ \text{m/s} \approx 1.44\times10^{2}\ \text{m/s}.$$

(b) From the Figure, we find the wavelength of the standing wave to be λ = (2/3)(90.0 cm) = 60.0 cm.

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times10^{2}\ \text{m/s}}{0.600\,\text{m}} = 241\,\text{Hz}.$$

44. Use Eq. 16–66 (for the resonant frequencies) and Eq. 16–26 $(v = \sqrt{\tau/\mu})$ to find f_n:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$.

(a) When $\tau_f = 4\tau_i$, we get the new frequency

$$f_3' = \frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}} = 2f_3.$$

(b) And we get the new wavelength

$$\lambda_3' = \frac{v'}{f_3'} = \frac{2L}{3} = \lambda_3.$$

45. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with $n + 1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n + 1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n}.$$

The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315\text{ Hz}}{420\text{ Hz} - 315\text{ Hz}} = 3.$$

The lowest possible resonant frequency is $f = v/2L = f_1/n = (315\text{ Hz})/3 = 105\text{ Hz}$.

(b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then

$$v = \lambda f = 2Lf = 2(0.75\text{ m})(105\text{ Hz}) = 158\text{ m/s}.$$

46. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus, f_1 = (390 Hz – 325 Hz) = 65 Hz. This further implies that the next higher resonance above 195 Hz should be (195 Hz + 65 Hz) = 260 Hz.

47. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3\ \text{cm}^{-1}$. The wave speed is

$$v = \omega/k = (40\pi\ \text{rad/s})/(\pi/3\ \text{cm}^{-1}) = 1.2\times10^{2}\ \text{cm/s}.$$

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3\ \text{cm}^{-1}) = 3.0$ cm. Here $2\pi/k$ was substituted for λ.

(d) The string speed is given by $u(x, t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t)$. For the given coordinate and time,

$$u = -(40\pi\ \text{rad/s})\ (0.50\,\text{cm})\ \sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)(1.5\,\text{cm})\right]\sin\left[\left(40\pi\ \text{s}^{-1}\right)\left(\frac{9}{8}\ \text{s}\right)\right] = 0.$$

48. Since the rope is fixed at both ends, then the phrase "second-harmonic standing wave pattern" describes the oscillation shown in Figure 16–23(b), where

$$\lambda = L \quad \text{and} \quad f = \frac{v}{L}$$

(see Eq. 16–65 and Eq. 16–69).

(a) Comparing the given function with Eq. 17–47, we obtain $k = \pi/2$ and $\omega = 12\pi$ (SI units understood). Since $k = 2\pi/\lambda$ then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \quad \Rightarrow \quad \lambda = 4.0\,\text{m} \quad \Rightarrow \quad L = 4.0\,\text{m}.$$

(b) Since $\omega = 2\pi f$ then $2\pi f = 12\pi \quad \Rightarrow \quad f = 6.0\,\text{Hz} \quad \Rightarrow \quad v = f\lambda = 24\,\text{m/s}.$

(c) Using Eq. 17–25, we have

$$v = \sqrt{\frac{\tau}{\mu}} \quad \Rightarrow \quad 24 = \sqrt{\frac{200}{m/L}}$$

which leads to $m = 1.4$ kg.

(d) With

$$f = \frac{3v}{2L} = \frac{3(24)}{2(4.0)} = 9.0\,\text{Hz}$$

The period is $T = 1/f = 0.11$ s.

49. (a) The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions. We take them to be $y_1 = y_m \sin(kx - \omega t)$ and $y_2 = y_m \sin(kx + \omega t)$. The amplitude y_m is half the maximum displacement of the standing wave, or 5.0×10^{-3} m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With L = 3.0m, λ = 2.0 m. The angular wave number is $k = 2\pi/\lambda = 2\pi/(2.0\text{ m}) = 3.1\text{ m}^{-1}$.

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\,\text{m/s})}{2(3.0\,\text{m})} = 50\text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or $\omega = 2\pi f = 2\pi(50\text{ Hz})$ = 314 rad/s.

(d) The two waves are

$$y_1 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x - \left(314\,\text{s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x + \left(314\,\text{s}^{-1}\right)t\right].$$

Thus, if one of the waves has the form $y(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y'(x,t) = y_m \sin(kx - \omega t)$. The sign in front of ω for $y'(x,t)$ is minus.

50. The nodes are located from vanishing of the spatial factor sin $5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \cdots$$

(a) The smallest value of x which corresponds to a node is $x = 0$.

(b) The second smallest value of x which corresponds to a node is $x = 0.20$ m.

(c) The third smallest value of x which corresponds to a node is $x = 0.40$ m.

(d) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(e) Comparing the given function with Eq. 16–58 through Eq. 16–60, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(f) And we see the amplitude is $y_m = 0.020$ m.

(g) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -(0.040)(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all x) at times such as $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \cdots$$

Thus, the first time in which all points on the string have zero transverse velocity is when $t = 0$ s.

(h) The second time in which all points on the string have zero transverse velocity is when $t = 1/40$ s $= 0.025$ s.

(i) The third time in which all points on the string have zero transverse velocity is when t = 2/40 s = 0.050 s.

51. From the $x = 0$ plot (and the requirement of an anti-node at $x = 0$), we infer a standing wave function of the form $y(x,t) = -(0.04)\cos(kx)\sin(\omega t)$, where $\omega = 2\pi / T = \pi$ rad/s, with length in meters and time in seconds. The parameter k is determined by the existence of the node at $x = 0.10$ (presumably the *first* node that one encounters as one moves from the origin in the positive x direction). This implies $k(0.10) = \pi/2$ so that $k = 5\pi$ rad/m.

(a) With the parameters determined as discussed above and $t = 0.50$ s, we find

$$y(0.20 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0.040 \text{ m}.$$

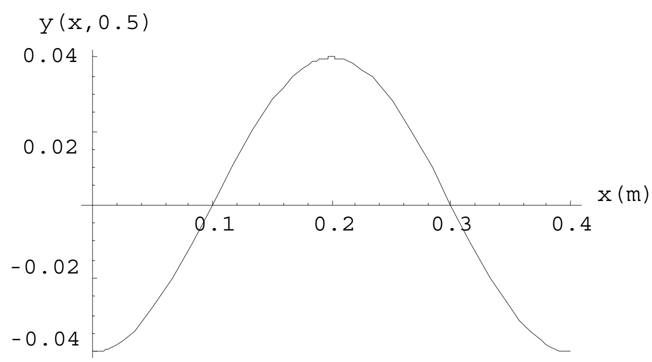
(b) The above equation yields $y(0.30 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0$.

(c) We take the derivative with respect to time and obtain, at $t = 0.50$ s and $x = 0.20$ m,

$$u = \frac{dy}{dt} = -0.04\omega \cos(kx)\cos(\omega t) = 0.$$

d) The above equation yields $u = -0.13$ m/s at $t = 1.0$ s.

