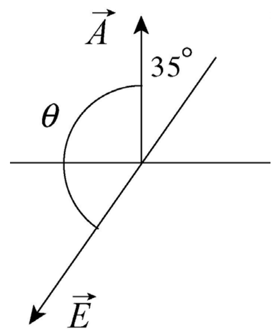
1. The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram below. The angle θ between them is 180° – 35° = 145°, so the electric flux through the area is

$$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta = (1800\ \text{N/C})(3.2\times10^{-3}\ \text{m})^2\cos145° = -1.5\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}.$$



2. We use $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\hat{\mathrm{j}} = (1.40\mathrm{m})^2\,\hat{\mathrm{j}}$.

(a) $\Phi = (6.00\ \ \mathrm{N/C})\hat{\mathrm{i}} \cdot (1.40\ \mathrm{m})^2\,\hat{\mathrm{j}} = 0.$

(b) $\Phi = (-2.00\ \ \mathrm{N/C})\hat{\mathrm{j}} \cdot (1.40\ \mathrm{m})^2\,\hat{\mathrm{j}} = -3.92\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}.$

(c) $\Phi = \left[(-3.00\ \ \mathrm{N/C})\hat{\mathrm{i}} + (400\,\mathrm{N/C})\hat{\mathrm{k}}\right] \cdot (1.40\ \mathrm{m})^2\,\hat{\mathrm{j}} = 0$.

(d) The total flux of a uniform field through a closed surface is always zero.

3. We use $\Phi = \int \vec{E} \cdot d\vec{A}$ and note that the side length of the cube is (3.0 m–1.0 m) = 2.0 m.

(a) On the top face of the cube y = 2.0 m and $d\vec{A} = (dA)\hat{\mathrm{j}}$. Therefore, we have $\vec{E} = 4\hat{\mathrm{i}} - 3\left((2.0)^2 + 2\right)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}$. Thus the flux is

$$\Phi = \int_{\text{top}} \vec{E} \cdot d\vec{A} = \int_{\text{top}} \left(4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}\right) \cdot (dA)\hat{\mathrm{j}} = -18\int_{\text{top}} dA = (-18)(2.0)^2 \ \mathrm{N \cdot m^2/C} = -72\,\mathrm{N \cdot m^2/C}.$$

(b) On the bottom face of the cube y = 0 and $d\vec{A} = (dA)\left(-\hat{\mathrm{j}}\right)$. Therefore, we have $E = 4\hat{\mathrm{i}} - 3\left(0^2 + 2\right)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 6\hat{\mathrm{j}}$. Thus, the flux is

$$\Phi = \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = \int_{\text{bottom}} \left(4\hat{\mathrm{i}} - 6\hat{\mathrm{j}}\right) \cdot (dA)\left(-\hat{\mathrm{j}}\right) = 6\int_{\text{bottom}} dA = 6(2.0)^2 \ \mathrm{N \cdot m^2/C} = +24 \ \mathrm{N \cdot m^2/C}.$$

(c) On the left face of the cube $d\vec{A} = (dA)\left(-\hat{\mathrm{i}}\right)$. So

$$\Phi = \int_{\text{left}} \hat{E} \cdot d\vec{A} = \int_{\text{left}} \left(4\hat{\mathrm{i}} + E_y\hat{\mathrm{j}}\right) \cdot (dA)\left(-\hat{\mathrm{i}}\right) = -4\int_{\text{bottom}} dA = -4(2.0)^2 \ \mathrm{N \cdot m^2/C} = -16\,\mathrm{N \cdot m^2/C}.$$

(d) On the back face of the cube $d\vec{A} = (dA)\left(-\hat{\mathrm{k}}\right)$. But since $\vec{E}$ has no z component $\vec{E} \cdot d\vec{A} = 0$. Thus, $\Phi = 0$.

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or +16 $\mathrm{N \cdot m^2/C}$. Thus the net flux through the cube is

$$\Phi = (-72 + 24 - 16 + 0 + 0 + 16)\ \mathrm{N \cdot m^2/C} = -48\ \mathrm{N \cdot m^2/C}.$$

4. The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E$. Thus, the flux through the netting is

$$\Phi' = -\Phi = -\pi a^2 E = -\pi (0.11 \text{ m})^2 (3.0\times10^{-3} \text{ N/C}) = -1.1\times10^{-4} \text{ N}\cdot\text{m}^2/\text{C}.$$

5. We use Gauss' law: $\varepsilon_0 \Phi = q$, where Φ is the total flux through the cube surface and q is the net charge inside the cube. Thus,

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.8\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 2.0\times10^{5}\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

6. There is no flux through the sides, so we have two "inward" contributions to the flux, one from the top (of magnitude $(34)(3.0)^2$) and one from the bottom (of magnitude $(20)(3.0)^2$). With "inward" flux being negative, the result is $\Phi = -\ 486\ \text{N}\cdot\text{m}^2/\text{C}$. Gauss' law then leads to $q_{\text{enc}} = \varepsilon_0\, \Phi = -4.3 \times 10^{-9}$ C.

7. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length *d*, with a proton of charge $q = +1.6\times10^{-19}$ C situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{net} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus, $\Phi = q/6\varepsilon_0 = 3.01\times 10^{-9}$ N·m^2/C.

8. (a) The total surface area bounding the bathroom is

$$A = 2(2.5\times3.0)+2(3.0\times2.0)+2(2.0\times2.5)=37\ \text{m}^2.$$

The absolute value of the total electric flux, with the assumptions stated in the problem, is

$$|\Phi| = |\sum \vec{E}\cdot\vec{A}| = |\vec{E}|\,A = (600)(37) = 22\times10^3\ \text{N}\cdot\text{m}^2/\text{C}.$$

By Gauss' law, we conclude that the enclosed charge (in absolute value) is $|q_{\text{enc}}| = \varepsilon_0 |\Phi| = 2.0\times10^{-7}$ C. Therefore, with volume V = 15 m^3, and recognizing that we are dealing with negative charges (see problem), the charge density is $q_{\text{enc}}/V = -1.3\times10^{-8}$ C/m^3.

(b) We find $(|q_{\text{enc}}|/e)/V = (2.0\times10^{-7}/1.6\times10^{-19})/15 = 8.2\times10^{10}$ excess electrons per cubic meter.

9. Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_l be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero, so the total flux through the cube surface is $\Phi = A(E_\ell - E_u)$. The net charge inside the cube is given by Gauss' law:

$$q = \varepsilon_0 \Phi = \varepsilon_0 A(E_\ell - E_u) = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(100\ \text{m})^2(100\ \text{N/C} - 60.0\ \text{N/C})$$
$$= 3.54\times10^{-6}\ \text{C} = 3.54\,\mu\text{C}.$$

10. We note that only the smaller shell contributes a (non-zero) field at the designated point, since the point is inside the radius of the large sphere (and E = 0 inside of a spherical charge), and the field points towards the $-x$ direction. Thus,

$$\vec{E} = E(-\hat{\text{j}}) = -\frac{q}{4\pi\varepsilon_o r^2}\,\hat{\text{j}} = -\frac{\sigma_2\, 4\pi R^2}{4\pi\varepsilon_o (L-x)^2}\,\hat{\text{j}} = -(2.8\times 10^4\ \text{N/C})\,\hat{\text{j}},$$

where R = 0.020 m (the radius of the smaller shell), d = 0.10 m and x = 0.020 m.

11. The total flux through any surface that completely surrounds the point charge is q/ε_0.

(a) If we stack identical cubes side by side and directly on top of each other, we will find that eight cubes meet at any corner. Thus, one-eighth of the field lines emanating from the point charge pass through a cube with a corner at the charge, and the total flux through the surface of such a cube is $q/8\varepsilon_0$. Now the field lines are radial, so at each of the three cube faces that meet at the charge, the lines are parallel to the face and the flux through the face is zero.

(b) The fluxes through each of the other three faces are the same, so the flux through each of them is one-third of the total. That is, the flux through each of these faces is $(1/3)(q/8\varepsilon_0) = q/24\varepsilon_0$. Thus, the multiple is 1/24 = 0.0417.

12. Eq. 23-6 (Gauss' law) gives $\varepsilon_o\Phi = q_{enclosed}$.

(a) Thus, the value $\Phi = 2.0 \times 10^5$ (in SI units) for small r leads to $q_{central} = +1.77 \times 10^{-6}$ C or roughly 1.8 μC.

(b) The next value that Φ takes is -4.0×10^5 (in SI units), which implies $q_{enc} = -3.54\times10^{-6}$ C. But we have already accounted for some of that charge in part (a), so the result for part (b) is $q_A = q_{enc} - q_{central} = -5.3 \times 10^{-6}$ C.

