1. (a) An Ampere is a Coulomb per second, so

$$84\ \text{A}\cdot\text{h} = \left(84\,\frac{\text{C}\cdot\text{h}}{\text{s}}\right)\left(3600\,\frac{\text{s}}{\text{h}}\right) = 3.0\times10^{5}\ \text{C}.$$

(b) The change in potential energy is $\Delta U = q\Delta V = (3.0\times10^{5}\ \text{C})(12\ \text{V}) = 3.6\times10^{6}$ J.

2. The magnitude is $\Delta U = e\Delta V = 1.2 \times 10^9$ eV = 1.2 GeV.

3. The electric field produced by an infinite sheet of charge has magnitude $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. The field is normal to the sheet and is uniform. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by $\Delta V = E\Delta x = (\sigma/2\varepsilon_0)\Delta x$. Thus,

$$\Delta x = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(50\ \mathrm{V})}{0.10\times10^{-6}\ \mathrm{C/m^2}} = 8.8\times10^{-3}\,\mathrm{m}.$$

4. (a) $V_B - V_A = \Delta U/q = -W/(-e) = -(3.94 \times 10^{-19}\ \text{J})/(-1.60 \times 10^{-19}\ \text{C}) = 2.46\ \text{V}$.

(b) $V_C - V_A = V_B - V_A = 2.46\ \text{V}$.

(c) $V_C - V_B = 0$ (Since C and B are on the same equipotential line).

5. (a) $E = F/e = (3.9\times10^{-15}\ \text{N})/(1.60\times10^{-19}\ \text{C}) = 2.4\times10^{4}\ \text{N/C}$.

(b) $\Delta V = E\Delta s = (2.4\times10^{4}\ \text{N/C})(0.12\ \text{m}) = 2.9\times10^{3}\ \text{V}$.

6. (a) By Eq. 24-18, the change in potential is the negative of the "area" under the curve. Thus, using the area-of-a-triangle formula, we have

$$V - 10 = -\int_0^{x=2} \vec{E} \cdot d\vec{s} = \frac{1}{2}(2)(20)$$

which yields $V = 30$ V.

(b) For any region within $0 < x < 3\,\text{m}, -\int \vec{E} \cdot d\vec{s}$ is positive, but for any region for which $x > 3$ m it is negative. Therefore, $V = V_{max}$ occurs at $x = 3$ m.

$$V - 10 = -\int_0^{x=3} \vec{E} \cdot d\vec{s} = \frac{1}{2}(3)(20)$$

which yields $V_{max} = 40$ V.

(c) In view of our result in part (b), we see that now (to find $V = 0$) we are looking for some $X > 3$ m such that the "area" from $x = 3$ m to $x = X$ is 40 V. Using the formula for a triangle ($3 < x < 4$) and a rectangle ($4 < x < X$), we require

$$\frac{1}{2}(1)(20) + (X - 4)(20) = 40.$$

Therefore, $X = 5.5$ m.

7. (a) The work done by the electric field is (in SI units)

$$W = \int_i^f q_0 \vec{E} \cdot d\vec{s} = \frac{q_0 \sigma}{2\varepsilon_0} \int_0^d dz = \frac{q_0 \sigma d}{2\varepsilon_0} = \frac{(1.60\times10^{-19})(5.80\times10^{-12})(0.0356)}{2(8.85\times10^{-12})} = 1.87\times10^{-21}\ \text{J}.$$

(b) Since $V - V_0 = -W/q_0 = -\sigma z/2\varepsilon_0$, with V_0 set to be zero on the sheet, the electric potential at P is (in SI units)

$$V = -\frac{\sigma z}{2\varepsilon_0} = -\frac{(5.80\times10^{-12})(0.0356)}{2(8.85\times10^{-12})} = -1.17\times10^{-2}\ \text{V}.$$

8. We connect A to the origin with a line along the y axis, along which there is no change of potential (Eq. 24-18: $\int \vec{E} \cdot d\vec{s} = 0$). Then, we connect the origin to B with a line along the x axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4} \vec{E} \cdot d\vec{s} = -4.00\int_0^4 x\,dx = -4.00\left(\frac{4^2}{2}\right)$$

which yields $V_B - V_A = -32.0$ V.

9. (a) The potential as a function of *r* is (in SI units)

$$V(r) = V(0) - \int_0^r E(r)dr = 0 - \int_0^r \frac{qr}{4\pi\varepsilon_0 R^3} dr = -\frac{qr^2}{8\pi\varepsilon_0 R^3}$$

$$= -\frac{(8.99\times10^9)(3.50\times10^{-15})(0.0145)^2}{2(0.0231)^3} = -2.68\times10^{-4}\text{ V.}$$

(b) Since $\Delta V = V(0) - V(R) = q/8\pi\varepsilon_0 R$, we have (in SI units)

$$V(R) = -\frac{q}{8\pi\varepsilon_0 R} = -\frac{(8.99\times10^9)(3.50\times10^{-15})}{2(0.0231)} = -6.81\times10^{-4}\text{ V.}$$

10. The charge is

$$q = 4\pi\varepsilon_0 RV = \frac{(10\,\text{m})(-1.0\,\text{V})}{8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2} = -1.1\times10^{-9}\,\text{C}.$$

11. (a) The charge on the sphere is

$$q = 4\pi\varepsilon_0 VR = \frac{(200\text{ V})(0.15\text{ m})}{8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = 3.3\times 10^{-9}\text{ C}.$$

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma = \frac{q}{4\pi R^2} = \frac{3.3\times 10^{-9}\text{ C}}{4\pi(0.15\text{ m})^2} = 1.2\times 10^{-8}\text{ C}/\text{m}^2.$$

12. (a) The potential difference is

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0 r_A} - \frac{q}{4\pi\varepsilon_0 r_B} = \left(1.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{1}{2.0\ \text{m}} - \frac{1}{1.0\ \text{m}}\right)$$

$$= -4.5\times10^3\ \text{V}.$$

(b) Since $V(r)$ depends only on the magnitude of $\vec{r}$, the result is unchanged.

13. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$. With $d = 24.0$ cm, we have $x = 6.00$ cm.

(b) Similarly, for $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. With $d = 24.0$ cm, we have $x = -12.0$ cm.

14. Since according to the problem statement there is a point in between the two charges on the x axis where the net electric field is zero, the fields at that point due to q_1 and q_2 must be directed opposite to each other. This means that q_1 and q_2 must have the same sign (i.e., either both are positive or both negative). Thus, the potentials due to either of them must be of the same sign. Therefore, the net electric potential cannot possibly be zero anywhere except at infinity.

15. A charge $-5q$ is a distance $2d$ from P, a charge $-5q$ is a distance d from P, and two charges $+5q$ are each a distance d from P, so the electric potential at P is (in SI units)

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{2d} - \frac{1}{d} + \frac{1}{d} + \frac{1}{d}\right] = \frac{q}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^{9})(5.00\times10^{-15})}{2(4.00\times10^{-2})} = 5.62\times10^{-4}\,\text{V}.$$

The zero of the electric potential was taken to be at infinity.

16. In applying Eq. 24-27, we are assuming $V \to 0$ as $r \to \infty$. All corner particles are equidistant from the center, and since their total charge is

$$2q_1 - 3q_1 + 2\,q_1 - q_1 = 0,$$

then their contribution to Eq. 24-27 vanishes. The net potential is due, then, to the two $+4q_2$ particles, each of which is a distance of $a/2$ from the center. In SI units, it is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} + \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} = \frac{16q_2}{4\pi\varepsilon_0 a} = \frac{16(8.99\times10^9)(6.00\times10^{-12})}{0.39} = 2.21 \text{ V}.$$

17. (a) The electric potential V at the surface of the drop, the charge q on the drop, and the radius R of the drop are related by $V = q/4\pi\varepsilon_0 R$. Thus

$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(30\times10^{-12}\ \text{C}\right)}{500\ \text{V}} = 5.4\times10^{-4}\ \text{m}.$$

(b) After the drops combine the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of original drop: $q' = 2q$. Thus,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500\ \text{V}) \approx 790\ \text{V}.$$

18. When the charge q_2 is infinitely far away, the potential at the origin is due only to the charge q_1 :

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 d} = 5.76 \times 10^{-7}\ \text{V}.$$

Thus, $q_1/d = 6.41 \times 10^{-17}$ C/m. Next, we note that when q_2 is located at $x = 0.080$ m, the net potential vanishes ($V_1 + V_2 = 0$). Therefore,

$$0 = \frac{kq_2}{0.08\ \text{m}} + \frac{kq_1}{d}$$

Thus, we find $q_2 = -(q_1/d)(0.08\ \text{m}) = -5.13 \times 10^{-18}\ \text{C} = -32\,e$.

19. We use Eq. 24-20:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.47\times3.34\times10^{-30}\ \text{C}\cdot\text{m}\right)}{\left(52.0\times10^{-9}\ \text{m}\right)^2} = 1.63\times10^{-5}\ \text{V}.$$

20. From Eq. 24-30 and Eq. 24-14, we have (for $\theta_i = 0º$)

$$W_a = q\Delta V = e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2} - \frac{p\cos\theta_i}{4\pi\varepsilon_0 r^2}\right) = \frac{e\,p}{4\pi\varepsilon_0 r^2}(\cos\theta - 1)\ .$$

where $r = 20 \times 10^{-9}$ m. For $\theta = 180º$ the graph indicates $W_a = -4.0 \times 10^{-30}$ J, from which we can determine p. The magnitude of the dipole moment is therefore 5.6×10^{-37} C·m.

21. (a) From Eq. 24-35, in SI units,

$$V = 2\frac{\lambda}{4\pi\varepsilon_0}\ \ln\left[\frac{L/2+\sqrt{(L^2/4)+d^2}}{d}\right]$$

$$= 2(8.99\times10^{9})(3.68\times10^{-12})\ln\left[\frac{(0.06/2)+\sqrt{(0.06)^2/4+(0.08)^2}}{0.08}\right]$$

$$= 2.43\times10^{-2}\ \text{V}.$$

(b) The potential at P is $V = 0$ due to superposition.

22. The potential is (in SI units)

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_{\text{rod}} \frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\varepsilon_0 R} = -\frac{(8.99\times10^9)(25.6\times10^{-12})}{3.71\times10^{-2}} = -6.20 \text{ V}.$$

We note that the result is exactly what one would expect for a point-charge $-Q$ at a distance R. This "coincidence" is due, in part, to the fact that V is a scalar quantity.