(e) The sketch of this function at $t = 0.50$ s for $0 \le x \le 0.40$ m is shown below:



52. Recalling the discussion in section 16-12, we observe that this problem presents us with a standing wave condition with amplitude 12 cm. The angular wave number and frequency are noted by comparing the given waves with the form $y = y_m \sin(k\,x \pm \omega\,t)$. The anti-node moves through 12 cm in simple harmonic motion, just as a mass on a vertical spring would move from its upper turning point to its lower turning point – which occurs during a half-period. Since the period T is related to the angular frequency by Eq. 15-5, we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.00\,\pi} = 0.500 \text{ s} .$$

Thus, in a time of $t = \frac{1}{2}T = 0.250$ s, the wave moves a distance $\Delta x = vt$ where the speed of the wave is $v = \frac{\omega}{k} = 1.00$ m/s. Therefore, $\Delta x =$ (1.00 m/s)(0.250 s) = 0.250 m.

53. (a) The angular frequency is ω = 8.00π/2 = 4.00π rad/s, so the frequency is $f = \omega/2\pi$ = (4.00π rad/s)/2π = 2.00 Hz.

(b) The angular wave number is k = 2.00π/2 = 1.00π m^{-1}, so the wavelength is $\lambda = 2\pi/k$ = 2π/(1.00π m^{-1}) = 2.00 m.

(c) The wave speed is

$$v = \lambda f = (2.00\,\text{m})(2.00\,\text{Hz}) = 4.00\text{ m/s}.$$

(d) We need to add two cosine functions. First convert them to sine functions using cos α = sin (α + π/2), then apply

$$\cos\alpha + \cos\beta = \sin\left(\alpha + \frac{\pi}{2}\right) + \sin\left(\beta + \frac{\pi}{2}\right) = 2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m\cos(kx+\omega t) + y_m\cos(kx-\omega t) = 2y_m\cos(kx)\cos(\omega t).$$

Nodes occur where cos(kx) = 0 or $kx = n\pi + \pi/2$, where n is an integer (including zero). Since k = 1.0π m^{-1}, this means $x = \left(n+\tfrac{1}{2}\right)(1.00\,\text{m})$. Thus, the smallest value of x which corresponds to a node is x = 0.500 m (n=0).

(e) The second smallest value of x which corresponds to a node is x = 1.50 m (n=1).

(f) The third smallest value of x which corresponds to a node is x = 2.50 m (n=2).

(g) The displacement is a maximum where cos(kx) = ±1. This means $kx = n\pi$, where n is an integer. Thus, $x = n$(1.00 m). The smallest value of x which corresponds to an anti-node (maximum) is x = 0 (n=0).

(h) The second smallest value of x which corresponds to an anti-node (maximum) is $x = 1.00$ m (n=1).

(i) The third smallest value of x which corresponds to an anti-node (maximum) is $x = 2.00$ m (n=2).

54. Reference to point A as an anti-node suggests that this is a standing wave pattern and thus that the waves are traveling in opposite directions. Thus, we expect one of them to be of the form $y = y_m \sin(kx + \omega t)$ and the other to be of the form $y = y_m \sin(kx - \omega t)$.

(a) Because of Eq. 16-60, we conclude that $y_m = \frac{1}{2}(9.0 \text{ mm}) = 4.5$ mm due to the fact that the amplitude of the standing wave is $\frac{1}{2}(1.80 \text{ cm}) = 0.90$ cm = 9.0 mm.

(b) Since one full cycle of the wave (one wavelength) is 40 cm, $k = 2\pi/\lambda \approx 16 \text{ m}^{-1}$.

(c) The problem tells us that the time of half a full period of motion is 6.0 ms, so T = 12 ms and Eq. 16-5 gives $\omega = 5.2 \times 10^2$ rad/s.

(d) The two waves are therefore

$$y_1(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x + (520 \text{ s}^{-1})t] \quad \text{and}$$

$$y_2(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x - (520 \text{ s}^{-1})t] \quad .$$

If one wave has the form $y(x,t) = y_m \sin(kx + \omega t)$ as in y_1, then the other wave must be of the form $y'(x,t) = y_m \sin(kx - \omega t)$ as in y_2. Therefore, the sign in front of ω is minus.

55. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are n_1 loops in the aluminum section of the wire. Then, $L_1 = n_1\lambda_1/2 = n_1v_1/2f$, where λ_1 is the wavelength and v_1 is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where f is the frequency. Thus $f = n_1v_1/2L_1$. A similar expression holds for the steel section: $f = n_2v_2/2L_2$. Since the frequency is the same for the two sections, $n_1v_1/L_1 = n_2v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\tau/\mu_1}$, where μ_1 is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1AL_1$, where ρ_1 is the mass density (mass per unit volume) for aluminum and A is the cross-sectional area of the wire. Thus $\mu_1 = \rho_1AL_1/L_1 = \rho_1A$ and $v_1 = \sqrt{\tau/\rho_1A}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\tau/\rho_2A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$, where A has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\text{m})\sqrt{7.80\times10^3\ \text{kg/m}^3}}{(0.600\,\text{m})\sqrt{2.60\times10^3\ \text{kg/m}^3}} = 2.50.$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is $f = n_1v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1A}$. The tension is provided by the hanging block and is $\tau = mg$, where m is the mass of the block. Thus

$$f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1A}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{(10.0\,\text{kg})(9.80\,\text{m/s}^2)}{(2.60\times10^3\text{kg/m}^3)(1.00\times10^{-6}\,\text{m}^2)}} = 324\,\text{Hz}.$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

56. According to Eq. 16-69, the block mass is inversely proportional to the harmonic number squared. Thus, if the 447 gram block corresponds to harmonic number n then

$$\frac{447}{286.1} = \frac{(n+1)^2}{n^2} = \frac{n^2+2n+1}{n^2} = 1+\frac{2n+1}{n^2} .$$

Therefore, $\frac{447}{286.1} - 1 = 0.5624$ must equal an odd integer ($2n + 1$) divided by a squared integer (n^2). That is, multiplying 0.5624 by a square (such as 1, 4, 9, 16, etc) should give us a number very close (within experimental uncertainty) to an odd number (1, 3, 5, …). Trying this out in succession (starting with multiplication by 1, then by 4, …), we find that multiplication by 16 gives a value very close to 9; we conclude $n = 4$ (so $n^2 = 16$ and $2n + 1 = 9$). Plugging $m = 0.447$ kg, $n = 4$, and the other values from Sample Problem 16-8 into Eq. 16-69, we find $\mu = 0.000845$ kg/m, or 0.845 g/m.

57. Setting $x = 0$ in $y = y_m \sin(kx - \omega t + \phi)$ gives $y = y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a positive "slope" (referring to its t-derivative) at $t = 0$:

$$\frac{dy}{dt} = \frac{d\, y_m \sin(-\omega t + \phi)}{dt} = -y_m \omega \cos(-\omega t + \phi) \quad > 0 \ \text{ at } \ t = 0.$$

This implies that $-\cos(\phi) > 0$ and consequently that ϕ is in either the second or third quadrant. The graph shows (at $t = 0$) $y = 2.00$ mm, and (at some later t) $y_m = 6.00$ mm. Therefore,

$$y = y_m \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi \ = \ \sin^{-1}\left(\tfrac{1}{3}\right) = \ 0.34 \text{ rad} \quad \text{or} \quad 2.8 \text{ rad}$$

(bear in mind that $\sin(\theta) = \sin(\pi - \theta)$), and we must choose $\phi = 2.8$ rad because this is about 161° and is in second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.8 - 2\pi = -3.48$ rad is also an acceptable result.