(c) Finally, the large r value for Φ is 6.0×10^5 (in SI units), which implies $q_{total\ enc} = 5.31\times10^{-6}$ C. Considering what we have already found, then the result is $q_{total\ enc} - q_A - q_{central} = +8.9\mu C$.

13. (a) Let $A = (1.40\text{ m})^2$. Then

$$\Phi = \left(3.00y\,\hat{\text{j}}\right)\cdot\left(-A\,\hat{\text{j}}\right)\Big|_{y=0} + \left(3.00y\,\hat{\text{j}}\right)\cdot\left(\text{A}\,\hat{\text{j}}\right)\Big|_{y=1.40} = (3.00)(1.40)(1.40)^2 = 8.23\ \text{N}\cdot\text{m}^2/\text{C}.$$

(b) The charge is given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

(c) The electric field can be re-written as $\vec{E} = 3.00y\,\hat{\text{j}} + \vec{E}_0$, where $\vec{E}_0 = -4.00\hat{\text{i}} + 6.00\hat{\text{j}}$ is a constant field which does not contribute to the net flux through the cube. Thus Φ is still $8.23\ \text{N}\cdot\text{m}^2/\text{C}$.

(d) The charge is again given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

14. The total electric flux through the cube is $\Phi = \oint \vec{E} \cdot d\vec{A}$. The net flux through the two faces parallel to the *yz* plane is

$$\Phi_{yz} = \iint [E_x(x=x_2) - E_x(x=x_1)]dydz = \int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz[10+2(4)-10-2(1)]$$
$$= 6\int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz = 6(1)(2) = 12.$$

Similarly, the net flux through the two faces parallel to the *xz* plane is

$$\Phi_{xz} = \iint [E_y(y=y_2) - E_y(y=y_1)]dxdz = \int_{x_1=1}^{x_2=4} dy \int_{z_1=1}^{z_2=3} dz[-3-(-3)] = 0,$$

and the net flux through the two faces parallel to the *xy* plane is

$$\Phi_{xy} = \iint [E_z(z=z_2) - E_z(z=z_1)]dxdy = \int_{x_1=1}^{x_2=4} dx \int_{y_1=0}^{y_2=1} dy\left(3b-b\right) = 2b(3)(1) = 6b.$$

Applying Gauss' law, we obtain

$$q_{\text{enc}} = \varepsilon_0\Phi = \varepsilon_0(\Phi_{xy} + \Phi_{xz} + \Phi_{yz}) = \varepsilon_0(6.00b + 0 + 12.0) = 24.0\varepsilon_0$$

which implies that $b = 2.00\ \text{N/C}\cdot\text{m}$.

15. (a) The charge on the surface of the sphere is the product of the surface charge density σ and the surface area of the sphere (which is $4\pi r^2$, where r is the radius). Thus,

$$q = 4\pi r^2 \sigma = 4\pi \left(\frac{1.2\ \text{m}}{2} \right)^2 \left(8.1\times10^{-6}\ \text{C/m}^2 \right) = 3.7\times10^{-5}\ \text{C}.$$

(b) We choose a Gaussian surface in the form of a sphere, concentric with the conducting sphere and with a slightly larger radius. The flux is given by Gauss's law:

$$\Phi = \frac{q}{\varepsilon_0} = \frac{3.66\times10^{-5}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 4.1\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C}\ .$$

16. Using Eq. 23-11, the surface charge density is

$$\sigma = E\varepsilon_0 = \left(2.3\times10^5\ \text{N/C}\right)\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right) = 2.0\times10^{-6}\ \text{C/m}^2.$$

17. (a) The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

$$\sigma = \frac{q}{\pi D^2} = \frac{2.4\times10^{-6}\ \text{C}}{\pi(1.3\ \text{m})^2} = 4.5\times10^{-7}\ \text{C/m}^2.$$

(b) Eq. 23-11 gives

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.5\times10^{-7}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N.m}^2} = 5.1\times10^{4}\ \text{N/C}.$$

18. Eq. 23-6 (Gauss' law) gives $\varepsilon_o \Phi = q_{enc}$.

(a) The value $\Phi = -9.0 \times 10^5$ (in SI units) for small r leads to $q_{central} = -7.97 \times 10^{-6}$ C or roughly – 8.0 μC.

(b) The next (non-zero) value that Φ takes is +4.0 × 10^5 (in SI units), which implies $q_{enc} = 3.54 \times 10^{-6}$ C. But we have already accounted for some of that charge in part (a), so the result is

$$q_A = q_{enc} - q_{central} = 11.5 \times 10^{-6}\ \text{C} \approx 12\ \mu\text{C}.$$

(c) Finally, the large r value for Φ is – 2.0 × 10^5 (in SI units), which implies $q_{total\ enc} = -1.77 \times 10^{-6}$ C. Considering what we have already found, then the result is

$$q_{total\ enc} - q_A - q_{central} = -5.3\ \mu\text{C}.$$

19. (a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it encloses is zero. The net charge is the sum of the charge q in the cavity and the charge q_w on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0 \times 10^{-6}$C.

(b) The net charge Q of the conductor is the sum of the charge on the cavity wall and the charge q_s on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_\omega = \left(10\times10^{-6}\text{ C}\right) - \left(-3.0\times10^{-6}\text{ C}\right) = +1.3\times10^{-5}\text{ C}.$$

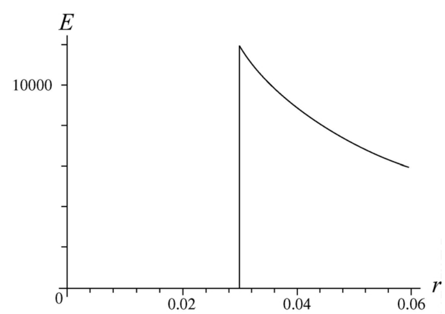
20. We imagine a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry $\oint_A \vec{E}\cdot d\vec{A} = 2\pi r E = \frac{q_{\text{enc}}}{\varepsilon_0}$.

(a) For $r < R$, $q_{\text{enc}} = 0$, so $E = 0$.

(b) For $r > R$, $q_{\text{enc}} = \lambda$, so $E(r) = \lambda / 2\pi r\varepsilon_0$. With $\lambda = 2.00\times10^{-8}$ C/m and $r = 2.00R = 0.0600$ m, we obtain

$$E = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.0600\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 5.99\times10^{3}\ \text{N/C}.$$

(c) The plot of E vs. r is shown below.



Here, the maximum value is

$$E_{\text{max}} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.030\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.2\times10^{4}\ \text{N/C}.$$

21. The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\varepsilon_0 r$, where λ is the linear charge density and r is the distance from the line to the point where the field is measured. See Eq. 23-12. Thus,

$$\lambda = 2\pi\varepsilon_0 Er = 2\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.5\times10^{4}\ \text{N/C}\right)\left(2.0\ \text{m}\right) = 5.0\times10^{-6}\ \text{C/m}.$$

22. We combine Newton's second law ($F = ma$) with the definition of electric field ($F = qE$) and with Eq. 23-12 (for the field due to a line of charge). In terms of magnitudes, we have (if $r = 0.080$ m and $\lambda = 6.0 \times 10^{-6}$ C/m)

$$ma = eE = \frac{e\lambda}{2\pi\varepsilon_0 r} \quad \Rightarrow \quad a = \frac{e\lambda}{2\pi\varepsilon_0 r\, m} = 2.1 \times 10^{17} \text{ m/s}^2 \ .$$

23. (a) The side surface area A for the drum of diameter D and length h is given by $A = \pi D h$. Thus

$$q = \sigma A = \sigma \pi D h = \pi \varepsilon_0 E D h = \pi\left(8.85\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}\right)\left(2.3\times10^{5}\ \mathrm{N/C}\right)\left(0.12\ \mathrm{m}\right)\left(0.42\ \mathrm{m}\right)$$

$$= 3.2\times10^{-7}\ \mathrm{C}.$$

(b) The new charge is

$$q' = q\left(\frac{A'}{A}\right) = q\left(\frac{\pi D' h'}{\pi D h}\right) = \left(3.2\times10^{-7}\,\mathrm{C}\right)\left[\frac{(8.0\ \mathrm{cm})(28\ \mathrm{cm})}{(12\ \mathrm{cm})(42\ \mathrm{cm})}\right] = 1.4\times10^{-7}\ \mathrm{C}.$$

24. We reason that point P (the point on the x axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that P is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for P. Using Eq. 23-12, we have

$$E_{net} = E_1 + E_2 = \frac{\lambda_1}{2\pi\varepsilon_o(x + L/2)} + \frac{\lambda_2}{2\pi\varepsilon_o(x - L/2)} .$$

Setting this equal to zero and solving for x we find

$$x = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\frac{L}{2}$$

which, for the values given in the problem, yields $x = 8.0$ cm.

