1. (a) With a understood to mean the magnitude of acceleration, Newton's second and third laws lead to

$$m_2 a_2 = m_1 a_1 \Rightarrow m_2 = \frac{\left(6.3\times10^{-7}\,\text{kg}\right)\left(7.0\,\text{m/s}^2\right)}{9.0\,\text{m/s}^2} = 4.9\times10^{-7}\,\text{kg}.$$

(b) The magnitude of the (only) force on particle 1 is

$$F = m_1 a_1 = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times10^9\right)\frac{|q|^2}{0.0032^2}.$$

Inserting the values for m_1 and a_1 (see part (a)) we obtain $|q| = 7.1 \times 10^{-11}$ C.

2. The magnitude of the mutual force of attraction at r = 0.120 m is

$$F = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times10^9\right)\frac{\left(3.00\times10^{-6}\right)\left(1.50\times10^{-6}\right)}{0.120^2} = 2.81\,\text{N}.$$

3. Eq. 21-1 gives Coulomb's Law, $F = k\frac{|q_1||q_2|}{r^2}$, which we solve for the distance:

$$r = \sqrt{\frac{k|q_1||q_2|}{F}} = \sqrt{\frac{\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(26.0\times10^{-6}\,\mathrm{C}\right)\left(47.0\times10^{-6}\,\mathrm{C}\right)}{5.70\,\mathrm{N}}} = 1.39\,\mathrm{m}.$$

4. The fact that the spheres are identical allows us to conclude that when two spheres are in contact, they share equal charge. Therefore, when a charged sphere (q) touches an uncharged one, they will (fairly quickly) each attain half that charge ($q/2$). We start with spheres 1 and 2 each having charge q and experiencing a mutual repulsive force $F = kq^2/r^2$. When the neutral sphere 3 touches sphere 1, sphere 1's charge decreases to $q/2$. Then sphere 3 (now carrying charge $q/2$) is brought into contact with sphere 2, a total amount of $q/2 + q$ becomes shared equally between them. Therefore, the charge of sphere 3 is $3q/4$ in the final situation. The repulsive force between spheres 1 and 2 is finally

$$F' = k\frac{(q/2)(3q/4)}{r^2} = \frac{3}{8}k\frac{q^2}{r^2} = \frac{3}{8}F \quad \Rightarrow \quad \frac{F'}{F} = \frac{3}{8} = 0.375.$$

5. The magnitude of the force of either of the charges on the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$$

where r is the distance between the charges. We want the value of q that maximizes the function $f(q) = q(Q - q)$. Setting the derivative df/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$. Thus, $q/Q = 0.500$.

6. For ease of presentation (of the computations below) we assume $Q > 0$ and $q < 0$ (although the final result does not depend on this particular choice).

(a) The x-component of the force experienced by $q_1 = Q$ is

$$F_{1x} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{(Q)(Q)}{\left(\sqrt{2}a\right)^2}\cos 45° + \frac{(|q|)\,(Q)}{a^2}\right) = \frac{Q|q|}{4\pi\varepsilon_0 a^2}\left(-\frac{Q/|q|}{2\sqrt{2}}+1\right)$$

which (upon requiring $F_{1x} = 0$) leads to $Q/|q| = 2\sqrt{2}$, or $Q/q = -2\sqrt{2} = -2.83$.

(b) The y-component of the net force on $q_2 = q$ is

$$F_{2y} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q|^2}{\left(\sqrt{2}a\right)^2}\sin 45° - \frac{(|q|)\,(Q)}{a^2}\right) = \frac{|q|^2}{4\pi\varepsilon_0 a^2}\left(\frac{1}{2\sqrt{2}} - \frac{Q}{|q|}\right)$$

which (if we demand $F_{2y} = 0$) leads to $Q/q = -1/2\sqrt{2}$. The result is inconsistent with that obtained in part (a). Thus, we are unable to construct an equilibrium configuration with this geometry, where the only forces present are given by Eq. 21-1.

7. The force experienced by q_3 is

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{|q_3||q_1|}{a^2}\hat{\mathrm{j}} + \frac{|q_3||q_2|}{(\sqrt{2}a)^2}(\cos 45°\hat{\mathrm{i}} + \sin 45°\hat{\mathrm{j}}) + \frac{|q_3||q_4|}{a^2}\hat{\mathrm{i}} \right)$$

(a) Therefore, the x-component of the resultant force on q_3 is

$$F_{3x} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(\frac{|q_2|}{2\sqrt{2}} + |q_4|\right) = \left(8.99\times10^9\right)\frac{2\left(1.0\times10^{-7}\right)^2}{(0.050)^2}\left(\frac{1}{2\sqrt{2}} + 2\right) = 0.17\,\mathrm{N}.$$

(b) Similarly, the y-component of the net force on q_3 is

$$F_{3y} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(-|q_1| + \frac{|q_2|}{2\sqrt{2}}\right) = \left(8.99\times10^9\right)\frac{2\left(1.0\times10^{-7}\right)^2}{(0.050)^2}\left(-1 + \frac{1}{2\sqrt{2}}\right) = -0.046\,\mathrm{N}.$$

8. (a) The individual force magnitudes (acting on Q) are, by Eq. 21-1,

$$k\frac{|q_1|Q}{\left(-a-\frac{a}{2}\right)^2}=k\frac{|q_2|Q}{\left(a-\frac{a}{2}\right)^2}$$

which leads to $|q_1| = 9.0\ |q_2|$. Since Q is located between q_1 and q_2, we conclude q_1 and q_2 are like-sign. Consequently, $q_1/q_2 = 9.0$.

(b) Now we have

$$k\frac{|q_1|Q}{\left(-a-\frac{3a}{2}\right)^2}=k\frac{|q_2|Q}{\left(a-\frac{3a}{2}\right)^2}$$

which yields $|q_1| = 25\ |q_2|$. Now, Q is not located between q_1 and q_2, one of them must push and the other must pull. Thus, they are unlike-sign, so $q_1/q_2 = -25$.

9. We assume the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges. We choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Then, the force on q_2 is

$$F_a = -\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = -k\frac{q_1q_2}{r^2}$$

where r = 0.500 m. The negative sign indicates that the spheres attract each other. After the wire is connected, the spheres, being identical, acquire the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1 + q_2)/2$. The force is now one of repulsion and is given by

$$F_b = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{r^2} = k\frac{(q_1+q_2)^2}{4r^2}.$$

We solve the two force equations simultaneously for q_1 and q_2. The first gives the product

$$q_1q_2 = -\frac{r^2F_a}{k} = -\frac{(0.500\,\text{m})^2(0.108\,\text{N})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = -3.00\times10^{-12}\,\text{C}^2,$$

and the second gives the sum

$$q_1 + q_2 = 2r\sqrt{\frac{F_b}{k}} = 2(0.500\,\text{m})\sqrt{\frac{0.0360\,\text{N}}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}} = 2.00\times10^{-6}\,\text{C}$$

where we have taken the positive root (which amounts to assuming $q_1 + q_2 \geq 0$). Thus, the product result provides the relation

$$q_2 = \frac{-\left(3.00\times10^{-12}\,\text{C}^2\right)}{q_1}$$

which we substitute into the sum result, producing

$$q_1 - \frac{3.00\times10^{-12}\,\text{C}^2}{q_1} = 2.00\times10^{-6}\,\text{C}.$$

Multiplying by q_1 and rearranging, we obtain a quadratic equation

$$q_1^2 - \left(2.00\times10^{-6}\,\text{C}\right)q_1 - 3.00\times10^{-12}\,\text{C}^2 = 0.$$

The solutions are

$$q_1 = \frac{2.00\times10^{-6}\,\text{C} \pm \sqrt{\left(-2.00\times10^{-6}\,\text{C}\right)^2 - 4\left(-3.00\times10^{-12}\,\text{C}^2\right)}}{2}.$$

If the positive sign is used, $q_1 = 3.00 \times 10^{-6}$ C, and if the negative sign is used, $q_1 = -1.00\times10^{-6}\ \text{C}$.

(a) Using $q_2 = (-3.00 \times 10^{-12})/q_1$ with $q_1 = 3.00 \times 10^{-6}$ C, we get $q_2 = -1.00\times10^{-6}\ \text{C}$.

(b) If we instead work with the $q_1 = -1.00 \times 10^{-6}$ C root, then we find $q_2 = 3.00\times10^{-6}\ \text{C}$.

Note that since the spheres are identical, the solutions are essentially the same: one sphere originally had charge -1.00×10^{-6} C and the other had charge $+3.00 \times 10^{-6}$ C.