58. Setting $x = 0$ in $a_y = -\omega^2 y$ (see the solution to part (b) of Sample Problem 16-2) where $y = y_m \sin(kx - \omega t + \phi)$ gives $a_y = -\omega^2 y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a negative "slope" (referring to its t-derivative) at $t = 0$:

$$\frac{d a_y}{dt} = \frac{d(-\omega^2 y_m \sin(-\omega t + \phi))}{dt} = y_m \omega^3 \cos(-\omega t + \phi) < 0 \text{ at } t = 0.$$

This implies that $\cos\phi < 0$ and consequently that ϕ is in either the second or third quadrant. The graph shows (at t = 0) $a_y = -100$ m/s², and (at another t) $a_{max} = 400$ m/s². Therefore,

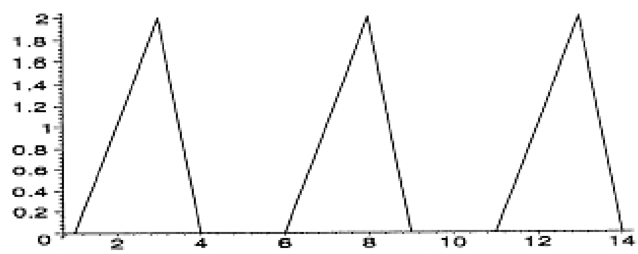
$$a_y = -a_{max} \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \sin^{-1}\left(\tfrac{1}{4}\right) = 0.25 \text{ rad} \quad \text{or} \quad 2.9 \text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi = 2.9$ rad because this is about 166° and is in the second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.9 - 2\pi = -3.4$ rad is also an acceptable result.

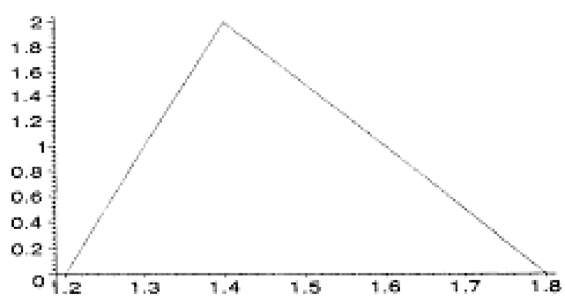
59. (a) Recalling the discussion in §16-5, we see that the speed of the wave given by a function with argument $x - 5.0t$ (where x is in centimeters and t is in seconds) must be 5.0 cm/s.

(b) In part (c), we show several “snapshots” of the wave: the one on the left is as shown in Figure 16–45 (at $t = 0$), the middle one is at $t = 1.0$ s, and the rightmost one is at $t = 2.0$ s. It is clear that the wave is traveling to the right (the $+x$ direction).

(c) The third picture in the sequence below shows the pulse at 2.0 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.



(d) The leading edge of the pulse reaches $x = 10$ cm at $t = (10 - 4.0)/5 = 1.2$ s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement $h = 2$ cm at $t = (10 - 3.0)/5 = 1.4$ s. Finally, the trailing edge of the pulse departs from $x = 10$ cm at $t = (10 - 1.0)/5 = 1.8$ s. Thus, we find for $h(t)$ at $x = 10$ cm (with the horizontal axis, t, in seconds):



60. We compare the resultant wave given with the standard expression (Eq. 16–52) to obtain $k = 20\,\text{m}^{-1} = 2\pi/\lambda, 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 3.0\,\text{mm}$, and $\tfrac{1}{2}\phi = 0.820\,\text{rad}$.

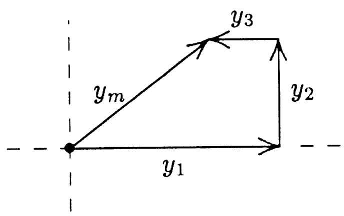
(a) Therefore, $\lambda = 2\pi/k = 0.31$ m.

(b) The phase difference is $\phi = 1.64$ rad.

(c) And the amplitude is $y_m = 2.2$ mm.

61. (a) The phasor diagram is shown here: y_1, y_2, and y_3 represent the original waves and y_m represents the resultant wave. The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$. The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1.$$



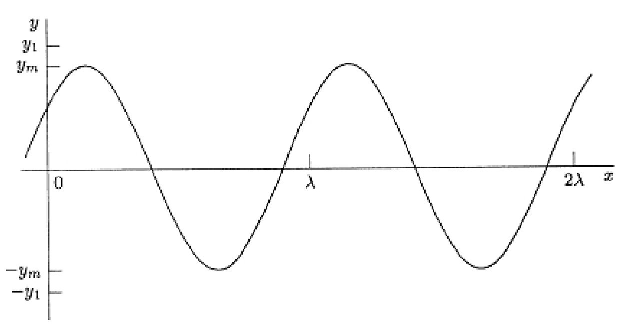
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\frac{y_{mv}}{y_{mh}} = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\frac{3}{4}$$
$$= 0.644 \text{ rad} = 37°.$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644 \text{ rad}).$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



62. We use Eq. 16-52 in interpreting the figure.

(a) Since y' = 6.0 mm when $\phi = 0$, then Eq. 16-52 can be used to determine y_m = 3.0 mm.

(b) We note that y' = 0 when the shift distance is 10 cm; this occurs because $\cos(\phi/2) = 0$ there $\Rightarrow$ $\phi = \pi$ rad or ½ cycle. Since a full cycle corresponds to a distance of one full wavelength, this ½ cycle shift corresponds to a distance of $\lambda/2$. Therefore, λ = 20 cm $\Rightarrow$ $k = 2\pi/\lambda = 31\ \text{m}^{-1}$.

(c) Since f = 120 Hz, $\omega = 2\pi f$ = 754 rad/s $\approx 7.5\times10^2$ rad/s.

(d) The sign in front of ω is minus since the waves are traveling in the $+x$ direction.

The results may be summarized as $y = (3.0\ \text{mm})\sin[(31.4\ \text{m}^{-1})x - (754\ \text{s}^{-1})t]]$ (this applies to each wave when they are in phase).

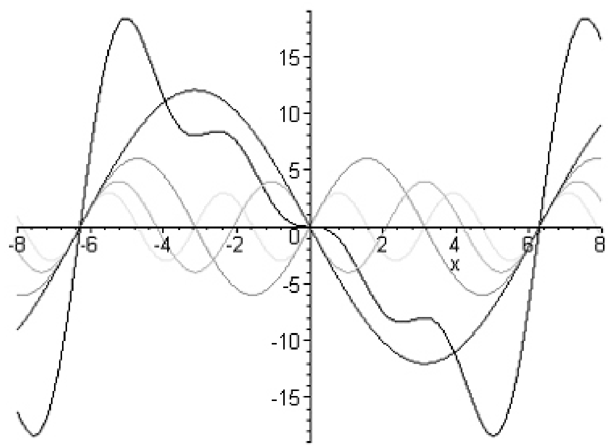
63. We note that $dy/dt = -\omega\cos(kx - \omega t + \phi)$, which we will refer to as $u(x,t)$. so that the ratio of the function $y(x,t)$ divided by $u(x,t)$ is $-\tan(kx - \omega t + \phi)/\omega$. With the given information (for $x = 0$ and $t = 0$) then we can take the inverse tangent of this ratio to solve for the phase constant:

$$\phi = \tan^{-1}\left(\frac{-\omega\, y(0,0)}{u(0,0)}\right) = \tan^{-1}\left(\frac{-(440)(0.0045)}{-0.75}\right) = 1.2 \text{ rad.}$$