25. We denote the inner and outer cylinders with subscripts *i* and *o*, respectively.

(a) Since $r_i < r = 4.0$ cm $< r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\ \text{m})} = 2.3\times10^{6}\ \text{N/C}.$$

(b) The electric field $\vec{E}(r)$ points radially outward.

(c) Since $r > r_o$,

$$E(r) = \frac{\lambda_i + \lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m} - 7.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\ \text{m})} = -4.5\times10^{5}\ \text{N/C},$$

or $|E(r)| = 4.5\times10^{5}$ N/C.

(d) The minus sign indicates that $\vec{E}(r)$ points radially inward.

26. As we approach $r = 3.5$ cm from the inside, we have

$$E_{\text{internal}} = \frac{\lambda}{2\pi\varepsilon_o r} = 1000 \text{ N/C} .$$

And as we approach $r = 3.5$ cm from the outside, we have

$$E_{\text{external}} = \frac{\lambda}{2\pi\varepsilon_o r} + \frac{\lambda'}{2\pi\varepsilon_o r} = -3000 \text{ N/C} .$$

Considering the difference ($E_{\text{external}} - E_{\text{internal}}$) allows us to find λ' (the charge per unit length on the larger cylinder). Using $r = 0.035$ m, we obtain $\lambda' = -5.8 \times 10^{-9}$ C/m.

27. We assume the charge density of both the conducting cylinder and the shell are uniform, and we neglect fringing effect. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

(a) We take the Gaussian surface to be a cylinder of length L, coaxial with the given cylinders and of larger radius r than either of them. The flux through this surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Now, the charge enclosed by the Gaussian surface is $q_{enc} = Q1 + Q_2 = -Q_1 = -3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{enc}$, or

$$E = \frac{q_{enc}}{2\pi\varepsilon_0 Lr} = \frac{-3.40\times10^{-12}\ \text{C}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(11.0\ \text{m})(20.0\times1.30\times10^{-3}\text{m})} = -0.214\ \text{N/C},$$

or $|E| = 0.214$ N/C.

(b) The negative sign in E indicates that the field points inward.

(c) Next, for r = 5.00 R_1, the charge enclosed by the Gaussian surface is $q_{enc} = Q_1 = 3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{enc}$, or

$$E = \frac{q_{enc}}{2\pi\varepsilon_0 Lr} = \frac{3.40\times10^{-12}\ \text{C}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(11.0\ \text{m})(5.00\times1.30\times10^{-3}\text{m})} = 0.855\ \text{N/C}.$$

(d) The positive sign indicates that the field points outward.

(e) we consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod has charge Q_1, the inner surface of the shell must have charge $Q_{in} = -Q_1 = -3.40\times10^{-12}$ C.

(f) Since the shell is known to have total charge $Q_2 = -2.00Q_1$, it must have charge $Q_{out} = Q_2 - Q_{in} = -Q_1 = -3.40\times10^{-12}$ C on its outer surface.

28. (a) In Eq. 23-12, $\lambda = q/L$ where q is the net charge enclosed by a cylindrical Gaussian surface of radius r. The field is being measured outside the system (the charged rod coaxial with the neutral cylinder) so that the net enclosed charge is only that which is on the rod. Consequently,

$$\left|\vec{E}\right| = \frac{\lambda}{2\pi\varepsilon_0\, r} = \frac{2.0\times10^{-9}}{2\pi\varepsilon_0(0.15)} = 2.4\times10^{2}\ \text{N/C}.$$

(b) Since the field is zero inside the conductor (in an electrostatic configuration), then there resides on the inner surface charge $-q$, and on the outer surface, charge $+q$ (where q is the charge on the rod at the center). Therefore, with $r_i = 0.05$ m, the surface density of charge is

$$\sigma_{\text{inner}} = \frac{-q}{2\pi r_i L} = -\frac{\lambda}{2\pi r_i} = -6.4\times10^{-9}\ \text{C/m}^2$$

for the inner surface.

(c) With $r_o = 0.10$ m, the surface charge density of the outer surface is

$$\sigma_{\text{outer}} = \frac{+q}{2\pi r_o L} = \frac{\lambda}{2\pi r_o} = +3.2\times10^{-9}\ \text{C/m}^2.$$

29. We denote the radius of the thin cylinder as R = 0.015 m. Using Eq. 23-12, the net electric field for $r > R$ is given by

$$E_{\text{net}} = E_{\text{wire}} + E_{\text{cylinder}} = \frac{-\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda'}{2\pi\varepsilon_0 r}$$

where $-\lambda = -3.6$ nC/m is the linear charge density of the wire and λ' is the linear charge density of the thin cylinder. We note that the surface and linear charge densities of the thin cylinder are related by

$$q_{\text{cylinder}} = \lambda' L = \sigma(2\pi RL) \Rightarrow \lambda' = \sigma(2\pi R).$$

Now, E_{net} outside the cylinder will equal zero, provided that $2\pi R\sigma = \lambda$, or

$$\sigma = \frac{\lambda}{2\pi R} = \frac{3.6\times10^{-6}\ \text{C/m}}{(2\pi)(0.015\ \text{m})} = 3.8\times10^{-8}\ \text{C/m}^2.$$

30. To evaluate the field using Gauss' law, we employ a cylindrical surface of area $2\pi\, r\, L$ where L is very large (large enough that contributions from the ends of the cylinder become irrelevant to the calculation). The volume within this surface is $V = \pi\, r^2\, L$, or expressed more appropriate to our needs: $dV = 2\pi\, r\, L\, dr$. The charge enclosed is, with $A = 2.5\times10^{-6}\,\text{C/m}^5$,

$$q_{\text{enc}} = \int_0^r A\, r^2\, 2\pi\, r\, L\, dr = \frac{\pi}{2} A\, L\, r^4 .$$

By Gauss' law, we find $\Phi = |\vec{E}|\,(2\pi r L) = q_{\text{enc}}/\varepsilon_0$; we thus obtain $\left|\vec{E}\right| = \dfrac{A\, r^3}{4\,\varepsilon_0}$.

(a) With $r = 0.030$ m, we find $|\vec{E}| = 1.9$ N/C.

(b) Once outside the cylinder, Eq. 23-12 is obeyed. To find $\lambda = q/L$ we must find the total charge q. Therefore,

$$\frac{q}{L} = \frac{1}{L}\int_0^{0.04} A\, r^2\, 2\pi\, r\, L\, dr = 1.0\times10^{-11}\ \text{C/m}.$$

And the result, for $r = 0.050$ m, is $|\vec{E}| = \lambda/2\pi\varepsilon_0\, r = 3.6$ N/C.

31. (a) To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\varepsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus,

$$\sigma = \frac{q}{2A} = \frac{6.0\times10^{-6}\text{ C}}{2(0.080\text{ m})^2} = 4.69\times10^{-4}\text{ C/m}^2.$$

The magnitude of the field is

$$E = \frac{4.69\times10^{-4}\text{ C/m}^2}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 5.3\times10^7\text{ N/C}.$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\varepsilon_0 r^2 = kq/r^2$, where r is the distance from the plate. Thus,

$$E = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\text{ C}\right)}{\left(30\text{ m}\right)^2} = 60\text{ N/C}.$$

32. According to Eq. 23-13 the electric field due to either sheet of charge with surface charge density $\sigma = 1.77\times 10^{-22}$ C/m^2 is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0 = (1.77 \times 10^{-22})/(8.85 \times 10^{-12}) = 2.00\times10^{-11}$ N/C, pointing in the upward direction, or $\vec{E} = (2.00\times10^{-11}\text{ N/C})\hat{\text{j}}$.

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down, or $\vec{E} = -(2.00\times10^{-11}\text{ N/C})\hat{\text{j}}$.

33. In the region between sheets 1 and 2, the net field is $E_1 - E_2 + E_3 = 2.0 \times 10^5$ N/C .

In the region between sheets 2 and 3, the net field is at its greatest value:

$$E_1 + E_2 + E_3 = 6.0 \times 10^5 \text{ N/C} .$$

The net field vanishes in the region to the right of sheet 3, where $E_1 + E_2 = E_3$. We note the implication that σ_3 is negative (and is the largest surface-density, in magnitude). These three conditions are sufficient for finding the fields:

$$E_1 = 1.0 \times 10^5 \text{ N/C} , \; E_2 = 2.0 \times 10^5 \text{ N/C} , \; E_3 = 3.0 \times 10^5 \text{ N/C} .$$

From Eq. 23-13, we infer (from these values of E)

$$\frac{|\sigma_3|}{|\sigma_2|} = \frac{3.0 \text{ x } 10^5 \text{ N/C}}{2.0 \text{ x } 10^5 \text{ N/C}} = 1.5 .$$

Recalling our observation, above, about σ_3, we conclude $\dfrac{\sigma_3}{\sigma_2} = -1.5$.