23. (a) All the charge is the same distance R from C, so the electric potential at C is (in SI units)

$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{R}-\frac{6Q_1}{R}\right]=-\frac{5Q_1}{4\pi\varepsilon_0 R}=-\frac{5(8.99\times10^9)(4.20\times10^{-12})}{8.20\times10^{-2}}=-2.30\text{ V},$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from P. That distance is $\sqrt{R^2+D^2}$, so the electric potential at P is (in SI units)

$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{R^2+D^2}}-\frac{6Q_1}{\sqrt{R^2+D^2}}\right]=-\frac{5Q_1}{4\pi\varepsilon_0\sqrt{R^2+D^2}}$$

$$=-\frac{5(8.99\times10^9)(4.20\times10^{-12})}{\sqrt{(8.20\times10^{-2})^2+(6.71\times10^{-2})^2}}=-1.78\text{ V}.$$

24. Since the charge distribution on the arc is equidistant from the point where V is evaluated, its contribution is identical to that of a point charge at that distance. We assume $V \to 0$ as $r \to \infty$ and apply Eq. 24-27:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+4Q_1}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R}$$

$$= \frac{(8.99\times10^9)(7.21\times10^{-12})}{2.00} = 3.24\times10^{-2}\ \text{V}.$$

25. The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P, so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk. We consider a ring of charge with radius r and (infinitesimal) width dr. Its area is $2\pi r\,dr$ and it contains charge $dq = 2\pi\sigma r\,dr$. All the charge in it is a distance $\sqrt{r^2+D^2}$ from P, so the potential it produces at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma r dr}{\sqrt{r^2+D^2}} = \frac{\sigma r dr}{2\varepsilon_0\sqrt{r^2+D^2}}.$$

The total potential at P is

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r dr}{\sqrt{r^2+D^2}} = \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+D^2}\,\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right].$$

The potential V_{sq} at P due to a single quadrant is (in SI units)

$$V_{sq} = \frac{V}{4} = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right] = \frac{(7.73\times10^{-15})}{8(8.85\times10^{-12})}\left[\sqrt{(0.640)^2+(0.259)^2}-0.259\right]$$
$$= 4.71\times10^{-5}\text{ V}.$$

26. The dipole potential is given by Eq. 24-30 (with θ = 90º in this case)

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = 0$$

since cos(90º) = 0 . The potential due to the short arc is $q_1/4\pi\varepsilon_0 r_1$ and that caused by the long arc is $q_2/4\pi\varepsilon_0 r_2$. Since q_1 = +2 µC, r_1 = 4.0 cm, q_2 = –3 µC, and r_2 = 6.0 cm, the potentials of the arcs cancel. The result is zero.

27. Letting *d* denote 0.010 m, we have (in SI units)

$$V = \frac{Q_1}{4\pi\varepsilon_0 d} + \frac{3Q_1}{8\pi\varepsilon_0 d} - \frac{3Q_1}{16\pi\varepsilon_0 d} = \frac{Q_1}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^9)(30\times10^{-9})}{2(0.01)} = 1.3\times10^4 \text{ V}.$$

28. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain (in SI units):

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right)$$

$$= \frac{(8.99\times10^{9})(56.1\times10^{-15})}{0.12}\ln\left(1+\frac{0.12}{0.025}\right) = 7.39\times10^{-3}\ \text{V}.$$

29. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx = cx\, dx$. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain (in SI units):

$$V = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x\,dx}{d+x} = \frac{c}{4\pi\varepsilon_0}[x - d\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\varepsilon_0}\left[L - d\ln\left(1+\frac{L}{d}\right)\right]$$
$$= (8.99\times10^9)(28.9\times10^{-12})\left[0.12 - (0.03)\ln\left(1+\frac{0.12}{0.03}\right)\right] = 1.86\times10^{-2}\ \text{V}.$$

30. The magnitude of the electric field is given by

$$|E| = \left| -\frac{\Delta V}{\Delta x} \right| = \frac{2(5.0\text{V})}{0.015\text{m}} = 6.7\times10^{2}\ \text{V/m}.$$

At any point in the region between the plates, $\vec{E}$ points away from the positively charged plate, directly towards the negatively charged one.

31. We use Eq. 24-41:

$$E_x(x,y) = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = -2(2.0\text{V}/\text{m}^2)x;$$
$$E_y(x,y) = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = 2(3.0\text{V}/\text{m}^2)y.$$

We evaluate at x = 3.0 m and y = 2.0 m to obtain

$$\vec{E} = (-12\text{ V/m})\hat{\text{i}} + (12\text{ V/m})\hat{\text{j}}.$$

32. We use Eq. 24-41. This is an ordinary derivative since the potential is a function of only one variable.

$$\vec{E} = -\left(\frac{dV}{dx}\right)\hat{\mathrm{i}} = -\frac{d}{dx}(1500x^2)\hat{\mathrm{i}} = (-3000x)\hat{\mathrm{i}}$$
$$= (-3000\,\mathrm{V/m^2})(0.0130\,\mathrm{m})\hat{\mathrm{i}} = (-39\,\mathrm{V/m})\hat{\mathrm{i}}.$$

(a) Thus, the magnitude of the electric field is E = 39 V/m.

(b) The direction of $\vec{E}$ is $-\hat{\mathrm{i}}$, or toward plate 1.

33. We apply Eq. 24-41:

$$E_x = -\frac{\partial V}{\partial x} = -2.00 yz^2$$
$$E_y = -\frac{\partial V}{\partial y} = -2.00 xz^2$$
$$E_z = -\frac{\partial V}{\partial z} = -4.00 xyz$$

which, at $(x, y, z) = (3.00, –2.00, 4.00)$, gives $(E_x, E_y, E_z) = (64.0, –96.0, 96.0)$ in SI units. The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 150\,\text{V/m} = 150\,\text{N/C}.$$

34. (a) According to the result of problem 28, the electric potential at a point with coordinate x is given by

$$V=\frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x-L}{x}\right).$$

At $x = -d$ we obtain (in SI units)

$$V=\frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{d+L}{d}\right)=\frac{(8.99\times10^9)(43.6\times10^{-15})}{0.135}\ln\left(1+\frac{0.135}{d}\right)$$
$$=(2.90\times10^{-3}\text{ V})\ln\left(1+\frac{0.135}{d}\right).$$

(b) We differentiate the potential with respect to x to find the x component of the electric field (in SI units):

$$E_x=-\frac{\partial V}{\partial x}=-\frac{Q}{4\pi\varepsilon_0 L}\frac{\partial}{\partial x}\ln\left(\frac{x-L}{x}\right)=-\frac{Q}{4\pi\varepsilon_0 L}\frac{x}{x-L}\left(\frac{1}{x}-\frac{x-L}{x^2}\right)=-\frac{Q}{4\pi\varepsilon_0 x(x-L)}$$
$$=-\frac{(8.99\times10^9)(43.6\times10^{-15})}{x(x+0.135)}=-\frac{(3.92\times10^{-4})}{x(x+0.135)},$$

or

$$|E_x|=\frac{(3.92\times10^{-4})}{x(x+0.135)}.$$

(c) Since $E_x<0$, its direction relative to the positive x axis is 180°.

(d) At $x = -d$ we obtain (in SI units)

$$|E_x|=\frac{(3.92\times10^{-4})}{(0.0620)(0.0620+0.135)}=0.0321\text{ N/C}.$$

(e) Consider two points an equal infinitesimal distance on either side of P_1, along a line that is perpendicular to the x axis. The difference in the electric potential divided by their separation gives the transverse component of the electric field. Since the two points are situated symmetrically with respect to the rod, their potentials are the same and the potential difference is zero. Thus, the transverse component of the electric field E_y is zero.

35. The electric field (along some axis) is the (negative of the) derivative of the potential V with respect to the corresponding coordinate. In this case, the derivatives can be read off of the graphs as slopes (since the graphs are of straight lines). Thus,

$$E_x = -\frac{dV}{dx} = -\left(\frac{-500\ \text{V}}{0.20\ \text{m}}\right) = 2500\ \text{V/m} = 2500\ \text{N/C}$$

$$E_y = -\frac{dV}{dy} = -\left(\frac{300\ \text{V}}{0.30\ \text{m}}\right) = -1000\ \text{V/m} = -1000\ \text{N/C}\ .$$

These components imply the electric field has a magnitude of 2693 N/C and a direction of –21.8º (with respect to the positive x axis). The force on the electron is given by $\vec{F} = q\vec{E}$ where $q = -e$. The minus sign associated with the value of q has the implication that $\vec{F}$ points in the opposite direction from $\vec{E}$ (which is to say that its angle is found by adding 180º to that of $\vec{E}$). With $e = 1.60 \times 10^{-19}$ C, we obtain

$$\vec{F} = (-1.60\times10^{-19}\,\text{C})[(2500\ \text{N/C})\hat{\text{i}} - (1000\ \text{N/C})\hat{\text{j}}] = (-4.0\times10^{-16}\,\text{N})\hat{\text{i}} + (1.60\times10^{-16}\ \text{N})\hat{\text{j}}\ .$$

36. (a) Consider an infinitesimal segment of the rod from x to $x + dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}dx.$$

Thus, (in SI units)

$$\begin{aligned} V &= \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}dx = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2}-y\right) \\ &= (8.99\times10^9)(49.9\times10^{-12})\left(\sqrt{(0.100)^2+(0.0356)^2}-0.0356\right) \\ &= 3.16\times10^{-2}\ \text{V}. \end{aligned}$$

(b) The y component of the field there is

$$\begin{aligned} E_y &= -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}\left(\sqrt{L^2+y^2}-y\right) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right). \\ &= (8.99\times10^9)(49.9\times10^{-12})\left(1-\frac{0.0356}{\sqrt{(0.100)^2+(0.0356)^2}}\right) \\ &= 0.298\ \text{N/C}. \end{aligned}$$

(c) We obtained above the value of the potential at any point P strictly on the y-axis. In order to obtain $E_x(x, y)$ we need to first calculate $V(x, y)$. That is, we must find the potential for an arbitrary point located at (x, y). Then $E_x(x, y)$ can be obtained from $E_x(x,y) = -\partial V(x,y)/\partial x$.

37. We choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$U_f = \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right) = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right).$$

Thus the amount of work required to set up the system is given by (in SI units)

$$W = \Delta U = U_f - U_i = U_f = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right) = \frac{2(8.99\times10^9)(2.30\times10^{-12})^2}{(0.640)}\left(\frac{1}{\sqrt{2}}-2\right)$$
$$= -1.92\times10^{-13}\,\text{J}.$$

38. The work done must equal the change in the electric potential energy. From Eq. 24-14 and Eq. 24-26, we find (with $r = 0.020$ m)

$$W = \frac{(3e)\,(7e)}{4\pi\varepsilon_0\, r} = 2.1 \times 10^{-25}\ \text{J} .$$

39. (a) We use Eq. 24-43 with $q_1 = q_2 = -e$ and $r = 2.00$ nm:

$$U = k\frac{q_1 q_2}{r} = k\frac{e^2}{r} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\ \text{C})^2}{2.00\times10^{-9}\ \text{m}} = 1.15\times10^{-19}\ \text{J}.$$

(b) Since $U > 0$ and $U \propto r^{-1}$ the potential energy U decreases as r increases.

40. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 Q}{2d} + \frac{q_2 Q}{d}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 Q}{2d} + \frac{(-q_1/2)Q}{d}\right] = 0.$$

41. (a) Let $\ell = 0.15\,\text{m}$ be the length of the rectangle and $w = 0.050$ m be its width. Charge q_1 is a distance ℓ from point A and charge q_2 is a distance w, so the electric potential at A is

$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{\ell} + \frac{q_2}{w}\right] = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left[\frac{-5.0\times10^{-6}\,\text{C}}{0.15\,\text{m}} + \frac{2.0\times10^{-6}\,\text{C}}{0.050\text{ m}}\right]$$
$$= 6.0\times10^4\,\text{V}.$$

(b) Charge q_1 is a distance w from point b and charge q_2 is a distance ℓ, so the electric potential at B is

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{w} + \frac{q_2}{\ell}\right] = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left[\frac{-5.0\times10^{-6}\,\text{C}}{0.050\text{ m}} + \frac{2.0\times10^{-6}\,\text{C}}{0.15\,\text{m}}\right]$$
$$= -7.8\times10^5\,\text{V}.$$

(c) Since the kinetic energy is zero at the beginning and end of the trip, the work done by an external agent equals the change in the potential energy of the system. The potential energy is the product of the charge q_3 and the electric potential. If U_A is the potential energy when q_3 is at A and U_B is the potential energy when q_3 is at B, then the work done in moving the charge from B to A is

$$W = U_A - U_B = q_3(V_A - V_B) = (3.0\times10^{-6}\text{ C})(6.0\times10^4\text{ V} + 7.8\times10^5\text{ V}) = 2.5\text{ J}.$$

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter which path is used.

