1. Each atom has a mass of $m = M/N_A$, where M is the molar mass and N_A is the Avogadro constant. The molar mass of arsenic is 74.9 g/mol or 74.9×10^{-3} kg/mol. 7.50×10^{24} arsenic atoms have a total mass of (7.50×10^{24}) $(74.9 \times 10^{-3}$ kg/mol$)/(6.02 \times 10^{23}$ mol$^{-1})$ = 0.933 kg.

2. (a) Eq. 19-3 yields $n = M_{\text{sam}}/M = 2.5/197 = 0.0127$ mol.

(b) The number of atoms is found from Eq. 19-2:

$$N = nN_{\text{A}} = (0.0127)(6.02 \times 10^{23}) = 7.64 \times 10^{21}.$$

3. (a) We solve the ideal gas law $pV = nRT$ for n:

$$n = \frac{pV}{RT} = \frac{(100\,\text{Pa})(1.0\times 10^{-6}\,\text{m}^3)}{(8.31\,\text{J/mol}\cdot\text{K})(220\,\text{K})} = 5.47\times 10^{-8}\,\text{mol}.$$

(b) Using Eq. 19-2, the number of molecules N is

$$N = nN_\text{A} = (5.47\times 10^{-6}\,\text{mol})\ (6.02\times 10^{23}\,\text{mol}^{-1}) = 3.29\times 10^{16}\ \text{molecules}.$$

4. With $V = 1.0 \times 10^{-6}$ m^3, $p = 1.01 \times 10^{-13}$ Pa, and $T = 293$ K, the ideal gas law gives

$$n = \frac{pV}{RT} = \frac{\left(1.01 \times 10^{-13}\ \text{Pa}\right)\left(1.0 \times 10^{-6}\ \text{m}^3\right)}{\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(293\ \text{K}\right)} = 4.1 \times 10^{-23}\ \text{mole}.$$

Consequently, Eq. 19-2 yields $N = nN_A = 25$ molecules. We can express this as a ratio (with V now written as 1 cm^3) $N/V = 25$ molecules/cm^3.

5. (a) In solving $pV = nRT$ for n, we first convert the temperature to the Kelvin scale: $T = (40.0 + 273.15)\,\text{K} = 313.15\ \text{K}$. And we convert the volume to SI units: 1000 cm^3 = $1000 \times 10^{-6}\ \text{m}^3$. Now, according to the ideal gas law,

$$n = \frac{pV}{RT} = \frac{\left(1.01 \times 10^5\ \text{Pa}\right)\left(1000 \times 10^{-6}\ \text{m}^3\right)}{\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(313.15\,\text{K}\right)} = 3.88 \times 10^{-2}\ \text{mol}.$$

(b) The ideal gas law $pV = nRT$ leads to

$$T = \frac{pV}{nR} = \frac{\left(1.06 \times 10^5\ \text{Pa}\right)\left(1500 \times 10^{-6}\ \text{m}^3\right)}{\left(3.88 \times 10^{-2}\ \text{mol}\right)\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 493\,\text{K}.$$

We note that the final temperature may be expressed in degrees Celsius as 220°C.

6. Since (standard) air pressure is 101 kPa, then the initial (absolute) pressure of the air is p_i = 266 kPa. Setting up the gas law in ratio form (where $n_i = n_f$ and thus cancels out — see Sample Problem 19-1), we have

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \Rightarrow p_f = (266\,\text{kPa})\left(\frac{1.64\times10^{-2}\,\text{m}^3}{1.67\times10^{-2}\,\text{m}^3}\right)\left(\frac{300\,\text{K}}{273\,\text{K}}\right)$$

which yields p_f = 287 kPa. Expressed as a gauge pressure, we subtract 101 kPa and obtain 186 kPa.

7. (a) With $T = 283$ K, we obtain

$$n = \frac{pV}{RT} = \frac{\left(100 \times 10^3\,\text{Pa}\right)\left(2.50\,\text{m}^3\right)}{\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(283\,\text{K}\right)} = 106\,\text{mol.}$$

(b) We can use the answer to part (a) with the new values of pressure and temperature, and solve the ideal gas law for the new volume, or we could set up the gas law in ratio form as in Sample Problem 19-1 (where $n_i = n_f$ and thus cancels out):

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \Rightarrow V_f = \left(2.50\,\text{m}^3\right)\left(\frac{100\,\text{kPa}}{300\,\text{kPa}}\right)\left(\frac{303\,\text{K}}{283\,\text{K}}\right)$$

which yields a final volume of $V_f = 0.892$ m^3.

8. (a) Eq. 19-45 (which gives 0) implies $Q = W$. Then Eq. 19-14, with $T = (273 + 30.0)$K leads to gives $Q = -3.14 \times 10^3$ J, or $|Q| = 3.14 \times 10^3$ J.

(b) That negative sign in the result of part (a) implies the transfer of heat is *from* the gas.

9. The pressure p_1 due to the first gas is $p_1 = n_1RT/V$, and the pressure p_2 due to the second gas is $p_2 = n_2RT/V$. So the total pressure on the container wall is

$$p = p_1 + p_2 = \frac{n_1RT}{V} + \frac{n_2RT}{V} = \left(n_1 + n_2\right)\frac{RT}{V}.$$

The fraction of P due to the second gas is then

$$\frac{p_2}{p} = \frac{n_2RT/V}{\left(n_1+n_2\right)\left(RT/V\right)} = \frac{n_2}{n_1+n_2} = \frac{0.5}{2+0.5} = 0.2.$$

10. Using Eq. 19-14, we note that since it is an isothermal process (involving an ideal gas) then

$$Q = W = nRT\ln(V_f/V_i)$$

applies at any point on the graph. An easy one to read is Q = 1000 J and V_f = 0.30 m^3, and we can also infer from the graph that V_i = 0.20 m^3. We are told that n = 0.825 mol, so the above relation immediately yields T = 360 K.

11. Since the pressure is constant the work is given by $W = p(V_2 - V_1)$. The initial volume is $V_1 = (AT_1 - BT_1^2)/p$, where T_1=315 K is the initial temperature, A =24.9 J/K and B=0.00662 J/K^2. The final volume is $V_2 = (AT_2 - BT_2^2)/p$, where T_2=315 K. Thus

$$
\begin{aligned}
W &= A(T_2 - T_1) - B(T_2^2 - T_1^2) \\
&= (24.9\text{ J/K})(325\text{ K} - 315\text{ K}) - (0.00662\text{ J/K}^2)[(325\text{ K})^2 - (315\text{ K})^2] = 207\text{ J.}
\end{aligned}
$$

12. (a) At point *a*, we know enough information to compute *n*:

$$n = \frac{pV}{RT} = \frac{(2500\,\text{Pa})(1.0\,\text{m}^3)}{(8.31\text{ J/mol}\cdot\text{K})\,(200\,\text{K})} = 1.5\,\text{mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and volume, and solve the ideal gas law for the new temperature, or we could set up the gas law as in Sample Problem 19-1 in terms of ratios (note: $n_a = n_b$ and cancels out):

$$\frac{p_b V_b}{p_a V_a} = \frac{T_b}{T_a} \Rightarrow T_b = (200\,\text{K})\left(\frac{7.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at *b* of $T_b = 1.8\times10^3$ K.

(c) As in the previous part, we choose to approach this using the gas law in ratio form (see Sample Problem 19-1):

$$\frac{p_c V_c}{p_a V_a} = \frac{T_c}{T_a} \Rightarrow T_c = (200\,\text{K})\left(\frac{2.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at *c* of $T_c = 6.0\times10^2$ K.

(d) The net energy added to the gas (as heat) is equal to the net work that is done as it progresses through the cycle (represented as a right triangle in the *pV* diagram shown in Fig. 19-19). This, in turn, is related to ± "area" inside that triangle (with area = $\frac{1}{2}$(base)(height)), where we choose the plus sign because the volume change at the largest pressure is an *increase*. Thus,

$$Q_\text{net} = W_\text{net} = \frac{1}{2}\,(2.0\,\text{m}^3)\,(5.0\times10^3\,\text{Pa}) = 5.0\times10^3\ \text{J}.$$

13. Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i},$$

where the ideal gas law $pV = nRT$ was used to replace p with nRT/V. Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with p_iV_i to obtain

$$W = p_iV_i \ln \frac{p_i}{p_f}.$$

Since the initial gauge pressure is 1.03×10^5 Pa, $p_i = 1.03 \times 10^5 \text{ Pa} + 1.013 \times 10^5 \text{ Pa} = 2.04 \times 10^5$ Pa. The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus

$$W = \left(2.04 \times 10^5 \text{ Pa}\right)\left(0.14 \text{ m}^3\right) \ln\left(\frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}}\right) = 2.00 \times 10^4 \text{ J}.$$

During the constant pressure portion of the process the work done by the gas is $W = p_f(V_i - V_f)$. The gas starts in a state with pressure p_f, so this is the pressure throughout this portion of the process. We also note that the volume decreases from V_f to V_i. Now $V_f = p_iV_i/p_f$, so

$$\begin{aligned} W &= p_f\left(V_i - \frac{p_iV_i}{p_f}\right) = \left(p_f - p_i\right)V_i = \left(1.013 \times 10^5 \text{ Pa} - 2.04 \times 10^5 \text{ Pa}\right)\left(0.14 \text{ m}^3\right) \\ &= -1.44 \times 10^4 \text{ J}. \end{aligned}$$

The total work done by the gas over the entire process is

$$W = 2.00 \times 10^4 \text{ J} - 1.44 \times 10^4 \text{ J} = 5.60 \times 10^3 \text{ J}.$$

14. We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If d is the depth of the lake and ρ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho g d$, where p_0 is atmospheric pressure. Since $p_1 V_1 = nRT_1$, the number of moles of gas in the bubble is $n = p_1 V_1 / RT_1 = (p_0 + \rho g d) V_1 / RT_1$, where V_1 is the volume of the bubble at the bottom of the lake and T_1 is the temperature there. At the surface of the lake the pressure is p_0 and the volume of the bubble is $V_2 = nRT_2/p_0$. We substitute for n to obtain

$$\begin{aligned}
V_2 &= \frac{T_2}{T_1}\,\frac{p_0 + \rho g d}{p_0}\,V_1 \\
&= \left(\frac{293\,\text{K}}{277\,\text{K}}\right)\left(\frac{1.013\times10^5\ \text{Pa} + \left(0.998\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(40\,\text{m}\right)}{1.013\times10^5\ \text{Pa}}\right)\left(20\,\text{cm}^3\right) \\
&= 1.0\times10^2\ \text{cm}^3.
\end{aligned}$$

15. When the valve is closed the number of moles of the gas in container A is $n_A = p_A V_A / R T_A$ and that in container B is $n_B = 4 p_B V_A / R T_B$. The total number of moles in both containers is then

$$n = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4 p_B}{T_B}\right) = \text{const.}$$