34. The charge distribution in this problem is equivalent to that of an infinite sheet of charge with surface charge density $\sigma = 4.50 \times 10^{-12}$ C/m^2 plus a small circular pad of radius R = 1.80 cm located at the middle of the sheet with charge density $-\sigma$. We denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. Using Eq. 22-26 for $\vec{E}_2$, the net electric field $\vec{E}$ at a distance z = 2.56 cm along the central axis is then

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathrm{k}} + \frac{(-\sigma)}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{\mathrm{k}} = \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+R^2}}\hat{\mathrm{k}}$$

$$= \frac{(4.50\times10^{-12})(2.56\times10^{-2})}{2(8.85\times10^{-12})\sqrt{(2.56\times10^{-2})^2+(1.80\times10^{-2})^2}}\hat{\mathrm{k}} = (0.208 \text{ N/C})\,\hat{\mathrm{k}}$$

35. We use Eq. 23-13.

(a) To the left of the plates:

$$\vec{E} = (\sigma/2\varepsilon_0)(-\hat{\mathrm{i}}) \text{ (from the right plate) } + (\sigma/2\varepsilon_0)\hat{\mathrm{i}} \text{ (from the left one)} = 0.$$

(b) To the right of the plates:

$$\vec{E} = (\sigma/2\varepsilon_0)\hat{\mathrm{i}} \text{ (from the right plate)} + (\sigma/2\varepsilon_0)(-\hat{\mathrm{i}}) \text{ (from the left one)} = 0.$$

(c) Between the plates:

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathrm{i}}) + \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathrm{i}}) = \left(\frac{\sigma}{\varepsilon_0}\right)(-\hat{\mathrm{i}}) = -\left(\frac{7.00\times10^{-22}\ \mathrm{C/m^2}}{8.85\times10^{-12}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}}\right)\hat{\mathrm{i}} = \left(-7.91\times10^{-11}\ \mathrm{N/C}\right)\hat{\mathrm{i}}.$$

36. The field due to the sheet is $E = \frac{\sigma}{2\varepsilon_o}$. The force (in magnitude) on the electron (due to that field) is $F = eE$, and assuming it's the *only* force then the acceleration is

$$a = \frac{e\sigma}{2\varepsilon_o m} = \text{slope of the graph} \ \ (= 2.0 \times 10^5 \text{ m/s divided by } 7.0 \times 10^{-12} \text{ s}) \ .$$

Thus we obtain $\sigma = 2.9 \times 10^{-6}$ C/m^2.

37. The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. We take the initial direction of motion of the electron to be positive. Then, the electric field is given by $E = \sigma/\varepsilon_0$, where σ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\varepsilon_0$ and the acceleration is

$$a = \frac{F}{m} = -\frac{e\sigma}{\varepsilon_0 m}$$

where m is the mass of the electron. The force is constant, so we use constant acceleration kinematics. If v_0 is the initial velocity of the electron, v is the final velocity, and x is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax$. Set $v = 0$ and replace a with $-e\sigma/\varepsilon_0 m$, then solve for x. We find

$$x = -\frac{v_0^2}{2a} = \frac{\varepsilon_0 m v_0^2}{2e\sigma}.$$

Now $\frac{1}{2}mv_0^2$ is the initial kinetic energy K_0, so

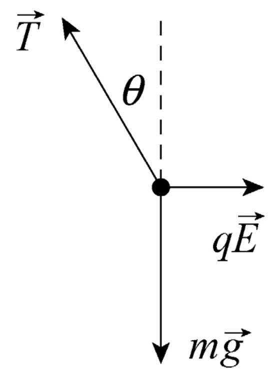
$$x = \frac{\varepsilon_0 K_0}{e\sigma} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.60\times10^{-17}\ \text{J}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.0\times10^{-6}\ \text{C/m}^2\right)} = 4.4\times10^{-4}\ \text{m}.$$

38. We use the result of part (c) of problem 35 to obtain the surface charge density.

$$E = \sigma / \varepsilon_0 \Rightarrow \sigma = \varepsilon_0 E = \left(8.85\times10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(55 \text{ N/C}) = 4.9\times10^{-10} \text{ C/m}^2.$$

Since the area of the plates is $A = 1.0 \text{ m}^2$, the magnitude of the charge on the plate is $Q = \sigma A = 4.9\times10^{-10} \text{ C}$.

39. The forces acting on the ball are shown in the diagram below. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the magnitude of the electric field at the position of the ball; and, the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical.



Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields $qE - T\sin\theta = 0$ and the sum of the vertical components yields $T\cos\theta - mg = 0$. The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Thus,

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and

$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N.m}^2\right)\left(1.0\times10^{-6}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\tan 30°}{2.0\times10^{-8}\ \text{C}}$$
$$= 5.0\times10^{-9}\ \text{C/m}^2.$$

40. The point where the individual fields cancel cannot be in the region between the sheet and the particle ($-d < x < 0$) since the sheet and the particle have opposite-signed charges. The point(s) could be in the region to the right of the particle ($x > 0$) and in the region to the left of the sheet ($x < d$); this is where the condition

$$\frac{|\sigma|}{2\varepsilon_o} = \frac{Q}{4\pi\varepsilon_o r^2}$$

must hold. Solving this with the given values, we find $r = x = \pm\sqrt{3/2\pi} \approx \pm\, 0.691$ m.

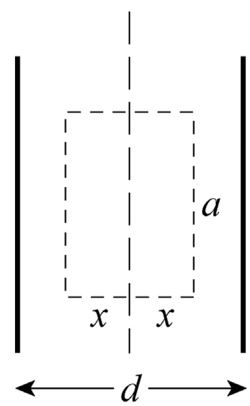
If d = 0.20 m (which is less than the magnitude of r found above), then neither of the points ($x \approx \pm\, 0.691$ m) is in the "forbidden region" between the particle and the sheet. Thus, both values are allowed. Thus, we have

(a) x = 0.691 m on the positive axis, and

(b) x = – 0.691 m on the negative axis.

(c) If, however, d = 0.80 m (greater than the magnitude of r found above), then one of the points ($x \approx -0.691$ m) is in the "forbidden region" between the particle and the sheet and is disallowed. In this part, the fields cancel only at the point $x \approx +0.691$ m.

41. We use a Gaussian surface in the form of a box with rectangular sides. The cross section is shown with dashed lines in the diagram below.



It is centered at the central plane of the slab, so the left and right faces are each a distance x from the central plane. We take the thickness of the rectangular solid to be a, the same as its length, so the left and right faces are squares. The electric field is normal to the left and right faces and is uniform over them. Since ρ = 5.80 fC/m^3 is positive, it points outward at both faces: toward the left at the left face and toward the right at the right face. Furthermore, the magnitude is the same at both faces. The electric flux through each of these faces is Ea^2. The field is parallel to the other faces of the Gaussian surface and the flux through them is zero. The total flux through the Gaussian surface is $\Phi = 2Ea^2$. The volume enclosed by the Gaussian surface is $2a^2x$ and the charge contained within it is $q = 2a^2x\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = 2a^2x\rho$. We solve for the magnitude of the electric field: $E = \rho x / \varepsilon_0$.

(a) For x =0, E =0.

(b) For x = 2.00 mm = 2.00 × 10^{-3} m,

$$E = (5.80\times10^{-15})(2.00\times10^{-3})/(8.85\times10^{-12}) = 1.31\times10^{-6}\ \text{N/C}.$$

(c) For $x = d/2$ = 4.70 mm = 4.70 × 10^{-3} m,

$$E = (5.80\times10^{-15})(4.70\times10^{-3})/(8.85\times10^{-12}) = 3.08\times10^{-6}\ \text{N/C}.$$

(d) For x = 26.0 mm = 2.60 × 10^{-2} m, we take a Gaussian surface of the same shape and orientation, but with $x > d/2$, so the left and right faces are outside the slab. The total flux through the surface is again $\Phi = 2Ea^2$ but the charge enclosed is now $q = a^2d\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = a^2 d\rho$, so

$$E = \frac{\rho d}{2\varepsilon_0} = \frac{(5.80\times10^{-15})(9.40\times10^{-3})}{2(8.85\times10^{-12})} = 3.08\times10^{-6}\ \text{N/C}.$$

42. We determine the (total) charge on the ball by examining the maximum value ($E = 5.0 \times 10^7$ N/C) shown in the graph (which occurs at $r = 0.020$ m). Thus,

$$E = \frac{q}{4\pi\varepsilon_0 r^2} \quad \Rightarrow \quad q = 2.2 \times 10^{-6}\ \text{C}.$$

43. Charge is distributed uniformly over the surface of the sphere and the electric field it produces at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = q/4\pi\varepsilon_0 r^2$, where q is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured. Thus,

$$q = 4\pi\varepsilon_0 r^2 E = \frac{(0.15\text{ m})^2(3.0\times10^3\text{ N/C})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 7.5\times10^{-9}\text{ C}.$$

The field points inward, toward the sphere center, so the charge is negative: -7.5×10^{-9} C.