What if we had not made the assumption, above, that $q_1 + q_2 \geq 0$? If the signs of the charges were reversed (so $q_1 + q_2 < 0$), then the forces remain the same, so a charge of $+1.00 \times 10^{-6}$ C on one sphere and a charge of -3.00×10^{-6} C on the other also satisfies the conditions of the problem.

10. With rightwards positive, the net force on q_3 is

$$F_3 = F_{13} + F_{23} = k\frac{q_1 q_3}{(L_{12} + L_{23})^2} + k\frac{q_2 q_3}{L_{23}^2}.$$

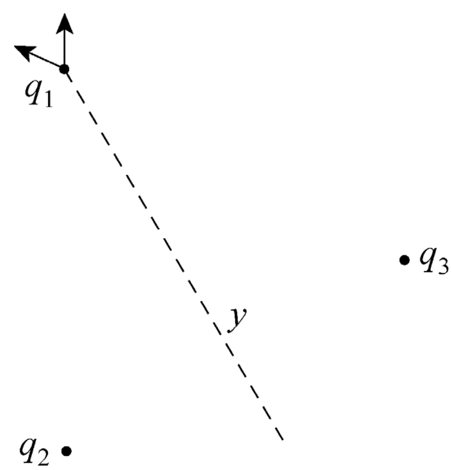
We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on q_3 by q_1) is negative if they are unlike charges, indicating that q_3 is being pulled toward q_1, and it is positive if they are like charges (so q_3 would be repelled from q_1). Setting the net force equal to zero $L_{23} = L_{12}$ and canceling k, q_3 and L_{12} leads to

$$\frac{q_1}{4.00} + q_2 = 0 \quad \Rightarrow \quad \frac{q_1}{q_2} = -4.00.$$

11. (a) Eq. 21-1 gives

$$F_{12} = k\frac{q_1 q_2}{d^2} = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(20.0\times10^{-6}\,\text{C}\right)^2}{\left(1.50\,\text{m}\right)^2} = 1.60\,\text{N}.$$

(b) A force diagram is shown as well as our choice of *y* axis (the dashed line).



The *y* axis is meant to bisect the line between q_2 and q_3 in order to make use of the symmetry in the problem (equilateral triangle of side length *d*, equal-magnitude charges $q_1 = q_2 = q_3 = q$). We see that the resultant force is along this symmetry axis, and we obtain

$$\left|F_y\right| = 2\left(k\frac{q^2}{d^2}\right)\cos 30° = 2.77\,\text{N}.$$

12. (a) According to the graph, when q_3 is very close to q_1 (at which point we can consider the force exerted by particle 1 on 3 to dominate) there is a (large) force in the positive x direction. This is a repulsive force, then, so we conclude q_1 has the same sign as q_3. Thus, q_3 is a positive-valued charge.

(b) Since the graph crosses zero and particle 3 is *between* the others, q_1 must have the same sign as q_2, which means it is also positive-valued. We note that it crosses zero at r = 0.020 m (which is a distance d = 0.060 m from q_2). Using Coulomb's law at that point, we have

$$\frac{q_1\, q_3}{4\pi\varepsilon_o\, r^2} = \frac{q_3\, q_2}{4\pi\varepsilon_o\, d^2} \quad \Rightarrow \quad q_2 = \left(\frac{d^2}{r^2}\right) q_1 \;=\; 9.0\, q_1\,,$$

or $q_2/q_1 = 9.0$.

13. (a) There is no equilibrium position for q_3 *between* the two fixed charges, because it is being pulled by one and pushed by the other (since q_1 and q_2 have different signs); in this region this means the two force arrows on q_3 are in the same direction and cannot cancel. It should also be clear that off-axis (with the axis defined as that which passes through the two fixed charges) there are no equilibrium positions. On the semi-infinite region of the axis which is nearest q_2 and furthest from q_1 an equilibrium position for q_3 cannot be found because $|q_1| < |q_2|$ and the magnitude of force exerted by q_2 is everywhere (in that region) stronger than that exerted by q_1 on q_3. Thus, we must look in the semi-infinite region of the axis which is nearest q_1 and furthest from q_2, where the net force on q_3 has magnitude

$$\left| k\frac{|q_1 q_3|}{x^2} - k\frac{|q_2 q_3|}{(d+x)^2} \right|$$

with d = 10 cm and x assumed positive. We set this equal to zero, as required by the problem, and cancel k and q_3. Thus, we obtain

$$\frac{|q_1|}{x^2} - \frac{|q_2|}{(d+x)^2} = 0 \Rightarrow \left(\frac{d+x}{x}\right)^2 = \left|\frac{q_2}{q_1}\right| = 3$$

which yields (after taking the square root)

$$\frac{d+x}{x} = \sqrt{3} \Rightarrow x = \frac{d}{\sqrt{3}-1} \approx 14\,\text{cm}$$

for the distance between q_3 and q_1.

(b) As stated above, $y = 0$.

14. Since the forces involved are proportional to q, we see that the essential difference between the two situations is $F_a \propto q_B + q_C$ (when those two charges are on the same side) versus $F_b \propto -q_B + q_C$ (when they are on opposite sides). Setting up ratios, we have

$$\frac{F_a}{F_b} = \frac{q_B + q_C}{-q_B + q_C} \quad \Rightarrow \quad \frac{20.14}{-2.877} = \frac{1 + r}{-1 + r}$$

where in the last step we have canceled (on the left hand side) 10^{-24} N from the numerator and the denominator, and (on the right hand side) introduced the symbol $r = q_C / q_B$. After noting that the ratio on the left hand side is very close to – 7, then, after a couple of algebra steps, we are led to

$$r = \frac{7+1}{7-1} = \frac{8}{6} = 1.333.$$

15. (a) The distance between q_1 and q_2 is

$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-0.020 - 0.035)^2 + (0.015 - 0.005)^2} = 0.056 \text{ m}.$$

The magnitude of the force exerted by q_1 on q_2 is

$$F_{21} = k\frac{|q_1 q_2|}{r_{12}^2} = \frac{\left(8.99\times10^9\right)\left(3.0\times10^{-6}\right)\left(4.0\times10^{-6}\right)}{(0.056)^2} = 35 \text{ N}.$$

(b) The vector $\vec{F}_{21}$ is directed towards q_1 and makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \tan^{-1}\left(\frac{1.5 - 0.5}{-2.0 - 3.5}\right) = -10.3° \approx -10°.$$

(c) Let the third charge be located at (x_3, y_3), a distance r from q_2. We note that q_1, q_2 and q_3 must be collinear; otherwise, an equilibrium position for any one of them would be impossible to find. Furthermore, we cannot place q_3 on the same side of q_2 where we also find q_1, since in that region both forces (exerted on q_2 by q_3 and q_1) would be in the same direction (since q_2 is attracted to both of them). Thus, in terms of the angle found in part (a), we have $x_3 = x_2 - r\cos\theta$ and $y_3 = y_2 - r\sin\theta$ (which means $y_3 > y_2$ since θ is negative). The magnitude of force exerted on q_2 by q_3 is $F_{23} = k|q_2 q_3|/r^2$, which must equal that of the force exerted on it by q_1 (found in part (a)). Therefore,

$$k\frac{|q_2 q_3|}{r^2} = k\frac{|q_1 q_2|}{r_{12}^2} \Rightarrow r = r_{12}\sqrt{\frac{q_3}{q_1}} = 0.0645 \text{cm}.$$

Consequently, $x_3 = x_2 - r\cos\theta = -2.0$ cm – (6.45 cm) cos(–10°) = –8.4 cm,

(d) and $y_3 = y_2 - r\sin\theta = 1.5$ cm – (6.45 cm) sin(–10°) = 2.7 cm.

16. (a) For the net force to be in the $+x$ direction, the y components of the individual forces must cancel. The angle of the force exerted by the q_1 = 40 μC charge on $q_3 = 20\mu C$ is 45°, and the angle of force exerted on q_3 by Q is at $-\theta$ where

$$\theta = \tan^{-1}\left(\frac{2.0}{3.0}\right) = 33.7°.$$

Therefore, cancellation of y components requires

$$k\frac{q_1 q_3}{\left(0.02\sqrt{2}\right)^2}\sin 45° = k\frac{|Q|\, q_3}{\left(\sqrt{(0.030)^2+(0.020)^2}\right)^2}\sin\theta$$

from which we obtain $|Q|$ = 83 μC. Charge Q is "pulling" on q_3, so (since $q_3 > 0$) we conclude Q = –83 μC.