64. The plot (at $t = 0$) is shown below. The curve that peaks around $x = -5$ and then descends like a staircase until about $x = +5$ is the resultant wave. This general shape is maintained as time increases, but moves towards the right at the wave speed (which in this example is set at v = 2 units). The individual waves shown in this example are of the form:

$$y_1 = -12 \sin(\tfrac{1}{2} x - t), \qquad y_2 = 6 \sin(x - 2t)$$

$$y_3 = -4 \sin(3/2\, x - 3t), \qquad y_4 = 3 \sin(2x - 4t)\ .$$



65. (a) From the frequency information, we find $\omega = 2\pi f = 10\pi$ rad/s. A point on the rope undergoing simple harmonic motion (discussed in Chapter 15) has maximum speed as it passes through its "middle" point, which is equal to $y_m\omega$. Thus,

$$5.0\text{ m/s} = y_m\omega \;\Rightarrow\; y_m = 0.16\text{ m} \quad .$$

(b) Because of the oscillation being in the *fundamental* mode (as illustrated in Fig. 16-23(a) in the textbook), we have $\lambda = 2L = 4.0$ m. Therefore, the speed of waves along the rope is $v = f\lambda = 20$ m/s. Then, with $\mu = m/L = 0.60$ kg/m, Eq. 16-26 leads to

$$v = \sqrt{\frac{\tau}{\mu}} \;\Rightarrow\; \tau = \mu v^2 = 240\text{ N} \approx 2.4\times10^2\text{ N}.$$

(c) We note that for the fundamental, $k = 2\pi/\lambda = \pi/L$, and we observe that the anti-node having zero displacement at $t = 0$ suggests the use of sine instead of cosine for the simple harmonic motion factor. Now, *if* the fundamental mode is the only one present (so the amplitude calculated in part (a) is indeed the amplitude of the fundamental wave pattern) then we have

$$y = (0.16\text{ m})\sin\left(\frac{\pi x}{2}\right)\sin(10\pi t) = (0.16\text{ m})\sin[(1.57\text{ m}^{-1})x]\sin[(31.4\text{ rad/s})t]$$

66. (a) The displacement of the string is assumed to have the form $y(x, t) = y_m \sin(kx - \omega t)$. The velocity of a point on the string is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and its maximum value is $u_m = \omega y_m$. For this wave the frequency is f = 120 Hz and the angular frequency is $\omega = 2\pi f = 2\pi$ (120 Hz) = 754 rad/s. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00 \times 10^{-3}$ m. The maximum speed is

$$u_m = (754 \text{ rad/s})(5.00 \times 10^{-3} \text{ m}) = 3.77 \text{ m/s}.$$

(b) Consider the string at coordinate x and at time t and suppose it makes the angle θ with the x axis. The tension is along the string and makes the same angle with the x axis. Its transverse component is $\tau_{\text{trans}} = \tau \sin\theta$. Now θ is given by $\tan\theta = \partial y/\partial x = k y_m \cos(kx - \omega t)$ and its maximum value is given by $\tan\theta_m = k y_m$. We must calculate the angular wave number k. It is given by $k = \omega/v$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1}.$$

Thus

$$\tan\theta_m = (27.5\,\text{m}^{-1})(5.00\times10^{-3}\,\text{m}) = 0.138$$

and θ = 7.83°. The maximum value of the transverse component of the tension in the string is τ_{trans} = (90.0 N) sin 7.83° = 12.3 N. We note that $\sin\theta$ is nearly the same as $\tan\theta$ because θ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at x. The transverse component of the tension pulling on it due to the string to the left is $-\tau(\partial y/\partial x) = -\tau k y_m \cos(kx - \omega t)$ and it reaches its maximum value when $\cos(kx - \omega t) = -1$. The wave speed is $u = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and it also reaches its maximum value when $\cos(kx - \omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx - \omega t) = -1$ the value of $\sin(kx - \omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement Δy, the tension does work $\Delta W = \tau_{\text{trans}}\,\Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}}\frac{\Delta y}{\Delta t} = \tau_{\text{trans}}u.$$

P has its maximum value when the transverse component τ_{trans} of the tension and the string speed u have their maximum values. Hence the maximum power is (12.3 N)(3.77 m/s) = 46.4 W.

(e) As shown above $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx - \omega t) = 0$ and $\sin(kx - \omega t) = \pm 1$ at that time. The string displacement is $y = \pm y_m = \pm 0.50$ cm.

67. (a) We take the form of the displacement to be $y(x, t) = y_m \sin(kx - \omega t)$. The speed of a point on the cord is $u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$. The ratio is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on the ratio of the amplitude to the wavelength. Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

68. Let the cross-sectional area of the wire be A and the density of steel be ρ. The tensile stress is given by τ/A where τ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$v_{\max} = \sqrt{\frac{\tau_{\max}}{\mu}} = \sqrt{\frac{\tau_{\max}/A}{\rho}} = \sqrt{\frac{7.00\times10^{8}\ \text{N/m}^2}{7800\,\text{kg/m}^3}} = 3.00\times10^{2}\ \text{m/s}$$

which is indeed independent of the diameter of the wire.

69. (a) The amplitude is y_m = 1.00 cm = 0.0100 m, as given in the problem.

(b) Since the frequency is f = 550 Hz, the angular frequency is $\omega = 2\pi f = 3.46\times10^3$ rad/s.

(c) The angular wave number is $k = \omega / v = (3.46\times10^3 \text{ rad/s})/(330 \text{ m/s}) = 10.5 \text{ rad/m}$.

(d) Since the wave is traveling in the $-x$ direction, the sign in front of ω is plus and the argument of the trig function is $kx + \omega t$.

The results may be summarized as

$$y(x,t) = y_\text{m} \sin(kx+\omega t) = y_\text{m} \sin\left[2\pi f\left(\frac{x}{v}+t\right)\right] = (0.010\,\text{m})\sin\left[2\pi(550\,\text{Hz})\left(\frac{x}{330\,\text{m/s}}+t\right)\right]$$

$$= (0.010\,\text{m}) \sin[(10.5 \text{ rad/s})\, x + (3.46\times10^3 \text{ rad/s})t].$$

70. We write the expression for the displacement in the form $y(x, t) = y_m \sin(kx - \omega t)$.

(a) The amplitude is y_m = 2.0 cm = 0.020 m, as given in the problem.

(b) The angular wave number k is $k = 2\pi/\lambda = 2\pi/(0.10 \text{ m}) = 63 \text{ m}^{-1}$

(c) The angular frequency is $\omega = 2\pi f = 2\pi(400 \text{ Hz}) = 2510 \text{ rad/s} = 2.5\times10^3 \text{ rad/s}$.

(d) A minus sign is used before the ωt term in the argument of the sine function because the wave is traveling in the positive x direction.