After the valve is opened the pressure in container A is $p'_A = R n'_A T_A / V_A$ and that in container B is $p'_B = R n'_B T_B / 4 V_A$. Equating p'_A and p'_B, we obtain $R n'_A T_A / V_A = R n'_B T_B / 4 V_A$, or $n'_B = (4 T_A / T_B) n'_A$. Thus,

$$n = n'_A + n'_B = n'_A\left(1 + \frac{4 T_A}{T_B}\right) = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4 p_B}{T_B}\right).$$

We solve the above equation for n'_A:

$$n'_A = \frac{V}{R}\frac{\left(p_A / T_A + 4 p_B / T_B\right)}{\left(1 + 4 T_A / T_B\right)}.$$

Substituting this expression for n'_A into $p' V_A = n'_A R T_A$, we obtain the final pressure:

$$p' = \frac{n'_A R T_A}{V_A} = \frac{p_A + 4 p_B T_A / T_B}{1 + 4 T_A / T_B} = 2.0 \times 10^5 \text{ Pa}.$$

16. The molar mass of argon is 39.95 g/mol. Eq. 19–22 gives

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31\,\text{J/mol}\cdot\text{K})(313\,\text{K})}{39.95\times10^{-3}\,\text{kg/mol}}} = 442\,\text{m/s}.$$

17. According to kinetic theory, the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where T is the temperature and M is the molar mass. See Eq. 19-34. According to Table 19-1, the molar mass of molecular hydrogen is 2.02 g/mol = 2.02×10^{-3} kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3\,(8.31\,\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{2.02\times10^{-3}\ \text{kg/mol}}} = 1.8\times10^{2}\ \text{m/s}.$$

18. Appendix F gives $M = 4.00 \times 10^{-3}$ kg/mol (Table 19-1 gives this to fewer significant figures). Using Eq. 19-22, we obtain

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\,(8.31\ \text{J/mol}\cdot\text{K})(1000\ \text{K})}{4.00\times 10^{-3}\ \text{kg/mol}}} = 2.50\times 10^{3}\ \text{m/s}.$$

19. Table 19-1 gives M = 28.0 g/mol for nitrogen. This value can be used in Eq. 19-22 with T in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using v for v_{rms}:

$$\frac{v_2}{v_1} = \frac{\sqrt{3RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}.$$

(a) With $T_2 = (20.0 + 273.15)\ \text{K} \approx 293\ \text{K}$, we obtain

$$v_2 = (517\ \text{m/s})\sqrt{\frac{293\ \text{K}}{300\ \text{K}}} = 511\ \text{m/s}.$$

(b) In this case, we set $v_3 = \frac{1}{2}v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for T_3:

$$T_3 = T_2\left(\frac{v_3}{v_2}\right)^2 = (293\ \text{K})\left(\frac{1}{2}\right)^2 = 73.0\ \text{K}$$

which we write as 73.0 – 273 = – 200°C.

(c) Now we have $v_4 = 2v_2$ and obtain

$$T_4 = T_2\left(\frac{v_4}{v_2}\right)^2 = (293\ \text{K})(4) = 1.17 \times 10^3\ \text{K}$$

which is equivalent to 899°.

20. First we rewrite Eq. 19-22 using Eq. 19-4 and Eq. 19-7:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(kN_{\text{A}}\right)T}{\left(mN_{\text{A}}\right)}} = \sqrt{\frac{3kT}{M}}.$$

The mass of the electron is given in the problem, and $k = 1.38 \times 10^{-23}$ J/K is given in the textbook. With $T = 2.00 \times 10^6$ K, the above expression gives $v_{\text{rms}} = 9.53 \times 10^6$ m/s. The pressure value given in the problem is not used in the solution.

21. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is $2mv\cos\theta$, where m is the mass of the molecule, v is its speed, and θ is the angle between its velocity and the normal to the wall. If N molecules collide with the wall, then the change in their total momentum is $2Nmv\cos\theta$, and if the total time taken for the collisions is Δt, then the average rate of change of the total momentum is $2(N/\Delta t)mv\cos\theta$. This is the average force exerted by the N molecules on the wall, and the pressure is the average force per unit area:

$$\begin{aligned}
p &= \frac{2}{A}\left(\frac{N}{\Delta t}\right) mv\cos\theta \\
&= \left(\frac{2}{2.0\times10^{-4}\,\mathrm{m}^2}\right)\left(1.0\times10^{23}\,\mathrm{s}^{-1}\right)\left(3.3\times10^{-27}\,\mathrm{kg}\right)\left(1.0\times10^{3}\,\mathrm{m/s}\right)\cos 55^\circ \\
&= 1.9\times10^{3}\,\mathrm{Pa}.
\end{aligned}$$

We note that the value given for the mass was converted to kg and the value given for the area was converted to m^2.

22. We can express the ideal gas law in terms of density using $n = M_{\text{sam}}/M$:

$$pV = \frac{M_{\text{sam}}RT}{M} \Rightarrow \rho = \frac{pM}{RT} .$$

We can also use this to write the rms speed formula in terms of density:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(pM/\rho)}{M}} = \sqrt{\frac{3p}{\rho}} .$$

(a) We convert to SI units: $\rho = 1.24 \times 10^{-2}$ kg/m^3 and $p = 1.01 \times 10^3$ Pa. The rms speed is $\sqrt{3(1010)/0.0124} = 494$ m/s.

(b) We find M from $\rho = pM/RT$ with $T = 273$ K.

$$M = \frac{\rho RT}{p} = \frac{(0.0124\,\text{kg/m}^3)\,(8.31\,\text{J/mol}\cdot\text{K})(273\,\text{K})}{1.01 \times 10^3\,\text{Pa}} = 0.0279\ \text{kg/mol} = 27.9\ \text{g/mol}.$$

(c) From Table 19.1, we identify the gas to be N_2.

23. The average translational kinetic energy is given by $K_{\text{avg}} = \frac{3}{2}kT$, where k is the Boltzmann constant $(1.38 \times 10^{-23}$ J/K) and T is the temperature on the Kelvin scale. Thus

$$K_{\text{avg}} = \frac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(1600\,\text{K}) = 3.31 \times 10^{-20}\ \text{J} .$$

24. (a) Eq. 19-24 gives $K_{\text{avg}} = \frac{3}{2}\left(1.38\times10^{-23}\ \frac{\text{J}}{\text{K}}\right)(273\,\text{K}) = 5.65\times10^{-21}\ \text{J}$.

(b) Similarly, for T = 373 K, the average translational kinetic energy is $K_{\text{avg}} = 7.72\times10^{-21}\ \text{J}$.

(c) The unit mole may be thought of as a (large) collection: 6.02×10^{23} molecules of ideal gas, in this case. Each molecule has energy specified in part (a), so the large collection has a total kinetic energy equal to

$$K_{\text{mole}} = N_{\text{A}} K_{\text{avg}} = (6.02\times10^{23})\ (5.65\times10^{-21}\ \text{J}) = 3.40\times10^{3}\ \text{J}.$$

(d) Similarly, the result from part (b) leads to

$$K_{\text{mole}} = (6.02\times10^{23})\ (7.72\times10^{-21}\ \text{J}) = 4.65\times10^{3}\ \text{J}.$$

25. (a) We use $\varepsilon = L_V/N$, where L_V is the heat of vaporization and N is the number of molecules per gram. The molar mass of atomic hydrogen is 1 g/mol and the molar mass of atomic oxygen is 16 g/mol so the molar mass of H_2O is (1.0 + 1.0 + 16) = 18 g/mol. There are $N_A = 6.02 \times 10^{23}$ molecules in a mole so the number of molecules in a gram of water is $(6.02 \times 10^{23}\ \text{mol}^{-1})/(18\ \text{g/mol}) = 3.34 \times 10^{22}$ molecules/g. Thus

$$\varepsilon = (539\ \text{cal/g})/(3.34 \times 10^{22}/\text{g}) = 1.61 \times 10^{-20}\ \text{cal} = 6.76 \times 10^{-20}\ \text{J}.$$

(b) The average translational kinetic energy is

$$K_{\text{avg}} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})[(32.0 + 273.15)\ \text{K}] = 6.32 \times 10^{-21}\ \text{J}.$$

The ratio $\varepsilon/K_{\text{avg}}$ is $(6.76 \times 10^{-20}\ \text{J})/(6.32 \times 10^{-21}\ \text{J}) = 10.7$.

26. Using $v = f\lambda$ with v = 331 m/s (see Table 17-1) with Eq. 19-2 and Eq. 19-25 leads to

$$\begin{aligned} f &= \frac{v}{\left(\dfrac{1}{\sqrt{2}\pi d^2 (N/V)}\right)} = (331\,\text{m/s})\,\pi\sqrt{2}\;(3.0\times10^{-10}\ \text{m})^2\left(\frac{nN_\text{A}}{V}\right) \\ &= \left(8.0\times10^7\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{n}{V}\right) = \left(8.0\times10^7\ \frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{1.01\times10^5\ \text{Pa}}{(8.31\ \text{J/mol}\cdot\text{K})\,(273.15\,\text{K})}\right) \\ &= 3.5\times10^9\ \text{Hz}. \end{aligned}$$

where we have used the ideal gas law and substituted $n/V = p/RT$. If we instead use v = 343 m/s (the “default value” for speed of sound in air, used repeatedly in Ch. 17), then the answer is 3.7×10^9 Hz.

27. (a) According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where d is the diameter of a molecule and N is the number of molecules in volume V. Substitute $d = 2.0 \times 10^{-10}$ m and $N/V = 1 \times 10^6$ molecules/m^3 to obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10}\ \text{m})^2\,(1 \times 10^6\ \text{m}^{-3})} = 6 \times 10^{12}\ \text{m}.$$

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

28. We solve Eq. 19-25 for *d*:

$$d = \sqrt{\frac{1}{\lambda\pi\sqrt{2}\,(N/V)}} = \sqrt{\frac{1}{(0.80\times 10^{5}\ \text{cm})\ \pi\sqrt{2}\,(2.7\times 10^{19}\ /\text{cm}^{3})}}$$

which yields $d = 3.2\times 10^{-8}$ cm, or 0.32 nm.