(b) Now, we require that the x components cancel, and we note that in this case, the angle of force on q_3 exerted by Q is $+\theta$ (it is repulsive, and Q is positive-valued). Therefore,

$$k\frac{q_1 q_3}{\left(0.02\sqrt{2}\right)^2}\cos 45° = k\frac{Q q_3}{\left(\sqrt{(0.030)^2+(0.020)^2}\right)^2}\cos\theta$$

from which we obtain Q = 55.2 μC $\approx$ 55 μC .

17. (a) If the system of three charges is to be in equilibrium, the force on each charge must be zero. The third charge q_3 must lie between the other two or else the forces acting on it due to the other charges would be in the same direction and q_3 could not be in equilibrium. Suppose q_3 is at a distance x from q, and $L - x$ from $4.00q$. The force acting on it is then given by

$$F_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} - \frac{4qq_3}{(L-x)^2}\right)$$

where the positive direction is rightward. We require $F_3 = 0$ and solve for x. Canceling common factors yields $1/x^2 = 4/(L - x)^2$ and taking the square root yields $1/x = 2/(L - x)$. The solution is $x = L/3$.

The force on q is

$$F_q = \frac{-1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} + \frac{4.00q^2}{L^2}\right).$$

The signs are chosen so that a negative force value would cause q to move leftward. We require $F_q = 0$ and solve for q_3:

$$q_3 = -\frac{4qx^2}{L^2} = -\frac{4}{9}q \ \Rightarrow \ \frac{q_3}{q} = -\frac{4}{9} = -0.444$$

where $x = L/3$ is used. We may easily verify that the force on $4.00q$ also vanishes:

$$F_{4q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4qq_0}{(L-x)^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2}\right) = 0.$$

(b) As seen above, q_3 is located at $x = L/3$. With $L = 9.00$ cm, we have $x = 3.00$ cm.

(c) Similarly, the y coordinate of q_3 is $y = 0$.

18. (a) We note that cos(30º) $= \frac{1}{2}\sqrt{3}$, so that the dashed line distance in the figure is $r = 2d/\sqrt{3}$. We net force on q_1 due to the two charges q_3 and q_4 (with $|q_3| = |q_4| = 1.60 \times 10^{-19}$ C) on the y axis has magnitude

$$2\frac{|q_1 q_3|}{4\pi\varepsilon_0 r^2}\cos(30°) = \frac{3\sqrt{3}\,|q_1 q_3|}{16\pi\varepsilon_0 d^2}.$$

This must be set equal to the magnitude of the force exerted on q_1 by $q_2 = 8.00 \times 10^{-19}$ C $= 5.00\,|q_3|$ in order that its net force be zero:

$$\frac{3\sqrt{3}\,|q_1 q_3|}{16\pi\varepsilon_0 d^2} = \frac{|q_1 q_2|}{4\pi\varepsilon_0 (D+d)^2} \quad\Rightarrow\quad D = d\left(2\sqrt{\frac{5}{3\sqrt{3}}} - 1\right) = 0.9245\,d\,.$$

Given d = 2.00 cm, then this leads to D = 1.92 cm.

(b) As the angle decreases, its cosine increases, resulting in a larger contribution from the charges on the y axis. To offset this, the force exerted by q_2 must be made stronger, so that it must be brought closer to q_1 (keep in mind that Coulomb's law is *inversely* proportional to distance-squared). Thus, D must be decreased.

19. The charge dq within a thin shell of thickness dr is $\rho A\, dr$ where $A = 4\pi r^2$. Thus, with $\rho = b/r$, we have

$$q = \int dq = 4\pi b \int_{r_1}^{r_2} r\, dr = 2\pi b \left(r_2^2 - r_1^2\right).$$

With $b = 3.0\ \mu\text{C/m}^2$, $r_2 = 0.06$ m and $r_1 = 0.04$ m, we obtain $q = 0.038\ \mu\text{C} = 3.8 \times 10^{-8}$ C.

20. If θ is the angle between the force and the x-axis, then

$$\cos\theta = \frac{x}{\sqrt{x^2 + d^2}} .$$

We note that, due to the symmetry in the problem, there is no y component to the net force on the third particle. Thus, F represents the magnitude of force exerted by q_1 or q_2 on q_3. Let $e = +1.60 \times 10^{-19}$ C, then $q_1 = q_2 = +2e$ and $q_3 = 4.0e$ and we have

$$F_{\text{net}} = 2F\cos\theta = \frac{2(2e)(4e)}{4\pi\varepsilon_o (x^2 + d^2)} \frac{x}{\sqrt{x^2 + d^2}} = \frac{4e^2 x}{\pi\varepsilon_o (x^2 + d^2)^{3/2}} .$$

(a) To find where the force is at an extremum, we can set the derivative of this expression equal to zero and solve for x, but it is good in any case to graph the function for a fuller understanding of its behavior – and as a quick way to see whether an extremum point is a maximum or a miminum. In this way, we find that the value coming from the derivative procedure is a maximum (and will be presented in part (b)) and that the minimum is found at the lower limit of the interval. Thus, the net force is found to be zero at $x = 0$, which is the smallest value of the net force in the interval $5.0 \text{ m} \geq x \geq 0$.

(b) The maximum is found to be at $x = d/\sqrt{2}$ or roughly 12 cm.

(c) The value of the net force at $x = 0$ is $F_{\text{net}} = 0$.

(d) The value of the net force at $x = d/\sqrt{2}$ is $F_{\text{net}} = 4.9 \times 10^{-26}$ N.

21. (a) The magnitude of the force between the (positive) ions is given by

$$F = \frac{(q)(q)}{4\pi\varepsilon_0 r^2} = k\frac{q^2}{r^2}$$

where q is the charge on either of them and r is the distance between them. We solve for the charge:

$$q = r\sqrt{\frac{F}{k}} = \left(5.0\times10^{-10}\ \text{m}\right)\sqrt{\frac{3.7\times10^{-9}\ \text{N}}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 3.2\times10^{-19}\ \text{C}.$$

(b) Let N be the number of electrons missing from each ion. Then, $Ne = q$, or

$$N = \frac{q}{e} = \frac{3.2\times10^{-9}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 2.$$

22. The magnitude of the force is

$$F = k\frac{e^2}{r^2} = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{\left(2.82\times10^{-10}\ \mathrm{m}\right)^2} = 2.89\times10^{-9}\ \mathrm{N}.$$

23. Eq. 21-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{1.0 \times 10^{-7}\,\text{C}}{1.6 \times 10^{-19}\,\text{C}} = 6.3 \times 10^{11}.$$

24. (a) Eq. 21-1 gives

$$F = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.00\times10^{-16}\ \text{C}\right)^2}{\left(1.00\times10^{-2}\ \text{m}\right)^2} = 8.99\times10^{-19}\ \text{N}.$$

(b) If n is the number of excess electrons (of charge $-e$ each) on each drop then

$$n = -\frac{q}{e} = -\frac{-1.00\times10^{-16}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 625.$$

25. The unit Ampere is discussed in §21-4. The proton flux is given as 1500 protons per square meter per second, where each proton provides a charge of $q = +e$. The current through the spherical area $4\pi R^2 = 4\pi\,(6.37 \times 10^6 \text{ m})^2 = 5.1 \times 10^{14} \text{ m}^2$ would be

$$i = \left(5.1 \times 10^{14}\,\text{m}^2\right)\left(1500\,\frac{\text{protons}}{\text{s}\cdot\text{m}^2}\right)\left(1.6 \times 10^{-19}\ \text{C/proton}\right) = 0.122\,\text{A}\,.$$

26. The volume of 250 cm^3 corresponds to a mass of 250 g since the density of water is 1.0 g/cm^3. This mass corresponds to 250/18 = 14 moles since the molar mass of water is 18. There are ten protons (each with charge $q = +e$) in each molecule of H_2O, so

$$Q = 14N_A q = 14\left(6.02\times10^{23}\right)(10)\left(1.60\times10^{-19}\,\text{C}\right) = 1.3\times10^{7}\,\text{C}.$$

27. Since the graph crosses zero, q_1 must be positive-valued: $q_1 = +8.00e$. We note that it crosses zero at $r = 0.40$ m. Now the asymptotic value of the force yields the magnitude and sign of q_2:

$$\frac{q_1\, q_2}{4\pi\varepsilon_o r^2} = F \qquad \Rightarrow \qquad q_2 = \left(\frac{1.5 \times 10^{-25}}{k\, q_1}\right) r^2 = 2.086 \times 10^{-18}\,\text{C} = 13e\,.$$

28. Let d be the vertical distance from the coordinate origin to $q_3 = -q$ and $q_4 = -q$ on the $+y$ axis, where the symbol q is assumed to be a positive value. Similarly, d is the (positive) distance from the origin $q_4 = -$ on the $-y$ axis. If we take each angle θ in the figure to be positive, then we have $\tan\theta = d/R$ and $\cos\theta = R/r$ (where r is the dashed line distance shown in the figure). The problem asks us to consider θ to be a variable in the sense that, once the charges on the x axis are fixed in place (which determines R), d can then be arranged to some multiple of R, since $d = R \tan\theta$. The aim of this exploration is to show that if q is bounded then θ (and thus d) is also bounded.