Using the results above, the wave may be written as

$$y(x,t) = (2.00\,\text{cm})\sin\left(\left(62.8\,\text{m}^{-1}\right)x - \left(2510\,\text{s}^{-1}\right)t\right).$$

(e) The (transverse) speed of a point on the cord is given by taking the derivative of y:

$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

which leads to a maximum speed of $u_m = \omega y_m$ = (2510 rad/s)(0.020 m) = 50 m/s.

(f) The speed of the wave is

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2510\,\text{rad/s}}{62.8\,\text{rad/m}} = 40\,\text{m/s}.$$

71. We orient one phasor along the x axis with length 3.0 mm and angle 0 and the other at 70° (in the first quadrant) with length 5.0 mm. Adding the components, we obtain

$$(3.0\ \text{mm}) + (5.0\ \text{mm})\cos(70°) = 4.71\,\text{mm}\ \text{ along } x \text{ axis}$$
$$(5.0\ \text{mm})\sin(70°) = 4.70\ \text{mm}\ \text{ along } y \text{ axis.}$$

(a) Thus, amplitude of the resultant wave is $\sqrt{(4.71\ \text{mm})^2 + (4.70\ \text{mm})^2} = 6.7\,\text{mm}$.

(b) And the angle (phase constant) is $\tan^{-1}(4.70/4.71) = 45°$.

72. (a) With length in centimeters and time in seconds, we have

$$u = \frac{dy}{dt} = -60\pi\cos\left(\frac{\pi x}{8} - 4\pi t\right).$$

Thus, when $x = 6$ and $t = \frac{1}{4}$, we obtain

$$u = -60\pi\cos\frac{-\pi}{4} = \frac{-60\pi}{\sqrt{2}} = -133$$

so that the *speed* there is 1.33 m/s.

(b) The numerical coefficient of the cosine in the expression for u is -60π. Thus, the maximum *speed* is 1.88 m/s.

(c) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2\sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

so that when $x = 6$ and $t = \frac{1}{4}$ we obtain $a = -240\pi^2 \sin(-\pi/4)$ which yields $a = 16.7$ m/s^2.

(d) The numerical coefficient of the sine in the expression for a is $-240\pi^2$. Thus, the maximum acceleration is 23.7 m/s^2.

73. (a) Using $v = f\lambda$, we obtain

$$f = \frac{240\,\text{m/s}}{3.2\,\text{m}} = 75\ \text{Hz}.$$

(b) Since frequency is the reciprocal of the period, we find

$$T = \frac{1}{f} = \frac{1}{75\,\text{Hz}} = 0.0133\,\text{s} \approx 13\,\text{ms}.$$

74. By Eq. 16–69, the higher frequencies are integer multiples of the lowest (the fundamental).

(a) The frequency of the second harmonic is $f_2 = 2(440) = 880$ Hz.

(b) The frequency of the third harmonic is and $f_3 = 3(440) = 1320$ Hz.

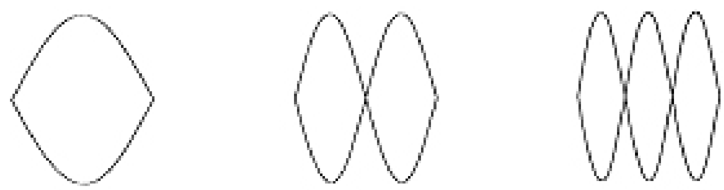
75. We make use of Eq. 16–65 with $L = 120$ cm.

(a) The longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_1 = 2L/1 = 240$ cm.

(b) The second longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_2 = 2L/2 = 120$ cm.

(c) The third longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_3 = 2L/3 = 80.0$ cm.

The three standing waves are shown below:



76. (a) At x = 2.3 m and t = 0.16 s the displacement is

$$y(x,t) = 0.15\sin[(0.79)(2.3) - 13(0.16)]\,\text{m} = -0.039\,\text{m}.$$

(b) We choose y_m = 0.15 m, so that there would be nodes (where the wave amplitude is zero) in the string as a result.

(c) The second wave must be traveling with the same speed and frequency. This implies $k = 0.79\ \text{m}^{-1}$,

(d) and $\omega = 13\ \text{rad/s}$.

(e) The wave must be traveling in $-x$ direction, implying a plus sign in front of ω.

Thus, its general form is $y'(x,t) = (0.15\ \text{m})\sin(0.79x + 13t)$.

(f) The displacement of the standing wave at x = 2.3 m and t = 0.16 s is

$$y(x,t) = -0.039\,\text{m} + (0.15\,\text{m})\sin[(0.79)(2.3) + 13(0.16)] = -0.14\,\text{m}.$$

77. (a) The wave speed is

$$v=\sqrt{\frac{\tau}{\mu}}=\sqrt{\frac{120\,\text{N}}{8.70\times10^{-3}\,\text{kg}/1.50\,\text{m}}}=144\ \text{m/s}.$$

(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50\text{ m}) = 3.00\text{ m}$.

(c) For the two-loop standing wave $\lambda_2 = L = 1.50\text{ m}$.

(d) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144\text{ m/s})/(3.00\text{ m}) = 48.0\text{ Hz}$.

(e) The frequency for the two-loop wave is $f_2 = v/\lambda_2 = (144\text{ m/s})/(1.50\text{ m}) = 96.0\text{ Hz}$.

78. We use $P = \frac{1}{2}\mu v\omega^2 y_m^2 \propto vf^2 \propto \sqrt{\tau} f^2$.

(a) If the tension is quadrupled, then

$$P_2 = P_1\sqrt{\frac{\tau_2}{\tau_1}} = P_1\sqrt{\frac{4\tau_1}{\tau_1}} = 2P_1.$$

(b) If the frequency is halved, then

$$P_2 = P_1\left(\frac{f_2}{f_1}\right)^2 = P_1\left(\frac{f_1/2}{f_1}\right)^2 = \frac{1}{4}P_1.$$

79. We use Eq. 16-2, Eq. 16-5, Eq. 16-9, Eq. 16-13, and take the derivative to obtain the transverse speed u.

(a) The amplitude is y_m = 2.0 mm.

(b) Since ω = 600 rad/s, the frequency is found to be f = 600/2π ≈ 95 Hz.

(c) Since k = 20 rad/m, the velocity of the wave is $v = \omega/k$ = 600/20 = 30 m/s in the +x direction.

(d) The wavelength is λ = 2π/k ≈ 0.31 m, or 31 cm.

(e) We obtain

$$u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t) \Rightarrow u_m = \omega y_m$$

so that the maximum transverse speed is u_m = (600)(2.0) = 1200 mm/s, or 1.2 m/s.

80. (a) The frequency is $f = 1/T = 1/4$ Hz, so $v = f\lambda = 5.0$ cm/s.

(b) We refer to the graph to see that the maximum transverse speed (which we will refer to as u_m) is 5.0 cm/s. Recalling from Ch. 11 the simple harmonic motion relation $u_m = y_m\omega = y_m 2\pi f$, we have

$$5.0 = y_m\left(2\pi\frac{1}{4}\right) \quad \Rightarrow \quad y_m = 3.2 \text{ cm.}$$

(c) As already noted, $f = 0.25$ Hz.