29. (a) We use the ideal gas law $pV = nRT = NkT$, where p is the pressure, V is the volume, T is the temperature, n is the number of moles, and N is the number of molecules. The substitutions $N = nN_A$ and $k = R/N_A$ were made. Since 1 cm of mercury = 1333 Pa, the pressure is $p = (10^{-7})(1333\text{ Pa}) = 1.333 \times 10^{-4}$ Pa. Thus,

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.333\times10^{-4}\text{ Pa}}{(1.38\times10^{-23}\text{ J/K})(295\text{ K})} = 3.27\times10^{16}\text{ molecules/m}^3 = 3.27\times10^{10}\text{ molecules/cm}^3 .$$

(b) The molecular diameter is $d = 2.00 \times 10^{-10}$ m, so, according to Eq. 19-25, the mean free path is

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} = \frac{1}{\sqrt{2}\pi(2.00\times10^{-10}\text{ m})^2(3.27\times10^{16}\text{ m}^{-3})} = 172\text{ m}.$$

30. (a) We set up a ratio using Eq. 19-25:

$$\frac{\lambda_{\mathrm{Ar}}}{\lambda_{\mathrm{N_2}}}=\frac{1/\left(\pi\sqrt{2}d_{\mathrm{Ar}}^2\,(N/V)\right)}{1/\left(\pi\sqrt{2}d_{\mathrm{N_2}}^2\,(N/V)\right)}=\left(\frac{d_{\mathrm{N_2}}}{d_{\mathrm{Ar}}}\right)^2.$$

Therefore, we obtain

$$\frac{d_{\mathrm{Ar}}}{d_{\mathrm{N_2}}}=\sqrt{\frac{\lambda_{\mathrm{N_2}}}{\lambda_{\mathrm{Ar}}}}=\sqrt{\frac{27.5}{9.9}}=1.7.$$

(b) Using Eq. 19-2 and the ideal gas law, we substitute $N/V = N_A n/V = N_A p/RT$ into Eq. 19–25 and find

$$\lambda=\frac{RT}{\pi\sqrt{2}d^2pN_{\mathrm{A}}}.$$

Comparing (for the same species of molecule) at two different pressures and temperatures, this leads to

$$\frac{\lambda_2}{\lambda_1}=\left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right).$$

With $\lambda_1 = 9.9 \times 10^{-6}$ cm, $T_1 = 293$ K (the same as T_2 in this part), $p_1 = 750$ torr and $p_2 = 150$ torr, we find $\lambda_2 = 5.0 \times 10^{-5}$ cm.

(c) The ratio set up in part (b), using the same values for quantities with subscript 1, leads to $\lambda_2 = 7.9 \times 10^{-6}$ cm for $T_2 = 233$ K and $p_2 = 750$ torr.

31. (a) The average speed is

$$\bar{v} = \frac{\sum v}{N},$$

where the sum is over the speeds of the particles and N is the number of particles. Thus

$$\bar{v} = \frac{(2.0+3.0+4.0+5.0+6.0+7.0+8.0+9.0+10.0+11.0)\,\text{km/s}}{10} = 6.5\,\text{km/s}.$$

(b) The rms speed is given by

$$v_{\text{rms}} = \sqrt{\frac{\sum v^2}{N}}.$$

Now

$$\begin{aligned}\sum v^2 &= [(2.0)^2+(3.0)^2+(4.0)^2+(5.0)^2+(6.0)^2 \\ &\quad +(7.0)^2+(8.0)^2+(9.0)^2+(10.0)^2+(11.0)^2]\ \text{km}^2/\text{s}^2 = 505\ \text{km}^2/\text{s}^2\end{aligned}$$

so

$$v_{\text{rms}} = \sqrt{\frac{505\ \text{km}^2/\text{s}^2}{10}} = 7.1\,\text{km/s}.$$

32. (a) The average speed is

$$v_{\text{avg}} = \frac{\sum n_i v_i}{\sum n_i} = \frac{[2(1.0)+4(2.0)+6(3.0)+8(4.0)+2(5.0)]\ \text{cm/s}}{2+4+6+8+2} = 3.2\,\text{cm/s}.$$

(b) From $v_{\text{rms}} = \sqrt{\sum n_i v_i^2 / \sum n_i}$ we get

$$v_{\text{rms}} = \sqrt{\frac{2(1.0)^2+4(2.0)^2+6(3.0)^2+8(4.0)^2+2(5.0)^2}{2+4+6+8+2}}\ \text{cm/s} = 3.4\,\text{cm/s}.$$

(c) There are eight particles at v = 4.0 cm/s, more than the number of particles at any other single speed. So 4.0 cm/s is the most probable speed.

33. (a) The average speed is

$$v_{\text{avg}} = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{10}[4(200 \text{ m/s}) + 2(500 \text{ m/s}) + 4(600 \text{ m/s})] = 420 \text{ m/s}.$$

(b) The rms speed is

$$v_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200 \text{ m/s})^2 + 2(500 \text{ m/s})^2 + 4(600 \text{ m/s})^2]} = 458 \text{ m/s}$$

(c) Yes, $v_{\text{rms}} > v_{\text{avg}}$.

34. (a) From the graph we see that $v_p = 400$ m/s. Using the fact that $M = 28$ g/mol = 0.028 kg/mol for nitrogen (N_2) gas, Eq. 19-35 can then be used to determine the absolute temperature. We obtain $T = \frac{1}{2} M v_p^2 / R = 2.7 \times 10^2$ K.

(b) Comparing with Eq. 19-34, we conclude $v_{rms} = \sqrt{3/2}\, v_p = 4.9 \times 10^2$ m/s.

35. The rms speed of molecules in a gas is given by $v_{rms} = \sqrt{3RT/M}$, where T is the temperature and M is the molar mass of the gas. See Eq. 19-34. The speed required for escape from Earth's gravitational pull is $v = \sqrt{2gr_e}$, where g is the acceleration due to gravity at Earth's surface and r_e (= 6.37×10^6 m) is the radius of Earth. To derive this expression, take the zero of gravitational potential energy to be at infinity. Then, the gravitational potential energy of a particle with mass m at Earth's surface is $U = -GMm/r_e^2 = -mgr_e$, where $g = GM/r_e^2$ was used. If v is the speed of the particle, then its total energy is $E = -mgr_e + \frac{1}{2}mv^2$. If the particle is just able to travel far away, its kinetic energy must tend toward zero as its distance from Earth becomes large without bound. This means $E = 0$ and $v = \sqrt{2gr_e}$. We equate the expressions for the speeds to obtain $\sqrt{3RT/M} = \sqrt{2gr_e}$. The solution for T is $T = 2gr_eM/3R$.

(a) The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so for that gas

$$T = \frac{2\left(9.8\,\text{m/s}^2\right)\left(6.37\times10^6\,\text{m}\right)\left(2.02\times10^{-3}\,\text{kg/mol}\right)}{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 1.0\times10^4\ \text{K}.$$

(b) The molar mass of oxygen is 32.0×10^{-3} kg/mol, so for that gas

$$T = \frac{2\left(9.8\,\text{m/s}^2\right)\left(6.37\times10^6\,\text{m}\right)\left(32.0\times10^{-3}\,\text{kg/mol}\right)}{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 1.6\times10^5\ \text{K}.$$

(c) Now, $T = 2g_mr_mM / 3R$, where r_m (= 1.74×10^6 m) is the radius of the Moon and g_m (= $0.16g$) is the acceleration due to gravity at the Moon's surface. For hydrogen

$$T = \frac{2\left(0.16\right)\left(9.8\,\text{m/s}^2\right)\left(1.74\times10^6\,\text{m}\right)\left(2.02\times10^{-3}\,\text{kg/mol}\right)}{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 4.4\times10^2\ \text{K}.$$

(d) For oxygen

$$T = \frac{2\left(0.16\right)\left(9.8\,\text{m/s}^2\right)\left(1.74\times10^6\,\text{m}\right)\left(32.0\times10^{-3}\,\text{kg/mol}\right)}{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 7.0\times10^3\ \text{K}.$$

(e) The temperature high in Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result the atmosphere is depleted of hydrogen.

(f) On the other hand, very few oxygen atoms escape. So there should be much oxygen high in Earth’s upper atmosphere.

36. We divide Eq. 19-35 by Eq. 19-22:

$$\frac{v_P}{v_{\text{rms}}}=\frac{\sqrt{2RT_2/M}}{\sqrt{3RT_1/M}}=\sqrt{\frac{2T_2}{3T_1}}$$

which leads to

$$\frac{T_2}{T_1}=\frac{3}{2}\left(\frac{v_P}{v_{\text{rms}}}\right)^2=\frac{3}{2} \qquad \text{if } v_P=v_{\text{rms}}.$$

37. (a) The root-mean-square speed is given by $v_{\text{rms}} = \sqrt{3RT/M}$. See Eq. 19-34. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31\,\text{J/mol}\cdot\text{K})(4000\,\text{K})}{2.02\times10^{-3}\,\text{kg/mol}}} = 7.0\times10^3\ \text{m/s}.$$

(b) When the surfaces of the spheres that represent an H_2 molecule and an Ar atom are touching, the distance between their centers is the sum of their radii:

$$d = r_1 + r_2 = 0.5 \times 10^{-8}\ \text{cm} + 1.5 \times 10^{-8}\text{cm} = 2.0 \times 10^{-8}\ \text{cm}.$$

(c) The argon atoms are essentially at rest so in time t the hydrogen atom collides with all the argon atoms in a cylinder of radius d and length vt, where v is its speed. That is, the number of collisions is $\pi d^2 vtN/V$, where, N/V is the concentration of argon atoms. The number of collisions per unit time is

$$\frac{\pi d^2 vN}{V} = \pi\left(2.0\times10^{-10}\ \text{m}\right)^2\left(7.0\times10^3\ \text{m/s}\right)\left(4.0\times10^{25}\ \text{m}^{-3}\right) = 3.5\times10^{10}\ \text{collisions/s}.$$

38. We divide Eq. 19-31 by Eq. 19-22:

$$\frac{v_{\text{avg2}}}{v_{\text{rms1}}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}$$

which leads to

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8}\left(\frac{v_{\text{avg2}}}{v_{\text{rms1}}}\right)^2 = \frac{3\pi}{2} = 4.7 \qquad \text{if } v_{\text{avg2}} = 2v_{\text{rms1}}.$$

39. (a) The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v)\,dv = 1$. Evaluate the integral by calculating the area under the curve in Fig. 19-22. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0. Thus $\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0$, so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3 = 0.67$.

(b) The average speed is given by $v_{\text{avg}} = \int vP(v)\,dv$. For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2 dv = \frac{a}{3v_0}v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0,$$

where $2/3v_0$ was substituted for a. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\,dv = \frac{a}{2}\left(4v_0^2 - v_0^2\right) = \frac{3a}{2}v_0^2 = v_0.$$

Therefore,

$$v_{\text{avg}} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \quad \Rightarrow \quad \frac{v_{\text{avg}}}{v_0} = 1.22\,.$$

(c) The mean-square speed is given by

$$v_{\text{rms}}^2 = \int v^2 P(v)\,dv.$$

The contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3 dv = \frac{a}{4v_0}v_0^4 = \frac{1}{6}v_0^2.$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2 dv = \frac{a}{3}\left(8v_0^3 - v_0^3\right) = \frac{7a}{3}v_0^3 = \frac{14}{9}v_0^2.$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0 \;\Rightarrow\; \frac{v_{\text{rms}}}{v_0} = 1.31 \,.$$

(d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is $N\,a(2.0v_0 - 1.5v_0) = 0.5N\,av_0 = N/3$, where $2/3v_0$ was substituted for a. In other words, the fraction of particles in this range is 1/3 or 0.33.