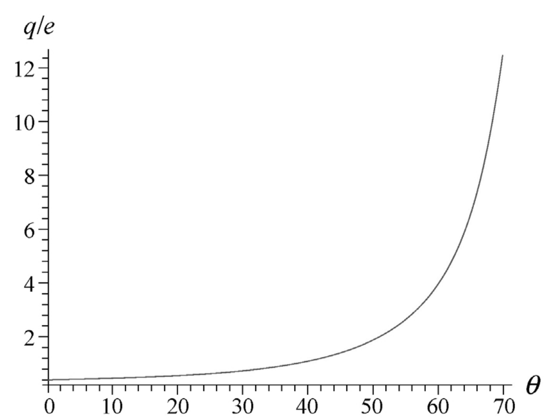
From symmetry, we see that there is no net force in the vertical direction on $q_2 = -e$ sitting at a distance R to the left of the coordinate origin. We note that the net x force caused by q_3 and q_4 on the y axis will have a magnitude equal to

$$2\frac{q\,e}{4\pi\varepsilon_o r^2}\cos(\theta) = \frac{2\,q\,e\cos(\theta)}{4\pi\varepsilon_o (R/\cos(\theta))^2} = \frac{2\,q\,e\cos^3(\theta)}{4\pi\varepsilon_o R^2} \; .$$

Consequently, to achieve a zero net force along the x axis, the above expression must equal the magnitude of the repulsive force exerted on q_2 by $q_1 = -e$. Thus,

$$\frac{2\,q\,e\cos^3(\theta)}{4\pi\varepsilon_o R^2} = \frac{e^2}{4\pi\varepsilon_o R^2} \quad \Rightarrow \quad q = \frac{e}{2\cos^3(\theta)} \; .$$

Below we plot q/e as a function of the angle (in degrees):



The graph suggests that $q/e < 5$ for $\theta < 60^\circ$, roughly. We can be more precise by solving the above equation. The requirement that $q \le 5e$ leads to

$$\frac{e}{2\cos^3(\theta)} \le 5e \quad \Rightarrow \quad \frac{1}{(10)^{1/3}} \le \cos\theta$$

which yields $\theta \leq 62.34°$. The problem asks for "physically possible values," and it is reasonable to suppose that only positive-integer-multiple values of e are allowed for q. If we let $q = Ne$, for $N = 1 \ldots 5$, then θ_N will be found by taking the inverse cosine of the cube root of $(1/2N)$.

(a) The smallest value of angle is $\theta_1 = 37.5°$ (or 0.654 rad).

(b) The second smallest value of angle is $\theta_2 = 50.95°$ (or 0.889 rad).

(c) The third smallest value of angle is $\theta_3 = 56.6°$ (or 0.988 rad).

29. (a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) Rather than remove a cesium ion, we superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3}a$, where a is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = \left(\sqrt{3}/2\right)a$. The force has magnitude

$$F = k\frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{(3/4)\left(0.40\times10^{-9}\ \mathrm{m}\right)^2} = 1.9\times10^{-9}\ \mathrm{N}.$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

30. (a) Since the proton is positively charged, the emitted particle must be a positron (as opposed to the negatively charged electron) in accordance with the law of charge conservation.

(b) In this case, the initial state had zero charge (the neutron is neutral), so the sum of charges in the final state must be zero. Since there is a proton in the final state, there should also be an electron (as opposed to a positron) so that $\Sigma q = 0$.

31. None of the reactions given include a beta decay, so the number of protons, the number of neutrons, and the number of electrons are each conserved. Atomic numbers (numbers of protons and numbers of electrons) and molar masses (combined numbers of protons and neutrons) can be found in Appendix F of the text.

(a) ^{1}H has 1 proton, 1 electron, and 0 neutrons and ^{9}Be has 4 protons, 4 electrons, and 9 – 4 = 5 neutrons, so X has 1 + 4 = 5 protons, 1 + 4 = 5 electrons, and 0 + 5 – 1 = 4 neutrons. One of the neutrons is freed in the reaction. X must be boron with a molar mass of 5 + 4 = 9 g/mol: ^{9}B.

(b) ^{12}C has 6 protons, 6 electrons, and 12 – 6 = 6 neutrons and ^{1}H has 1 proton, 1 electron, and 0 neutrons, so X has 6 + 1 = 7 protons, 6 + 1 = 7 electrons, and 6 + 0 = 6 neutrons. It must be nitrogen with a molar mass of 7 + 6 = 13 g/mol: ^{13}N.

(c) ^{15}N has 7 protons, 7 electrons, and 15 – 7 = 8 neutrons; ^{1}H has 1 proton, 1 electron, and 0 neutrons; and ^{4}He has 2 protons, 2 electrons, and 4 – 2 = 2 neutrons; so X has 7 + 1 – 2 = 6 protons, 6 electrons, and 8 + 0 – 2 = 6 neutrons. It must be carbon with a molar mass of 6 + 6 = 12: ^{12}C.

32. We note that the problem is examining the force <u>on</u> charge A, so that the respective distances (involved in the Coulomb force expressions) between B and A, and between C and A, do not change as particle B is moved along its circular path. We focus on the endpoints ($\theta = 0°$ and $180°$) of each graph, since they represent cases where the forces (on A) due to B and C are either parallel or antiparallel (yielding maximum or minimum force magnitudes, respectively). We note, too, that since Coulomb's law is inversely proportional to r^2 then the (if, say, the charges were all the same) force due to C would be one-fourth as big as that due to B (since C is twice as far away from A). The charges, it turns out, are not the same, so there is also a factor of the charge ratio ξ (the charge of C divided by the charge of B), as well as the aforementioned ¼ factor. That is, the force exerted by C is, by Coulomb's law equal to $\pm\frac{1}{4}\xi$ multiplied by the force exerted by B.

(a) The maximum force is $2F_0$ and occurs when $\theta = 180°$ (B is to the left of A, while C is the right of A). We choose the minus sign and write

$$2\,F_0 = (1 - \tfrac{1}{4}\xi)\,F_0 \quad \Rightarrow \quad \xi = -4\,.$$

One way to think of the minus sign choice is $\cos(180°) = -1$. This is certainly consistent with the minimum force ratio (zero) at $\theta = 0°$ since that would also imply

$$0 = 1 + \tfrac{1}{4}\xi \quad \Rightarrow \quad \xi = -4\,.$$

(b) The ratio of maximum to minimum forces is $1.25/0.75 = 5/3$ in this case, which implies

$$\frac{5}{3} = \frac{1 + \frac{1}{4}\xi}{1 - \frac{1}{4}\xi} \quad \Rightarrow \quad \xi = 16\,.$$

Of course, this could also be figured as illustrated in part (a), looking at the maximum force ratio by itself and solving, or looking at the minimum force ratio (¾) at $\theta = 180°$ and solving for ξ.

33. We note that, as result of the fact that the Coulomb force is inversely proportional to r^2, a particle of charge Q which is distance d from the origin will exert a force on some charge q_o at the origin of equal strength as a particle of charge $4Q$ at distance $2d$ would exert on q_o. Therefore, $q_6 = +8e$ on the $-y$ axis could be replaced with a $+2e$ closer to the origin (at half the distance); this would add to the $q_5 = +2e$ already there and produce $+4e$ below the origin which exactly cancels the force due to $q_2 = +4e$ above the origin.

Similarly, $q_4 = +4e$ to the far right could be replaced by a $+e$ at half the distance, which would add to $q_3 = +e$ already there to produce a $+2e$ at distance d to the right of the central charge q_7. The horizontal force due to this $+2e$ is cancelled exactly by that of $q_1 = +2e$ on the $-x$ axis, so that the net force on q_7 is zero.