44. (a) The flux is still $-750\ \mathrm{N\cdot m^2/C}$, since it depends only on the amount of charge enclosed.

(b) We use $\Phi = q/\varepsilon_0$ to obtain the charge q:

$$q = \varepsilon_0 \Phi = \left(8.85\times10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)\left(-750\ \mathrm{N\cdot m^2/C}\right) = -6.64\times10^{-9}\ \mathrm{C}.$$

45. (a) Since r_1 = 10.0 cm < r = 12.0 cm <r_2 = 15.0 cm,

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2\text{C}^2\right)\left(4.00\times10^{-8}\ \text{C}\right)}{\left(0.120\ \text{m}\right)^2} = 2.50\times10^4\ \text{N/C}.$$

(b) Since $r_1 < r_2 < r$ = 20.0 cm,

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1+q_2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00+2.00\right)\left(1\times10^{-8}\ \text{C}\right)}{\left(0.200\ \text{m}^2\right)} = 1.35\times10^4\ \text{N/C}.$$

46. The point where the individual fields cancel cannot be in the region between the shells since the shells have opposite-signed charges. It cannot be inside the radius R of one of the shells since there is only one field contribution there (which would not be canceled by another field contribution and thus would not lead to zero net field). We note shell 2 has greater magnitude of charge ($|\sigma_2|A_2$) than shell 1, which implies the point is not to the right of shell 2 (any such point would always be closer to the larger charge and thus no possibility for cancellation of equal-magnitude fields could occur). Consequently, the point should be in the region to the left of shell 1 (at a distance $r > R_1$ from its center); this is where the condition

$$E_1 = E_2 \Rightarrow \frac{|q_1|}{4\pi\varepsilon_o r^2} = \frac{|q_2|}{4\pi\varepsilon_o (r+L)^2}$$

or

$$\frac{\sigma_1 A_1}{4\pi\varepsilon_o r^2} = \frac{|\sigma_2| A_2}{4\pi\varepsilon_o (r+L)^2} .$$

Using the fact that the area of a sphere is $A = 4\pi R^2$, this condition simplifies to

$$r = \frac{L}{(R_2/R_1)\sqrt{|\sigma_2|/\sigma_1} - 1} = 3.3 \text{ cm} .$$

We note that this value satisfies the requirement $r > R_1$. The answer, then, is that the net field vanishes at $x = -r = -3.3$ cm.

47. To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance. We use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center. The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho \, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2 \, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 dr = 4\pi A \int_a^{r_g} r \, dr = 2\pi A \left(r_g^2 - a^2\right).$$

The total charge inside the Gaussian surface is $q + q_s = q + 2\pi A \left(r_g^2 - a^2\right)$. The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields $4\pi\varepsilon_0 E r_g^2 = q + 2\pi A\left(r_g^2 - a^2\right)$. We solve for E:

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2}\right].$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$. With $a = 2.00 \times 10^{-2}$ m and $q = 45.0 \times 10^{-15}$ C, we have $A = 1.79 \times 10^{-11} \text{ C/m}^2$.

48. Let E_A designate the magnitude of the field at r = 2.4 cm. Thus $E_A = 2.0 \times 10^7$ N/C, and is totally due to the particle. Since

$$E_{\text{particle}} = \frac{q}{4\pi\varepsilon_o r^2}$$

then the field due to the particle at any other point will relate to E_A by a ratio of distances squared. Now, we note that at r = 3.0 cm the total contribution (from particle and shell) is 8.0×10^7 N/C. Therefore,

$$E_{\text{shell}} + E_{\text{particle}} = E_{\text{shell}} + (2.4/3)^2 E_A = 8.0 \times 10^7 \text{ N/C}$$

Using the value for E_A noted above, we find $E_{\text{shell}} = 6.6 \times 10^7$ N/C. Thus, with r = 0.030 m, we find the charge Q using

$$E_{\text{shell}} = \frac{Q}{4\pi\varepsilon_o r^2} \quad \Rightarrow \quad Q = 6.6 \times 10^{-6}\ \text{C} .$$

49. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E$, where r is the radius of the Gaussian surface.

For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For $r = 0$, the above equation implies $E = 0$.

(b) For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{2(2.00\times10^{-2}\,\text{m})^2} = 5.62\times10^{-2}\ \text{N/C}.$$

(c) For $r = a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{(2.00\times10^{-2}\,\text{m})^2} = 0.112\ \text{N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \Rightarrow E = \frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{(1.50\times2.00\times10^{-2}\,\text{m})^2} = 0.0499\ \text{N/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0$. Thus, $E = 0$ at $r = 3.50a$.

(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E} \cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -5.00$ fC.

(h) Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means $Q_o = -q_1 - Q_i = -q_1 -(-q_1) = 0$.

50. The field is zero for $0 \le r \le a$ as a result of Eq. 23-16. Thus,

(a) $E = 0$ at $r = 0$,

(b) $E = 0$ at $r = a/2.00$, and

(c) $E = 0$ at $r = a$.

For $a \le r \le b$ the enclosed charge q_{enc} (for $a \le r \le b$) is related to the volume by

$$q_{\text{enc}} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right).$$

Therefore, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right) = \frac{\rho}{3\varepsilon_0}\frac{r^3 - a^3}{r^2}$$

for $a \le r \le b$.

(d) For $r = 1.50a$, we have

$$E = \frac{\rho}{3\varepsilon_0}\frac{(1.50a)^3 - a^3}{(1.50a)^2} = \frac{\rho a}{3\varepsilon_0}\frac{2.375}{2.25} = \frac{(1.84\times10^{-9})(0.100)}{3(8.85\times10^{-12})}\frac{2.375}{2.25} = 7.32 \text{ N/C}.$$

(e) For $r = b = 2.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(2.00a)^2} = \frac{\rho a}{3\varepsilon_0}\frac{7}{4} = \frac{(1.84\times10^{-9})(0.100)}{3(8.85\times10^{-12})}\frac{7}{4} = 12.1 \text{ N/C}.$$

(f) For $r \ge b$ we have $E = q_{\text{total}}/4\pi\varepsilon_0 r^2$ or

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2}.$$

Thus, for $r = 3.00b = 6.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(6.00a)^2} = \frac{\rho a}{3\varepsilon_0}\frac{7}{36} = \frac{(1.84\times10^{-9})(0.100)}{3(8.85\times10^{-12})}\frac{7}{4} = 1.35 \text{ N/C}.$$

51. (a) We integrate the volume charge density over the volume and require the result be equal to the total charge:

$$\int dx \int dy \int dz\, \rho = 4\pi \int_0^R dr\ r^2\ \rho = Q.$$

Substituting the expression $\rho = \rho_s r/R$, with ρ_s= 14.1 pC/m^3, and performing the integration leads to

$$4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{R^4}{4}\right) = Q$$

or

$$Q = \pi\rho_s R^3 = \pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})^3 = 7.78\times10^{-15}\ \text{C}.$$

(b) At r = 0, the electric field is zero (E = 0) since the enclosed charge is zero.