40. The internal energy is

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(1.0\,\text{mol})(8.31\ \text{J/mol}\cdot\text{K})(273\,\text{K}) = 3.4\times10^{3}\ \text{J}.$$

41. (a) The work is zero in this process since volume is kept fixed.

(b) Since $C_V = \frac{3}{2}R$ for an ideal monatomic gas, then Eq. 19-39 gives $Q = +374$ J.

(c) $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) Two moles are equivalent to $N = 12 \text{ x } 10^{23}$ particles. Dividing the result of part (c) by N gives the average translational kinetic energy change per atom: 3.11×10^{-22} J.

42. (a) Since the process is a constant-pressure expansion,

$$W = p\Delta V = nR\Delta T = (2.02\,\text{mol})(8.31\ \text{J/mol}\cdot\text{K})(15\,\text{K}) = 249\,\text{J}.$$

(b) Now, $C_p = \frac{5}{2}R$ in this case, so $Q = nC_p\Delta T =$ +623 J by Eq. 19-46.

(c) The change in the internal energy is $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) The change in the average kinetic energy per atom is $\Delta K_{\text{avg}} = \Delta E_{\text{int}}/N = +3.11 \times 10^{-22}$ J.

43. When the temperature changes by ΔT the internal energy of the first gas changes by $n_1C_1\ \Delta T$, the internal energy of the second gas changes by $n_2C_2\ \Delta T$, and the internal energy of the third gas changes by $n_3C_3\ \Delta T$. The change in the internal energy of the composite gas is

$$\Delta E_{\text{int}} = (n_1\, C_1 + n_2\, C_2 + n_3\, C_3)\, \Delta T.$$

This must be $(n_1 + n_2 + n_3)\ C_V\ \Delta T$, where C_V is the molar specific heat of the mixture. Thus

$$C_V = \frac{n_1C_1 + n_2C_2 + n_3C_3}{n_1 + n_2 + n_3}.$$

With n_1=2.40 mol, C_{V1}=12.0 J/mol·K for gas 1, n_2=1.50 mol, C_{V2}=12.8 J/mol·K for gas 2, and n_3=3.20 mol, C_{V3}=20.0 J/mol·K for gas 3, we obtain C_V=15.8 J/mol·K for the mixture.

44. (a) According to the first law of thermodynamics $Q = \Delta E_{\text{int}} + W$. When the pressure is a constant $W = p\,\Delta V$. So

$$\Delta E_{\text{int}} = Q - p\Delta V = 20.9\ \text{J} - \left(1.01\times10^5\ \text{Pa}\right)\left(100\ \text{cm}^3 - 50\ \text{cm}^3\right)\left(\frac{1\times10^{-6}\ \text{m}^3}{1\ \text{cm}^3}\right) = 15.9\ \text{J}.$$

(b) The molar specific heat at constant pressure is

$$C_p = \frac{Q}{n\Delta T} = \frac{Q}{n\left(p\Delta V/nR\right)} = \frac{R}{p}\frac{Q}{\Delta V} = \frac{\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(20.9\,\text{J}\right)}{\left(1.01\times10^5\ \text{Pa}\right)\left(50\times10^{-6}\,\text{m}^3\right)} = 34.4\,\text{J/mol}\cdot\text{K}.$$

(c) Using Eq. 19-49, $C_V = C_p - R = 26.1$ J/mol·K.

45. Argon is a monatomic gas, so $f = 3$ in Eq. 19-51, which provides

$$C_V = \frac{3}{2}R = \frac{3}{2}(8.31\ \text{J/mol}\cdot\text{K})\left(\frac{1\ \text{cal}}{4.186\ \text{J}}\right) = 2.98\frac{\text{cal}}{\text{mol}\cdot\text{C}°}$$

where we have converted Joules to calories, and taken advantage of the fact that a Celsius degree is equivalent to a unit change on the Kelvin scale. Since (for a given substance) M is effectively a conversion factor between grams and moles, we see that c_V (see units specified in the problem statement) is related to C_V by $C_V = c_V\ M$ where $M = mN_A$, and m is the mass of a single atom (see Eq. 19-4).

(a) From the above discussion, we obtain

$$m = \frac{M}{N_A} = \frac{C_V / c_V}{N_A} = \frac{2.98/0.075}{6.02\times 10^{23}} = 6.6\times 10^{-23}\ \text{g}.$$

(b) The molar mass is found to be $M = C_V/c_V = 2.98/0.075 = 39.7$ g/mol which should be rounded to 40 since the given value of c_V is specified to only two significant figures.

46. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a *straight line* on the pV diagram — the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Further, Eq. 19-44 with Eq. 19-51 gives

$$E_{\text{int}} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in Joules, pressure should be in Pascals (N / m^2) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$\begin{aligned} E_{\text{int}\,c} - E_{\text{int}\,a} &= \frac{5}{2}(p_cV_c - p_aV_a) = \frac{5}{2}\left(\left(2.0\times10^3\ \text{Pa}\right)\left(4.0\,\text{m}^3\right) - \left(5.0\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right)\right) \\ &= -5.0\times10^3\ \text{J}. \end{aligned}$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a) = \left(3.5\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right)$$

which yields $W_{\text{diag}} = 7.0\times10^3$ J. Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = (-5.0\times10^3 + 7.0\times10^3)\ \text{J} = 2.0\times10^3\ \text{J}.$$

(c) The fact that ΔE_{int} only depends on the initial and final states, and not on the details of the "path" between them, means we can write $\Delta E_{\text{int}} = E_{\text{int}\,c} - E_{\text{int}\,a} = -5.0\times10^3$ J for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = \left(5.0\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right) + 0 = 1.0\times10^4\ \text{J}.$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = (-5.0\times10^{3} + 1.0\times10^{4})\ \text{J} = 5.0\times10^{3}\ \text{J}.$$

47. To model the "uniform rates" described in the problem statement, we have expressed the volume and the temperature functions as follows:

$$V = V_i + \left(\frac{V_f - V_i}{\tau_f}\right) t \qquad \text{and} \qquad T = T_i + \left(\frac{T_f - T_i}{\tau_f}\right) t$$

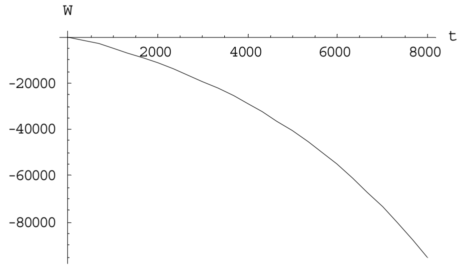
where $V_i = 0.616\ \text{m}^3$, $V_f = 0.308\ \text{m}^3$, $\tau_f = 7200\ \text{s}$, $T_i = 300\ \text{K}$ and $T_f = 723\ \text{K}$.

(a) We can take the derivative of V with respect to t and use that to evaluate the cumulative work done (from $t = 0$ until $t = \tau$):

$$W = \int p\, dV = \int \left(\frac{nRT}{V}\right)\left(\frac{dV}{dt}\right) dt = 12.2\ \tau + 238113 \ln(14400 - \tau) - 2.28 \times 10^6$$

with SI units understood. With $\tau = \tau_f$ our result is $W = -77169\ \text{J} \approx -77.2\ \text{kJ}$, or $|W| \approx 77.2\ \text{kJ}$.

The graph of cumulative work is shown below. The graph for work done is purely negative because the gas is being compressed (work is being done *on* the gas).

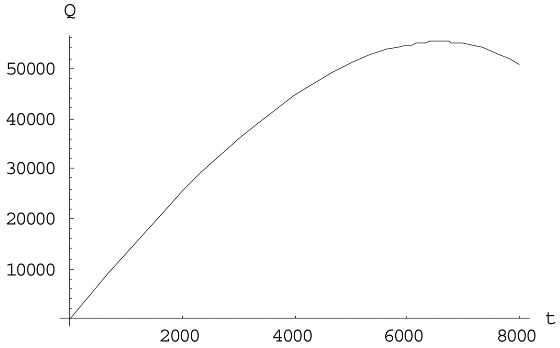


(b) With $C_V = \frac{3}{2}R$ (since it's a monatomic ideal gas) then the (infinitesimal) change in internal energy is $nC_V\, dT = \frac{3}{2}nR\left(\frac{dT}{dt}\right) dt$ which involves taking the derivative of the temperature expression listed above. Integrating this and adding this to the work done gives the cumulative heat absorbed (from $t = 0$ until $t = \tau$):

$$Q = \int \left(\frac{nRT}{V}\right)\left(\frac{dV}{dt}\right) + \frac{3}{2}nR\left(\frac{dT}{dt}\right) dt = 30.5\ \tau + 238113 \ln(14400 - \tau) - 2.28 \times 10^6$$

with SI units understood. With $\tau = \tau_f$ our result is $Q_{\text{total}} = 54649\ \text{J} \approx 5.46 \times 10^4\ \text{J}$.

The graph cumulative heat is shown below. We see that $Q > 0$ since the gas is absorbing heat.



(c) Defining $C = \dfrac{Q_{\text{total}}}{n(T_f - T_i)}$ we obtain C = 5.17 J/mol·K. We note that this is considerably smaller than the constant-volume molar heat C_V.

We are now asked to consider this to be a two-step process (time dependence is no longer an issue) where the first step is isothermal and the second step occurs at constant volume (the ending values of pressure, volume and temperature being the same as before).

(d) Eq. 19-14 readily yields $W = -43222$ J $\approx -4.32 \times 10^4$ J (or $|W| \approx 4.32 \times 10^4$ J), where it is important to keep in mind that no work is done in a process where the volume is held constant.

(e) In step 1 the heat is equal to the work (since the internal energy does not change during an isothermal ideal gas process), and step 2 the heat is given by Eq. 19-39. The total heat is therefore 88595 $\approx 8.86 \times 10^4$ J.

(f) Defining a molar heat capacity in the same manner as we did in part (c), we now arrive at C = 8.38 J/ mol·K.

48. Referring to Table 19-3, Eq. 19-45 and Eq. 19-46, we have

$$\Delta E_{\text{int}} = nC_V \Delta T = \frac{5}{2} nR\Delta T$$
$$Q = nC_p \Delta T = \frac{7}{2} nR\Delta T.$$

Dividing the equations, we obtain

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{5}{7}.$$

Thus, the given value $Q = 70$ J leads to $\Delta E_{\text{int}} = 50$ J.