34. For the Coulomb force to be sufficient for circular motion at that distance (where r = 0.200 m and the acceleration needed for circular motion is a = v^2/r) the following equality is required:

$$\frac{Q\,q}{4\pi\varepsilon_0 r^2} = -\frac{m\,v^2}{r} \;.$$

With $q = 4.00 \times 10^{-6}$ C, m = 0.000800 kg, v = 50.0 m/s, this leads to $Q = -1.11 \times 10^{-5}$ C.

35. Let $\vec{F}_{12}$ denotes the force on q_1 exerted by q_2 and F_{12} be its magnitude.

(a) We consider the net force on q_1. $\vec{F}_{12}$ points in the $+x$ direction since q_1 is attracted to q_2. $\vec{F}_{13}$ and $\vec{F}_{14}$ both point in the $-x$ direction since q_1 is repelled by q_3 and q_4. Thus, using $d = 0.0200$ m, the net force is

$$F_1 = F_{12} - F_{13} - F_{14} = \frac{2e|-e|}{4\pi\varepsilon_0 d^2} - \frac{(2e)(e)}{4\pi\varepsilon_0 (2d)^2} - \frac{(2e)(4e)}{4\pi\varepsilon_0 (3d)^2} = +3.52 \times 10^{-25}\,\text{N},$$

or $\vec{F}_1 = (3.52 \times 10^{-25}\ \text{N})\hat{\text{i}}$.

(b) We now consider the net force on q_2. We note that $\vec{F}_{21} = -\vec{F}_{12}$ points in the $-x$ direction, and $\vec{F}_{23}$ and $\vec{F}_{24}$ both point in the $+x$ direction. The net force is

$$F_{23} + F_{24} - F_{21} = \frac{4e|-e|}{4\pi\varepsilon_0 (2d)^2} + \frac{e|-e|}{4\pi\varepsilon_0 d^2} - \frac{2e|-e|}{4\pi\varepsilon_0 d^2} = 0$$

36. As a result of the first action, both sphere W and sphere A possess charge $\frac{1}{2}q_A$, where q_A is the initial charge of sphere A. As a result of the second action, sphere W has charge

$$\frac{1}{2}\left(\frac{1}{2}q_A - 32e\right) .$$

As a result of the final action, sphere W now has charge equal to

$$\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}q_A - 32e\right) + 48e\right] .$$

Setting this final expression equal to $+18e$ as required by the problem leads (after a couple of algebra steps) to the answer: $q_A = +16e$.

37. If θ is the angle between the force and the x axis, then

$$\cos\theta = \frac{d_2}{\sqrt{d_1^2 + d_2^2}} .$$

Thus, using Coulomb's law for F, we have

$$F_x = F\cos\theta = \frac{q_1 q_2}{4\pi\varepsilon_0 (d_1^2 + d_2^2)} \frac{d_2}{\sqrt{d_1^2 + d_2^2}} = 1.31 \times 10^{-22}\ \text{N} .$$

38. (a) We note that $\tan(30°) = 1/\sqrt{3}$. In the initial (highly symmetrical) configuration, the net force on the central bead is in the $-y$ direction and has magnitude $3F$ where F is the Coulomb's law force of one bead on another at distance d = 10 cm. This is due to the fact that the forces exerted on the central bead (in the initial situation) by the beads on the x axis cancel each other; also, the force exerted "downward" by bead 4 on the central bead is four times larger than the "upward" force exerted by bead 2. This net force along the y axis does not change as bead 1 is now moved, though there is now a nonzero x-component F_x. The components are now related by

$$\tan(30°) = \frac{F_x}{F_y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{F_x}{3F}$$

which implies $F_x = \sqrt{3}\,F$. Now, bead 3 exerts a "leftward" force of magnitude F on the central bead, while bead 1 exerts a "rightward" force of magnitude F'. Therefore,

$$F' - F = \sqrt{3}\,F. \quad \Rightarrow \quad F' = (\sqrt{3} + 1)\,F\,.$$

The fact that Coulomb's law depends inversely on distance-squared then implies

$$r^2 = \frac{d^2}{\sqrt{3}+1} \quad \Rightarrow \quad r = \frac{d}{\sqrt{\sqrt{3}+1}}$$

where r is the distance between bead 1 and the central bead. Thus r = 6.05 cm.

(b) To regain the condition of high symmetry (in particular, the cancellation of x-components) bead 3 must be moved closer to the central bead so that it, too, is the distance r (as calculated in part(a)) away from it.

39. (a) Charge Q_1 = +80 × 10^{-9} C is on the y axis at y = 0.003 m, and charge $Q_2 = +80\times10^{-9}$ C is on the y axis at y = –0.003 m. The force on particle 3 (which has a charge of q = +18 × 10^{-9} C) is due to the vector sum of the repulsive forces from Q_1 and Q_2. In symbols, $\vec{F}_{31} + \vec{F}_{32} = \vec{F}_3$, where

$$|\vec{F}_{31}| = k\frac{q_3|q_1|}{r_{31}^2} \quad \text{and} \quad |\vec{F}_{32}| = k\frac{q_3 q_2}{r_{32}^2}.$$

Using the Pythagorean theorem, we have r_{31} = r_{32} = 0.005 m. In magnitude-angle notation (particularly convenient if one uses a vector-capable calculator in polar mode), the indicated vector addition becomes

$$\vec{F}_3 = (0.518\angle -37°) + (0.518\angle 37°) = (0.829\angle 0°).$$

Therefore, the net force is $\vec{F}_3 = (0.829\text{ N})\hat{\text{i}}$.

(b) Switching the sign of Q_2 amounts to reversing the direction of its force on q. Consequently, we have

$$\vec{F}_3 = (0.518\angle -37°) + (0.518\angle -143°) = (0.621\angle -90°).$$

Therefore, the net force is $\vec{F}_3 = -(0.621\text{ N})\hat{\text{j}}$.

40. (a) Let x be the distance between particle 1 and particle 3. Thus, the distance between particle 3 and particle 2 is $L - x$. Both particles exert leftward forces on q_3 (so long as it is on the line between them), so the magnitude of the net force on q_3 is

$$F_{\text{net}} = |\vec{F}_{13}| + |\vec{F}_{23}| = \frac{|q_1 q_3|}{4\pi\varepsilon_o x^2} + \frac{|q_2 q_3|}{4\pi\varepsilon_o (L-x)^2} = \frac{e^2}{\pi\varepsilon_o}\left(\frac{1}{x^2} + \frac{27}{(L-x)^2}\right)$$

with the values of the charges (stated in the problem) plugged in. Finding the value of x which minimizes this expression leads to $x = \frac{1}{4} L$. Thus, $x = 2.00$ cm.

(b) Substituting $x = \frac{1}{4} L$ back into the expression for the net force magnitude and using the standard value for e leads to $F_{\text{net}} = 9.21 \times 10^{-24}$ N.

41. The individual force magnitudes are found using Eq. 21-1, with SI units (so $a = 0.02$ m) and k as in Eq. 21-5. We use magnitude-angle notation (convenient if one uses a vector-capable calculator in polar mode), listing the forces due to +4.00q, +2.00q, and –2.00q charges:

$$\left(4.60\times10^{-24}\ \angle 180°\right)+\left(2.30\times10^{-24}\ \angle -90°\right)+\left(1.02\times10^{-24}\ \angle -145°\right)=\left(6.16\times10^{-24}\ \angle -152°\right)$$

(a) Therefore, the net force has magnitude 6.16×10^{-24} N.

(b) The direction of the net force is at an angle of –152° (or 208° measured counterclockwise from the +x axis).

42. The charge dq within a thin section of the rod (of thickness dx) is $\rho\, A\, dx$ where $A = 4.00\times10^{-4}\,\text{m}^2$ and ρ is the charge per unit volume. The number of (excess) electrons in the rod (of length L = 2.00 m) is $N = q/(-e)$ where e is given in Eq. 21-14.