42. Let r = 1.5 m, x = 3.0 m, q_1 = –9.0 nC, and q_2 = –6.0 pC. The work done by an external agent is given by

$$W = \Delta U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + x^2}}\right)$$

$$= \left(-9.0\times10^{-9}\ \text{C}\right)\left(-6.0\times10^{-12}\,\text{C}\right)\left(8.99\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\cdot\left[\frac{1}{1.5\ \text{m}} - \frac{1}{\sqrt{(1.5\ \text{m})^2 + (3.0\ \text{m})^2}}\right]$$

$$= 1.8\times10^{-10}\,\text{J}.$$

43. We use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\varepsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\varepsilon_0 r_2$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the final speed of the particle. Conservation of energy yields

$$\frac{q^2}{4\pi\varepsilon_0 r_1} = \frac{q^2}{4\pi\varepsilon_0 r_2} + \frac{1}{2}mv^2 .$$

The solution for v is

$$v = \sqrt{\frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \sqrt{\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2)(3.1\times10^{-6}\,\mathrm{C})^2}{20\times10^{-6}\,\mathrm{kg}}\left(\frac{1}{0.90\times10^{-3}\,\mathrm{m}} - \frac{1}{2.5\times10^{-3}\,\mathrm{m}}\right)}$$

$$= 2.5\times10^3\ \mathrm{m/s}.$$

44. The change in electric potential energy of the electron-shell system as the electron starts from its initial position and just reaches the shell is $\Delta U = (-e)(-V) = eV$. Thus from $\Delta U = K = \frac{1}{2} m_e v_i^2$ we find the initial electron speed to be (in SI units)

$$v_i = \sqrt{\frac{2\Delta U}{m_e}} = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.6\times10^{-19})(125)}{9.11\times10^{-31}}} = 6.63\times10^{6} \text{ m/s.}$$

45. We use conservation of energy, taking the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then $U_f = 2e^2/4\pi\varepsilon_0 d$, where d is half the distance between the fixed electrons. The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, where m is the mass of an electron and v is the initial speed of the moving electron. The final kinetic energy is zero. Thus $K_i = U_f$ or $\frac{1}{2}mv^2 = 2e^2/4\pi\varepsilon_0 d$. Hence

$$v = \sqrt{\frac{4e^2}{4\pi\varepsilon_0 dm}} = \sqrt{\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(0.010\ \text{m})\left(9.11\times10^{-31}\ \text{kg}\right)}} = 3.2\times10^2\ \text{m/s}.$$

46. (a) The electric field between the plates is leftward in Fig, 24-50 since it points towards lower values of potential. The force (associated with the field, by Eq. 23-28) is evidently leftward, from the problem description (indicating deceleration of the rightward moving particle), so that $q > 0$ (ensuring that $\vec{F}$ is parallel to $\vec{E}$); it is a proton.

(b) We use conservation of energy:

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2} m_p v_0^2 + qV_1 = \frac{1}{2} m_p v^2 + qV_2 \ .$$

Using $q = +1.6 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ kg, $v_0 = 90 \times 10^3$ m/s, $V_1 = -70$ V and $V_2 = -50$ V, we obtain the final speed $v = 6.53 \times 10^4$ m/s. We note that the value of d is not used in the solution.

47. Let the distance in question be r. The initial kinetic energy of the electron is $K_i = \frac{1}{2} m_e v_i^2$, where $v_i = 3.2 \times 10^5$ m/s. As the speed doubles, K becomes $4K_i$. Thus

$$\Delta U = \frac{-e^2}{4\pi\varepsilon_0 r} = -\Delta K = -(4K_i - K_i) = -3K_i = -\frac{3}{2} m_e v_i^2,$$

or

$$r = \frac{2e^2}{3(4\pi\varepsilon_0) m_e v_i^2} = \frac{2\left(1.6\times10^{-19}\ \text{C}\right)^2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}{3\left(9.11\times10^{-19}\ \text{kg}\right)\left(3.2\times10^5\ \text{m/s}\right)^2} = 1.6\times10^{-9}\,\text{m}.$$

48. When particle 3 is at $x = 0.10$ m, the total potential energy vanishes. Using Eq. 24-43, we have (with meters understood at the length unit)

$$0 = \frac{q_1 q_2}{4\pi\varepsilon_o d} + \frac{q_1 q_3}{4\pi\varepsilon_o (d + 0.10)} + \frac{q_3 q_2}{4\pi\varepsilon_o (0.10)} .$$

This leads to

$$q_3 \left(\frac{q_1}{(d + 0.10)} + \frac{q_2}{0.10} \right) = -\frac{q_1 q_2}{d}$$

which yields $q_3 = -5.7\ \mu C$.

49. We apply conservation of energy for particle 3 (with $q' = -15 \times 10^{-6}$ C):

$$K_0 + U_0 = K_f + U_f$$

where (letting $x = \pm 3$ m and $q_1 = q_2 = 50 \times 10^{-6}$ C $= q$)

$$U = \frac{q_1 q'}{4\pi\varepsilon_o\sqrt{x^2+y^2}} + \frac{q_2 q'}{4\pi\varepsilon_o\sqrt{x^2+y^2}} = \frac{q\,q'}{2\pi\varepsilon_o\sqrt{x^2+y^2}} \ .$$

(a) We solve for K_f (with $y_0 = 4$ m):

$$K_f = K_0 + U_0 - U_f = 1.2\text{ J} + \frac{q\,q'}{2\pi\varepsilon_o} + \left(\frac{1}{\sqrt{x^2+y_0^2}} - \frac{1}{|x|}\right) = 3.0\text{ J} \ .$$

(b) We set $K_f = 0$ and solve for y (choosing the negative root, as indicated in the problem statement):

$$K_0 + U_0 = U_f \Rightarrow \quad 1.2\text{ J} + \frac{q\,q'}{2\pi\varepsilon_o\sqrt{x^2+y_0^2}} = \frac{q\,q'}{2\pi\varepsilon_o\sqrt{x^2+y^2}}$$

This yields $y = -8.5$ m.

50. From Eq. 24-30 and Eq. 24-7, we have (for $\theta = 180°$)

$$U = qV = -e\left(\frac{p\cos\theta}{4\pi\varepsilon_o r^2}\right) = \frac{e\,p}{4\pi\varepsilon_o r^2}$$

where r = 0.020 m. Appealing to energy conservation, we set this expression equal to 100 eV and solve for p. The magnitude of the dipole moment is therefore 4.5×10^{-12} C·m.

51. (a) Using $U = qV$ we can "translate" the graph of voltage into a potential energy graph (in eV units). From the information in the problem, we can calculate its kinetic energy (which is its total energy at $x = 0$) in those units: $K_i = 284$ eV. This is less than the "height" of the potential energy "barrier" (500 eV high once we've translated the graph as indicated above). Thus, it must reach a turning point and then reverse its motion.

(b) Its final velocity, then, is in the negative x direction with a magnitude equal to that of its initial velocity. That is, its speed (upon leaving this region) is 1.0×10^7 m/s.

52. (a) The work done results in a potential energy gain:

$$W = q\,\Delta V = (-e)\left(\frac{Q}{4\pi\varepsilon_0 R}\right) = +2.16\times 10^{-13}\text{ J}.$$

With R = 0.0800 m, we find $Q = -1.20\times 10^{-5}$ C.

(b) The work is the same, so the increase in the potential energy is $\Delta U = +2.16\times 10^{-13}$ J.

53. If the electric potential is zero at infinity, then the potential at the surface of the sphere is given by $V = q/4\pi\varepsilon_0 r$, where q is the charge on the sphere and r is its radius. Thus

$$q = 4\pi\varepsilon_0 rV = \frac{(0.15\text{ m})(1500\text{ V})}{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.5\times10^{-8}\,\text{C}.$$

54. Since the electric potential throughout the entire conductor is a constant, the electric potential at its center is also +400 V.

55. (a) The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ (= 1.0 m) from the center of each sphere, so the potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\varepsilon_0\, d/2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.0\times10^{-8}\,\text{C} - 3.0\times10^{-8}\,\text{C}\right)}{1.0\ \text{m}} = -1.8\times10^2\,\text{V}.$$

(b) The distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{1.0\times10^{-8}\,\text{C}}{0.030\,\text{m}} - \frac{3.0\times10^{-8}\,\text{C}}{2.0\,\text{m} - 0.030\,\text{m}}\right] = 2.9\times10^3\,\text{V}.$$

(c) The potential at the surface of sphere 2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{1.0\times10^{-8}\,\text{C}}{2.0\,\text{m} - 0.030\,\text{m}} - \frac{3.0\times10^{-8}\,\text{C}}{0.030\,\text{m}}\right] = -8.9\times10^3\,\text{V}.$$

56. (a) Since the two conductors are connected V_1 and V_2 must be equal to each other.

Let $V_1 = q_1/4\pi\varepsilon_0 R_1 = V_2 = q_2/4\pi\varepsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. We solve for q_1 and q_2: $q_1 = q/3$, $q_2 = 2q/3$, or

(b) $q_1/q = 1/3 = 0.333$,

(c) and $q_2/q = 2/3 = 0.667$.

(d) The ratio of surface charge densities is

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \left(\frac{q_1}{q_2}\right)\left(\frac{R_2}{R_1}\right)^2 = 2.00.$$

57. (a) The magnitude of the electric field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\left(3.0\times10^{-8}\,\text{C}\right)\left(8.99\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}{\left(0.15\,\text{m}\right)^2} = 1.2\times10^{4}\ \text{N/C}.$$

(b) $V = RE = (0.15\ \text{m})(1.2 \times 10^4\ \text{N/C}) = 1.8 \times 10^3\ \text{V}$.

(c) Let the distance be x. Then

$$\Delta V = V(x) - V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R+x} - \frac{1}{R}\right) = -500\,\text{V},$$

which gives

$$x = \frac{R\Delta V}{-V - \Delta V} = \frac{(0.15\,\text{m})(-500\,\text{V})}{-1800\,\text{V} + 500\,\text{V}} = 5.8\times10^{-2}\,\text{m}.$$

58. Since the charge distribution is spherically symmetric we may write

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{encl}}}{r},$$

where q_{encl} is the charge enclosed in a sphere of radius r centered at the origin.

(a) For $r = 4.00$ m, $R_2 = 1.00$ m and $R_1 = 0.500$ m, with $r > R_2 > R_1$ we have (in SI units)

$$E(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9)(2.00\times10^{-6} + 1.00\times10^{-6})}{(4.00)^2} = 1.69\times10^3 \text{ V/m}.$$

(b) For $R_2 > r = 0.700$ m $> R_2$

$$E(r) = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9)(2.00\times10^{-6})}{(0.700)^2} = 3.67\times10^4 \text{ V/m}.$$

(c) For $R_2 > R_1 > r$, the enclosed charge is zero. Thus, $E = 0$.