49. The fact that they rotate but do not oscillate means that the value of *f* given in Table 19-3 is relevant. Thus, Eq. 19-46 leads to

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)\left(T_f - T_i\right) = nRT_i\left(\frac{7}{2}\right)\left(\frac{T_f}{T_i} - 1\right)$$

where T_i = 273 K and n = 1.0 mol. The ratio of absolute temperatures is found from the gas law in ratio form (see Sample Problem 19-1). With $p_f = p_i$ we have

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = 2.$$

Therefore, the energy added as heat is

$$Q = \left(1.0\,\text{mol}\right)\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(273\,\text{K}\right)\left(\frac{7}{2}\right)(2-1) \approx 8.0\times10^3\ \text{J}.$$

50. (a) Using M = 32.0 g/mol from Table 19-1 and Eq. 19-3, we obtain

$$n = \frac{M_{\text{sam}}}{M} = \frac{12.0\ \text{g}}{32.0\ \text{g/mol}} = 0.375\ \text{mol}.$$

(b) This is a constant pressure process with a diatomic gas, so we use Eq. 19-46 and Table 19-3. We note that a change of Kelvin temperature is numerically the same as a change of Celsius degrees.

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)\Delta T = (0.375\ \text{mol})\left(\frac{7}{2}\right)(8.31\ \text{J/mol}\cdot\text{K})(100\,\text{K}) = 1.09\times10^3\ \text{J}.$$

(c) We could compute a value of ΔE_{int} from Eq. 19-45 and divide by the result from part (b), or perform this manipulation algebraically to show the generality of this answer (that is, many factors will be seen to cancel). We illustrate the latter approach:

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{n\left(\frac{5}{2}R\right)\ \Delta T}{n\left(\frac{7}{2}R\right)\ \Delta T} = \frac{5}{7} \approx 0.714.$$

51. (a) Since the process is at constant pressure, energy transferred as heat to the gas is given by $Q = nC_p\,\Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. For a diatomic ideal gas $C_p = \frac{7}{2}R$. Thus

$$Q = \frac{7}{2}nR\Delta T = \frac{7}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 6.98\times10^3\,\text{J}.$$

(b) The change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\,\Delta T$, where C_V is the specific heat at constant volume. For a diatomic ideal gas $C_V = \frac{5}{2}R$, so

$$\Delta E_{\text{int}} = \frac{5}{2}nR\Delta T = \frac{5}{2}(4.00\,\text{mol})(8.31\,\text{J/mol.K})(60.0\,\text{K}) = 4.99\times10^3\,\text{J}.$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so

$$W = Q - \Delta E_{\text{int}} = 6.98\times10^3\,\text{J} - 4.99\times10^3\,\text{J} = 1.99\times10^3\,\text{J}.$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2}nR\Delta T = \frac{3}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 2.99\times10^3\,\text{J}.$$

52. The fact that they rotate but do not oscillate means that the value of *f* given in Table 19-3 is relevant. In §19-11, it is noted that $\gamma = C_p/C_V$ so that we find $\gamma = 7/5$ in this case. In the state described in the problem, the volume is

$$.V = \frac{nRT}{p} = \frac{(2.0\,\text{mol})(8.31\ \text{J/mol}\cdot\text{K})(300\,\text{K})}{1.01\times10^5\ \text{N/m}^2} = 0.049\ \text{m}^3$$

Consequently,

$$pV^\gamma = (1.01\times10^5\ \text{N/m}^2)(0.049\,\text{m}^3)^{1.4} = 1.5\times10^3\ \text{N}\cdot\text{m}^{2.2}.$$

53. (a) Let p_i, V_i, and T_i represent the pressure, volume, and temperature of the initial state of the gas. Let p_f, V_f, and T_f represent the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_i V_i^\gamma = p_f V_f^{\,\gamma}$, so

$$p_f = \left(\frac{V_i}{V_f}\right)^\gamma p_i = \left(\frac{4.3\text{ L}}{0.76\text{ L}}\right)^{1.4} (1.2\,\text{atm}) = 13.6\,\text{atm} \approx 14\text{ atm.}$$

We note that since V_i and V_f have the same units, their units cancel and p_f has the same units as p_i.

(b) The gas obeys the ideal gas law $pV = nRT$, so $p_iV_i/p_fV_f = T_i/T_f$ and

$$T_f = \frac{p_f V_f}{p_i V_i} T_i = \left[\frac{(13.6\,\text{atm})(0.76\,\text{L})}{(1.2\,\text{atm})(4.3\,\text{L})}\right](310\,\text{K}) = 6.2\times 10^2\text{ K.}$$

54. (a) We use Eq. 19-54 with $V_f / V_i = \frac{1}{2}$ for the gas (assumed to obey the ideal gas law).

$$p_i V_i^\gamma = p_f V_f^\gamma \Rightarrow \frac{p_f}{p_i} = \left(\frac{V_i}{V_f}\right)^\gamma = (2.00)^{1.3}$$

which yields $p_f = (2.46)(1.0 \text{ atm}) = 2.46 \text{ atm}$.

(b) Similarly, Eq. 19-56 leads to

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma - 1} = (273\,\text{K})(1.23) = 336\,\text{K}.$$

(c) We use the gas law in ratio form (see Sample Problem 19-1) and note that when $p_1 = p_2$ then the ratio of volumes is equal to the ratio of (absolute) temperatures. Consequently, with the subscript 1 referring to the situation (of small volume, high pressure, and high temperature) the system is in at the end of part (a), we obtain

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{273\,\text{K}}{336\,\text{K}} = 0.813.$$

The volume V_1 is half the original volume of one liter, so

$$V_2 = 0.813(0.500\,\text{L}) = 0.406\,\text{L}.$$

55. Since ΔE_{int} does not depend on the type of process,

$$\left(\Delta E_{\text{int}}\right)_{\text{path 2}} = \left(\Delta E_{\text{int}}\right)_{\text{path 1}}.$$

Also, since (for an ideal gas) it only depends on the temperature variable (so $\Delta E_{\text{int}} = 0$ for isotherms), then

$$\left(\Delta E_{\text{int}}\right)_{\text{path 1}} = \sum\left(\Delta E_{\text{int}}\right)_{\text{adiabat}}.$$

Finally, since $Q = 0$ for adiabatic processes, then (for path 1)

$$\begin{aligned}\left(\Delta E_{\text{int}}\right)_{\text{adiabatic expansion}} &= -W = -40\text{ J}\\ \left(\Delta E_{\text{int}}\right)_{\text{adiabatic compression}} &= -W = -(-25)\text{ J} = 25\text{ J}.\end{aligned}$$

Therefore, $\left(\Delta E_{\text{int}}\right)_{\text{path 2}} = -40\text{ J} + 25\text{ J} = -15\text{ J}$.

56. (a) Eq. 19-54 leads to

$$4 = \left(\frac{200}{74.3}\right)^{\gamma} \Rightarrow \gamma = \log(4)/\log(200/74.3) = 1.4 = 7/5.$$

This implies that the gas is diatomic (see Table 19-3).

(b) One can now use either Eq. 19-56 (as illustrated in part (a) of Sample Problem 19-9) or use the ideal gas law itself. Here we illustrate the latter approach:

$$\frac{P_f V_f}{P_i V_i} = \frac{nRT_f}{nRT_i} \quad \Rightarrow \quad T_f = 446 \text{ K} .$$

(c) Again using the ideal gas law: $n = P_i V_i / RT_i = 8.10$ moles. The same result would, of course, follow from $n = P_f V_f / RT_f$.

57. The aim of this problem is to emphasize what it means for the internal energy to be a state function. Since path 1 and path 2 start and stop at the same places, then the internal energy change along path 1 is equal to that along path 2. Now, during isothermal processes (involving an ideal gas) the internal energy change is zero, so the only step in path 1 that we need to examine is step 2. Eq. 19-28 then immediately yields –20 J as the answer for the internal energy change.

58. (a) In the free expansion from state 0 to state 1 we have $Q = W = 0$, so $\Delta E_{\text{int}} = 0$, which means that the temperature of the ideal gas has to remain unchanged. Thus the final pressure is

$$p_1 = \frac{p_0 V_0}{V_1} = \frac{p_0 V_0}{3.00 V_0} = \frac{1}{3.00} p_0 \Rightarrow \frac{p_1}{p_0} = \frac{1}{3.00} = 0.333.$$

(b) For the adiabatic process from state 1 to 2 we have $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$, i.e.,

$$\frac{1}{3.00} p_0 \left(3.00 V_0\right)^{\gamma} = \left(3.00\right)^{\frac{1}{3}} p_0 V_0^{\gamma}$$

which gives $\gamma = 4/3$. The gas is therefore polyatomic.

(c) From $T = pV/nR$ we get

$$\frac{\bar{K}_2}{\bar{K}_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1} = \left(3.00\right)^{\frac{1}{3}} = 1.44.$$

59. In the following $C_V = \frac{3}{2}R$ is the molar specific heat at constant volume, $C_p = \frac{5}{2}R$ is the molar specific heat at constant pressure, ΔT is the temperature change, and n is the number of moles.

The process 1 → 2 takes place at constant volume.

(a) The heat added is

$$Q = nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = 3.74\times10^3\ \text{J}.$$

(b) Since the process takes place at constant volume the work W done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74\times10^3\ \text{J}.$$

(c) The work W done by the gas is zero.

The process 2 → 3 is adiabatic.

(d) The heat added is zero.

(e) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K} - 600\,\text{K}) = -1.81\times10^3\ \text{J}.$$

(f) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81\times10^3\,\text{J}.$$

The process 3 → 1 takes place at constant pressure.