(a) In the case where $\rho = -4.00 \times 10^{-6}$ C/m^3, we have

$$N = \frac{q}{-e} = \frac{\rho A}{-e}\int_0^L dx = \frac{|\rho|\,AL}{e} = 2.00\times10^{10}\,.$$

(b) With $\rho = bx^2$ ($b = -2.00 \times 10^{-6}$ C/m^5) we obtain

$$N = \frac{b\,A}{-e}\int_0^L x^2\,dx = \frac{|\,b\,|\,A\,L^3}{3e} = 1.33\times10^{10}.$$

43. The magnitude of the net force on the $q = 42 \times 10^{-6}$ C charge is

$$k\frac{q_1\, q}{0.28^2} + k\frac{|q_2|q}{0.44^2}$$

where $q_1 = 30 \times 10^{-9}$ C and $|q_2| = 40 \times 10^{-9}$ C. This yields 0.22 N. Using Newton's second law, we obtain

$$m = \frac{F}{a} = \frac{0.22\,\text{N}}{100\times10^3\ \text{m/s}^2} = 2.2\times10^{-6}\,\text{kg}.$$

44. Let q_1 be the charge of one part and q_2 that of the other part; thus, $q_1 + q_2 = Q = 6.0$ μC. The repulsive force between them is given by Coulomb's law:

$$F = \frac{q_1\, q_2}{4\pi\varepsilon_o\, r^2} = \frac{q_1(Q - q_1)}{4\pi\varepsilon_o\, r^2} \quad .$$

If we maximize this expression by taking the derivative with respect to q_1 and setting equal to zero, we find $q_1 = Q/2$, which might have been anticipated (based on symmetry arguments). This implies $q_2 = Q/2$ also. With $r = 0.0030$ m and $Q = 6.0 \times 10^{-6}$ C, we find

$$F = \frac{(Q/2)(Q/2)}{4\pi\varepsilon_o\, r^2} \approx 9.0 \times 10^3 \text{ N} .$$

45. For the net force on $q_1 = +Q$ to vanish, the x force component due to $q_2 = q$ must exactly cancel the force of attraction caused by $q_4 = -2Q$. Consequently,

$$\frac{Qq}{4\pi\varepsilon_0 a^2} = \frac{Q|2Q|}{4\pi\varepsilon_0(\sqrt{2}a)^2}\cos 45° = \frac{Q^2}{4\pi\varepsilon_0\sqrt{2}a^2}$$

or $q = Q/\sqrt{2}$. This implies that $q/Q = 1/\sqrt{2} = 0.707$.

46. We are looking for a charge q which, when placed at the origin, experiences $\vec{F}_{\text{net}} = 0$, where

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 .$$

The magnitude of these individual forces are given by Coulomb's law, Eq. 21-1, and without loss of generality we assume $q > 0$. The charges q_1 (+6 μC), q_2 (–4 μC), and q_3 (unknown), are located on the $+x$ axis, so that we know $\vec{F}_1$ points towards $-x$, $\vec{F}_2$ points towards $+x$, and $\vec{F}_3$ points towards $-x$ if $q_3 > 0$ and points towards $+x$ if $q_3 < 0$. Therefore, with $r_1 = 8$ m, $r_2 = 16$ m and $r_3 = 24$ m, we have

$$0 = -k\frac{q_1 q}{r_1^2} + k\frac{|q_2| q}{r_2^2} - k\frac{q_3 q}{r_3^2} .$$

Simplifying, this becomes

$$0 = -\frac{6}{8^2} + \frac{4}{16^2} - \frac{q_3}{24^2}$$

where q_3 is now understood to be in μC. Thus, we obtain $q_3 = -45\ \mu$C.

47. There are two protons (each with charge $q = +e$) in each molecule, so

$$Q = N_A q = \left(6.02\times10^{23}\right)(2)\left(1.60\times10^{-19}\ \text{C}\right) = 1.9\times10^{5}\ \text{C} = 0.19\ \text{MC}.$$

48. (a) Since the rod is in equilibrium, the net force acting on it is zero, and the net torque about any point is also zero. We write an expression for the net torque about the bearing, equate it to zero, and solve for x. The charge Q on the left exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\,(qQ/h^2)$, at a distance $L/2$ from the bearing. We take the torque to be negative. The attached weight exerts a downward force of magnitude W, at a distance $x-L/2$ from the bearing. This torque is also negative. The charge Q on the right exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\,(2qQ/h^2)$, at a distance $L/2$ from the bearing. This torque is positive. The equation for rotational equilibrium is

$$\frac{-1}{4\pi\varepsilon_0}\frac{qQ}{h^2}\frac{L}{2}-W\left(x-\frac{L}{2}\right)+\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2}\frac{L}{2}=0.$$

The solution for x is

$$x=\frac{L}{2}\left(1+\frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2W}\right).$$

(b) If F_N is the magnitude of the upward force exerted by the bearing, then Newton's second law (with zero acceleration) gives

$$W-\frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2}-\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2}-F_N=0.$$

We solve for h so that $F_N=0$. The result is

$$h=\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{3qQ}{W}}.$$

49. Charge $q_1 = -80 \times 10^{-6}$ C is at the origin, and charge $q_2 = +40 \times 10^{-6}$ C is at $x = 0.20$ m. The force on $q_3 = +20 \times 10^{-6}$ C is due to the attractive and repulsive forces from q_1 and q_2, respectively. In symbols, $\vec{F}_{3\,\text{net}} = \vec{F}_{3\,1} + \vec{F}_{3\,2}$, where

$$\left|\vec{F}_{31}\right| = k\frac{q_3\,|\,q_1\,|}{r_{3\,1}^2} \quad \text{and} \quad |\vec{F}_{32}\,| = k\frac{q_3 q_2}{r_{3\,2}^2}.$$

(a) In this case r_{31} = 0.40 m and r_{32} = 0.20 m, with $\vec{F}_{3\,1}$ directed towards $-x$ and $\vec{F}_{3\,2}$ directed in the $+x$ direction. Using the value of k in Eq. 21-5, we obtain $\vec{F}_{3\,\text{net}} = (89.9\text{ N})\hat{\text{i}}$.

(b) In this case r_{31} = 0.80 m and r_{32} = 0.60 m, with $\vec{F}_{3\,1}$ directed towards $-x$ and $\vec{F}_{3\,2}$ towards $+x$. Now we obtain $\vec{F}_{3\,\text{net}} = (-2.50\text{ N})\hat{\text{i}}$.

(c) Between the locations treated in parts (a) and (b), there must be one where $\vec{F}_{3\,\text{net}} = 0$. Writing $r_{31} = x$ and $r_{32} = x - 0.20$ m, we equate $\left|\vec{F}_{3\,1}\right|$ and $\left|\vec{F}_{3\,2}\right|$, and after canceling common factors, arrive at

$$\frac{|q_1|}{x^2} = \frac{q_2}{(x-0.2)^2}.$$

This can be further simplified to

$$\frac{(x-0.2)^2}{x^2} = \frac{q_2}{|q_1|} = \frac{1}{2}.$$

Taking the (positive) square root and solving, we obtain x = 0.683 m. If one takes the negative root and ‘solves’, one finds the location where the net force *would* be zero *if* q_1 and q_2 were of like sign (which is not the case here).

(d) From the above, we see that $y = 0$.

50. We are concerned with the charges in the nucleus (not the "orbiting" electrons, if there are any). The nucleus of Helium has 2 protons and that of Thorium has 90.

(a) Eq. 21-1 gives

$$F = k\frac{q^2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(2(1.60\times10^{-19}\ \mathrm{C}))(90(1.60\times10^{-19}\ \mathrm{C}))}{(9.0\times10^{-15}\ \mathrm{m})^2} = 5.1\times10^2\ \mathrm{N}.$$

(b) Estimating the helium nucleus mass as that of 4 protons (actually, that of 2 protons and 2 neutrons, but the neutrons have approximately the same mass), Newton's second law leads to

$$a = \frac{F}{m} = \frac{5.1\times10^2\ \mathrm{N}}{4\left(1.67\times10^{-27}\ \mathrm{kg}\right)} = 7.7\times10^{28}\ \mathrm{m/s^2}\ .$$

51. Coulomb's law gives

$$F = \frac{|q| \cdot |q|}{4\pi\varepsilon_0 r^2} = \frac{k(e/3)^2}{r^2} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(1.60\times10^{-19}\text{ C})^2}{9(2.6\times10^{-15}\text{ m})^2} = 3.8\text{ N}.$$

52. (a) Since q_A = –2.00 nC and q_C = +8.00 nC Eq. 21-4 leads to

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-2.00\times10^{-9}\ \text{C})(8.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 3.60\times10^{-6}\ \text{N}.$$

(b) After making contact with each other, both *A* and *B* have a charge of

$$\frac{q_A + q_B}{2} = \left(\frac{-2.00+(-4.00)}{2}\right)\ \text{nC} = -3.00\ \text{nC}.$$

When *B* is grounded its charge is zero. After making contact with *C*, which has a charge of +8.00 nC, *B* acquires a charge of [0 + (–8.00 nC)]/2 = –4.00 nC, which charge *C* has as well. Finally, we have Q_A = –3.00 nC and Q_B = Q_C = –4.00 nC. Therefore,

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-3.00\times10^{-9}\ \text{C})(-4.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 2.70\times10^{-6}\ \text{N}.$$

(c) We also obtain

$$|\vec{F}_{BC}| = \frac{|q_B q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4.00\times10^{-9}\ \text{C})(-4.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 3.60\times10^{-6}\ \text{N}.$$

53. Let the two charges be q_1 and q_2. Then $q_1 + q_2 = Q = 5.0 \times 10^{-5}$ C. We use Eq. 21-1:

$$1.0\,\text{N} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right) q_1 q_2}{(2.0\,\text{m})^2}.$$

We substitute $q_2 = Q - q_1$ and solve for q_1 using the quadratic formula. The two roots obtained are the values of q_1 and q_2, since it does not matter which is which. We get 1.2×10^{-5} C and 3.8×10^{-5} C. Thus, the charge on the sphere with the smaller charge is 1.2×10^{-5} C.