The electric potential may be obtained using Eq. 24-18: $V(r) - V(r') = \int_r^{r'} E(r)\,dr$.

(d) For $r = 4.00$ m $> R_2 > R_1$, we have

$$V(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9)(2.00\times10^{-6} + 1.00\times10^{-6})}{(4.00)} = 6.74\times10^3 \text{ V}.$$

(e) For $r = 1.00$ m $= R_2 > R_1$, we have

$$V(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9)(2.00\times10^{-6} + 1.00\times10^{-6})}{(1.00)} = 2.70\times10^4 \text{ V}.$$

(f) For $R_2 > r = 0.700$ m $> R_2$,

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9)\left(\frac{2.00\times10^{-6}}{0.700} + \frac{1.00\times10^{-6}}{1.00}\right) = 3.47\times10^4 \text{ V}.$$

(g) For $R_2 > r = 0.500$ m $= R_2$,

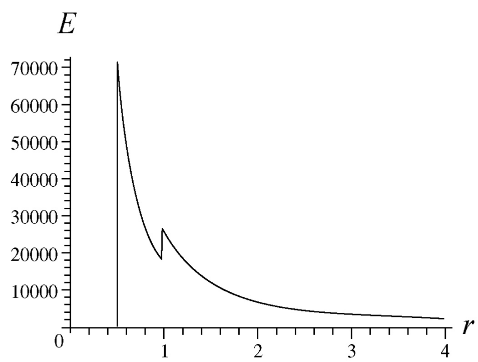
$$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9)\left(\frac{2.00\times10^{-6}}{0.500}+\frac{1.00\times10^{-6}}{1.00}\right)=4.50\times10^4\text{ V}.$$

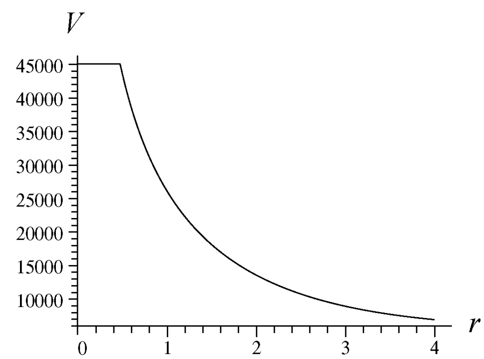
(h) For $R_2 > R_1 > r$,

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1}+\frac{q_2}{R_2}\right)=(8.99\times10^9)\left(\frac{2.00\times10^{-6}}{0.500}+\frac{1.00\times10^{-6}}{1.00}\right)=4.50\times10^4\text{ V}.$$

(i) At $r = 0$, the potential remains constant, $V=4.50\times10^4$ V.

(j) The electric field and the potential as a function of r are depicted below:





59. Using Gauss' law, $q = \varepsilon_o \Phi = +495.8$ nC. Consequently, $V = \dfrac{q}{4\pi\varepsilon_o r} = 37.1$ kV.

60. (a) We use Eq. 24-18 to find the potential: $V_{\text{wall}} - V = -\int_r^R E\,dr$, or

$$0 - V = -\int_r^R \left(\frac{\rho r}{2\varepsilon_0}\right) \quad \Rightarrow \quad -V = -\frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right).$$

Consequently, $V = \rho(R^2 - r^2)/4\varepsilon_0$.

(b) The value at $r = 0$ is

$$V_{\text{center}} = \frac{-1.1\times10^{-3}\ \text{C/m}^3}{4\left(8.85\times10^{-12}\ \text{C/V}\cdot\text{m}\right)}\left((0.05\,\text{m})^2 - 0\right) = -7.8\times10^4\ \text{V}.$$

Thus, the difference is $|V_{\text{center}}| = 7.8\times10^4\ \text{V}$.

61. The electric potential energy in the presence of the dipole is

$$U = qV_{\text{dipole}} = \frac{q\,p\cos\theta}{4\pi\varepsilon_o r^2} = \frac{(-e)(e\,d)\cos\theta}{4\pi\varepsilon_o r^2} \;.$$

Noting that $\theta_i = \theta_f = 0^\circ$, conservation of energy leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow v = \sqrt{\frac{2e^2}{4\pi\varepsilon_o m\,d}\left(\frac{1}{25} - \frac{1}{49}\right)} = 7.0 \times 10^5 \text{ m/s} \,.$$

62. (a) When the proton is released, its energy is $K + U = 4.0$ eV + 3.0 eV (the latter value is inferred from the graph). This implies that if we draw a horizontal line at the 7.0 Volt "height" in the graph and find where it intersects the voltage plot, then we can determine the turning point. Interpolating in the region between 1.0 cm and 3.0 cm, we find the turning point is at roughly $x = 1.7$ cm.

(b) There is no turning point towards the right, so the speed there is nonzero, and is given by energy conservation:

$$v = \sqrt{\frac{2(7.0\text{ eV})}{m}} = \sqrt{\frac{2(7.0\text{ eV})(1.6 \times 10^{-19}\text{ J/eV})}{1.67 \times 10^{-27}\text{ kg}}} = 20\text{ km/s}.$$

(c) The electric field at any point P is the (negative of the) slope of the voltage graph evaluated at P. Once we know the electric field, the force on the proton follows immediately from $\vec{F} = q\vec{E}$, where $q = +e$ for the proton. In the region just to the left of x = 3.0 cm, the field is $\vec{E} = (+300\text{ V/m})\hat{\text{i}}$ and the force is $F = +4.8 \times 10^{-17}$ N.

(d) The force $\vec{F}$ points in the $+x$ direction, as the electric field $\vec{E}$.

(e) In the region just to the right of x = 5.0 cm, the field is $\vec{E} = (-200\text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F = 3.2 \times 10^{-17}$ N.

(f) The force $\vec{F}$ points in the $-x$ direction, as the electric field $\vec{E}$.

63. Eq. 24-32 applies with $dq = \lambda\, dx = bx\, dx$ (along $0 \le x \le 0.20$ m).

(a) Here $r = x > 0$, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bx\,dx}{x} = \frac{b(0.20)}{4\pi\varepsilon_0} = 36\text{ V}.$$

(b) Now $r = \sqrt{x^2 + d^2}$ where $d = 0.15$ m, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bxdx}{\sqrt{x^2 + d^2}} = \frac{b}{4\pi\varepsilon_0}\left(\sqrt{x^2 + d^2}\right)\Bigg|_0^{0.20} = 18\text{ V}.$$

64. (a)When the electron is released, its energy is $K + U = 3.0$ eV $- 6.0$ eV (the latter value is inferred from the graph along with the fact that $U = qV$ and $q = -e$). Because of the minus sign (of the charge) it is convenient to imagine the graph multiplied by a minus sign so that it represents potential energy in eV. Thus, the 2 V value shown at $x = 0$ would become –2 eV, and the 6 V value at $x = 4.5$ cm becomes –6 eV, and so on. The total energy (– 3.0 eV) is constant and can then be represented on our (imagined) graph as a horizontal line at – 3.0 V. This intersects the potential energy plot at a point we recognize as the turning point. Interpolating in the region between 1.0 cm and 4.0 cm, we find the turning point is at $x = 1.75$ cm $\approx$ 1.8 cm.

(b) There is no turning point towards the right, so the speed there is nonzero. Noting that the kinetic energy at $x = 7.0$ cm is – 3.0 eV – (– 5.0 eV) = 2.0 eV, we find the speed using energy conservation:

$$v = \sqrt{\frac{2(2.0\text{ eV})}{m}} = \sqrt{\frac{2(2.0\text{ eV})(1.6 \times 10^{-19}\text{ J/eV})}{9.11 \times 10^{-31}\text{ kg}}} = 8.4\times10^{5}\text{ m/s}.$$

(c) The electric field at any point P is the (negative of the) slope of the voltage graph evaluated at P. Once we know the electric field, the force on the electron follows immediately from $\vec{F} = q\vec{E}$, where $q = -e$ for the electron. In the region just to the left of $x = 4.0$ cm, the field is $\vec{E} = (-133\text{ V/m})\,\hat{\text{i}}$ and the magnitude of the force is $F = 2.1\times10^{-17}$ N.

(d) The force points in the $+x$ direction.

(e) In the region just to the right of $x = 5.0$ cm, the field is $\vec{E} = +100\text{ V/m}\ \hat{\text{i}}$ and the force is $\vec{F} = (-1.6 \times 10^{-17}\text{ N})\ \hat{\text{i}}$. Thus, the magnitude of the force is $F = 1.6\times10^{-17}$ N.

(f) The minus sign indicates that $\vec{F}$ points in the $-x$ direction.

65. We treat the system as a superposition of a disk of surface charge density σ and radius R and a smaller, oppositely charged, disk of surface charge density $-\sigma$ and radius r. For each of these, Eq 24-37 applies (for $z > 0$)

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-z\right)+\frac{-\sigma}{2\varepsilon_0}\left(\sqrt{z^2+r^2}-z\right).$$

This expression does vanish as $r \to \infty$, as the problem requires. Substituting $r = 0.200R$ and $z = 2.00R$ and simplifying, we obtain

$$V = \frac{\sigma R}{\varepsilon_0}\left(\frac{5\sqrt{5}-\sqrt{101}}{10}\right) = \frac{(6.20\times10^{-12})(0.130)}{8.85\times10^{-12}}\left(\frac{5\sqrt{5}-\sqrt{101}}{10}\right) = 1.03\times10^{-2}\ \text{V}.$$

66. Since the electric potential energy is not changed by the introduction of the third particle, we conclude that the net electric potential evaluated at P caused by the original two particles must be zero:

$$\frac{q_1}{4\pi\varepsilon_o r_1} + \frac{q_2}{4\pi\varepsilon_o r_2} = 0 .$$

Setting $r_1 = 5d/2$ and $r_2 = 3d/2$ we obtain $q_1 = -5q_2/3$, or $q_1/q_2 = -5/3 \approx -1.7$.

67. The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged sphere:

$$V_A = V_S = \frac{q}{4\pi\varepsilon_0 R}$$

where $q = 30 \times 10^{-9}$ C and $R = 0.030$ m. For points beyond the surface of the sphere, the potential follows Eq. 24-26:

$$V_B = \frac{q}{4\pi\varepsilon_0 r}$$

where $r = 0.050$ m.

(a) We see that

$$V_S - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right) = 3.6 \times 10^3 \text{ V}.$$

(b) Similarly,

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right) = 3.6 \times 10^3 \text{ V}.$$

68. The *escape speed* may be calculated from the requirement that the initial kinetic energy (of *launch*) be equal to the absolute value of the initial potential energy (compare with the gravitational case in chapter 14). Thus,

$$\frac{1}{2} m v^2 = \frac{e\, q}{4\pi\varepsilon_o r}$$

where $m = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C, $q = 10000e$, and $r = 0.010$ m. This yields the answer $v = 22490$ m/s $\approx 2.2\times10^4$ m/s .

69. We apply conservation of energy for the particle with $q = 7.5 \times 10^{-6}$ C (which has zero initial kinetic energy):

$$U_0 = K_f + U_f \qquad \text{where } U = \frac{q\,Q}{4\pi\varepsilon_o r} .$$

(a) The initial value of r is 0.60 m and the final value is (0.6 + 0.4) m = 1.0 m (since the particles repel each other). Conservation of energy, then, leads to $K_f = 0.90$ J.