(g) The heat added is

$$Q = nC_p\Delta T = \frac{5}{2}nR\Delta T = \frac{5}{2}(1.00\ \text{mol})\,(8.31\,\text{J/mol}\cdot\text{K})\,(300\,\text{K} - 455\,\text{K}) = -3.22\times10^3\ \text{J}.$$

(h) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -1.93\times10^3\ \text{J}.$$

(i) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22\times10^3\ \text{J} + 1.93\times10^3\ \text{J} = -1.29\times10^3\ \text{J}.$$

(j) For the entire process the heat added is

$$Q = 3.74\times10^3\ \text{J} + 0 - 3.22\times10^3\ \text{J} = 520\ \text{J}.$$

(k) The change in the internal energy is

$$\Delta E_{\text{int}} = 3.74\times10^3\ \text{J} - 1.81\times10^3\ \text{J} - 1.93\times10^3\ \text{J} = 0.$$

(l) The work done by the gas is

$$W = 0 + 1.81\times10^3\ \text{J} - 1.29\times10^3\ \text{J} = 520\ \text{J}.$$

(m) We first find the initial volume. Use the ideal gas law $p_1V_1 = nRT_1$ to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{(1.013\times10^5\ \text{Pa})} = 2.46\times10^{-2}\ \text{m}^3.$$

(n) Since 1 → 2 is a constant volume process $V_2 = V_1 = 2.46\times10^{-2}\ \text{m}^3$. The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(600\,\text{K})}{2.46\times10^{-2}\,\text{m}^3} = 2.02\times10^5\,\text{Pa}.$$

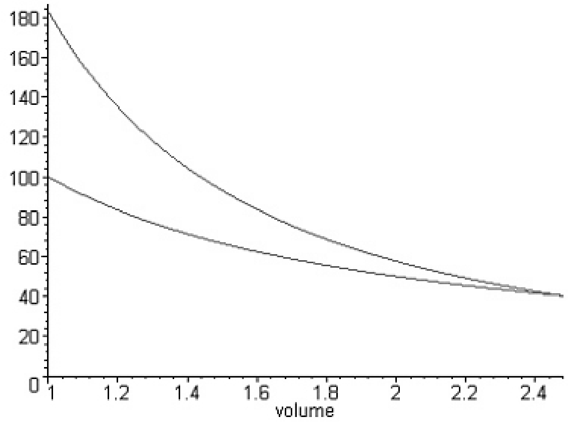
This is approximately equal to 2.00 atm.

(o) 3 → 1 is a constant pressure process. The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K})}{1.013\times10^5\ \text{Pa}} = 3.73\times10^{-2}\ \text{m}^3.$$

(p) The pressure for state 3 is the same as the pressure for state 1: $p_3 = p_1 = 1.013\times10^5$ Pa (1.00 atm)

60. (a) The *p*-*V* diagram is shown below:



Note that o obtain the above graph, we have chosen n = 0.37 moles for concreteness, in which case the horizontal axis (which we note starts not at zero but at 1) is to be interpreted in units of cubic centimeters, and the vertical axis (the absolute pressure) is in kilopascals. However, the constant volume temp-increase process described in the third step (see problem statement) is difficult to see in this graph since it coincides with the pressure axis.

(b) We note that the change in internal energy is zero for an ideal gas isothermal process, so (since the net change in the internal energy must be zero for the entire cycle) the increase in internal energy in step 3 must equal (in magnitude) its decease in step 1. By Eq. 19-28, we see this number must be 125 J.

(c) As implied by Eq. 19-29, this is equivalent to heat being added *to the gas*.

61. (a) The ideal gas law leads to

$$V=\frac{nRT}{p}=\frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})\,(273\,\text{K})}{1.01\times10^{5}\,\text{Pa}}$$

which yields V = 0.0225 m^3 = 22.5 L. If we use the standard pressure value given in Appendix D, 1 atm = 1.013×10^5 Pa, then our answer rounds more properly to 22.4 L.

(b) From Eq. 19-2, we have $N = 6.02 \times 10^{23}$ molecules in the volume found in part (a) (which may be expressed as $V = 2.24 \times 10^4$ cm^3), so that

$$\frac{N}{V}=\frac{6.02\times10^{23}}{2.24\times10^{4}\,\text{cm}^{3}}=2.69\times10^{19}\ \text{molecules/cm}^{3}\,.$$

62. Using the ideal gas law, one mole occupies a volume equal to

$$V = \frac{nRT}{p} = \frac{(1)(8.31)(50.0)}{1.00\times10^{-8}} = 4.16\times10^{10}\ \text{m}^3.$$

Therefore, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_\text{A}}{V} = \frac{(1)\left(6.02\times10^{23}\right)}{4.16\times10^{10}} = 1.45\times10^{13}\ \frac{\text{molecules}}{\text{m}^3}.$$

Using $d = 20.0 \times 10^{-9}$ m, Eq. 19-25 yields

$$\lambda = \frac{1}{\sqrt{2}\pi d^2\left(\frac{N}{V}\right)} = 38.8\ \text{m}.$$

63. In this solution we will use non-standard notation: writing ρ for *weight*-density (instead of mass-density), where ρ_c refers to the cool air and ρ_h refers to the hot air. Then the condition required by the problem is

$$F_{\text{net}} = F_{\text{buoyant}} - \text{hot-air-weight} - \text{balloon-weight}$$

$$2.67 \times 10^3 \text{ N} = \rho_c V - \rho_h V - 2.45 \times 10^3 \text{ N}$$

where $V = 2.18 \times 10^3$ m^3 and $\rho_c = 11.9$ N/m^3. This condition leads to $\rho_h = 9.55$ N/m^3. Using the ideal gas law to write ρ_h as PMg/RT where $P = 101000$ Pascals and $M = 0.028$ kg/m^3 (as suggested in the problem), we conclude that the temperature of the enclosed air should be 349 K.

64. (a) Using the atomic mass value in Appendix F, we compute that the molecular mass for the (diatomic) hydrogen gas is M = 2.016 g/mol = 0.002016 kg/mol. Eq. 19-35 then gives $v_p = 1.44 \times 10^3$ m/s.

(b) At that value of speed the Maxwell distribution (Eq. 19-27) has the value $P(v_p) = 5.78 \times 10^{-4}$.

(c) Eq. 19-29, with the limits indicated in the problem, yields frac = 0.707 = 71%.

(d) For T = 500 K, the result is $v_p = 2.03 \times 10^3$ m/s.

(e) Now the Maxwell distribution (Eq. 19-27) has the value $P(v_p) = 4.09 \times 10^{-4}$.

(f) As expected (from Eq. 19-35) the value of v_p increased.

(g) We also found that the value of $P(v_p)$ decreased. One way to think of this is that the curve (see Fig 19-7(b)) *widens* as temperature increases (but must maintain the same total area, by Eq. 19-28), thus causing the peak value to lower.

65. We note that $\Delta K = n\left(\frac{3}{2}R\right)\Delta T$ according to the discussion in §19-5 and §19-9. Also, $\Delta E_{\text{int}} = nC_V\Delta T$ can be used for each of these processes (since we are told this is an ideal gas). Finally, we note that Eq. 19-49 leads to $C_p = C_V + R \approx 8.0$ cal/mol·K after we convert Joules to calories in the ideal gas constant value (Eq. 19-6): $R \approx 2.0$ cal/mol·K. The first law of thermodynamics $Q = \Delta E_{\text{int}} + W$ applies to each process.

- Constant volume process with $\Delta T = 50$ K and $n = 3.0$ mol.

(a) Since the change in the internal energy is $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal, and the work done by the gas is $W = 0$ for constant volume processes, the first law gives $Q = 900 + 0 = 900$ cal.

(b) As shown in part (a), $W = 0$.

(c) The change in the internal energy is, from part (a), $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal.

(d) The change in the total translational kinetic energy is

$$\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$$

- Constant pressure process with $\Delta T = 50$ K and $n = 3.0$ mol.

(e) $W = p\Delta V$ for constant pressure processes, so (using the ideal gas law)

$$W = nR\Delta T = (3.0)(2.0)(50) = 300 \text{ cal}.$$

The first law gives $Q = (900 + 300)$ cal = 1200 cal.

(f) From (e), we have W=300 cal.

(g) The change in the internal energy is $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal.

(h) The change in the translational kinetic energy is $\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$

- Adiabiatic process with $\Delta T = 50$ K and $n = 3.0$ mol.

(i) $Q = 0$ by definition of “adiabatic.”

(j) The first law leads to $W = Q - E_{\text{int}} = 0 - 900 \text{ cal} = -900$ cal.

(k) The change in the internal energy is $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal.

(l) As in part (d) and (h), $\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}$.

66. (a) Since an ideal gas is involved, then $\Delta E_{\text{int}} = 0$ implies $T_1 = T_0$ (see Eq. 19-62). Consequently, the ideal gas law leads to

$$p_1 = p_0\left(\frac{V_0}{V_1}\right) = \frac{p_0}{5.00}$$

for the pressure at the end of the sudden expansion. Now, the (slower) adiabatic process is described by Eq. 19-54:

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = p_1(5.00)^{\gamma}$$

as a result of the fact that $V_2 = V_0$. Therefore,

$$p_2 = \left(\frac{p_0}{5.00}\right)(5.00)^{\gamma} = (5.00)^{\gamma-1}\,p_0$$

which is compared with the problem requirement that $p_2 = (5.00)^{0.4}\,p_0$. Thus, we find that $\gamma = 1.4 = 7/5$. Since $\gamma = C_p/C_V$, we see from Table 19-3 that this is a diatomic gas with rotation of the molecules.

(b) The direct connection between E_{int} and K_{avg} is explained at the beginning of §19-8. Since $\Delta E_{\text{int}} = 0$ in the free expansion, then $K_1 = K_0$, or $K_1/K_0 = 1.00$.

(c) In the (slower) adiabatic process, Eq. 19-56 indicates

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (5.00)^{0.4}T_0 \quad\Rightarrow\quad \frac{(E_{\text{int}})_2}{(E_{\text{int}})_0} = \frac{T_2}{T_0} = (5.00)^{0.4} \approx 1.90.$$

Therefore, $K_2/K_0 = 1.90$.

67. (a) Differentiating Eq. 19-53, we obtain

$$\frac{dp}{dV} = (\text{constant})\frac{-\gamma}{V^{\gamma+1}} \Rightarrow B = -V\frac{dp}{dV} = (\text{constant})\frac{\gamma}{V^{\gamma}}$$

which produces the desired result upon using Eq. 19-53 again (p = (constant)/V^{γ}).

(b) Due to the fact that $v = \sqrt{B/\rho}$ (from Chapter 17) and $p = nRT/V = (M_{\text{sam}}/M)RT/V$ (from this chapter) with $\rho = M_{\text{sam}}/V$ (the definition of density), the speed of sound in an ideal gas becomes

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma\left(M_{\text{sam}}/M\right)RT/V}{M_{\text{sam}}/V}} = \sqrt{\frac{\gamma RT}{M}}.$$

68. With $p = 1.01 \times 10^5$ Pa and $\rho = 1.29$ kg/m^3, we use the result of part (b) of the previous problem to obtain

$$\gamma = \frac{\rho v^2}{p} = \frac{\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)^2}{1.01\times10^5\ \text{Pa}} = 1.40.$$

69. (a) We use the result of exercise 58 to express γ in terms of the speed of sound $v = f\lambda$.

$$\gamma = \frac{Mv^2}{RT} = \frac{M\lambda^2 f^2}{RT}.$$

The distance between nodes is half of a wavelength $\lambda = 2 \times 0.0677$ m, and the molar mass in SI units is $M = 0.127$ kg/mol. Consequently,

$$\gamma = \frac{(0.127)(2\times 0.0677)^2 (1400)^2}{(8.31)(400)} = 1.37.$$

(b) Using Table 19-3, we find $\gamma = 5/3 \approx 1.7$ for monatomic gases, $\gamma = 7/5 = 1.4$ for diatomic gases, and $\gamma = 4/3 \approx 1.3$ for polyatomic gases. Our result in part (a) suggests that iodine is a diatomic gas.