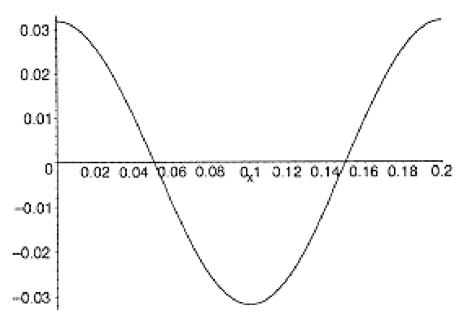
(d) Since $k = 2\pi/\lambda$, we have $k = 10\pi$ rad/m. There must be a sign difference between the t and x terms in the argument in order for the wave to travel to the right. The figure shows that at $x = 0$, the transverse velocity function is $0.050 \sin\frac{\pi}{2}t$. Therefore, the function $u(x,t)$ is

$$u(x,t) = 0.050\sin\left(\frac{\pi}{2}t - 10\pi x\right)$$

with lengths in meters and time in seconds. Integrating this with respect to time yields

$$y(x,t) = -\frac{2(0.050)}{\pi}\cos\left(\frac{\pi}{2}t - 10\pi x\right) + C$$

where C is an integration constant (which we will assume to be zero). The sketch of this function at $t = 2.0$ s for $0 \le x \le 0.20$ m is shown below.



81. Using Eq. 16-50, we have

$$y' = \left[0.60\cos\frac{\pi}{6}\right]\sin\left(5\pi x - 200\pi t + \frac{\pi}{6}\right)$$

with length in meters and time in seconds (see Eq. 16-55 for comparison).

(a) The amplitude is seen to be

$$0.60\cos\frac{\pi}{6} = 0.3\sqrt{3} = 0.52\,\text{m}.$$

(b) Since $k = 5\pi$ and $\omega = 200\pi$, then (using Eq. 16-12) $v = \frac{\omega}{k} = 40\,\text{m/s}$.

(c) $k = 2\pi/\lambda$ leads to $\lambda = 0.40$ m.

82. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where λ is the wavelength and L is the length of the string. The wavelength is related to the frequency f and wave speed v by $\lambda = v/f$, so $L/2 = v/f$ and

$$L = 2v/f = 2(400 \text{ m/s})/(600 \text{ Hz}) = 1.3 \text{ m}.$$

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx) \cos(\omega t)$, where y_m is the maximum displacement, k is the angular wave number, and ω is the angular frequency. The angular wave number is $k = 2\pi/\lambda = 2\pi f/v = 2\pi(600 \text{ Hz})/(400 \text{ m/s}) = 9.4\text{m}^{-1}$ and the angular frequency is $\omega = 2\pi f = 2\pi(600 \text{ Hz}) = 3800$ rad/s. y_m is 2.0 mm. The displacement is given by

$$y(x,t) = (2.0 \,\text{mm}) \sin[(9.4 \,\text{m}^{-1})x] \cos[(3800 \,\text{s}^{-1})t].$$

83. To oscillate in four loops means $n = 4$ in Eq. 16-65 (treating both ends of the string as effectively "fixed"). Thus, $\lambda = 2(0.90 \text{ m})/4 = 0.45$ m. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is $\mu = m/L = (0.044 \text{ kg})/(0.90 \text{ m}) = 0.049$ kg/m. Thus, using Eq. 16-26, we obtain the tension:

$$\tau = v^2 \mu = (27)^2(0.049) = 36 \text{ N}.$$

84. Repeating the steps of Eq. 16-47 → Eq. 16-53, but applying

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

(see Appendix E) instead of Eq. 16-50, we obtain $y' = [0.10\cos\pi x]\cos 4\pi t$, with SI units understood.

(a) For non-negative x, the smallest value to produce cos πx = 0 is x = 1/2, so the answer is x = 0.50 m.

(b) Taking the derivative,

$$u' = \frac{dy'}{dt} = [0.10\cos\pi x](-4\pi\sin 4\pi t)$$

We observe that the last factor is zero when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ Thus, the value of the first time the particle at x=0 has zero velocity is t = 0.

(c) Using the result obtained in (b), the second time where the velocity at x =0 vanishes would be t = 0.25 s,

(d) and the third time is t = 0.50 s.

85. (a) This distance is determined by the longitudinal speed:

$$d_\ell = v_\ell t = (2000\,\text{m/s})(40\times10^{-6}\,\text{s}) = 8.0\times10^{-2}\,\text{m}.$$

(b) Assuming the acceleration is constant (justified by the near-straightness of the curve $a = 300/40 \times 10^{-6}$) we find the stopping distance d:

$$v^2 = v_o^2 + 2ad \Rightarrow d = \frac{(300)^2 (40\times10^{-6})}{2(300)}$$

which gives $d = 6.0 \times 10^{-3}$ m. This and the radius r form the legs of a right triangle (where r is opposite from $\theta = 60°$). Therefore,

$$\tan 60° = \frac{r}{d} \Rightarrow r = d\tan 60° = 1.0\times10^{-2}\,\text{m}.$$

86. (a) Let the displacements of the wave at (y,t) be $z(y,t)$. Then $z(y,t) = z_m \sin(ky - \omega t)$, where $z_m = 3.0$ mm, $k = 60$ cm^{-1}, and $\omega = 2\pi/T = 2\pi/0.20$ s $= 10\pi$ s^{-1}. Thus

$$z(y,t) = (3.0\,\text{mm})\sin\left[\left(60\,\text{cm}^{-1}\right)y - \left(10\pi\,\text{s}^{-1}\right)t\right].$$

(b) The maximum transverse speed is $u_m = \omega z_m = (2\pi/0.20\,\text{s})(3.0\,\text{mm}) = 94\,\text{mm/s}$.

87. (a) The wave speed is

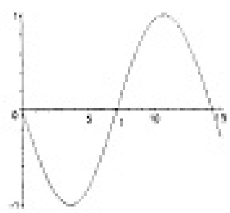
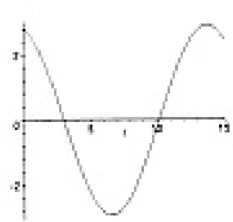
$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{k\Delta\ell}{m/(\ell+\Delta\ell)}}=\sqrt{\frac{k\Delta\ell(\ell+\Delta\ell)}{m}}.$$

(b) The time required is

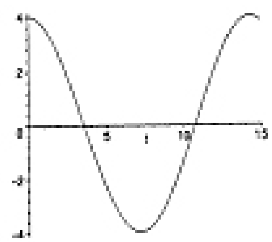
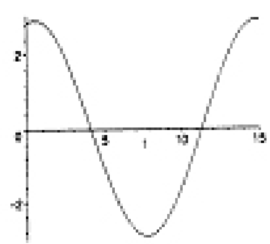
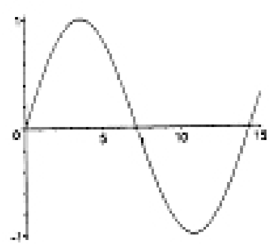
$$t=\frac{2\pi(\ell+\Delta\ell)}{v}=\frac{2\pi(\ell+\Delta\ell)}{\sqrt{k\Delta\ell(\ell+\Delta\ell)/m}}=2\pi\sqrt{\frac{m}{k}}\sqrt{1+\frac{\ell}{\Delta\ell}}.$$

Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$; and if $\ell/\Delta\ell \ll 1$, then $t \simeq 2\pi\sqrt{m/k} = \text{const.}$

88. (a) The wave number for each wave is $k = 25.1/\text{m}$, which means $\lambda = 2\pi/k = 250.3$ mm. The angular frequency is $\omega = 440/\text{s}$; therefore, the period is $T = 2\pi/\omega = 14.3$ ms. We plot the superposition of the two waves $y = y_1 + y_2$ over the time interval $0 \le t \le 15$ ms. The first two graphs below show the oscillatory behavior at $x = 0$ (the graph on the left) and at $x = \lambda/8 \approx 31$ mm. The time unit is understood to be the millisecond and vertical axis (y) is in millimeters.