(b) Now the particles attract each other so that the final value of r is 0.60 − 0.40 = 0.20 m. Use of energy conservation yields $K_f = 4.5$ J in this case.

70. (a) Using $d = 2$ m, we find the potential at P:

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{+2e}{d} + \frac{1}{4\pi\varepsilon_0}\frac{-2e}{2d} = \frac{1}{4\pi\varepsilon_0}\frac{e}{d} \ .$$

Thus, with $e = 1.60 \times 10^{-19}$ C, we find $V_P = 7.19 \times 10^{-10}$ V. Note that we are implicitly assuming that $V \to 0$ as $r \to \infty$.

(b) Since $U = qV$, then the movable particle's contribution of the potential energy when it is at $r = \infty$ is zero, and its contribution to U_{system} when it is at P is $(2e)V_P = 2.30 \times 10^{-28}$ J. Thus, we obtain $W_{\text{app}} = 2.30 \times 10^{-28}$ J.

(c) Now, combining the contribution to U_{system} from part (b) and from the original pair of fixed charges

$$U_{\text{fixed}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(-2e)}{\sqrt{4^2+2^2}} = -2.1 \times 10^{-28}\ \text{J} \ ,$$

we obtain

$$U_{\text{system}} = U_{\text{part (b)}} + U_{\text{fixed}} = 2.43 \times 10^{-29}\ \text{J} \ .$$

71. The derivation is shown in the book (Eq. 24-33 through Eq. 24-35) except for the change in the lower limit of integration (which is now $x = D$ instead of $x = 0$). The result is therefore (cf. Eq. 24-35)

$$V = \frac{\lambda}{4\pi\varepsilon_o} \ln\left(\frac{L + \sqrt{L^2 + d^2}}{D + \sqrt{D^2 + d^2}}\right) = \frac{2.0 \times 10^{-6}}{4\pi\varepsilon_o} \ln\left(\frac{4 + \sqrt{17}}{1 + \sqrt{2}}\right) = 2.18 \times 10^4 \text{ V}.$$

72. Using Eq. 24-18, we have

$$\Delta V = -\int_2^3 \frac{A}{r^4}\, dr = \frac{A}{3}\left(\frac{1}{2^3} - \frac{1}{3^3}\right) = A(0.029/\text{m}^3).$$

73. The work done results in a change of potential energy:

$$W = \Delta U = \frac{2(0.12\text{ C})^2}{4\pi\varepsilon_o\left(\dfrac{1.7\text{ m}}{2}\right)} - \frac{2(0.12\text{ C})^2}{4\pi\varepsilon_o(1.7\text{ m})} = 1.5\times 10^8\text{ J} \; .$$

At a rate of $P = 0.83 \times 10^3$ Joules per second, it would take $W/P = 1.8 \times 10^5$ seconds or about 2.1 days to do this amount of work.

74. The charges are equidistant from the point where we are evaluating the potential — which is computed using Eq. 24-27 (or its integral equivalent). Eq. 24-27 implicitly assumes $V \to 0$ as $r \to \infty$. Thus, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+3Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2Q_1}{R} = \frac{2(8.99\times10^{9})(4.52\times10^{-12})}{0.0850}$$

$$= 0.956 \text{ V}.$$

75. The radius of the cylinder (0.020 m, the same as r_B) is denoted R, and the field magnitude there (160 N/C) is denoted E_B. The electric field beyond the surface of the sphere follows Eq. 23-12, which expresses inverse proportionality with r:

$$\frac{|\vec{E}|}{E_B} = \frac{R}{r} \quad \text{for } r \geq R \ .$$

(a) Thus, if $r = r_C = 0.050$ m, we obtain $|\vec{E}| = (160)(0.020)/(0.050) = 64\,\text{N/C}$.

(b) Integrating the above expression (where the variable to be integrated, r, is now denoted ρ) gives the potential difference between V_B and V_C.

$$V_B - V_C = \int_R^r \frac{E_B R}{\rho}\, d\rho = E_B R \ln\left(\frac{r}{R}\right) = 2.9\,\text{V} \ .$$

(c) The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged cylinder: $V_A - V_B = 0$.

76. We note that for two points on a circle, separated by angle θ (in radians), the direct-line distance between them is $r = 2R\sin(\theta/2)$. Using this fact, distinguishing between the cases where N = odd and N = even, and counting the pair-wise interactions very carefully, we arrive at the following results for the total potential energies. We use $k = 1/4\pi\varepsilon_0$. For configuration 1 (where all N electrons are on the circle), we have

$$U_{1,N=\text{even}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta/2)}+\frac{1}{2}\right), \quad U_{1,N=\text{odd}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N-1}{2}}\frac{1}{\sin(j\theta/2)}\right)$$

where $\theta = \dfrac{2\pi}{N}$. For configuration 2, we find

$$U_{2,N=\text{even}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta'/2)}+2\right)$$

$$U_{2,N=\text{odd}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N-3}{2}}\frac{1}{\sin(j\theta'/2)}+\frac{5}{2}\right)$$

where $\theta' = \dfrac{2\pi}{N-1}$. The results are all of the form

$$U_{1\,\text{or}\,2}\,\frac{ke^2}{2R}\times\text{a pure number.}$$

In our table, below, we have the results for those "pure numbers" as they depend on N and on which configuration we are considering. The values listed in the U rows are the potential energies divided by $ke^2/2R$.

N	4	5	6	7	8	9	10	11	12	13	14	15
U_1	3.83	6.88	10.96	16.13	22.44	29.92	38.62	48.58	59.81	72.35	86.22	101.5
U_2	4.73	7.83	11.88	16.96	23.13	30.44	39.92	48.62	59.58	71.81	85.35	100.2

We see that the potential energy for configuration 2 is greater than that for configuration 1 for $N < 12$, but for $N \geq 12$ it is configuration 1 that has the greatest potential energy.

(a) $N = 12$ is the smallest value such that $U_2 < U_1$.

(b) For $N = 12$, configuration 2 consists of 11 electrons distributed at equal distances around the circle, and one electron at the center. A specific electron e_0 on the circle is R distance from the one in the center, and is

$$r = 2R\sin\left(\frac{\pi}{11}\right) \approx 0.56R$$

distance away from its nearest neighbors on the circle (of which there are two — one on each side). Beyond the nearest neighbors, the next nearest electron on the circle is

$$r = 2R\sin\left(\frac{2\pi}{11}\right) \approx 1.1R$$

distance away from e_0. Thus, we see that there are only two electrons closer to e_0 than the one in the center.

77. We note that the net potential (due to the "fixed" charges) is zero at the first location ("at ∞") being considered for the movable charge q (where $q = +2e$). Thus, the work required is equal to the potential energy in the final configuration: qV where

$$V = \frac{1}{4\pi\varepsilon_0}\frac{(+2e)}{2D} + \frac{1}{4\pi\varepsilon_0}\frac{+e}{D} \; .$$

Using $D = 4.00$ m and $e = 1.60 \times 10^{-19}$ C, we obtain

$$W_{\text{app}} = qV = (2e)(7.20 \times 10^{-10}\ \text{V}) = 2.30 \times 10^{-28}\ \text{J}.$$

78. Since the electric potential is a scalar quantity, this calculation is far simpler than it would be for the electric field. We are able to simply take half the contribution that would be obtained from a complete (whole) sphere. If it were a whole sphere (of the same density) then its charge would be $q_{whole} = 8.00\ \mu C$. Then

$$V = \frac{1}{2} V_{whole} = \frac{1}{2} \frac{q_{whole}}{4\pi\varepsilon_o r} = \frac{1}{2} \frac{8.00 \text{ x } 10^{-6}\text{ C}}{4\pi\varepsilon_o(0.15\text{ m})} = 2.40 \times 10^5\text{ V} .$$

79. The net potential at point P (the place where we are to place the third electron) due to the fixed charges is computed using Eq. 24-27 (which assumes $V \to 0$ as $r \to \infty$):

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{-e}{d} + \frac{1}{4\pi\varepsilon_0}\frac{-e}{d} = \frac{-e}{2\pi\varepsilon_0 d} \quad .$$

Thus, with $d = 2.00 \times 10^{-6}$ m and $e = 1.60 \times 10^{-19}$ C, we find $V_P = -1.438 \times 10^{-3}$ V. Then the required "applied" work is, by Eq. 24-14,

$$W_{\text{app}} = (-e)\, V_P = 2.30 \times 10^{-22} \text{ J} \quad .$$

80. The work done is equal to the change in the (total) electric potential energy U of the system, where

$$U = \frac{q_1 q_2}{4\pi\varepsilon_o r_{12}} + \frac{q_3 q_2}{4\pi\varepsilon_o r_{23}} + \frac{q_1 q_3}{4\pi\varepsilon_o r_{13}}$$

and the notation r_{13} indicates the distance between q_1 and q_3 (similar definitions apply to r_{12} and r_{23}).

(a) We consider the difference in U where initially $r_{12} = b$ and $r_{23} = a$, and finally $r_{12} = a$ and $r_{23} = b$ (r_{13} doesn't change). Converting the values given in the problem to SI units (μC to C, cm to m), we obtain $\Delta U = -24$ J.

(b) Now we consider the difference in U where initially $r_{23} = a$ and $r_{13} = a$, and finally r_{23} is again equal to a and r_{13} is also again equal to a (and of course, r_{12} doesn't change in this case). Thus, we obtain $\Delta U = 0$.

81. (a) Clearly, the net voltage

$$V = \frac{q}{4\pi\varepsilon_o |x|} + \frac{2q}{4\pi\varepsilon_o |d - x|}$$

is not zero for any finite value of x.

(b) The electric field cancels at a point between the charges:

$$\frac{q}{4\pi\varepsilon_o x^2} = \frac{2q}{4\pi\varepsilon_o (d - x)^2}$$

which has the solution: $x = (\sqrt{2} - 1)\, d = 0.41$ m.

82. (a) The potential on the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(4.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}{0.10\,\text{m}} = 3.6\times10^{5}\ \text{V}\ .$$

(b) The field just outside the sphere would be

$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{V}{R} = \frac{3.6\times10^{5}\ \text{V}}{0.10\,\text{m}} = 3.6\times10^{6}\ \text{V/m}\ ,$$

which would have exceeded 3.0 MV/m. So this situation cannot occur.

83. This can be approached more than one way, but the simplest is to observe that the net potential (using Eq. 24-27) due to $q_1 = +2e$ and $q_3 = -2e$ is zero at both the initial and final positions of the movable charge $q_2 = +5q$. This implies that no work is necessary to effect its change of position, which, in turn, implies there is no resulting change in potential energy of the configuration. Hence, the ratio is unity.

84. We use $E_x = -dV/dx$, where dV/dx is the local slope of the V vs. x curve depicted in Fig. 24-54. The results are:

(a) $E_x(ab) = -6.0$ V/m,

(b) $E_x(bc) = 0$,

(c) $E_x(cd)$ 3.0 V/m,

(d) $E_x(de) = 3.0$ V/m,

(e) $E_x(ef) = 15$ V/m,

(f) $E_x(fg) = 0$,

(g) $E_x(gh) = -3.0$ V/m.

Since these values are constant during their respective time-intervals, their graph consists of several disconnected line-segments (horizontal) and is not shown here.