At a certain point within the sphere, at some distance r from the center, the field (see Eq. 23-8 through Eq. 23-10) is given by Gauss' law:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2}$$

where q_{enc} is given by an integral similar to that worked in part (a):

$$q_{\text{enc}} = 4\pi \int_0^r dr\, r^2 \rho = 4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{r^4}{4}\right).$$

Therefore,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^4}{Rr^2} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^2}{R}.$$

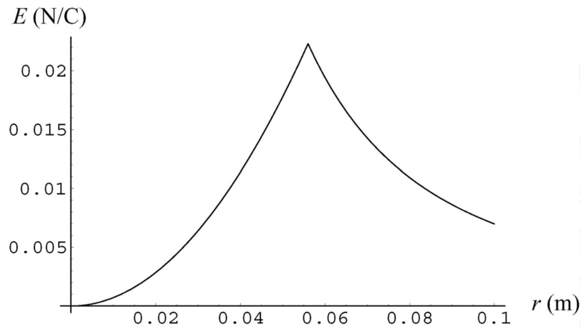
(c) For $r = R/2.00$, where R = 5.60 cm, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s (R/2.00)^2}{R} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R}{4.00} = \frac{(8.99\times10^9)\pi(14.1\times10^{-12})(0.0560)}{4.00} = 5.58\times10^{-3}\,\text{N/C}.$$

(d) For $r = R$, the electric field is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R^2}{R}=\frac{\pi\rho_s R}{4\pi\varepsilon_0}=(8.99\times10^9)\pi(14.1\times10^{-12})(0.0560)=2.23\times10^{-2}\ \text{N/C}.$$

(e) The electric field strength as a function of r is depicted below:



52. Applying Eq. 23-20, we have

$$E_1 = \frac{|q_1|}{4\pi\varepsilon_o R^3} r_1 = \frac{|q_1|}{4\pi\varepsilon_o R^3}\left(\frac{R}{2}\right) = \frac{|q_1|}{8\pi\varepsilon_o R^2} .$$

Also, outside sphere 2 we have

$$E_2 = \frac{|q_2|}{4\pi\varepsilon_o r^2} = \frac{|q_2|}{4\pi\varepsilon_o (1.5\, R)^2} .$$

Equating these and solving for the ratio of charges, we arrive at $\frac{q_2}{q_1} = \frac{9}{8} = 1.125.$

53. We use

$$E(r) = \frac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\int_0^r \rho(r)4\pi r^2 dr$$

to solve for $\rho(r)$:

$$\rho(r) = \frac{\varepsilon_0}{r^2}\frac{d}{dr}\left[r^2 E(r)\right] = \frac{\varepsilon_0}{r^2}\frac{d}{dr}\left(Kr^6\right) = 6K\varepsilon_0 r^3.$$

54. (a) We consider the radial field produced at points within a uniform cylindrical distribution of charge. The volume enclosed by a Gaussian surface in this case is $L\pi r^2$. Thus, Gauss' law leads to

$$E = \frac{|q_{\text{enc}}|}{\varepsilon_0 A_{\text{cylinder}}} = \frac{|\rho|\left(L\pi r^2\right)}{\varepsilon_0 (2\pi r L)} = \frac{|\rho| r}{2\varepsilon_0}.$$

(b) We note from the above expression that the magnitude of the radial field grows with r.

(c) Since the charged powder is negative, the field points radially inward.

(d) The largest value of r which encloses charged material is $r_{\text{max}} = R$. Therefore, with $|\rho| = 0.0011\ \text{C/m}^3$ and $R = 0.050$ m, we obtain

$$E_{\text{max}} = \frac{|\rho| R}{2\varepsilon_0} = 3.1\times 10^6\ \text{N/C}.$$

(e) According to condition 1 mentioned in the problem, the field is high enough to produce an electrical discharge (at $r = R$).

55. (a) The cube is totally within the spherical volume, so the charge enclosed is

$$q_{enc} = \rho V_{cube} = (500 \times 10^{-9})(0.0400)^3 = 3.20 \times 10^{-11} \text{ C}.$$

By Gauss' law, we find $\Phi = q_{enc}/\varepsilon_0 = 3.62 \text{ N}\cdot\text{m}^2/\text{C}$.

(b) Now the sphere is totally contained within the cube (note that the radius of the sphere is less than half the side-length of the cube). Thus, the total charge is

$$q_{enc} = \rho V_{sphere} = 4.5 \times 10^{-10} \text{ C}.$$

By Gauss' law, we find $\Phi = q_{enc}/\varepsilon_0 = 51.1 \text{ N}\cdot\text{m}^2/\text{C}$.

56. (a) Since the volume contained within a radius of $\frac{1}{2}R$ is one-eighth the volume contained within a radius of R, so the charge at $0 < r < R/2$ is $Q/8$. The fraction is 1/8 = 0.125.

(b) At $r = R/2$, the magnitude of the field is

$$E = \frac{Q/8}{4\pi\varepsilon_o (R/2)^2} = \frac{Q}{8\pi\varepsilon_o R^2}$$

and is equivalent to *half* the field at the surface. Thus, the ratio is 0.500.

57. (a) We use $m_e g = eE = e\sigma/\varepsilon_0$ to obtain the surface charge density.

$$\sigma = \frac{m_e g \varepsilon_0}{e} = \frac{(9.11\times10^{-31}\,\text{kg})(9.8\,\text{m/s})(8.85\times10^{-12}\,\frac{C^2}{\text{N.m}^2})}{1.60\times10^{-19}\,\text{C}} = 4.9\times10^{-22}\,\text{C/m}^2\,.$$

(b) Downward (since the electric force exerted on the electron must be upward).

58. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only. In Si units, we have

$$E_{\text{non-constant}} = 3x\,\hat{\mathrm{i}} \ .$$

The face of the cube located at $x = 0$ (in the yz plane) has area $A = 4\ \mathrm{m}^2$ (and it "faces" the $+\hat{\mathrm{i}}$ direction) and has a "contribution" to the flux equal to $E_{\text{non-constant}}A = (3)(0)(4) = 0$. The face of the cube located at $x = -2$ m has the same area A (and this one "faces" the $-\hat{\mathrm{i}}$ direction) and a contribution to the flux: $-E_{\text{non-constant}}A = -(3)(-2)(4) = 24$ (in SI units). Thus, the net flux is $\Phi = 0 + 24 = 24\ \mathrm{N{\cdot}m/C^2}$. According to Gauss' law, we therefore have $q_{\text{enc}} = \varepsilon_o \Phi = 2.13 \times 10^{-10}\ \mathrm{C}$.

59. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only:

$$E_{\text{non-constant}} = (-4.00y^2)\ \hat{\text{i}}\ \text{(in SI units)}\ .$$

The face of the cube located at y = 4.00 has area A = 4.00 m^2 (and it "faces" the $+\hat{\text{j}}$ direction) and has a "contribution" to the flux equal to $E_{\text{non-constant}}A = (-4)(4^2)(4) = -256$ (in SI units). The face of the cube located at y = 2.00 m has the same area A (and this one "faces" the $-\hat{\text{j}}$ direction) and a contribution to the flux: $-E_{\text{non-constant}}A = -(-4)(2^2)(4) = 64$ (in SI units). Thus, the net flux is $\Phi = -256 + 64 = -192$ N·m/C^2. According to Gauss's law, we therefore have $q_{\text{enc}} = \varepsilon_{\text{o}}\Phi = -1.70 \times 10^{-9}$ C .

60. (a) The field maximum occurs at the outer surface:

$$E_{\text{max}} = \left(\frac{|q|}{4\pi\varepsilon_o r^2}\right)_{\text{at } r = R} = \frac{|q|}{4\pi\varepsilon_o R^2}$$

Applying Eq. 23-20, we have

$$E_{\text{internal}} = \frac{|q|}{4\pi\varepsilon_o R^3} r = \frac{1}{4} E_{\text{max}} \quad \Rightarrow \quad r = \frac{R}{4} = 0.25\, R\,.$$

(b) Outside sphere 2 we have

$$E_{\text{external}} = \frac{|q|}{4\pi\varepsilon_o r^2} = \frac{1}{4} E_{\text{max}} \quad \Rightarrow \quad r = 2.0R\,.$$

61. The initial field (evaluated "just outside the outer surface" which means it is evaluated at $r = 0.20$ m) is related to the charge q on the hollow conductor by Eq. 23-15. After the point charge Q is placed at the geometric center of the hollow conductor, the final field at that point is a combination of the initial and that due to Q (determined by Eq. 22-3).

(a) $q = 4\pi\varepsilon_o r^2 E_{\text{initial}} = +2.0 \times 10^{-9}$ C.

(b) $Q = 4\pi\varepsilon_o r^2(E_{\text{final}} - E_{\text{initial}}) = -1.2 \times 10^{-9}$ C.

(c) In order to cancel the field (due to Q) within the conducting material, there must be an amount of charge equal to $-Q$ distributed uniformly on the inner surface. Thus, the answer is $+1.2 \times 10^{-9}$ C.

(d) Since the total excess charge on the conductor is q and is located on the surfaces, then the outer surface charge must equal the total minus the inner surface charge. Thus, the answer is $2.0 \times 10^{-9}\text{ C} - 1.2 \times 10^{-9}\text{ C} = +0.80 \times 10^{-9}$ C.

62. Since the charge distribution is uniform, we can find the total charge q by multiplying ρ by the spherical volume ($\frac{4}{3}\pi r^3$) with $r = R =$ 0.050 m. This gives q = 1.68 nC.

(a) Applying Eq. 23-20 with r = 0.035 m, we have

$$E_{\text{internal}} = \frac{|q|}{4\pi\varepsilon_0 R^3} r = 4.2 \times 10^3 \text{ N/C}.$$

(b) Outside the sphere we have (with r = 0.080 m)

$$E_{\text{external}} = \frac{|q|}{4\pi\varepsilon_0 r^2} = 2.4 \times 10^3 \text{ N/C}.$$

63. (a) In order to have net charge –10 μC when –14 μC is known to be on the outer surface, then there must be +4.0 μC on the inner surface (since charges reside on the surfaces of a conductor in electrostatic situations).