70. The ratio is

$$\frac{mgh}{\frac{1}{2}m\text{v}_{\text{rms}}^{\;2}} = \frac{2gh}{\text{v}_{\text{rms}}^{\;2}} = \frac{2Mgh}{3RT}$$

where we have used Eq. 19-22 in that last step. With T = 273 K, h = 0.10 m and M = 32 g/mol = 0.032 kg/mol, we find the ratio equals 9.2×10^{-6}.

71. (a) By Eq. 19-28, $W = -374$ J (since the process is an adiabatic compression).

(b) $Q = 0$ since the process is adiabatic.

(c) By first law of thermodynamics, the change in internal energy is $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) The change in the average kinetic energy per atom is $\Delta K_{\text{avg}} = \Delta E_{\text{int}}/N = +3.11 \times 10^{-22}$ J.

72. Using Eq. 19-53 in Eq. 18-25 gives

$$W = P_i V_i^{\gamma} \int V^{-\gamma}\, dV = P_i V_i^{\gamma} \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma}.$$

Using Eq. 19-54 we can write this as

$$W = P_i V_i \left(\frac{1 - \left(\frac{P_f}{P_i}\right)^{1-1/\gamma}}{1-\gamma} \right).$$

In this problem, $\gamma = 7/5$ (see Table 19-3) and $P_f/P_i = 2$. Converting the initial pressure to Pascals we find $P_i V_i = 24240$ J. Plugging in, then, we obtain $W = -1.33 \times 10^4$ J.

73. (a) With work being given by $W = p\Delta V = (250)(-0.60)$ J $= -150$ J, and the heat transfer given as –210 J, then the change in internal energy is found from the first law of thermodynamics to be [–210 – (–150)] J = –60 J.

(b) Since the pressures (and also the number of moles) don't change in this process, then the volume is simply proportional to the (absolute) temperature. Thus, the final temperature is ¼ of the initial temperature. The answer is 90 K.

74. Eq. 19-25 gives the mean free path:

$$\lambda = \frac{1}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, (N/V)} = \frac{n\,R\,T}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\; P\,N}$$

where we have used the ideal gas law in that last step. Thus, the change in the mean free path is

$$\Delta\lambda = \frac{n\,R\,\Delta T}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\; P\,N} = \frac{R\,Q}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\; P\,N\,C_p}$$

where we have used Eq. 19-46. The constant pressure molar heat capacity is (7/2)R in this situation, so (with $N = 9 \times 10^{23}$ and $d = 250 \times 10^{-12}$ m) we find

$$\Delta\lambda = 1.52 \times 10^{-9}\,\text{m} = 1.52\ \text{nm}\ .$$

75. This is very similar to Sample Problem 19-4 (and we use similar notation here) except for the use of Eq. 19-31 for v_{avg} (whereas in that Sample Problem, its value was just assumed). Thus,

$$f = \frac{\text{speed}}{\text{distance}} = \frac{v_{\text{avg}}}{\lambda} = \frac{p\,d^2}{k}\left(\frac{16\pi R}{MT}\right) .$$

Therefore, with $p = 2.02 \times 10^3$ Pa, $d = 290 \times 10^{-12}$ m and $M = 0.032$ kg/mol (see Table 19-1), we obtain $f = 7.03 \times 10^9\ \text{s}^{-1}$.

76. (a) The volume has increased by a factor of 3, so the pressure must decrease accordingly (since the temperature does not change in this process). Thus, the final pressure is one-third of the original 6.00 atm. The answer is 2.00 atm.

(b) We note that Eq. 19-14 can be written as $P_iV_i \ln(V_f/V_i)$. Converting "atm" to "Pa" (a Pascal is equivalent to a N/m^2) we obtain $W = 333$ J.

(c) The gas is monatomic so $\gamma = 5/3$. Eq. 19-54 then yields $P_f = 0.961$ atm.

(d) Using Eq. 19-53 in Eq. 18-25 gives

$$W = P_i V_i^{\gamma} \int V^{-\gamma} dV = P_i V_i^{\gamma} \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma} = \frac{P_f V_f - P_i V_i}{1-\gamma}$$

where in the last step Eq. 19-54 has been used. Converting "atm" to "Pa", we obtain $W = 236$ J.

77. (a) With $P_1 = (20.0)(1.01 \times 10^5 \text{ Pa})$ and $V_1 = 0.0015 \text{ m}^3$, the ideal gas law gives

$$P_1V_1 = nRT_1 \quad \Rightarrow \quad T_1 = 121.54 \text{ K} \approx 122 \text{ K}.$$

(b) From the information in the problem, we deduce that $T_2 = 3T_1 = 365$ K.

(c) We also deduce that $T_3 = T_1$ which means $\Delta T = 0$ for this process. Since this involves an ideal gas, this implies the change in internal energy is zero here.

78. (a) We use $p_i V_i^\gamma = p_f V_f^{\ \gamma}$ to compute γ.

$$\gamma = \frac{\ln\left(p_i/p_f\right)}{\ln\left(V_f/V_i\right)} = \frac{\ln\left(1.0\,\text{atm}/1.0\times10^5\,\text{atm}\right)}{\ln\left(1.0\times10^3\,\text{L}/1.0\times10^6\,\text{L}\right)} = \frac{5}{3}.$$

Therefore the gas is monatomic.

(b) Using the gas law in ratio form (see Sample Problem 19-1), the final temperature is

$$T_f = T_i\frac{p_f V_f}{p_i V_i} = \left(273\,\text{K}\right)\frac{\left(1.0\times10^5\,\text{atm}\right)\left(1.0\times10^3\,\text{L}\right)}{\left(1.0\,\text{atm}\right)\left(1.0\times10^6\text{L}\right)} = 2.7\times10^4\ \text{K}.$$

(c) The number of moles of gas present is

$$n = \frac{p_i V_i}{RT_i} = \frac{\left(1.01\times10^5\ \text{Pa}\right)\left(1.0\times10^3\ \text{cm}^3\right)}{\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(273\,\text{K}\right)} = 4.5\times10^4\ \text{mol}.$$

(d) The total translational energy per mole before the compression is

$$K_i = \frac{3}{2}RT_i = \frac{3}{2}\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(273\,\text{K}\right) = 3.4\times10^3\ \text{J}.$$

(e) After the compression,

$$K_f = \frac{3}{2}RT_f = \frac{3}{2}\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(2.7\times10^4\ \text{K}\right) = 3.4\times10^5\ \text{J}.$$

(f) Since $v_{\text{rms}}^2 \propto T$, we have

$$\frac{v_{\text{rms},i}^2}{v_{\text{rms},f}^2} = \frac{T_i}{T_f} = \frac{273\,\text{K}}{2.7\times10^4\ \text{K}} = 0.010.$$

79. (a) The final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32\,\text{atm})(1.0\,\text{L})}{4.0\,\text{L}} = 8.0\,\text{atm},$$

(b) For the isothermal process the final temperature of the gas is $T_f = T_i = 300$ K.

(c) The work done is

$$W = nRT_i \ln\left(\frac{V_f}{V_i}\right) = p_i V_i \ln\left(\frac{V_f}{V_i}\right) = (32\,\text{atm})\left(1.01\times10^5\,\text{Pa/atm}\right)\left(1.0\times10^{-3}\,\text{m}^3\right)\ln\left(\frac{4.0\,\text{L}}{1.0\,\text{L}}\right)$$
$$= 4.4\times10^3\,\text{J}.$$

For the adiabatic process $p_i V_i^\gamma = p_f V_f^\gamma$. Thus,

(d) The final pressure is

$$p_f = p_f\left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{5/3} = 3.2\,\text{atm}.$$

(e) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 120\,\text{K}\ .$$

(f) The work done is

$$W = Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}} = -\frac{3}{2}nR\Delta T = -\frac{3}{2}\left(p_f V_f - p_i V_i\right)$$
$$= -\frac{3}{2}\left[(3.2\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right]\left(1.01\times10^5\,\text{Pa/atm}\right)\left(10^{-3}\,\text{m}^3/\text{L}\right)$$
$$= 2.9\times10^3\,\text{J}\ .$$

If the gas is diatomic, then $\gamma = 1.4$.

(g) The final pressure is

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{1.4} = 4.6\,\text{atm}\,.$$

(h) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 170\,\text{K}\ .$$

(i) The work done is

$$\begin{aligned} W &= Q - \Delta E_{\text{int}} = -\frac{5}{2} nR\Delta T = -\frac{5}{2}\left(p_f V_f - p_i V_i\right) \\ &= -\frac{5}{2}\left[(4.6\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right]\left(1.01\times 10^5\ \text{Pa/atm}\right)\left(10^{-3}\ \text{m}^3/\text{L}\right) \\ &= 3.4\times 10^3\ \text{J}. \end{aligned}$$

80. We label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with $V_2 = 4\ \text{m}^3$; then continues on to state 3 isothermally, with $V_3 = 10\ \text{m}^3$; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process $1 \to 2$ $p_1V_1^\gamma = p_2V_2^\gamma$, for the isothermal process $2 \to 3$ $p_2V_2 = p_3V_3$, and finally for the adiabatic process $3 \to 4$ $p_3V_3^\gamma = p_4V_4^\gamma$. These equations yield

$$p_4 = p_3\left(\frac{V_3}{V_4}\right)^\gamma = p_2\left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma = p_1\left(\frac{V_1}{V_2}\right)^\gamma\left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma .$$

We substitute this expression for p_4 into the equation $p_1V_1 = p_4V_4$ (since $T_1 = T_4$) to obtain $V_1V_3 = V_2V_4$. Solving for V_4 we obtain

$$V_4 = \frac{V_1V_3}{V_2} = \frac{\left(2.0\,\text{m}^3\right)\left(10\,\text{m}^3\right)}{4.0\,\text{m}^3} = 5.0\,\text{m}^3.$$

81. We write $T = 273$ K and use Eq. 19-14:

$$W = (1.00\,\text{mol})\,(8.31\ \text{J/mol}\cdot\text{K})\,(273\,\text{K})\ \ln\left(\frac{16.8}{22.4}\right)$$

which yields $W = -653$ J. Recalling the sign conventions for work stated in Chapter 18, this means an external agent does 653 J of work *on* the ideal gas during this process.