85. (a) We denote the surface charge density of the disk as σ_1 for $0 < r < R/2$, and as σ_2 for $R/2 < r < R$. Thus the total charge on the disk is given by

$$q = \int_{\text{disk}} dq = \int_0^{R/2} 2\pi\sigma_1 r\,dr + \int_{R/2}^{R} 2\pi\sigma_2 r\,dr = \frac{\pi}{4}R^2\left(\sigma_1 + 3\sigma_2\right)$$
$$= \frac{\pi}{4}\left(2.20\times10^{-2}\text{ m}\right)^2\left[1.50\times10^{-6}\text{ C/m}^2 + 3\left(8.00\times10^{-7}\text{ C/m}^2\right)\right]$$
$$= 1.48\times10^{-9}\text{ C}.$$

(b) We use Eq. 24-36:

$$V(z) = \int_{\text{disk}} dV = k\left[\int_0^{R/2} \frac{\sigma_1(2\pi R')dR'}{\sqrt{z^2+R'^2}} + \int_{R/2}^{R} \frac{\sigma_2(2\pi R')dR'}{\sqrt{z^2+R'^2}}\right]$$
$$= \frac{\sigma_1}{2\varepsilon_0}\left(\sqrt{z^2+\frac{R^2}{4}} - z\right) + \frac{\sigma_2}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - \sqrt{z^2+\frac{R^2}{4}}\right).$$

Substituting the numerical values of σ_1, σ_2, R and z, we obtain $V(z) = 7.95\times10^2$ V.

86. The net potential (at point *A* or *B*) is computed using Eq. 24-27. Thus, using *k* for $1/4\pi\varepsilon_0$, the difference is

$$V_A - V_B = \left(\frac{ke}{d} + \frac{k(-5e)}{5d}\right) - \left(\frac{ke}{2d} + \frac{k(-5e)}{2d}\right) = \frac{2ke}{d} = \frac{2(8.99\times10^{9})(1.6\times10^{-19})}{5.60\times10^{-6}} = 5.14\times10^{-4}\ \text{V}.$$

87. We denote $q = 25 \times 10^{-9}$ C, $y = 0.6$ m, $x = 0.8$ m, with V = the net potential (assuming $V \to 0$ as $r \to \infty$). Then,

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{y} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{x}$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{y}$$

leads to

$$V_B - V_A = \frac{2}{4\pi\varepsilon_0}\frac{q}{x} - \frac{2}{4\pi\varepsilon_0}\frac{q}{y} = \frac{q}{2\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{y}\right)$$

which yields $\Delta V = -187\,\text{V}$.

88. In the "inside" region between the plates, the individual fields (given by Eq. 24-13) are in the same direction ($-\hat{\mathrm{i}}$):

$$\vec{E}_{\text{in}} = -\left(\frac{50\times10^{-9}}{2\varepsilon_0} + \frac{25\times10^{-9}}{2\varepsilon_0}\right)\hat{\mathrm{i}} = -4.2\times10^3\,\hat{\mathrm{i}}$$

in SI units (N/C or V/m). And in the "outside" region where $x > 0.5$ m, the individual fields point in opposite directions:

$$\vec{E}_{\text{out}} = -\frac{50\times10^{-9}}{2\varepsilon_0}\hat{\mathrm{i}} + \frac{25\times10^{-9}}{2\varepsilon_0}\hat{\mathrm{i}} = -1.4\times10^3\,\hat{\mathrm{i}}\,.$$

Therefore, by Eq. 24-18, we have

$$\Delta V = -\int_0^{0.8} \vec{E}\cdot d\vec{s} = -\int_0^{0.5} \left|\vec{E}\right|_{\text{in}} dx - \int_{0.5}^{0.8} \left|\vec{E}\right|_{\text{out}} dx = -\left(4.2\times10^3\right)(0.5) - \left(1.4\times10^3\right)(0.3)$$
$$= 2.5\times10^3 \text{ V}.$$

89. (a) The charges are equal and are the same distance from C. We use the Pythagorean theorem to find the distance $r=\sqrt{(d/2)^2+(d/2)^2}=d/\sqrt{2}$. The electric potential at C is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$V=\frac{2q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{d}=\frac{2\sqrt{2}q}{4\pi\varepsilon_0 d}=\frac{\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)(2)\sqrt{2}\left(2.0\times10^{-6}\,\mathrm{C}\right)}{0.020\text{ m}}=2.5\times10^6\,\mathrm{V}.$$

(b) As you move the charge into position from far away the potential energy changes from zero to qV, where V is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in:

$$W=qV=\left(2.0\times10^{-6}\,\mathrm{C}\right)\left(2.54\times10^6\,\mathrm{V}\right)=5.1\text{ J}.$$

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is d so this potential energy is $q^2/4\pi\varepsilon_0 d$. The total potential energy is

$$U=W+\frac{q^2}{4\pi\varepsilon_0 d}=5.1\text{ J}+\frac{\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(2.0\times10^{-6}\,\mathrm{C}\right)^2}{0.020\,\mathrm{m}}=6.9\text{ J}.$$

90. The potential energy of the two-charge system is

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}\right] = \frac{\left(8.99\times 10^9 \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\left(3.00\times 10^{-6}\,\mathrm{C}\right)\left(-4.00\times 10^{-6}\,\mathrm{C}\right)}{\sqrt{(3.50+2.00)^2 + (0.500-1.50)^2}\ \mathrm{cm}}$$

$$= -1.93\ \mathrm{J}.$$

Thus, –1.93 J of work is needed.

91. For a point on the axis of the ring the potential (assuming $V \to 0$ as $r \to \infty$) is

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2 + R^2}}$$

where $q = 16 \times 10^{-6}$ C and $R = 0.0300$ m. Therefore,

$$V_B - V_A = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{z_B^2 + R^2}} - \frac{1}{R}\right)$$

where $z_B = 0.040$ m. The result is -1.92×10^6 V.

92. The initial speed v_i of the electron satisfies $K_i = \frac{1}{2} m_e v_i^2 = e\Delta V$, which gives

$$v_i = \sqrt{\frac{2e\Delta V}{m_e}} = \sqrt{\frac{2\left(1.60\times10^{-19}\ \text{J}\right)\left(625\ \text{V}\right)}{9.11\times10^{-31}\ \text{kg}}} = 1.48\times10^{7}\ \text{m/s}.$$

93. (a) The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-6}\ \text{C}\right)^2}{1.00\ \text{m}} = 0.225\ \text{J}$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-6}\ \text{C}\right)^2}{\left(1.00\ \text{m}\right)^2} = 0.225\ \text{N}.$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let m_A and m_B be the masses of the spheres. The acceleration of sphere A is

$$a_A = \frac{F}{m_A} = \frac{0.225\ \text{N}}{5.0\times10^{-3}\ \text{kg}} = 45.0\ \text{m/s}^2$$

and the acceleration of sphere B is

$$a_B = \frac{F}{m_B} = \frac{0.225\ \text{N}}{10\times10^{-3}\ \text{kg}} = 22.5\ \text{m/s}^2.$$

(c) Energy is conserved. The initial potential energy is $U = 0.225$ J, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, where v_A and v_B are the final velocities. Thus,

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2.$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B.$$

These equations may be solved simultaneously for v_A and v_B. Substituting $v_B = -(m_A/m_B)v_A$, from the momentum equation into the energy equation, and collecting terms, we obtain $U = \frac{1}{2}(m_A/m_B)(m_A + m_B)v_A^2$. Thus,

$$v_A = \sqrt{\frac{2Um_B}{m_A(m_A + m_B)}} = \sqrt{\frac{2(0.225 \text{ J})(10\times10^{-3} \text{ kg})}{(5.0\times10^{-3} \text{ kg})(5.0\times10^{-3} \text{ kg} + 10\times10^{-3} \text{ kg})}} = 7.75 \text{ m/s}.$$

We thus obtain

$$v_B = -\frac{m_A}{m_B} v_A = -\left(\frac{5.0\times10^{-3} \text{ kg}}{10\times10^{-3} \text{ kg}}\right)(7.75 \text{ m/s}) = -3.87 \text{ m/s},$$

or $|v_B| = 3.87$ m/s.

94. The particle with charge $-q$ has both potential and kinetic energy, and both of these change when the radius of the orbit is changed. We first find an expression for the total energy in terms of the orbit radius r. Q provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\varepsilon_0 r^2$. The acceleration of $-q$ is v^2/r, where v is its speed. Newton's second law yields

$$\frac{Q_q}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{Qq}{4\pi\varepsilon_0 r},$$

and the kinetic energy is $K = \frac{1}{2}mv^2 = Qq/8\pi\varepsilon_0 r$. The potential energy is $U = -Qq/4\pi\varepsilon_0 r$, and the total energy is

$$E = K + U = \frac{Qq}{8\pi\varepsilon_0 r} - \frac{Qq}{4\pi\varepsilon_0 r} = -\frac{Qq}{8\pi\varepsilon_0 r}.$$

When the orbit radius is r_1 the energy is $E_1 = -Qq/8\pi\varepsilon_0 r_1$ and when it is r_2 the energy is $E_2 = -Qq/8\pi\varepsilon_0 r_2$. The difference $E_2 - E_1$ is the work W done by an external agent to change the radius:

$$W = E_2 - E_1 = -\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

95. (a) The total electric potential energy consists of three equal terms:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_o r} + \frac{q_2 q_3}{4\pi\varepsilon_o r} + \frac{q_1 q_3}{4\pi\varepsilon_o r}$$

where $q_1 = q_2 = q_3 = -\frac{e}{3}$, and r as given in the problem. The result is $U = 2.72 \times 10^{-14}$ J.

(b) Dividing by the square of the speed of light (roughly 3.0×10^8 m/s), we obtain a value in kilograms (about a third of the correct electron mass value): 3.02×10^{-31} kg.

96. A positive charge q is a distance $r - d$ from P, another positive charge q is a distance r from P, and a negative charge $-q$ is a distance $r + d$ from P. Sum the individual electric potentials created at P to find the total:

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-d} + \frac{1}{r} - \frac{1}{r+d}\right].$$

We use the binomial theorem to approximate $1/(r - d)$ for r much larger than d:

$$\frac{1}{r-d} = (r-d)^{-1} \approx (r)^{-1} - (r)^{-2}(-d) = \frac{1}{r} + \frac{d}{r^2}.$$

Similarly,

$$\frac{1}{r+d} \approx \frac{1}{r} - \frac{d}{r^2}.$$

Only the first two terms of each expansion were retained. Thus,

$$V \approx \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{d}{r^2} + \frac{1}{r} - \frac{1}{r} + \frac{d}{r^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{2d}{r^2}\right] = \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{2d}{r}\right].$$

97. Assume the charge on Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of Earth it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on Earth and $R = 6.37 \times 10^6$ m is the radius of Earth. The magnitude of the electric field at the surface is $E = q/4\pi\varepsilon_0 R^2$, so

$$V = ER = (100 \text{ V/m})\,(6.37 \times 10^6 \text{ m}) = 6.4 \times 10^8 \text{ V}.$$