(b) In order to cancel the electric field inside the conducting material, the contribution from the +4 μC on the inner surface must be canceled by that of the charged particle in the hollow. Thus, the particle's charge is –4.0 μC.

64. The field at the proton's location (but not caused by the proton) has magnitude E. The proton's charge is e. The ball's charge has magnitude q. Thus, as long as the proton is at $r \geq R$ then the force on the proton (caused by the ball) has magnitude

$$F = eE = e\left(\frac{q}{4\pi\varepsilon_o r^2}\right) = \frac{e\,q}{4\pi\varepsilon_o r^2}$$

where r is measured from the center of the ball (to the proton).This agrees with Coulomb's law from Chapter 22. We note that if $r = R$ then this expression becomes

$$F_R = \frac{e\,q}{4\pi\varepsilon_o R^2} .$$

(a) If we require $F = \frac{1}{2}F_R$, and solve for r, we obtain $r = \sqrt{2}\,R$. Since the problem asks for the measurement from the surface then the answer is $\sqrt{2}\,R - R = 0.41R$.

(b) Now we require $F_{\text{inside}} = \frac{1}{2}F_R$ where $F_{\text{inside}} = eE_{\text{inside}}$ and E_{inside} is given by Eq. 23-20. Thus,

$$e\left(\frac{q}{4\pi\varepsilon_o R^2}\right)r = \frac{1}{2}\,\frac{e\,q}{4\pi\varepsilon_o R^2} \qquad \Rightarrow \qquad r = \tfrac{1}{2}R = 0.50\,R .$$

65. (a) At $x = 0.040$ m, the net field has a rightward ($+x$) contribution (computed using Eq. 23-13) from the charge lying between $x = -0.050$ m and $x = 0.040$ m, and a leftward ($-x$) contribution (again computed using Eq. 23-13) from the charge in the region from $x = 0.040$ m to $x = 0.050$ m. Thus, since $\sigma = q/A = \rho V/A = \rho\Delta x$ in this situation, we have

$$|\vec{E}| = \frac{\rho(0.090\,\text{m})}{2\varepsilon_0} - \frac{\rho(0.010\,\text{m})}{2\varepsilon_0} = 5.4\,\text{N/C}.$$

(b) In this case, the field contributions from all layers of charge point rightward, and we obtain

$$|\vec{E}| = \frac{\rho(0.100\,\text{m})}{2\varepsilon_0} = 6.8\,\text{N/C}.$$

66. From Gauss's law, we have

$$\Phi = \frac{q_{\text{enclosed}}}{\varepsilon_o} = \frac{\sigma\,\pi r^2}{\varepsilon_o} = \frac{(8.0 \times 10^{-9}\ \text{C/m}^2)\ \pi\ (0.050\ \text{m})^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 7.1\ \text{N}\cdot\text{m}^2/\text{C}\ .$$

67. (a) For $r < R$, $E = 0$ (see Eq. 23-16).

(b) For r slightly greater than R,

$$E_R = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \approx \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-7}\ \text{C}\right)}{\left(0.250\text{m}\right)^2} = 2.88\times10^4\ \text{N/C}.$$

(c) For $r > R$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = E_R\left(\frac{R}{r}\right)^2 = \left(2.88\times10^4\ \text{N/C}\right)\left(\frac{0.250\ \text{m}}{3.00\ \text{m}}\right)^2 = 200\,\text{N/C}.$$

68. (a) There is no flux through the sides, so we have two contributions to the flux, one from the $x = 2$ end (with $\Phi_2 = +(2 + 2)(\pi (0.20)^2) = 0.50$ N·m^2/C) and one from the $x = 0$ end (with $\Phi_0 = -(2)(\pi (0.20)^2)$).

(b) By Gauss' law we have $q_{enc} = \varepsilon_0 (\Phi_2 + \Phi_0) = 2.2 \times 10^{-12}$ C.

69. (a) Outside the sphere, we use Eq. 23-15 and obtain $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 15.0\,\text{N/C}$.

(b) With $q = +6.00 \times 10^{-12}$ C, Eq. 23-20 leads to $E = 25.3\,\text{N/C}$.

70. Since the fields involved are uniform, the precise location of *P* is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward $(+\hat{\text{j}})$, and (from Eq. 23-13) its magnitude is

$$|\vec{E}| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0\times10^{-6}}{2\times8.85\times10^{-12}} = 5.65\times10^4 \text{ N/C}.$$

In unit-vector notation, we have $\vec{E} = (5.65\times10^4 \text{ N/C})\hat{\text{j}}$.

71. Let $\Phi_0 = 10^3\,\mathrm{N\cdot m^2/C}$. The net flux through the entire surface of the dice is given by

$$\Phi = \sum_{n=1}^{6}\Phi_n = \sum_{n=1}^{6}(-1)^n n\,\Phi_0 = \Phi_0(-1+2-3+4-5+6) = 3\Phi_0\,.$$

Thus, the net charge enclosed is

$$q = \varepsilon_0\Phi = 3\varepsilon_0\Phi_0 = 3\left(8.85\times10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)\left(10^3\,\mathrm{N\cdot m^2/C}\right) = 2.66\times10^{-8}\,\mathrm{C}.$$

72. (a) From Gauss' law,

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{encl}}}{r^3}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{\left(4\pi\rho r^3/3\right)\vec{r}}{r^3} = \frac{\rho\vec{r}}{3\varepsilon_0}.$$

(b) The charge distribution in this case is equivalent to that of a whole sphere of charge density ρ plus a smaller sphere of charge density $-\rho$ which fills the void. By superposition

$$\vec{E}(\vec{r}) = \frac{\rho\vec{r}}{3\varepsilon_0} + \frac{(-\rho)(\vec{r}-\vec{a})}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0}.$$

73. We choose a coordinate system whose origin is at the center of the flat base, such that the base is in the *xy* plane and the rest of the hemisphere is in the $z > 0$ half space.

(a) $\Phi = \pi R^2 \left(-\hat{\mathrm{k}}\right) \cdot E\hat{\mathrm{k}} = -\pi R^2 E = -\pi (0.0568 \text{ m})^2 (2.50 \text{ N/C}) = -0.0253 \text{ N} \cdot \text{m}^2/\text{C}.$

(b) Since the flux through the entire hemisphere is zero, the flux through the curved surface is $\vec{\Phi}_c = -\Phi_{\text{base}} = \pi R^2 E = 0.0253 \text{ N} \cdot \text{m}^2/\text{C}.$

74. (a) The direction of the electric field at P_1 is away from q_1 and its magnitude is

$$\left|\vec{E}\right| = \frac{q}{4\pi\varepsilon_0 r_1^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-7}\ \text{C})}{(0.015\,\text{m})^2} = 4.0\times10^6\ \text{N/C}.$$

(b) $\vec{E} = 0$, since P_2 is inside the metal.

75. The field due to a sheet of charge is given by Eq. 23-13. Both sheets are horizontal (parallel to the *xy* plane), producing vertical fields (parallel to the *z* axis). At points above the $z = 0$ sheet (sheet *A*), its field points upward (towards $+z$); at points above the $z = 2.0$ sheet (sheet *B*), its field does likewise. However, below the $z = 2.0$ sheet, its field is oriented downward.

(a) The magnitude of the net field in the region between the sheets is

$$|\vec{E}| = \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0} = 2.82\times10^2 \text{ N/C}.$$

(b) The magnitude of the net field at points above both sheets is

$$|\vec{E}| = \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0} = 6.21\times10^2 \text{ N/C}.$$

76. Since the fields involved are uniform, the precise location of *P* is not relevant. Since the sheets are oppositely charged (though not equally so), the field contributions are additive (since *P* is between them). Using Eq. 23-13, we obtain

$$\vec{E} = \frac{\sigma_1}{2\varepsilon_0} + \frac{3\sigma_1}{2\varepsilon_0} = \frac{2\sigma_1}{\varepsilon_0}$$

directed towards the negatively charged sheet. The multiple is 2.00.

77. We use Eqs. 23-15, 23-16 and the superposition principle.

(a) $E = 0$ in the region inside the shell.

(b) $E = q_a/4\pi\varepsilon_0 r^2$.

(c) $E = (q_a + q_b)/4\pi\varepsilon_0 r^2$.