The following three graphs show the oscillation at $x = \lambda/4 = 62.6$ mm ≈ 63 mm (graph on the left), at $x = 3\lambda/8 \approx 94$ mm (middle graph), and at $x = \lambda/2 \approx 125$ mm.

(b) We can think of wave y_1 as being made of two smaller waves going in the same direction, a wave y_{1a} of amplitude 1.50 mm (the same as y_2) and a wave y_{1b} of amplitude 1.00 mm. It is made clear in §16-12 that two equal-magnitude oppositely-moving waves form a standing wave pattern. Thus, waves y_{1a} and y_2 form a standing wave, which leaves y_{1b} as the remaining traveling wave. Since the argument of y_{1b} involves the subtraction $kx - \omega t$, then y_{1b} travels in the $+x$ direction.

(c) If y_2 (which travels in the $-x$ direction, which for simplicity will be called "leftward") had the larger amplitude, then the system would consist of a standing wave plus a leftward moving wave. A simple way to obtain such a situation would be to interchange the amplitudes of the given waves.

(d) Examining carefully the vertical axes, the graphs above certainly suggest that the largest amplitude of oscillation is $y_{max} = 4.0$ mm and occurs at $x = \lambda/4 = 62.6$ mm.

(e) The smallest amplitude of oscillation is $y_{\min} = 1.0$ mm and occurs at $x = 0$ and at $x = \lambda/2 = 125$ mm.

(f) The largest amplitude can be related to the amplitudes of y_1 and y_2 in a simple way: $y_{\max} = y_{1m} + y_{2m}$, where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

(g) The smallest amplitudes is $y_{\min} = y_{1m} - y_{2m}$, where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

89. (a) For visible light

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^{8}\ \text{m/s}}{700\times10^{-9}\,\text{m}} = 4.3\times10^{14}\ \text{Hz}$$

and

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^{8}\ \text{m/s}}{400\times10^{-9}\,\text{m}} = 7.5\times10^{14}\ \text{Hz.}$$

(b) For radio waves

$$\lambda_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^{8}\ \text{m/s}}{300\times10^{6}\,\text{Hz}} = 1.0\,\text{m}$$

and

$$\lambda_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.5\times10^{6}\,\text{Hz}} = 2.0\times10^{2}\,\text{m.}$$

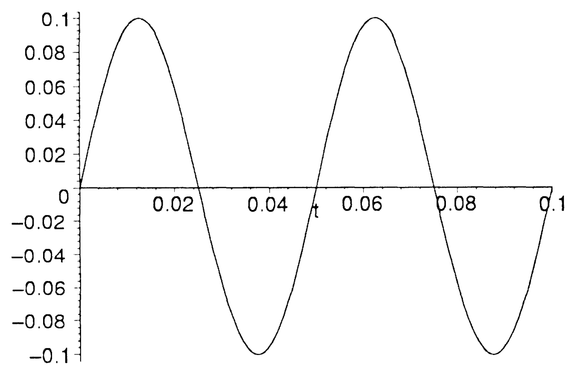
(c) For X rays

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^{8}\ \text{m/s}}{5.0\times10^{-9}\,\text{m}} = 6.0\times10^{16}\,\text{Hz}$$

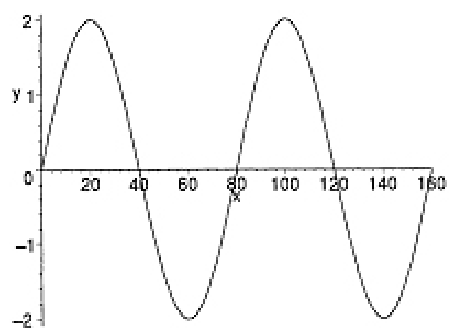
and

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.0\times10^{-11}\,\text{m}} = 3.0\times10^{19}\ \text{Hz.}$$

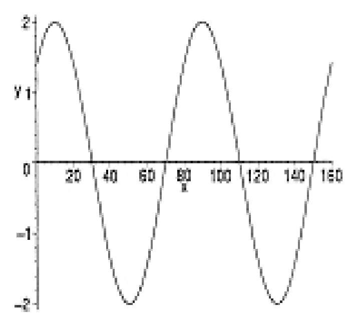
90. It is certainly possible to simplify (in the trigonometric sense) the expressions at x = 3 m (since k = 1/2 in inverse-meters), but there is no particular need to do so, if the goal is to plot the time-dependence of the wave superposition at this value of x. Still, it is worth mentioning the end result of such simplification if it provides some insight into the nature of the graph (shown below): $y_1 + y_2 = (0.10\text{ m})\sin(40\pi t)$ with t in seconds.



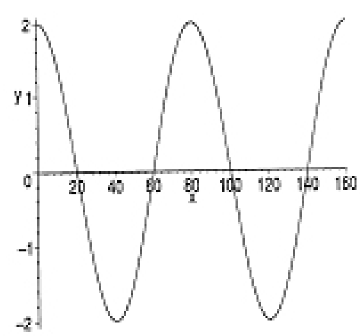
91. (a) Centimeters are to be understood as the length unit and seconds as the time unit. Making sure our (graphing) calculator is in radians mode, we find



(b) The previous graph is at $t = 0$, and this next one is at $t = 0.050$ s.



And the final one, shown below, is at $t = 0.010$ s.



(c) The wave can be written as $y(x,t) = y_m \sin(kx + \omega t)$, where $v = \omega / k$ is the speed of propagation. From the problem statement, we see that $\omega = 2\pi / 0.40 = 5\pi$ rad/s and $k = 2\pi / 80 = \pi / 40$ rad/cm. This yields $v = 2.0 \times 10^2$ cm/s $= 2.0$ m/s

(d) These graphs (as well as the discussion in the textbook) make it clear that the wave is traveling in the $-x$ direction.