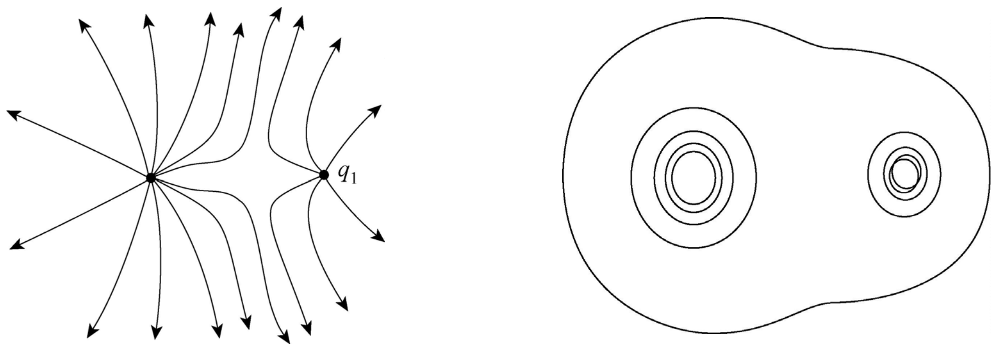
98. The net electric potential at point P is the sum of those due to the six charges:

$$V_P = \sum_{i=1}^{6} V_{Pi} = \sum_{i=1}^{6} \frac{q_i}{4\pi\varepsilon_0 r_i} = \frac{10^{-15}}{4\pi\varepsilon_0}\left[\frac{5.00}{\sqrt{d^2+(d/2)^2}} + \frac{-2.00}{d/2} + \frac{-3.00}{\sqrt{d^2+(d/2)^2}}\right.$$

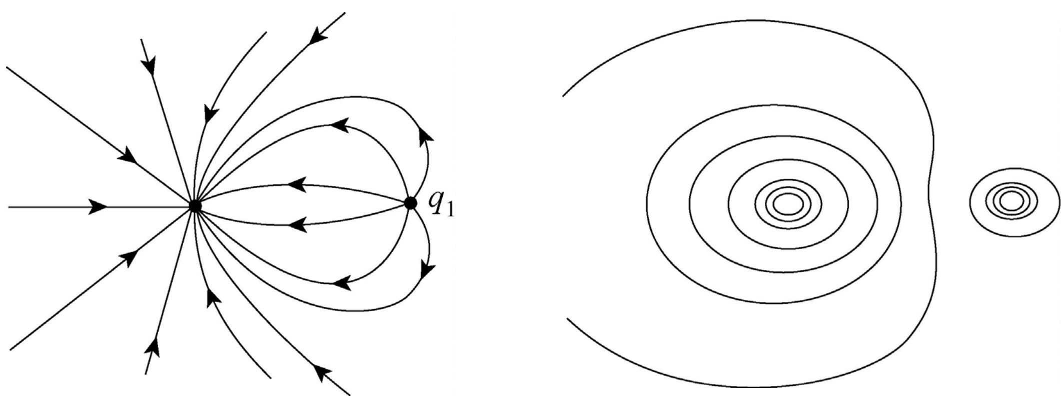
$$\left. + \frac{3.00}{\sqrt{d^2+(d/2)^2}} + \frac{-2.00}{d/2} + \frac{+5.00}{\sqrt{d^2+(d/2)^2}}\right] = \frac{9.4\times10^{-16}}{4\pi\varepsilon_0(2.54\times10^{-2})} = 3.34\times10^{-4}\ \text{V}.$$

99. In the sketches shown next, the lines with the arrows are field lines and those without are the equipotentials (which become more circular the closer one gets to the individual charges). In all pictures, q_2 is on the left and q_1 is on the right (which is reversed from the way it is shown in the textbook).

(a)



(b)



100. (a) We use Gauss' law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside. Outside the charge distribution the magnitude of the field is $E = q/4\pi\varepsilon_0 r^2$ and the potential is $V = q/4\pi\varepsilon_0 r$, where r is the distance from the center of the distribution. This is the same as the field and potential of a point charge at the center of the spherical distribution. To find an expression for the magnitude of the field inside the charge distribution, we use a Gaussian surface in the form of a sphere with radius r, concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is $4\pi r^2 E$. The charge enclosed is qr^3/R^3. Gauss' law becomes

$$4\pi\varepsilon_0 r^2 E = \frac{qr^3}{R^3},$$

so

$$E = \frac{qr}{4\pi\varepsilon_0 R^3}.$$

If V_s is the potential at the surface of the distribution ($r = R$) then the potential at a point inside, a distance r from the center, is

$$V = V_s - \int_R^r E\,dr = V_s - \frac{q}{4\pi\varepsilon_0 R^3}\int_R^r r\,dr = V_s - \frac{qr^2}{8\pi\varepsilon_0 R^3} + \frac{q}{8\pi\varepsilon_0 R}.$$

The potential at the surface can be found by replacing r with R in the expression for the potential at points outside the distribution. It is $V_s = q/4\pi\varepsilon_0 R$. Thus,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right] = \frac{q}{8\pi\varepsilon_0 R^3}\left(3R^2 - r^2\right).$$

(b) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\varepsilon_0 R} - \frac{3q}{8\pi\varepsilon_0 R} = -\frac{q}{8\pi\varepsilon_0 R},$$

or $|\Delta V| = q/8\pi\varepsilon_0 R$.

101. (a) For $r > r_2$ the field is like that of a point charge and

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r},$$

where the zero of potential was taken to be at infinity.

(b) To find the potential in the region $r_1 < r < r_2$, first use Gauss's law to find an expression for the electric field, then integrate along a radial path from r_2 to r. The Gaussian surface is a sphere of radius r, concentric with the shell. The field is radial and therefore normal to the surface. Its magnitude is uniform over the surface, so the flux through the surface is $\Phi = 4\pi r^2 E$. The volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)},$$

and the charge enclosed by the Gaussian surface is

$$q = \left(\frac{4\pi}{3}\right)(r^3 - r_1^3)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right).$$

Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)}.$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$V = V_s - \int_{r_2}^{r} E\,dr = V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr$$
$$= V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right).$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\varepsilon_0 r_2$. We make this substitution and collect terms to find

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right).$$

Since $\rho = 3Q/4\pi\left(r_2^3 - r_1^3\right)$ this can also be written

$$V = \frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right).$$

(c) The electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. We put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{3\left(r_2^2 - r_1^2\right)}{2\left(r_2^3 - r_1^3\right)},$$

or in terms of the charge density $V = \frac{\rho}{2\varepsilon_0}\left(r_2^2 - r_1^2\right).$

(d) The solutions agree at $r = r_1$ and at $r = r_2$.

102. The distance r being looked for is that where the alpha particle has (momentarily) zero kinetic energy. Thus, energy conservation leads to

$$K_0 + U_0 = K + U \quad \Rightarrow (0.48 \times 10^{-12}\ \text{J}) + \frac{(2e)(92e)}{4\pi\varepsilon_0\, r_0} = 0 + \frac{(2e)(92e)}{4\pi\varepsilon_0\, r} \quad .$$

If we set $r_0 = \infty$ (so $U_0 = 0$) then we obtain $r = 8.8 \times 10^{-14}$ m.

103. (a) The net potential is

$$V = V_1 + V_2 = \frac{q_1}{4\pi\varepsilon_o r_1} + \frac{q_2}{4\pi\varepsilon_o r_2}$$

where $r_1 = \sqrt{x^2 + y^2}$ and $r_2 = \sqrt{(x-d)^2 + y^2}$. The distance d is 8.6 nm. To find the locus of points resulting in $V = 0$, we set V_1 equal to the (absolute value of) V_2 and square both sides. After simplifying and rearranging we arrive at an equation for a circle:

$$y^2 + \left(x + \frac{9d}{16}\right)^2 = \frac{225}{256}\, d^2 .$$

From this form, we recognize that the center of the circle is $-9d/16 = -4.8$ nm.

(b) Also from this form, we identify the radius as the square root of the right-hand side: $R = 15d/16 = 8.1$ nm.

(c) If one uses a graphing program with “implicitplot” features, it is certainly possible to set $V = 5$ volts in the expression (shown in part (a)) and find its (or one of its) equipotential curves in the xy plane. In fact, it will look very much like a circle. Algebraically, attempts to put the expression into any standard form for a circle will fail, but that can be a frustrating endeavor. Perhaps the easiest way to show that it is not truly a circle is to find where its “horizontal diameter” D_x and its “vertical diameter” D_y (not hard to do); we find $D_x = 2.582$ nm and $D_y = 2.598$ nm. The fact that $D_x \neq D_y$ is evidence that it is not a true circle.

104. The electric field (along the radial axis) is the (negative of the) derivative of the voltage with respect to r. There are no other components of $\vec{E}$ in this case, so (noting that the derivative of a constant is zero) we conclude that the magnitude of the field is

$$E = -\frac{dV}{dr} = -\frac{Ze}{4\pi\varepsilon_o}\left(\frac{d\,r^{-1}}{dr} + 0 + \frac{1}{2R^3}\frac{d\,r^2}{dr}\right) = \frac{Ze}{4\pi\varepsilon_o}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

for $r \le R$. This agrees with the Rutherford field expression shown in exercise 37 (in the textbook). We note that he has designed his voltage expression to be zero at $r = R$. Since the zero point for the voltage of this system (in an otherwise empty space) is arbitrary, then choosing $V = 0$ at $r = R$ is certainly permissible.

105. If the electric potential is zero at infinity then at the surface of a uniformly charged sphere it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on the sphere and R is the sphere radius. Thus $q = 4\pi\varepsilon_0 RV$ and the number of electrons is

$$N = \frac{|q|}{e} = \frac{4\pi\varepsilon_0 R|V|}{e} = \frac{(1.0\times10^{-6}\ \text{m})(400\ \text{V})}{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})} = 2.8\times10^{5}\ .$$

106. We imagine moving all the charges on the surface of the sphere to the center of the the sphere. Using Gauss' law, we see that this would not change the electric field *outside* the sphere. The magnitude of the electric field E of the uniformly charged sphere as a function of r, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\varepsilon_0 r^2)$ for $r > R$. Here R is the radius of the sphere. Thus, the potential V at the surface of the sphere (where $r = R$) is given by

$$V(R) = V\big|_{r=\infty} + \int_R^\infty E(r)\,dr = \int_\infty^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.50\times10^8\ \text{C}\right)}{0.160\text{m}}$$

$$= 8.43\times10^2\ \text{V}.$$

107. On the dipole axis $\theta = 0$ or π, so $|\cos\theta| = 1$. Therefore, magnitude of the electric field is

$$|E(r)| = \left|-\frac{\partial V}{\partial r}\right| = \frac{p}{4\pi\varepsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\varepsilon_0 r^3}\ .$$

108. The potential difference is

$$\Delta V = E\Delta s = (1.92 \times 10^5 \text{ N/C})(0.0150 \text{ m}) = 2.90 \times 10^3 \text{ V}.$$

109. (a) Using Eq. 24-26, we calculate the radius r of the sphere representing the 30 V equipotential surface:

$$r = \frac{q}{4\pi\varepsilon_0 V} = 4.5\,\text{m}.$$

(b) If the potential were a linear function of r then it would have equally spaced equipotentials, but since $V \propto 1/r$ they are spaced more and more widely apart as r increases.