(d) Since $E = 0$ for $r < a$ the charge on the inner surface of the inner shell is always zero. The charge on the outer surface of the inner shell is therefore q_a. Since $E = 0$ inside the metallic outer shell the net charge enclosed in a Gaussian surface that lies in between the inner and outer surfaces of the outer shell is zero. Thus the inner surface of the outer shell must carry a charge $-q_a$, leaving the charge on the outer surface of the outer shell to be $q_b + q_a$.

78. The net enclosed charge q is given by

$$q = \varepsilon_0 \Phi = \left(8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N} \cdot \mathrm{m}^2} \right) \left(-48 \ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C} \right) = -4.2 \times 10^{-10} \, \mathrm{C}.$$

79. (a) At A, the only field contribution is from the +5.00 pC particle in the hollow (this follows from Gauss' law — it is the only charge enclosed by a Gaussian spherical surface passing through point A, concentric with the shell). Thus, using k for $1/4\pi\varepsilon_0$, we have $|\vec{E}| = k(5.00\times10^{-12})/(0.5)^2 = 0.180$.

(b) The direction is radially outward.

(c) Point B is in the conducting material, where the field must be zero in any electrostatic situation.

(d) Point C is outside the sphere where the net charge at smaller values of radius is $(-3.00\text{ pC} + 5.00\text{ pC}) = 2.00\text{ pC}$. Therefore, we have

$$|\vec{E}| = k(2.00\times10^{-12})/(2)^2 = 4.50\times10^{-3}\text{ N/C}$$

directed radially outward.

80. We can express Eq. 23-17 in terms of the charge density ρ as follows:

$$E = \frac{q}{4\pi\varepsilon_o r^2} = \frac{\rho \frac{4}{3}\pi R^3}{4\pi\varepsilon_o r^2} = \frac{\rho R^3}{3\,\varepsilon_o r^2} .$$

Thus, at $r = 2R$, we have (when the ball is solid)

$$E_1 = \frac{\rho R^3}{3\,\varepsilon_o (2R)^2} = \frac{\rho}{12\,\varepsilon_o R} .$$

Now, with the hollow core of radius $R/2$, we have a similar field but without the contribution from those charges that would have been in that core:

$$E_{\text{new}} = E_1 - \left(\frac{\rho (R/2)^3}{3\,\varepsilon_o r^2}\right)_{\text{at } r = 2R} = \frac{\rho}{12\,\varepsilon_o R} - \frac{\rho}{96\,\varepsilon_o R} = \frac{7\rho}{96\,\varepsilon_o R}$$

which is equivalent to $\frac{7}{8}E_1$. Thus, the fraction is 7/8 = 0.875.

81. The proton is in uniform circular motion, with the electrical force of the sphere on the proton providing the centripetal force. According to Newton's second law, $F = mv^2/r$, where F is the magnitude of the force, v is the speed of the proton, and r is the radius of its orbit, essentially the same as the radius of the sphere. The magnitude of the force on the proton is $F = eq/4\pi\varepsilon_0 r^2$, where q is the magnitude of the charge on the sphere. Thus,

$$\frac{1}{4\pi\varepsilon_0}\frac{eq}{r^2} = \frac{mv^2}{r}$$

so

$$q = \frac{4\pi\varepsilon_0 mv^2 r}{e} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.00\times10^{5}\text{ m/s}\right)^2\left(0.0100\text{ m}\right)}{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-9}\text{ C}\right)} = 1.04\times10^{-9}\text{ C}.$$

The force must be inward, toward the center of the sphere, and since the proton is positively charged, the electric field must also be inward. The charge on the sphere is negative: $q = -1.04 \times 10^{-9}$ C.

82. We interpret the question as referring to the field *just* outside the sphere (that is, at locations roughly equal to the radius r of the sphere). Since the area of a sphere is $A = 4\pi r^2$ and the surface charge density is $\sigma = q/A$ (where we assume q is positive for brevity), then

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{q}{4\pi r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

which we recognize as the field of a point charge (see Eq. 22-3).

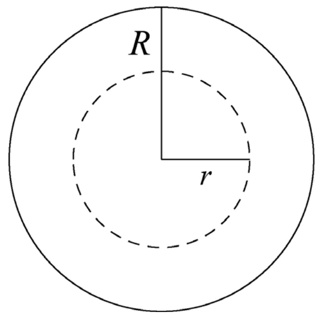
83. The field is radially outward and takes on equal magnitude-values over the surface of any sphere centered at the atom's center. We take the Gaussian surface to be such a sphere (of radius r). If E is the magnitude of the field, then the total flux through the Gaussian sphere is $\Phi = 4\pi r^2 E$. The charge enclosed by the Gaussian surface is the positive charge at the center of the atom plus that portion of the negative charge within the surface. Since the negative charge is uniformly distributed throughout the large sphere of radius R, we can compute the charge inside the Gaussian sphere using a ratio of volumes. That is, the negative charge inside is $-Zer^3/R^3$. Thus, the total charge enclosed is $Ze - Zer^3/R^3$ for $r \le R$. Gauss' law now leads to

$$4\pi\varepsilon_0 r^2 E = Ze\left(1 - \frac{r^3}{R^3}\right) \Rightarrow E = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right).$$

84. The electric field is radially outward from the central wire. We want to find its magnitude in the region between the wire and the cylinder as a function of the distance r from the wire. Since the magnitude of the field at the cylinder wall is known, we take the Gaussian surface to coincide with the wall. Thus, the Gaussian surface is a cylinder with radius R and length L, coaxial with the wire. Only the charge on the wire is actually enclosed by the Gaussian surface; we denote it by q. The area of the Gaussian surface is $2\pi RL$, and the flux through it is $\Phi = 2\pi RLE$. We assume there is no flux through the ends of the cylinder, so this Φ is the total flux. Gauss' law yields $q = 2\pi\varepsilon_0 RLE$. Thus,

$$q = 2\pi\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.014\ \text{m})(0.16\ \text{m})\ (2.9\times10^4\ \text{N/C}) = 3.6\times10^{-9}\ \text{C}.$$

85. (a) The diagram below shows a cross section (or, perhaps more appropriately, "end view") of the charged cylinder (solid circle).



Consider a Gaussian surface in the form of a cylinder with radius r and length ℓ, coaxial with the charged cylinder. An "end view" of the Gaussian surface is shown as a dotted circle. The charge enclosed by it is $q = \rho V = \pi r^2 \ell \rho$, where $V = \pi r^2 \ell$ is the volume of the cylinder.

If ρ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it. Thus, the total flux through the Gaussian cylinder is $\Phi = EA_{\text{cylinder}} = E(2\pi r\ell)$. Now, Gauss' law leads to

$$2\pi\varepsilon_0 r\ell E = \pi r^2 \ell\rho \Rightarrow E = \frac{\rho r}{2\varepsilon_0}.$$

(b) Next, we consider a cylindrical Gaussian surface of radius $r > R$. If the external field E_{ext} then the flux is $\Phi = 2\pi r\ell E_{\text{ext}}$. The charge enclosed is the total charge in a section of the charged cylinder with length ℓ. That is, $q = \pi R^2 \ell\rho$. In this case, Gauss' law yields

$$2\pi\varepsilon_0 r\ell E_{\text{ext}} = \pi R^2 \ell\rho \Rightarrow E_{\text{ext}} = \frac{R^2\rho}{2\varepsilon_0 r}.$$

86. (a) The mass flux is $wd\rho v$ = (3.22 m) (1.04 m) (1000 kg/m^3) (0.207 m/s) = 693 kg/s.

(b) Since water flows only through area wd, the flux through the larger area is still 693 kg/s.

(c) Now the mass flux is $(wd/2)\rho v$ = (693 kg/s)/2 = 347 kg/s.

(d) Since the water flows through an area (wd/2), the flux is 347 kg/s.

(e) Now the flux is $(wd\cos\theta)\,\rho v = (693\,\text{kg/s})(\cos 34°) = 575\ \text{kg/s}$.

87. (a) We note that the symbol "e" stands for the elementary charge in the manipulations below. From

$$-e = \int_0^\infty \rho(r)\, 4\pi r^2 dr = \int_0^\infty A \exp(-2r/a_0)\, 4\pi r^2\, dr = \pi a_0^3 A$$

we get $A = -e/\pi a_0^3$.

(b) The magnitude of the field is

$$E = \frac{q_{\text{encl}}}{4\pi\varepsilon_0 a_0^2} = \frac{1}{4\pi\varepsilon_0 a_0^2}\left(e + \int_0^{a_0} \rho(r) 4\pi r^2 dr\right) = \frac{e}{4\pi\varepsilon_0 a_0^2}\left(1 - \frac{4}{a_0^3}\int_0^{a_0} \exp(-2r/a_0)\, r^2 dr\right)$$
$$= \frac{5\, e \exp(-2)}{4\pi\varepsilon_0 a_0^2}.$$

We note that $\vec{E}$ points radially outward.