54. The unit Ampere is discussed in §21-4. Using i for current, the charge transferred is

$$q = it = \left(2.5\times10^{4}\,\text{A}\right)\left(20\times10^{-6}\,\text{s}\right) = 0.50\ \text{C}.$$

55. (a) Using Coulomb's law, we obtain

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.00\,\mathrm{C}\right)^2}{\left(1.00\,\mathrm{m}\right)^2} = 8.99\times10^9\,\mathrm{N}.$$

(b) If $r = 1000$ m, then

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.00\,\mathrm{C}\right)^2}{\left(1.00\times10^3\,\mathrm{m}\right)^2} = 8.99\times10^3\,\mathrm{N}.$$

56. Keeping in mind that an Ampere is a Coulomb per second, and that a minute is 60 seconds, the charge (in absolute value) that passes through the chest is

$$|q| = (0.300 \frac{\text{Coulomb}}{\text{second}})(120 \text{ seconds}) = 36.0 \text{ Coulombs} .$$

This charge consists of a number N of electrons (each of which has an absolute value of charge equal to e). Thus,

$$N = \frac{|q|}{e} = \frac{36.0 \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 2.25 \times 10^{20} .$$

57. When sphere C touches sphere A, they divide up their total charge ($Q/2$ plus Q) equally between them. Thus, sphere A now has charge $3Q/4$, and the magnitude of the force of attraction between A and B becomes

$$F = k\frac{(3Q/4)(Q/4)}{d^2} = 4.68\times10^{-19}\,\text{N}.$$

58. In experiment 1, sphere *C* first touches sphere *A*, and they divided up their total charge ($Q/2$ plus Q) equally between them. Thus, sphere *A* and sphere *C* each acquired charge $3Q/4$. Then, sphere *C* touches *B* and those spheres split up their total charge ($3Q/4$ plus $-Q/4$) so that *B* ends up with charge equal to $Q/4$. The force of repulsion between *A* and *B* is therefore

$$F_1 = k\frac{(3Q/4)(Q/4)}{d^2}$$

at the end of experiment 1. Now, in experiment 2, sphere *C* first touches *B* which leaves each of them with charge $Q/8$. When *C* next touches *A*, sphere *A* is left with charge $9Q/16$. Consequently, the force of repulsion between *A* and *B* is

$$F_2 = k\frac{(9Q/16)(Q/8)}{d^2}$$

at the end of experiment 2. The ratio is

$$\frac{F_2}{F_1} = \frac{(9/16)(1/8)}{(3/4)(1/4)} = 0.375.$$

59. If the relative difference between the proton and electron charges (in absolute value) were

$$\frac{q_p - |q_e|}{e} = 0.0000010$$

then the actual difference would be $q_p - |q_e| = 1.6\times10^{-25}\,\mathrm{C}$. Amplified by a factor of 29 × 3 × 10^{22} as indicated in the problem, this amounts to a deviation from perfect neutrality of

$$\Delta q = \left(29\times3\times10^{22}\right)\left(1.6\times10^{-25}\,\mathrm{C}\right) = 0.14\,\mathrm{C}$$

in a copper penny. Two such pennies, at r = 1.0 m, would therefore experience a very large force. Eq. 21-1 gives

$$F = k\frac{(\Delta q)^2}{r^2} = 1.7\times10^{8}\,\mathrm{N}.$$

60. With $F = m_e g$, Eq. 21-1 leads to

$$y^2 = \frac{ke^2}{m_e g} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(9.8\text{m/s}^2\right)}$$

which leads to $y = \pm 5.1$ m. We choose $y = -5.1$ m since the second electron must be below the first one, so that the repulsive force (acting on the first) is in the direction opposite to the pull of Earth's gravity.

61. Letting $kq^2/r^2 = mg$, we get

$$r = q\sqrt{\frac{k}{mg}} = \left(1.60\times10^{-19}\ \text{C}\right)\sqrt{\frac{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)}} = 0.119\,\text{m}.$$

62. The net charge carried by John whose mass is m is roughly

$$
\begin{aligned}
q &= (0.0001)\frac{mN_A Ze}{M} \\
&= (0.0001)\frac{(90\,\text{kg})(6.02\times10^{23}\ \text{molecules/mol})(18\ \text{electron proton pairs/molecule})(1.6\times10^{-19}\,\text{C})}{0.018\,\text{kg/mol}} \\
&= 8.7\times10^{5}\,\text{C},
\end{aligned}
$$

and the net charge carried by Mary is half of that. So the electrostatic force between them is estimated to be

$$F \approx k\frac{q(q/2)}{d^2} = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(8.7\times10^{5}\,\text{C})^2}{2(30\text{m})^2} \approx 4\times10^{18}\,\text{N}.$$

Thus, the order of magnitude of the electrostatic force is 10^{18} N .

63. (a) Eq. 21-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{2.00 \times 10^{-6}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = 1.25 \times 10^{13}\ \text{electrons}.$$

(b) Since you have the excess electrons (and electrons are lighter and more mobile than protons) then the electrons "leap" from you to the faucet instead of protons moving from the faucet to you (in the process of neutralizing your body).

(c) Unlike charges attract, and the faucet (which is grounded and is able to gain or lose any number of electrons due to its contact with Earth's large reservoir of mobile charges) becomes positively charged, especially in the region closest to your (negatively charged) hand, just before the spark.

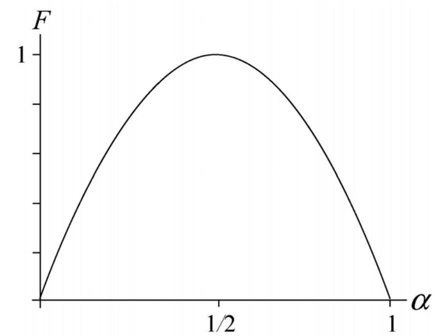
(d) The cat is positively charged (before the spark), and by the reasoning given in part (b) the flow of charge (electrons) is from the faucet to the cat.

(e) If we think of the nose as a conducting sphere, then the side of the sphere closest to the fur is of one sign (of charge) and the side furthest from the fur is of the opposite sign (which, additionally, is oppositely charged from your bare hand which had stroked the cat's fur). The charges in your hand and those of the furthest side of the "sphere" therefore attract each other, and when close enough, manage to neutralize (due to the "jump" made by the electrons) in a painful spark.

64. The two charges are $q = \alpha Q$ (where α is a pure number presumably less than 1 and greater than zero) and $Q - q = (1 - \alpha)Q$. Thus, Eq. 21-4 gives

$$F = \frac{1}{4\pi\varepsilon_0} \frac{(\alpha Q)((1-\alpha)Q)}{d^2} = \frac{Q^2\alpha(1-\alpha)}{4\pi\varepsilon_0 d^2}.$$

The graph below, of F versus α, has been scaled so that the maximum is 1. In actuality, the maximum value of the force is $F_{max} = Q^2/16\pi\varepsilon_0 d^2$.



(a) It is clear that $\alpha = \frac{1}{2} = 0.5$ gives the maximum value of F.

(b) Seeking the half-height points on the graph is difficult without grid lines or some of the special tracing features found in a variety of modern calculators. It is not difficult to algebraically solve for the half-height points (this involves the use of the quadratic formula). The results are

$$\alpha_1 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right) \approx 0.15 \quad \text{and} \quad \alpha_2 = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) \approx 0.85.$$

Thus, the smaller value of α is $\alpha_1 = 0.15$,

(c) and the larger value of α is $\alpha_2 = 0.85$.