110. (a) Let the quark-quark separation be r. To "naturally" obtain the eV unit, we only plug in for one of the e values involved in the computation:

$$U_{\text{up-up}} = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2e}{3}\right)\left(\frac{2e}{3}\right)}{r} = \frac{4ke}{9r}e = \frac{4\left(8.99\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\,\text{C}\right)}{9\left(1.32\times10^{-15}\,\text{m}\right)}e$$

$$= 4.84\times10^{5}\,\text{eV} = 0.484\,\text{MeV}.$$

(b) The total consists of all pair-wise terms:

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{\left(\frac{2e}{3}\right)\left(\frac{2e}{3}\right)}{r} + \frac{\left(\frac{-e}{3}\right)\left(\frac{2e}{3}\right)}{r} + \frac{\left(\frac{-e}{3}\right)\left(\frac{2e}{3}\right)}{r}\right] = 0.$$

111. (a) At the smallest center-to-center separation d_p the initial kinetic energy K_i of the proton is entirely converted to the electric potential energy between the proton and the nucleus. Thus,

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{eq_{\text{lead}}}{d_p} = \frac{82e^2}{4\pi\varepsilon_0 d_p}.$$

In solving for d_p using the eV unit, we note that a factor of e cancels in the middle line:

$$d_p = \frac{82e^2}{4\pi\varepsilon_0 K_i} = k\frac{82e^2}{4.80\times10^6\, e\text{V}} = \left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{82\left(1.6\times10^{-19}\,\text{C}\right)}{4.80\times10^6\,\text{V}}$$
$$= 2.5\times10^{-14}\,\text{m} = 25\,\text{fm}\,.$$

It is worth recalling that a volt is a newton·meter/coulomb, in making sense of the above manipulations.

(b) An alpha particle has 2 protons (as well as 2 neutrons). Therefore, using r'_{min} for the new separation, we find

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{lead}}}{d_\alpha} = 2\left(\frac{82e^2}{4\pi\varepsilon_0 d_\alpha}\right) = \frac{82e^2}{4\pi\varepsilon_0 d_p}$$

which leads to $d_\alpha / d_p = 2.00$.

112. (a) The potential would be

$$V_e = \frac{Q_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e^2 \sigma_e}{4\pi\varepsilon_0 R_e} = 4\pi R_e \sigma_e k$$

$$= 4\pi\left(6.37\times10^6\,\text{m}\right)\left(1.0\,\text{electron/m}^2\right)\left(-1.6\times10^{-9}\,\text{C/electron}\right)\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)$$

$$= -0.12\,\text{V}.$$

(b) The electric field is

$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\,\text{V}}{6.37\times10^6\,\text{m}} = -1.8\times10^{-8}\,\text{N/C},$$

or $|E| = 1.8\times10^{-8}\,\text{N/C}$.

(c) The minus sign in E indicates that $\vec{E}$ is radially inward.

113. The electric potential energy is

$$U = k\sum_{i\neq j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0 d}\left(q_1q_2 + q_1q_3 + q_2q_4 + q_3q_4 + \frac{q_1q_4}{\sqrt{2}} + \frac{q_2q_3}{\sqrt{2}}\right)$$

$$= \frac{\left(8.99\times10^{9}\right)}{1.3}\left[(12)(-24)+(12)(31)+(-24)(17)+(31)(17)+\frac{(12)(17)}{\sqrt{2}}+\frac{(-24)(31)}{\sqrt{2}}\right](10^{-19})^2$$

$$= -1.2\times10^{-6}\ \text{J}.$$

114. (a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r=\sqrt{z^2+R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}}=\frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq=\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2+R^2}}.$$

(b) The electric field is along the axis and its component is given by

$$E=-\frac{\partial V}{\partial z}=-\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2}=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z)=\frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}.$$

This agrees with Eq. 23-16.

115. From the previous chapter, we know that the radial field due to an infinite line-source is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

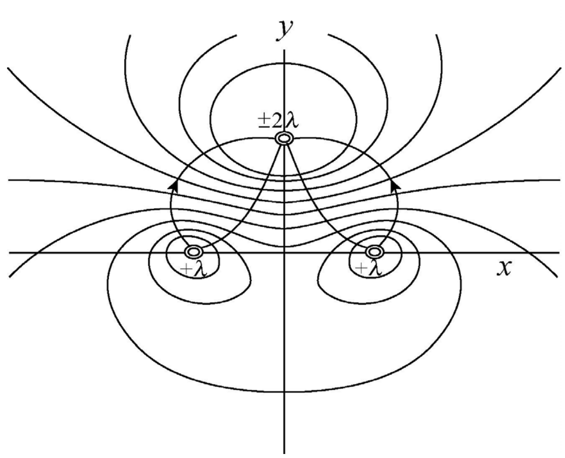
which integrates, using Eq. 24-18, to obtain

$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f}\frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right).$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_o$ be the potential along some reference axis (which intersects the plane of our figure, shown next, at the xy coordinate origin, placed midway between the bottom two line charges — that is, the midpoint of the bottom side of the equilateral triangle) at a distance $r_f = a$ from each of the bottom wires (and a distance $a\sqrt{3}$ from the topmost wire). Thus, each side of the triangle is of length $2a$. Skipping some steps, we arrive at an expression for the net potential created by the three wires (where we have set $V_o = 0$):

$$V_{\text{net}} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\left(x^2+\left(y-a\sqrt{3}\right)^2\right)^2}{\left((x+a)^2+y^2\right)\left((x-a)^2+y^2\right)}\right)$$

which forms the basis of our contour plot shown below. On the same plot we have shown four electric field lines, which have been sketched (as opposed to rigorously calculated) and are not meant to be as accurate as the equipotentials. The ±2λ by the top wire in our figure should be –2λ (the ± typo is an artifact of our plotting routine).



116. From the previous chapter, we know that the radial field due to an infinite line-source is

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$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f} \frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right).$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_o$ be the potential along some reference axis (which will be the z axis described in this problem) at a distance $r_f = a$ from the wire. In the "end-view" presented here, the wires and the z axis appear as points as they intersect the xy plane. The potential due to the wire on the left (intersecting the plane at $x = -a$) is

$$V_{\text{negative wire}} = V_o + \frac{(-\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right),$$

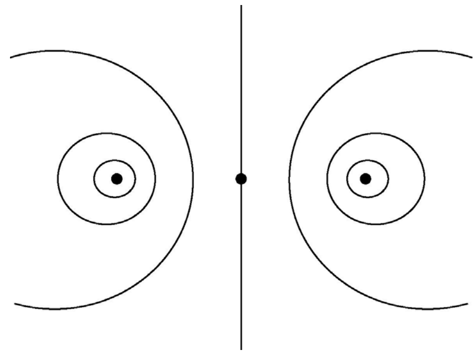
and the potential due to the wire on the right (intersecting the plane at $x = +a$) is

$$V_{\text{positive wire}} = V_o + \frac{(+\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right).$$

Since potential is a scalar quantity, the net potential at point P is the addition of $V_{-\lambda}$ and $V_{+\lambda}$ which simplifies to

$$V_{\text{net}} = 2V_0 + \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right) - \ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right)\right) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right)$$

where we have set the potential along the z axis equal to zero ($V_o = 0$) in the last step (which we are free to do). This is the expression used to obtain the equipotentials shown next. The center dot in the figure is the intersection of the z axis with the xy plane, and the dots on either side are the intersections of the wires with the plane.



117. (a) With $V = 1000$ V, we solve

$$V = \frac{q}{4\pi\varepsilon_o R} \qquad \text{where } R = 0.010 \text{ m}$$

for the net charge on the sphere, and find $q = 1.1 \times 10^{-9}$ C. Dividing this by e yields 6.95 $\times$ 10^9 electrons that entered the copper sphere. Now, half of the 3.7×10^8 decays per second resulted in electrons entering the sphere, so the time required is

$$\frac{6.95 \times 10^9}{\frac{1}{2}(3.7 \times 10^8)} = 38 \text{ seconds.}$$

(b) We note that 100 keV is 1.6×10^{-14} J (per electron that entered the sphere). Using the given heat capacity, we note that a temperature increase of $\Delta T = 5.0$ K = 5.0 C° required 71.5 J of energy. Dividing this by 1.6×10^{-14} J, we find the number of electrons needed to enter the sphere (in order to achieve that temperature change); since this is half the number of decays, we multiply to 2 and find

$$N = 8.94 \times 10^{15} \text{ decays.}$$

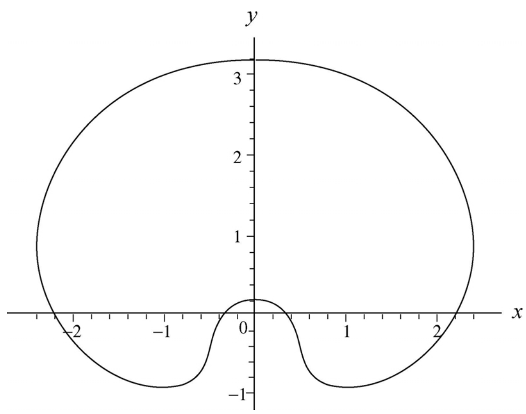
We divide N by 3.7×10^8 to obtain the number of seconds. Converting to days, this becomes roughly 280 days.

118. The (implicit) equation for the pair (x,y) in terms of a specific V is

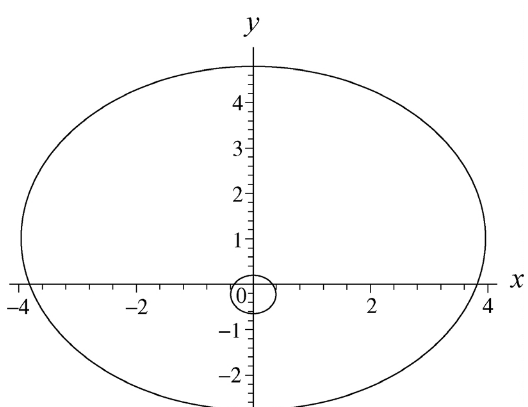
$$V = \frac{q_1}{4\pi\varepsilon_o\sqrt{x^2+y^2}} + \frac{q_2}{4\pi\varepsilon_o\sqrt{x^2+(y-d)^2}}$$

where $d = 0.50$ m. The values of q_1 and q_2 are given in the problem.

(a) We set $V = 5.0$ V and plotted (using MAPLE's implicit plotting routine) those points in the xy plane which (when plugged into the above expression for V) yield 5.0 volts. The result is



(b) In this case, the same procedure yields these two equipotential lines:



(c) One way to search for the "crossover" case (from a single equipotential line, to two) is to "solve" for a point on the y axis (chosen here to be an absolute distance ξ below q_1 – that is, the point is at a negative value of y, specifically at $y = -\xi$) in terms of V (or more conveniently, in terms of the parameter $\eta = 4\pi\varepsilon_o V \times 10^{10}$). Thus, the above expression for V becomes simply

$$\eta = \frac{-12}{\xi} + \frac{25}{d+\xi} .$$

This leads to a quadratic equation with the (formal) solution

$$\xi = \frac{13 - d\eta \pm \sqrt{d^2\eta^2 + 169 - 74\, d\eta}}{2\,\eta} .$$

Clearly there is the possibility of having two solutions (implying two intersections of equipotential lines with the –y axis) when the square root term is nonzero. This suggests that we explore the special case where the square root term is zero; that is,

$$\sqrt{d^2\eta^2 + 169 - 74\, d\eta} = 0 .$$

Squaring both sides, using the fact that $d = 0.50$ m and recalling how we have defined the parameter η, this leads to a "critical value" of the potential (corresponding to the crossover case, between one and two equipotentials):

$$\eta_{\text{critical}} = \frac{37 - 20\sqrt{3}}{d} \quad \Rightarrow \quad V_{\text{critical}} = \frac{\eta_{\text{critical}}}{4\pi\varepsilon_o \times 10^{10}} = 4.2 \text{ V}.$$