82. (a) We use $pV = nRT$. The volume of the tank is

$$V = \frac{nRT}{p} = \frac{\left(\frac{300\,\text{g}}{17\ \text{g/mol}}\right)(8.31\ \text{J/mol}\cdot\text{K})(350\,\text{K})}{1.35\times10^{6}\ \text{Pa}} = 3.8\times10^{-2}\ \text{m}^3 = 38\,\text{L}.$$

(b) The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{(8.7\times10^{5}\ \text{Pa})(3.8\times10^{-2}\ \text{m}^3)}{(8.31\ \text{J/mol}\cdot\text{K})(293\,\text{K})} = 13.5\,\text{mol}.$$

The mass of the gas that leaked out is then Δm = 300 g – (13.5 mol)(17 g/mol) = 71 g.

83. From Table 19-3, $C_V = \frac{3}{2}R = 12.5\ \text{J/mol}\cdot\text{K}$ for a monatomic gas such as helium. To obtain the desired result c_V we need to effectively "convert" mol → kg, which can be done using the molar mass M expressed in kilograms per mole. Although we could look up M for helium in Table 19-1 or Appendix F, the problem gives us m so that we can use Eq. 19-4 to find M. That is,

$$M = mN_\text{A} = \left(6.66\times10^{-27}\ \text{kg}\right)\left(6.02\times10^{23}\ /\,\text{mol}\right) = 4.01\times10^{-3}\ \frac{\text{kg}}{\text{mol}}.$$

Therefore, $c_V = C_V/M = 3.11 \times 10^3$ J/kg·K.

84. (a) When $n = 1$, $V = V_m = RT/p$, where V_m is the molar volume of the gas. So

$$V_m = \frac{RT}{p} = \frac{(8.31\,\text{J/mol}\cdot\text{K})(273.15\,\text{K})}{1.01\times10^5\ \text{Pa}} = 22.5\,\text{L}\ .$$

(b) We use $v_{\text{rms}} = \sqrt{3RT/M}$. The ratio is given by

$$\frac{v_{\text{rms, He}}}{v_{\text{rms, Ne}}} = \sqrt{\frac{M_{\text{Ne}}}{M_{\text{He}}}} = \sqrt{\frac{20\,\text{g}}{4.0\,\text{g}}} = 2.25.$$

(c) We use $\lambda_{\text{He}} = (\sqrt{2}\pi d^2\, N/V)^{-1}$, where the number of particles per unit volume is given by $N/V = N_A n/V = N_A p/RT = p/kT$. So

$$\lambda_{\text{He}} = \frac{1}{\sqrt{2}\pi d^2\,(p/kT)} = \frac{kT}{\sqrt{2}\pi d^2 p}$$
$$= \frac{(1.38\times10^{-23}\ \text{J/K})(273.15\,\text{K})}{1.414\pi(1\times10^{-10}\ \text{m})^2\,(1.01\times10^5\ \text{Pa})} = 0.840\,\mu\text{m}\ .$$

(d) $\lambda_{\text{Ne}} = \lambda_{\text{He}} = 0.840\ \mu\text{m}$.

85. For convenience, the "int" subscript for the internal energy will be omitted in this solution. Recalling Eq. 19-28, we note that

$$\sum_{\text{cycle}} E = 0$$

$$\Delta E_{A\to B} + \Delta E_{B\to C} + \Delta E_{C\to D} + \Delta E_{D\to E} + \Delta E_{E\to A} = 0.$$

Since a gas is involved (assumed to be ideal), then the internal energy does not change when the temperature does not change, so

$$\Delta E_{A\to B} = \Delta E_{D\to E} = 0.$$

Now, with $\Delta E_{E\to A} = 8.0$ J given in the problem statement, we have

$$\Delta E_{B\to C} + \Delta E_{C\to D} + 8.0\text{ J} = 0.$$

In an adiabatic process, $\Delta E = -W$, which leads to $-5.0\text{ J} + \Delta E_{C\to D} + 8.0\text{ J} = 0$, and we obtain $\Delta E_{C\to D} = -3.0$ J.

86. We solve

$$\sqrt{\frac{3RT}{M_{\text{helium}}}} = \sqrt{\frac{3R(293\,\text{K})}{M_{\text{hydrogen}}}}$$

for T. With the molar masses found in Table 19-1, we obtain

$$T = (293\,\text{K})\left(\frac{4.0}{2.02}\right) = 580\,\text{K}$$

which is equivalent to 307°C.

87. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a straight line on the pV diagram, the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Also, from the ideal gas law in ratio form (see Sample Problem 1), we find the final temperature:

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) = 4T_1 .$$

(a) With $\Delta V = V_2 - V_1 = 2V_1 - V_1 = V_1$ and $p_1 + p_2 = p_1 + 2p_1 = 3p_1$, we obtain

$$W = \frac{3}{2}\left(p_1 V_1\right) = \frac{3}{2} nRT_1 \;\Rightarrow\; \frac{W}{nRT_1} = \frac{3}{2} = 1.5$$

where the ideal gas law is used in that final step.

(b) With $\Delta T = T_2 - T_1 = 4T_1 - T_1 = 3T_1$ and $C_V = \frac{3}{2}R$, we find

$$\Delta E_{\text{int}} = n\left(\frac{3}{2}R\right)\left(3T_1\right) = \frac{9}{2}nRT_1 \;\Rightarrow\; \frac{\Delta E_{\text{int}}}{nRT_1} = \frac{9}{2} = 4.5.$$

(c) The energy added as heat is $Q = \Delta E_{\text{int}} + W = 6nRT_1$, or $Q / nRT_1 = 6$.

(d) The molar specific heat for this process may be defined by

$$C = \frac{Q}{n\Delta T} = \frac{6nRT_1}{n(3T_1)} = 2R \;\Rightarrow\; \frac{C}{R} = 2.$$

88. The gas law in ratio form (see Sample Problem 19-1) leads to

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = (5.67\,\text{Pa})\left(\frac{4.00\,\text{m}^3}{7.00\,\text{m}^3}\right)\left(\frac{313\,\text{K}}{217\,\text{K}}\right) = 4.67\,\text{Pa} \ .$$

89. It is recommended to look over §19-7 before doing this problem.

(a) We normalize the distribution function as follows:

$$\int_0^{v_o} P(v)\,dv = 1 \Rightarrow C = \frac{3}{v_o^3}.$$

(b) The average speed is

$$\int_0^{v_o} vP(v)\,dv = \int_0^{v_o} v\left(\frac{3v^2}{v_o^3}\right)dv = \frac{3}{4}v_o\ .$$

(c) The rms speed is the square root of

$$\int_0^{v_o} v^2\ P(v)\,dv = \int_0^{v_o} v^2\left(\frac{3v^2}{v_o^3}\right)dv = \frac{3}{5}v_o^2\ .$$

Therefore, $v_{\text{rms}} = \sqrt{3/5}v_{\circ} \approx 0.775v_{\circ}$.

90. (a) From Table 19-3, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. Thus, Eq. 19-46 yields

$$Q = nC_p\Delta T = (3.00)\left(\frac{7}{2}(8.31)\right)(40.0) = 3.49\times10^3\ \text{J}.$$

(b) Eq. 19-45 leads to

$$\Delta E_{\text{int}} = nC_V\Delta T = (3.00)\left(\frac{5}{2}(8.31)\right)(40.0) = 2.49\times10^3\ \text{J}.$$

(c) From either $W = Q - \Delta E_{\text{int}}$ or $W = p\Delta T = nR\Delta T$, we find $W = 997$ J.

(d) Eq. 19-24 is written in more convenient form (for this problem) in Eq. 19-38. Thus, we obtain

$$\Delta K_{\text{trans}} = \Delta\left(NK_{\text{avg}}\right) = n\left(\frac{3}{2}R\right)\Delta T \approx 1.50\times10^3\ \text{J}.$$

91. (a) The temperature is 10.0°C → $T = 283$ K. Then, with $n = 3.50$ mol and $V_f/V_0 = 3/4$, we use Eq. 19-14:

$$W = nRT\ \ln\left(\frac{V_f}{V_0}\right) = -2.37\,\text{kJ}.$$

(b) The internal energy change ΔE_{int} vanishes (for an ideal gas) when $\Delta T = 0$ so that the First Law of Thermodynamics leads to $Q = W = -2.37$ kJ. The negative value implies that the heat transfer is from the sample to its environment.

92. (a) Since $n/V = p/RT$, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_\text{A}}{V} = N_\text{A}\left(\frac{p}{RT}\right)(6.02\times10^{23})\frac{1.01\times10^5\text{ Pa}}{\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\left(293\text{ K}\right)} = 2.5\times10^{25}\,\frac{\text{molecules}}{\text{m}^3}.$$

(b) Three-fourths of the 2.5×10^{25} value found in part (a) are nitrogen molecules with $M = 28.0$ g/mol (using Table 19-1), and one-fourth of that value are oxygen molecules with $M = 32.0$ g/mol. Consequently, we generalize the $M_\text{sam} = NM/N_\text{A}$ expression for these two species of molecules and write

$$\frac{3}{4}(2.5\times10^{25})\frac{28.0}{6.02\times10^{23}} + \frac{1}{4}(2.5\times10^{25})\frac{32.0}{6.02\times10^{23}} = 1.2\times10^3\text{ g}.$$

93. (a) The work done in a constant-pressure process is $W = p\Delta V$. Therefore,

$$W = \left(25\,\mathrm{N/m^2}\right)(1.8\,\mathrm{m^3} - 3.0\,\mathrm{m^3}) = -30\,\mathrm{J}.$$

The sign conventions discussed in the textbook for Q indicate that we should write –75 J for the energy which leaves the system in the form of heat. Therefore, the first law of thermodynamics leads to

$$\Delta E_{\mathrm{int}} = Q - W = (-75\ \mathrm{J}) - (-30\ \mathrm{J}) = -45\ \mathrm{J}.$$

(b) Since the pressure is constant (and the number of moles is presumed constant), the ideal gas law in ratio form (see Sample Problem 19-1) leads to

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (300\,\mathrm{K})\left(\frac{1.8\,\mathrm{m^3}}{3.0\,\mathrm{m^3}}\right) = 1.8\times10^2\ \mathrm{K}.$$

It should be noted that this is consistent with the gas being monatomic (that is, if one assumes $C_V = \frac{3}{2}R$ and uses Eq. 19-45, one arrives at this same value for the final temperature).

94. Since no heat is transferred in an adiabatic process, then

$$Q_{\text{total}} = Q_{\text{isotherm}} = W_{\text{isotherm}} = nRT\ln\left(\frac{3}{12}\right)$$

where the First Law of Thermodynamics (with $\Delta E_{\text{int}} = 0$ during the isothermal process) and Eq. 19-14 have been used. With $n = 2.0$ mol and $T = 300$ K, we obtain $Q = -6912$ J ≈ -6.9 kJ.