65. (a) The magnitudes of the gravitational and electrical forces must be the same:

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = G\frac{mM}{r^2}$$

where q is the charge on either body, r is the center-to-center separation of Earth and Moon, G is the universal gravitational constant, M is the mass of Earth, and m is the mass of the Moon. We solve for q:

$$q = \sqrt{4\pi\varepsilon_0 GmM}.$$

According to Appendix C of the text, $M = 5.98 \times 10^{24}$ kg, and $m = 7.36 \times 10^{22}$ kg, so (using $4\pi\varepsilon_0 = 1/k$) the charge is

$$q = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 5.7\times10^{13}\ \text{C}.$$

(b) The distance r cancels because both the electric and gravitational forces are proportional to $1/r^2$.

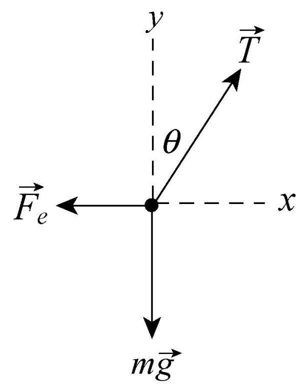
(c) The charge on a hydrogen ion is $e = 1.60 \times 10^{-19}$ C, so there must be

$$\frac{q}{e} = \frac{5.7\times10^{13}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 3.6\times10^{32}\ \text{ions}.$$

Each ion has a mass of 1.67×10^{-27} kg, so the total mass needed is

$$\left(3.6\times10^{32}\right)\left(1.67\times10^{-27}\ \text{kg}\right) = 6.0\times10^{5}\ \text{kg}.$$

66. (a) A force diagram for one of the balls is shown below. The force of gravity $m\vec{g}$ acts downward, the electrical force $\vec{F}_e$ of the other ball acts to the left, and the tension in the thread acts along the thread, at the angle θ to the vertical. The ball is in equilibrium, so its acceleration is zero. The y component of Newton's second law yields $T\cos\theta - mg = 0$ and the x component yields $T\sin\theta - F_e = 0$. We solve the first equation for T and obtain $T = mg/\cos\theta$. We substitute the result into the second to obtain $mg\tan\theta - F_e = 0$.



Examination of the geometry of Figure 21-43 leads to

$$\tan\theta = \frac{x/2}{\sqrt{L^2 - (x/2)^2}}.$$

If L is much larger than x (which is the case if θ is very small), we may neglect $x/2$ in the denominator and write $\tan\theta \approx x/2L$. This is equivalent to approximating $\tan\theta$ by $\sin\theta$. The magnitude of the electrical force of one ball on the other is

$$F_e = \frac{q^2}{4\pi\varepsilon_0 x^2}$$

by Eq. 21-4. When these two expressions are used in the equation $mg\tan\theta = F_e$, we obtain

$$\frac{mgx}{2L} \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} \Rightarrow x \approx \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}.$$

(b) We solve $x^3 = 2kq^2L/mg$) for the charge (using Eq. 21-5):

$$q=\sqrt{\frac{mgx^3}{2kL}}=\sqrt{\frac{(0.010\,\text{kg})(9.8\,\text{m/s}^2)(0.050\,\text{m})^3}{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.20\,\text{m})}}=\pm2.4\times10^{-8}\ \text{C}.$$

Thus, the magnitude is $|q|=2.4\times10^{-8}$ C.

67. (a) If one of them is discharged, there would no electrostatic repulsion between the two balls and they would both come to the position $\theta = 0$, making contact with each other.

(b) A redistribution of the remaining charge would then occur, with each of the balls getting $q/2$. Then they would again be separated due to electrostatic repulsion, which results in the new equilibrium separation

$$x' = \left[\frac{(q/2)^2 L}{2\pi\varepsilon_0 mg}\right]^{1/3} = \left(\frac{1}{4}\right)^{1/3} x = \left(\frac{1}{4}\right)^{1/3} (5.0 \text{ cm}) = 3.1 \text{ cm}.$$

68. Regarding the forces on q_3 exerted by q_1 and q_2, one must "push" and the other must "pull" in order that the net force is zero; hence, q_1 and q_2 have opposite signs. For individual forces to cancel, their magnitudes must be equal:

$$k\frac{|q_1||q_3|}{(L_{12}+L_{23})^2} = k\frac{|q_2||q_3|}{(L_{23})^2}.$$

With $L_{23} = 2.00L_{12}$, the above expression simplifies to $\frac{|q_1|}{9} = \frac{|q_2|}{4}$. Therefore, $q_1 = -9q_2/4$, or $q_1/q_2 = -2.25$.

69. (a) The charge q placed at the origin is a distance r from Q (which is the positive charge on which the forces are being evaluated), and the charge q placed at $x = d$ is a distance r' from Q. Depending on what region Q is located in, the relation between r, r' and d will be either

$r' = r + d$ if Q is along the $-x$ axis (region **A**)
$r' = d - r$ if Q is between the charges (region **B**)
$r' = r - d$ if Q is at $x > d$ (region **C**).

Since all charges in this problem are taken to be positive, then the net force in region **A** will in the $-x$ direction; its magnitude will consist of the individual force magnitudes *added* together. In region **C** the net force will be in the $+x$ direction and will consist again of the individual force magnitudes *added* together. It is in region **B** where the individual force magnitudes must be *subtracted*, and in order for the result to exhibit the correct sign (positive when the net force $\vec{F}$ should point in the $+x$ direction, and so forth), we must write

$$\vec{F}_{\rm B} = \frac{q\,Q}{4\pi\varepsilon_0 r^2} - \frac{q\,Q}{4\pi\varepsilon_0 r'^2} = \frac{q\,Q}{4\pi\varepsilon_0 r^2} - \frac{q\,Q}{4\pi\varepsilon_0 (d-r)^2}\ .$$

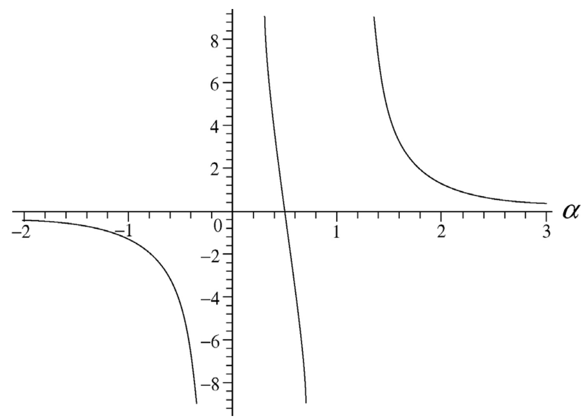
If we further adopt the notation suggested in the problem, then $r = \alpha d$ in regions **B** and **C**, and $r = -\alpha d$ in region **A**.(since r must by definition be a positive number, yet α is negative-valued in region **A**). Using this notation, too, it is clear that we can factor out a common $qQ/4\pi\varepsilon_0 d^2$ from our expressions. For brevity we will use the notation

$$J = \frac{q\,Q}{4\pi\varepsilon_0 d^2}\ .$$

Then, using the observations noted above, we are able to write down the expressions for the force in each region:

$$\vec{F}_{\rm A} = -J\left(\frac{1}{\alpha^2} + \frac{1}{(1-\alpha)^2}\right)$$
$$\vec{F}_{\rm B} = J\left(\frac{1}{\alpha^2} - \frac{1}{(1-\alpha)^2}\right)$$
$$\vec{F}_{\rm C} = J\left(\frac{1}{\alpha^2} + \frac{1}{(\alpha-1)^2}\right)$$

(b) We set $J = 1$ in our plot of the force, below.



70. The mass of an electron is $m = 9.11 \times 10^{-31}$ kg, so the number of electrons in a collection with total mass $M = 75.0$ kg is

$$N = \frac{M}{m} = \frac{75.0\,\text{kg}}{9.11\times10^{-31}\,\text{kg}} = 8.23\times10^{31}\ \text{electrons}.$$

The total charge of the collection is

$$q = -Ne = -\left(8.23\times10^{31}\right)\left(1.60\times10^{-19}\,\text{C}\right) = -1.32\times10^{13}\,\text{C}.$$

71. (a) If a (negative) charged particle is placed a distance x to the right of the $+2q$ particle, then its attraction to the $+2q$ particle will be exactly balanced by its repulsion from the $-5q$ particle is we require

$$\frac{5}{(L+x)^2} = \frac{2}{x^2}$$

which is obtained by equating the Coulomb force magnitudes and then canceling common factors. Cross-multiplying and taking the square root, we obtain

$$\frac{x}{L+x} = \sqrt{\frac{2}{5}}$$

which can be rearranged to produce

$$x = \frac{L}{\sqrt{\frac{2}{5}} - 1} \approx 1.72\ L$$

(b) The y coordinate of particle 3 is $y = 0$.