92. We consider an infinitesimal segment of a string oscillating in a standing wave pattern. Its length is dx and its mass is dm = μdx, where μ is its linear mass density. If it is moving with speed u its kinetic energy is $\mathrm{d}K = \frac{1}{2}u^2\mathrm{d}m = \frac{1}{2}\mu u^2\mathrm{d}x$. If the segment is located at x its displacement at time t is $y = 2y_m \sin(kx)\cos(\omega t)$ and its velocity is $u = \partial y/\partial t = -2\omega y_m \sin(kx)\sin(\omega t)$, so its kinetic energy is

$$\mathrm{d}K = \left(\frac{1}{2}\right)\left(4\mu\omega^2 y_m^2\right)\sin^2(kx)\ \sin^2(\omega t) = 2\mu\omega^2 y_m^2 \sin^2(kx)\sin^2(\omega t).$$

Here y_m is the amplitude of each of the traveling waves that combine to form the standing wave. The infinitesimal segment has maximum kinetic energy when $\sin^2(\omega t) = 1$ and the maximum kinetic energy is given by the differential amount

$$dK_m = 2\mu\omega^2 y_m^2 \sin^2(kx).$$

Note that every portion of the string has its maximum kinetic energy at the same time although the values of these maxima are different for different parts of the string. If the string is oscillating with n loops, the length of string in any one loop is L/n and the kinetic energy of the loop is given by the integral

$$K_m = 2\mu\omega^2 y_m^2 \int_0^{L/n} \sin^2(kx)\ dx.$$

We use the trigonometric identity $\sin^2(kx) = \frac{1}{2}[1 + 2\cos(2kx)]$ to obtain

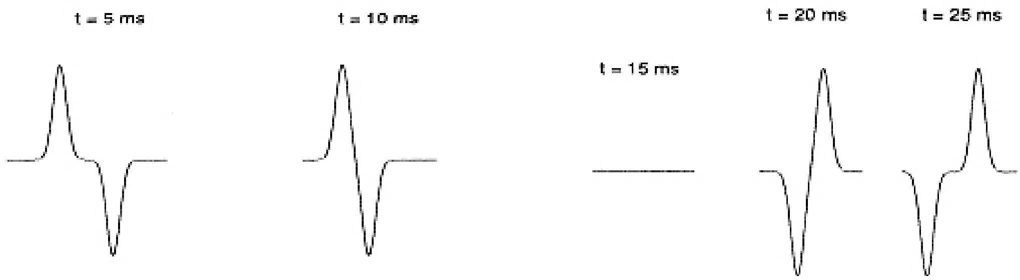
$$K_m = \mu\omega^2 y_m^2 \int_0^{L/n} [1 + 2\cos(2kx)]\mathrm{d}x = \mu\omega^2 y_m^2\left[\frac{L}{n} + \frac{1}{k}\sin\frac{2kL}{n}\right].$$

For a standing wave of n loops the wavelength is $\lambda = 2L/n$ and the angular wave number is $k = 2\pi/\lambda = n\pi/L$, so $2kL/n = 2\pi$ and $\sin(2kL/n) = 0$, no matter what the value of n. Thus,

$$K_m = \frac{\mu\omega^2 y_m^2 L}{n}.$$

To obtain the expression given in the problem statement, we first make the substitutions $\omega = 2\pi f$ and $L/n = \lambda/2$, where f is the frequency and λ is the wavelength. This produces $K_m = 2\pi^2\mu y_m^2 f^2\lambda$. We now substitute the wave speed v for $f\lambda$ and obtain $K_m = 2\pi^2\mu y_m^2 fv$.

93. (a) We note that each pulse travels 1 cm during each Δt = 5 ms interval. Thus, in these first two pictures, their peaks are closer to each other by 2 cm, successively. And the next pictures show the (momentary) complete cancellation of the visible pattern at t = 15 ms, and the pulses moving away from each other after that.



(b) The particles of the string are moving rapidly as they pass (transversely) through their equilibrium positions; the energy at t = 15 ms is purely kinetic.

94. We refer to the points where the rope is attached as A and B, respectively. When A and B are not displaced horizontal, the rope is in its initial state (neither stretched (under tension) nor slack). If they are displaced away from each other, the rope is clearly stretched. When A and B are displaced in the same direction, by amounts (in absolute value) $|\xi_A|$ and $|\xi_B|$, then if $|\xi_A| < |\xi_B|$ then the rope is stretched, and if $|\xi_A| > |\xi_B|$ the rope is slack. We must be careful about the case where one is displaced but the other is not, as will be seen below.

(a) The standing wave solution for the shorter cable, appropriate for the initial condition $\xi = 0$ at $t = 0$, and the boundary conditions $\xi = 0$ at $x = 0$ and $x = L$ (the x axis runs vertically here), is $\xi_A = \xi_m \sin(k_A x) \sin(\omega_A t)$. The angular frequency is $\omega_A = 2\pi/T_A$, and the wave number is $k_A = 2\pi/\lambda_A$ where $\lambda_A = 2L$ (it begins oscillating in its fundamental mode) where the point of attachment is $x = L/2$. The displacement of what we are calling point A at time $t = \eta T_A$ (where η is a pure number) is

$$\xi_A = \xi_m \sin\left(\frac{2\pi}{2L}\frac{L}{2}\right)\sin\left(\frac{2\pi}{T_A}\eta T_A\right) = \xi_m \sin(2\pi\eta).$$

The fundamental mode for the longer cable has wavelength $\lambda_B = 2\lambda_A = 2(2L) = 4L$, which implies (by $v = f\lambda$ and the fact that both cables support the same wave speed v) that $f_B = \frac{1}{2} f_A$ or $\omega_B = \frac{1}{2}\omega_A$. Thus, the displacement for point B is

$$\xi_B = \xi_m \sin\left(\frac{2\pi}{4L}\frac{L}{2}\right)\sin\left(\frac{1}{2}\left(\frac{2\pi}{T_A}\right)\eta T_A\right) = \frac{\xi_m}{\sqrt{2}}\sin(\pi\eta).$$

Running through the possibilities $\left(\eta = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, \text{and } 2\right)$ we find the rope is under tension in the following cases. The first case is one we must be very careful about in our reasoning, since A is not displaced but B is displaced in the positive direction; we interpret that as the direction *away from A* (rightwards in the figure) — thus making the rope stretch.

$$\eta = \frac{1}{2} \qquad \xi_A = 0 \qquad \xi_B = \frac{\xi_m}{\sqrt{2}} > 0$$

$$\eta = \frac{3}{4} \qquad \xi_A = -\xi_m < 0 \qquad \xi_B = \frac{\xi_m}{2} > 0$$

$$\eta = \frac{7}{4} \qquad \xi_A = -\xi_m < 0 \qquad \xi_B = -\frac{\xi_m}{2} < 0$$

where in the last case they are both displaced leftward but A more so than B so that the rope is indeed stretched.

(b) The values of η (where we have defined $\eta = t/T_A$) which reproduce the initial state are

$$\eta = 1 \quad \xi_A = 0 \quad \xi_B = 0 \quad \text{and}$$
$$\eta = 2 \quad \xi_B = 0 \quad \xi_B = 0.$$

(c) The values of η for which the rope is slack are given below. In the first case, both displacements are to the right, but point A is farther to the right than B. In the second case, they are displaced towards each other.

$$\eta = \frac{1}{4} \quad \xi_A = x_m > 0 \quad \xi_B = \frac{\xi_m}{\sqrt{2}} > 0$$
$$\eta = \frac{5}{4} \quad \xi_A = \xi_m > 0 \quad \xi_B = -\frac{\xi_m}{2} < 0$$
$$\eta = \frac{3}{2} \quad \xi_A = 0 \qquad \xi_B = -\frac{\xi_m}{\sqrt{2}} < 0$$

where in the third case B is displaced leftward toward the undisplaced point A.

(d) The first design works effectively to damp fundamental modes of vibration in the two cables (especially in the shorter one which would have an anti-node at that point), whereas the second one only damps the fundamental mode in the longer cable.